Comparative studies for baryon interactions with HAL QCD method and Luscher's method

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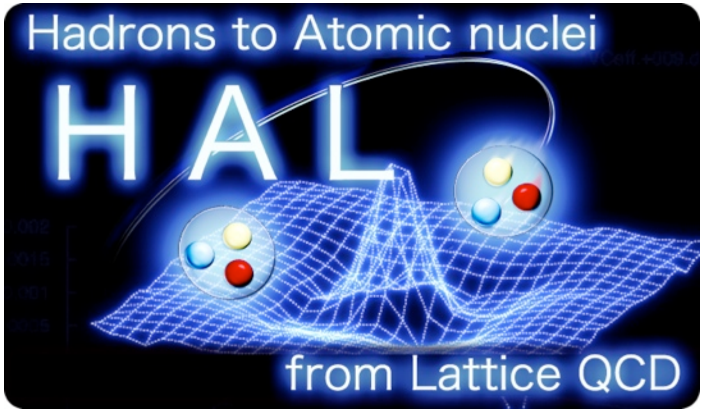
for HAL QCD Collaboration



S. Aoki, S. Gongyo, D. Kawai, T. Miyamato (YITP)

T. Doi, T.Hatsuda, Y. Ikeda (RIKEN)

F. Etminan (Univ. of Birjand)

T. Inoue (Nihon Univ.)

T. Iritani (Stony Brook Univ.)

N. Ishii, K. Murano (RCNP)

H. Nemura, K. Sasaki (Univ. of Tsukuba)

Interactions on the Lattice

- Luscher's method

M.Luscher (1986, 91)

  - Energy spectrum in finite V ➔ phase shift by Luscher's formula

$$\Delta E = 2\sqrt{m^2 + k^2} - 2m$$

- HAL QCD method

Ishii-Aoki-Hatsuda (2007), Ishii et al. (HAL) (2012)

  - NBS wave func. ⇔ E-indep & non-local "potential"
    - ➔ phase shifts by solving Schrodinger eq in infinite V

$$\left(-\frac{\partial}{\partial t} + \frac{1}{4m}\frac{\partial^2}{\partial t^2} - H_0\right) R(\boldsymbol{r}, t) = \int d\boldsymbol{r}' \underline{U(\boldsymbol{r}, \boldsymbol{r}')} R(\boldsymbol{r}', t)$$

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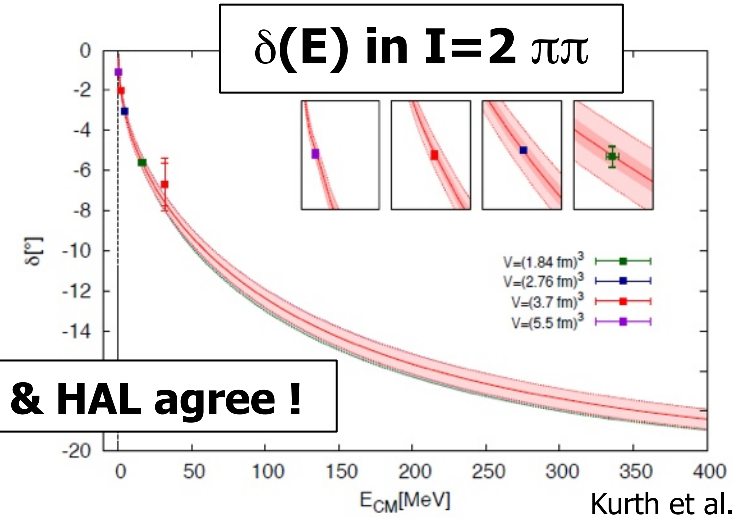

Kurth et al. (2013)

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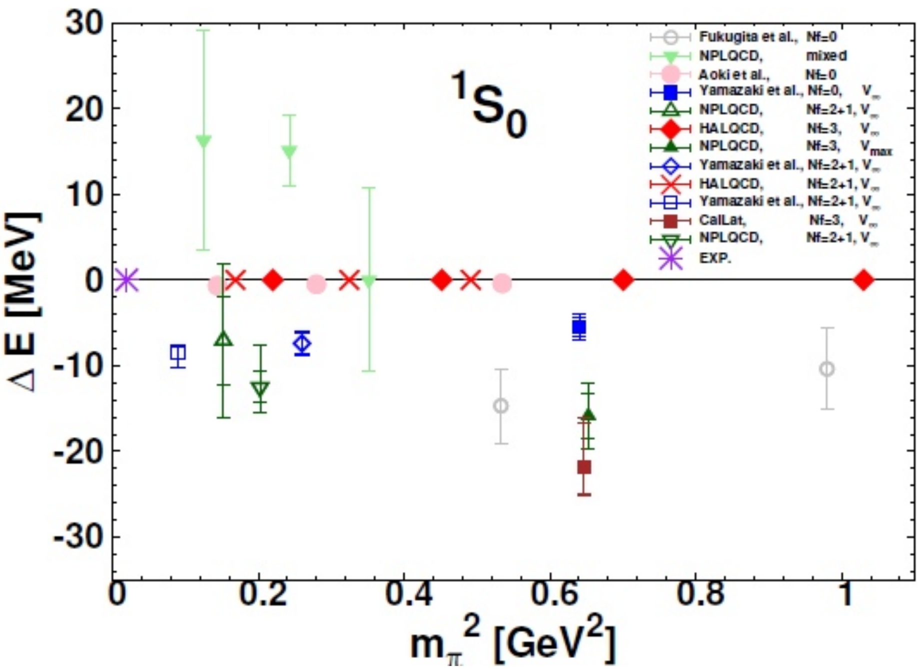
E-indep & non-local pot

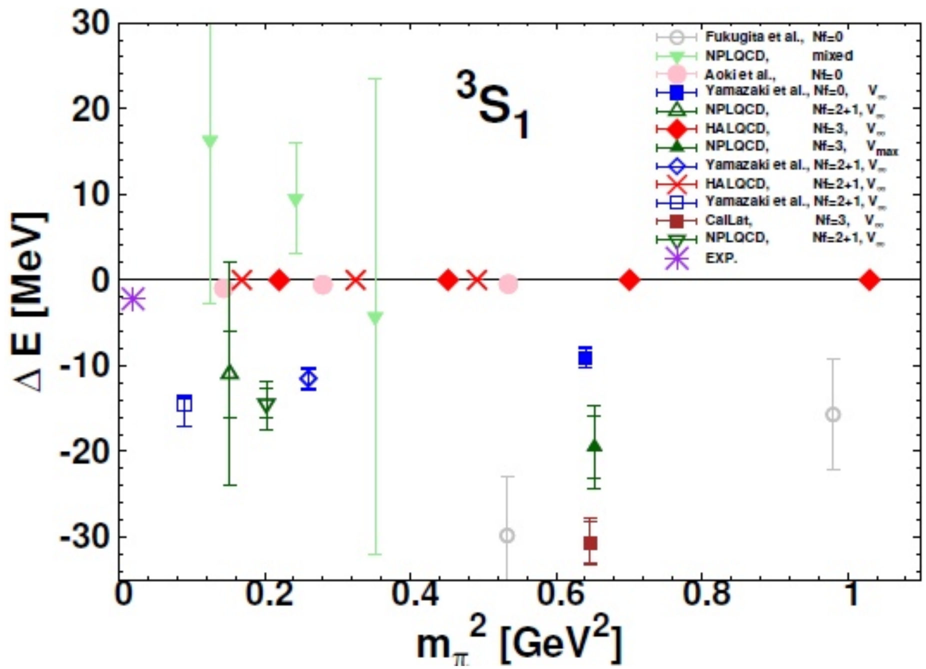
Luscher vs HAL : NN systems

Reviewed in T.D. PoS LAT2012,009 (+ updates)

“di-neutron”



“deuteron”



HAL method (HAL) : **unbound**

Lushcer’s method (PACS-CS (Yamazaki et al.)/NPL/CalLat): **bound**

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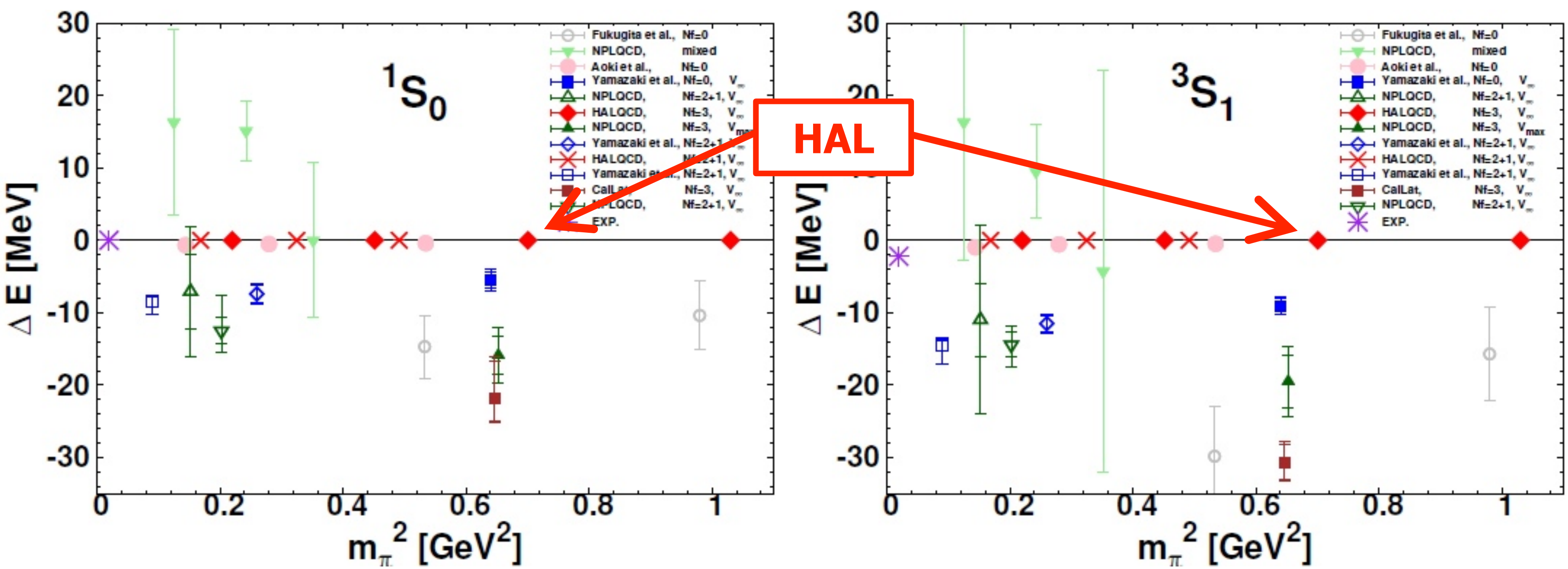

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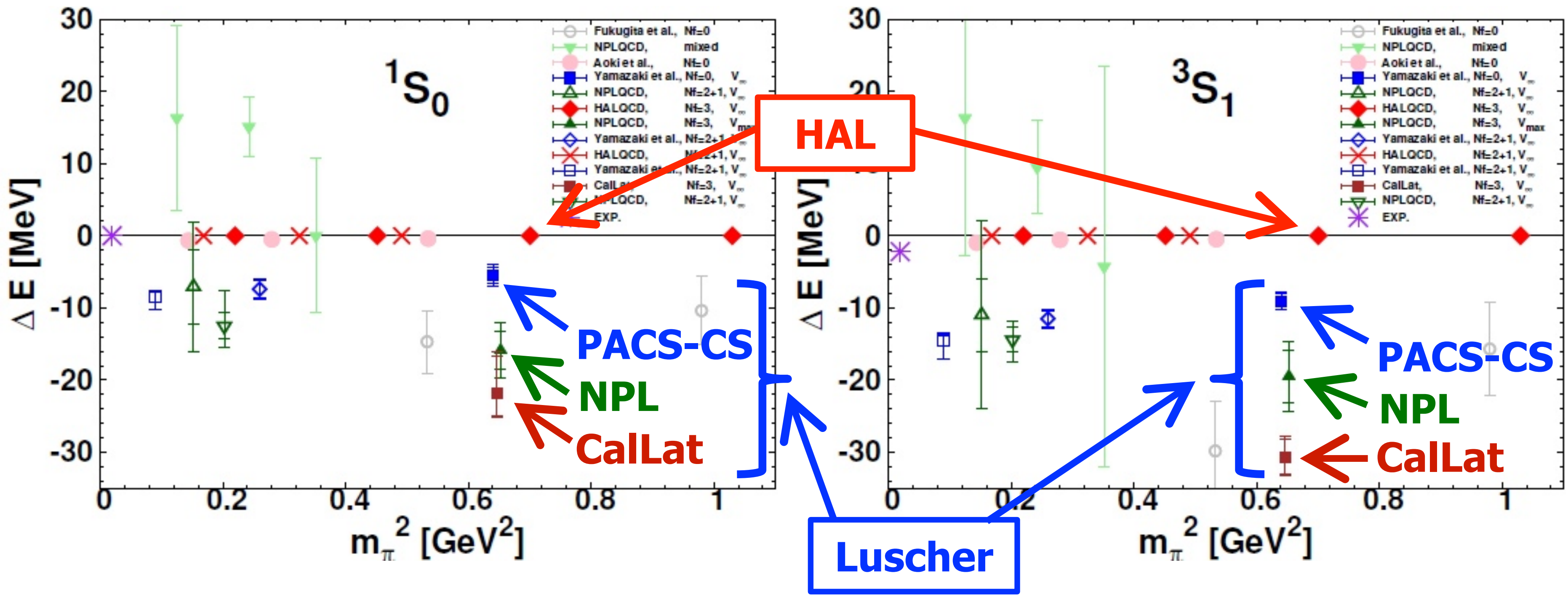

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Examine the systematics

Luscher’s method

(Yamazaki et al. / NPL / CalLat)

HAL method

(HAL Coll.)

