中法核工程与技术学院
Institut Franco—Chinois de l'Energie Nucléaire

YITP
YUKAWA INSTITUTE FOR
THEORETICAL PHYSICS

Recent shell model progress on exotic nuclei

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Special thanks for my coming baby because he/she is on time (Oct. 19th).

ADS, HIAF, CSNS,
Daya Bay, Jiangmen neutrino
~20 power plant

Located and managed
by SYSU

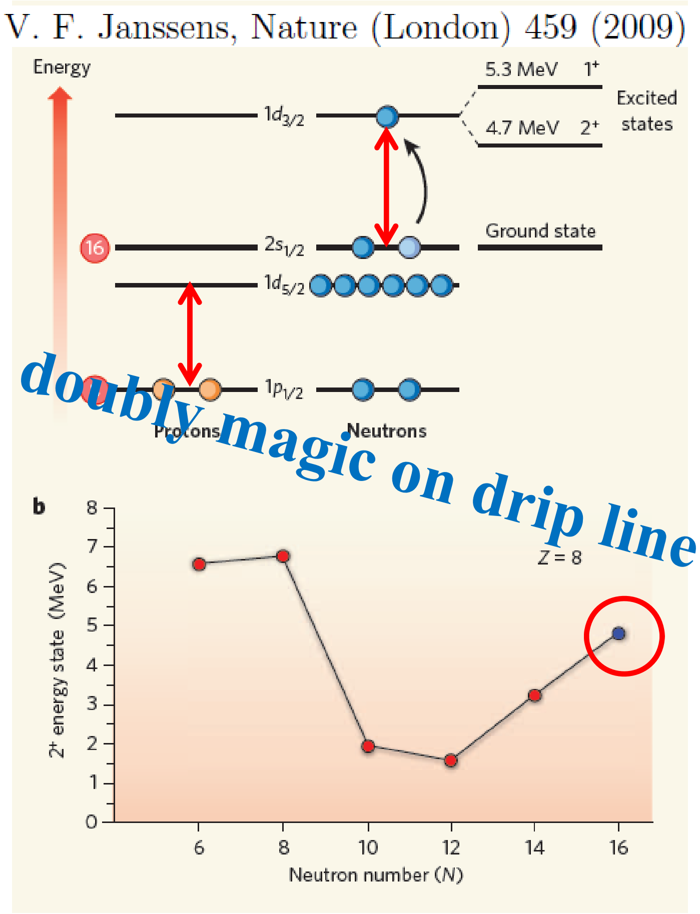
Outline

- Introduction
- Hamiltonian YSOX for *psd* region
  - Drip line of light nuclei
  - Neutron rich carbon isotopes
  - EM properties and GT transition
- Proton-rich nuclei around A=20
- Heavier nuclei (^{132}Sn)
- Summary

Introduction

From stability line to drip line, many exotic properties are found.

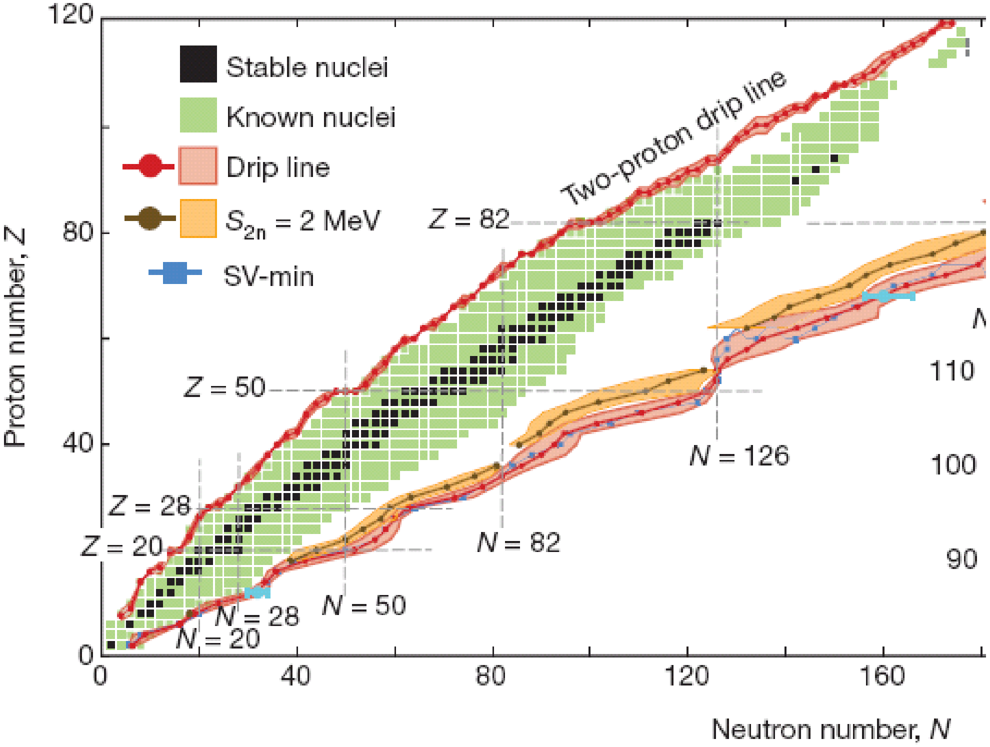
R. V. F. Janssens, Nature (London) 459 (2009) 1069



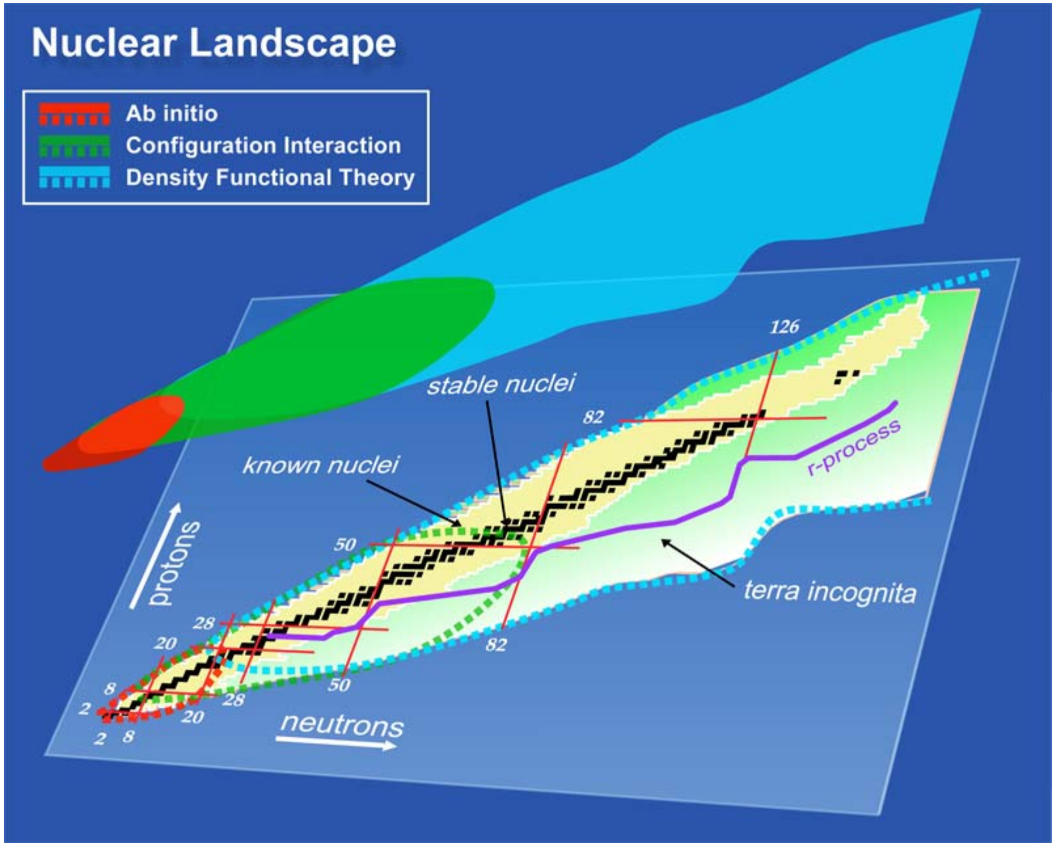
doubly magic on drip line

It is hard to describe both stable and drip line nuclei.

J. Erler *et al.* Nature (London) 486 (2012) 509



The existing shell-model interactions can not describe the drip line of C and O isotopes. Describe both stable nuclei and nuclei far from stability.



Nuclear Models

Ab initio

Mean Field

Shell Model

configurations mixing;

g.s. and excited states are given at the same time;

easy to deal with transitions

Monte Carlo Shell Model for Atomic Nuclei
T. Otsuka, M. Honma, T. Mizusaki, N. Shimizu,
Y. Utsuno, Prog. Part. Nucl. Phys. 47 (2001) 319.

The Nuclear Shell Model Towards the Drip Lines
B.A. Brown, Prog. Part. Nucl. Phys. 47 (2001) 517.

The shell model as a unified view of nuclear structure
E. Caurier, G. Martínez-Pinedo, F. Nowacki, A. Poves,
and A. P. Zuker, Rev. Mod. Phys. 77, 427 (2005).

Single Particle Basis (normally H.O.)

Model Space

Effective Hamiltonian

$$H = \sum_i \epsilon_i n_i + \sum_{i,j,k,l} v_{ij,kl} a_i^\dagger a_j^\dagger a_l a_k,$$

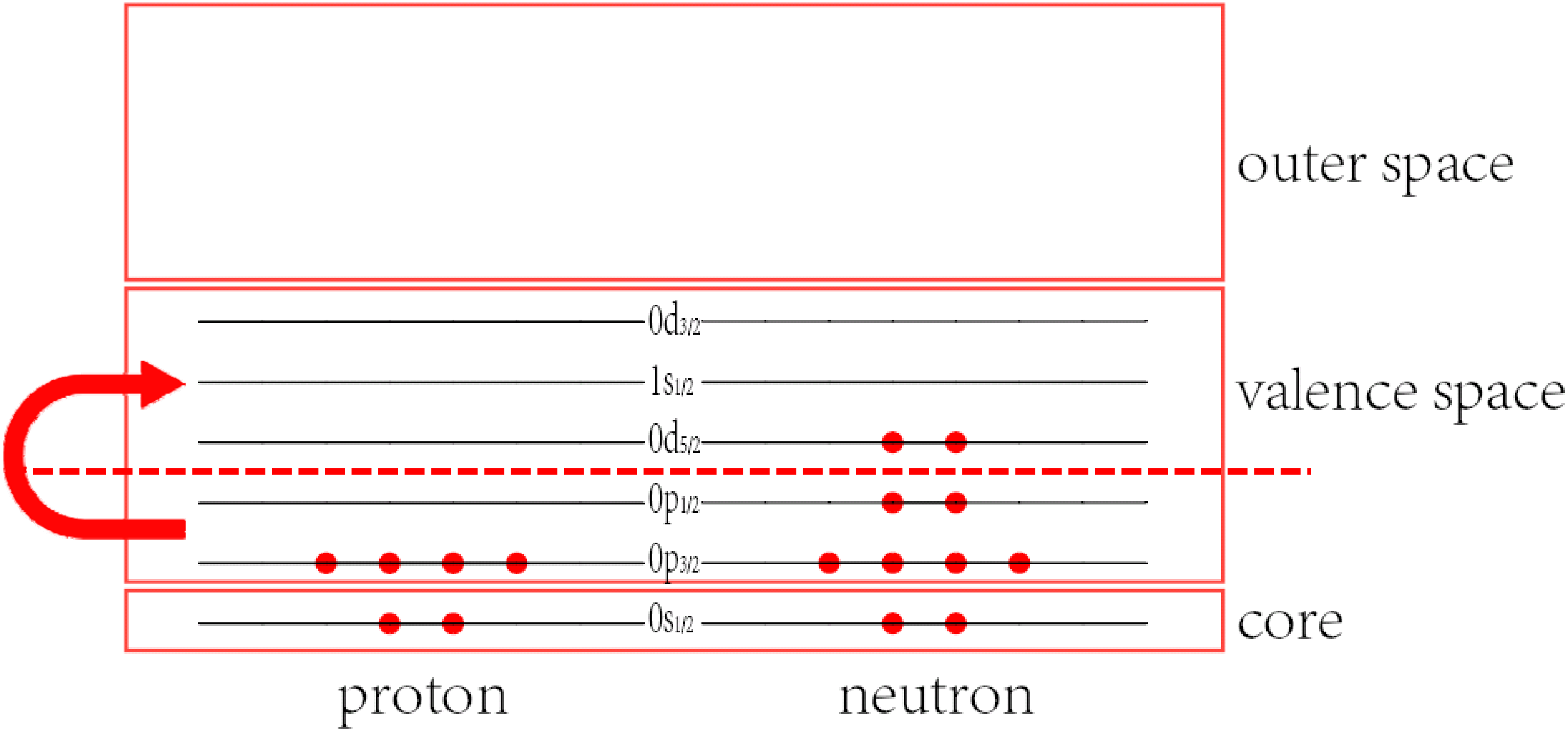
Wave Functions (configuration mixing)

Solve Schrödinger Equation (energies, wave functions)

$$\boldsymbol{H}\Psi = E\Psi \rightarrow \boldsymbol{H}_{\text{eff}}\Psi_{\text{eff}} = E\Psi_{\text{eff}}$$

Other Properties: electromagnetic moments and transitions, beta decay, spectroscopic factors…

Model Space

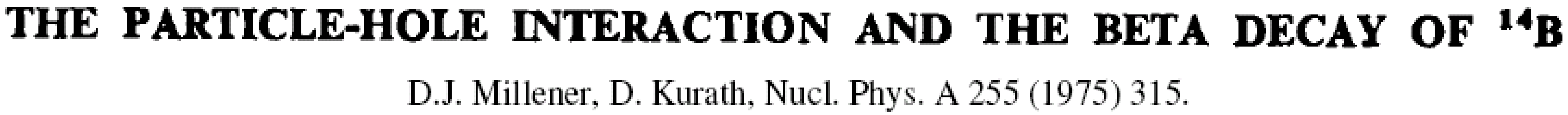


0, 1, 2, 3… nucleons are allowed to be excited from *p* to *sd*.
So called 0hw, 1hw, 2hw, 3hw… model space.
For MK, WBT and WBP, 0-1hw. For present, 0-3hw.

