Role of tensor force in light nuclei studied with tensor-optimized shell model

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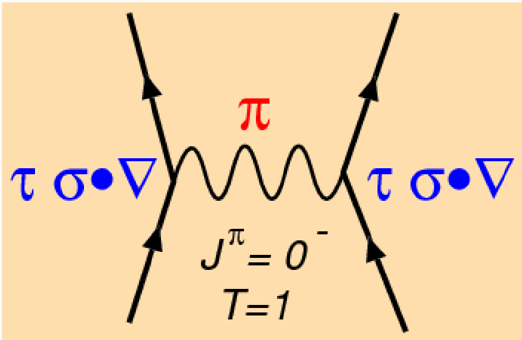
Outline

- **Role of V_{tensor}** in the nuclear structure by decribing strong tensor correlation explicitly.
- Tensor Optimized Shell Model (**TOSM**)
  - Importance of 2p2h excitation involving high-k
  - Application to He, Li, Be isotopes
- Tensor Optimized AMD (**TOAMD**) ***preliminary***
  - Clustering & tensor force
  - Formulation given in PTEP 2015, 073D02

Pion exchange interaction & V_{tensor}

$$3(\vec{\sigma}_1 \cdot \hat{q})(\vec{\sigma}_2 \cdot \hat{q})\frac{q^2}{m^2+q^2} = (\vec{\sigma}_1 \cdot \vec{\sigma}_2)\frac{q^2}{m^2+q^2} + S_{12}\frac{q^2}{m^2+q^2}$$

$$= (\vec{\sigma}_1 \cdot \vec{\sigma}_2)\left[\frac{m^2+q^2}{m^2+q^2} - \frac{m^2}{m^2+q^2}\right] + S_{12}\frac{q^2}{m^2+q^2}$$



δ interaction

Yukawa interaction

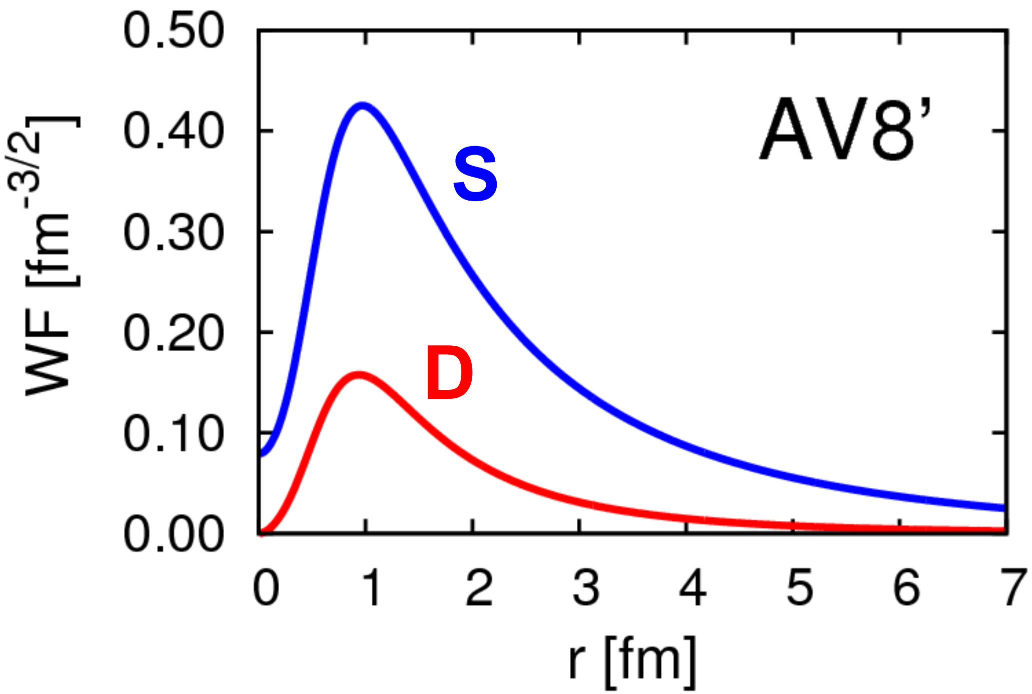
involve large momentum

Tensor operator

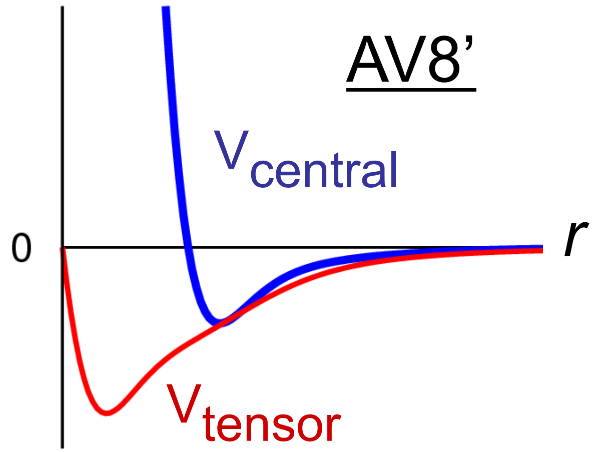
$$S_{12} = 3(\vec{\sigma}_1 \cdot \hat{q})(\vec{\sigma}_2 \cdot \hat{q}) - (\vec{\sigma}_1 \cdot \vec{\sigma}_2)$$

- V_{tensor} produces the high momentum component.

Deuteron properties & tensor force



Energy	-2.24 MeV
Kinetic	19.88
Central	-4.46
Tensor	**-16.64**
LS	-1.02
P(L=2)	5.77%
Radius	1.96 fm



$R_m(s)$=2.00 fm

$R_m(d)$=1.22 fm

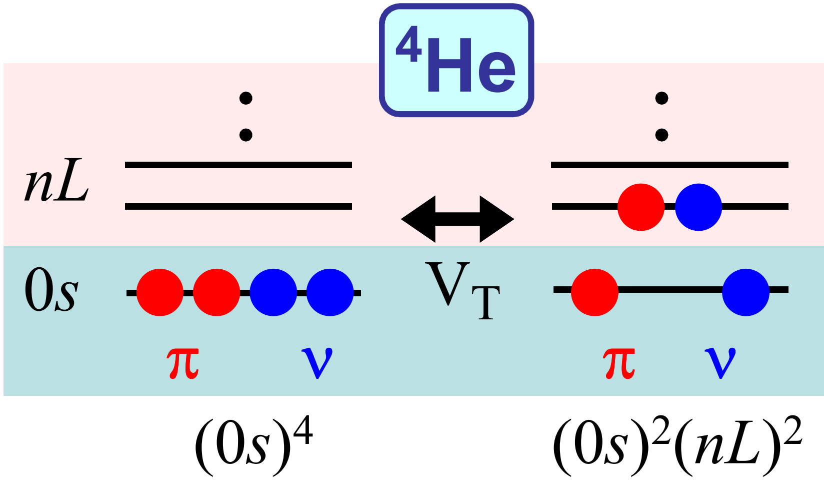
d-wave is
"**spatially compact**"
(high momentum)

Tensor-optimized shell model (TOSM)

TM, Sugimoto, Kato, Toki, Ikeda   PTP117(2007)257

- 2p2h excitations with high-L orbits.
- V_{tensor} is **NOT** treated as residual interactions

$$\frac{V_\pi}{V_{NN}} \sim 80\% \text{ in GFMC}$$



- Radial WF of nucleon are optimized variationally by superposing Gaussian bases.
  - Spatial shrinkage (high momentum) of ***D*-wave** owing to $\mathbf{V_{tensor}}$ as like deuteron. cf. HF/RMF (Toki, Ogawa et al., NPA740, PRC73).
- Satisfy few-body results with Minnesota central force (^{4,6}He)

TM, A. Umeya, H. Toki, K. Ikeda PRC84 (2011) 034315

Hamiltonian and variational equations in TOSM

$$H = \sum_{i=1}^{A} t_i - T_G + \sum_{i<j}^{A} v_{ij},$$

(0p0h+1p1h+2p2h)

$$\Phi(A) = \sum_k C_k \cdot \psi_k(A)$$

Shell model type configuration with mass number A

particle state : Gaussian expansion for each orbit

$$\varphi_{lj}^{n'}(\mathbf{r}) = \sum_{n=1}^{N} C_{lj,n}^{n'} \cdot \phi_{lj,n}(\mathbf{r})$$

$$\phi_{lj,n}(\mathbf{r}) \propto r^l \exp\left[-\frac{1}{2}\left(\frac{r}{b_{lj,n}}\right)^2\right]\left[Y_l(\hat{\mathbf{r}}), \chi_{1/2}^{\sigma}\right]_j$$

$$\left\langle \varphi_{lj}^{n'} \middle| \varphi_{lj}^{n''} \right\rangle = \delta_{n',n''}$$

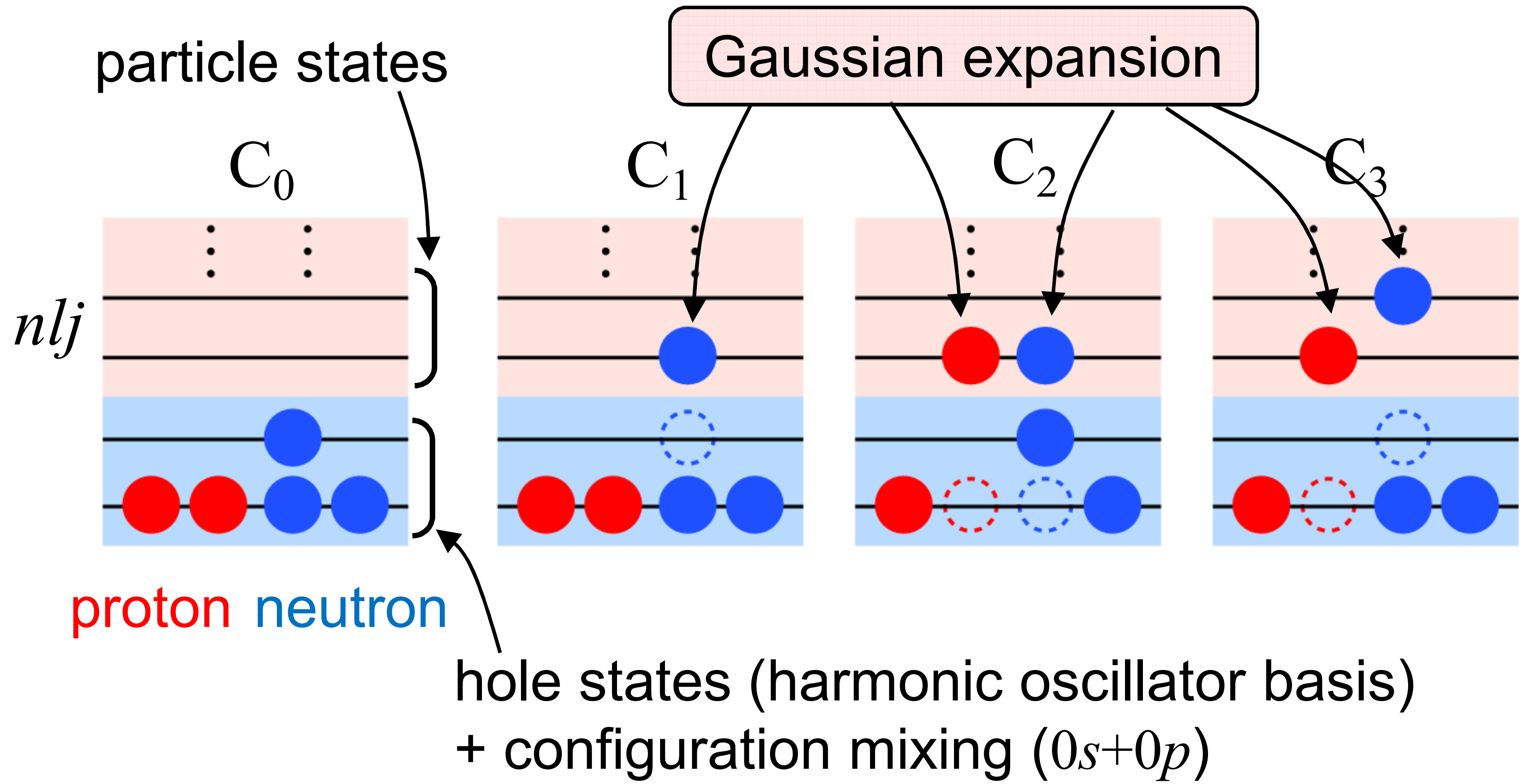
Gaussian basis function

Hiyama, Kino, Kamimura
PPNP51(2003)223

$$\frac{\partial \langle H-E \rangle}{\partial C_k} = 0, \qquad \frac{\partial \langle H-E \rangle}{\partial b_{lj,n}} = 0$$

