

# Equilibrium sampling of hard spheres up to the jamming density and beyond

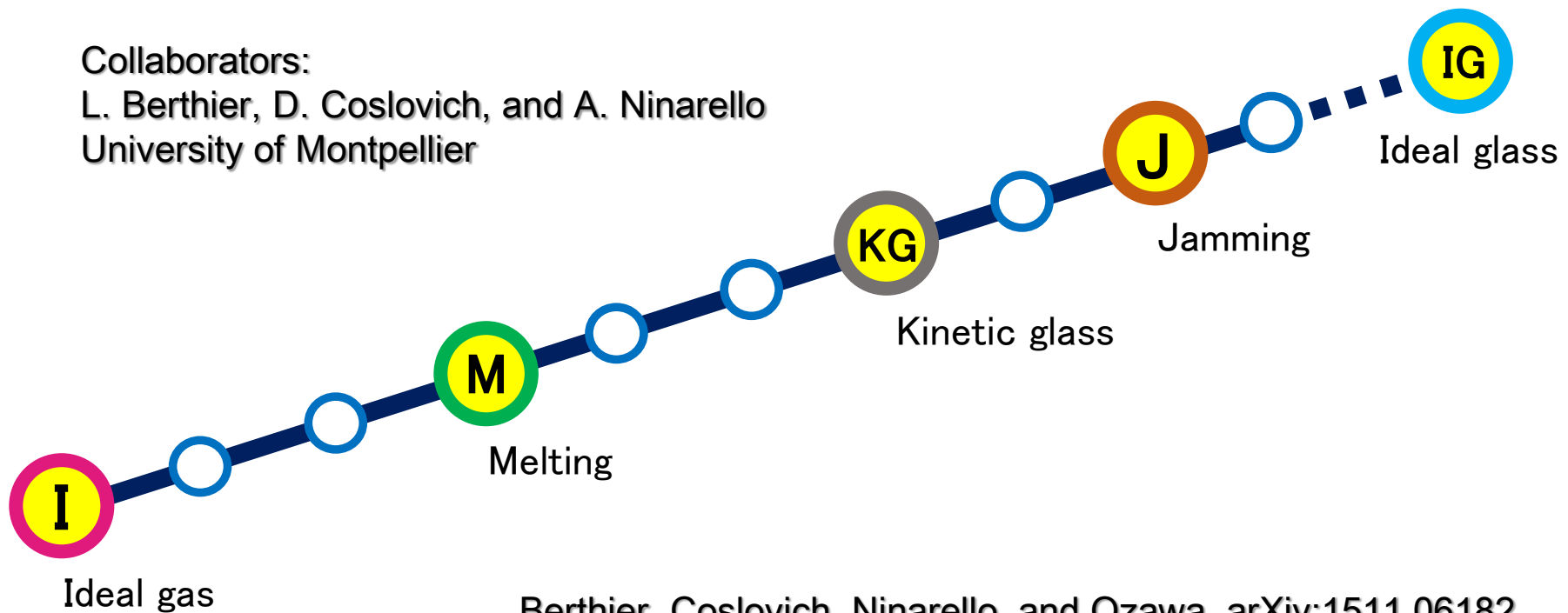
Misaki Ozawa

Nagoya University

Collaborators:

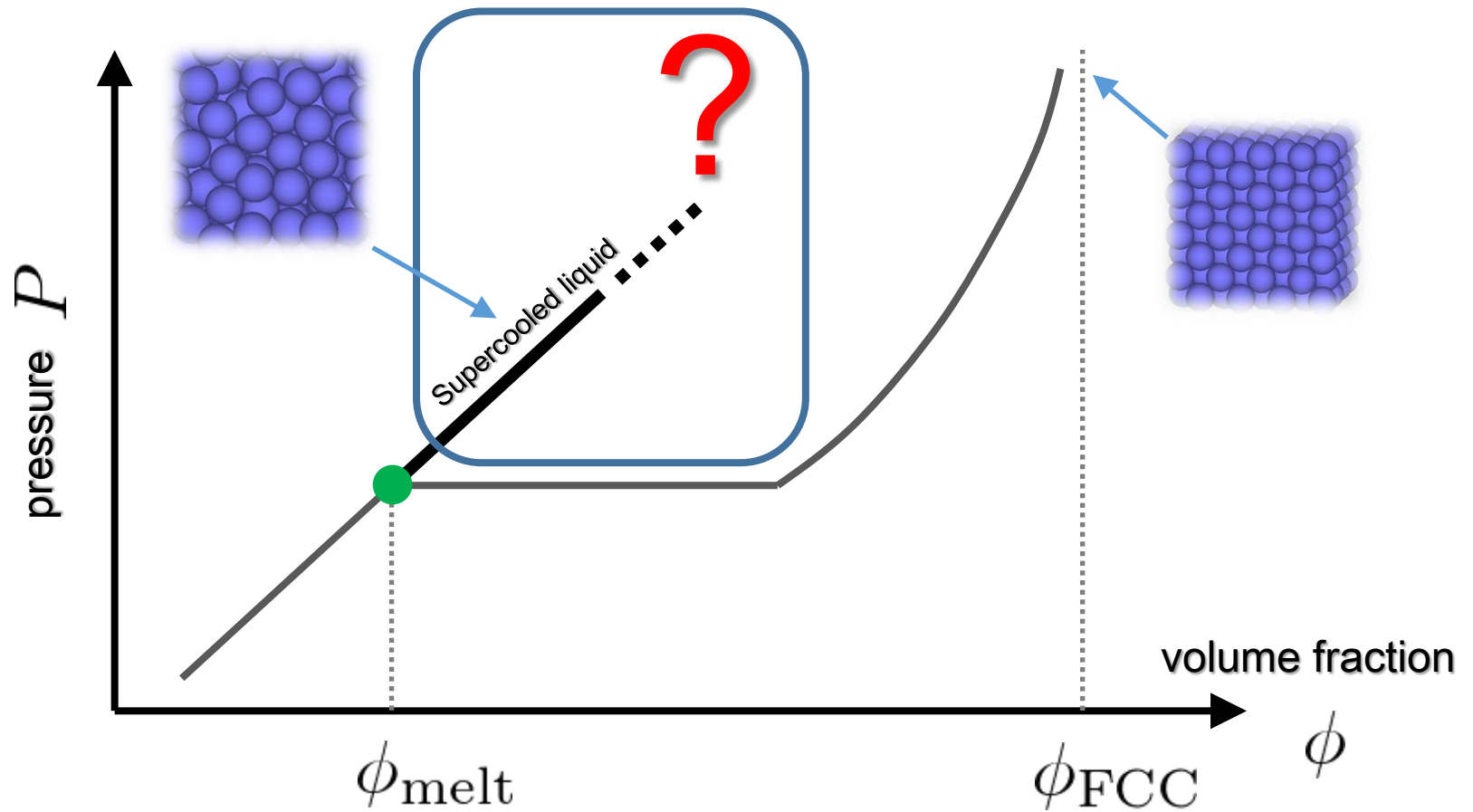
L. Berthier, D. Coslovich, and A. Ninarello