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G.S. saturation: NECESSARY

Tune quark source for better saturation ?

HAL method

(HAL Coll.)

G.S. saturation: NOT necessary

E-independence of U(r,r')

➔ (elastic) excited scattering states share the same U(r,r')

➔ Excited states give signals

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Convergence of derivative expansion of U(r,r')

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wall source
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(favor (spurious?) bound states ?)

wall source
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Crucial test to establish a reliable LQCD method

Luscher & HAL w/ wall & smeared src

- **Employ the same config** used in previous Luscher method study
  - Confs by Yamazaki et al. : Claimed that NN are bound (Luscher w/ (exp-)smeared src)

T. Yamazaki et al. PRD86(2012)074514

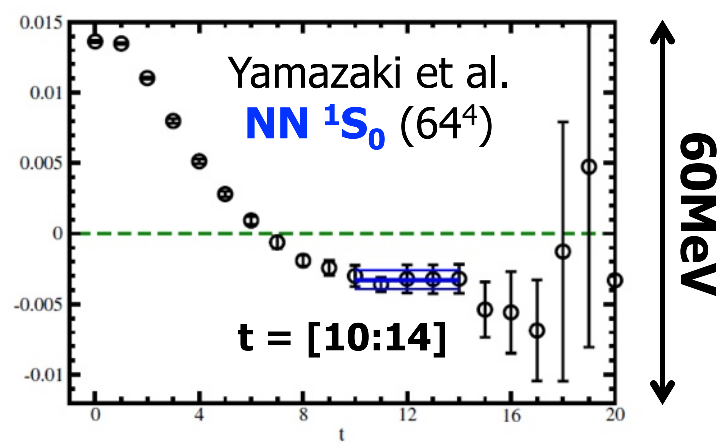
  - **High statistics (e.g., 48^4 smeared: x5 #stat of Yamazaki et al.)**

- Nf=2+1 clover, m_π = 0.51GeV, m_N = 1.32GeV, m_Ξ = 1.46GeV, 1/a=2.2GeV

L	volume	smeared src.	wall src.	
3.6 fm	$40^3 \times 48$	200 conf. $\times$ 256 meas.	200 conf. $\times$ 48 meas.	
4.3 fm	$48^3 \times 48$	800 conf. $\times$ 256 meas.	800 conf. $\times$ 48 meas.	← Figs in this talk
5.8 fm	$64^3 \times 64$	327 conf. $\times$ 64 meas.	327 conf. $\times$ 128 meas.	

  - First study: we use **ΞΞ $^1S_0$ system** (~ NN $^1S_0$, but much better S/N)

- (1) Luscher's method: wall vs smeared
- (2) HAL method: wall vs smeared
- (3) Comparison of Luscher vs HAL



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$$\Delta E = m_{\Xi\Xi} - 2m_{\Xi}$$

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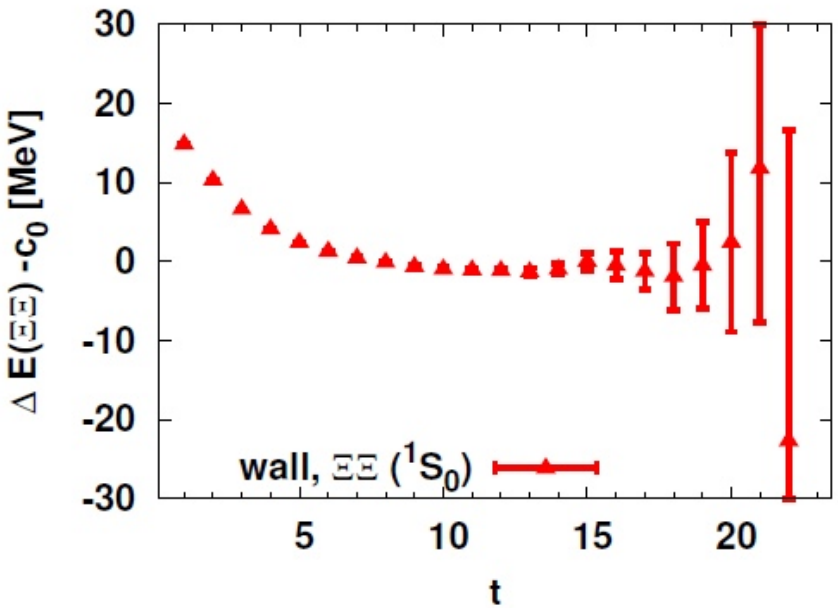
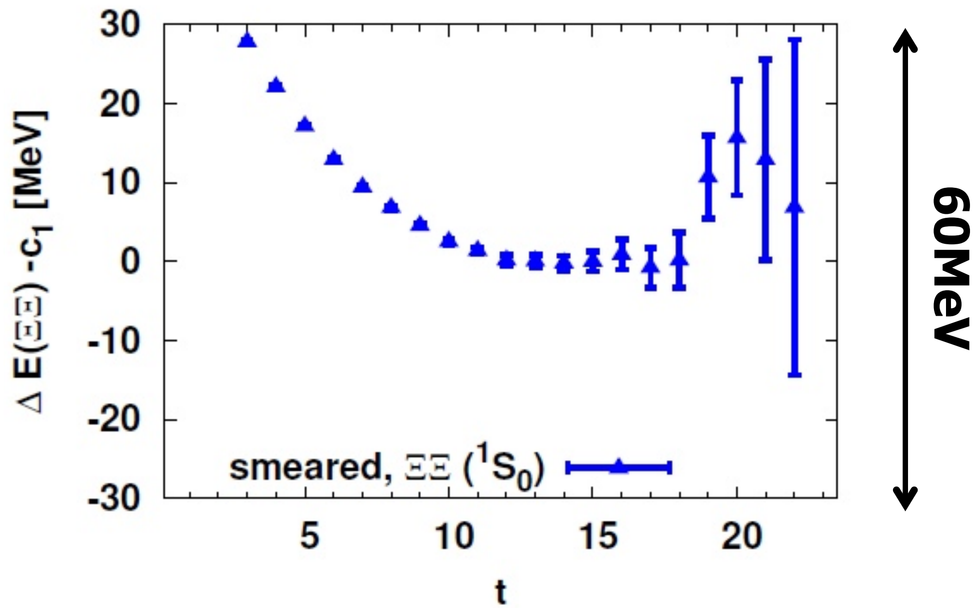
wall



smeared



Excellent plateaux for both cases ?

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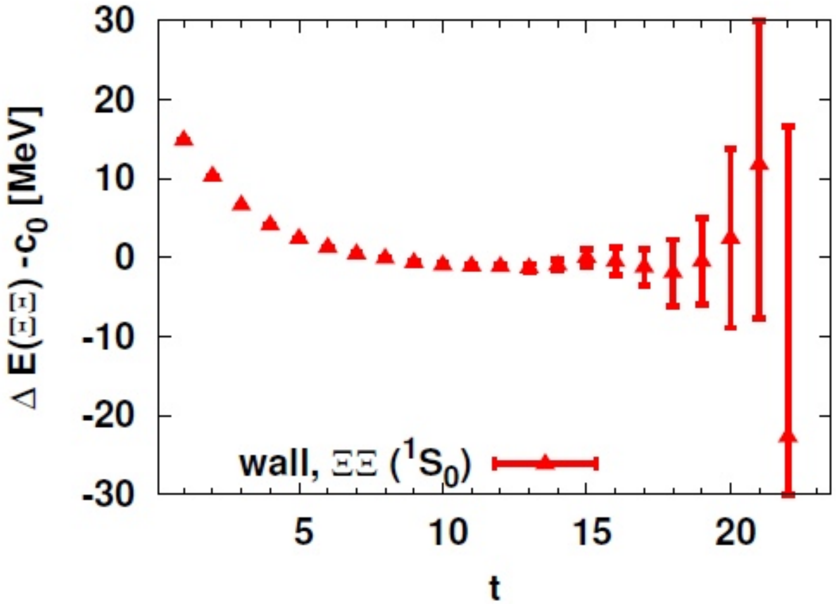

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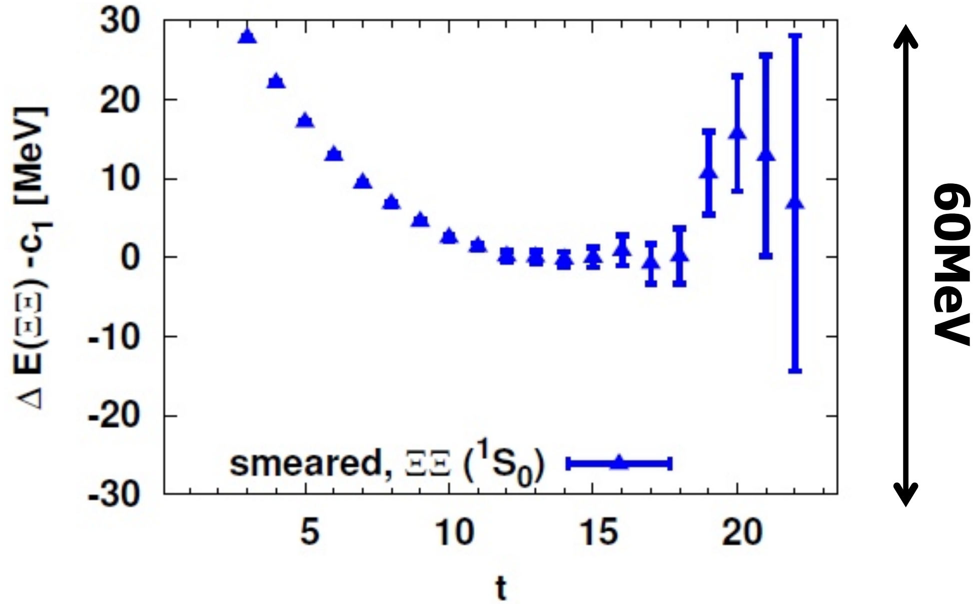

Excellent plateaux for both cases ?

However, we need a few – 10 MeV precision

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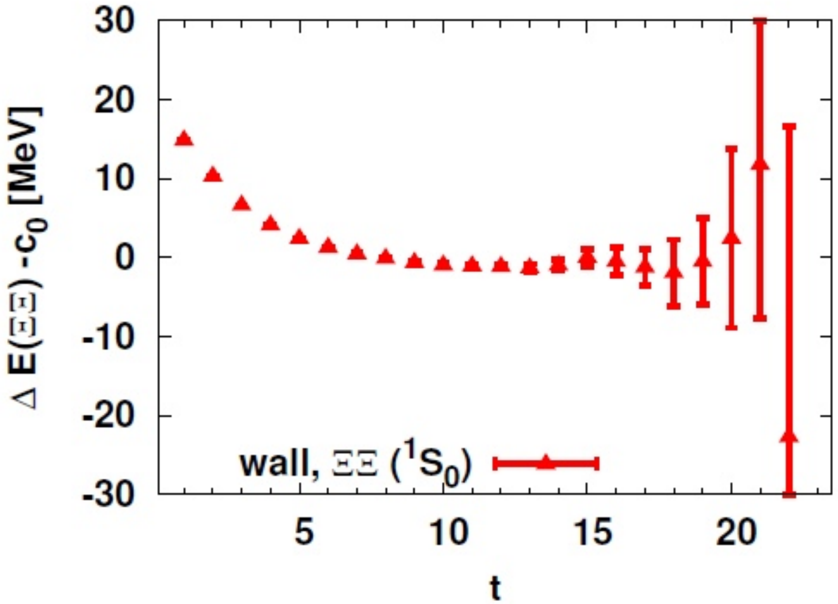

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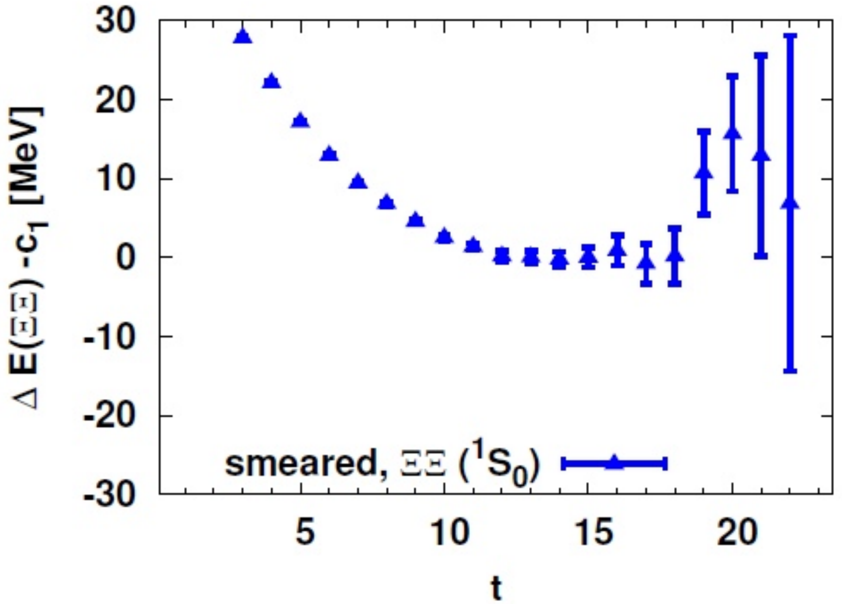

