2017/09/ 20-22
研究会「生物から宇宙までの非線形現象」

@ 京大人環

宇宙の大規模構造の記述：観測から理論まで

樽家篤史

（基礎物理学研究所）

阪上さんとの関わり

1995年(D1)　天体核中間発表会で阪上さんと初めて会う

カオスと量子デコヒーレンスの話に興味

1996年(D2)　基研アトム型研究員で京都滞在

インフレーション後の宇宙の再加熱期に
起こるパラメーター共鳴について議論

1998年4月~1999年3月　研修員として人環の研究室に所属

日頃接する機会が増えたが、研究まで
にはつながらなかった

2001年　東大宇宙理論研（佐藤・須藤研）助教

ついに共同研究の機会が訪れる（→次）

非加法エントロピー

$$S_\mathrm{q} = -\frac{1}{q-1}\int d^6\tau\left[\{p(\mathrm{x},\mathrm{v})\}^q - p(\mathrm{x},\mathrm{v})\right]$$

$$\left(\text{q}\rightarrow\text{1: Boltzmann-Gibbs}\quad -\int d^6\tau\ p(\mathrm{x},\mathrm{v})\ln p(\mathrm{x},\mathrm{v})\right)$$

Tsallis (1988)；Tsallis, Mendes & Plastino (1998)

- probability $p(\mathrm{x},\mathrm{v})$ s.t. $\int d^6\tau\ p(\mathrm{x},\mathrm{v}) = 1$
- escort distribution $P_q(\mathrm{x},\mathrm{v}) \equiv \dfrac{\{p(\mathrm{x},\mathrm{v})\}^q}{\int d^6\tau\ \{p(\mathrm{x},\mathrm{v})\}^q}$
- normalized q-value $\langle O_i \rangle_q = \int d^6\tau\ \ O_i\, P_q(\mathrm{x},\mathrm{v})$

特徴

擬加法性

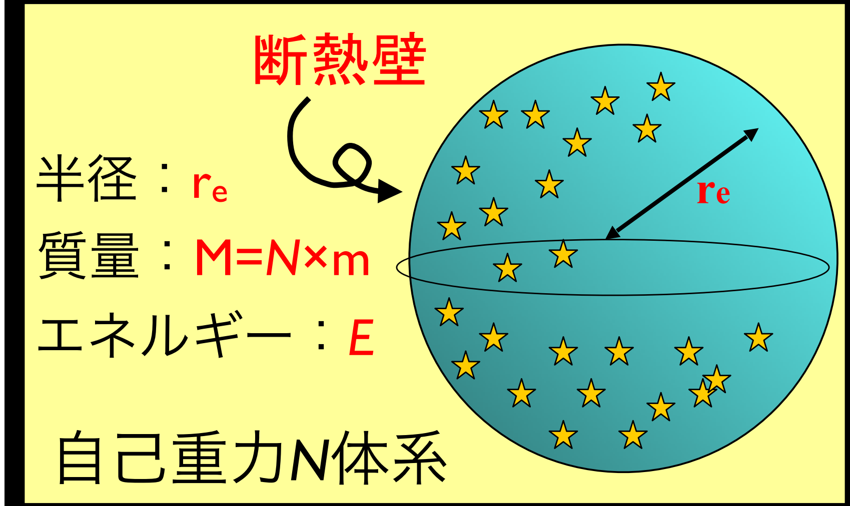
(pseudo-additivity)

$$S_\mathrm{q}(A,B) = S_\mathrm{q}(A) + S_\mathrm{q}(B) + (1-q)\,S_\mathrm{q}(A)\,S_\mathrm{q}(B)$$

Antonov problem

恒星系力学の古典的問題

自己重力系の熱平衡状態は安定か？



Antonov (‘62), Lynden-Bell & Wood (’68)

エネルギー・質量一定の下で

系のサイズが臨界半径を超えると不安定

$$r_{\mathrm{crit}} = 0.335 \frac{GM^2}{(-E)}$$

（壁がない現実的な）

自己重力多体系には安定な熱平衡系は存在しない

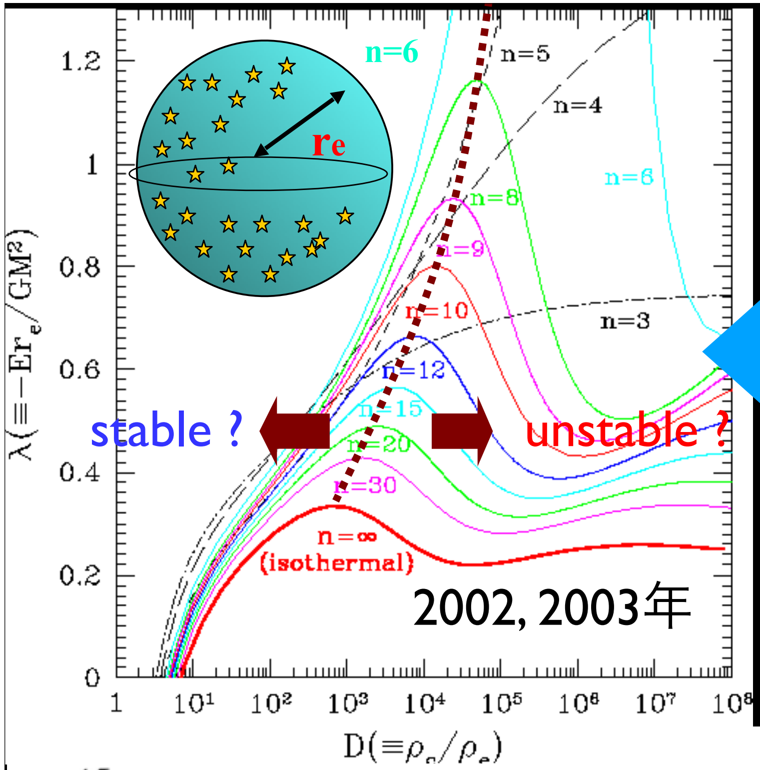
Non-equilibrium extension of Antonov problem

（非加法エントロピーの平衡状態）

恒星ポリトロープ（べき分布）の解系列

n>5では

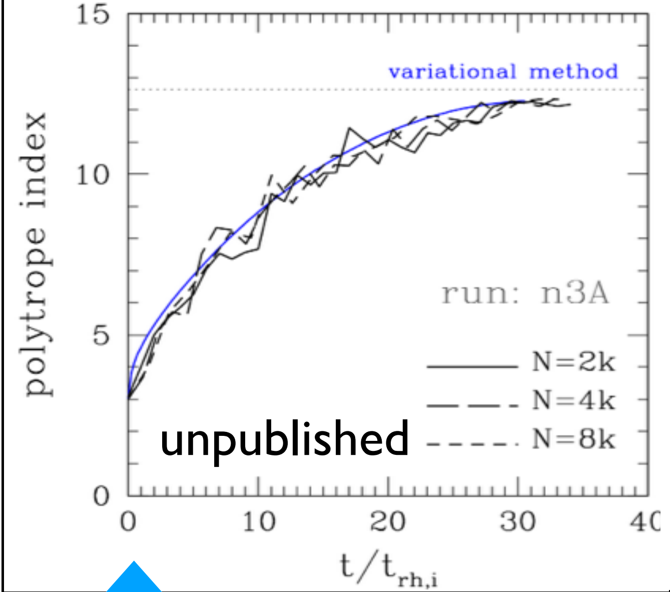
臨界線を境に安定・不安定相に分離？



2002, 2003年

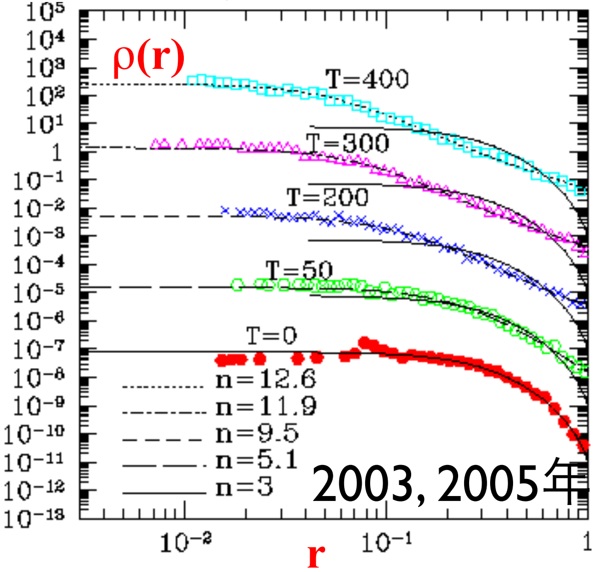
GRAPEによるN体シミュレーション

ポリトロープの解系列に沿って進化



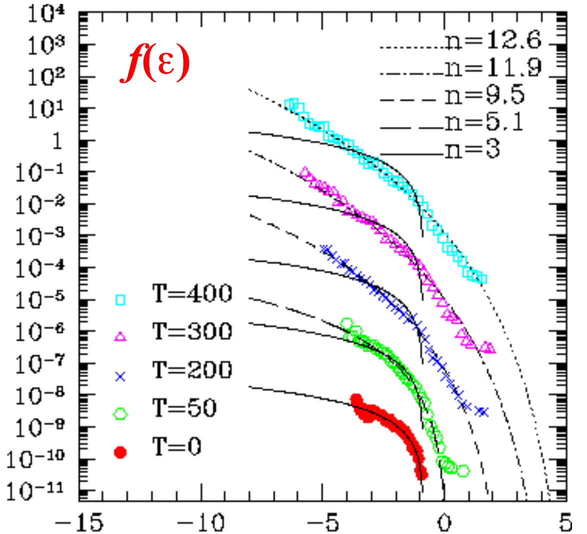
unpublished

密度プロファイル



2003, 2005年

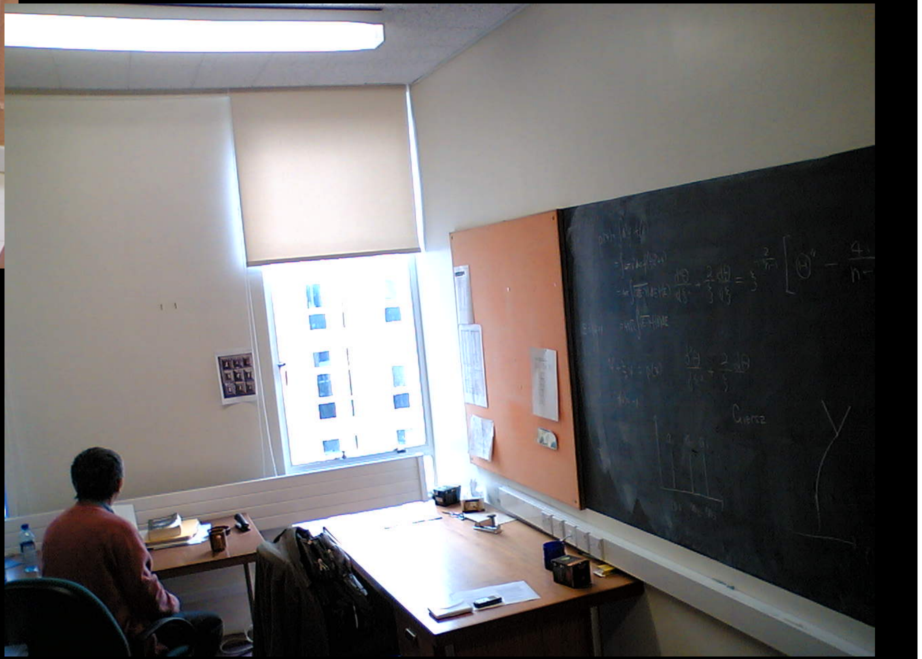
分布関数



$\varepsilon \equiv \frac{1}{2}v^2 + \Phi(x)$

変分法によるポリトロープ指数の変化の解析的記述

Sardinia



Edinburgh

自己重力多体系の運命

e.g., Binney & Tremaine ('87, '08)

初期条件 $\xrightarrow{\text{free-fall time}}$ 力学平衡 (ビリアル平衡) $\xrightarrow{\text{relaxation time}}$ 熱平衡 (不安定!!)

$t_{\rm ff} \sim (G\,\rho)^{-1/2}$

$t_{\rm relax} \sim (N/\ln N)\,t_{\rm ff}$

collisionless

楕円銀河

銀河団

$\boxed{\text{宇宙大規模構造}}$

以後のお話

$t_{\rm ff} \lesssim t_{\rm age} (= 13.8\,{\rm G\,yr})$

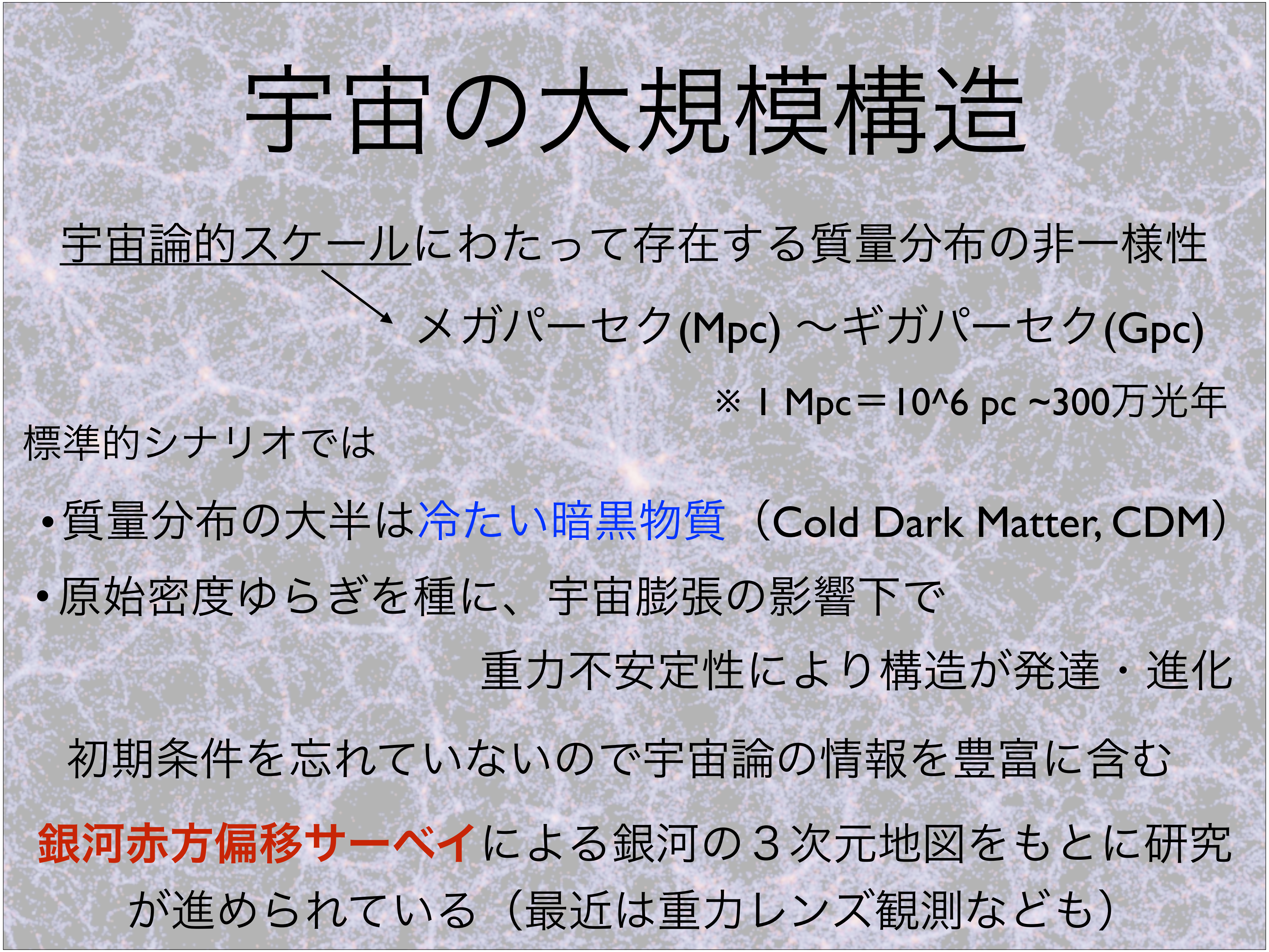
collisional

球状星団

ρ :質量密度
N :粒子数

・・・初期条件に依る

・・・初期条件を忘れて行く



宇宙の大規模構造

宇宙論的スケールにわたって存在する質量分布の非一様性

→ メガパーセク(Mpc) 〜ギガパーセク(Gpc)

※ 1 Mpc＝10^6 pc ~300万光年

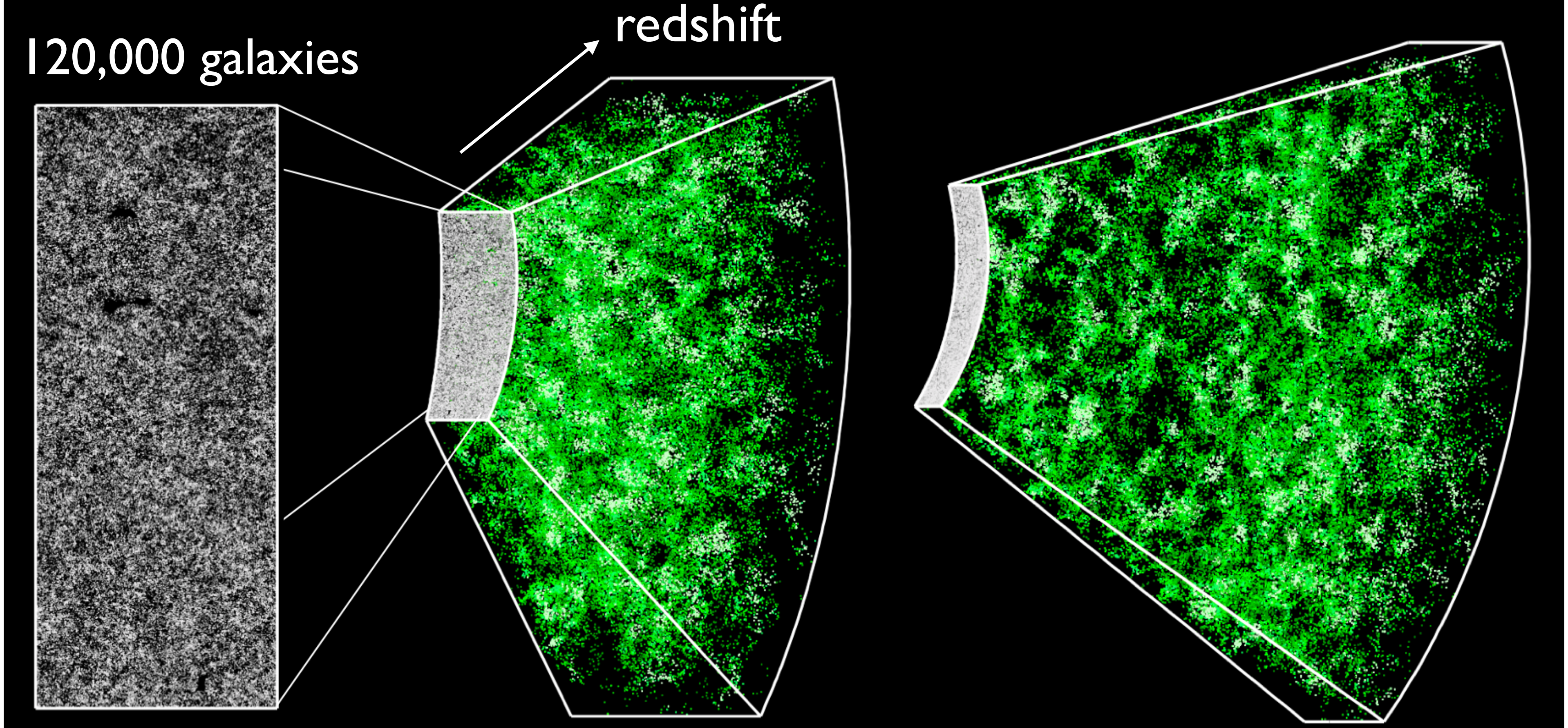
標準的シナリオでは

- 質量分布の大半は冷たい暗黒物質（Cold Dark Matter, CDM）
- 原始密度ゆらぎを種に、宇宙膨張の影響下で
  重力不安定性により構造が発達・進化

