Dynamical Approach to Synthesis of Superheavy Elements

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<table>
<thead>
<tr>
<th>Structure</th>
<th>Light Nuclei</th>
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<tbody>
<tr>
<td>Reaction</td>
<td>Superheavy Nuclei</td>
</tr>
</tbody>
</table>
Key Words

1. Shell Correction Energy
   Two-Center Shell Model

2. Dynamical Approach
1. Introduction
   - Super Heavy Elements
   - Stability of Nuclei, Shell effects
   - Synthesis of SHE

2. Experimental methods

3. Theoretical calculation
   - Dynamical model (Fluctuation Dissipation model)
   - Langevin equation

4. Calculation results
   - fusion-fission process

5. Further study
### Periodic Table

<table>
<thead>
<tr>
<th>Element</th>
<th>Symbol</th>
<th>Atomic Number</th>
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<tr>
<td>Hydrogen</td>
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<tr>
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<td>3</td>
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<tr>
<td>Beryllium</td>
<td>Be</td>
<td>4</td>
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<tr>
<td>Boron</td>
<td>B</td>
<td>5</td>
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<tr>
<td>Carbon</td>
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<tr>
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<tr>
<td>Oxygen</td>
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<tr>
<td>Fluorine</td>
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<td>9</td>
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<tr>
<td>Neon</td>
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<td>10</td>
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<tr>
<td>Iodine</td>
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<td>53</td>
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<tr>
<td>Xenon</td>
<td>Xe</td>
<td>54</td>
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</tbody>
</table>

#### Hyper Heavy Elements

- **Flerovium (Fl)**: atomic number 113
- **Livermorium (Lv)**: atomic number 116

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### Super Heavy Elements

- **Super Heavy Elements**: less stable

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**Mendeleev (1834-1907)**

- **1869**: periodic table proposed

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**May 2012 IUPAC**

- **Fl**: Flerovium
- **Lv**: Livermorium

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**La**, **Ac**

- **Lanthanides**
- **Actinides**

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**Taka**, **Fuji**

- Photograph of two children, likely representing Mendeleev's children.
1. Introduction

Our Interest
- Next magic number $\leftrightarrow Z=82$, $N=126$
- Verification of ‘Island of Stability’ (predicted by macroscopic-microscopic model in 1960’s)
- Synthesis of new elements

Nuclear Chart
Stability of nuclei
Experimental setup for synthesis of SHE

<table>
<thead>
<tr>
<th>Lab</th>
<th>Country</th>
<th>City</th>
<th>Accelerator</th>
<th>Separator</th>
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<tbody>
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<td>Russia</td>
<td>Dubna</td>
<td>U400</td>
<td>DGFRS VASSILISSA</td>
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<tr>
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<td>Germany</td>
<td>Darmstadt</td>
<td>UNILAC</td>
<td>SHIP TASCA</td>
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<td>Japan</td>
<td>Wako</td>
<td>RILAC</td>
<td>GALIS</td>
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<td>LBNL</td>
<td>USA</td>
<td>Berkeley</td>
<td>88-inch Cyclotron</td>
<td>BGS</td>
</tr>
<tr>
<td>GANIL</td>
<td>France</td>
<td>Caen</td>
<td>SPIRAL2's LINAC</td>
<td>S3 (Super Separator Spectrometer)</td>
</tr>
</tbody>
</table>
Fusion process in Superheavy mass region

FUSION

TRANSFER, QUASI-FISSION

Nuclear Molecule

Compound Nucleus (CN)

FUSION-FISSION

Evaporation Residue (ER)

Fission Fragments

90~99%
Experimental data

Evaporation residue cross sections

Pb target
Actinide target

Graph showing evaporation residue cross sections for cold fusion reaction (1n) and hot fusion reaction (3n-5n) with respect to element number.
\[ \sigma_{ER} = \frac{\pi \hbar^2}{2\mu_0 E_{cm}} \sum_{\ell=0}^{\infty} \left(2\ell + 1\right) T_\ell \left(E_{cm}, \ell\right) P_{CN} \left(E^*, \ell\right) W \left(E^*, \ell\right) \]
\[ \sigma_{ER} = \frac{\pi \hbar^2}{2 \mu_0 E_{cm}} \sum_{\ell=0}^{\infty} (2\ell + 1) T_\ell (E_{cm}, \ell) P_{CN} (E^*, \ell) W (E^*, \ell) \]

**Formation probability**

**Survival probability**

**Reaction time**

- \( t < 10^{-22} \text{ s} \)
- \( 10^{-22} < t < 10^{-18} \text{ s} \)
- \( \sim 10^{-18} < t \text{ s} \)