University of Montpellier

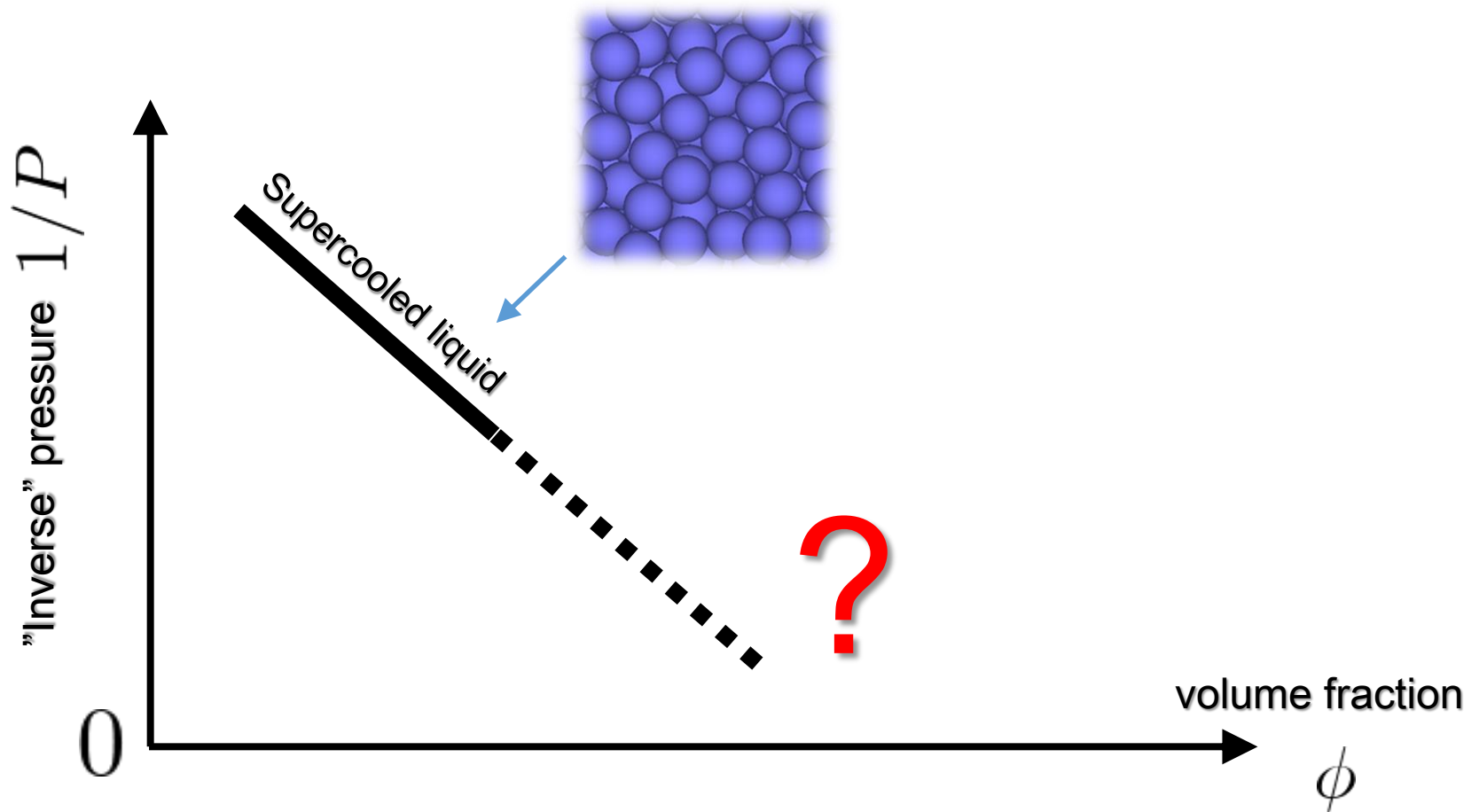


Berthier, Coslovich, Ninarello, and Ozawa, arXiv:1511.06182

# Introduction: Equation of state of hard spheres



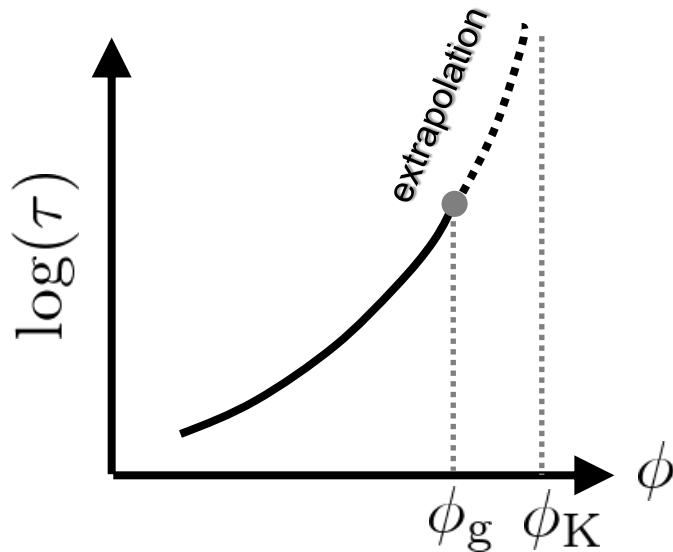
# Introduction: Equation of state of hard spheres



- What is the ultimate fate of the supercooled liquid state?

