YIPQS Long-term workshop
Computational Advances in Nuclear and Hadron Physics (CANHP 2015)
21th September - 30th October, 2015
Yukawa Institute for Theoretical Physics, Kyoto, Japan

Large-scale configuration interaction description of the structure of nuclei around ^{100}Sn

Chong Qi (亓冲)
KTH Royal institute of Technology, Stockholm

R. Liotta, R. Wyss, T. Bäck, A. Johnson, B. Cederwall (KTH)
Zhen-Xiang Xu, F.R. Xu (Beijing)
H. Jiang, YM Zhao (Shanghai)

PHYSICAL REVIEW C **80**, 064323 (2009)

New effective interaction for f_5pg_9-shell nuclei

M. Honma,[1] T. Otsuka,[2,3,4] T. Mizusaki,[5] and M. Hjorth-Jensen[6]

[1]*Center for Mathematical Sciences, University of Aizu, Tsuruga, Ikki-machi, Aizu-Wakamatsu, Fukushima 965-8580, Japan*

[2]*Department of Physics and Center for Nuclear Study, University of Tokyo, Hongo, Bunkyo-ku, Tokyo 113-0033, Japan*

[3]*RIKEN, Hirosawa, Wako-shi, Saitama 351-0198, Japan*

[4]*National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824-1321, USA*

[5]*Institute of Natural Sciences, Senshu University, Higashimita, Tama, Kawasaki, Kanagawa 214-8580, Japan*

[6]*Department of Physics and Center of Mathematics for Applications, University of Oslo, N-0316 Oslo, Norway*

Stochastic extension of the Lanczos method for nuclear shell-model calculations with variational Monte Carlo method

Noritaka Shimizu [a,*], Takahiro Mizusaki [b], Kazunari Kaneko [c]

$$|\Psi(\mathbf{c})\rangle = \left(\sum_{q=0}^{p} c_q H^q\right)|\psi^t\rangle \qquad (1)$$

where H is the Hamiltonian and $\mathbf{c} = \{c_0, c_1, c_2, \ldots, c_p\}$ is a set of variational parameters, which are determined by minimizing

赤・緑・黄の食品の摂り方

Outline

- Brief introduction to the nuclear shell model/full configuration interaction approach
  - ❖Quest for the effective interaction
  - ❖A short tour to Sn isotopes
- Truncation methods
- System with identical particles: Novel aspects of the seniority coupling
  Unexpected strong E1 transitions
- Structure of nuclei around Z=50 and the stability of the Z=N=50 closed shell
  Breaking the presumed ^{100}Sn inert core

Nuclear theory: guiding principles

The quest for the mean field

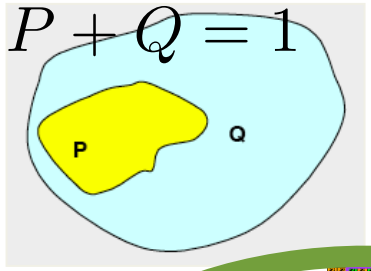
$$\hat{H} = \sum_{i=1}^{A} \frac{\hat{p}_i^2}{2m_i} + \sum_{i<j}^{A} \hat{V}(r_i, r_j)$$

$$\hat{H} = \sum_{i=1}^{A} \left[\frac{\hat{p}_i^2}{2m_i} + \hat{V}(r_i) \right] + \left[\sum_{i<j}^{A} \hat{V}(r_i, r_j) - \sum_{i=1}^{A} \hat{V}(r_i) \right]$$

$$\left[-\frac{\hbar^2}{2 \cdot m} \nabla^2 + V(r) - \varepsilon \right] \Psi(r) = 0$$

$$\Psi(r) = \frac{u_\ell(r)}{r} \cdot Y_{\ell m}(\vartheta, \varphi) \cdot X_{m_s}$$

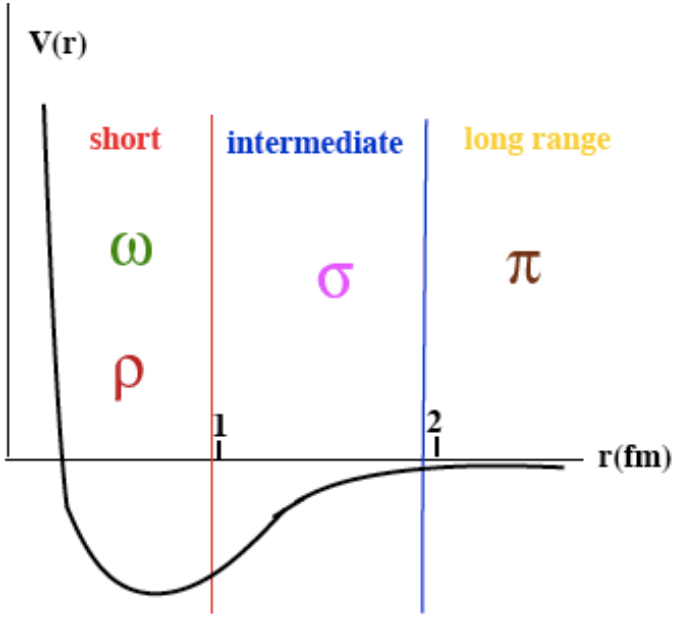
$P + Q = 1$



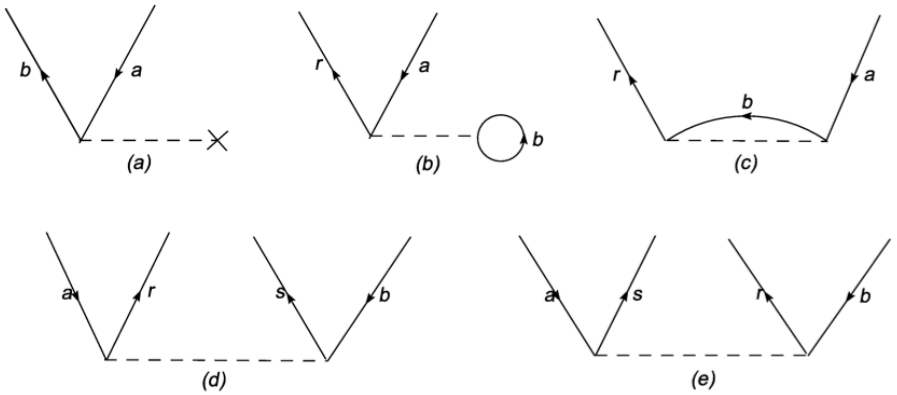
The nuclear shell model, as we call it, is a *full* configuration interaction approach. That is, it considers the mixing effect of all possible configurations within a given model space.

"Bare" Nucleon-Nucleon Potentia

- Argonne V18: PRC 56, 1720 (1997)
- CD-Bonn 2000: PRC 63, 024001 (2000)
- *N³LO: PRC 68, 041001 (2003)*
- *INOY: PRC 69, 054001 (2004)*



Perturbation treatment



and many others.

Usually we stop at the second or third order (and do the dirty trick).

Binding energies of Ca isotopes

A phenomenological view

WS + constant pairing

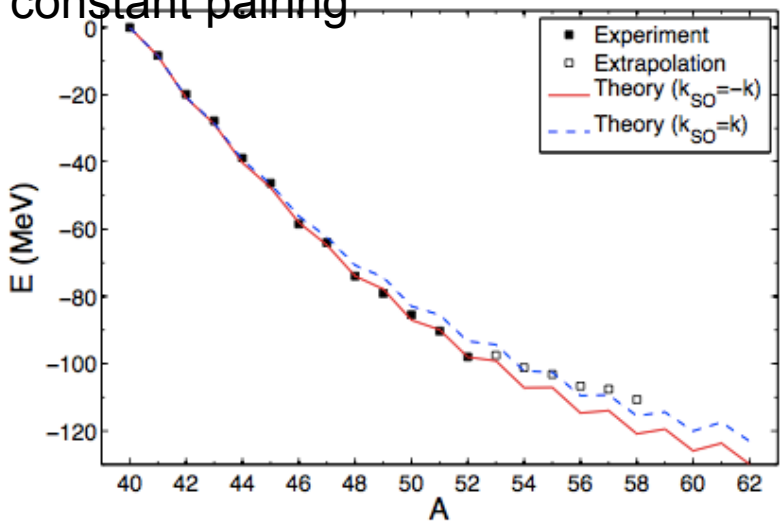
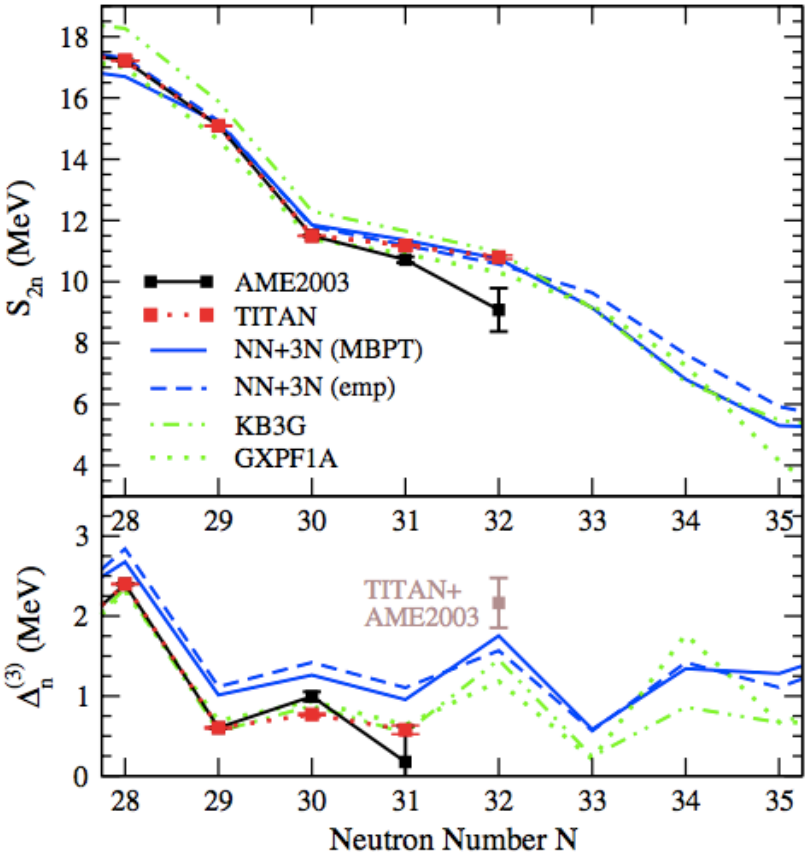


FIG. 6. (Color online) Experimental [34, 37] and calculated ground-state energies of Ca isotopes, relative to that of ^{40}Ca, as a function of mass number A.

- Easier to handle
- Quantitatively good

Z. Xu, CQ, Phys. Lett. B (2013)

The Microscopic picture



A. T. Gallant et al., PRL 109, 032506 (2012)

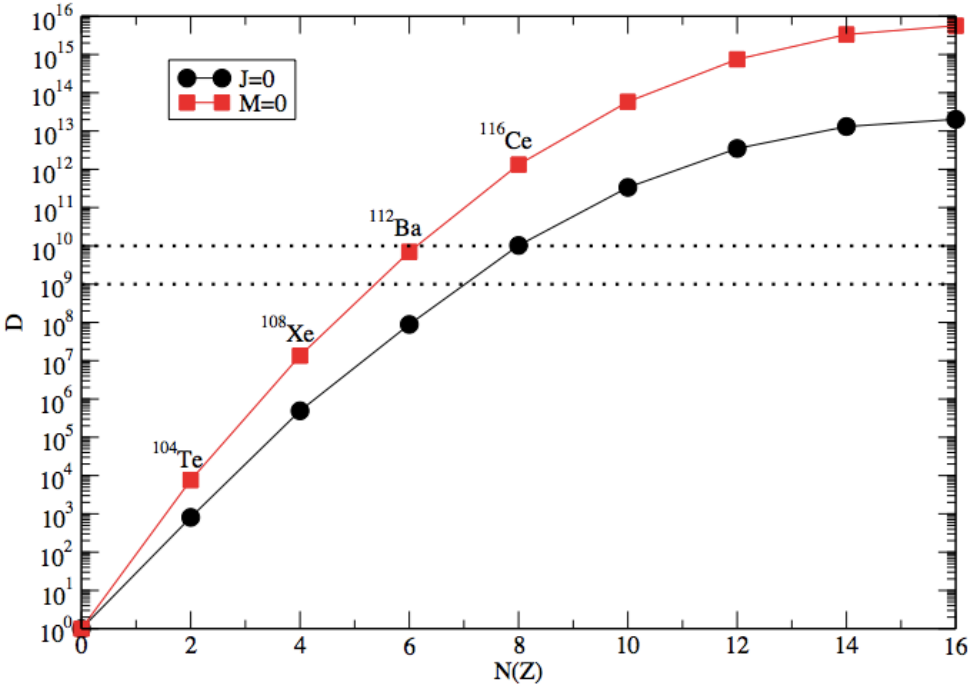
Calculations with three-body interaction

J.D. Holt, T. Otsuka, A. Schwenk, and T. Suzuki, J. Phys. G 39, 085111 (2012).

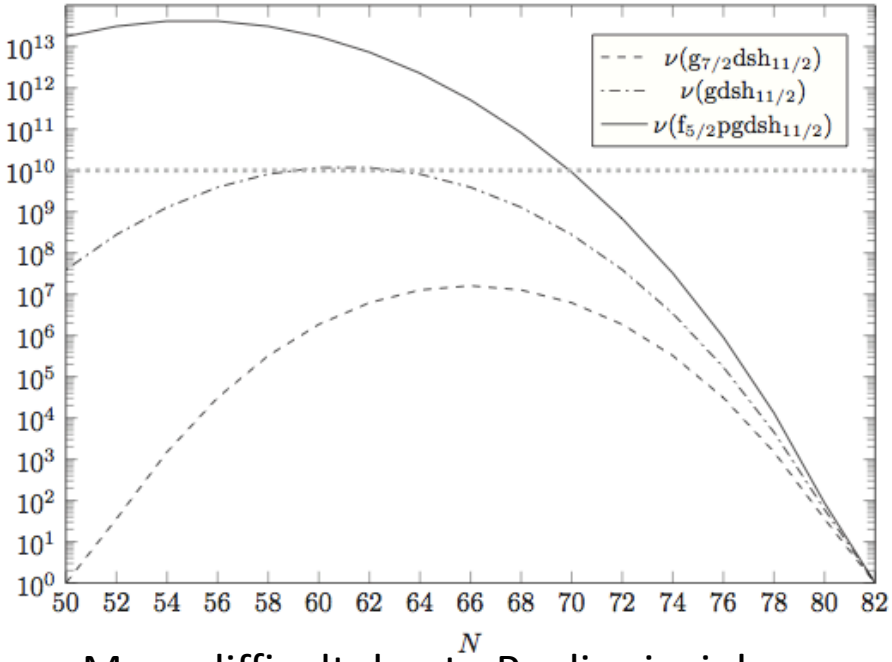
G. Hagen, M. Hjorth-Jensen, G.R. Jansen, R. Machleidt, T. Papenbrock, Phys.Rev.Lett. 109, 032502 (2012).

