

# Shell-model study of strength function in the *sd-pf* shell region

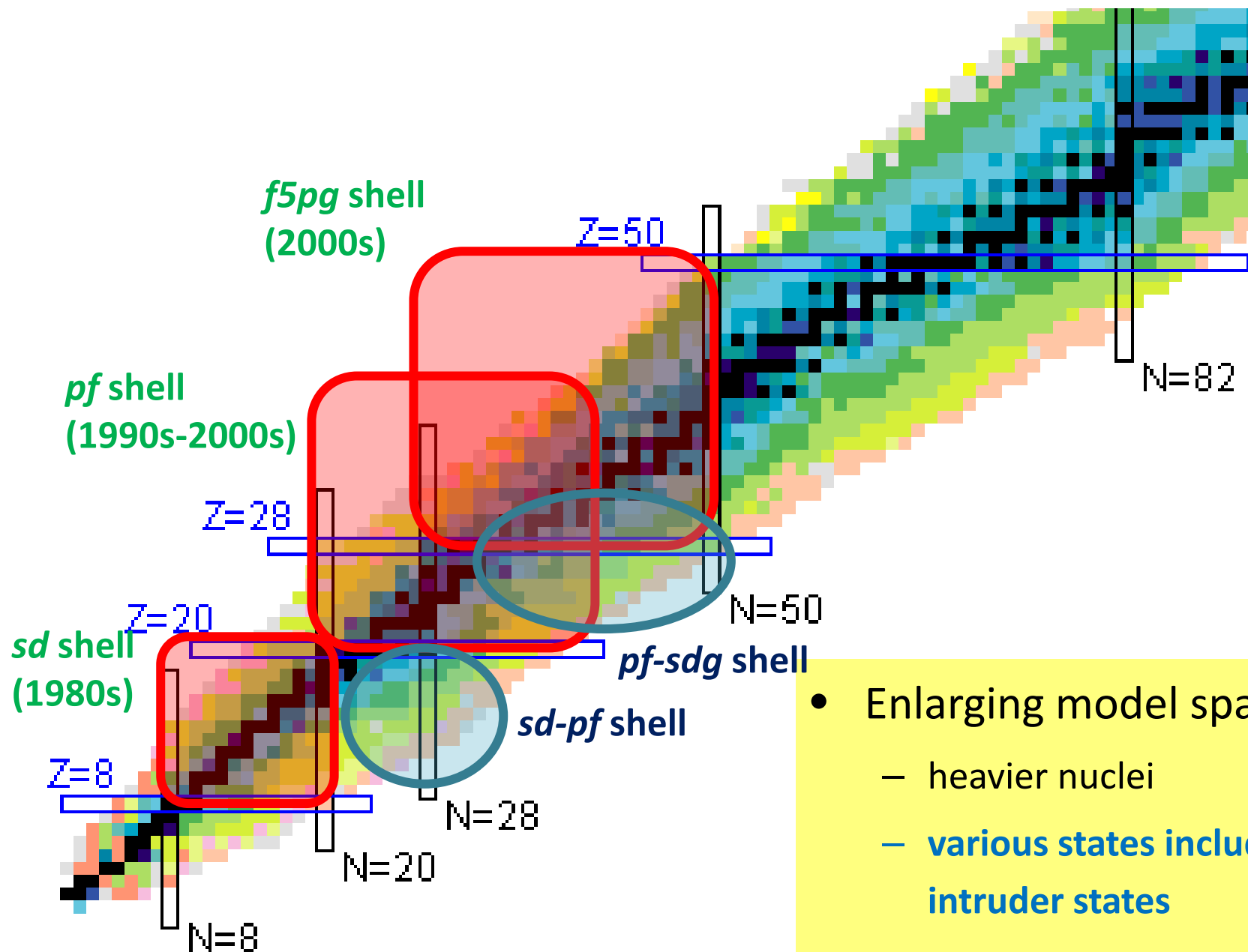
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# Collaborators

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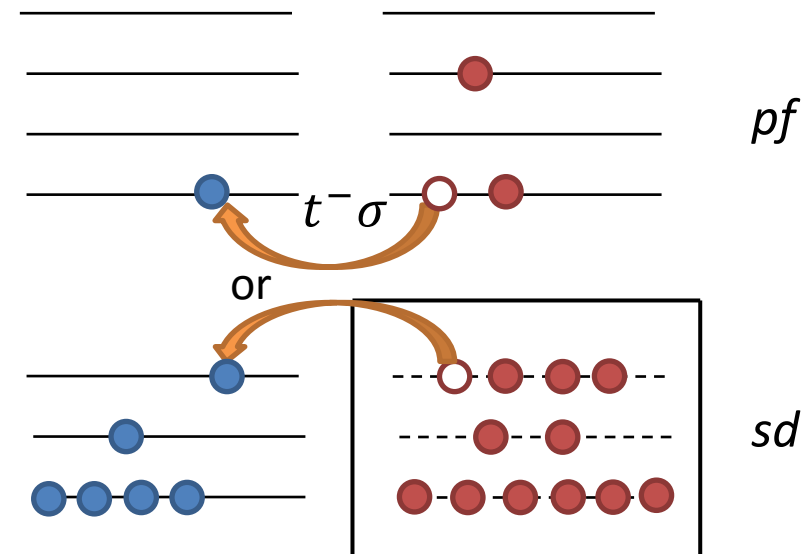
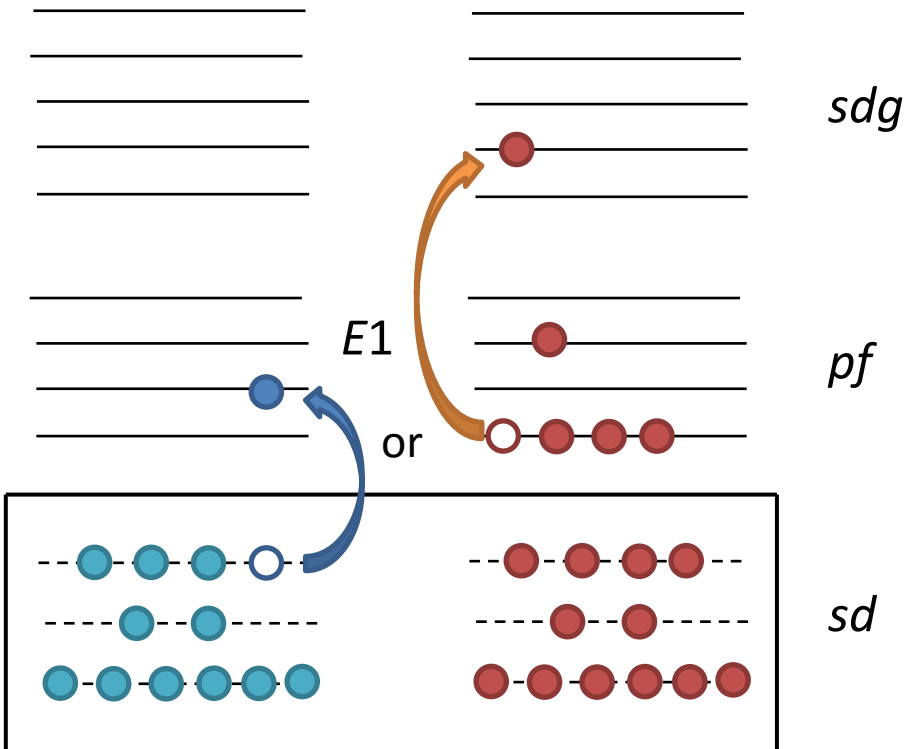
# Frontier of large-scale shell-model calculations



- Enlarging model space
  - heavier nuclei
  - various states including intruder states

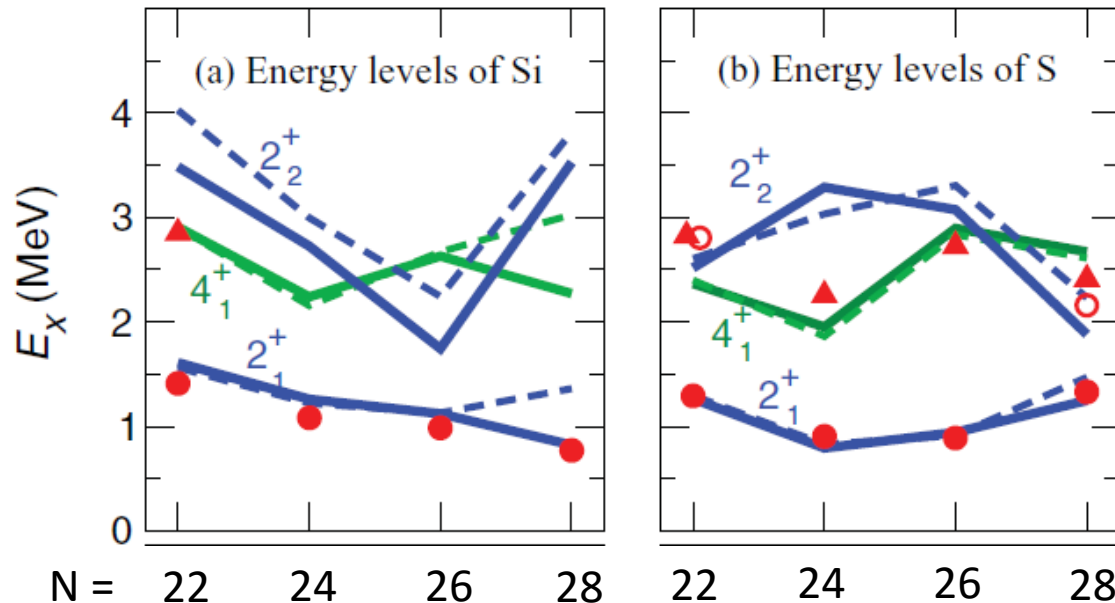
# Objectives of this study

- Unnatural-parity states and strength function in the  $sd$ - $pf$  shell
    - More than one major shell are required.
1. Systematics of unnatural-parity states and  $E1$  strength function in Ca isotopes
  2. Gamow-Teller strength function of neutron-rich nuclei



# Model space and effective interaction

- Model space
  - Full *sd-pf-sdg* shell for E1 calc. or Full *sd-pf* shell for GT calc.
  - $1\hbar\omega$  [or  $(1+3)\hbar\omega$ ] calculation in the given model space
- Effective interaction
  - SDPF-MU for the *sd-pf* shell or its natural extension to the *sd-pf-sdg* shell:
    - USD (*sd*) + GXPF1B (*pf*) + the refined  $V_{\text{MU}}$  for the remaining

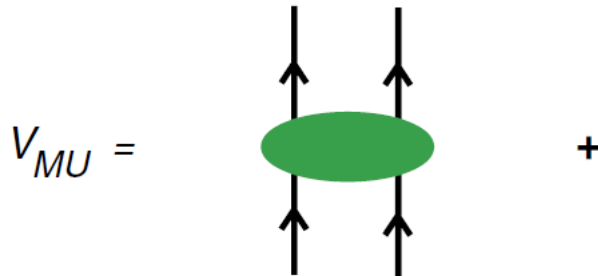


Y. Utsuno et al., Phys. Rev. C 86, 051301(R) (2012).  
 Y. Utsuno et al., Phys. Rev. Lett. 114, 032501 (2015).

# Monopole-based universal interaction $V_{MU}$

(a) central force :  
Gaussian  
(strongly renormalized)