# Introduction: Glass and Jamming transitions

## Glass transition



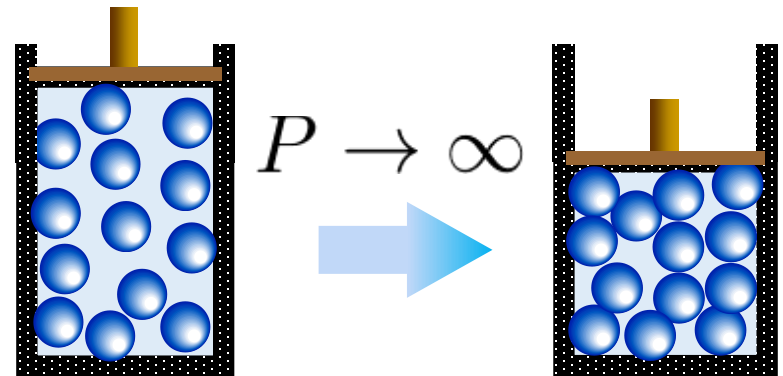
- Kinetic glass transition

$$\tau(\phi_g) = 10^2 \text{ sec.}$$

- Ideal glass transition

$$\tau(\phi_K) = \infty$$

## Jamming transition



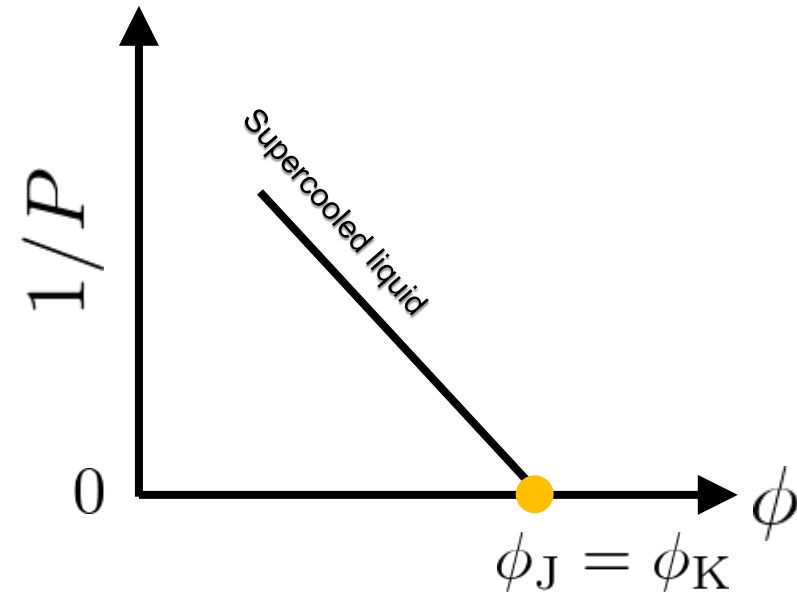
- Jamming transition density

$$\phi_J \sim 0.64$$

- Average contact number

$$\bar{z} = 2d \text{ (Isostaticity)}$$

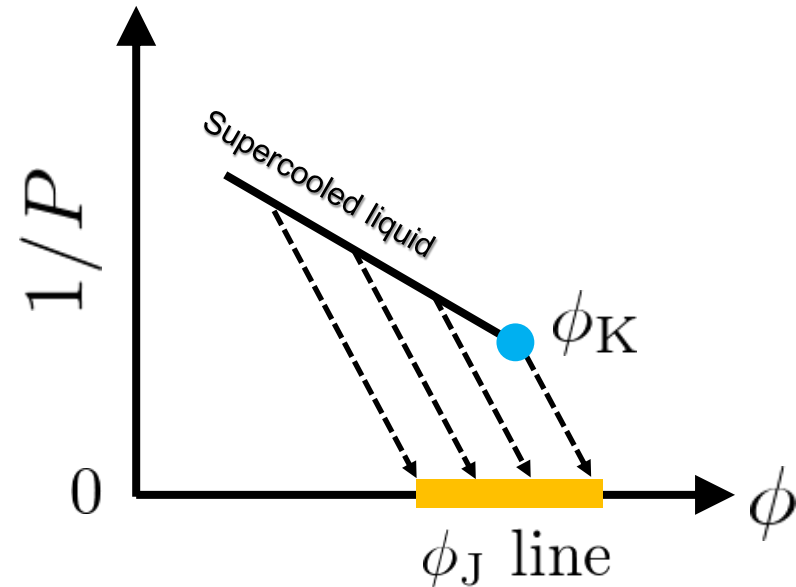
# Introduction: Two scenarios



Kamien and Liu, PRL 2007

Aste and Coniglio, EPL 2004

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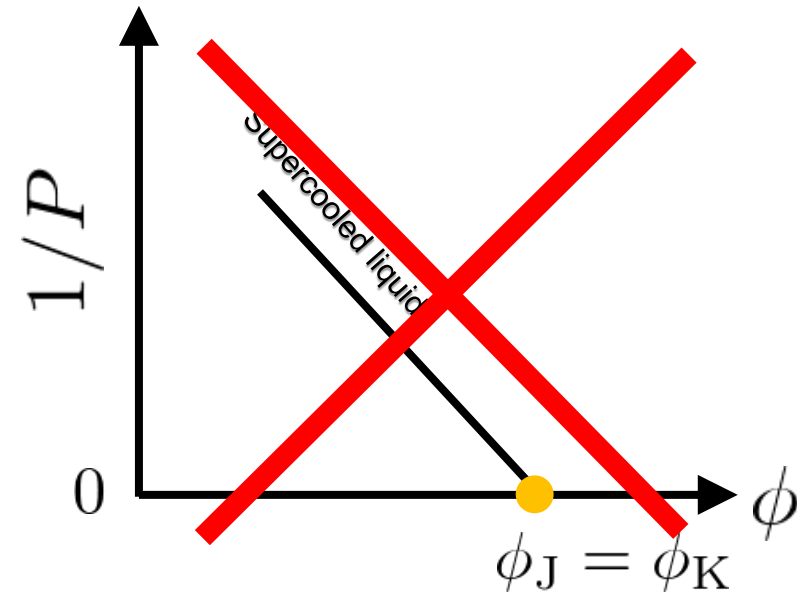


Mari, Krzakala, and Kurchan, PRL 2009

Parisi and Zamponi, RMP 2010

## Motivation of this work

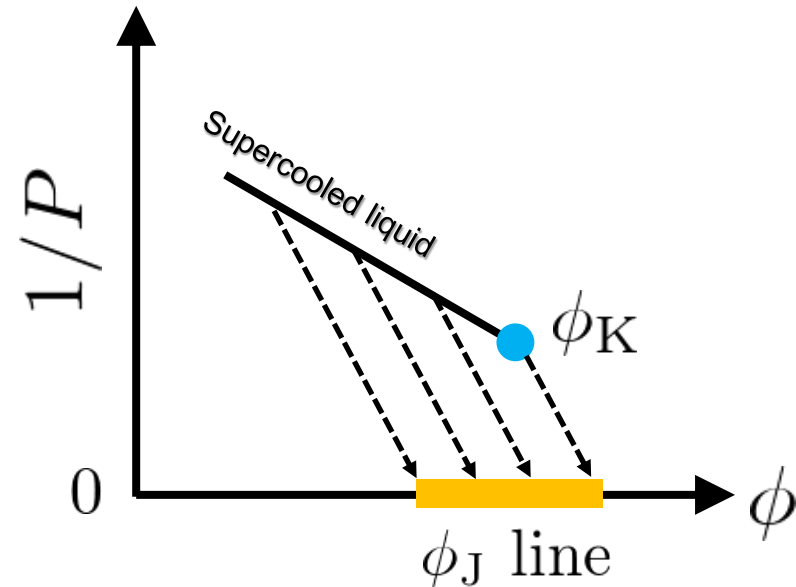
Making equilibrium states  $\phi > \phi_J$  !