初期条件を忘れていないので宇宙論の情報を豊富に含む

銀河赤方偏移サーベイによる銀河の３次元地図をもとに研究が進められている（最近は重力レンズ観測なども）

A section of galaxy 3D map



http://www.sdss.org/press-releases/astronomers-map-a-record-breaking-1-2-million-galaxies-to-study-the-properties-of-dark-energy/

バリオン音響振動 (BAO)

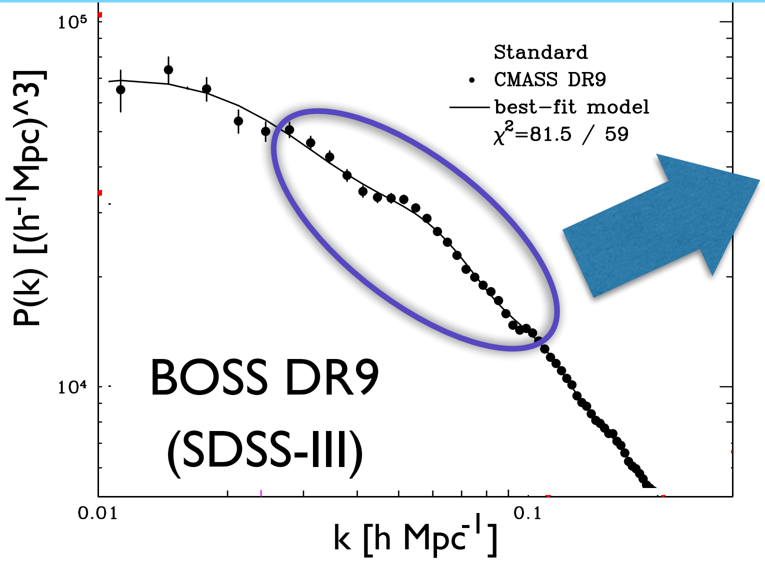
(Baryon Acoustic Oscillations)

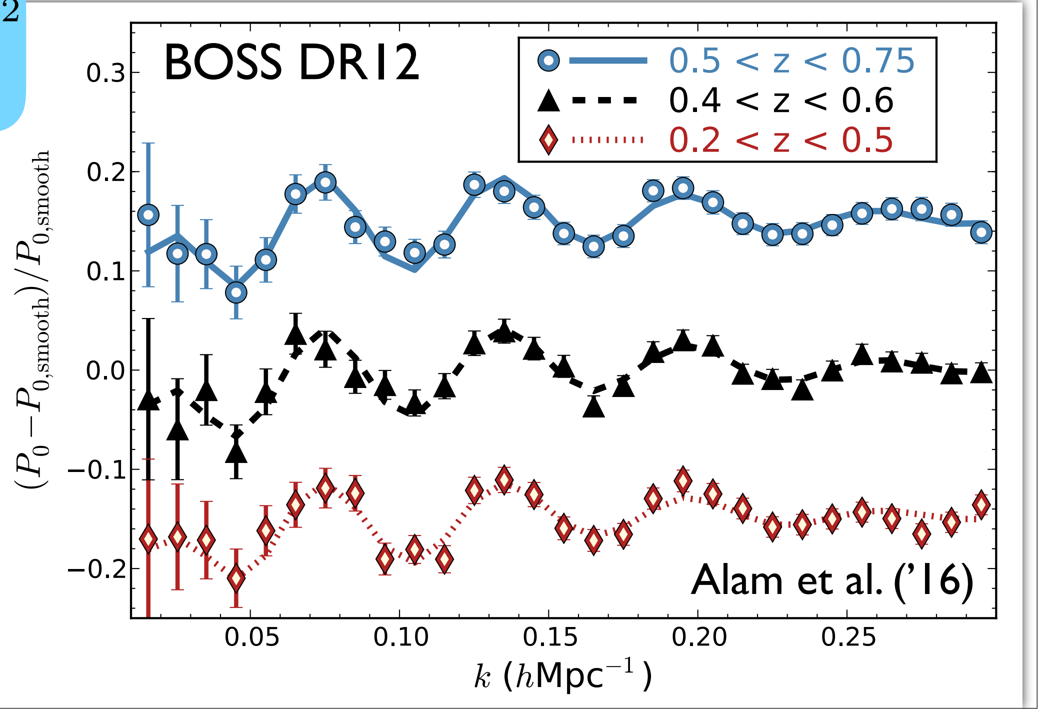
- 宇宙晴れ上がり前のバリオン-光子流体の痕跡

  (⇔ 宇宙マイクロ波背景放射の音響振動)
- 振動スケール は「標準ものさし」になる

  → 遠方宇宙の宇宙膨張診断（加速膨張の起源に迫る手がかり）

ゆらぎのパワースペクトル $$P(k) = \frac{1}{N_k} \sum_{|\vec{k}|=k} |\delta(\vec{k})|^2$$

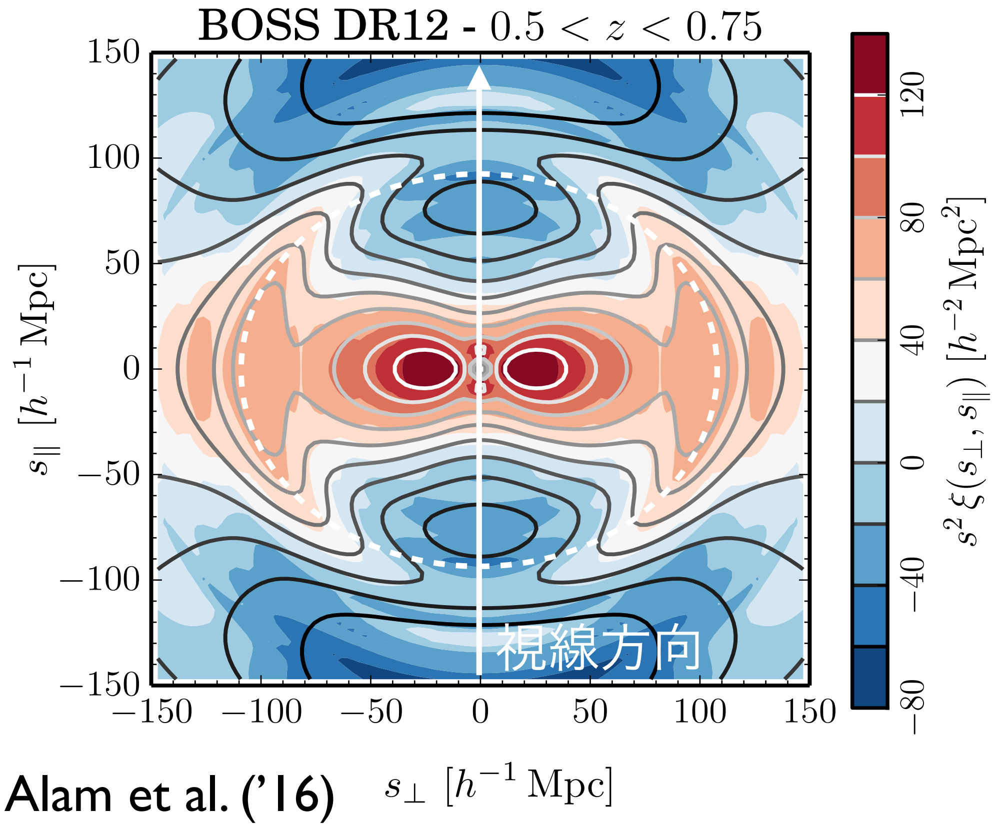




赤方偏移空間ゆがみ (RSD)

(Redshift-Space Distortions)

銀河の特異速度場がドップラー効果を通じて赤方偏移測定に影響、銀河クラスタリングの統計的等方性が破れる



Alam et al. ('16)

大スケールでは、

ゆがみの強さは重力由来の密度ゆらぎの成長率に比例

$$f(z) \equiv \frac{d\ln D_+}{d\ln a}$$

線形成長因子

スケール因子

→ 宇宙論的スケールでの重力のテスト

e.g., Linder ('08); Guzzo et al. ('08); Yamamoto et al. ('08); Percival & White ('09)

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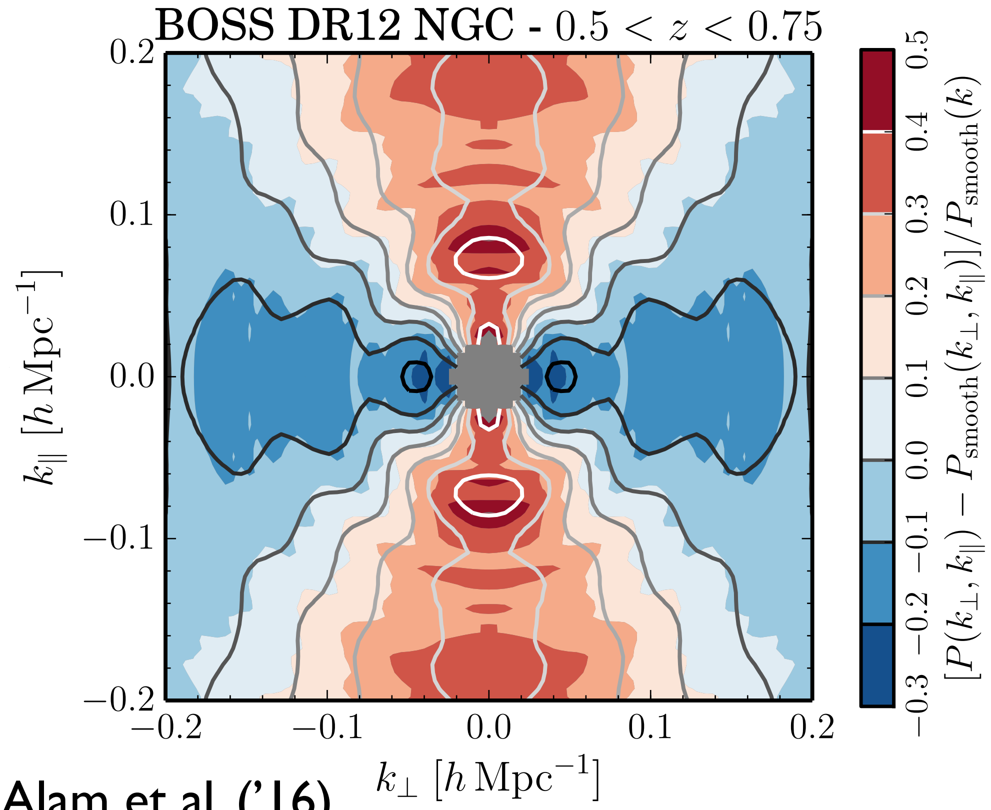

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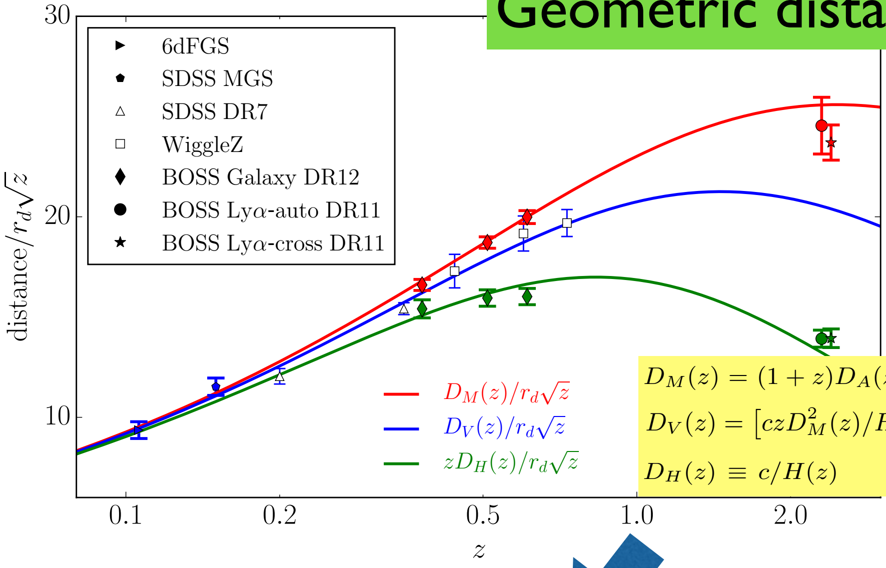
Cosmological constraints

Alam et al. ('16)

Geometric distances

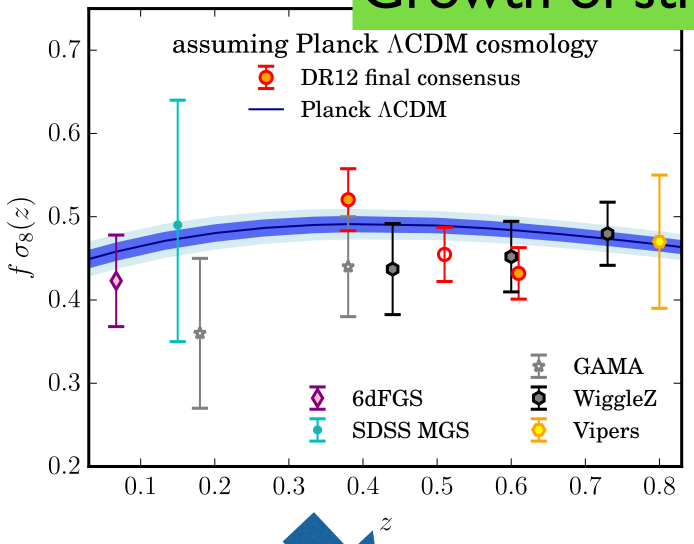


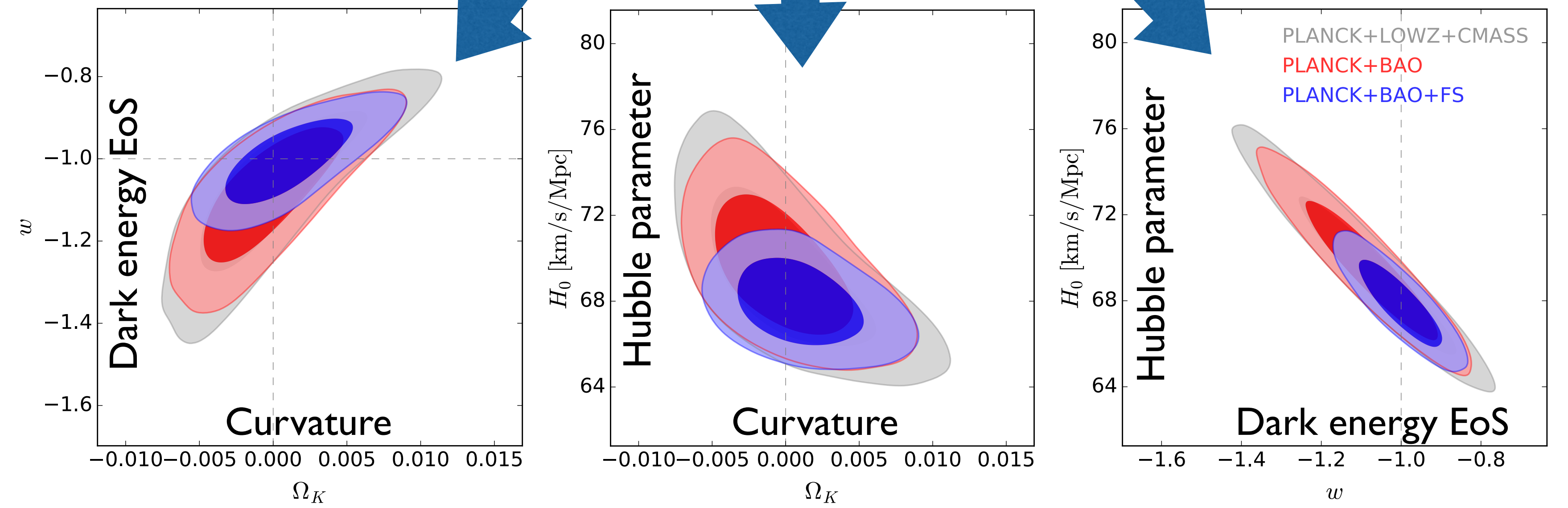
$$D_M(z) = (1+z)D_A(z)$$

$$D_V(z) = \left[czD_M^2(z)/H(z)\right]^{1/3}$$

$$D_H(z) \equiv c/H(z)$$

Growth of structure





Cosmological constraints

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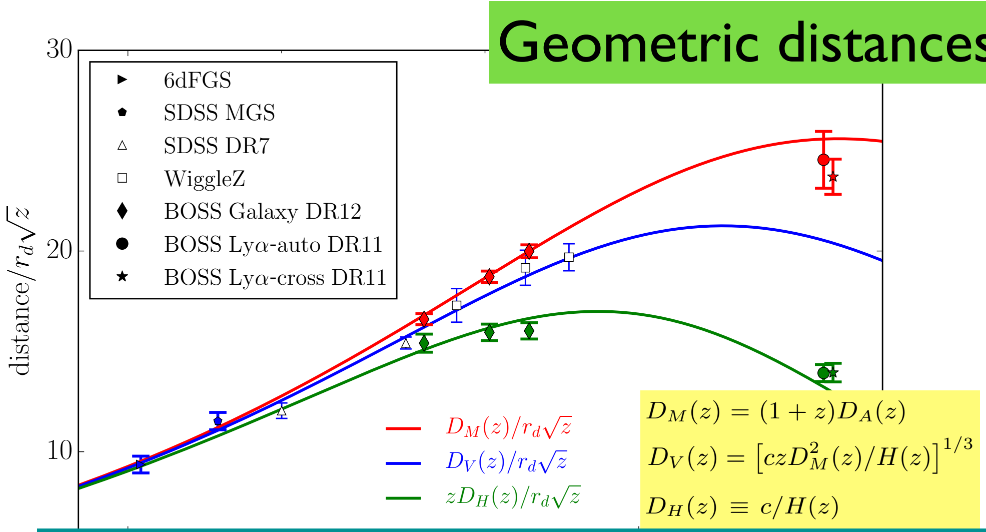

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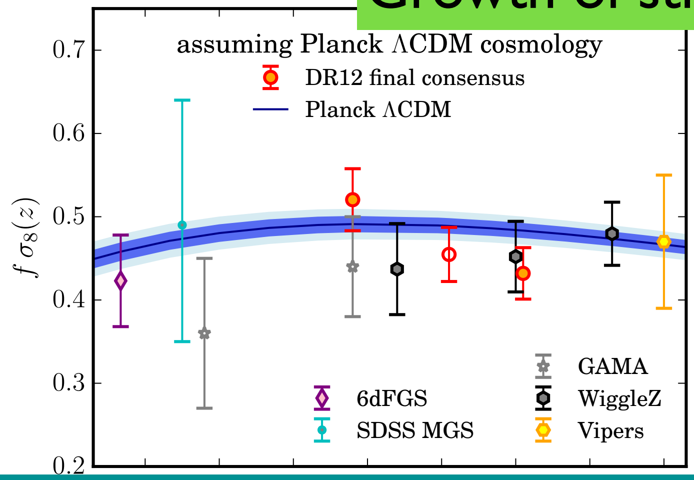
Growth of structure



Summary of BAO/RSD measurements

Planck + BAO +FS

- Curvature $\Omega_K = 0.0003 \pm 0.0026$
- Dark energy EoS $w = -1.01 \pm 0.06,$
- Hubble parameter $H_0 = 67.3 \pm 1.0\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (+SNe data)
  (at z=0)

⟶ consistent with Planck **ΛCDM model**

but tension with local Hubble measurement & weak lensing

Large-scale structure as self-gravitating collisionless system

Main ingredients

Cold dark matter (CDM)

(invisible matter component)

(Newtonian)

Gravity & Cosmic expansion

Cosmological many-body system $t_{\rm ff} \lesssim t_{\rm age}(=13.8\,{\rm G\,yr})$

$$\frac{\vec{p}_i}{dt} = -\frac{Gm^2}{a}\sum_{j\neq i}^{N}\frac{\vec{x}_i-\vec{x}_j}{|\vec{x}_i-\vec{x}_j|^3} \qquad \vec{p}_i = ma^2\frac{d\vec{x}_i}{dt}$$

$N \rightarrow \infty$

$(i=1,2,\cdots,N)$

$a(t)$: cosmic scale factor
(expansion of the Universe)

m : mass of CDM particle

G : Newton const.

