Glass-to-glass transition of hard sphere glasses under external perturbations

Yuliang Jin
Cybermedia Center, Osaka University

Group leader: Prof. Hajime Yoshino

Collaborators:

- Ludovic Berthier (Montpellier, France)
- Patrick Charbonneau (Duke, USA)
- Giorgio Parisi (Sapienza, Italy)
- Corrado Rainone (ENS, France)
- Beatriz Seoane (ENS, France)
- Francesco Zamponi (ENS, France)

Related talks:

- Misaki Ozawa
- Hajime Yoshino

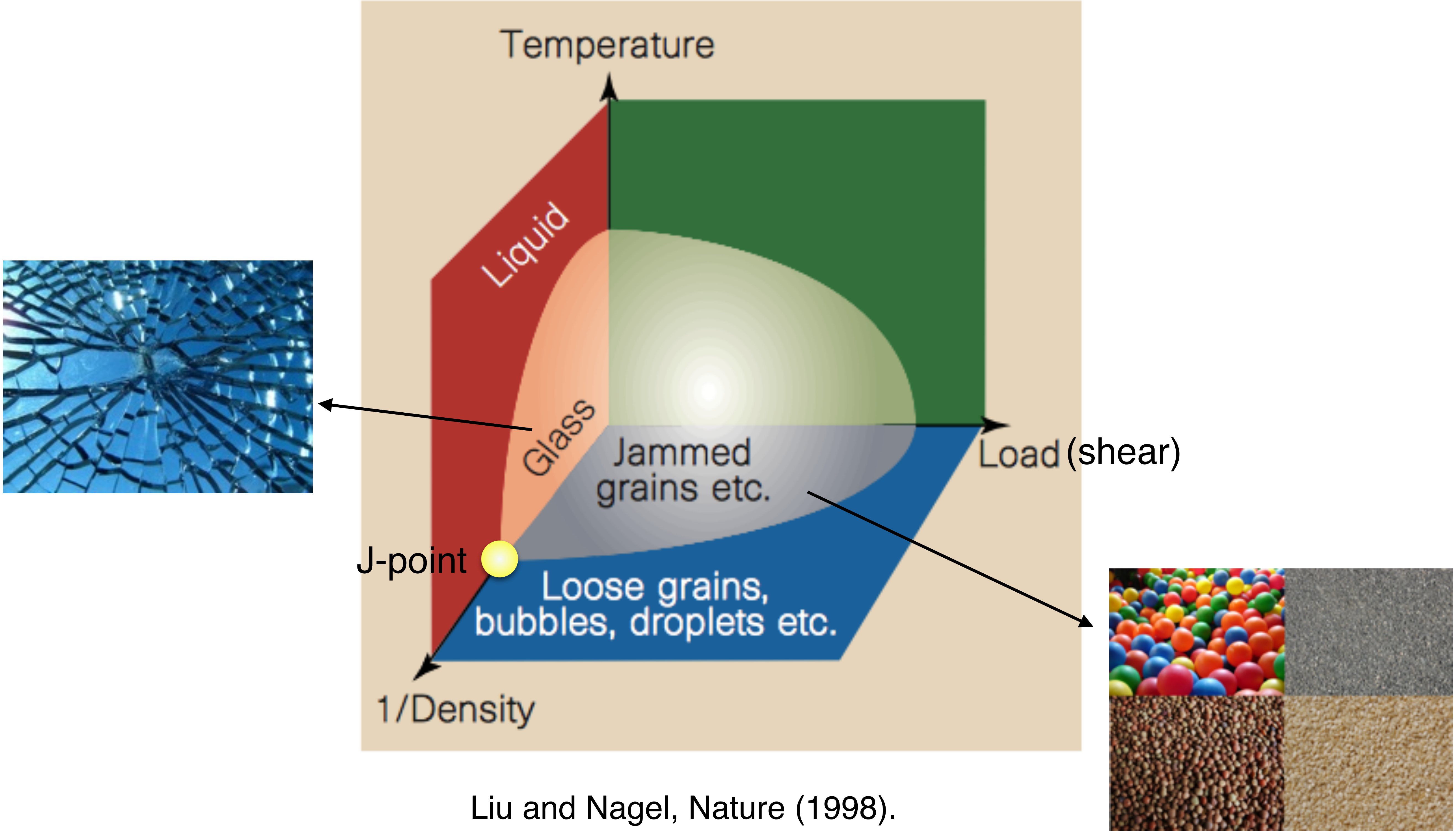
Fluctuation & Structure

大阪大学
OSAKA UNIVERSITY

Workshop on “Avalanches, plasticity, and nonlinear response in nonequilibrium solids”
Yukawa Institute for Theoretical Physics • Kyoto University • March 9th, 2016

Unifying glass and jamming transitions

Initial proposal by Liu and Nagel



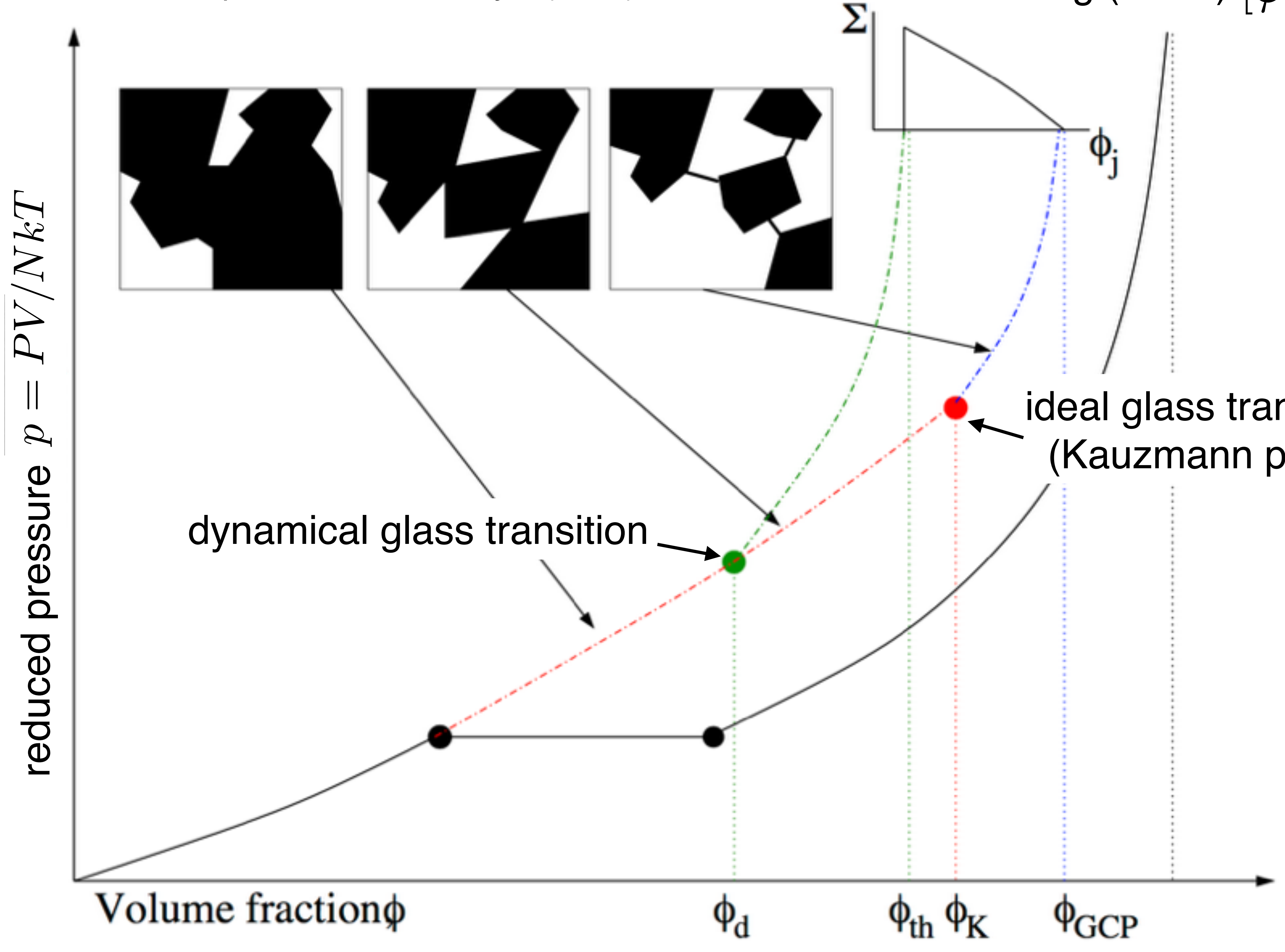
Liu and Nagel, Nature (1998).

Mean-field theory of hard spheres

1-step replica symmetry breaking (1RSB) theory

G. Parisi and F. Zamponi, Rev. Mod. Phys. (2010)

Jamming (J-line) $[\varphi_{\rm th}, \varphi_{\rm GCP}]$



ideal glass transition
(Kauzmann point)

dynamical glass transition

How good is the 1RSB theory?

Success:

- The predicted transition densities are basically consistent with simulations in large dimensions.

P. Charbonneau, A. Ikeda, G. Parisi, and F. Zamponi, PRL (2011)

- The prediction of J-line is confirmed by simulations — the jamming transition does not occur at a unique density.

P. Chaudhuri, L. Berthier, and S. Sastry, PRL (2010)

- The jammed states are isostatic: $Z = 2d$.

Open:

- Does the ideal glass transition (Kauzmann point) exist?

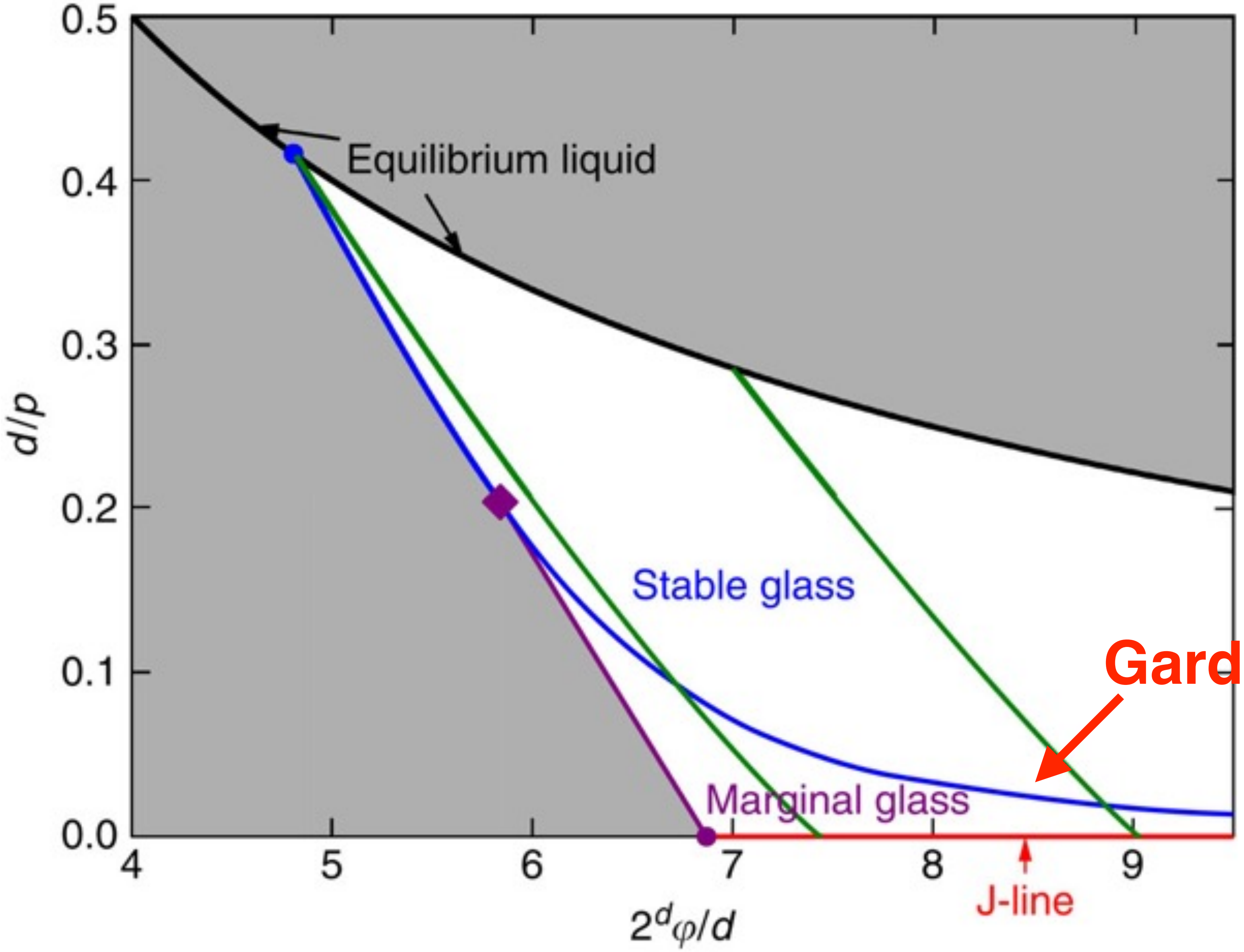
Failure:

- The predicted critical jamming exponents are inconsistent with simulations.