Kamien and Liu, PRL 2007

Aste and Coniglio, EPL 2004

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Mari, Krzakala, and Kurchan, PRL 2009

Parisi and Zamponi, RMP 2010

# Model and simulation method

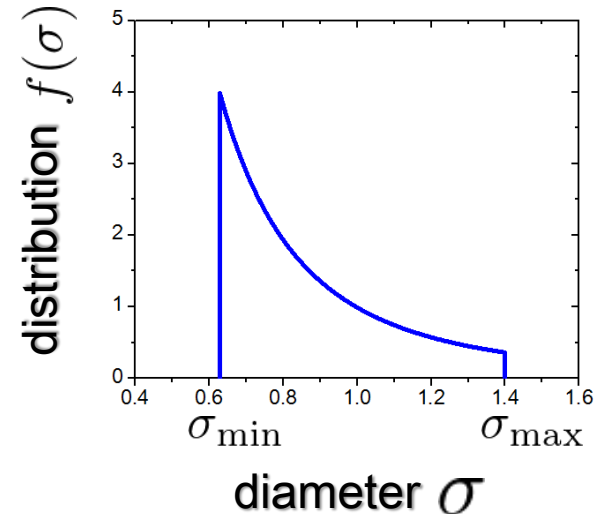
## ■ Continuous polydisperse hard spheres

$d = 3$  with periodic boundary condition

$$f(\sigma) \propto \sigma^{-3}$$

$$\delta = \frac{\sqrt{\overline{\sigma^2} - \bar{\sigma}^2}}{\bar{\sigma}} = 0.23$$

