Hyperonic lattice QCD potentials and hypernuclear few-body problems

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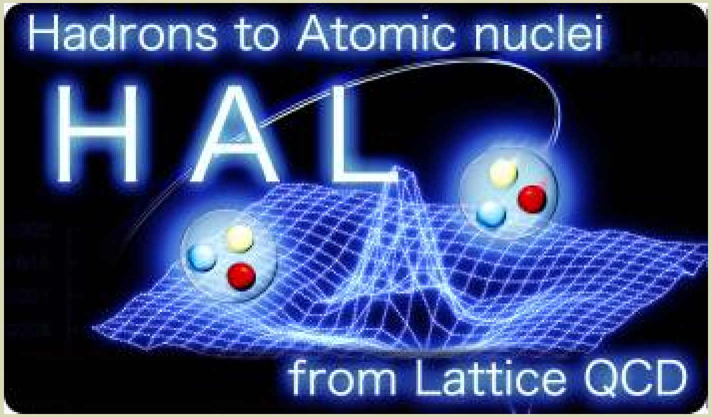

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Plan of research

QCD

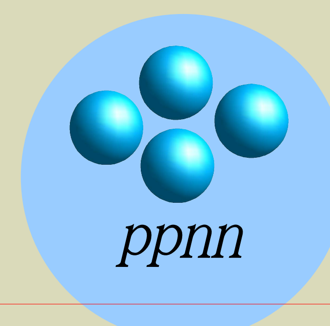
Baryon interaction

J-PARC
hyperon-nucleon (YN) scattering

Structure and reaction of (hyper)nuclei

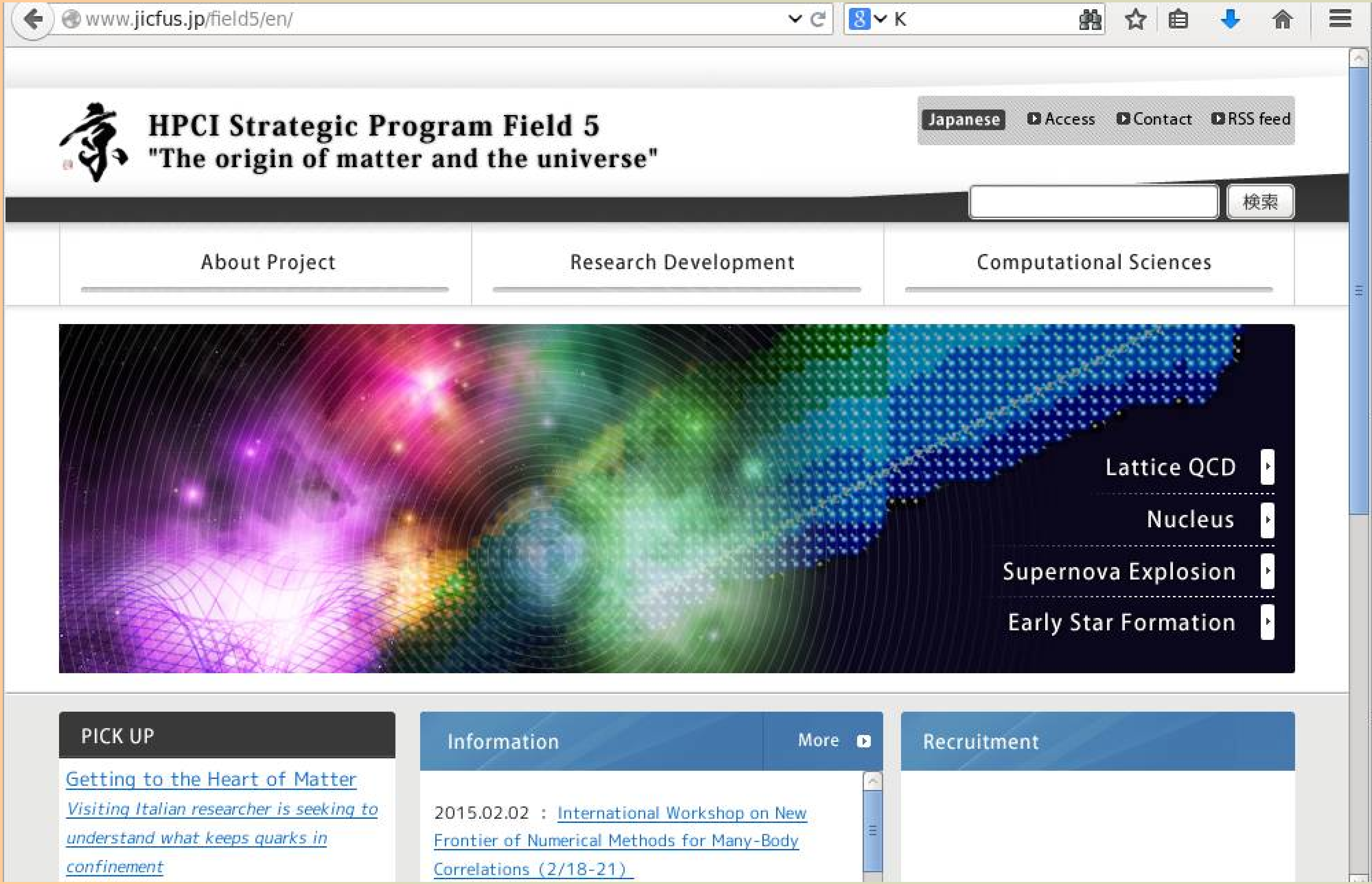
Equation of State (EoS) of nuclear matter

This talk: Stochastic variational calculation of 4He

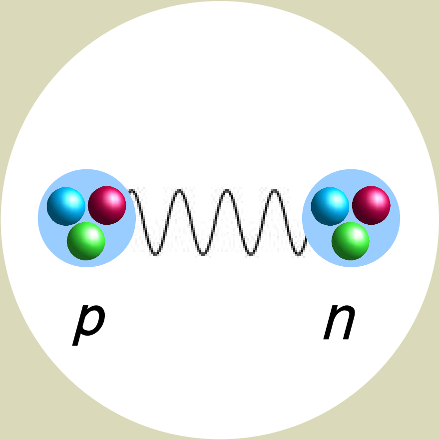


Neutron star and supernova

Determination of the baryon-baryon interactions using lattice QCD at the physical point