(b) tensor force :  
 $\pi + \rho$  meson  
exchange



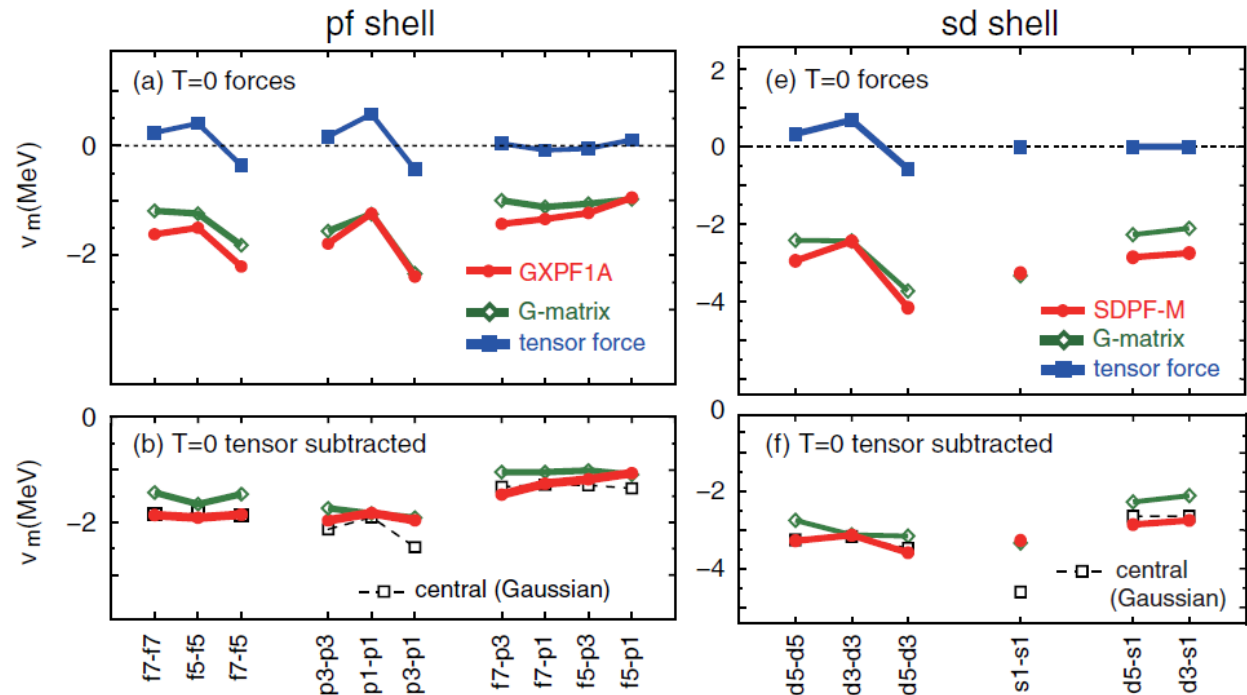
T. Otsuka et al., Phys. Rev. Lett. 104, 012501 (2010).

- Bare tensor

- Renormalization persistency

- Phenomenological Gaussian central

- Supported by empirical interactions



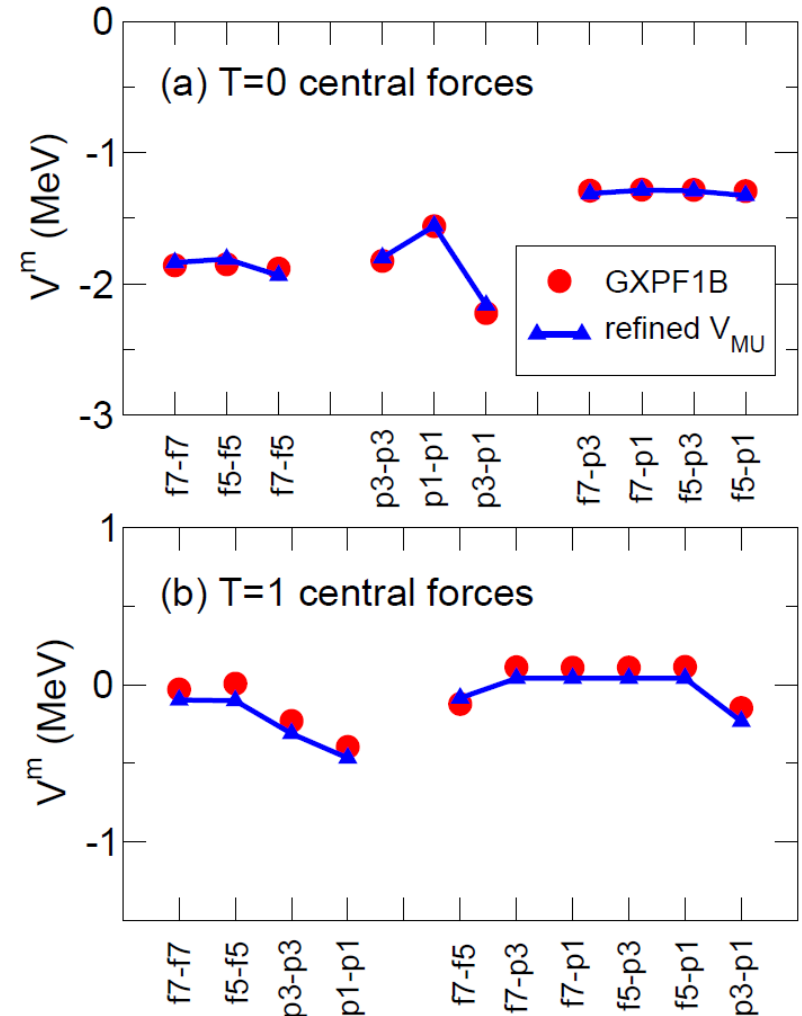
# Refined $V_{\text{MU}}$ for the shell-model

- tensor:  $\pi+\rho$
- spin-orbit: M3Y
  - Works in some cases
- central: to be close to GXPF1
  - Including “density dependence” to better fit empirical interactions



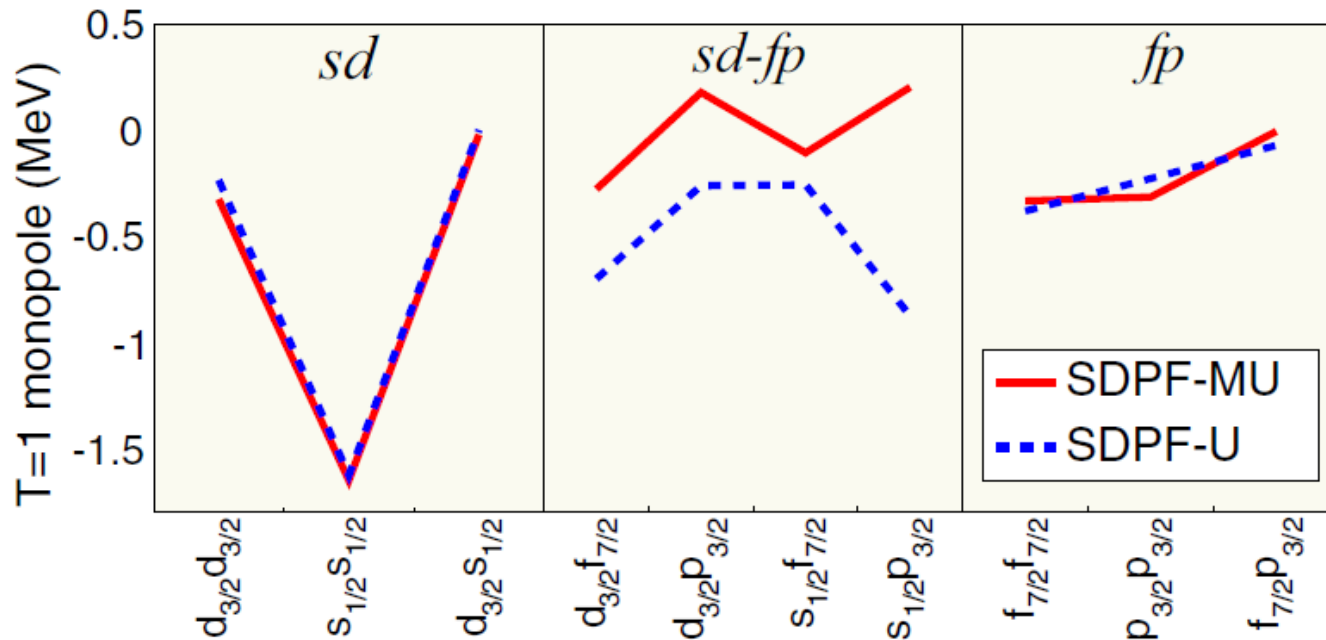
a good guide for a shell-model interaction without direct fitting to experiment

Central force fitted with six parameters



# $T=1$ monopole: case of $sd$ - $pf$ shell

- SDPF-MU interaction based on the refined  $V_{\text{MU}}$ 
  - USD for the  $sd$  shell and GXPF1B for the  $pf$  shell
  - Refined  $V_{\text{MU}}$  for the cross-shell

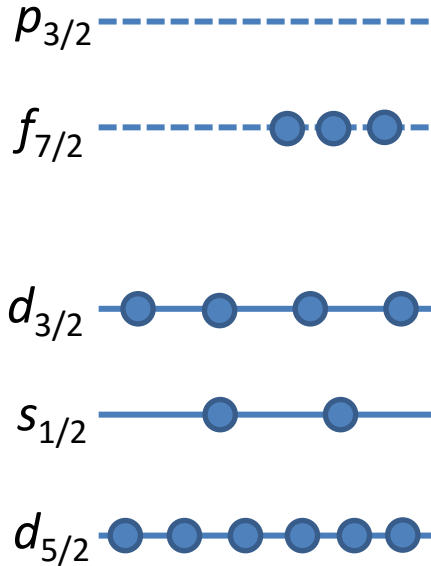


S. R. Stroberg, A. Gade et al., Phys. Rev. C 91, 041302(R) (2015).

Cross-shell of SDPF-U: two-body G matrix



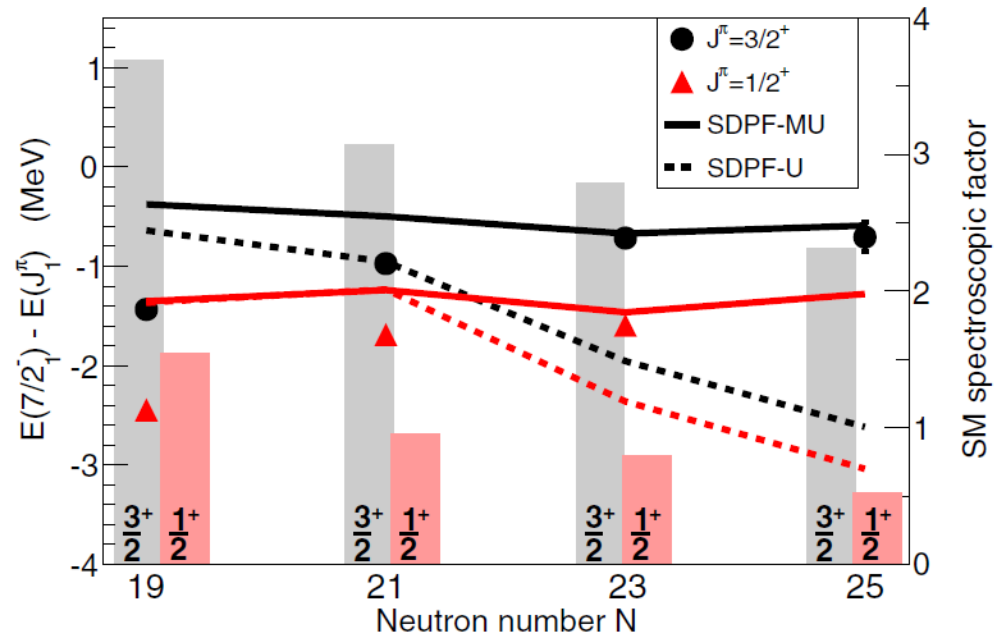
# Evolution of unnatural-parity states in Si



The gap changes with increasing neutrons in  $f_{7/2}$  depending on the  $T=1$  monopole strength.

Unnatural-parity states are good indicators of the gap.