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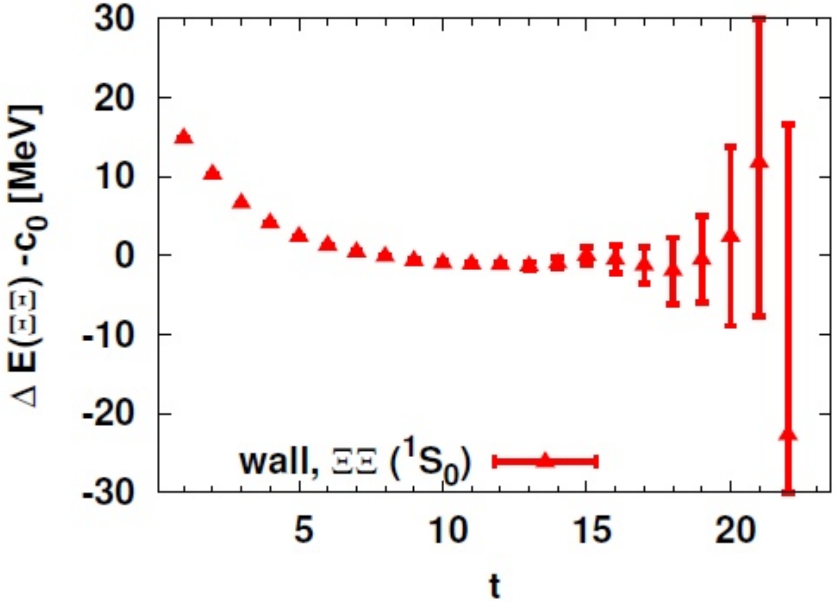

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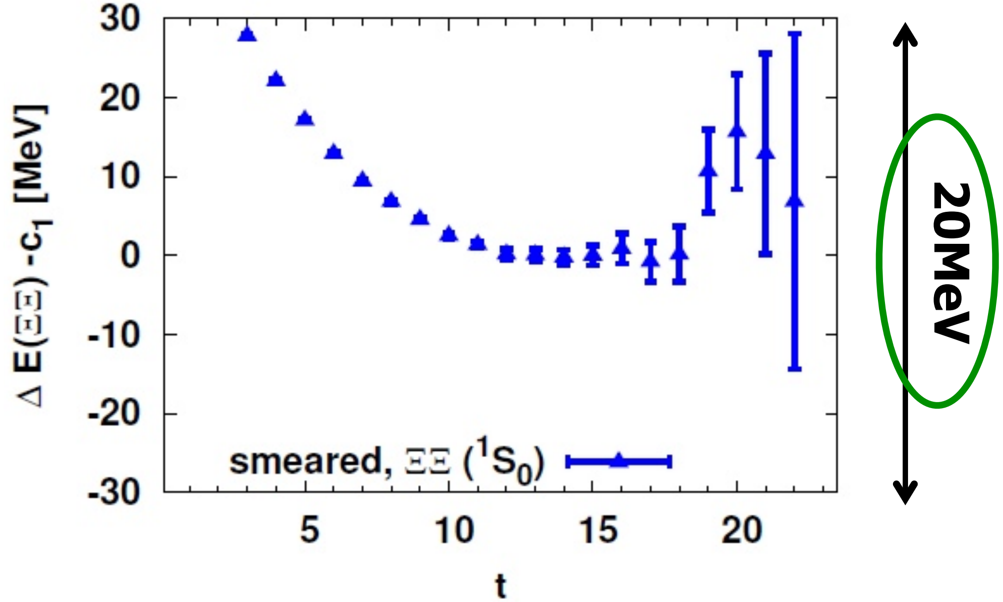

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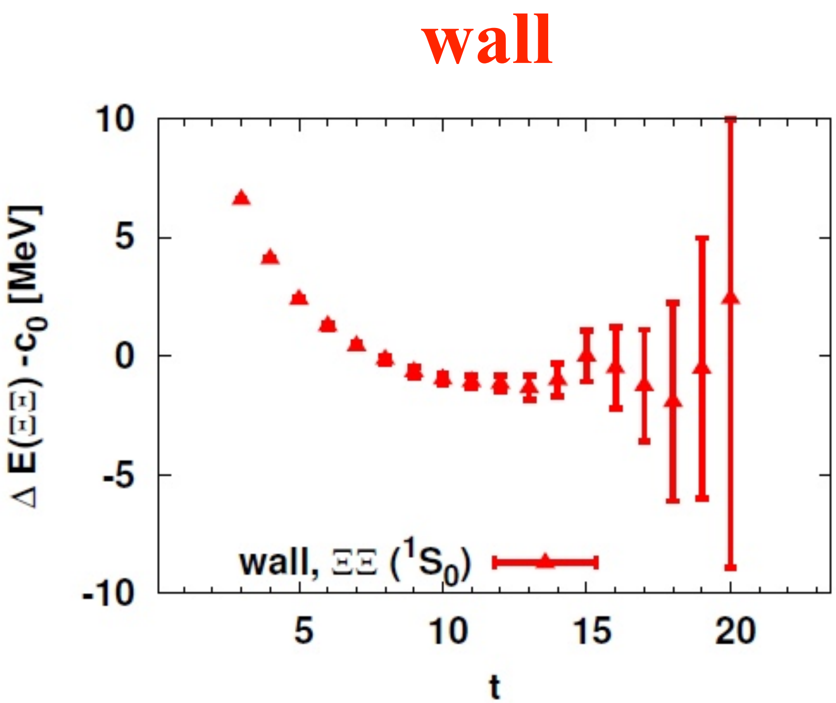

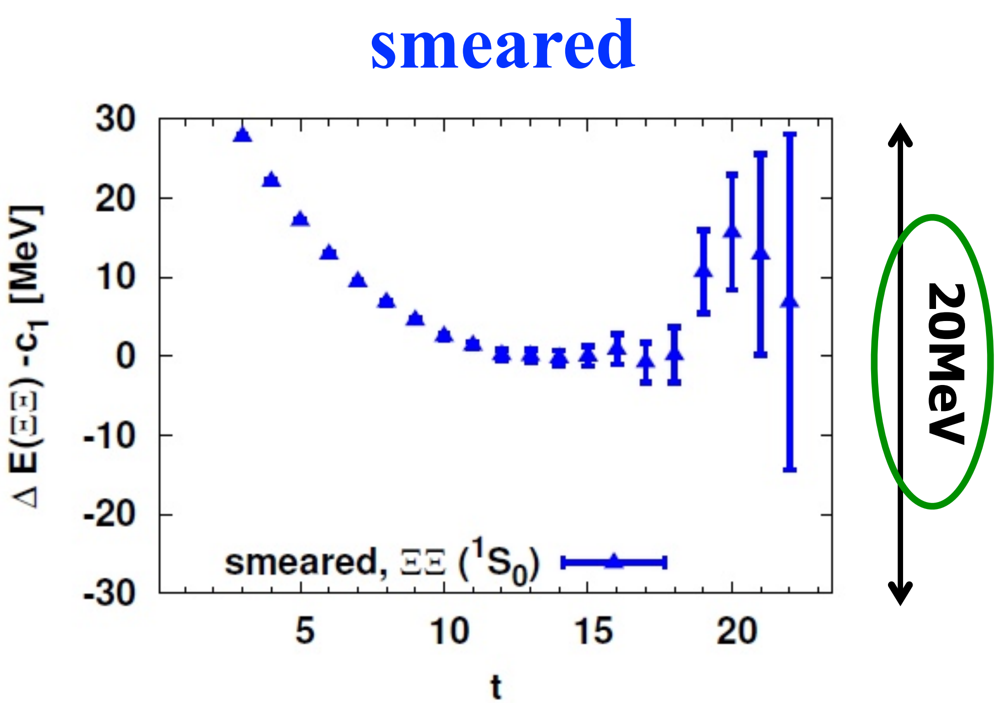

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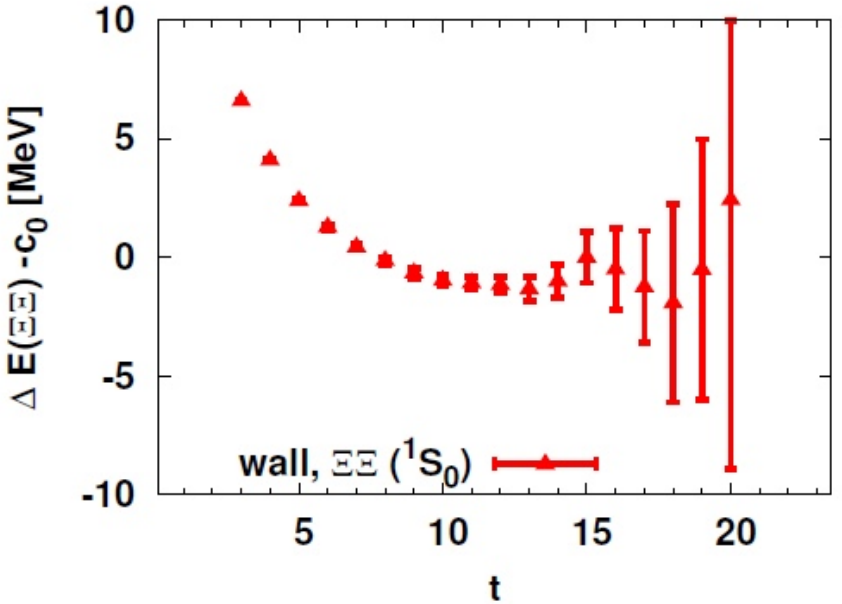

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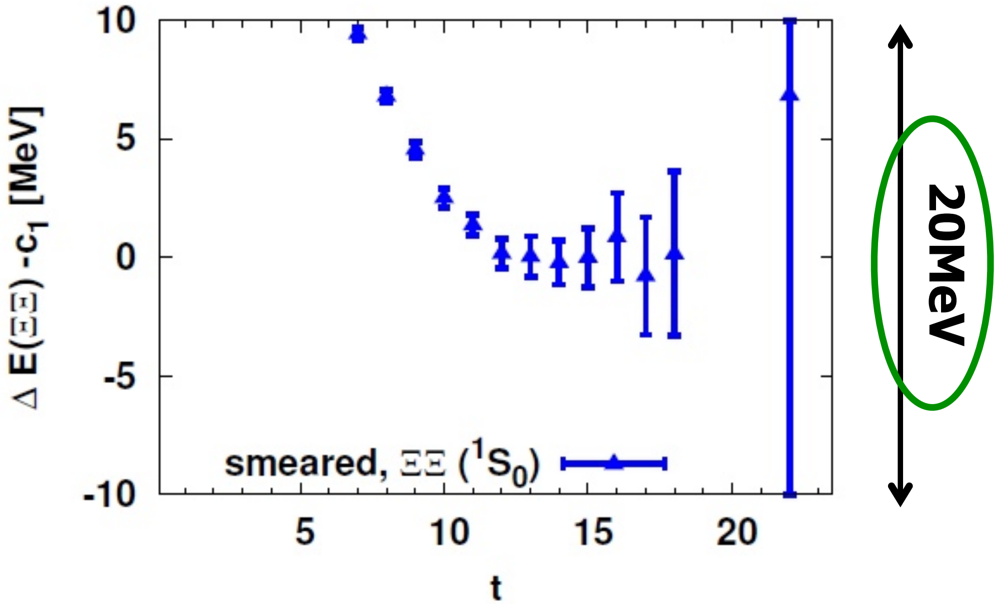

