YITP Workshop on "Avalanches, plasticity, and nonlinear response in nonequilibrium solids", 7th-9th March 2016

Friction and Stick-Slip Motion of Solids and Granular Systems

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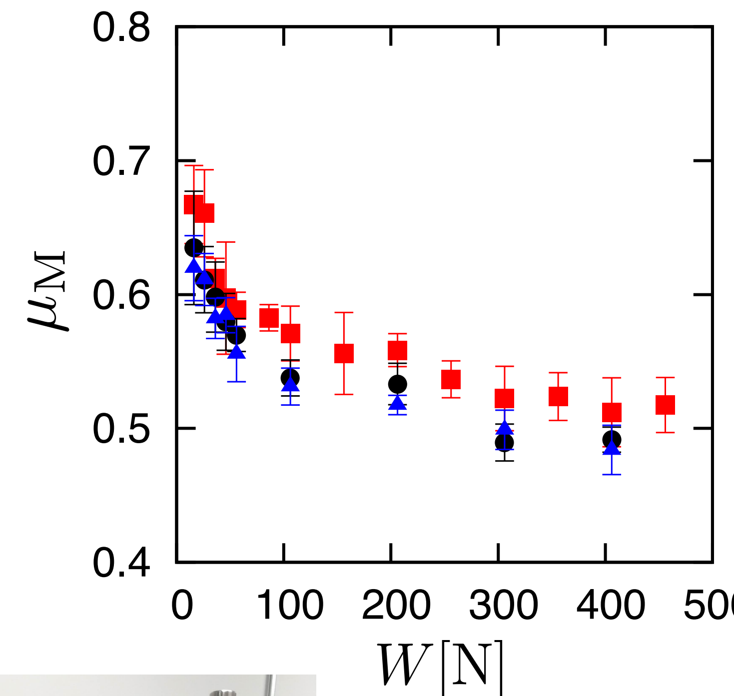
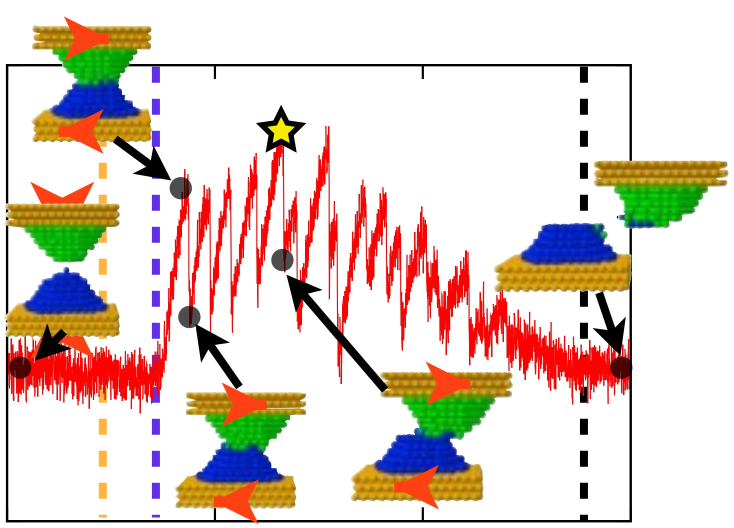
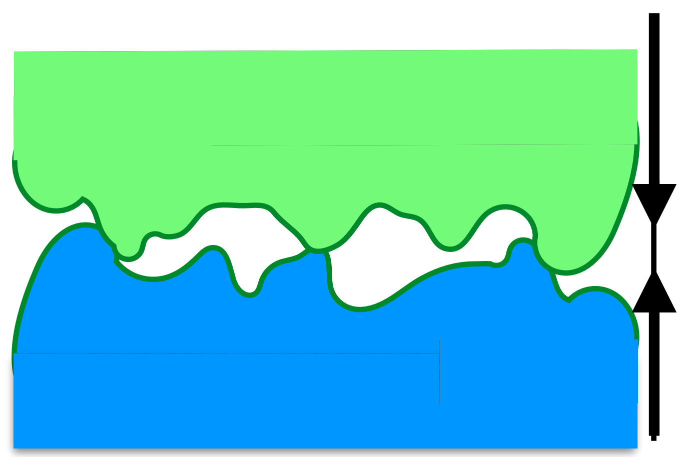
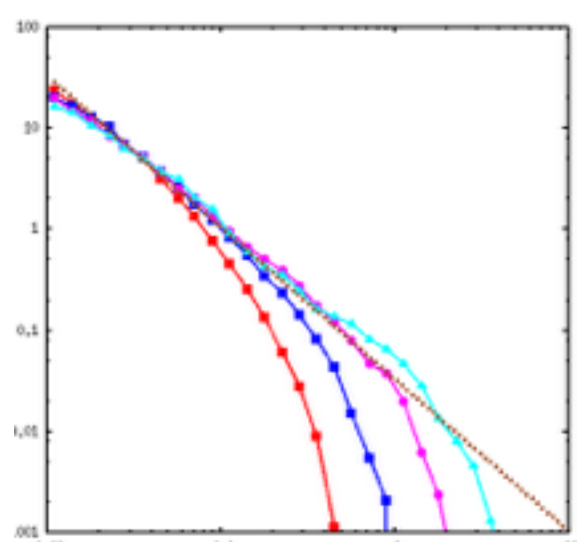
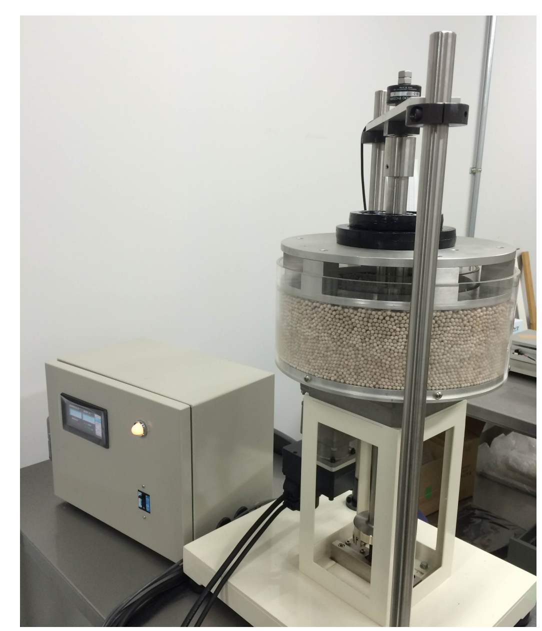
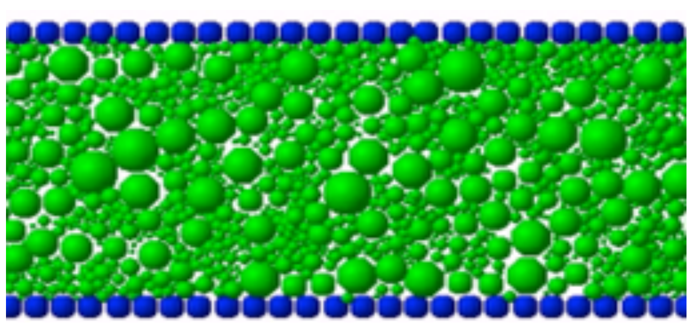












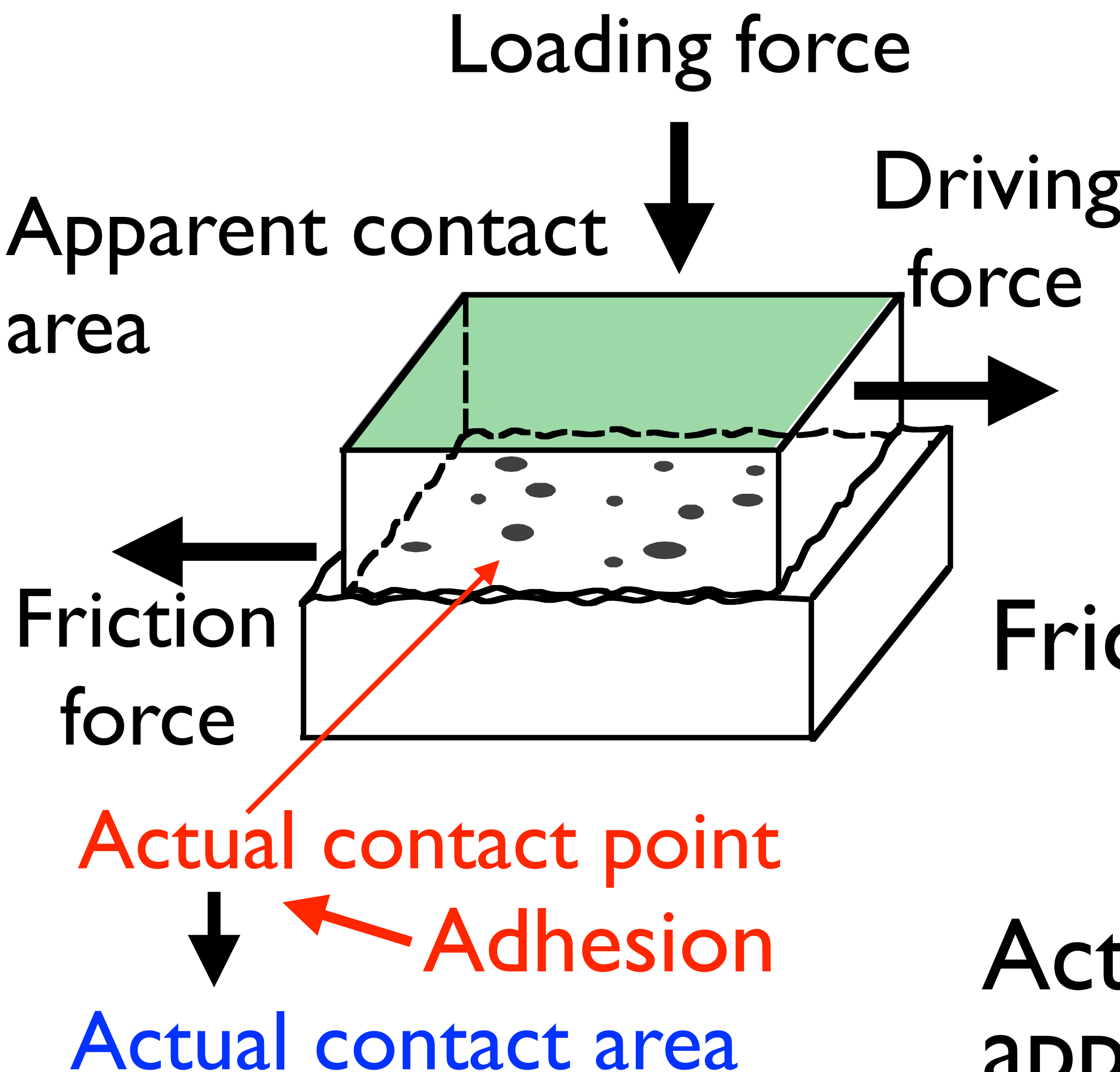
Amontons and Coulomb's Law

- Friction force is independent of apparent contact area.
- Friction force is proportional to the loading force.
- Kinetic friction force is less than maximum static friction force and independent of sliding velocity.

Friction Force/ Loading Force
=Friction Coefficient : constant

Amontons' Law

Why friction force is independent of apparent contact area and proportional to the loading force ?



Apparent contact area
≠ ← surface roughness
Actual contact area

Adhesion occurs at actual contact points. → Friction

Friction force = Actual contact area
X
Adhesion force per unit area

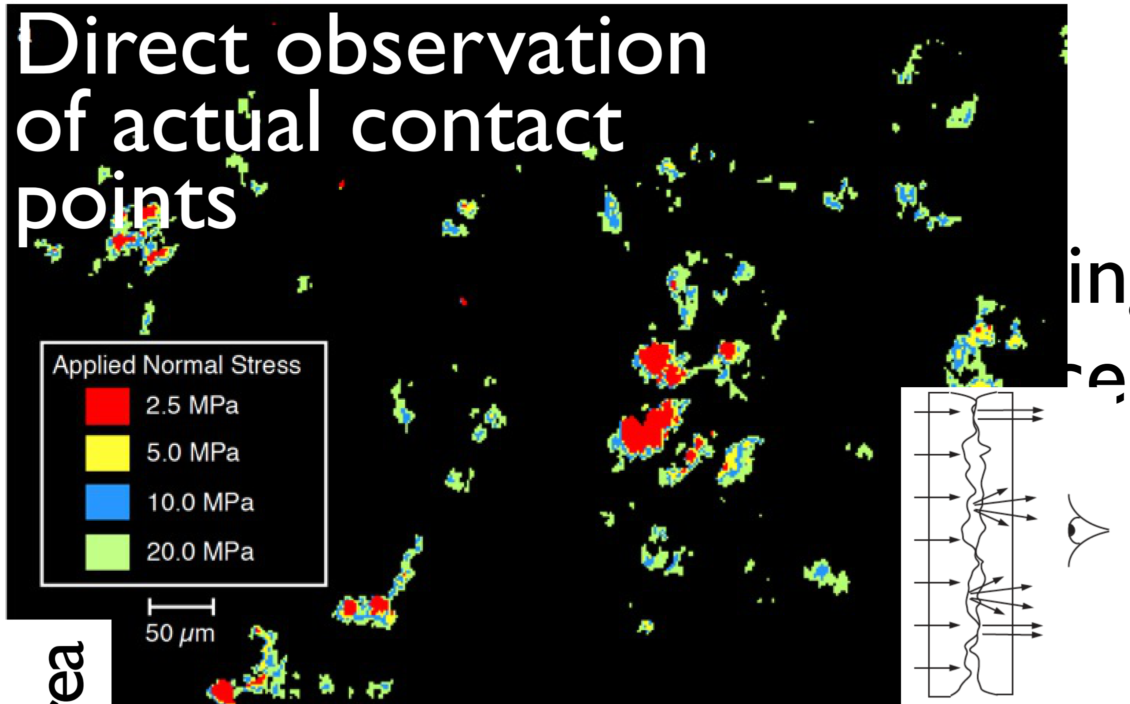
Actual contact area is independent of apparent contact area and proportional to the loading force.

↓

Friction force is independent of apparent contact area and proportional to the loading force.

Bowden & Tabor

Why friction force is independent of apparent contact area and proportional to the loading force ?



Apparent contact area
≠ ← surface roughness
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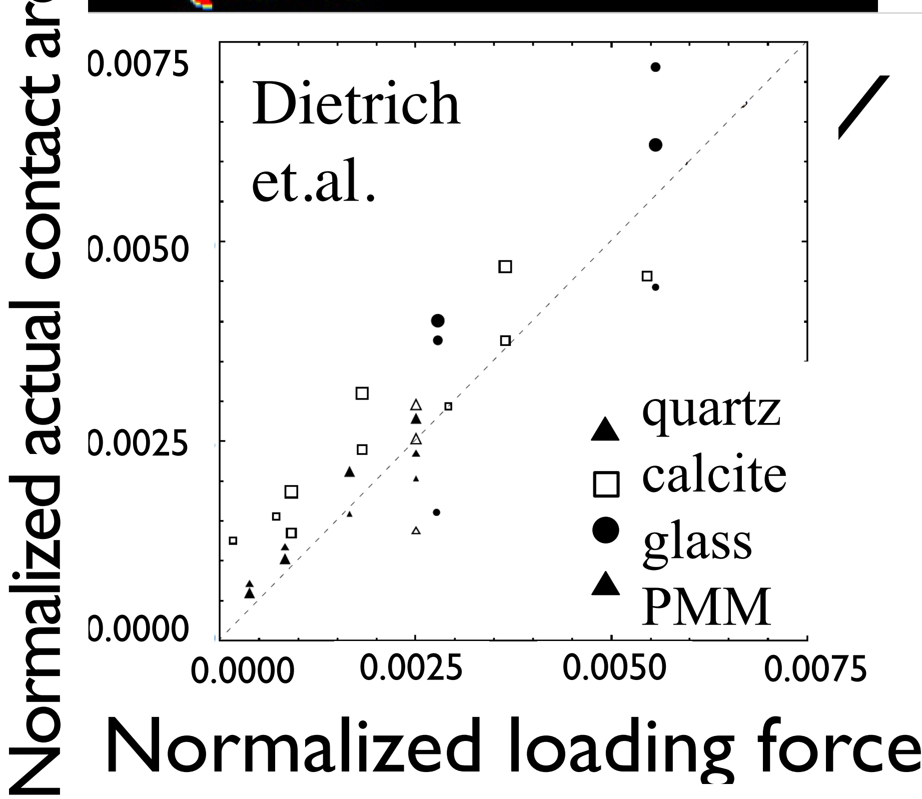


Friction force = Actual contact area X Adhesion force per unit area

Actual contact area is independent of apparent contact area and proportional to the loading force.

↓

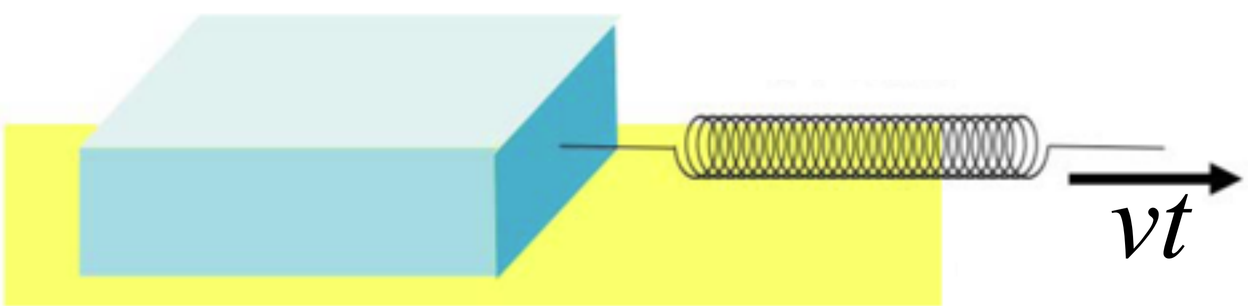
Friction force is independent of apparent contact area and proportional to the loading force.

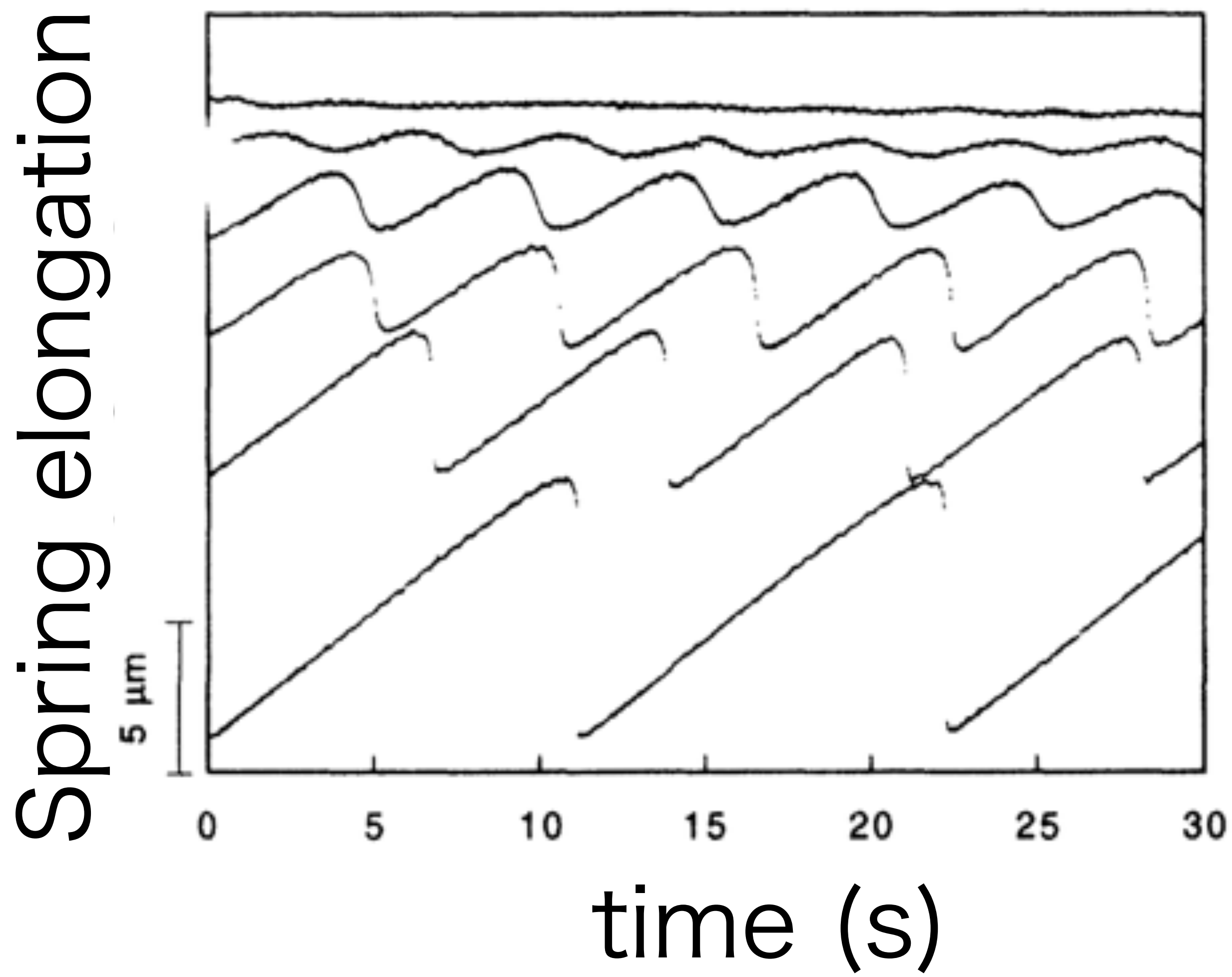
Bowden & Tabor

Max. Static Friction Force > Kinetic Friction Force

→ Steady sliding motion becomes unstable.