Large-scale structure as self-gravitating collisionless system

Main ingredients

Cold dark matter (CDM)

(invisible matter component)

(Newtonian)

Gravity & Cosmic expansion

Cosmological Vlasov-Poisson system

Vlasov equation
$$\left[\frac{\partial}{\partial t}+\frac{\boldsymbol{p}}{ma^2}\frac{\partial}{\partial\boldsymbol{x}}-m\frac{\partial\Psi}{\partial\boldsymbol{x}}\frac{\partial}{\partial\boldsymbol{p}}\right]f(\boldsymbol{x},\boldsymbol{p})=0,$$

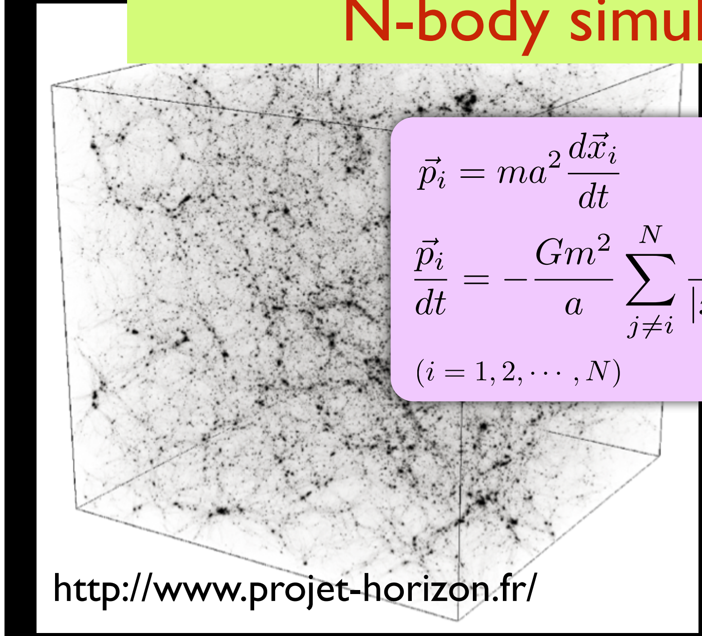
Poisson equation
$$\nabla^2\Psi(\boldsymbol{x})=4\pi\,G\,a^2\left[\frac{m}{a^3}\int d^3\boldsymbol{p}\,f(\boldsymbol{x},\boldsymbol{p})-\rho_{\rm m}\right]$$

Single-stream flow
(initial condition)
$$f(\boldsymbol{x},\boldsymbol{p})=\overline{n}\,a^3\,\{1+\delta_{\rm m}(\boldsymbol{x})\}\,\delta_{\rm D}\left[\boldsymbol{p}-m\,a\,\boldsymbol{v}(\boldsymbol{x})\right]$$

Theoretical tools of LSS

cold dark matter (CDM)
(Newtonian)
gravity & cosmic expansion
→
statistical properties
of LSS

Cosmological
N-body simulation



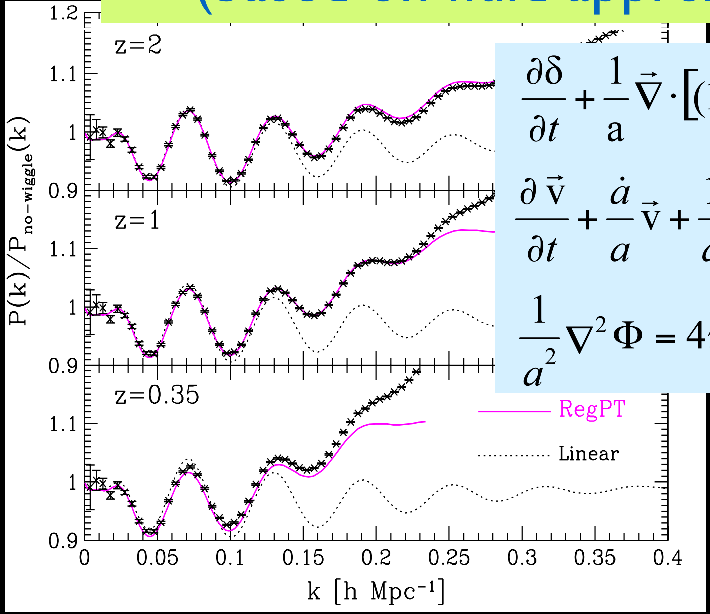
$$\vec{p}_i = ma^2\frac{d\vec{x}_i}{dt}$$

$$\frac{\vec{p}_i}{dt} = -\frac{Gm^2}{a}\sum_{j\neq i}^{N}\frac{\vec{x}_i-\vec{x}_j}{|\vec{x}_i-\vec{x}_j|^3}$$

$$(i = 1, 2, \cdots, N)$$

http://www.projet-horizon.fr/

Perturbation theory (PT)
(based on fluid approx.)



$$\frac{\partial\delta}{\partial t}+\frac{1}{\mathrm{a}}\vec{\nabla}\cdot\left[(1+\delta)\vec{\mathrm{v}}\right]=0$$

$$\frac{\partial\,\vec{\mathrm{v}}}{\partial t}+\frac{\dot{a}}{a}\vec{\mathrm{v}}+\frac{1}{a}(\vec{\mathrm{v}}\cdot\vec{\nabla})\,\vec{\mathrm{v}}=-\frac{1}{a}\vec{\nabla}\Phi$$

$$\frac{1}{a^2}\nabla^2\Phi=4\pi G\,\overline{\rho}_{\mathrm{m}}\,\delta$$

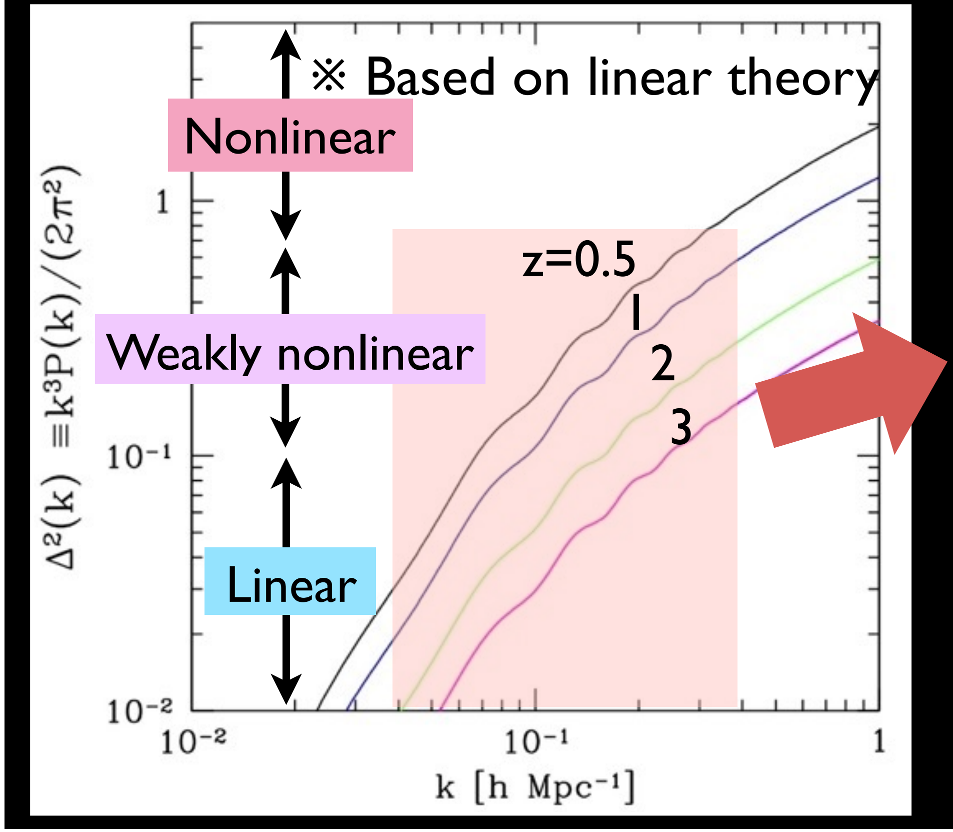
AT et al. ('12)

On top of the gravitational evolution, observational effects (redshift-space distortions, galaxy bias, …) also need to be considered

Regime of our interest

Most of interesting cosmological information (BAO, RSD, signature of massive neutrinos, …) lies at k<0.2-0.3 h/Mpc

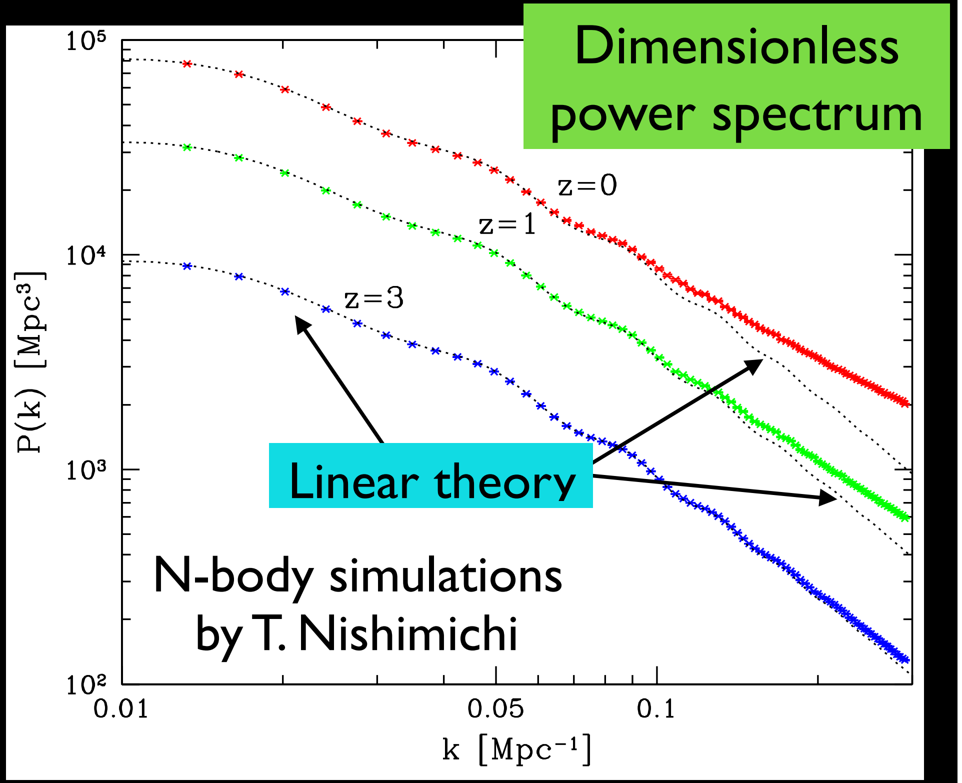
→ Weakly nonlinear regime


※ Based on linear theory

Nonlinear

Weakly nonlinear

Linear


Dimensionless power spectrum

Linear theory

N-body simulations by T. Nishimichi

Perturbation theory (PT): reloaded

Single-stream approx. of Vlasov-Poisson system

CDM + baryon → pressureless & irrotational fluid

Basic eqs.

$$\frac{\partial\delta}{\partial t}+\frac{1}{\mathrm{a}}\vec{\nabla}\cdot\left[(1+\delta)\vec{\mathrm{v}}\right]=0$$

$$\frac{\partial\,\vec{\mathrm{v}}}{\partial t}+\frac{\dot{a}}{a}\vec{\mathrm{v}}+\frac{1}{a}(\vec{\mathrm{v}}\cdot\vec{\nabla})\,\vec{\mathrm{v}}=-\frac{1}{a}\vec{\nabla}\,\Phi$$

$$\frac{1}{a^2}\nabla^2\Phi=4\pi\,G\,\bar{\rho}_{\mathrm{m}}\,\delta$$

Juszkiewicz ('81), Vishniac ('83), Goroff et al. ('86), Suto & Sasaki ('91), Makino, Sasaki & Suto ('92), Jain & Bertschinger ('94), …

Standard PT $(\delta_1 \ll 1)$

$$\delta=\delta_1+\delta_2+\delta_3+\cdots$$

Recent progress

- Improving accuracy by resummation or renormalized PT treatment
- Higher-order calculation & fast PT code (*RegPT*)

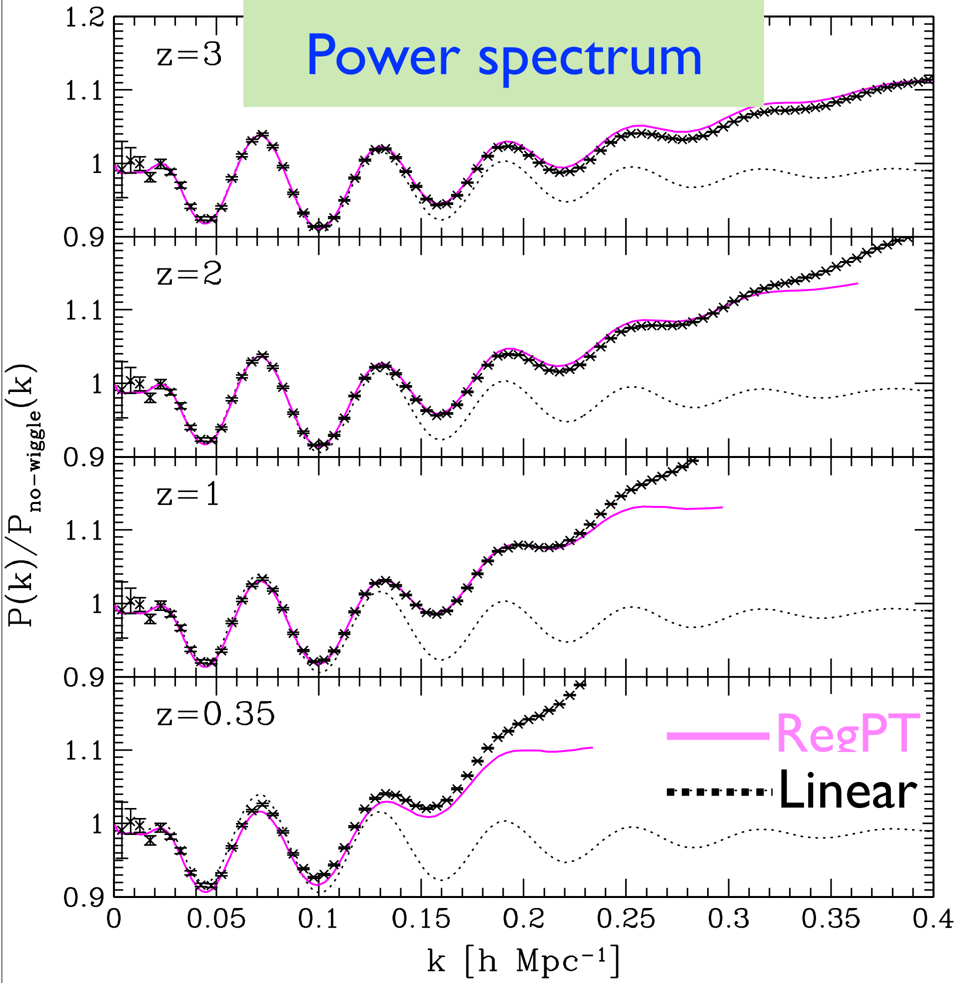
2-loop (next-to-next-to leading order)

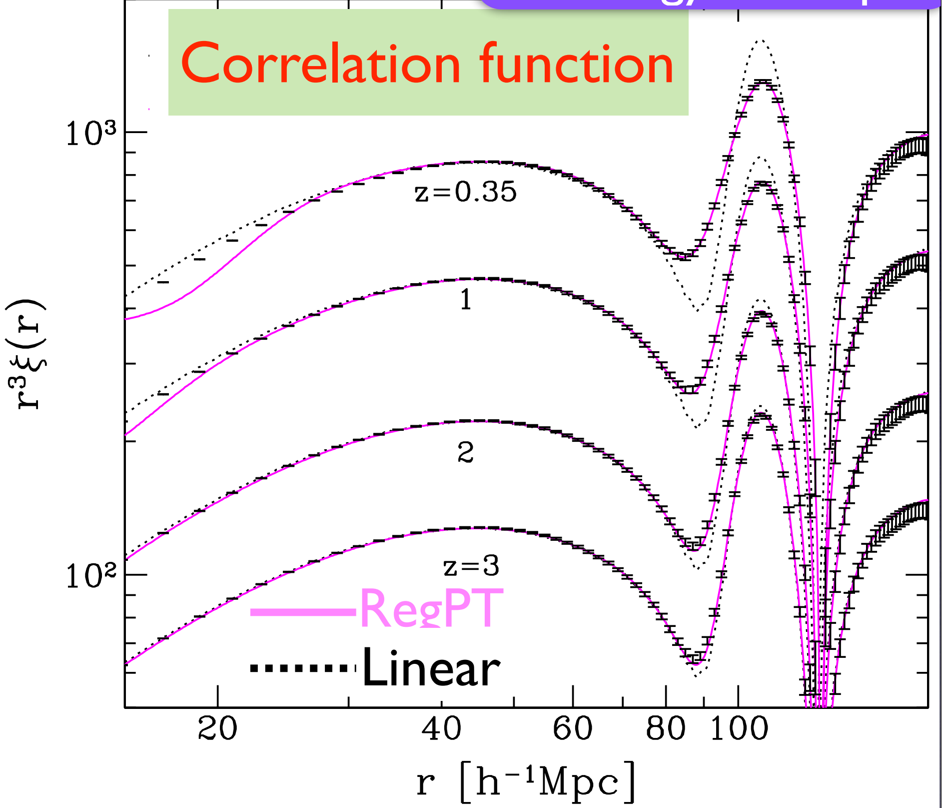
- Incorporating other systematics (massive ν, modified gravity, halo bias,…)

Performance of resummed PT

RegPT fast resummed PT code (http://ascl.net/1404.012)
including 2-loop (*next-to-next-to-leading*) *order*

$L_{\rm box}=2,048\,h^{-1}\,{\rm Mpc}$
# of particles : $1,024^3$
# of runs : 60
cosmology : wmap5


Power spectrum


Correlation function

AT, Bernardeau, Nishimichi & Codis ('12)

Cosmic propagators

Propagator should carry information on
non-linear evolution & statistical properties

Crocce & Scoccimarro ('06)

Evolved (non-linear) density field

$$\left\langle \frac{\delta\, \delta_{\rm m}(\boldsymbol{k};t)}{\delta\, \delta_0(\boldsymbol{k}')} \right\rangle \equiv \delta_{\rm D}(\boldsymbol{k}-\boldsymbol{k}')\,\Gamma^{(1)}(k;t)$$

Propagator

Initial density field

Ensemble w.r.t randomness of initial condition

Contain statistical information on *full-nonlinear* evolution

(Non-linear extension of Green's function)

Multi-point propagators

Bernardeau, Crocce & Scoccimarro ('08)

Matsubara ('11) ⟶ *integrated PT*

As a natural generalization,

Multi-point propagator

$$\left\langle \frac{\delta^n \, \delta_{\rm m}(\boldsymbol{k};t)}{\delta\,\delta_0(\boldsymbol{k}_1)\cdots\delta\,\delta_0(\boldsymbol{k}_n)} \right\rangle = (2\pi)^{3(1-n)}\,\delta_{\rm D}(\boldsymbol{k}-\boldsymbol{k}')\,\Gamma^{(n)}(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n;t)$$

With this multi-point prop.