P. Charbonneau, E. I. Corwin, G. Parisi, and F. Zamponi, PRL (2012)

Full-step replica symmetry breaking (fullRSB) theory

Compression



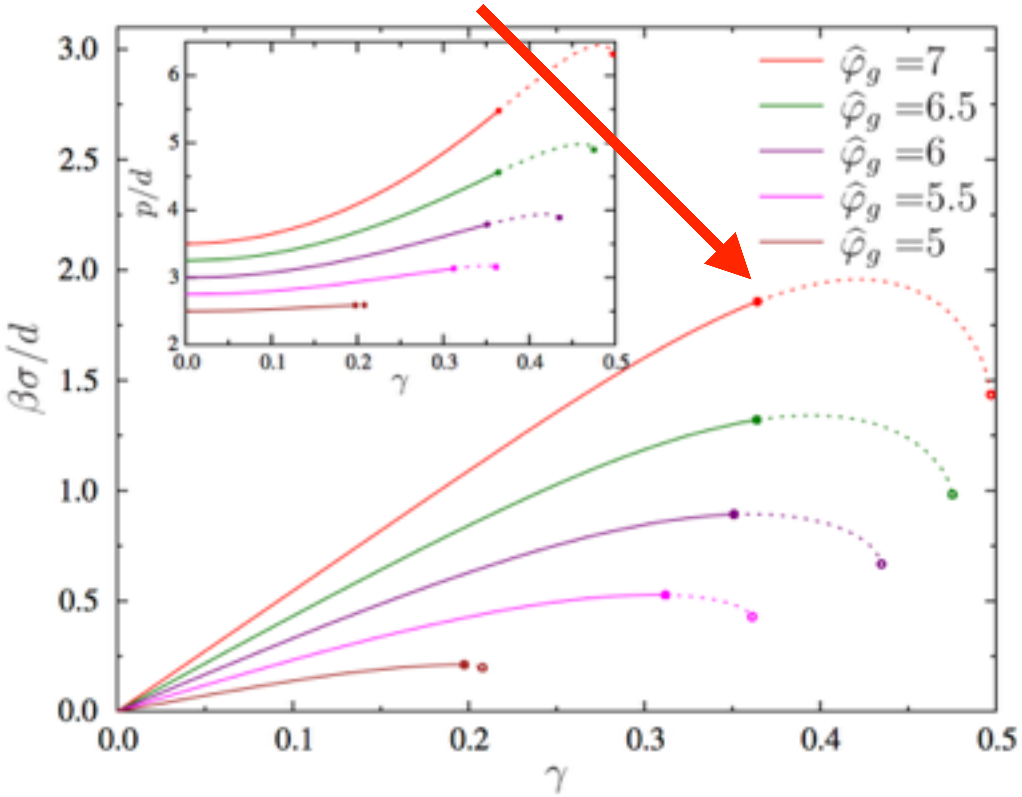
P. Charbonneau, J. Kurchan, G. Parisi, P. Urbani, and F. Zamponi, Nat. Comm. (2014)

Predict correct critical exponents at jamming.

P. Charbonneau, E. I. Corwin, G. Parisi, F. Zamponi, PRL (2015)

shear

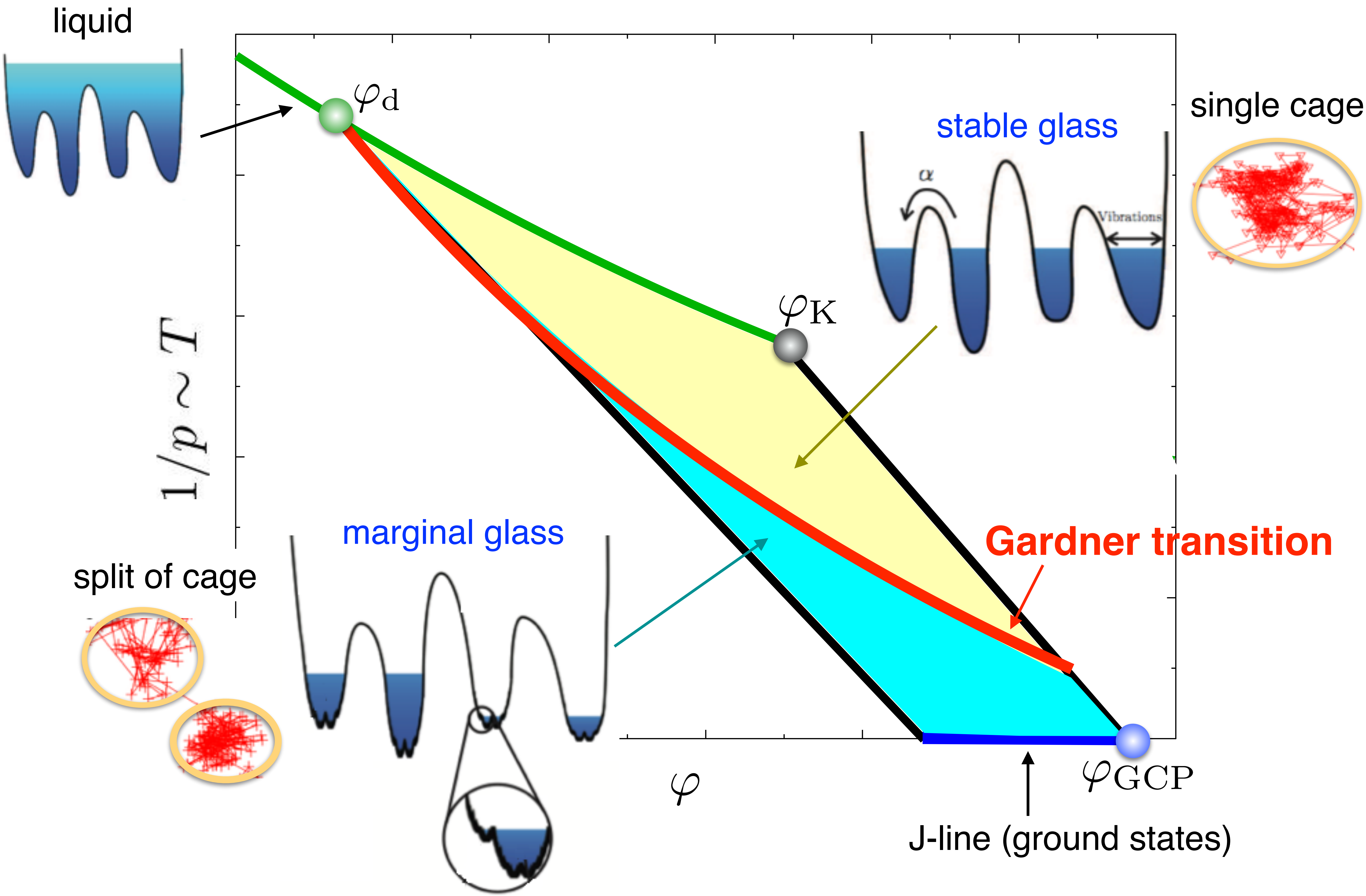
Gardner transition



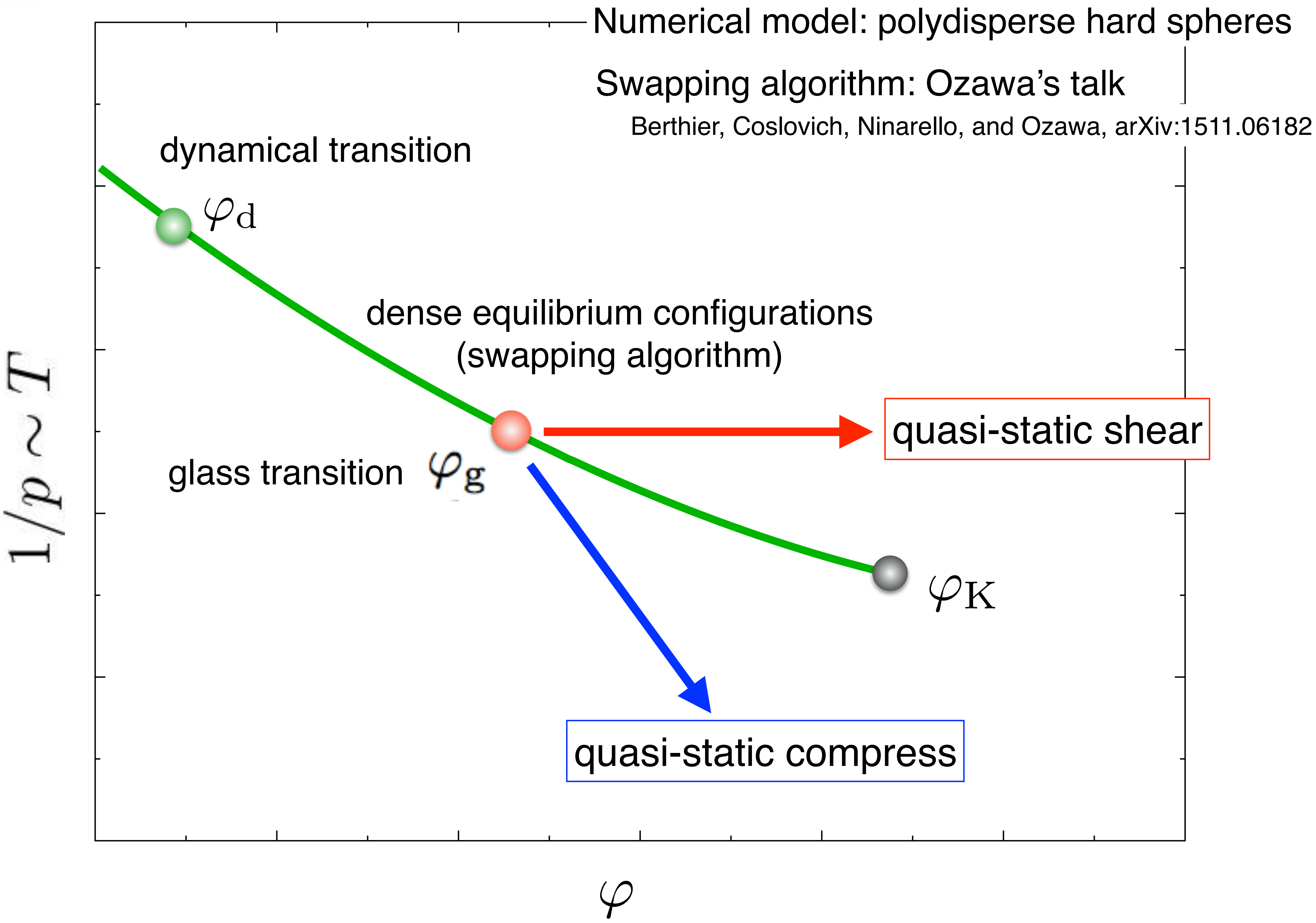
State following calculation: Yoshino's talk

C. Rainone, P. Urbani, H. Yoshino, and F. Zamponi, PRL (2015)

Basic physical picture



Numerical protocols: following the evolution of hard sphere (HS) glasses under compression and shear



Numerical model: polydisperse hard spheres

Swapping algorithm: Ozawa's talk

Berthier, Coslovich, Ninarello, and Ozawa, arXiv:1511.06182

dynamical transition

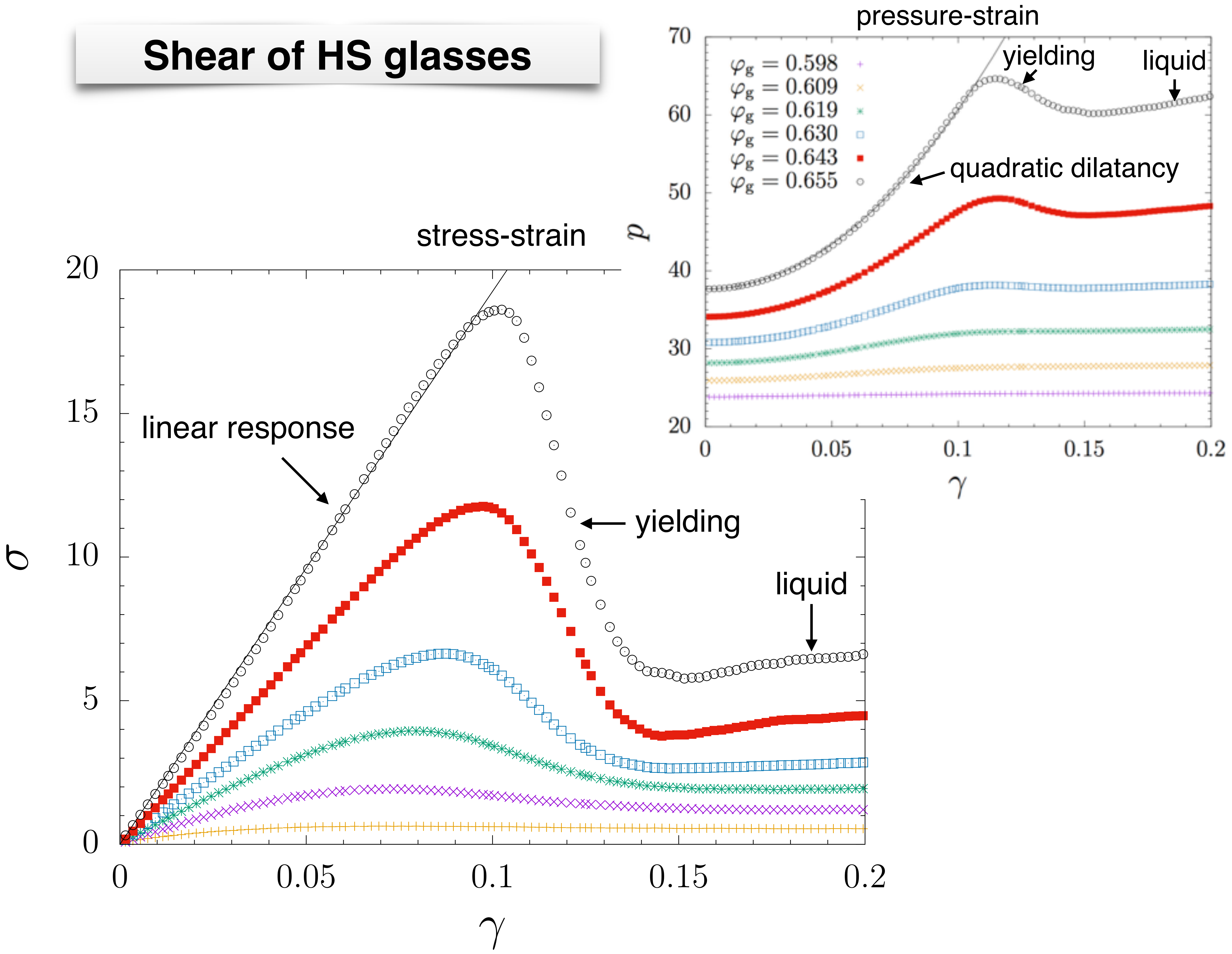
dense equilibrium configurations (swapping algorithm)