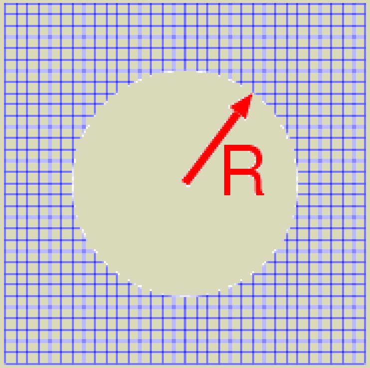


Lattice QCD calculation



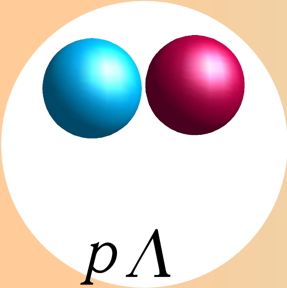
Formulation

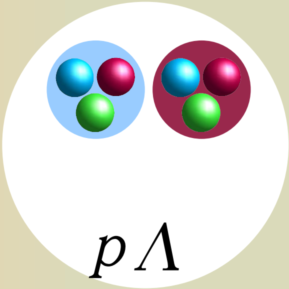
Lattice QCD simulation

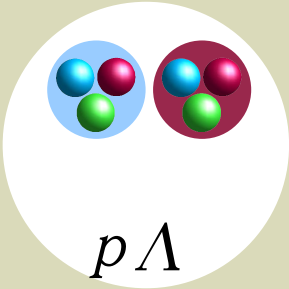


$$L = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \bar{q}\gamma^\mu (i\partial_\mu - g t^a A^a_\mu) q - m\bar{q} q$$

$$\begin{aligned}
\langle O(\bar{q}, q, U)\rangle &= \int dU\, d\bar{q}\, dq\, \mathrm{e}^{-S(\bar{q}, q, U)} O(\bar{q}, q, U) \\
&= \int dU \det D(U)\, \mathrm{e}^{-S_U(U)} O(D^{-1}(U)) \\
&= \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} O(D^{-1}(U_i))
\end{aligned}$$



$\rightarrow \langle$ 

(t) 

$\overline{(t_0)} \rangle$

Formulation

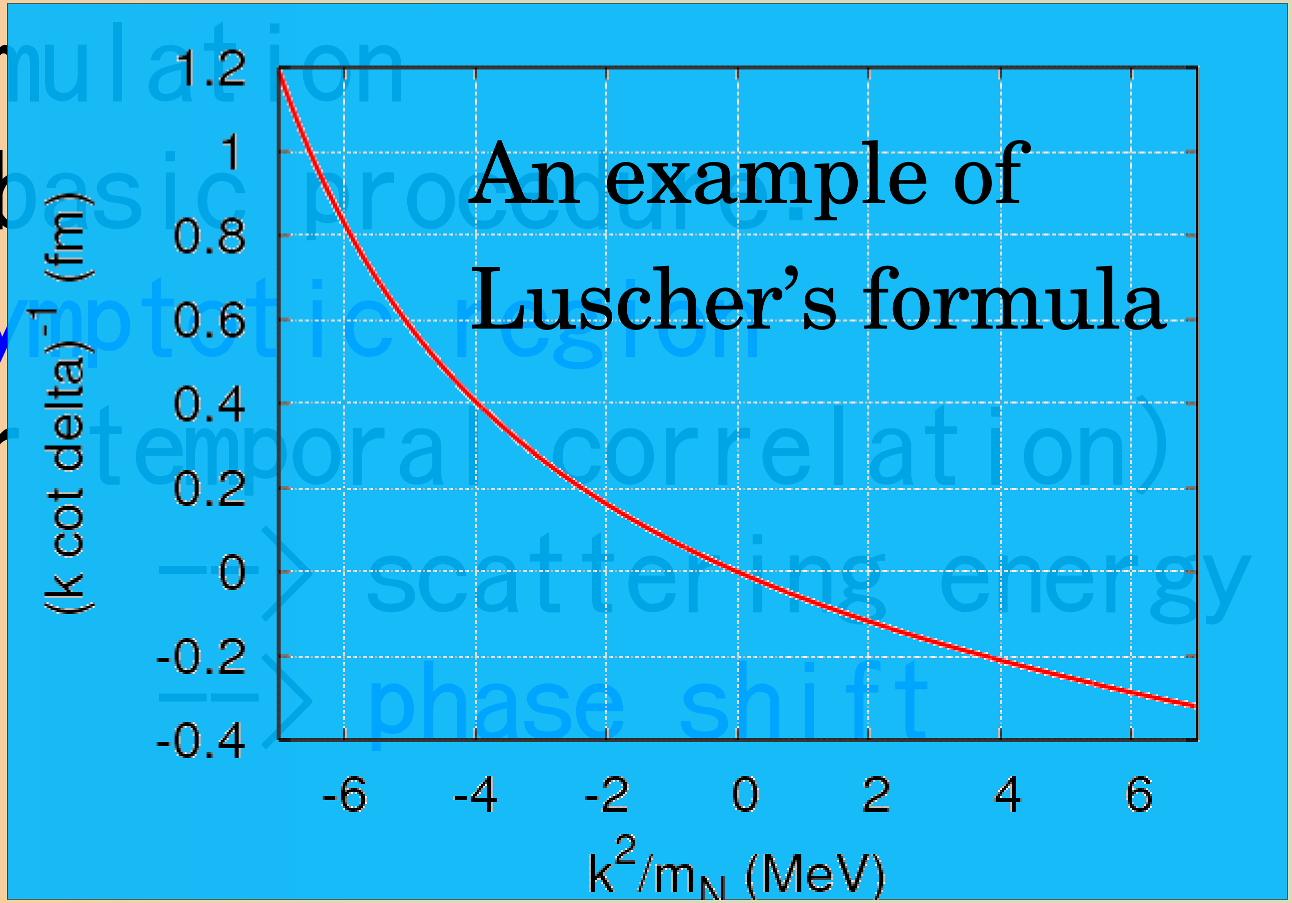
i) basic procedure:

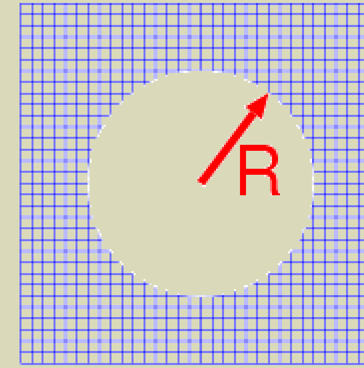
asymptotic region

(or temporal correlation)

