

STATPHYS 26

JULY 18-22, 2016 *Lyon*



**4th INTERNATIONAL SOFT
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**12-16 SEPTEMBER 2016
GRENOBLE, FRANCE**

8th International Conference on Multiscale Materials Modeling

9-14 October 2016, Dijon, France





UNIVERSITÉ
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Elastoplastic models for amorphous plasticity

Jean-Louis Barrat
Université Grenoble Alpes
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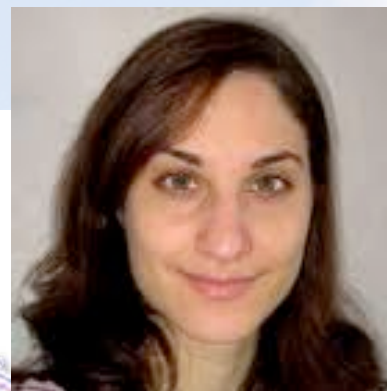
Acknowledgements

Kirsten
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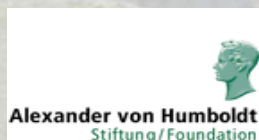
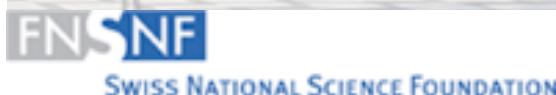
Lydéric
Bocquet



Chen Liu

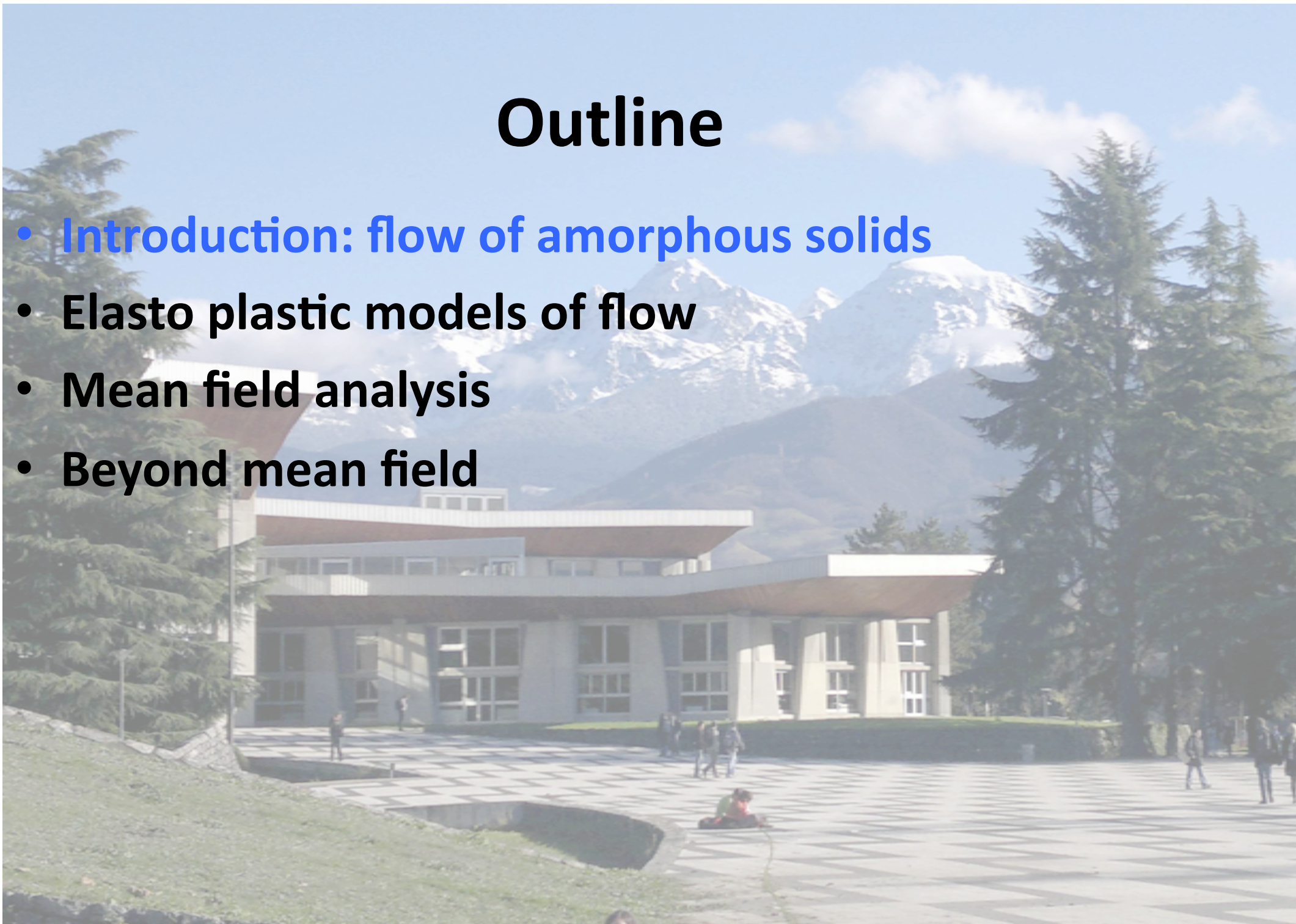
Also Anne Tanguy (Lyon), Fabien Leonforte (Göttingen), Claus Heussinger (Göttingen), Michel Tsamados (UC London), Jörg Rottler (Vancouver), Ludovic Berthier (Montpellier), Hideyuki Mizuno (Kyoto) Francesco Puosi (Lyon)

FUNDING



Outline

- **Introduction: flow of amorphous solids**
- **Elasto plastic models of flow**
- **Mean field analysis**
- **Beyond mean field**



Amorphous materials

Amorphous solids



Metallic glasses



Polymer glasses

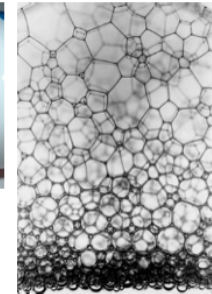
Granular media



Complex Fluids

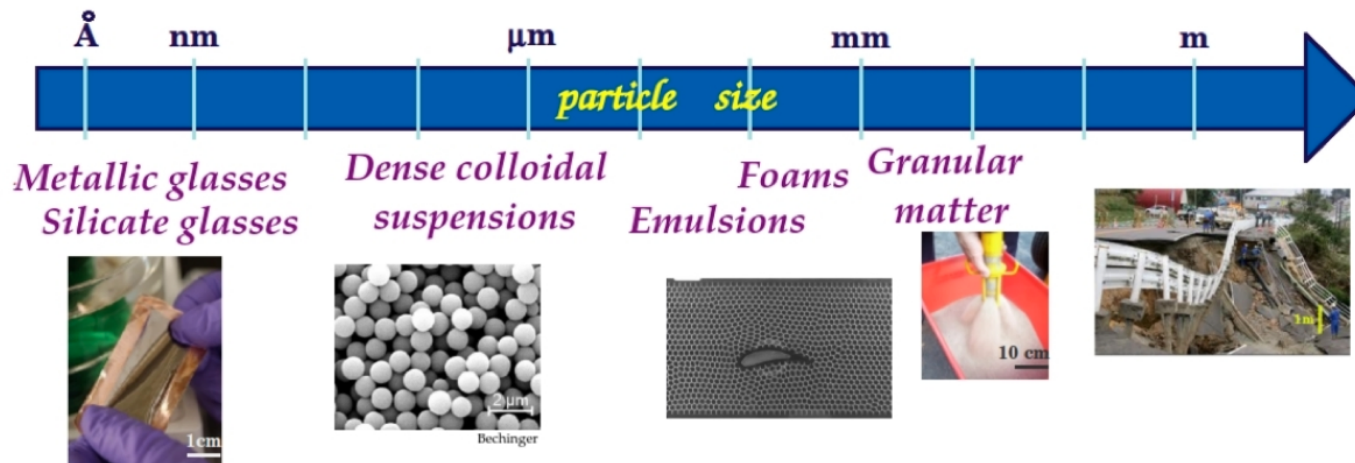


Gels



Foams

...spanning many orders of magnitude in terms of particle size...



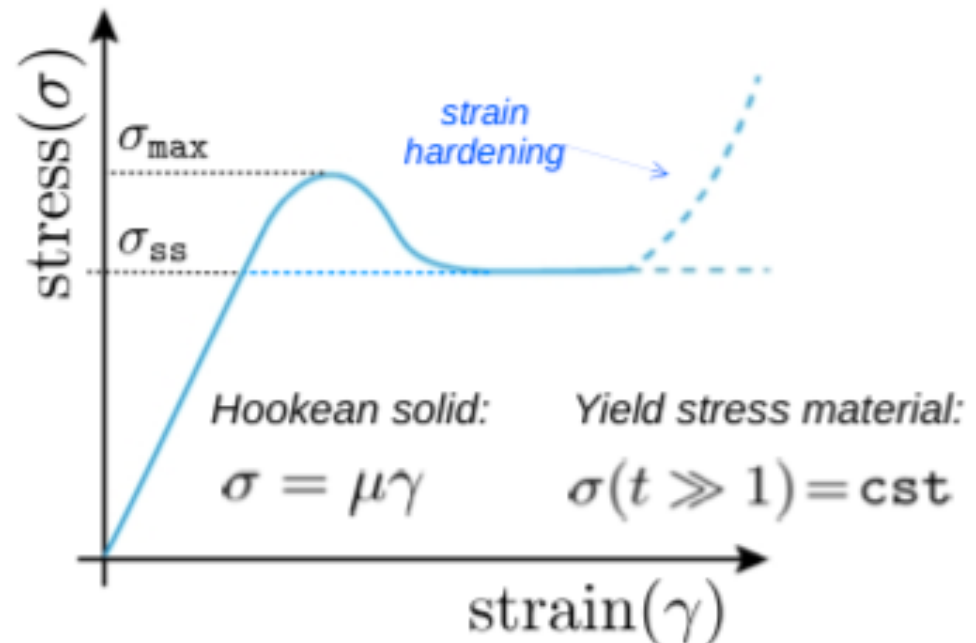
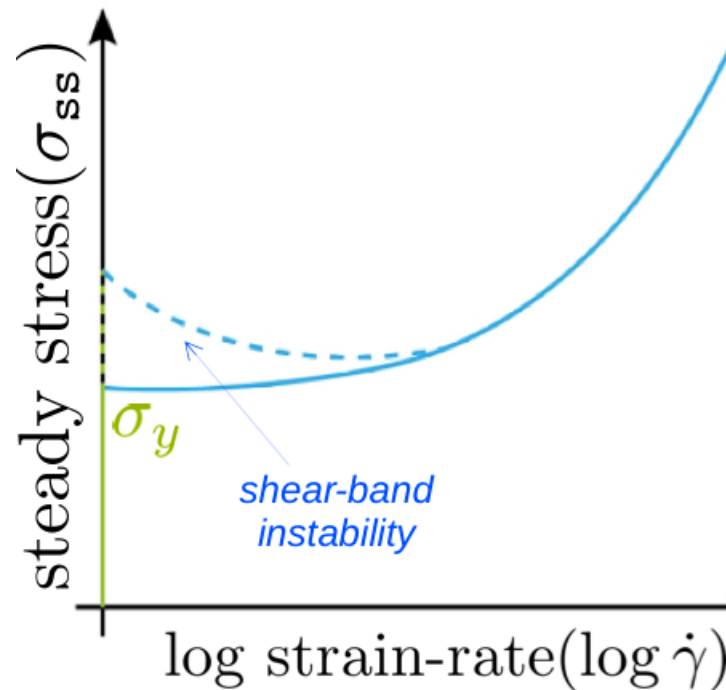
- Disordered lastic solids, far below/above any « glass transition temperature/density
- Extremely diverse in terms of scales (nm-cm) and strength (100Pa-100GPa).
- Still share common features in their deformation mechanisms