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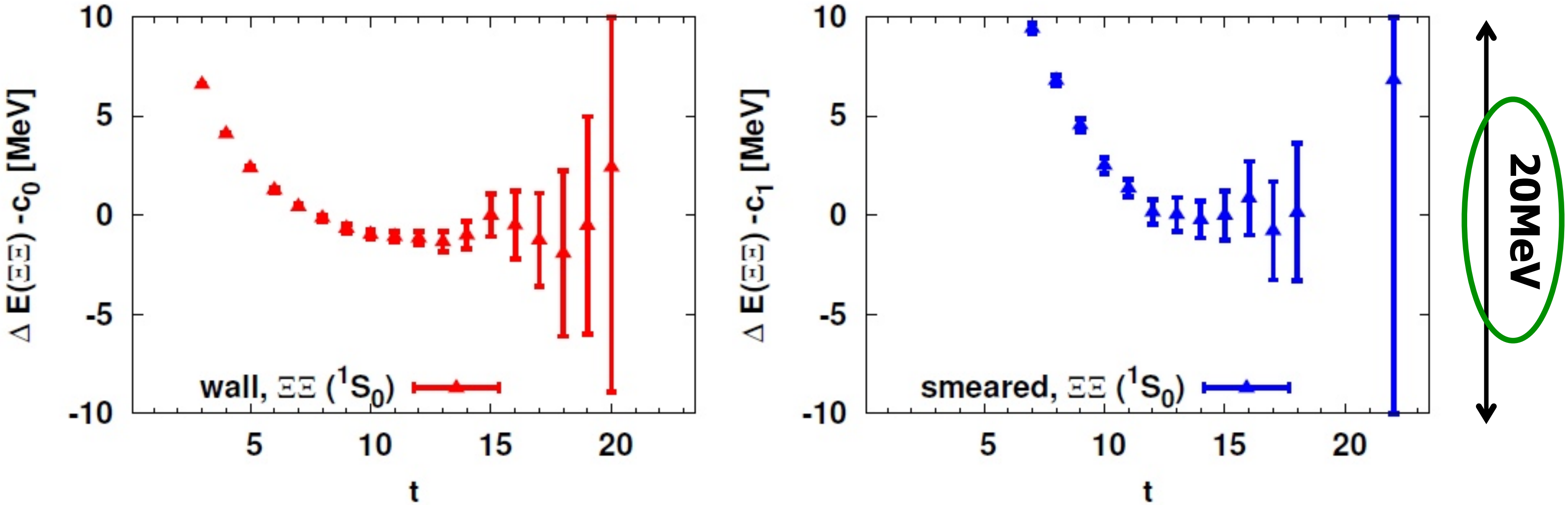

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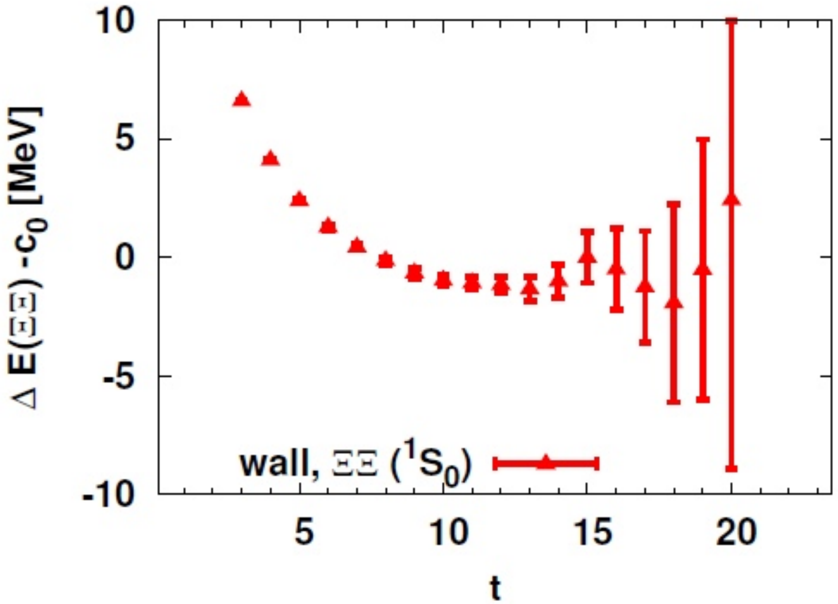

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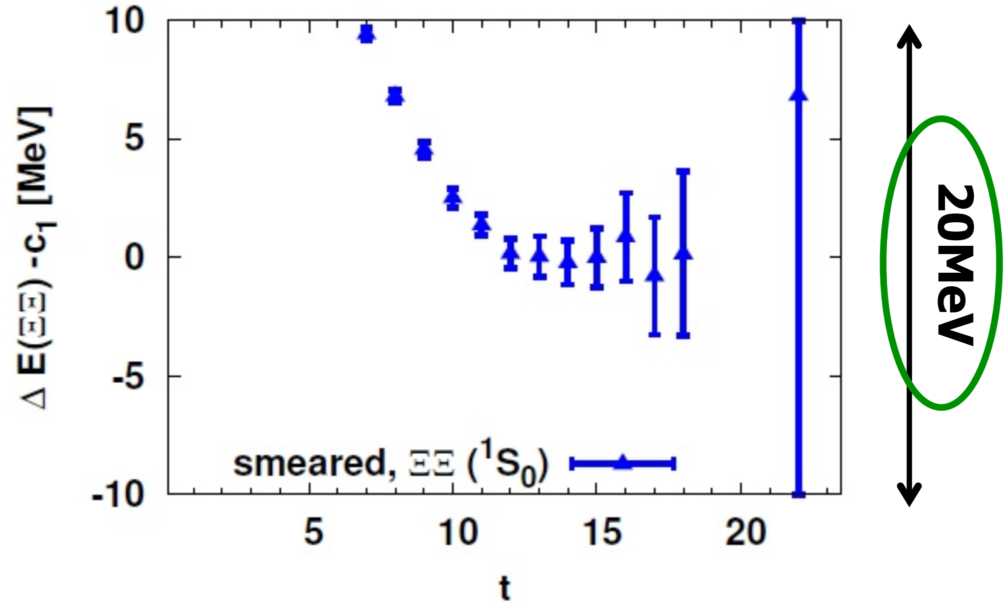

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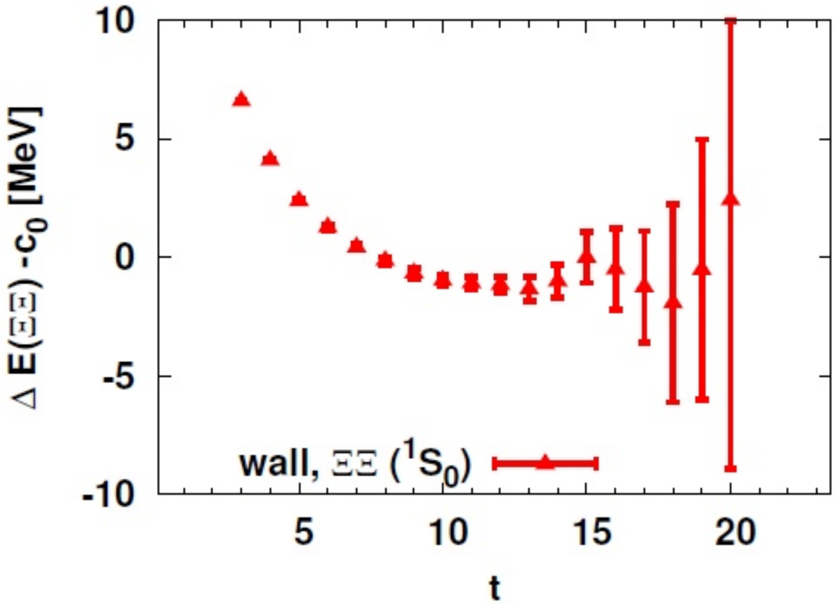

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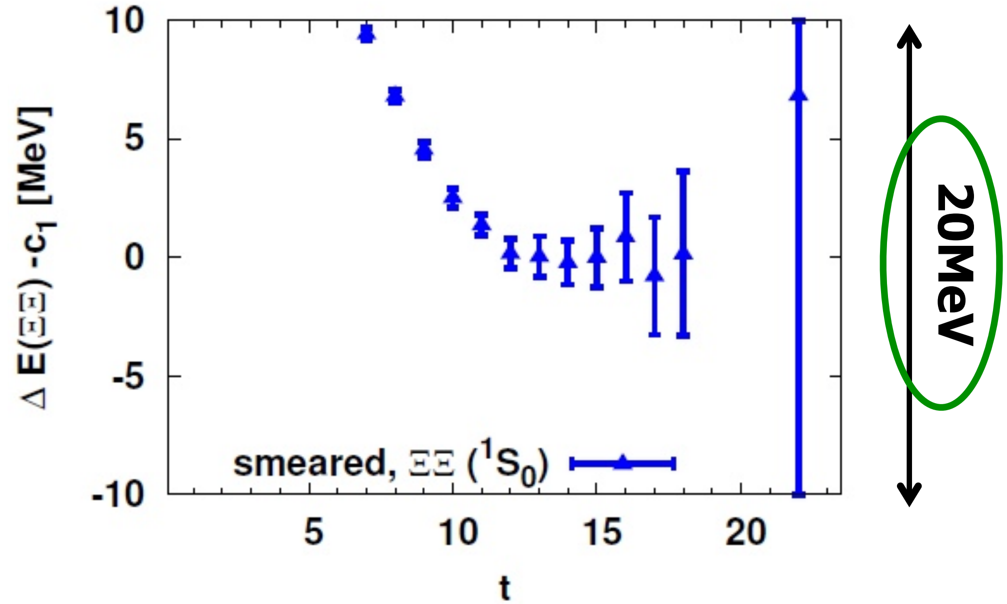

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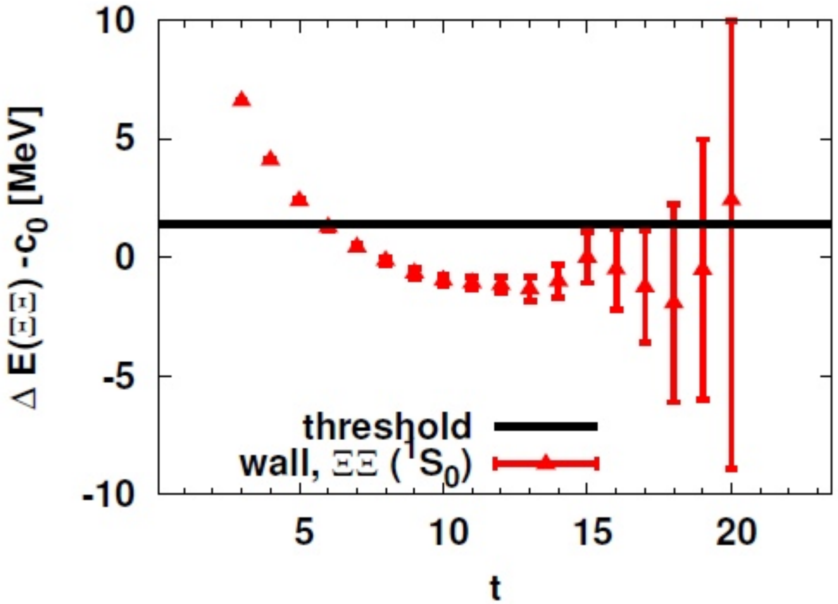

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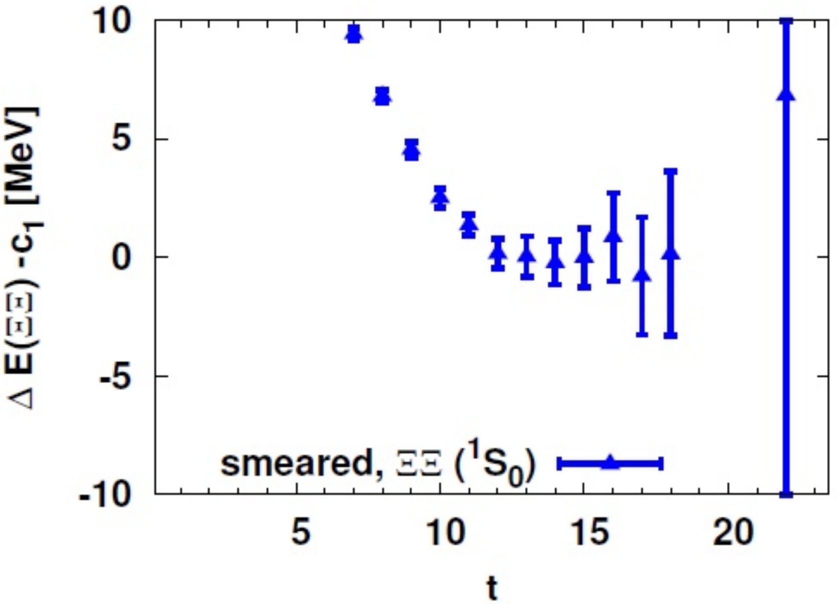

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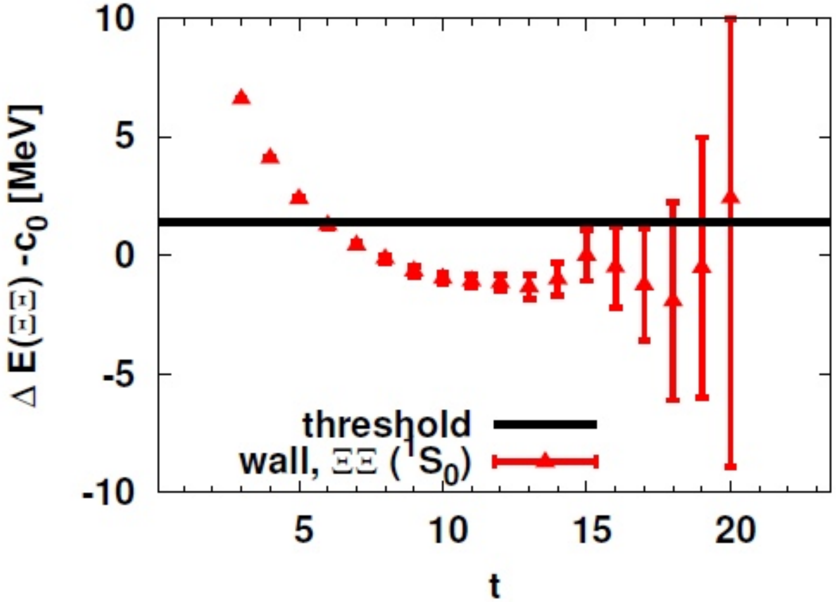