--> scattering energy

--> phase shift





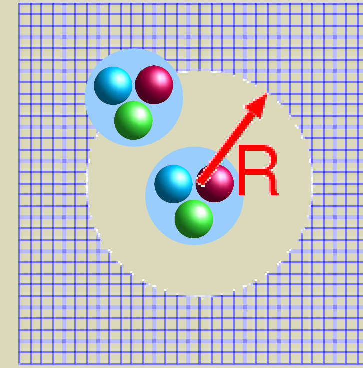
$$E = \frac{k^2}{2\mu}$$

$$k \cot \delta_0(k) = \frac{2}{\sqrt{\pi} L} Z_{00}(1; (kL/(2\pi))^2) = \frac{1}{a_0} + O(k^2)$$

$$Z_{00}(1; q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\vec{n} \in Z^3} \frac{1}{(n^2 - q^2)^s} \qquad \Re\, s > \frac{3}{2}$$

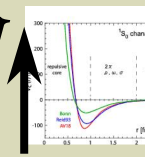
Luscher, NPB354, 531 (1991).
Aoki, et al., PRD71, 094504 (2005).

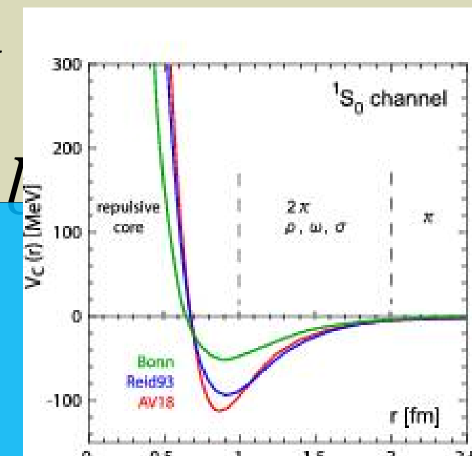
Formulation



Lattice QCD simulation

$$L=-\frac{1}{4}G^{a}_{\mu\nu}G^{a\mu\nu}+\bar{q}\gamma^{\mu}(i\partial_{\mu}-g\,t^{a}A^{a}_{\mu})q-m\bar{q}q$$



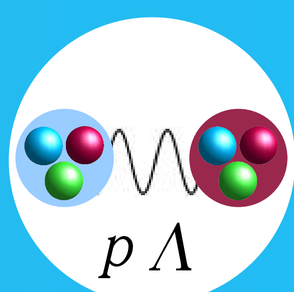


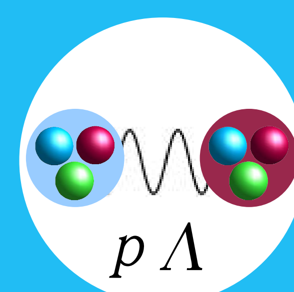
$$\langle O(\bar{q},q,U)\rangle=\int dU\,d\bar{q}\,dq\,e^{-S(\bar{q},q,U)}O(\bar{q},q$$

$$=\int dU\det D(U)e^{-S_U(U)}O(D^{-1}(U$$

$$=\lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}O(D^{-1}(U_i))$$

$$F^{(JM)}_{\alpha\beta}(\vec{r},t-t_0)$$

$\rightarrow\langle$ 

$(\vec{r},t)\;\overline{}$ 

$(t_0)\rangle$

Calculate the scattering state

Formulation

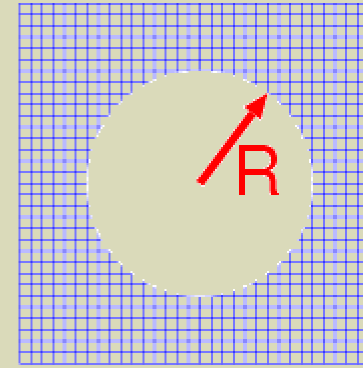
i) basic procedure:

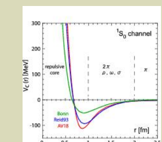
asymptotic region

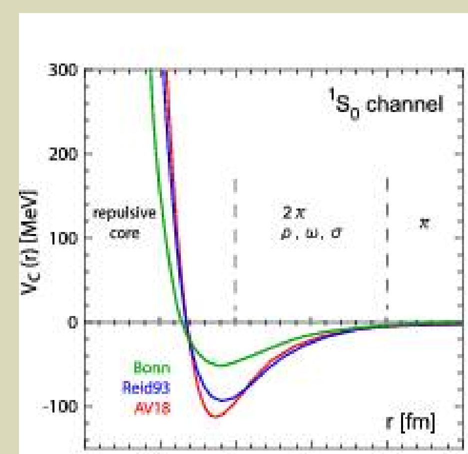
--> phase shift

ii) advanced (HAL's) procedure: interacting region

--> potential

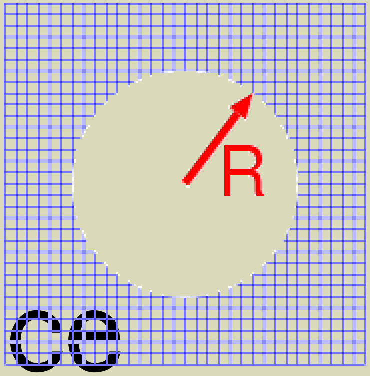






HAL formulation

ii) advanced procedure:



make better use of the lattice output ! (wave function)

interacting region

--> potential

Ishii, Aoki, Hatsuda, PRL99, 022001 (2007); ibid., arXiv:0805.2462[hep-ph].

NOTE:

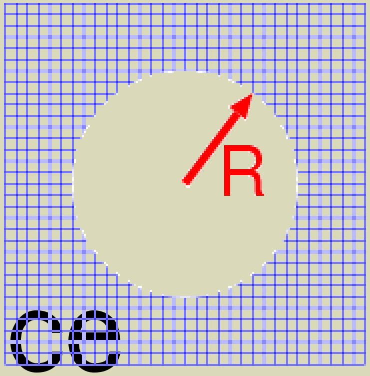
> Potential is not a direct experimental observable.

> Potential is a useful tool to give (and to reproduce) the physical quantities. (e.g., phase shift)

....

HAL formulation

ii) advanced procedure:
make better use of the lattice output ! (wave function)



interacting region
--> potential

Ishii, Aoki, Hatsuda,
PRL99, 022001 (2007);
ibid., arXiv:0805.2462[hep-ph].

> Phase shift
> Nuclear many-body problems

An improved recipe for NY potential:

- cf. Ishii (HAL QCD), PLB712 (2012) 437.