- A recent experiment at NSCL supports nearly zero value of  $T=1$  cross-shell monopole matrix elements.

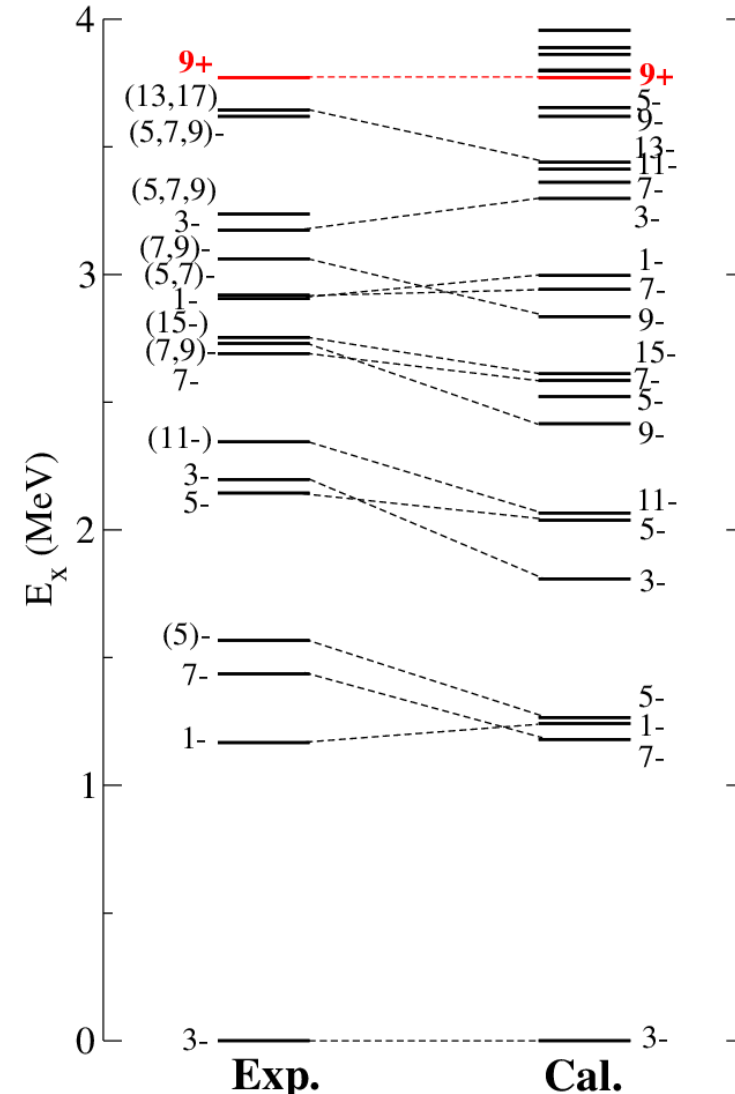


# Position of $g_{9/2}$ in $n$ -rich Ca isotopes

- $g_{9/2}$  orbit in neutron-rich Ca isotopes
  - Plays a crucial role in determining the drip line and the double magicity in  $^{60}\text{Ca}$
  - Unnatural-parity states are examined.
- Determining SPE of  $sdg$ 
  - $g_{9/2}$ : to reproduce the  $9/2^+_{1}$  of  $^{51}\text{Ti}$
  - other  $sdg$ : to follow schematic spin-orbit splitting

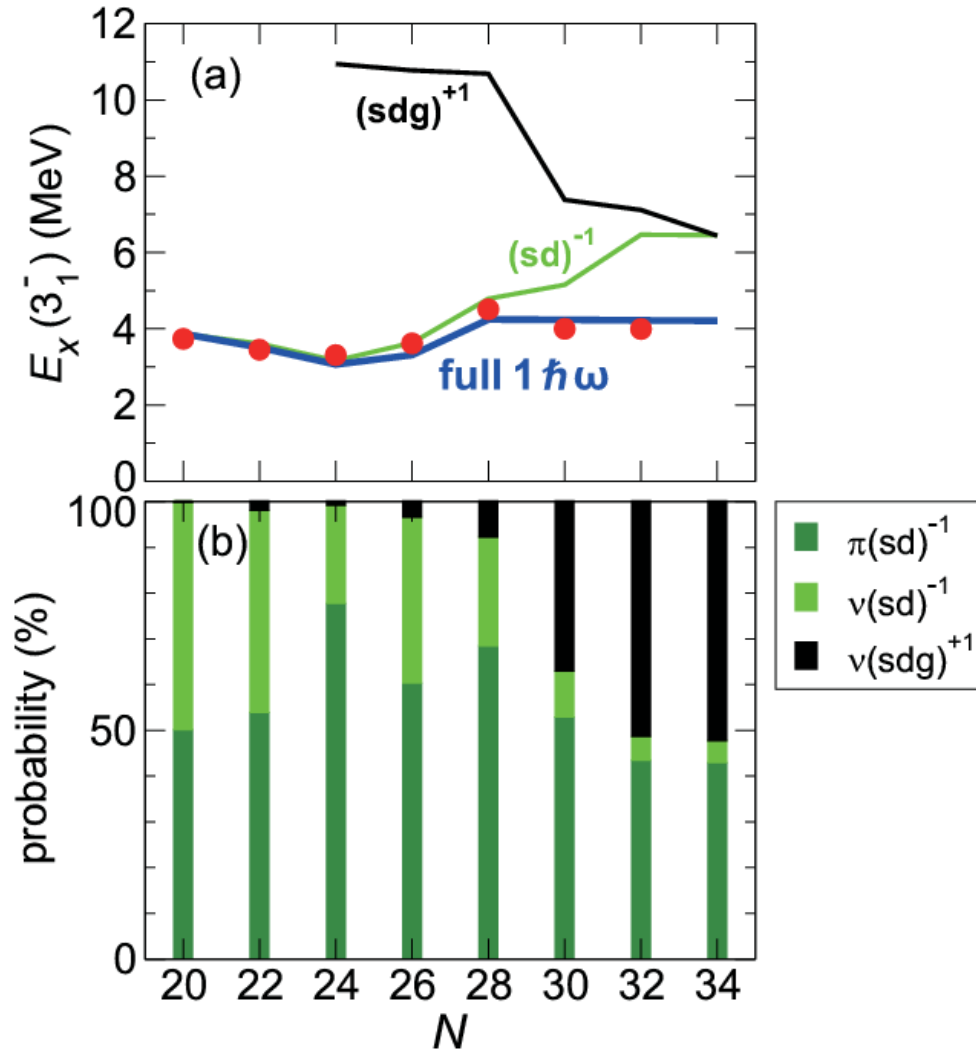
		Expt.	Calc.
Optical pot.	B4	CA	
$C^2S(g_{9/2})$	0.54	0.37	0.47

What happens in Ca levels?



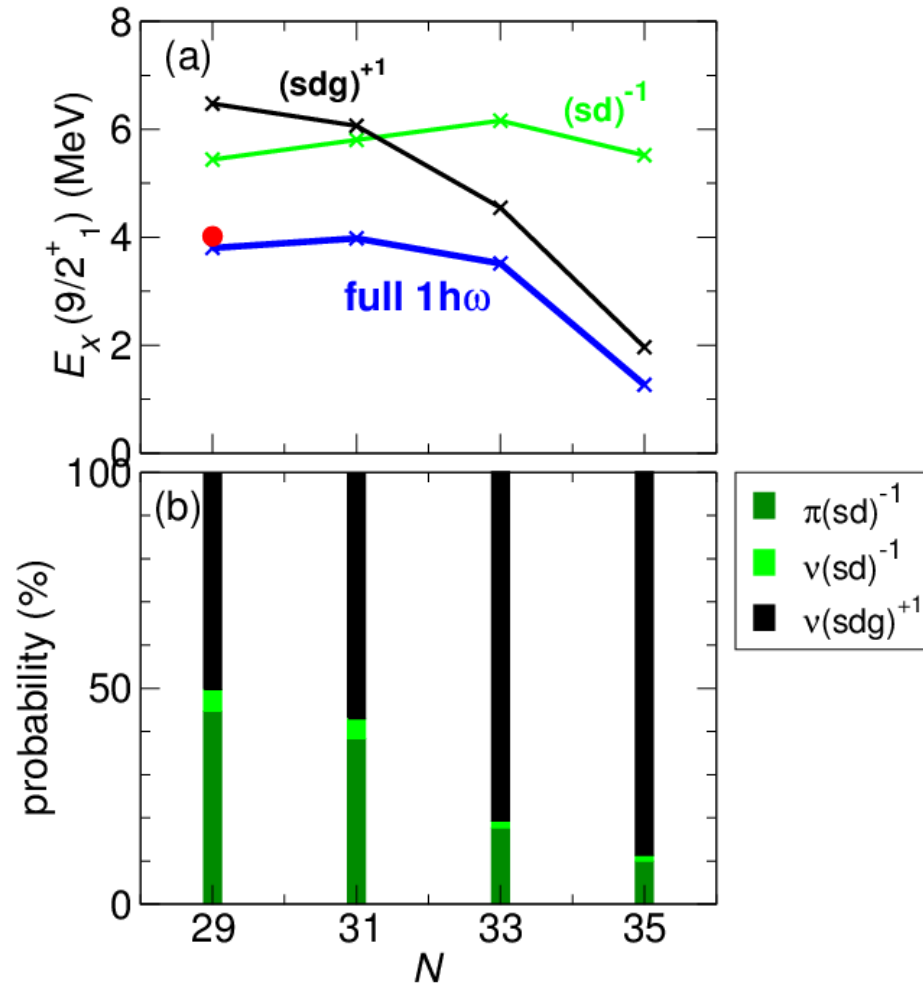
# Systematics of the $3^-_1$ state in even- $A$ Ca

- Three calculations
  - A) excitations from  $sd$  to  $pf$  only
  - B) excitations from  $pf$  to  $sdg$  only
  - C) full  $1\hbar\omega$  configurations
- $3^-_1$  levels
  - $sd$ - $pf$  calc.
    - good agreement for  $N \leq 28$
    - large deviation for  $N > 28$
  - full  $1\hbar\omega$  calc.
    - Strong mixing with the  $sdg$  configuration accounts for the stable positioning of the  $3^-$  levels.



# Systematics of the $9/2^+_1$ state in odd-A Ca

- $9/2^+_1$  in the *sd-pf* calculation
  - Core-coupled state
  - Located stably at 5-6 MeV
- $9/2^+_1$  in the *pf-sdg* calculation
  - Sharply decreasing due to the shift of the Fermi level
- $9/2^+_1$  in the *full  $1\hbar\omega$*  calculation
  - 3-4 MeV up to  $N=33$  but drops considerably at  $N=35$
  - The state at  $N=55$  is nearly a single-particle character.



