Phase Transition and Power Law Appearing in Friction Behavior in the Medium with Heat, Fluid Pressure and Dilatancy

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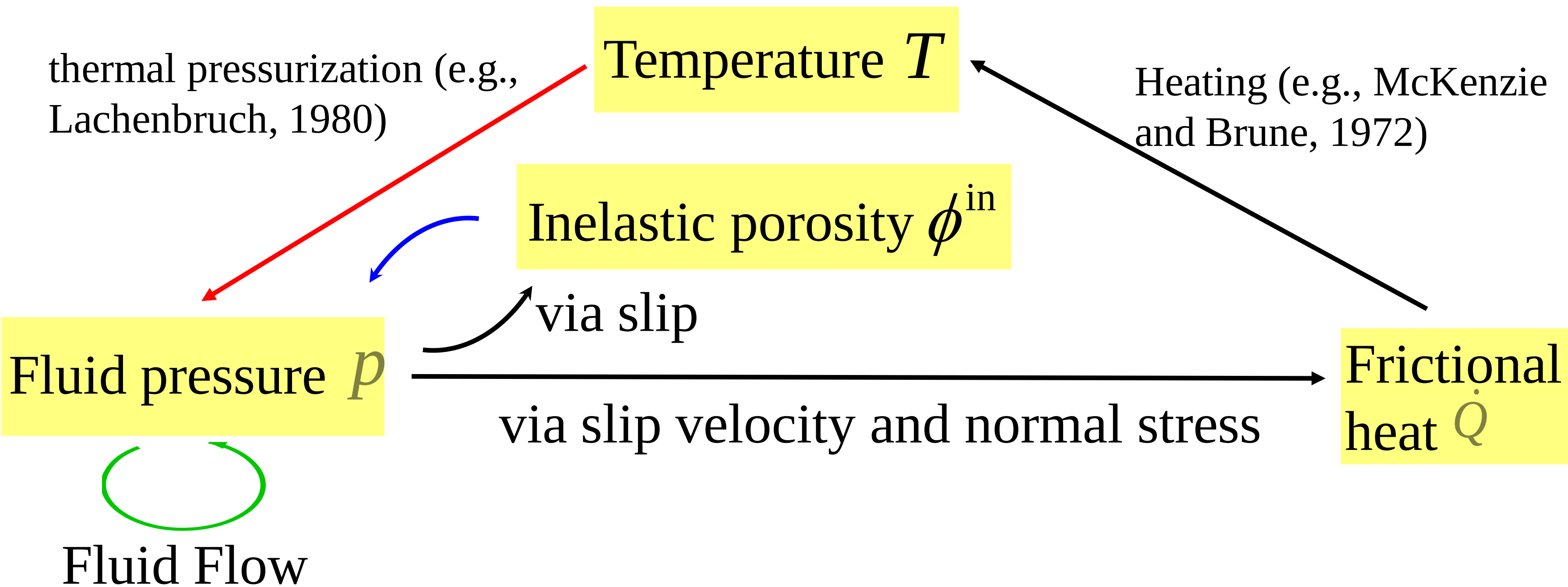
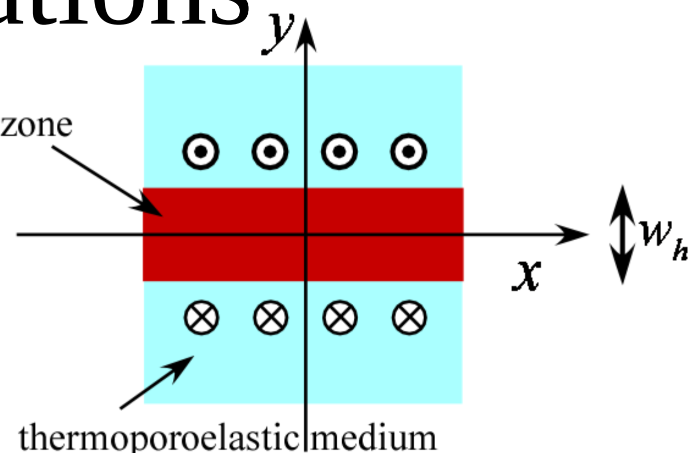
Model Setup

- A one-dimensional (1-D) mode III fault is assumed. The interaction among heat, fluid pressure and inelastic pore creation is investigated. See details in Suzuki and Yamashita (2010, 2014).
- In particular, temporal evolution of the porosity is investigated because it produces an important universality and analytical study of such a behavior gives some implications for understanding the slip behavior.



