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ICG Seminar
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Perturbation Theory of Large-scale Structure and Future Galaxy Surveys

Atsushi Taruya

(RESCEU, Univ.Tokyo)

RESCEU=RESearch Center for the Early Universe

Contents

- Introduction: LSS and precision cosmology
- Modeling non-linear evolution of P(k)
- Improving perturbation theory prediction
- Summary

Refs.

AT & Hiramatsu, ApJ 674 (2008) 617

Nishimichi et al. arXiv:0810.0813

AT et al. (2008) in prep.

Cosmology with Large-scale Structure (LSS)

Large-scale structure (LSS)

Spatial inhomogeneities of mass distribution inferred from observed galaxy clustering

Growth of structure in the Universe reveals

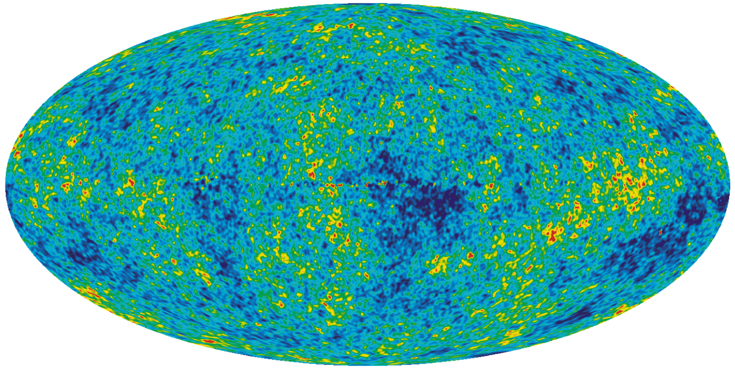
- Dynamics of cosmic expansion
- Information of primordial fluctuations

LSS study is a major traditional method in cosmology to determine/constrain cosmological parameters

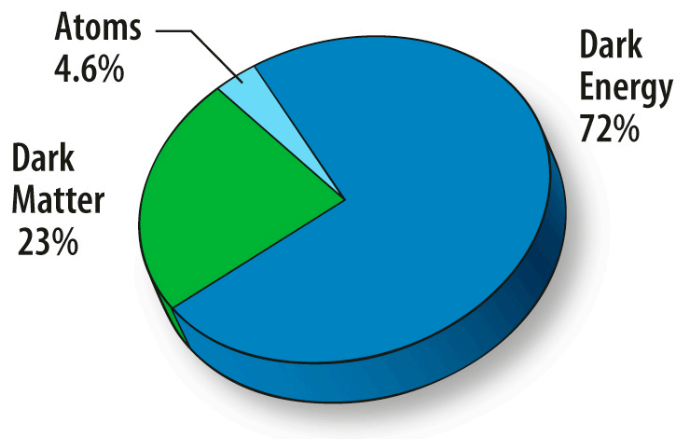
New Era of LSS Study

Entering era of precision cosmology

Precise measurement of CMB anisotropies by WMAP (2003~)



Joint constraints with LSS, SNe



Standard Model (**LCDMmodel**)

Universe with flat geometry consisting of cold dark matter (CDM), baryon and dark energy (characterized by **6** parameters)

Targets of Future Galaxy Surveys

- **Detection of primordial non-Gaussianity**

  Narrowing the window of early universe physics

- **Determination of Dark energy equation-of-state**

  Whether the nature of dark energy is cosmological constant or not?

- **Constraint on neutrino masses**

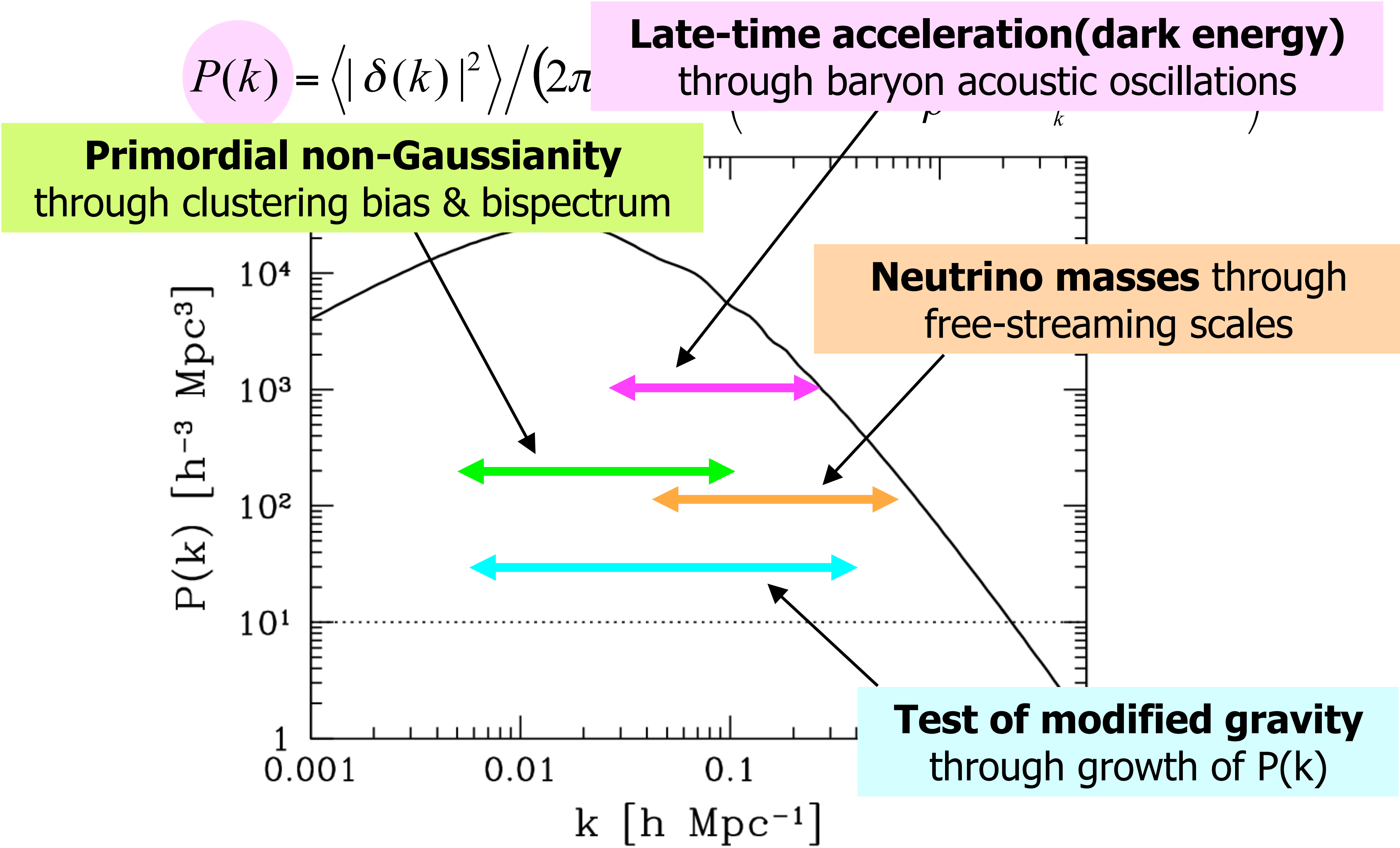
  A step beyond standard model of particle physics

- **Test of general theory of gravity (GR)**

  Validity of GR on cosmological scales (alternative to dark energy)

Signals of these targets are primarily very weak, and need precise measurement. Observation of LSS plays a key role.

P(k): A Map of Treasure Hunting



Confronting Theory with Precision Measurement

Scale of interest: **$0 < k < 0.4 \sim 0.6$ [h/Mpc]**

Linear ~ weakly non-linear regime
of gravitational evolution

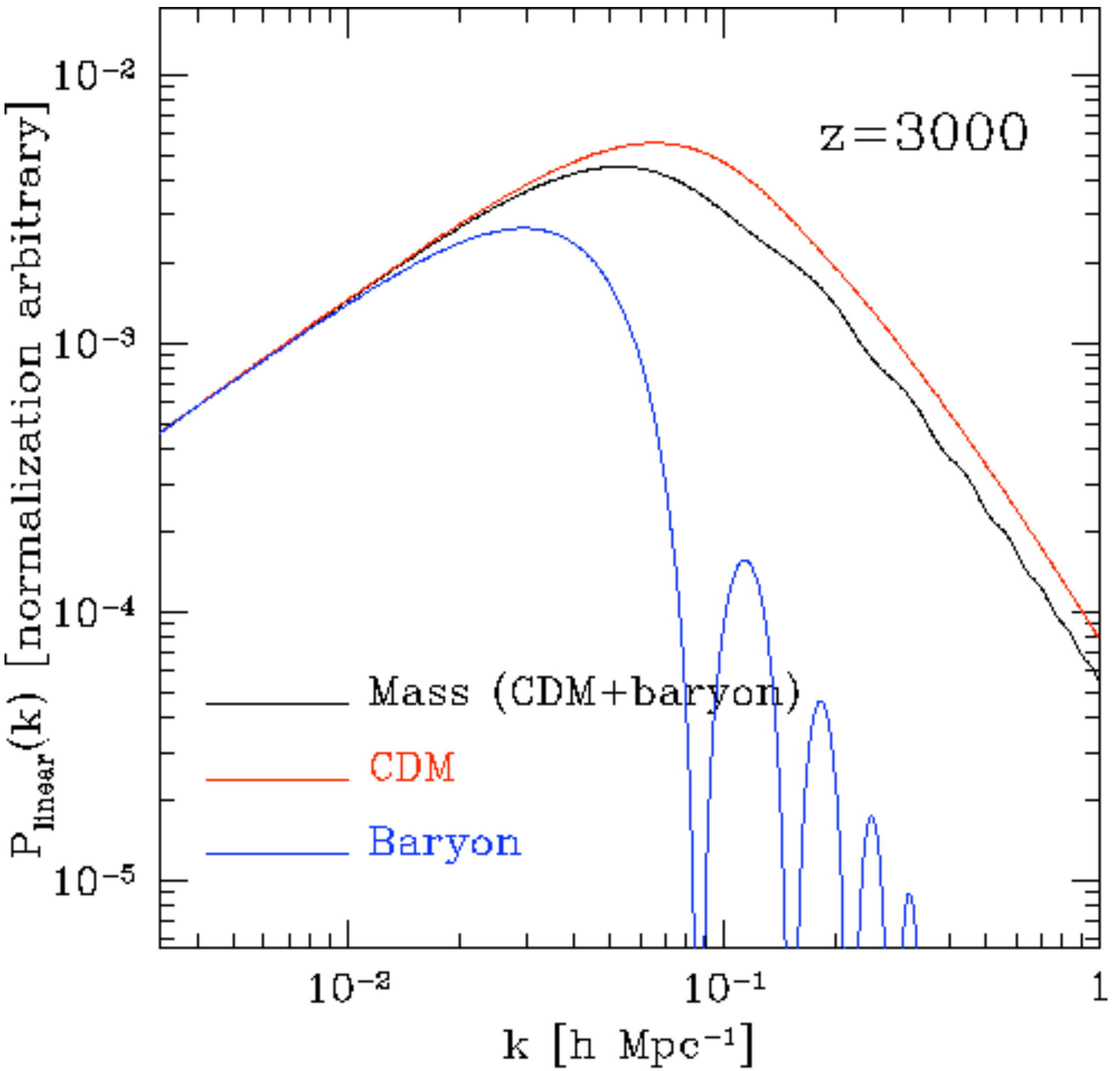
Accurate modeling for gravitational evolution of P(k) is crucial

Precise measurement with future galaxy surveys requires a percent-level precision for template modeling

There are also other systematic effects that should be considered (i.e., redshift-space distortion, galaxy biasing)