- Take account of not only the spatial correlation but also the temporal correlation in terms of the R-correlator:

$$-\frac{1}{2\mu}\nabla^2 R(t,\vec{r})+\int d^3r' \, U(\vec{r},\vec{r}')R(t,\vec{r}')=-\frac{\partial}{\partial t}R(t,\vec{r})$$

$$\rightarrow \frac{k^2}{2\mu}R(t,\vec{r})$$

$$U(\vec{r},\vec{r}')=V_{NY}(\vec{r},\nabla)\delta(\vec{r}-\vec{r}')$$

- A general expression of the potential:

$$V_{NY}=V_0(r)+V_\sigma(r)(\vec{\sigma}_N\cdot\vec{\sigma}_Y)$$
$$+V_T(r)S_{12}+V_{LS}(r)(\vec{L}\cdot\vec{S}_+)$$
$$+V_{ALS}(r)(\vec{L}\cdot\vec{S}_-)+O(\nabla^2)$$

Effective block algorithm for various baryon-baryon calculations

arXiv:1510.00903(hep-lat)

$$R^{(J,M)}_{\alpha\beta}(\vec{r},t-t_0) = \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X}+\vec{r},t) B_{2,\beta}(\vec{X},t) \overline{\mathcal{J}^{(J,M)}_{B_3B_4}(t_0)} \right| 0 \right\rangle / \exp\{-(m_{B_1}+m_{B_2})(t-t_0)\},$$

$$p = \varepsilon_{abc}\left(u_a C\gamma_5 d_b\right) u_c, \qquad n = -\varepsilon_{abc}\left(u_a C\gamma_5 d_b\right) d_c, \tag{2}$$

$$\Sigma^+ = -\varepsilon_{abc}\left(u_a C\gamma_5 s_b\right) u_c, \qquad \Sigma^- = -\varepsilon_{abc}\left(d_a C\gamma_5 s_b\right) d_c, \tag{3}$$

$$\Sigma^0 = \frac{1}{\sqrt{2}}\left(X_u - X_d\right), \qquad \Lambda = \frac{1}{\sqrt{6}}\left(X_u + X_d - 2X_s\right), \tag{4}$$

$$\Xi^0 = \varepsilon_{abc}\left(u_a C\gamma_5 s_b\right) s_c, \qquad \Xi^- = -\varepsilon_{abc}\left(d_a C\gamma_5 s_b\right) s_c, \tag{5}$$

where

$$X_u = \varepsilon_{abc}\left(d_a C\gamma_5 s_b\right) u_c, \quad X_d = \varepsilon_{abc}\left(s_a C\gamma_5 u_b\right) d_c, \quad X_s = \varepsilon_{abc}\left(u_a C\gamma_5 d_b\right) s_c, \tag{6}$$

$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) = \int d^3r'\, U(\vec{r},\vec{r}') R(\vec{r}',t) + O(k^4) = V_{\rm LO}(\vec{r}) R(\vec{r},t) + \cdots \tag{8}$$

$$p = \varepsilon_{abc}\left(u_a C\gamma_5 d_b\right) u_c, \qquad n = -\varepsilon_{abc}\left(u_a C\gamma_5 d_b\right) d_c, \qquad (2)$$

$$\Sigma^+ = \varepsilon_{abc}\left(u_a C\gamma_5 s_b\right) u_c, \qquad \Sigma^- = -\varepsilon_{abc}\left(d_a C\gamma_5 s_b\right) d_c, \qquad (3)$$

$$\Sigma^0 = \frac{1}{\sqrt{2}}\left(X_u - X_d\right), \qquad \Lambda = \frac{1}{\sqrt{6}}\left(X_u + X_d - 2X_s\right), \qquad (4)$$

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where

$$X_u = \varepsilon_{abc}\left(d_a C\gamma_5 s_b\right) u_c, \quad X_d = \varepsilon_{abc}\left(s_a C\gamma_5 u_b\right) d_c, \quad X_s = \varepsilon_{abc}\left(u_a C\gamma_5 d_b\right) s_c,$$

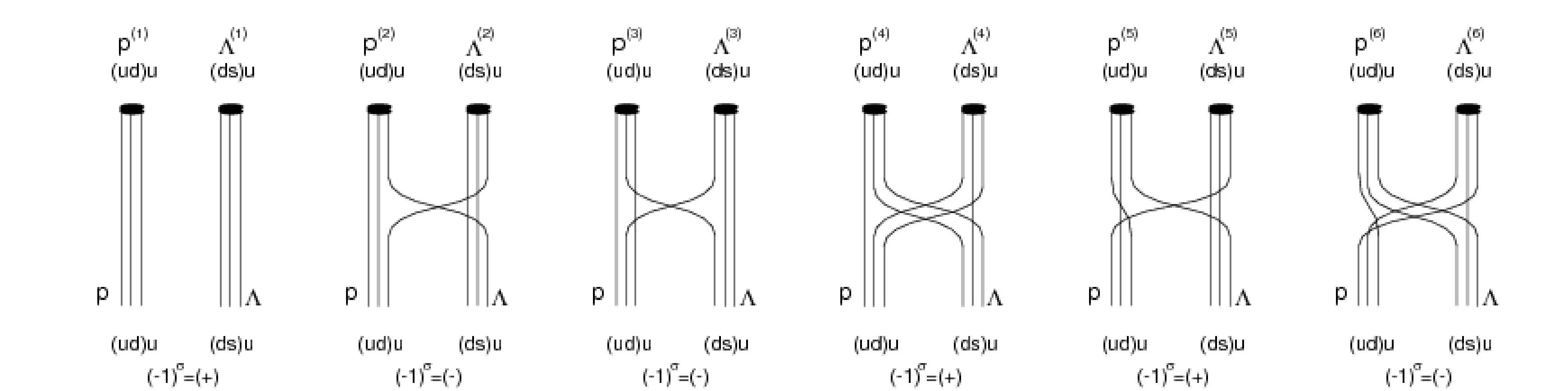
$$\begin{aligned} p_\alpha(x) &= \varepsilon(c_1, c_2, c_3)(C\gamma_5)(\alpha_1, \alpha_2)\delta(\alpha, \alpha_3)u(\xi_1)d(\xi_2)u(\xi_3), \qquad (\xi_i = x_i\alpha_i c_i) \\ &= \varepsilon(1,2,3)(C\gamma_5)(1,2)\delta(\alpha,3)u(1)d(2)u(3). \end{aligned} \qquad (11)$$