→ Stick-slip motion appears.





Heslot et al.

In this talk I discuss the following two problems.

1, Friction law at micro, meso and macro-scopic scales

Analytical, numerical and experimental methods.

We try to bridge the gaps between micro, meso and macroscopic scales.

Friction law depends on the scale.

2, Frictional Behavior and the Stick-Slip Dynamics of Sheared Granular Systems

Computer simulation of 2D system and real experiment of 3D system.

Slip size distribution and friction law.

Frictional behaviors of each element and the assembly of elements are different.

1, Friction Law in Micro, Meso and Macro-scopic Scales

Friction Law at Microscopic Scale

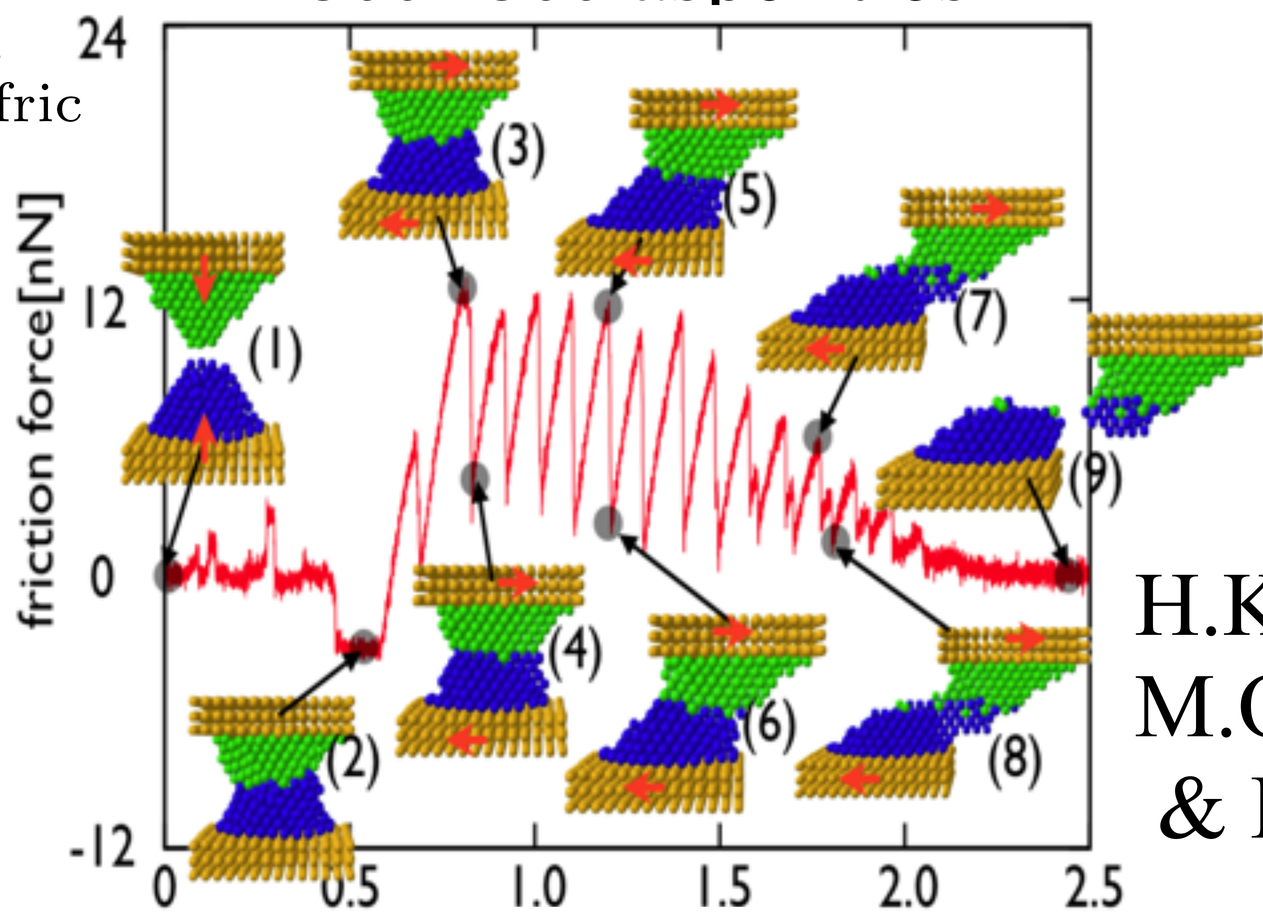
Simulation of shearing a single nano-junction

shearing velocity : 1.54[m/s]

fp[nN] : 6.0[nN]

bcc- bcc asperities

F_{fric}



time [ns]

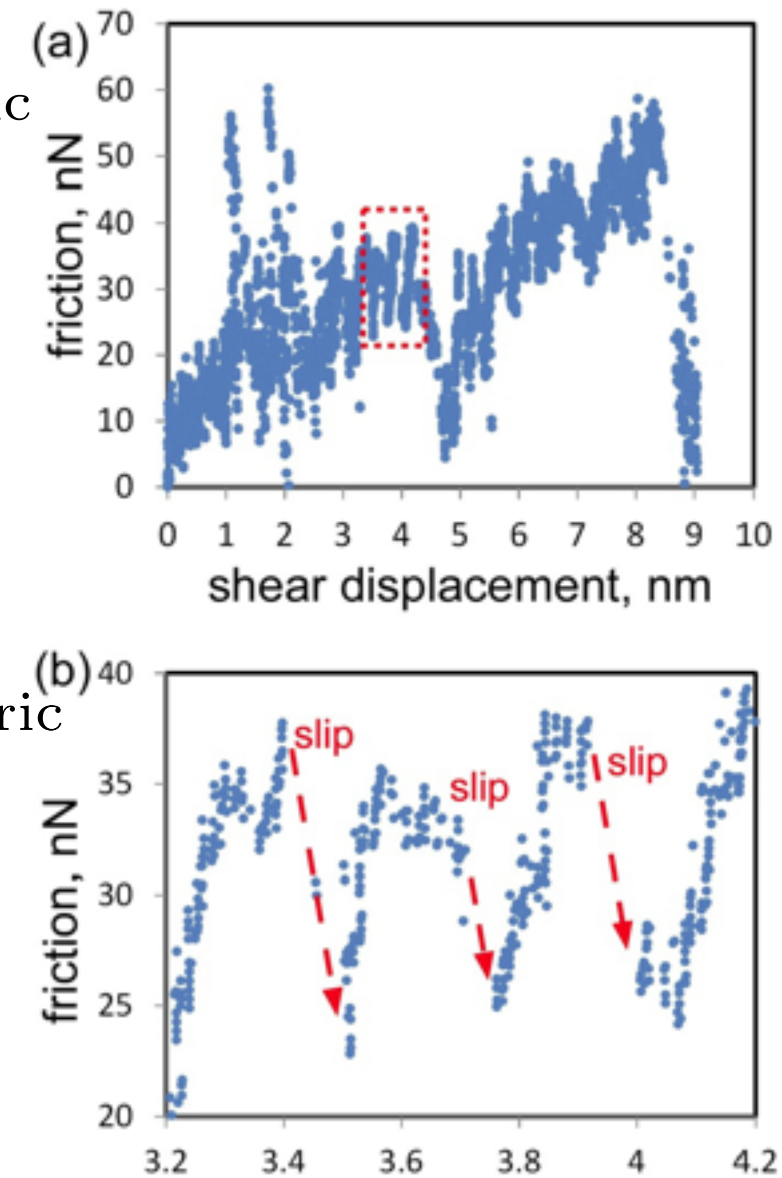
H.Kato, M.Otsuki & H.M.

Real experiment of Au nano junction

Ishida et al.

F_{fric}

F_{fric}

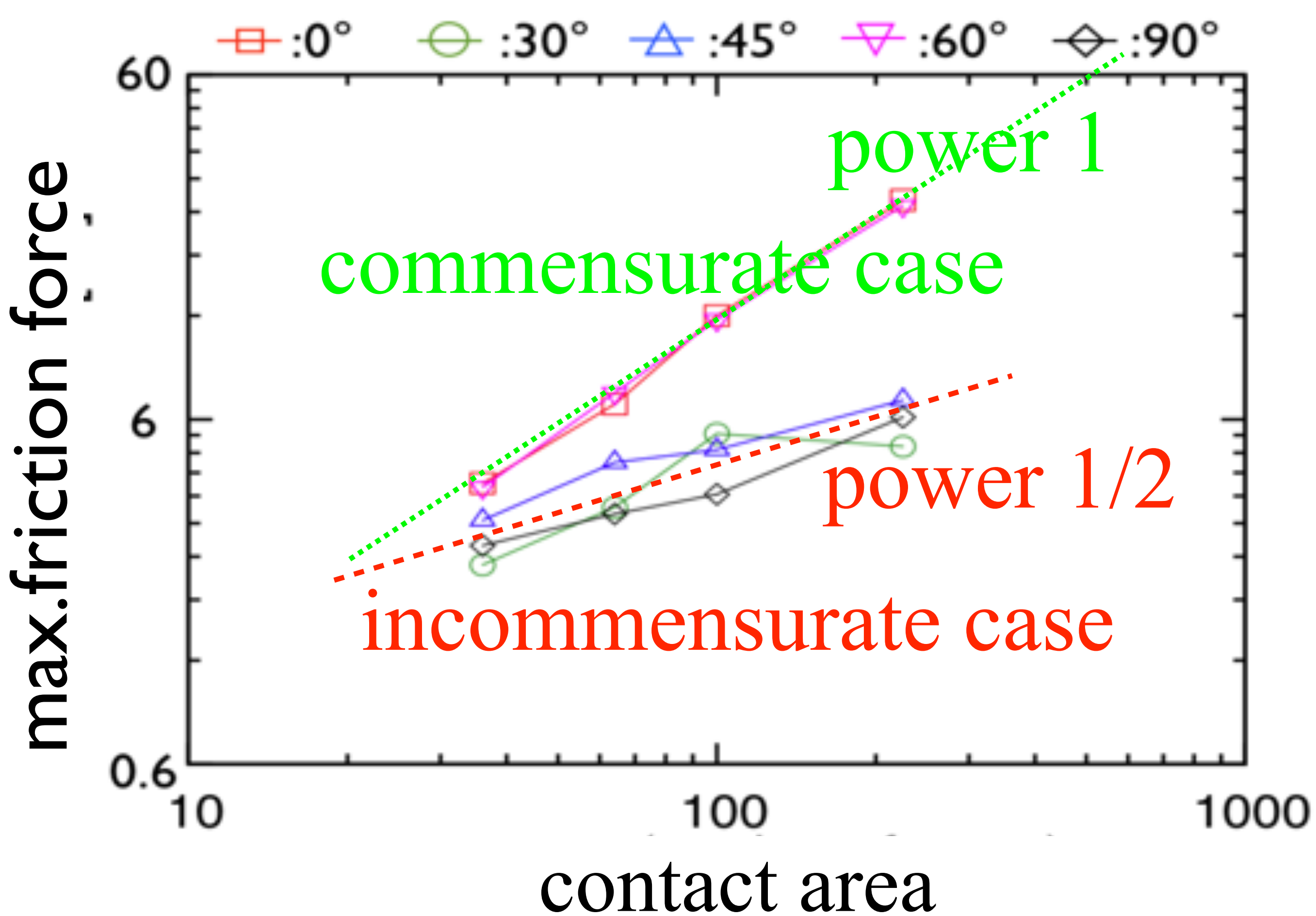


shear dipacement [nm]

Drops of friction force are accompanied by the slip about one lattice constant at the sliding plane.

Actual contact area and relative angle of crystal axes dependences of max. friction force between hcp - hcp pillars

temperature:300[K]



The max. static friction force is proportional to (the actual contact area)$^{1/2}$, for the incommensurate case (general case).

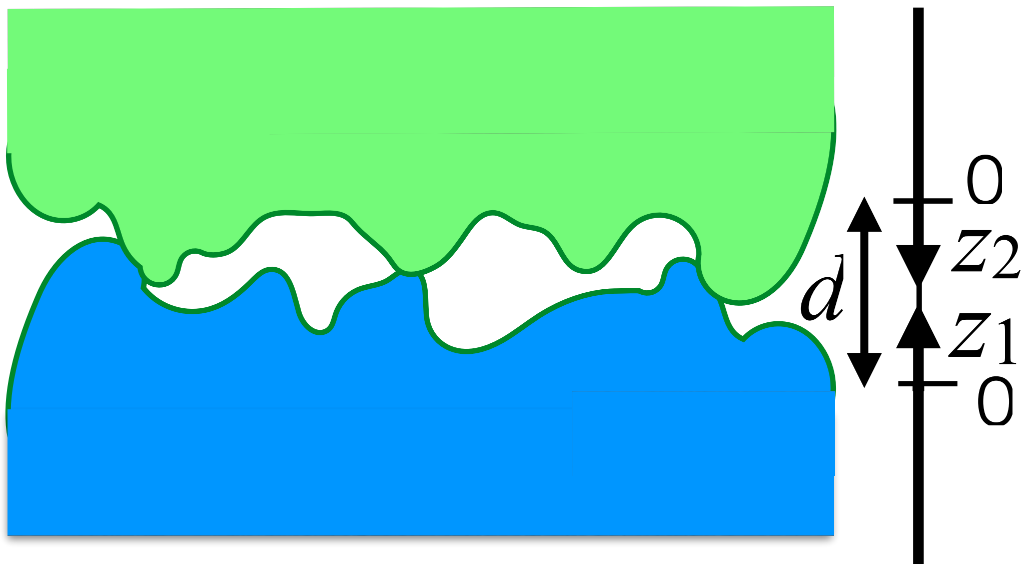
Similar behavior is observed for glass asperities.

Amontons' law does not hold for a single nano-junction in general.

Friction Law at Mesoscopic Scale

Multi-Junction Systems

Extension of Greenwood -Williamson Theory



$P_1(z_1), P_2(z_2)$: height distributions of two surfaces, respectively.

$z = z_1 + z_2 - d$: overlap distance

Asperities of z>0 is in contact.

For contacting asperithis with z >0

Hertz contact

actual contact area of a single junction $\delta A_{\mathrm{r}} = C_1 z^{\alpha}$

$\alpha = 1$

load of a single junction $\delta W_{\mathrm{r}} = C_2 z^{\beta}$

$\beta = 3/2$

contribution of a single junction to the max. static friction force of the whole system $\delta F_{\mathrm{r}} = C_3 z^{\gamma}$

$\gamma = \alpha$ or $\alpha/2$

comme. or incomme.

total actual contact area $$A_r = N\int_0^\infty dz P(z)\delta A_r = N\int_0^\infty dz P(z) C_1 z^\alpha$$

total load $$W_r = N\int_0^\infty dz P(z)\delta W_r = N\int_0^\infty dz P(z) C_2 z^\beta$$

max. static friction force of the whole system $$F_r = N\int_0^\infty dz P(z)\delta F_r = N\int_0^\infty dz P(z) C_3 z^\gamma$$

N: number of asperities

We assume that $P_1(z_1)$ and $P_2(z_2)$ are Gaussian distributions.

$$P_{1,2}(z_{1,2}) = \sqrt{\frac{1}{2\pi\ell_{1,2}^2}}\exp\left[-\frac{z_{1,2}^2}{2\ell_{1,2}^2}\right] \rightarrow P(z) = \sqrt{\frac{1}{2\pi\ell^2}}\exp\left[-\frac{(z+d)^2}{2\ell^2}\right]$$

$$\ell^2 = \ell_1^2 + \ell_2^2$$

The main contribution to the above integrals comes from the region around $z \gtrsim 0$.

So we can expand the function in the exponential function and get,

$$P(z) = C\exp\left[-\lambda z\right]$$

$$C = \sqrt{\frac{1}{2\pi\ell^2}}\exp\left[-\frac{d^2}{2\ell^2}\right] \qquad \lambda = \frac{d}{\ell^2}$$

Now we have,

$$A_\mathrm{r} = NCC_1 \int_0^\infty dz \exp[-\lambda z] z^\alpha = NCC_1 \underline{\Gamma(\alpha+1)}/\lambda^{\alpha+1}$$

$$W_\mathrm{r} = NCC_2 \int_0^\infty dz \exp[-\lambda z] z^\beta = NCC_2 \underline{\Gamma(\beta+1)}/\lambda^{\beta+1}$$

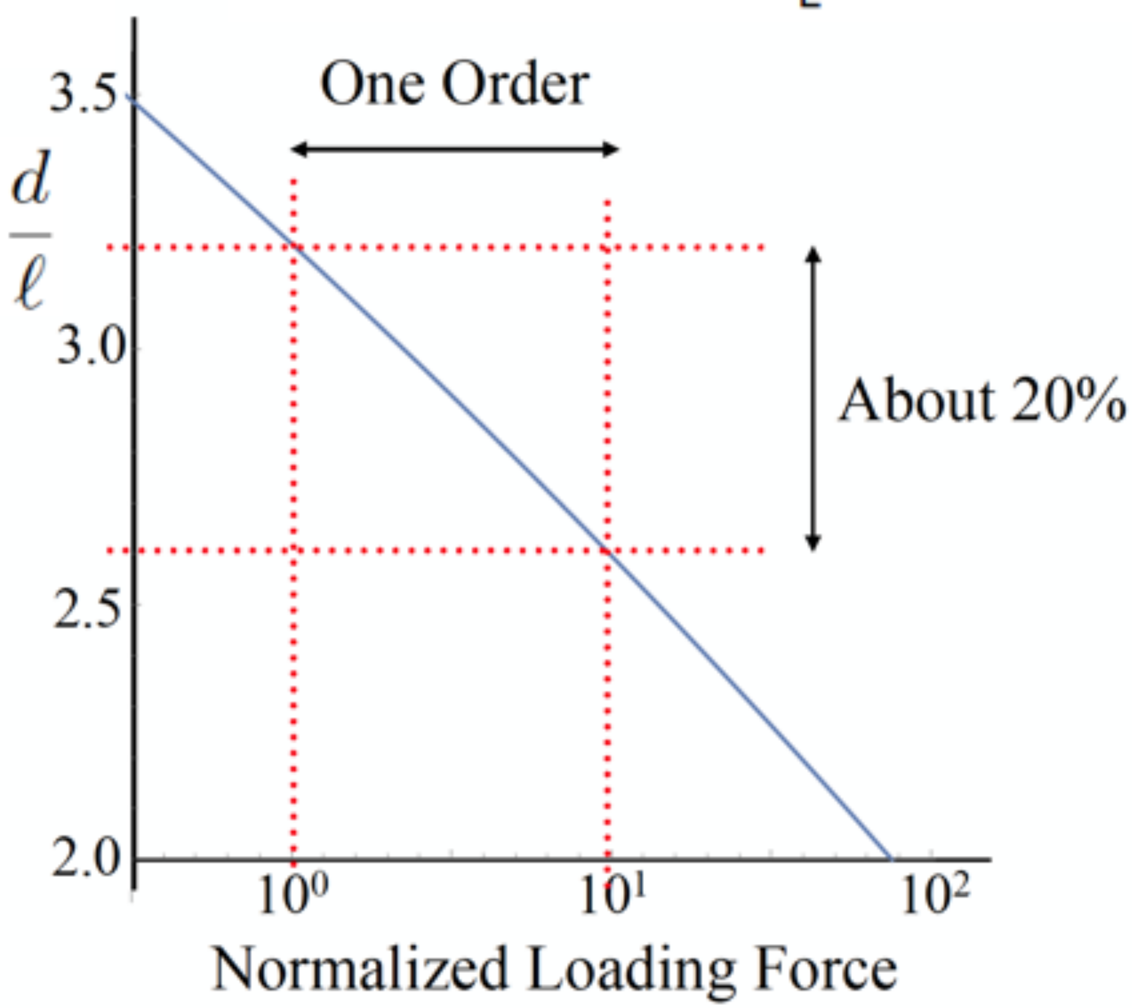
$$F_\mathrm{r} = NCC_3 \int_0^\infty dz \exp[-\lambda z] z^\alpha = NCC_3 \underline{\Gamma(\gamma+1)}/\lambda^{\gamma+1}$$

$$C = \sqrt{\frac{1}{2\pi\ell^2}} \exp\left[-\frac{d^2}{2\ell^2}\right]$$

$$\lambda = \frac{d}{\ell^2}$$

$$(d/\ell)^2 = -2\ln\left[\sqrt{2\pi\ell^2}\lambda^{\beta+1}\ W_\mathrm{r}/NC_2\underline{\Gamma(\beta+1)}\right]$$

Gamma function



d depends on W_r only logarithmically. As a result the changes of d and λ with the change of W_r are small enough.

Persson 1997

When W_r is changed, we can treat d and λ as constants, and the change of only C is important.

$$C = \sqrt{\frac{1}{2\pi\ell^2}} \exp\left[-\frac{d^2}{2\ell^2}\right]$$

Then we get,

$$\frac{F_r}{W_r} = \frac{C_3\Gamma(\gamma+1)}{C_2\Gamma(\beta+1)}\lambda^{\beta-\gamma} = \text{const.}$$

Amontons' law holds for multi-junction systems for any values of α, β and γ.