Non-linear rheology and yield stress:

$$\sigma = \sigma_Y + A\dot{\gamma}^{1/\beta}$$

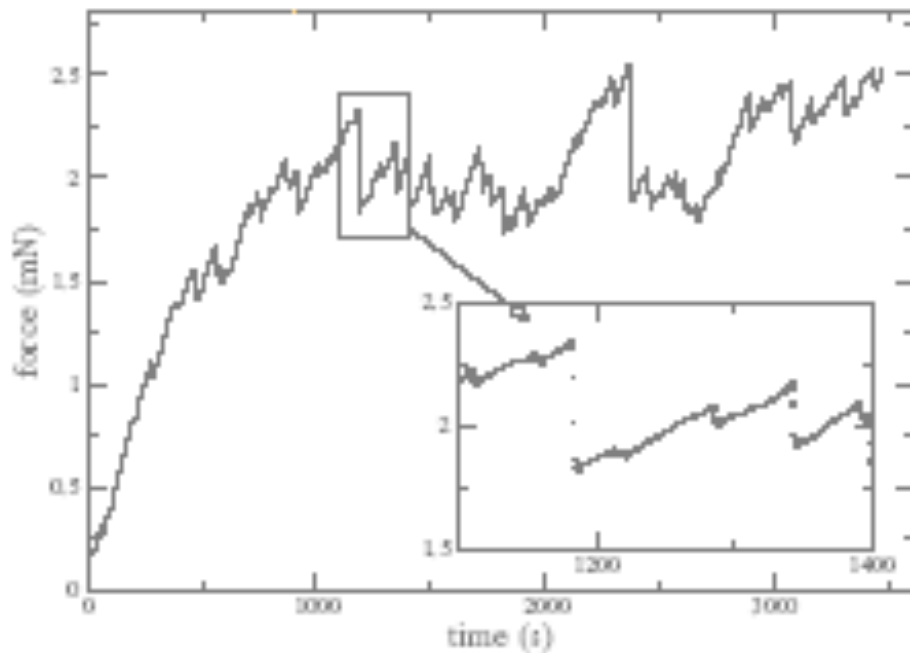
Herschel Bulkley equation

Stress-strain curves at fixed rate

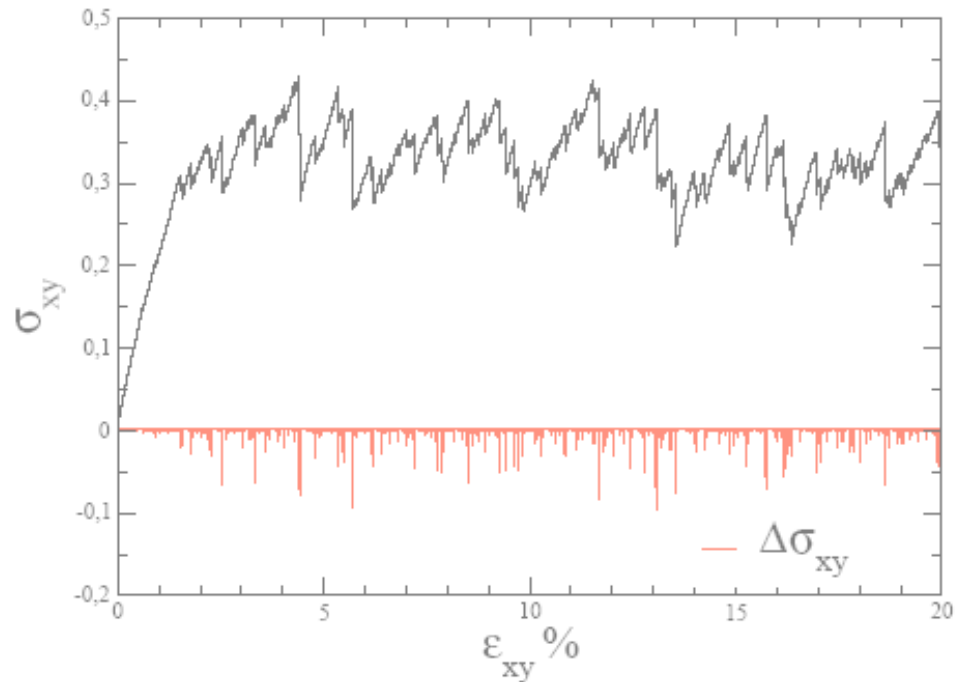


Deformation at low T proceeds through well identified *plastic events* or *shear transformations* (Argon and Kuo, 1976)

Stress-strain curve at low strain rate, low temperature, small systems



Plastic response of a foam (I. Cantat, O. Pitois, Phys. of fluids 2006)



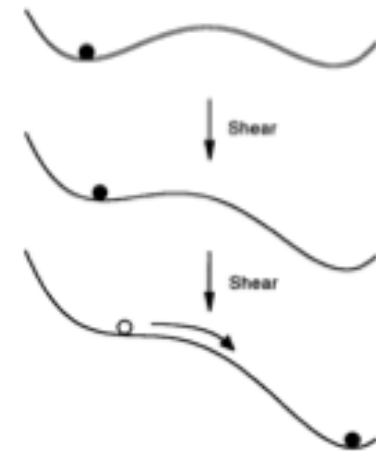
Plastic response of a simulated Lennard-Jones glass (Tanguy, Leonforte, JLB, EPJ E 2006)

Events are **shear transformations** of Eshelby type

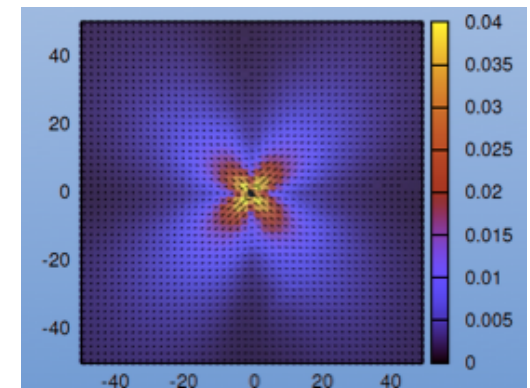
- Plastic instability in a very local region of the medium (irreversible) under the influence of the local stress.

- Instability involves typically a few tens of particles and small shear strains (1 to 10%)

- Surroundings respond essentially as an homogeneous elastic medium (incompressible). Quadrupolar symmetry of the response.



Malandro, Lacks, PRL 1998



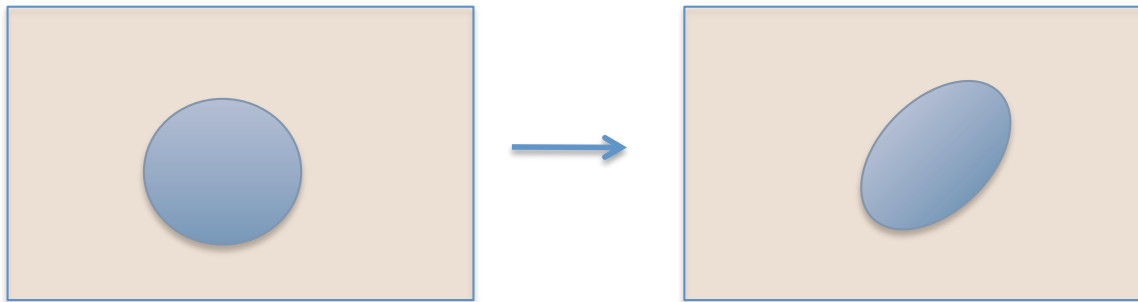
Puosi, Rottler, JLB, PRE 2014

Events are **shear transformations** of Eshelby type

Eshelby transformation: an inclusion within an elastic material undergoes a spontaneous change of shape (eigenstrain): circular to elliptical.

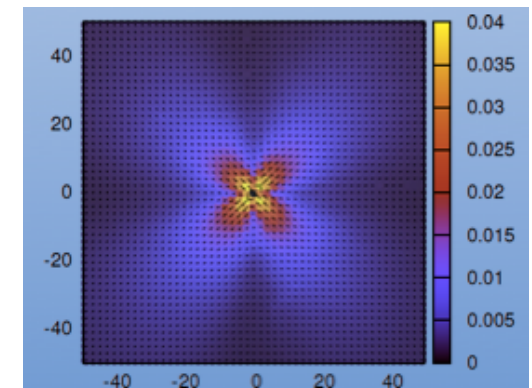


J.D. Eshelby



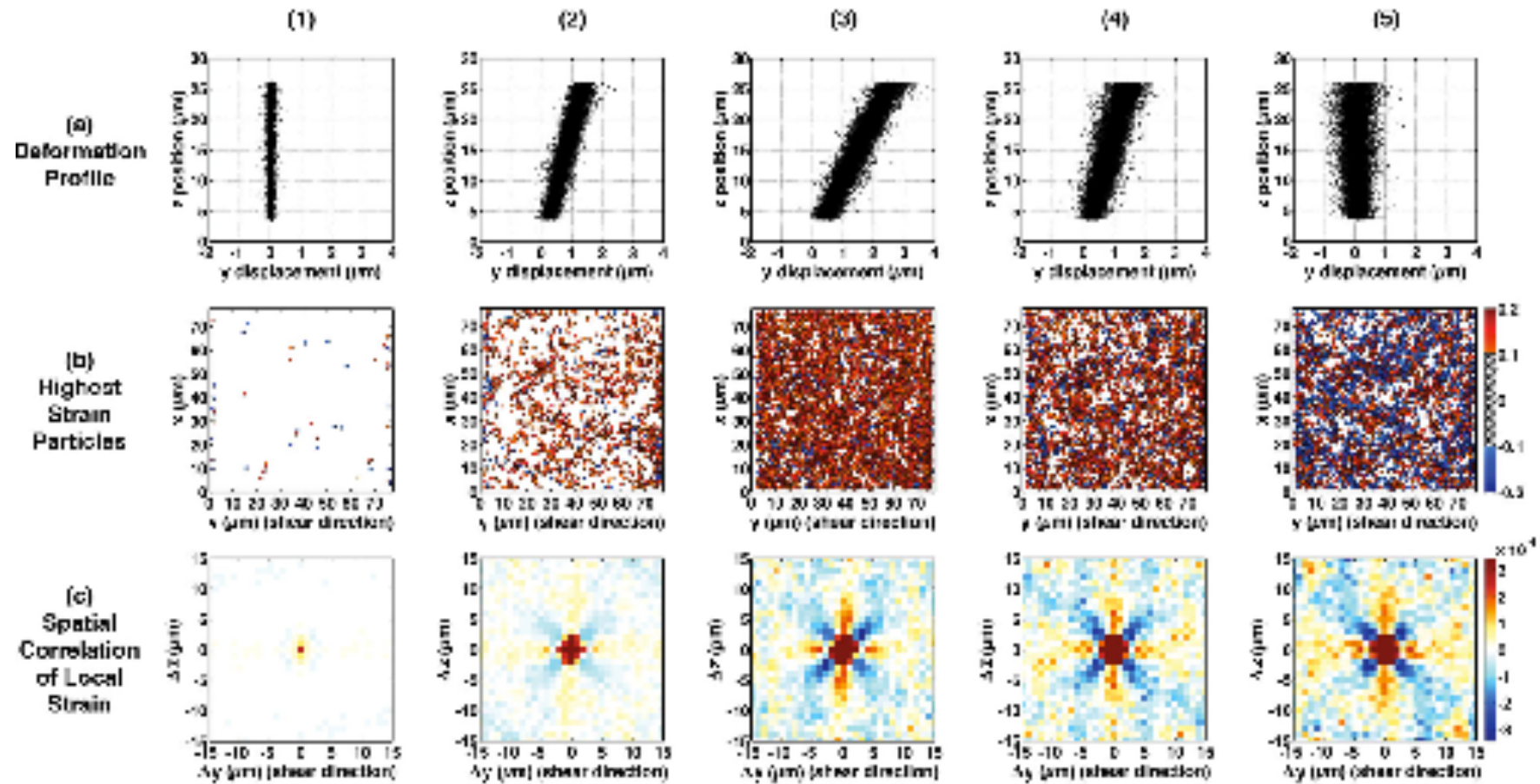
In an homogeneous, linear elastic solid, the Induced shear stress outside the inclusion is proportional to the inclusion transformation strain and to the Eshelby propagator:

$$G(r, \theta) = \frac{1}{\pi r^2} \cos(4\theta)$$



Events are shear transformations of Eshelby type

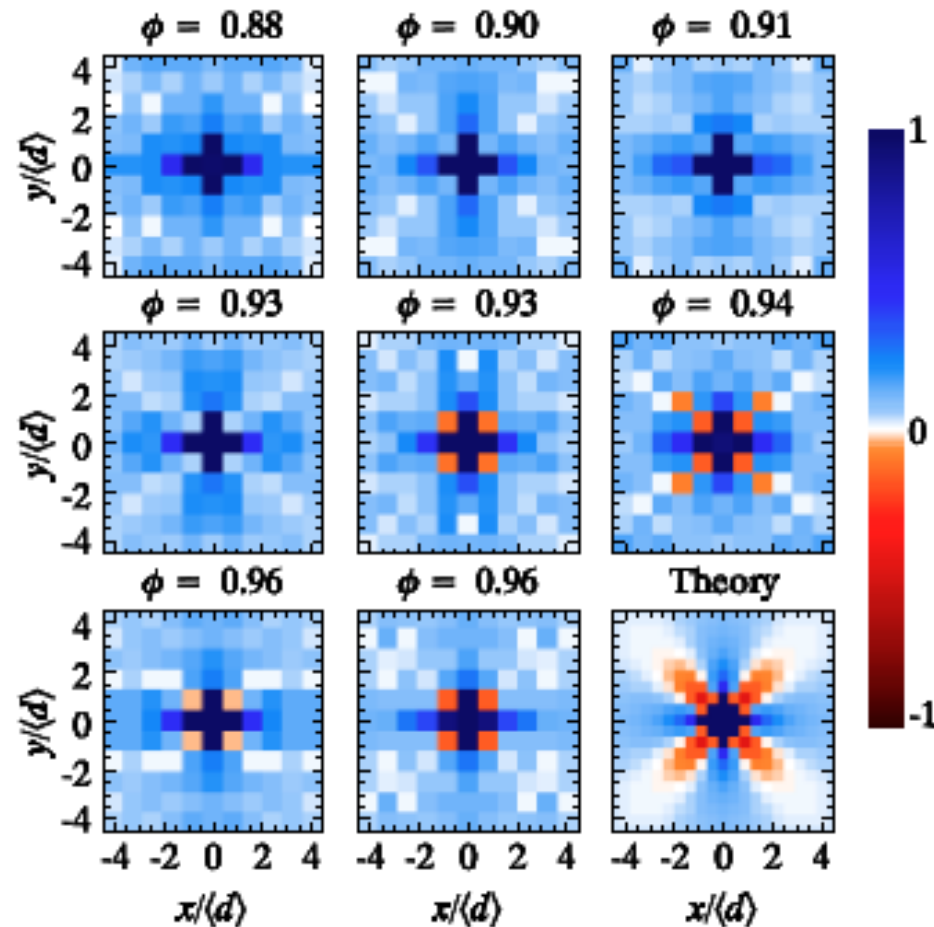
Best seen in experiments through correlation patterns



Colloidal paste under simple shear
(Jensen, Weitz, Spaepen, PRE 2014)

Events are shear transformations of Eshelby type

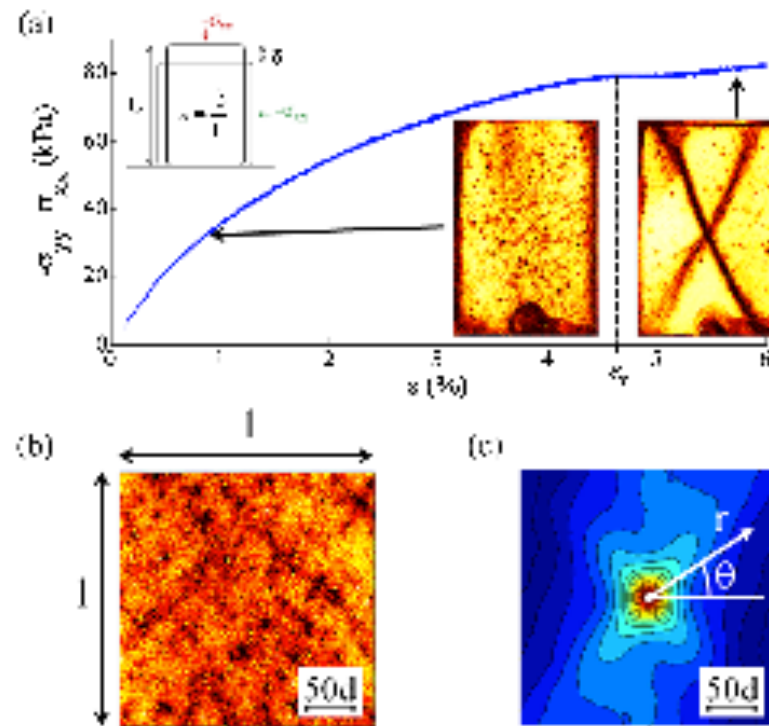
Best seen in experiments through correlation patterns



Stress redistribution in foams –quadrupole + isotropic term
(Desmond and Weeks, PRE 2013)

Events are shear transformations of Eshelby type

Best seen in experiments through correlation patterns

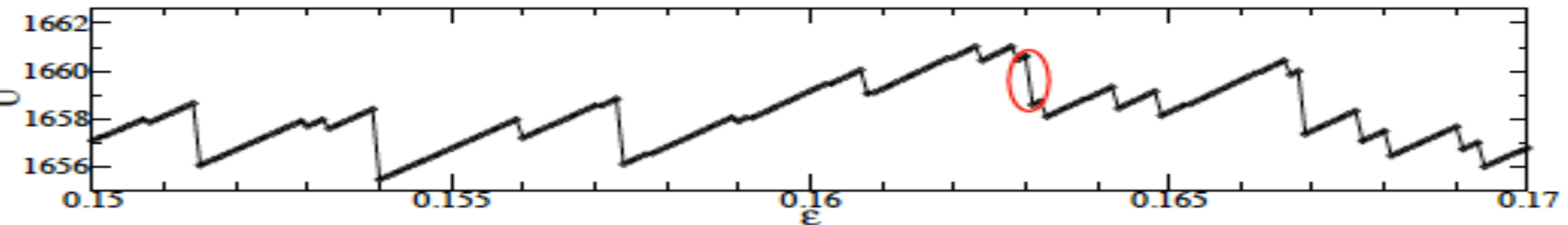


Granular medium under uniaxial deformation
(Le Bouil, Amon, Crassous, PRL 2014)

Stationnary plastic flow regime:

At large deformation, individual flow events interact and organize in the form of « avalanches » with a very broad distribution of amplitudes.

« Barkhausen » type behavior, encountered in many condensed matter systems with disorder (magnetic domain walls, contact lines, vortices in superconductors, earthquakes, friction)



Build generic, coarse grained models, in the spirit of Ising model for magnetism, with the individual shear transformation (involving $O(100)$ atoms) as the building block.

Flow of amorphous solids:

- Statistical physics problem: elementary events identified, space time interactions and correlations between events lead to avalanches and noise.
- Jamming (from flowing suspension to a solid paste) has features of a dynamical phase transition (depinning ?)

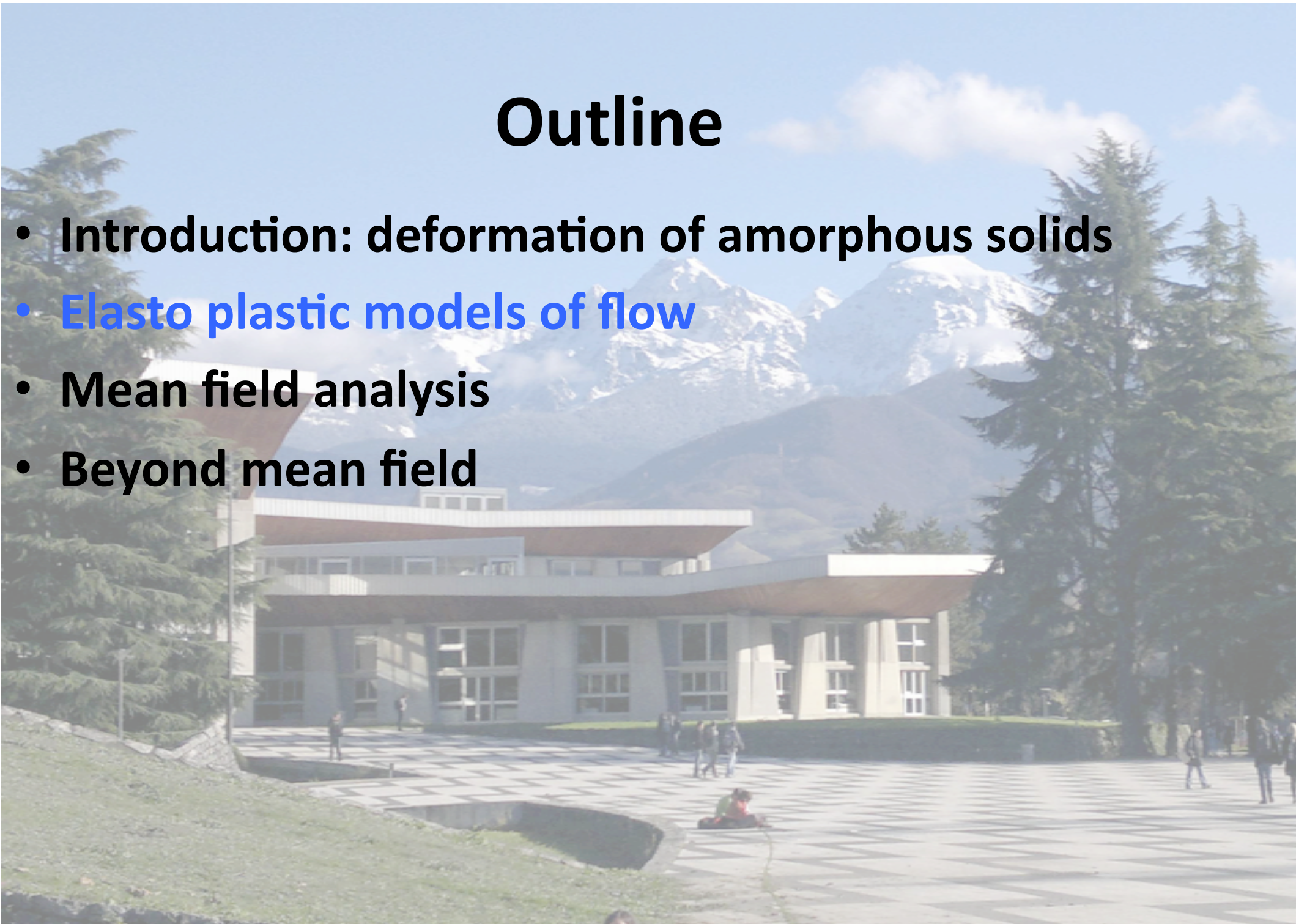
$$\sigma = \sigma_{\text{yield}} + A\dot{\gamma}^{\alpha} \longleftrightarrow \dot{\gamma} \propto (\sigma - \sigma_{\text{yield}})^{1/\alpha}$$

- Universal features from granular media to metallic glasses
- Different (simpler ?) from extensively studied crystalline plasticity which involves topological flow defects

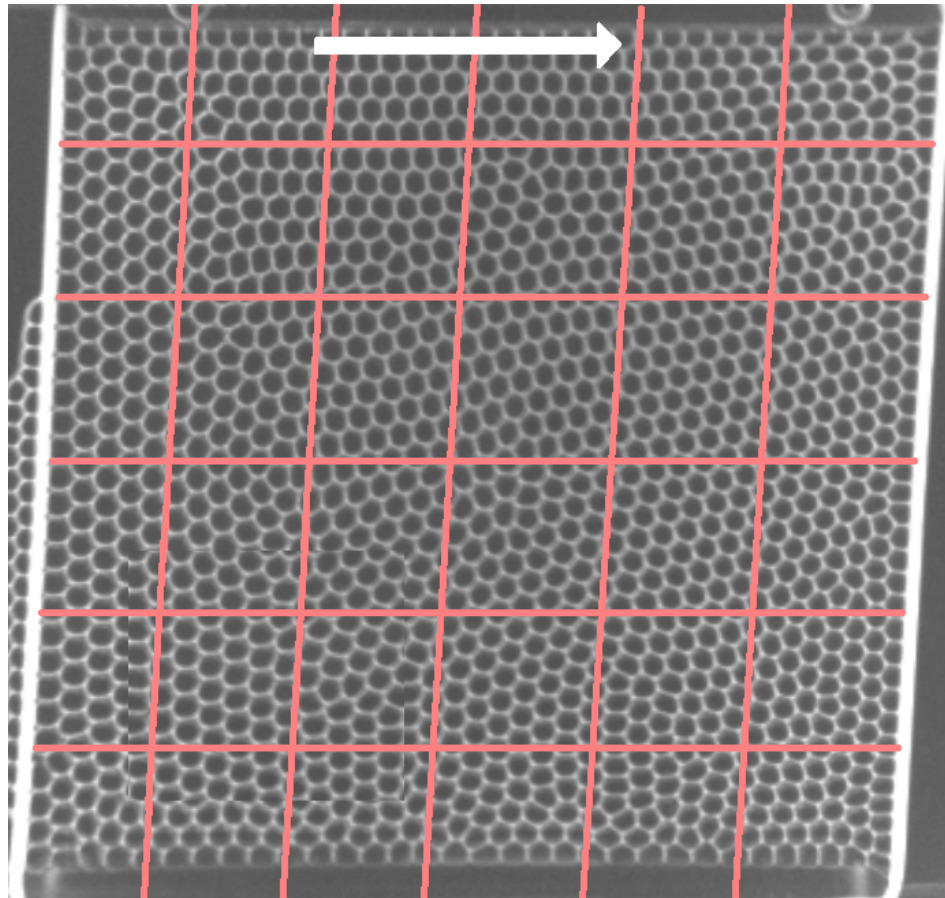
=> Gain qualitative understanding of phenomena at different scales with simple models, and tune model parameters for quantitative predictions.