$$\begin{aligned} &\sum_{\vec{X}} \left\langle 0 \left| p_\alpha(\vec{X}+\vec{r},t)\Lambda_\beta(\vec{X},t)\overline{\mathcal{J}_{p_{\alpha'}\Lambda_{\beta'}}(t_0)} \right| 0 \right\rangle \\ = &\sum_{\vec{X}} \tfrac{1}{6}\varepsilon(1,4,2)\varepsilon(5,6,3)\varepsilon(1',4',2')\varepsilon(5',6',3')(C\gamma_5)(1,4)\delta(\alpha,2)(C\gamma_5)(1',4')\delta(\alpha',2') \\ &\times \{(C\gamma_5)(5,6)\delta(\beta,3) + (C\gamma_5)(6,3)\delta(\beta,5) - 2(C\gamma_5)(3,5)\delta(\beta,6)\} \\ &\times \{(C\gamma_5)(5',6')\delta(\beta',3') + (C\gamma_5)(6',3')\delta(\beta',5') - 2(C\gamma_5)(3',5')\delta(\beta',6')\} \\ &\times \langle u(1)d(4)u(2)d(5)s(6)u(3)\bar{u}(3')\bar{s}(6')\bar{d}(5')\bar{u}(2')\bar{d}(4')\bar{u}(1')\rangle. \end{aligned} \qquad (12)$$

$$\sum_{c_1,\ldots,c_6} \sum_{\alpha_1,\ldots,\alpha_6} \sum_{c_1',\ldots,c_6'} \sum_{\alpha_1',\ldots,\alpha_6'}$$

E.g., 3981312
for Lambda-Nucleon →

$$(N_c!\,N_\alpha)^{2B} \times N_u!\,N_d!\,N_s!$$



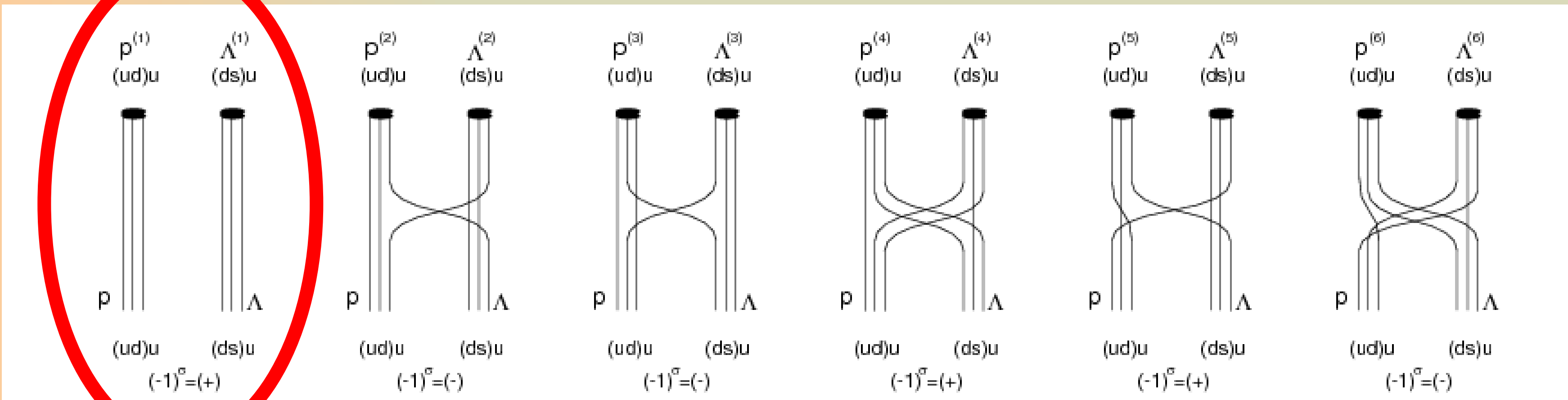
$$\begin{aligned} p_\alpha(x) &= \varepsilon(c_1,c_2,c_3)(C\gamma_5)(\alpha_1,\alpha_2)\delta(\alpha,\alpha_3)u(\xi_1)d(\xi_2)u(\xi_3), \qquad (\xi_i = x_i\alpha_i c_i) \\ &= \varepsilon(1,2,3)(C\gamma_5)(1,2)\delta(\alpha,3)u(1)d(2)u(3). \end{aligned} \tag{11}$$

$$\begin{aligned} &\sum_{\vec{X}} \left\langle 0 \left| p_\alpha(\vec{X}+\vec{r},t)\Lambda_\beta(\vec{X},t)\overline{\mathcal{J}_{p_{\alpha'}\Lambda_{\beta'}}(t_0)} \right| 0 \right\rangle \\ = \;& \sum_{\vec{X}} \tfrac{1}{6}\varepsilon(1,4,2)\varepsilon(5,6,3)\varepsilon(1',4',2')\varepsilon(5',6',3')(C\gamma_5)(1,4)\delta(\alpha,2)(C\gamma_5)(1',4')\delta(\alpha',2') \\ & \times \{(C\gamma_5)(5,6)\delta(\beta,3)+(C\gamma_5)(6,3)\delta(\beta,5)-2(C\gamma_5)(3,5)\delta(\beta,6)\} \\ & \times \{(C\gamma_5)(5',6')\delta(\beta',3')+(C\gamma_5)(6',3')\delta(\beta',5')-2(C\gamma_5)(3',5')\delta(\beta',6')\} \\ & \times \langle u(1)d(4)u(2)d(5)s(6)u(3)\bar{u}(3')\bar{s}(6')\bar{d}(5')\bar{u}(2')\bar{d}(4')\bar{u}(1')\rangle. \end{aligned} \tag{12}$$

$$\sum_{c_1,\dots,c_6}\;\sum_{\alpha_1,\dots,\alpha_6}\;\sum_{c_1',\dots,c_6'}\;\sum_{\alpha_1',\dots,\alpha_6'}$$