Further extension

Actual contact area of a single junction $\delta A_\mathrm{r} = a(z, \boldsymbol{\eta})$

$\boldsymbol{\eta}$: set of parameters which characterize the junction, e.g, curvature, shape, plasticity and so on.

Load of a single junction $\delta W_\mathrm{r} = w(z, \boldsymbol{\eta})$

Contribution of a single junction to the max. static friction force $\delta F_\mathrm{r} = f(z, \boldsymbol{\eta})$

Joint probability density of z and $\boldsymbol{\eta}$ $P_\mathrm{J}(z, \boldsymbol{\eta})$

Actual contact area A_r, load W_r, and the max. static friction force F_r are expressed as follows.

$$A_\mathrm{r} = N\int_0^\infty dz \int d\boldsymbol{\eta} P_\mathrm{J}(z,\eta)\delta A_\mathrm{r} = N\int_0^\infty dz \int d\boldsymbol{\eta} P_\mathrm{J}(z,\eta) a(z,\eta)$$

$$W_\mathrm{r} = N\int_0^\infty dz \int d\boldsymbol{\eta} P_\mathrm{J}(z,\eta)\delta W_\mathrm{r} = N\int_0^\infty dz \int d\boldsymbol{\eta} P_\mathrm{J}(z,\eta) w(z,\eta)$$

$$F_\mathrm{r} = N\int_0^\infty dz \int d\boldsymbol{\eta} P_\mathrm{J}(z,\eta)\delta F_\mathrm{r} = N\int_0^\infty dz \int d\boldsymbol{\eta} P_\mathrm{J}(z,\eta) f(z,\eta)$$

We can also take into account the effects of non-contact force.

We set $P_{\mathrm{J}}(z,\boldsymbol{\eta}) = P(z)\tilde{P}(z,\boldsymbol{\eta})$, $P(z) = \sqrt{\frac{1}{2\pi\ell^2}}\exp\left[-\frac{(z+d)^2}{2\ell^2}\right]$,

and approximate,

$$P(z) = C\exp[-\lambda z] \ , \quad C = \sqrt{\frac{1}{2\pi\ell^2}}\exp\left[-\frac{d^2}{2\ell^2}\right] \qquad \lambda = \frac{d}{\ell^2}$$

We assume $z-$dependences of $a(z,\boldsymbol{\eta})$, $w(z,\ \boldsymbol{\eta})$, $f(z,\ \boldsymbol{\eta})$ and $\tilde{P}(z,\boldsymbol{\eta})$ are weaker than the exp. function.

We get,

$$W_{\mathrm{r}} = N\int_0^\infty dz \int d\boldsymbol{\eta} P(z) \times \left\{\tilde{P}(z,\boldsymbol{\eta})w(z,\ \boldsymbol{\eta})\right\} = NCH_w(\lambda)$$

(d/ℓ) is a log. function of W_{r}. $(d/\ell)^2 = -2\ln\left[\sqrt{2\pi\ell^2}\, W_{\mathrm{r}}/NH_w(\lambda)\right]$

When W_{r} is changed, we can treat d and λ as constants.

$$F_{\mathrm{r}} = \int_0^\infty dz \int d\boldsymbol{\eta} P(z) \times \left\{\tilde{P}(z,\boldsymbol{\eta})f(z,\ \boldsymbol{\eta})\right\} = NCH_f(\lambda)$$

We get,

$$\frac{F_{\mathrm{r}}}{W_{\mathrm{r}}} = \frac{H_f(\lambda)}{H_w(\lambda)} = \text{const.}$$

Amontons' law holds.

Taking into account the correlation among asperities

$\{z(\boldsymbol{r})\}$ and $\{\boldsymbol{\eta}(\boldsymbol{r})\} \longleftarrow$ $z(\boldsymbol{r})$ and $\boldsymbol{\eta}(\boldsymbol{r})$ in whole interface.

Total actual contact area $\tilde{a}$, load $\tilde{w}$, and the max. static friction force $\tilde{f}$ are functionals of $\{z(\boldsymbol{r})\}$ and $\{\boldsymbol{\eta}(\boldsymbol{r})\}$.

$$\tilde{a} = \tilde{a}(\{z(\boldsymbol{r})\}, \{\boldsymbol{\eta}(\boldsymbol{r})\}), \cdots \longleftarrow \text{correlation effect}$$

Here we consider ensemble averages of $\tilde{a}$, $\tilde{w}$, and $\tilde{f}$ and the joint probability density, $P_{\rm J}(\{z(\boldsymbol{r})\}, \{\boldsymbol{\eta}(\boldsymbol{r})\})$

$$A_r = \left\{\prod_{\boldsymbol{r}} \int dz(\boldsymbol{r}) \int d\boldsymbol{\eta}(\boldsymbol{r})\right\} \tilde{a}(\{z(\boldsymbol{r})\}, \{\boldsymbol{\eta}(\boldsymbol{r})\}) P_{\rm J}(\{z(\boldsymbol{r})\}, \{\boldsymbol{\eta}(\boldsymbol{r})\})$$

$$W_r = \left\{\prod_{\boldsymbol{r}} \int dz(\boldsymbol{r}) \int d\boldsymbol{\eta}(\boldsymbol{r})\right\} \tilde{w}(\{z(\boldsymbol{r})\}, \{\boldsymbol{\eta}(\boldsymbol{r})\}) P_{\rm J}(\{z(\boldsymbol{r})\}, \{\boldsymbol{\eta}(\boldsymbol{r})\})$$

$$F_r = \left\{\prod_{\boldsymbol{r}} \int dz(\boldsymbol{r}) \int d\boldsymbol{\eta}(\boldsymbol{r})\right\} \tilde{f}(\{z(\boldsymbol{r})\}, \{\boldsymbol{\eta}(\boldsymbol{r})\}) P_{\rm J}(\{z(\boldsymbol{r})\}, \{\boldsymbol{\eta}(\boldsymbol{r})\})$$

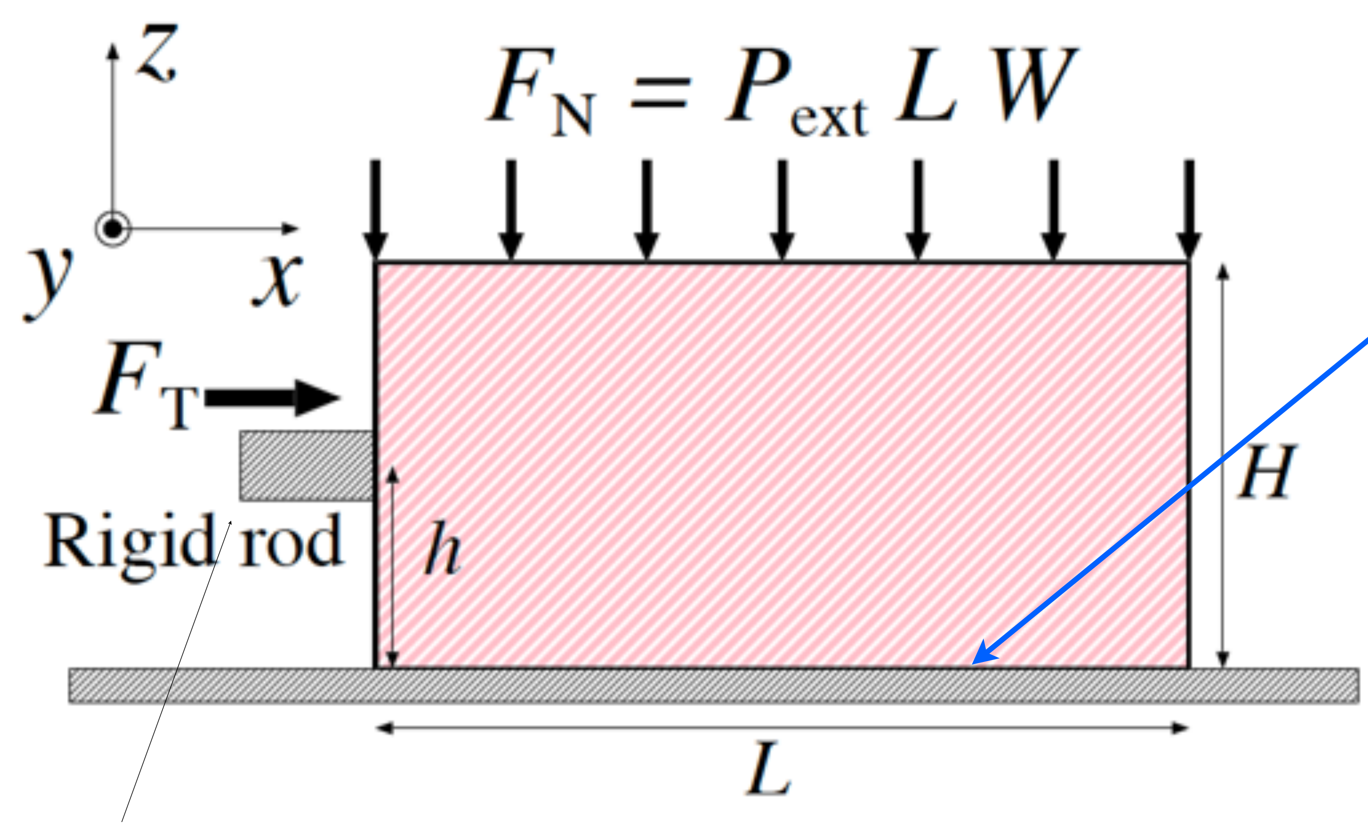
We obtain $F_{\rm r}/W_{\rm r}$=const.

Amontons' law holds for general multi-junction systems, which has correlations among asperities.

Friction Law at Macroscopic Scale

Model of FEM Calculation

2D Sheared-Viscoelastic Block

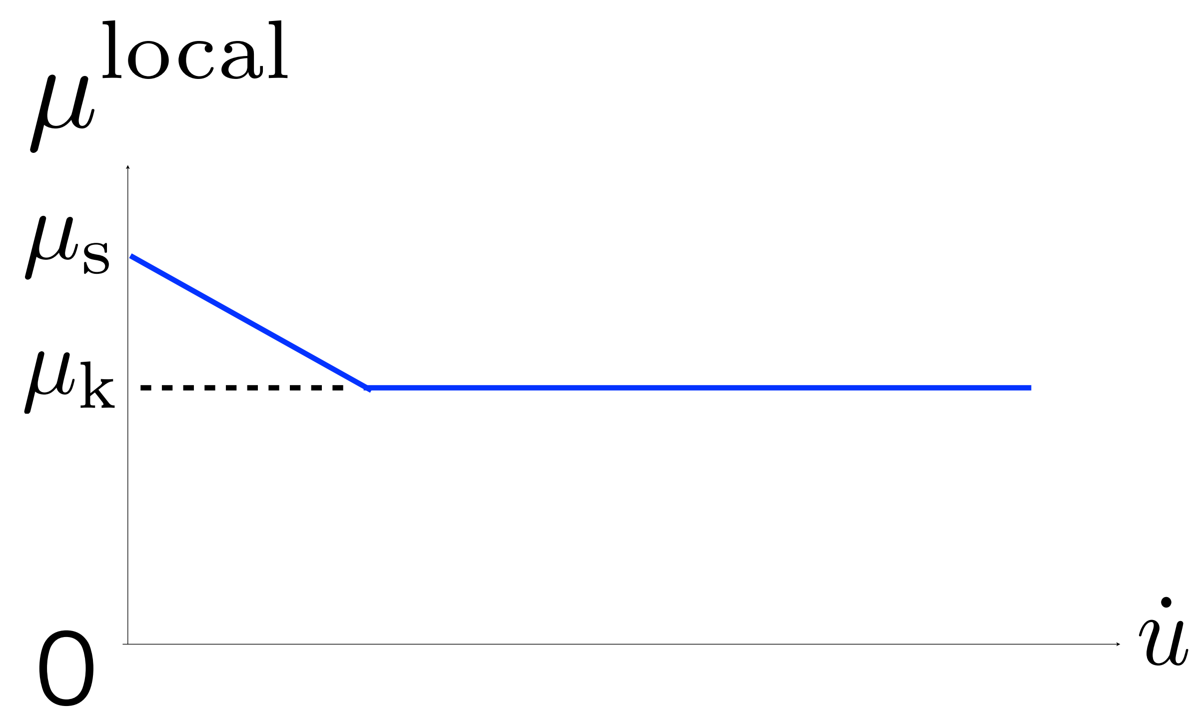


displacement of the rod U

$$U = vt, \ \ v \ll 1$$

Amotons' friction force works at the bottom.

$$\sigma_{xz}^{\mathrm{fric}} = \mu^{\mathrm{local}}(\dot{u}) \mid \sigma_{zz} \mid$$



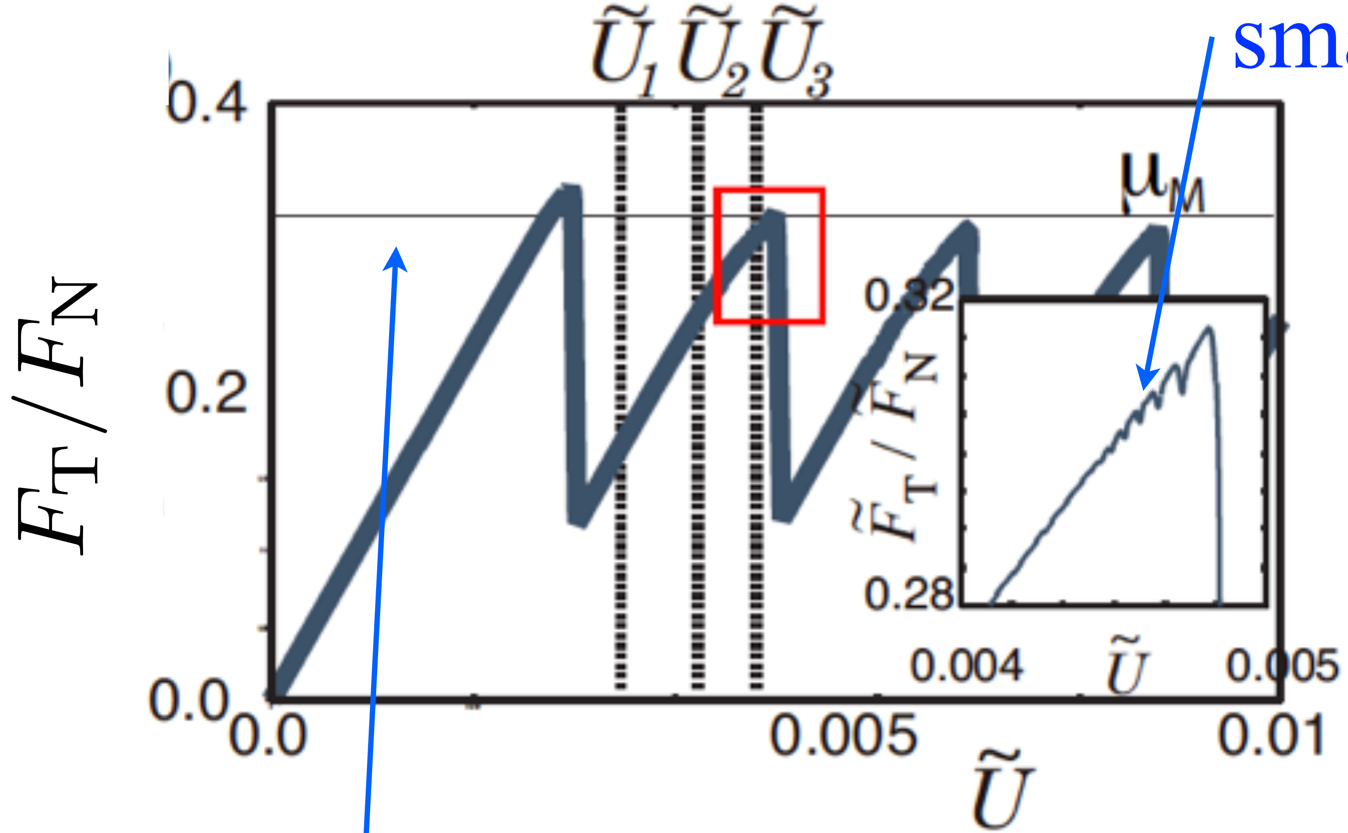
M. Otsuki & H. M.,

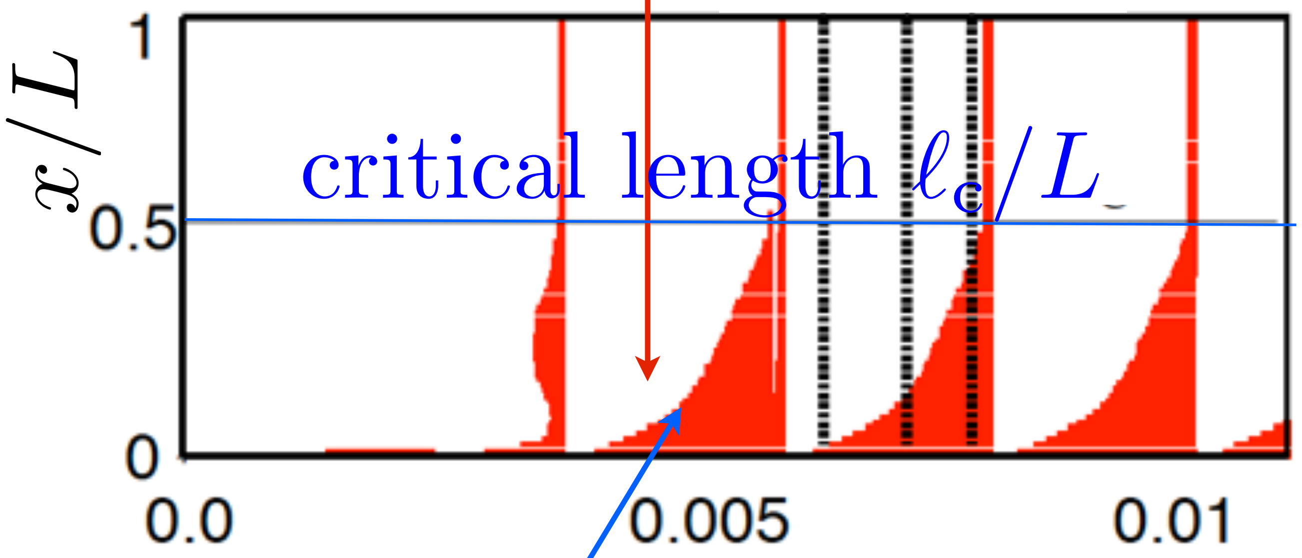
Sci. Rep. 3, 1586 (2013)

μ_s : local static friction coefficient

μ_k : local kinetic friction coefficient small drops

local slip region





displacement of the rod $U \propto$ time

displacement of the rod $U \propto$ time

bulk static friction coefficient

local slip front

slow (adiabatic) motion

at ℓ_c ↓

rapid motion

↓

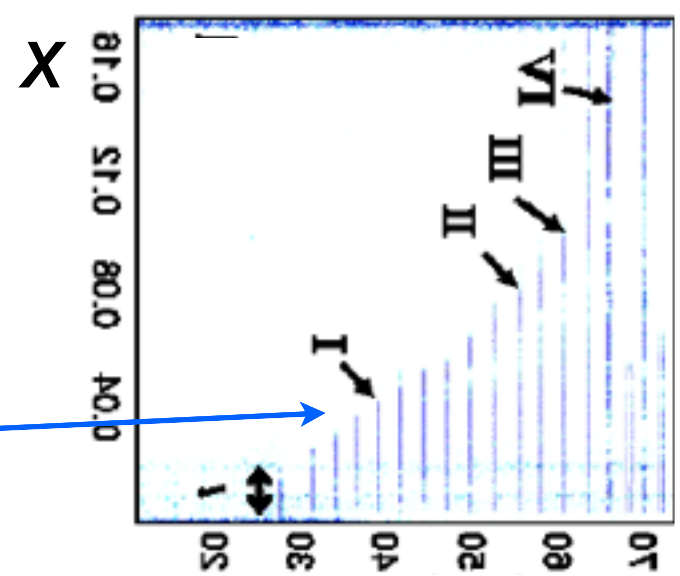
leading edge

Whole block slides.

Similar behavior is observed in experiment.