New Hamiltonian YSOX for *psd* region

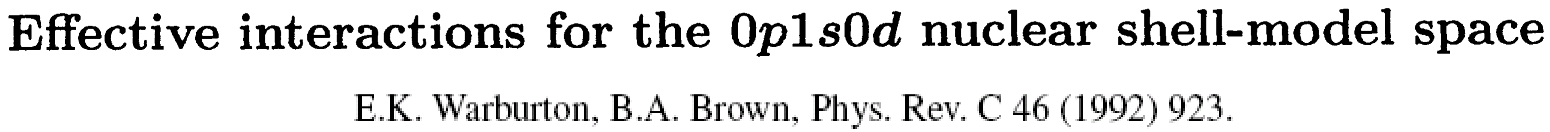
Effective Hamiltonian in *psd* shell

	$\langle pp\vert V\vert pp\rangle$	$\langle sdsd\vert V\vert sdsd\rangle$	$\langle psd\vert V\vert psd\rangle$	$\langle pp\vert V\vert sdsd\rangle$
MK(1975)	fit	fit	fit	not
WBT(1992)	fit	fit	fit	not
WBP(1992)	fit	fit	fit	not



THE PARTICLE-HOLE INTERACTION AND THE BETA DECAY OF ^{14}B

D.J. Millener, D. Kurath, Nucl. Phys. A 255 (1975) 315.



Effective interactions for the 0*p*1*s*0*d* nuclear shell-model space

E.K. Warburton, B.A. Brown, Phys. Rev. C 46 (1992) 923.

One part of the interaction has not been well studied.

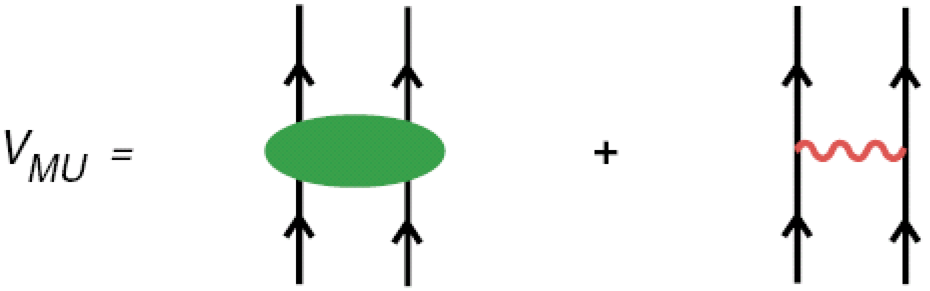
We enlarge the model space and consider the strength of this part of interaction.

- **<pp|V|pp>: SFO** T. Suzuki, R. Fujimoto and T. Otsuka, Phys. Rev. C **67**, 044302 (2003).
- **<sdsd|V|sdsd>: SDPF-M** Y. Utsuno. T. Otsuka. T. Mizusaki, and M. Honma, Phys Rev. C **60**, 054315 (1999).
- **<psd|V|psd>: V_{MU} (0.85central)+LS (M3Y)**
- **<pp|V|sdsd>: V_{MU} (0.55central)+LS (M3Y)**

(a) central force :
Gaussian
(strongly renormalized)

(b) tensor force :
π + ρ meson
exchange

spin-orbit force (M3Y)
Yukawa type



+ $$V = \sum_i V_i Y(r_{12}/R_i) L \cdot S,$$

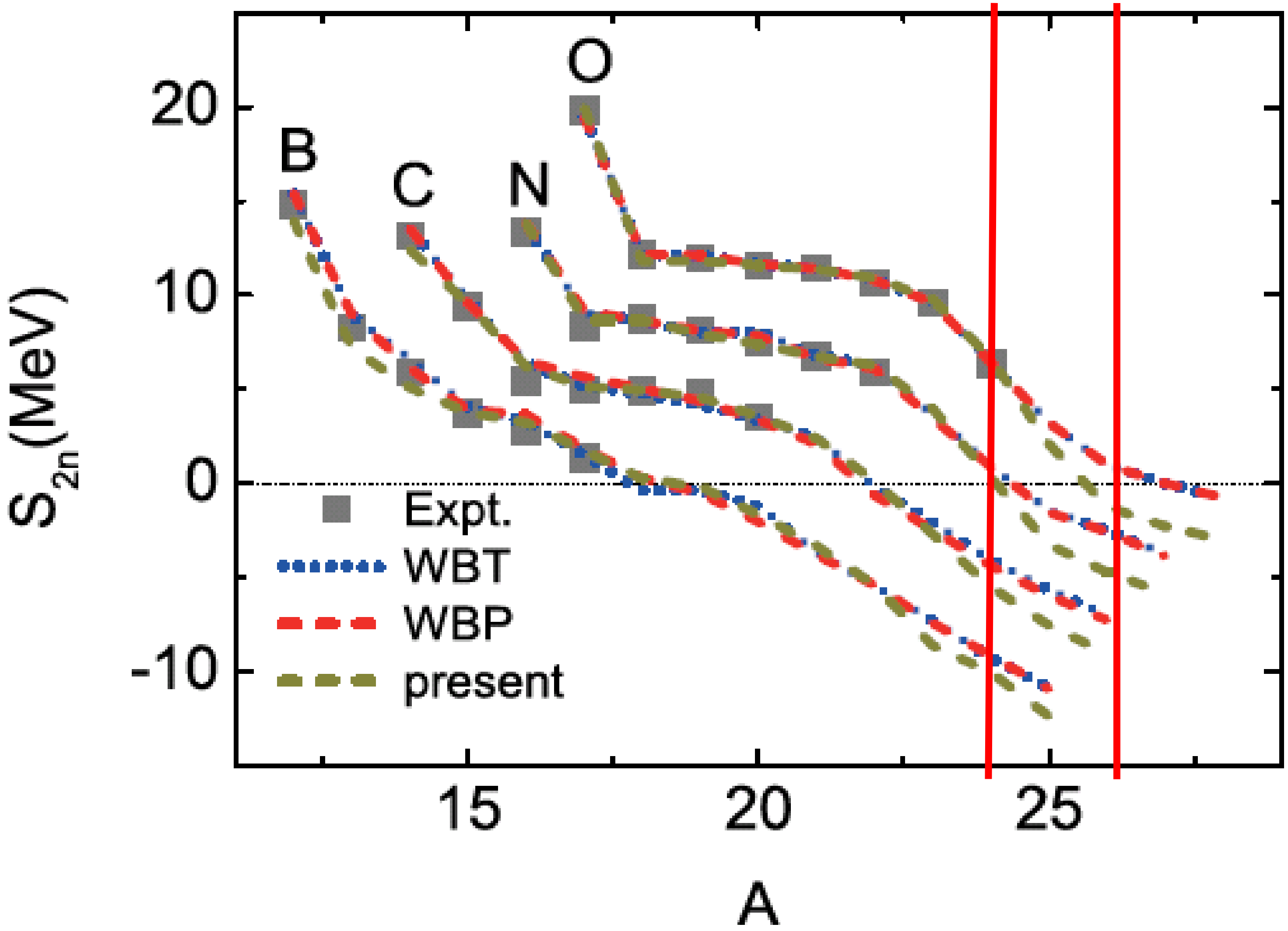
FIG. 2 (color online). Diagrams for the V_{MU} interaction.

T. Otsuka *et al.* PRL 104(2010)012501

G. Bertsch, *et al.* NPA 284(1977)399

- **Present LS and Tensor are determined in free space.**
  We keep them unchanged to use in *psd* region.

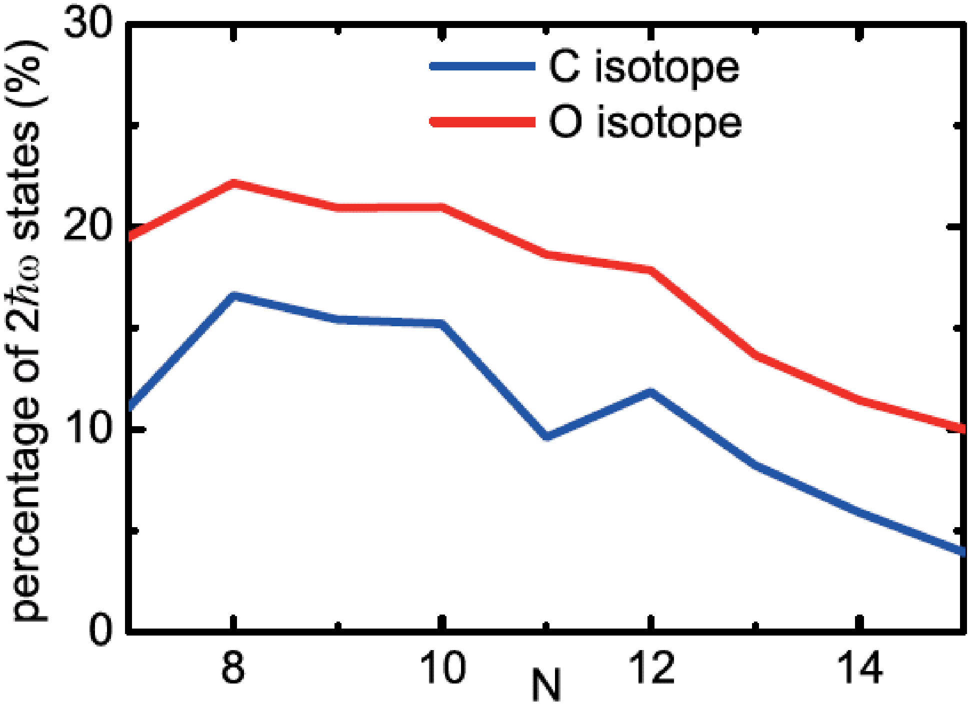
Drip line of light nuclei



CXY, T. Suzuki, T. Otsuka, F.R. Xu, N. Tsunoda, PRC 85 (2012) 064324

2hw vs Mass Dependence

- **Present** WBT, WBP
- **Ψ=aΨ(0hw)+ bΨ (2hw)** $V(A)=(18/A)^{1/3}$ V

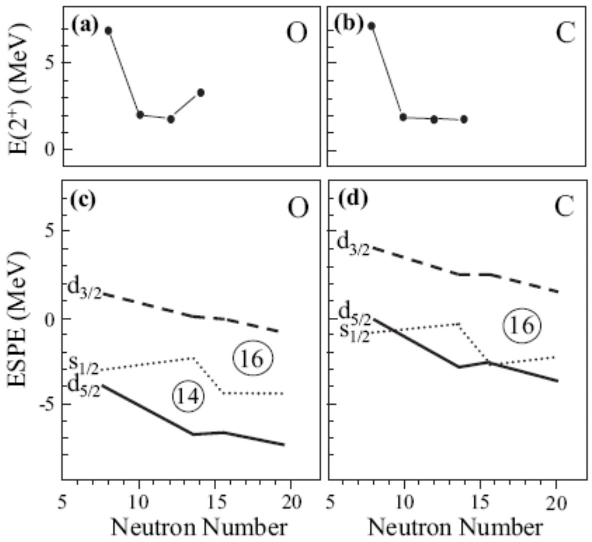


We enlarge the model space, mass dependent term is partially included automatically.