quasi-static shear

glass transition

quasi-static compress

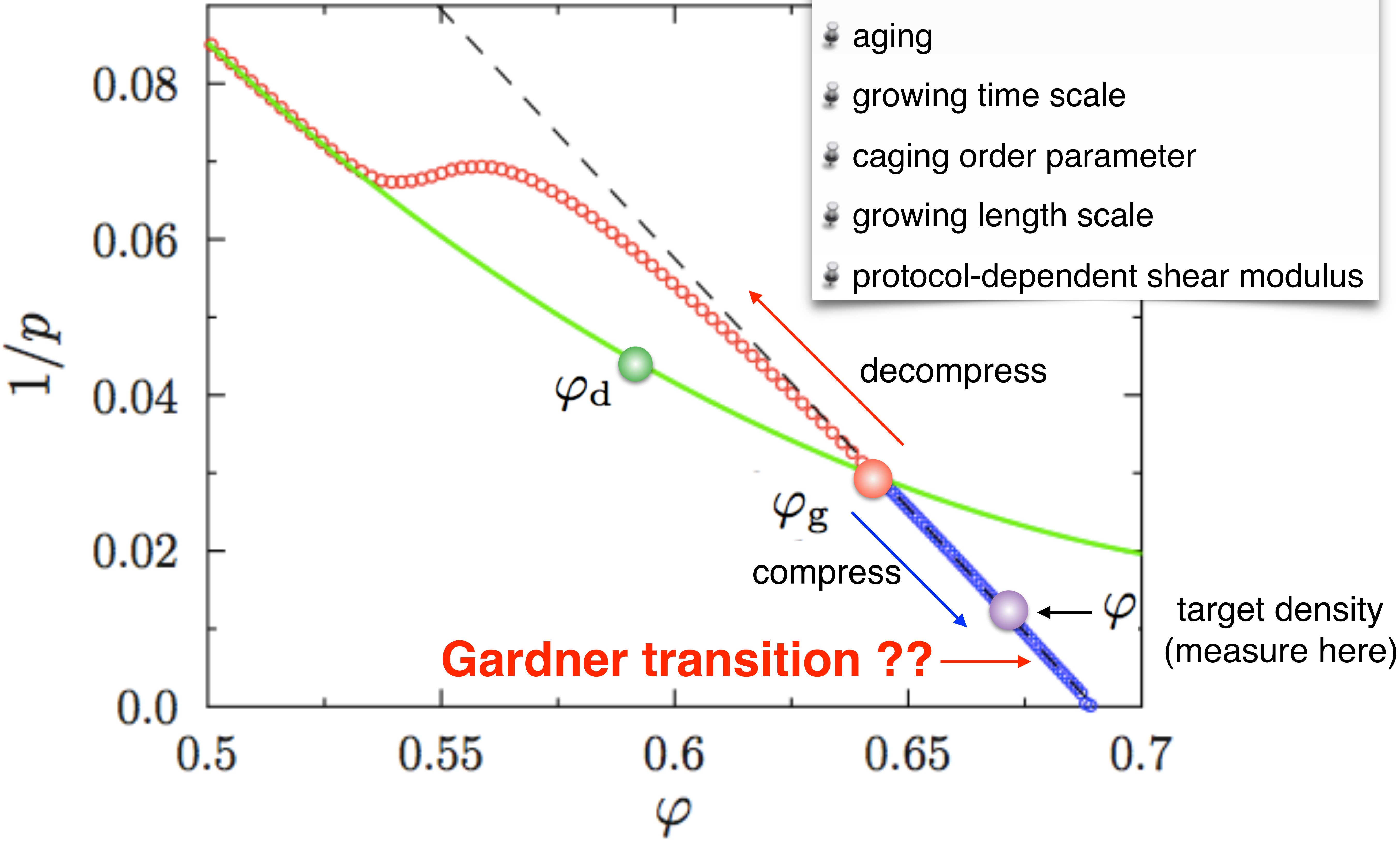
Shear of HS glasses



Compression of HS glasses

Gardner transition

- aging
- growing time scale
- caging order parameter
- growing length scale
- protocol-dependent shear modulus



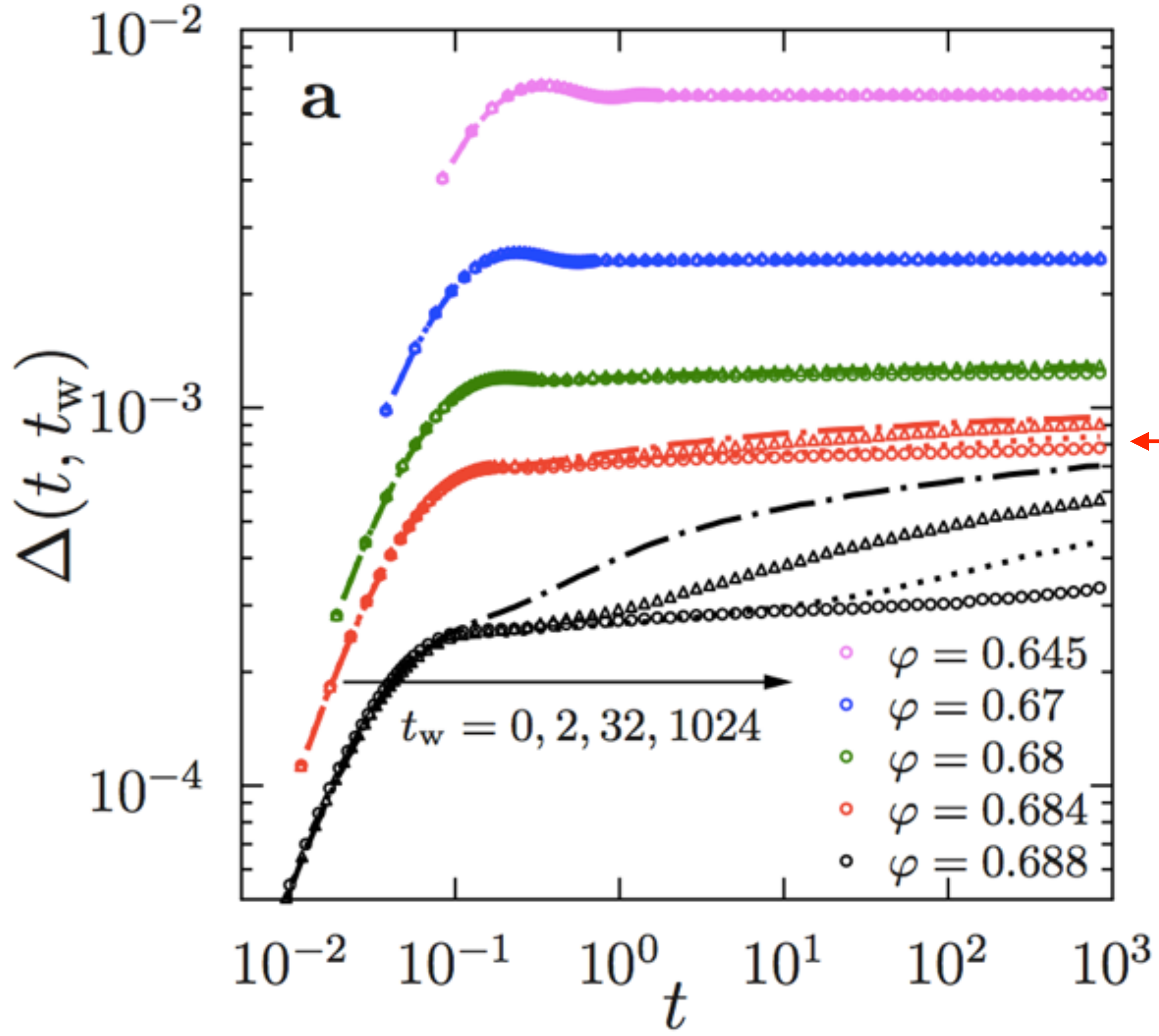
Gardner transition ??

target density (measure here)

Aging effect

Mean-squared displacement (MSD): $\Delta(t, t_{\mathrm{w}}) = \frac{1}{N}\sum_{i=1}^{N}\langle|\boldsymbol{r}_i(t+t_{\mathrm{w}}) - \boldsymbol{r}_i(t_{\mathrm{w}})|^2\rangle$

waiting time



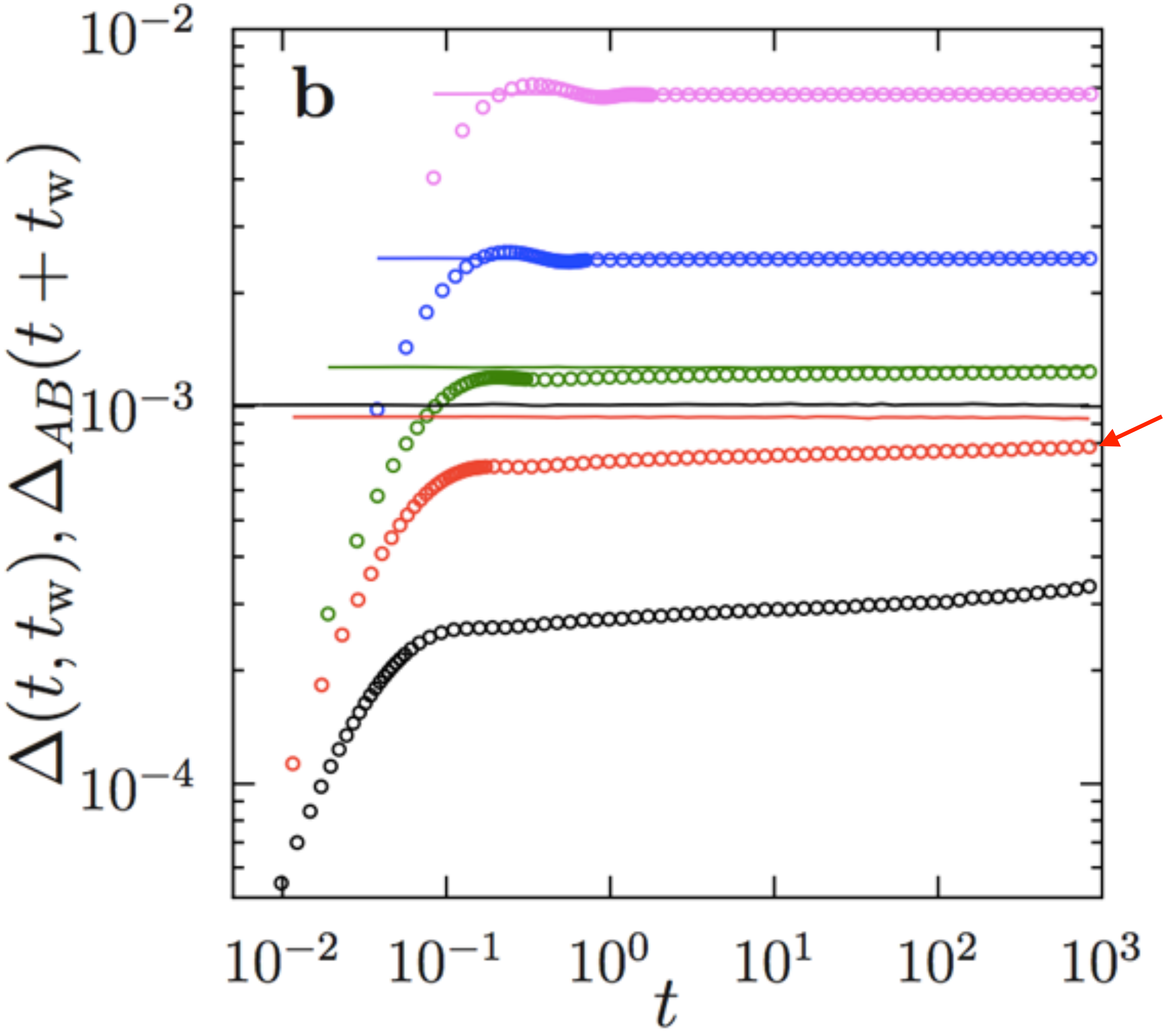
Gardner transition

Growing time scale

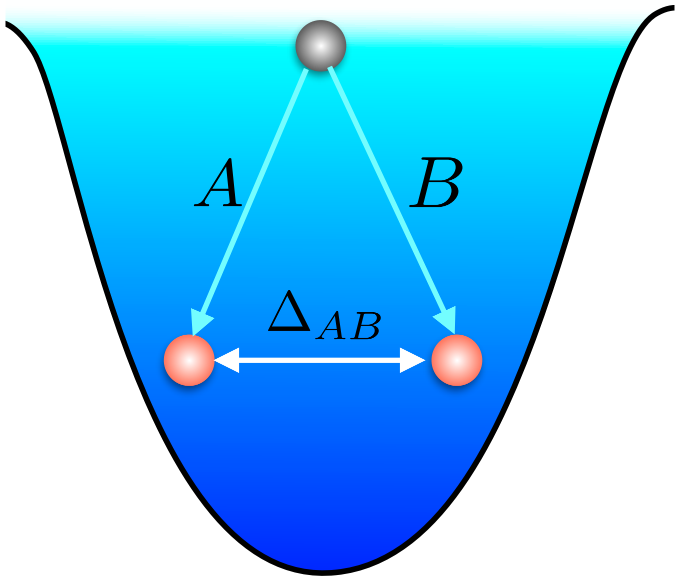
State cloning

$$\Delta_{AB}(t) = \frac{1}{N}\sum_{i=1}^{N}\langle|\boldsymbol{r}_i^A(t) - \boldsymbol{r}_i^B(t)|^2\rangle$$