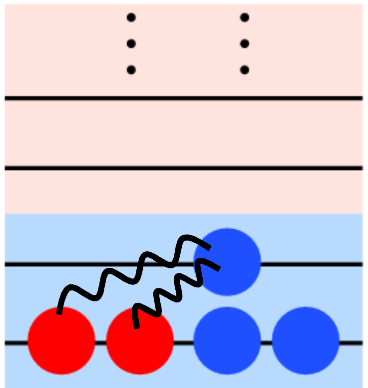
c.m. excitation is excluded by Lawson's method

Configurations in TOSM



Application to Hypernuclei to investigate ΛN-ΣN coupling by **Umeya** (NIT), **Hiyama** (RIKEN)

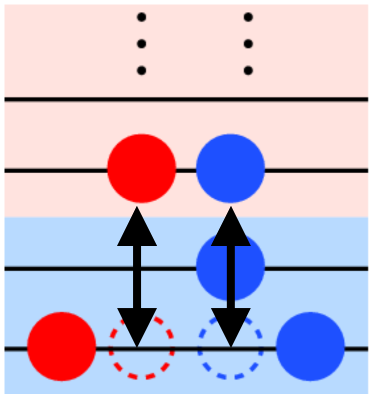
Tensor force matrix elements



$b_D \sim b_S$

$V_T = V_{residual}$

1st order



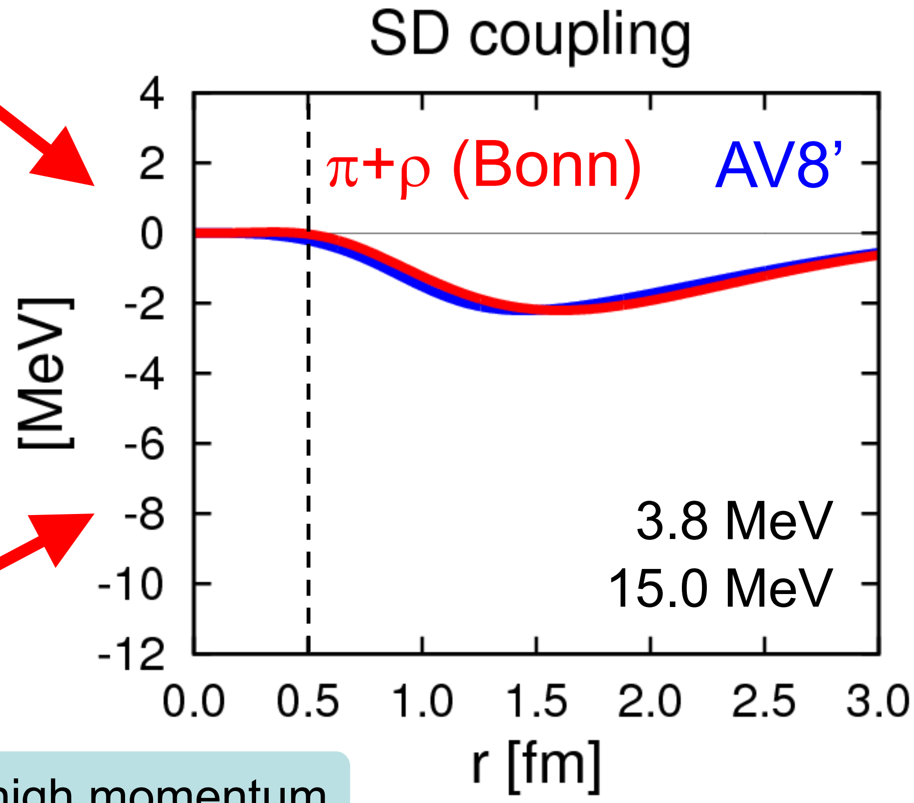
$b_D \sim b_S \times 0.5$

$V_T \neq V_{residual}$

0p0h-2p2h

$$M_{SD}(r) = r^2 \cdot \phi_S(r,b_S) \cdot V_T \cdot \phi_D(r,b_D)$$

Integrand of Tensor ME

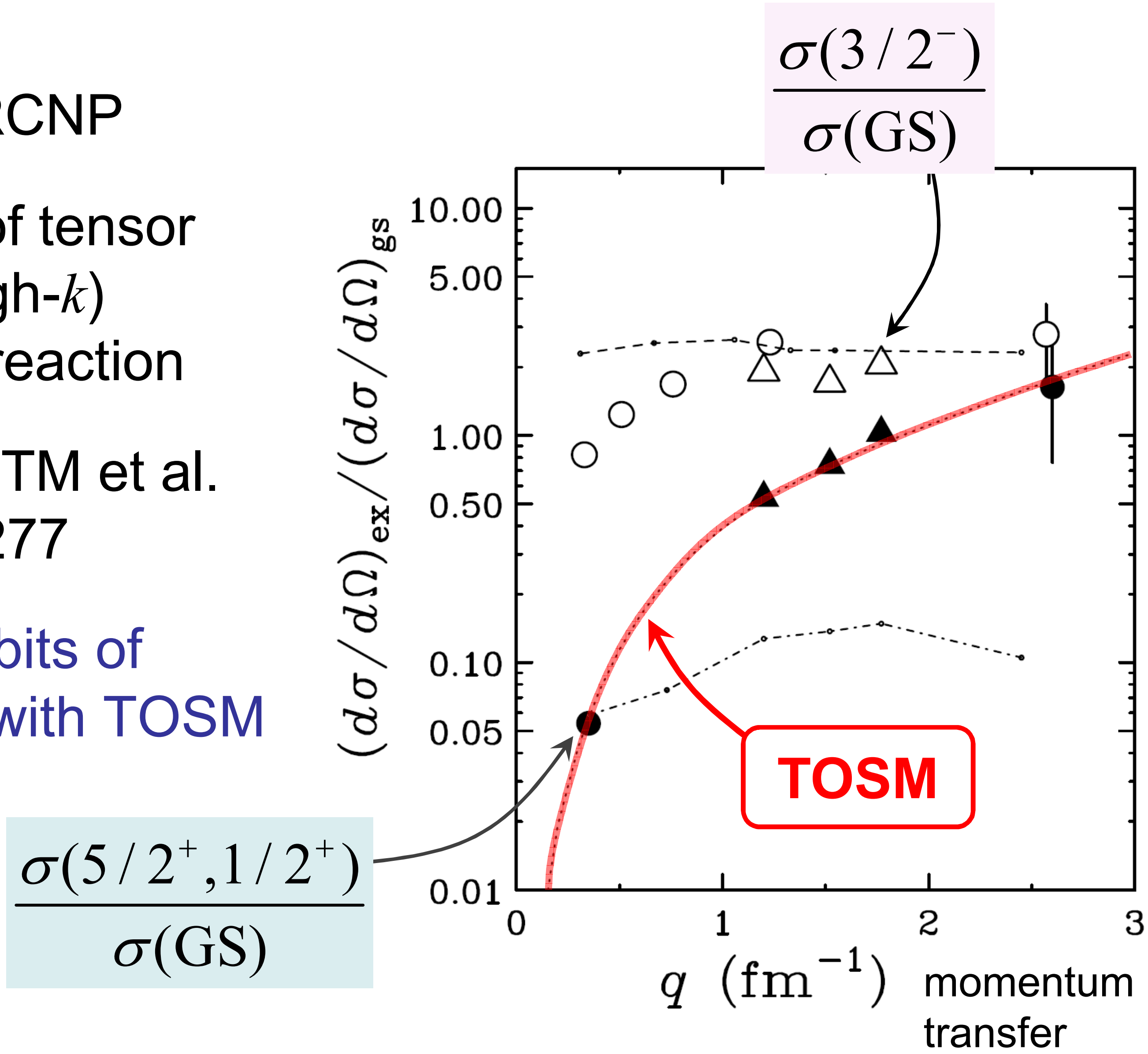


high momentum

- Centrifugal potential (1GeV@0.5fm) pushes away *D*-wave.

High-k component of nucleon in nucleus

- $^{16}O(p,d)^{15}O$ @RCNP
- Probing effect of tensor interactions (high-k) in ^{16}O via (p,d) reaction
- Ong, Tanihata, TM et al. PLB725(2013)277
- Compact *SD*-orbits of nucleon in ^{16}O with TOSM



Unitary Correlation Operator Method

(short-range part)

$$\Psi_{\text{corr.}} = C \cdot \Phi_{\text{uncorr.}}$$

TOSM

short-range correlator

$$C^{\dagger} = C^{-1} \quad \text{(Unitary trans.)}$$

$$H\Psi = E\Psi \;\rightarrow\; C^{\dagger} H C \Phi \equiv \hat{H}\Phi = E\Phi$$

Bare Hamiltonian

Shift operator depending on the relative distance

$$C = \exp(-i\sum_{i<j} g_{ij}), \qquad g_{ij} = \tfrac{1}{2}\left\{ p_r s(r_{ij}) + s(r_{ij}) p_r \right\} \qquad \vec{p} = \vec{p}_r + \vec{p}_{\Omega}$$

Amount of shift, variationally determined.

$$C^{\dagger} r C = r + s(r) + \tfrac{1}{2} s(r) s'(r) \cdots$$

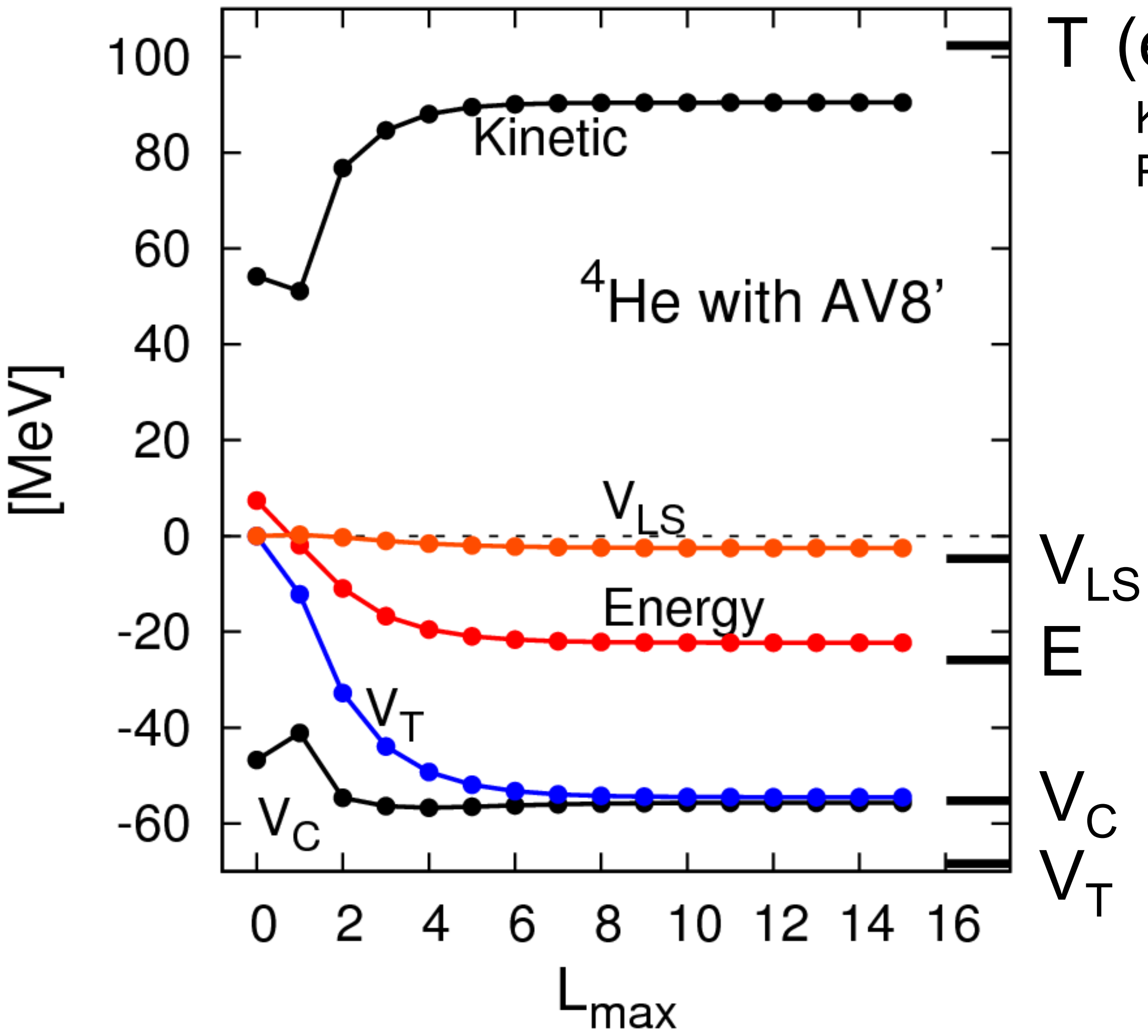
2-body cluster expansion

H. Feldmeier, T. Neff, R. Roth, J. Schnack, NPA632(1998)61

He, Li isotopes

TM, A. Umeya, H. Toki, K. Ikeda PRC84 (2011) 034315
TM, A. Umeya, H. Toki, K. Ikeda PRC86 (2012) 024318