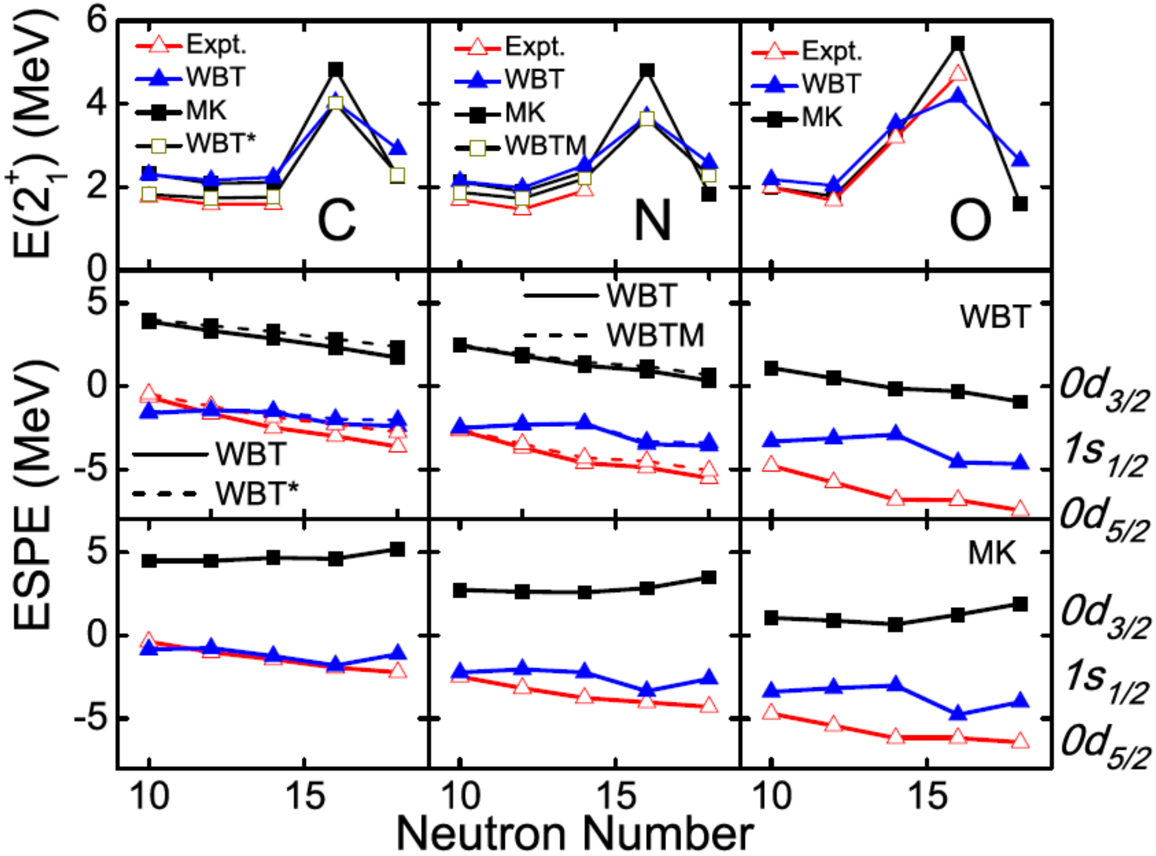
CXY, T. Suzuki, T. Otsuka, F.R. Xu, N. Tsunoda, PRC 85 (2012) 064324

Neutron rich carbon isotopes

M. Stanoiu *et al.*, Phys. Rev. C **78**, 034315 (2008)



CXY, C. Qi, F.R. Xu, NPA 883 (2012) 25

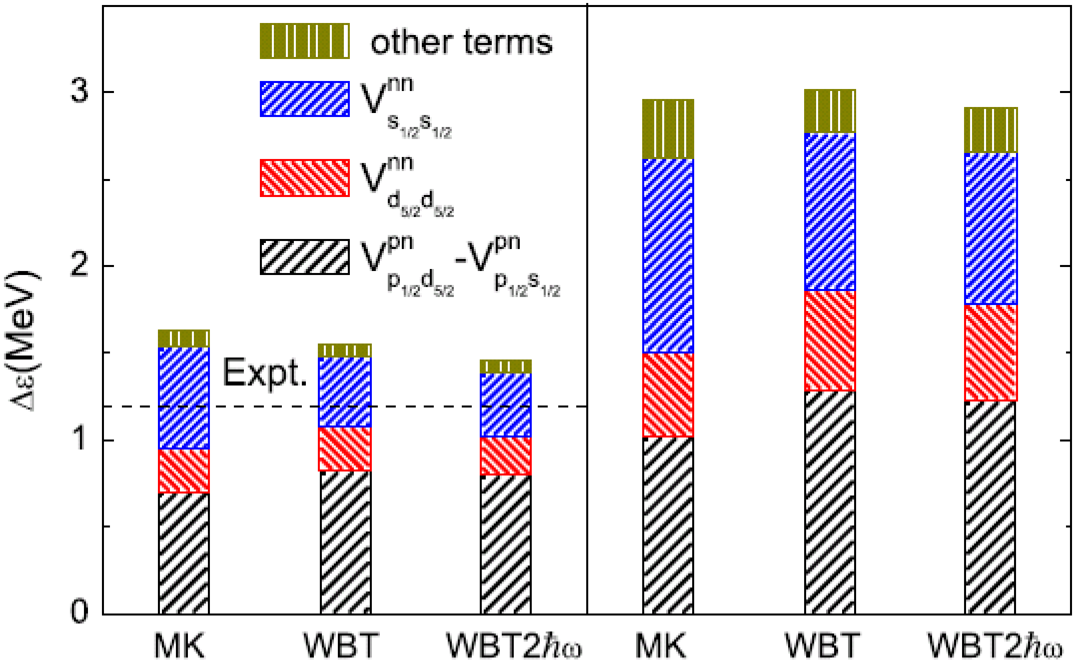


- **From O to C, two protons are removed, proton-neutron interaction contribute to shell evolution**

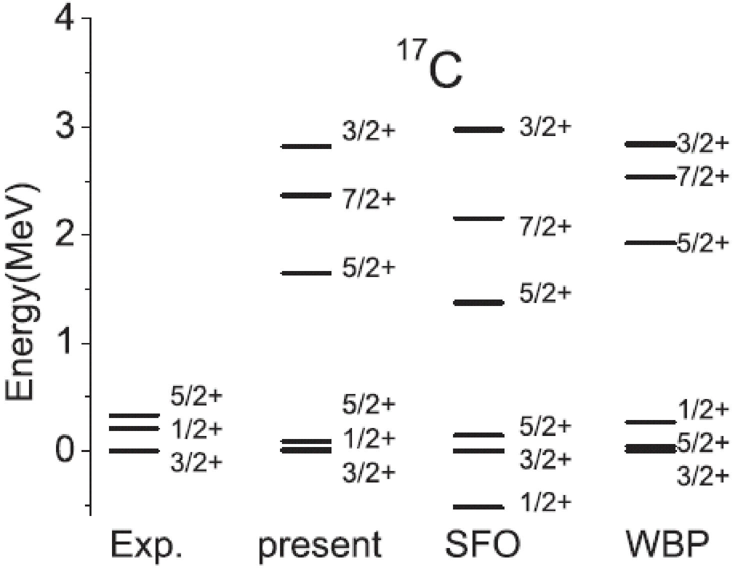
Neutron-Neutron Interaction in Shell Evolution