- Building blocks of a new perturbative theory (PT) expansion

--------- Γ-expansion or Wiener-Hermite expansion

- A good convergence of PT expansion is expected

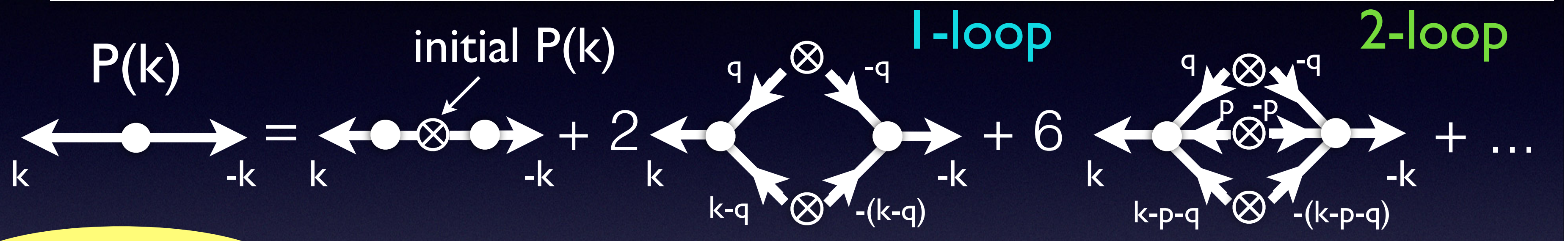
(c.f. standard PT)

Power spectrum

Initial power spectrum

$$P(k;t) = \left[\Gamma^{(1)}(k;t)\right]^2 P_0(k) + 2\int \frac{d^3\boldsymbol{q}}{(2\pi)^3} \left[\Gamma^{(2)}(\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q};t)\right]^2 P_0(q) P_0(|\boldsymbol{k}-\boldsymbol{q}|)$$

$$+ 6\int \frac{d^6\boldsymbol{p}d^3\boldsymbol{q}}{(2\pi)^6} \left[\Gamma^{(3)}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q};t)\right]^2 P_0(p)P_0(q)P_0(|\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q}|) + \cdots$$

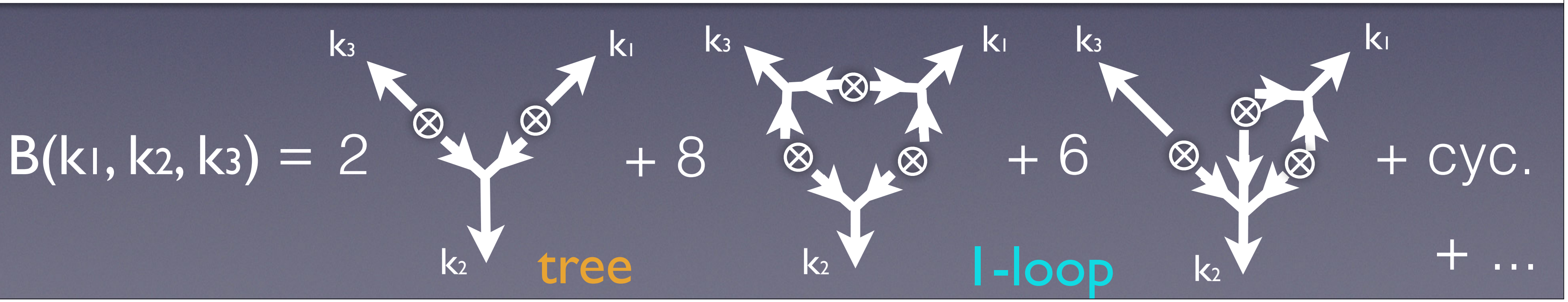


Bispectrum

$$B(k_1,k_2,k_3) = 2\,\Gamma^{(2)}(\mathbf{k}_1,\mathbf{k}_2)\,\Gamma^{(1)}(k_1)\,\Gamma^{(1)}(k_2)\,P_0(k_1)P_0(k_2) + \text{cyc.}$$

$$+ \Bigg[8\int d^3q\,\Gamma^{(2)}(\mathbf{k}_1-\mathbf{q},\mathbf{q})\Gamma^{(2)}(\mathbf{k}_2+\mathbf{q},-\mathbf{q})\Gamma^{(2)}(\mathbf{q}-\mathbf{k}_1,-\mathbf{k}_2-\mathbf{q})P_0(|\mathbf{k}_1-\mathbf{q}|)P_0(|\mathbf{k}_2+\mathbf{q}|)P_0(q)$$

$$+ 6\int d^3q\,\Gamma^{(3)}(-\mathbf{k}_3,-\mathbf{k}_2+\mathbf{q},-\mathbf{q})\Gamma^{(2)}(\mathbf{k}_2-\mathbf{q},\mathbf{q})\Gamma^{(1)}(\mathbf{k}_3)P_0(|\mathbf{k}_2-\mathbf{q}|)P_0(q)P_0(k_3) + \text{cyc.}\Bigg].$$

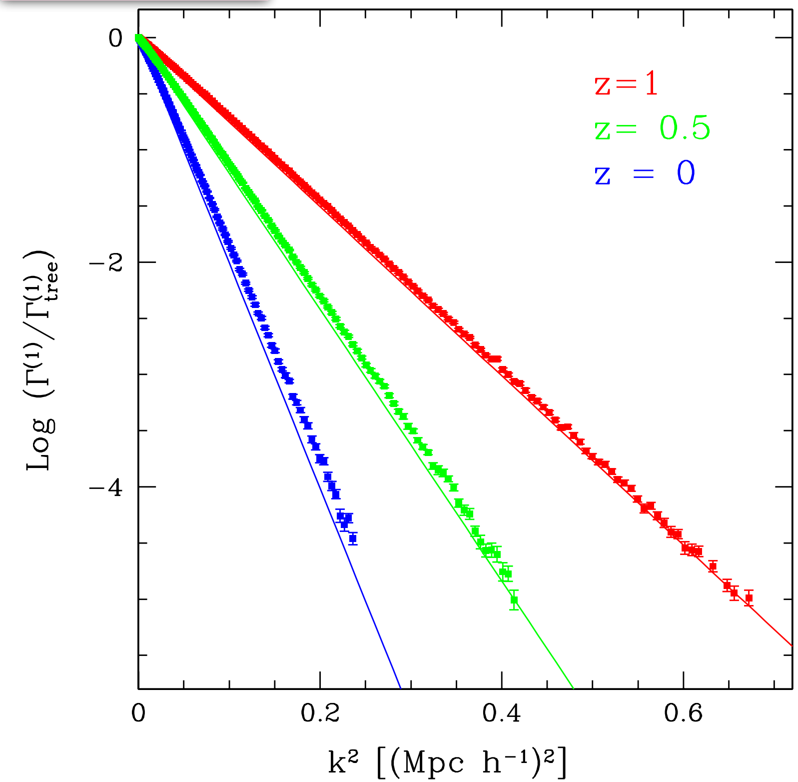


Generic property of propagators

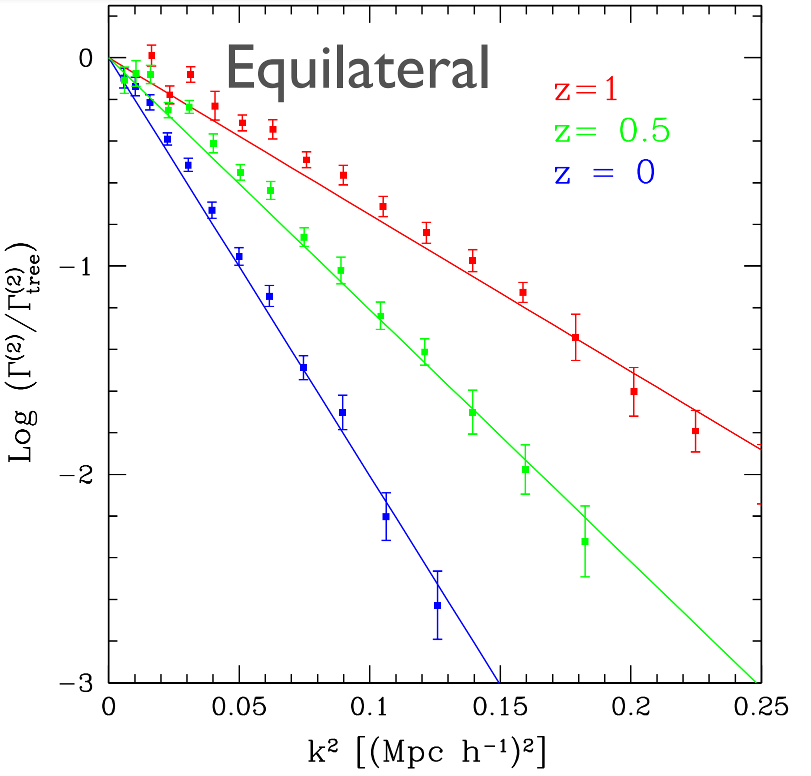
Crocce & Scoccimarro ’06, Bernardeau et al. ’08

$$\Gamma^{(n)} \xrightarrow{k\to+\infty} {}^{s}F_n(\boldsymbol{k}_1,\cdots\boldsymbol{k}_n)\,e^{-k^2\sigma_{\rm v}^2/2}\quad ; \quad \sigma_{\rm v}^2=\int\frac{dq}{6\pi^2}P_{\theta\theta}(q)$$

$\Gamma^{(1)}(k)$



$\Gamma^{(2)}(k_1,k_2,k_3)$



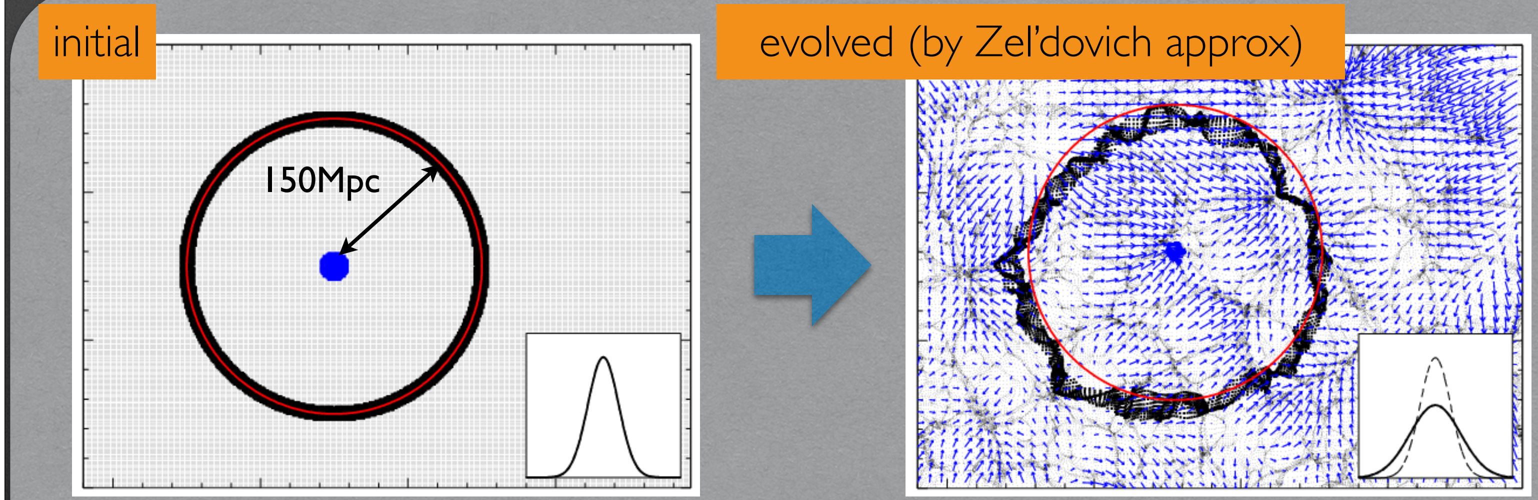
Origin of Exp. damping

For Gaussian initial condition,

$$\langle \delta_{\rm m}(\boldsymbol{k};t)\,\delta_0(\boldsymbol{k}')\rangle = \boxed{\Gamma^{(1)}(k;t)}\;\underline{\underline{\langle \delta_0(\boldsymbol{k})\delta_0(\boldsymbol{k}')\rangle}} \;\; /\!/ \;\; P_0(k)$$

initial power spectrum

→ Cross correlation between initial & evolved density fields



Padmanabhan et al. ('12)

Initial structure becomes blurred by the *local* cosmic flow

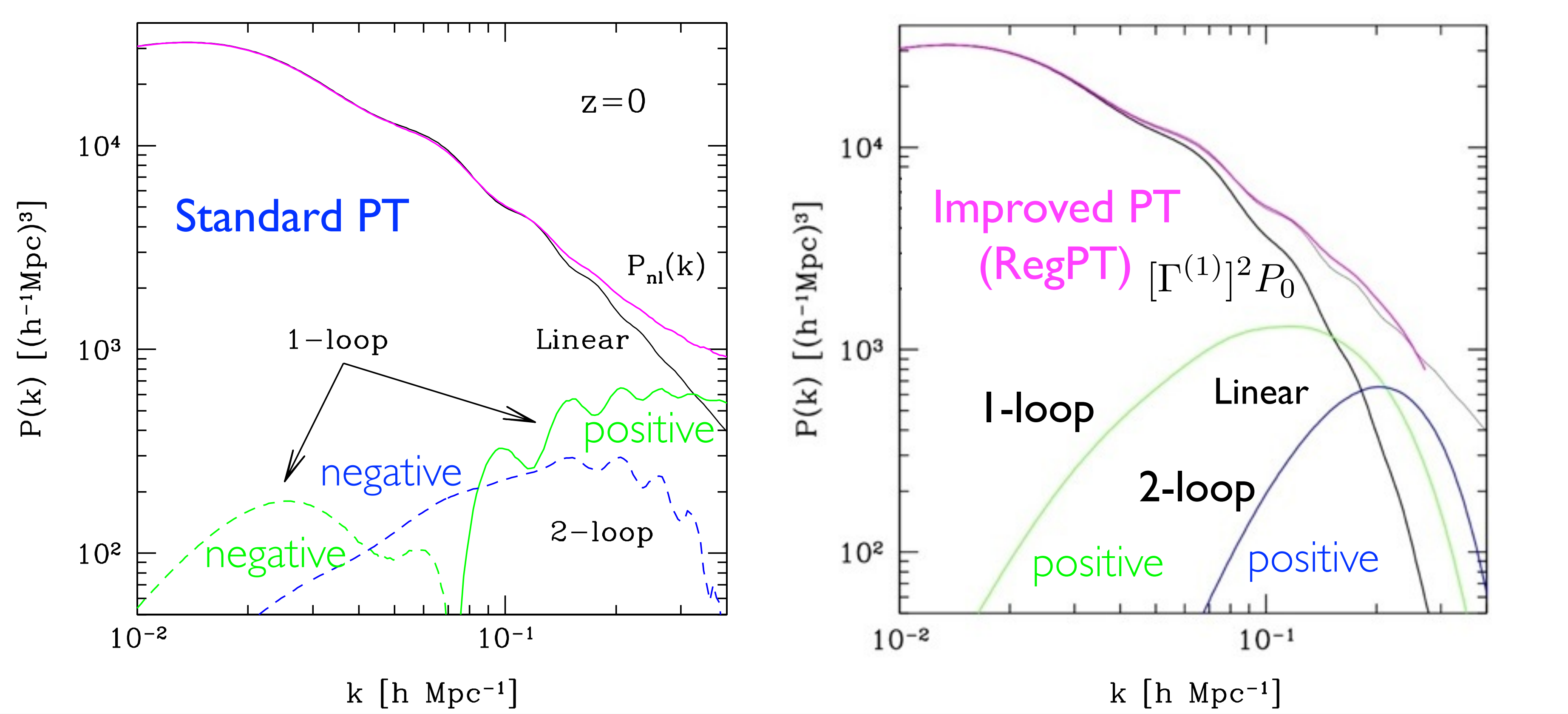
----- origin of Gaussian damping in propagator

Why improved PT works well?

AT, Bernardeau, Nishimichi, Codis ('12)
AT et al. ('09)

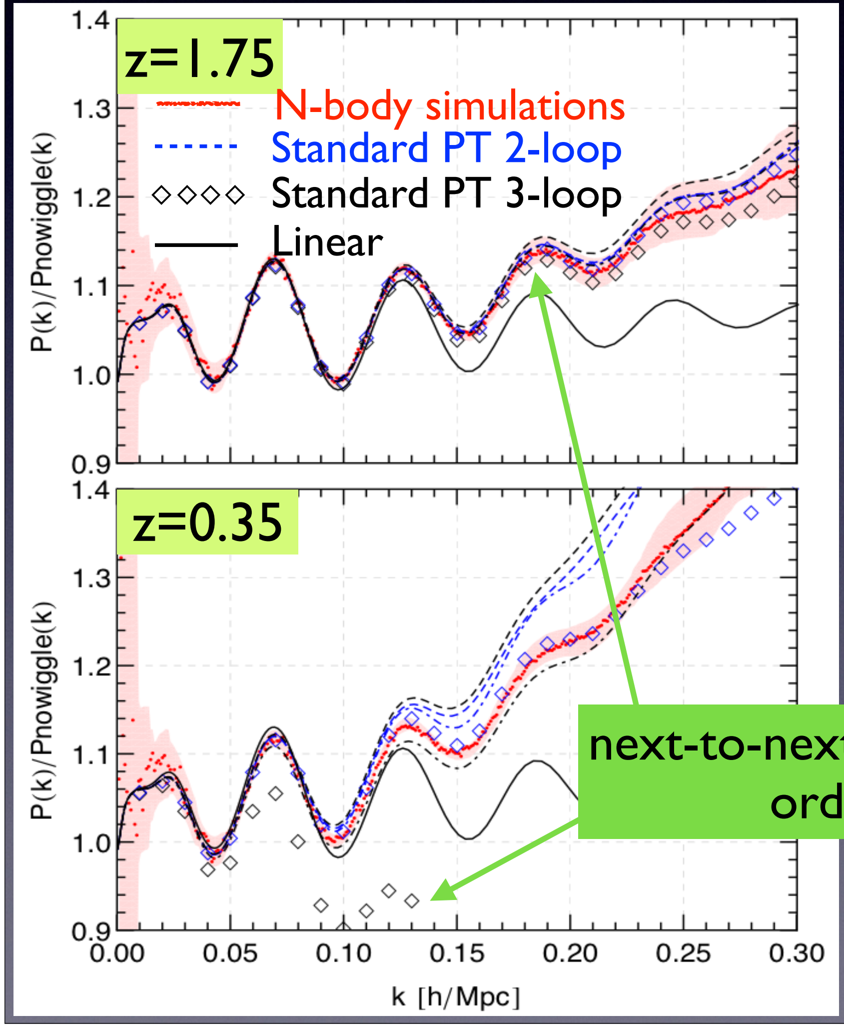
- All corrections become comparable at low-z.
- Positivity is not guaranteed.