E.g., 3981312
for Lambda-Nucleon →

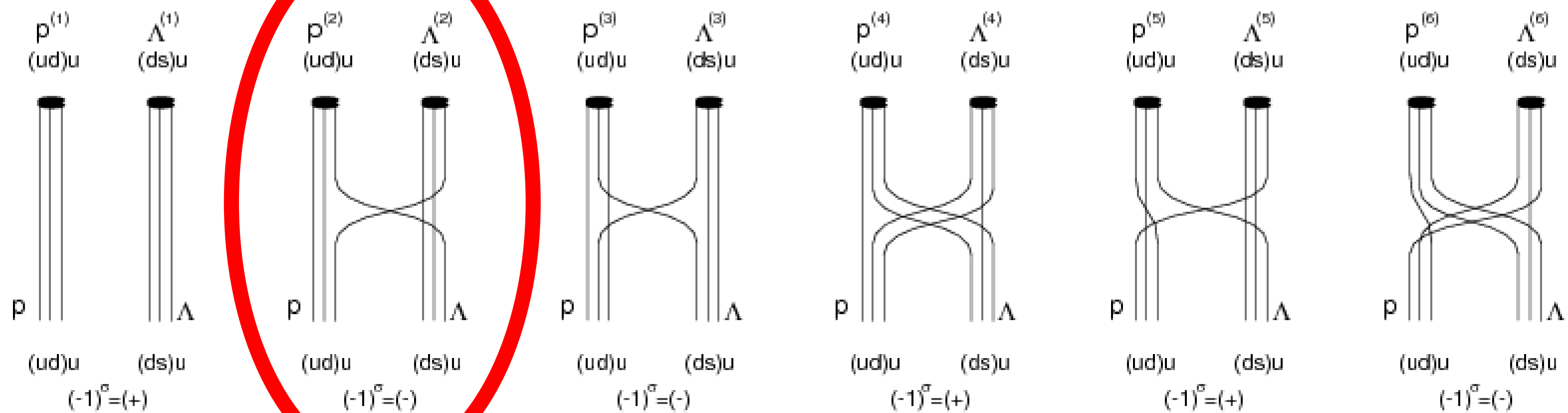
$$(N_c!\,N_\alpha)^{2B}\times N_u!\,N_d!\,N_s!$$



$$
\begin{aligned}
[p^{(1)}_{\alpha\alpha'}](\vec{x}) &= \varepsilon(1,4,2)(C\gamma_5)(1,4)\delta(\alpha,2)\varepsilon(1',4',2')(C\gamma_5)(1',4')\delta(\alpha',2') \\
&\times \det \begin{vmatrix} \langle u(1)\bar{u}(1')\rangle & \langle u(1)\bar{u}(2')\rangle \\ \langle u(2)\bar{u}(1')\rangle & \langle u(2)\bar{u}(2')\rangle \end{vmatrix} \langle d(4)\bar{d}(4')\rangle,
\end{aligned}
$$

$$
\begin{aligned}
[\Lambda^{(1)}_{\beta\beta'}](\vec{y}) &= \langle u(3)\bar{u}(3')\rangle\langle d(5)\bar{d}(5')\rangle\langle s(6)\bar{s}(6')\rangle \\
&\times\varepsilon(5,6,3)\left\{(C\gamma_5)(5,6)\delta(\beta,3) + (C\gamma_5)(6,3)\delta(\beta,5) - 2(C\gamma_5)(3,5)\delta(\beta,6)\right\} \\
&\times\varepsilon(5',6',3')\left\{(C\gamma_5)(5',6')\delta(\beta',3')\right\}.
\end{aligned} \tag{18}
$$

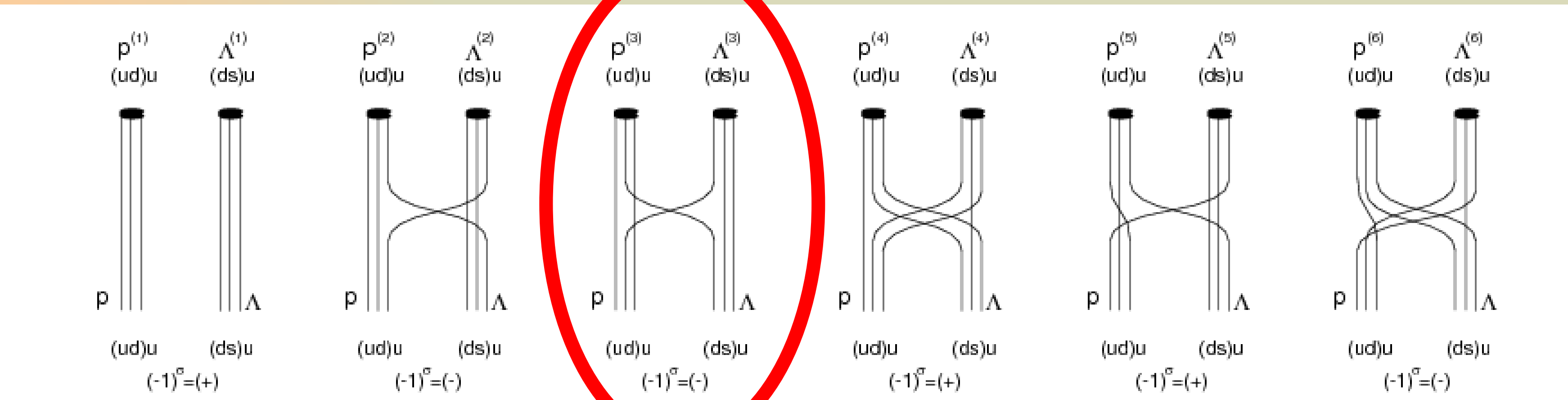
This fact significantly slashes in the computational cost: The reduction factor at the first diagram is $(N_c!N_\alpha)^2 \times 2^{B-N_\Lambda-N_{\Sigma^0}}/1 = 1152$.



$$
\begin{aligned}
[p^{(2)}_{\alpha\beta'}](\vec{x};c'_2,c'_3) &= \varepsilon(1,4,2)(C\gamma_5)(1,4)\delta(\alpha,2)\varepsilon(1',4',2')(C\gamma_5)(1',4')\times\delta(\beta',3') \\
&\times \det\begin{vmatrix} \langle u(1)\bar{u}(1')\rangle & \langle u(1)\bar{u}(3')\rangle \\ \langle u(2)\bar{u}(1')\rangle & \langle u(2)\bar{u}(3')\rangle \end{vmatrix} \langle d(4)\bar{d}(4')\rangle,
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
[\Lambda^{(2)}_{\beta;\alpha'}](\vec{y};c'_2,c'_3) &= \langle u(3)\bar{u}(2')\rangle\langle d(5)\bar{d}(5')\rangle\langle s(6)\bar{s}(6')\rangle\times\delta(\alpha',2') \\
&\times\varepsilon(5,6,3)\left\{(C\gamma_5)(5,6)\delta(\beta,3)+(C\gamma_5)(6,3)\delta(\beta,5)-2(C\gamma_5)(3,5)\delta(\beta,6)\right\} \\
&\times\varepsilon(5',6',3')\left\{(C\gamma_5)(5',6')\right\}.
\end{aligned} \tag{22}
$$

are crossed as $[p^{(2)}_{\alpha\beta'}]$ and $[\Lambda^{(2)}_{\beta\alpha'}]$. Performed these manipulations, the number of explicit summations of indices reduces to only two colors which makes the reduction factor $(N_c!N_\alpha)^2\times 2^{B-N_\Lambda-N_{\Sigma^0}}/(N_c^2) = 128$.