$$N = 300, 1000, 8000$$



## ■ Equilibration algorithm

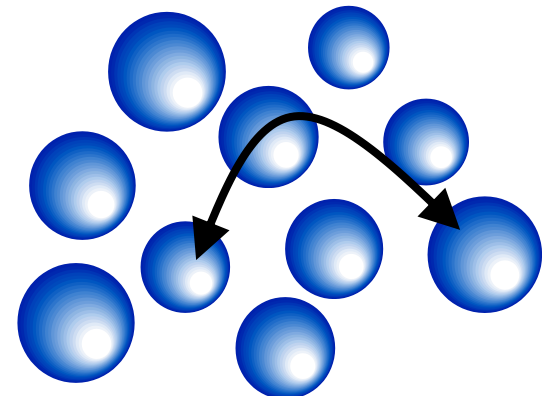
Standard Monte Carlo

+ non-local swap displacement

Grigera and Parisi, PRE 2001

Brumer and Reichman, JPCB 2004

Fernandez, Martin-Mayer, and Verrocchio, PRL 2007



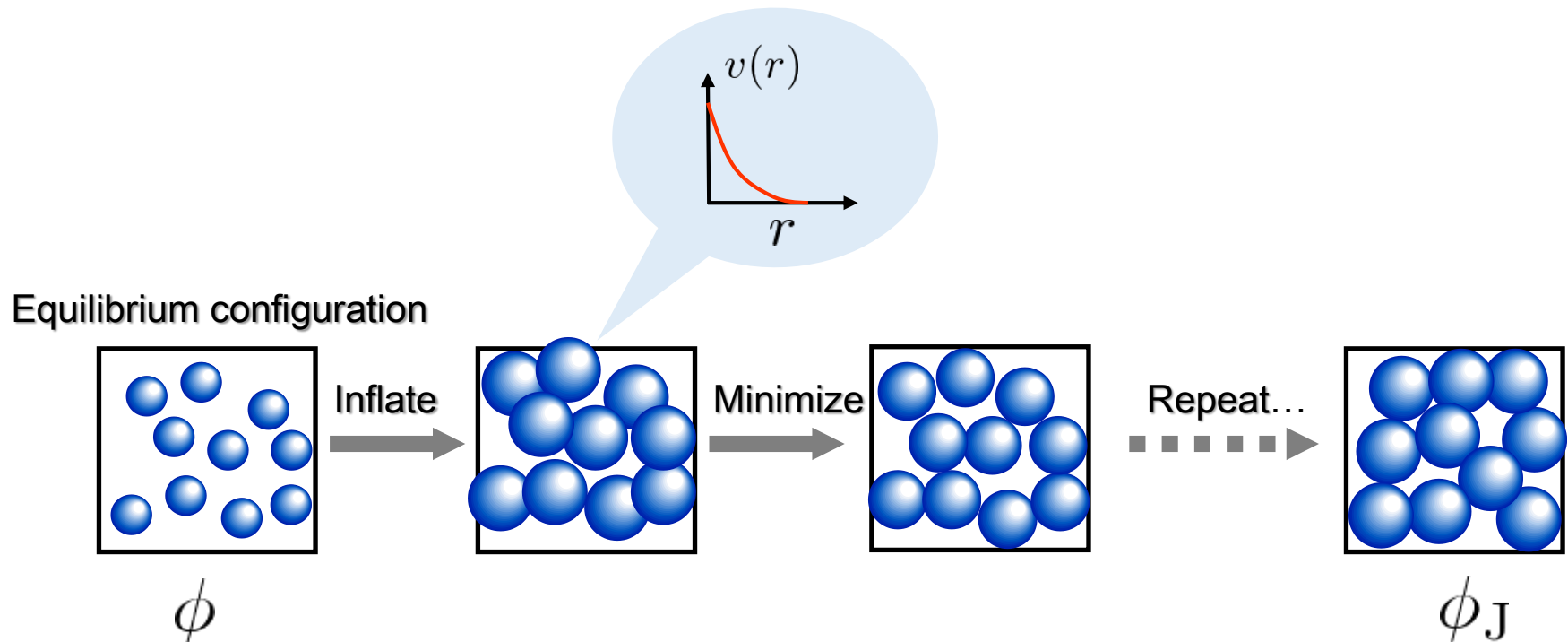
# Model and simulation method

## ■ Non-equilibrium compression algorithm for $\phi_J$

O'Hern et al., PRE 2003