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- **Elasto plastic models of flow**
- Mean field analysis
- Beyond mean field



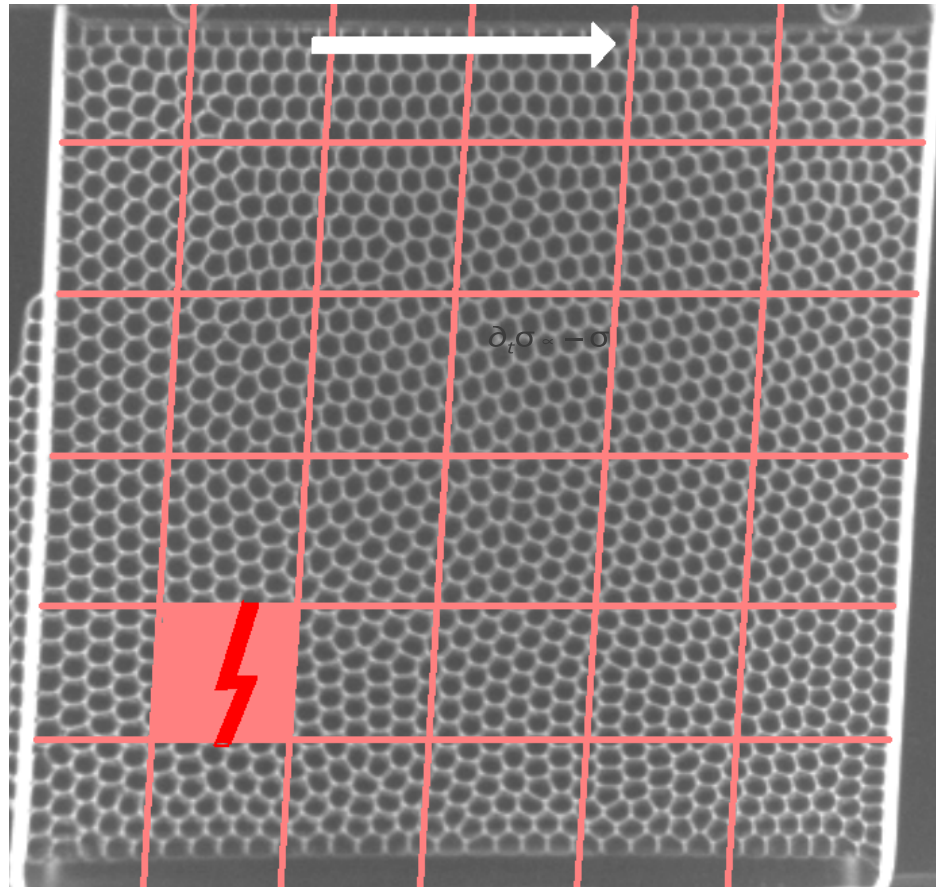
Mesoscopic elastoplastic model: « Ising model » of amorphous deformation



Linear elastic response at low shear

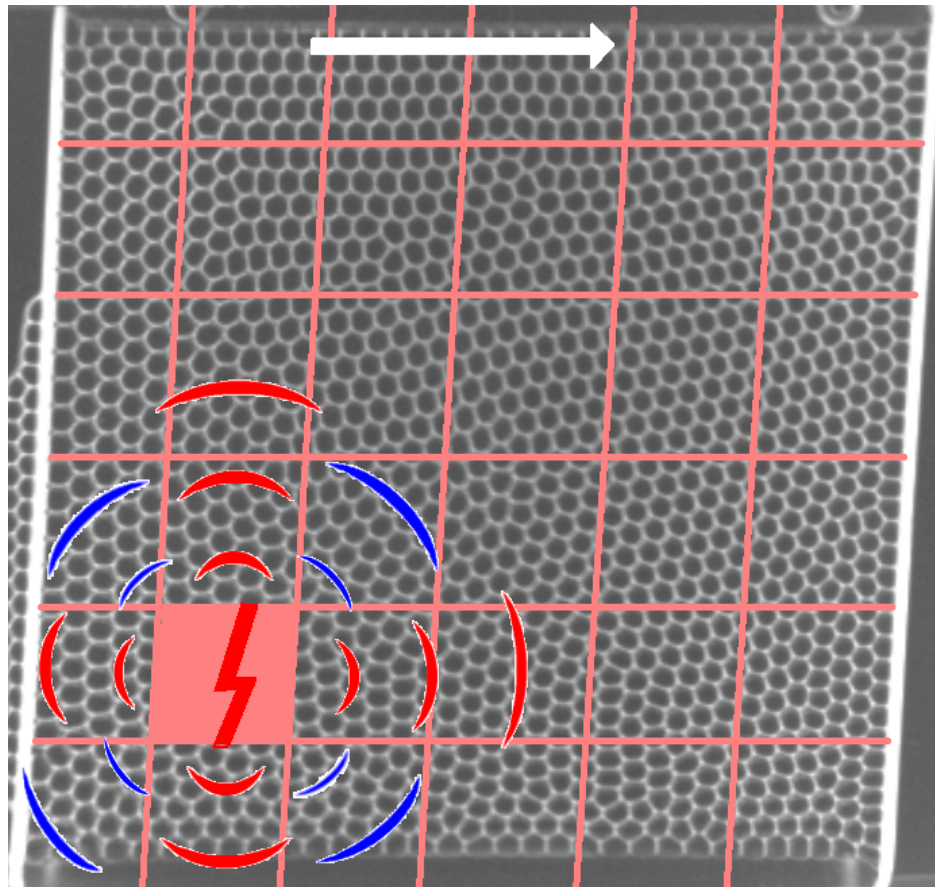
Related models by Bulatov and Argon, Picard et al

At some point (yield criterion), the material **yields** locally and the energy stored locally is dissipated → plastic event



Related models by Bulatov and Argon, Picard et al

Stress is redistributed during plastic event, owing to the presence of an elastic medium



Then elastic regime again, etc.

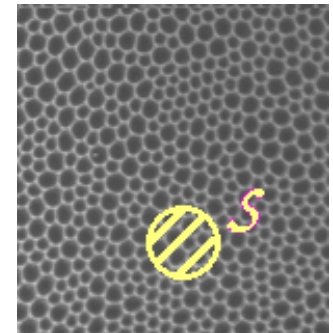
How to describe a single plastic event ?

(Eshelby's inclusion problem)

1. Start with incompressible elastic medium. The medium yields (becomes fluid) within some region S ("plastic inclusion")
2. In plastic inclusion, driving elastic force acting on the boundary is opposed by dissipative forces

$$\sigma = 2\mu\epsilon = \eta_{eff}\dot{\epsilon}_{pl}$$

$$\dot{\epsilon}_{pl} = \frac{\epsilon}{\tau} \text{ on } S, \text{ with } \tau = \eta_{eff}/\mu.$$



3. Local deformation on S induces long range elastic matrix deformation → elastic propagator **G**
4. Stop when deformation within inclusion reaches a threshold value (typically 1%).

Elastic propagator $G(r, r')$

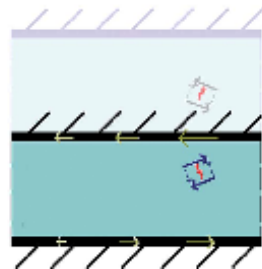
Linear elasticity, homogeneous isotropic medium

In an unbounded, isotropic linear elastic medium:

$$G(r, r') \sim \frac{\cos(4\theta)}{|r - r'|^d}$$

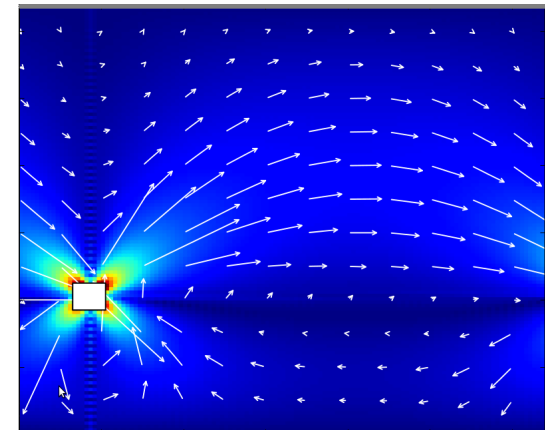
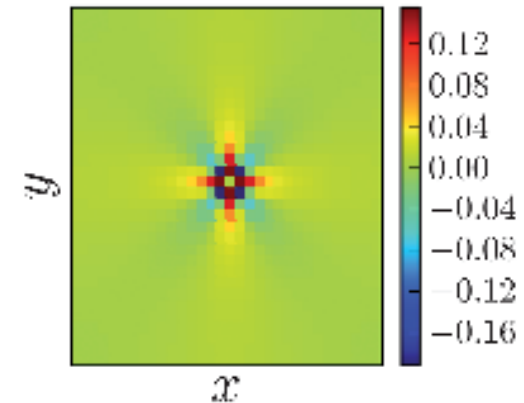
Quadrupolar symmetry (two force dipoles)

-More complex in the presence of walls
(image method to enforce boundary conditions, or FE calculation)



-Long range: nonlocal aspects of the flow.

-Very important : G changes sign in space



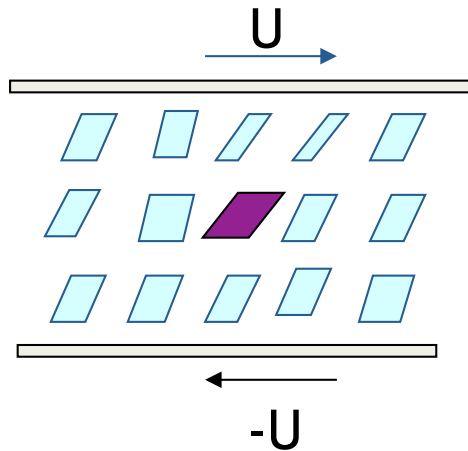
Stress dynamics (easily implemented on a lattice)

$$\partial_t \sigma(\vec{r}, t) = \mu \dot{\gamma} + \int d\vec{r}' G(\vec{r} - \vec{r}') \dot{\epsilon}^{plast}(\vec{r}', t)$$

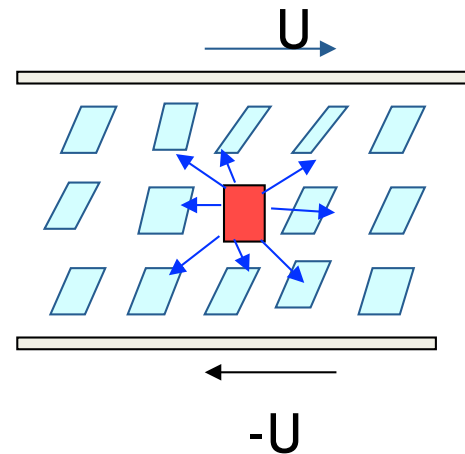
$G(r, \theta) \sim \cos(4\theta)/r^2$
elastic propagator

Plastic activity (number of sites that reach yield point)