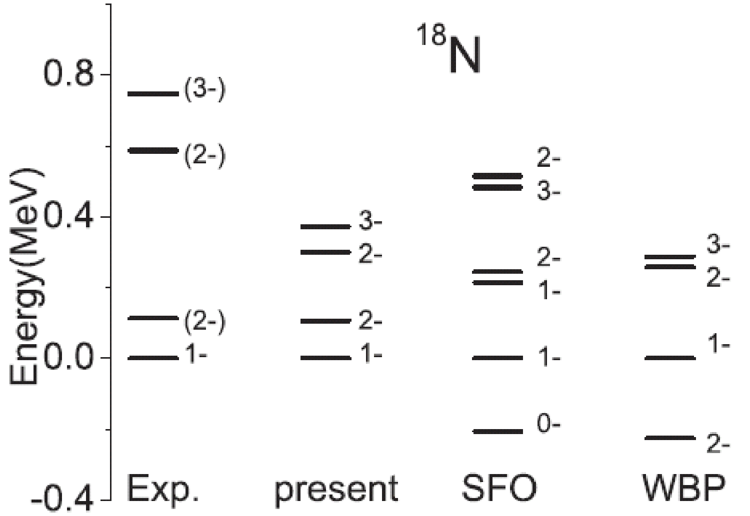
$N = 14$ gap difference between

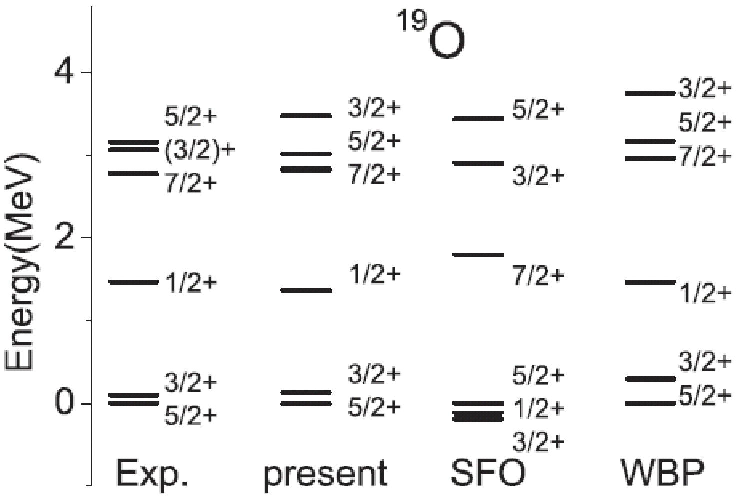
^{22}O and ^{21}N ^{22}O and ^{20}C

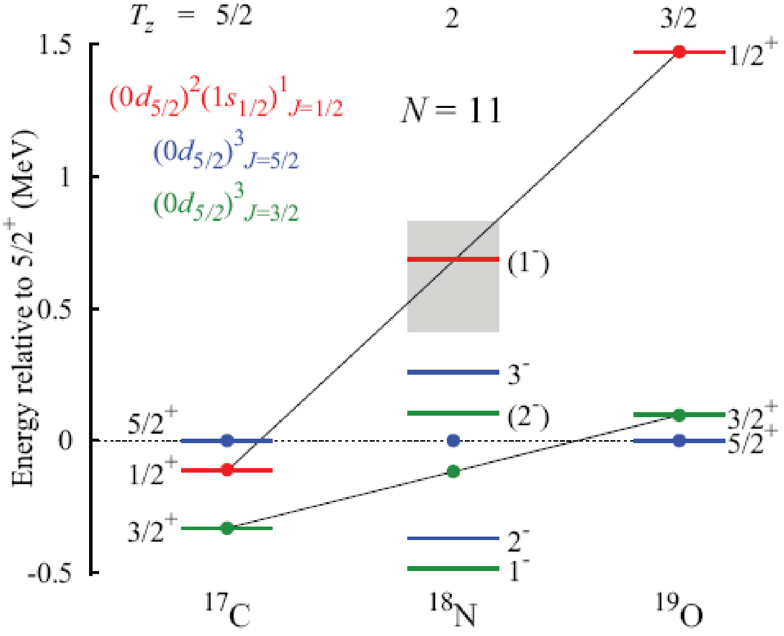


CXY, C. Qi, F.R. Xu, NPA 883 (2012) 25

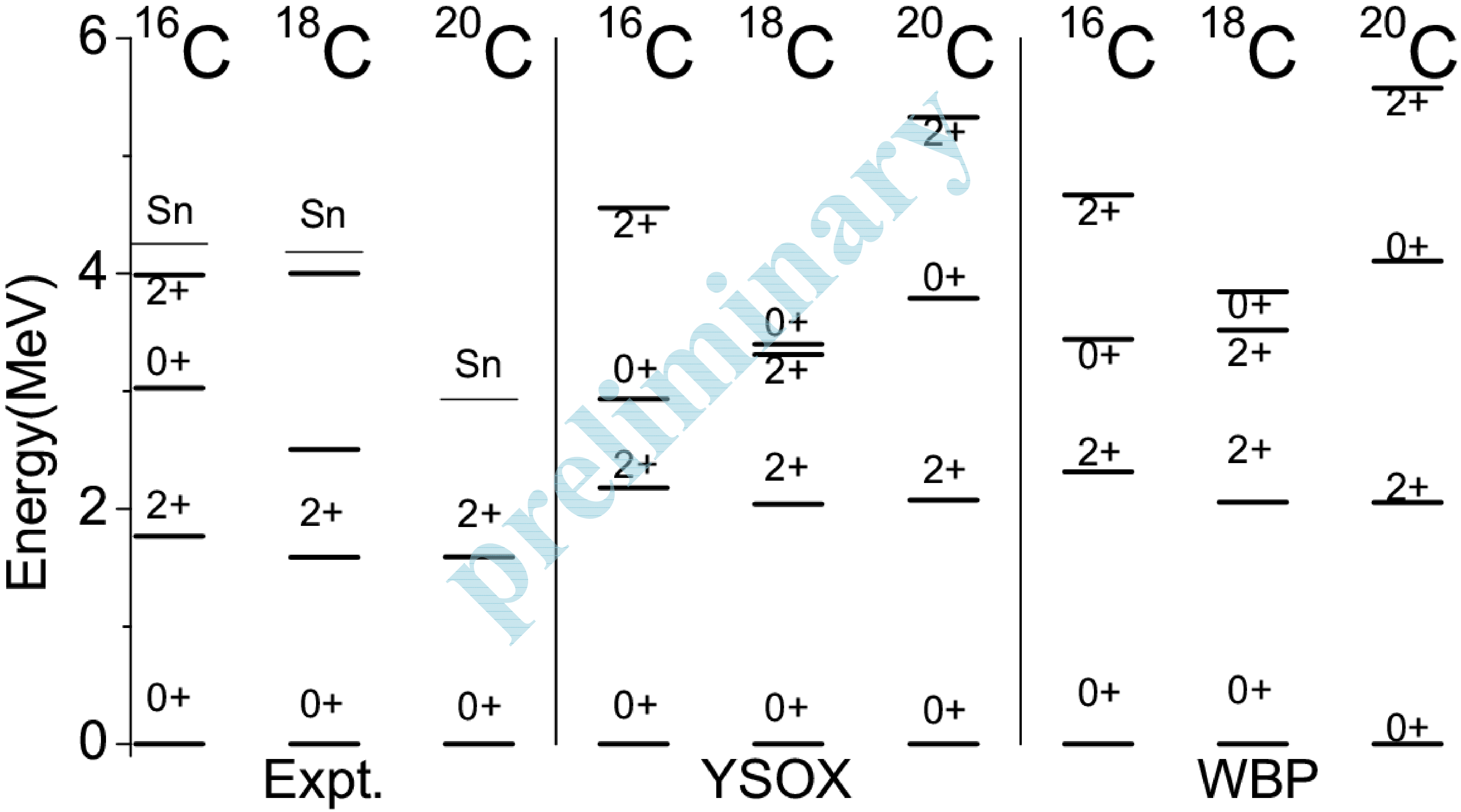




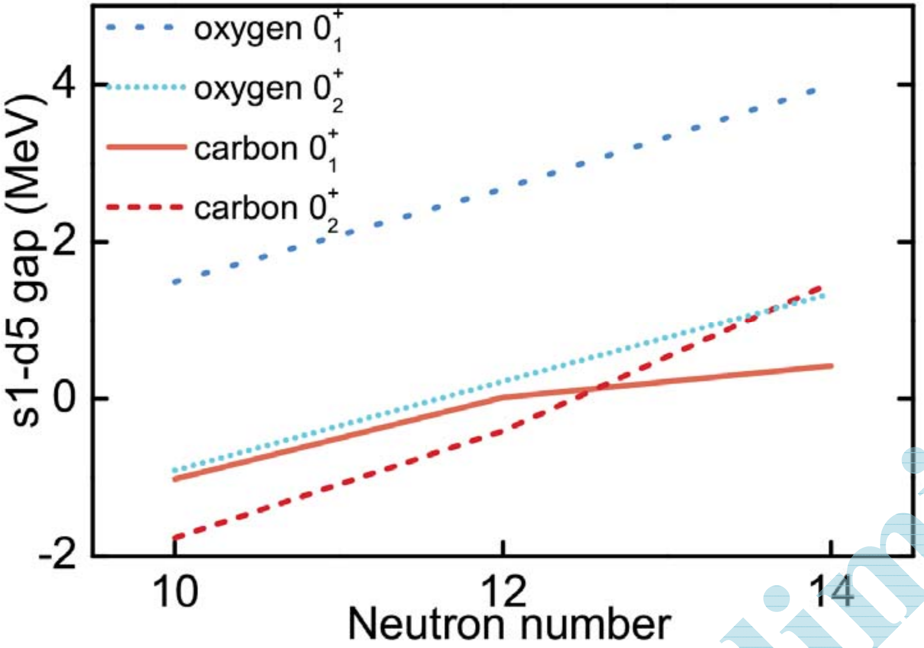


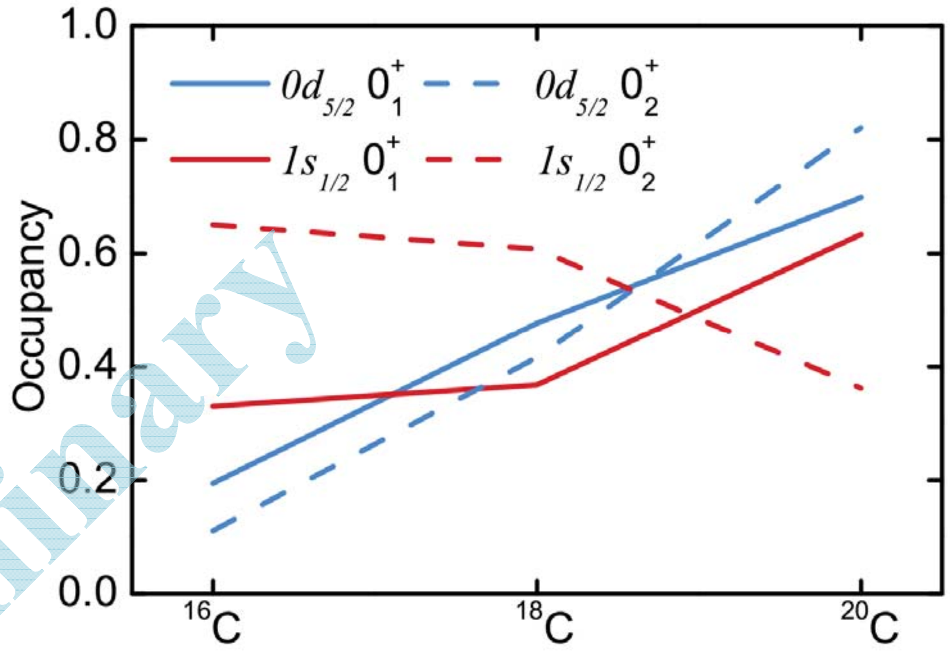


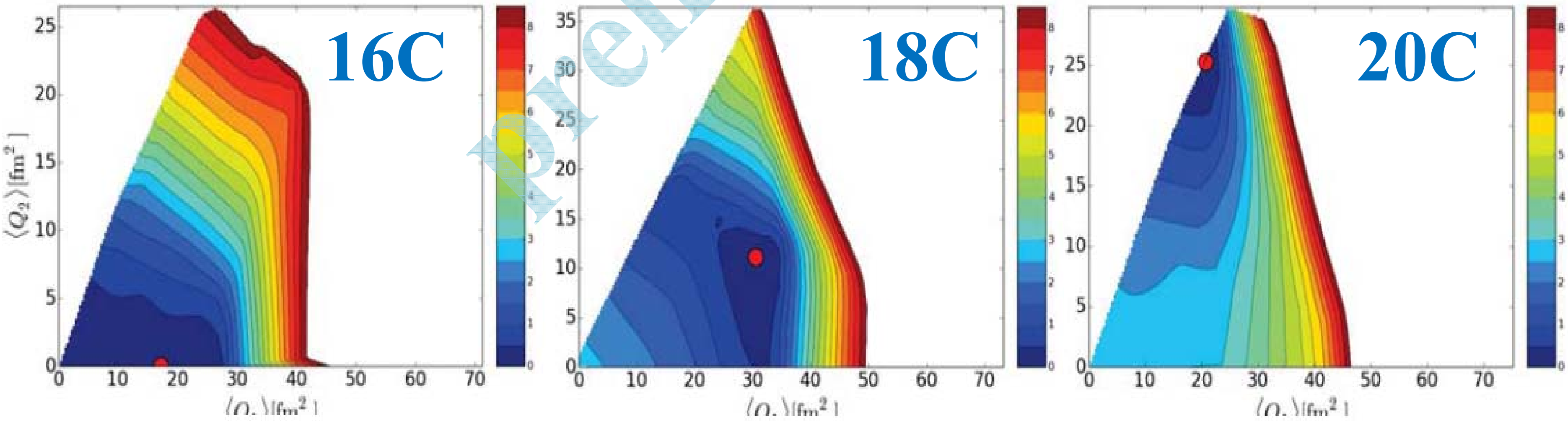
CXY, T. Suzuki, T. Otsuka, F.R. Xu, N. Tsunoda, PRC 85 (2012) 064324
C.R. Hoffman et al. PRC 88 (2013) 044317



CXY, T. Suzuki, N. Shimizu, *et al.* in preparation







g.s. and the second 0+ of neutron rich carbon isotopes

CXY, T. Suzuki, N. Shimizu, *et al.* in preparation

Deformation from B(E2) and Q

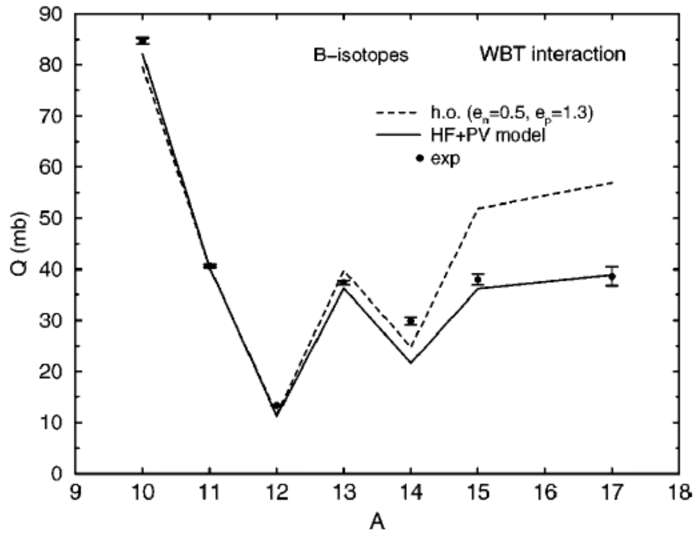
Nuclei	$J_i^\pi \to J_f^\pi$	A_p	A_n	$B(E2)_{YSOX}$	A_p	A_n	$B(E2)_{WBP}$	$B(E2)_{Expt.}$
^{16}C	$0_1^+ \to 2_1^+$	-1.25	-9.39	14.046	-1.53	-9.21	16.514	20.75
	$0_1^+ \to 2_2^+$	-1.63	0.32	3.979	1.78	-0.81	4.293	
	$0_2^+ \to 2_1^+$	0.36	-1.04	0.050	-0.38	1.35	0.030	
	$0_2^+ \to 2_2^+$	1.35	4.77	7.884	1.34	4.91	7.995	
	$2_1^+ \to 2_2^+$	0.05	-0.56	0.001	-0.10	0.63	0.000	
^{18}C	$0_1^+ \to 2_1^+$	-1.67	-11.01	21.688	1.88	11.09	24.437	21.50
	$0_1^+ \to 2_2^+$	1.77	3.46	9.272	-2.02	-3.15	10.789	
	$0_2^+ \to 2_1^+$	0.24	1.20	0.336	0.21	0.84	0.212	
	$0_2^+ \to 2_2^+$	0.97	4.19	4.838	1.09	4.22	5.531	
	$2_1^+ \to 2_2^+$	-2.40	-6.98	4.319	-2.72	-6.63	4.954	
^{20}C	$0_1^+ \to 2_1^+$	-2.94	-11.51	40.654	3.32	11.64	47.469	37.50
	$0_1^+ \to 2_2^+$	-0.27	0.42	0.059	-0.25	0.49	0.041	
	$0_2^+ \to 2_1^+$	0.16	0.72	0.139	-0.21	-1.31	0.329	
	$0_2^+ \to 2_2^+$	1.09	5.59	7.095	1.19	5.37	7.511	
	$2_1^+ \to 2_2^+$	-0.48	-1.85	0.214	0.40	1.19	0.123	