Computational challenge

Dimensions of N=Z nuclei above ^{100}Sn



M=0 states in Sn isotopes in different model spaces



More difficult due to Pauli principle

The Swedish e-science project

This strategic research area (SRA) funded by Vetenskapsrådet (VR) brings together a core of nationally leading IT research teams (tool makers) and leading scientists in selected strategic application areas

PDC's supercomputer Beskow

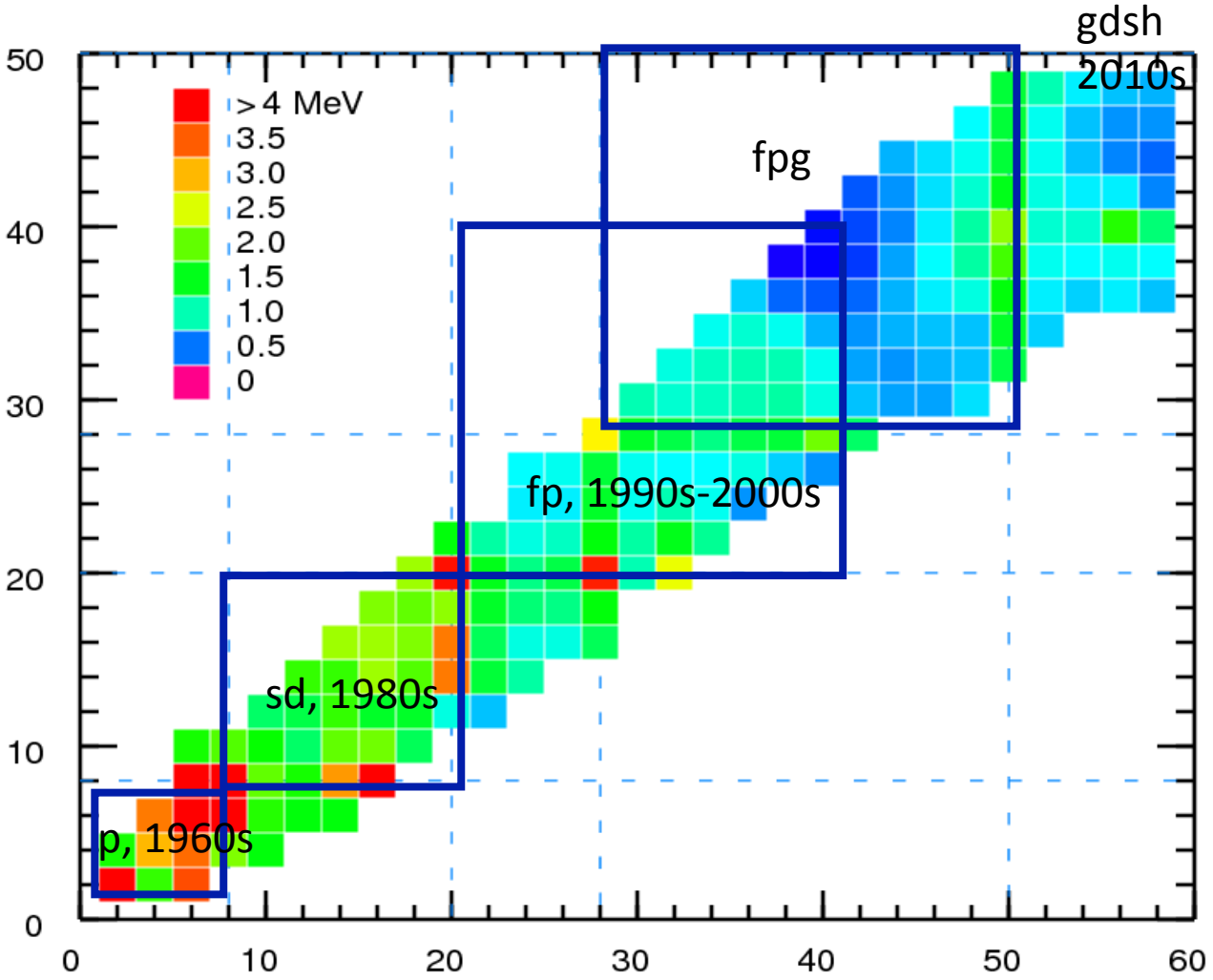
Quest of the effective interaction

- Empirical effective interaction
  - USD, B. A. Brown and W. A. Richter, Phys. Rev. C 74, 034315 (2006).
  - fp (KB3, gxpf), 1990s
  - fpg, M. Honma et al., Phys. Rev. C 80, 064323 (2009)
  - **gdsh, CQ, Z. Xu, Phys. Rev. C 86, 044323 (2012)**
  - **Cross-shell fpg+gdsh to understand the effect of the N=50 shell**

T. Bäck, CQ et al.PRC 87, 031306 (2013).

One has to consider:

- The core polarization effects induced by the assumed inert core
- Optimization of the monopole interaction due to the neglect of three-body and other effects



jj coupling instead of LS coupling

General properties of the effective interaction

The two-body interaction matrix elements in a single j shell $\langle j^2;JT|V|j^2;JT\rangle$

- Isovector (T=1): J=0,2,..,2J-1, J=0 term attractive *(pairing),* others close to zero
- Isoscalar (T=0): J=1,3,..,2j
  - **strongly attractive monopole interaction (mean field)**
  - ***Strong Quadrupole-Quadrupole correlation (which induces 'deformation')***
  - ***The np interaction breaks the seniority coupling***

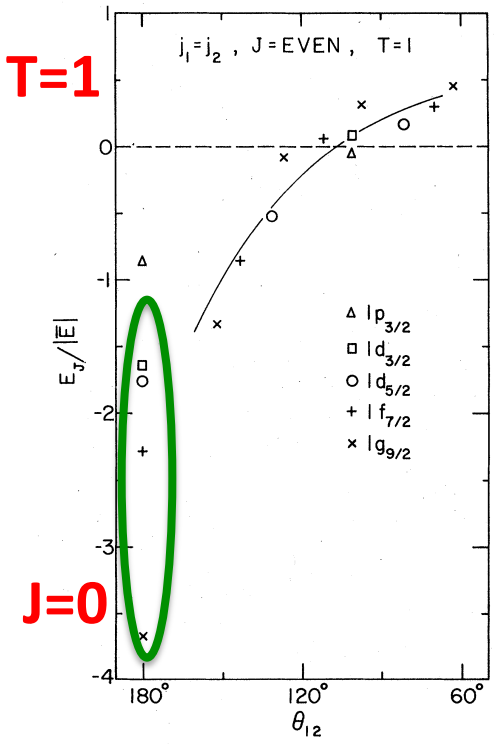


FIG. 3. Comparison of data from various multiplets with $j_1 = j_2$ and $T = 1$. The values of the matrix elements are divided by $\bar{E} \equiv \sum_J [J] E_J / \sum_J [J]$ to display the similarities in the J dependence (or θ dependence) of the various multiplets.

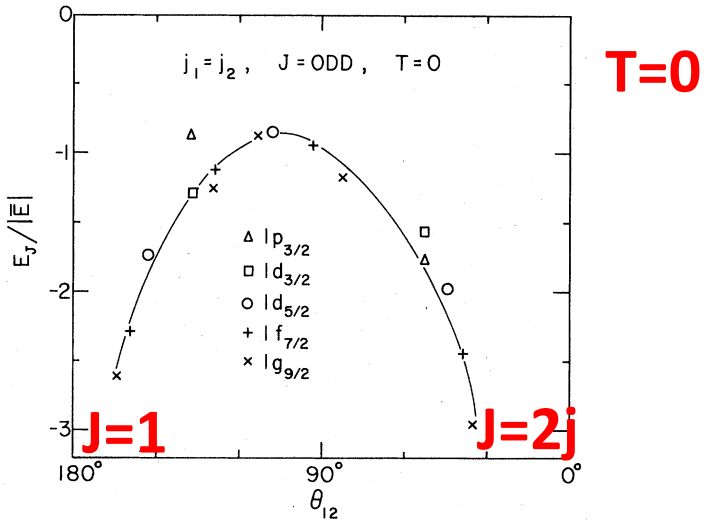


FIG. 2. Comparison of data from various multiplets with $j_1 = j_2$ and $T = 0$. The values of the matrix elements are divided by $\bar{E} \equiv \sum_J [J] E_J / \sum_J [J]$ to display the similarities in the J dependence (or θ dependence) of the various multiplets.

$$\cos\theta_{12} = \frac{J(J+1)}{2j(j+1)} - 1$$

J.P. Schiffer and W.W. True, Rev.Mod.Phys. 48,191 (1976)

Monopole Hamiltonian

Determines average energy of eigenstates in a given configuration.

- Important for binding energies, shell gaps

$$H_m = \sum_a \varepsilon_a n_a + \sum_{a \le b} \frac{1}{1+\delta_{ab}} \left[\frac{3V_{ab}^1 + V_{ab}^0}{4} n_a (n_a - \delta_{ab}) + (V_{ab}^1 - V_{ab}^0)(T_a \cdot T_b - \frac{3}{4} n_a \delta_{ab}) \right]$$

n_a, T_a … number, isospin operators of orbit *a*

Monopole centroids

- Angular-momentum averaged effects of two-body interaction
- **The monopole interaction itself does not induce mixing between different configurations.**
- **Strong mixture of the wave function is mainly induced by the residual J=0 *pairing* and *QQ* np interaction**

$$V_{ab}^T = \frac{\sum_J (2J+1) V_{abab}^{JT}}{\sum_J (2J+1)}$$

$$E_i^{\rm cal} = C + N\varepsilon_0 + \frac{N(N-1)}{2}V_m + \langle\Psi_I|H|\Psi_I\rangle,$$

$$\begin{aligned} E^{\rm SM} &= \langle\Psi_I|H|\Psi_I\rangle \\ &= \sum_\alpha \varepsilon_\alpha < \hat{N}_\alpha > + \sum_{\alpha\le\beta} V_{m;\alpha\beta} \left\langle \frac{\hat{N}_\alpha(\hat{N}_\beta - \delta_{\alpha\beta})}{1+\delta_{\alpha\beta}} \right\rangle \\ &\quad + \langle\Psi_I|H_M|\Psi_I\rangle, \end{aligned} \tag{4}$$

where $\sum_\alpha < \hat{N}_\alpha >= N$ and

$$\sum_{\alpha\le\beta} \left\langle \frac{\hat{N}_\alpha(\hat{N}_\beta - \delta_{\alpha\beta})}{1+\delta_{\alpha\beta}} \right\rangle = \frac{N(N-1)}{2}. \tag{5}$$

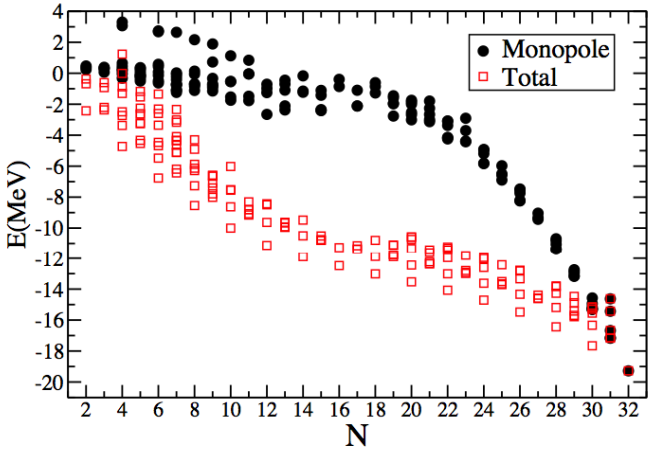
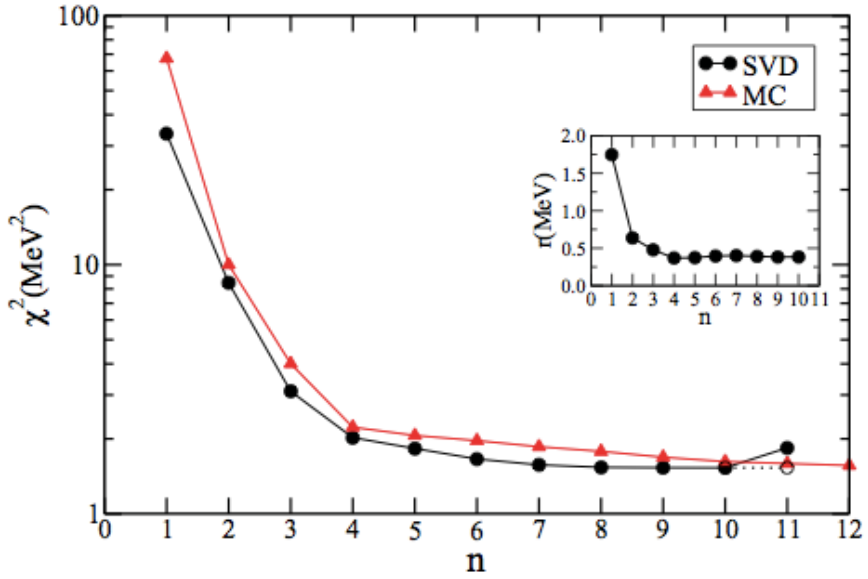


FIG. 5. (Color Online) The calculated shell model energies (solid), $E^{SM} =< H >$, and contributions from monopole Hamiltonian $< H_m >$ (open).

Optimization of the monopole interaction



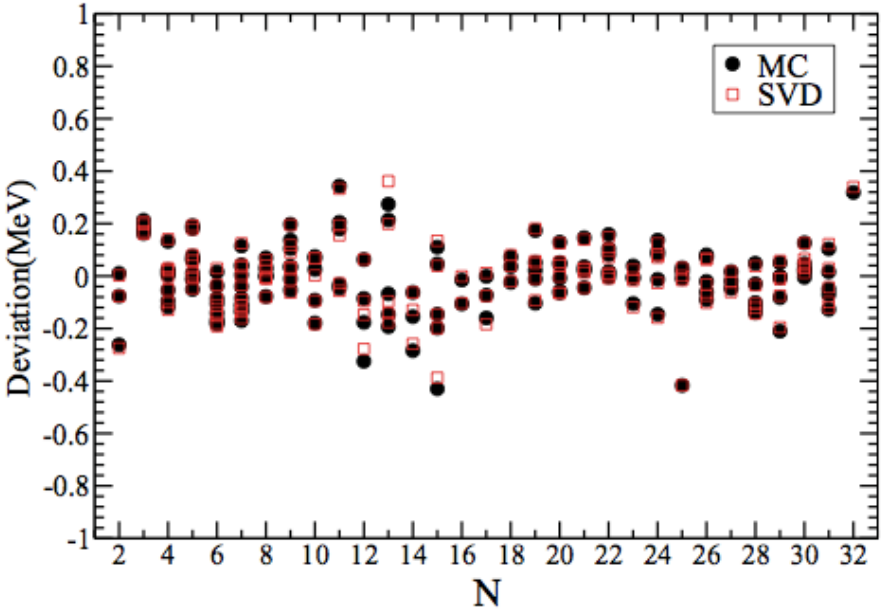
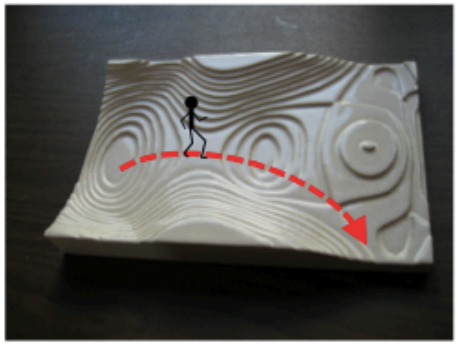


FIG. 4. (Color online) Differences between experimental and calculated binding energies $E^{\text{Expt.}} - E^{\text{Cal.}}$ as a function of valence neutron number.

The ground and yrast excited states in Sn isotopes can be reproduced within an average deviation of about 130 keV.



Random walk (Metropolis)

CQ, Z. Xu, Phys. Rev. C 86, 044323 (2012)

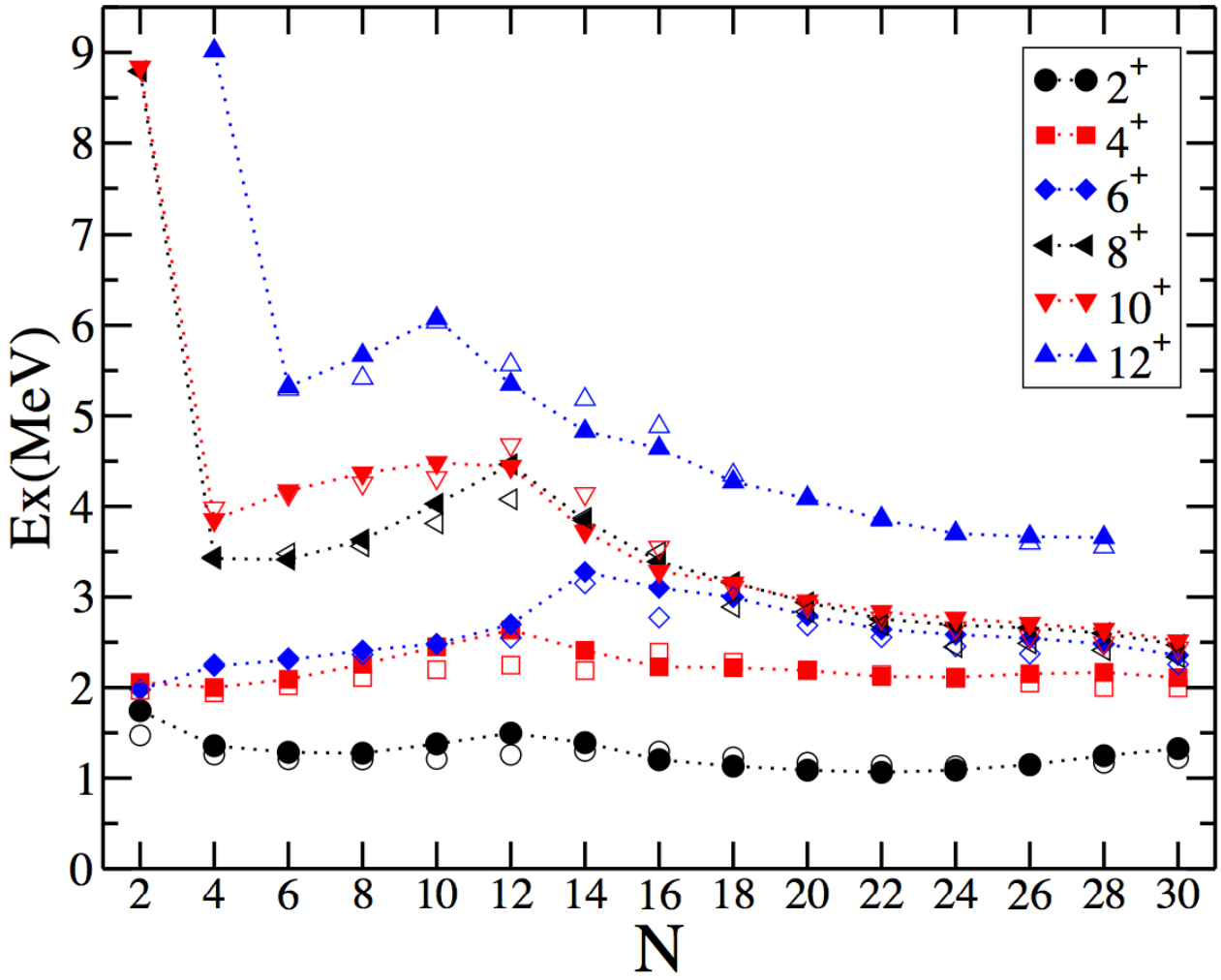
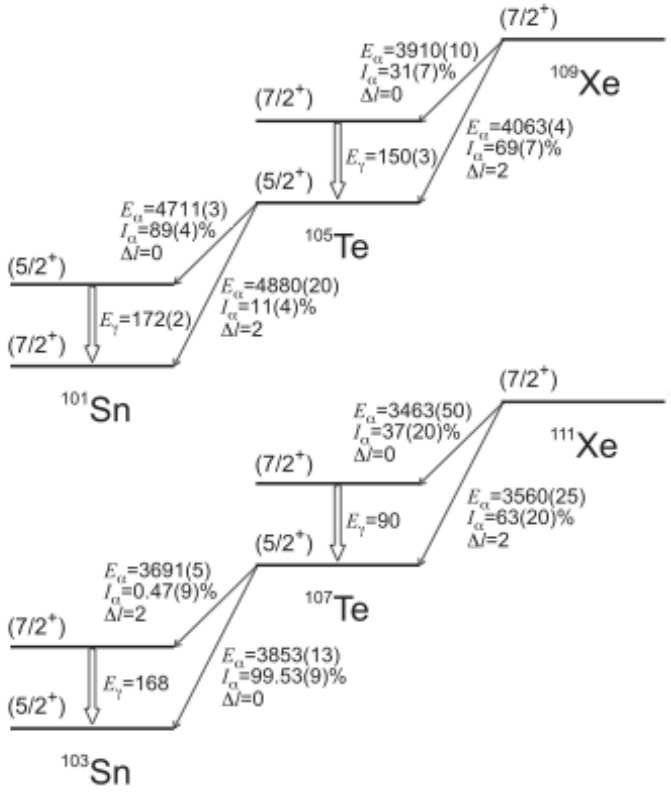