# Application to photonuclear reaction

N. Shimizu et al., in preparation; Y. Utsuno et al., Prog. Nucl. Ener. 82, 102 (2015).

- A good Hamiltonian for the full  $1\hbar\omega$  space is constructed.
- It is expected that photonuclear reaction, dominated by  $E1$  excitation, is well described with this shell-model calculation:

$$\sigma_{\text{abs}}(E) = \frac{16\pi^3 E}{9\hbar c} S_{E1}(E)$$

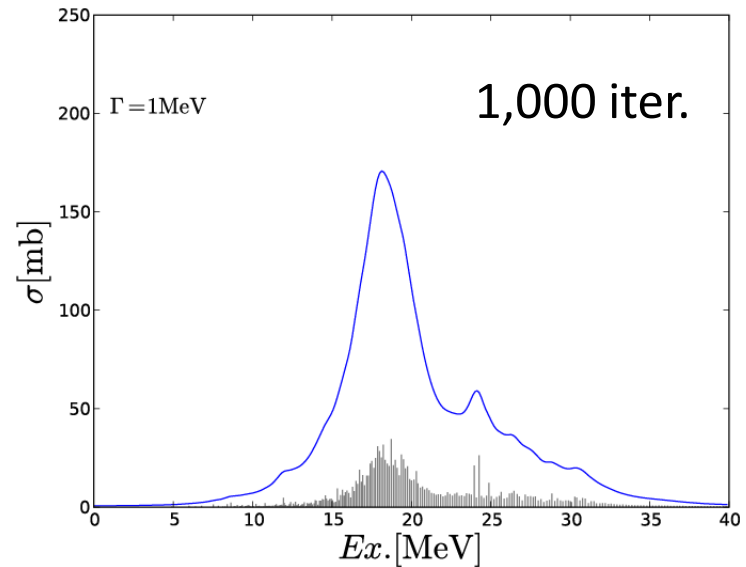
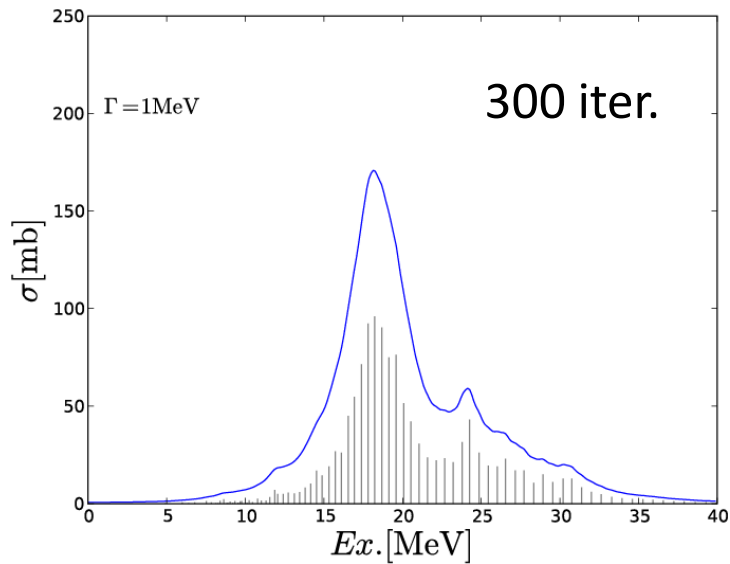
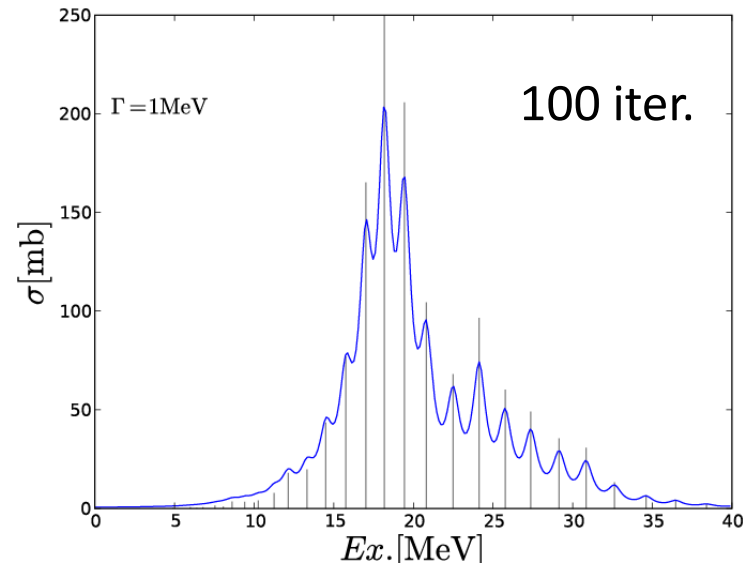
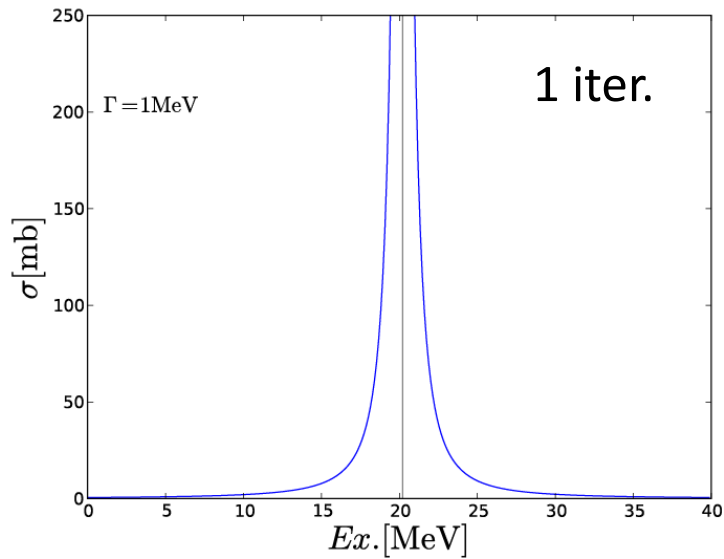
with  $S_{E1}(E) = \sum_{\nu} B(E1; g.s. \rightarrow \nu) \delta(E - E_{\nu} + E_0)$

- Shell-model calculation provides good level density, including non-collective levels, the coupling to which leads to the width of GDR.
- Application of shell model to photonuclear reaction has been very limited due to computational limitation.
  - Sagawa & Suzuki (O isotopes), Brown ( $^{208}\text{Pb}$ ), Ormand & Johnson (ab initio)

# Lanczos strength function method

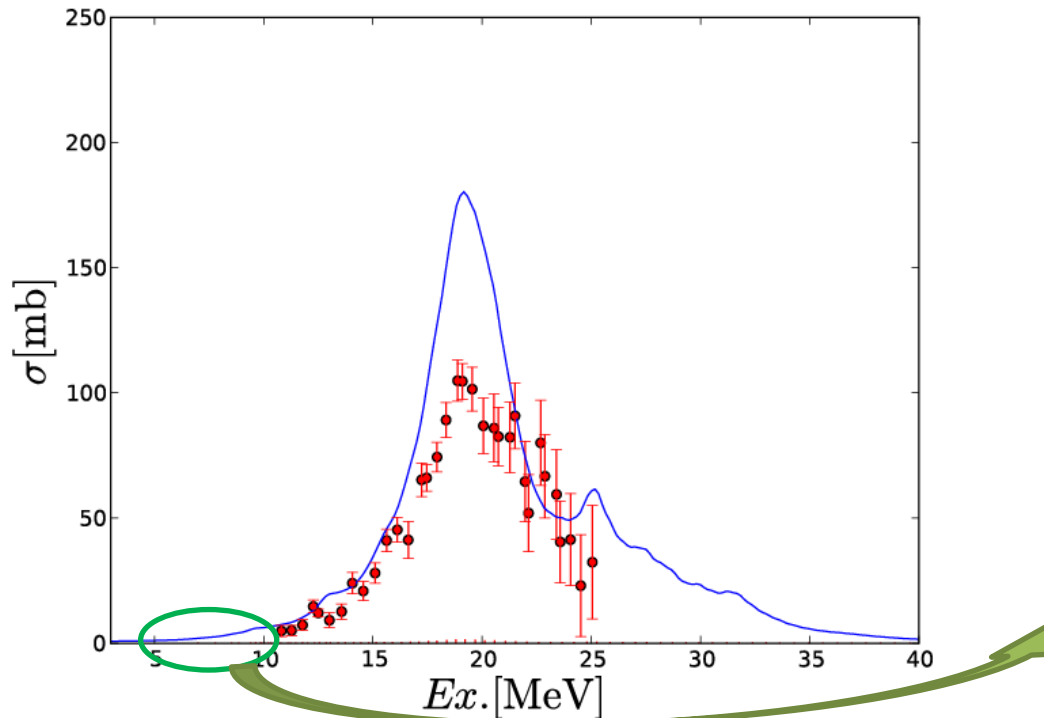
- It is almost impossible to calculate all the eigenstates concerned using the exact diagonalization.
- Moment method of Whitehead [Phys. Lett. B 89, 313 (1980)]
  - The shape of the strength function can be obtained with much less Lanczos iterations.
    1. Take an initial vector:  $\vec{v}_1 = T(E1)|g. s. \rangle$
    2. Follow the usual Lanczos procedure
    3. Calculate the strength function  $\sum_{\nu} B(E1; g. s. \rightarrow \nu) \frac{1}{\pi} \frac{\Gamma/2}{(E-E_{\nu}+E_0)^2+(\Gamma/2)^2}$  by summing up all the eigenstates  $\nu$  in the Krylov subspace with an appropriate smoothing factor  $\Gamma$  until good convergence is achieved.
  - See Caurier et al., Rev. Mod. Phys. 77, 427 (2005), for application to Gamow-Teller.