$$[p^{(3)}_{\alpha;\alpha'}](\vec{x}; c'_4, c'_5, \alpha'_4, \alpha'_5) \tag{25}$$

$$= \varepsilon(1,4,2)(C\gamma_5)(1,4)\delta(\alpha,2)\varepsilon(1',4',2')(C\gamma_5)(1',4')\delta(\alpha',2') \tag{26}$$

$$\times \det \begin{vmatrix} \langle u(1)\bar{u}(1')\rangle & \langle u(1)\bar{u}(2')\rangle \\ \langle u(2)\bar{u}(1')\rangle & \langle u(2)\bar{u}(2')\rangle \end{vmatrix} \langle d(4)\bar{d}(5')\rangle, \tag{27}$$

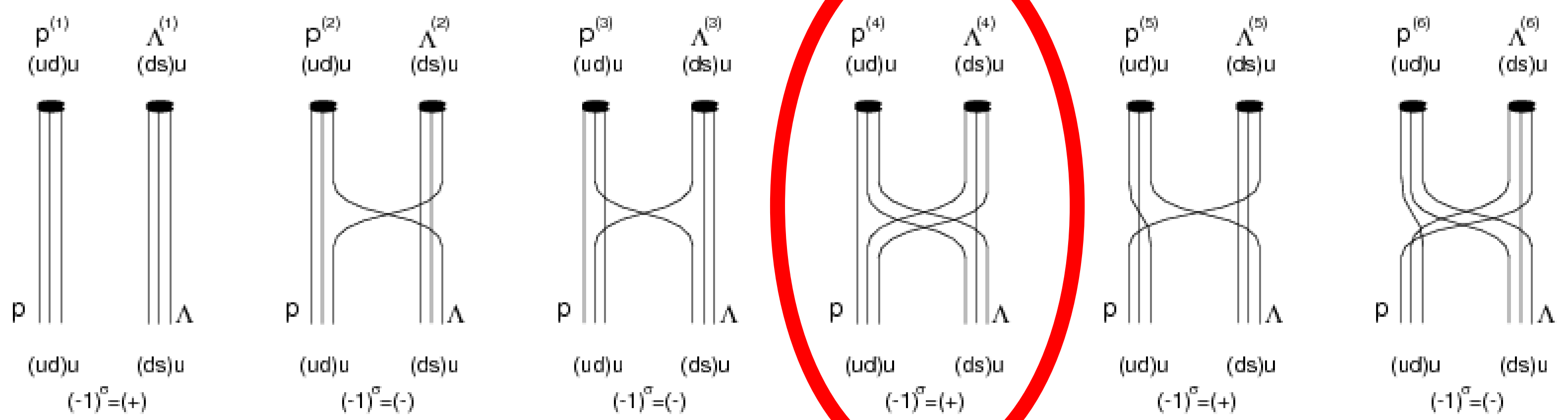
$$[\Lambda^{(3)}_{\beta;\beta'}](\vec{y}; c'_4, c'_5, \alpha'_4, \alpha'_5) \tag{28}$$

$$= \varepsilon(5,6,3)\left\{(C\gamma_5)(5,6)\delta(\beta,3) + (C\gamma_5)(6,3)\delta(\beta,5) - 2(C\gamma_5)(3,5)\delta(\beta,6)\right\} \tag{29}$$

$$\times\varepsilon(5',6',3')\left\{(C\gamma_5)(5',6')\delta(\beta',3')\right\}$$

$$\times\langle u(3)\bar{u}(3')\rangle\langle d(5)\bar{d}(4')\rangle\langle s(6)\bar{s}(6')\rangle. \tag{30}$$

The number of explicit summations of indices reduces to two colors and two spinors which makes the reduction factor $(N_c!N_\alpha)^2 \times 2^{B-N_\Lambda-N_{\Sigma^0}}/(N_c^2 N_\alpha^2) = 8$.



$$[p^{(4)}_{\alpha;\beta'}](\vec{x};c'_1,c'_6,\alpha'_1,\alpha'_6) \tag{33}$$

$$= \varepsilon(1,4,2)(C\gamma_5)(1,4)\delta(\alpha,2) \tag{34}$$

$$\times\varepsilon(5',6',3')\left\{(C\gamma_5)(5',6')\delta(\beta',3')\right\}$$

$$\times \det\begin{vmatrix} \langle u(1)\bar{u}(1')\rangle & \langle u(1)\bar{u}(3')\rangle \\ \langle u(2)\bar{u}(1')\rangle & \langle u(2)\bar{u}(3')\rangle \end{vmatrix} \langle d(4)\bar{d}(5')\rangle, \tag{35}$$