Linear Theory Prediction

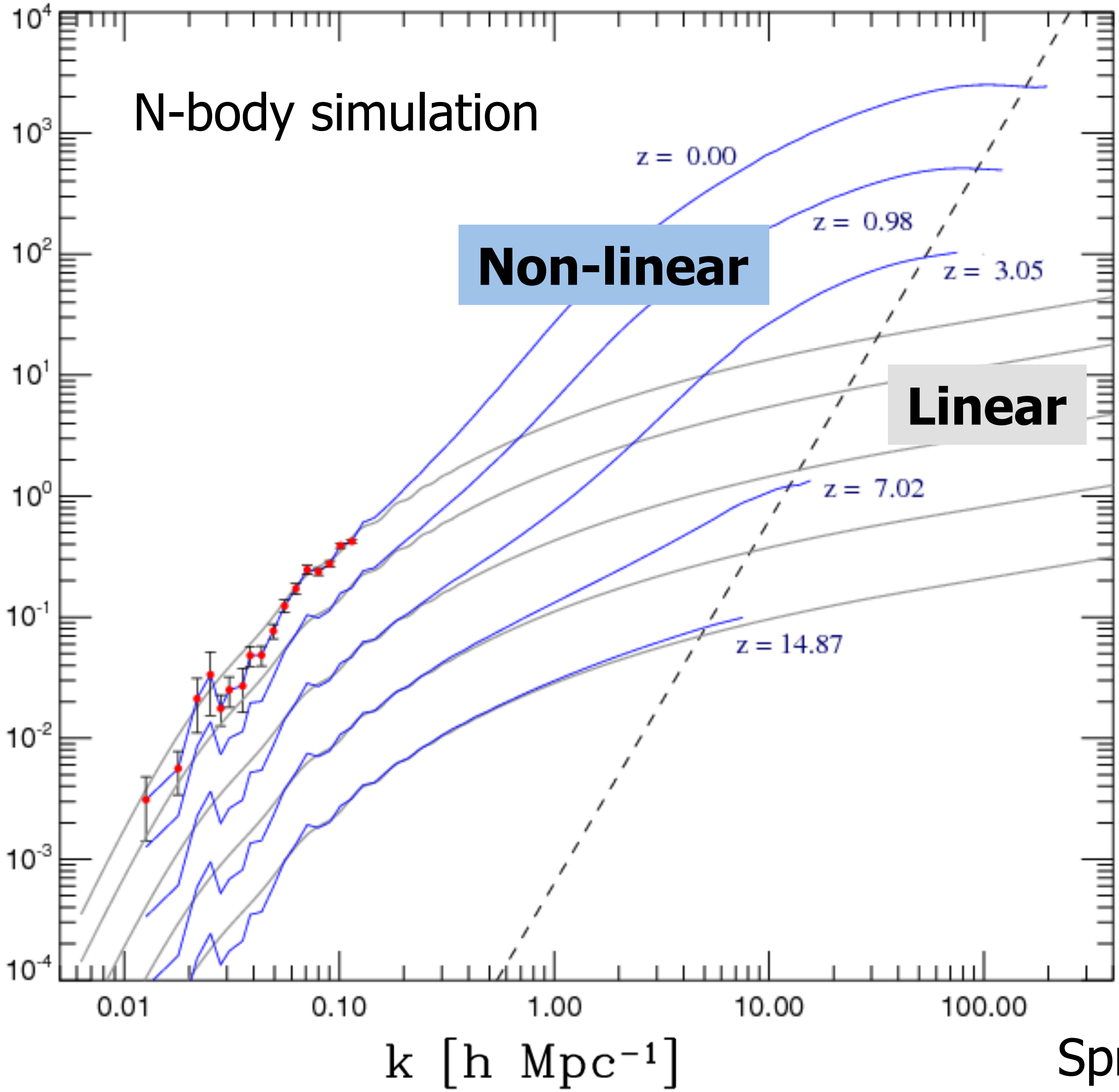
Accurate prediction is available for given cosmological parameters



computed by
CMBFAST4.5.1

Non-linear Evolution of P(k)

$$\frac{k^3 P(k)}{2\pi^2}$$



Springel et al. (2005)

Toward Accurate Modeling of P(k)

first-principle calculation

N-body simulations

Complementary approach

Fitting formulae to transform from linear P(k)
to non-linear P(k)

Halo model prescription

Peacock & Dodds 1996
Smith et al. 2001
Cooray & Sheth 2003

Independent approach

Analytical calculation based on Perturbation Theory(PT)

Since the formulation more than 20 years ago,

It has been reloaded

PT: Formalism

Perturbative calculation under the fluid treatment of LSS

Juszkiewicz (1981), Vishniac (1983), Goroff. et al. (1986)
Suto & Sasaki (1991), Makino et al. (1992), Jain & Bertschinger (1994)

「CDM+baryon」=「pressureless & irrotational perfect fluid」

Basic equations

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\vec{\nabla}\cdot\left[(1+\delta)\vec{v}\right] = 0$$

$$\frac{\partial \vec{v}}{\partial t} + \frac{\dot{a}}{a}\vec{v} + \frac{1}{a}(\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{1}{a}\vec{\nabla}\Phi$$

$$\frac{1}{a^2}\nabla^2\Phi = 4\pi G\,\bar{\rho}_{\rm m}\,\delta$$

Perturbative expansion

$$\delta = \delta^{(1)} + \delta^{(2)} + \delta^{(3)} + \cdots$$

$$\left\langle \delta(\vec{k};z)\delta(\vec{k}';z)\right\rangle = (2\pi)^3\,\delta_{\rm D}(\vec{k}+\vec{k}')\,P(k;z)$$

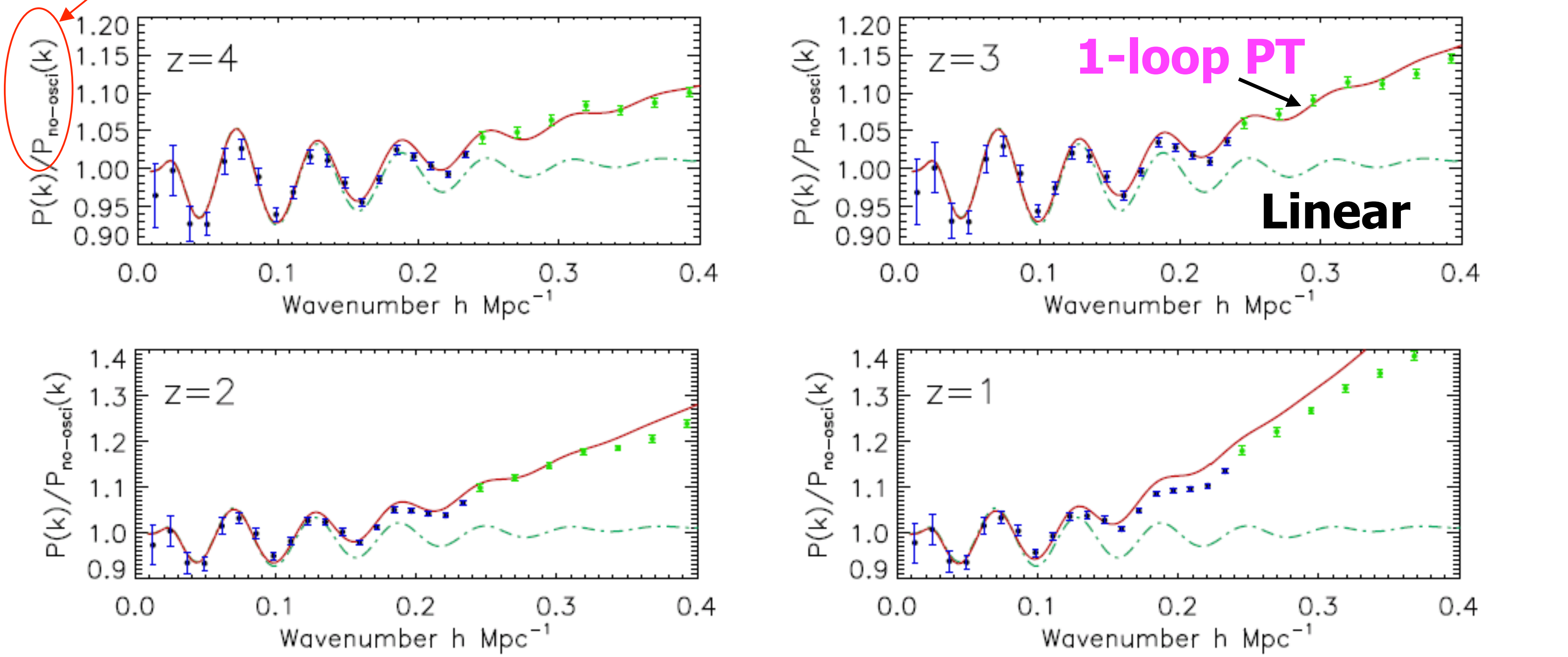
PT vs. N-body Simulations

PT prediction

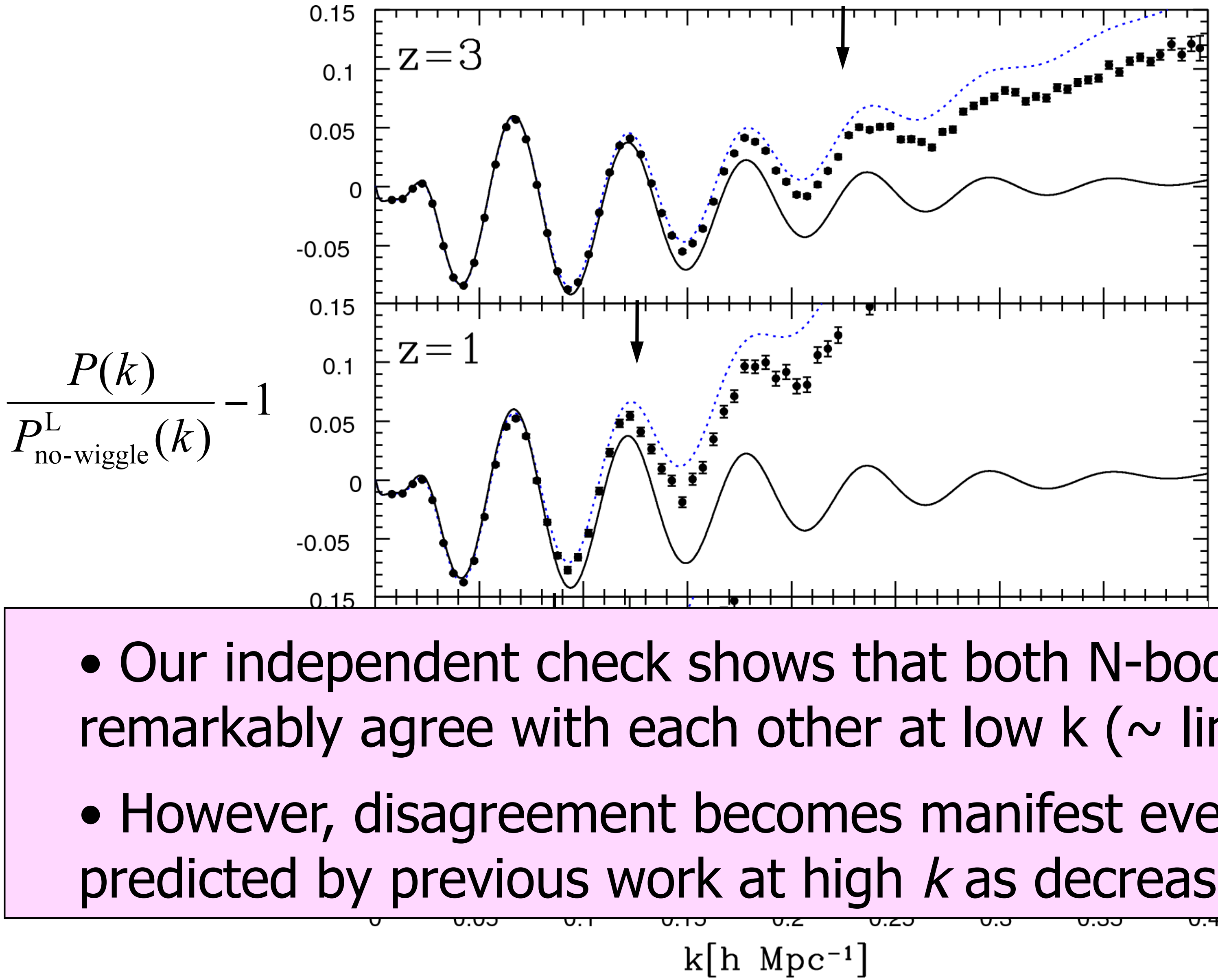
$$P(k;z) = D^2(z)\underbrace{P^L(k)}_{\text{linear}} + D^4(z)[\underbrace{P^{(22)}(k) + P^{(13)}(k)}_{\text{Leading-order correction (1-loop)}}] + \cdots$$

no-wiggle TF of Eisenstein & Hu (1998)

Jeong & Komatsu (2006)



Limitation of PT



Nishimichi et al.
arXiv:0810.0813

- Our independent check shows that both N-body and PT remarkably agree with each other at low k (~ linear scales)
- However, disagreement becomes manifest even faster than predicted by previous work at high *k* as decreasing redshift

Improving PT

Attempts to extend reliability range of PT

Renormalized PT (RPT)	Crocce & Scoccimarro (2006ab,2008)
CLosure Approx. (CLA)	Valageas (2007) AT & Hiramatsu (2008) (c.f. Pietroni 2008)
PT based on Lagrangian Picture	Matsubara (2008ab)
Path integral formulation	Matarrese & Pietroni (2007) Izumi & Soda (2007)

Non-perturbative Formulation

Crocce & Scoccimarro (2006ab,2008)

AT & Hiramatsu (2008)

1. Diagramatic representation of (naïve) perturbation series

2. Renormalizaed expressions of perturbation series
   in terms of non-perturbative quantities

   (Power spectrum, propagator, vertex function)

Tree-level approx. of vertex func.