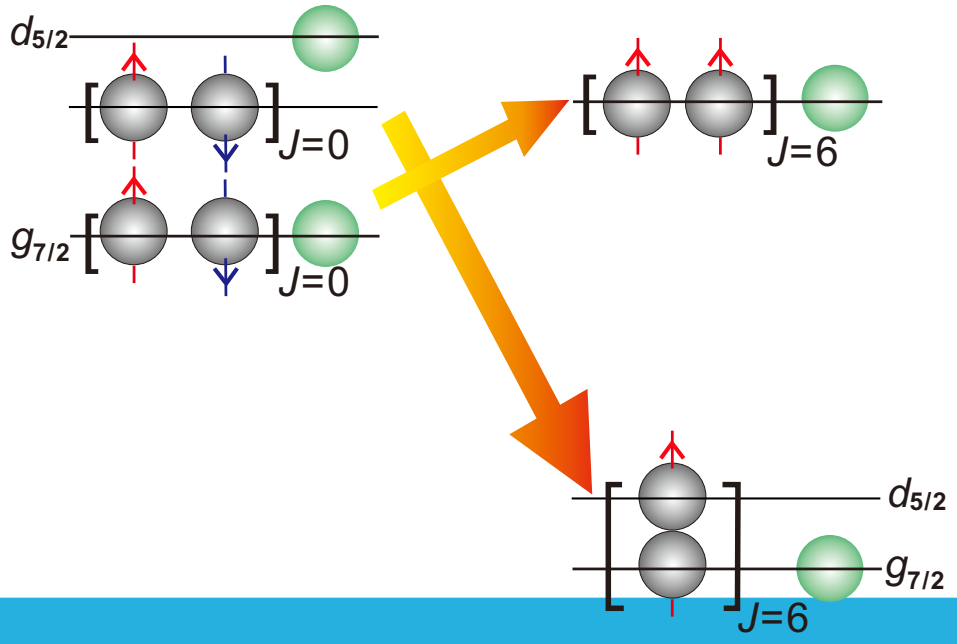
FIG. 6. (Color Online) Experimental [17, 18, 67, 76] (open symbols) and calculated (solid symbols with dotted lines) excitation energies of the low-lying even-spin states in nuclei $^{102-130}$Sn.

Two low-lying 9- states in 102Sn: One from the coupling between 3- and 6+

Orbital Dependent Nucleonic Pairing in the Lightest Known Isotopes of Tin

I. G. Darby,[1,2] R. K. Grzywacz,[1,3] J. C. Batchelder,[4] C. R. Bingham,[1,3] L. Cartegni,[1] C. J. Gross,[3] M. Hjorth-Jensen,[5] D. T. Joss,[6] S. N. Liddick,[1] W. Nazarewicz,[1,3,7] S. Padgett,[1] R. D. Page,[6] T. Papenbrock,[1,3] M. M. Rajabali,[1] J. Rotureau,[1] and K. P. Rykaczewski[3]





CQ, Z. Xu, Phys. Rev. C 86, 044323 (2012)

Seniority coupling involving many shells

- A way to solve the pairing Hamiltonian exactly
- Low-seniority configurations are dominant

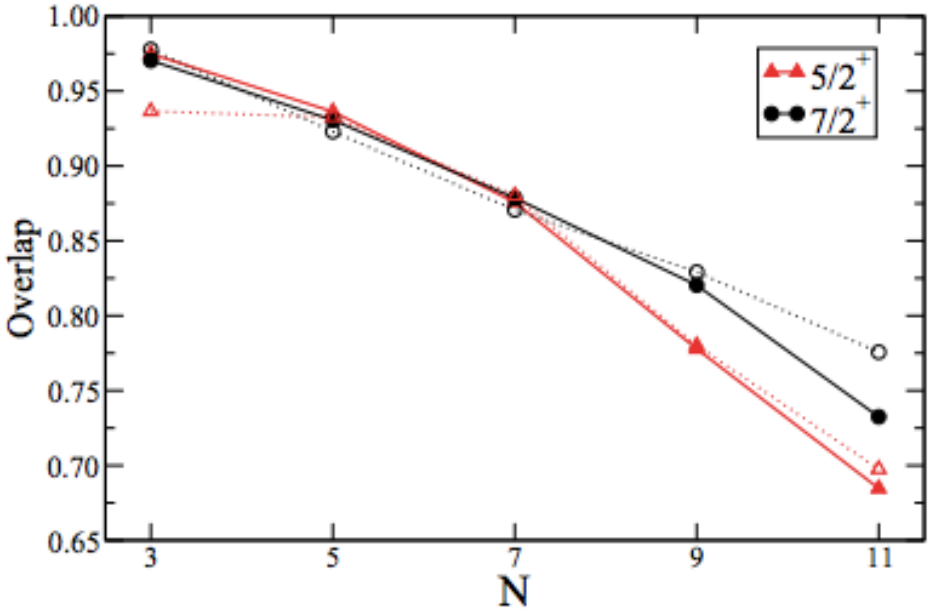


FIG. 11. (Color Online) Solid symbol: The overlaps between the wave functions $|\Psi_I\rangle$ of the full Hamiltonian H and those of the pairing Hamiltonian with $J_{\max} = 0$ for the first $5/2^+$ and $7/2^+$ states in light odd-A Sn isotopes. Open symbol: Same as above but only the non-diagonal pairing matrix elements are considered.

CQ, Z. Xu, Phys. Rev. C 86, 044323 (2012)

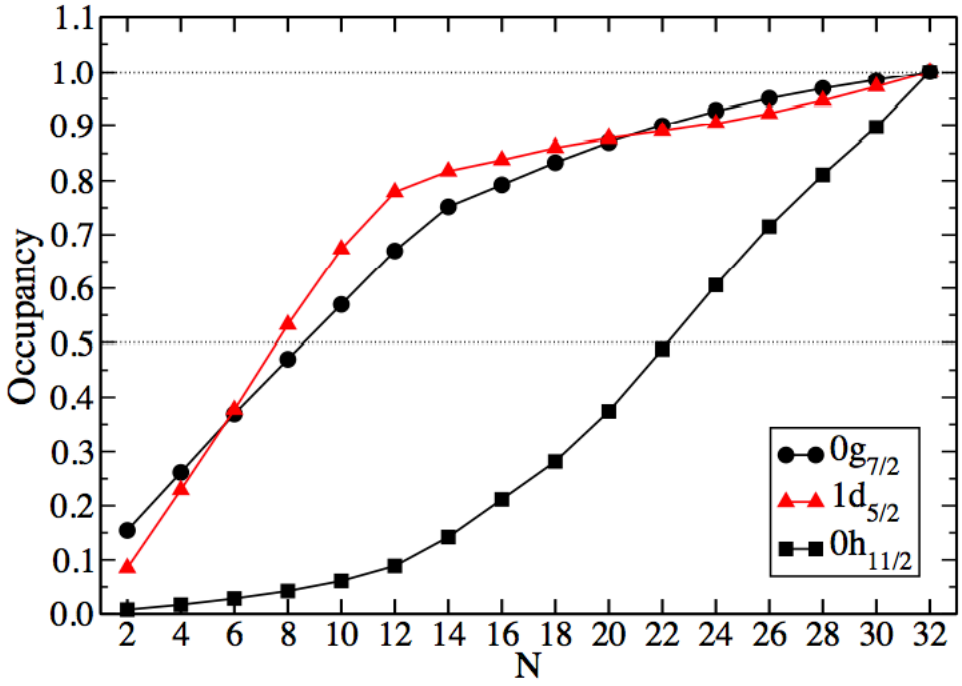


FIG. 7. (Color Online) Shell model occupancies, $< \hat{N}_j >$ $/(2j+1)$, of the three higher-j shells in the ground states of even tin isotopes.

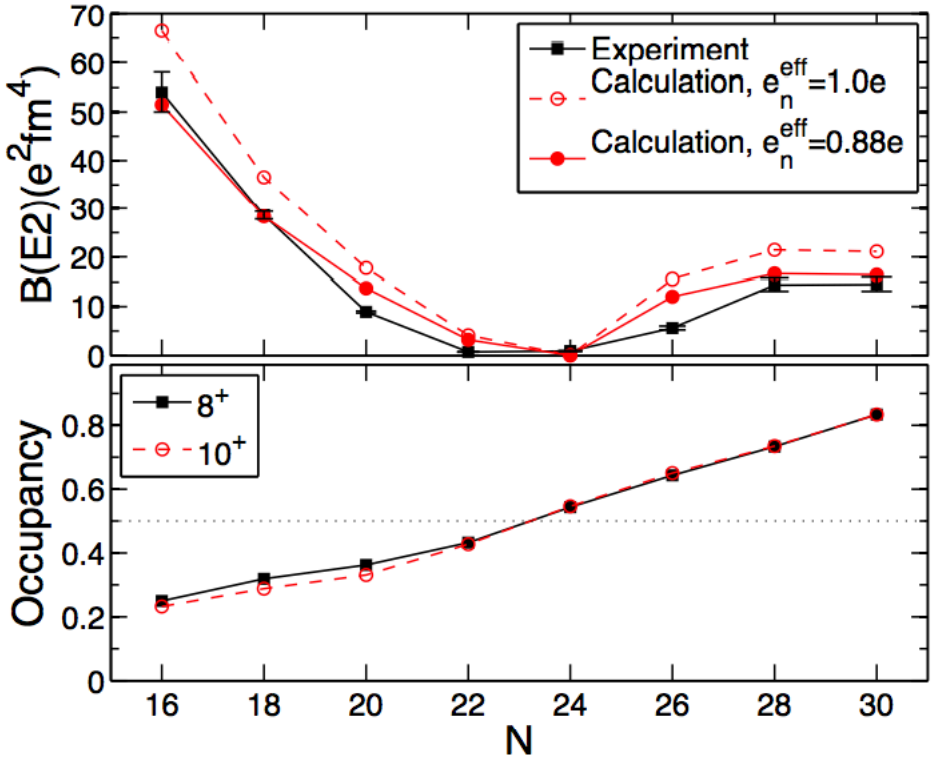


FIG. 8. (Color Online) Experimental [79, 80] and calculated B(E2) values on the transitions of the 10_1^+ states in even Sn isotopes. The lower panel gives the calculated occpancies of the $0h_{11/2}$ orbital in the 10^+ and 8^+ states.

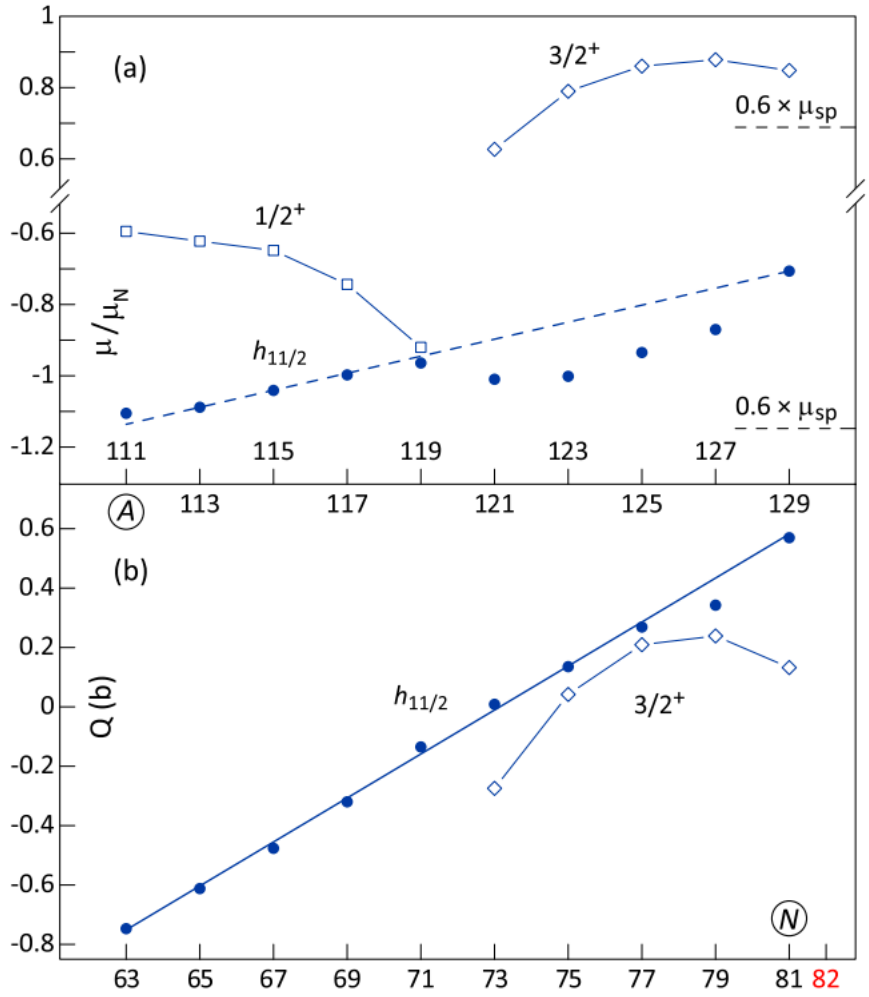


FIG. 2 (color online). Magnetic (a) and quadrupole (b) moments of $^{111-129}$Cd from this work. The experimental error bars are smaller than the markers. A straight line is fitted through the $h_{11/2}$ quadrupole moments, consistent with Eq. (2). The dashed line indicates the effect of core polarization.

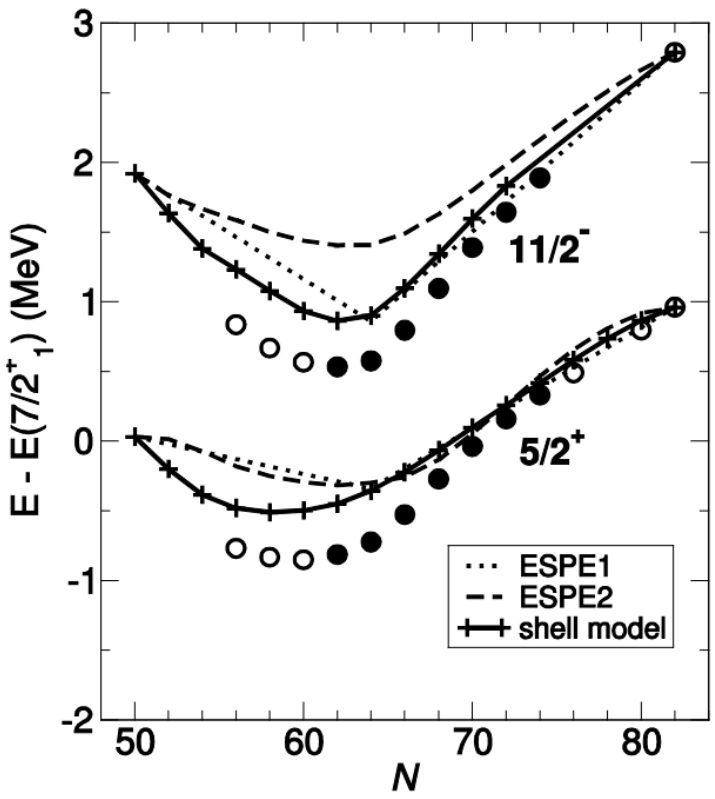


Fig. 1 Evolution of the $5/2^+_1$ and $11/2^-_1$ levels in antimony isotopes relative to $7/2^+_1$ compared between experiment (circles) and theory (lines).