Governing Equations

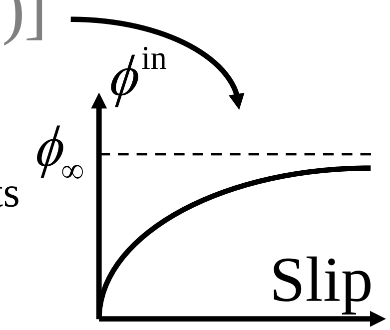


- From the interaction mentioned above, …

Fluid pressure
$$\frac{1}{M}\frac{\partial \bar{p}_f}{\partial t} = ((b-\phi_t)\alpha_s + \phi_t\alpha_f)\frac{\partial \bar{T}}{\partial t} + \frac{k}{\eta}\nabla^2 \bar{p}_f - \frac{\partial \phi^{\text{in}}}{\partial t}$$

Temperature
$$\left[(1-\phi_t)\rho_s C_s + \phi_t \rho_f C_f\right]\frac{\partial \bar{T}}{\partial t} = \sigma_{\text{fric}}\frac{v}{w_h}[H(y+\frac{w_h}{2}) - H(y-\frac{w_h}{2})]$$

Porosity
$$\frac{\partial \phi^{\text{in}}}{\partial t} = \alpha_0 v\left(1-\frac{\phi^{\text{in}}}{\phi_\infty}\right)[H(y+\frac{w_h}{2}) - H(y-\frac{w_h}{2})]$$

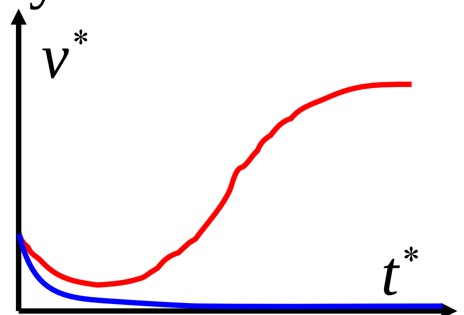


e.o.m.
$$\rho_B \frac{\partial^2}{\partial t^2}(\bar{u}_s)_z = \mu_v \nabla^2 (\bar{u}_s)_z$$

The fluid pressure affects the boundary condition for e.o.m.

Two Qualitatively Different Slip Behaviors

- If the fluid flow is neglected, governing equations can be rewritten in terms of the normalized slip velocity and inelastic porosity:



$$\dot{v}^* = v^*(1-v^*) - S_u(1-\phi^*)v^* \qquad \dot{\phi}^* = T_a(1-\phi^*)v^*$$

S_u, T_a : Nondimensional numbers

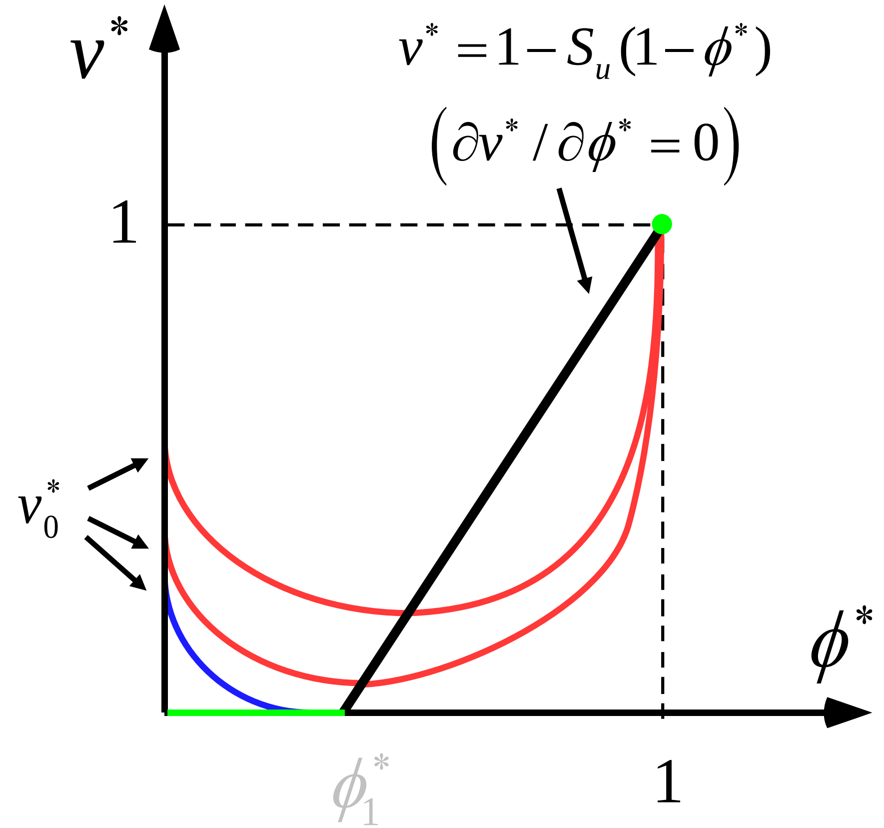
- We assume $S_u > 1 - v_0^*$ (v_0^* is the initial value of v^*). At the slip onset ($t^* = 0$), we can easily show $\dot{v}^* < 0$.
- From brief consideration, we clearly find two qualitatively different slip behaviors.
  - First case: $S_u(1-\phi^*)$ decreases with increasing time and $\dot{v}^*$ can become positive. → Acceleration
  - Second case: $\dot{v}^*$ is always negative. → Spontaneous slip cessation
- The steady state solutions for the governing equations are $(v_s^*, \phi_s^*) =$ (0, indeterminate) and (1, 1).