# Convergence of strength distribution

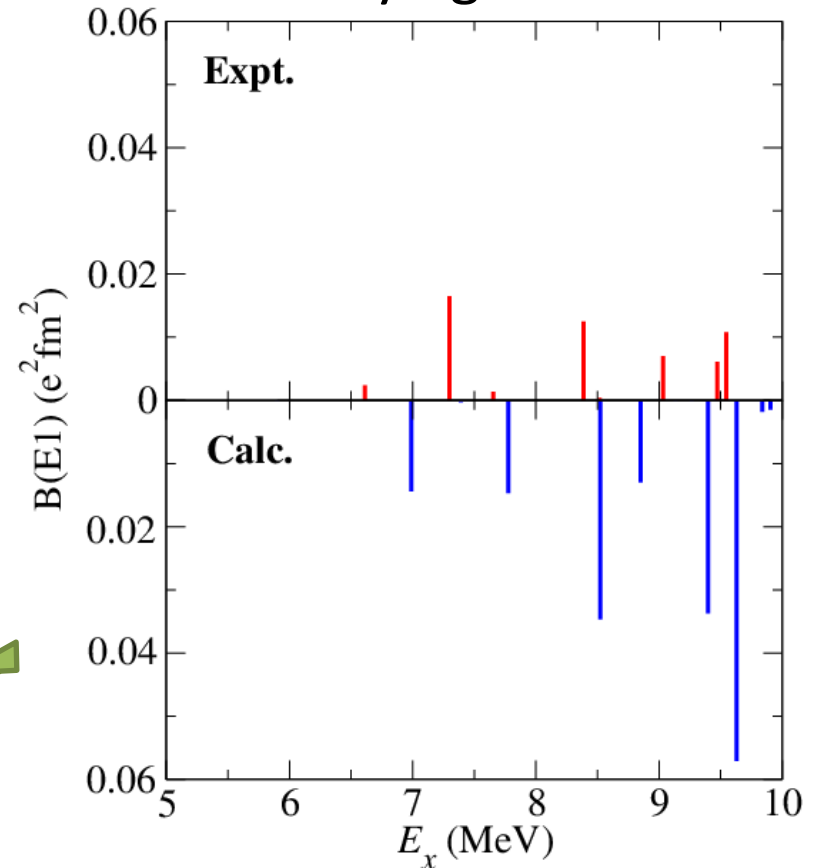


# Comparison with experiment for $^{48}\text{Ca}$

## GDR with $\Gamma=1$ MeV



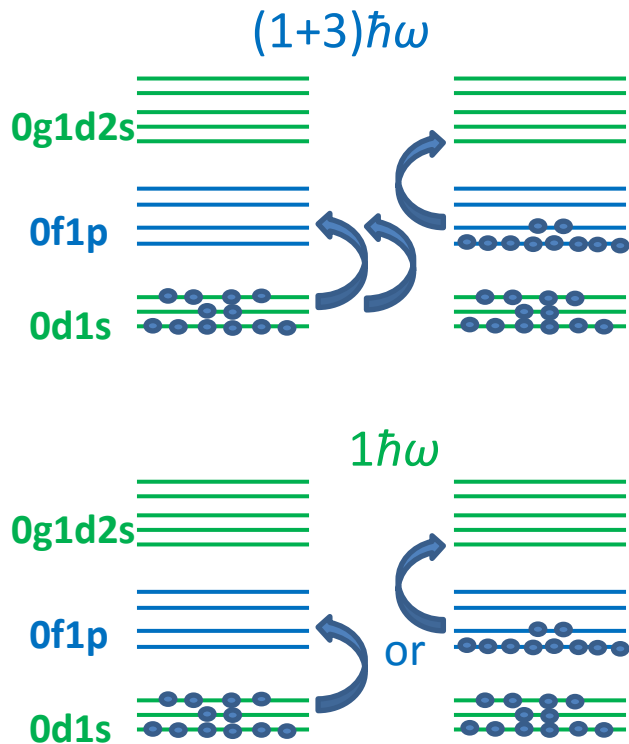
## Low-lying $1^-$ states



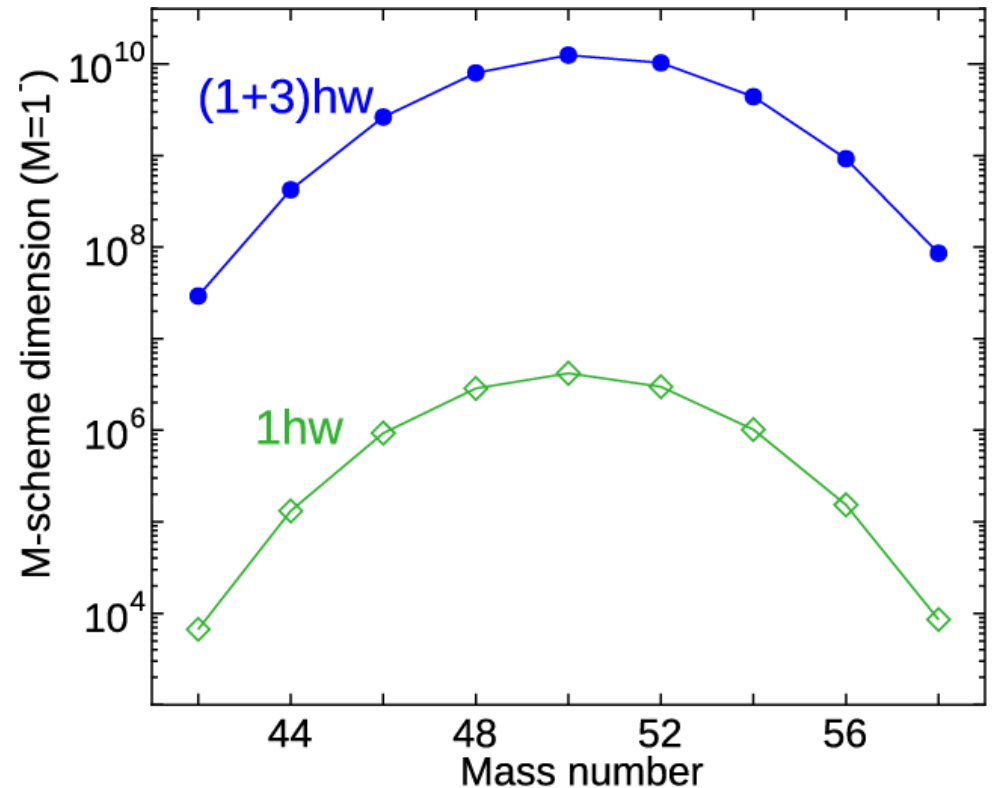
- GDR peak position: good
- GDR peak height: overestimated
- Low-lying states: about 0.7 MeV shifted



# Beyond $1\hbar\omega$ calculation



M-Scheme dimension for Ca isotopes

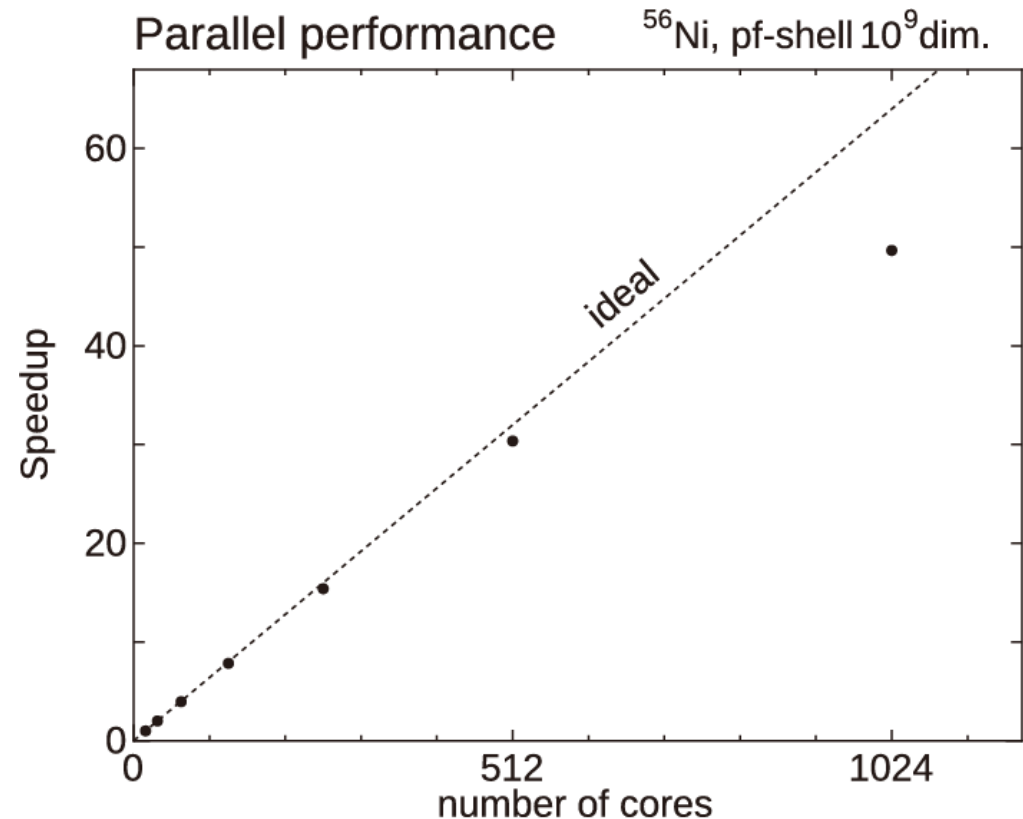


- $3\hbar\omega$  states in the *sd-pf-sdg* shell are included.
  - No single-nucleon excitation to the  $3\hbar\omega$  above shell
- Dimension becomes terrible!

# KSHELL: MPI + OpenMP hybrid code

N. Shimizu, arXiv:1310.5431 [nucl-th]

- *M*-scheme code
  - “On the fly”: Matrix elements are not stored in memory (analogous to ANTOINE and MSHELL64)
- Good parallel efficiency
  - Owing to categorizing basis states into “partition”, which stands for a set of basis states with the same sub-shell occupancies



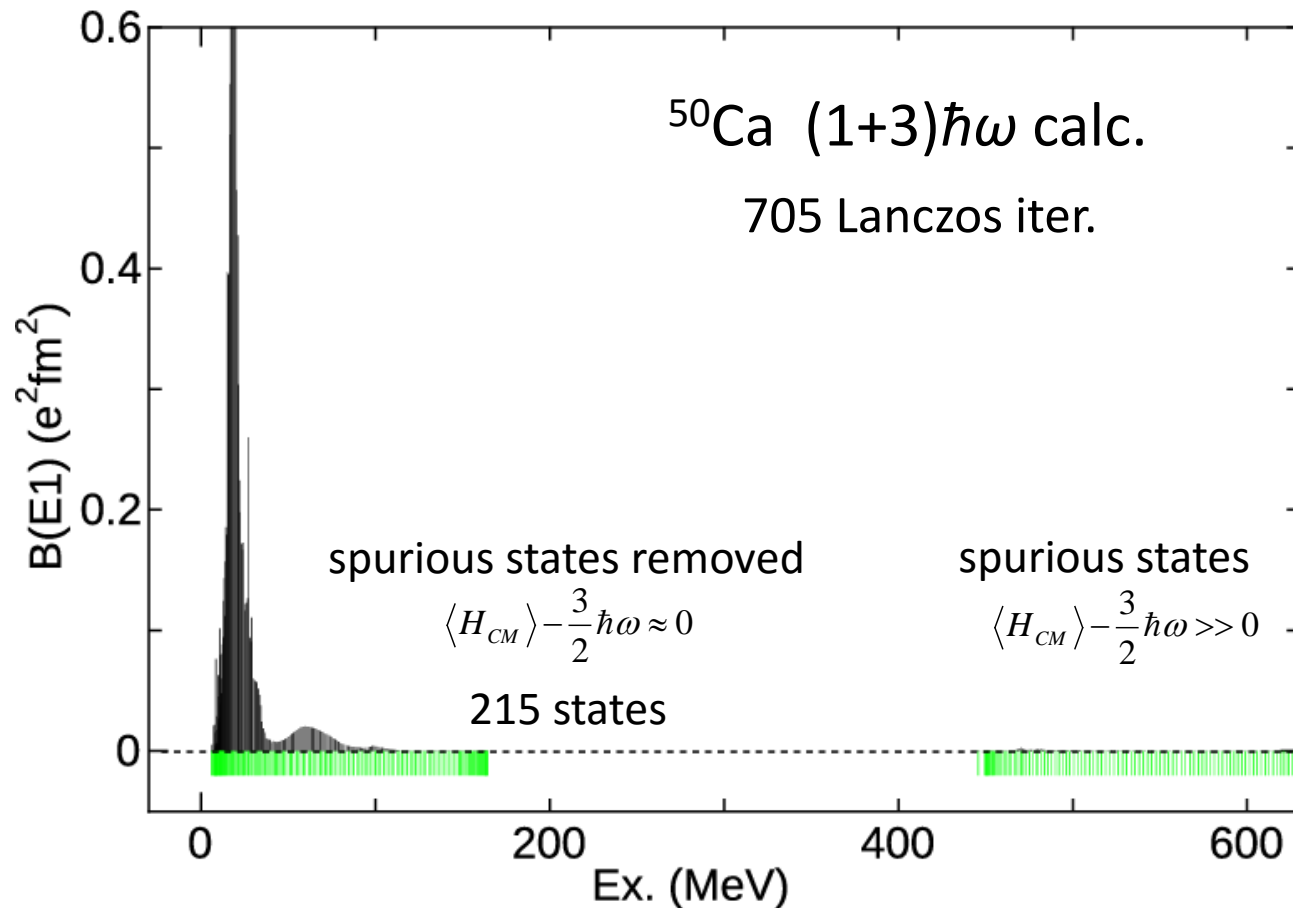
time/iteration : 25 min. (16 cores) ➡ 30 sec. (1024 cores)