Nuclei	$J_i^\pi \to J_f^\pi$	A_p	A_n	$Q_{0,YSOX}$	A_p	A_n	$Q_{0,WBP}$	$Q_{0,Expt.}$
^{16}C	2_1^+	-1.30	-10.86	11.01	-1.06	-10.47	9.97	
	2_2^+	1.57	7.60	-9.93	1.37	7.17	-9.00	
^{18}C	2_1^+	-1.41	-11.99	12.07	-1.37	-12.37	12.18	
	2_2^+	1.45	11.26	-11.75	1.67	11.63	-12.73	
^{18}C	2_1^+	3.44	13.57	-19.88	4.00	13.74	-21.85	
	2_2^+	-1.15	-9.80	9.84	-1.45	-9.88	10.92	

CXY, T. Suzuki, N. Shimizu, *et al.* in preparation

EM properties and GT transition

E2 Properties

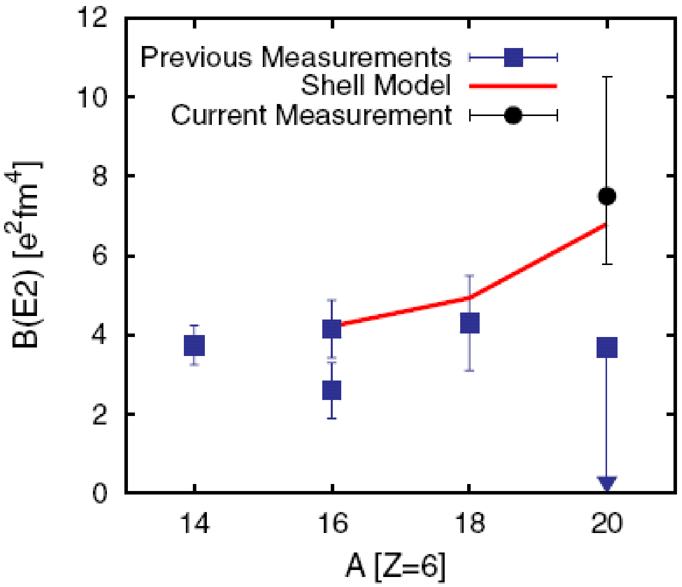


H. Sagawa and K. Asahi, Phys. Rev. C **63**, 064310 (2001).

$$e_{pol}(\mathrm{PV})/e = a\frac{Z}{A} + b\frac{N-Z}{A} + \left(c + d\frac{Z}{A}\frac{N-Z}{A}\right)\tau_z,$$

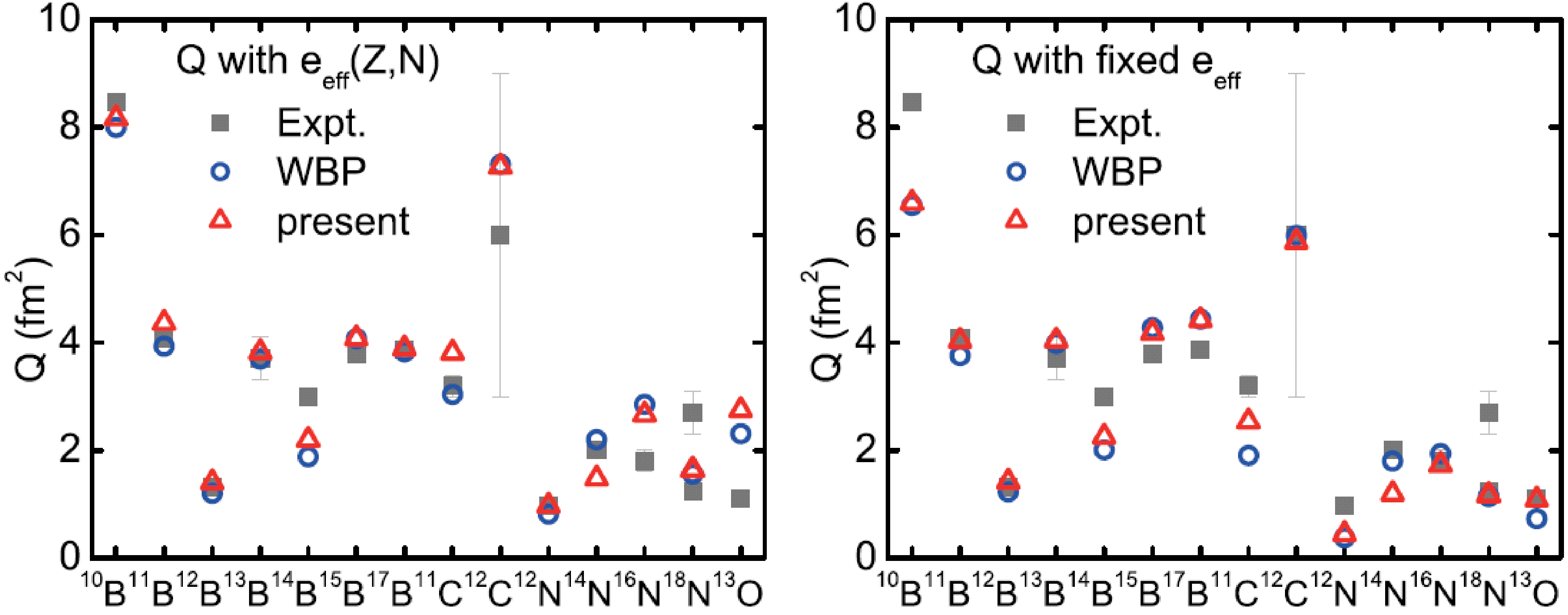
$$\tau_z = 1\ (-1)\ \text{for}\ \nu(\pi),$$

H. Sagawa *et al.*, Phys. Rev. C 70 (2004) 054316.



- **Is such e_{eff} suitable for present H?**
- **Can we use a constant e_{eff} ?**

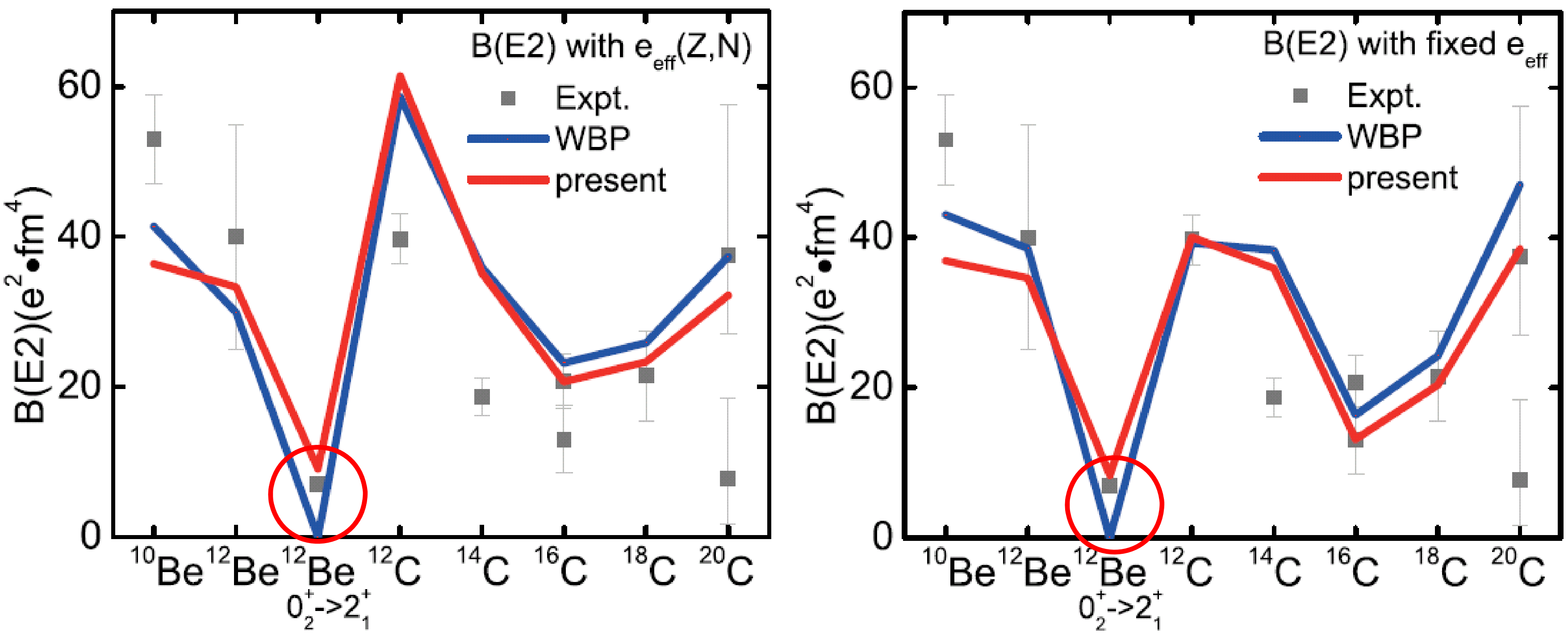
M. Petri, *et al.*, Phys. Rev. Lett. 107 (2011) 102501.



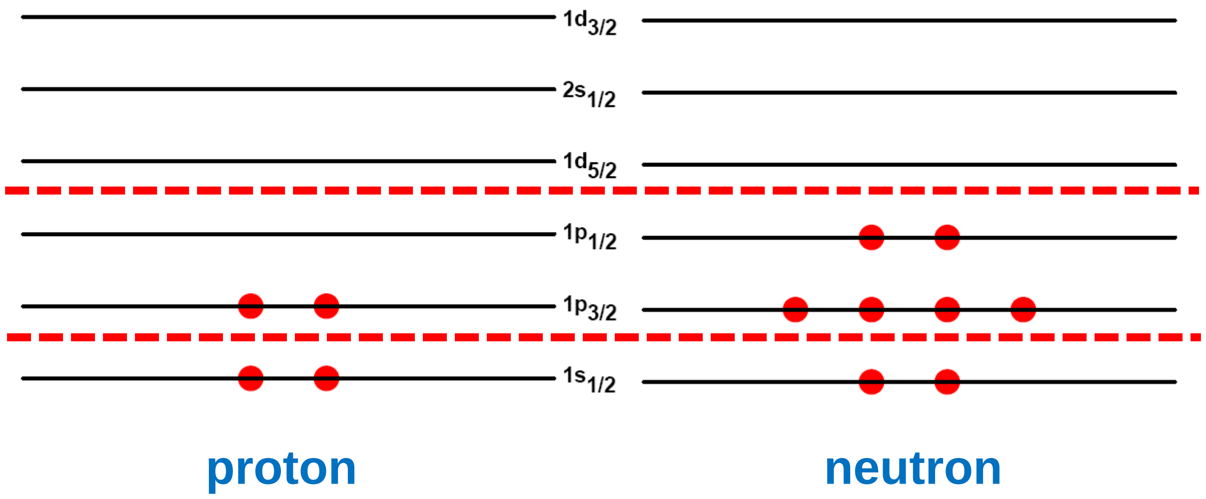
- **Z, N dependent e_{eff} is suitable for present H.**
- **A fixed e_{eff} also works well.**