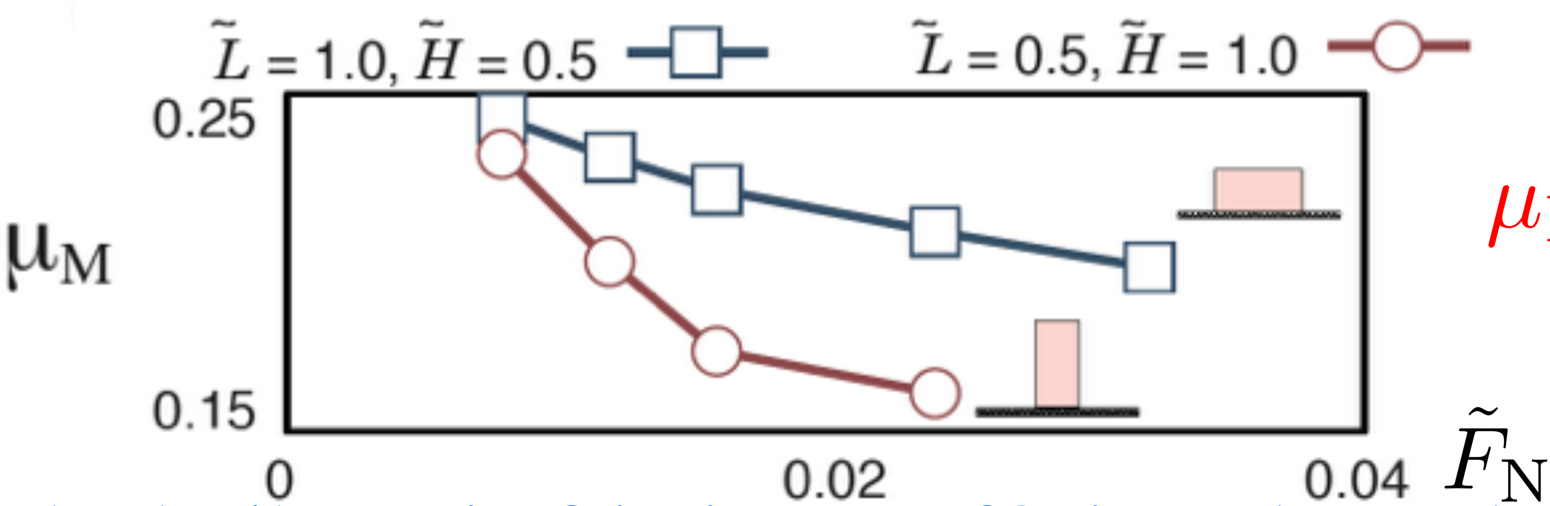
PMMA

local slip



Rubinstein et.al. 2007

Bulk Static Friction Coefficient $\mu_{\rm M}$

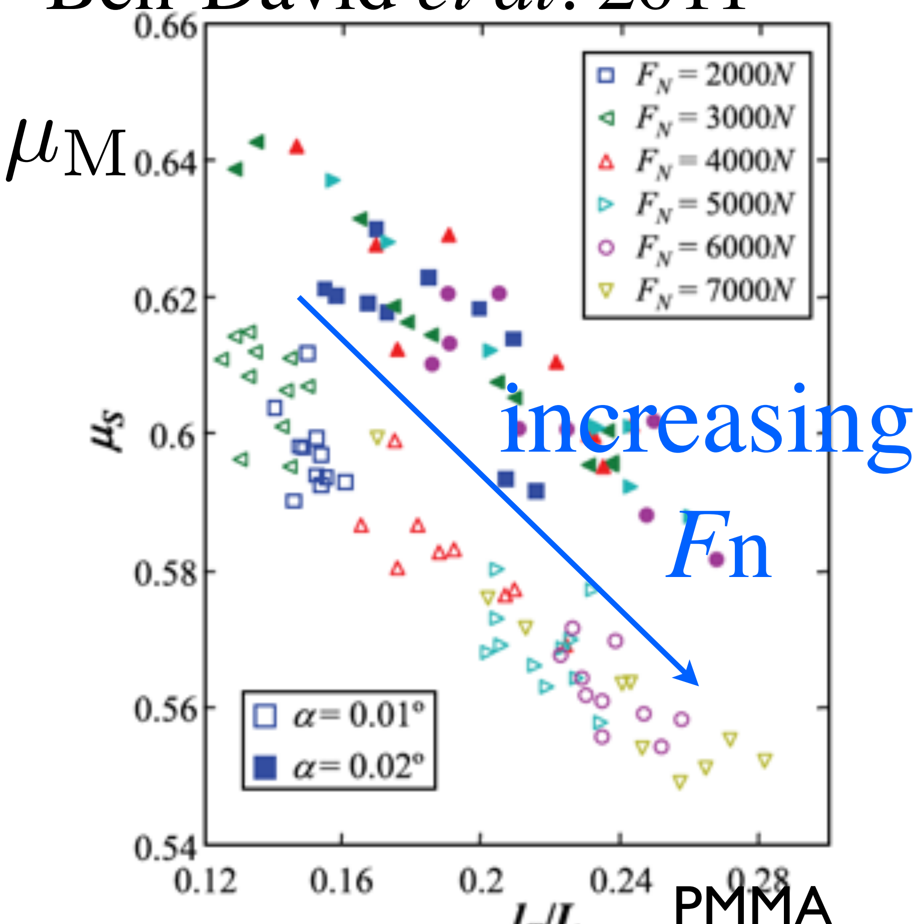


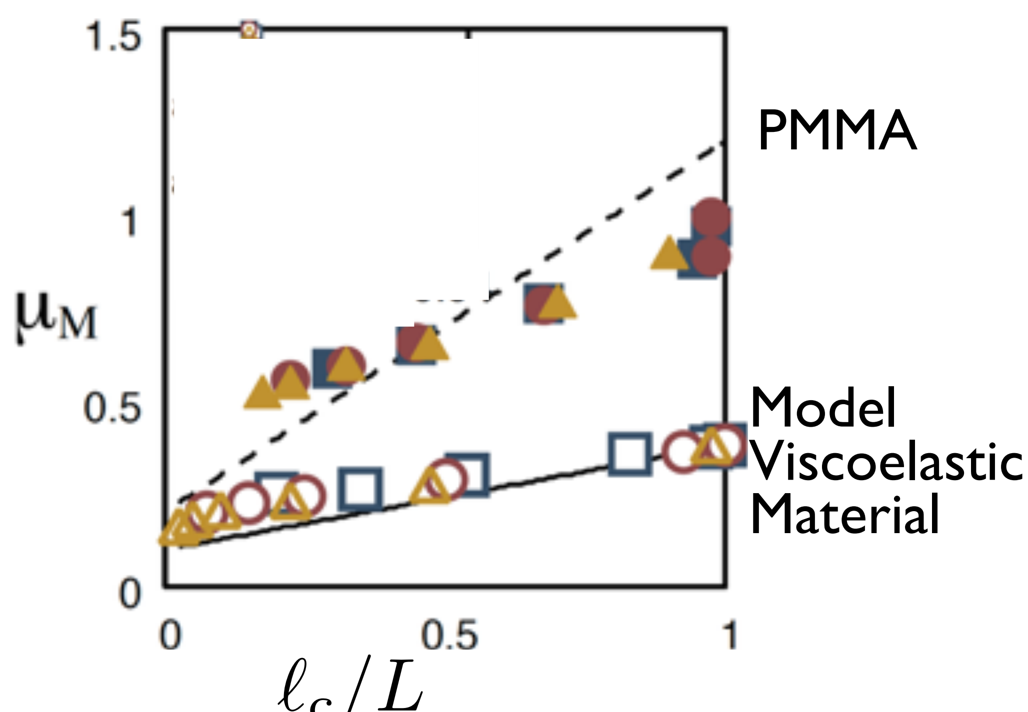
$$\mu_{\rm M} = \text{const.} + C F_{\rm N}^{-1/3}$$

The bulk static friction coefficient depends on the normal force and the apparent contact area.

Amontons' Law breaks and instead new friction law holds.

Ben-David *et al*. 2011



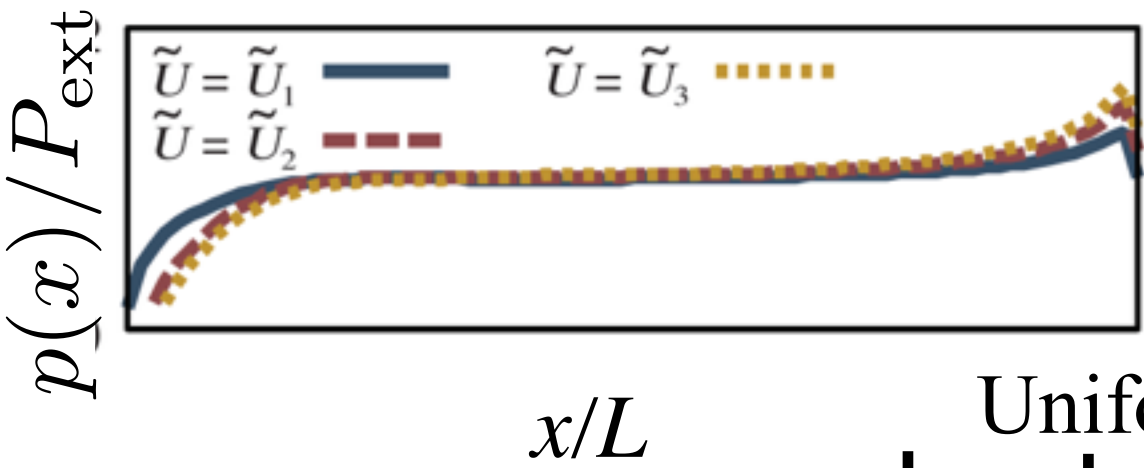


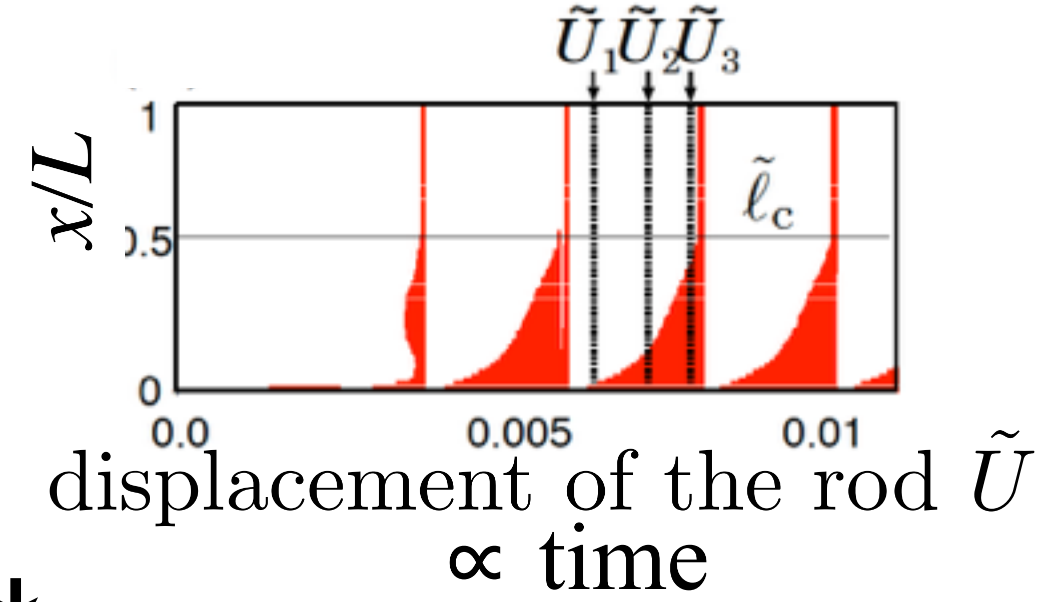
critical length/system length

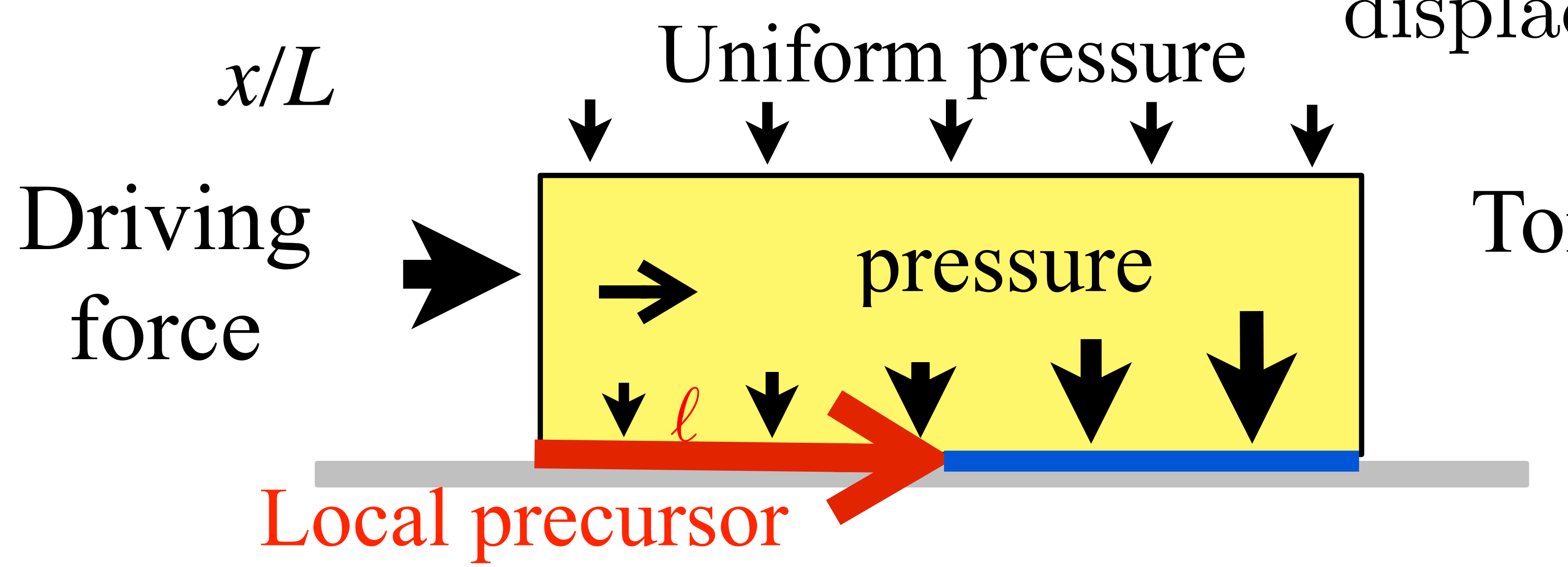
$\mu_{\rm M}$ is determined by the critical length of the local slip.