**Touching probability**

**Quasi-fission** 90~99 %

**Fusion-fission**
Synthesis of New Elements

Heavy ion reaction

Cold fusion reaction  Hot fusion reaction

1994
- 110 Ds $^{62}$Ni + $^{208}$Pb $\rightarrow$ $^{269}$110 + n (GSI)
- 111 Rg $^{64}$Ni + $^{209}$Bi $\rightarrow$ $^{272}$111 + n (GSI)

1996
- 112 Cn $^{70}$Zn + $^{208}$Pb $\rightarrow$ $^{277}$112 + n (GSI) $\leftarrow$ named in Feb. 2010

1999
- 114 Fl $^{48}$Ca + $^{244}$Pu $\rightarrow$ $^{292}$114 + 3n (FLNR) $\leftarrow$ named in May. 2012

2000
- 116 Lv $^{48}$Ca + $^{248}$Cm $\rightarrow$ $^{292}$116 + 4n (FLNR) $\leftarrow$ named in May. 2012

2002
- 118 $^{48}$Ca + $^{249}$Cf $\rightarrow$ $^{294}$118 + 3n (FLNR)

2003
- 115 $^{48}$Ca + $^{243}$Am $\rightarrow$ $^{288}$115 + 3n $\rightarrow$ $^{284}$113 + $\alpha$ (FLNR)

2004
- 113 $^{70}$Zn + $^{208}$Bi $\rightarrow$ $^{278}$113 + n (RIKEN)

2010
- 117 $^{48}$Ca + $^{249}$Bk $\rightarrow$ $^{294,293}$117 + 3-4n (FLNR)
2. Model

2-1. Estimation of cross sections
2-2. Dynamical Equation
Overview of Dynamical Process in reaction $^{36}S + ^{238}U$

**Time-evolution of nuclear shape in fusion-fission process**

1. Potential energy surface
2. Trajectory $\rightarrow$ described by equations
Nuclear shape

two-center parametrization \((z, \delta, \alpha)\)

(Maruhn and Greiner, Z. Phys. 251(1972) 431)

\[ q(z, \delta, \alpha) \]

\[ z = \frac{z_0}{BR} \]

\[ B = \frac{3 + \delta}{3 - 2\delta} \]

\( R \) : Radius of the spherical compound nucleus

\[ \delta = \frac{3(a - b)}{2a + b} \]  \( (\delta_1 = \delta_2) \)

\[ \alpha = \frac{A_1 - A_2}{A_{CN}} \]
Two Center Shell Model

\[ \hat{H} = -\frac{\hbar^2}{2m_0} \nabla^2 + V(r) + V_{LS}(r,p,s) + V_L^2(r,p). \]

Neck parameter is the ratio of smoothed potential height to the original one where two harmonic oscillator potential cross each other:

\[ \varepsilon = \frac{E}{E_0} \]

J. Maruhn and W. Greiner, Z. Phys, 1972
Potential Energy

\[ V(q, \ell, T) = V_{DM}(q) + \frac{\hbar^2 \ell(\ell + 1)}{2I(q)} + V_{SH}(q, T) \]

\[ V_{DM}(q) = E_S(q) + E_C(q) \]

\[ V_{SH}(q, T) = E_{shell}^0(q) \Phi(T) \]

\( T \): nuclear temperature

\( E^* = aT^2 \quad a \) : level density parameter

Toke and Swiatecki

\( E_S \): Generalized surface energy (finite range effect)

\( E_C \): Coulomb repulsion for diffused surface

\( E_{shell}^0 \): Shell correction energy at \( T=0 \)

\( I \): Moment of inertia for rigid body

\( \Phi(T) \): Temperature dependent factor

\[ \Phi(T) = \exp \left\{ - \frac{aT^2}{E_d} \right\} \]

\( E_d = 20 \text{ MeV} \)
Taking into account the fluctuation around the mean trajectory

Thermal fluctuation of nuclear shape
→ thermal fluctuation of collective motion
Multi-dimensional Langevin Equation

\[
\frac{dq_i}{dt} = (m^{-1})_{ij} p_j \\
\frac{dp_i}{dt} = -\frac{\partial V}{\partial q_i} - \frac{1}{2} \frac{\partial}{\partial q_i} (m^{-1})_{jk} p_j p_k - \gamma_{ij} (m^{-1})_{jk} p_k + g_{ij} R_j(t)
\]

\[
\langle R_i(t) \rangle = 0, \quad \langle R_i(t_1) R_j(t_2) \rangle = 2\delta_{ij}\delta(t_1 - t_2) : \text{white noise (Markovian process)}
\]

\[
\sum_k g_{ik} g_{jk} = T\gamma_{ij} \quad \text{Einstein relation}
\]

\[
E_{\text{int}} = E^* - \frac{1}{2} (m^{-1})_{ij} p_i p_j - V(q)
\]

\[
E_{\text{int}} : \text{intrinsic energy}, \quad E^* : \text{excitation energy}
\]