Corrections are positive & localized, shifted to higher-k for higher-loop



3-loop : source of trouble

Further including 3-loop (i.e., next-to-next-to-next-to-leading order), PT calculations start to get worse !!

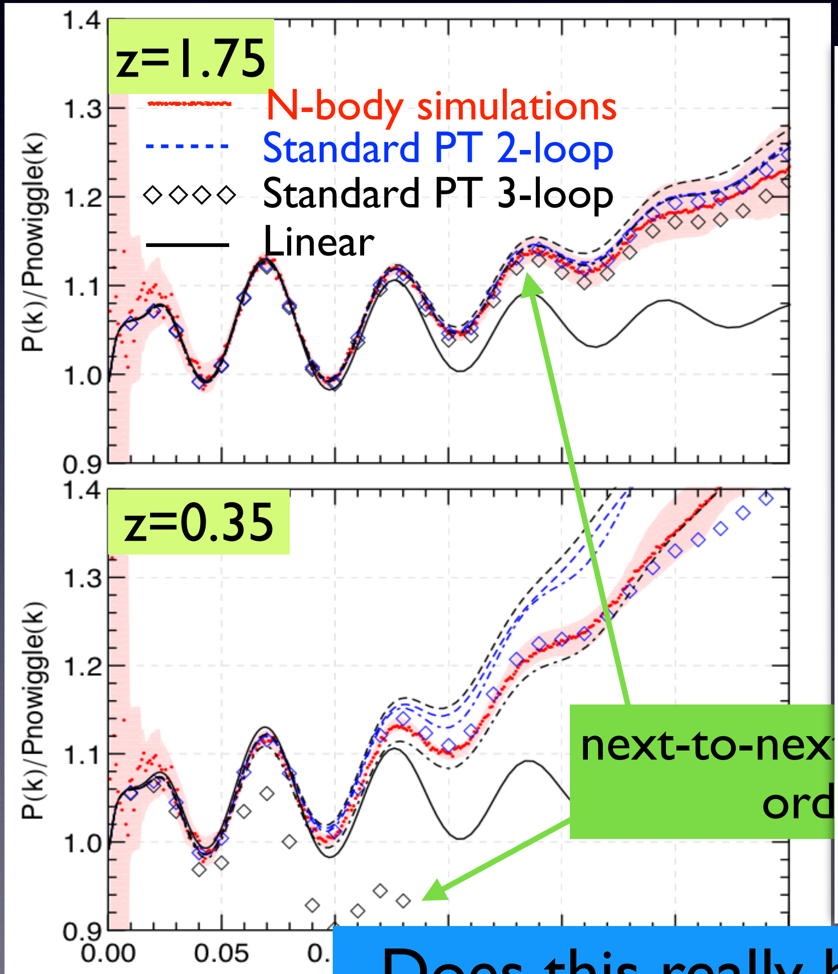


next-to-next-to-next-to-leading order (3-loop)

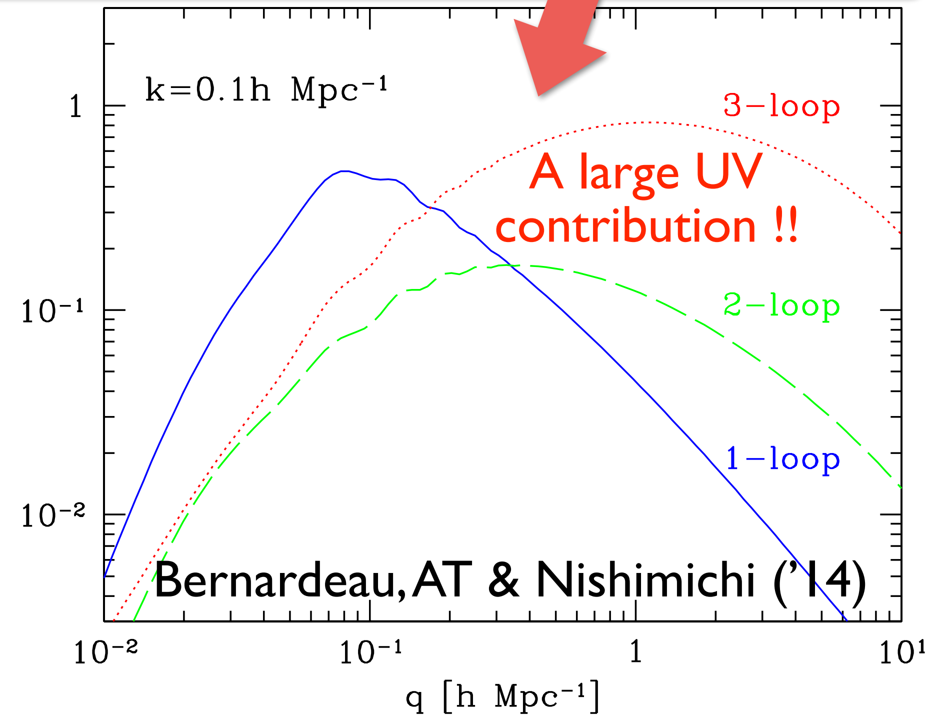
Blas et al. ('14)

3-loop : source of trouble

Further including 3-loop (i.e., next-to-next-to-next-to-leading order), PT calculations start to get worse !!



$$P_{\text{n-loop}}(k) \propto \int d\ln q\, K_{\text{n-loop}}(k,q)\, P_0(q)$$



A large UV contribution !!

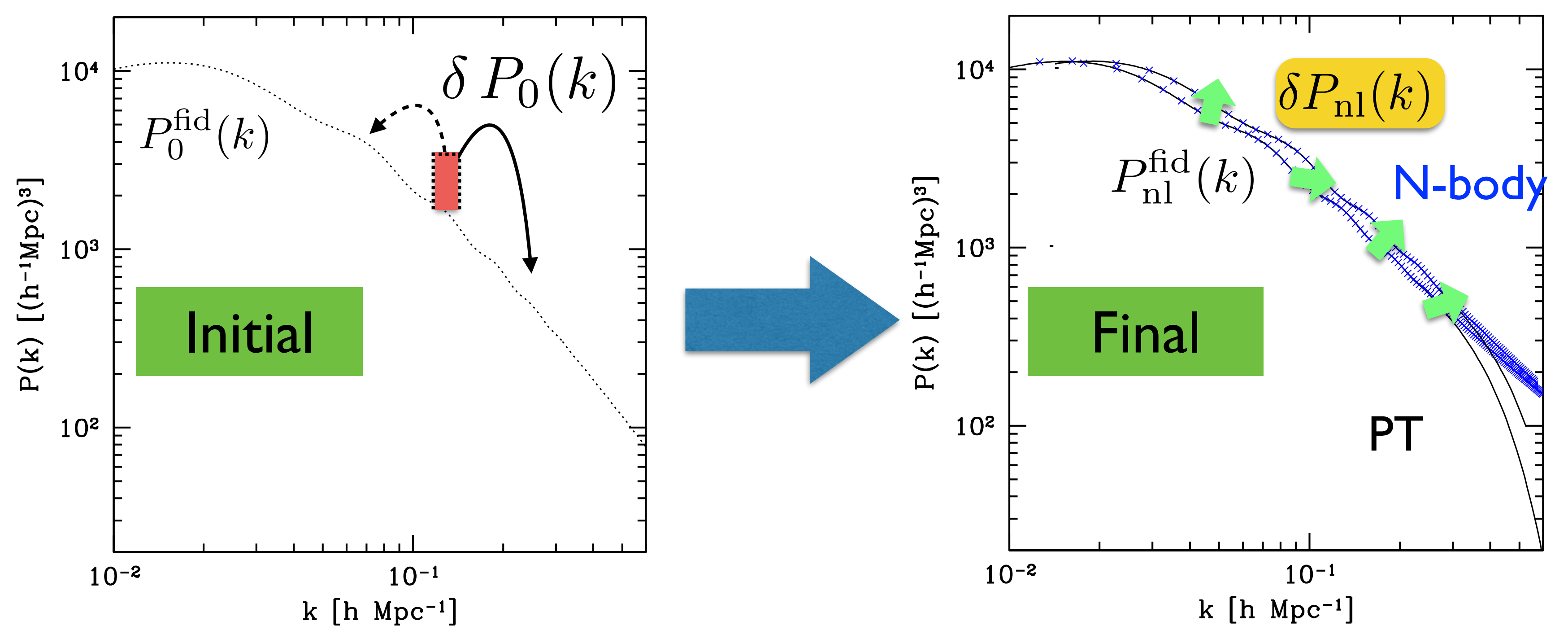
Bernardeau, AT & Nishimichi ('14)

Does this really happen in real universe ?

Nature of nonlinear mode-coupling

How the small-scale fluctuations affect the evolution of large-scale modes ? (or vice versa)

How the small disturbance added in <u>initial power spectrum</u> can contribute to each Fourier mode in <u>final power spectrum ?</u>



Nature of nonlinear mode-coupling

How the small-scale fluctuations affect the evolution of large-scale modes ? (or vice versa)

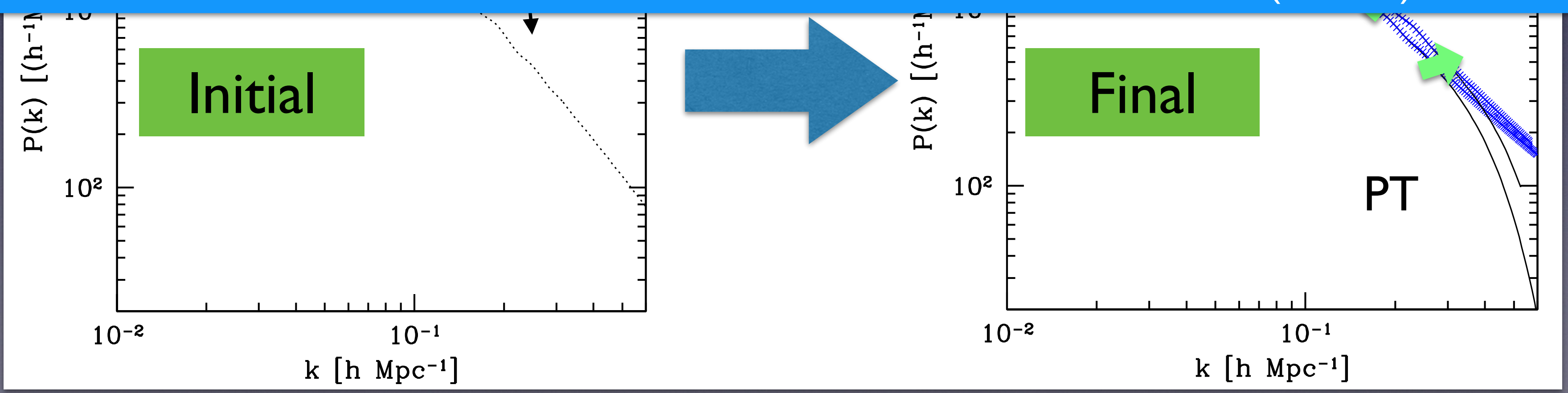
How the small disturbance added in initial power spectrum can contribute to each Fourier mode in final trum ?

Response function

$$\delta P_{\rm nl}(k) = \int d\ln q \, K(k,q) \, \delta P_0(q)$$

Final (nonlinear)

initial (linear)

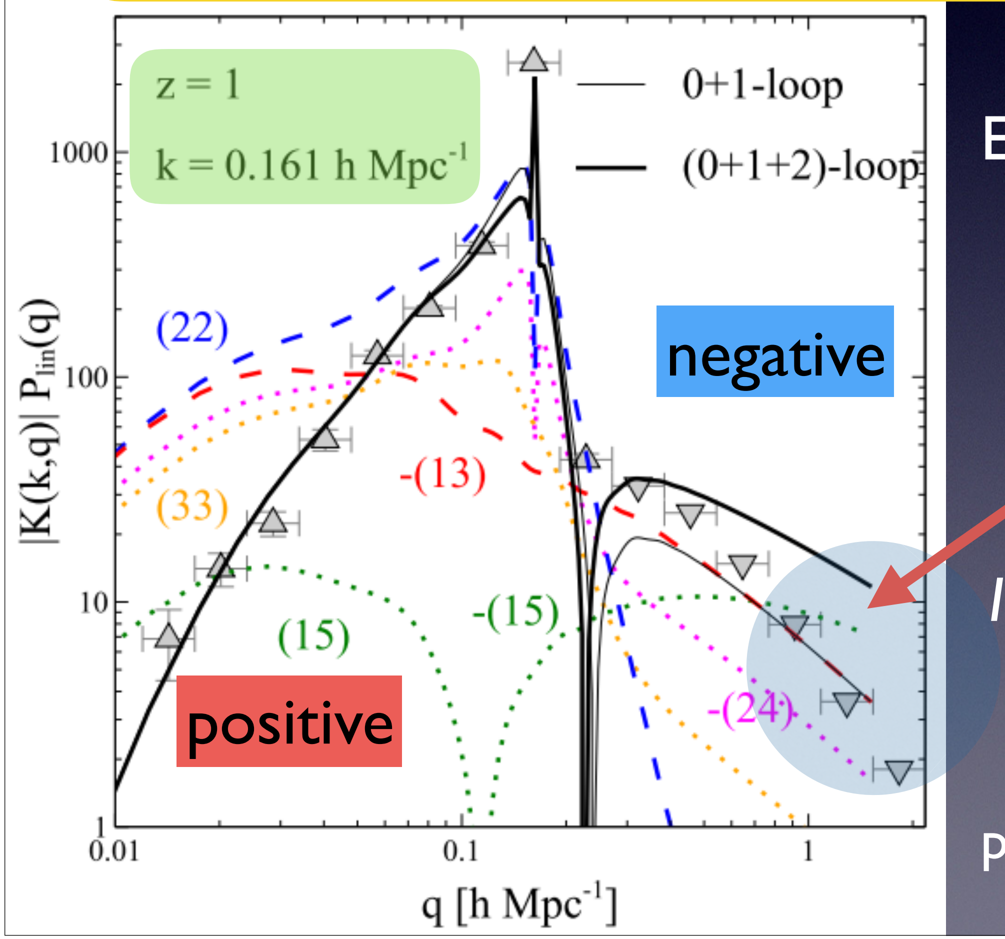


A measurement result

Nishimichi, Bernardeau & AT ('16)

Response of power spectrum at k
to a small initial variation at q

$$K(k,q;z) = q\frac{\delta P^{\rm nl}(k;z)}{\delta P^{\rm lin}(q;z)}$$



Even for *low-k* modes,

Standard PT gets a *large UV contribution* *(q-modes)*:

2-loop > 1-loop > N-body

In other words,

low-k mode in simulation is *UV-insensitive*

protected against small-scale uncertainty

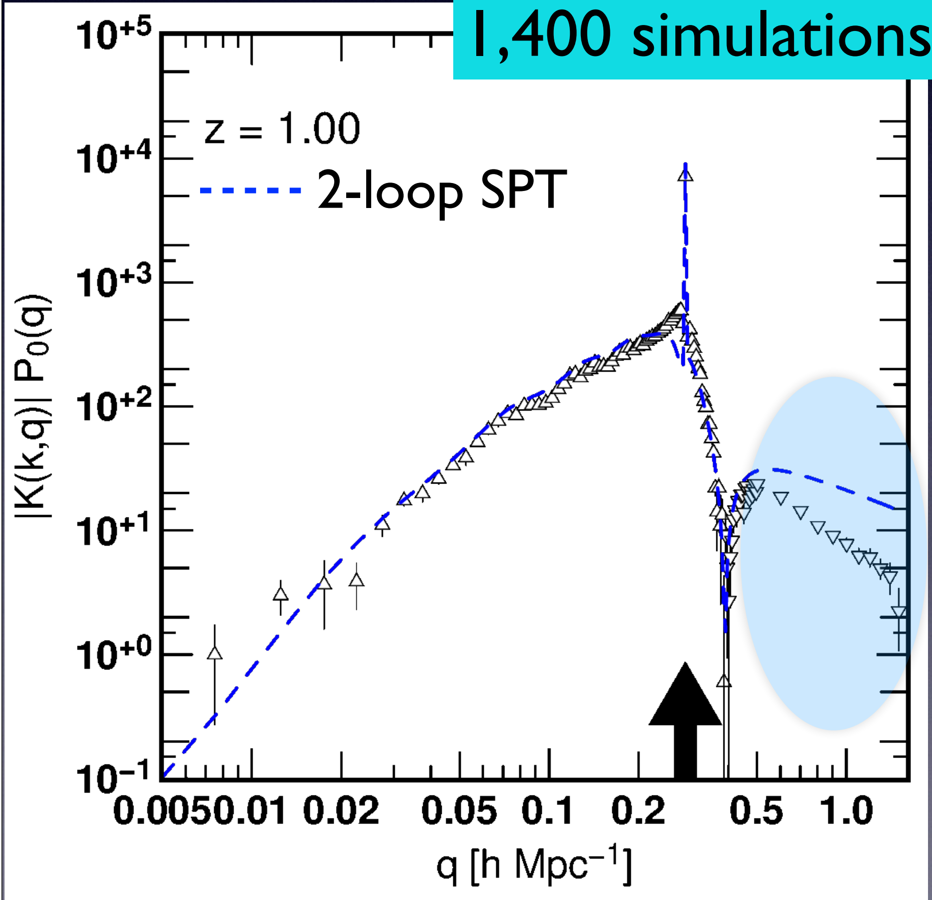
Refined measurement

Nishimichi, Bernardeau & AT ('16 &'17 in prep.)