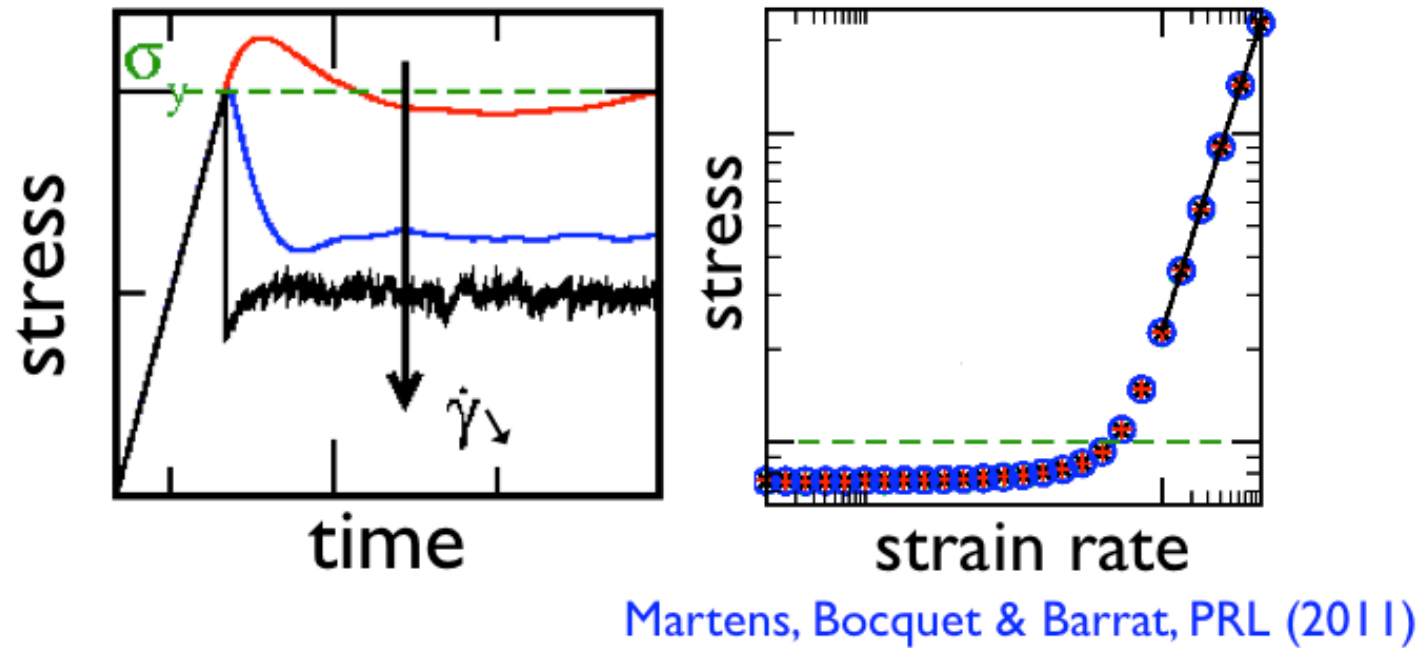
Global elastic forcing



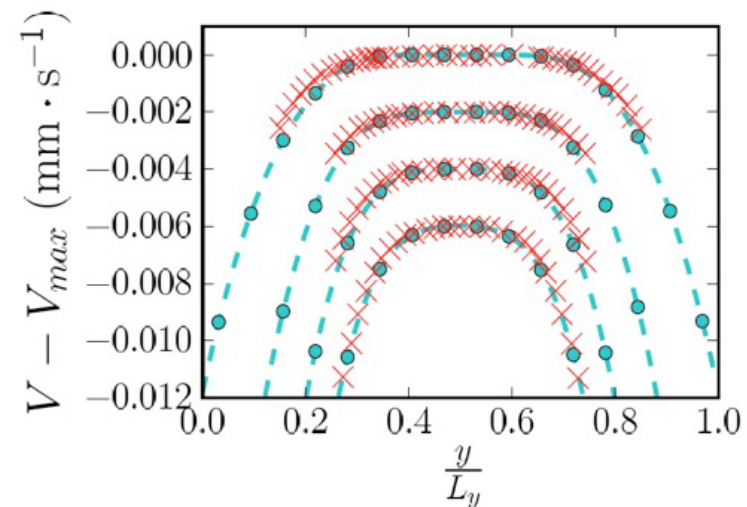
Plastic relaxations



Stress strain and flow curves

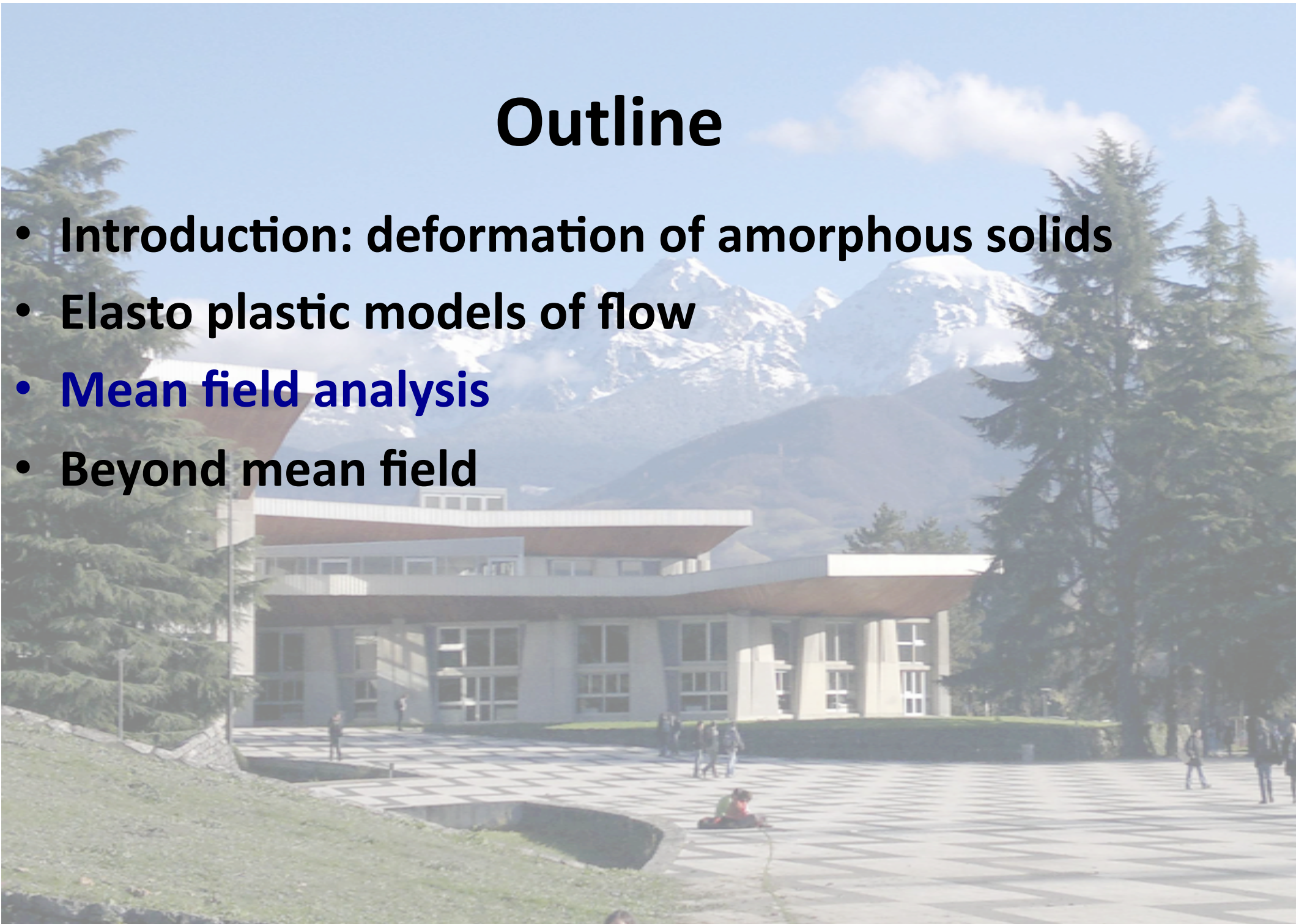


Model can reproduce quantitatively flow and fluctuations in microchannel geometries (Nicolas and Barrat, PRL 2013)



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Mean field analysis (Hébraud Lequeux model): Stress diffusion due to mechanical noise + self consistency

$P(\sigma, t)$ probability distribution of stress on a typical site (no disorder, single local yield stress)

$$\partial_t \mathcal{P}(\sigma, t) = \underbrace{-G_0 \dot{\gamma}(t) \partial_\sigma \mathcal{P}}_{\text{External drive}} + \underbrace{D_{\text{HL}}(t) \partial_\sigma^2 \mathcal{P}}_{\text{Stress diffusion}} - \underbrace{\nu_{\text{HL}}(\sigma, \sigma_c) \mathcal{P}}_{\text{Yield if } \sigma > \sigma_c} + \underbrace{\Gamma(t) \delta(\sigma)}_{\text{Reset to zero after yield}}$$

Yield rule and plastic activity

$$\nu_{\text{HL}}(\sigma, \sigma_c) \equiv \frac{1}{\tau} \theta(\sigma - \sigma_c)$$

$$\Gamma(t) = \frac{1}{\tau} \int_{\sigma' > \sigma_c} d\sigma' \mathcal{P}(\sigma', t)$$

Non linear feedback

$$D_{\text{HL}}(t) = \alpha \Gamma(t)$$

Mean field analysis (Hébraud Lequeux model): Stress diffusion due to mechanical noise + self consistency

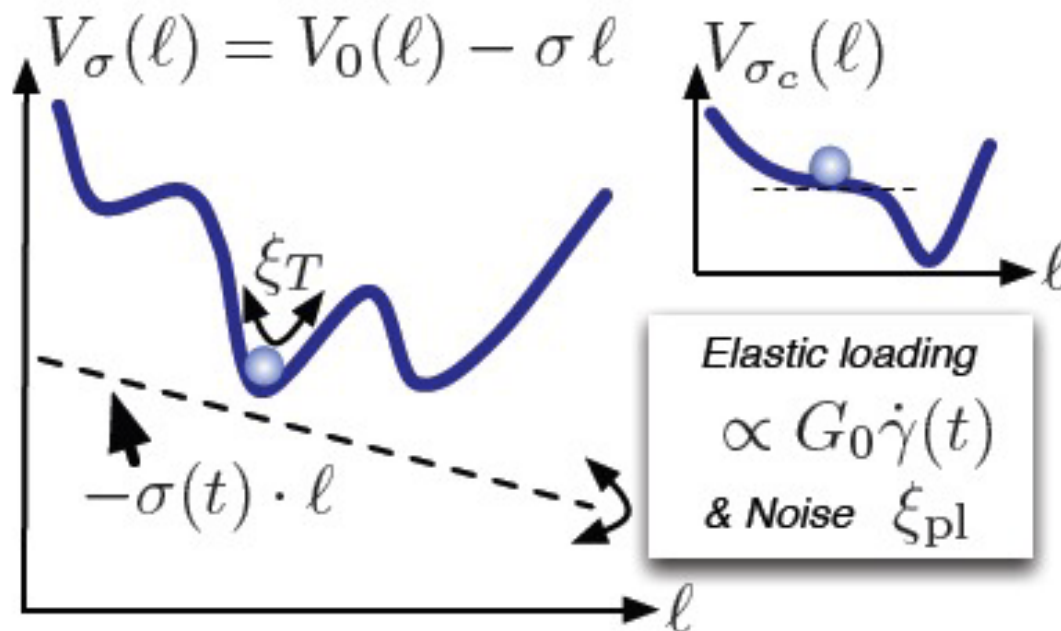
- $\alpha > \alpha_c = 2$ Newtonian behaviour
 $\sigma \sim \dot{\gamma}$
- $\alpha < \alpha_c = 2$ Herschel Bulkley law with exponent $1/2$:
 $\sigma = \sigma_Y + A\dot{\gamma}^{1/2}$

A spatial version (inhomogeneous stress or strain rate) can be derived (Bocquet et al, PRL 2012) and compares well with experiments in confined geometries (A. Nicolas, JLB, PRL2014)

Note: mechanical noise in athermal systems is different from thermal noise!