^{4}He in TOSM + short-range UCOM



Kamada et al.
PRC64 (Jacobi)

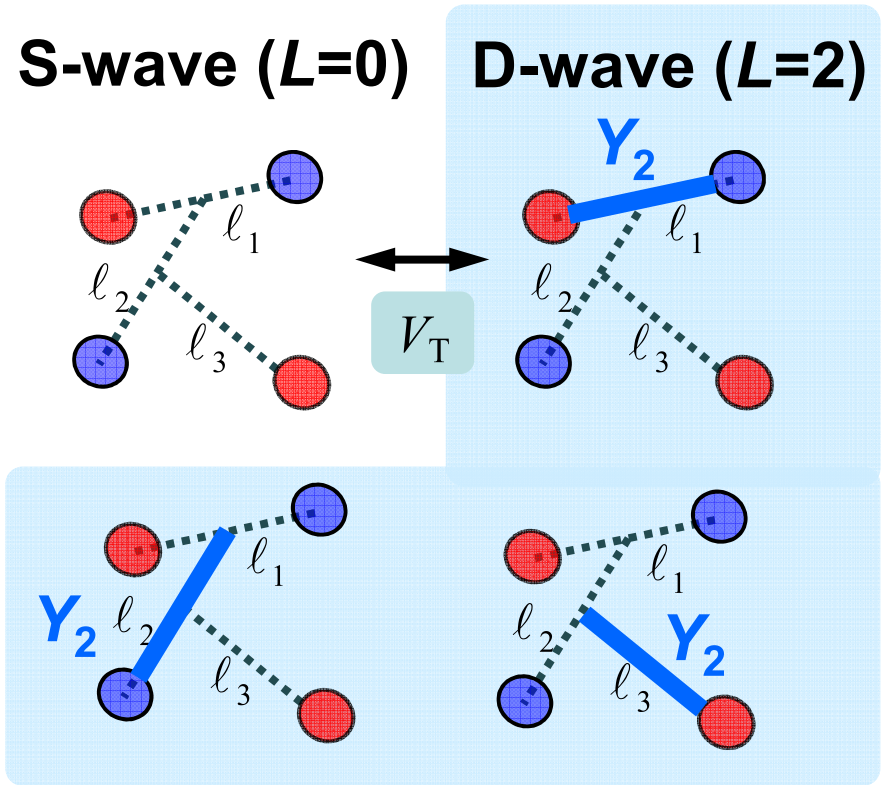
TM, H. Toki, K. Ikeda
PTP121(2009)511

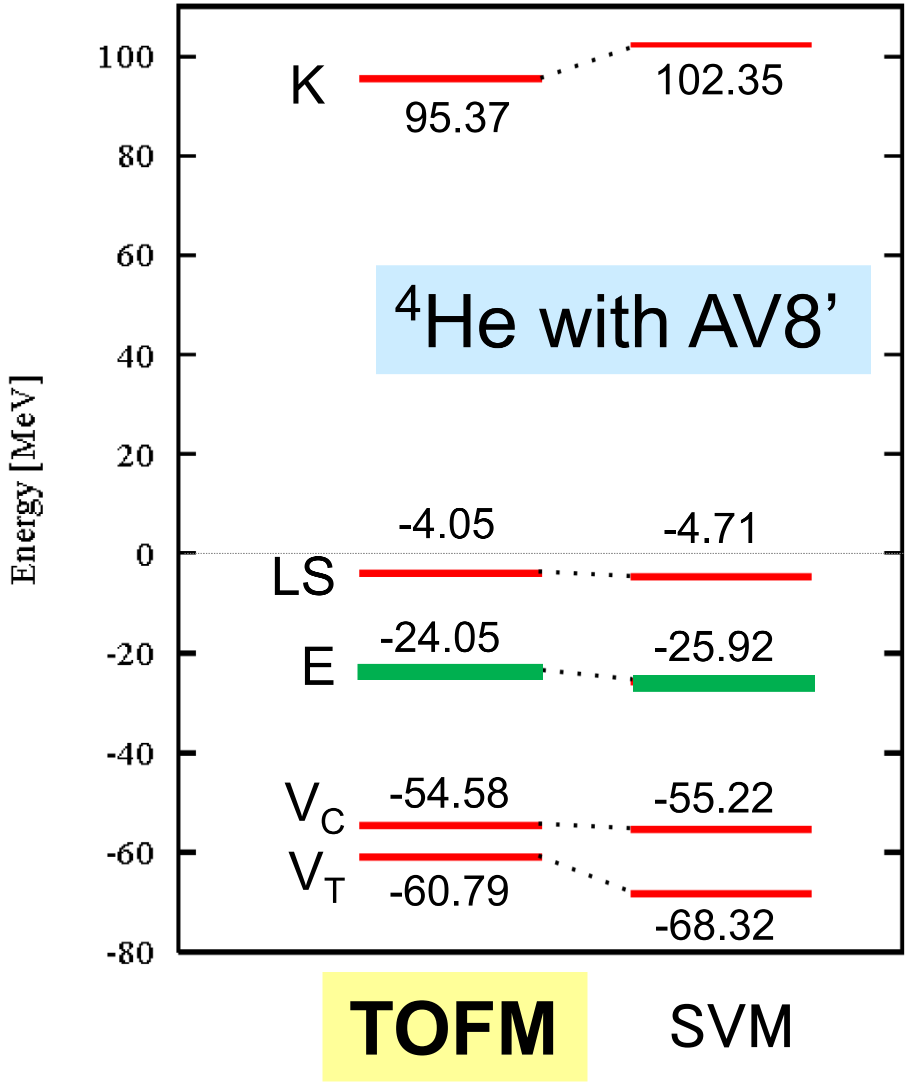
- variational calculation
- Gaussian expansion with 9 bases

good convergence

Tensor Optimized Few-body Model (TOFM)

- Same as TOSM concept
- No use of UCOM
- Correlated Gaussian basis
  + Global vector in SVM



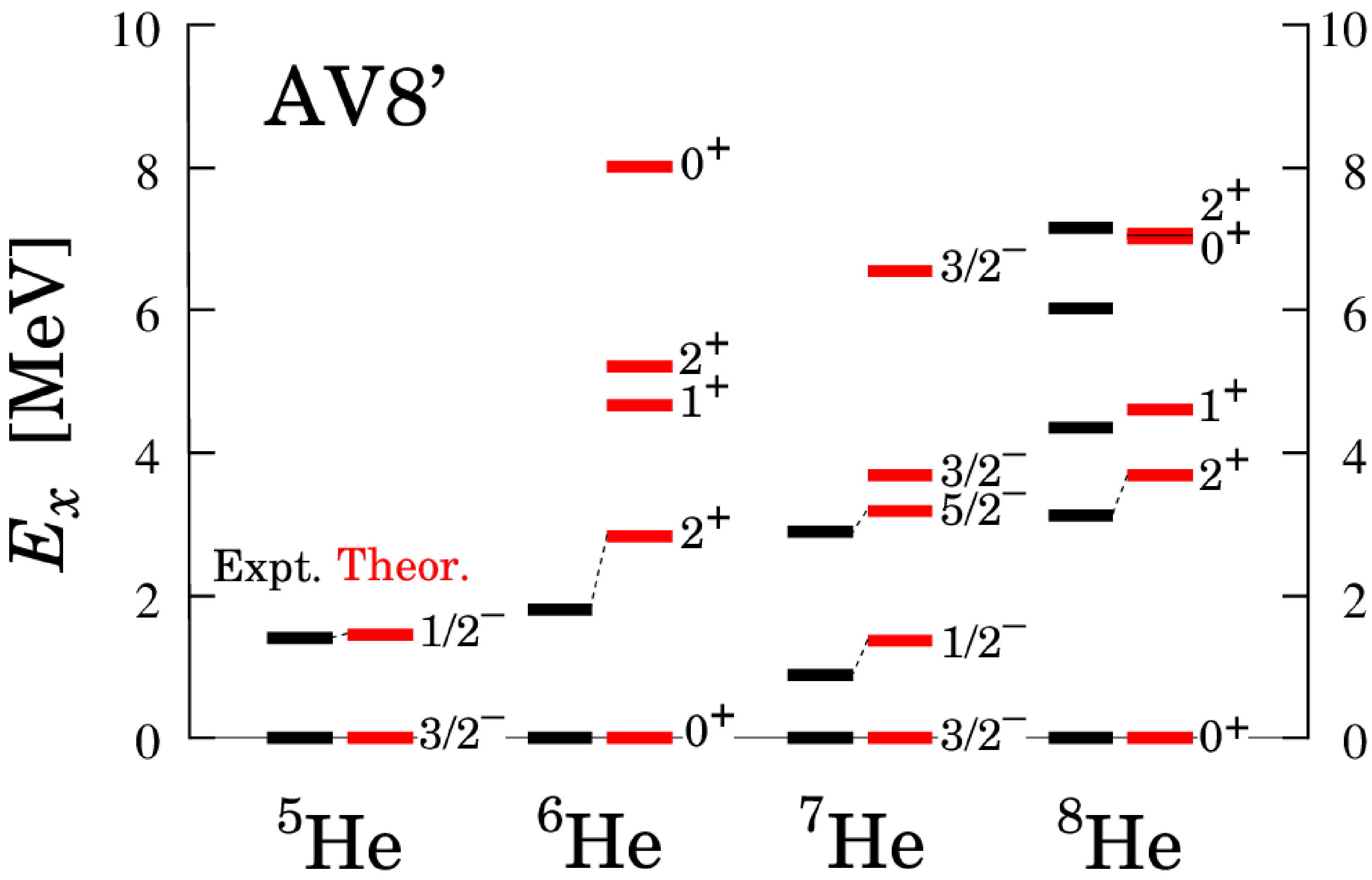


Horii, Toki, Myo, Ikeda PTP127(2012)1019

$^{5\text{-}8}$He with TOSM+UCOM

- Excitation energies in MeV

TM, A. Umeya, H. Toki, K. Ikeda
PRC84 (2011) 034315



- Excitation energy spectra are reproduced well

Configurations of ^{4}He with AV8'

$(0s_{1/2})^4$	83.0 %
$(0s_{1/2})^{-2}_{JT}(p_{1/2})^2_{JT}$ ***JT*=10**	2.6
JT=01	0.1
$(0s_{1/2})^{-2}_{10}(1s_{1/2})(d_{3/2})_{10}$	2.3
$(0s_{1/2})^{-2}_{10}(p_{3/2})(f_{5/2})_{10}$	1.9
Radius [fm]	1.54

TM, H. Toki, K. Ikeda
PTP121(2009)511

- deuteron correlation with $(J,T)=(1,0)$

Cf. R.Schiavilla et al. (VMC) PRL98(2007)132501
R. Subedi et al. (JLab) Science320(2008)1476