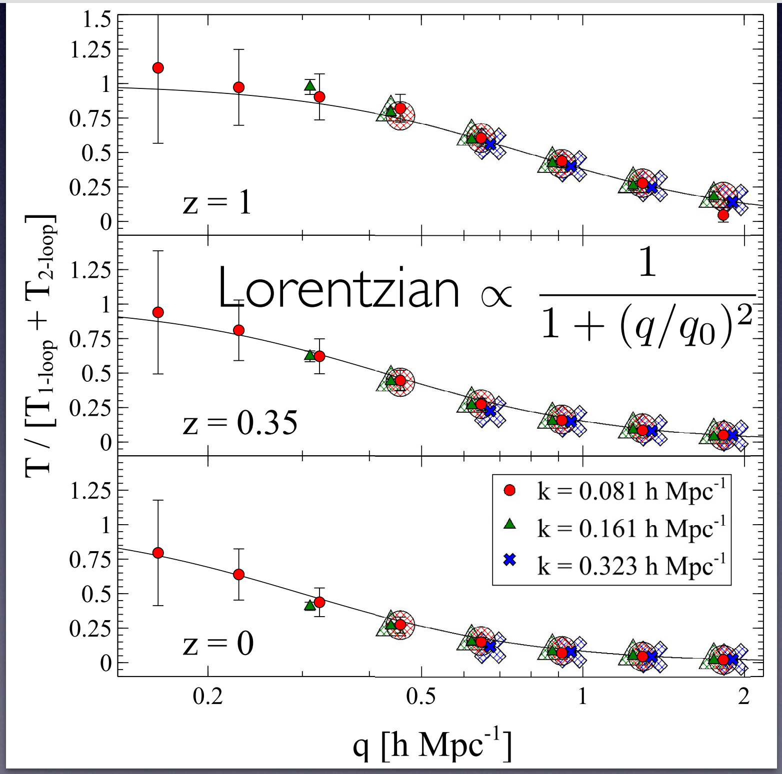
Response of power spectrum at k to a small initial variation at q

$$K(k,q;\,z)=q\frac{\delta P_{\rm nl}(k;\,z)}{\delta P_0(q;\,z)}$$

1,400 simulations



$$T(k,q)=[K(k,q)-K^{\rm lin}(k,q)]/[qP^{\rm lin}(k)]$$



Lorentzian $\propto \dfrac{1}{1+(q/q_0)^2}$

UV suppression is seen at various k

Refined measurement

Nishimichi, Bernardeau & AT ('16 &'17 in prep.)

Response of power spectrum at k
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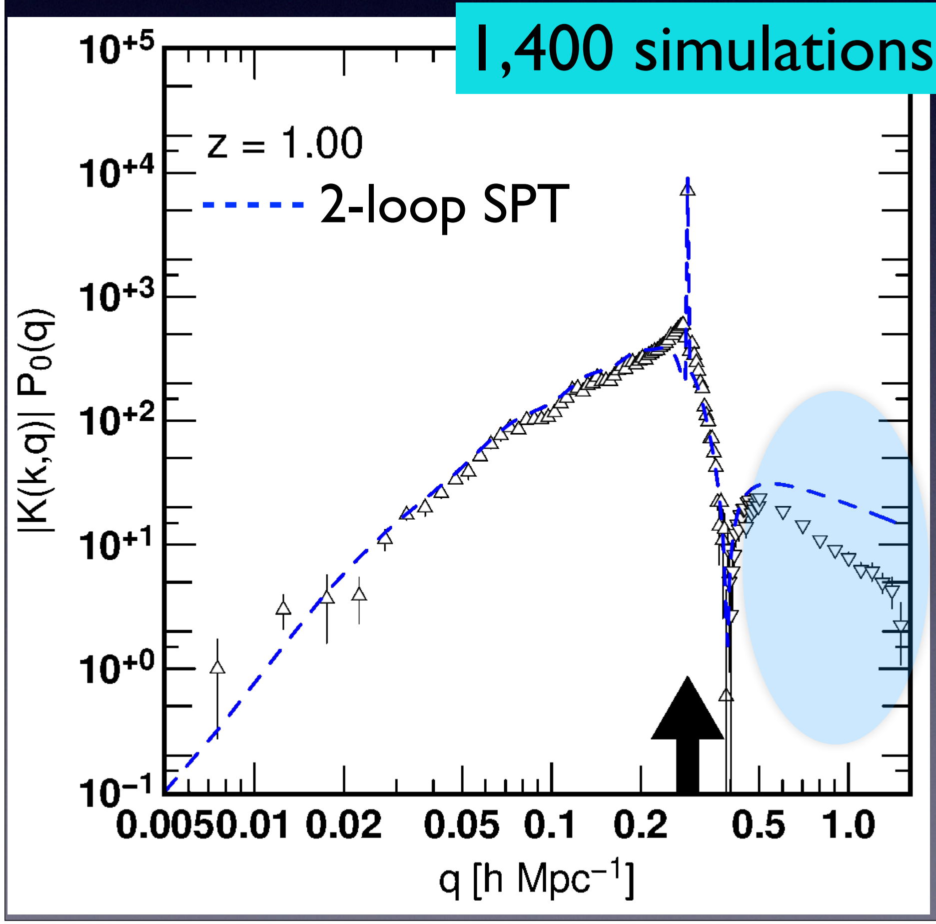

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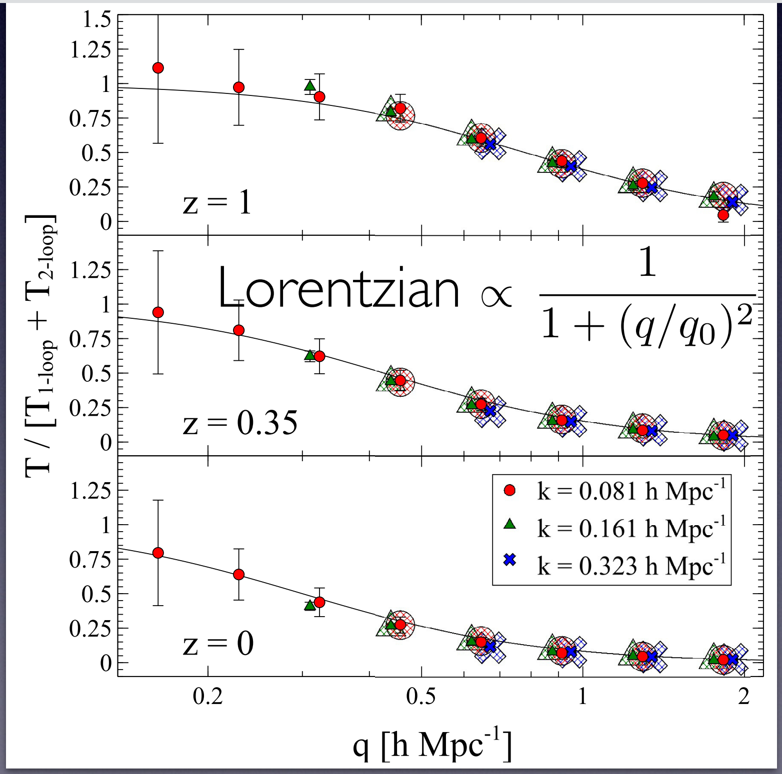


Lorentzian $\propto \dfrac{1}{1+(q/q_0)^2}$

UV suppression is seen at various k

What's wrong ?

Short summary

- Higher-order mode-coupling gets a larger UV contribution

However !

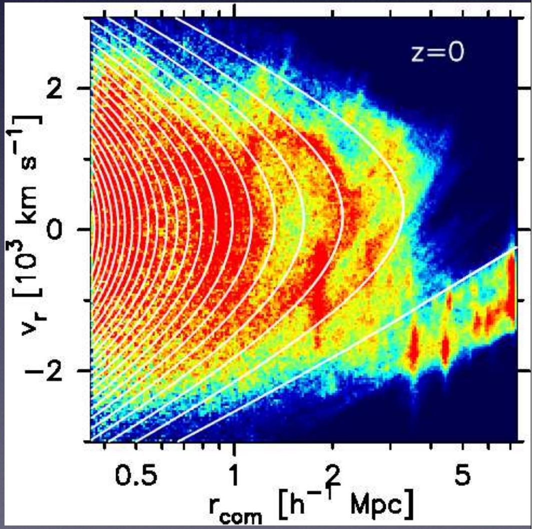
Blas, Garny & Konstandin ('14), Bernardeau, AT & Nishimichi ('14)

- In simulation, actual UV contribution is suppressed

Nishimichi, Bernardeau & AT ('16, '17 in prep.)

Most likely

Breakdown of single-stream PT treatment (even at large scales)



Suto et al. (2016)

What is a role of small-scale dynamics ?

Is there a way to go beyond single-stream PT ?

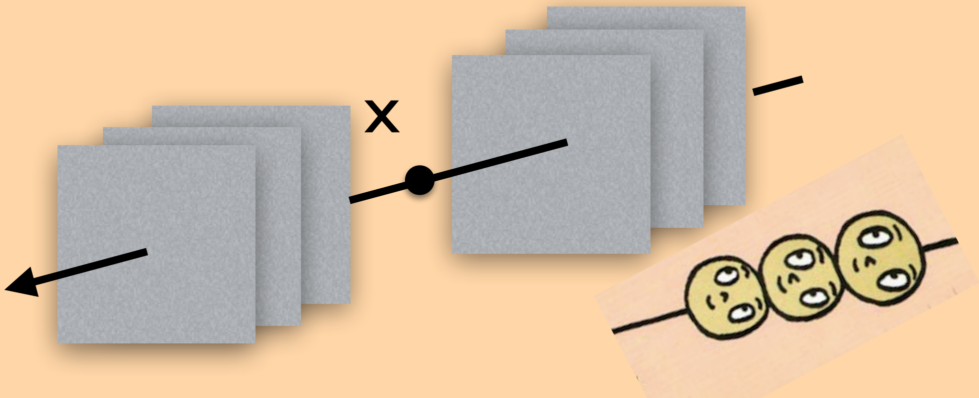
Multi-stream flows

(formation/merger of halos)

1D cosmology

Simplification may help us to understand what's going on

$$\nabla_x^2 \phi(x) = 4\pi\, G\overline{\rho}\, a^2\, \delta(x)$$

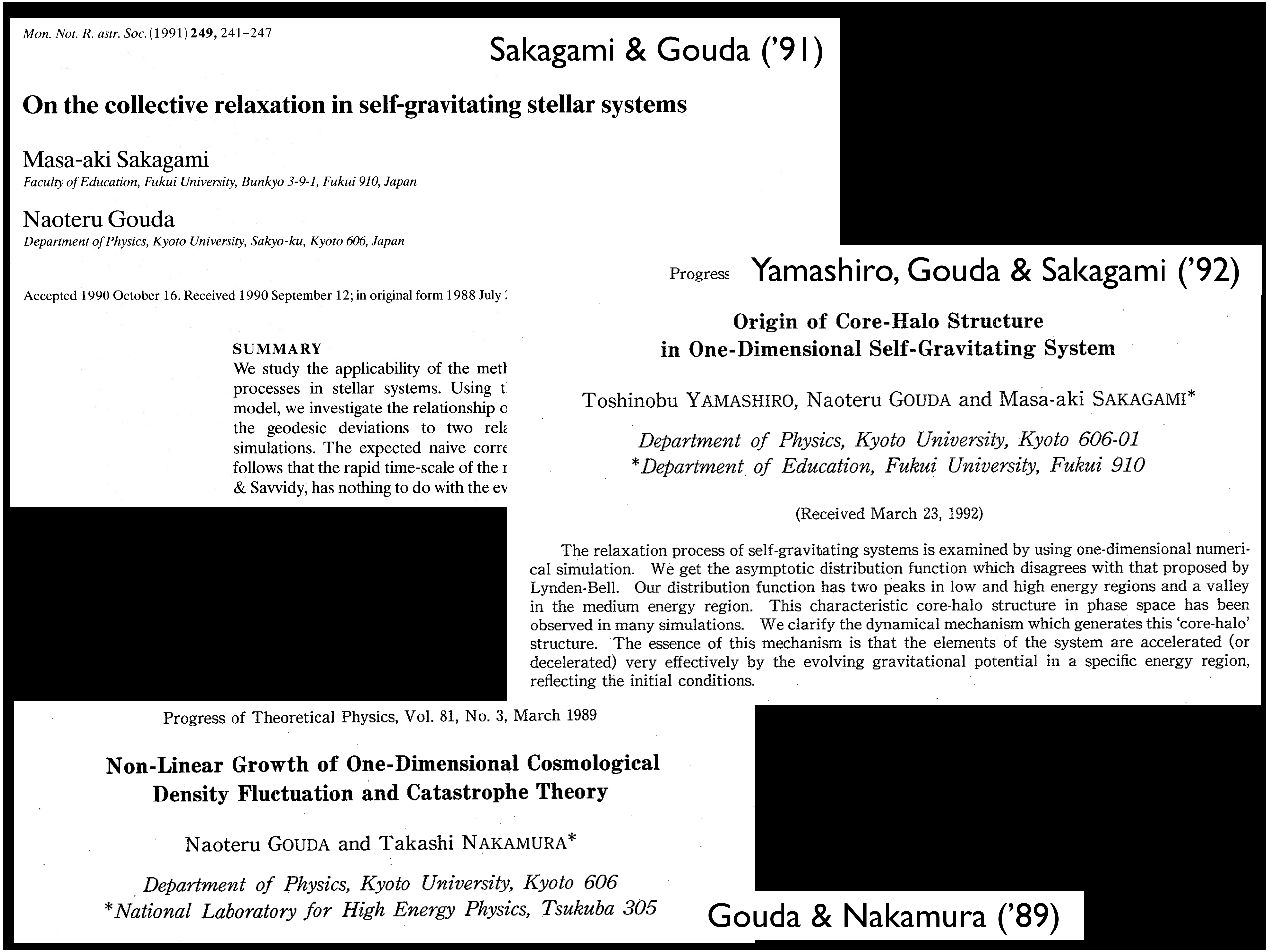


Force $\propto$ (# of sheets at RHS) - (# of sheets at LHS)

- Generic features of nonlinear mode-coupling :

  Response function
- Perturbative description beyond shell-crossing: Post-collapse PT

Learn something in simple *1D cosmology*



Sakagami & Gouda ('91)

Mon. Not. R. astr. Soc. (1991) **249,** 241–247

On the collective relaxation in self-gravitating stellar systems

Masa-aki Sakagami
Faculty of Education, Fukui University, Bunkyo 3-9-1, Fukui 910, Japan

Naoteru Gouda
Department of Physics, Kyoto University, Sakyo-ku, Kyoto 606, Japan

Accepted 1990 October 16. Received 1990 September 12; in original form 1988 July

SUMMARY

We study the applicability of the metl
processes in stellar systems. Using t
model, we investigate the relationship o
the geodesic deviations to two rel
simulations. The expected naive corre
follows that the rapid time-scale of the r
& Savvidy, has nothing to do with the ev

Progress Yamashiro, Gouda & Sakagami ('92)

Origin of Core-Halo Structure in One-Dimensional Self-Gravitating System

Toshinobu YAMASHIRO, Naoteru GOUDA and Masa-aki SAKAGAMI*

Department of Physics, Kyoto University, Kyoto 606-01
**Department of Education, Fukui University, Fukui 910*

(Received March 23, 1992)

The relaxation process of self-gravitating systems is examined by using one-dimensional numerical simulation. We get the asymptotic distribution function which disagrees with that proposed by Lynden-Bell. Our distribution function has two peaks in low and high energy regions and a valley in the medium energy region. This characteristic core-halo structure in phase space has been observed in many simulations. We clarify the dynamical mechanism which generates this 'core-halo' structure. The essence of this mechanism is that the elements of the system are accelerated (or decelerated) very effectively by the evolving gravitational potential in a specific energy region, reflecting the initial conditions.

Progress of Theoretical Physics, Vol. 81, No. 3, March 1989

Non-Linear Growth of One-Dimensional Cosmological Density Fluctuation and Catastrophe Theory

Naoteru GOUDA and Takashi NAKAMURA*

Department of Physics, Kyoto University, Kyoto 606
**National Laboratory for High Energy Physics, Tsukuba 305*

Gouda & Nakamura ('89)

1D Zel'dovich solution

(Zel'dovich '70)

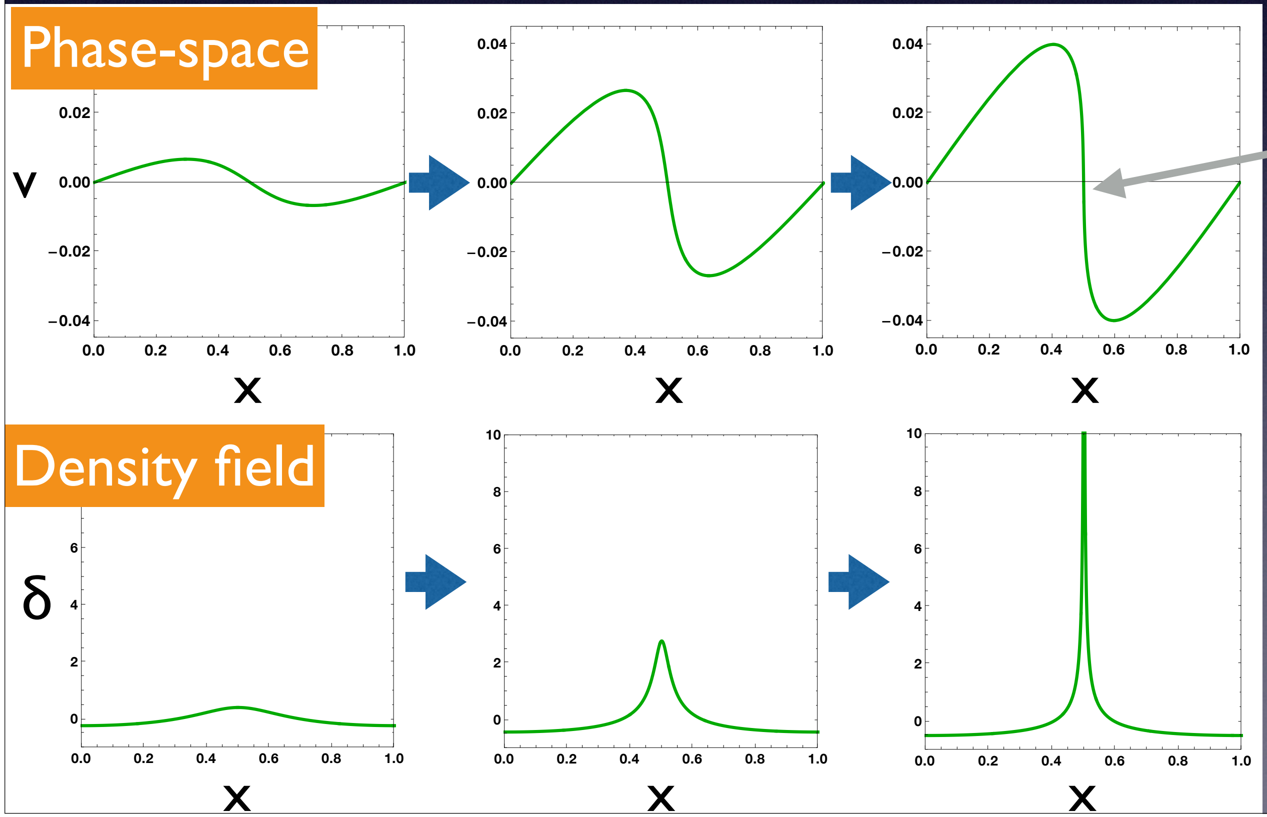
Exact single-stream solution

$$x(q;\tau) = q + \psi(q)\, D_+(\tau)$$

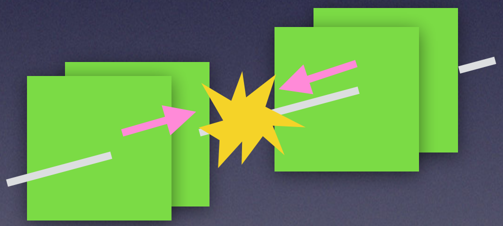
$$\mathrm{v}(q;\tau) = \psi(q)\, \frac{dD_+(\tau)}{d\tau}$$