RPT ◆ Calculation of each loop diagram by 1st Born approx.

CLA ◆ Truncating the diagrams at 1-loop order, iteratively evaluate 'renormalized' 1-loop diagram

Standard PT vs. Improved PT

$$P^{(mn)}(k) \sim \left\langle \delta^{(m)} \delta^{(n)} \right\rangle$$

Standard PT

$$P(k) = \underbrace{P^{(11)}(k)}_{\text{Linear (tree)}} + \underbrace{\left[P^{(13)}(k) + P^{(22)}(k) \right]}_{\text{1-loop}} + \underbrace{\left[P^{(33)}(k) + P^{(24)}(k) + P^{(15)}(k) \right]}_{\text{2-loop}} + \cdots$$

$\propto D^2(t)$ $\propto D^4(t)$ $\propto D^6(t)$

Straightforward calculation based on naïve expansion

Improved PT (CLA)

Initial P(k)

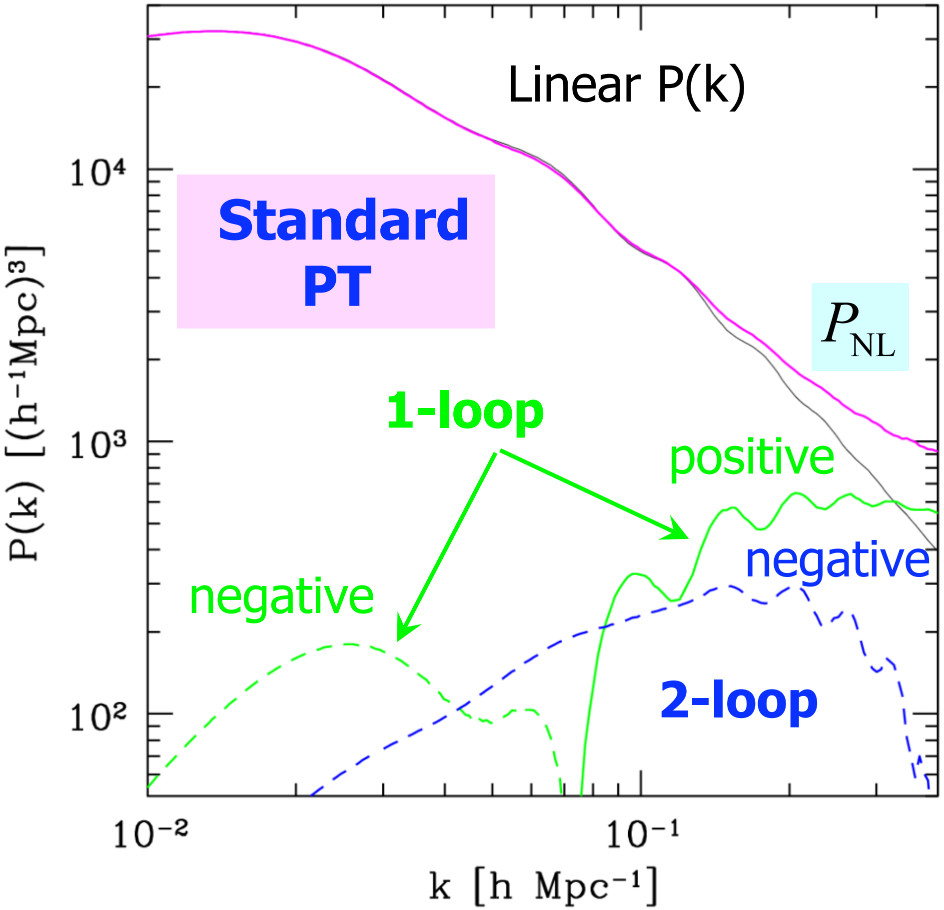
$$P(k;t) = \underbrace{G^2(k \,|\, t, t_0)}_{\text{propagator}} P(k;t_0) + \underbrace{\int\!\!\int ds\,dt\; G(k \,|\, s, t_0)\; G(k \,|\, t, t_0)\, \Phi_{1\text{-loop}}[\,P(k); s, t\,]}_{\text{Mode-coupling term}}$$

Non-perturbative effects is incorporated through **propagator**

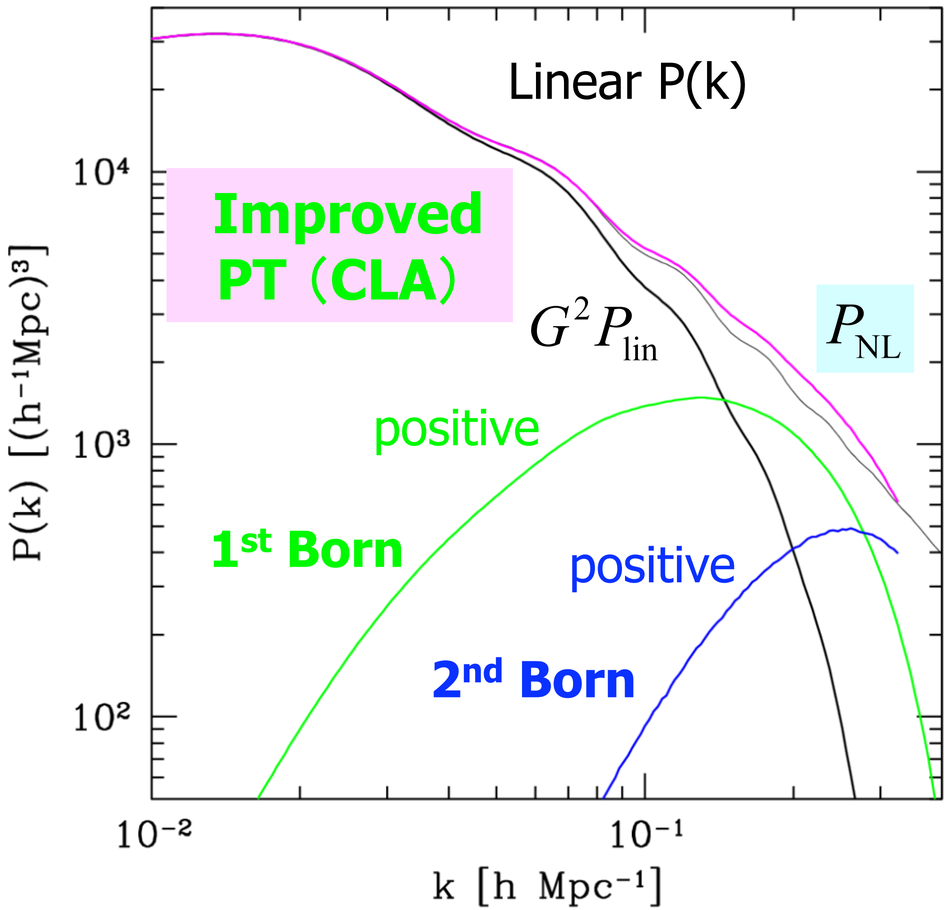
Iteratively evaluate mode-coupling term by Born approximation

Convergence Properties

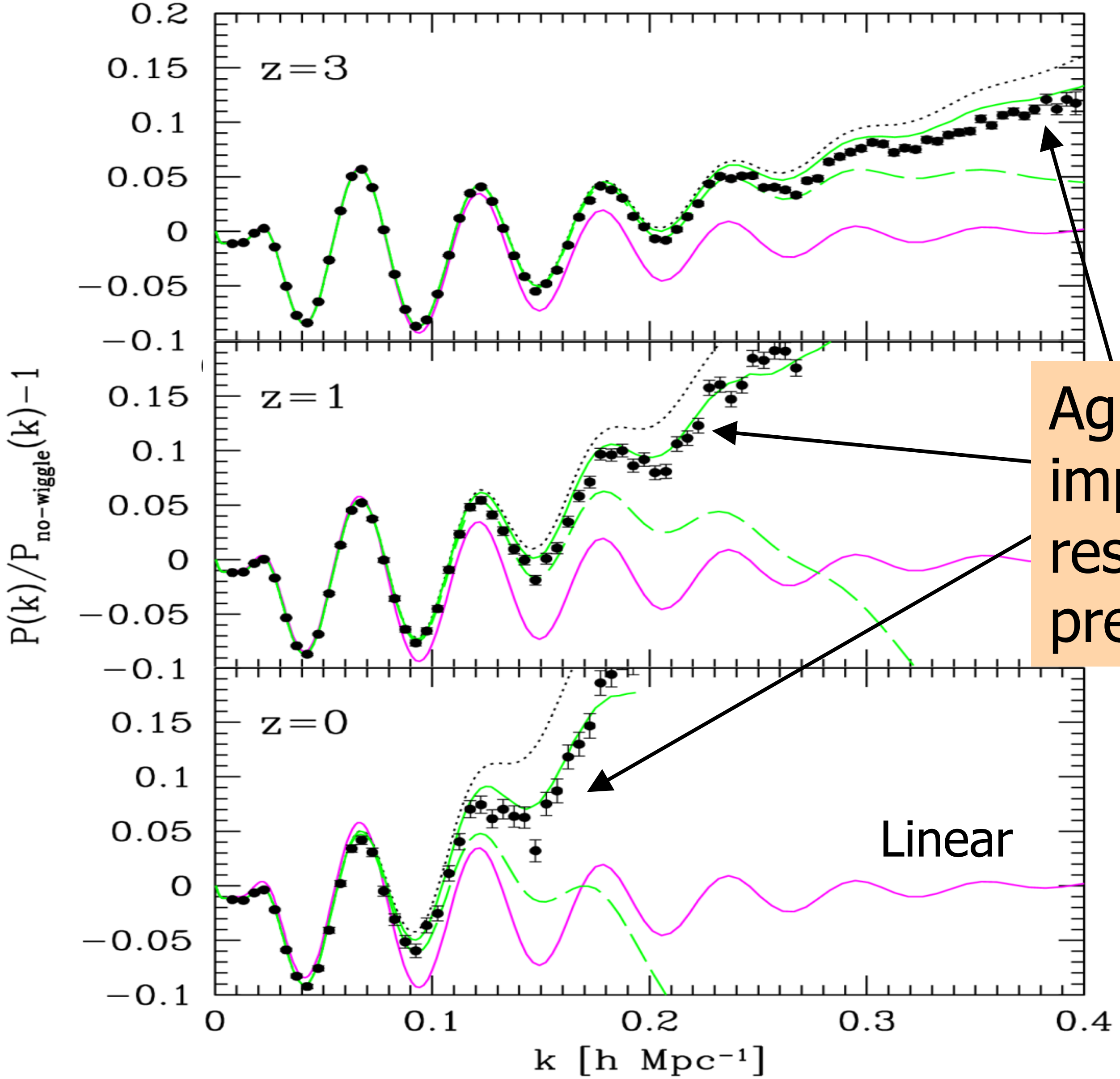
All contributions become comparable at low-z, positivity is not guaranteed