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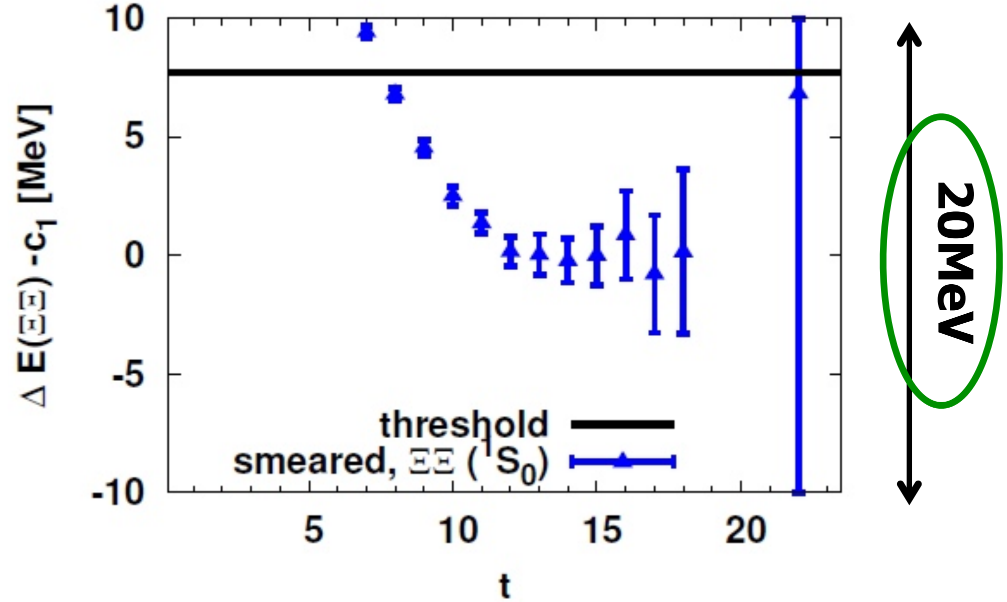

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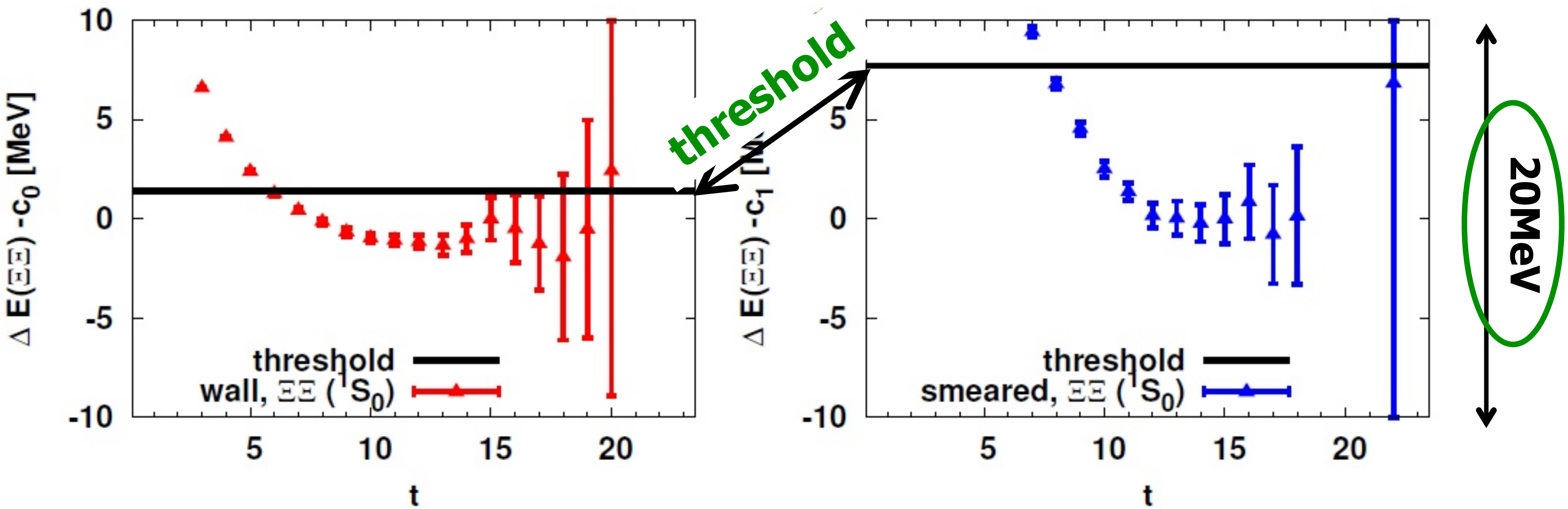

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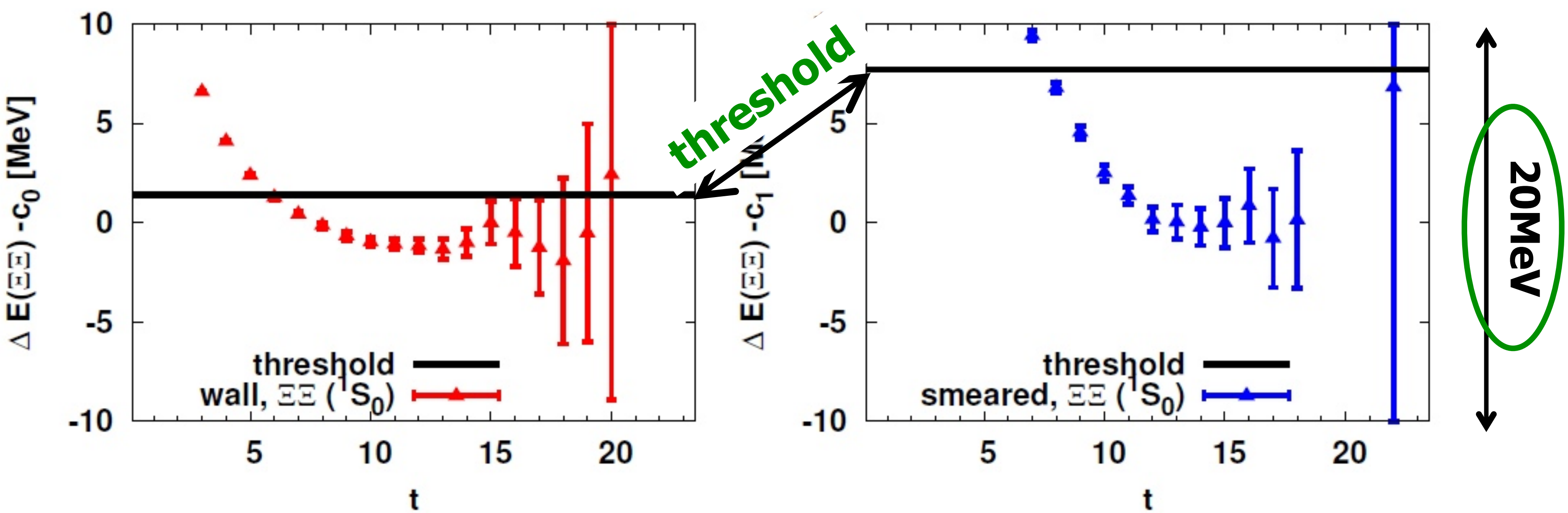

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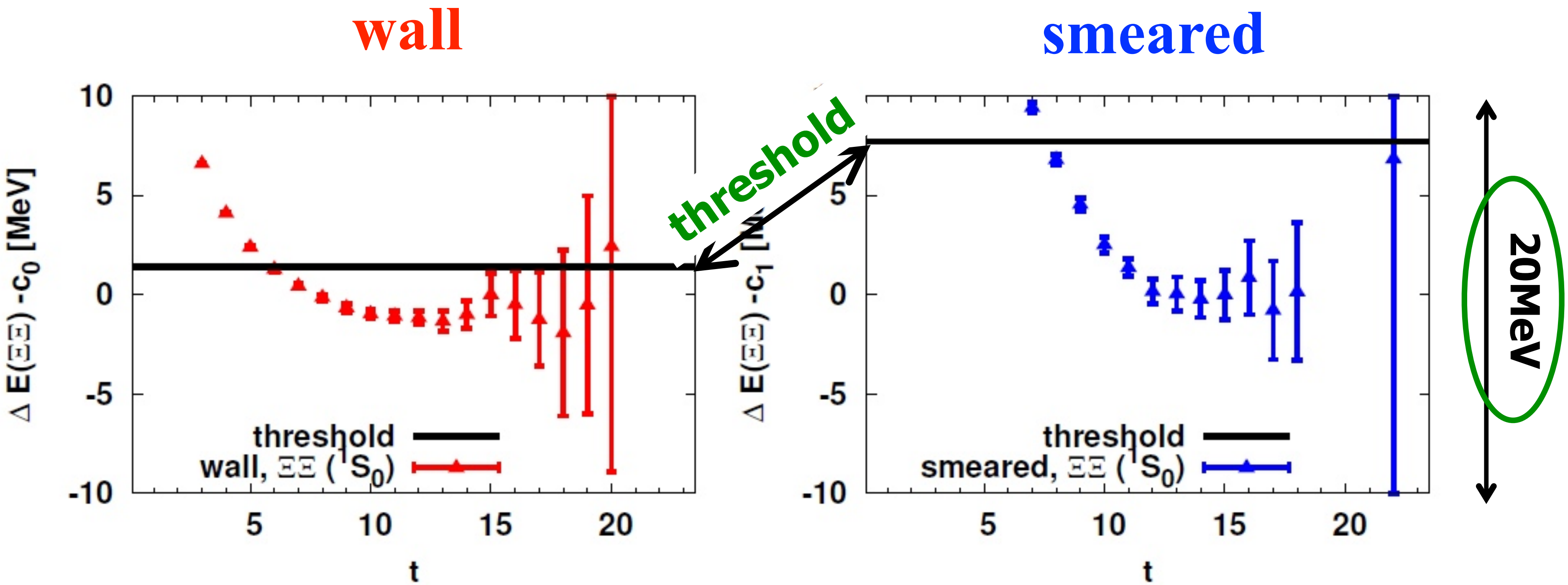

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Let's plot in the same figure

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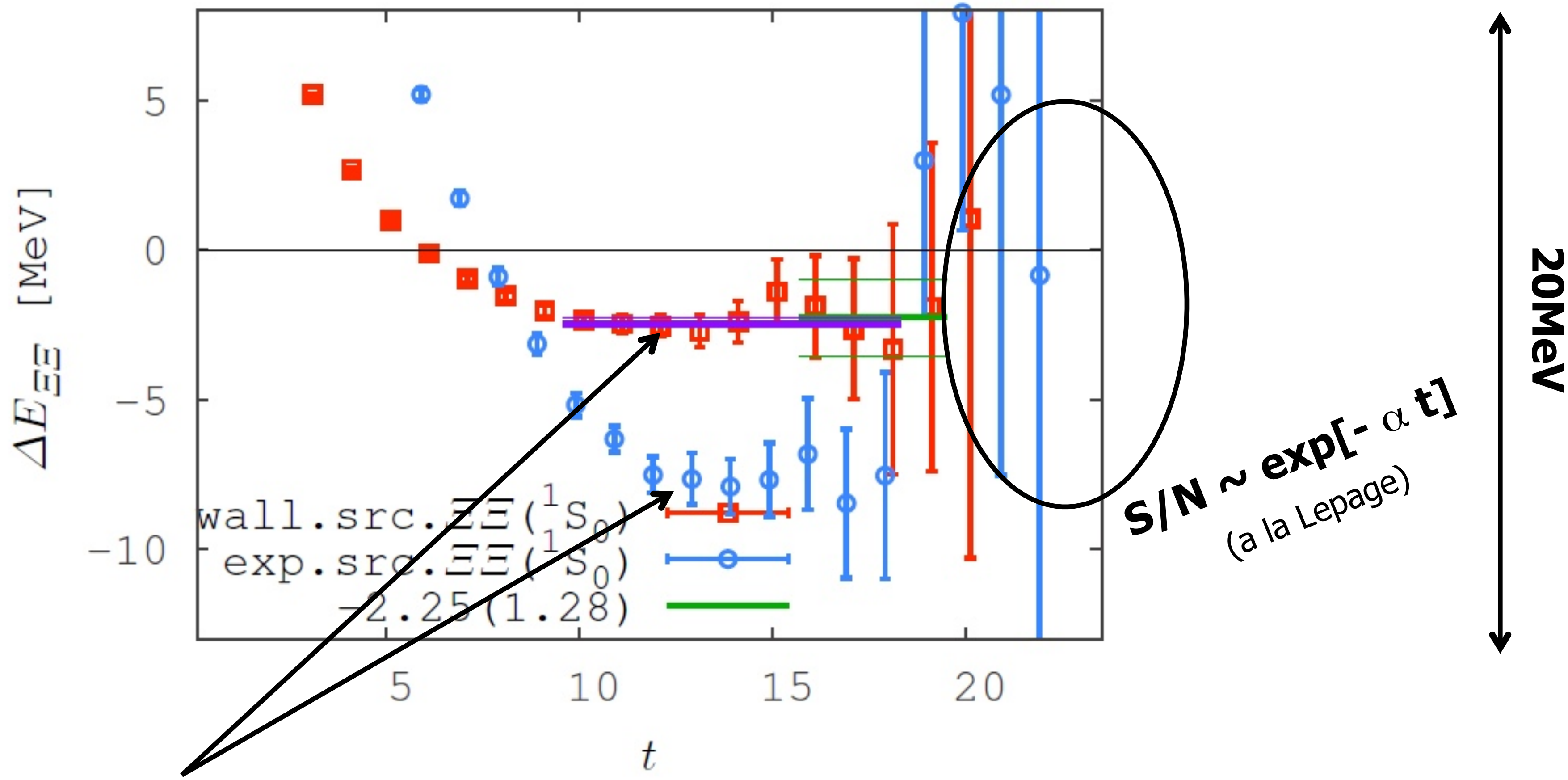