$D_+(\tau)$: linear growth factor

$\psi(q)$: displacement field



Shell crossing



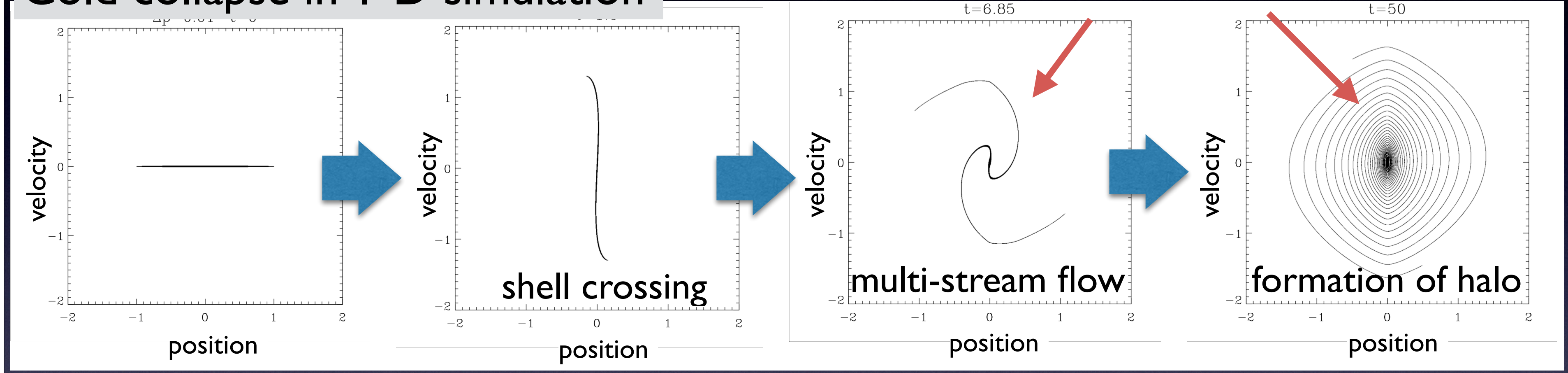
Solution is exact until shell crossing

Post-collapse PT: beyond shell-crossing

AT & Colombi ('17)

Cold collapse in 1-D simulation

Breakdown of Zel'dovich solution



Computing back-reaction to the Zel'dovich flow: *Lagrangian*

1. Expand the displacement field around *shell-crossing point*, q_0:

$$x(q;\tau) \simeq A(q_0;\tau) - B(q_0;\tau)(q-q_0) + C(q_0;\tau)(q-q_0)^3$$

2. Compute force $F(x(q;\tau)) = -\nabla_x \Phi(x(q;\tau))$ at multi-stream region

$$\Delta \mathrm{v}(Q;\tau,\,\tau_{\rm q}) = \int_{\tau_{\rm q}}^{\tau} d\tau'\, F(x(Q,\tau')), \quad \Delta x(Q;\tau,\,\tau_{\rm q}) = \int_{\tau_{\rm q}}^{\tau} d\tau'\, \Delta \mathrm{v}(Q;\tau',\tau_{\rm q})$$

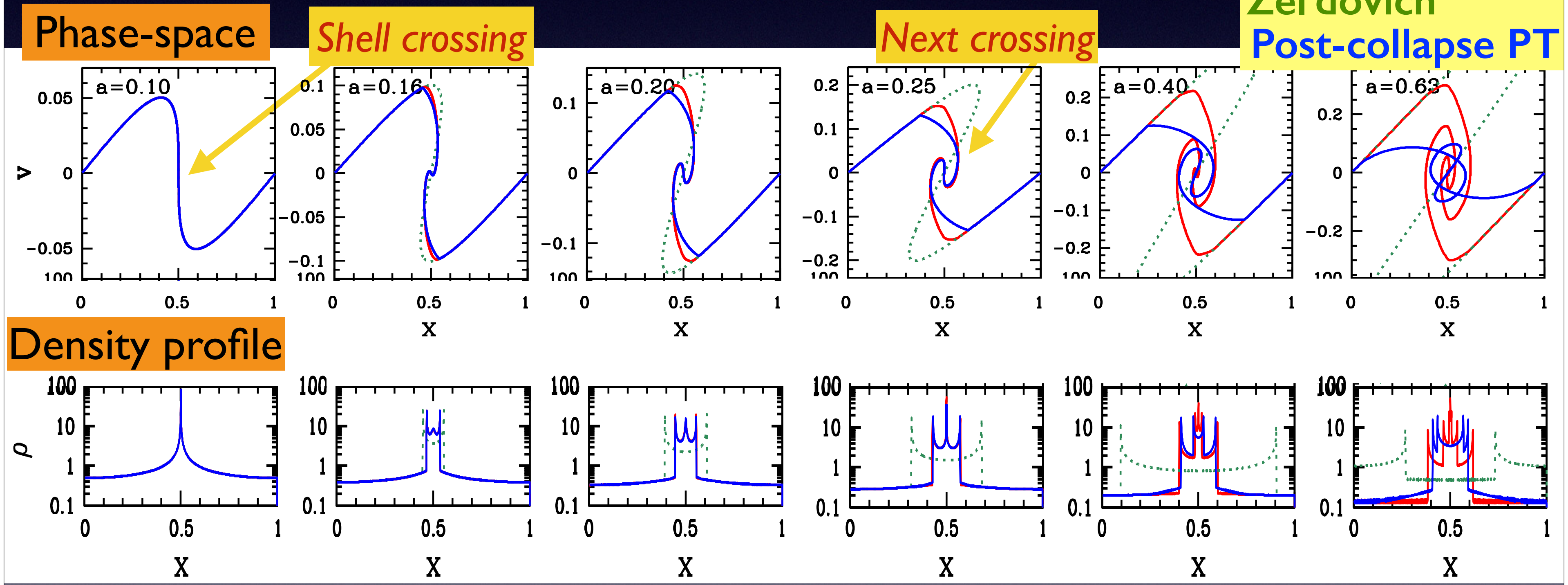
…… polynomial function of *$Q=q-q_0$ up to 7th order*

Post-collapse PT: single cluster

AT & Colombi ('17)

Post-collapse PT basically fails after next shell-crossing, but it still gives reasonable prediction for density profiles

Simulation
Zel'dovich
Post-collapse PT



Phase-space

Shell crossing

Next crossing

Density profile

Of course, this does not guarantee the accuracy of power spectrum prediction at small scales (→ next slide)

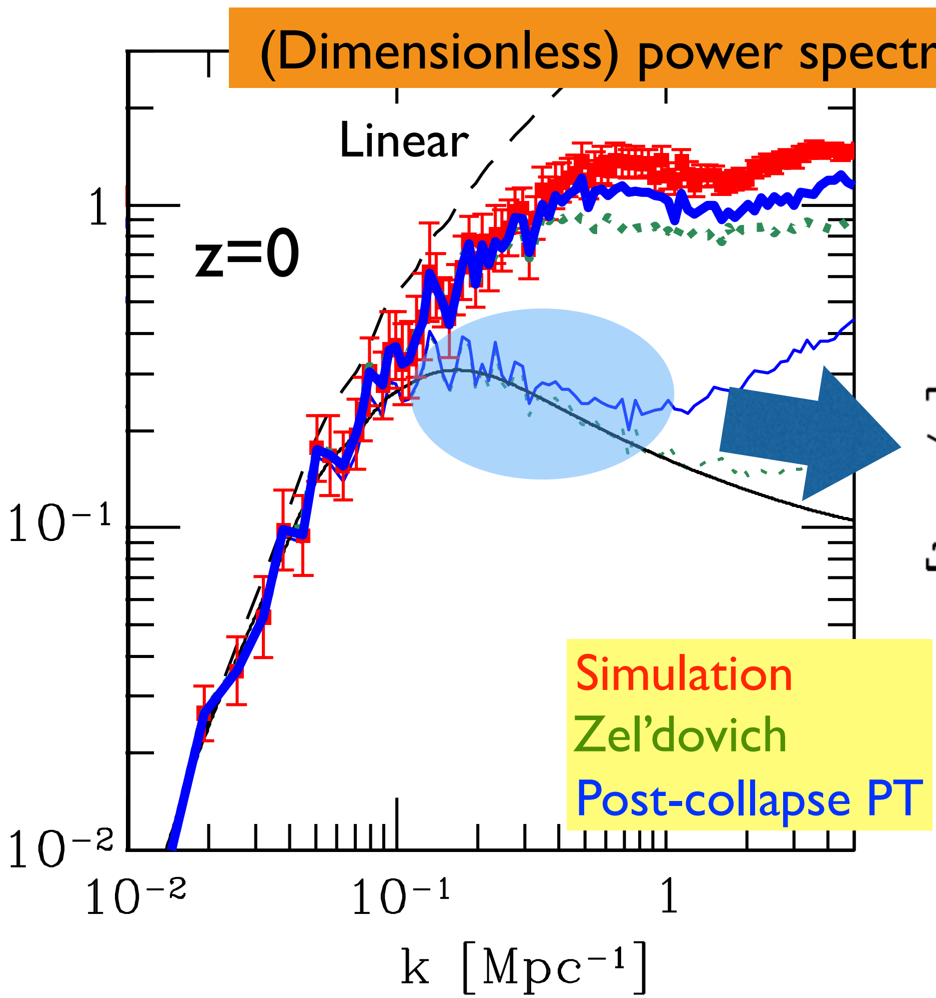
Post-collapse PT: ΛCDM

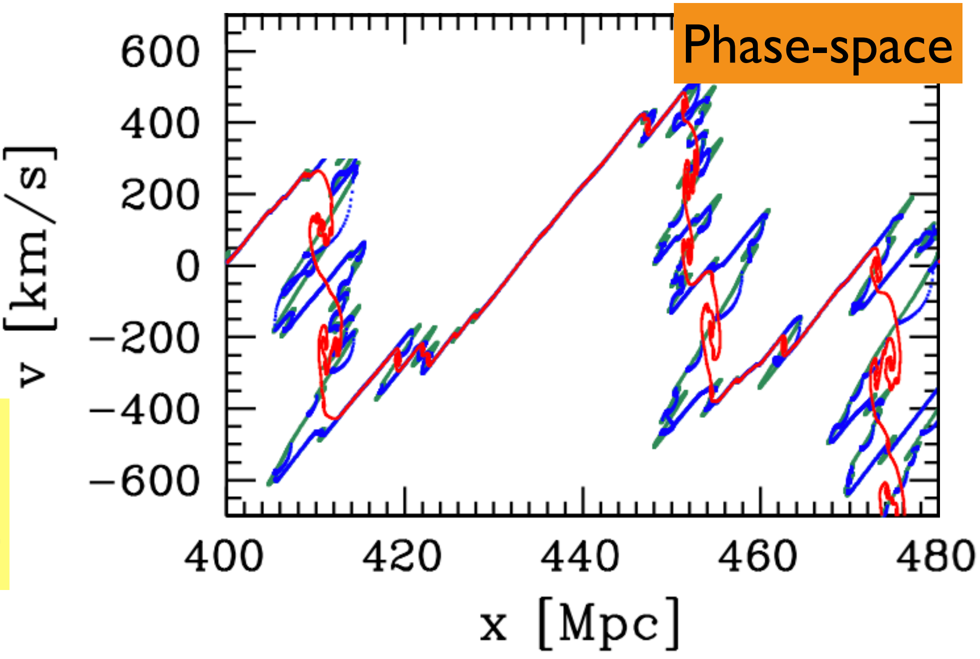
k P(k)/π

Planck ΛCDM

$$P_{\rm 1D}(k) = \frac{k^2}{2\pi}\,\underline{P_{\rm 3D}(k)}$$

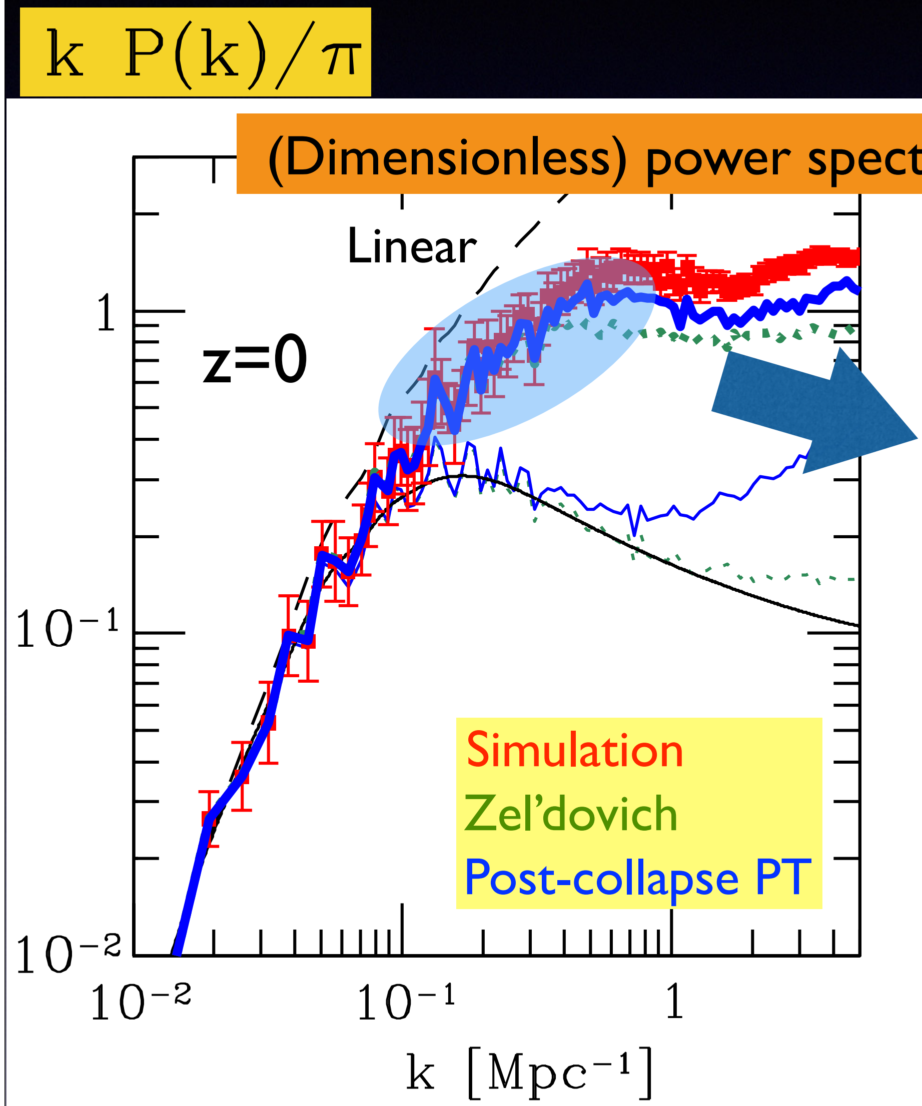
$L = 1{,}000\,{\rm Mpc}$
\# of particles (sheets) : 2×10^5
\# of runs : 50





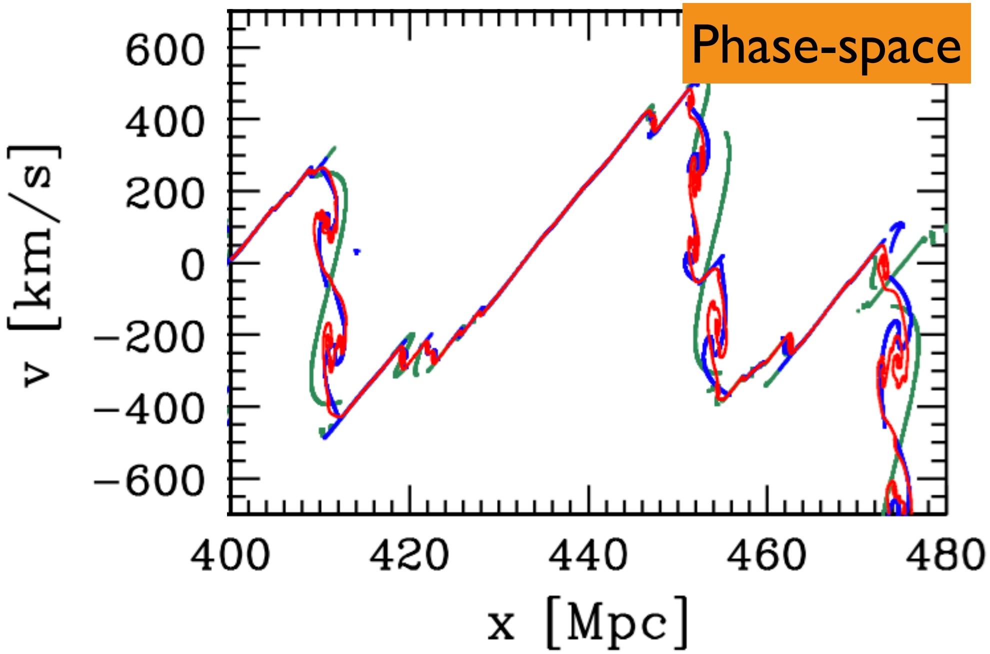
AT & Colombi ('17)

Post-collapse PT: ΛCDM



Adaptive smoothing

applied to initial density peaks
(with filter scales determined
by first-barrier crossing)



AT & Colombi ('17)

Implication to 3D

Combination of the two methods are rather crucial:

PT scheme beyond shell crossing & *Coarse-graining*

(post-collapse PT) *(adaptive smoothing)*

But, idea & technique are promising and can be extended to 3D

Issues to be addressed

- Accurate pre-collapse description
  - ✓ Zel'dovich approx. is inaccurate
  - ✓ Various topologies of shell crossing
- Tractable analytical calculation of statistical quantities

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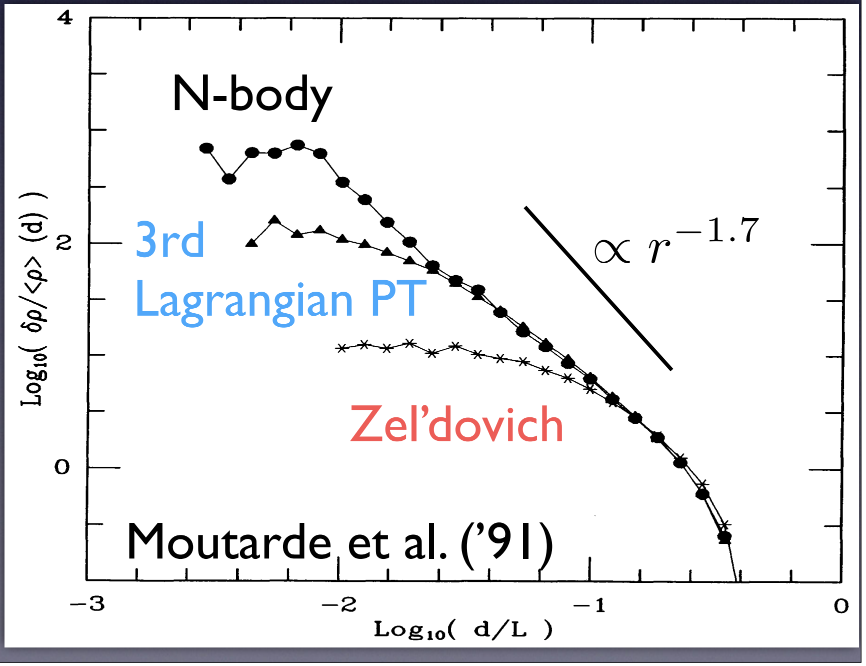
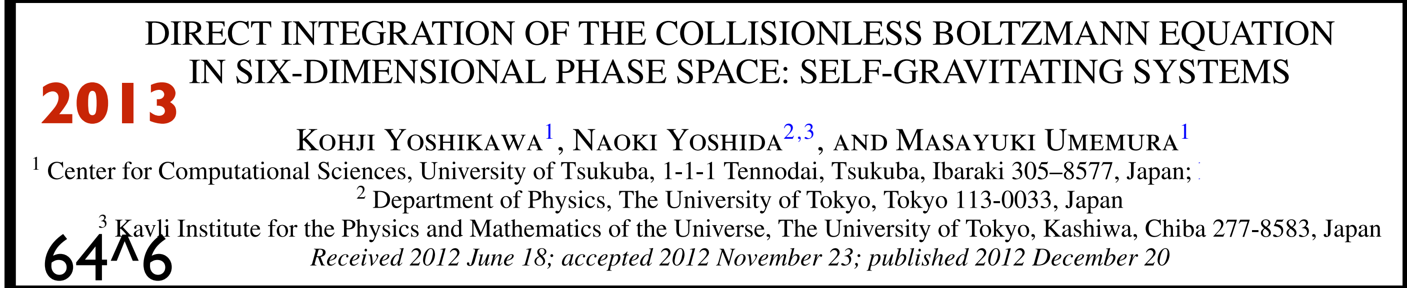