Contributions are all positive, shifted to higher k as increasing order of PT



Improved PT vs. N-body Simulation

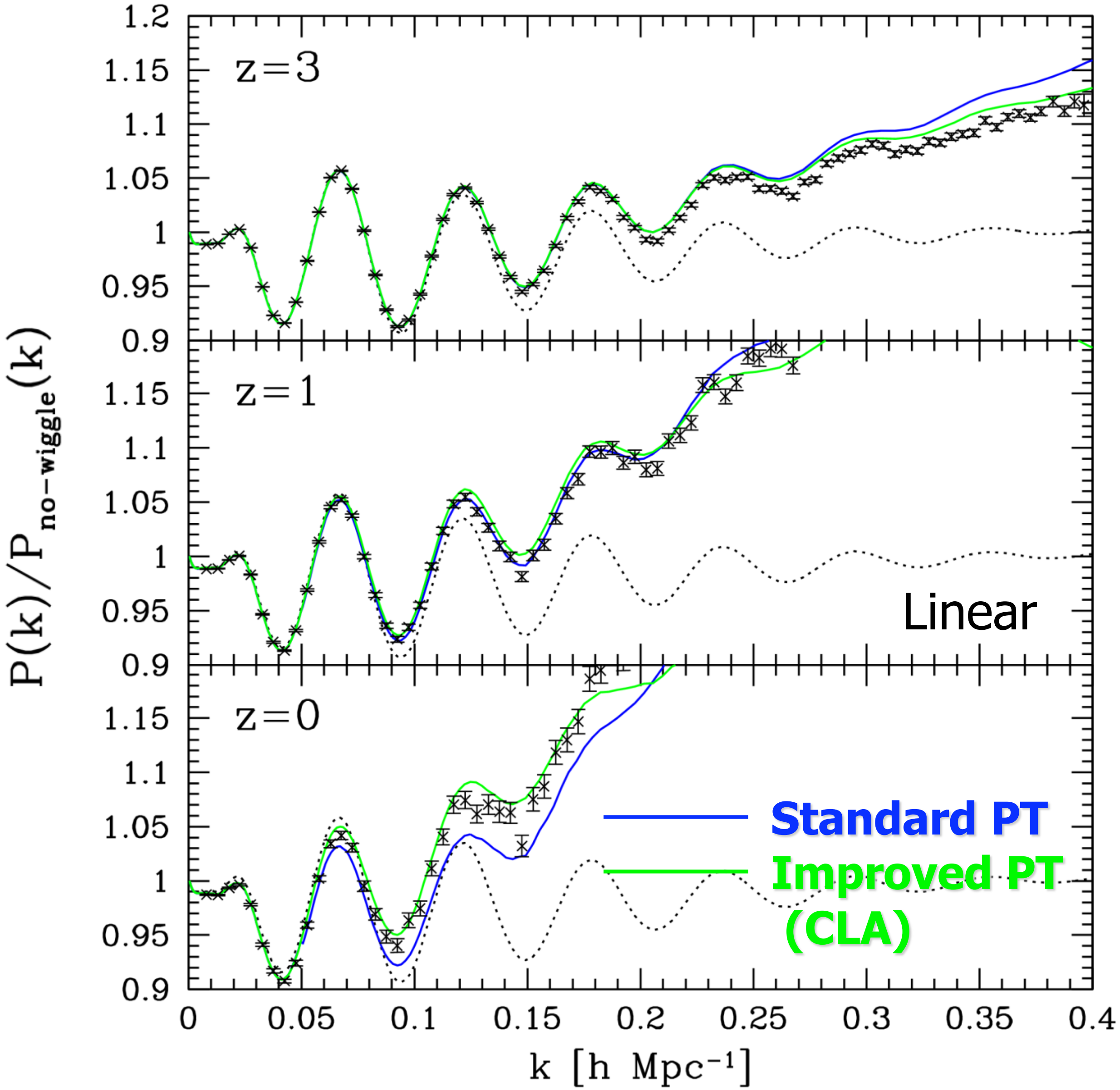


Agreement between improved PT and N-body results is very well with precision at 1~2%

Solid: 1st Born

Dashed: 2nd Born

Improved PT vs. Standard PT (1/2)



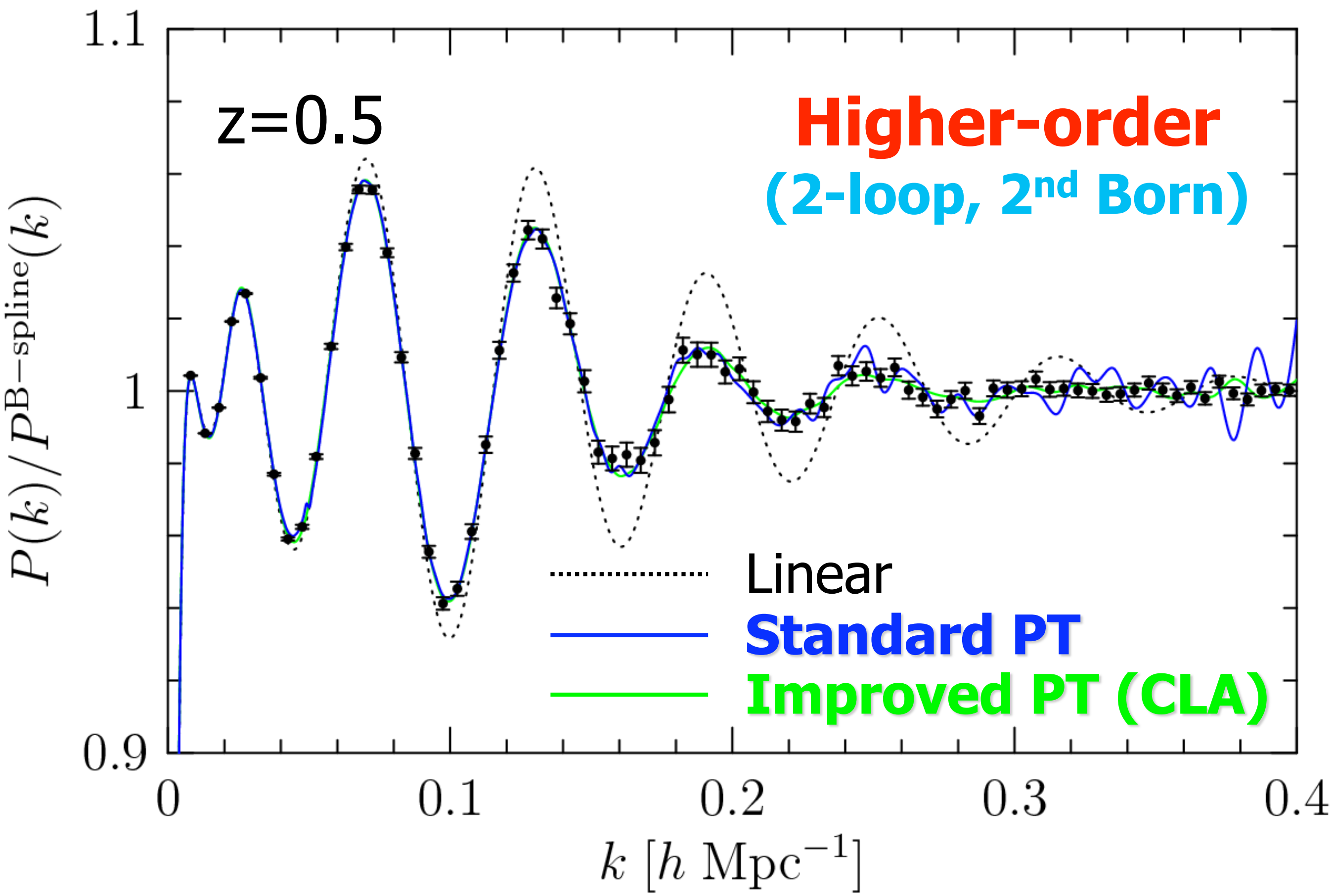
Higher-order correction:

SPT: 2-loop

CLA: 2^{nd} order Born

Improved PT vs. Standard PT (2/2)

Damping behavior of BAOs



$P^{\mathrm{B-spline}}(k)$:

Smoothed P(k) using B-spline fit

$P(k)/P^{\mathrm{B-spline}}(k)$:

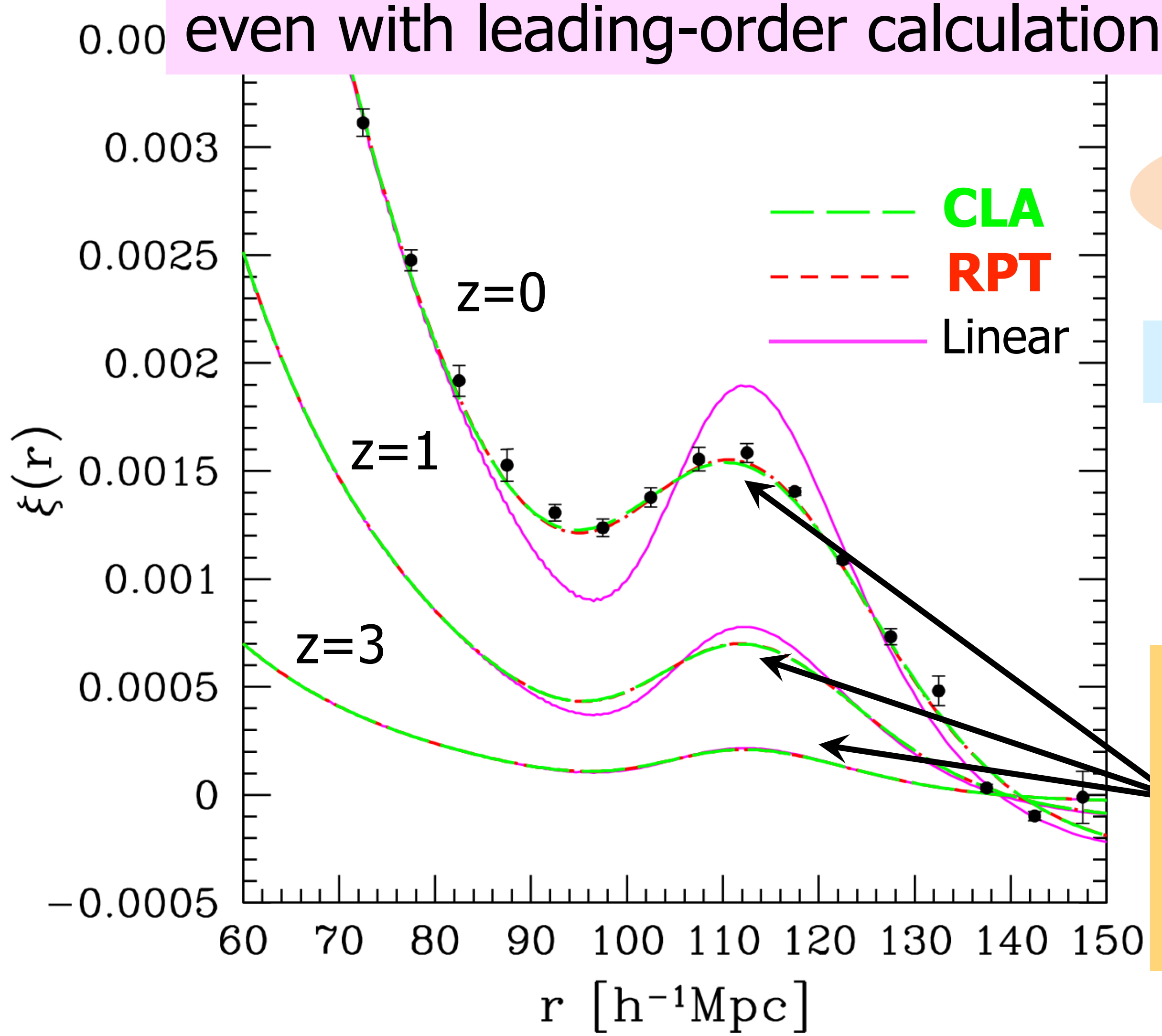
Useful to characterize phases of BAOs

(Percival et al. 2007)