Pressure and Shear Stress distribution

Normalized Pressure at the bottom



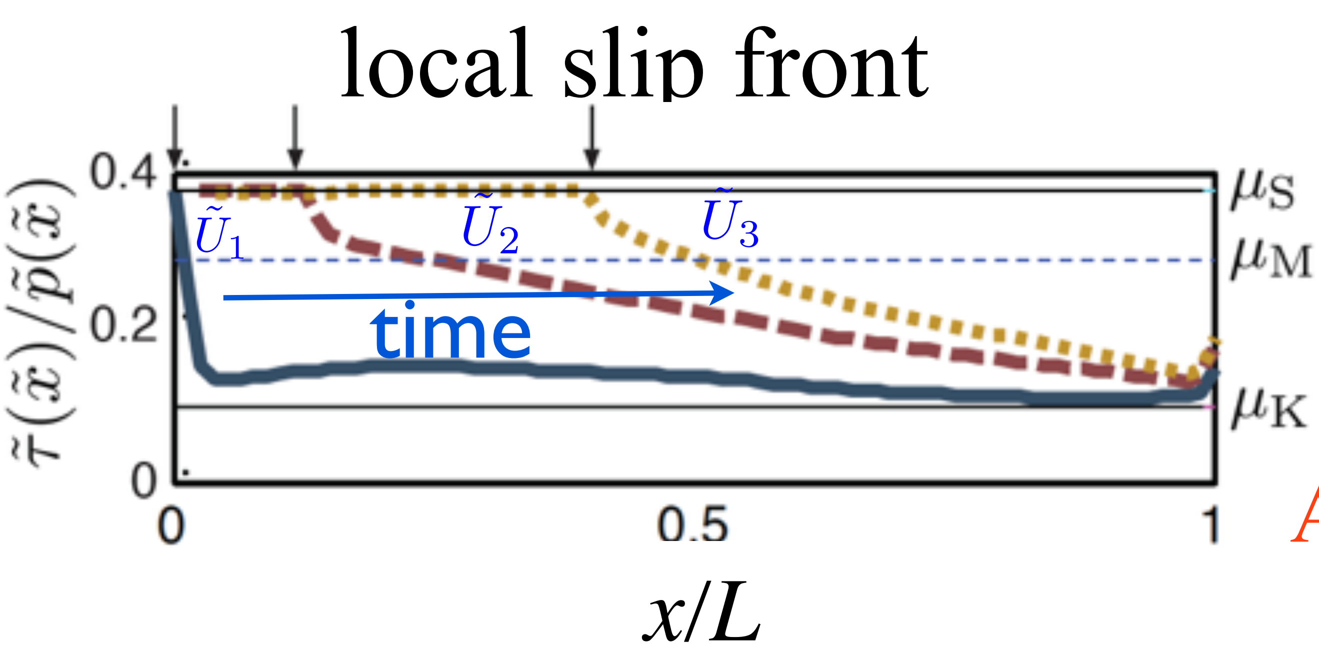


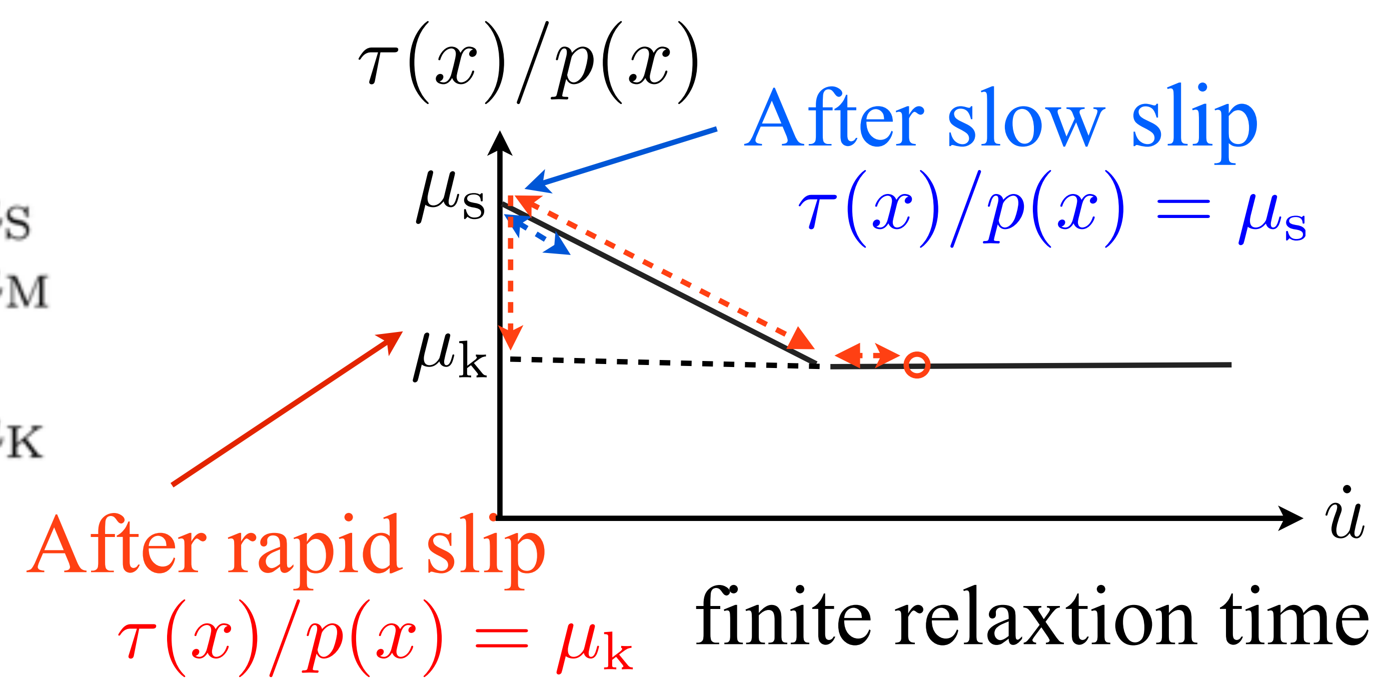


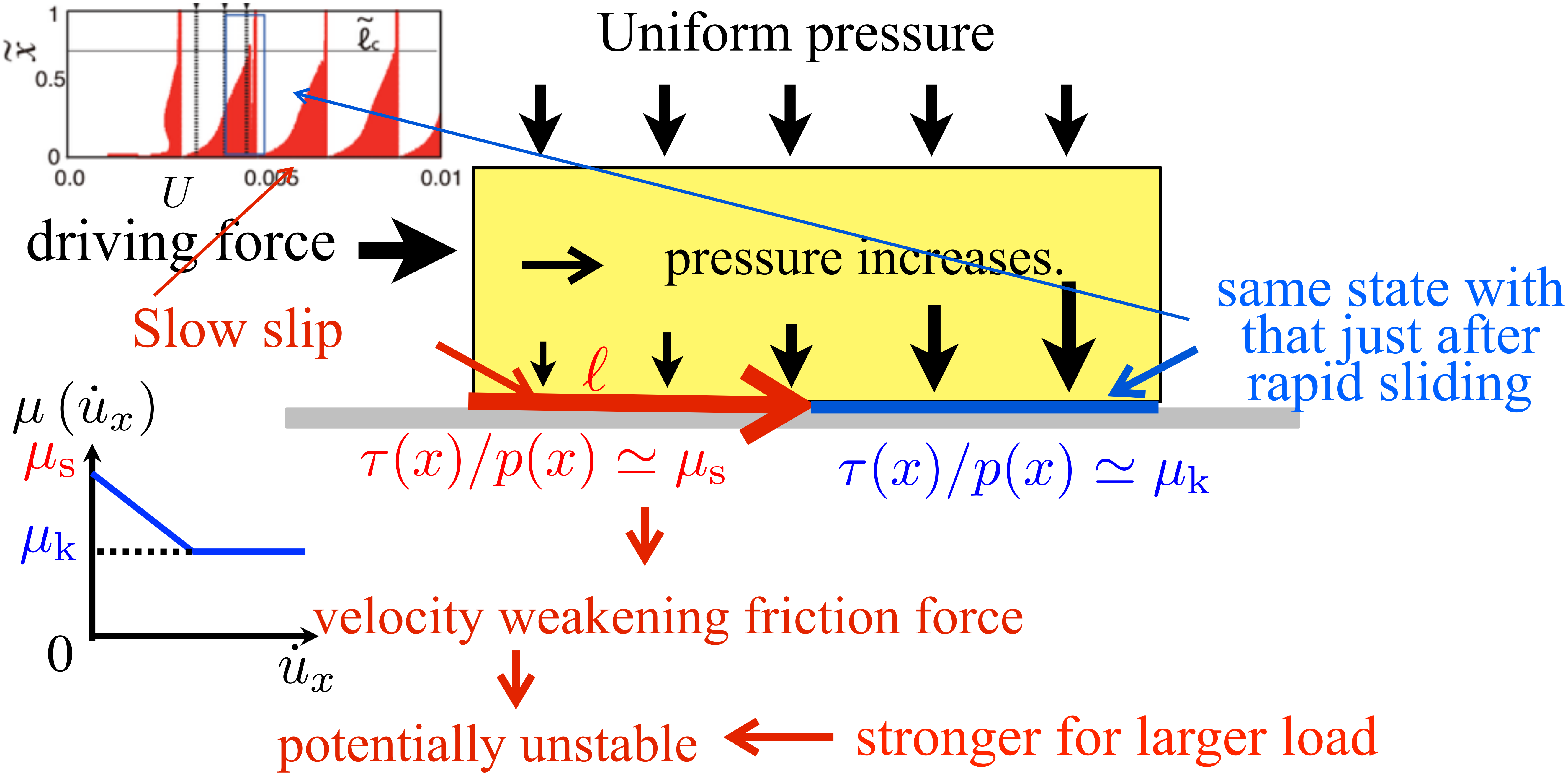
Torque effects

J. Scheibert

Ratio of Shear Stress to Pressure at the bottom





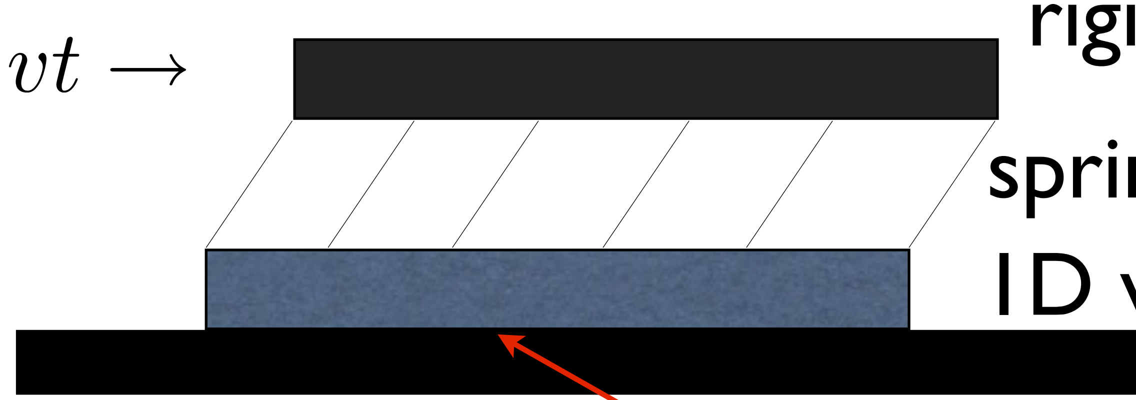


Slow slip becomes unstable when ℓ reaches at certain critical length ℓ_c.

The friction coefficient of the block becomes larger for larger ℓ_c.

ℓ_c is samller for larger load. —> The friction coefficient of the block becomes smaller for larger load.

1D Effective Model



We obtain the adiabatic solution of this model.
From the adiabatic solution and the expression of the shear force,
$F_{\mathrm{t}} = \int_0^L dx \sigma_{xz}(x)$, we obtain,

$$\mu_{\mathrm{M}} = \mu_{\mathrm{K}} + (\mu_{\mathrm{S}} - \mu_{\mathrm{K}})\ell_{\mathrm{c}}/L$$

Time evolution eq. of the fluctuation around the adiabatic solution, δu.

$$\delta\ddot{u} \doteqdot E\partial_{xx}\delta u - \underbrace{(\eta/L)\delta\dot{u}}_{\text{viscosity}} + \underbrace{\alpha P_{\mathrm{ext}}\ell^3\delta\dot{u}}_{\text{velocity weakening frictional force}}$$

traveling length of the local slip front

With adiabatic increases of U, ℓ increases adiabatically at first.
At ℓ_{c} the adiabatic motion becomes unstable.

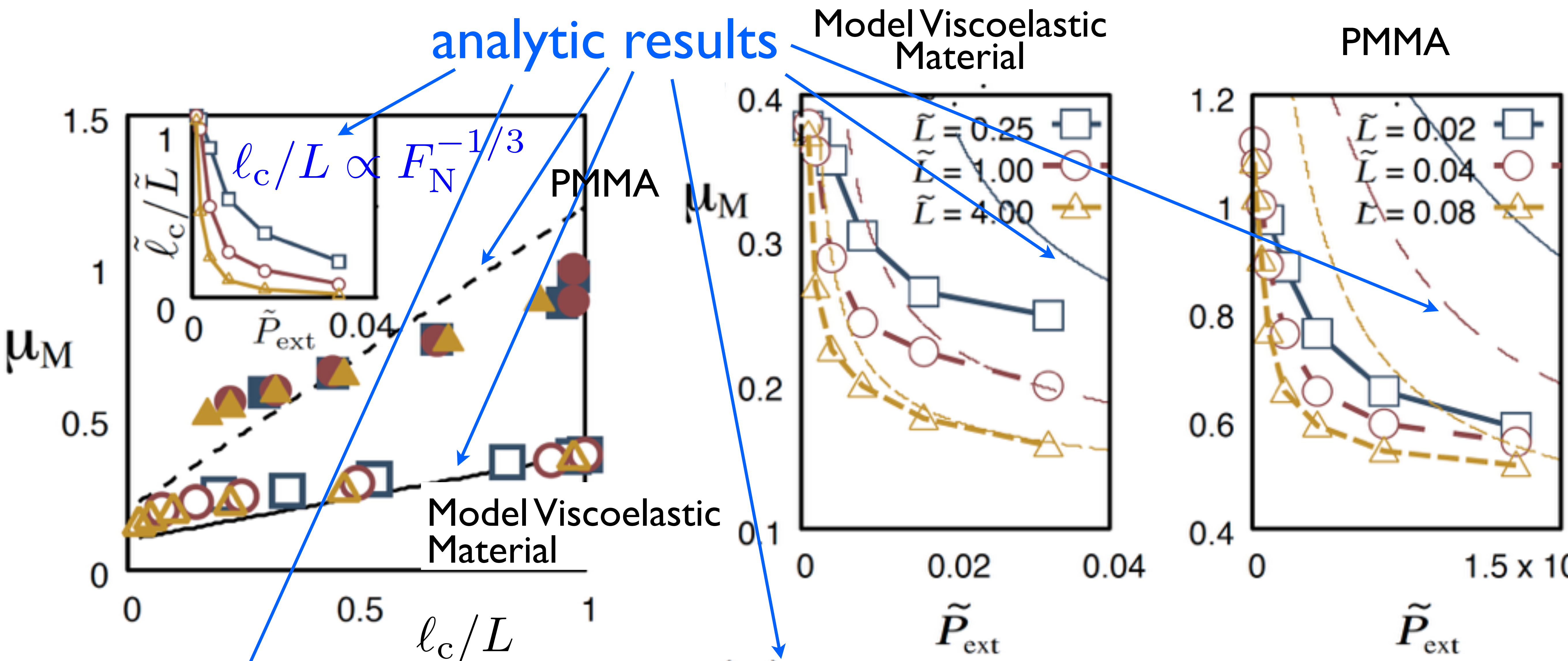
$$\rightarrow \quad \ell_{\mathrm{c}}/L \propto (\eta/P_{\mathrm{ext}}L)^{(1/3)} \longrightarrow \mu_{\mathrm{M}} = \mu_{\mathrm{k}} + CF_{\mathrm{N}}^{-1/3}$$

New friction law is derived analytically.

Bulk static friction coeff.

μ_{M} vs. critical length

μ_{M} vs. pressure



$\mu_{\mathrm{M}} = \mu_{\mathrm{k}} + (\mu_{\mathrm{s}} - \mu_{\mathrm{k}})\ell_{\mathrm{c}}/L$ $\mu_{\mathrm{M}} = \mu_{\mathrm{k}} + C F_{\mathrm{N}}^{-1/3}$ $C \propto \mu_{\mathrm{s}} - \mu_{\mathrm{k}}$

Amontons' law breaks and instead the new friction law holds.

This results from the local precursor slip before the onset of bulk sliding.

For large or small enough pressure and/or system length,

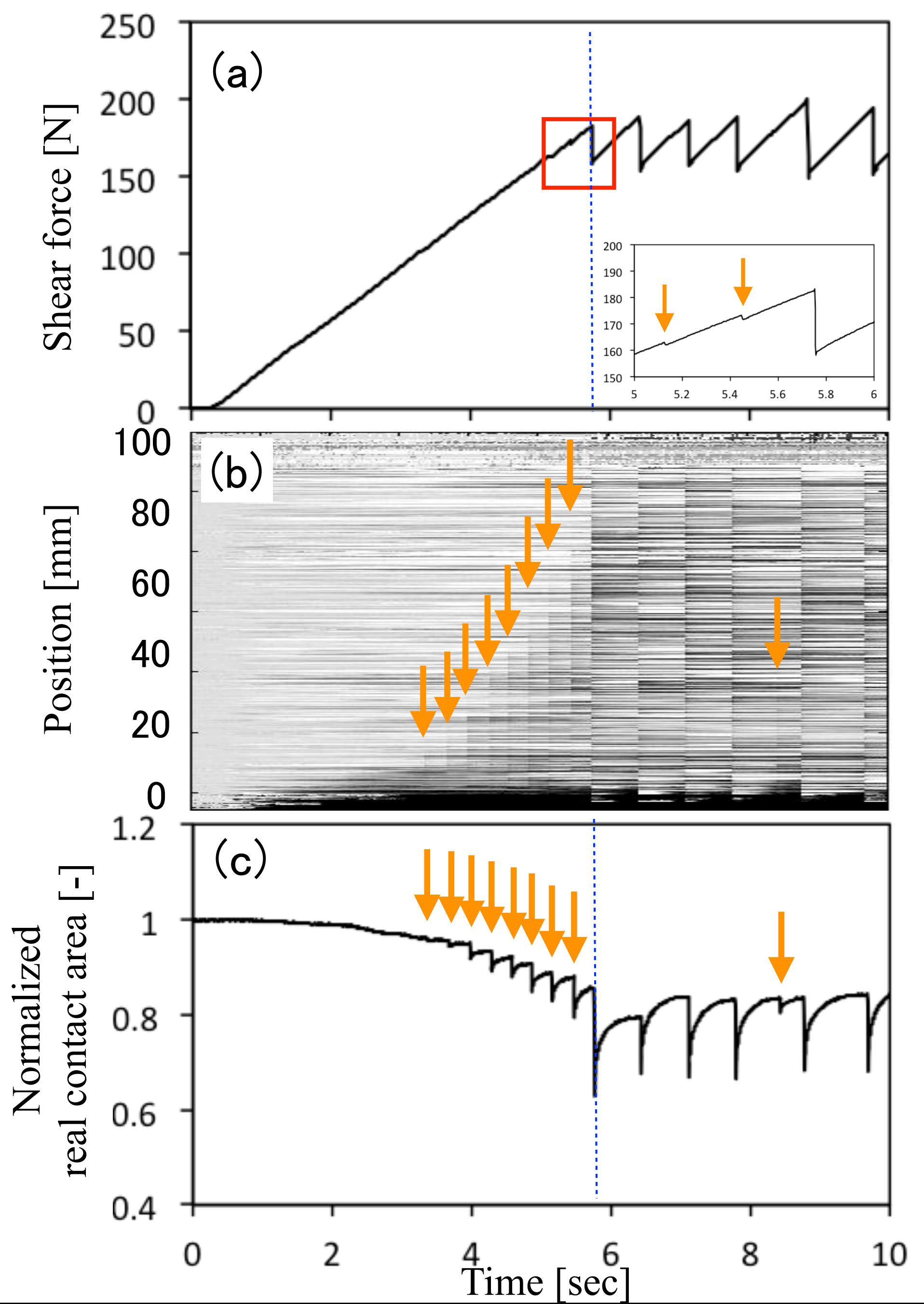
where $\ell_{\mathrm{c}}/L << 1$, or $\ell_{\mathrm{c}}/L \lesssim 1$, Amonton's law recovers.

Experimantal study of the load dependence of the friction coefficient.

Y. Katano, K. Nakano, M. Otsuki& H. M., Sci. Rep. 4, 6324 (2014)

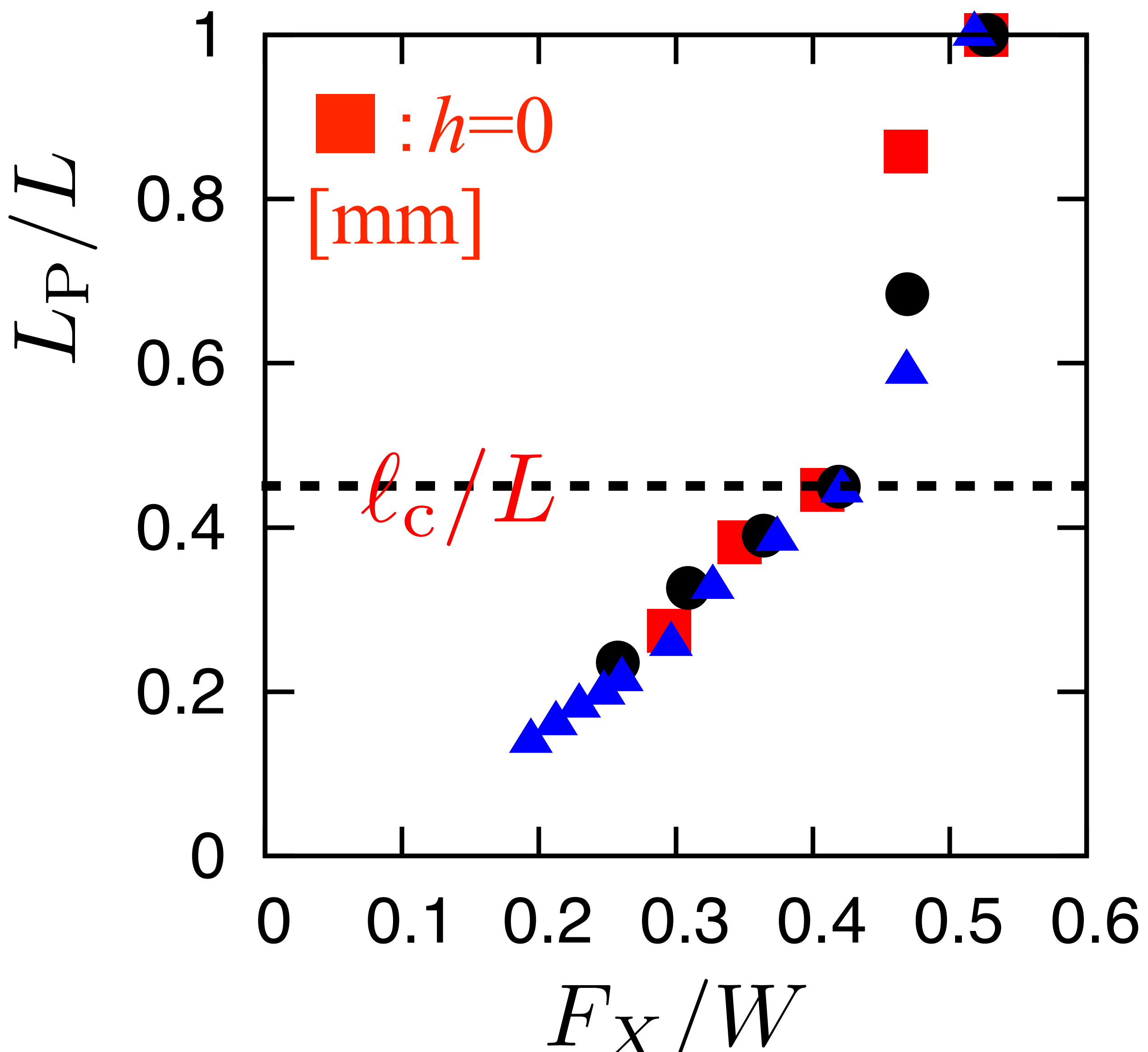
PMMA

Precursor Slip



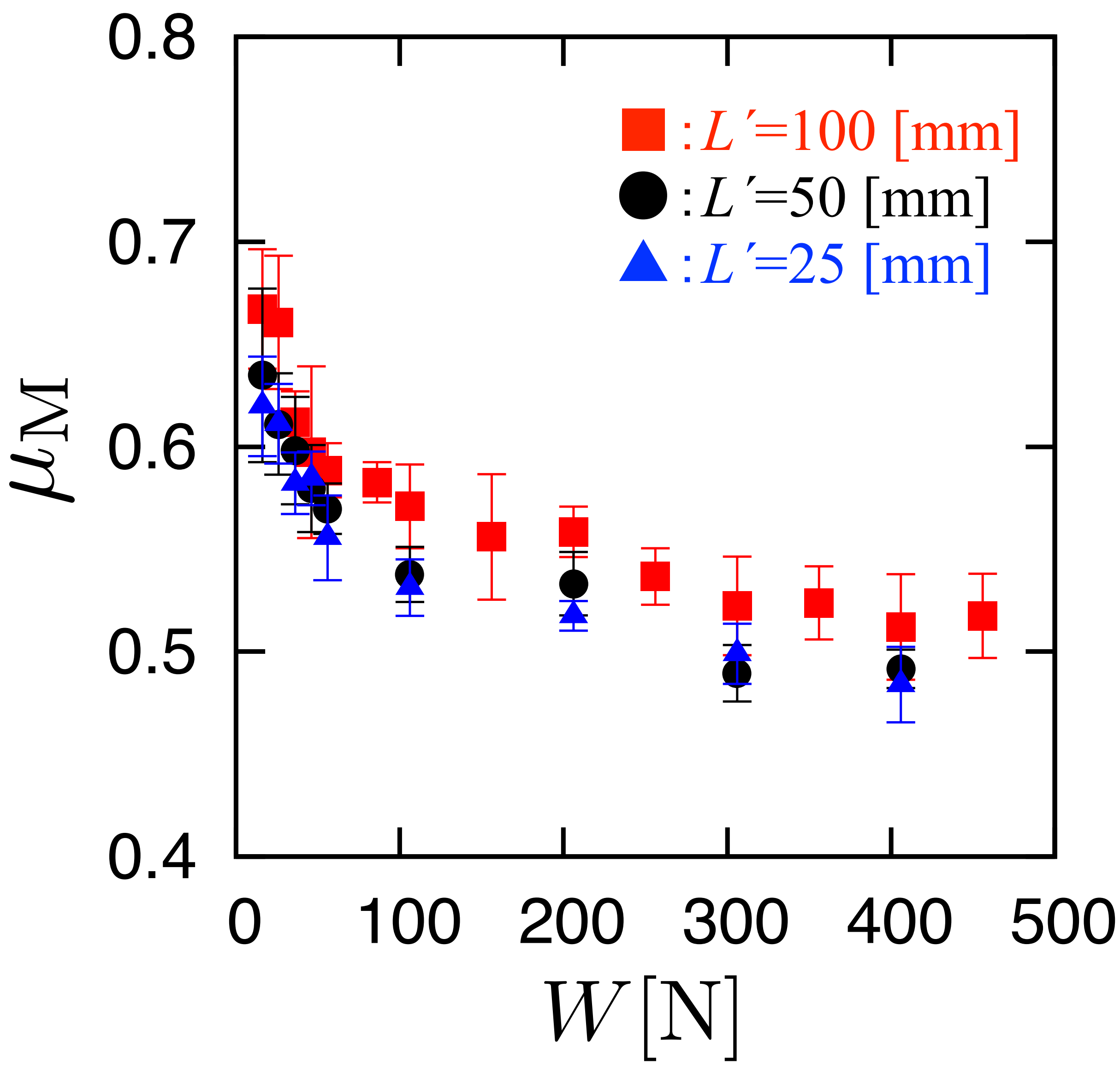
KV=40[N/s]
F_Z= 400[N]
h= 10[mm]

(KV = 40 [N/s], F_Z= 400[N])



Load and apparent contact area dependence of max. static fric. coef. μ_{M}

(KV = 40 [N/s], h = 0 [mm])



Max. static fric. coef. μ_{M} increases with increasing apparent contact area.

New friction law,

$$\mu_{\mathrm{M}} = \mu_{\mathrm{k}} + C F_{\mathrm{N}}^{-1/3}$$

is verified experimentally.

Summary of Friction Law at Micro, Meso and Macro-scopic Scales

We successfully bridged the gap between microscopic and macroscopic frictional phenomena.