# Removal of spurious center-of-mass motion

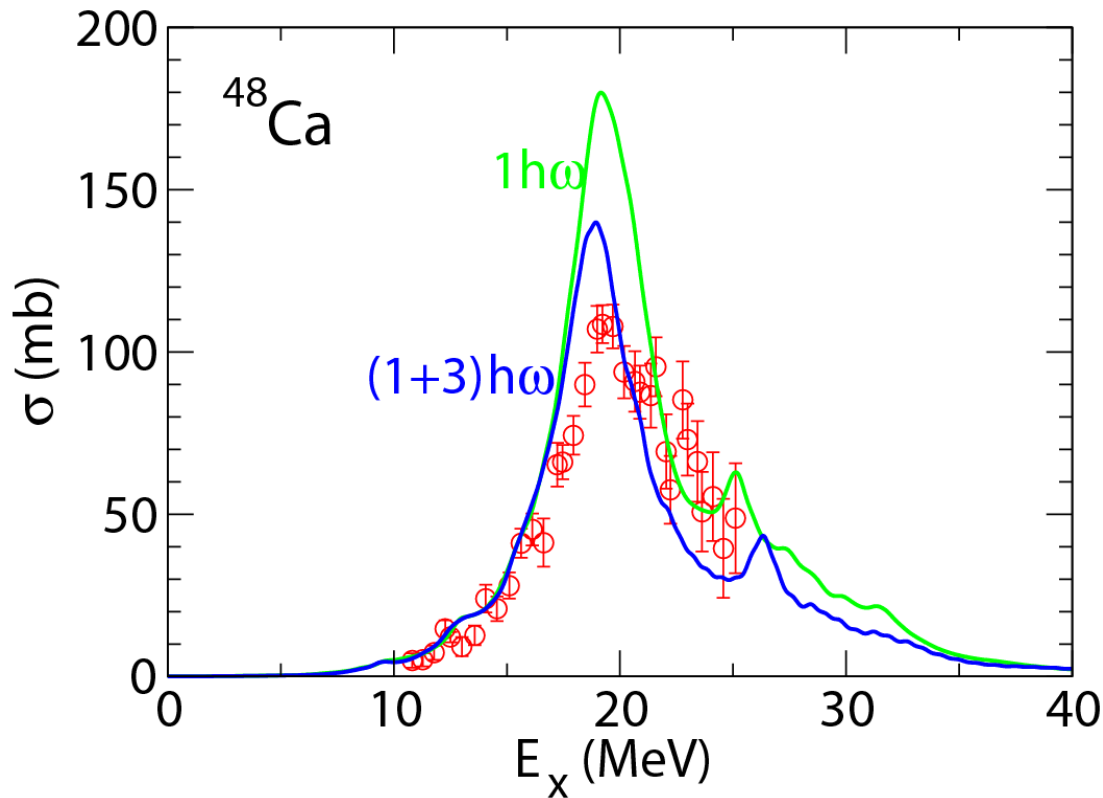
- Usual prescription of Lawson and Gloeckner

$$H' = H + \beta H_{CM} \text{ with } \beta = 10\hbar\omega/A \text{ MeV}$$

- Confirming that eigenstates are well separated

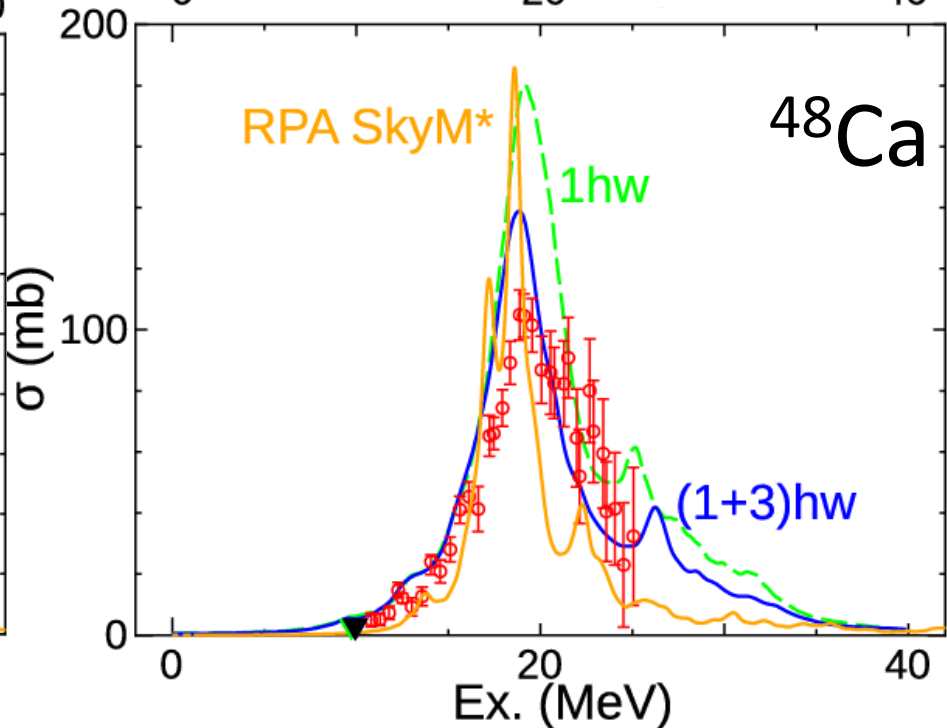
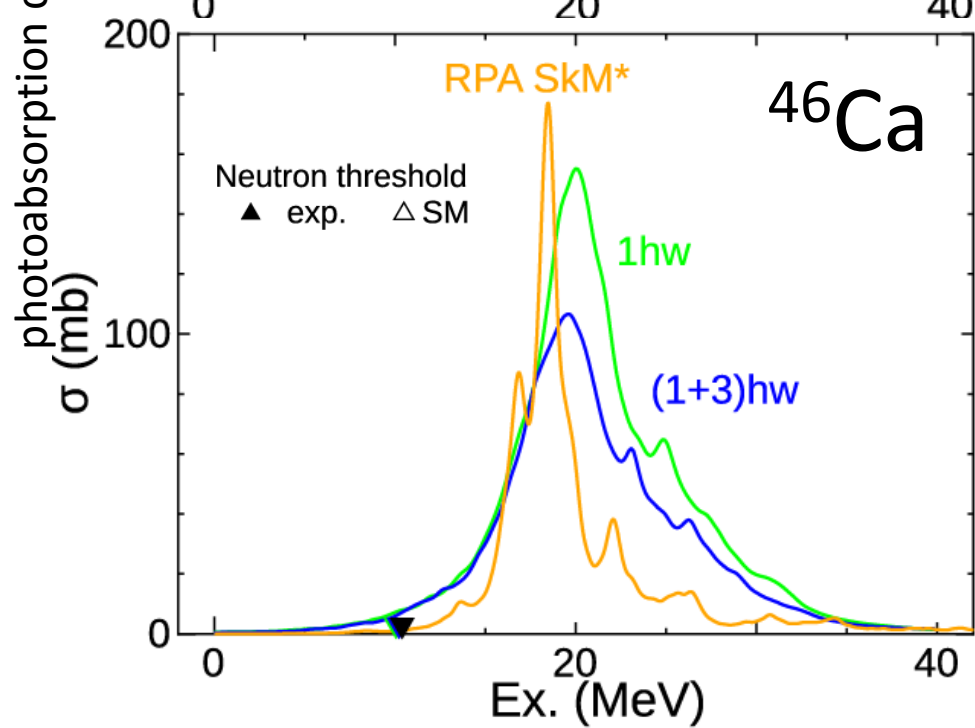
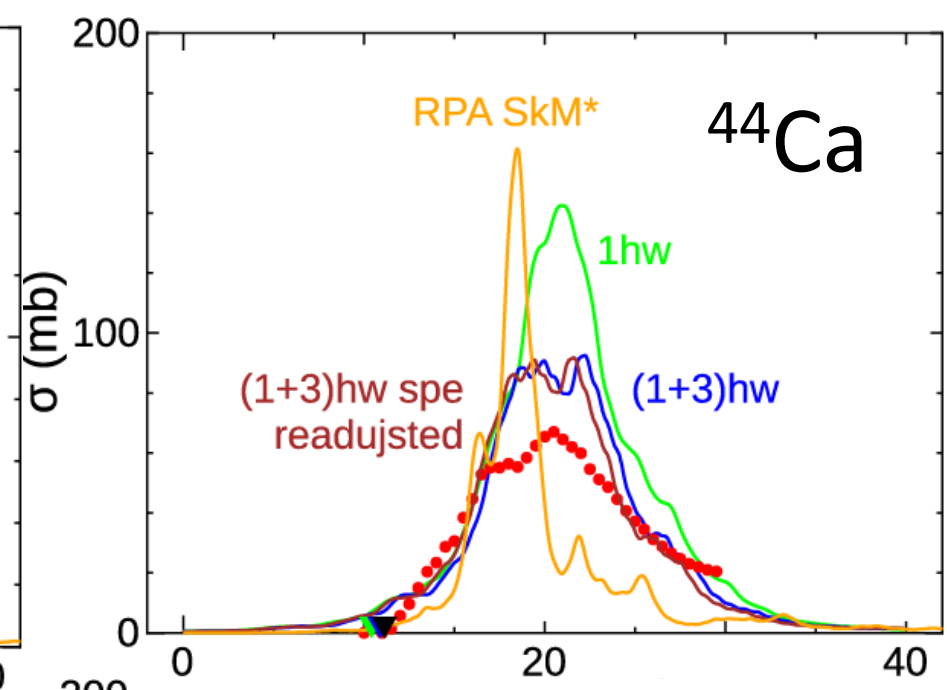
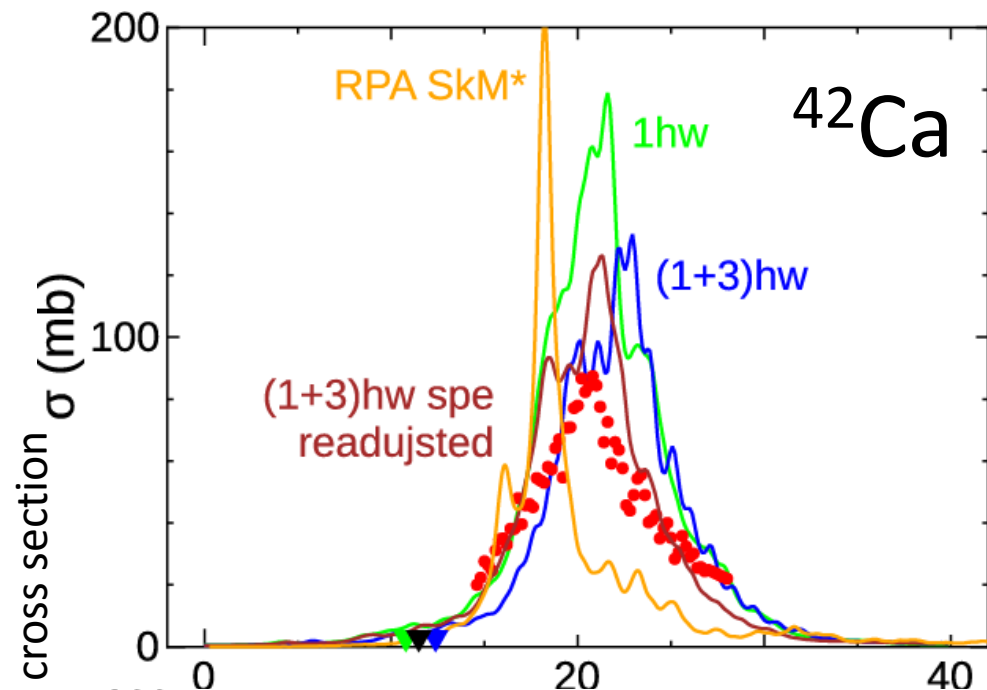