$\Delta_{AB}(t)$ (lines) $\Delta(t, t_{\mathrm{w}})$ (points)



Gardner transition



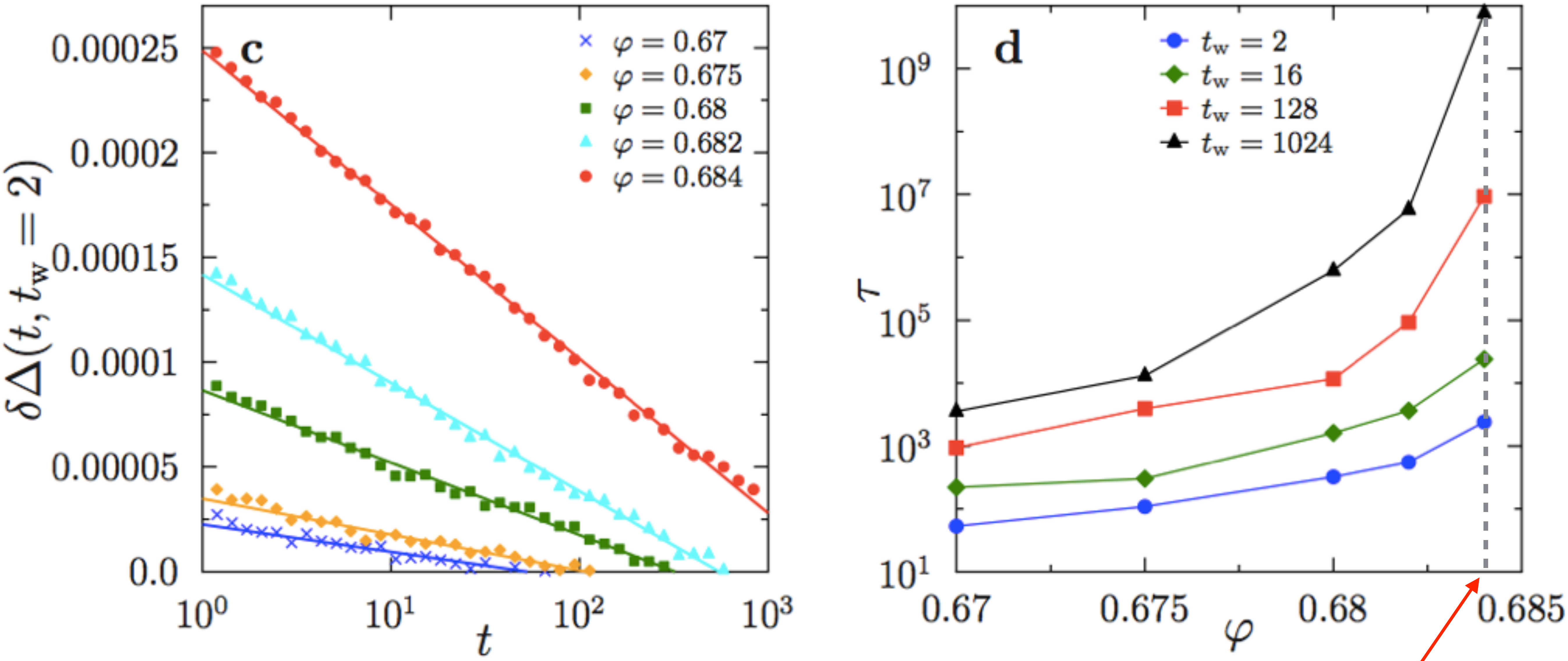
Δ_{AB} is nearly time independent

$$\Delta_{AB} \simeq \Delta(t \to \infty, t_{\mathrm{w}} \to \infty)$$

caging order parameter
(average cage size)

Growing time scale

$$\delta\Delta(t, t_{\mathrm{w}}) = \Delta_{AB}(t_{\mathrm{w}} + t) - \Delta(t, t_{\mathrm{w}})$$



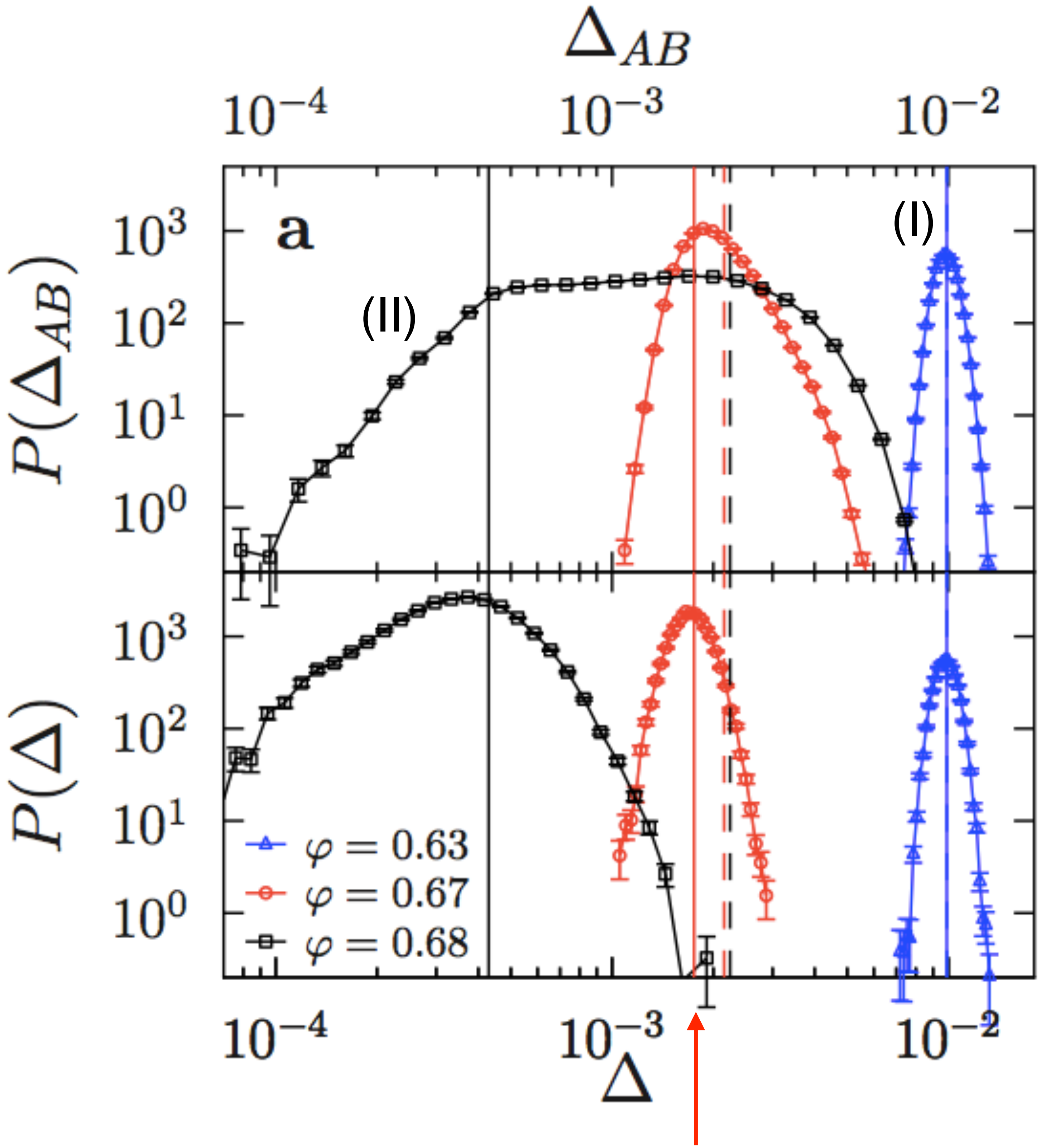
Gardner transition

$$\delta\Delta(t, t_{\mathrm{w}}) \sim 1 - \ln t / \ln \tau$$

relaxation timescale

Caging order parameter

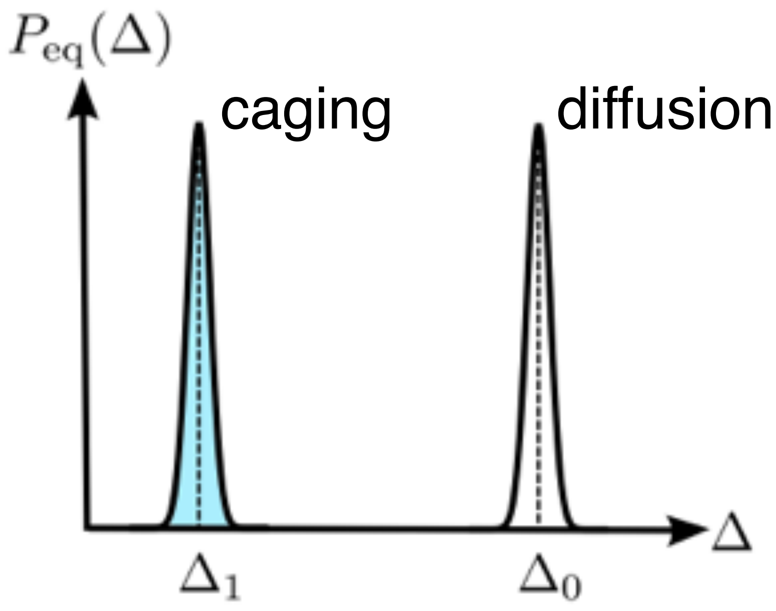
distributions



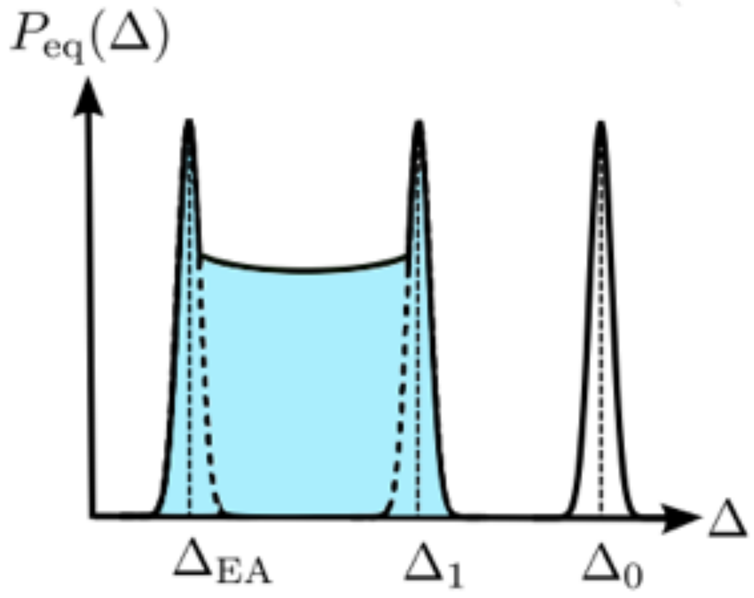
Gardner transition

theoretical prediction (qualitative)