At micro scale, friction force does not obey Amontons' law.

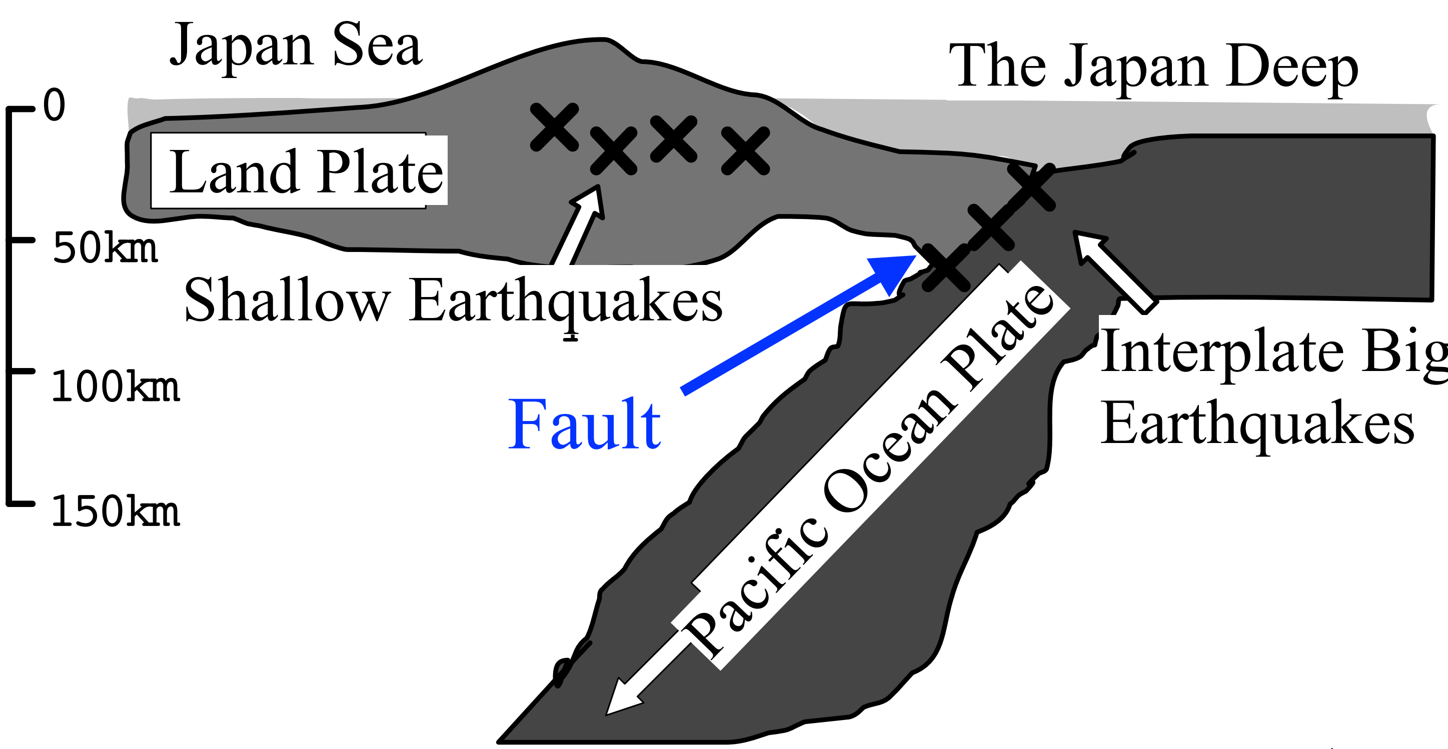
At meso scale, friction force obeys Amontons' law.

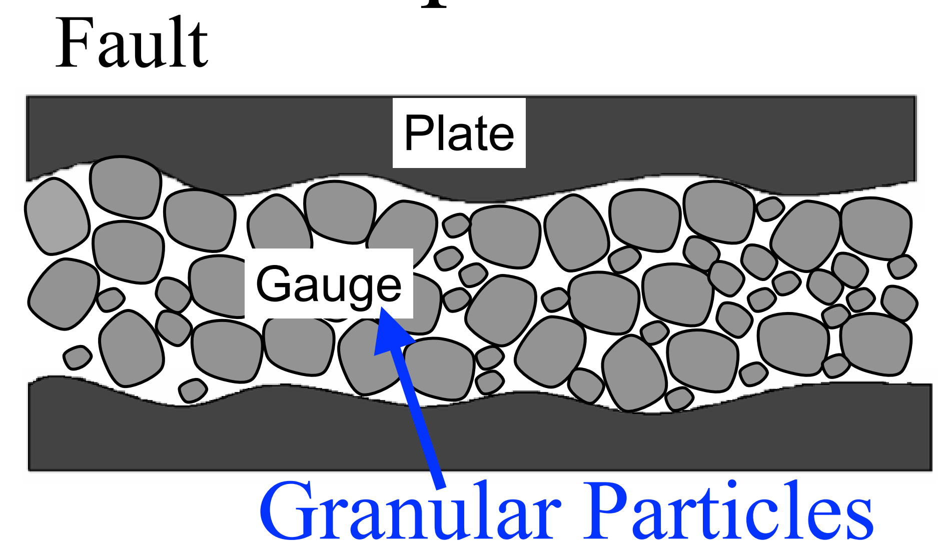
At macro scale, friction force can break Amontons' law and instead new friction law holds.

Friction law depends on the scale of the system.

2, Frictional Behavior and the Stick-slip Dynamics of Sheared Granular Systems

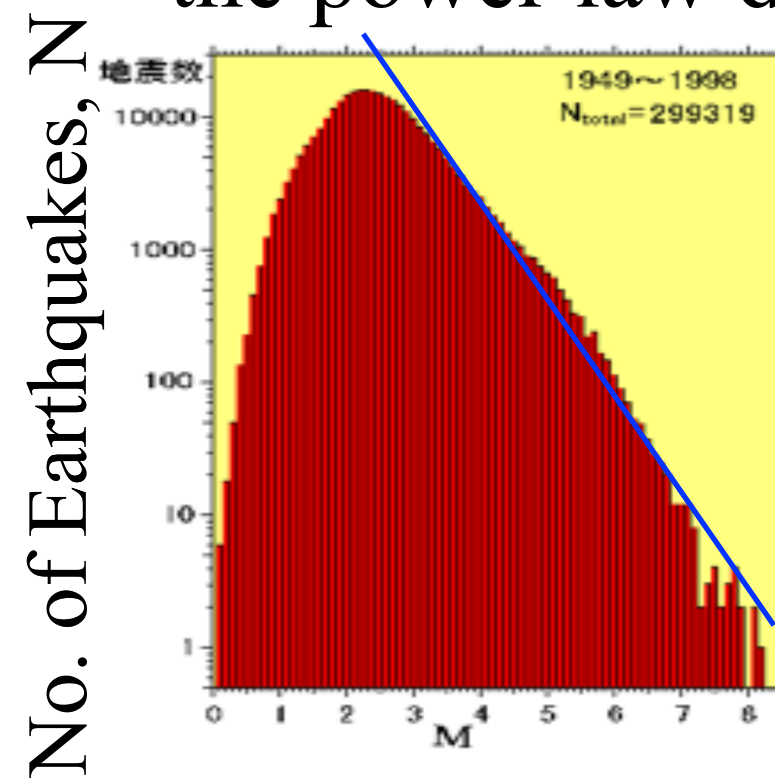
Largest stick-slip motion on the earth is earthquake.





The size of the gauge shows the power law distribution.

Size distribution of earthquakes obeys the power law; the Gutenberg-Richter's (GR) law.



Magnitude, M ∝ Log(Earthquake Size, S)

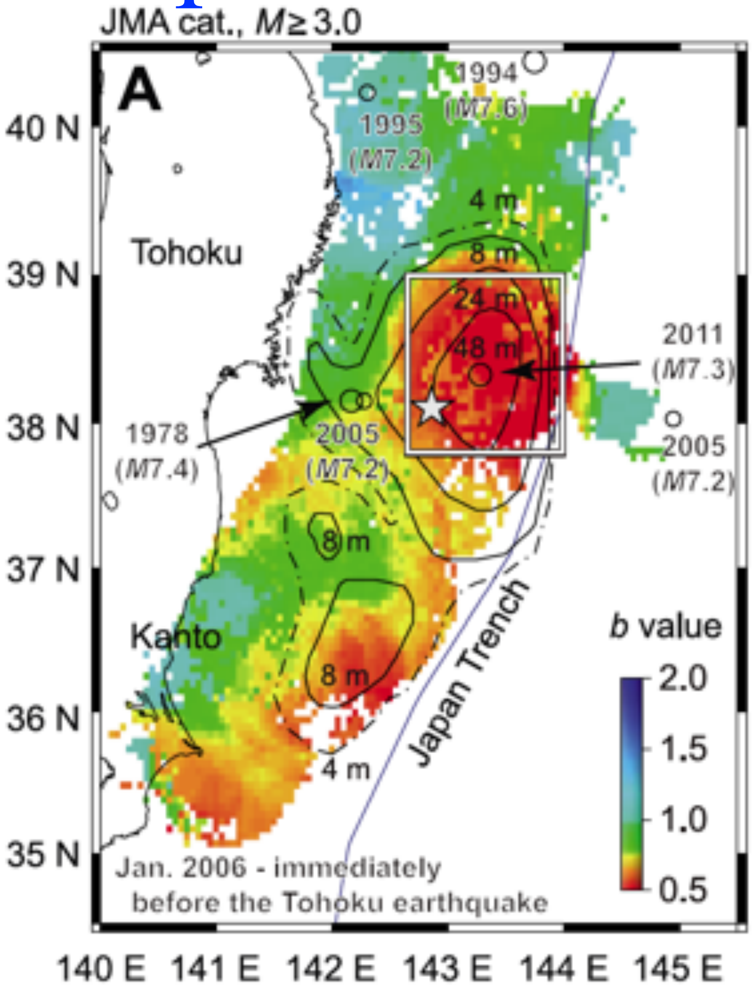
$$N \propto S^{-b}$$

$$b \simeq 1$$

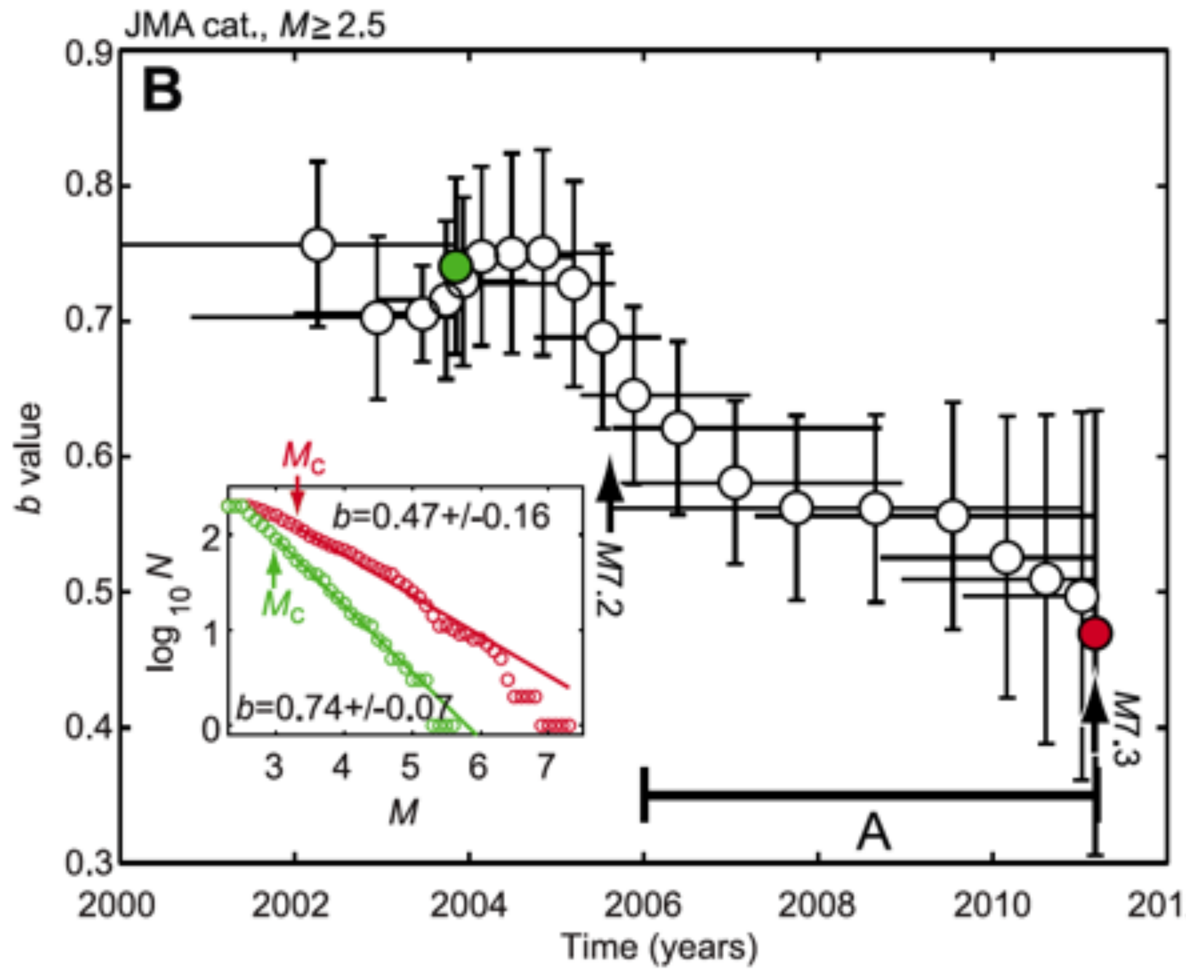
b-value decreases before the occurrence of large earthquake.

2011.03.11 Tohoku-oki M9 Earthquake

map of *b*-value



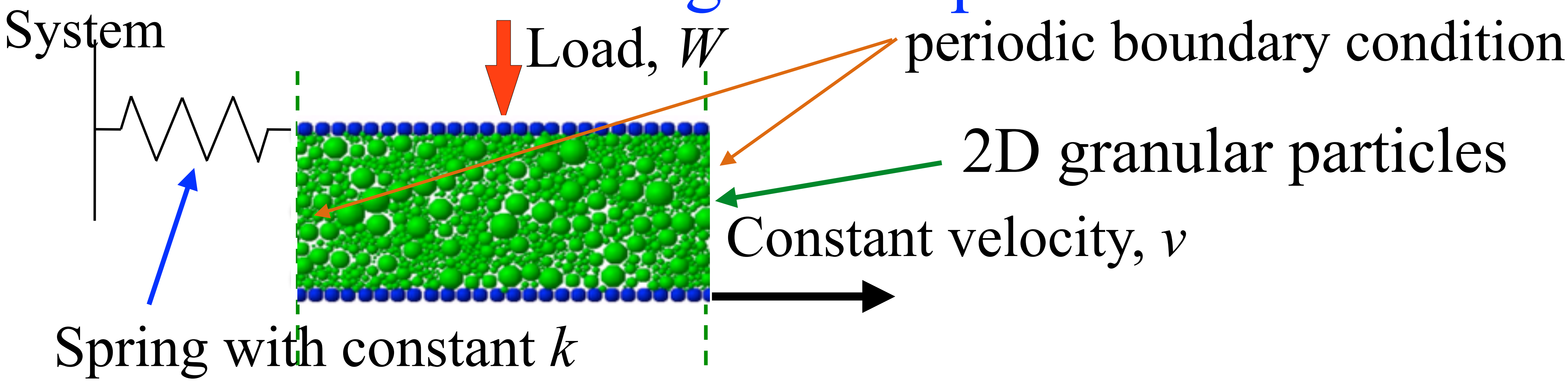
time dependence of *b*-value



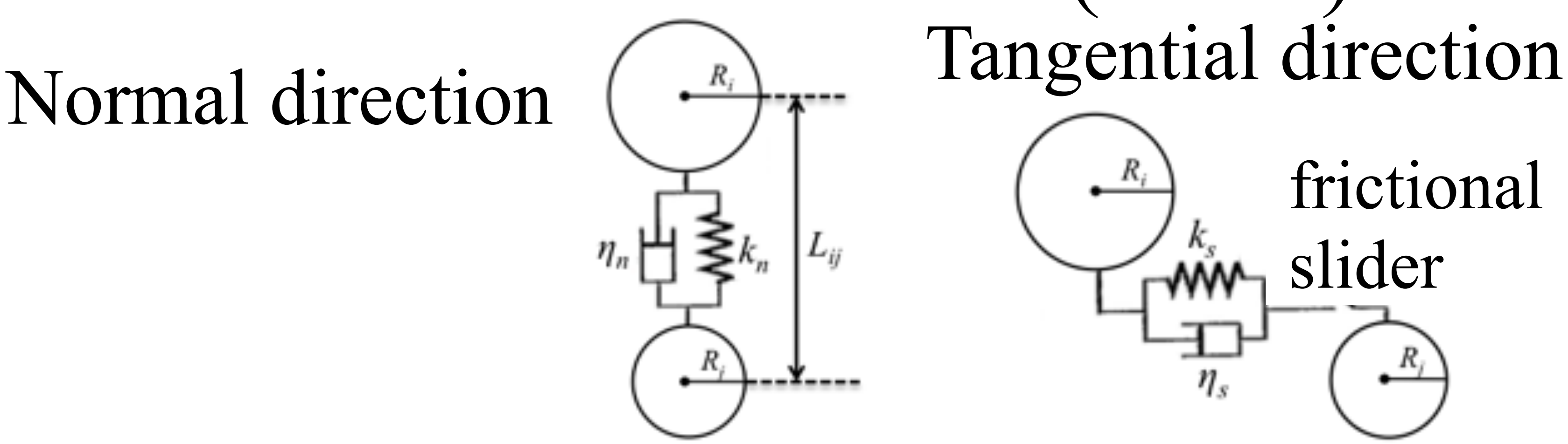
Nanjo et al.

What determines the *b*-value?

Numerical Study of the Frictional Behavior of 2D sheared granular particles



Discrete Element Method (DEM) is employed.



Coulomb friction works, but the static and kinetic friction coefficients are the same.

We examined 3 types of particle size distribution, monodispesre, uniform and power law distributions.

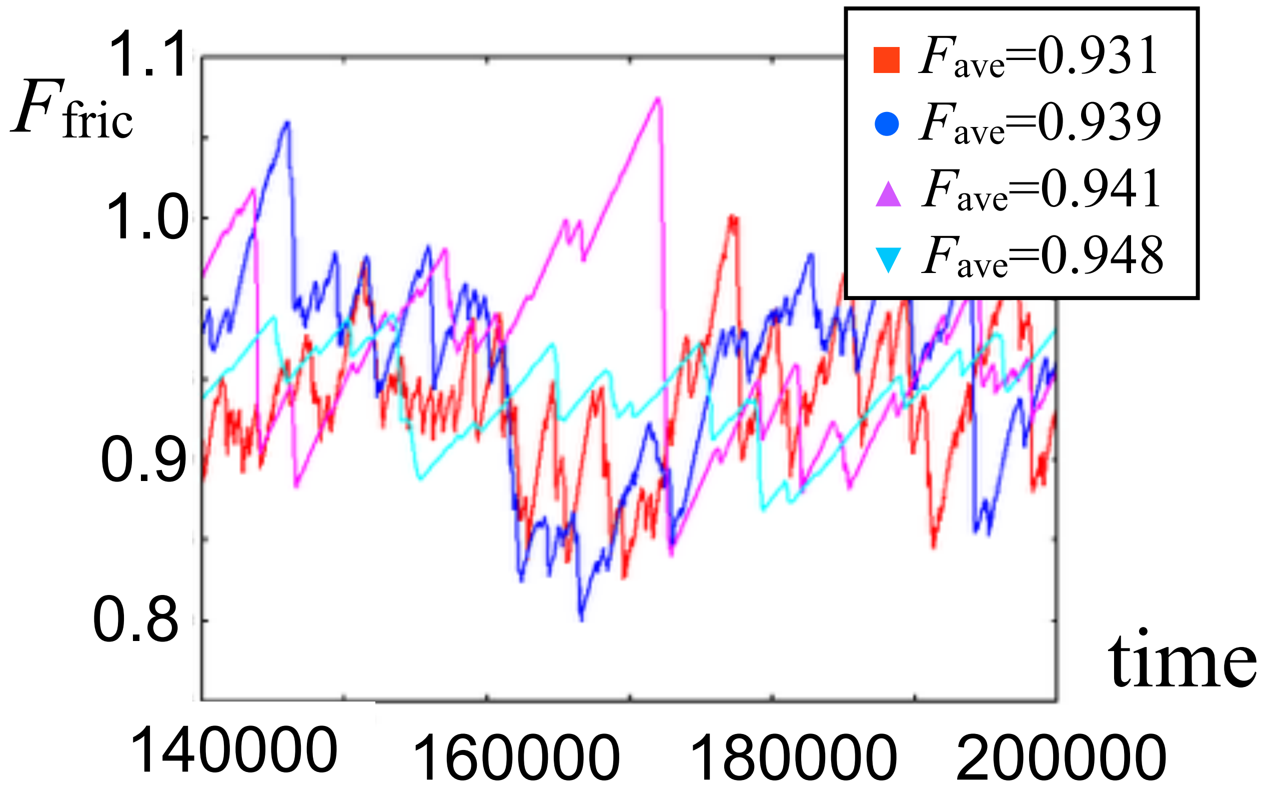
Frictional behaviors do not depend on the type of distribution.