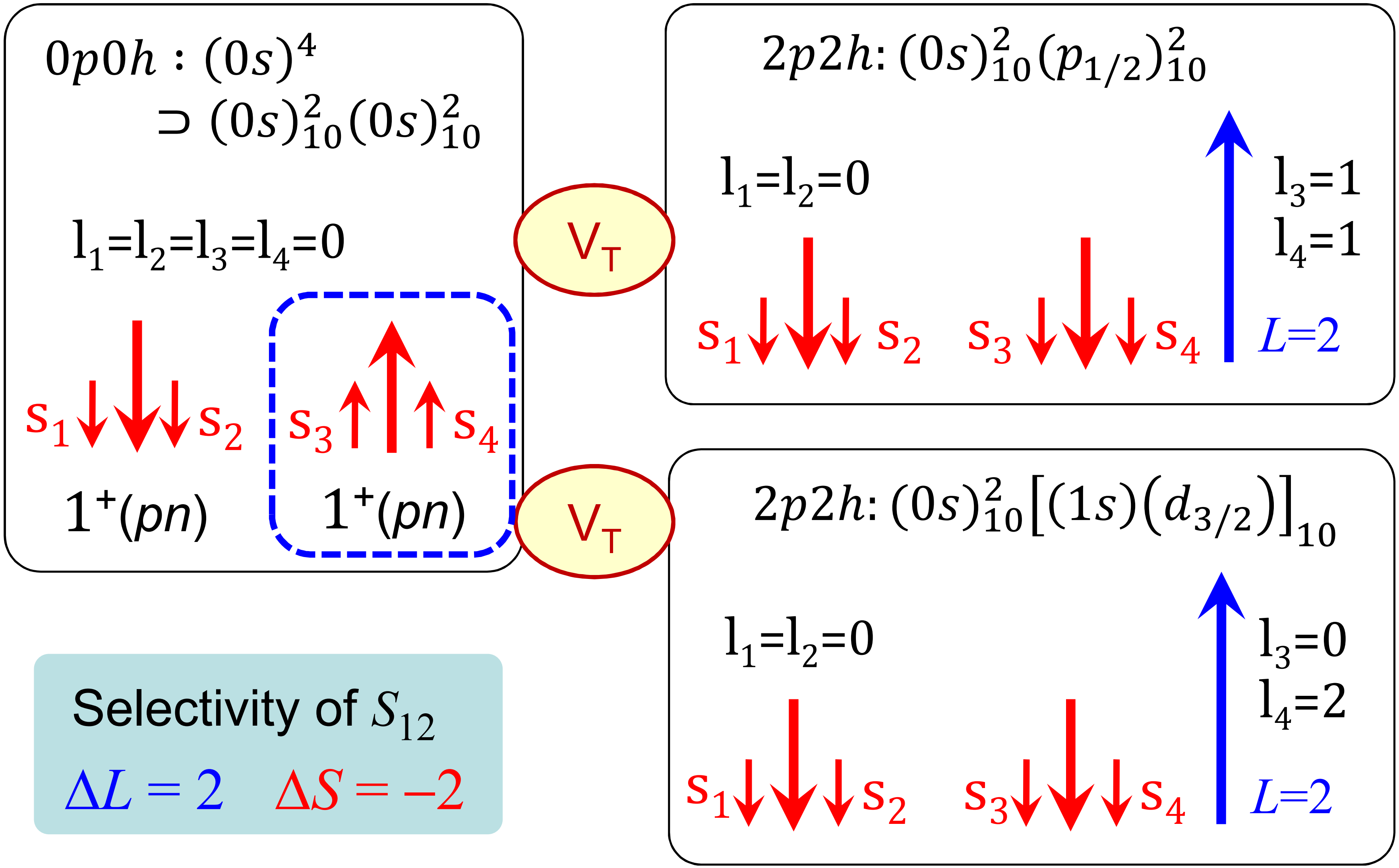
^{12}C(e,e'*p*N)

S.C.Simpson, J.A.Tostevin PRC83(2011)014605

$^{12}\mathrm{C} \rightarrow {}^{10}\mathrm{B}+pn$

- ^{4}He contains ***$p_{1/2}$*** of "***pn*-pair**"
  - Same feature in ^{5}He-^{8}He ground state

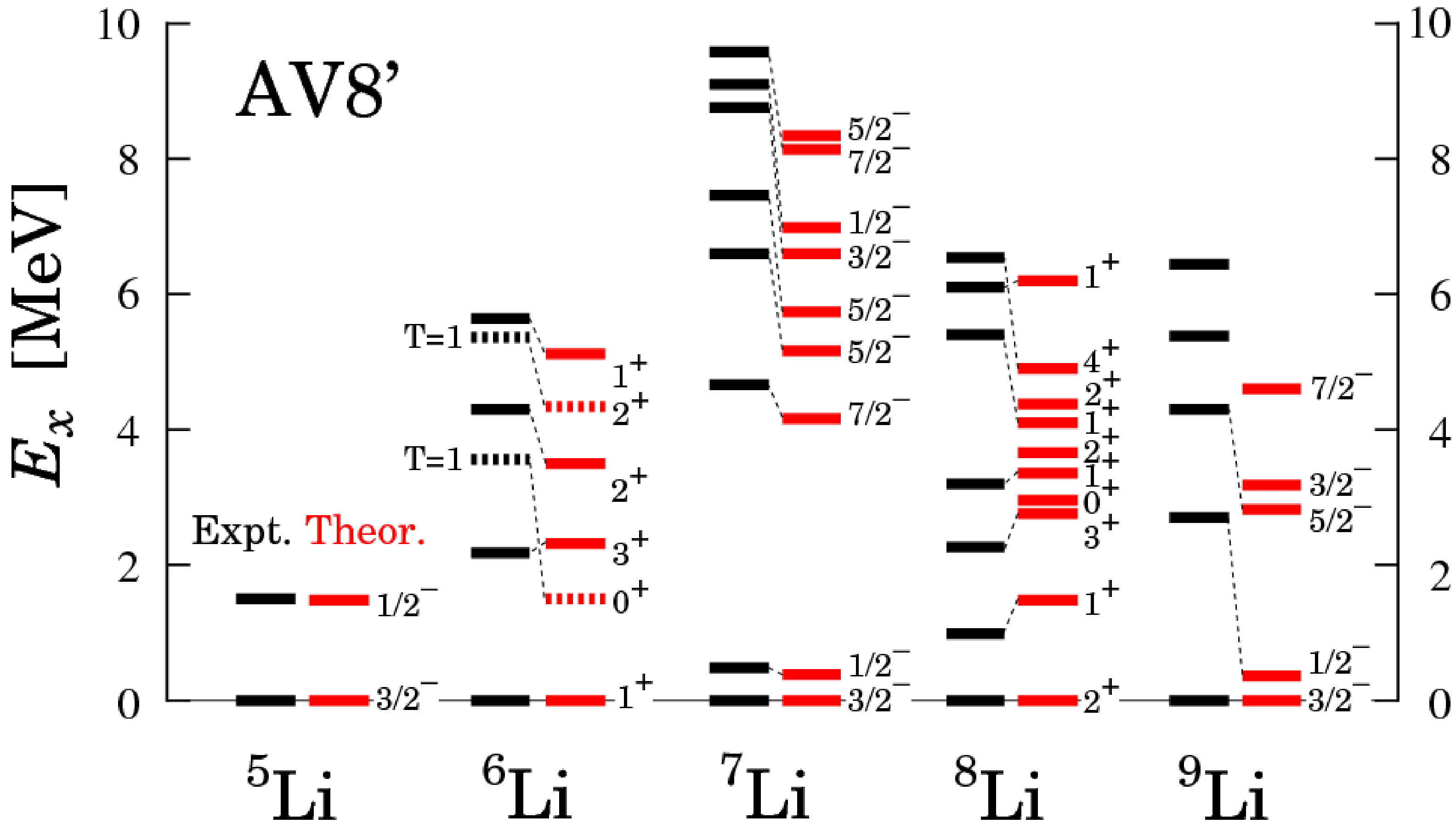
Selectivity of the tensor coupling in ^{4}He