Phase Diagram

- All orbits of solutions are first apparently absorbed into $v^* = 0$.
- If the orbits do not cross the straight line $\partial v^* / \partial \phi^* = 0$, they are finally absorbed to the stable steady state

$(v_s^*, \phi_s^*) = (0, \text{indeterminate}).$

- On the other hand, if the orbits cross the line $\partial v^* / \partial \phi^* = 0$, the relationship $\partial v^* / \partial \phi^* > 0$ is satisfied and they are finally absorbed into the stable steady state $(v_s^*, \phi_s^*) = (1,1).$



(1) The Function G Determining Slip Behavior Completely

- The function $G(S_u, T_a, v_0^*)$ determining the system behavior can be given by (Suzuki and Yamashita, 2014)

$$G(S_u, T_a, v_0^*) = 1 - \left(\frac{1}{S_u T_a} \left((1 - v_0^*) T_a + S_u - 1 + v_0^* \right) \right)^{\frac{T_a}{T_a - 1}} \cdot S_u$$

- If $G > 0$,
  - $\dot{v}^*$ becomes positive. → $(v_s^*, \phi_s^*) = (1, 1)$
- If $G < 0$,
  - $\dot{v}^*$ does not become positive and v^* approaches zero. → $(v_s^*, \phi_s^*) = (0, \text{indeterminate})$

(2) Critical Porosity ϕ_1^*

- For the acceleration case, the deceleration changes to the acceleration at $\dot{v}^* = 0$, and we define v_a^* and ϕ_a^* as the values of v^* and ϕ^* at that time:

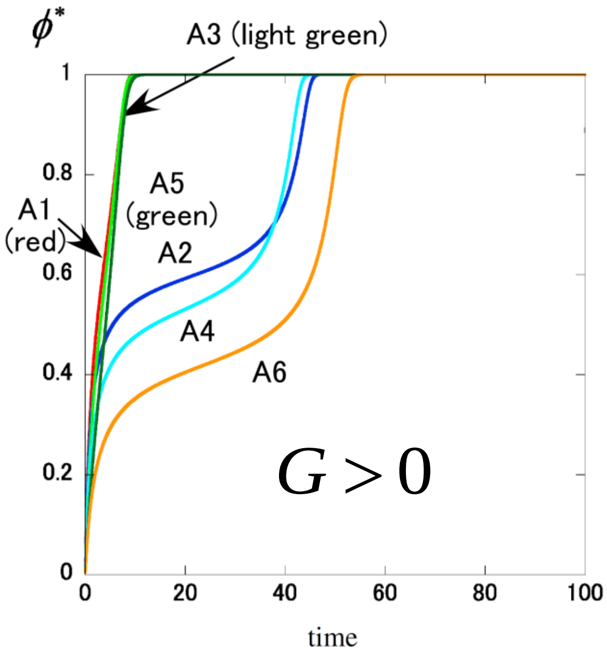
$$\dot{v}^* = (1 - v_a^* - S_u(1 - \phi_a^*))v_a^* = 0$$