Here we concentrate on the power law distribution system.

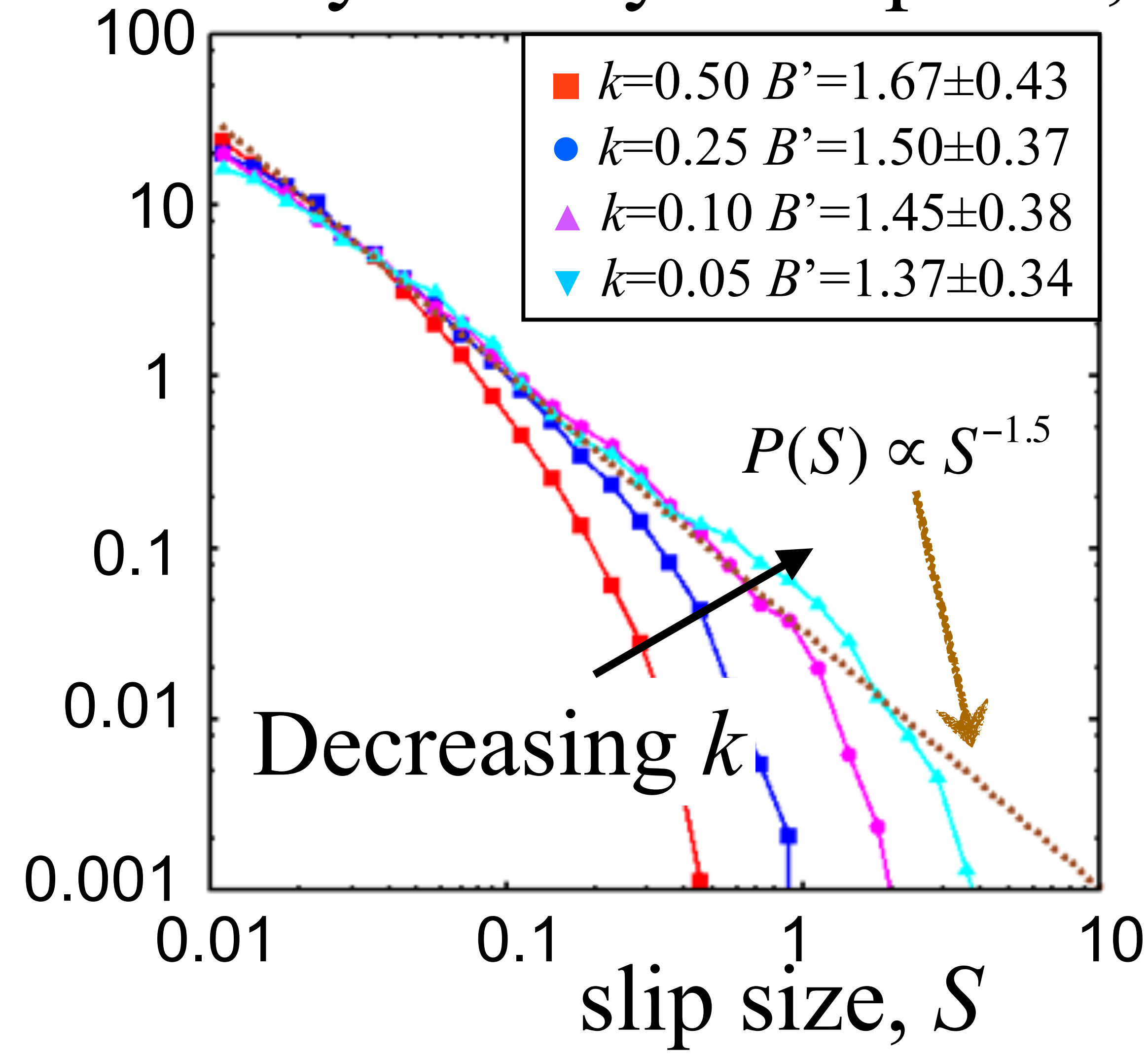
Spring constant k dependence

W=4.5 v=2.5×10^{-4} d_1=0.2~2.0

Time variation of friction force



Probability density of slip size, $P(S)$



Clear stick-slip motion appears.

Probability density of slip size, $P(S)$, depends on the spring constant k.

$P(S)$ approaches to $S^{-B'}$ with decreasing k, where B'=1.5.

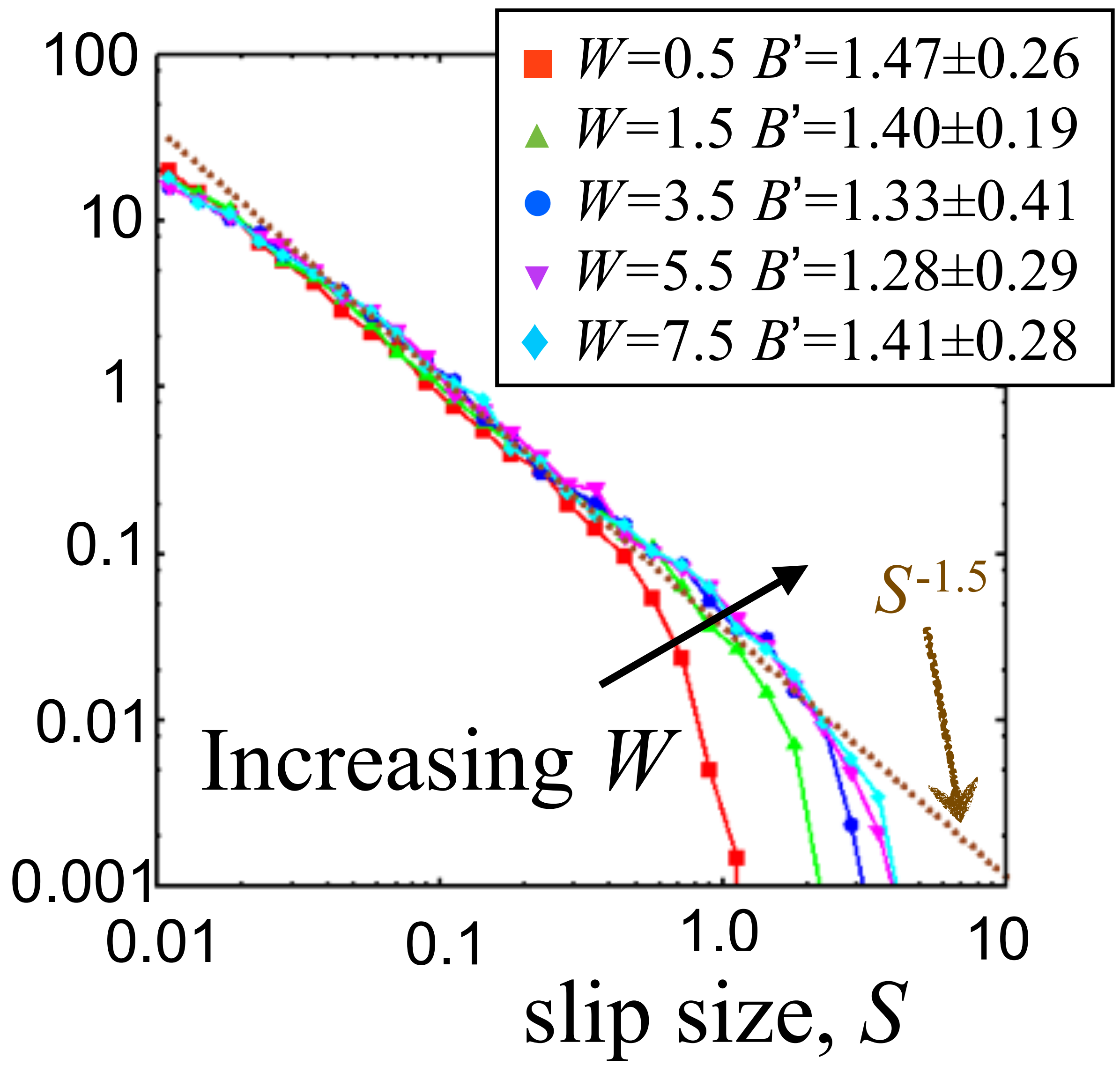
$$B' = 2/3b + 1$$

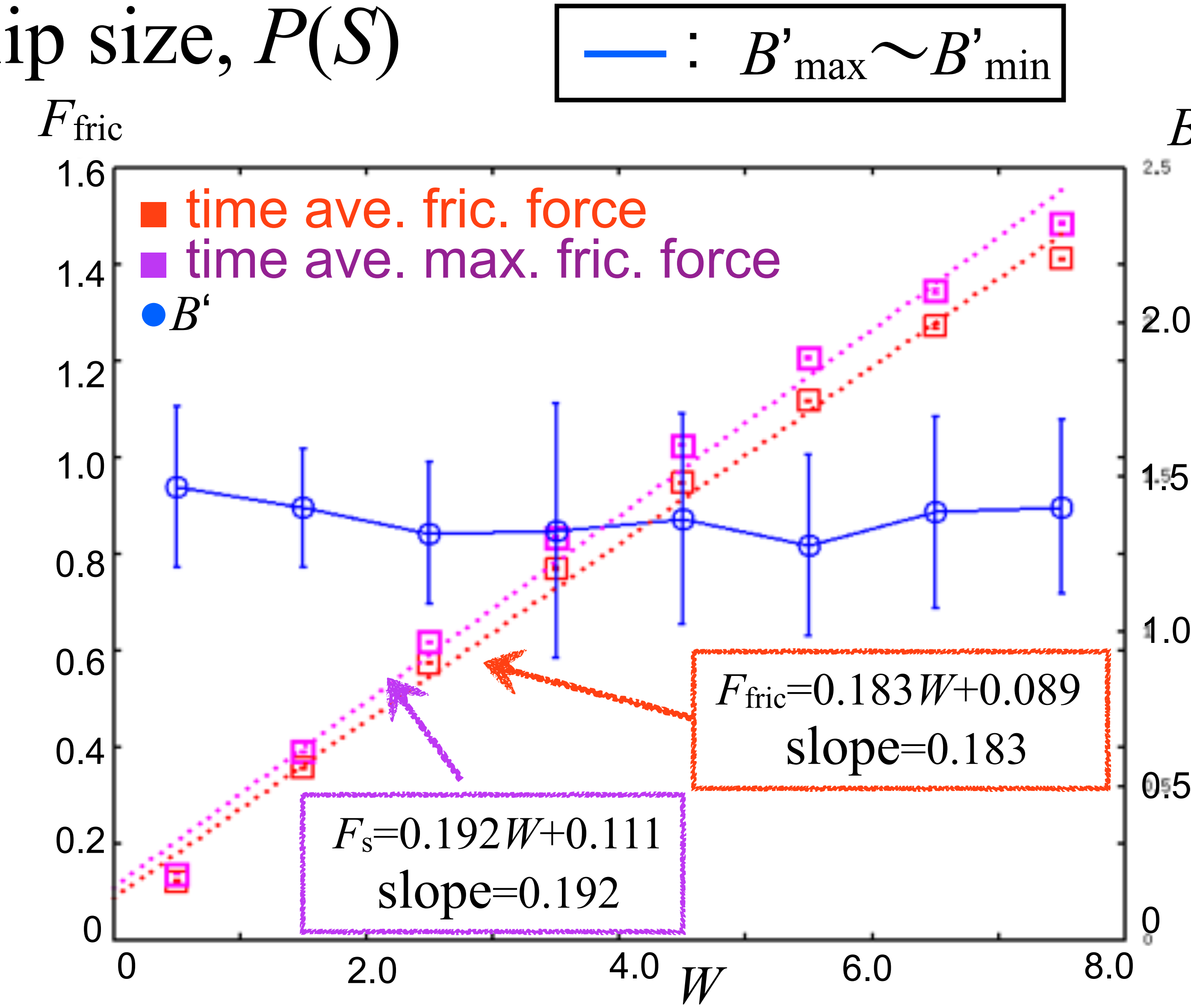
B'=1.5 coincides with the mean field theory by Dahmen et al.

Load W dependence

k=0.05 d_1=0.2~2.0 v=2.5×10^{-4}

Probability density of slip size, $P(S)$





Probability density of slip size, $P(S)$, depends on the load W.

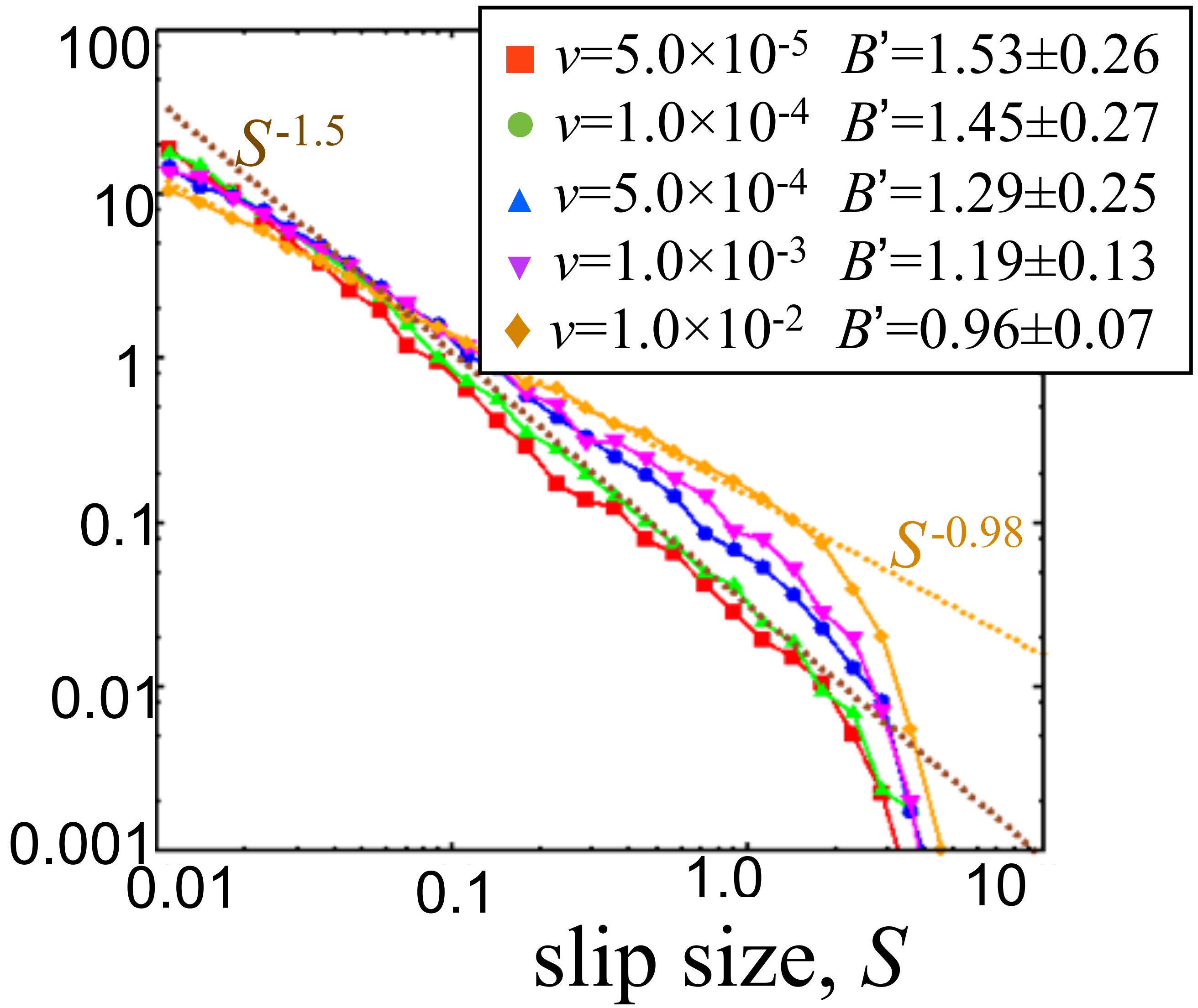
$P(S)$ approaches to $S^{-B'}$ with increasing W, where B'=1.5.

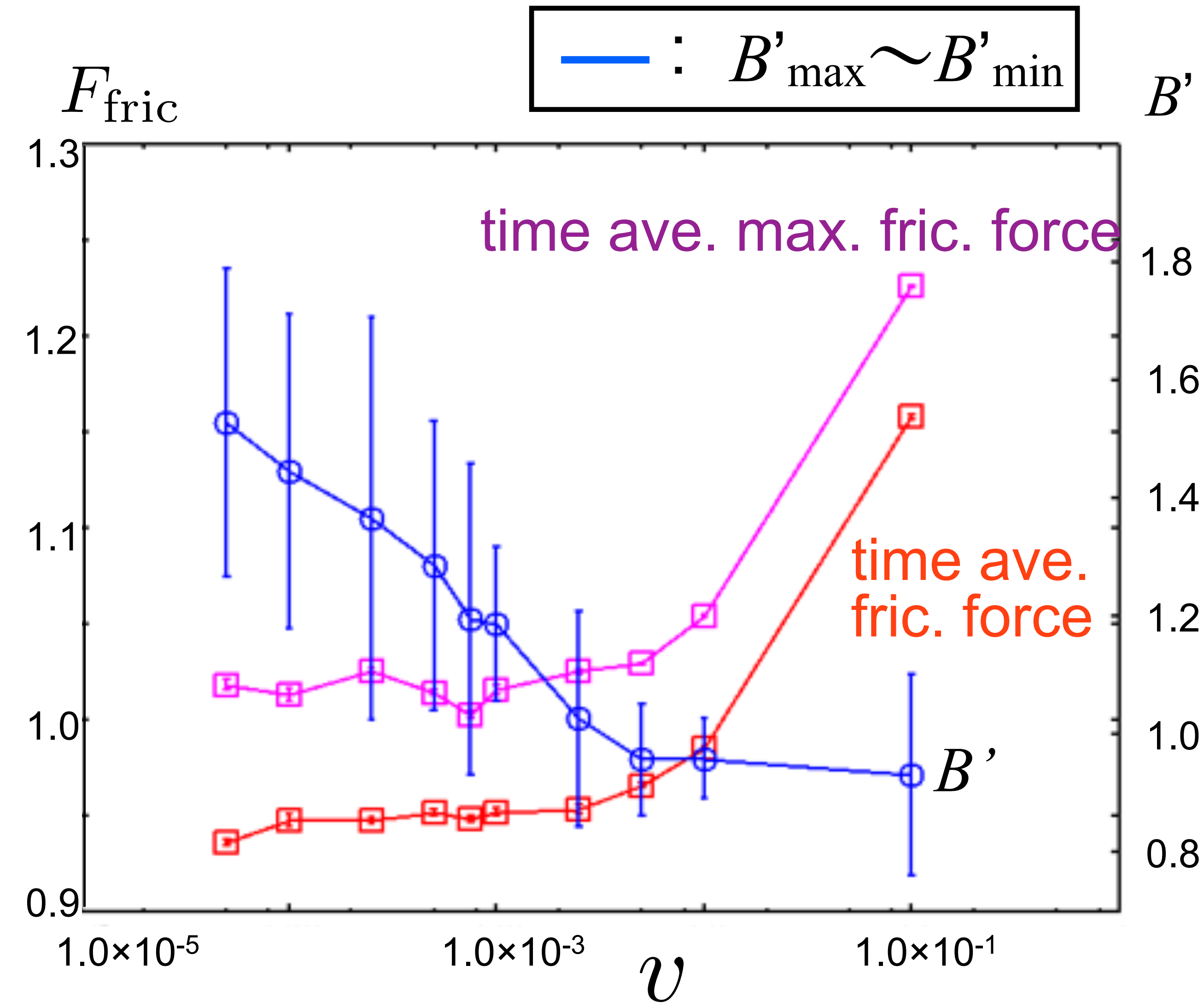
Time averaged friction force does not obey Amontons' law, but depends on W linearly.