With improved PT, the leading-order calculation is sufficient to reproduce the major trends in BAO phases

Two-point correlation function (1/2)

With improved PT, accurate prediction is now available even with leading-order calculation



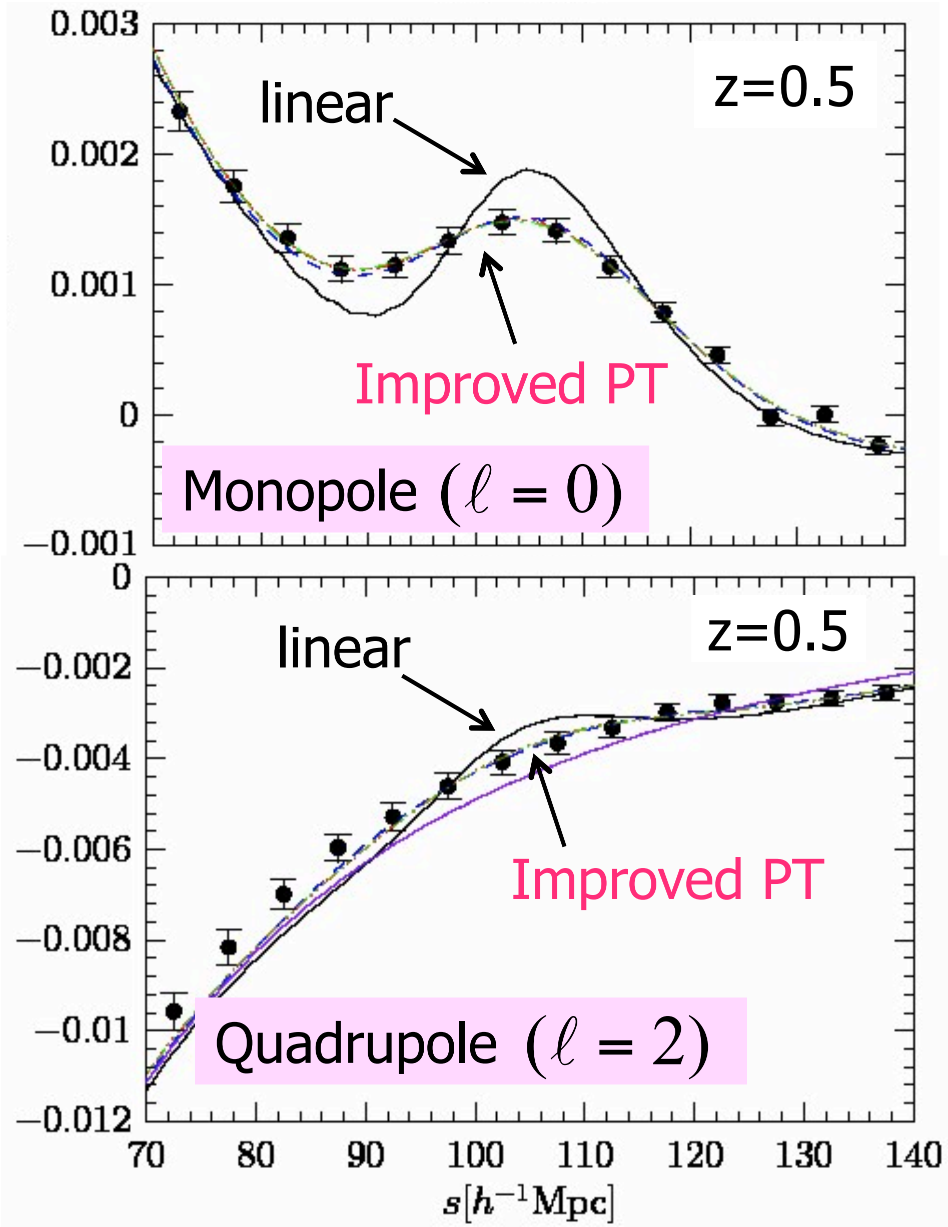
Real space

Def.

$$\xi(r)=\int\frac{dk\,k^2}{2\pi^2}P(k)\frac{\sin(k\,r)}{k\,r}$$

Agreement between CLA and RPT is remarkable, and they also reproduce N-body results quite well.

Two-point correlation function (2/2)



Redshift space

$$\xi^{(S)}(s_\perp, s_\parallel) = \sum_{\ell:\,\mathrm{even}} \xi_\ell^{(S)}(s)\, P_\ell(\mu)$$

$$\left(\mu = s_\parallel / s\right)$$

$$\left\{ \begin{array}{l} \text{Improved PT (leading-order)} \\ \text{Model of redshift-space P(k)} \end{array} \right.$$

(Scoccimarro 2004, Percival & White 2008)

Simple redshift-space modeling reproduces N-body trend well

Problem: Galaxy Biasing

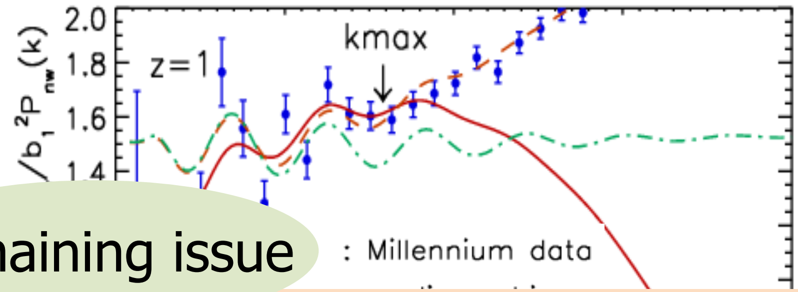
Incorporating the galaxy biasing scheme into PT prediction:

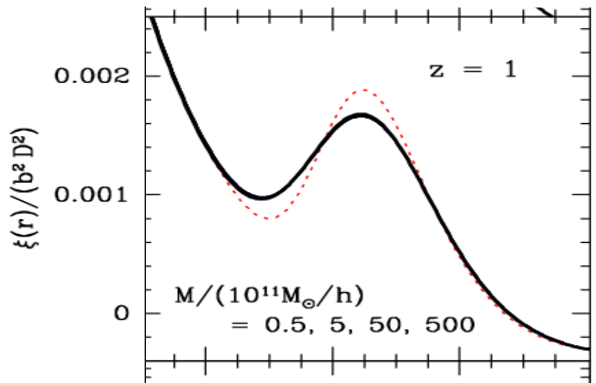
$$\delta_{\rm gal}(\vec{x}) = b_1\,\delta_{\rm mass}(\vec{x}) + \tfrac{1}{2}\left\{[\delta_{\rm mass}(\vec{x})]^2 - \left\langle[\delta_{\rm mass}(\vec{x})]^2\right\rangle\right\} + \tfrac{1}{3!}[\delta_{\rm mass}(\vec{x})]^3 + \cdots$$

Partial success

Characterization based on standard PT / Lagrangian PT

(McDonald 2006; Jeong & Komatsu 2008; Matsubara 2008b)





Remaining issue

How to incorporate the galaxy biasing scheme into the improved PT prediction (need further investigation)

Summary

Development of perturbation theory (PT) of large-scale structure

Reloading and ***improving***

PT-based approach can give an accurate prediction for matter P(k)/ x(r) in the weakly non-linear regime (~ BAO scales)

- Physically motivated modeling
- Low computational cost

Including other physical effects is relatively easy :

Neutrinos: Standard PT → Saito, Takada & AT (2008)

Modified gravity: Hiramatsu, Koyama & AT in prep.

PT-based modeling may play a central role in future LSS study

Fin.

Appendix

Preliminaries

- 「CDM+baryon」=「pressureless & irrotational perfect fluid」

- basic quantities $\Psi_a(k.\eta) = \left(\delta(k,\eta),\ -\frac{\theta(k,\eta)}{f(\eta)} \right)$;

$\eta = \ln D_+$ ← Growth rate

$$\theta(k,\eta) = \frac{\vec{\nabla}\cdot\vec{v}}{aH}$$

$$f(\eta) = \frac{d\ln D_+}{d\ln a}$$

Basic eq. (in Fourier space)

$$\partial_\eta \Psi_a(\mathbf{k},\eta) + \Omega_{ab}\Psi_b(\mathbf{k},\eta) = \int \frac{d^3k_1 d^3k_2}{(2\pi)^3}\ \gamma^{(S)}_{abc}(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2)\Psi_b(\mathbf{k}_1,\eta)\Psi_c(\mathbf{k}_2,\eta)$$

$(a,b=1,2)$

$$\Omega_{ab} \equiv \begin{bmatrix} 0 & -1 \\ -3/2 & 1/2 \end{bmatrix}$$

$$\gamma^{(S)}_{121}(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2) = \delta_D(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\left(1+\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{|\mathbf{k}_1|^2}\right)/2$$

$$\gamma^{(S)}_{112}(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2) = \delta_D(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\left(1+\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{|\mathbf{k}_2|^2}\right)/2$$

$$\gamma^{(S)}_{222}(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2) = \delta_D(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\frac{(\mathbf{k}_1\cdot\mathbf{k}_2)|\mathbf{k}_1+\mathbf{k}_2|^2}{2|\mathbf{k}_1|^2|\mathbf{k}_2|^2}$$

Naïve expansion

Perturbative evaluation of basic eq. regarding $\Psi_a(k,\eta)$ as small quantities

Gaussian variable

$$\Psi_a(\mathbf{k},\eta) = \sum^{\infty} \Psi_a^{(n)}(\mathbf{k},\eta)\ ;$$

$$\Psi_a^{(n)}(\mathbf{k},\eta) = \int \delta_{\mathrm{D}}(\mathbf{k}-\mathbf{k}_{1\ldots n})\underline{\underline{\mathcal{F}_a^{(n)}(\mathbf{k}_1,\ldots,\mathbf{k}_n;\eta)}}\,\delta_0(\mathbf{k}_1)\ldots\delta_0(\mathbf{k}_n),$$

Recursion formula

$$\mathcal{F}_a^{(n)}(\mathbf{k}_1,\ldots,\mathbf{k}_n;\eta)\delta_{\mathrm{D}}(\mathbf{k}-\mathbf{k}_{1\ldots n})$$

Initial condition : $\mathcal{F}_a^{(1)}(k;\eta) = g_{ab}(\eta)u_b$

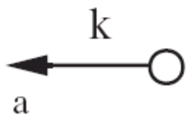
$$= \Bigg[\sum_{m=1}^{n}\int_0^{\eta} ds\,\underline{\underline{g_{ab}(\eta-s)}}\,\gamma_{bcd}^{(\mathrm{s})}(\mathbf{k},\mathbf{k}_{1\ldots m},\mathbf{k}_{m+1\ldots n}) \times \mathcal{F}_c^{(m)}(\mathbf{k}_{1\ldots m};s)\mathcal{F}_d^{(n-m)}(\mathbf{k}_{m+1\ldots n};s)\Bigg]_{\text{symmetrized}}$$

Linear propagator $g_{ab}(\eta) = \dfrac{e^{\eta}}{5}\begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix} - \dfrac{e^{-3\eta/2}}{5}\begin{bmatrix} -2 & 2 \\ 3 & -3 \end{bmatrix}$

Diagrammatic representation

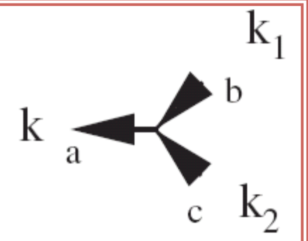
Basic quantities

$\phi_a(k)$:



$g_{ab}(\eta-\eta')$:

$\gamma^{(s)}_{abc}(k,k_1,k_2)$:

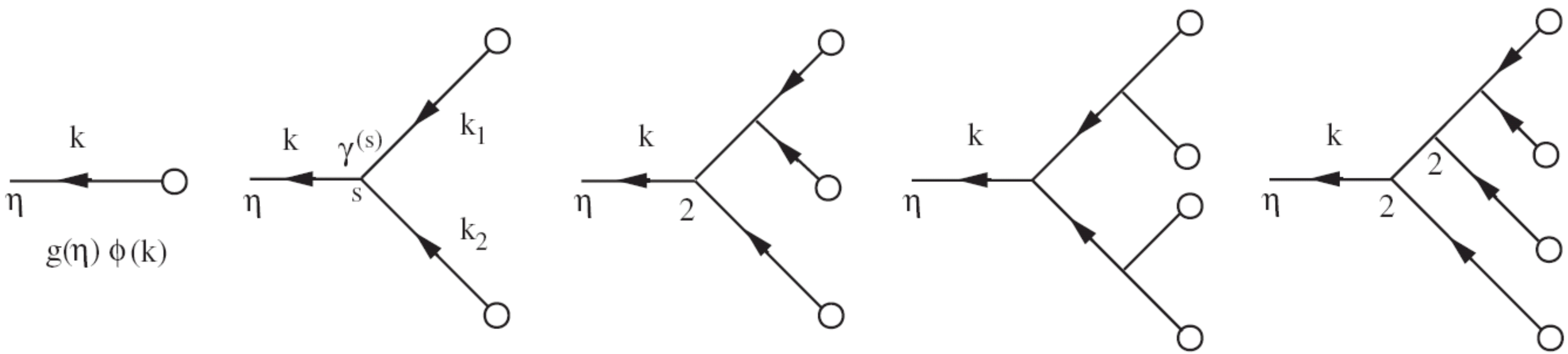


$$\phi_a(\mathbf{k}) = u_a \delta_0(\mathbf{k})$$

(1, 1) : growing mode

$k = k_1 + k_2$

Diagrams of perturbative solutions：



$\Psi^{(1)}(k,\eta)$ $\Psi^{(2)}(k,\eta)$ $\Psi^{(3)}(k,\eta)$ $\Psi^{(4)}(k,\eta)$

Power Spectrum

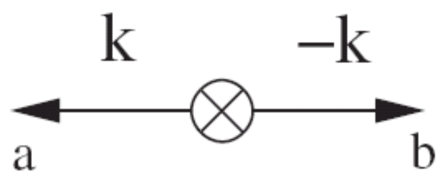
Definition $\langle \Psi_a(\mathbf{k}, \eta)\Psi_b(\mathbf{k}', \eta)\rangle = \delta_D(\mathbf{k} + \mathbf{k}')P_{ab}(\mathbf{k}, \eta)$

Statistical property of P(k) is determined via the ensemble average of random seeds $\phi_a(k)$ encoded in perturbative solutions

$$\langle \phi_a(\mathbf{k})\phi_b(\mathbf{k}')\rangle = \delta_D(\mathbf{k} + \mathbf{k}')u_a u_b P_0(k)$$

Diagram of inlinear P(k)

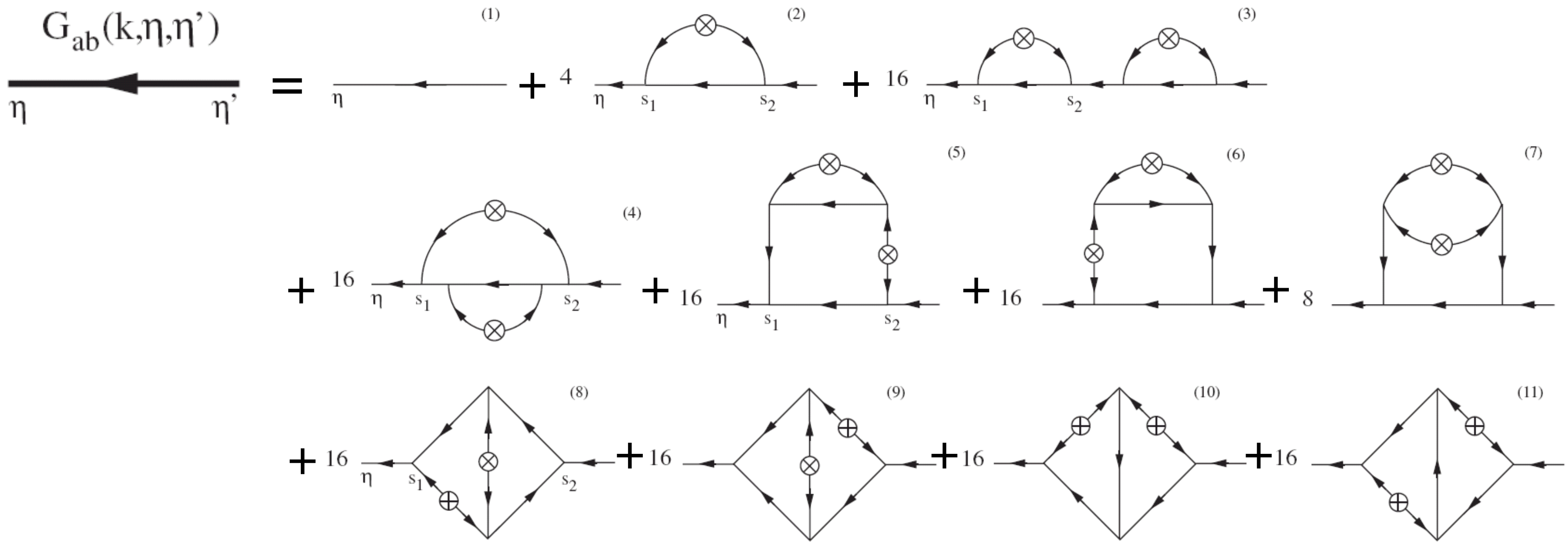
$u_a\ u_b\ P_o(k)$:



Propagator & Vertex: Perturbation

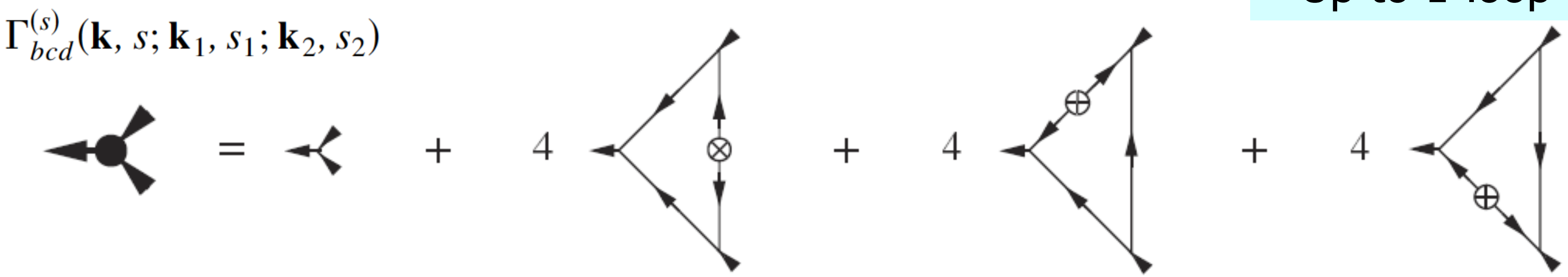
Propagator

Up to 2-loop



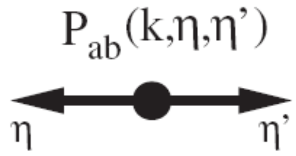
Vertex

Up to 1-loop



Non-perturbative quantities

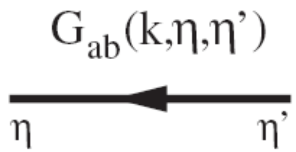
- **Power spectrum**



$$\left\langle \Phi_a(\vec{k})\Phi_b(\vec{k}')\right\rangle = (2\pi)^3\delta_D(\vec{k}+\vec{k}')\ P_{ab}(k)$$

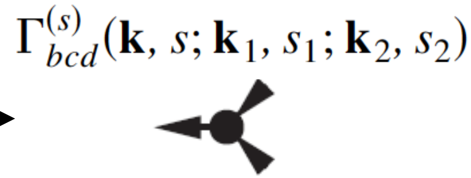
$$\Phi_a(\vec{k}) = \begin{pmatrix} \delta(\vec{k}) \\ -\theta(\vec{k}) \end{pmatrix}, \qquad \theta(\vec{k}) = \frac{i(\vec{k}\cdot\vec{\mathrm{v}})}{aH}$$

- **Propagator**



$$\left\langle \frac{\delta\Phi_a(\vec{k};t)}{\delta\Phi_b(\vec{k}';t')}\right\rangle = \delta_D(\vec{k}-\vec{k}')\ G_{ab}(k\,|\,t,t')$$

- **Vertex function**



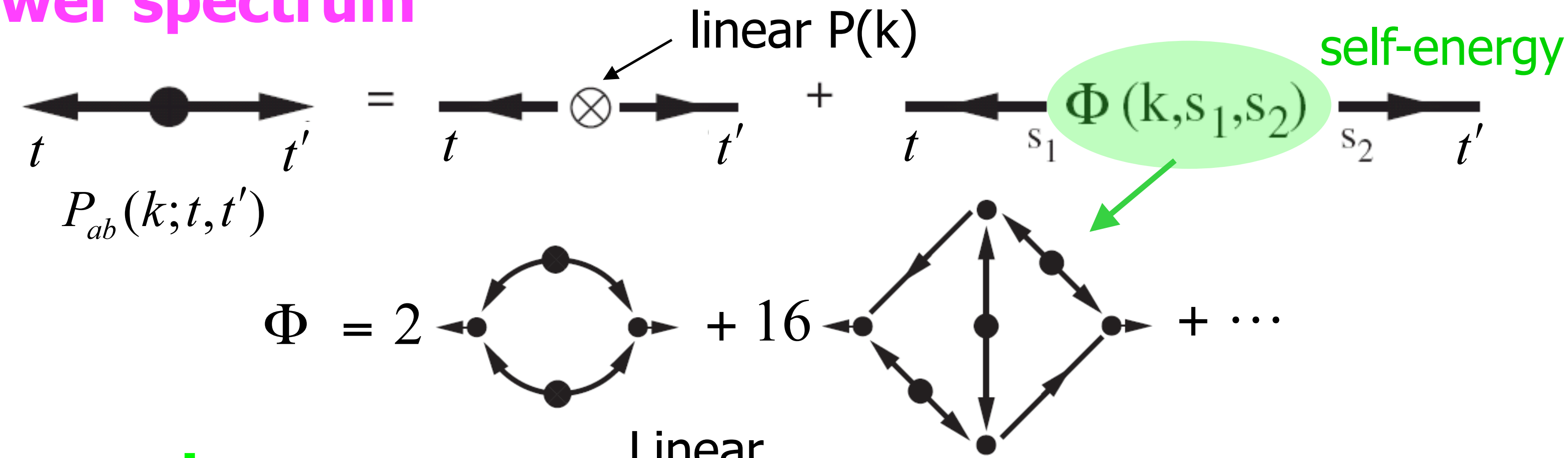
$$\left\langle \frac{\delta\Phi_a(\vec{k};t)}{\delta\Phi_b(\vec{k}_1;t_1)\delta\Phi_c(\vec{k}_2;t_2)}\right\rangle = \delta_D(\vec{k}-\vec{k}_1-\vec{k}_2)\ \Gamma_{abc}(\vec{k}_1,\vec{k}_2\,|\,t,t_1,t_2)$$

(a,b,c=1,2)