$^{5\text{-}9}$Li with TOSM+UCOM

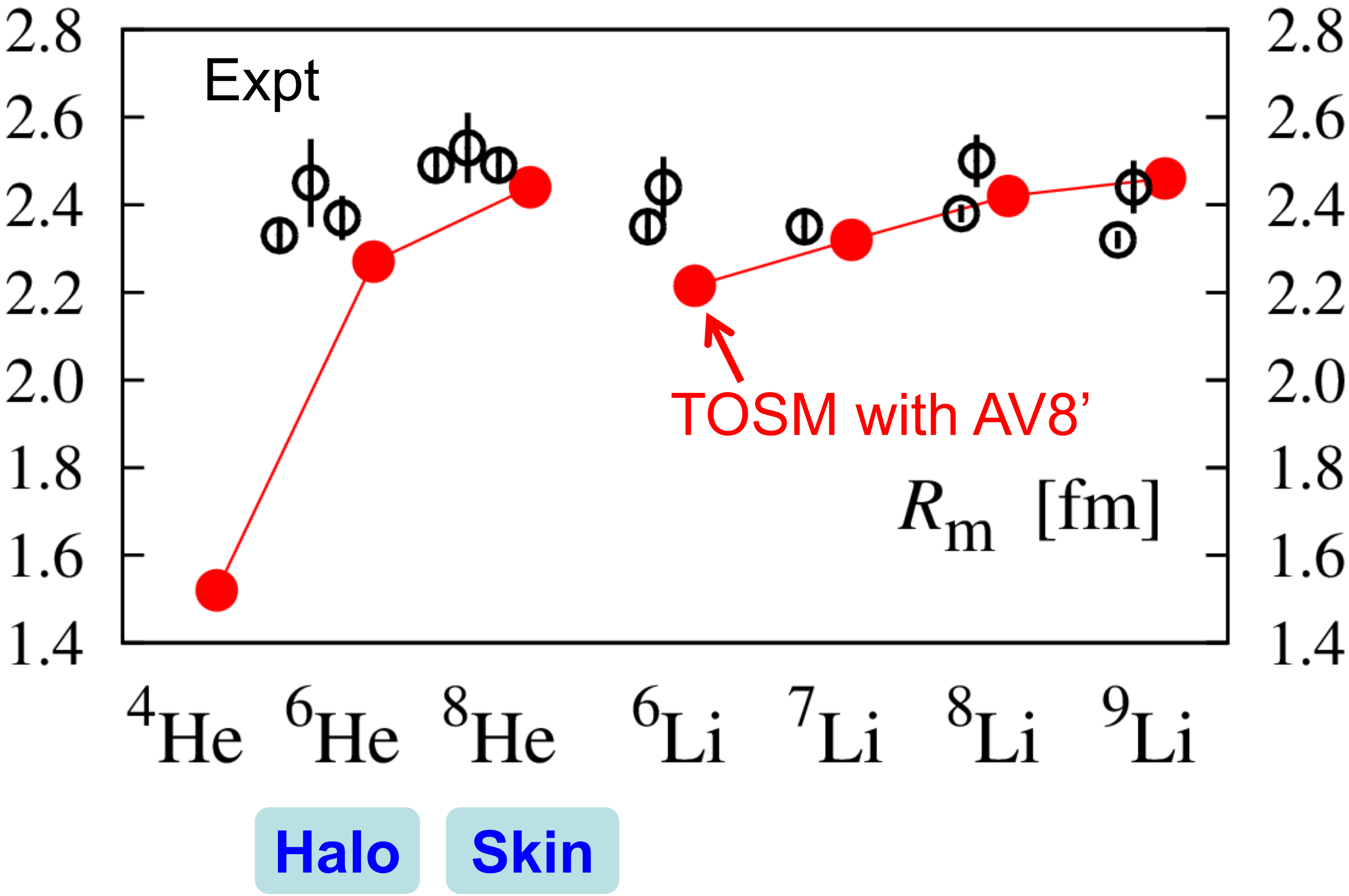
- Excitation energies in MeV

TM, A. Umeya, H. Toki, K. Ikeda
PRC86(2012) 024318



- Excitation energy spectra are reproduced well

Radius of He & Li isotopes

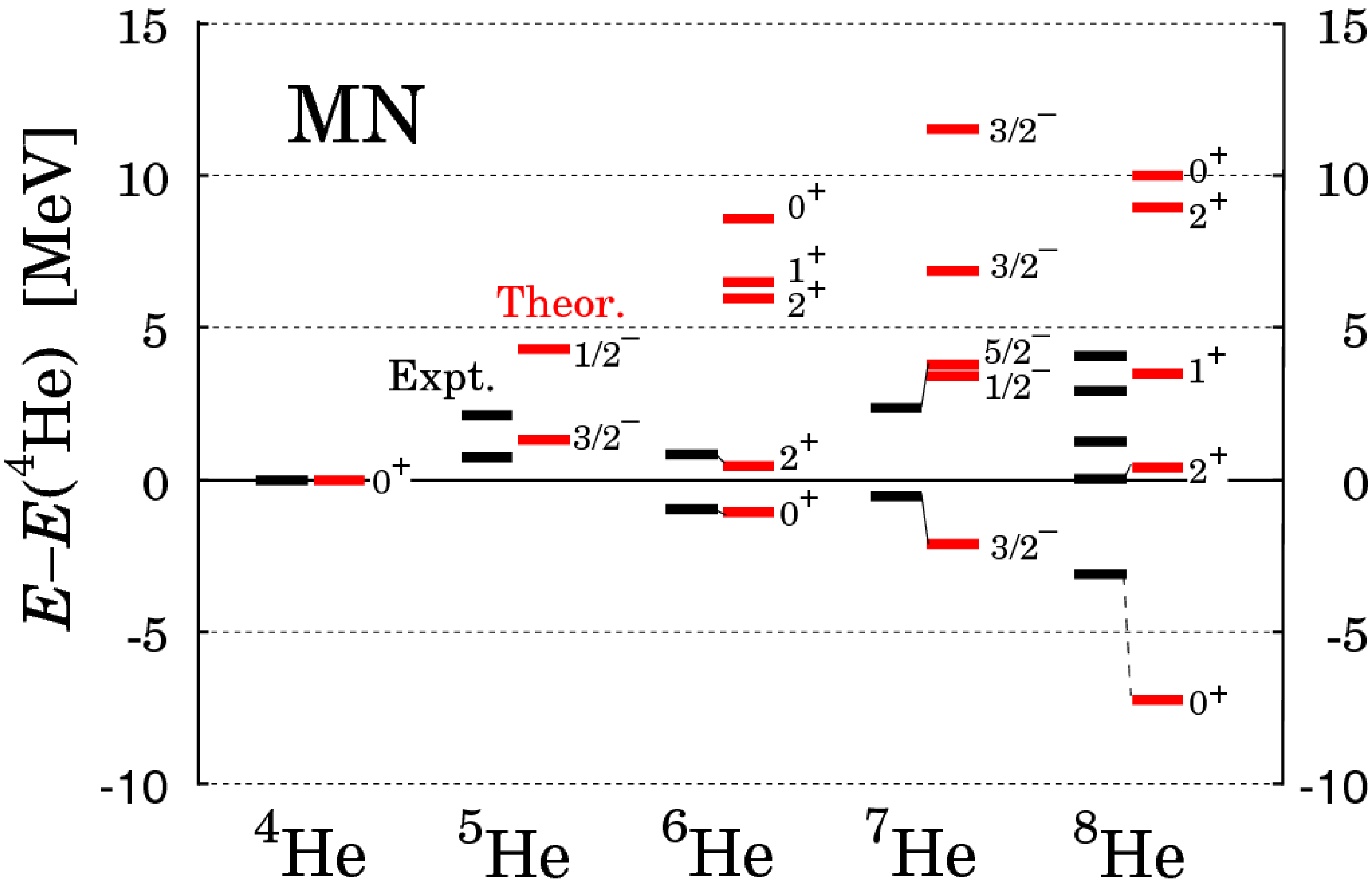


I. Tanihata et al., PLB289('92)261
O. A. Kiselev et al., EPJA 25, Suppl. 1('05)215.
A. Dobrovolsky, NPA 766(2006)1
G. D. Alkhazov et al., PRL78('97)2313
P. Mueller et al., PRL99(2007)252501

$^{4\text{-}8}$He with TOSM

Minnesota force (Central+LS)

- Difference from ^{4}He in MeV

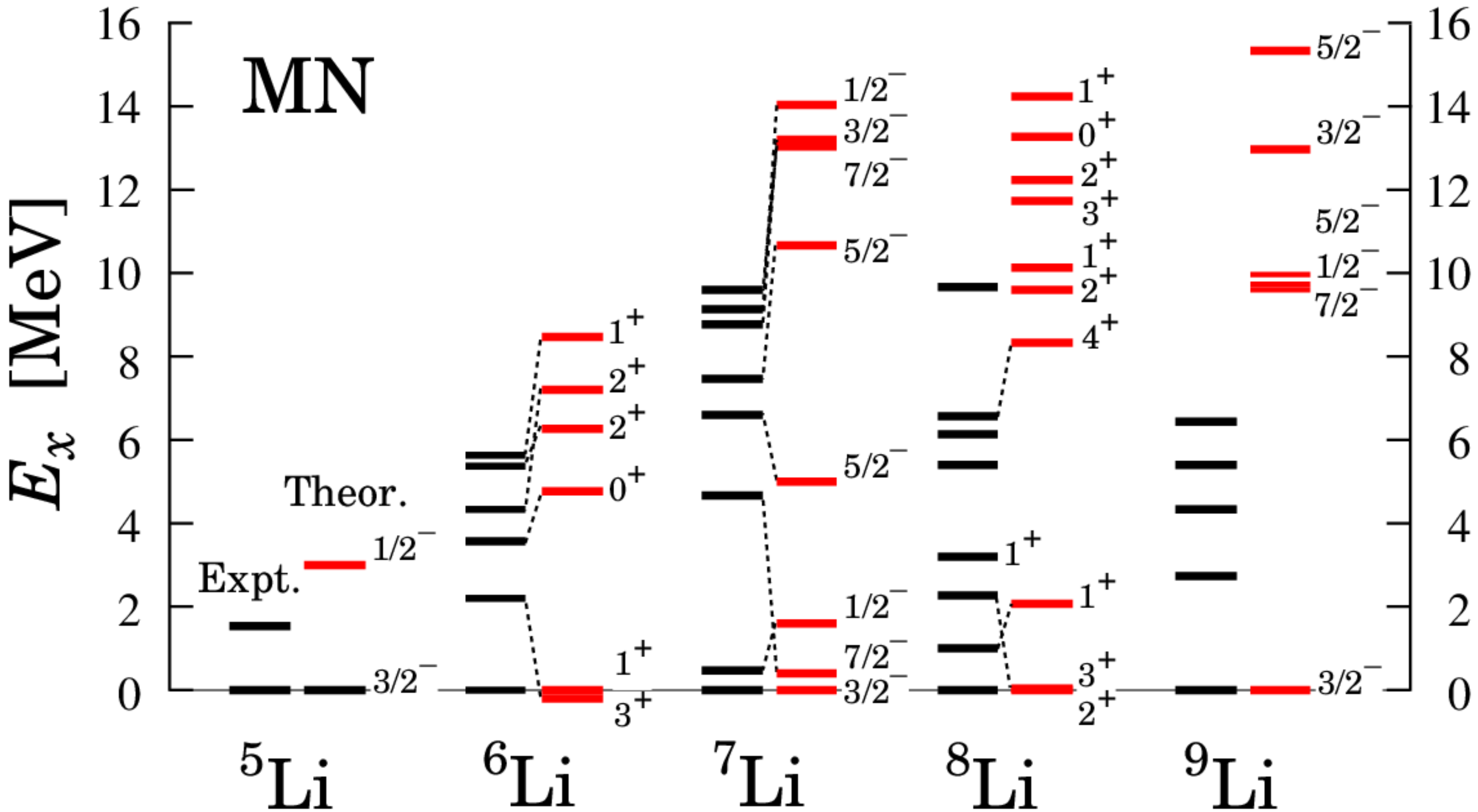


- No V_{NNN}
- No continuum

$^{5\text{-}9}$Li with TOSM

Minnesota force
NO tensor

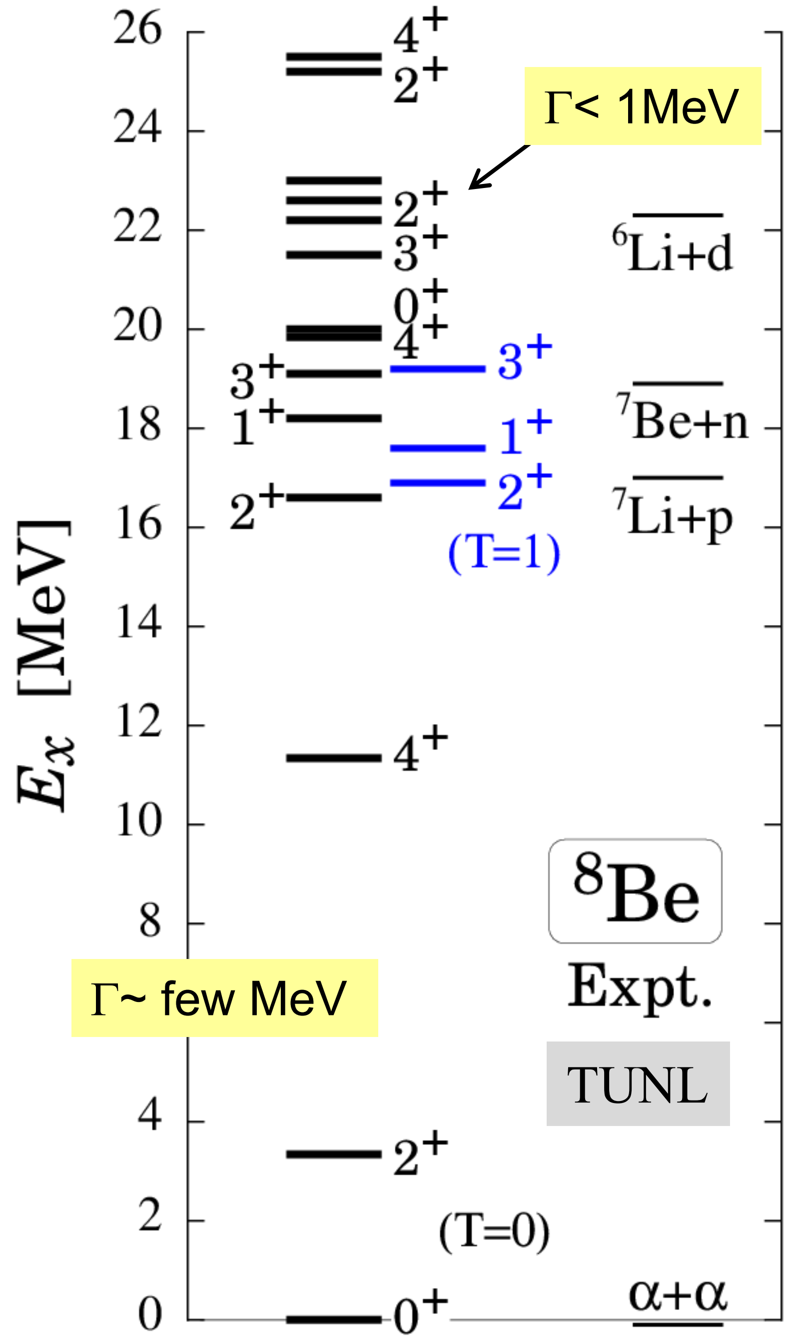
- Excitation energies in MeV



- Too large excitation energy

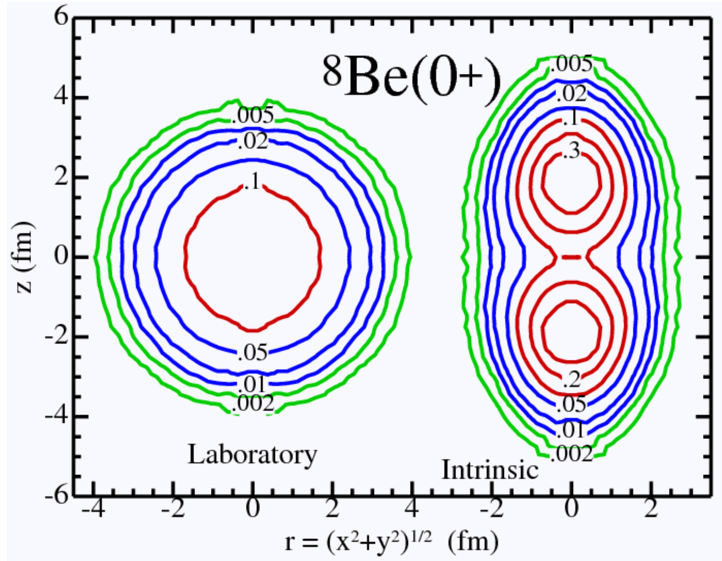
Be isotopes

TM, A. Umeya, K. Horii, H. Toki, K. Ikeda PTEP (2014) 033D01
TM, A. Umeya, H. Toki, K. Ikeda PTEP (2015) 073D02