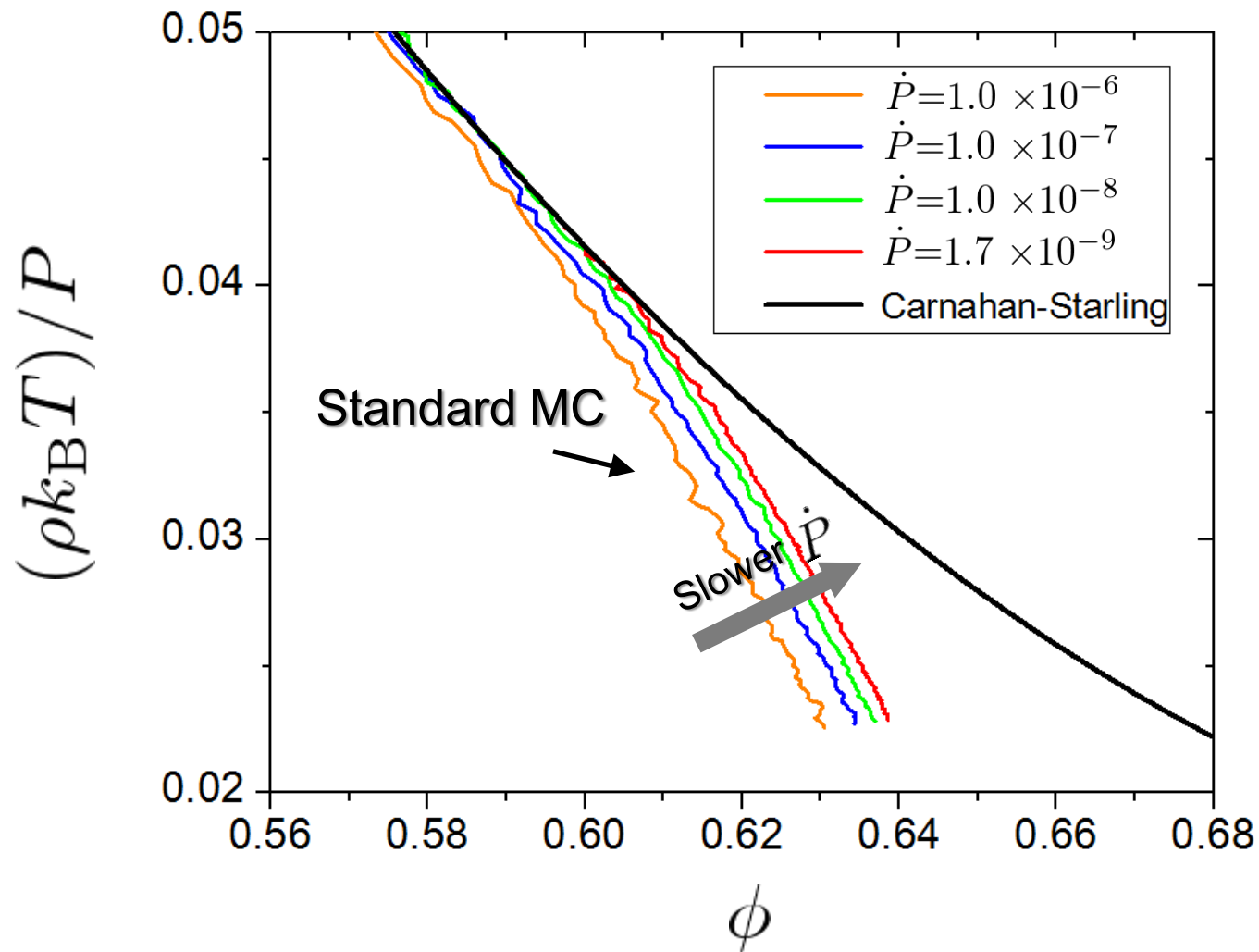
Xu, Blawdziewicz, and O'Hern, PRE 2005

Chaudhuri, Berhier, and Sastry, PRL 2010



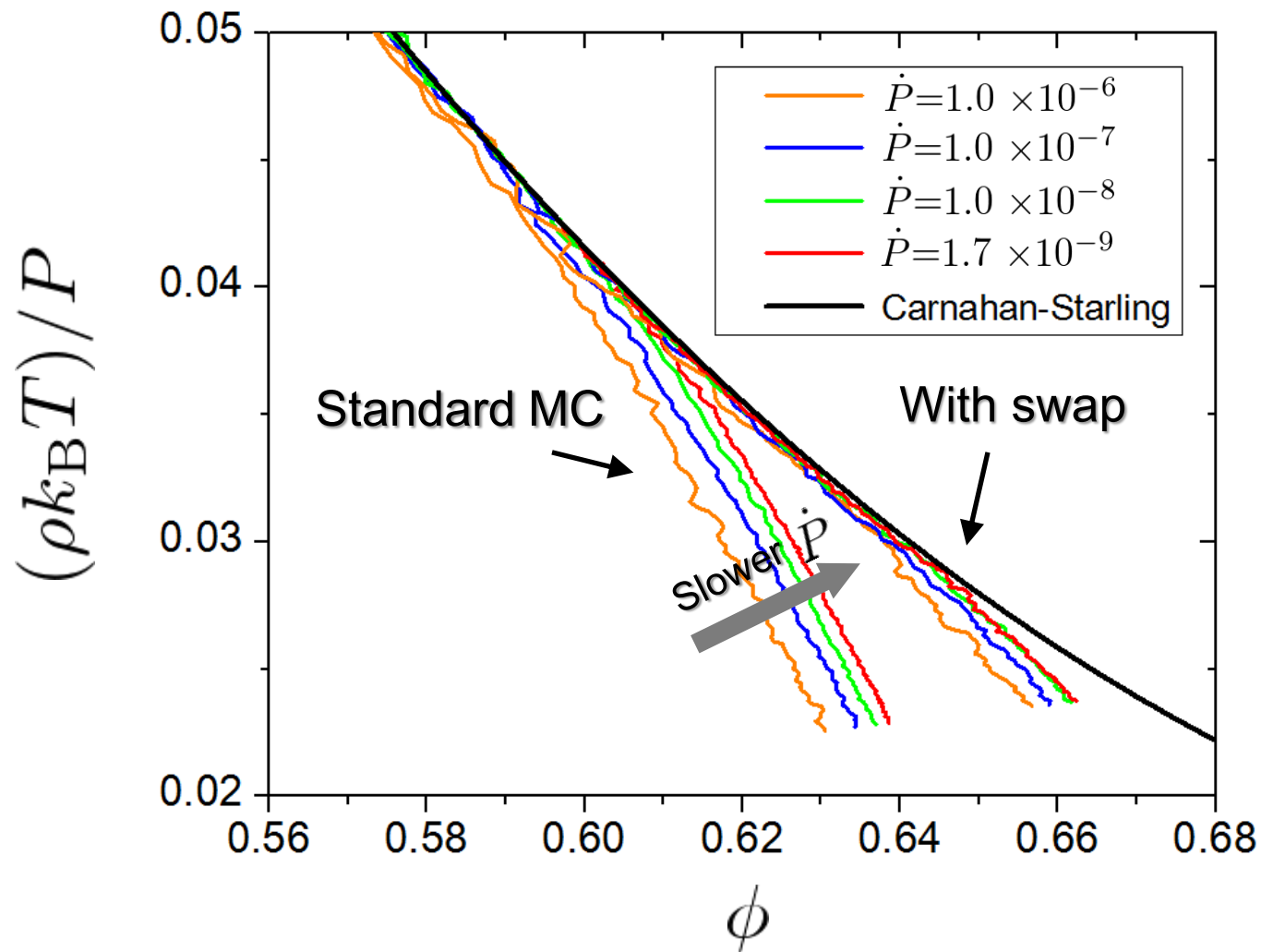


## Results: Performance of the swap displacement



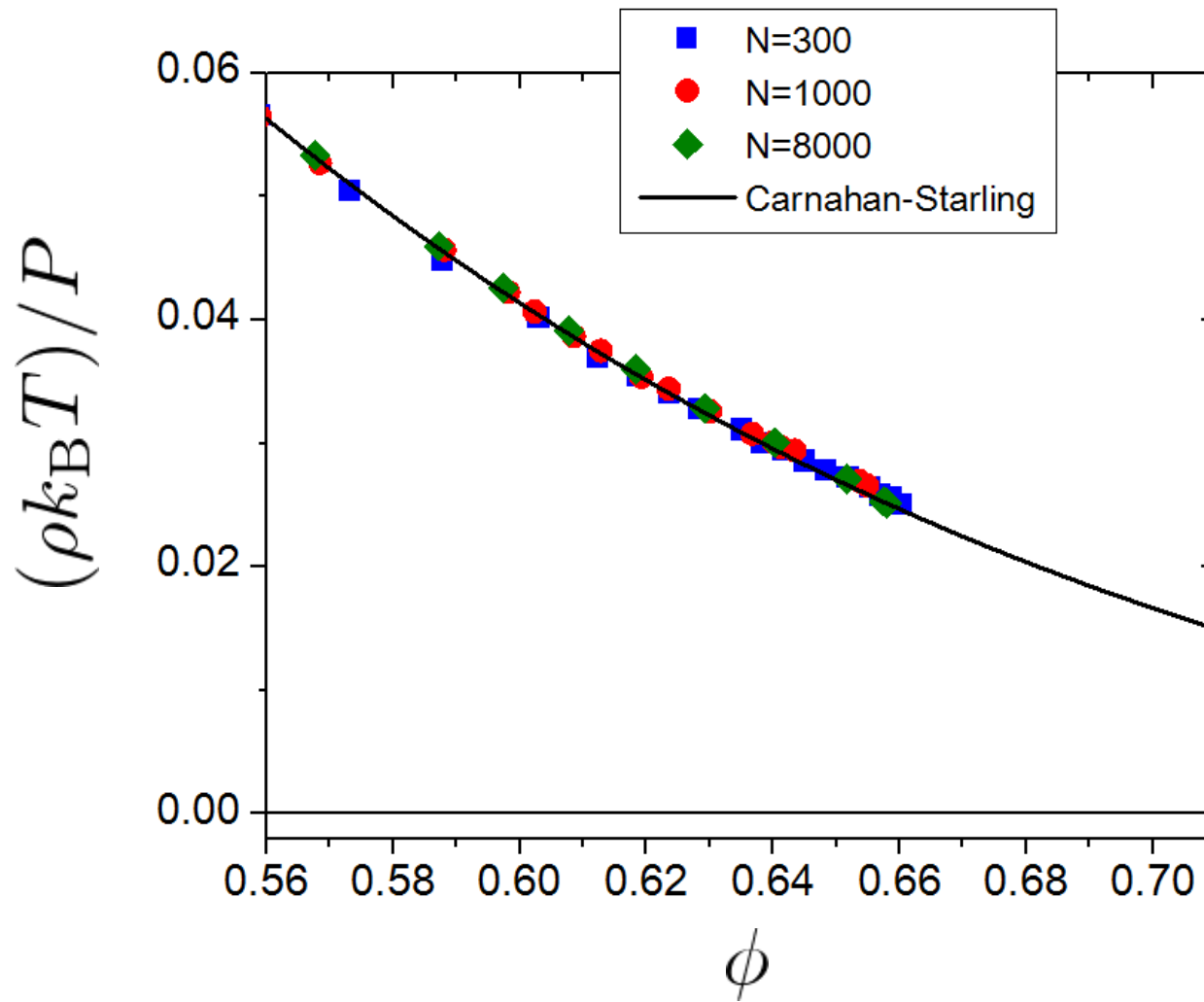
■ Standard MC falls out-of-equilibrium quickly

## Results: Performance of the swap displacement



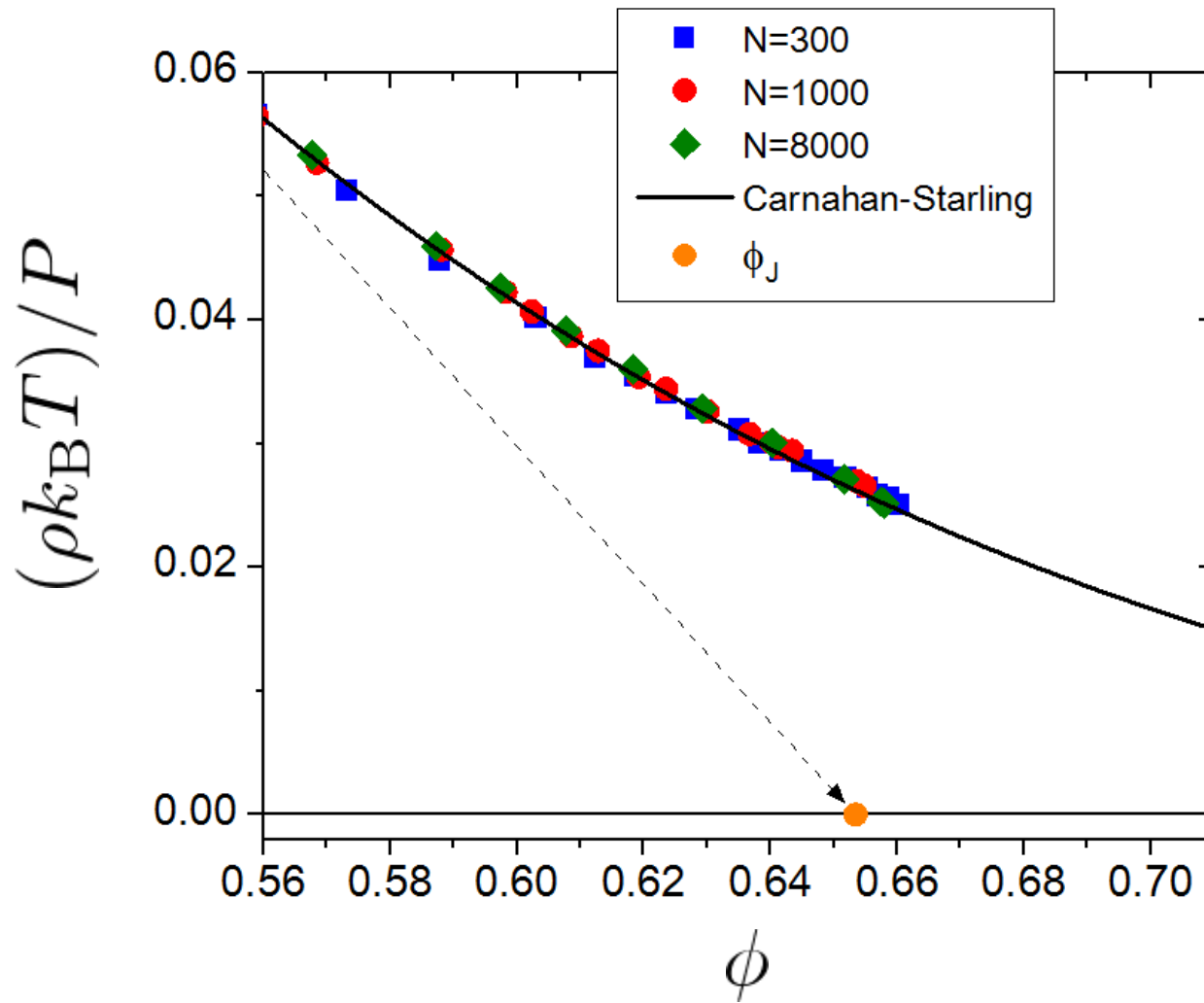
■ Non-local swap displacement accelerates equilibration!