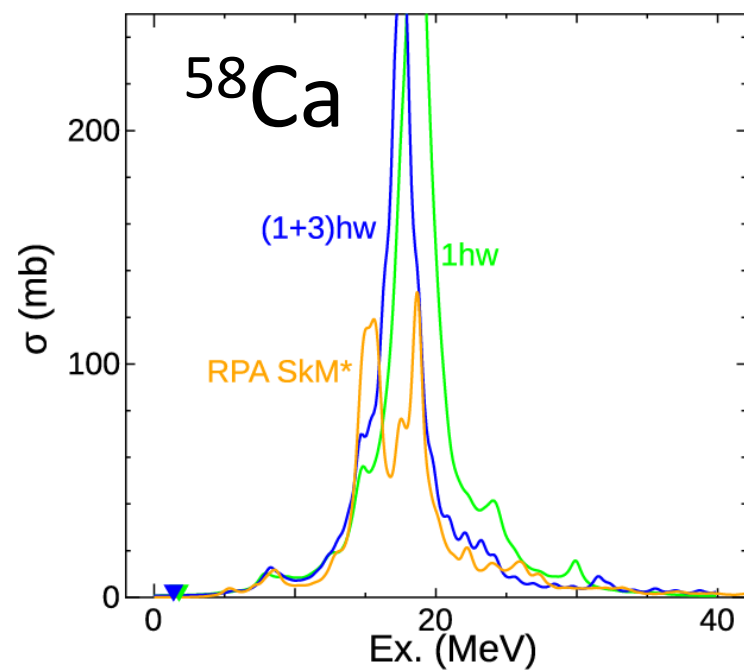
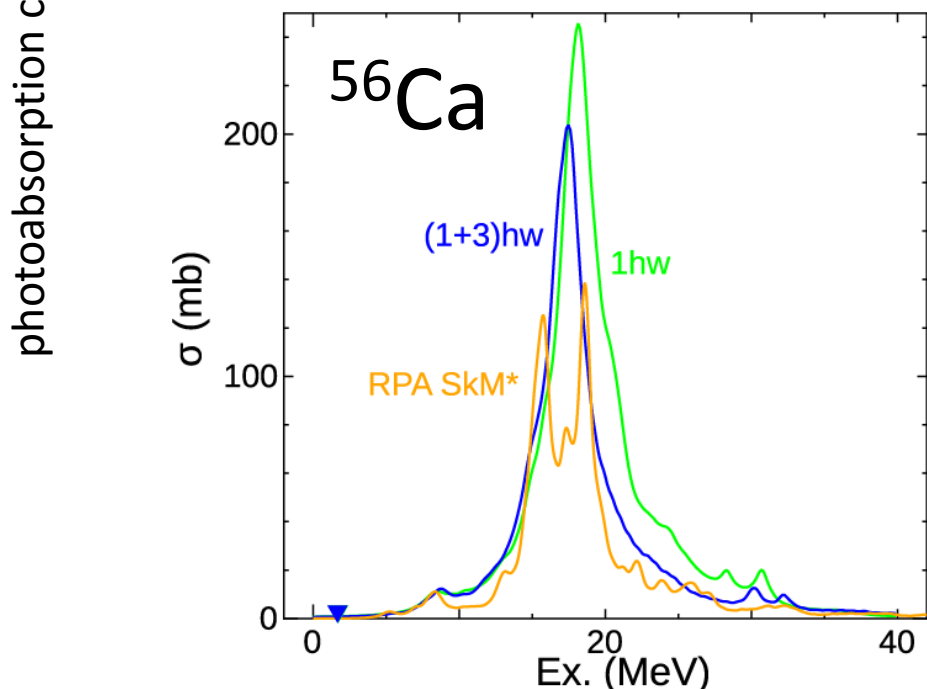
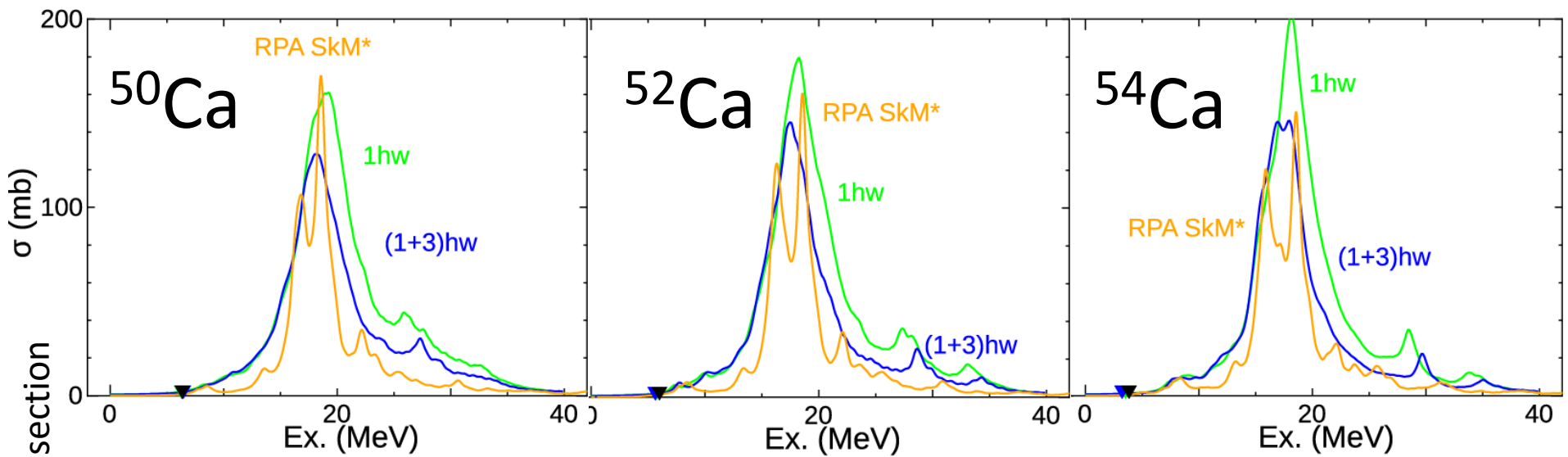
# Effect of correlation



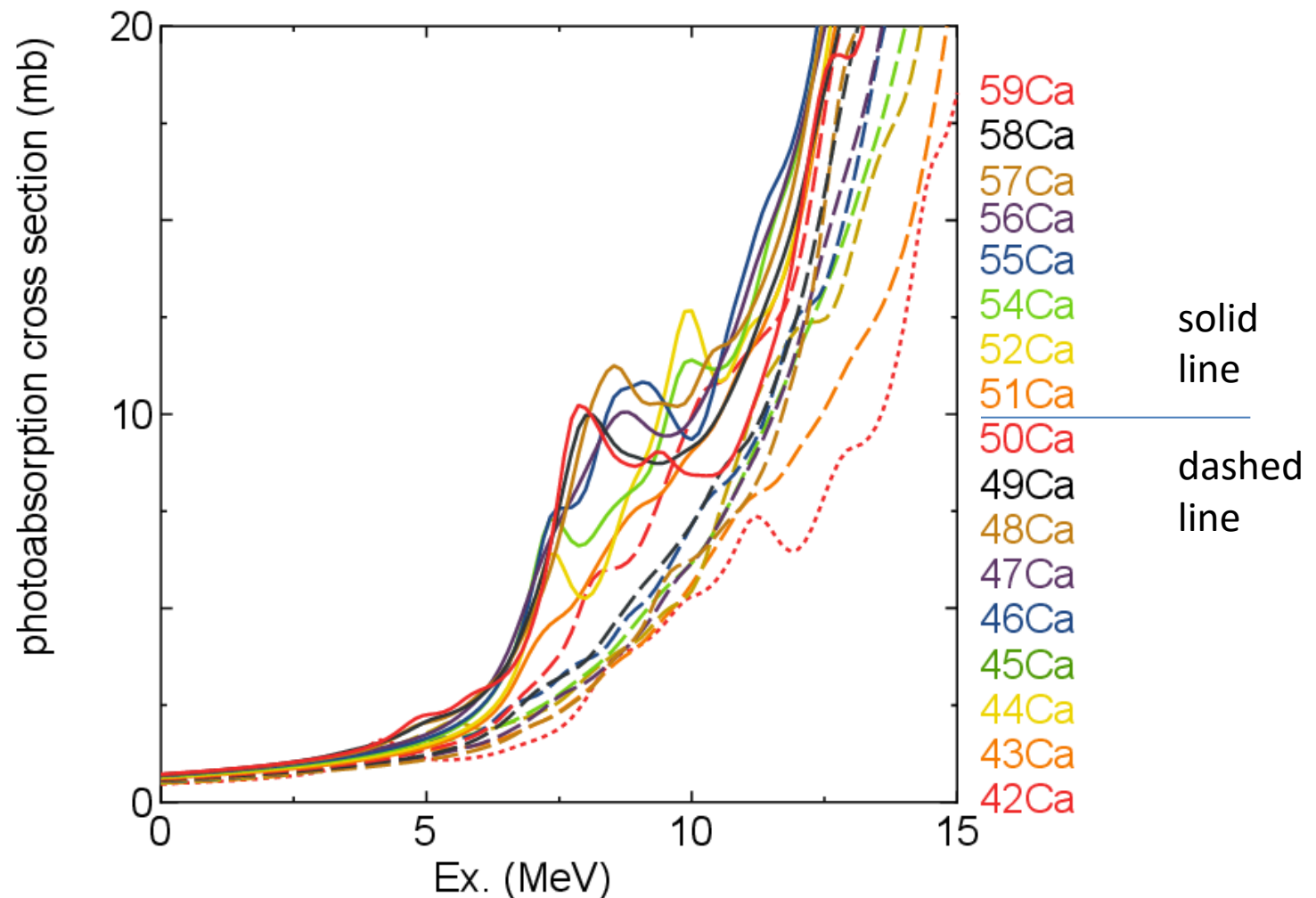
	B(E1) sum
$1\hbar\omega$	16.5
$(1+3)\hbar\omega$	13.6
MCSM 50 dim.	10.1

- GDR peak height is suppressed and improved with increasing ground-state correlation.
- Low-energy tail is almost unchanged.





# Development of pygmy dipole resonance

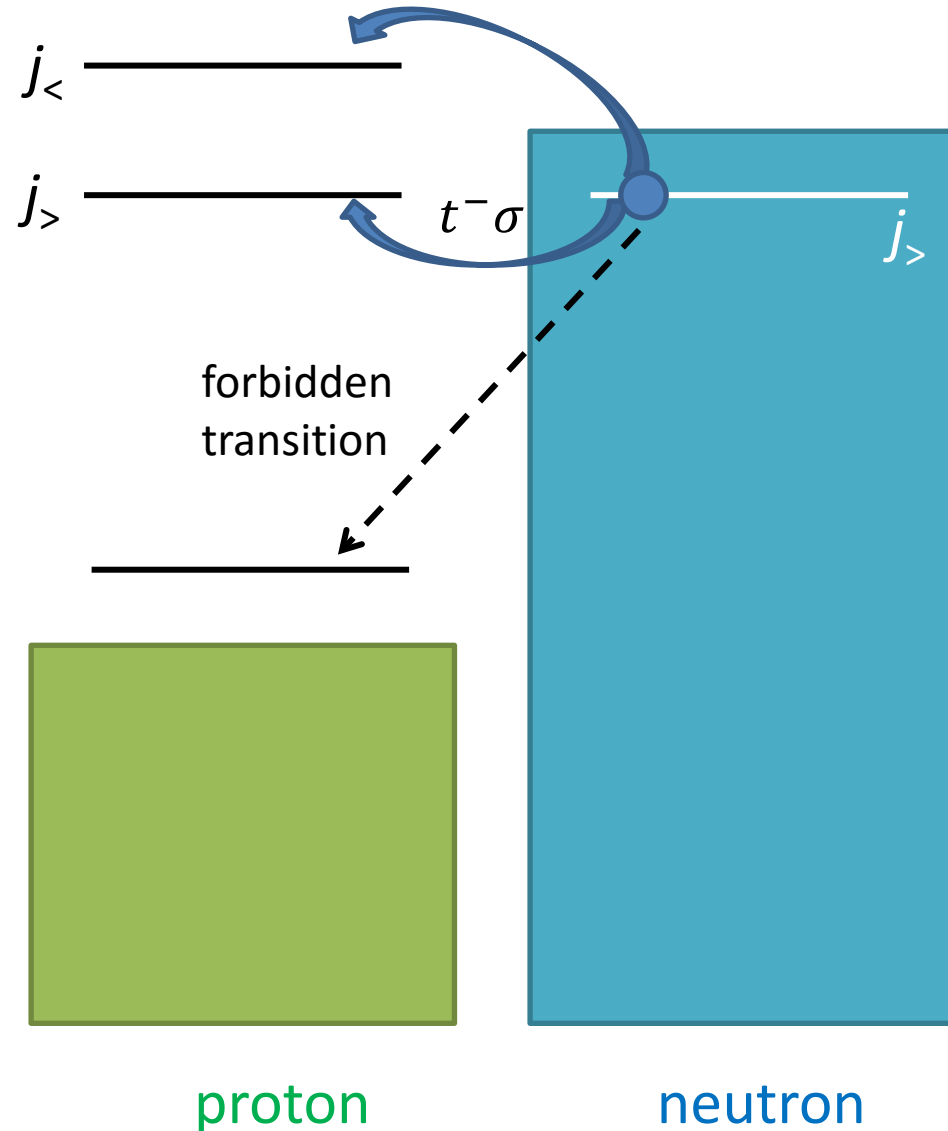


- PDR develops for  $A \geq 50$ , but the tail of GDR makes the peak less pronounced.