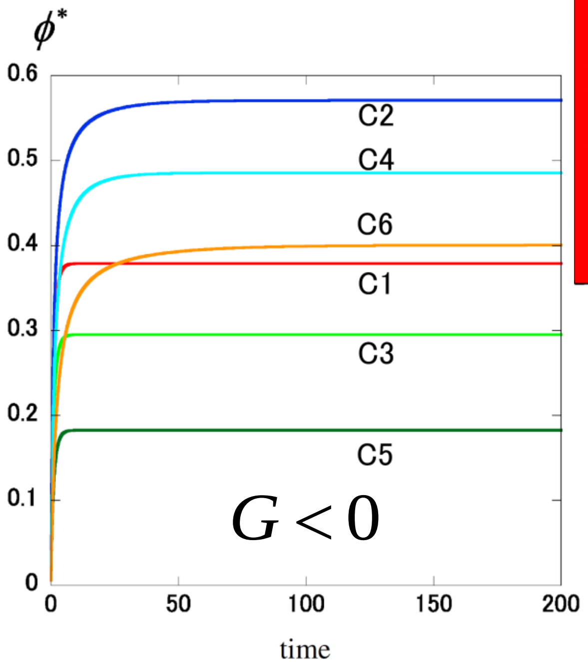
- Since the condition $v_a^* > 0$ must be satisfied, we obtain

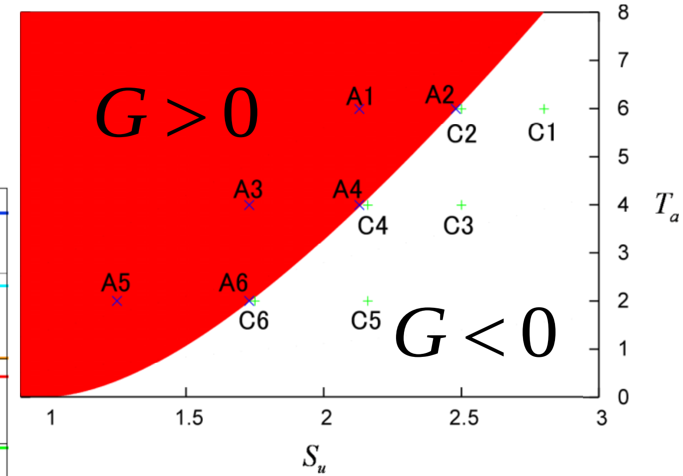
$$\phi_a^* > \frac{S_u - 1}{S_u} (\equiv \phi_1^*)$$

- We can conclude that if $\phi^* > \phi_1^*$ is satisfied, $(v_s^*, \phi_s^*) = (1,1)$ can emerge.
- In other words, it is the upper value of ϕ^* for the case $(v_s^*, \phi_s^*) = (0, \text{indeterminate})$: $\phi_s^* \le \phi_1^*$

Temporal Changes in Porosity







$(v_0^* = 0.1)$

ϕ_1^*

C1:0.64, C2:0.6

C3:0.6, C4:0.54

C5:0.54, C6:0.43

- If $G > 0$, $\phi^* \to 1$.
- If $G < 0$, ϕ^* approaches the constant values.
  - The constant values are below ϕ_1^*.

Phase Transition and Power Law

- We introduce the variables $w^* \equiv 1 - v^*$ and $\psi^* \equiv 1 - \phi^*$ and change $w_0^* \equiv 1 - v_0^*$ as a parameter.
  - If $G > 0,$ $(w^*, \psi^*) \to (0,0)$
  - If $G < 0,$ $(w^*, \psi^*) \to (1, 1 - \phi_s^*)$
- For $G < 0,$ the value ϕ_s^* changes continuously with changing v_0^*.

⬇

- The power law near (w_c^*, ψ_c^*) is expected for $G < 0:$

$$\psi_\infty^* - \psi_c^* \sim (w_0^* - w_c^*)^\alpha \qquad \left(G(S_u, T_a, 1 - w_c^*) = 0, \;\; w_c^* = \frac{S_u^{1/T_a} T_a - S_u}{T_a - 1} \right)$$

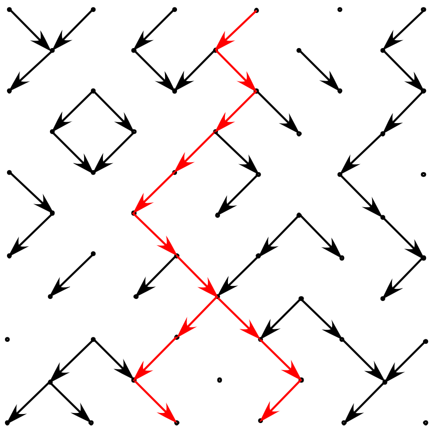
$$\psi_\infty^* = 1 - \phi_s^* \qquad \left(\psi_c^* \equiv 1 - \phi_1^* = 1 - \frac{S_u - 1}{S_u} = \frac{1}{S_u} \right)$$

Absorbing Phase Transition

- Once the system enters one state, the system cannot escape the state.
- ex.) Directed percolation
  - Percolation in a lattice with channels
  - Each channel is open with the possibility p , while it is closed with the possibility $1-p$.
  - Flow occurs from upstream to downstream.
  - If we define P_∞ as the possibility that the infinite percolation occurs, P_∞ is known to show a critical phenomenon:

$$P_\infty \sim \begin{cases} (p-p_c)^\beta & (p \geq p_c) \\ 0 & (p \leq p_c) \end{cases}$$

$(p_c$: critical value)



Power Law (Universality…?)