(I) stable (1RSB) glass phase

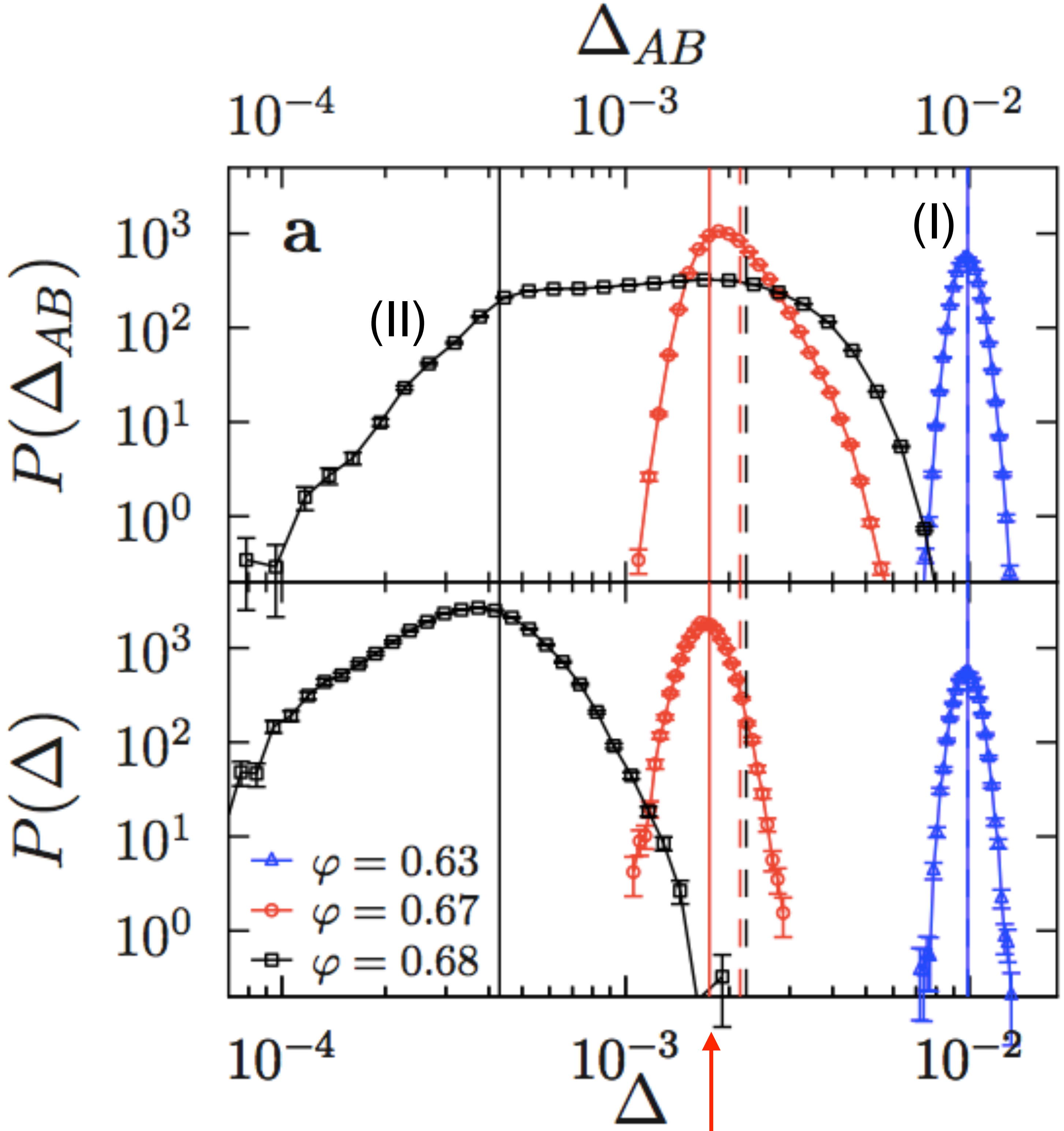


(II) marginal (fullRSB) glass phase



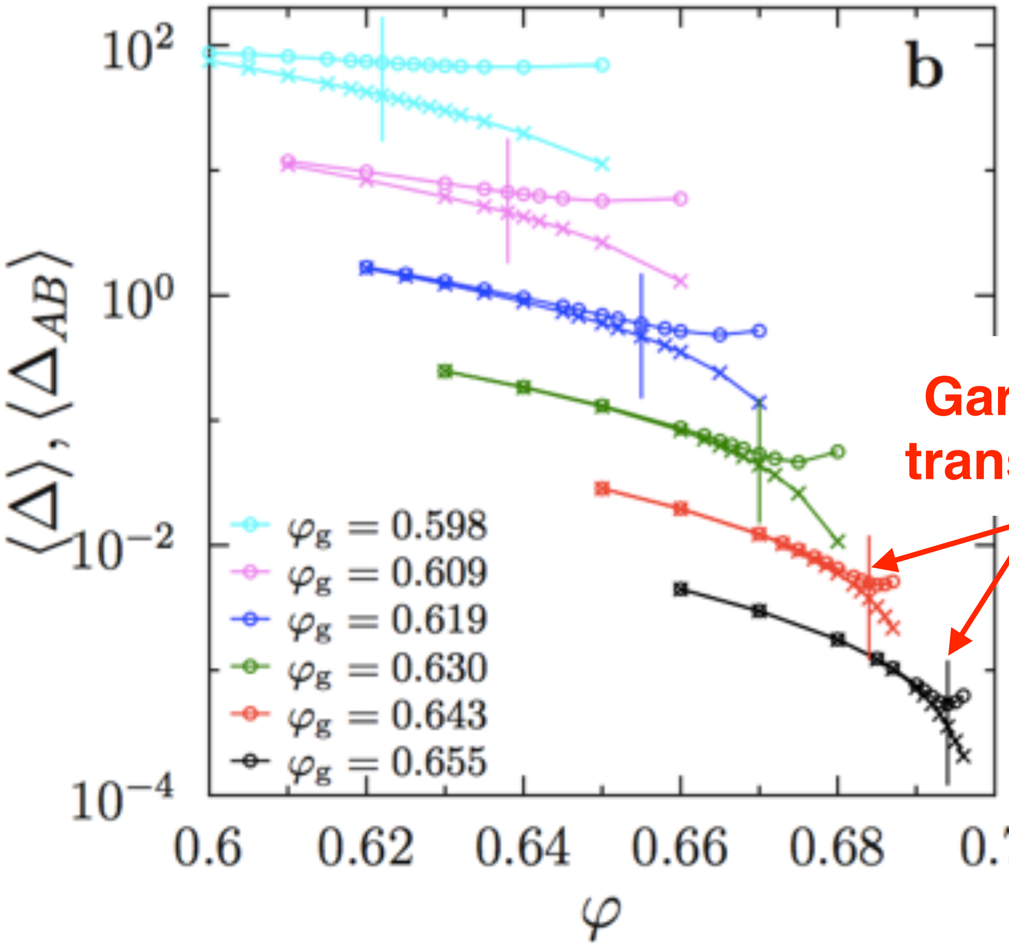
Caging order parameter

distributions

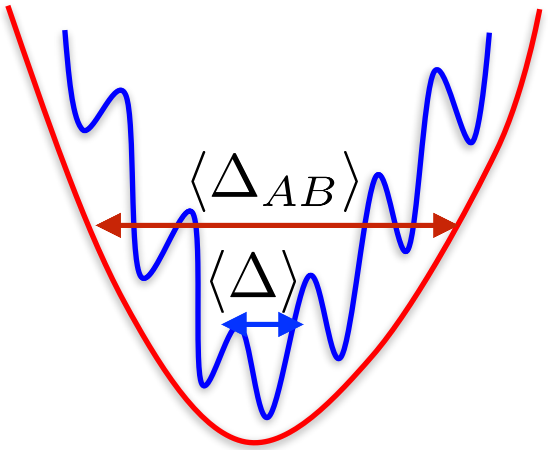


Gardner transition

mean cage size
(first moment)



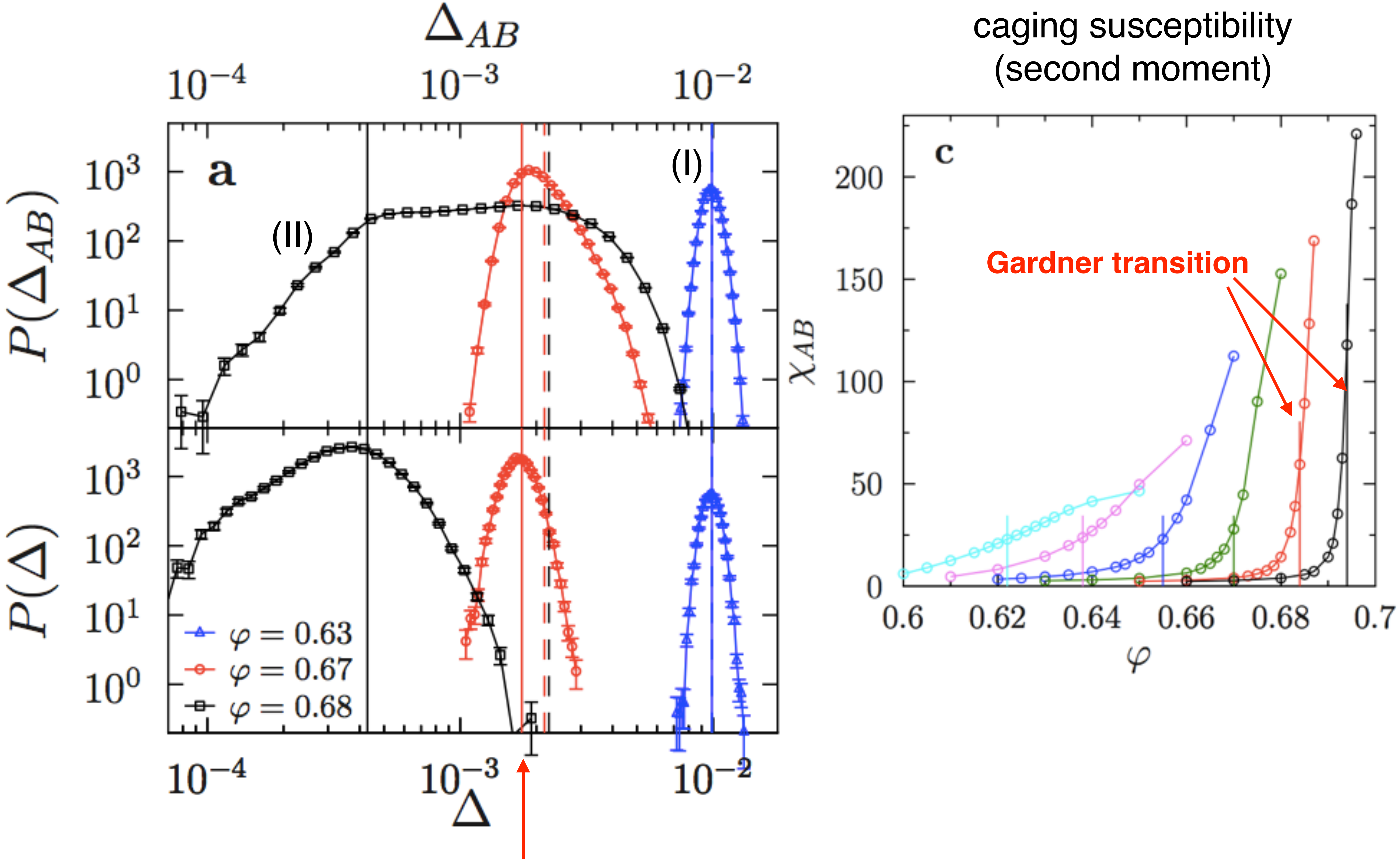
Gardner transition



Caging order parameter

distributions

caging susceptibility
(second moment)