Potential Energy Landscape

Picture for a small region (STZ):

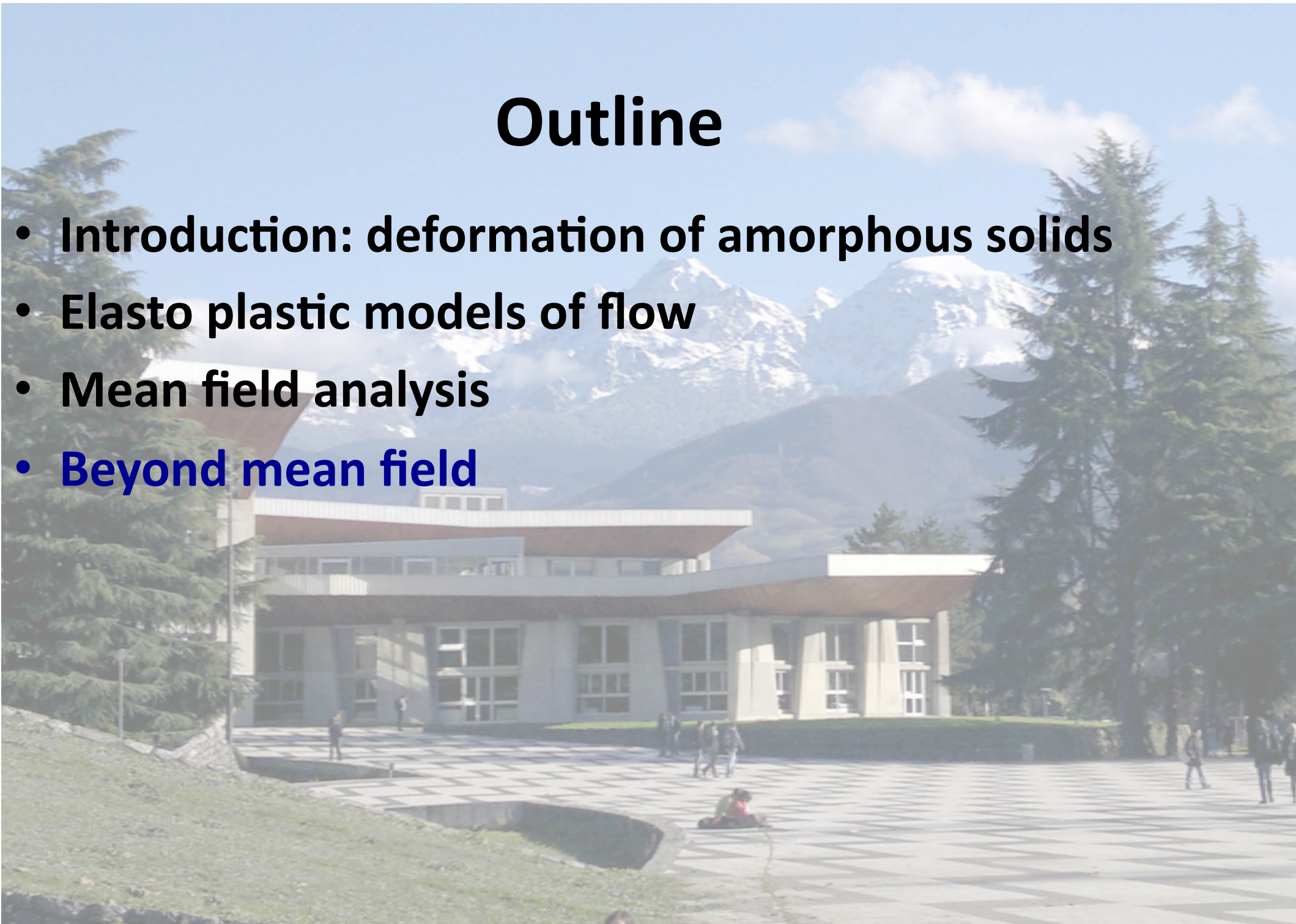


- Thermal noise acts on strain variable ℓ in a fixed landscape biased by the stress
- Mechanical noise acts a diffusive process on the stress bias itself

=> Very different escape times (Arrhenius vs diffusive)

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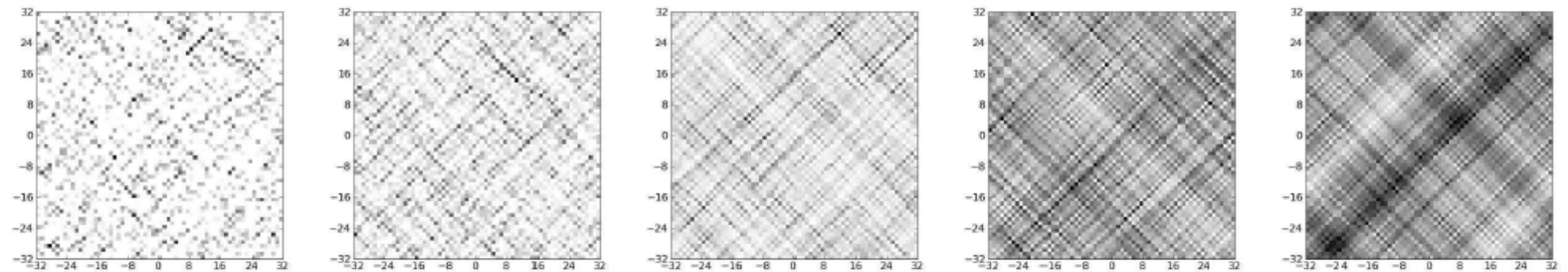


Understanding gained from numerical studies of elastoplastic models:

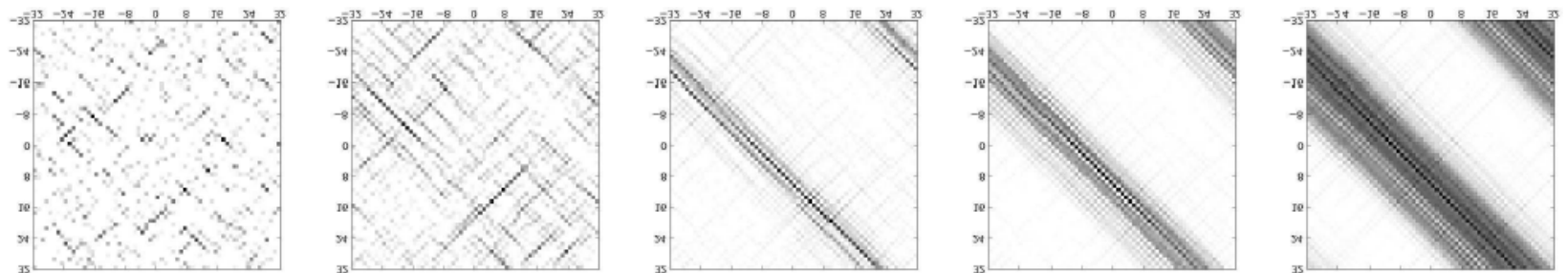
- Acceleration of diffusion by active deformation (Martens, Bocquet, JLB, PRL 2011)
- Interplay between aging and strain localisation (Vandembroucq and Roux, PRB 2011)
- Microscopic criteria for strain localisation in soft materials (Martens, Bocquet, JLB, Soft Matter 2012)
- Introducing thermal activation of local events allows one to reproduce « compressed exponential » relaxation (Ferrero, Martens, JLB, PRL 2014).

Accumulated plastic deformation (Roux Vandembroucq)

As quenched



Aged



But many questions remain open.....



Scaling description of the yielding transition in soft amorphous solids at zero temperature

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^aCenter for Soft Matter Research, Department of Physics, New York University, New York, NY 10003; and ^bLaboratoire de Physique Théorique et Modèles Statistiques (Centre National de la Recherche Scientifique, Unité Mixte de Recherche 8626), Université de Paris-Sud, 91405 Orsay Cedex, France

Edited by David A. Weitz, Harvard University, Cambridge, MA, and approved August 21, 2014 (received for review April 8, 2014)