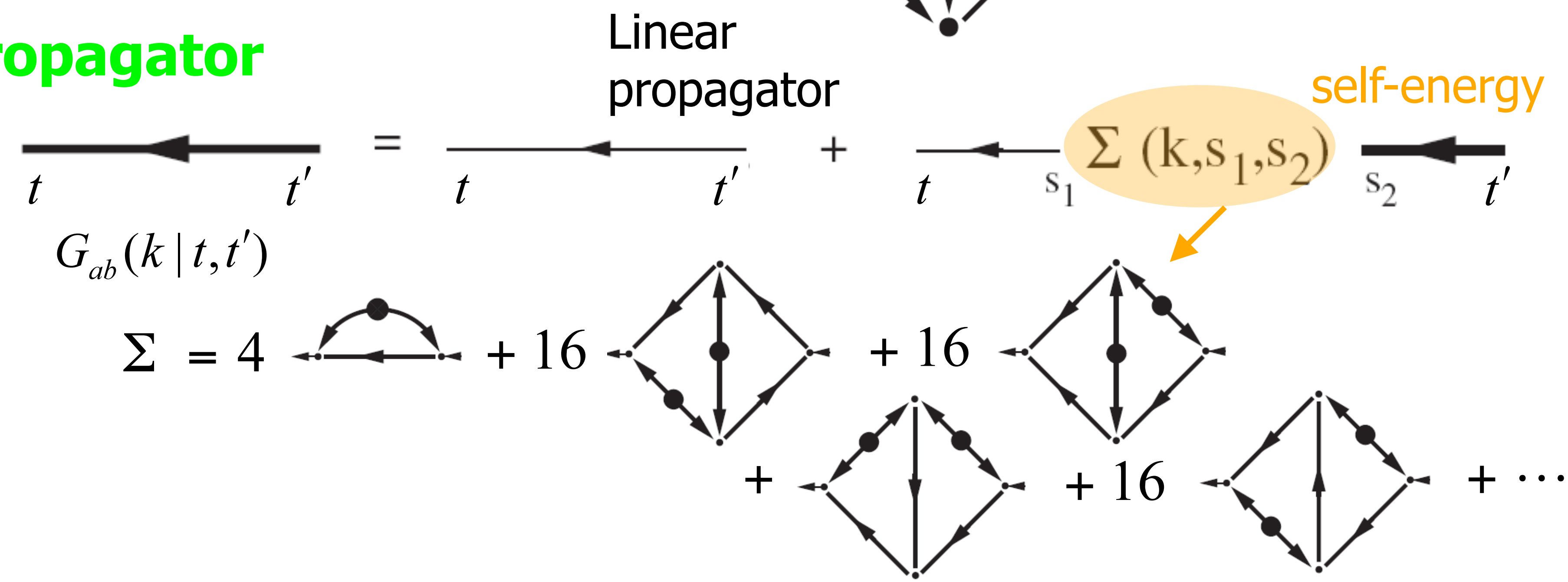
Renormalized expansion

Crocce & Scoccimarro (2006a)

Power spectrum



Propagator



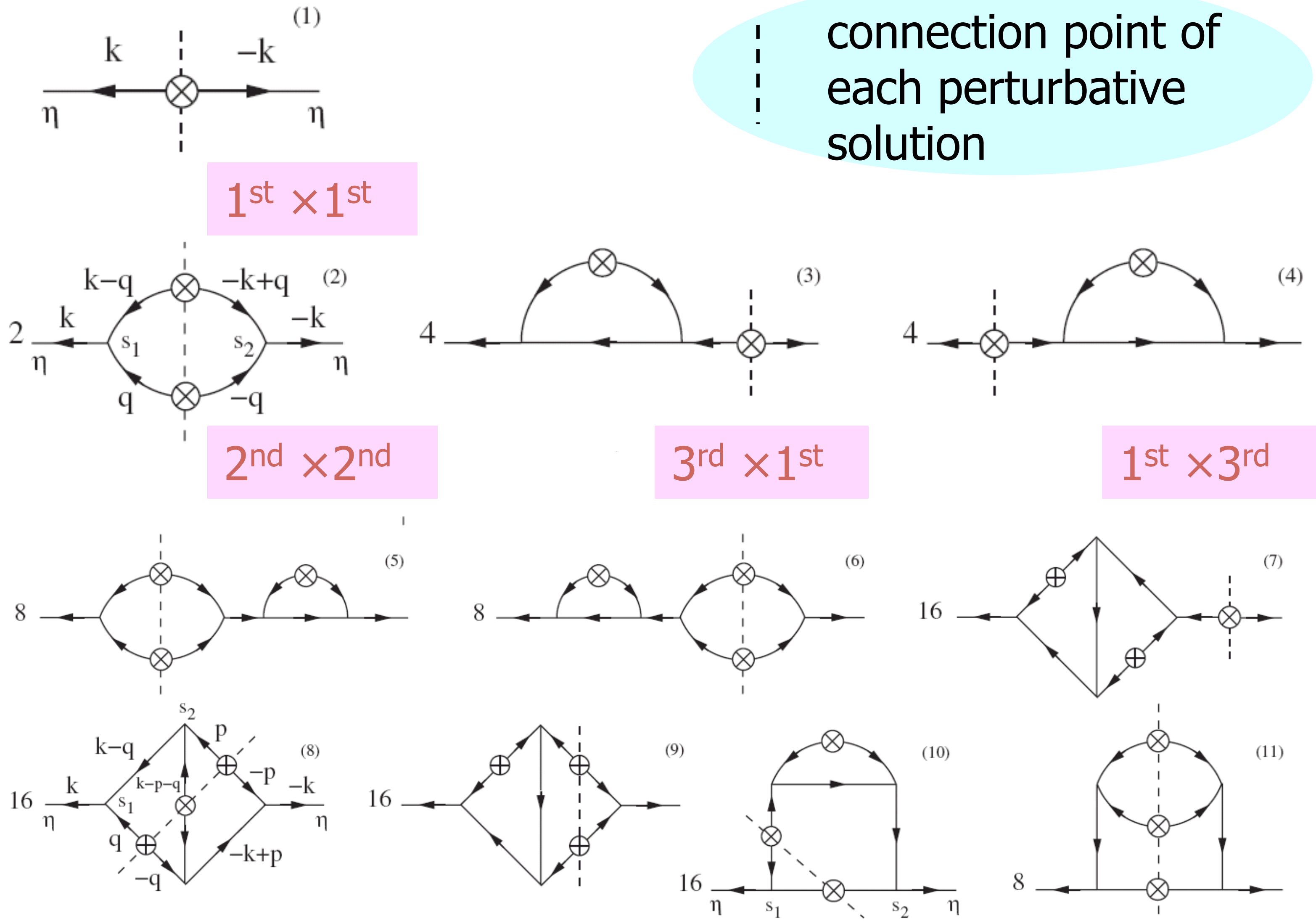
Diagrams of P(k)

connection point of each perturbative solution

tree

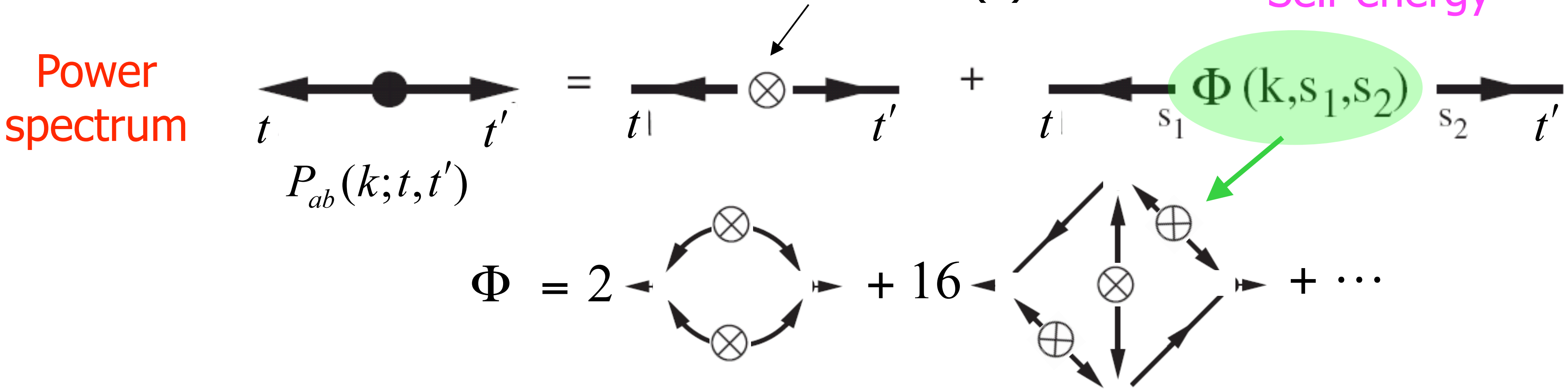
1-loop

2-loop
(partial)

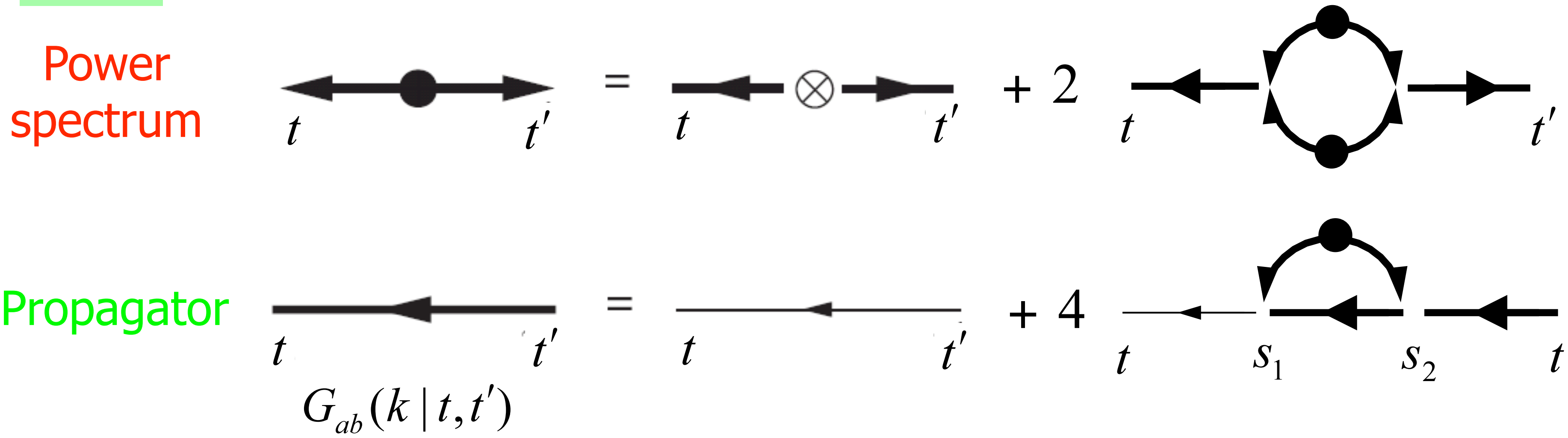


RPT vs. CLA

RPT (Crocce & Scoccimarro)



CLA (AT & Hiramatsu)



Closure Equations

Time variable

$\eta \equiv \ln D_+(z)$

Evolution equations corresponding to the truncated diagrams:

$$\widehat{\Lambda}_{ab}(\eta)\, P_{bc}(k;\eta,\eta') = \int \frac{d^3\boldsymbol{q}}{(2\pi)^3}\, \gamma_{apq}(\boldsymbol{q}, \boldsymbol{k}-\boldsymbol{q})\, K_{cpq}(-\boldsymbol{k}, \boldsymbol{q}, \boldsymbol{k}-\boldsymbol{q};\, \eta, \eta'),$$

$$\widehat{\Lambda}_{ab}(\eta)\, G_{bc}(\boldsymbol{k}|\eta,\eta') = 4\int_{\eta'}^{\eta} d\eta'' \int \frac{d^3\boldsymbol{q}}{(2\pi)^3}\, \gamma_{apq}(\boldsymbol{q}, \boldsymbol{k}-\boldsymbol{q})\, \gamma_{lrs}(-\boldsymbol{q}, \boldsymbol{k})$$
$$\times G_{ql}(|\boldsymbol{k}-\boldsymbol{q}||\eta,\eta'')\, P_{pr}(q;\eta,\eta'')\, G_{sc}(\boldsymbol{k}|\eta'',\eta').$$