PHYSICAL REVIEW C **92**, 034320 (2015)

Shell-model calculations of nuclei around mass 130

E. Teruya,[1,*] N. Yoshinaga,[1,†] K. Higashiyama,[2,‡] and A. Odahara[3,§]
[1]*Department of Physics, Saitama University, Saitama City 338-8570, Japan*
[2]*Department of Physics, Chiba Institute of Technology, Narashino, Chiba 275-0023, Japan*
[3]*Department of Physics, Osaka University, Osaka 560-0043, Japan*
(Received 26 May 2015; published 21 September 2015)

Shell-model calculations are performed for even-even, odd-mass, and doubly-odd nuclei of Sn, Sb, Te, I, X
Cs, and Ba isotopes around mass 130 using the single-particle space made up of valence nucleons occupyin
the $0g_{7/2}$, $1d_{5/2}$, $2s_{1/2}$, $0h_{11/2}$, and $1d_{3/2}$ orbitals. The calculated energies and electromagnetic transitions a
compared with the experimental data. In addition, several typical isomers in this region are investigated.

Y.Utsuno, N.Shimizu, M.Honma, T.Mizusaki, T.Otsuka
JAEA-Review 2013-057, p.39 (2013)

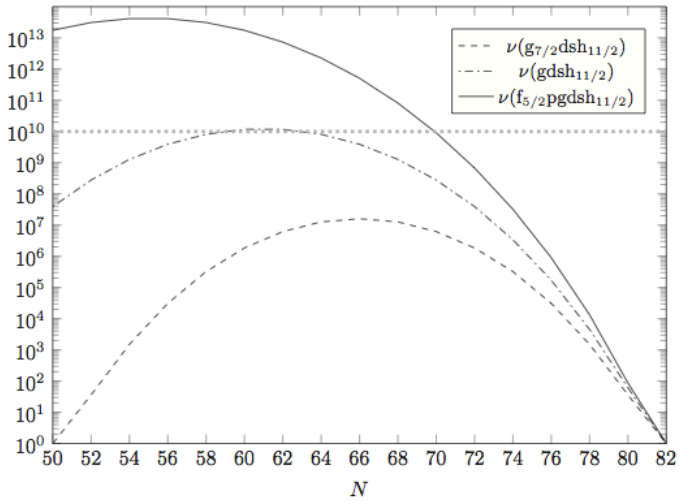
Truncation approaches

Construct a set of initial bases that can be as close to the desired one as possible.

- If the initial vector is closer to the physics ones, the Lanczos iteration will be much faster
- If one can construct one or a few initial vectors that are close enough to the desired one, diagonalization of the large matrix can be avoided.
- Angular momentum should be conserved

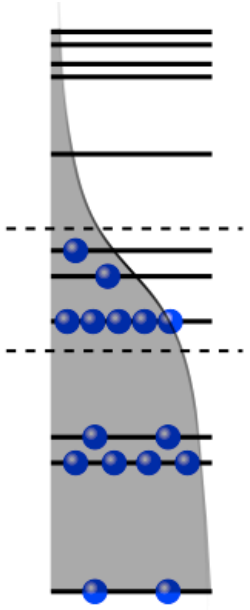
Seniority coupling for many shells

- Shell model calculations with a **pairing Hamiltonian**.
- The physical vector only spans the v=0 subspace
- **There are as many independent solutions as states in the v=0 space.**



10^2 in seniority space

Easier to include many shells



Particle number restriction

- A rather crude approximation
- Convergence can be slow, especially for systems with no clear subshell structure

Maximum number of nucleons can be excited from $0f_{7/2}$, t, in ^{56}Ni

$$(f_{7/2})^{n_1-t}(p_{3/2}\ \ f_{5/2}\ \ p_{1/2})^{n_2+t}$$

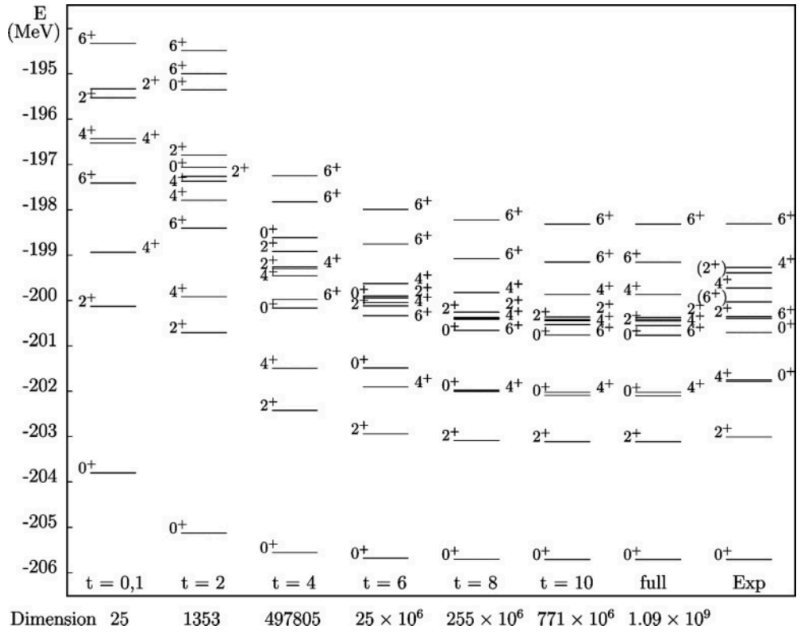


FIG. 1. The evolution of the first three 0^+, 2^+, 4^+, and 6^+ states as a function of the truncation level t.

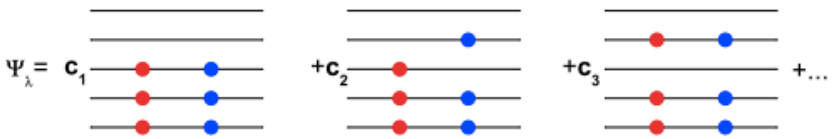
Importance truncation (perturbation)

R. Roth, P. Navratil, PRL 99, 092501 (2009); R. Roth, PRC 79, 064324 (2009)
D. Bianco et al, PRC 85, 034332 (2012)

$$\kappa_\nu = -\frac{\langle \Phi_\nu | H' | \Psi_{\rm ref} \rangle}{\epsilon_\nu - \epsilon_{\rm ref}},$$

'Monopole' truncation

$$\begin{aligned} E^{\rm SM} &= \langle \Psi_I | H | \Psi_I \rangle \\ &= \sum_\alpha \varepsilon_\alpha < \hat{N}_\alpha > + \sum_{\alpha \le \beta} V_{m;\alpha\beta} \left\langle \frac{\hat{N}_\alpha (\hat{N}_\beta - \delta_{\alpha\beta})}{1+\delta_{\alpha\beta}} \right\rangle \\ &\quad + \langle \Psi_I | H_M | \Psi_I \rangle, \end{aligned} \tag{4}$$



Construction of the basis vectors

M scheme

Angular momentum conservation is imporant

jj coupled scheme

Expansion of the bases with good angular momentum as a function of M-scheme bases with the help of projection or coefficients of fractional parentage.

$$|\Psi_i^J\rangle = \sum_{m \leq i} \mathcal{M}_{im} P^J |\alpha_m\rangle, \qquad (4)$$

where P^J is the projection operator and $|\alpha\rangle$ a set of specially chosen M-scheme bases. $\mathcal{M}$ is a lower triangle matrix.

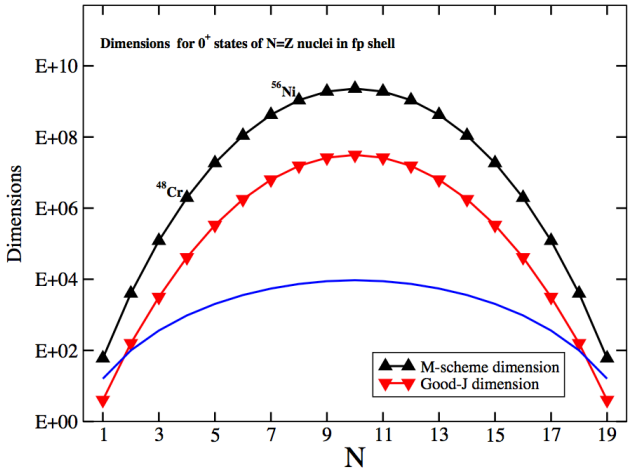


Figure: Dimensions for the 0^+ states of $N = Z$ nuclei in the *fp*-shell.

Projection operator

$$P^{JT} = P^J P^T = P^{J=J_z} P^T.$$

$$P^J = \prod_{J'=J_z, J' \neq J}^{J_M} \frac{\hat{J}^2 - J'(J'+1)}{J(J+1) - J'(J'+1)},$$

$$\hat{J}^2 = J^- J^+ + J_z(J_z+1)$$

P. O. Lodwin, Rev. Mod. Phys. 36, 966 (1964).

The projection can be prohibitive time consuming: N×M problem; Difficult to construct the Hamiltonian matrix; Does not contain any information about the Hamiltonian and, as a result, one still need to diagonalize a huge N×N matrix.

M scheme + Lanczos diagonalization is a cheaper choice. Dimension is much larger.

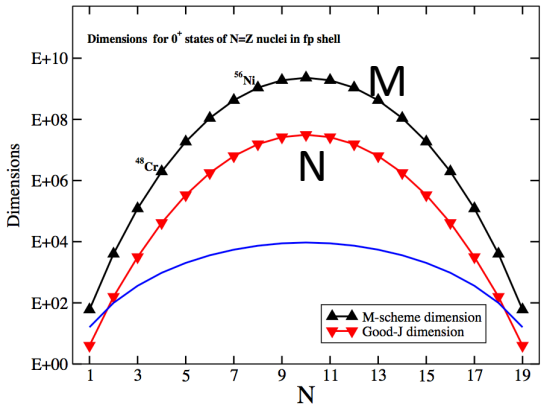


Figure: Dimensions for the 0^+ states of $N = Z$ nuclei in the *fp*-shell.

Lanczos approach

Usually the Lanczos iteration approach is used for the diagonalization since we only need the lowest a few eigenstates

- H operation enhances low-lying components.

$$|\psi_0\rangle, H|\psi_0\rangle, H^2|\psi_0\rangle, H^3|\psi_0\rangle, \cdots$$

$$|\psi_k\rangle \equiv \frac{1}{N_k} H^k |\psi_0\rangle$$

$$|\psi_0\rangle, |\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle, \cdots$$

- We diagonalize the hamiltonian by these basis vectors.
- As these vectors are non-orthgonal, we orthogonalize them.
- These vectors are called Lanczos vectors.

- Lanczos method can covert original matrix to tridiagonal one, which is easily diagonalized.

$$H \longrightarrow \begin{pmatrix} \alpha_1 & \beta_1 & & & & \\ \beta_1 & \alpha_2 & \beta_2 & & & \\ & \beta_2 & \alpha_3 & \beta_3 & & \\ & & \beta_3 & \alpha_4 & \beta_4 & \\ & & & \beta_4 & \alpha_5 & \beta_5 \\ & & & & \beta_5 & \alpha_6 \end{pmatrix}$$

$$|\phi_{L+1}\rangle = H|\phi_L\rangle - \alpha_L|\phi_L\rangle - \beta_L|\phi_{L-1}\rangle$$

$$\alpha_L = \langle\phi_L|H|\phi_L\rangle$$

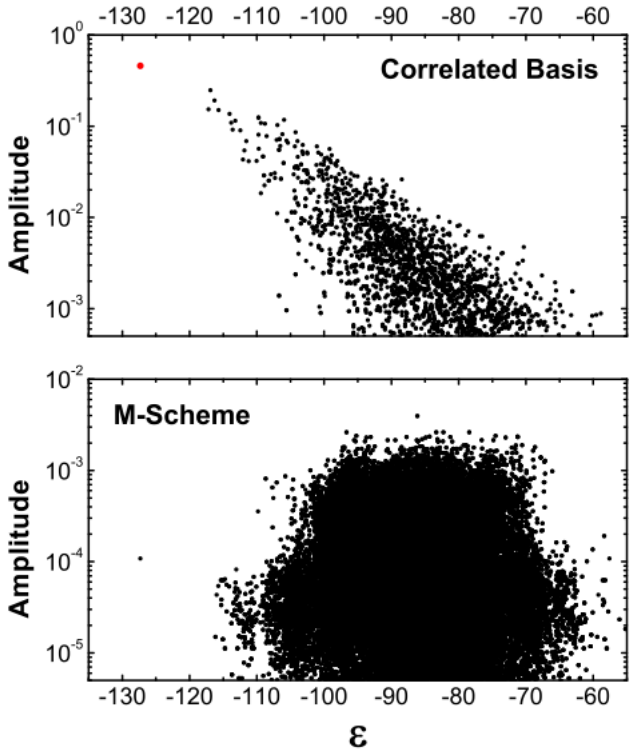
$$\beta_L = \langle\phi_{L-1}|H|\phi_L\rangle$$

$E_1 > E_2 > E_3 > E_4 > E_5 > E_6$

- Ground state energy can be obtained by L Lanczos vectors. We consider convergence of ground state energy as a function of L.

- Angular momentum conservation is important
- The Hamiltonian itself is a good angular momentum projector

Correlation between the diagonal matrix elements and the amplitudes in the final wave function



LF Jiao, ZH Sun, ZX Xu, FR Xu, CQ, PRC**90**, 024306 (2014)

$n \times M$ problem (instead of $M \times M$)

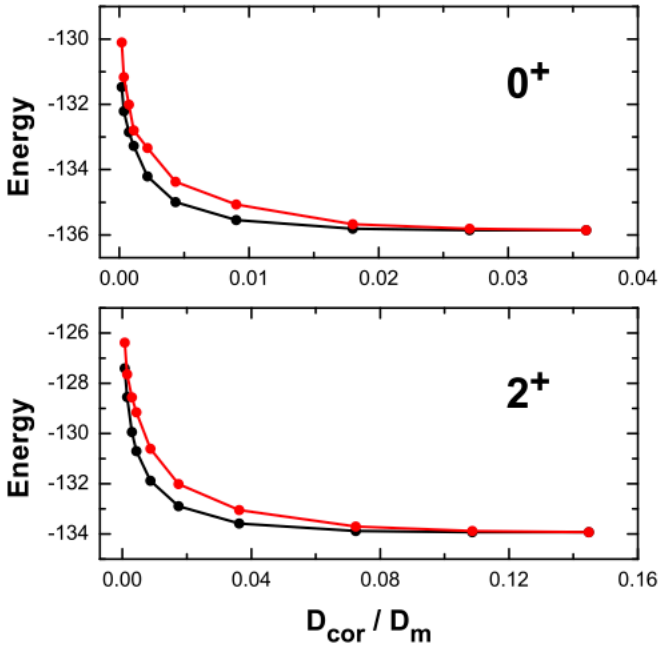


FIG. 5. Convergence of the correlated basis approach with different basis vector selection method for the lowest 0^+ and 2^+ states in ^{28}Si. The black and red lines show results based on Eq.(1) and (2), respectively. See text for details.

$$\chi_{ic} = \frac{|\langle \psi_i | H | \psi_c \rangle|}{\epsilon_i - \epsilon_c},$$

It is similar to what we seen in the importance truncation approach but angular momentum is conserved here.

The simpler choice works reasonably well

$$\Delta\epsilon_{ic} = \epsilon_i - \epsilon_c.$$

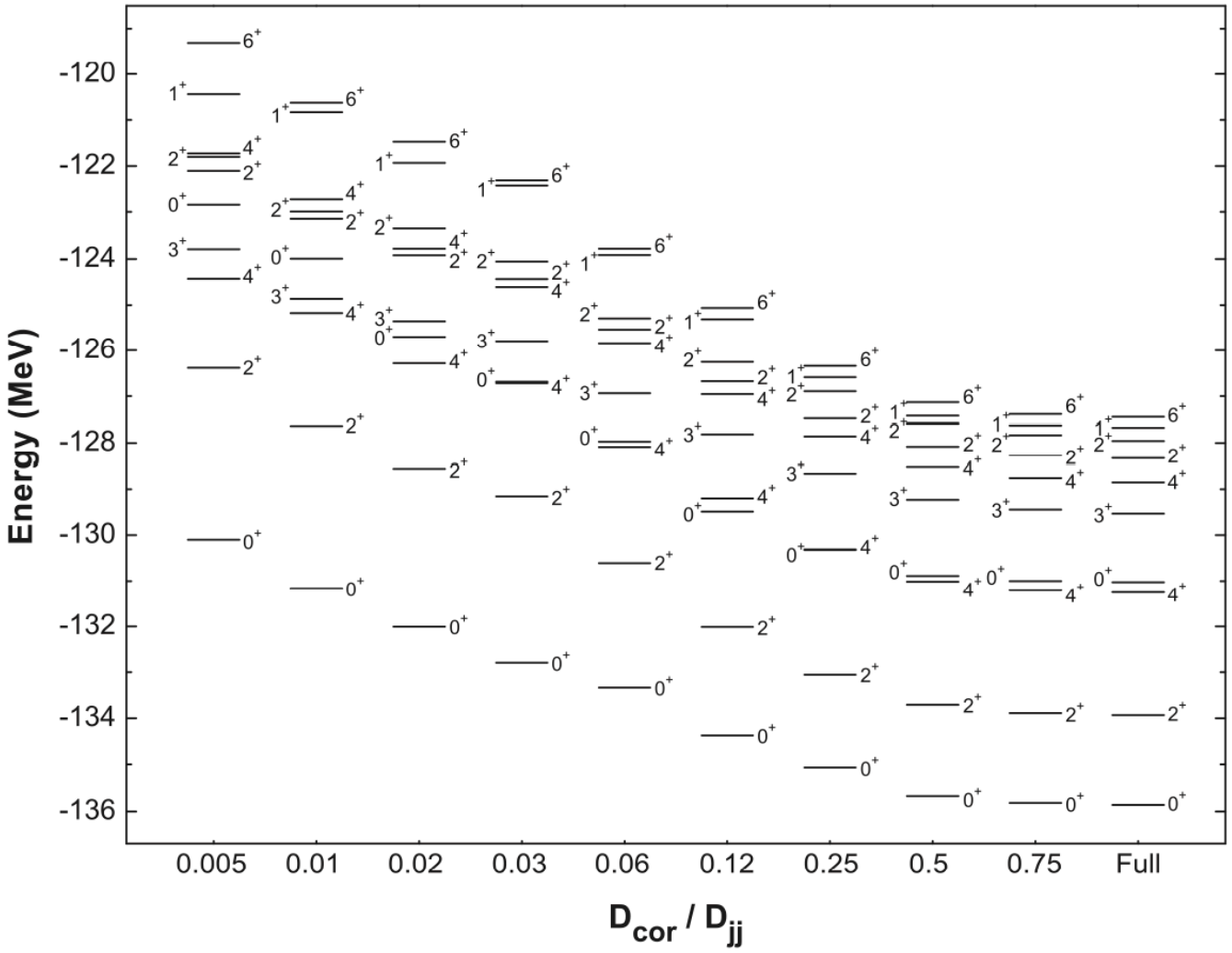


FIG. 1. Energies of low-lying states in ^{28}Si, calculated with the correlated-basis shell model at different truncations. $D_{\rm cor}$ and D_{jj} are the dimensions of the correlated-basis truncation and the "full" dimension of the jj-coupling scheme, respectively.