e_p=1.27, e_n=0.23

CXY, T. Suzuki, T. Otsuka, F.R. Xu, N. Tsunoda, PRC 85 (2012) 064324



^{12}Be



WBP: all block

Present: allow 2 nucleons excited to *sd*

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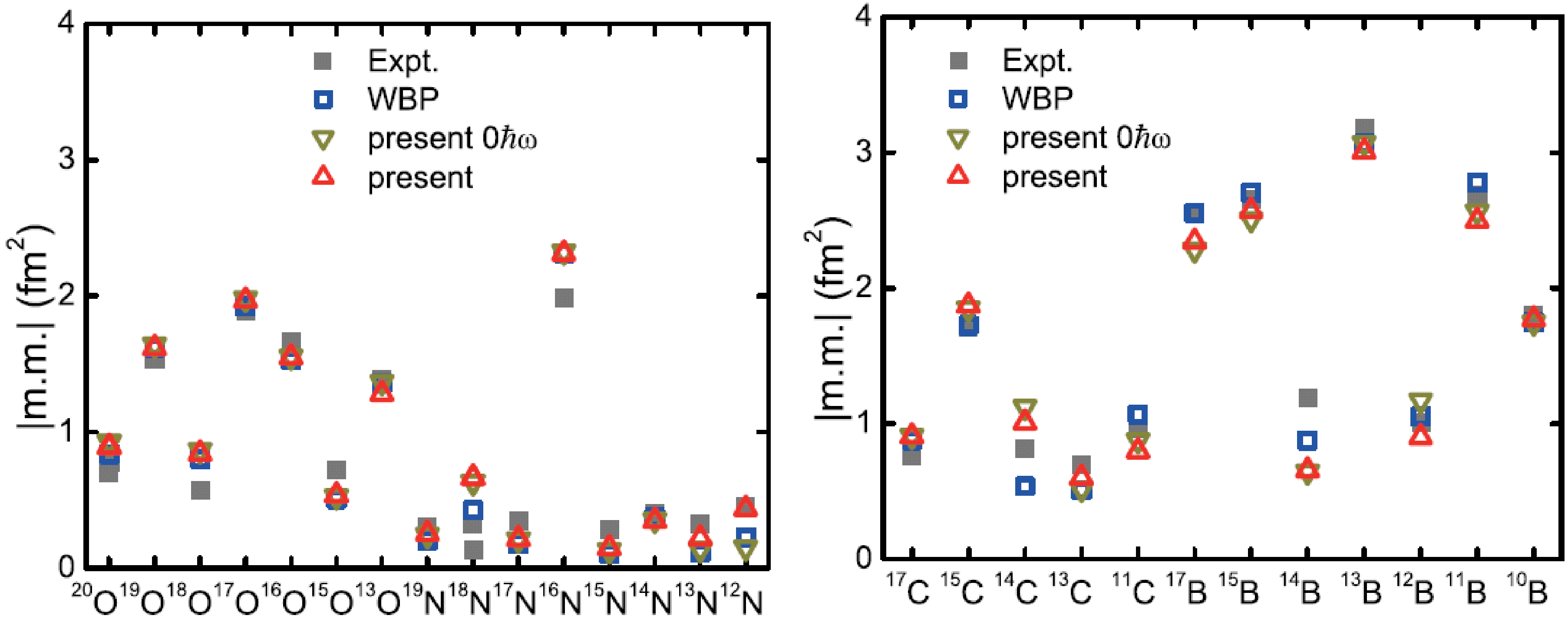
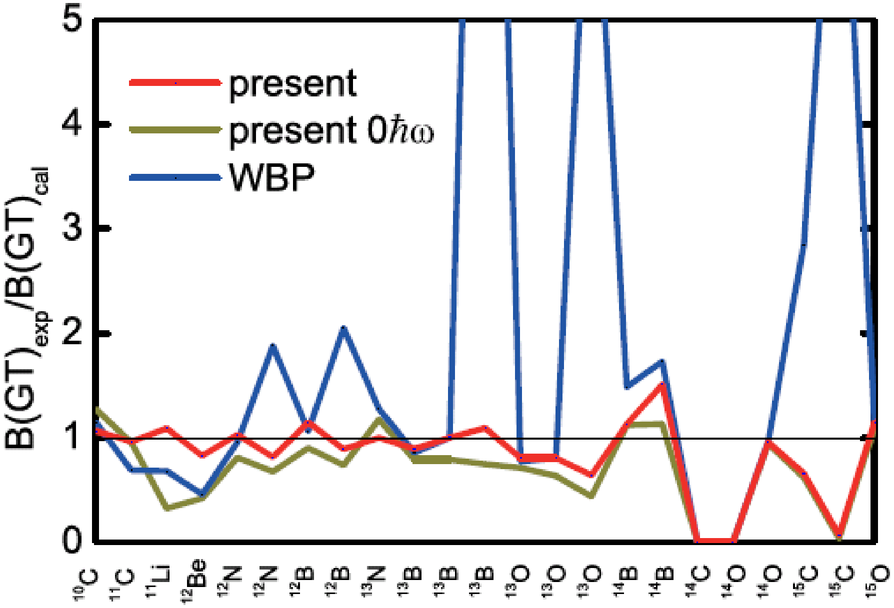
Magnetic Moments

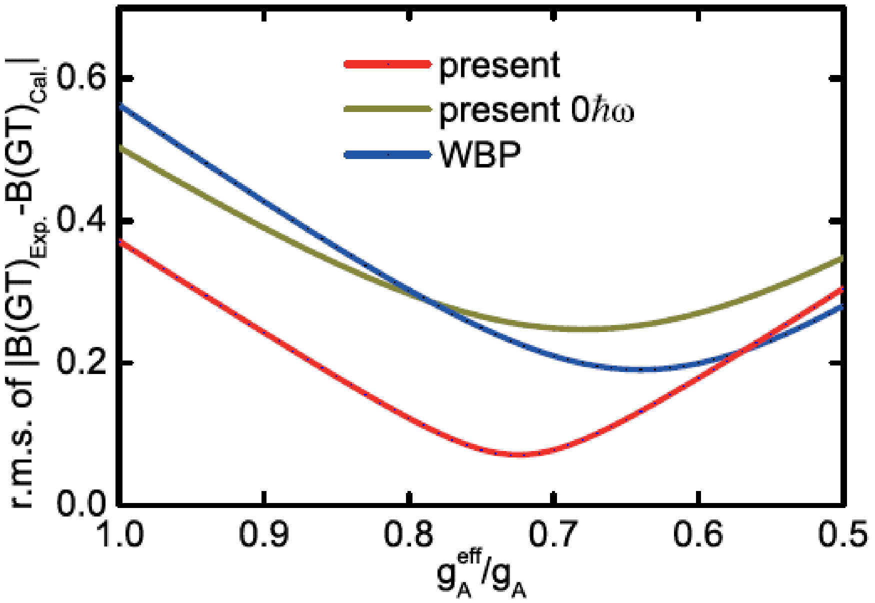


CXY, T. Suzuki, T. Otsuka, F.R. Xu, N. Tsunoda, PRC 85 (2012) 064324

Gamow-Teller Transitions

$B(GT)_{exp}/B(GT)_{cal}$





CXY, T. Suzuki, T. Otsuka, F.R. Xu, N. Tsunoda, PRC 85 (2012) 064324

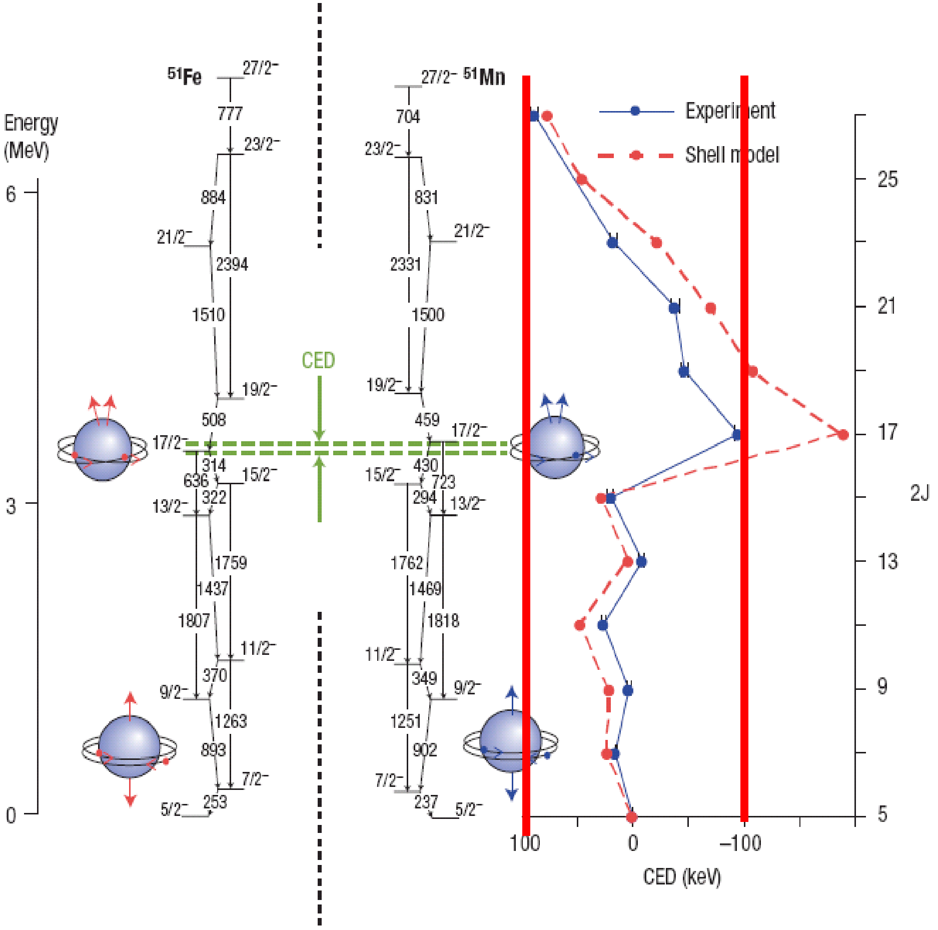
^{14}C $x|^1S> + y|^3P>$ $J=0, T=1$ **^{14}N** $a|^3S> + b|^1P> + c|^3D>$ $J=1, T=0$

B(GT) $(xa - yb/\sqrt{3})\sqrt{6}$ **10^{-7}**

I. Talmi, Fifty Years of the Shell Model-The Quest for the Effective Interaction, Adv. Nucl. Phys. **27**, 1 (2003).

Proton-rich nuclei around A=20

Understand proton-rich nuclei form their mirror nuclei



D. D. Warner, M. A. Bentley, and P. Van Isacker, Nat. Phys. **2**, 311 (2006).

- **Mirror energy difference, normally 0.1MeV caused by isospin asymmetric term in NN interaction. Shell model Hamiltonian is normally isospin symmetric.**

Weakly bound effect

- **In some case, the mirror energy difference is large (around 1MeV), caused by weakly bound effect.**

1, **The shift of single particle energies**

R. G. Thomas, Phys Rev. C **88**, 1109 (1952).

J. B. Ehrman, Phys Rev. C **81**, 412 (1951).

2, The modification of the residual interaction.

K. Ogawa, H. Nakada, S. Hino, and R. Motegi, Phys. Lett. **B464**, 157 (1999).

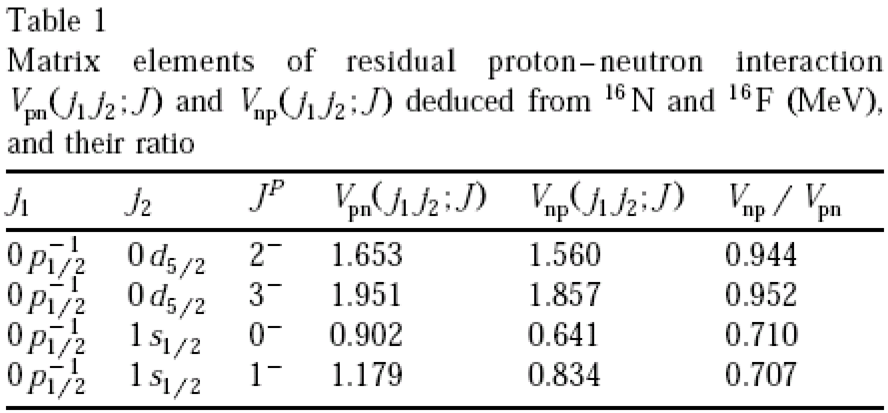


Table 1
Matrix elements of residual proton–neutron interaction $V_{pn}(j_1 j_2; J)$ and $V_{np}(j_1 j_2; J)$ deduced from ^{16}N and ^{16}F (MeV), and their ratio

j_1	j_2	J^P	$V_{pn}(j_1 j_2; J)$	$V_{np}(j_1 j_2; J)$	V_{np} / V_{pn}
$0\,p^{-1}_{1/2}$	$0\,d_{5/2}$	2^-	1.653	1.560	0.944
$0\,p^{-1}_{1/2}$	$0\,d_{5/2}$	3^-	1.951	1.857	0.952
$0\,p^{-1}_{1/2}$	$1\,s_{1/2}$	0^-	0.902	0.641	0.710
$0\,p^{-1}_{1/2}$	$1\,s_{1/2}$	1^-	1.179	0.834	0.707