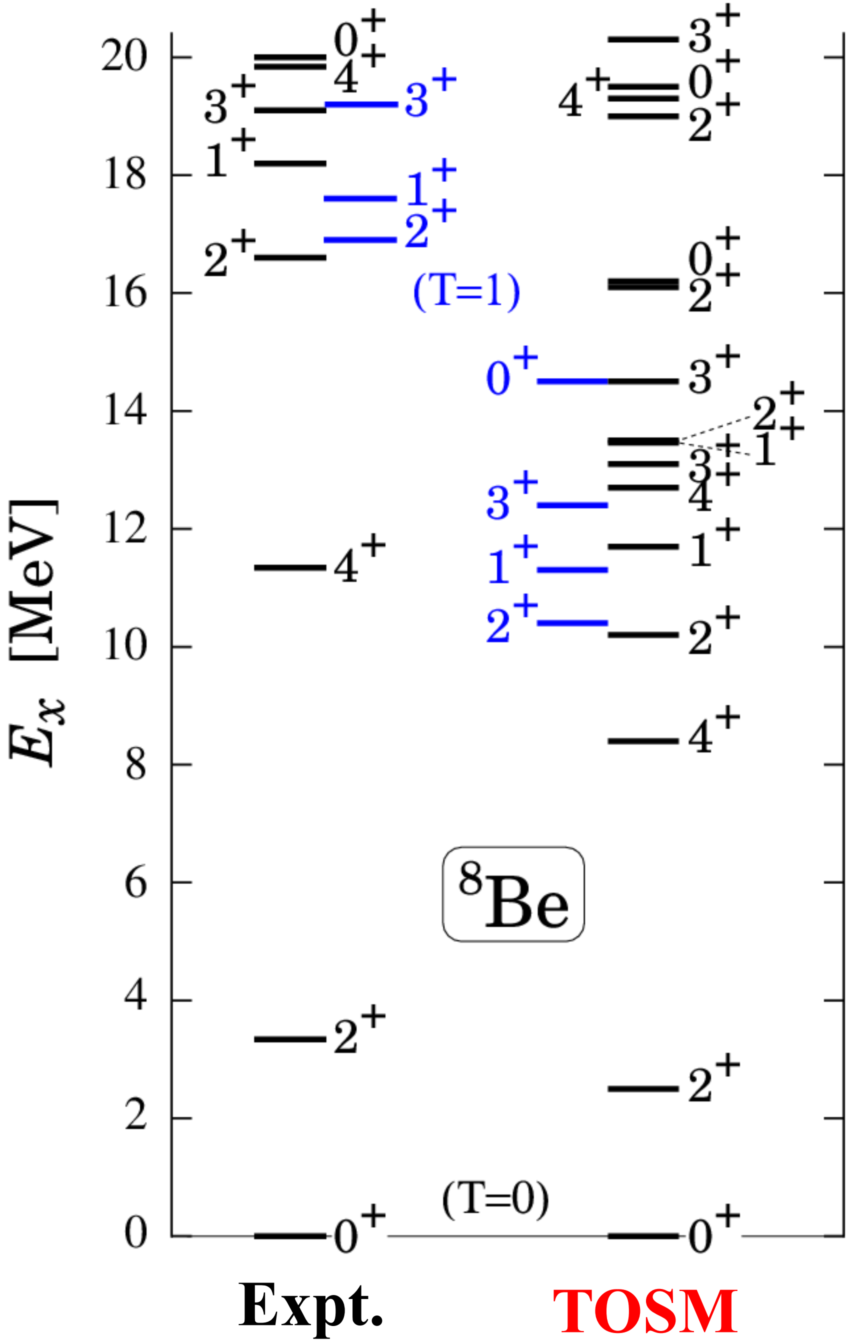


^{8}Be spectrum

- Argonne Group
  - Green's function Monte Carlo
    C. Pieper, R. B. Wiringa,
    Annu.Rev.Nucl.Part.Sci.51 (2001)

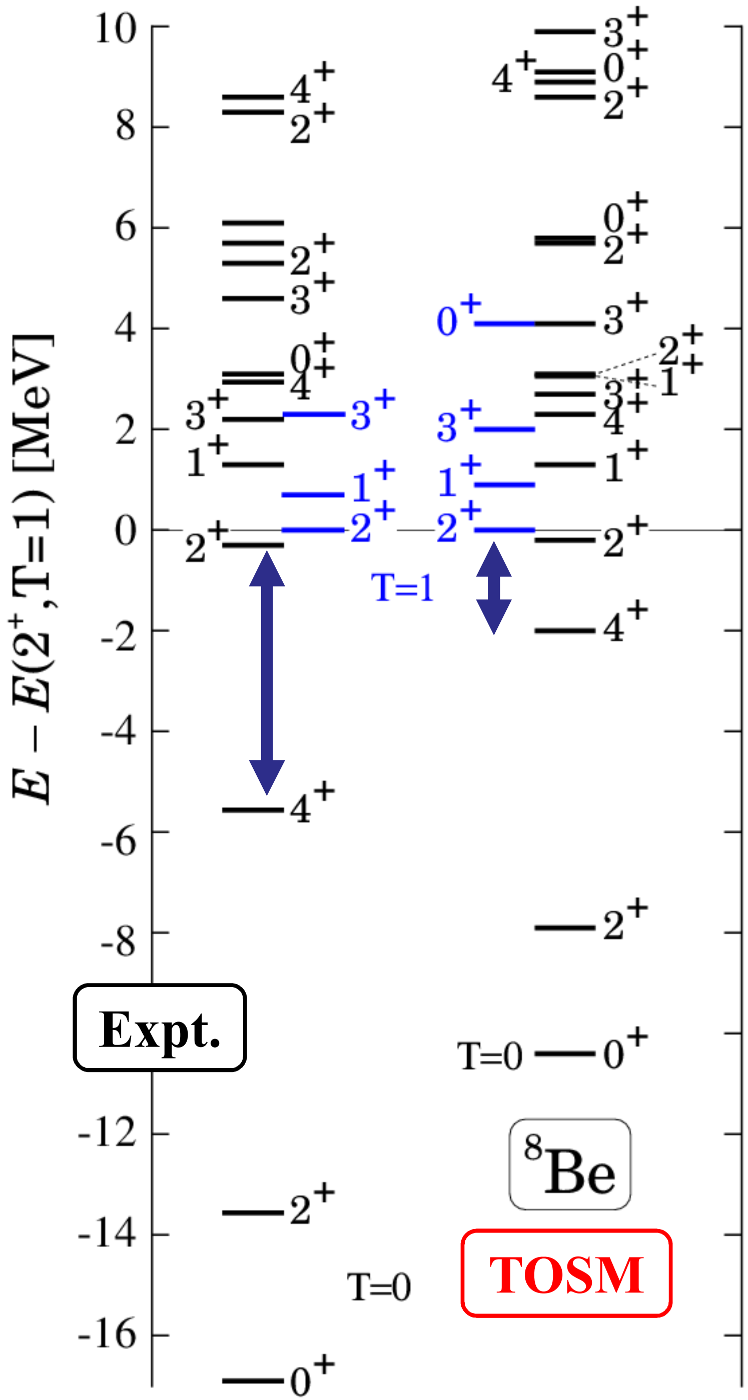


α-α structure



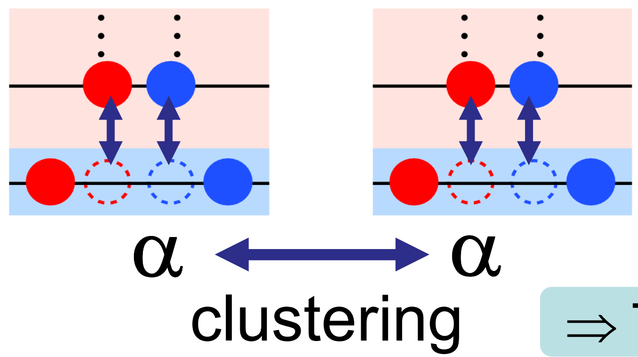
^{8}Be in TOSM – AV8' –

- $V_T \times 1.1$, $V_{LS} \times 1.4$
  - simulate ^{4}He benchmark (Kamada et al., PRC64)
- ground band
- highly excited states
  - small E_x
  - correct level order (T=0,1)
- R_m(^{8}Be)=2.21 fm
  - 2α THSR : 2.8 fm
  - ^{4}He :1.52 fm
  - ^{12}C :2.35 fm ~ R_m(exp)



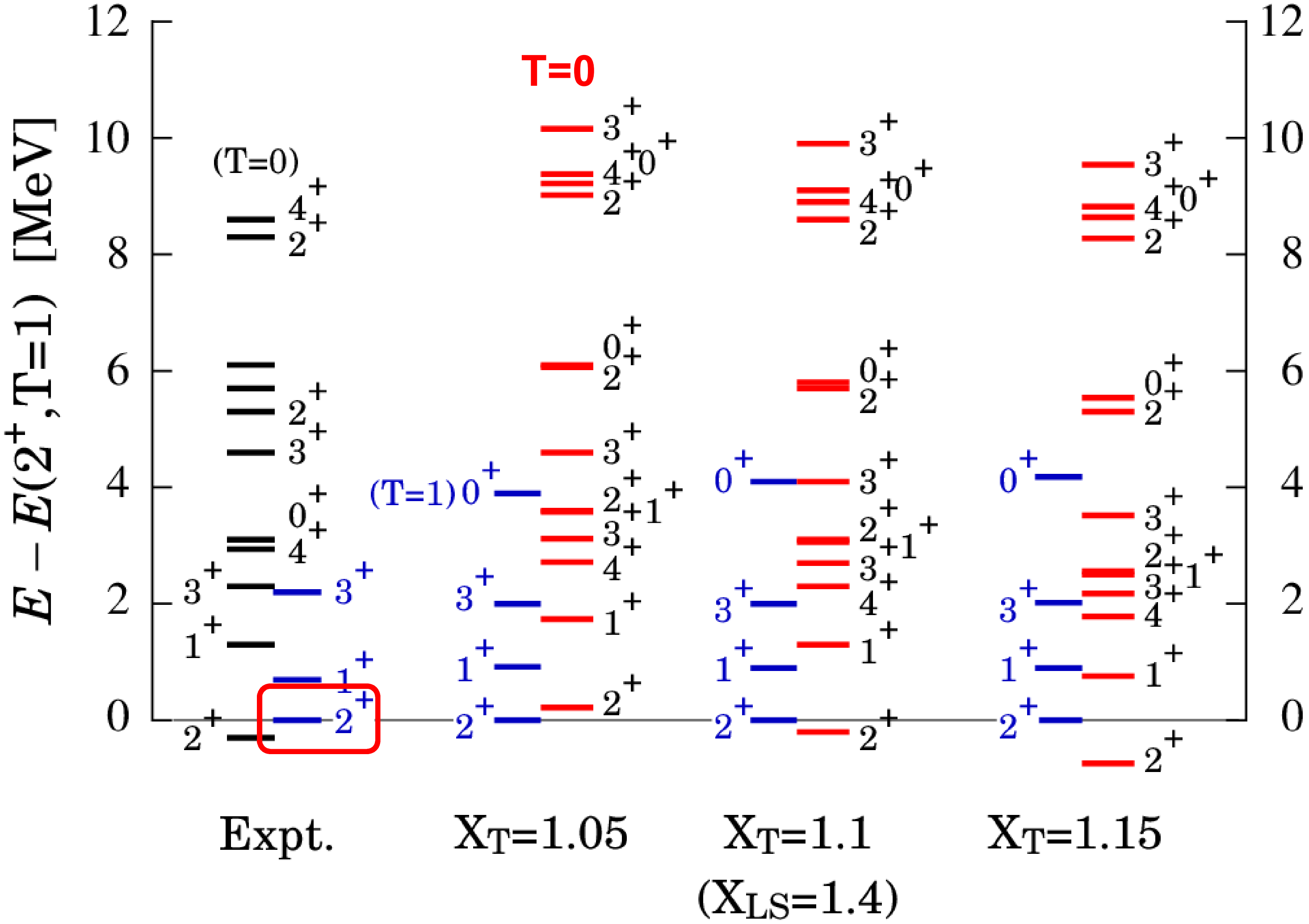
^{8}Be in TOSM
– AV8' –

- $V_T \times 1.1$, $V_{LS} \times 1.4$
  - simulate ^{4}He benchmark (Kamada et al., PRC64)
- correct level order (T=0,1)
- tensor contribution : T=0 > T=1
- α : 0p0h+2p2h with high-k
  - 2α needs 4p4h.
  - spatial asymptotic form of 2α