Ongoing works

Monte Carlo/Random sampling of the correlated bases

Now:

- Bases with a larger correlation energy are chosen sequentially

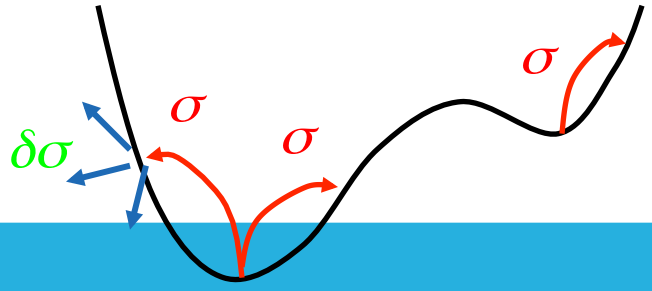
Next step:

Selection based on energy gain

- Bases with larger correlation energy be added with larger probability
- Bases contribute more to the total energy be kept.
- Bases contribute less be kept with certain probability

$$|\Psi(\mathbf{c})\rangle = \left(\sum_{q=0}^{p} c_q H^q\right)|\psi^t\rangle \qquad (1)$$

where H is the Hamiltonian and $\mathbf{c} = \{c_0, c_1, c_2, \ldots, c_p\}$ is a set of variational parameters, which are determined by minimizing

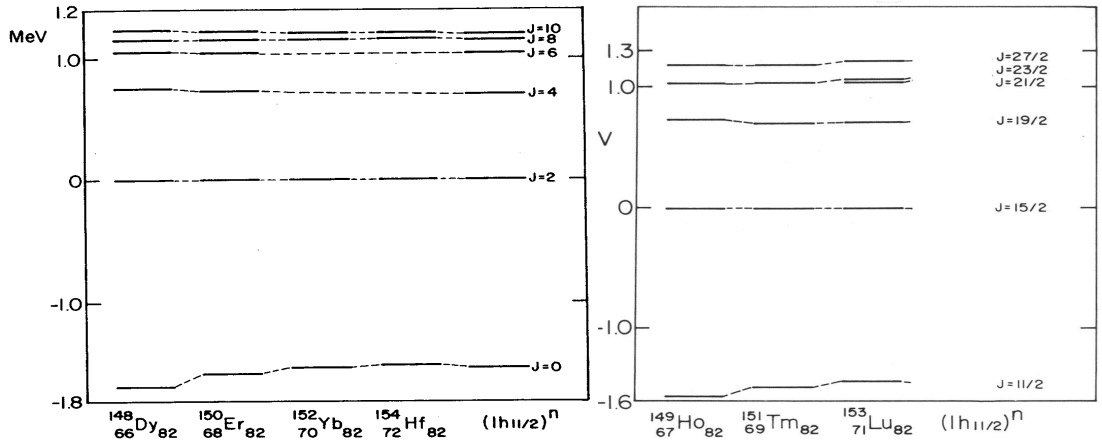


A good starting point is essential

Seniority coupling scheme with realistic interaction

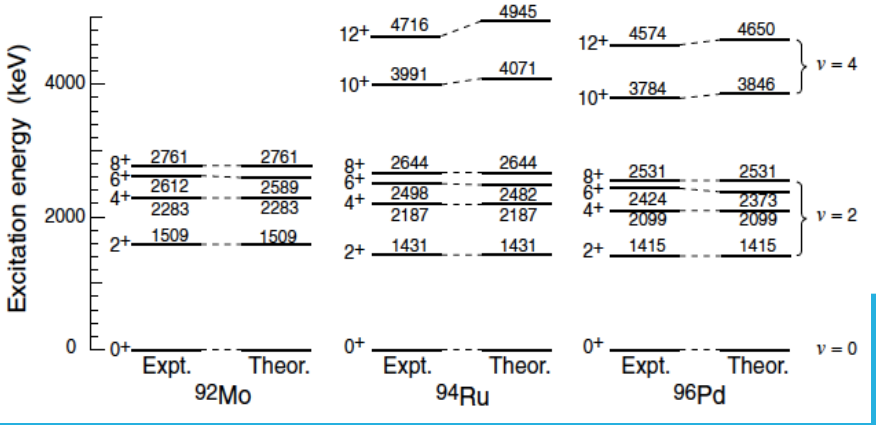
$$|\text{g.s.}\rangle = |\nu = 0; J = 0\rangle = (P_j^+)^{n/2}|\Phi_0\rangle$$
$$|\nu = 2; JM\rangle = (P_j^+)^{(n-2)/2} A^+(j^2 JM)|\Phi_0\rangle$$

Energy levels of $0h_{11/2}$ protons in N=82 isotones



I. Talmi, Simple models of complex nuclei

Energy levels of $0g_{9/2}$ protons in *N*=50 isotones



D.J. Rowe and G. Rosensteel, Phys. Rev. Lett. ***87*** *(2001) 172502*

The v=4 J=4 and 6 states in j=9/2 shell

In principle, the system is not solvable even if seniority is conserved in the interaction

- **It was found (through numerical calculations) that one of the v=4 states remain pure no matter the interaction conserve seniority or not!**

Escuderos and L. Zamick, Phys. Rev. C 73, 044302(2006).
L. Zamick, Phys. Rev. C 75, 064305 (2007).

- **This special v=4 (J=4,6) state is an eigenstate of any Hamiltonian!**

Ghost state?

$$H|Ghost\rangle = E(H)|Ghost\rangle$$

We don't consider the trivial cases, say unique states with dimension one.

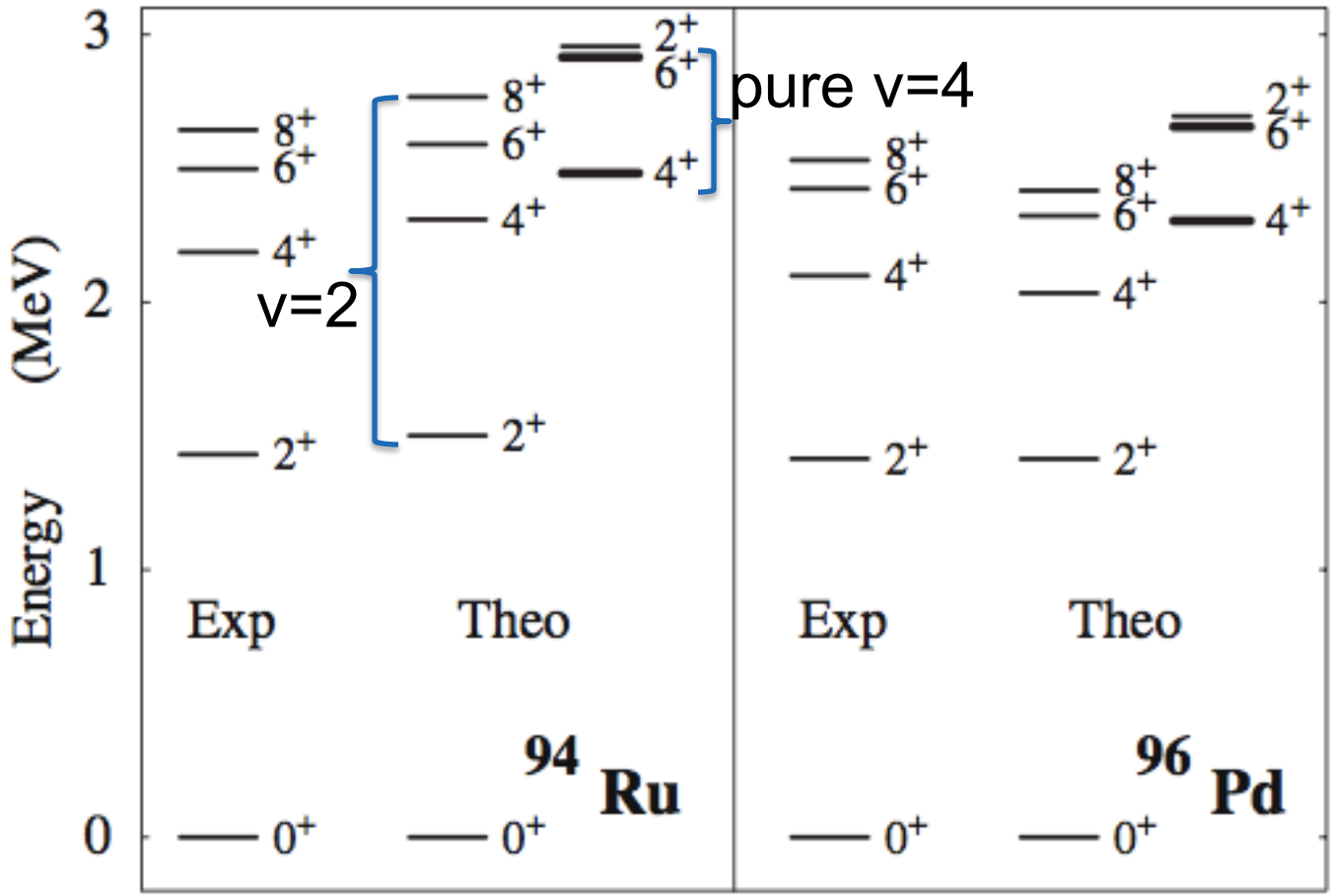
- **The energy expressions of these states are given as**

$$E_4(\upsilon = 4) = \frac{68}{33}V_2 + V_4 + \frac{13}{15}V_6 + \frac{114}{55}V_8$$

$$E_6(\nu = 4) = \frac{19}{11}V_2 + \frac{12}{13}V_4 + V_6 + \frac{336}{143}V_8.$$

- **These states show the partial dynamic conservation of seniority in nuclear systems.**

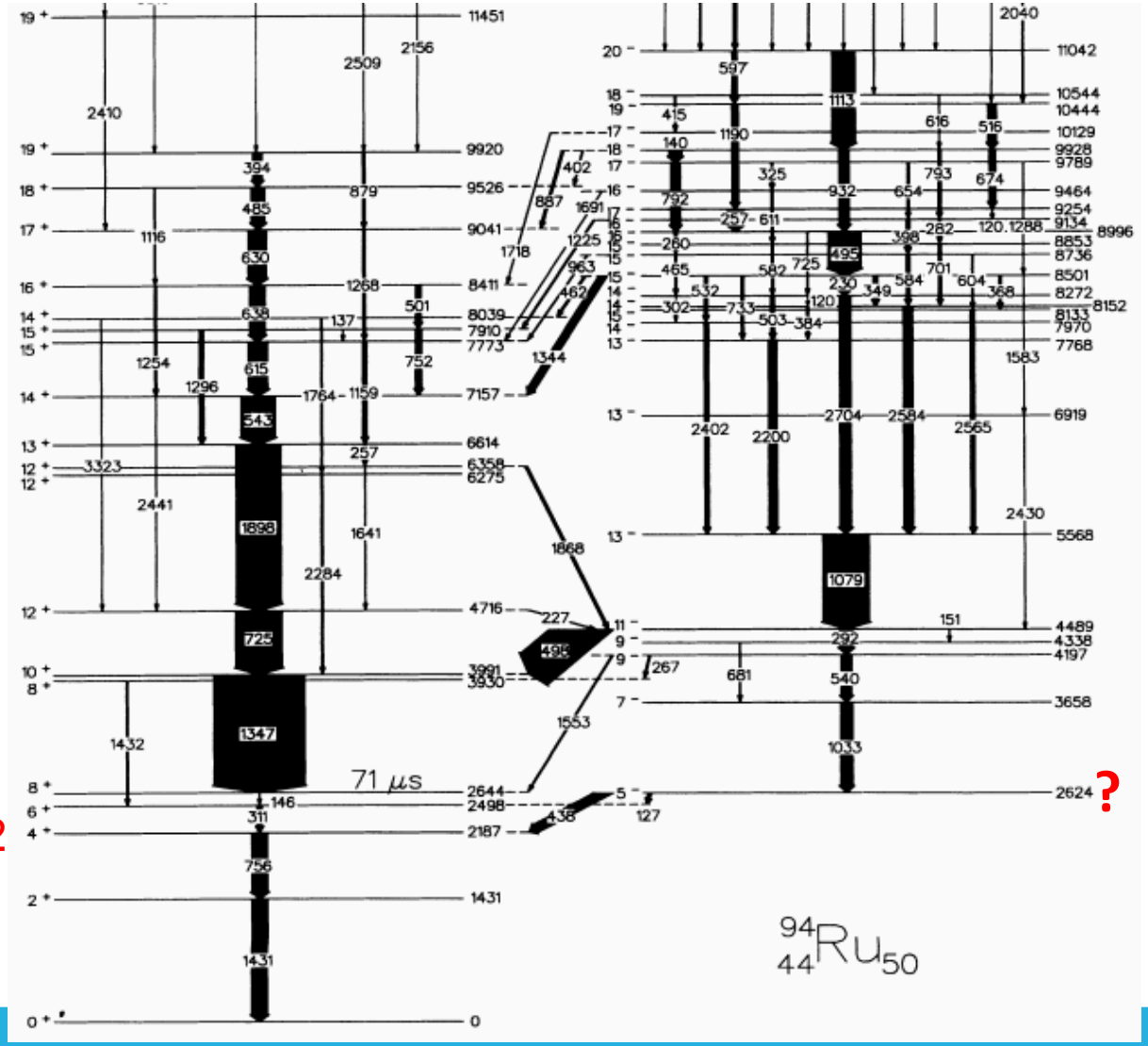
P. Van Isacker and S. Heinze, Phys. Rev. Lett. 100, 052501 (2008).



C. Qi, Z.X. Xu, R.J. Liotta.
Nucl. Phys. A 884–885, 21 (2012).
C. Qi.Phys. Rev. C 83, 014307 (2011).

P. Van Isacker and S. Heinze, Phys. Rev. Lett. 100, 052501 (2008).

Searching for the ghost states



HA Roth et al., PRC 50, 1330 (1994)