## Results: Equation of state



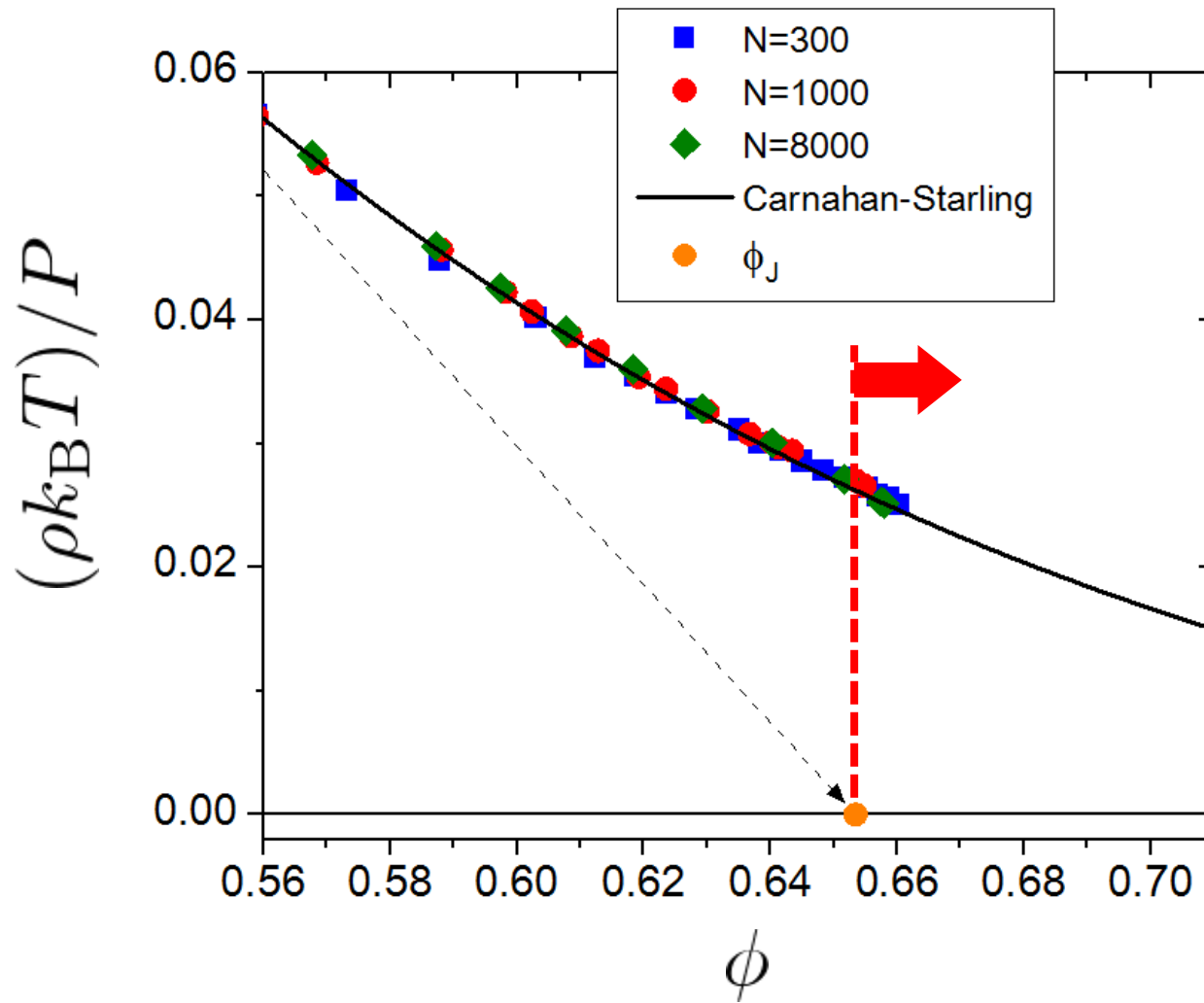
■ The system is equilibrated up to  $\phi \sim 0.66$

## Results: Equation of state



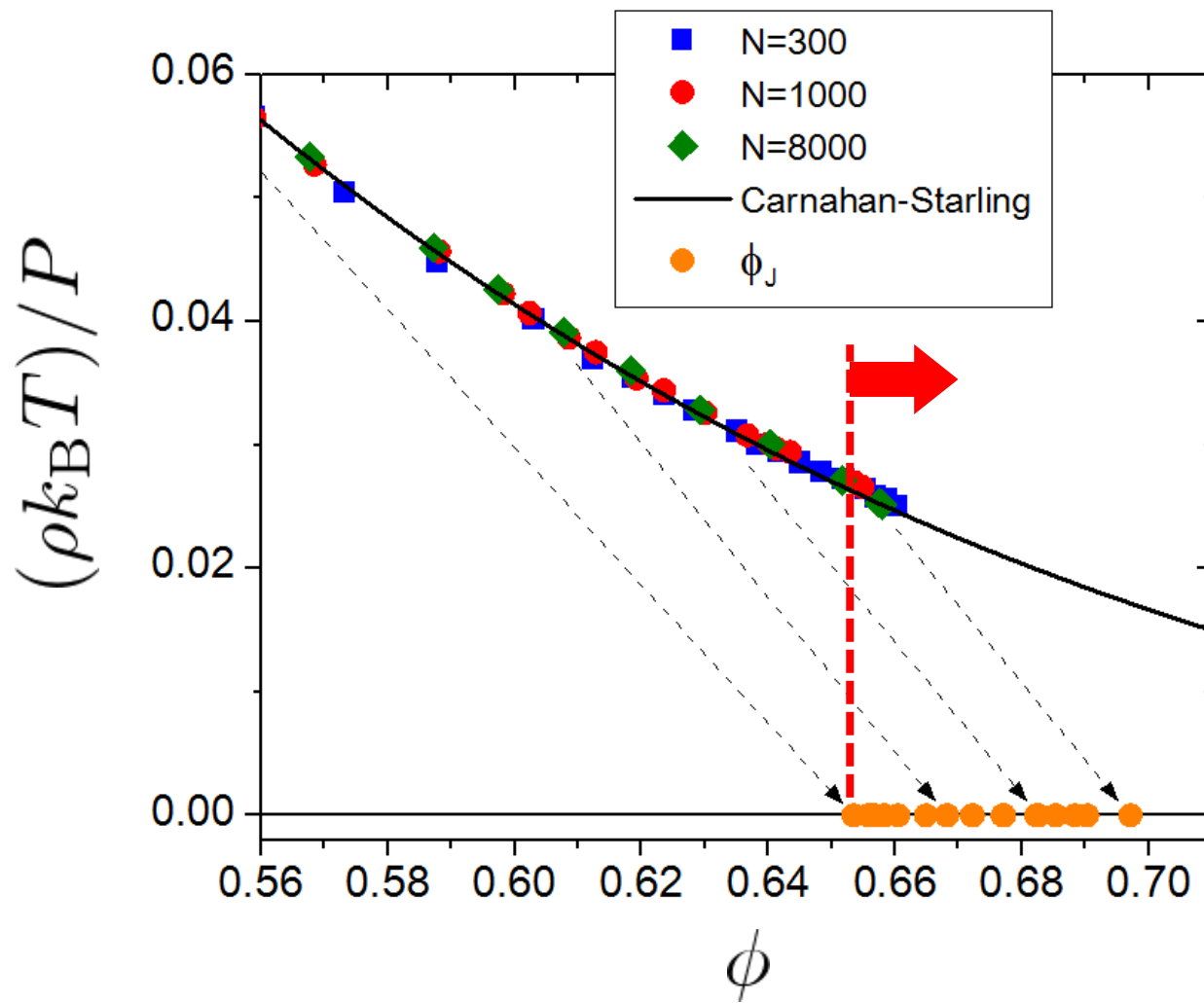
- $\phi_J \sim 0.655$  is obtained by compressing dilute liquids