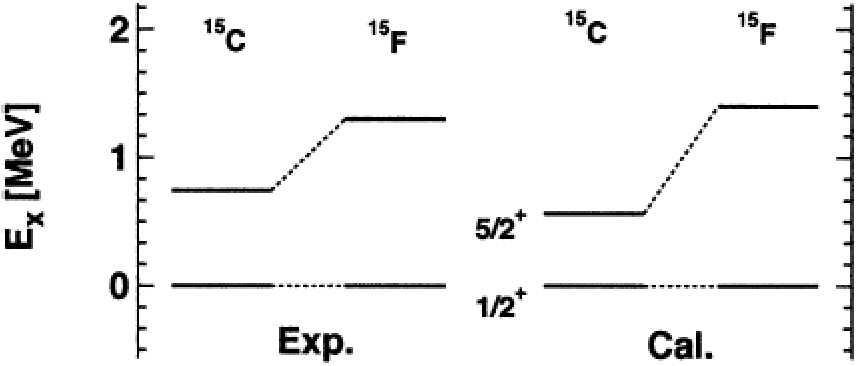
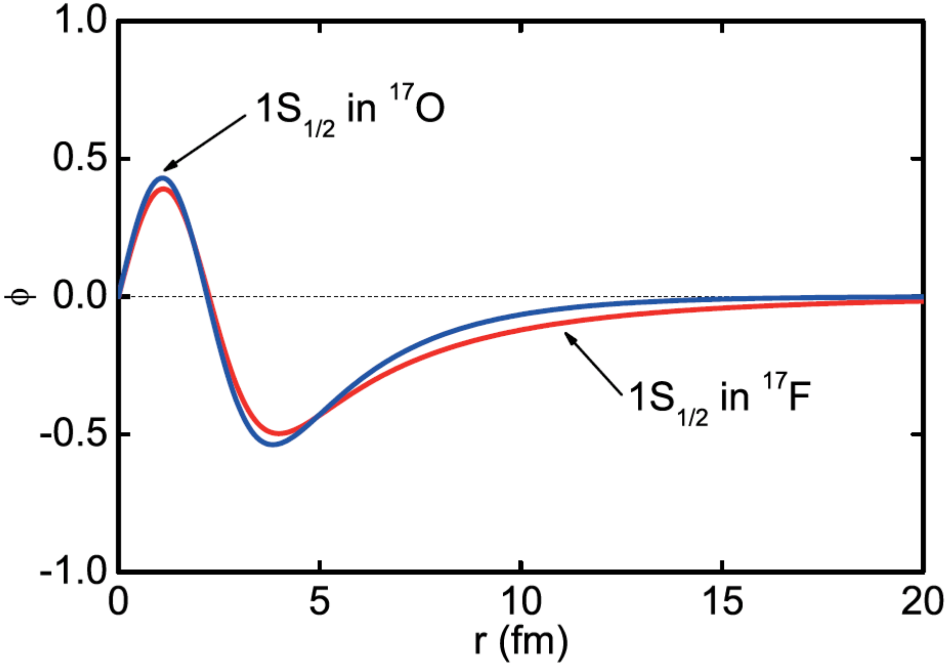


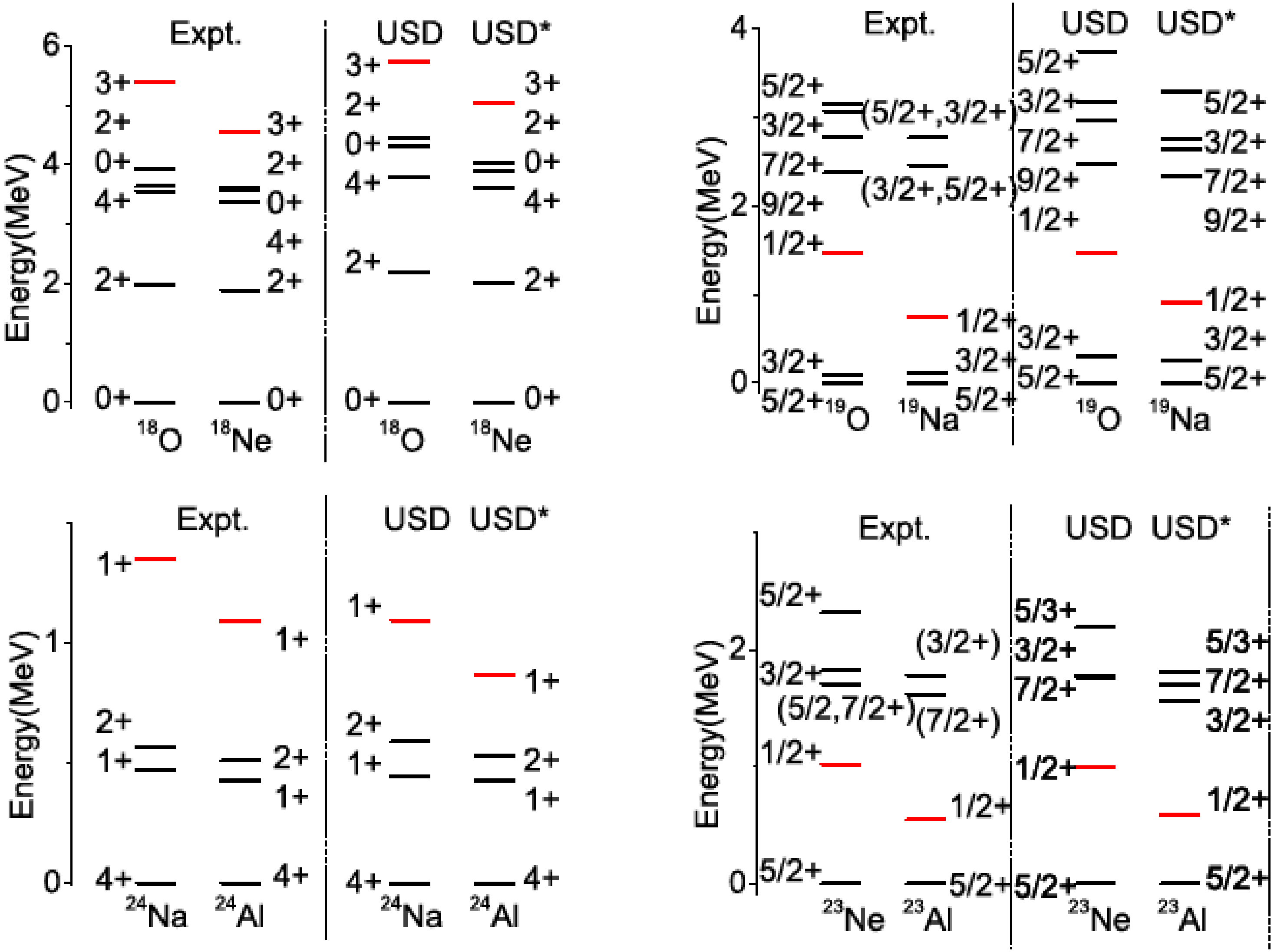
Fig. 1. Experimental and calculated energy spectra of the ^{15}C–^{15}F mirror nuclei.

deduce TBME ratio from experimental values and M3Y interaction. But did not apply in shell model calculations.

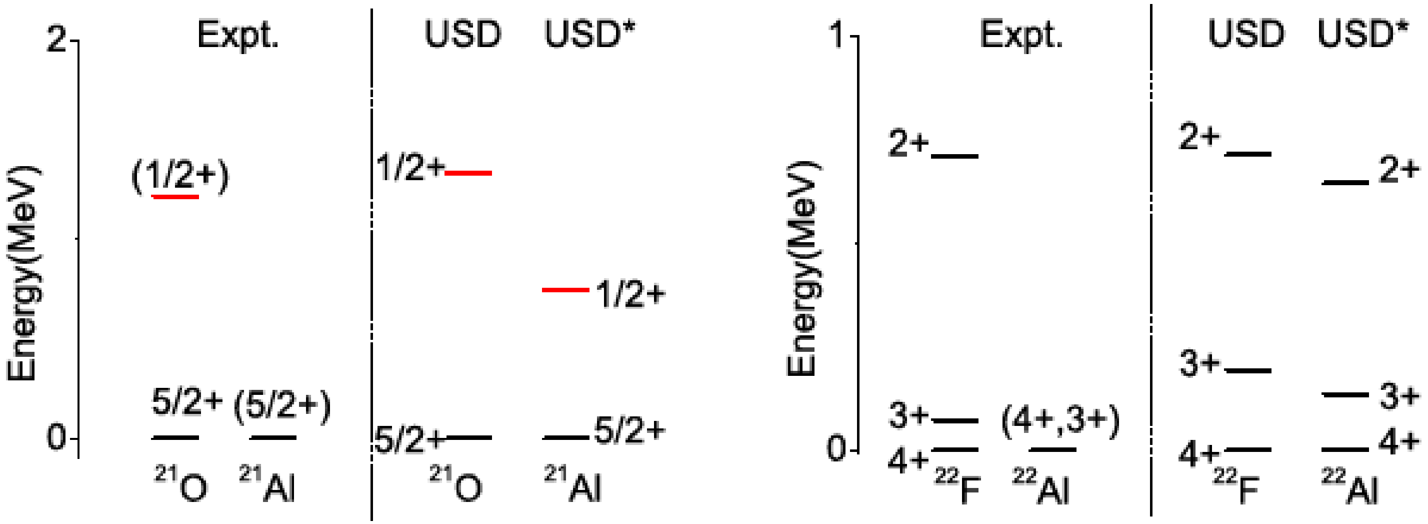
- **Including the weakly bound effect, we calculated the TBME ratio from V_{MU}+LS in WS basis. Then modify on the USD family.**

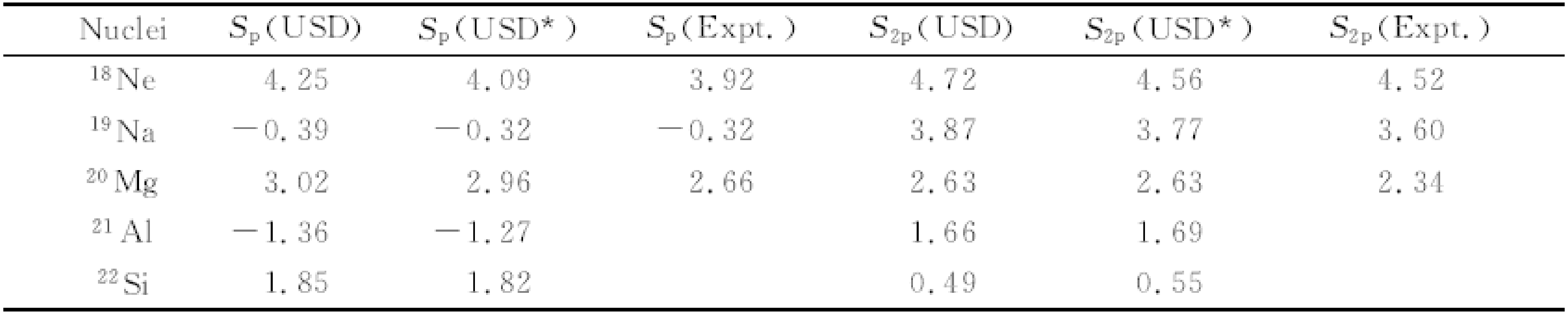


TBME ($\langle ij\|V\|kl\rangle^{pp}_{JT}$)	Reduction factor
$\langle(1s_{1/2})^2\|V\|(1s_{1/2})^2\rangle^{pp}_{01}$	0.68
$\langle 1s_{1/2}0d_{5/2}\|V\|1s_{1/2}0d_{5/2}\rangle^{pp}_{31}$	0.78
$\langle 1s_{1/2}0d_{5/2}\|V\|1s_{1/2}0d_{5/2}\rangle^{pp}_{21}$	0.84
$\langle(0d_{5/2})^2\|V\|(1s_{1/2})^2\rangle^{pp}_{01}$	0.80
$\langle 1s_{1/2}0d_{5/2}\|V\|(0d_{5/2})^2\rangle^{pp}_{21}$	0.87



CXY, C. Qi, F.R. Xu, T. Suzuki, T. Otsuka, PRC 89, 044327 (2014)

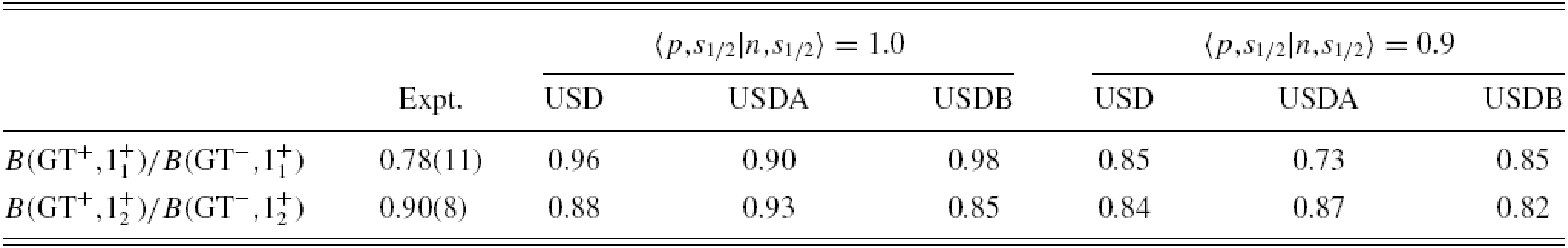




Nuclei	S_p(USD)	S_p(USD*)	S_p(Expt.)	S_{2p}(USD)	S_{2p}(USD*)	S_{2p}(Expt.)
^{18}Ne	4.25	4.09	3.92	4.72	4.56	4.52
^{19}Na	−0.39	−0.32	−0.32	3.87	3.77	3.60
^{20}Mg	3.02	2.96	2.66	2.63	2.63	2.34
^{21}Al	−1.36	−1.27		1.66	1.69	
^{22}Si	1.85	1.82		0.49	0.55	