Moutarde et al. (’91)

State-of-the-art cosmological Vlasov code



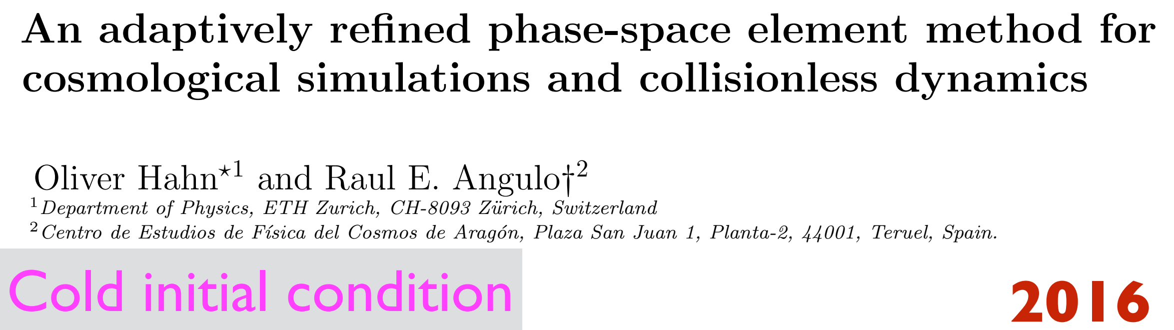
DIRECT INTEGRATION OF THE COLLISIONLESS BOLTZMANN EQUATION IN SIX-DIMENSIONAL PHASE SPACE: SELF-GRAVITATING SYSTEMS

2013

KOHJI YOSHIKAWA[1], NAOKI YOSHIDA[2,3], AND MASAYUKI UMEMURA[1]
[1] Center for Computational Sciences, University of Tsukuba, 1-1-1 Tennodai, Tsukuba, Ibaraki 305–8577, Japan;
[2] Department of Physics, The University of Tokyo, Tokyo 113-0033, Japan
[3] Kavli Institute for the Physics and Mathematics of the Universe, The University of Tokyo, Kashiwa, Chiba 277-8583, Japan

Received 2012 June 18; accepted 2012 November 23; published 2012 December 20

64^6

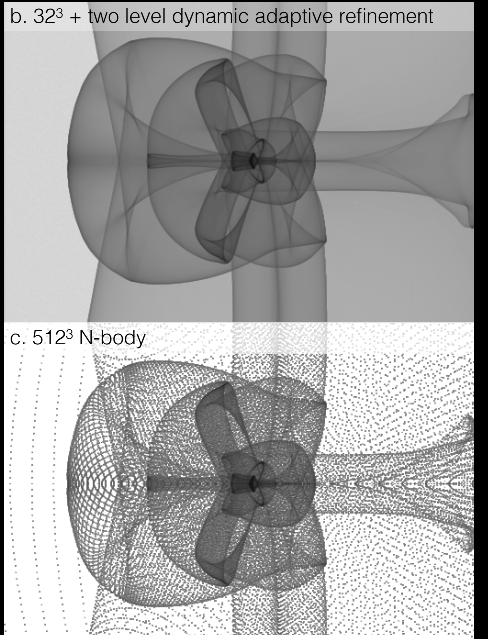


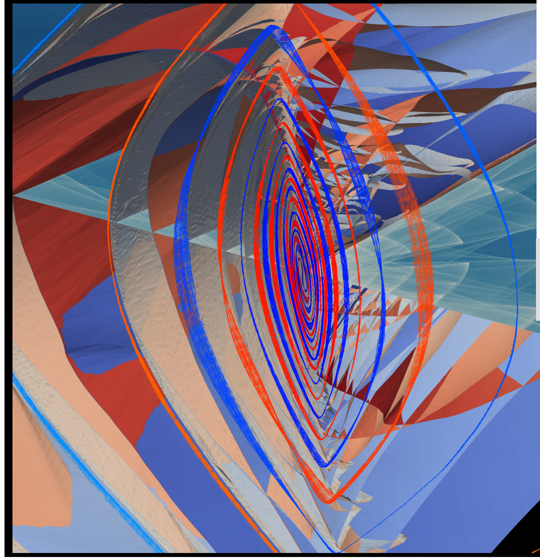
An adaptively refined phase-space element method for cosmological simulations and collisionless dynamics

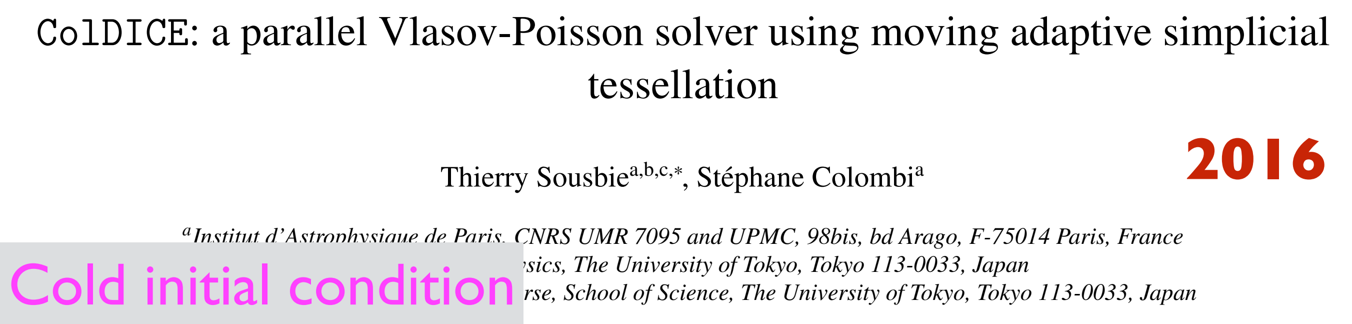
Oliver Hahn⋆[1] and Raul E. Angulo†[2]
[1] *Department of Physics, ETH Zurich, CH-8093 Zürich, Switzerland*
[2] *Centro de Estudios de Física del Cosmos de Aragón, Plaza San Juan 1, Planta-2, 44001, Teruel, Spain.*

Cold initial condition

2016







`ColDICE`: a parallel Vlasov-Poisson solver using moving adaptive simplicial tessellation

Thierry Sousbie[a,b,c,*], Stéphane Colombi[a]
[a] *Institut d'Astrophysique de Paris, CNRS UMR 7095 and UPMC, 98bis, bd Arago, F-75014 Paris, France*
sics, The University of Tokyo, Tokyo 113-0033, Japan
rse, School of Science, The University of Tokyo, Tokyo 113-0033, Japan

2016

Cold initial condition

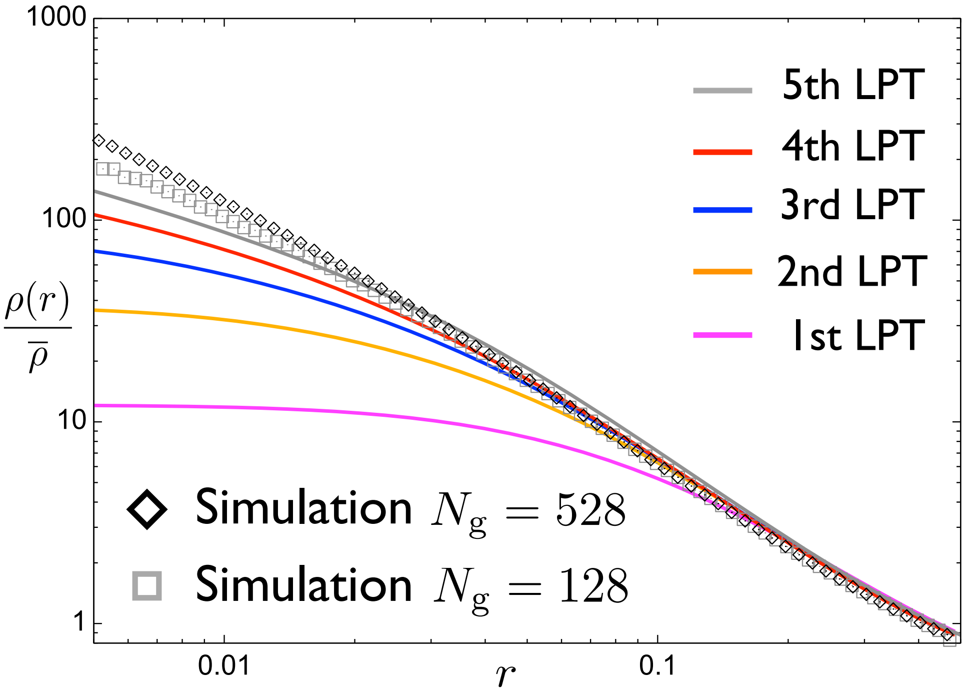
Analytic treatment helps to understand what is going on in Vlasov simulations

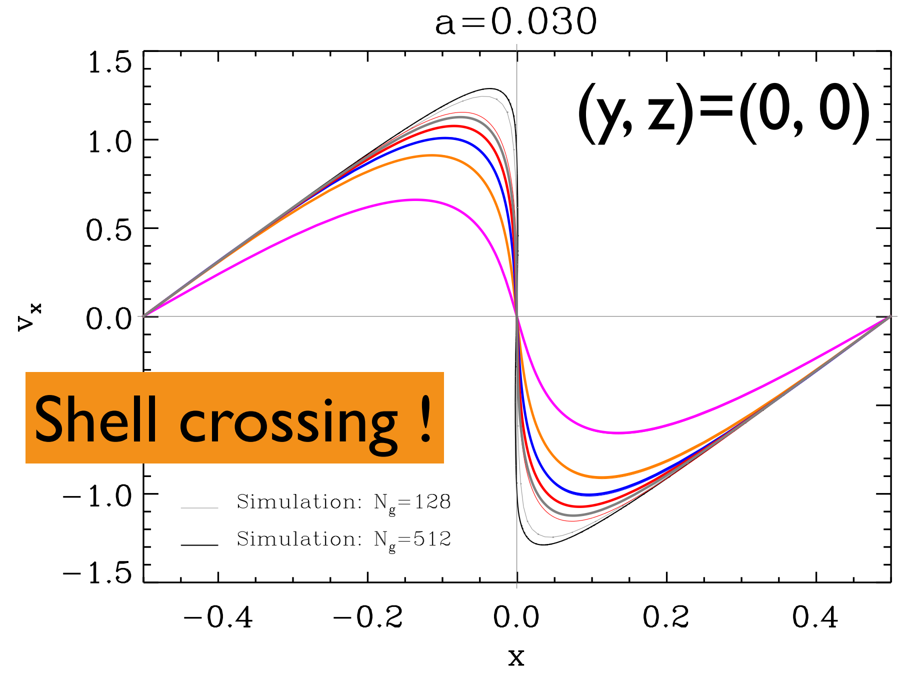
Approaching shell-crossing in 3D

W/ S. Saga & S. Colombi (in progress)

In 3D, even the description of pre-collapse phase is non-trivial

Lagrangian PT treatment is the only way to analytically capture the shell-crossing





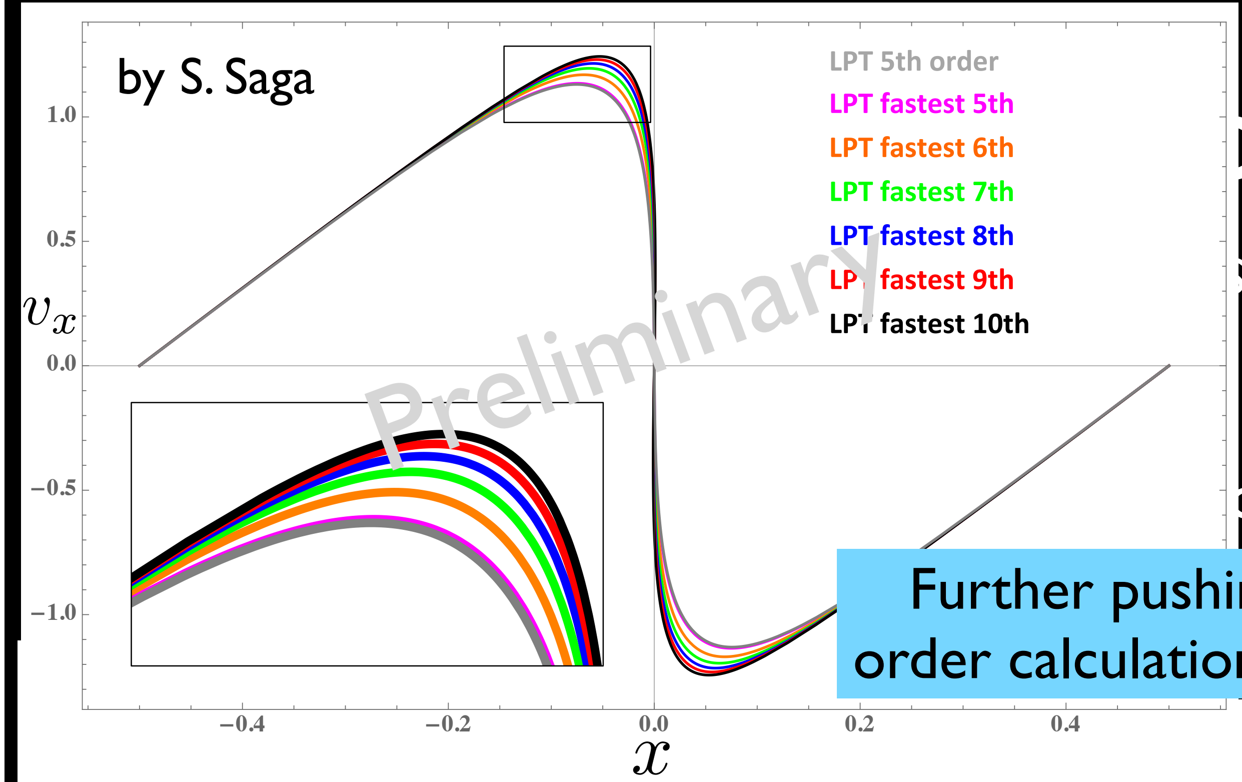
Shell crossing !

ossing in 3D

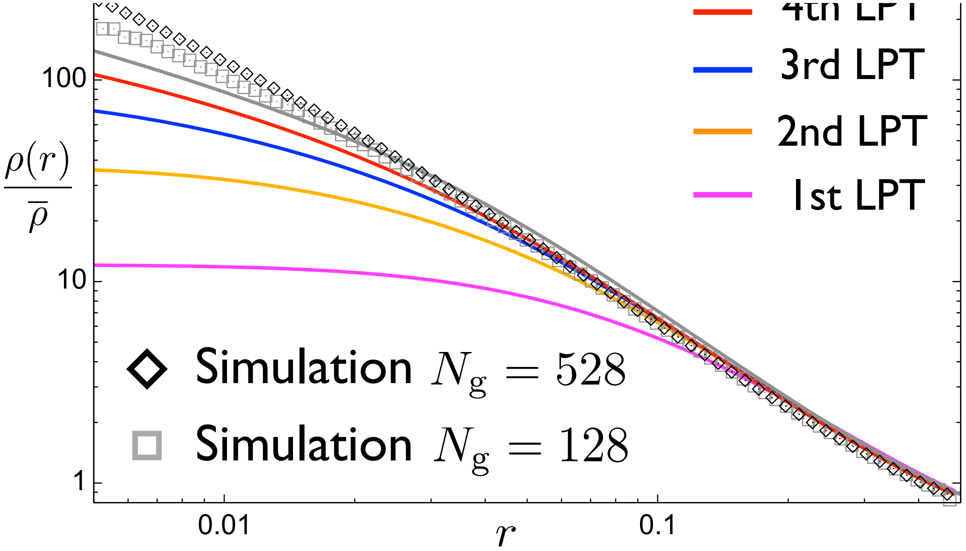
aga & S. Colombi (in progress)

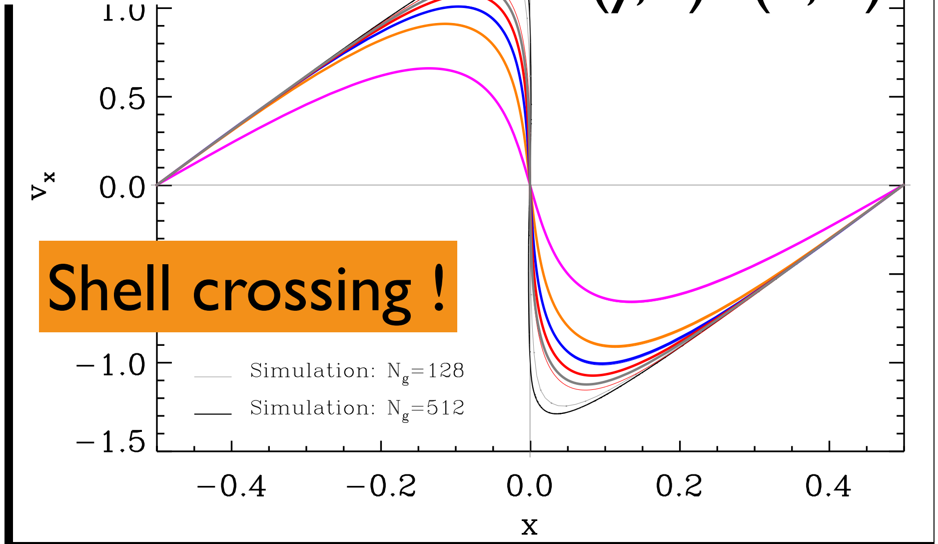
phase is non-trivial

o analytically capture



Further pushing higher-order calculation may help ?





Shell crossing !

Summary

宇宙論的（無衝突）自己重力多体系としての宇宙の大規模構造

その理論的記述をめぐる進展と混迷、あるいは

（観測の理論テンプレート）　摂動論的計算手法の再生と受難

- くりこみ・再和法の発展・観測的応用
- UV問題とその起源、応答関数による定量化
- 単一流近似を超える取り扱いと課題 → 今後に期待

（ポストコラプス摂動論）

大規模構造は、今後も理論・観測ともに目が離せない