## Results: Equation of state



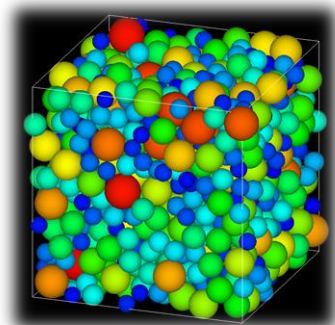
■ The system is equilibrated beyond  $\phi_J$

## Results: Equation of state

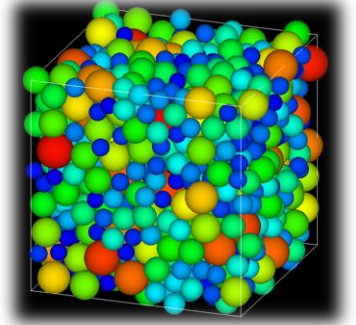
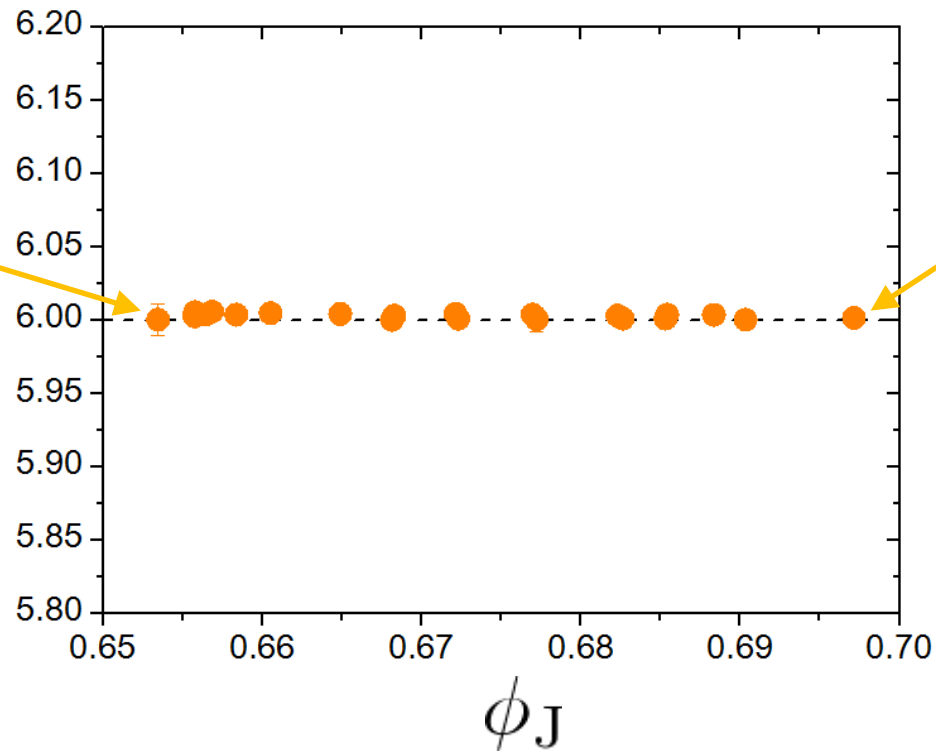


■ A continuous range of  $\phi_J$  line is obtained

## Results: $\phi_J$ - line



average contact number  $\bar{z}$

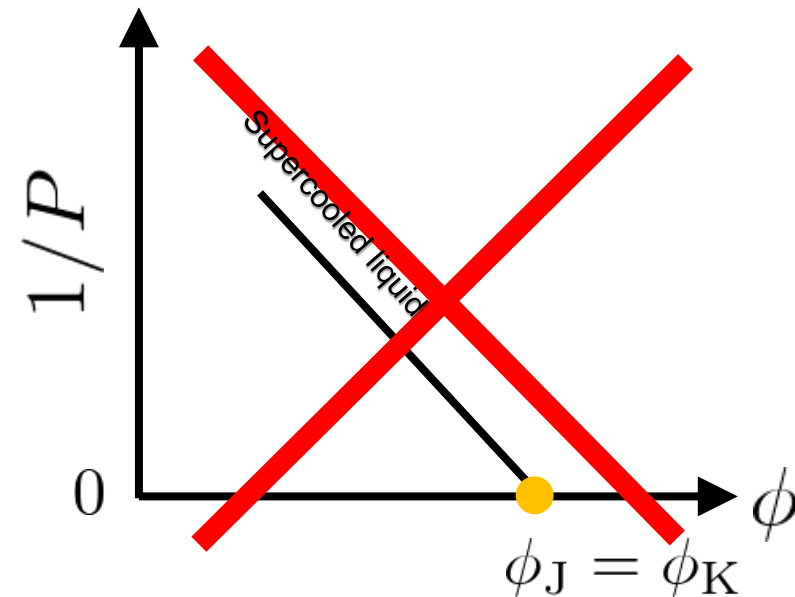


■  $\bar{z} = 2d$  holds at  $\phi_J$  - line

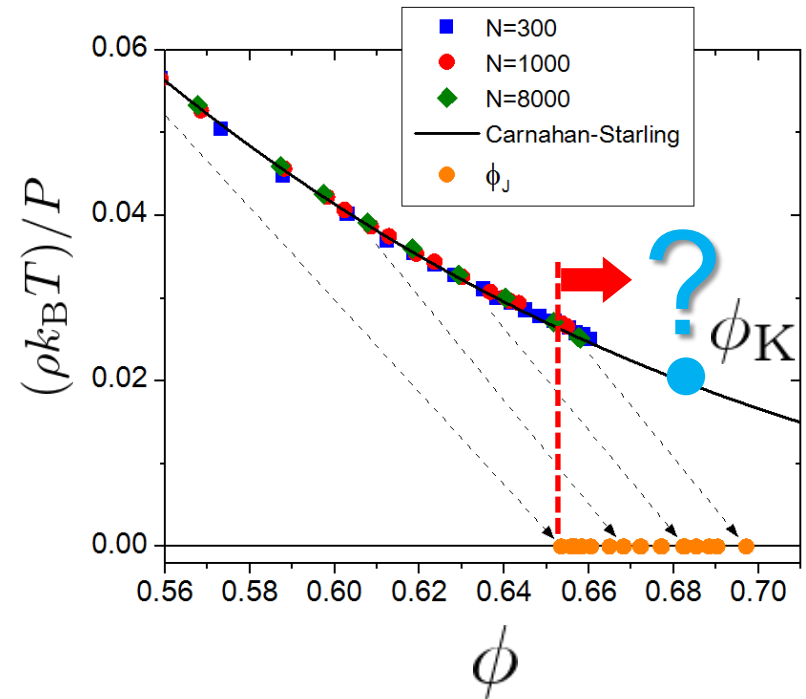
Chaudhuri, Berhier, and Sastry, PRL 2010

Ozawa, Kuroiwa, Ikeda, and Miyazaki, PRL 2012

# Conclusions



VS



- Related work:  
Glass-to-jamming transition
- Future work:  
Does the ideal glass transition density  $\phi_K$  exist?