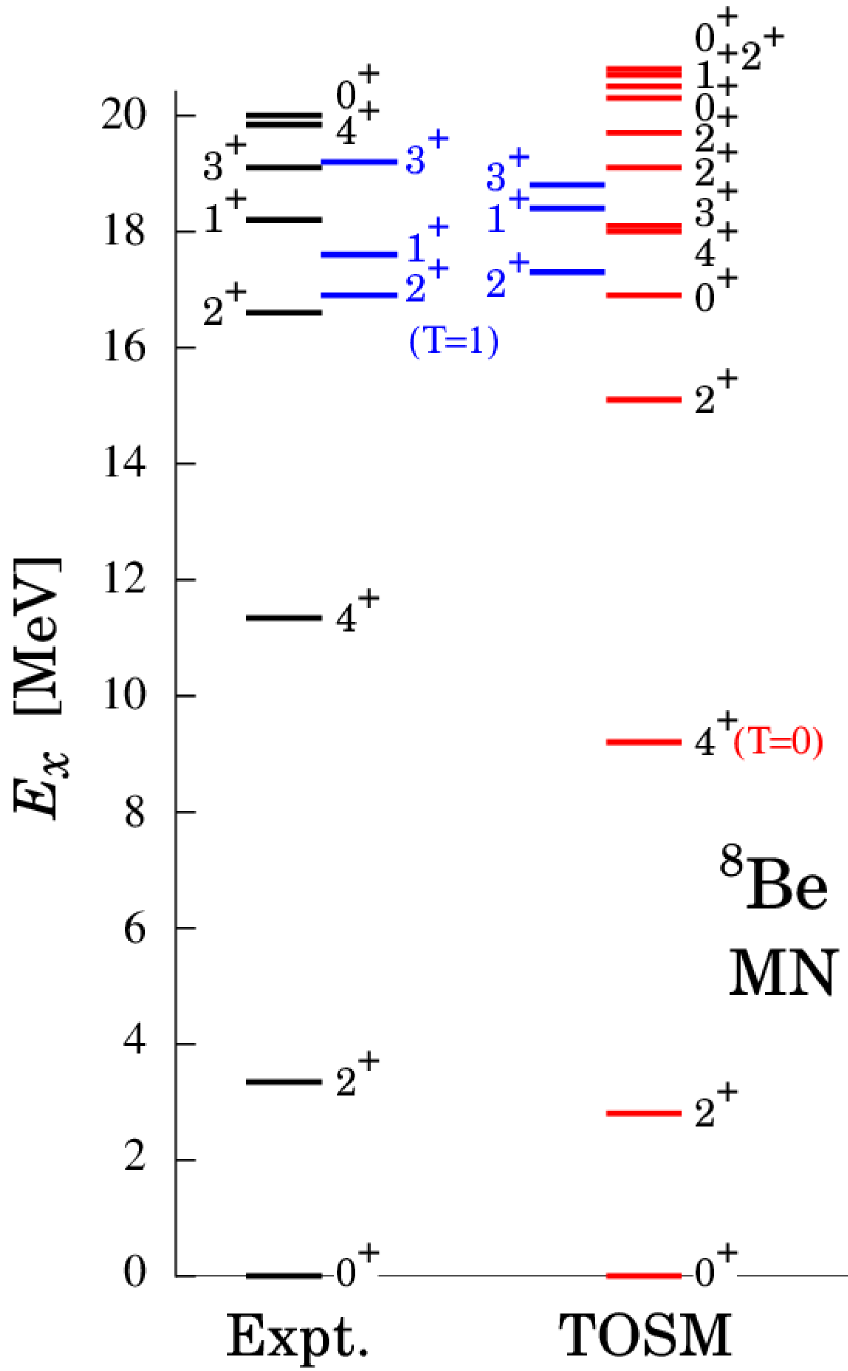


⇒ TOAMD

V_{tensor} dependence of ^{8}Be

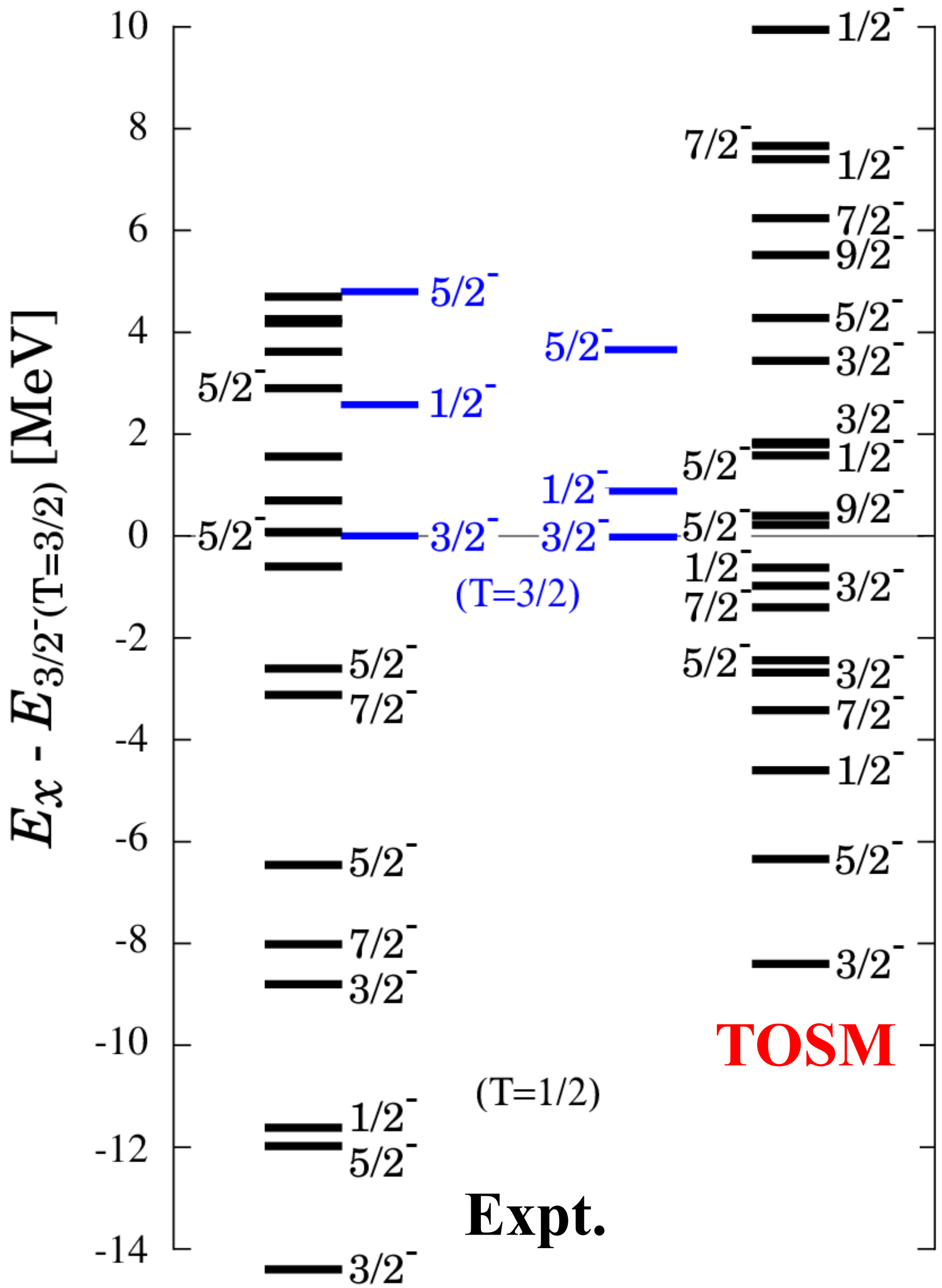


- *S*-wave UCOM can be simulated with X_T~1.1 (PTP121)
- Stronger tensor correlation in **<u>T=0 states</u>** than T=1 states.



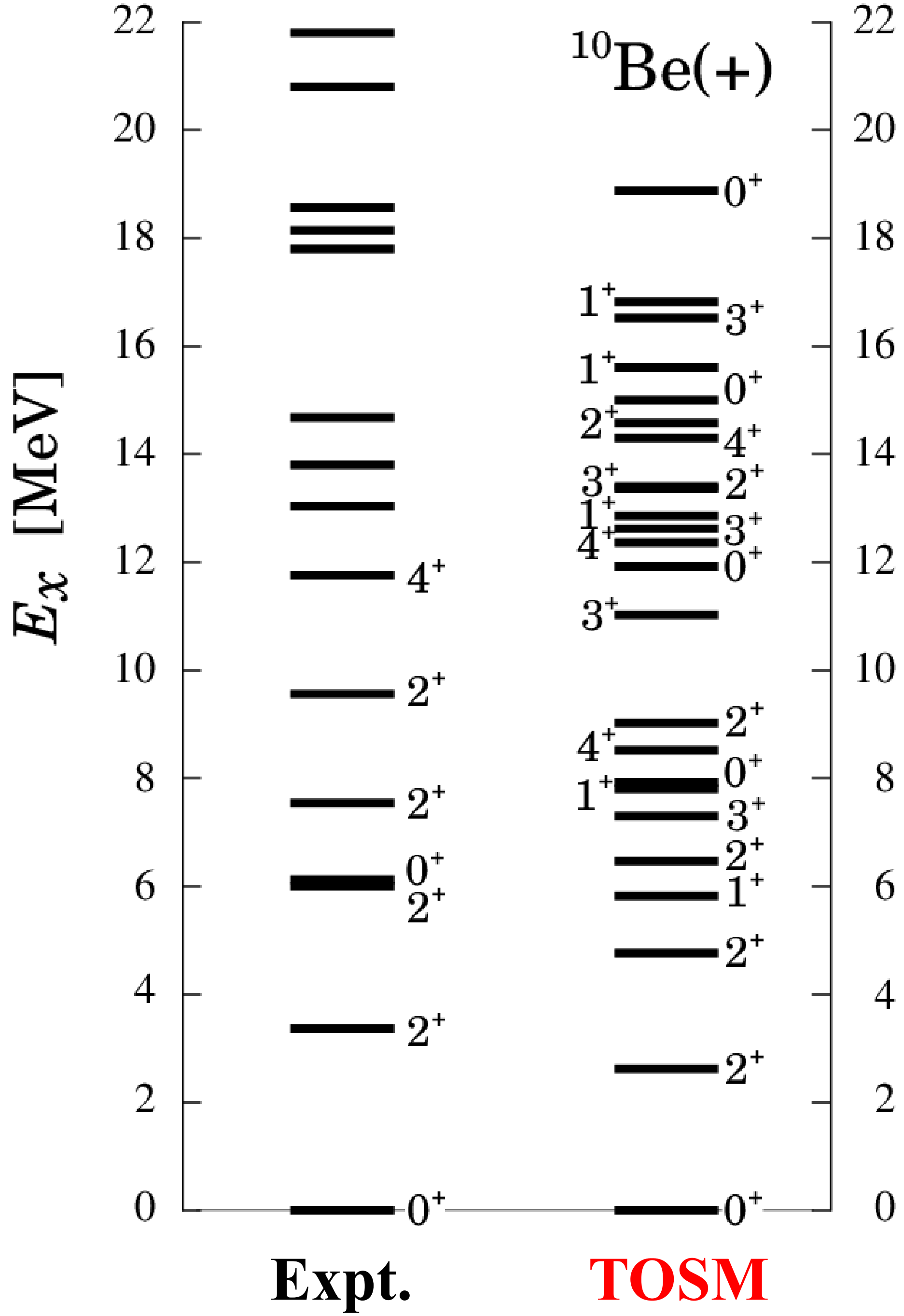
^{8}Be in TOSM – Minnesota –

- ground band
- highly excited states
  - good E_x
  - incorrect level order $(0^+, 1^+, 3^+)$
  - $E_x(T=0) < E_x(T=1)$
- Radius (R_m) is small
  - ^{4}He 1.39 fm with $(0s)^4$
  - ^{8}Be 1.89 fm
  - ^{12}C 1.85 fm