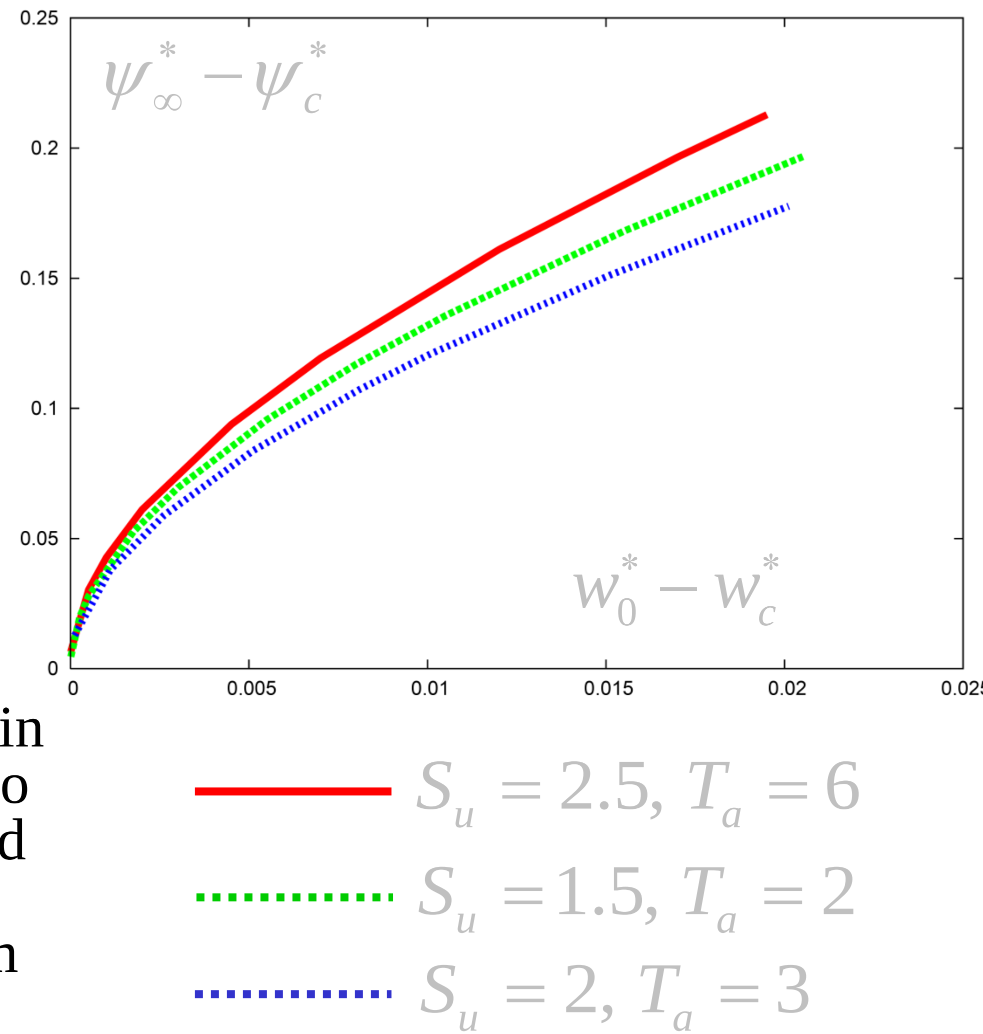
- Figure shows the power law near (w_c^*, ψ_c^*):

$$\psi_\infty^* - \psi_c^* = \beta (w_0^* - w_c^*)^\alpha$$

- Values for α are:

$$\alpha = 0.548 \ \ (S_u = 2.5, T_a = 6)$$
$$\alpha = 0.538 \ \ (S_u = 1.5, T_a = 2)$$
$$\alpha = 0.540 \ \ (S_u = 2, T_a = 3)$$

  - Universal?
  - If this value is universal, porosity values observed in natural faults are related to the initial slip velocity and we may presume the ancient slip behavior from the porosity.



Conclusions and Future Works

- Phase transition observed in porosity change within the framework including the heat, fluid pressure and dilatancy
- Universal value for α ?
  - New index characterizing the slip behavior?
- Analytical treatments to show the universality is required.