CXY, C. Qi, F.R. Xu, T. Suzuki, T. Otsuka, PRC 89, 044327 (2014)
CXY, China Science Paper 11 (2015) 1250 (in Chinese)

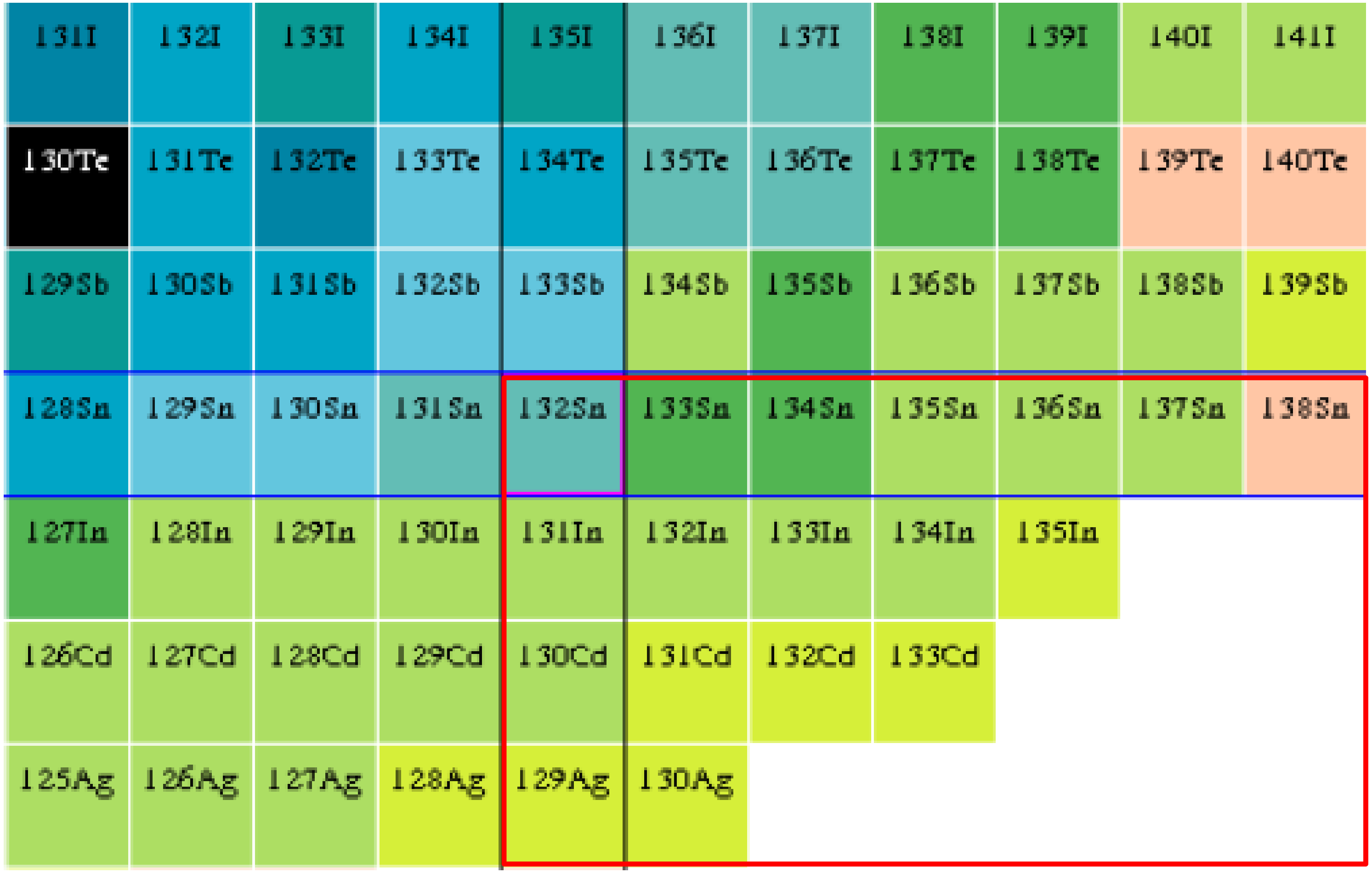
$B(GT^{+})(^{24}Si)/B(GT^{-})(^{24}Ne)$



		$\langle p,s_{1/2}\|n,s_{1/2}\rangle = 1.0$			$\langle p,s_{1/2}\|n,s_{1/2}\rangle = 0.9$		
	Expt.	USD	USDA	USDB	USD	USDA	USDB
$B(\mathrm{GT}^{+},1_1^{+})/B(\mathrm{GT}^{-},1_1^{+})$	0.78(11)	0.96	0.90	0.98	0.85	0.73	0.85
$B(\mathrm{GT}^{+},1_2^{+})/B(\mathrm{GT}^{-},1_2^{+})$	0.90(8)	0.88	0.93	0.85	0.84	0.87	0.82

CXY, C. Qi, F.R. Xu, T. Suzuki, T. Otsuka, PRC 89, 044327 (2014)

Heavier nuclei (^{132}Sn)



131I	132I	133I	134I	135I	136I	137I	138I	139I	140I	141I
130Te	131Te	132Te	133Te	134Te	135Te	136Te	137Te	138Te	139Te	140Te
129Sb	130Sb	131Sb	132Sb	133Sb	134Sb	135Sb	136Sb	137Sb	138Sb	139Sb
128Sn	129Sn	130Sn	131Sn	132Sn	133Sn	134Sn	135Sn	136Sn	137Sn	138Sn
127In	128In	129In	130In	131In	132In	133In	134In	135In		
126Cd	127Cd	128Cd	129Cd	130Cd	131Cd	132Cd	133Cd			
125Ag	126Ag	127Ag	128Ag	129Ag	130Ag					

- **Proton 28-50 ($1p_{3/2}$, $1p_{1/2}$, $0f_{5/2}$, $0g_{9/2}$)**
- **Neutron 82-126 ($1f_{7/2}$, $1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$, $0h_{9/2}$, $0i1_{3/2}$)**
- **Two major shell difference, cross shell interaction?**

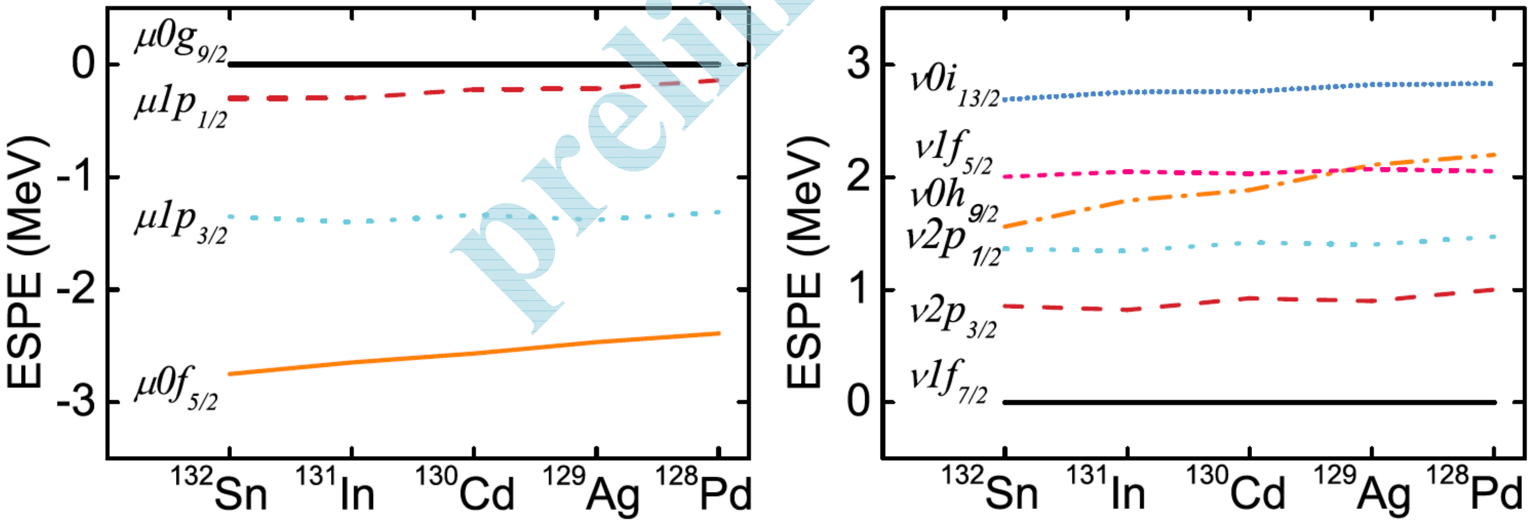
- **pp: jj45pna*0.74, M. Horth-Jensen**
- **nn: cwg, B.A. Brown, *et al.* Phys. Rev. C 71, 044317 (2005)**
- **pn: V_{MU}(1.07 central)+LS**

Nuclei	$S_n^{(expt)}$	$S_n^{(jj46)}$	$S_n^{(AME2012)}$	$S_n^{(Moller)}$
^{133}Sn	2.402	2.408		2.651
^{134}Sn	3.629	3.732		4.281
^{135}Sn	2.271	2.405		1.871
^{136}Sn		3.795	3.340	3.741
^{137}Sn		2.339	1.960	1.611
^{138}Sn		3.834	3.140	3.561
^{132}In	2.450	2.364		2.701
^{133}In		3.418	3.130	3.781
^{134}In		2.370	2.270	1.771
^{131}Cd		1.980	1.870	2.031
^{132}Cd		3.323	3.000	3.671
^{133}Cd		1.975	1.730	1.321
^{130}Ag		1.899	1.780	2.001
^{129}Pd		1.519		1.461

- **CXY, Z. Liu, F.R. Xu, in preparation**

possible isomer

Nuclei	$J_i^\pi \to J_f^\pi$	E_i	ΔE	$B(E2)_{th}$	τ_{E2}	$B(E2)_{Expt}$	Configuration
^{134}Sn	$6^+ \to 4^+$	1.267	0.149	41.72	0.266	35(6)	$\nu(1f_{7/2})^2(96.3\%)$
^{135}Sn	$21/2^- \to 17/2^-$	2.288	0.147	93.96	0.127		$\nu(1f_{7/2})^2(0h_{9/2})(96.7\%)$
^{136}Sn	$6^+ \to 4^+$	1.388	0.222	15.43	0.098	24(4)	$\nu(1f_{7/2})^4(75.5\%)$
^{138}Sn	$6^+ \to 4^+$	1.544	0.183	12.80	0.311	19(4)	$\nu(1f_{7/2})^6(53.13\%)$
^{132}In	$5^- \to 7^-$	0.067	0.067	1.75	345.693		$\mu(0g_{9/2})^{-1}\nu(1f_{7/2})(99.0\%)$
^{133}In	$17/2^- \to 13/2^-$	0.972	0.257	48.36	0.015		$\mu(0g_{9/2})^{-1}\nu(1f_{7/2})^2(93.9\%)$
^{134}In	$5^- \to 7^-$	0.074	0.074	27.69	13.282		$\mu(0g_{9/2})^{-1}\nu(1f_{7/2})^3(72.5\%)$
^{130}Cd	$8^+ \to 6^+$	2.123	0.106	59.07	1.032	66(13)/50(10)	$\mu(0g_{9/2})^{-2}(100.0\%)$
^{131}Cd	$19/2^- \to 15/2^-$	1.789	0.139	100.03	0.157		$\mu(0g_{9/2})^{-2}\nu(1f_{7/2})(99.7\%)$
^{130}Ag	$5^- \to 7^-$	0.072	0.072	18.06	23.350		$\mu(0g_{9/2})^{-3}\nu(1f_{7/2})(74.1\%)$
^{128}Pd	$8^+ \to 6^+$	2.198	0.100	13.43	6.076	8.43(0.25)	$\mu(0g_{9/2})^{-4}(69.3\%)$
^{129}Pd	$19/2^- \to 15/2^-$	1.913	0.146	13.70	0.898		$\mu(0g_{9/2})^{-4}\nu(1f_{7/2})(72.7\%)$
^{130}Pd	$6^+ \to 4^+$	1.328	0.211	184.24	0.011		$\mu(0g_{9/2})^{-4}\nu(1f_{7/2})^2(53.5\%)$



CXY, Z. Liu, F.R. Xu, in preparation

Summary

- Systematic study is performed on nuclei from stability line to both proton and neutron drip line in light region.
  - New shell model Hamiltonian YSOX is introduced for *psd* region and study B, C, N and O isotopes.
  - The weakly bound effect of proton 1s orbit is included in the shell model calculation
- V_{MU}+LS is applied to heavier region (^{132}Sn). The possible isomer is studied.

Collaboration

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Thank You for Attending