^{9}Be in TOSM – AV8' –

- correct level order
- highly excited states
  - small E_x with T=1/2,3/2
  - missing of α-clustering in the ground-band region
- $R_m(^{9}Be)$=2.32 fm
  - Exp: 2.38(1) fm
- $1/2^+$ locates at E_x=8 MeV.
- comparison of T=1/2 & 3/2 for $J^\pi=1/2^-_1, 3/2^-_1, 5/2^-_1$
  - $\Delta\langle T\rangle \sim$ 20 MeV
    $\Delta\langle V_T\rangle \sim -15$ MeV



^{10}Be in TOSM – AV8' –

- 0^+_1 : $(0p_{3/2})^6$ ~ 38%
- 0^+_2 : $(0p_{3/2})^4_\nu (0p_{1/2})^2_\pi$ ~ 39%
  - Largest $\langle V_T \rangle$= -110 MeV.
    c.f. ^{8}Be(gs) : -97 MeV
    ^{4}He(gs) : -62 MeV

    Suggest α clustering state?
- 0^+_3 : $(0p_{3/2})^4 (0p_{1/2})^2_\nu$ ~ 36%
- 0^+_4 : $(0p_{3/2})^4 (0p_{1/2})^2_{\pi\nu}$ ~ 30%
- $R_m(^{10}\text{Be})$=2.31 fm
  - Exp: 2.30(2) fm

Clustering with tensor correlation

PTEP
Prog. Theor. Exp. Phys. **2015**, 073D02 (38 pages)
DOI: 10.1093/ptep/ptv087

Tensor-optimized antisymmetrized molecular dynamics in nuclear physics (TOAMD)

Takayuki Myo[1,2,*], Hiroshi Toki[2], Kiyomi Ikeda[3], Hisashi Horiuchi[2], and Tadahiro Suhara[4]

$$\left|\Phi_{\mathrm{TOAMD}}\right\rangle = \left|\Phi_{\mathrm{AMD}}\right\rangle + F_D\left|\Phi_{\mathrm{AMD}}\right\rangle + F_S\left|\Phi_{\mathrm{AMD}}\right\rangle + \cdots$$

0p0h 2p2h

$$F_D = \sum_{t=0,1}\sum_{i<j}^{A} f_D^t(\vec{r}_i - \vec{r}_j)\ ,\ \ f_D(\vec{r}\,) = \sum_n C_n \exp(-a_n r^2)\, r^2 S_{12}$$

tensor correlation

$$F_S = \sum_{s,t}\sum_{i<j}^{A} f_S^{st}(\vec{r}_i - \vec{r}_j)\ ,\ \ f_S(\vec{r}\,) = \sum_n C_n \exp(-a_n r^2)$$

short-range correlation

Formulation of TOAMD

0p0h

2p2h

$$\left|\Phi_{\mathrm{TOAMD}}\right\rangle = C_0\left|\Phi_{\mathrm{AMD}}\right\rangle + F_D\left|\Phi^{D}_{\mathrm{AMD}}\right\rangle + F_S\left|\Phi^{S}_{\mathrm{AMD}}\right\rangle$$

$$+F_S F_D\left|\Phi^{SD}_{\mathrm{AMD}}\right\rangle + F_D F_D\left|\Phi^{DD}_{\mathrm{AMD}}\right\rangle + F_S F_S\left|\Phi^{SS}_{\mathrm{AMD}}\right\rangle + \cdots$$

4p4h

$$F_D = \sum_{t=0,1}\sum_{i<j}^{A} f_D^t(\vec{r}_i - \vec{r}_j)\ ,\ \ f_D(r) = r^2 S_{12}\sum_{n}^{N_G} C_n \exp(-a_n r^2)$$

- Variational parameters
  - ν, $\mathbf{Z}_i$ (i=1, ..., A) , spin-direction (up/down) in AMD
  - C_0 , C_n , a_n in Gaussian expansion
  - Solve cooling equation for $E = \dfrac{\langle\Phi_{\mathrm{TOAMD}}|H|\Phi_{\mathrm{TOAMD}}\rangle}{\langle\Phi_{\mathrm{TOAMD}}|\Phi_{\mathrm{TOAMD}}\rangle}$

Matrix elements of multi-body operator

$$|\Phi_{\mathrm{AMD}}\rangle = \frac{1}{\sqrt{A!}} \det\{\varphi_1, \ldots, \varphi_A\}$$

$$|\varphi\rangle = |\mathbf{Z}\rangle |\chi^{\sigma\tau}\rangle$$

$$\langle \mathbf{r} | \mathbf{Z} \rangle \propto \exp\left[-\nu (\mathbf{r} - \mathbf{Z})^2\right]$$

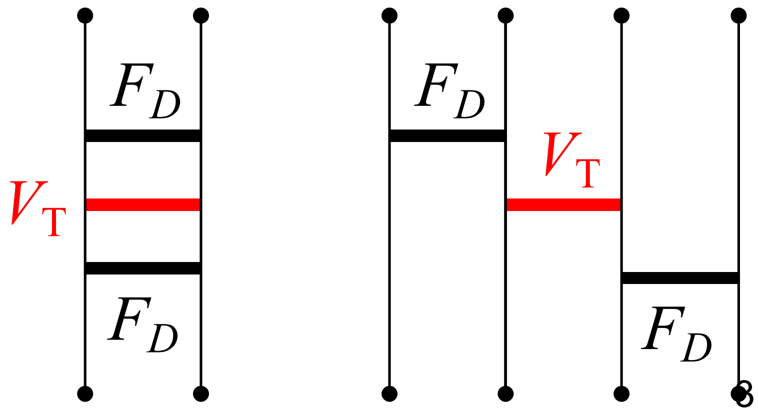
Matrix elements

$$\langle \varphi_i \varphi_j \ldots | \hat{O} | \varphi_{i'} \varphi_{j'} \ldots \rangle_{\mathrm{A}}$$

$$\hat{O} = F_D F_D \;,\; F_S F_S \;,\; F_S F_D \;,$$
$$F_D V_{\mathrm{T}} F_D \;,\; F_S V_{\mathrm{T}} F_S \;,\; \cdots$$

- $F_D V_{\mathrm{T}} F_D$ generates at most 6-body matrix elements.
- Classify the connection between the operators.

2-body　　　4-body



Matrix elements with Fourier trans.

- Fourier transformation of the interaction V & F_D, $F_{S.}$
  - Y. Goto and H. Horiuchi, Prog. Theor. Phys., **62** (1979) 662
  - Gaussian expansion of V, F_D, F_S
  - Multi-body operators are represented in the separable form for particle coordinates.
  - Three-body interaction can be treated in the same manner.

$$e^{-a(\vec{r}_i-\vec{r}_j)^2} = \frac{1}{(2\pi)^3}\left(\frac{\pi}{a}\right)^{3/2}\int d\vec{k}\ e^{-\vec{k}^2/4a}\ e^{i\vec{k}r_i}e^{-i\vec{k}r_j}$$

$$r_{ij}^2 S_{12}(\vec{r})\ e^{-a(\vec{r}_i-\vec{r}_j)^2} = \frac{1}{(2\pi)^3}\left(\frac{\pi}{a}\right)^{3/2}\left(\frac{i}{2a}\right)^2\int d\vec{k}\ e^{-\vec{k}^2/4a}\ e^{i\vec{k}r_i}e^{-i\vec{k}r_j}\ \vec{k}^2 S_{12}(\vec{k})$$

$$\left\langle \mathbf{Z}\middle| e^{i\vec{k}r} \middle| \mathbf{Z}'\right\rangle = \left\langle \mathbf{Z}\middle|\mathbf{Z}'\right\rangle \cdot \exp\left(i\vec{k}(\mathbf{Z}+\mathbf{Z}')/2 - \vec{k}^2/8\nu\right)$$

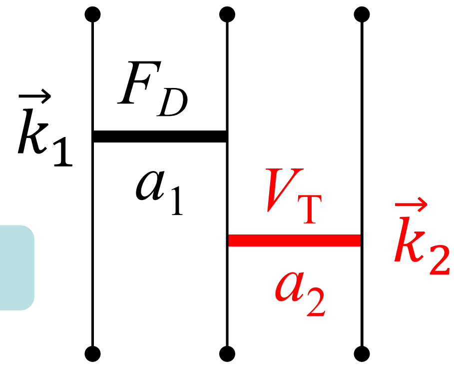
Matrix elements with Fourier trans.

- Example : $F_D \times V_\mathrm{T}$

  (2-body)×(2-body)

  = (2-body)+(3-body)+(4-body)

3-body



$$\langle \Phi_\mathrm{AMD} | (F_D V_\mathrm{T})_3 | \Phi_\mathrm{AMD} \rangle \propto \iint d\vec{k}_1 d\vec{k}_2 \; e^{-\vec{k}_1^2/4a_1} e^{-\vec{k}_2^2/4a_2}$$

k-integral

$$\times \sum_{\mathrm{ijk,\, i'j'k'}} \langle \mathbf{Z}_i | e^{i\vec{k}_1 r} | \mathbf{Z}'_i \rangle \langle \mathbf{Z}_j | e^{-i\vec{k}_1 r} e^{i\vec{k}_2 r} | \mathbf{Z}'_j \rangle \langle \mathbf{Z}_k | e^{-i\vec{k}_2 r} | \mathbf{Z}'_k \rangle$$

spatial part

$$\times \sum_{\substack{xyx'y' \\ uvu'v'}} k_{1x} k_{1y} k_{2u} k_{2v} \left(3\delta_{xx'}\delta_{yy'} - \delta_{xy}\delta_{x'y'} \right) \left(3\delta_{uu'}\delta_{vv'} - \delta_{uv}\delta_{u'v'} \right)$$

(tensor)^2

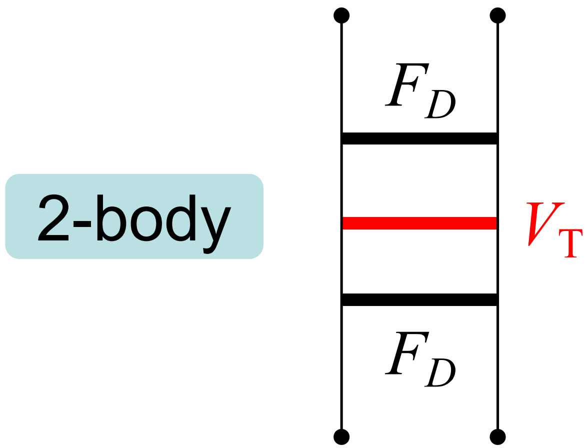
$$\times \langle \chi_i | \sigma_{x'} | \chi'_i \rangle \langle \chi_j | \sigma_{y'} \sigma_{u'} | \chi'_j \rangle \langle \chi_k | \sigma_{v'} | \chi'_k \rangle \cdot \det \left| B^{-1}_{i'i} B^{-1}_{j'j} B^{-1}_{k'k} \right|$$

spin-isospin part

antisymmetrization

Results

- ^3H, ^4He, ^8Be (preliminary)
- AV8' (central+LS+tensor)
- AMD state $|\Phi_{\mathrm{AMD}}\rangle$ is common in each basis state
- Within 2-body correlation from V, F_D, F_S
- No J^{π}-projection



Summary

- **Tensor-optimized shell model (TOSM)** using V_{bare}.
  - Strong tensor correlation from 0p0h-2p2h involving high-k.
- He, Li isotopes
  - ***pn*-pair of $p_{1/2}$** owing to S_{12} of V_{T}, affects the spectrum of n-rich nuclei.
- ^{8}Be, ^{9}Be
  - Two aspects : grand state region & highly excited states.
  - Indication of more configurations to describe α–α states.
- ^{10}Be : p-shell dominant configurations in $0^+_{1,2}$
  - Largest tensor contribution in 0^+_2 among 0^+ states
- **Tensor-Optimized AMD (TOAMD)**.
  - Two-kinds of correlation functions F_D (tensor) & F_S (short-range)