- Operator: $\widehat{\Lambda}_{ab}(\eta) \equiv \delta_{ab}\frac{\partial}{\partial\eta} + \begin{bmatrix} 0 & -1 \\ -3/2 & 1/2 \end{bmatrix}$

subscripts 1, 2 indicate δ, θ

//

$(\nabla\cdot\vec{v})/(aH)$

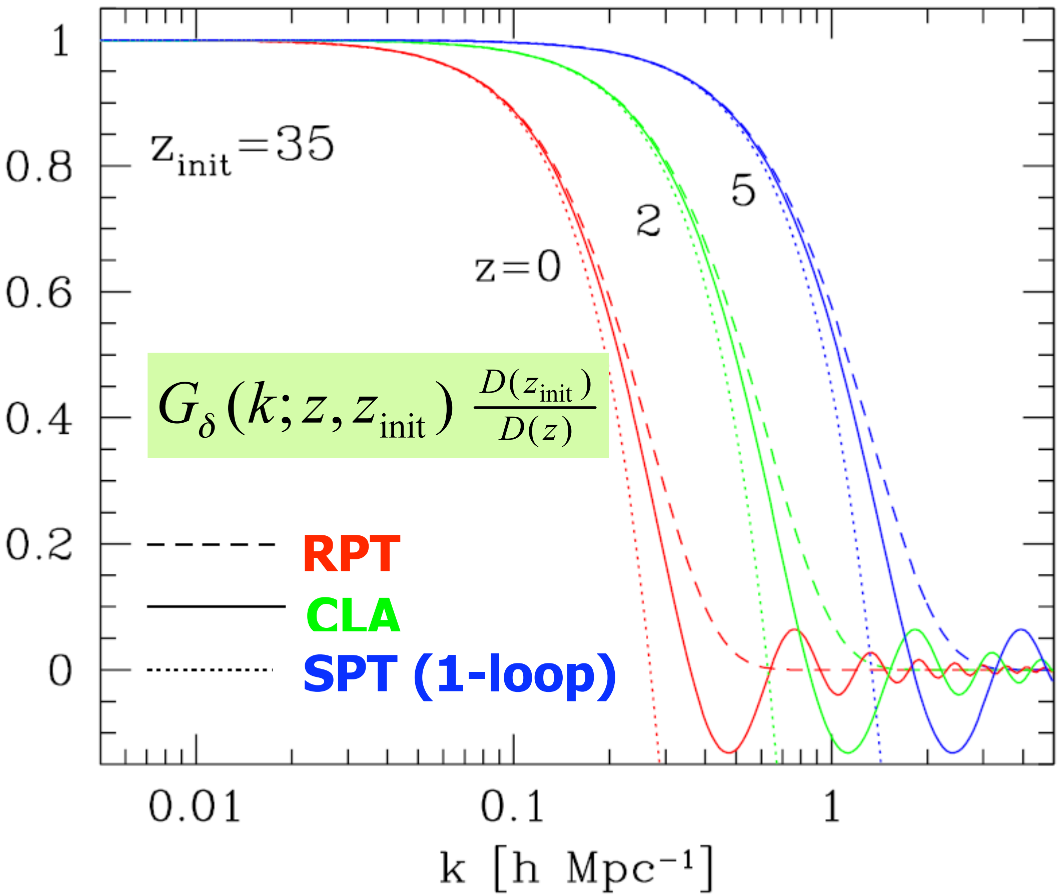
- Fourier kernel:

$$K_{cpq}(\boldsymbol{k}', \boldsymbol{k}_1, \boldsymbol{k}_2; \eta, \eta')$$
$$= 4\int_{\eta_0}^{\eta} d\eta''\, G_{ql}(k_2|\eta,\eta'')\, \gamma_{lrs}(\boldsymbol{k}', \boldsymbol{k}_1)\, P_{ps}(k_1;\eta,\eta'')$$
$$\times \left\{ P_{cr}(k';\eta',\eta'')\,\Theta(\eta'-\eta'') + P_{rc}(k';\eta'',\eta')\,\Theta(\eta''-\eta') \right\}$$
$$+ 2\int_{\eta_0}^{\eta'} d\eta''\, G_{cl}(k'|\eta',\eta'')\, \gamma_{lrs}(\boldsymbol{k}_1, \boldsymbol{k}_2)\, P_{pr}(k_1;\eta,\eta'')\, P_{qs}(k_2;\eta,\eta''),$$

Propagator

Decaying property of propagator is a non-perturbative result of resummation at infinite order

→ Improve convergence properties of PT



$$G_\delta(k\,|\,z_1,z_2)$$
$$\equiv G_{11}(k\,|\,z_1,z_2)+G_{12}(k;z_1,z_2)$$

At high-k,

- Damping oscillation (**CLA**)
- Gaussian damping (**RPT**)

※ SPT(1-loop) diverges at high k

Born approximation

Iterative evaluation based on the renormalized 1-loop diagram:

$$P_{ab}(k;\eta) = P_{ab}^{(\mathrm{A})}(k;\eta) + P_{ab}^{(\mathrm{B1})}(k;\eta) + P_{ab}^{(\mathrm{B2})}(k;\eta) + \cdots ;$$

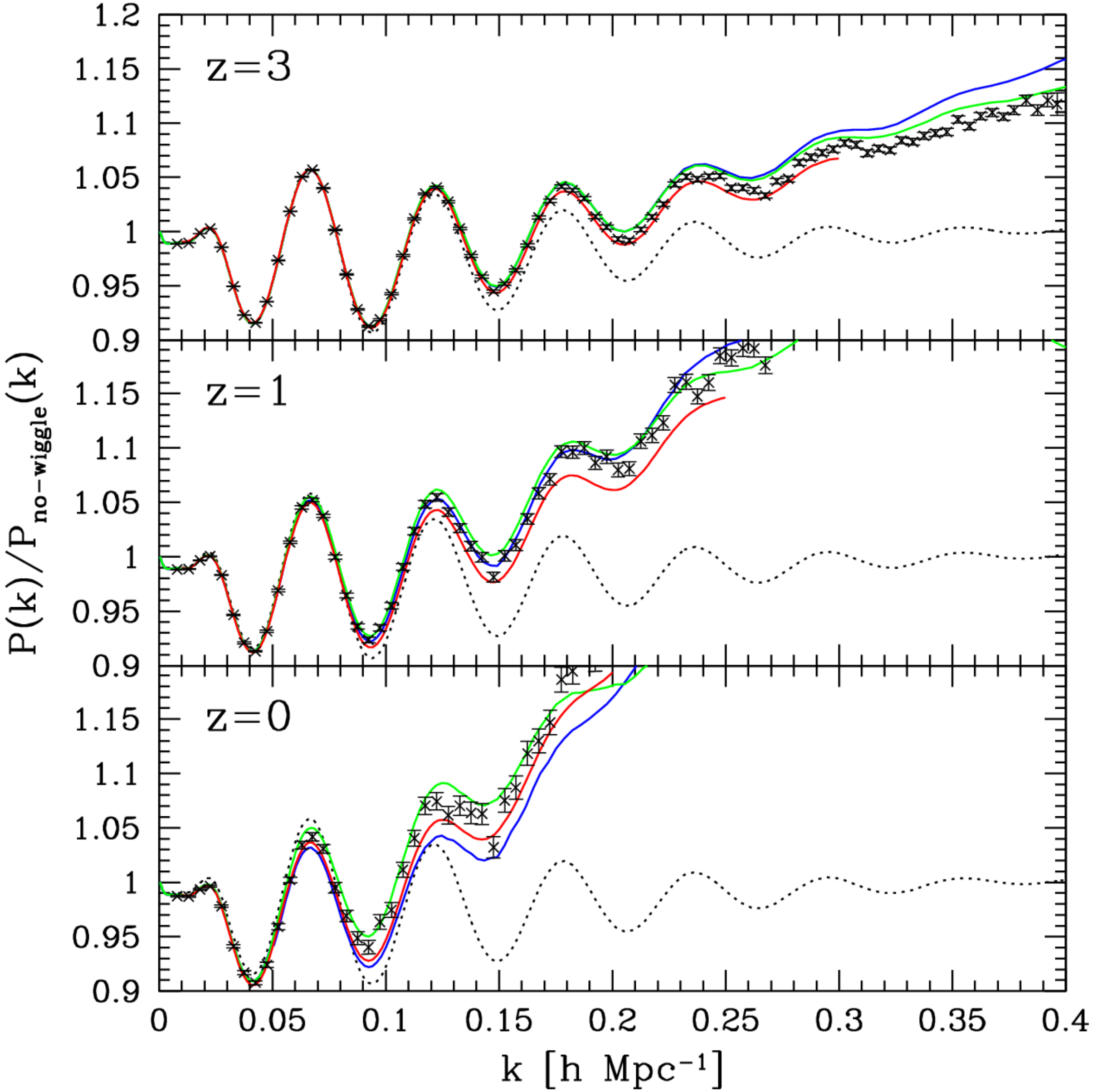
linear P(k)

$$\left\{\begin{aligned}
P_{ab}^{(\mathrm{A})}(k;\eta) &= \widetilde{G}_a(k|\eta,\eta_0)\widetilde{G}_b(k|\eta,\eta_0)e^{2\eta_0}P_0(k), \\
P_{ab}^{(\mathrm{B1})}(k;\eta) &= 2\int\frac{d^3\boldsymbol{q}}{(2\pi)^3}\,I_a(\boldsymbol{k},\,\boldsymbol{q};\eta,\,\eta_0)\,I_b(\boldsymbol{k},\,\boldsymbol{q};\eta,\,\eta_0)\;e^{4\eta_0}P_0(q)\,P_0(|\boldsymbol{k}-\boldsymbol{q}|), \\
P_{ab}^{(\mathrm{B2})}(k;\eta) &= 8\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\int\frac{d^3\boldsymbol{q}}{(2\pi)^3}J_a(\boldsymbol{k},\boldsymbol{p},\boldsymbol{q};\eta,\,\eta_0)J_b(\boldsymbol{k},\,\boldsymbol{p},\,\boldsymbol{q};\eta,\,\eta_0) \\
&\qquad\times\; e^{6\eta_0}\;P_0(|\boldsymbol{k}-\boldsymbol{p}|)P_0(r)P_0(|\boldsymbol{p}-\boldsymbol{q}|).
\end{aligned}\right.$$

$$I_a(\boldsymbol{k},\,\boldsymbol{q};\eta,\,\eta_0) = \int_{\eta_0}^{\eta}d\eta'\,G_{al}(k|\eta,\eta')\,\gamma_{lrs}(\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q})\,\widetilde{G}_r(q|\eta',\eta_0)\,\widetilde{G}_s(|\boldsymbol{k}-\boldsymbol{q}||\eta',\eta_0),$$

$$\begin{aligned}
J_a(\boldsymbol{k},\,\boldsymbol{p},\,\boldsymbol{q};\eta,\,\eta_0) &= \int_{\eta_0}^{\eta}d\eta_1\int_{\eta_0}^{\eta}d\eta_2 G_{al}(k|\eta,\eta_1)\,\gamma_{lrs}(\boldsymbol{p},\boldsymbol{k}-\boldsymbol{p})\,G_{rc}(p|\eta_1,\eta_2) \\
&\quad\times\;\gamma_{cpq}(\boldsymbol{q},\boldsymbol{p}-\boldsymbol{q})\,\widetilde{G}_p(q|\eta_2,\eta_0)\widetilde{G}_q(|\boldsymbol{p}-\boldsymbol{q}||\eta_2,\eta_0)\widetilde{G}_s(|\boldsymbol{k}-\boldsymbol{p}||\eta_1,\eta_0).
\end{aligned}$$

RPT, CLA, SPT



Higher-order correction

Blue: Stanrd PT (2-loop)

Green: CLA (2nd Born)

Red: RPT (2-loop)