Wall and Smeared are Inconsistent:

one cannot judge which (or neither) is reliable

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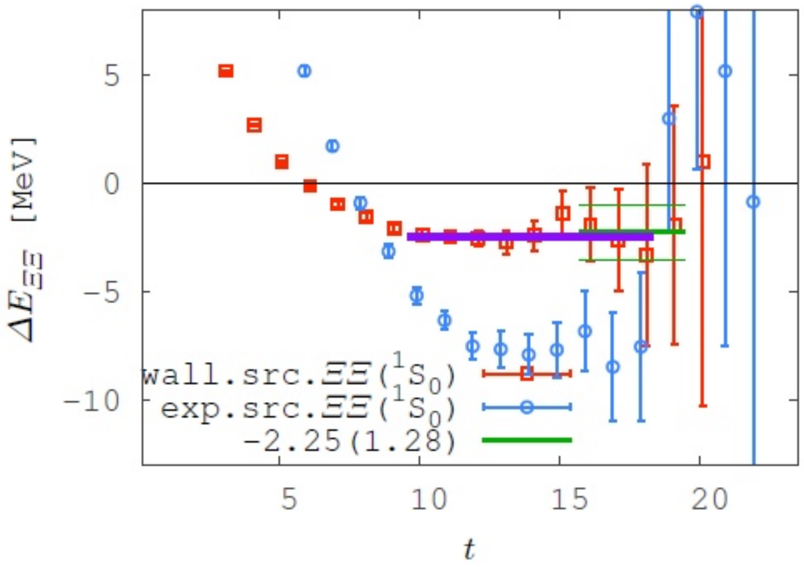

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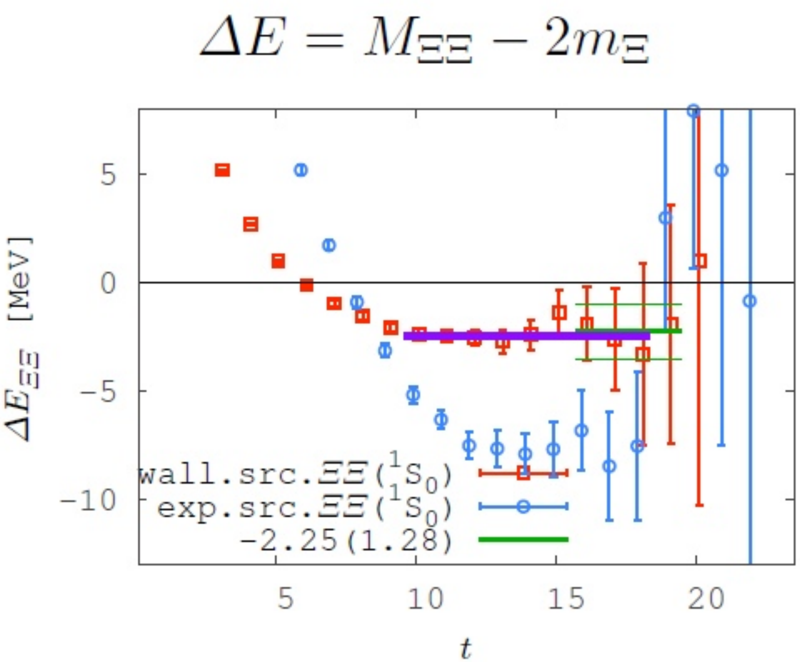
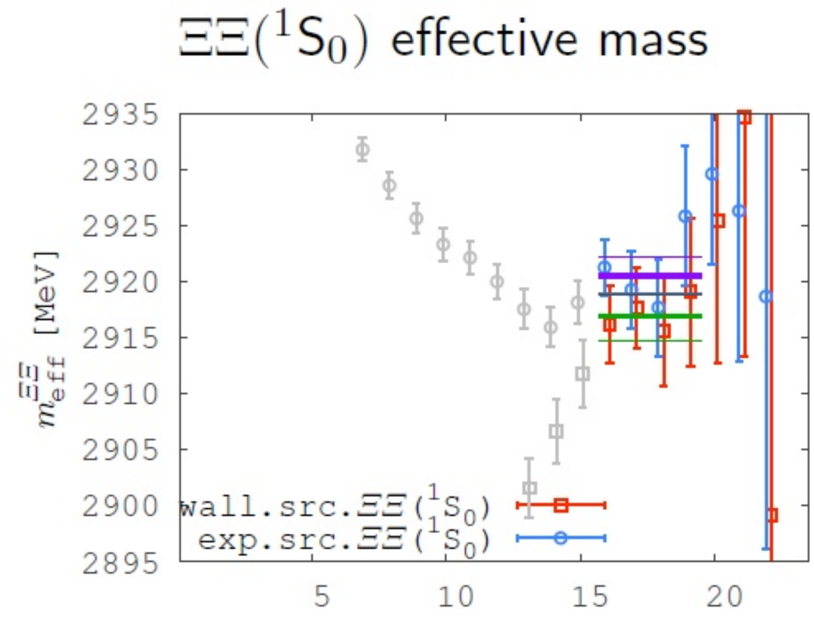
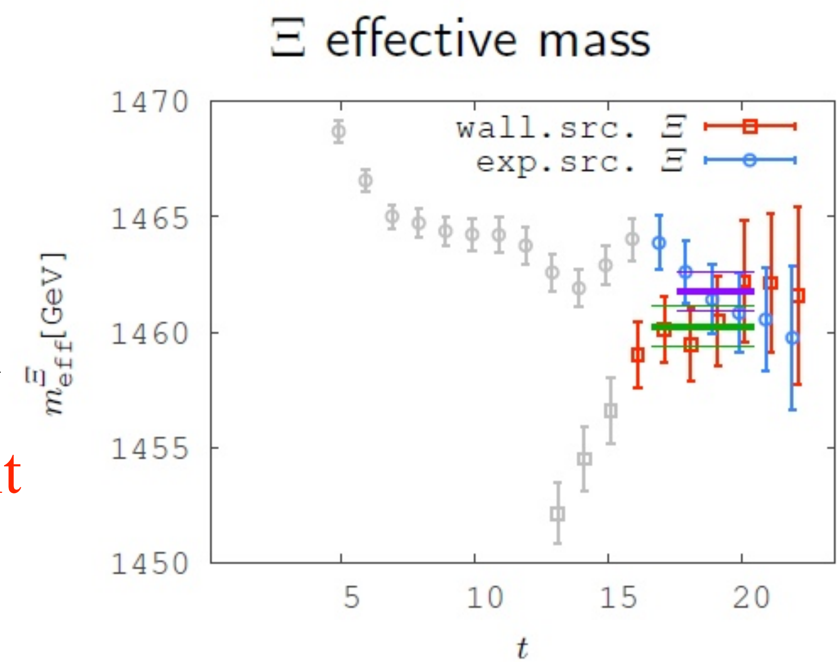




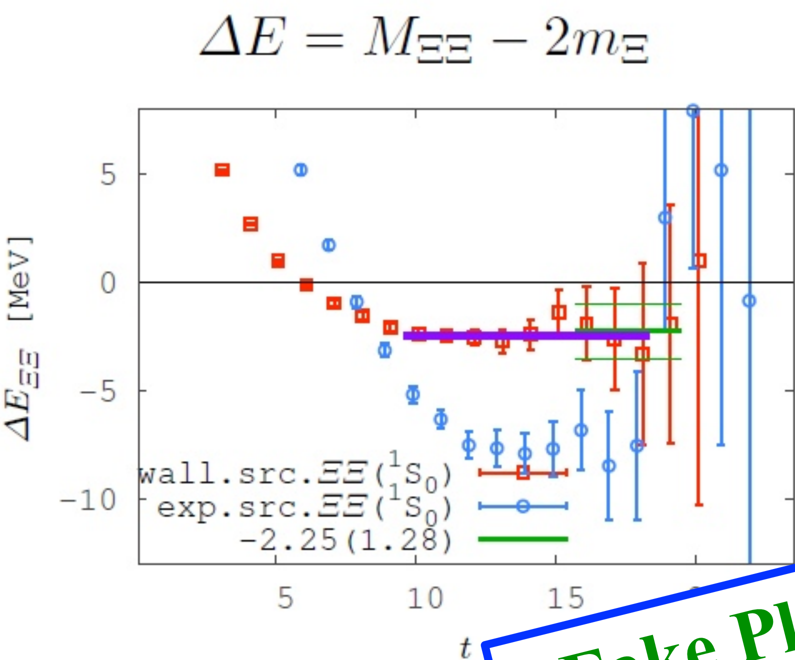


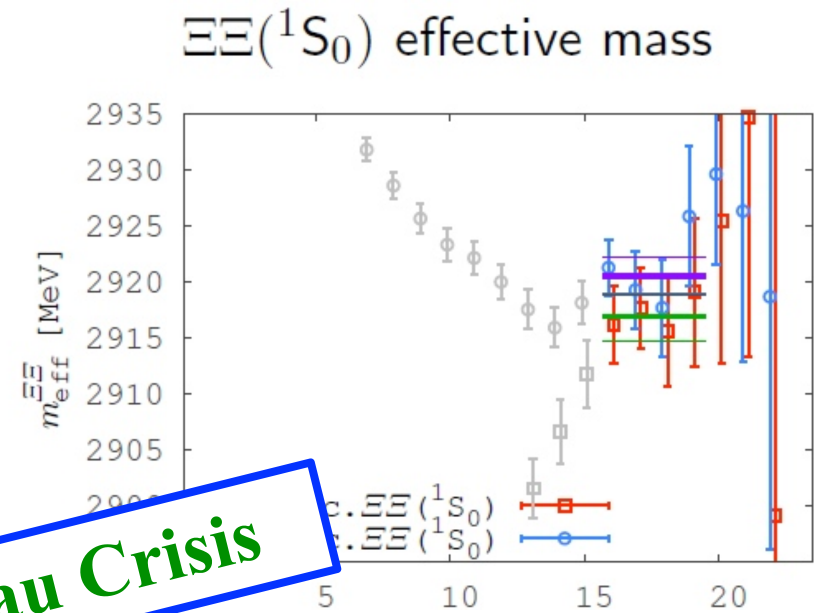
Effective mass for ΔE is dangerous

"Fake plateau" can easily appear due to 1-body and 2-body cancellation

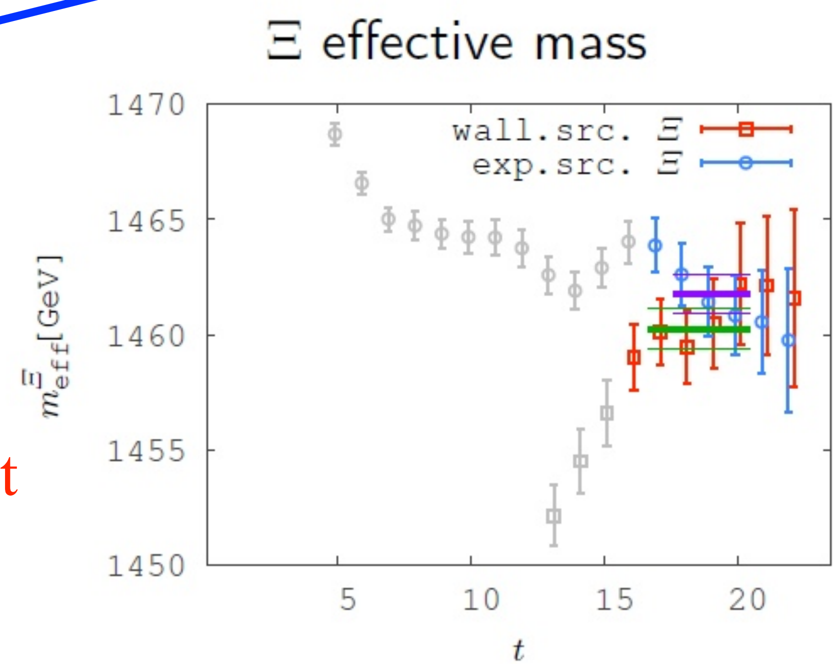
Ground state saturation is very difficult