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PHYSICAL REVIEW LETTERS

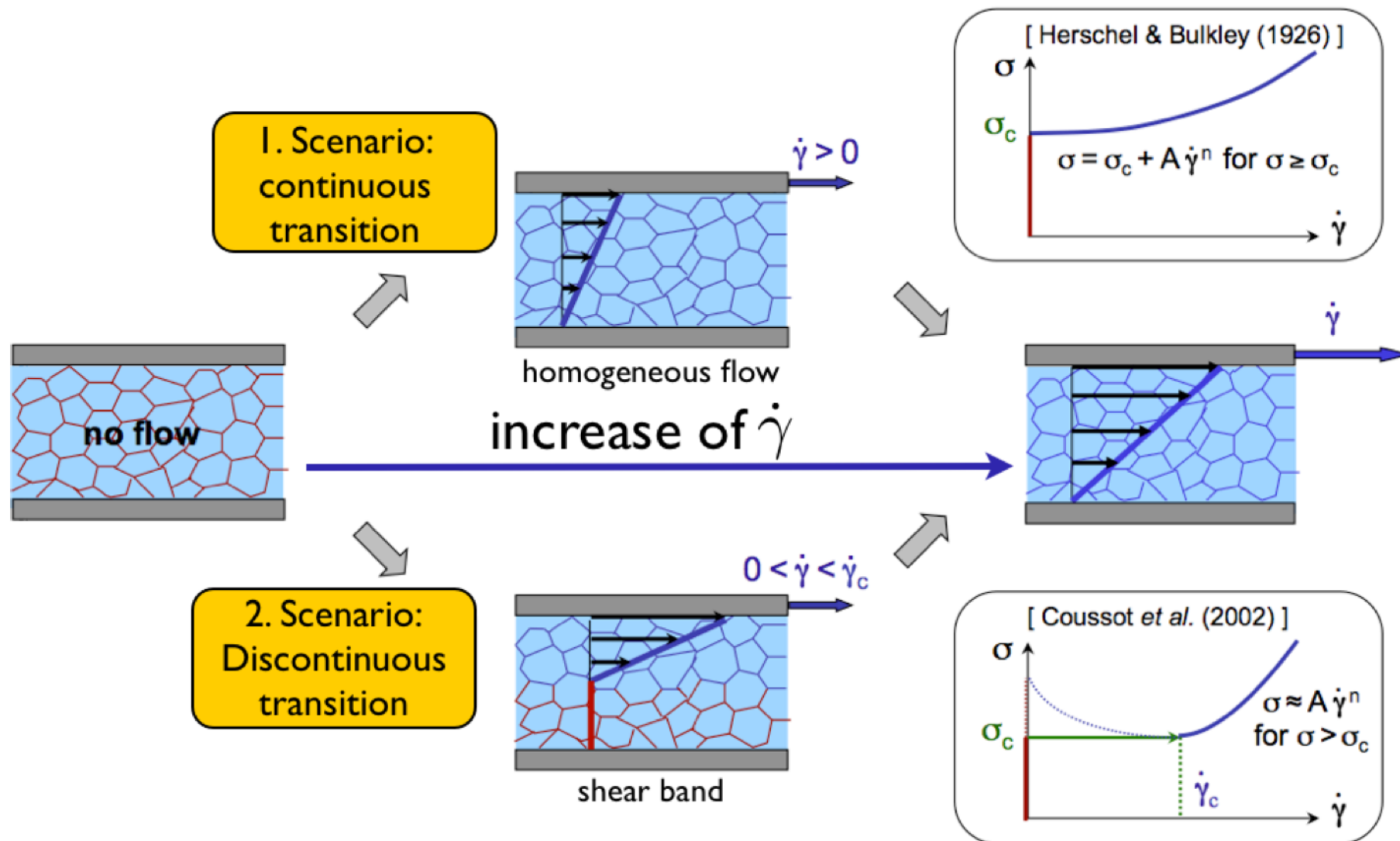
week ending
26 FEBRUARY 2016

Mechanical Yield in Amorphous Solids: A First-Order Phase Transition

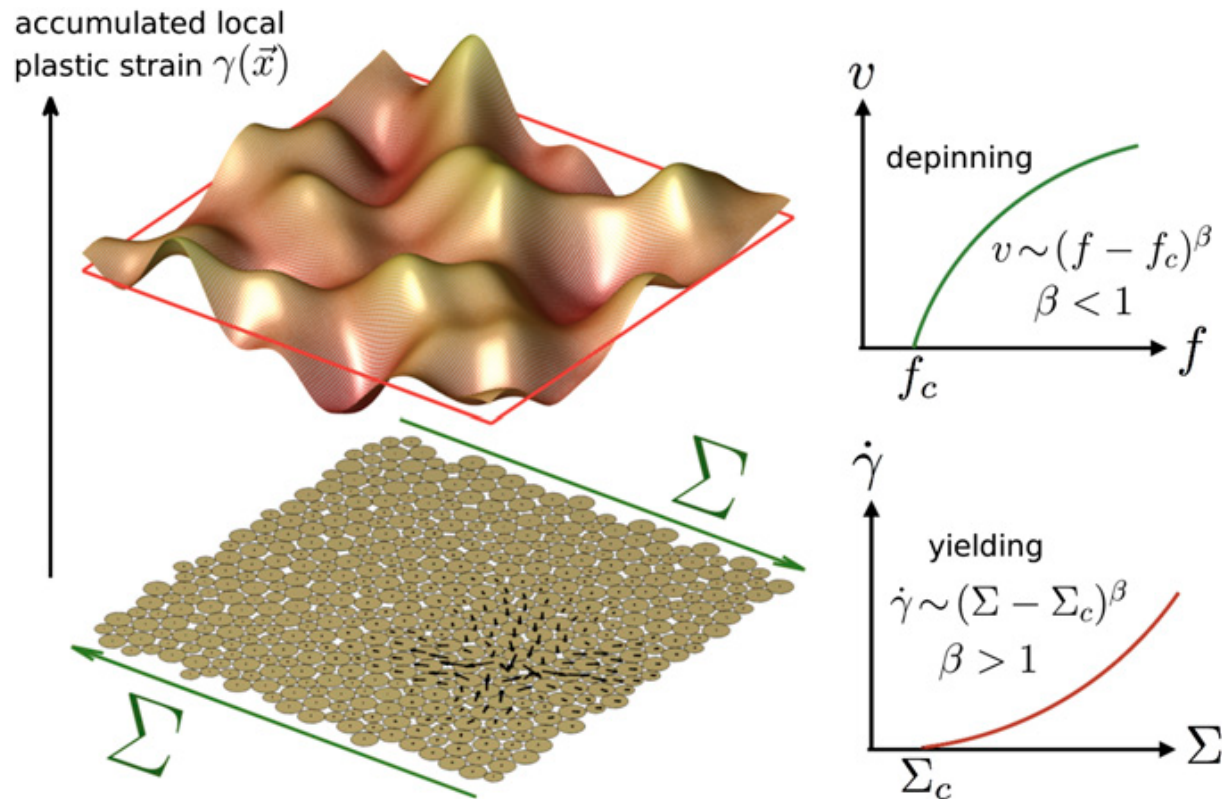
Prabhat K. Jaiswal, Itamar Procaccia, Corrado Rainone, and Murari Singh

Yielding transition

Is it a dynamical phase transition? Which is the order parameter? Is it continuous or discontinuous?



When continuous, how similar is the yielding transition to depinning ?



Lin et al, PNAS2014

Lin, Lerner, Rosso, Wyart,
EPL 2014, PNAS 2014

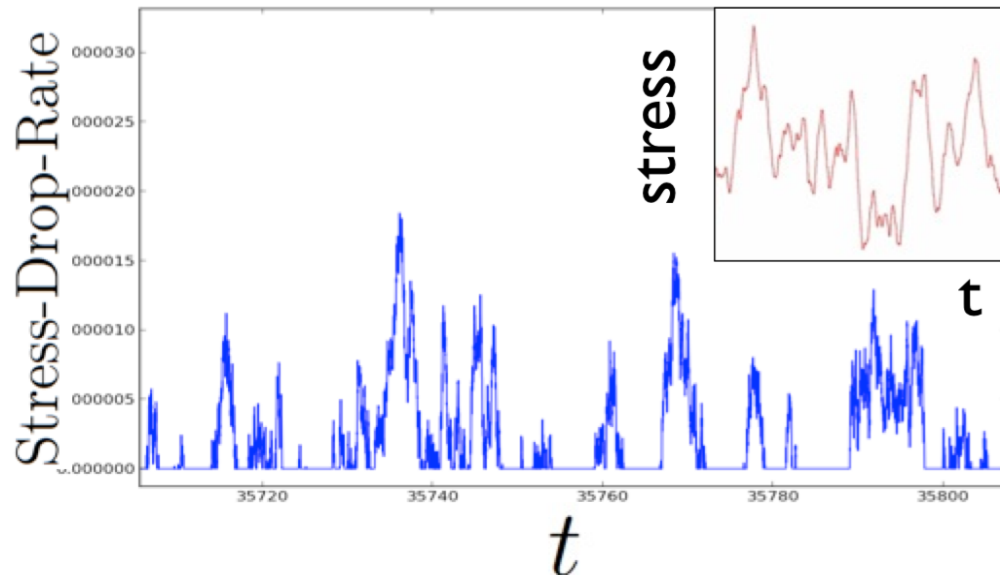
Major difference with
depinning problem: Kernel
 $G(r-r')$ is **not positive
everywhere**

⇒ New universality class(es)

Transition characterized by avalanche statistics (similar to Barkhausen noise, earthquakes) obtained from numerics/experiments
Avalanches visible as stress drops in the stress/strain curve

Relevant quantities at finite strain rate

Stress-Drop-Rate $H(-\langle\dot{\sigma}\rangle)|\langle\dot{\sigma}\rangle|$:

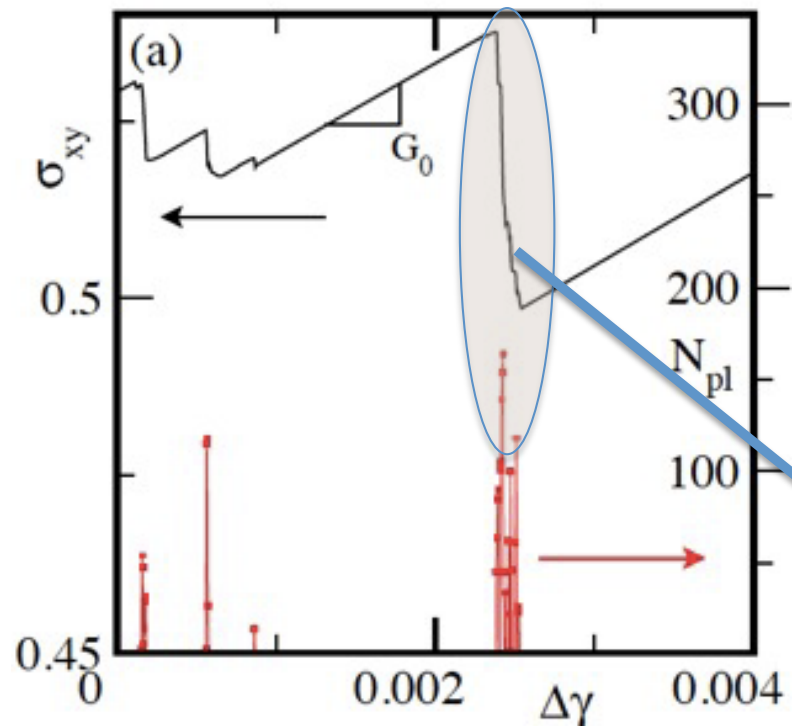


Stress drop amplitude:

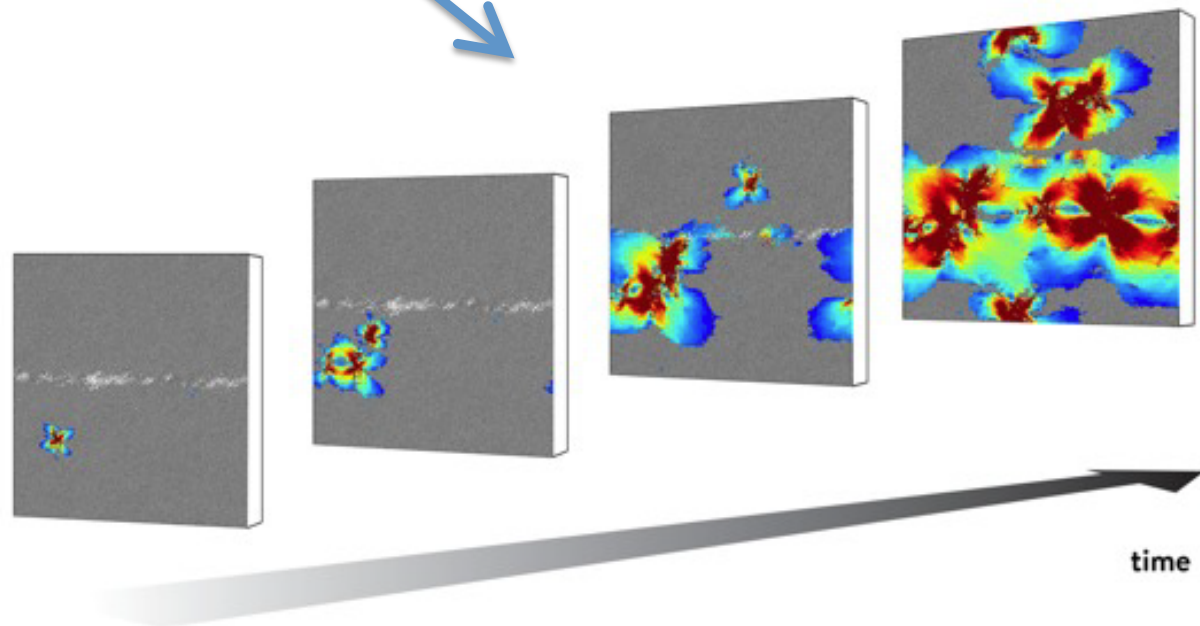
$$S \hat{=} |\Delta\langle\sigma\rangle|L^d = L^d \int_{aval} |\langle\dot{\sigma}\rangle| dt$$

Stress drop duration:

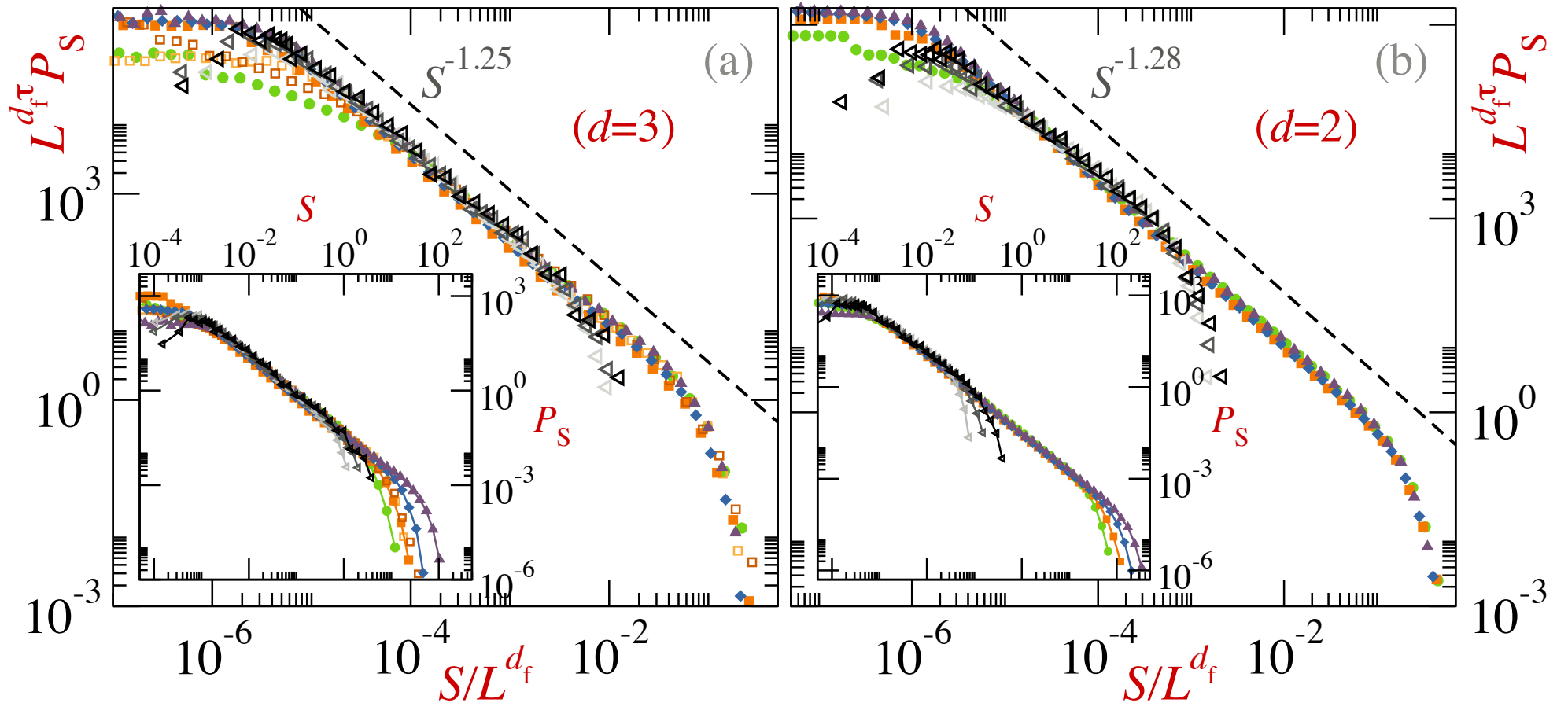
$$T \hat{=} \int_{aval} dt$$



**Spatial structure of a stress drop avalanche
(MD simulations, Binary Lennard Jones, F. Puosi)**

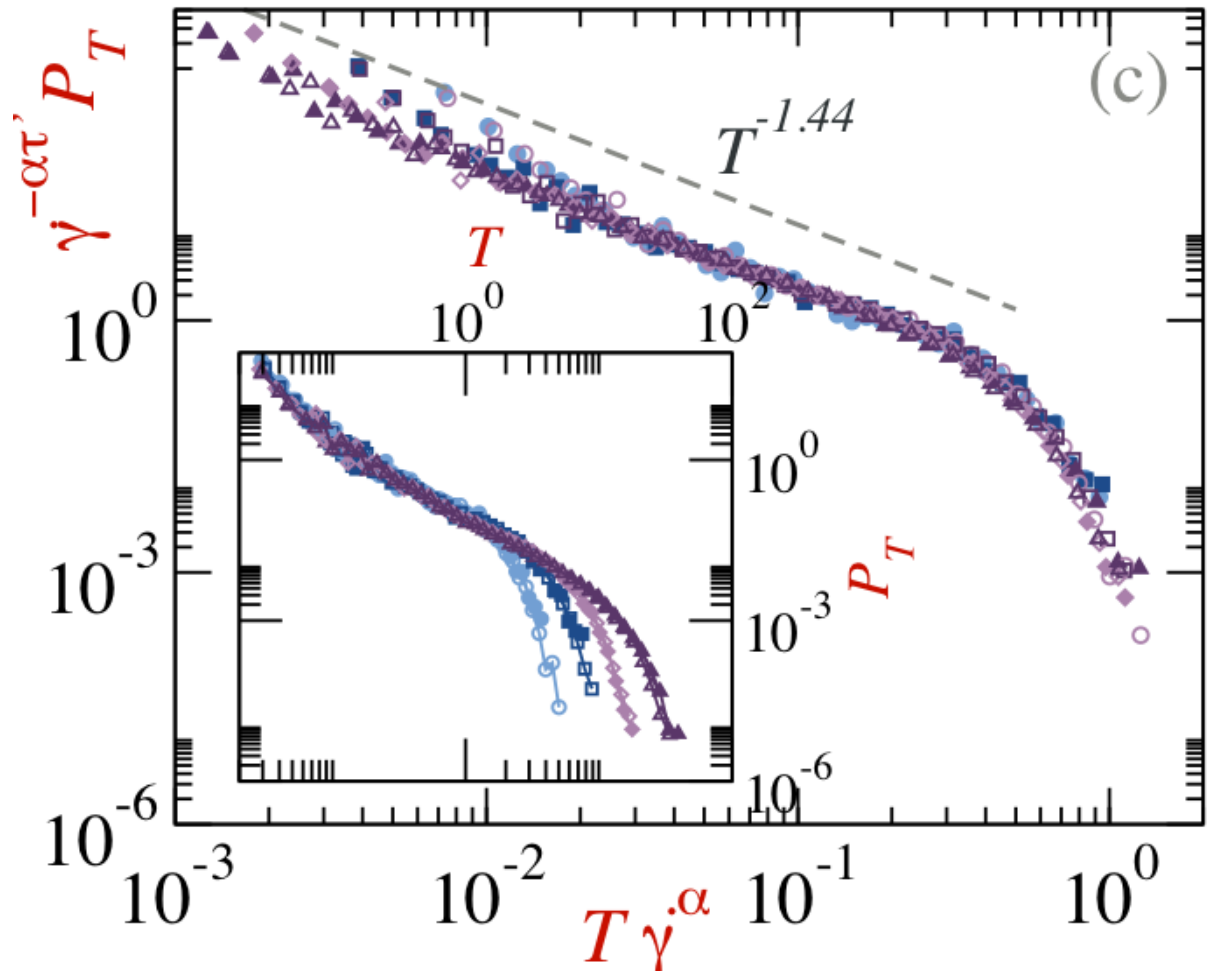


Distribution of stress drops



$$P(S) = S^{-\tau} P(S/L^{d_f})$$

Distribution of stress drops durations



$$P(T) = T^{-\tau'} g(T \dot{\gamma}^\alpha)$$

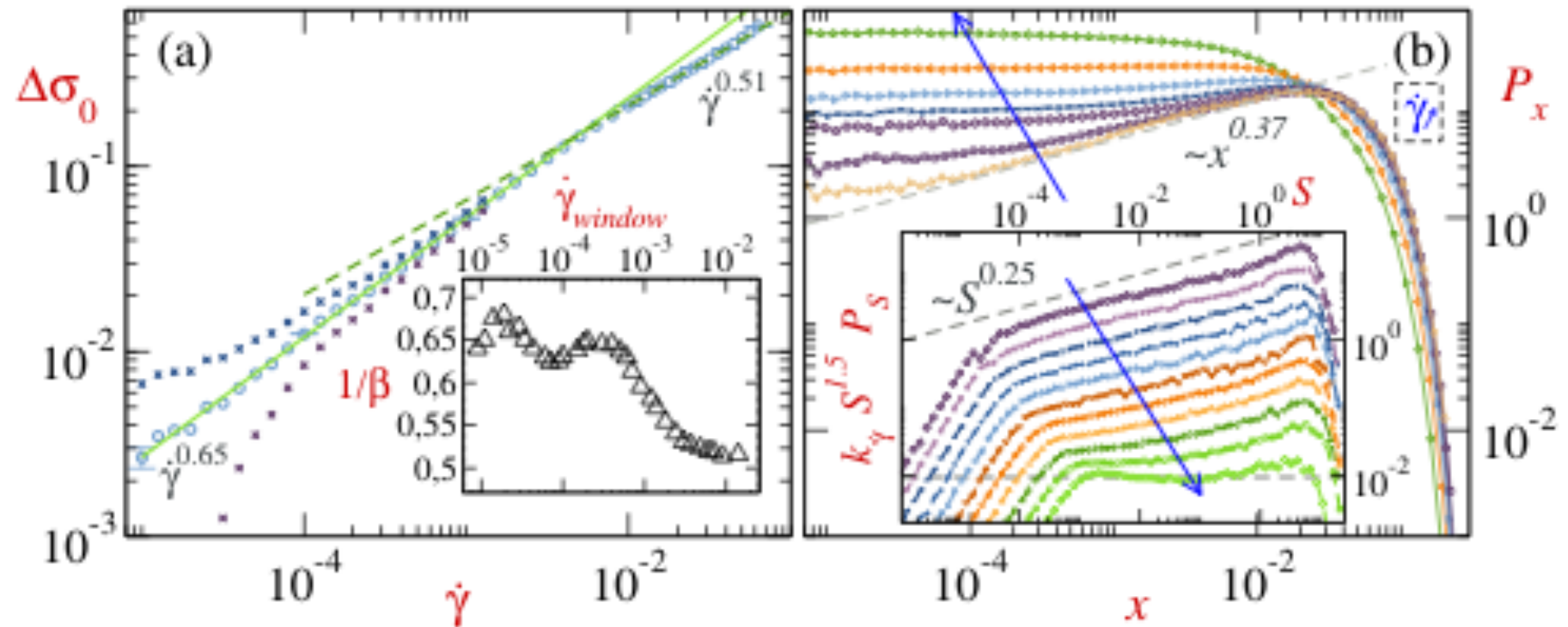
Critical exponents

(Chen, Ferrero, Puosi, JLB, Martens for a slowly sheared system,
Lin, Lerner, Rosso and Wyart for a quasistatic protocol)

	Expression	This work		MF
		2d	3d	∞
β	$\dot{\gamma} \sim (\Delta\sigma)^\beta$	1.54(2)	1.55(2)	2 [47]
τ	$P_S \sim S^{-\tau}$	1.28(5)	1.25(5)	1.5 [42]
d_f	$S_c \sim L^{d_f}$	0.90(7)	1.3(1)	—
τ'	$P_T \sim T^{-\tau'}$	1.41(4)	1.44(4)	2 [42]
α	$T_c \sim \dot{\gamma}^\alpha$	0.38(4)	0.30(4)	—
z	$T \sim \ell^z$	~ 0.57	~ 0.82	—
δ	$S \sim T^\delta$	1.58(7)	1.58(5)	2 [42]
θ	$P_x \sim x^\theta$	0.52(3)	0.37(5)	1 [48]

- Exponents are different from mean field
- Crossover to mean field like behaviour as the strain rate increases
(Chen et al, [arXiv:1506.08161](https://arxiv.org/abs/1506.08161), Phys Rev. Lett 2016)

Crossover to mean field behaviour with increasing strain rate (Chen et al, PRL 2016)



Conclusions

- Amorphous plasticity is a « simple » situation that exhibits complex, possibly universal behaviour.
- Can be described by simple elasto plastic models.
- Dynamical phase transition with similarities to depinning but with clear differences (interaction kernel changes sign)
- Exponents in 2d, 3d are different from mean field.
- Possibility of first order transition (strain localisation) ; conditions are not understood in detail. **Inertia matters (A. Nicolas, J. Röttler, JLB, PRL 2016)**
- Mostly numerics*, room for theory and experiments...

**From many different groups: Zapperi, Rosso, Jagla, Wyart, Procaccia, Robbins, Vandembroucq, Lemaître, Maloney, Falk, Benzi... Elastoplastic models but also molecular dynamics, exponents match.*

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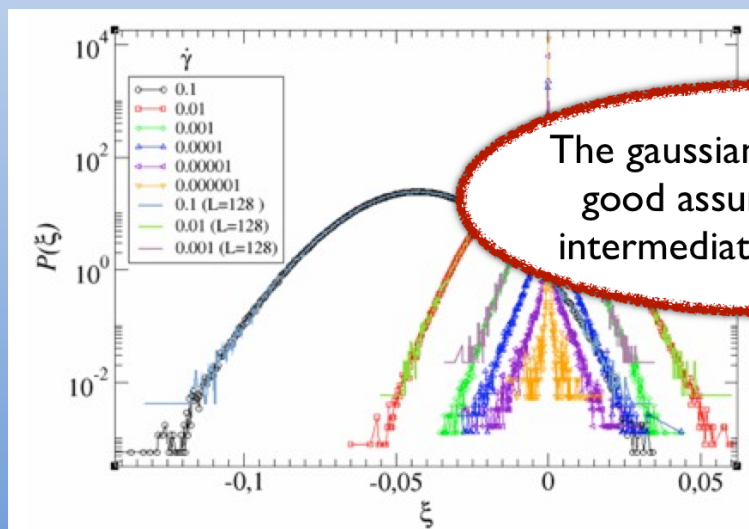
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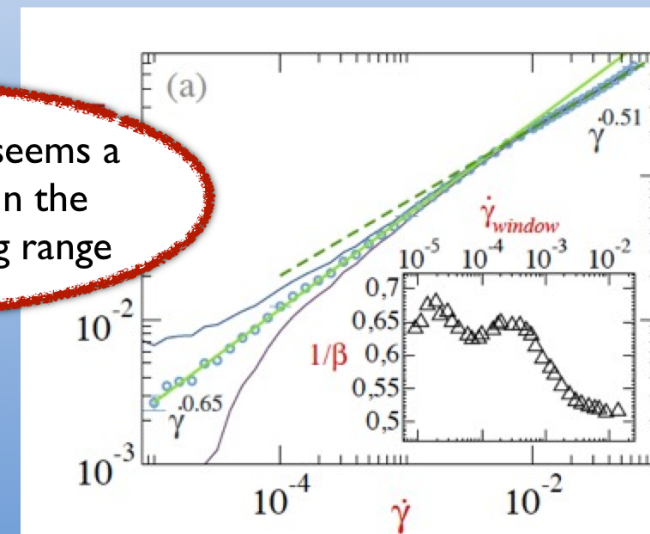
9-14 October 2016, Dijon, France





The gaussian noise seems a good assumption in the intermediate driving range

2d noise distributions
Ezequiel Ferrero



Flow curve exponents

Liu et al. PRL (2016)