^{94}Ru

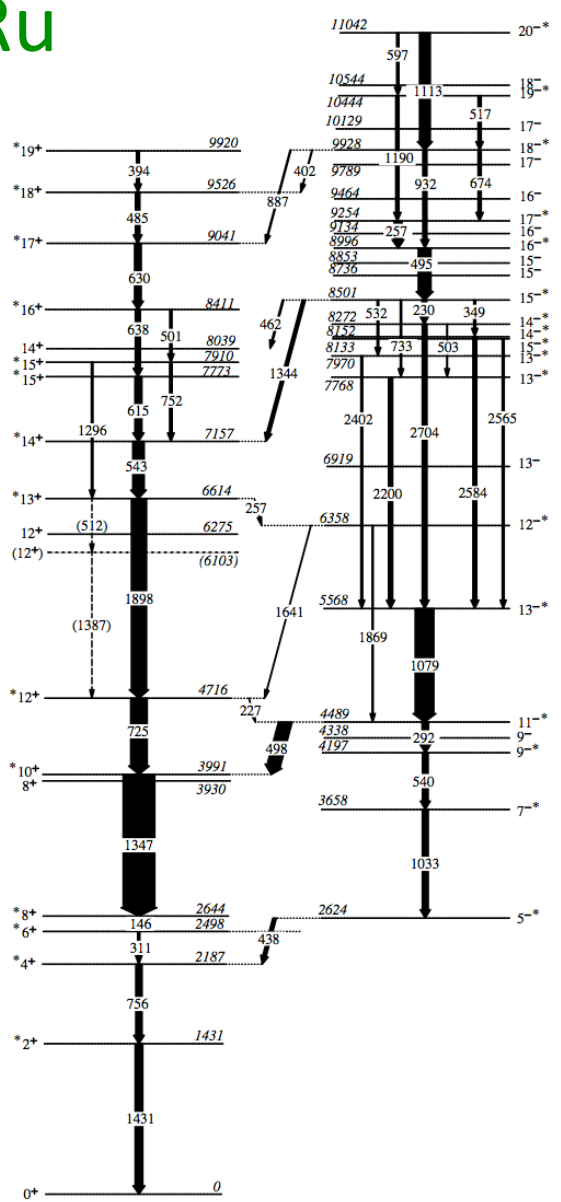


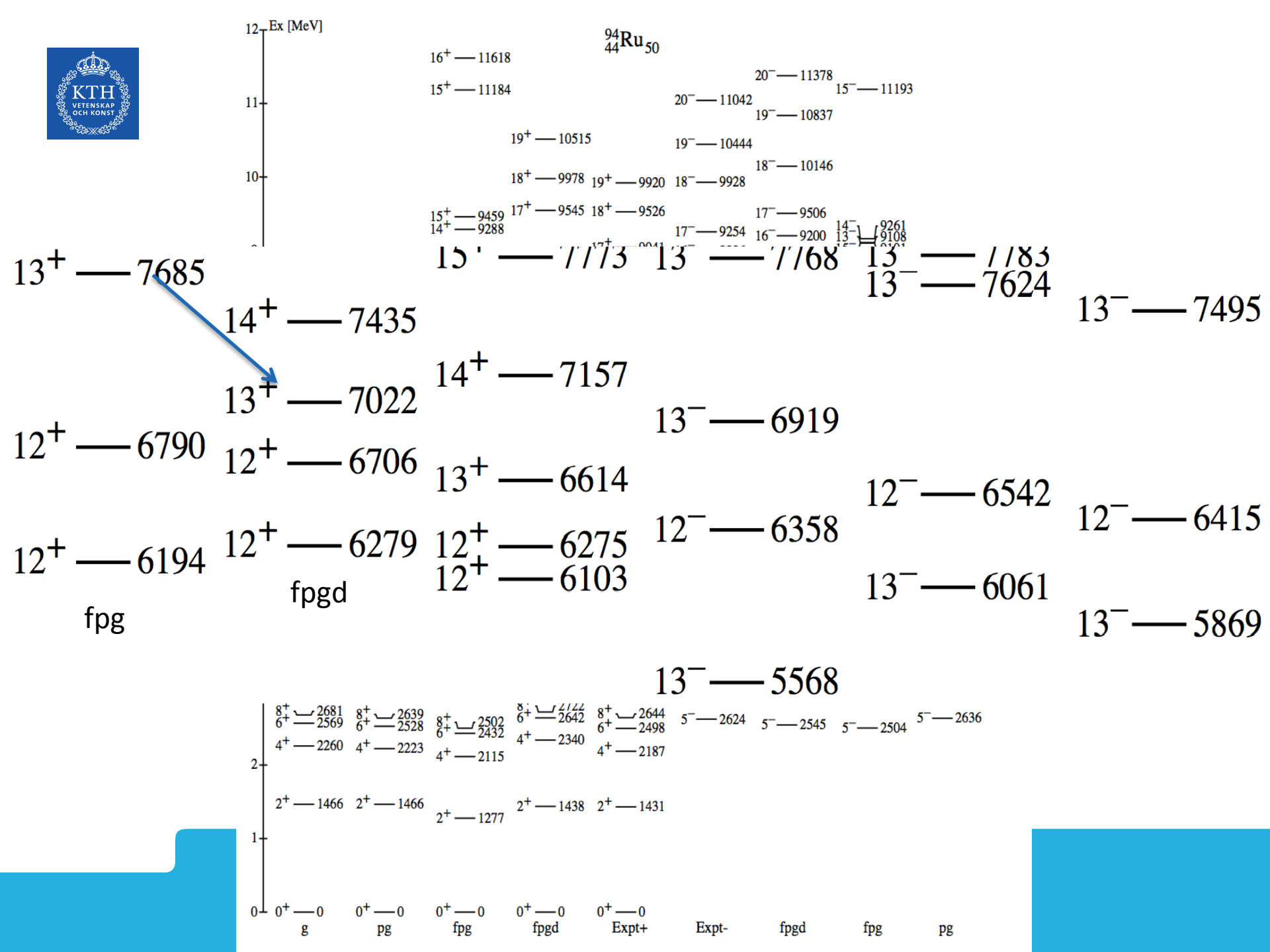
FIG. 2. Partial level scheme of ^{94}Ru showing spins and parities deduced in the present work and in Ref. [17]. The stars indicate spins and parities confirmed by our asymmetry-R_{DCO} measurement. The relative intensities were taken from Ref. [17].

TABLE II. The hindrance factors[a], H, for the observed E1 γ-ray transitions as deduced from the branching ratios and lifetimes [30] of the initial states. Uncertainties are given in parenthesis. $\Delta E_\gamma = \pm$ 0.5 keV.

E_γ (keV)	$J_i^\pi \rightarrow J_f^\pi$	$H \times 10^5$ (W.u.)$^{-1}$
257	$13^+ \rightarrow 12_1^-$	0.006(1)
462	$15_2^- \rightarrow 14_2^+$	0.051(5)
402	$18_1^- \rightarrow 18^+$	0.188(25)
1344	$15_2^- \rightarrow 14_1^+$	0.451(32)
227	$12_1^+ \rightarrow 11^-$	0.57(27)
887	$18_1^- \rightarrow 17^+$	1.09(12)
438	$5^- \rightarrow 4^+$	1.90(17)
498	$11^- \rightarrow 10^+$	4.27(19)

[a] $H = \frac{A^{2/3}}{(15.5 \times B(\mathrm{E1}))}$

FG Moradi et al Phys. Rev. C 89 (1), 014301 (2014)


Ex [MeV]
12
11
10
$^{94}_{44}Ru_{50}$
16^+ — 11618
15^+ — 11184
19^+ — 10515
18^+ — 9978
19^+ — 9920
15^+ — 9459
14^+ — 9288
17^+ — 9545
18^+ — 9526
20^- — 11378
20^- — 11042
15^- — 11193
19^- — 10837
19^- — 10444
18^- — 10146
18^- — 9928
17^- — 9506
17^- — 9254
16^- — 9200
14^- — 9261
13^- — 9108
13^+ — 7685
14^+ — 7435
13^+ — 7022
12^+ — 6790
12^+ — 6706
12^+ — 6279
12^+ — 6194
fpg
fpgd
14^+ — 7157
13^+ — 6614
12^+ — 6275
12^+ — 6103
13^- — 6919
12^- — 6358
13^- — 7624
12^- — 6542
13^- — 6061
13^- — 7495
12^- — 6415
13^- — 5869
13^- — 5568
2
1
0
8^+ 2681
6^+ 2569
4^+ — 2260
2^+ — 1466
0^+ — 0
g
8^+ 2639
6^+ 2528
4^+ — 2223
2^+ — 1466
0^+ — 0
pg
8^+ 2502
6^+ 2432
4^+ — 2115
2^+ — 1277
0^+ — 0
fpg
6^+ 2642
4^+ — 2340
2^+ — 1438
0^+ — 0
fpgd
8^+ 2644
6^+ — 2498
4^+ — 2187
2^+ — 1431
0^+ — 0
Expt+
5^- — 2624
Expt-
5^- — 2545
fpgd
5^- — 2504
fpg
5^- — 2636
pg

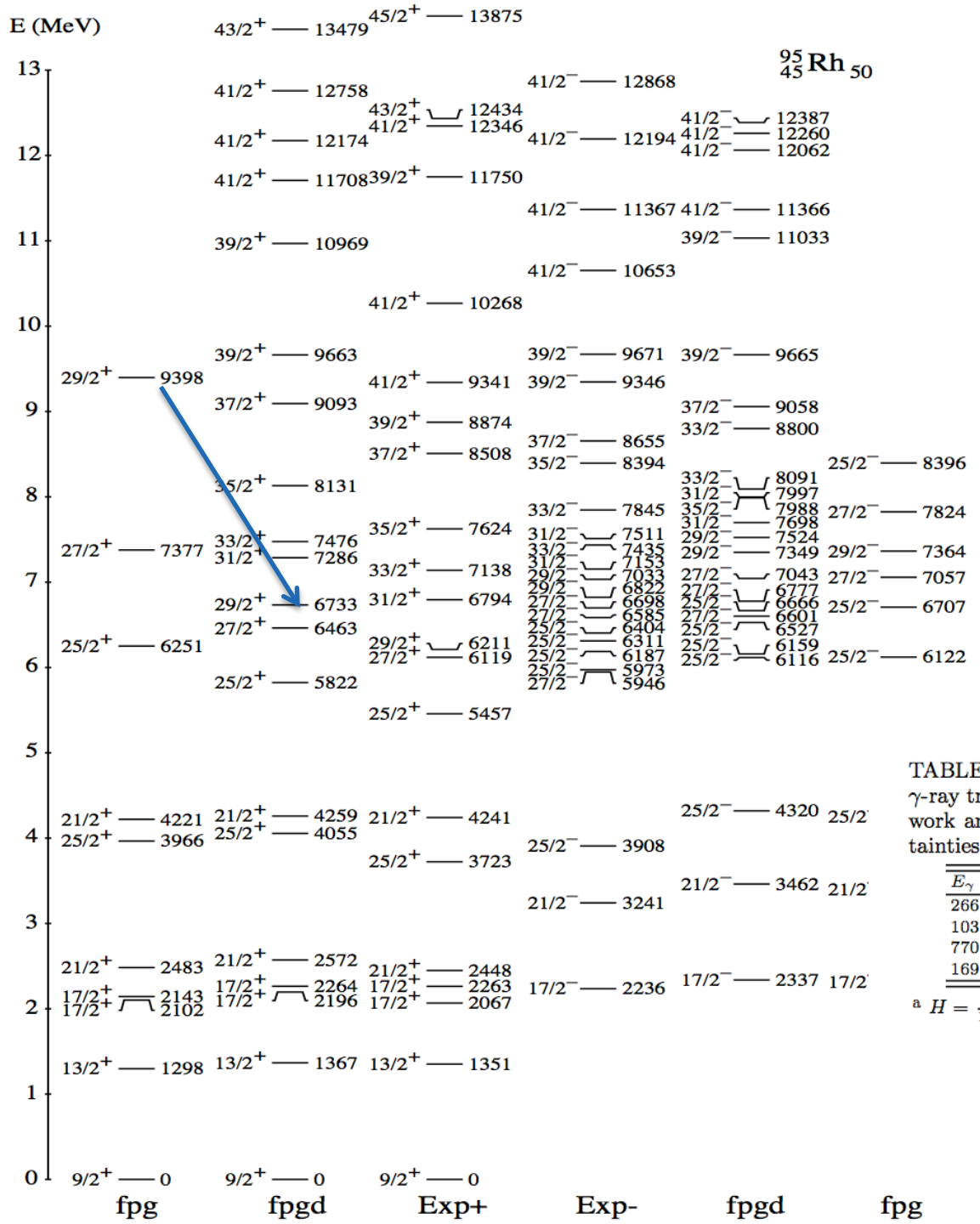


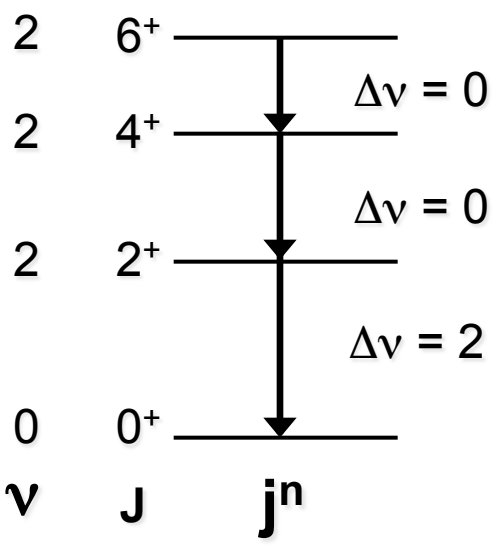
TABLE II. The hindrance factors[a], H, for the observed E1 γ-ray transitions as deduced from the branching ratios of this work and lifetimes of the initial states from Ref. [8]. Uncertainties are given in parenthesis. $\Delta E_\gamma = \pm$ 0.5 keV.

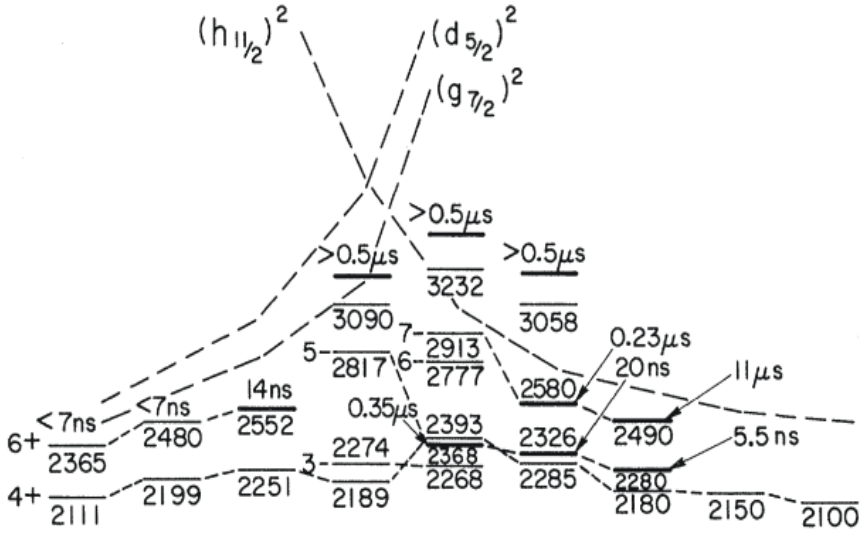
E_γ (keV)	$J_i^\pi \rightarrow J_f^\pi$	$H \times 10^5$ (W.u.)$^{-1}$
266	$29/2^+ \rightarrow 27/2^-$	0.11(1)
1031	$37/2^- \rightarrow 35/2^+$	0.29(1)
770	$35/2^- \rightarrow 35/2^+$	0.35(1)
169	$17/2^- \rightarrow 17/2^+_1$	2.8(2)

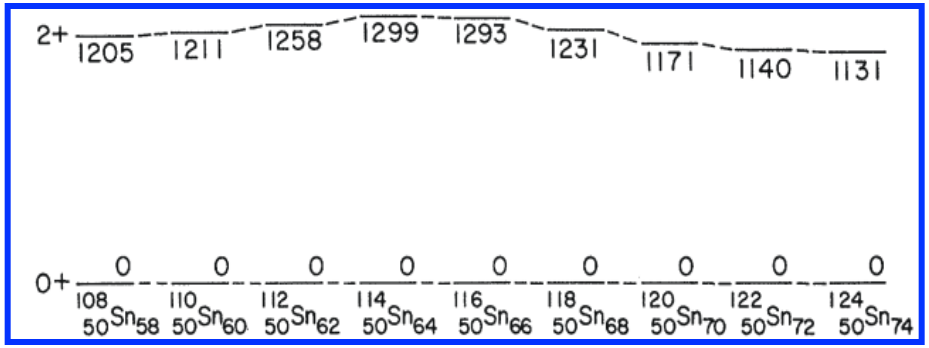
[a] $H = \frac{A^{2/3}}{15.5 \times B(\mathrm{E1})}$

F. Ghazi Moradi et al.
Phys. Rev. C 89, 044310 (2014).

Seniority scheme in Sn isotopes:







Generalized seniority scheme:

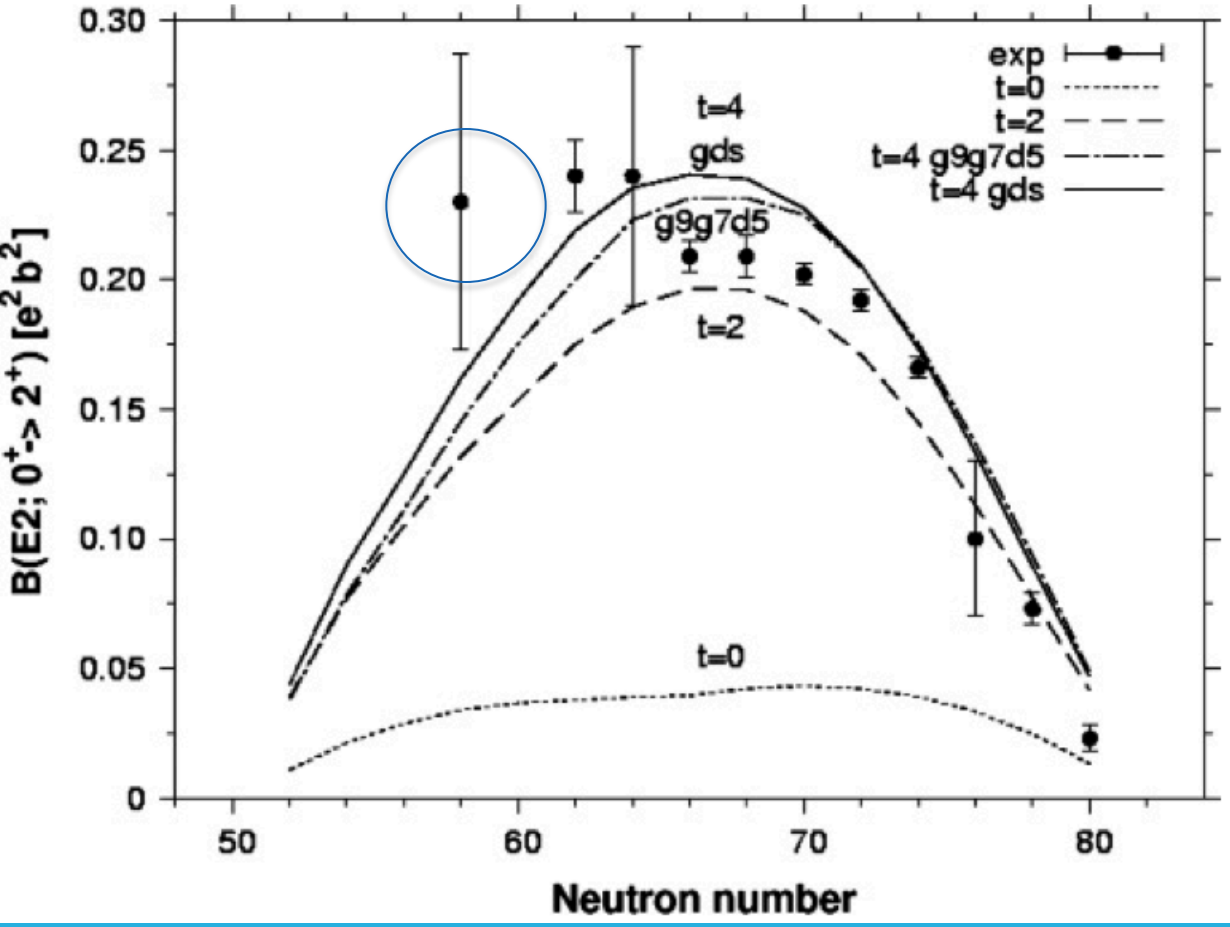
$B(E2;0^+ \rightarrow 2^+) \sim f(1-f)$ where