(1) Luscher’s method: wall vs smeared src (cont’d)





Fake Plateau Crisis



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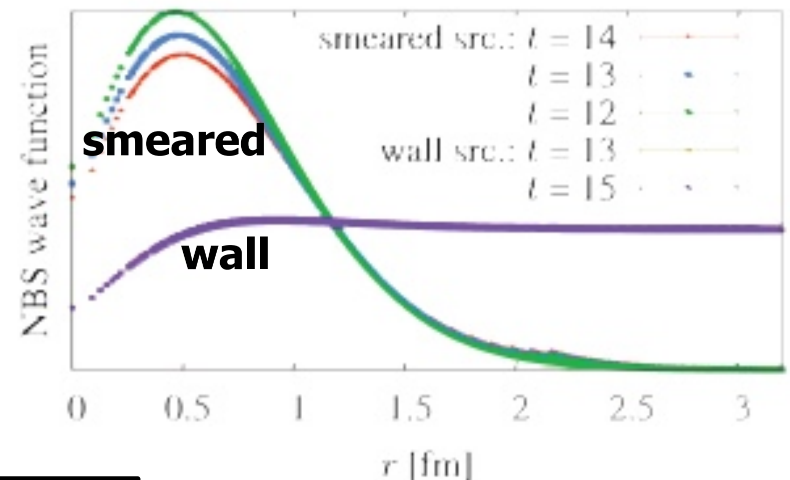
(2) HAL method: wall vs smeared src

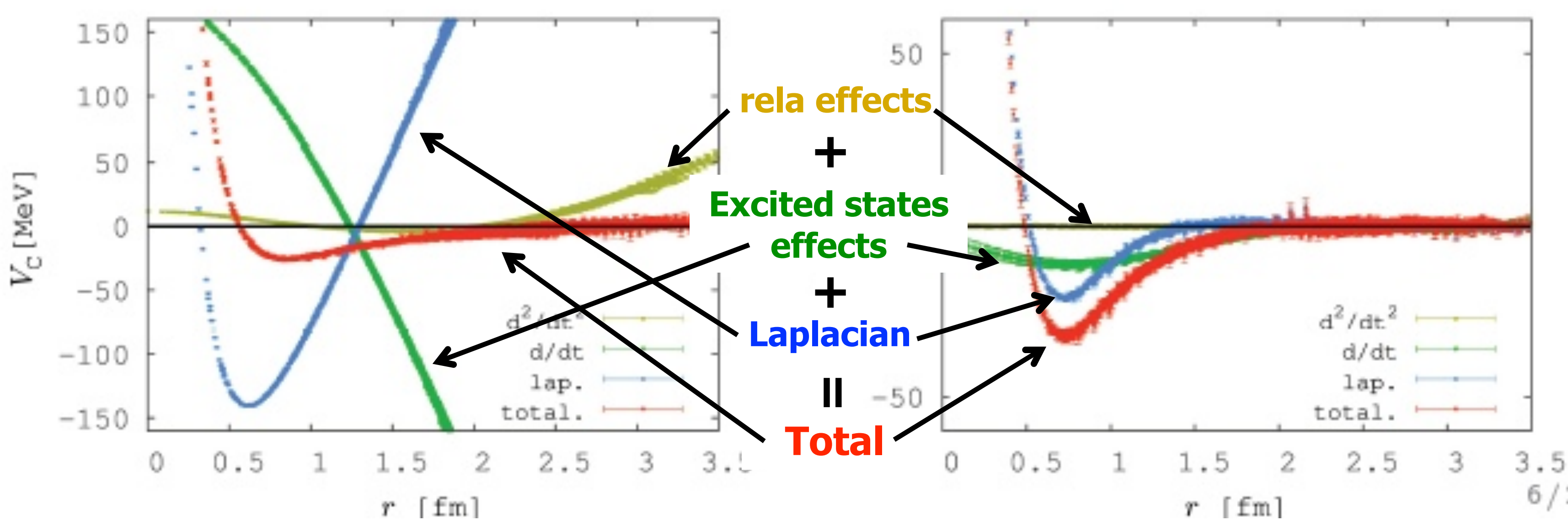
$$V(\vec{r}) = \frac{(\partial/\partial t)^2 R(\vec{r},t)}{4mR(\vec{r},t)} - \frac{(\partial/\partial t)R(\vec{r},t)}{R(\vec{r},t)} - \frac{H_0 R(\vec{r},t)}{R(\vec{r},t)}$$

- ∘ smeared. src. — strong t-dep.
  - $\Rightarrow \sim \mathcal{O}(100)$ MeV cancellation
- • t-dep. HAL method works well!!
- ∘ wall src. — weak t-dep.

NBS w.f.

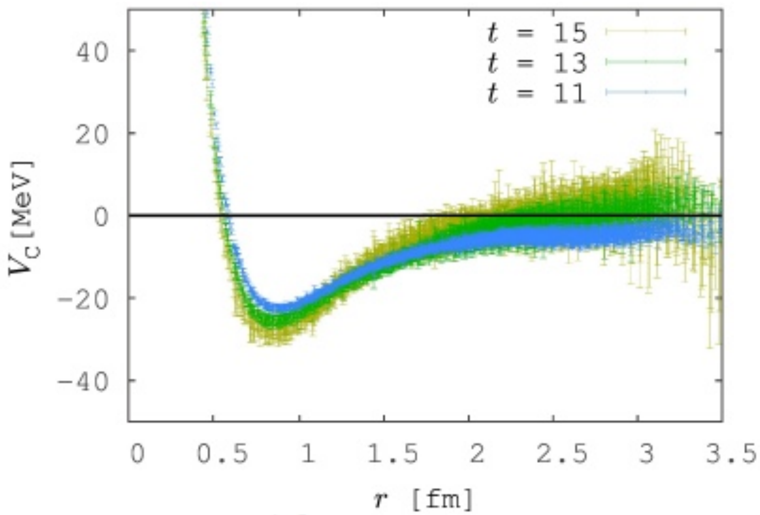


☐ smeared src. **Potential** ■ wall src.

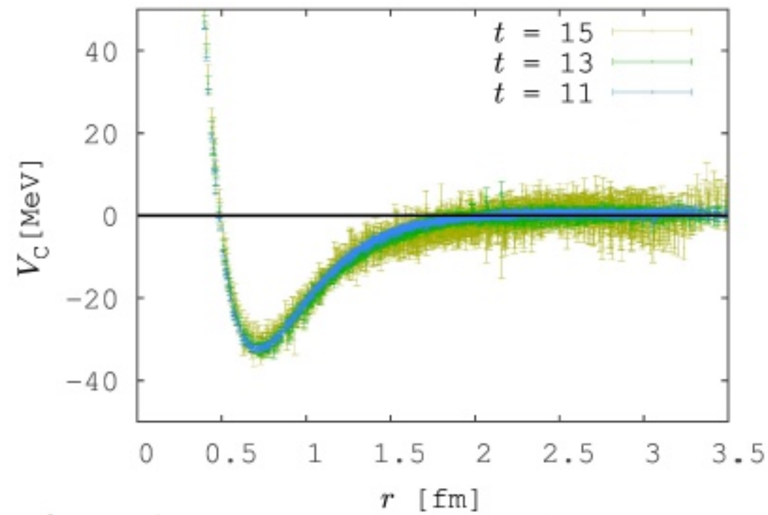


(2) HAL method: wall vs smeared src (cont'd)

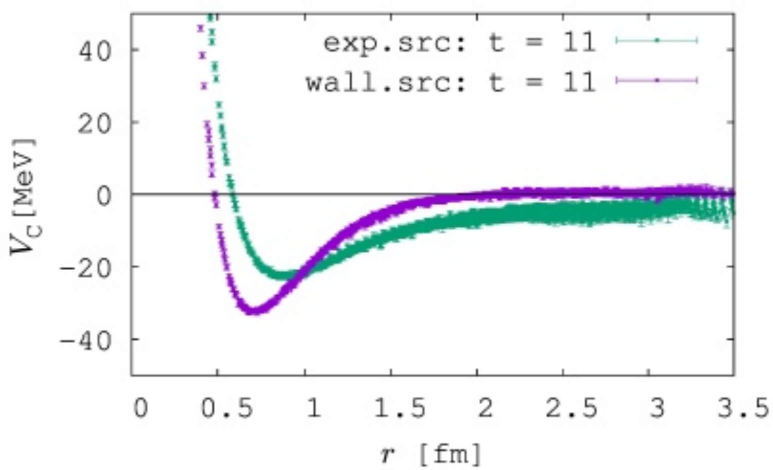
smeared: t-dep exists

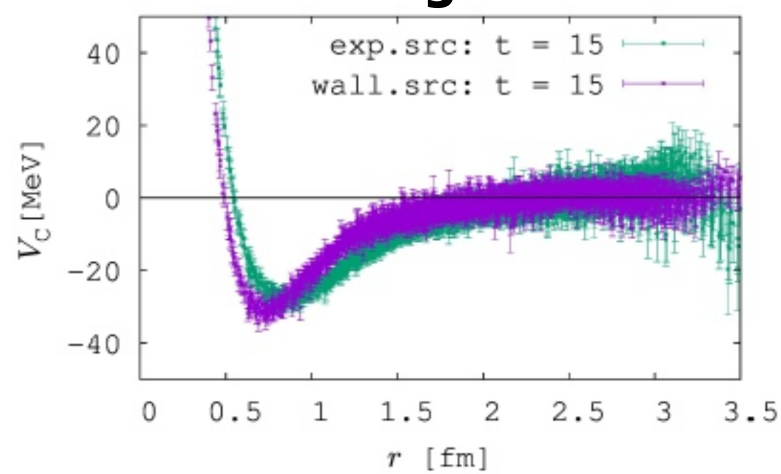


wall: t-dep negligible



smeared & wall in the same fig





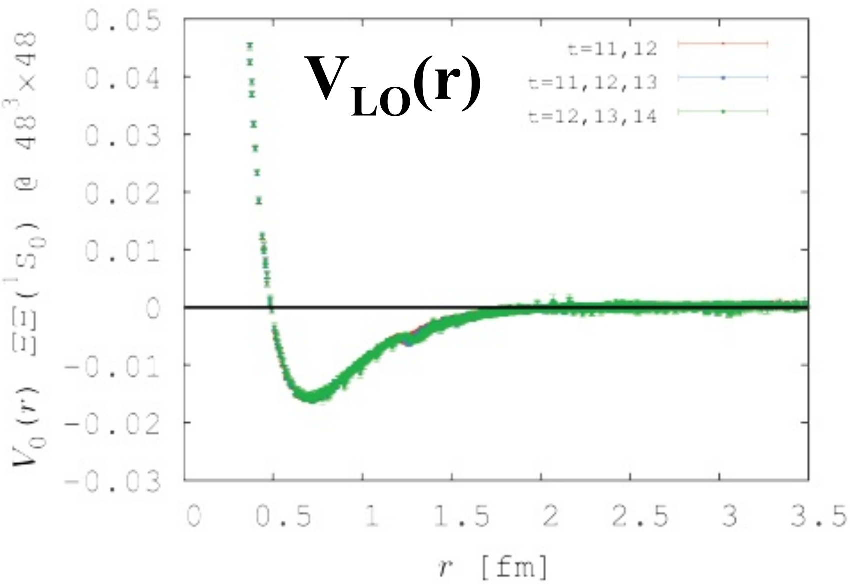
Smeared/Wall almost agree : t-dep HAL method works excellently

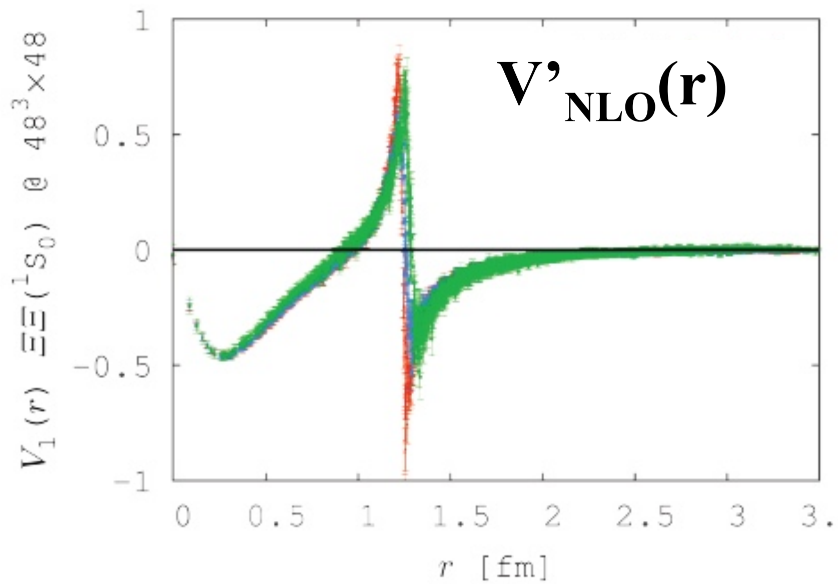
Smeared tends to converge to Wall w/ larger t, but deviation still exists

(2) HAL method: analysis w/ LO + NLO potentials

$$U(\vec{r},\vec{r}') = [V_{\rm LO}(\vec{r}) + V'_{\rm NLO}(\vec{r})\nabla^2]\,\delta(\vec{r}-\vec{r}')$$ (derivative expansion)

Combined analyses of wall & smeared data





The difference from wall / smeared are not fake but physics ($V_{\rm NLO}(r)$)

New method to obtain NLO potential !