Driving velocity v dependence.

k=0.05 d_1=0.2~2.0 W=4.5

Probability density of slip size, $P(S)$





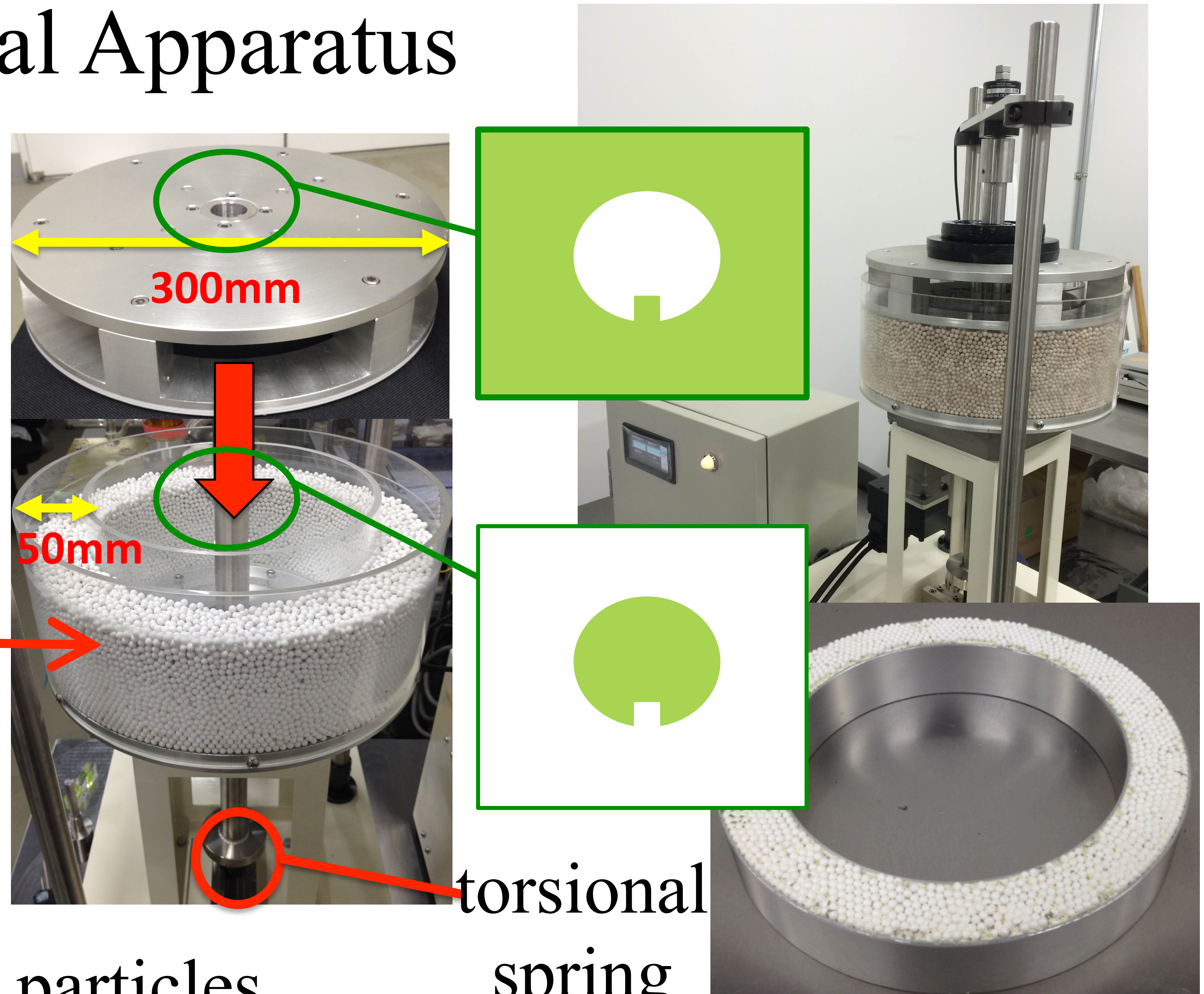
B' increases with decreasing v and approaches to 1.5.

Ave. friction force increases with v.

But the system shows stick slip motion.

Experimental Study of the Frictional Behavior of 3D Sheared Granular Particles

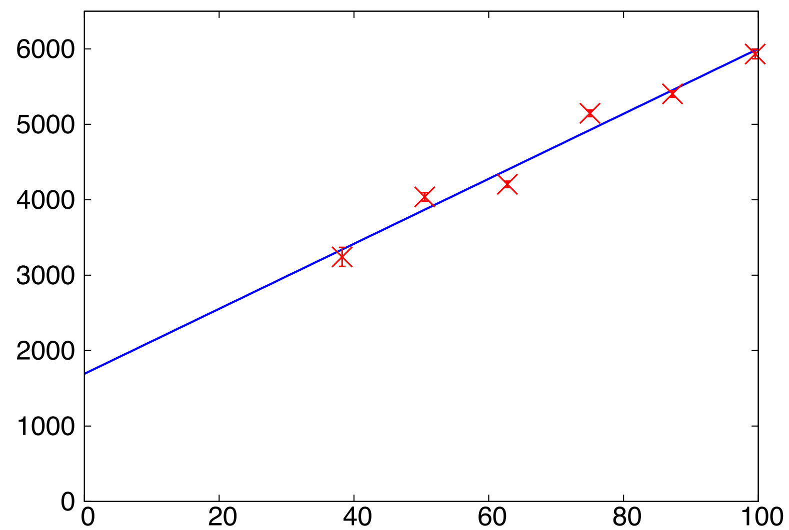
Experimental Apparatus



Granular particles
Alumina ball of 5mm
in diameter

Load W dependence

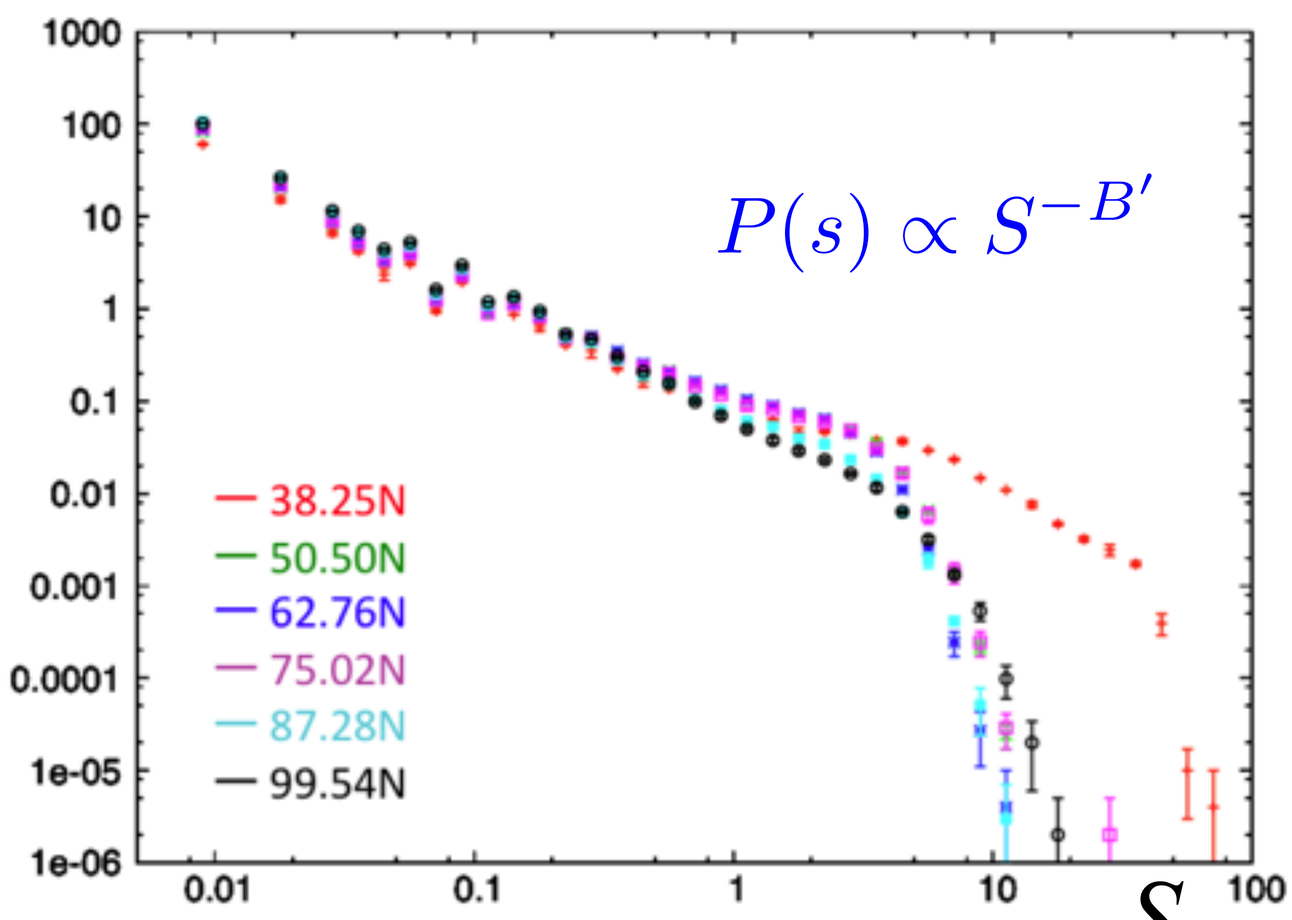
Ave. Max.Fric.Force

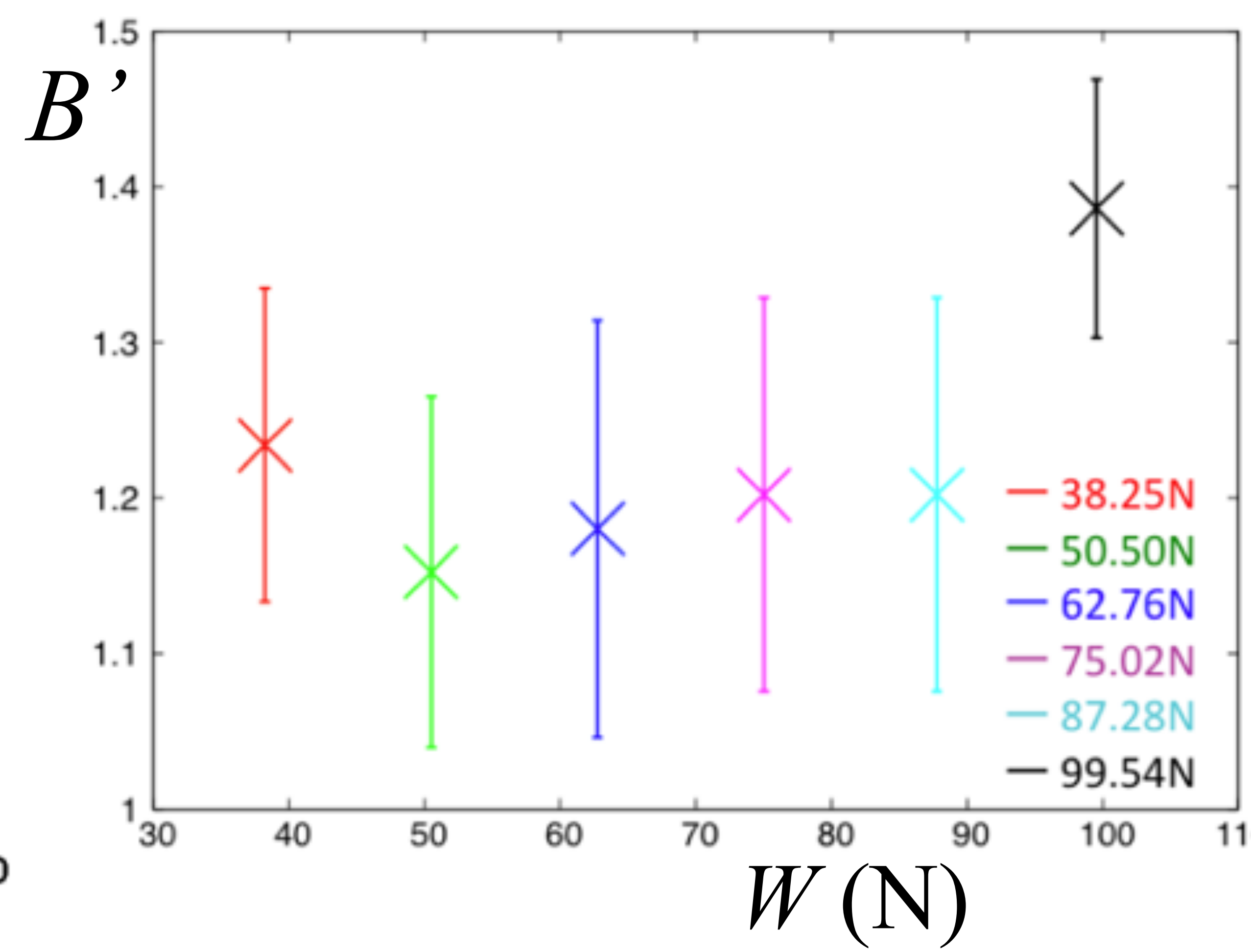


Amontons' law does not holds, but the friction force depends linearly on W.

S is the change of angle by the slip (degree).

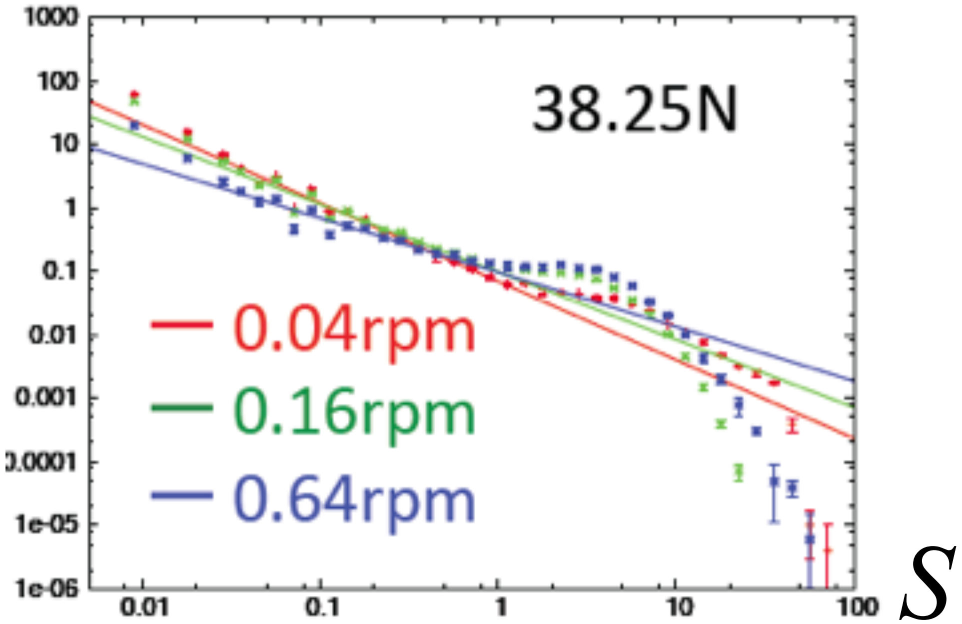
Probability density of slip size, $P(S)$

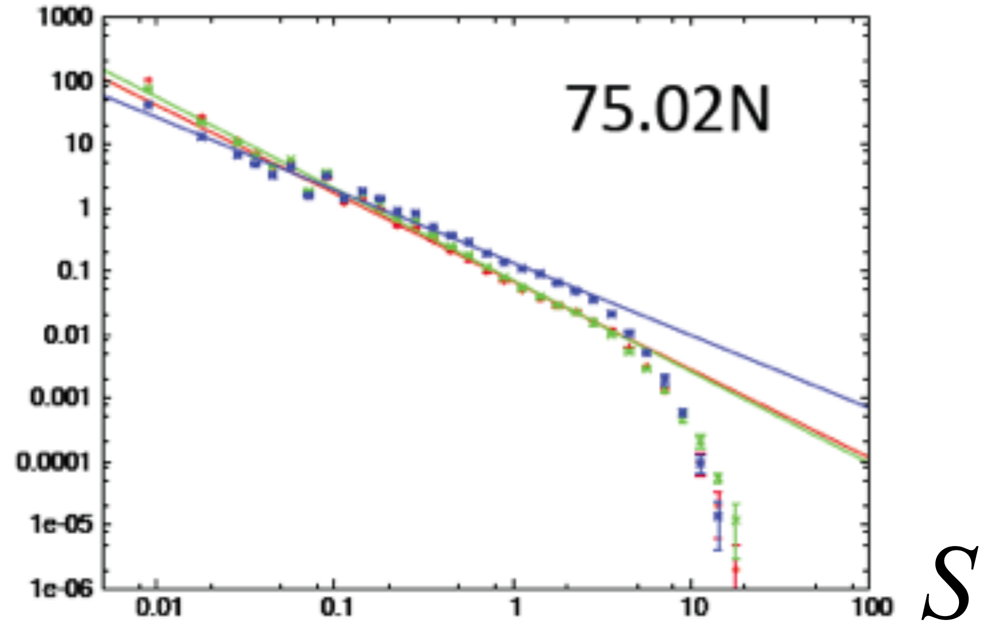




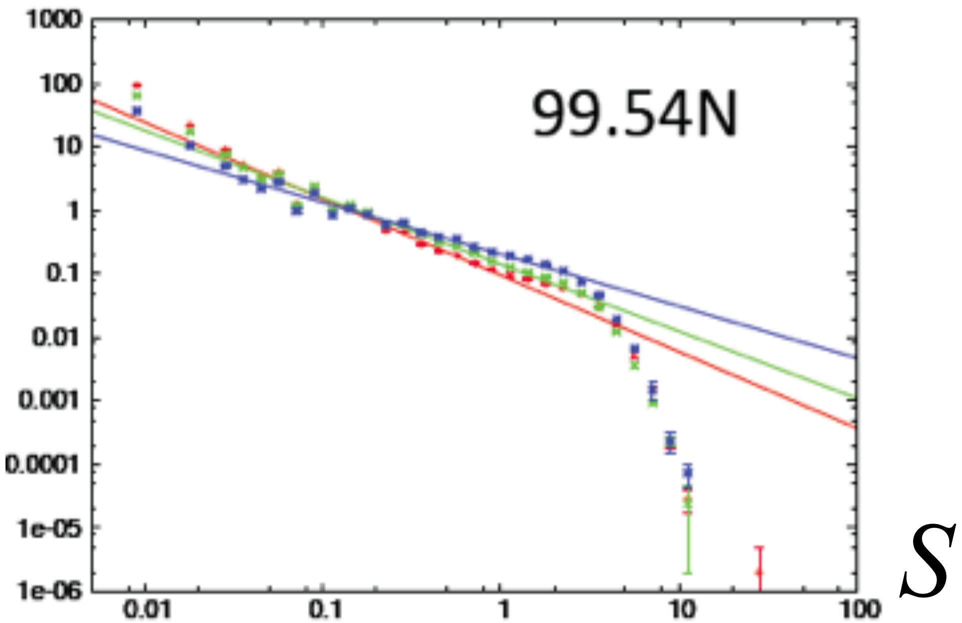
Driving velocity v (rpm) dependence

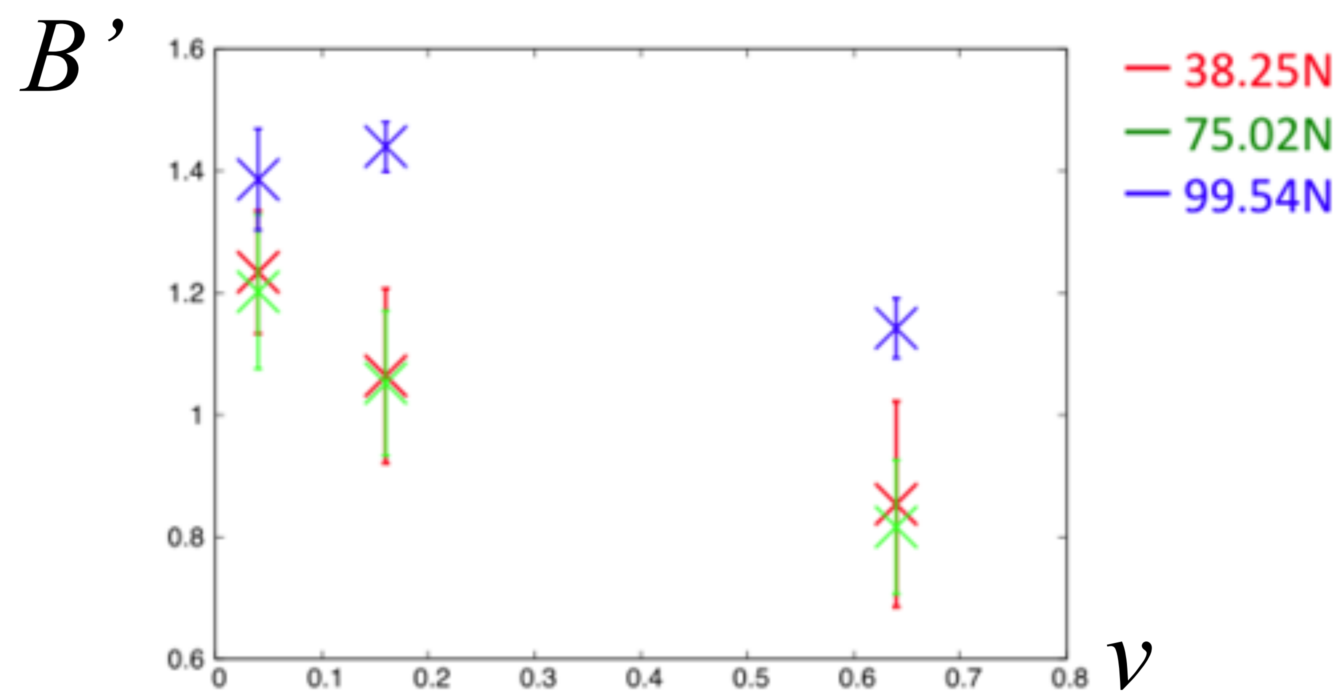
Probability density of slip size, $P(S)$





$$P(s) \propto S^{-B'}$$





Consistent with the 2D simulation → B' increases with decreasing v and approaches to 1.5.

Summary of Frictional Behavior and the Stick-slip Dynamics of Sheared Granular Systems

Amontons' law does not holds but the friction force depends on the load linearly.

Friction force increases with increasing driving velocity, but the system shows stick-slip motion in 2D simulation.

The slip size distribution obeys the power law in the whole range of slip in the limit of weak driving spring and large load in 2D simulation. ← The system does not show self-organized criticality.

The power *B*' increases with decreasing *v* and approaches to 1.5, which is the value obtained by the mean field theory.

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