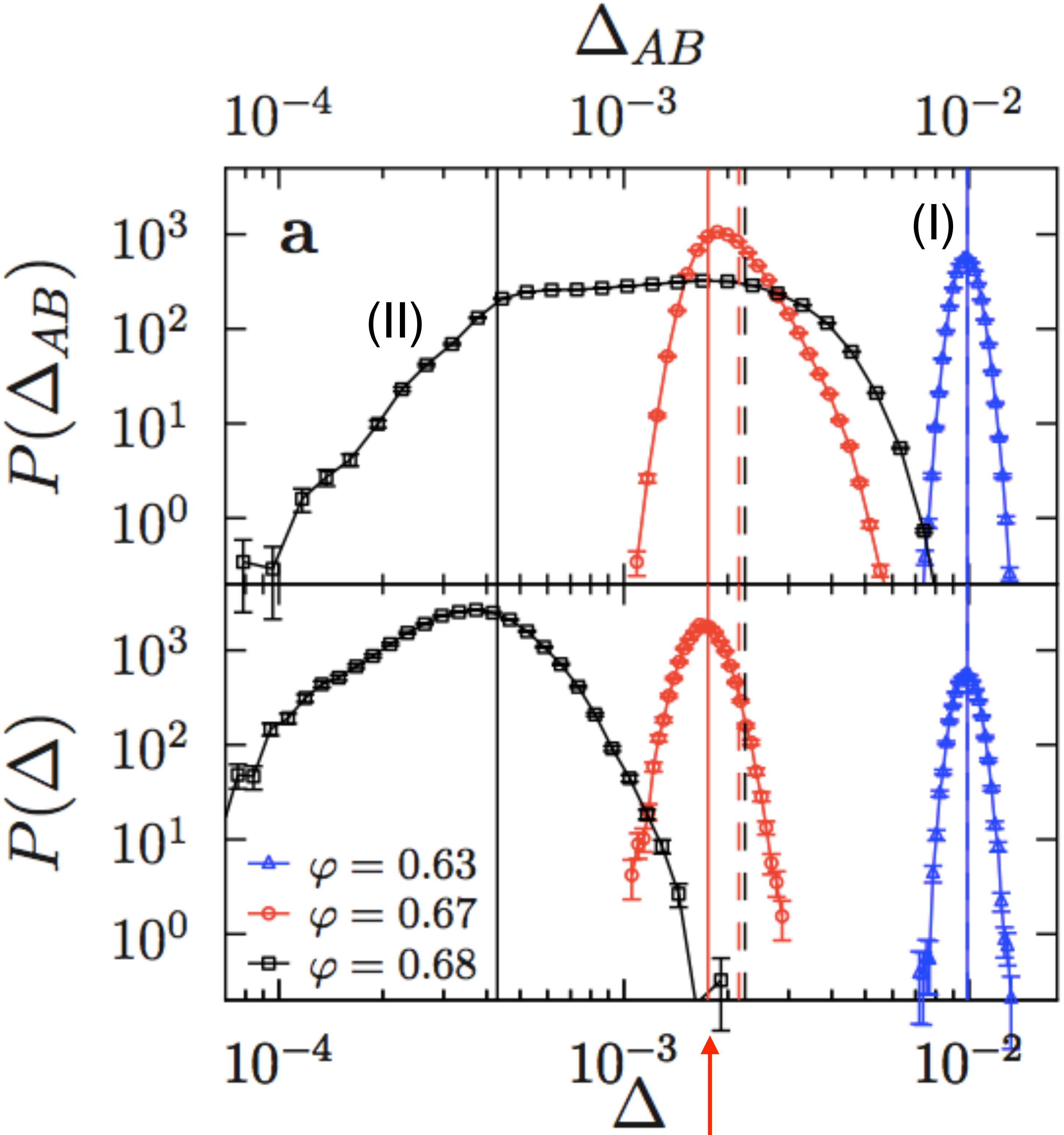


Gardner transition

Gardner transition

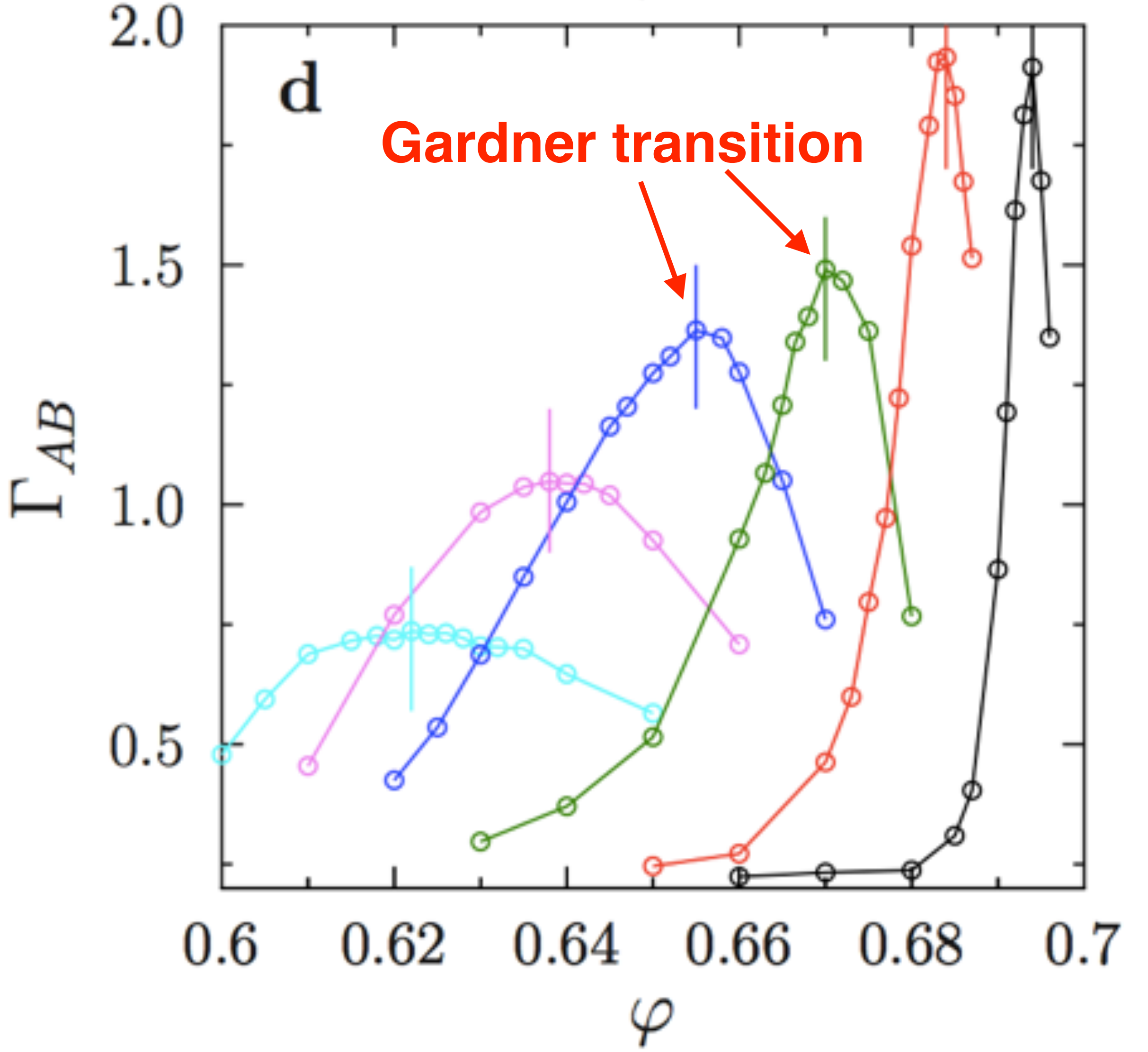
Caging order parameter

distributions



Gardner transition

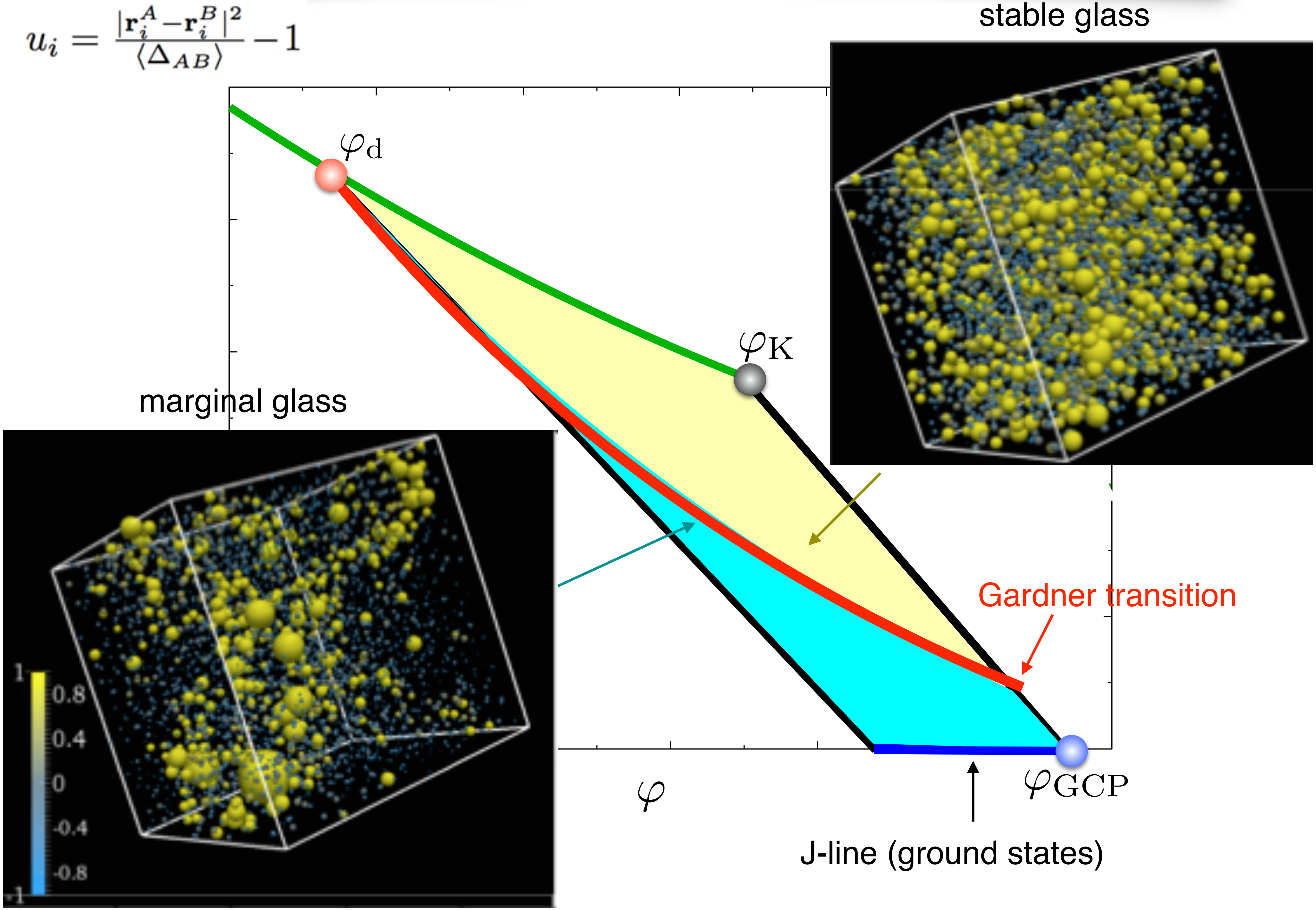
caging skewness
(third moment)



Gardner transition

Spatial organization of cages: visualization of cage fields

$$u_i = \frac{|\mathbf{r}_i^A - \mathbf{r}_i^B|^2}{\langle \Delta_{AB} \rangle} - 1$$



Growing correlation lengths

$$u_i = \frac{|\mathbf{r}_i^A - \mathbf{r}_i^B|^2}{\langle \Delta_{AB} \rangle} - 1$$