- \( q_i \): deformation coordinate (nuclear shape)
- \( p_i \): momentum
- \( m_{ij} \): Hydrodynamical mass (inertia mass)
- \( \gamma_{ij} \): Wall and Window (one-body) dissipation (friction)

Two-center parametrization \((z, \delta, \alpha)\)

(Maruhn and Greiner, Z. Phys. 251(1972) 431)
Fission process $^{240}\text{U}$, $E^* < 20\text{ MeV}$

Trajectory on potential energy surface
236U E*=20 MeV

揺動項なし Newton Eq

揺動項あり Langevin Eq.

質量分布
Overview of Dynamical Process in reaction $^{36}\text{S} + ^{238}\text{U}$
Projectile dependence of fragment mass distributions

Experiments by K. Nishio et al. (JAEA)
Calculated spectra for fusion-fission and quasi-fission

Experiments by K. Nishio et al. (JAEA)
4. Mechanism of Dynamical process

MDFF at Low incident energy

\( ^{30}\text{Si} + ^{238}\text{U} \) vs \( ^{36}\text{S} + ^{238}\text{U} \)

\[ E_{c.m.} = 129.0 \text{ MeV}, \quad \varepsilon^* = 35.5 \text{ MeV} \]

\[ E_{c.m.} = 154.0 \text{ MeV}, \quad \varepsilon^* = 39.5 \text{ MeV} \]

Clarify the origin of the difference
(b) Trajectory Analysis on Potential Energy Surface  \( z-A \) plane

\[ ^{30}\text{Si} + ^{238}\text{U} \quad E^* = 35.5 \text{ MeV} \]
\( L=0, \theta=0 \)

\[ ^{36}\text{S} + ^{238}\text{U} \quad E^* = 39.5 \text{ MeV} \]
\( L=0, \theta=0 \)
Time evolution of probability distribution

$^{30}\text{Si} + ^{238}\text{U} \rightarrow ^{268}\text{Sg} \ (E^* = 35.5 \text{ MeV})$

Try to clarify the origin of difference between the both cases →
Probability distribution on the z-A plane

(a) $E^*=35.5$ MeV, $L=0$, $\theta=0$

(b) $^{30}\text{Si}+^{238}\text{U}$

(a) $E^*=35.5$ MeV, $L=0$, $\theta=0$

(b) $^{36}\text{S}+^{238}\text{U}$
Probability distribution of total time on the $z$-$\delta$ plane

$^{30}$Si + $^{238}$U, $E^*$ = 35.5 MeV

$^{36}$S + $^{238}$U, $E^*$ = 39.5 MeV

The relation between the touching point and the ridge line is very important to decide the process -> fusion hindrance
(1) Origin of the reaction process
(2) Building times
(3) Deformation of fragments
Fission Process
Fission process $^{240}$U \hspace{1cm} E^* < 20 MeV

Trajectory on potential energy surface
$^{236}\text{U}^* \ (E^* = 20\text{MeV})$

Experiment

J. Katakura, JENDL FP Decay Data File 2011 and Fission Yields

calculation


Exp. Phys. Rev. 141(1966) 1146
Mass distribution of fission fragments

\[ E^* = 20 \text{ MeV} \]

\[ ^{234}\text{U} \]

\[ ^{240}\text{Pu} \]
Comparison between Cal. and Exp.

Cal. $E_D = 20$ MeV

Cal. $E_D = 30$ MeV

$234$Th $235$Pa $236$U

$E^*$

40–50 MeV

30–40 MeV

20–30 MeV

Fragment Mass (u)

Cal. $\Phi(T) = \exp \left\{ - \frac{aT^2}{E_d} \right\}$

Compiled by K. Nishio and JAEA group
5. Summary

1. In order to analyze the fusion-fission process in superheavy mass region, we apply the Couple channels method + Langevin calculation.

2. **Incident energy dependence** of mass distribution of fission fragments (MDFF) is reproduced in reaction $^{36}\text{S}+^{238}\text{U}$ and $^{30}\text{Si}+^{238}\text{U}$.

3. The shape of the MDFF is analyzed using *probability distribution*

4. The relation between the touching point and the ridge line is very important to decide the process $\rightarrow$ fusion hindrance

And....
Collaborators

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