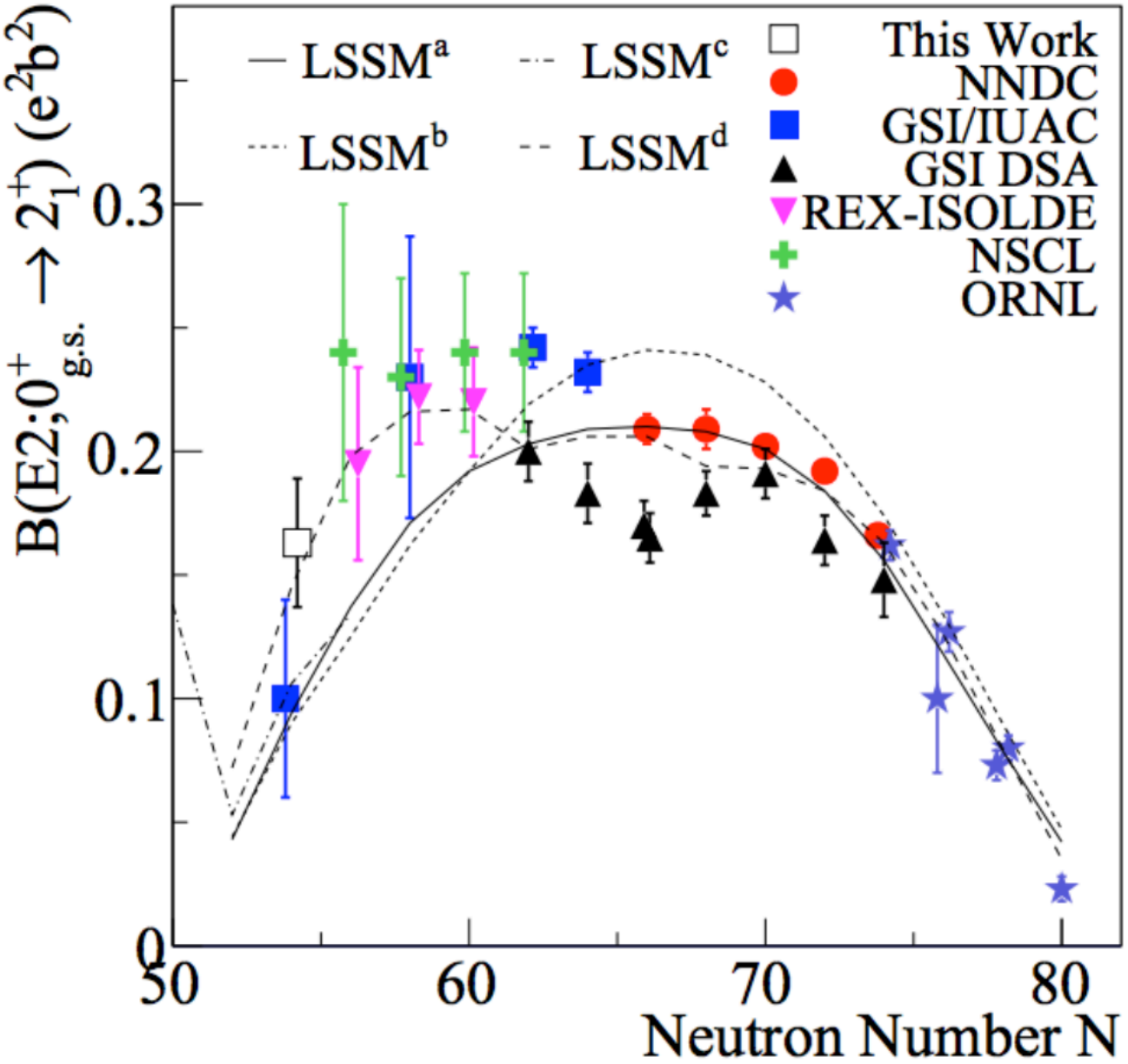
$f = (N - 50)/ 32$

PHYSICAL REVIEW C 72, 061305(R) (2005)

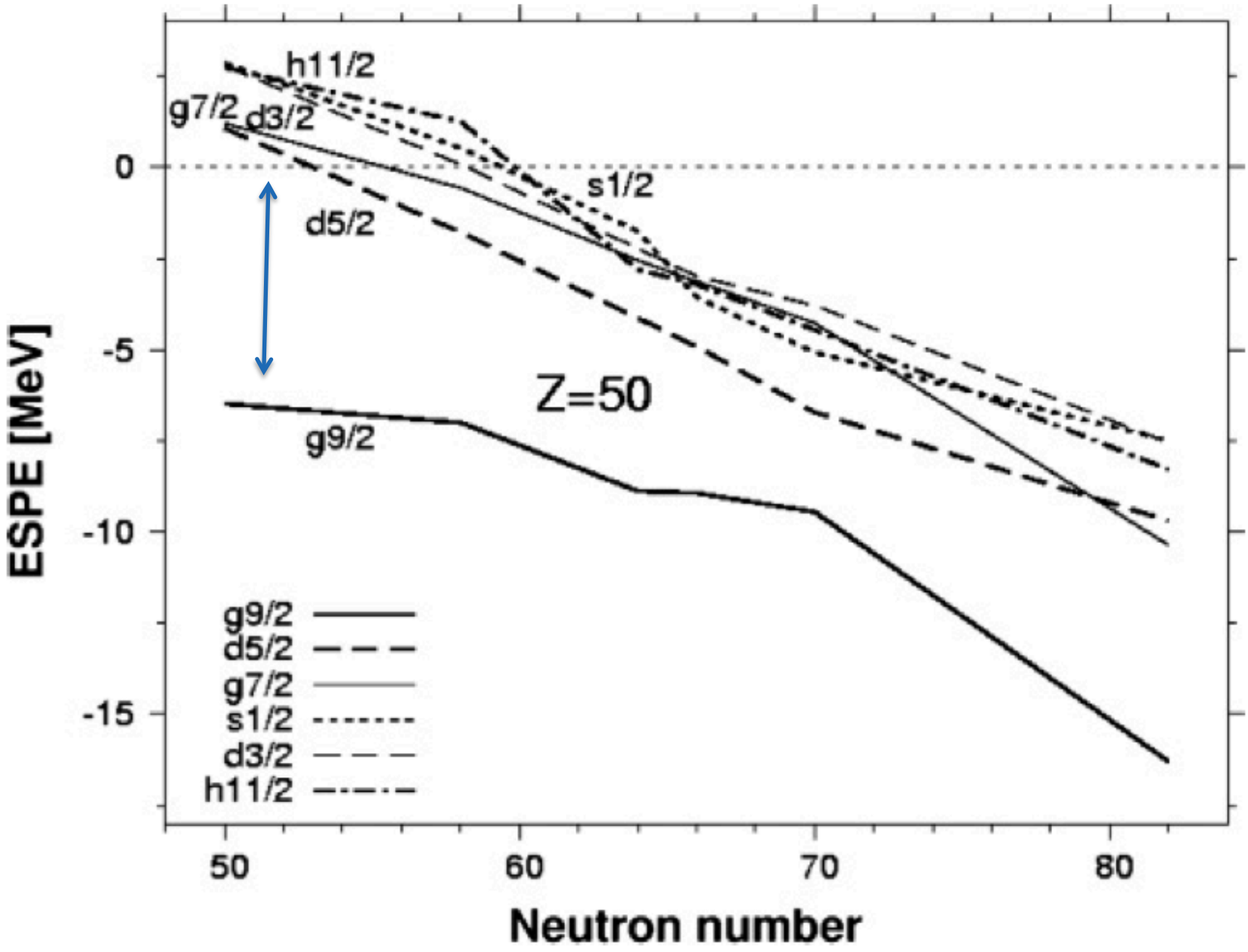
^{108}Sn studied with intermediate-energy Coulomb excitation

A. Banu,[1,2,*] J. Gerl,[1] C. Fahlander,[3] M. Górska,[1] H. Grawe,[1] T. R. Saito,[1] H.-J. Wollersheim,[1] E. Caurier,[4] T. Engeland,[5] A. Gniady,[4] M. Hjorth-Jensen,[5] F. Nowacki,[4] T. Beck,[1] F. Becker,[1] P. Bednarczyk,[1,6] M. A. Bentley,[7] A. Bürger,[8] F. Cristancho,[3,†] G. de Angelis,[9] Zs. Dombrádi,[10] P. Doornenbal,[1,11] H. Geissel,[1] J. Grębosz,[1,6] G. Hammond,[12,‡] M. Hellström,[1,§] J. Jolie,[11] I. Kojouharov,[1] N. Kurz,[1] R. Lozeva,[1,||] S. Mandal,[1,¶] N. Mărginean,[9] S. Muralithar,[1,**] J. Nyberg,[13] J. Pochodzalla,[2] W. Prokopowicz,[1,6] P. Reiter,[11] D. Rudolph,[3] C. Rusu,[9] N. Saito,[1] H. Schaffner,[1] D. Sohler,[10] H. Weick,[1] C. Wheldon,[1,††] and M. Winkler[1]



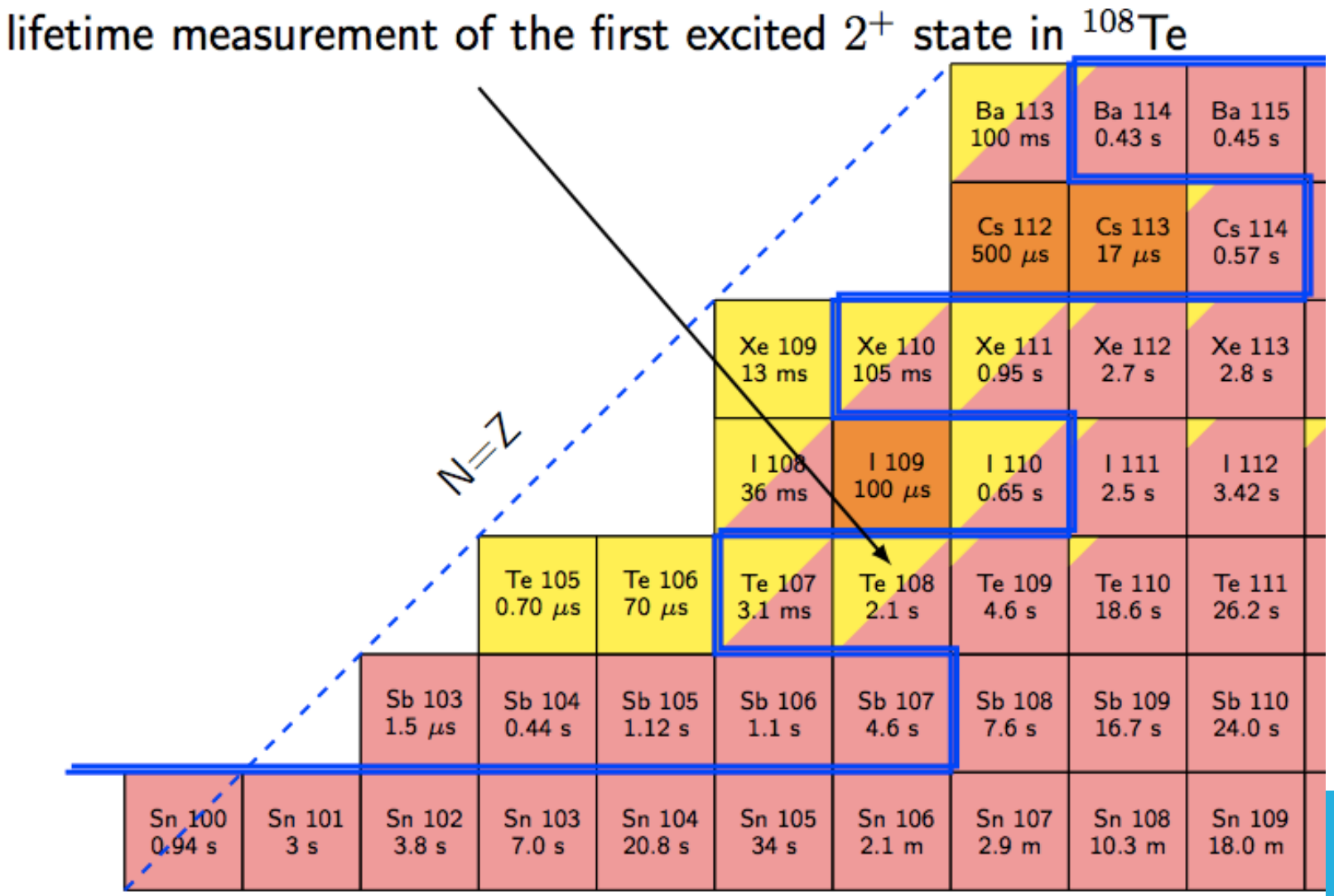


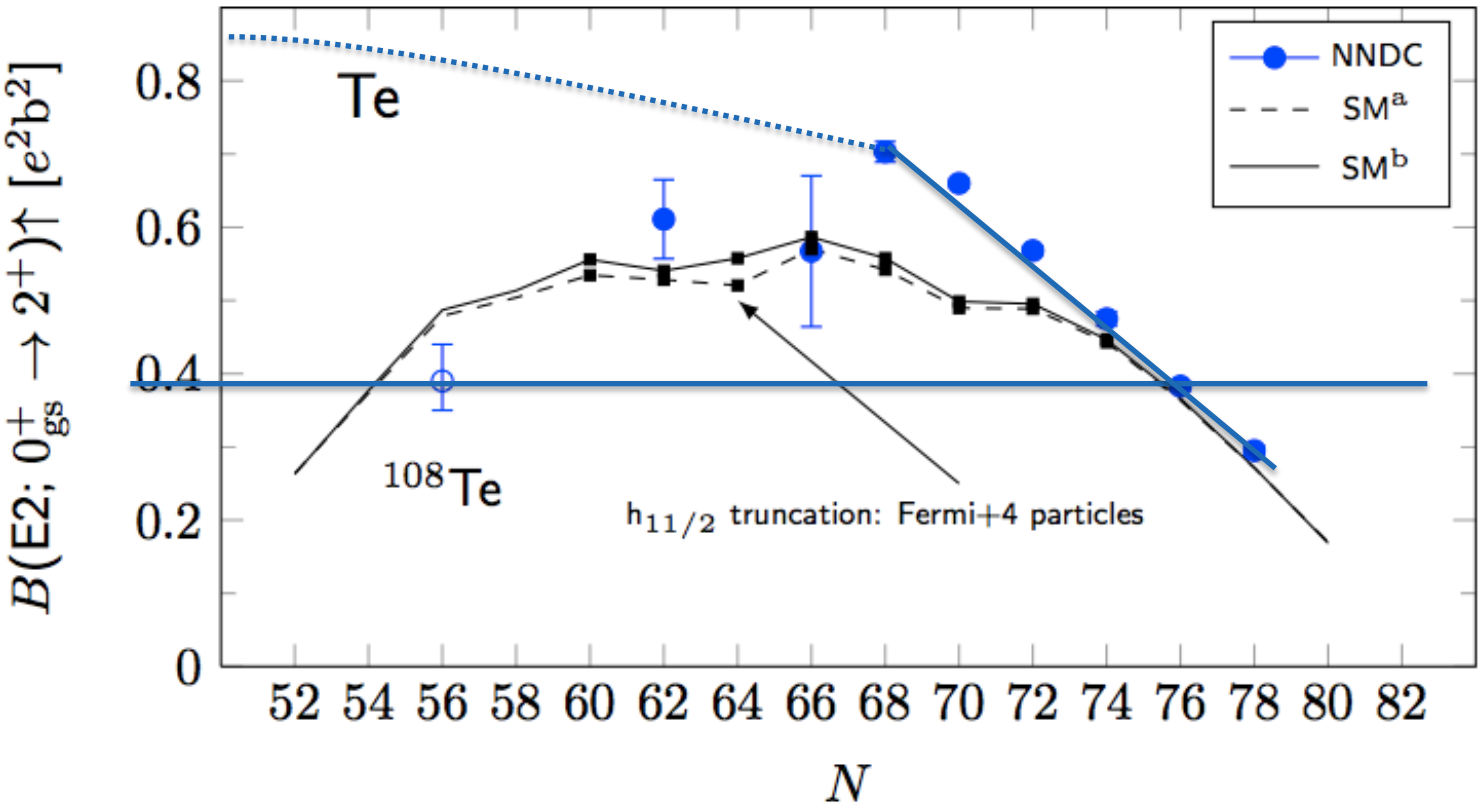
P. Doornenbal et al,arxiv.org/abs/1305.2877


h11/2
g7/2
d3/2
s1/2
d5/2
g9/2
Z=50
0
-5
-10
-15
ESPE [MeV]
50
60
70
80
Neutron number
g9/2
d5/2
g7/2
s1/2
d3/2
h11/2

What happens in other open-shell nuclei if the N=Z=50 shell was eroded?