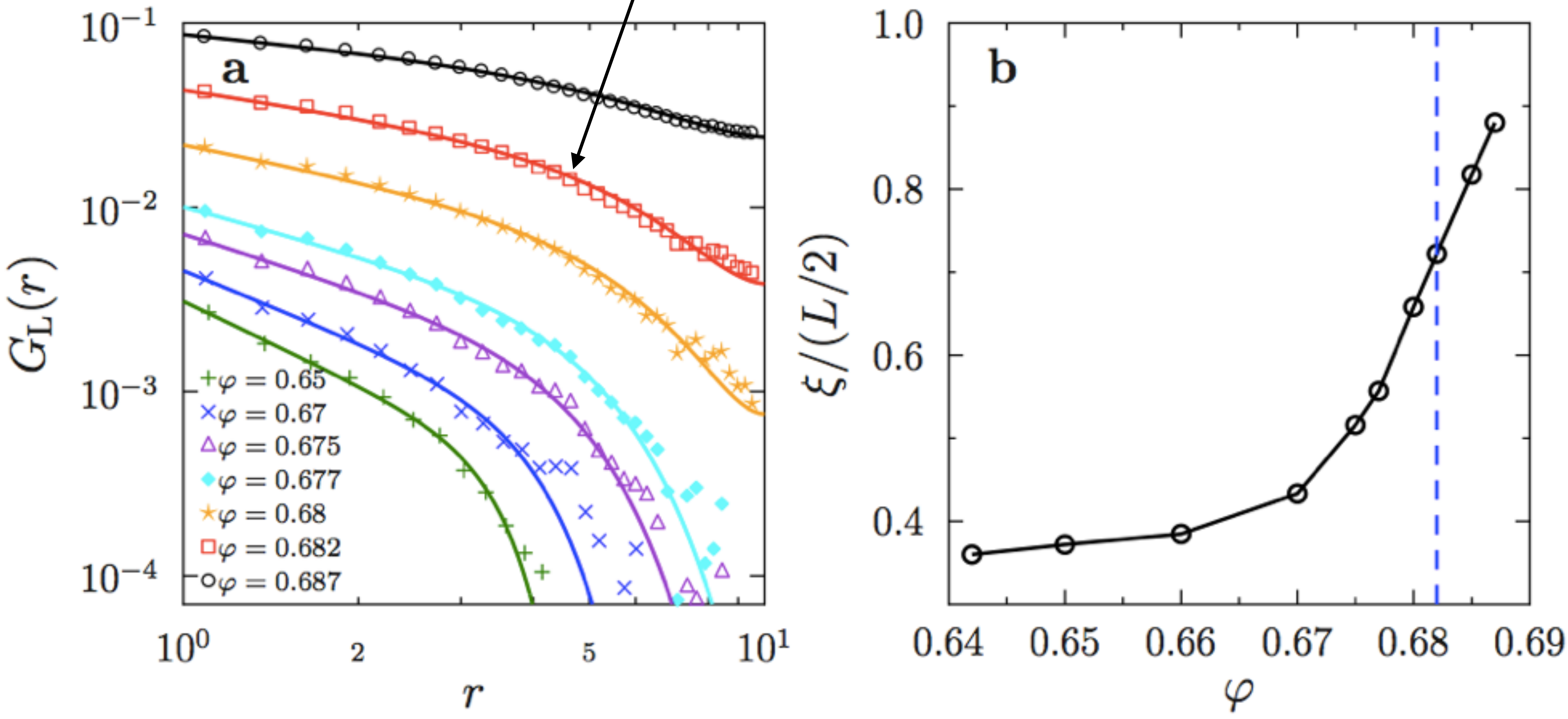
sizes of cages

positions of cages

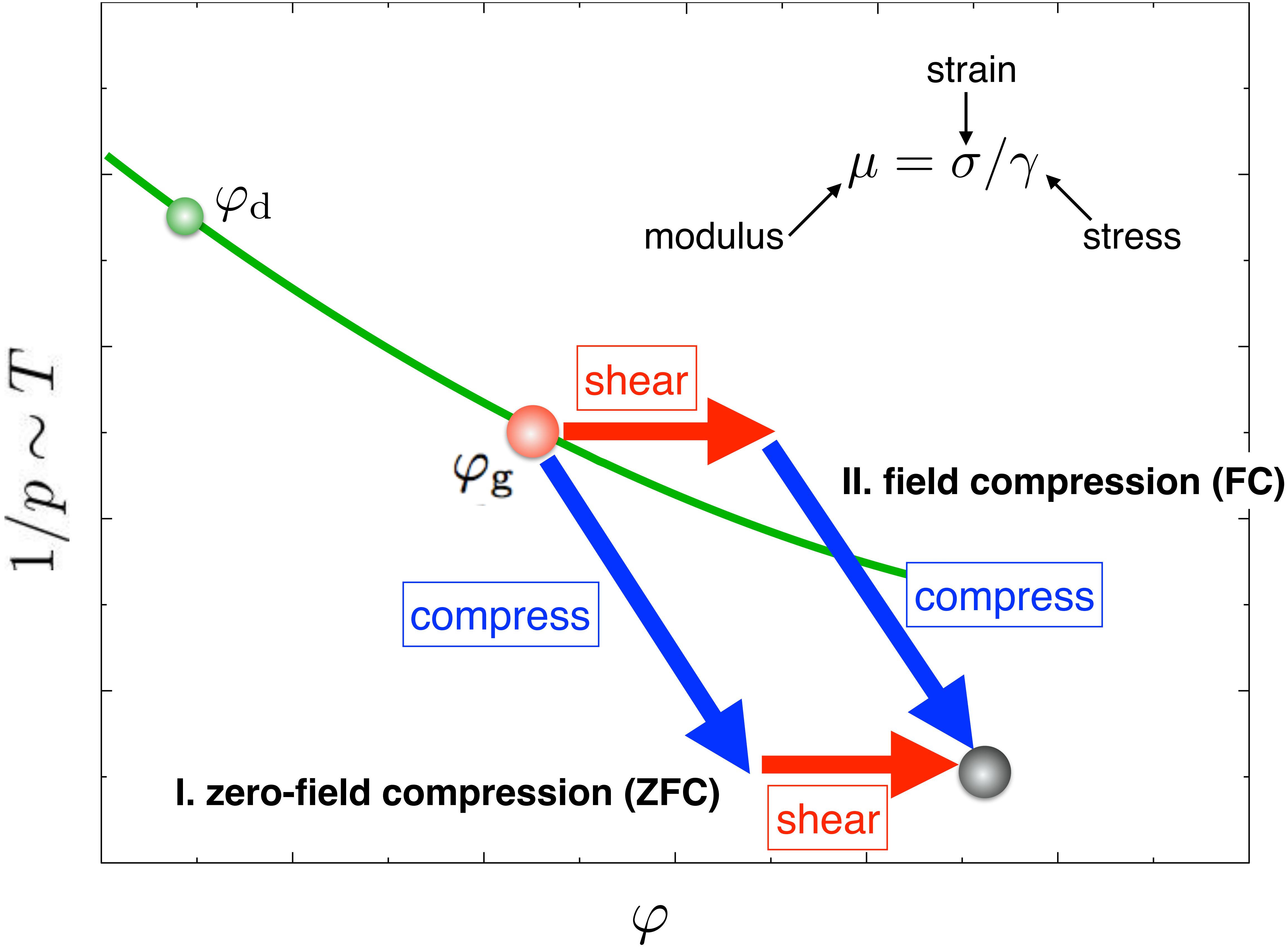
$$G_{\mathrm{L}}(r) \sim \langle \textstyle\sum_{\mu=1}^{3} \sum_{i \neq j} u_i u_j \delta(r - |\mathbf{r}_{i,\mu}^A - \mathbf{r}_{j,\mu}^A|) \rangle$$

$$G_{\mathrm{L}}(r) \sim \frac{1}{r^a} e^{-\left(\frac{r}{\xi}\right)^b}$$

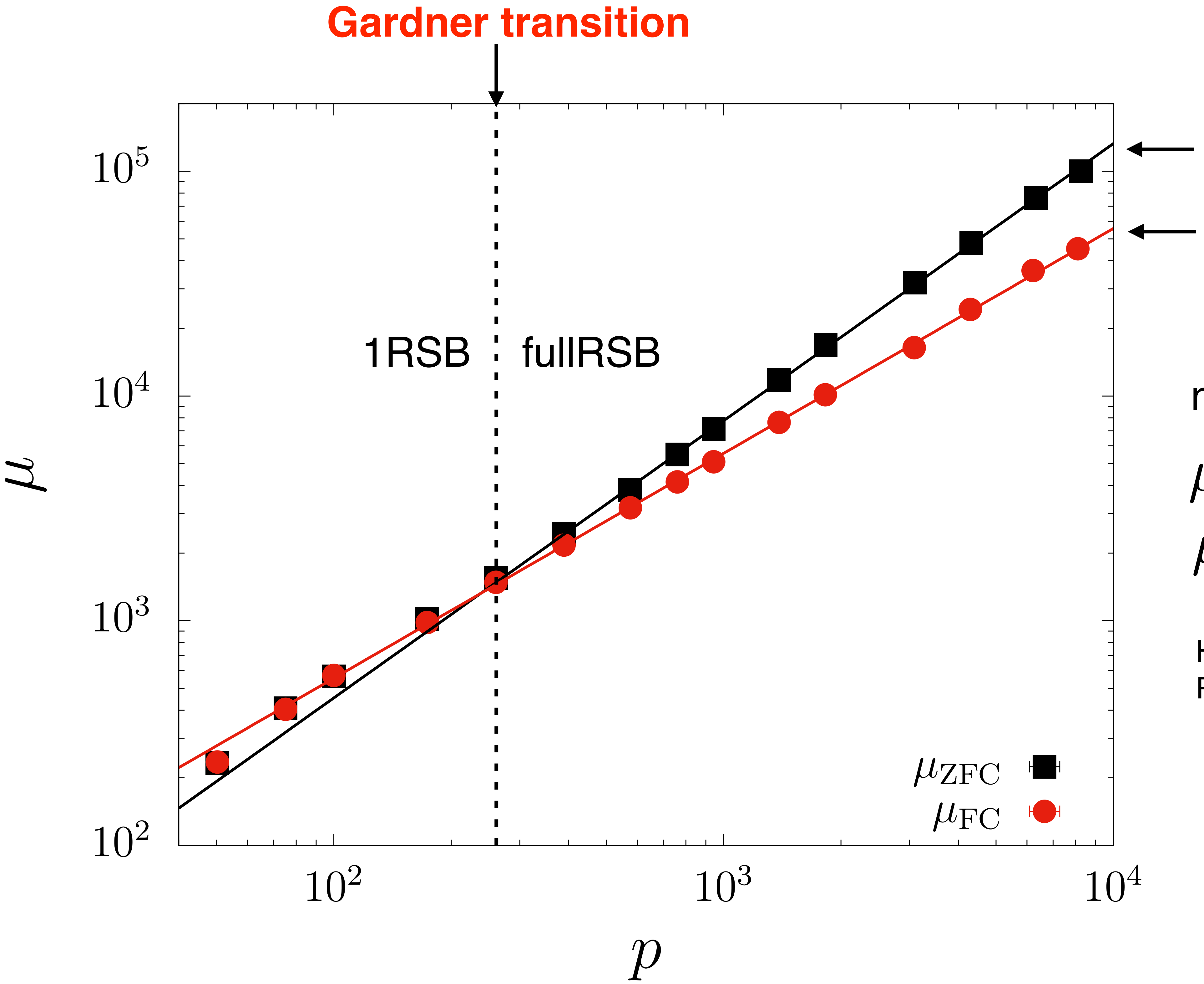
correlation length



Protocol-dependent shear modulus



Protocol-dependent shear modulus



$\mu_{\rm ZFC} \sim p^{1.234}$

$\mu_{\rm FC} \sim p^{1.001}$

mean-field theory:

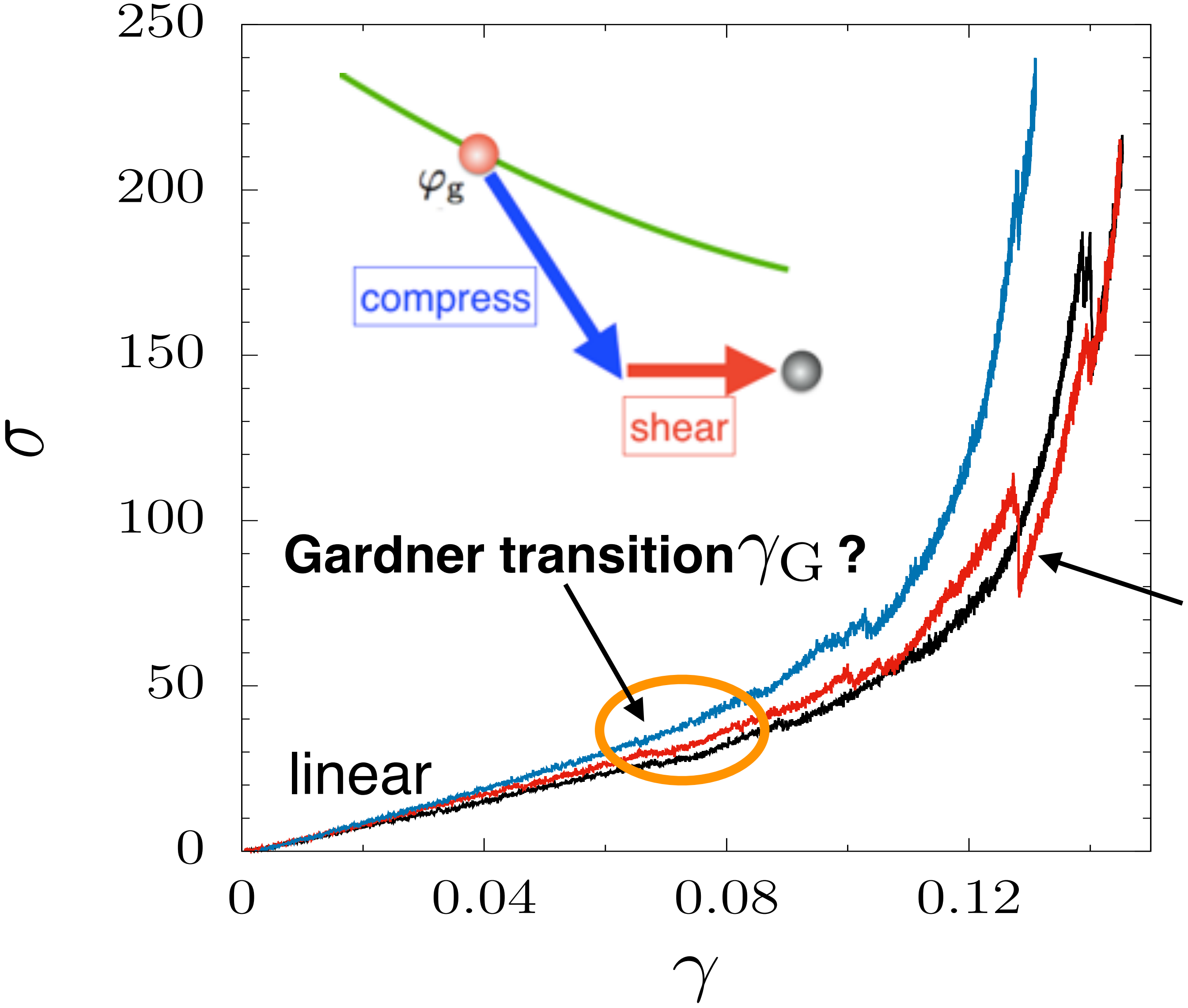
$\mu_{\rm ZFC} \sim p^{1.41574}$

$\mu_{\rm FC} \sim p$

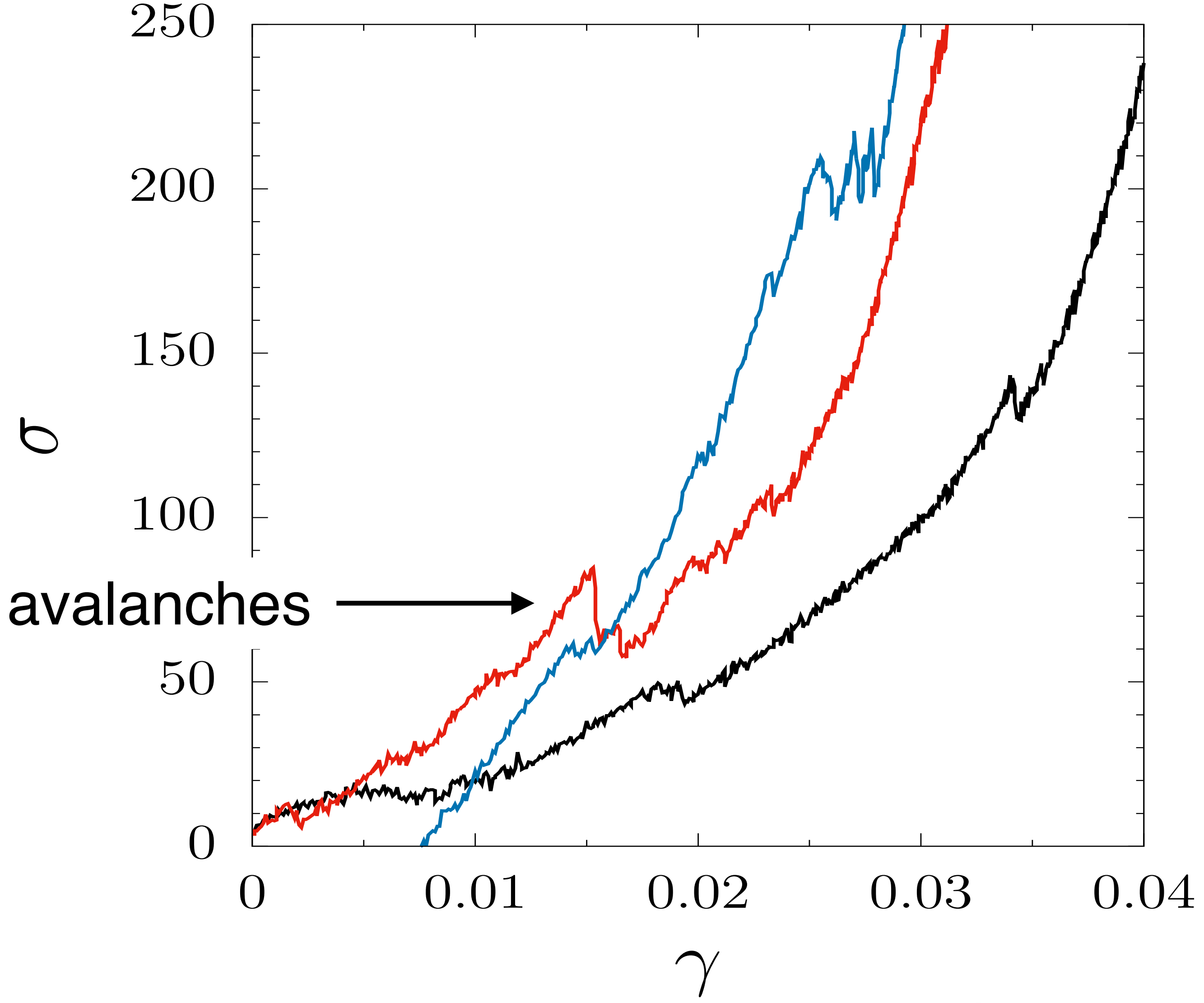
H. Yoshino and F. Zamponi,
PRE (2014)