We also found

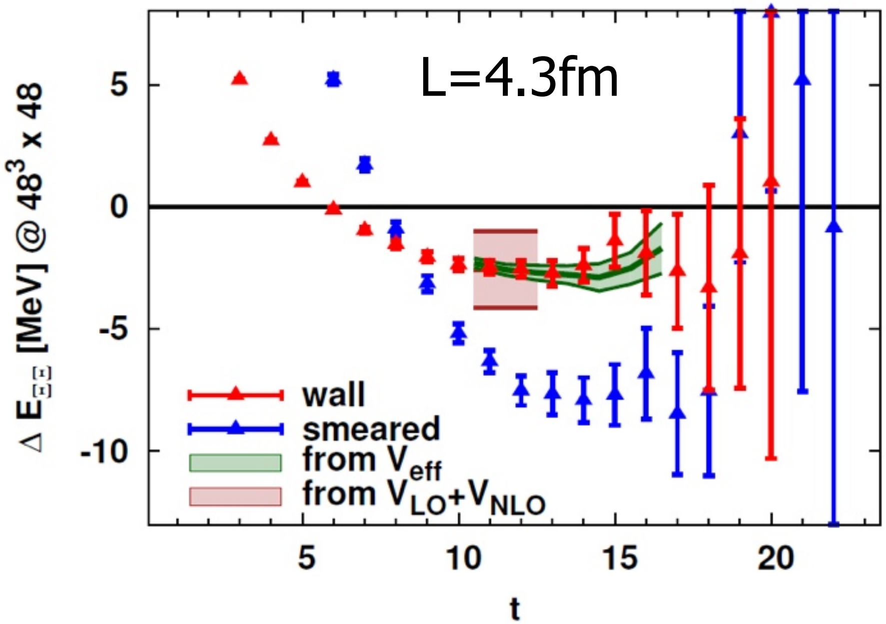
$V_{\rm eff}$ (r) from wall by (effective) LO analysis

$V_{\rm LO}$ (r) from wall & smeared by LO+NLO analysis

→ ! **consistent**

Smeared data contain much more excited states → more sensitive to NLO

(3) Comparison between Luscher and HAL



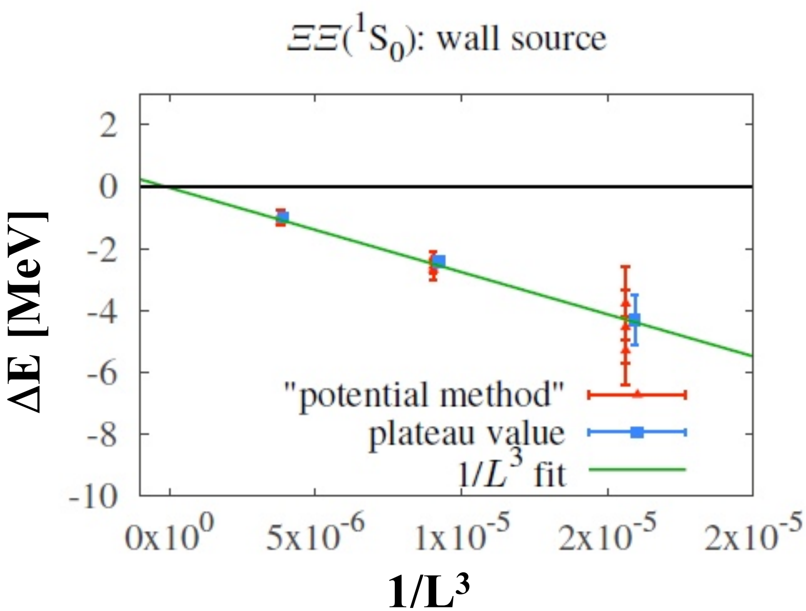
Finite V spectrum by V(r) are consistent btw:

(1) $V_{eff}(r)$ from wall

(2) $V_{LO}(r) + V_{NLO}(r)$ from wall & smeared

➔ **Good convergence in non-locality of V(r)**

➔ Indicate that previous HAL results ($V_{eff}(r)$ from wall) are reliable



FV spectrum by V(r) is consistent with "plateau" from wall in Luscher method (even for other volumes)

Volume dep of ΔE

➔ Not a bound state but a scattering state

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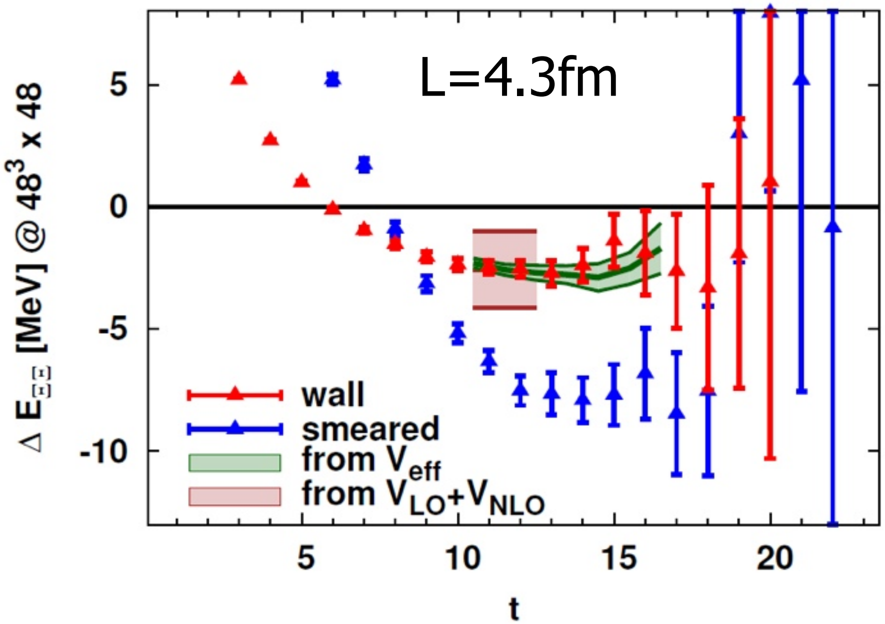

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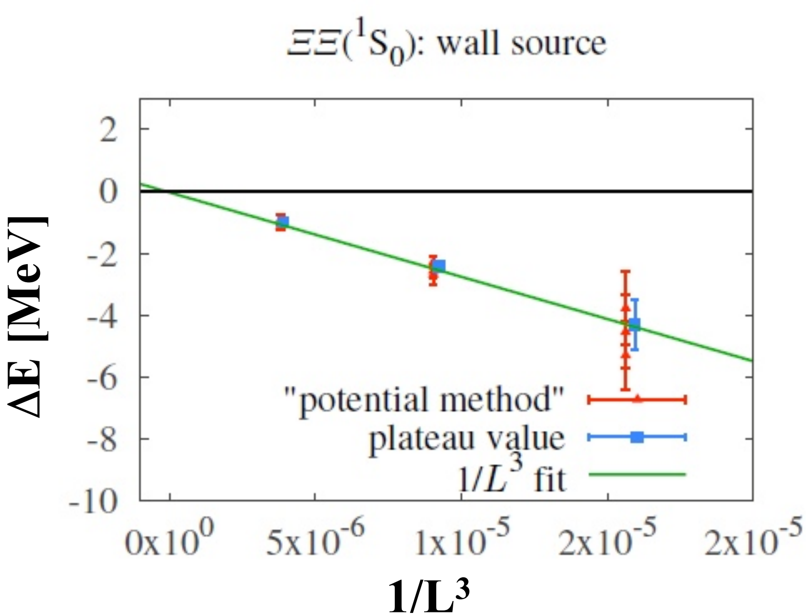

Volume dep of ΔE

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"Luscher vs HAL"

➔ **"Luscher (smeared) vs Luscher (wall)"**

Summary

- **Systematic study btw Luscher method and HAL method**
  - Nf=2+1 clover, m(pi) = 0.51 GeV, L = (2.9), 3.6, 4.3, 5.8fm
  - **wall & smeared src** for ΞΞ $^1S_0$ system
- **Luscher's method**
  - G.S. saturation is necessary, but difficult to achieve (**"Fake Plateau Crisis"**)
    - wall and smeared are inconsistent
- **HAL QCD method**
  - t-dep HAL method works well w/o G.S. saturation
  - V(r) (smeared) ➔ V(r) (wall) w/ larger t
  - LO + (small) NLO potential can explain the remaining difference
    - **New method to determine NLO potential**
  - FV spectra from V(r) are consistent w/ Luscher's method from wall src

"potential" is useful tool to reliably extract phase shifts in LQCD

- **Prospects / Comments**
  - We are increasing #stat ➔ NN ➔ direct comparison w/ Yamazaki et al.
  - **Luscher's method needs breakthrough or at least one should check src-dependence**

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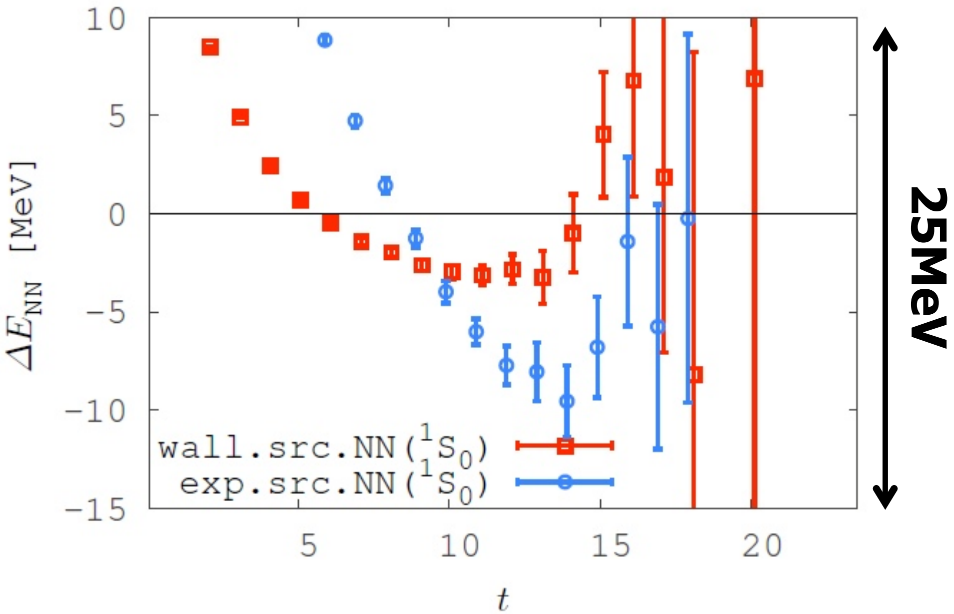
Backup Slides

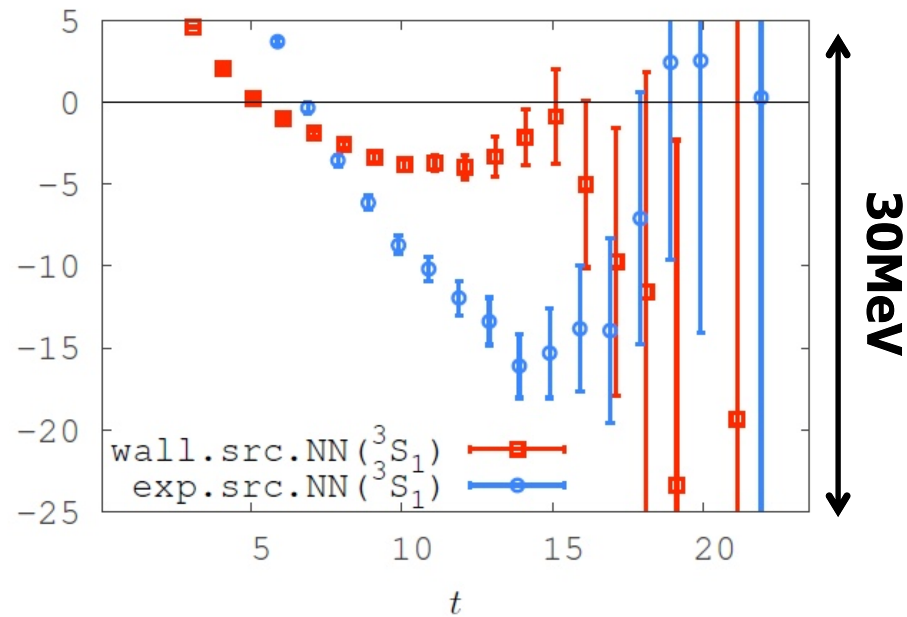
Preliminary results for NN

L = 4.3 fm (48^4)

$^1S_0$ "di-neutron"



$^3S_1$ "deuteron"



c.f. Yamazaki et al. (2012) by exp.src (smeared)

ΔE = 7.3(1.7)(0.5) MeV @ t=[10,14]

ΔE = 11.1(1.7)(0.3) MeV @ t=[10,14]

N.B. our #stat for smeared is > x5 of Yamazaki et al.