Combined recoil decay tagging and differential recoil distance Doppler shift technique (plunger)





T. Bäck, CQ et al, PRC (R), 84 (4), 041306 (2011)

Update 2015

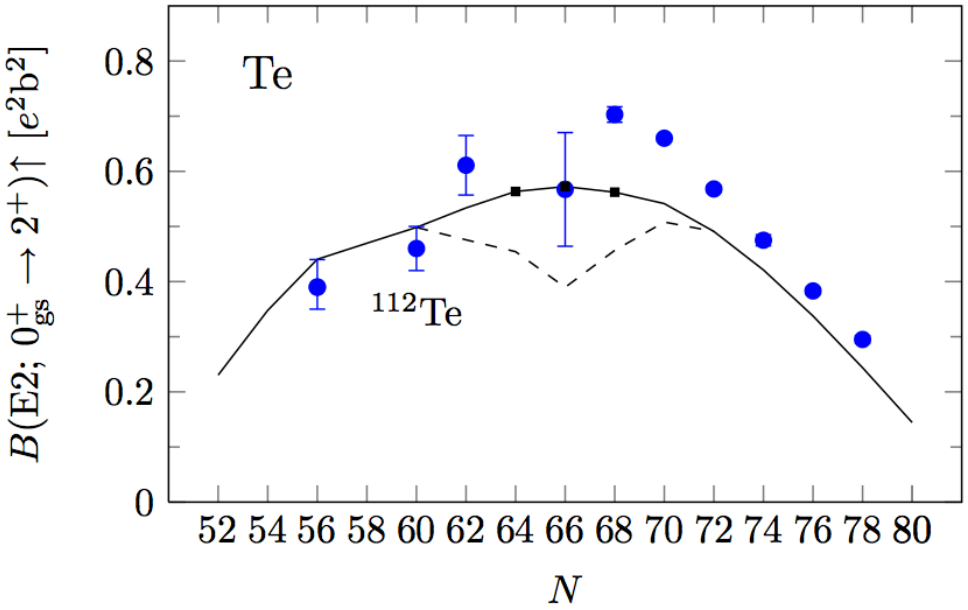
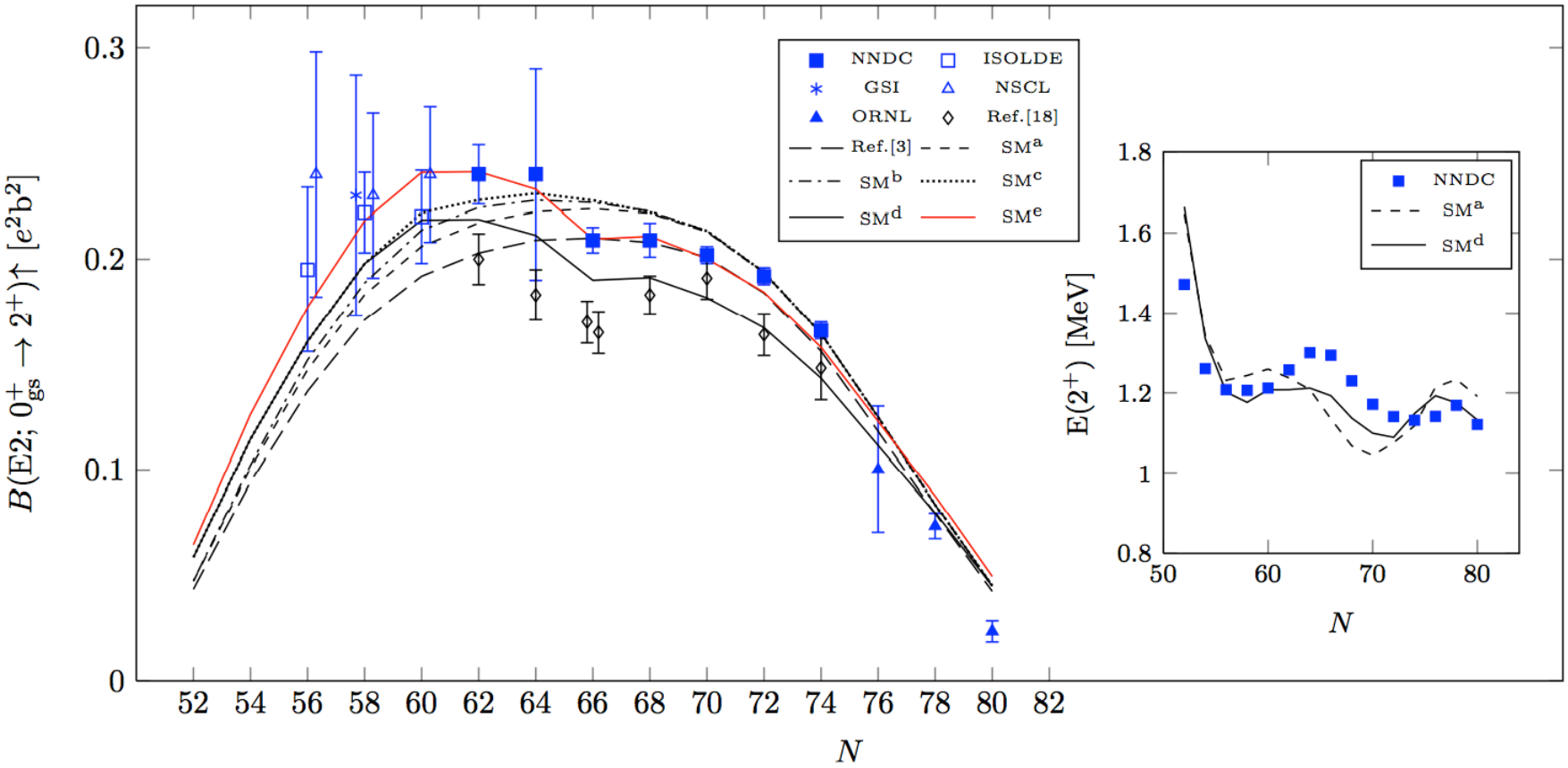


FIG. 3. (Color online) Comparison of the theoretical and experimental $B(E2;0^+_{\mathrm{g.s.}} \rightarrow 2^+)$ values for the Te isotopic chain. The solid line corresponds to the calculation in which the full model space ($g_{7/2}$, $d_{5/2}$, $d_{3/2}$, $s_{1/2}$, and $h_{11/2}$) has been considered for all nuclei except $^{116-120}$Te (marked by small solid squares) while the dashed line represents the calculation in the smaller space, see the main text for more details.

M. Doncel, CQ et al, PRC 91, 061304(R) (2015)

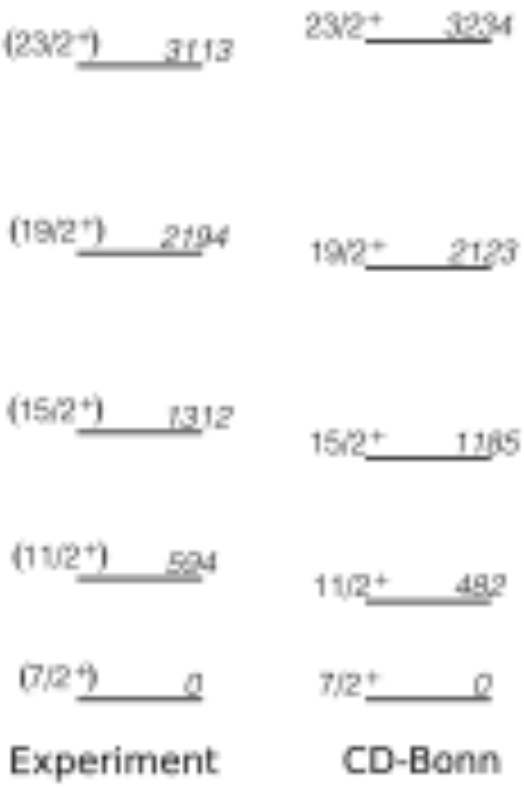
CQ, to be published

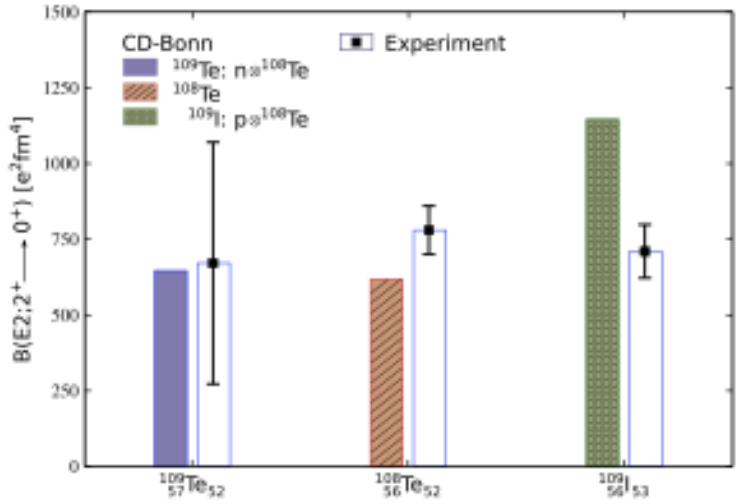
Pauli blocking effect?



T. Bäck, CQ et al, PRC **87**, 031306(R) (2013)

The unbound nucleus ^{109}I





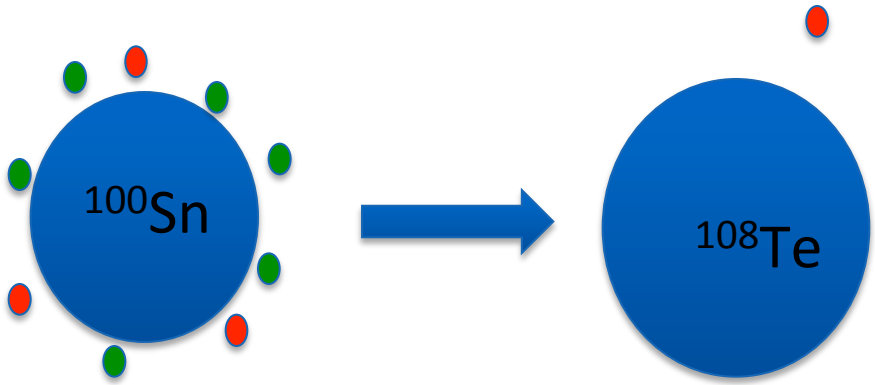
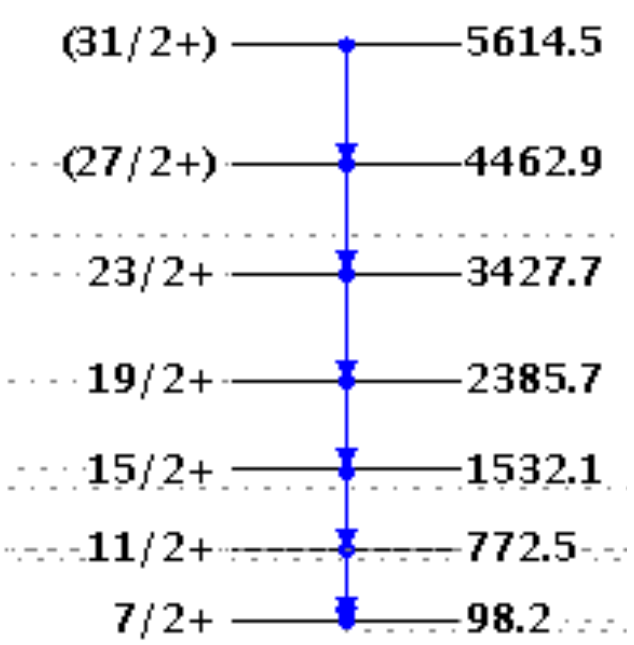


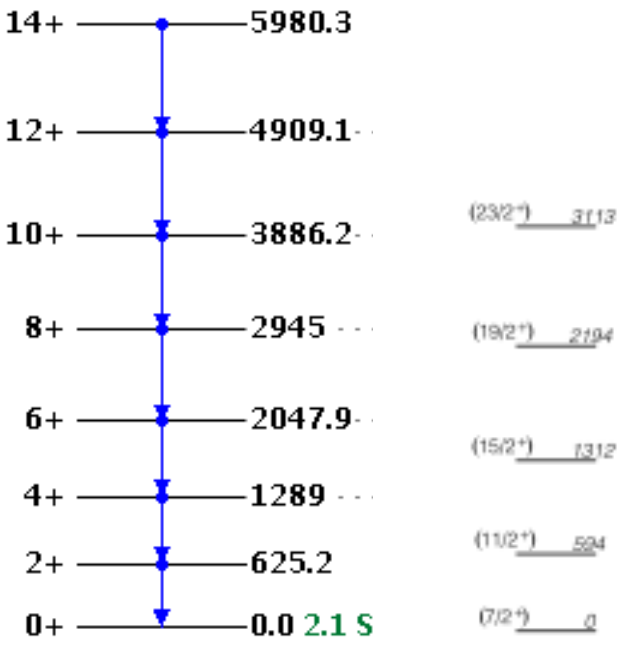
Fig. 3. Experimental excitation energies of ^{109}I (left). All excitation energies are quoted relative to the $(7/2^+)$ state. Theoretical CD-Bonn calculated excitation energies (right) are shown for comparison. The excitation energies of the ground-state band levels are shown to the right of each state.

M.G. Procter et al, Physics Letters B 704 (2011) 118–122

The level schemes



^{109}Te



^{108}Te

^{109}I

- The weak-coupling picture in Te109, I109

^{109}I			^{109}Te		
$\|^{109}\mathrm{I}(k)\rangle$	$\|(\pi j)\otimes^{108}\mathrm{Te}(\lambda);k\rangle$	Overlap	$\|^{109}\mathrm{Te}(k)\rangle$	$\|(\nu j)\otimes^{108}\mathrm{Te}(\lambda);k\rangle$	Overlap
$\|7/2_1^+\rangle$	$\|(\pi g_{7/2})\otimes(0_1^+)\rangle$	0.82	$\|5/2_1^+\rangle$	$\|(\nu d_{5/2})\otimes(0_1^+)\rangle$	0.94
$\|11/2_1^+\rangle$	$\|(\pi g_{7/2})\otimes(2_1^+)\rangle$	0.86	$\|9/2_2^+\rangle$	$\|(\nu d_{5/2})\otimes(2_1^+)\rangle$	0.89
$\|15/2_1^+\rangle$	$\|(\pi g_{7/2})\otimes(4_1^+)\rangle$	0.87	$\|5/2_2^+\rangle$	$\|(\nu g_{7/2})\otimes(2_1^+)\rangle$	0.81
$\|19/2_1^+\rangle$	$\|(\pi g_{7/2})\otimes(6_1^+)\rangle$	0.93	$\|9/2_1^+\rangle$	$\|(\nu g_{7/2})\otimes(2_1^+)\rangle$	0.89
$\|11/2_1^-\rangle$	$\|(\pi h_{11/2})\otimes(0_1^+)\rangle$	0.85	$\|13/2_1^+\rangle$	$\|(\nu g_{7/2})\otimes(4_1^+)\rangle$	0.77
$\|15/2_1^-\rangle$	$\|(\pi h_{11/2})\otimes(2_1^+)\rangle$	0.91	$\|7/2_1^+\rangle$	$\|(\nu g_{7/2})\otimes(0_1^+)\rangle$	0.93
$\|19/2_1^-\rangle$	$\|(\pi h_{11/2})\otimes(4_1^+)\rangle$	0.92	$\|11/2_1^+\rangle$	$\|(\nu g_{7/2})\otimes(2_1^+)\rangle$	0.92
$\|23/2_1^-\rangle$	$\|(\pi h_{11/2})\otimes(6_1^+)\rangle$	0.95	$\|15/2_1^+\rangle$	$\|(\nu g_{7/2})\otimes(4_1^+)\rangle$	0.89
			$\|19/2_1^+\rangle$	$\|(\nu g_{7/2})\otimes(6_1^+)\rangle$	0.93
			$\|11/2_1^-\rangle$	$\|(\nu h_{11/2})\otimes(0_1^+)\rangle$	0.95
			$\|15/2_1^-\rangle$	$\|(\nu h_{11/2})\otimes(2_1^+)\rangle$	0.98
			$\|19/2_1^-\rangle$	$\|(\nu h_{11/2})\otimes(4_1^+)\rangle$	0.99
			$\|23/2_1^-\rangle$	$\|(\nu h_{11/2})\otimes(6_1^+)\rangle$	0.99

H Jiang, C Qi, Y Lei, R Liotta, R Wyss, YM Zhao
Physical Review C 88 (4), 044332 (2013).

Summary

- Introduction to the nuclear shell model approach
  - ❖Properties of the effective interaction
  - ❖Truncation methods
  - ❖Truncation based an a correlated basis
- Applications in Sn and neighboring isotopes
- Future looks prosperous

どうも ありがとうございます

Thank you!