# $\beta$ decay

- Describing the Gamow-Teller strength for very neutron-rich nuclei using the shell model is a big challenge because a large model space is required to satisfy the sum rule.

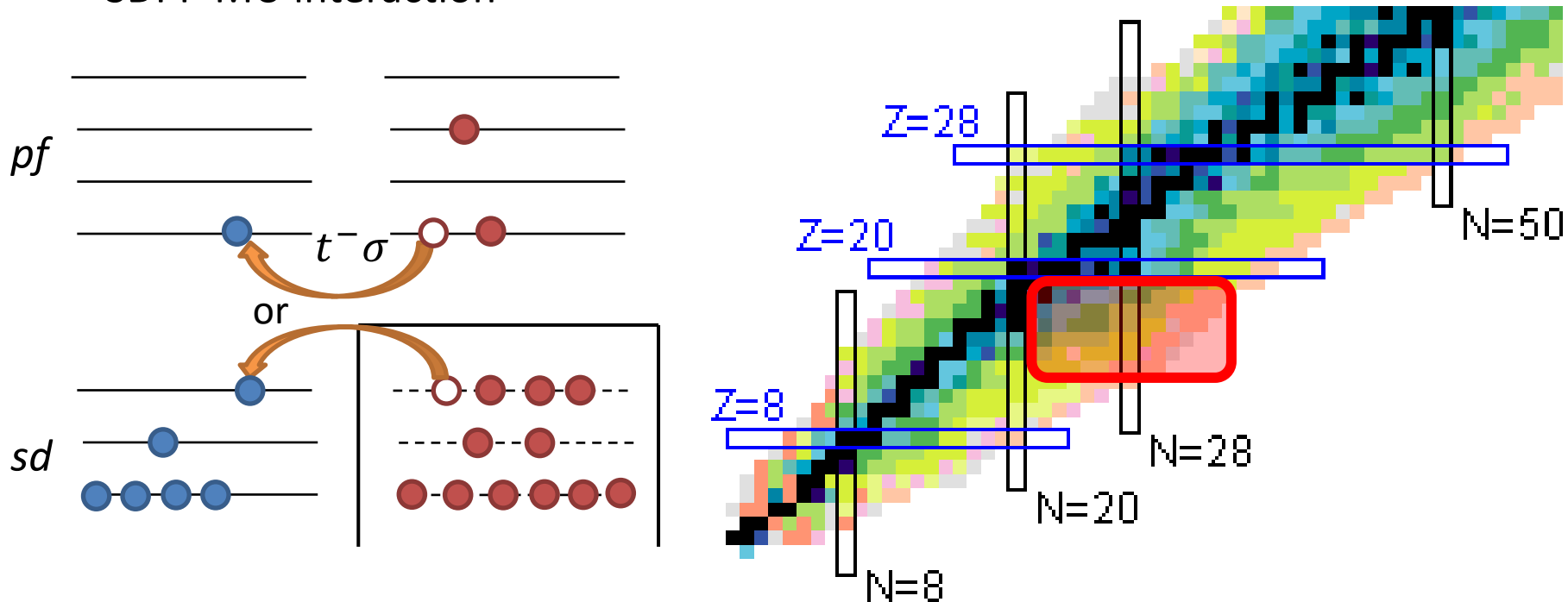
Most of previous shell-model studies were one-major-shell calculations such as the *pf*-shell calc.





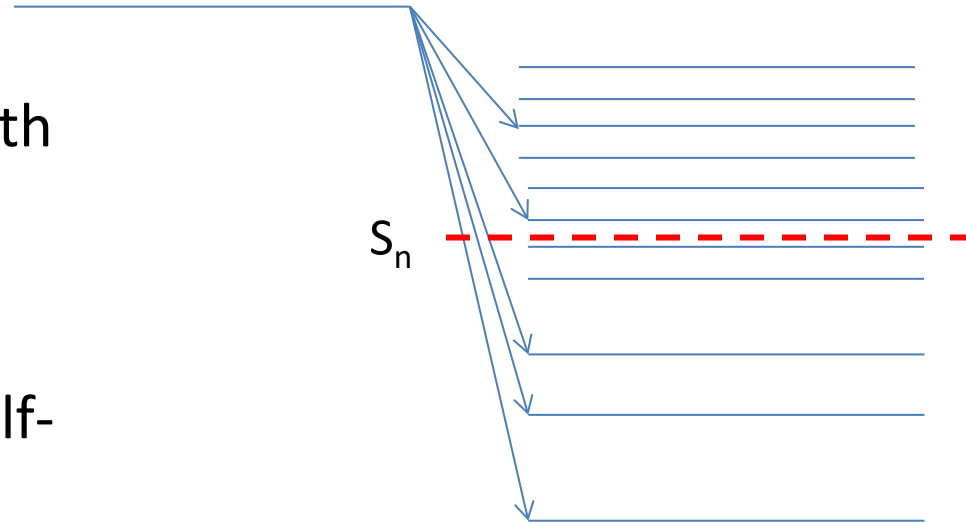
# *sd-pf* case: example of multi major shell

- Calculation for  $Z < 20$ ,  $N > 20$  nuclei
  - Model space:  $0\hbar\omega$  state for the parent state and  $1\hbar\omega$  states in the *sd-pf* shell for the daughter states
    - Satisfying the Ikeda sum rule
    - Applicable to all the nuclei except the “island of inversion”
  - SDPF-MU interaction



# Half lives and delayed neutron probabilities

- $\frac{1}{t_{1/2}} = \sum_i \frac{1}{t_{1/2}(i)}$
- Calculate the GT distribution with the Lanczos strength function method until convergence
- $P_n$  is evaluated by the partial half-lives with  $E_x > S_n$ .

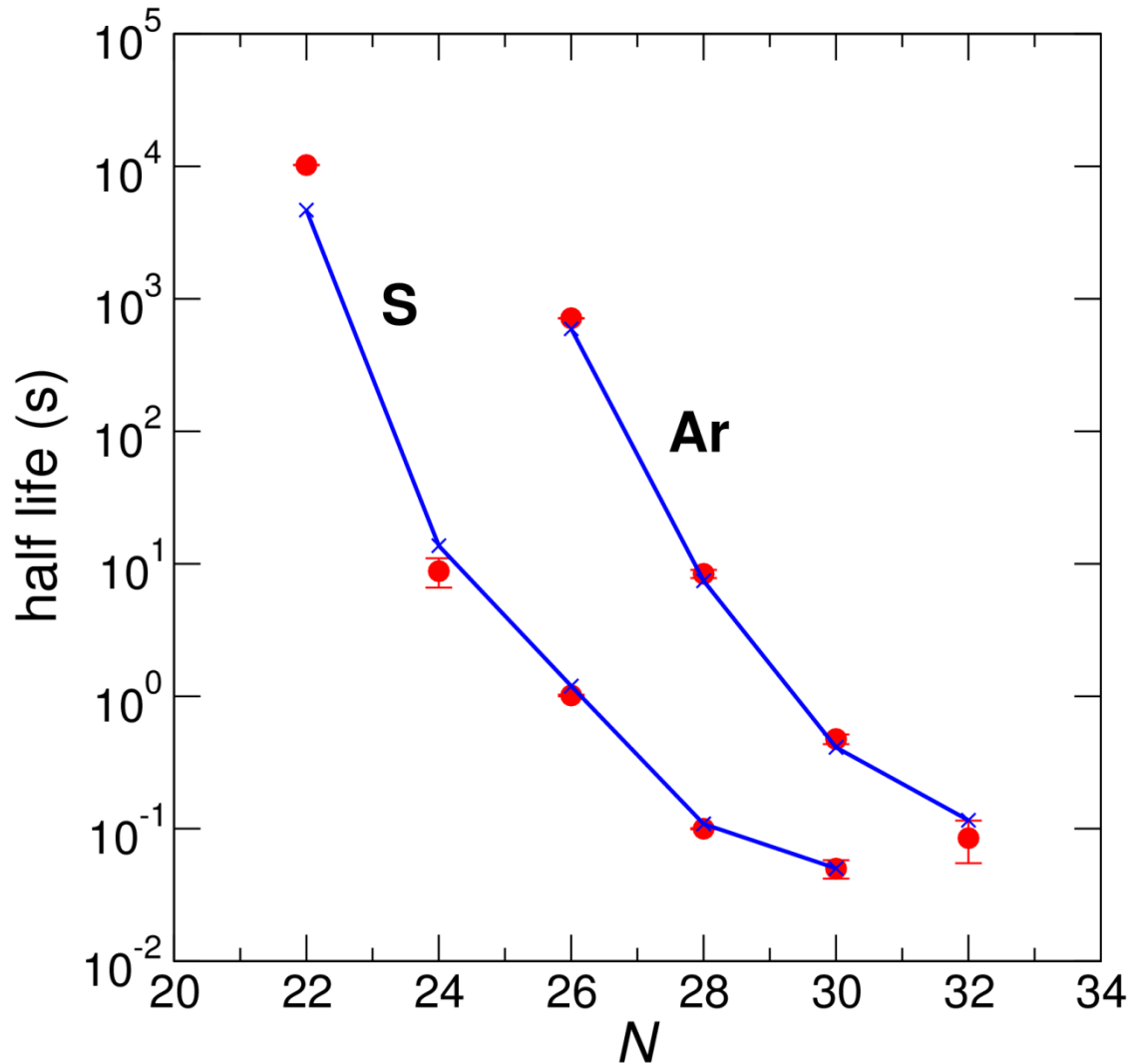


## Comparison with recent data

half lives	RIBF Expt.	Calc.	
$^{37}\text{Al}$	11.5(4) ms	11.0 ms ( $5/2^+$ )	
$^{38}\text{Al}$	9.0(7) ms	8.3 ms ( $5^-$ ), 8.8 ms ( $0^-$ )	quenching factor: 0.77

# Systematics of even-*A* S and Ar isotopes

S. Yoshida et al.



Delayed neutron emission probability ( $P_n$ )

	Expt.	Calc.
<sup>44</sup> S	18(3)%	16%
<sup>50</sup> Ar	35(10)%	48%

quenching factor: 0.77

Q values used: experimental or AME2012 evaluation (<sup>48,50</sup>Ar and <sup>46</sup>S)

# Summary

- Recent development in large-scale shell-model calculations (methodology, computing, effective interaction ...) allows to extend its frontier for heavier nuclei and higher excited states.
- We focus on unnatural-parity states and their  $E1$  and Gamow-Teller strength functions in exotic nuclei in the  $sd$ - $pf$  shell region, which also provide a good testing ground for effective interaction.
- Photonuclear cross sections are well reproduced in stable Ca isotopes, and pygmy dipole resonances are predicted for  $N > 28$ .
- The ground-state correlation works to reduce the  $B(E1)$  sum.
- Half lives and delayed neutron emission probabilities are excellently reproduced for  $N > 20$  exotic nuclei. More systematic calculations will be performed.