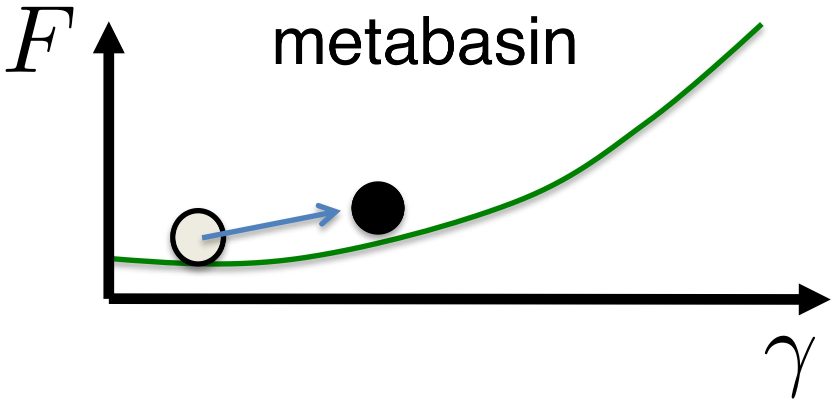
Compression followed by shear

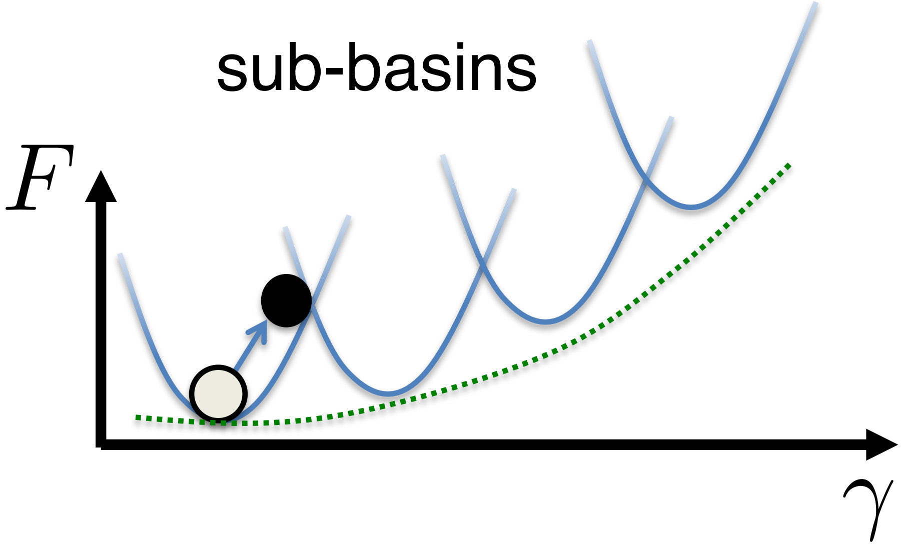
1RSB phase $\varphi < \varphi_G$



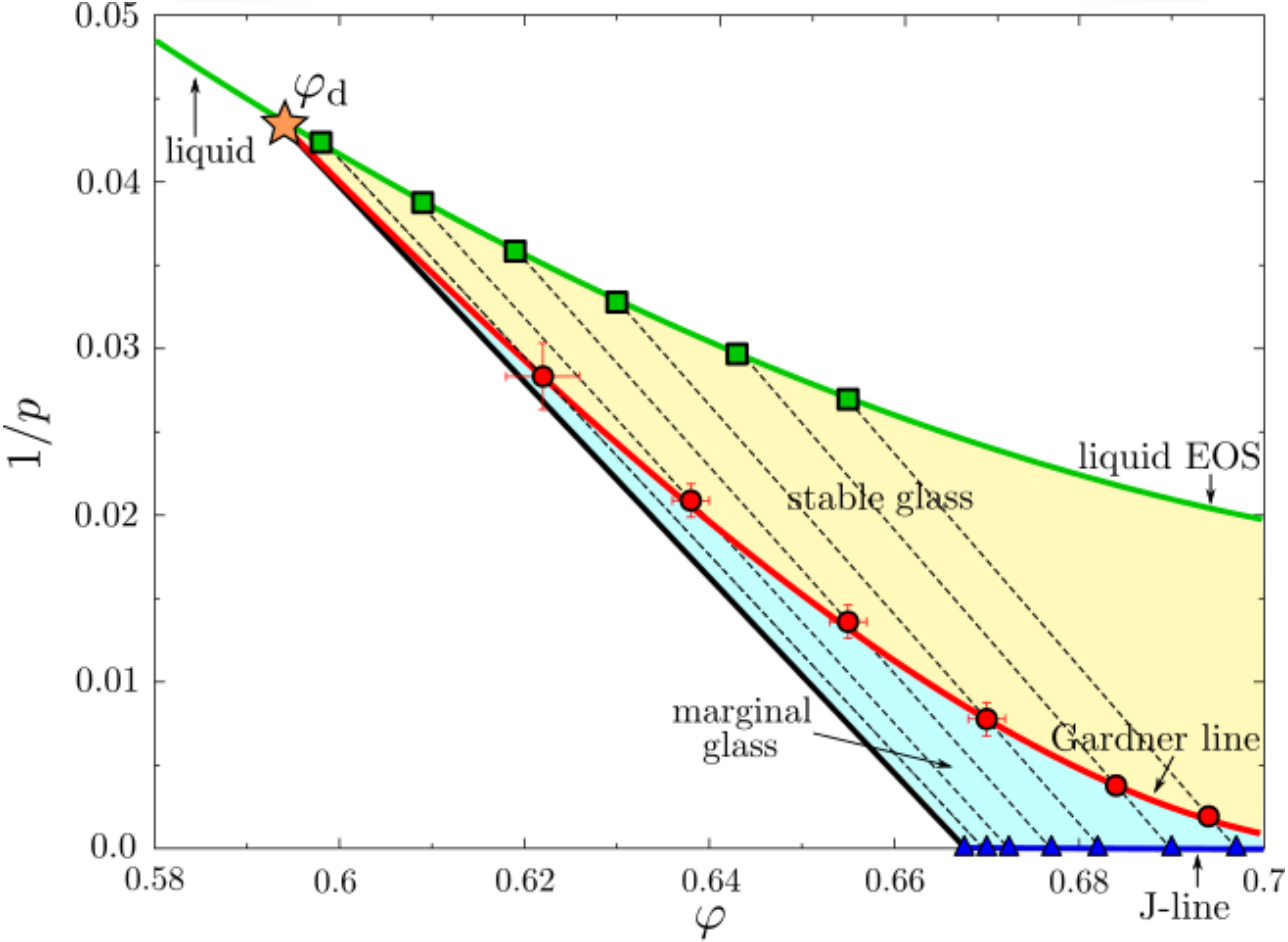
fullRSB phase $\varphi > \varphi_G$







Summary: phase diagram of HS glasses



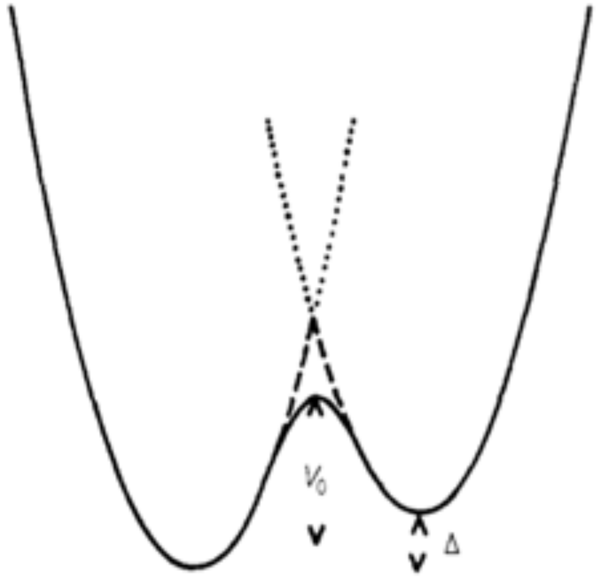
Berthier, Charbonneau, Jin, Parisi, Seoane, and Zamponi, arXiv:1511.04201
P. Charbonneau, Y. Jin, G. Parisi, C. Rainone, B. Seoane, and F. Zamponi, PRE (2015)

Experimental consequences (I)

- Anomalous transport properties: anomalous specific heat and thermal conductivity (tunneling two-level systems model).

W. A. Phillips, Rep. Prog. Phys (1987)

	glass	crystal (Debye theory)
specific heat:	$C_p \sim T$	$C_{\text{Debye}} \sim T^3$
thermal conductivity:	$\kappa \sim T^2$	$\kappa_{\text{Debye}} \sim T^3$



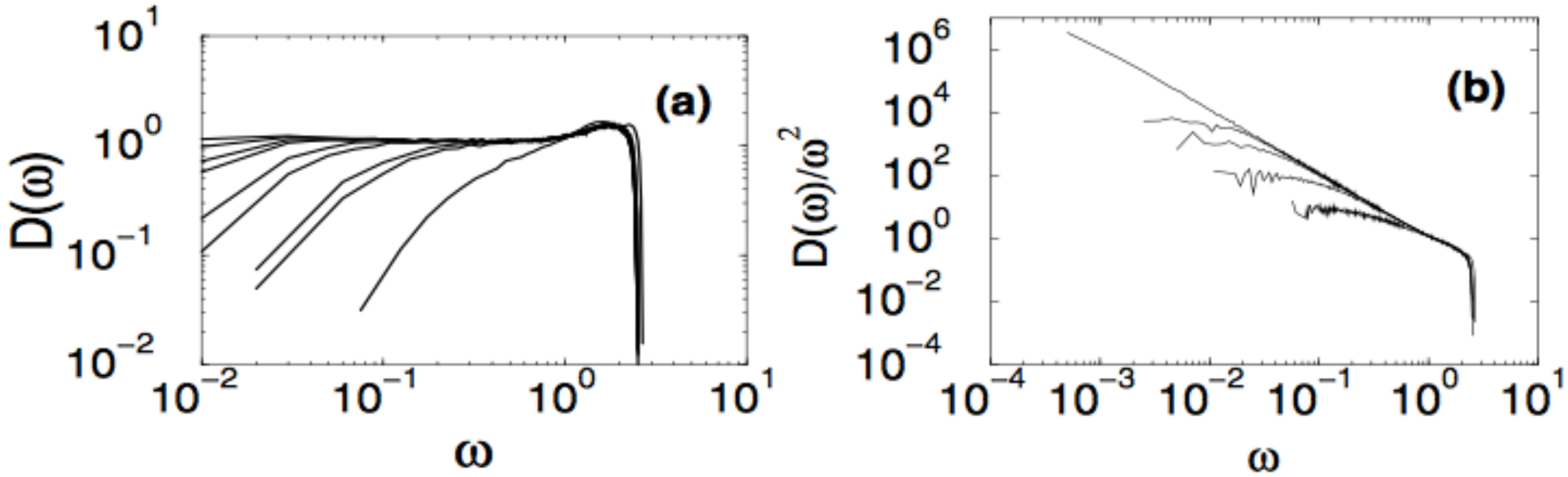
- Activated slow dynamics across barriers (Johari-Goldstein relaxation).

M. Goldstein, J. Chem. Phys (2010)

Experimental consequences (II)

- An abundance of soft vibration modes (Boson peak): isostaticity and marginal stability.

V. K. Malinovsky and A. P. Sokolov, Sol. St. Comm (1986)



Silbert, Liu, and Nagel, PRL (2005)

Debye density of vibrational modes

$$\mathcal{D}(\omega) \propto \omega^{D-1}$$

- Complex irreversible responses to small mechanical deformations.

Markus Muller and Matthieu Wyart, Annu. Rev. Condens. Matter Phys. (2015)

Conclusions

Four independent ways to detect the Gardner transition:

- The growth of the characteristic relaxation time.
- The growth of the correlation length.
- A non-trivial change in the probability distribution function of a global order parameter.
- Protocol-dependent shear modulus.

Other systems:

- Bidisperse hard disks.

  Berthier, Charbonneau, Jin, Parisi, Seoane, and Zamponi, arXiv:1511.04201

- Mari-Kurchan model (mean-field hard spheres)

  P. Charbonneau, Y. Jin, G. Parisi, C. Rainone, B. Seoane, and F. Zamponi, PRE (2015)

Thank you!