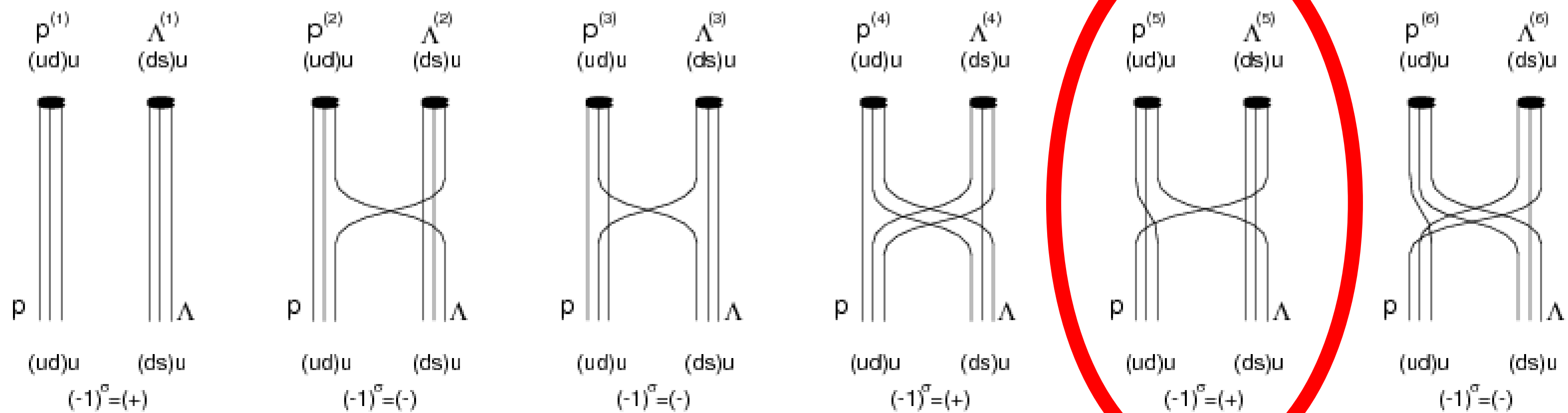
$$[\Lambda^{(4)}_{\beta;\alpha'}](\vec{y};c'_1,c'_6,\alpha'_1,\alpha'_6) \tag{36}$$

$$= \varepsilon(5,6,3)\left\{(C\gamma_5)(5,6)\delta(\beta,3)+(C\gamma_5)(6,3)\delta(\beta,5)-2(C\gamma_5)(3,5)\delta(\beta,6)\right\} \tag{37}$$

$$\times\varepsilon(1',4',2')(C\gamma_5)(1',4')\delta(\alpha',2') \tag{38}$$

$$\times\langle u(3)\bar{u}(2')\rangle\langle d(5)\bar{d}(4')\rangle\langle s(6)\bar{s}(6')\rangle. \tag{39}$$

exchanged, too. The number of explicit summations of indices reduces to two colors and two spinors which makes the reduction factor $(N_c!N_\alpha)^2 \times 2^{B-N_\Lambda-N_{\Sigma^0}}/(N_c^2N_\alpha^2) = 8$.



$$[p^{(5)}_{\alpha\alpha'\beta'}](\vec{x}; c'_1, c'_3, \alpha'_1) \tag{42}$$

$$= \varepsilon(1,4,2)(C\gamma_5)(1,4)\delta(\alpha,2)\varepsilon(1',4',2')(C\gamma_5)(1',4')\delta(\alpha',2') \times \delta(\beta',3') \tag{43}$$

$$\times \det \begin{vmatrix} \langle u(1)\bar{u}(2')\rangle & \langle u(1)\bar{u}(3')\rangle \\ \langle u(2)\bar{u}(2')\rangle & \langle u(2)\bar{u}(3')\rangle \end{vmatrix} \langle d(4)\bar{d}(4')\rangle, \tag{44}$$

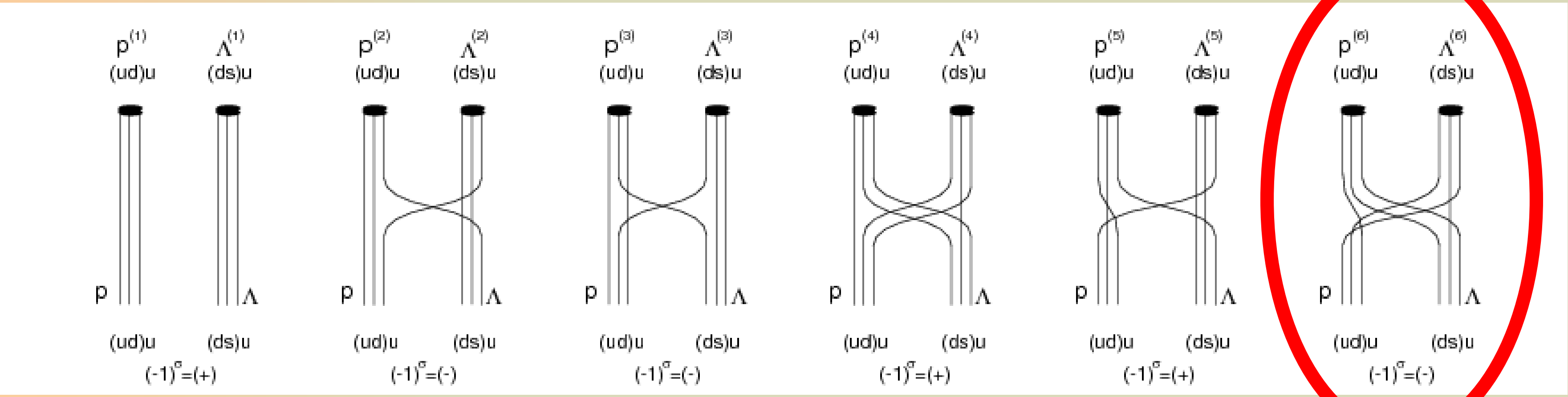
$$[\Lambda^{(5)}_{\beta}](\vec{y}; c'_1, c'_3, \alpha'_1) \tag{45}$$

$$= \varepsilon(5,6,3)\{(C\gamma_5)(5,6)\delta(\beta,3) + (C\gamma_5)(6,3)\delta(\beta,5) - 2(C\gamma_5)(3,5)\delta(\beta,6)\} \tag{46}$$

$$\times\varepsilon(5',6',3')\{(C\gamma_5)(5',6')\}$$

$$\times\langle u(3)\bar{u}(1')\rangle\langle d(5)\bar{d}(5')\rangle\langle s(6)\bar{s}(6')\rangle. \tag{47}$$

accompanied in the $[p^{(5)}_{\alpha\alpha'\beta'}]$. The number of explicit summations of indices reduces to two colors and one spinor which makes the reduction factor $(N_c!N_\alpha)^2 \times 2^{B-N_\Lambda-N_{\Sigma^0}}/(N_c^2 N_\alpha) = 32.$



$$[p^{(6)}_{\alpha\alpha'\beta'}](\vec{x}; c'_2, c'_6, \alpha'_6) \tag{50}$$

$$= \varepsilon(1,4,2)(C\gamma_5)(1,4)\delta(\alpha,2) \times \delta(\alpha',2') \tag{51}$$

$$\times \varepsilon(5',6',3') \{(C\gamma_5)(5',6')\delta(\beta',3')\}$$

$$\times \det \begin{vmatrix} \langle u(1)\bar{u}(2')\rangle & \langle u(1)\bar{u}(3')\rangle \\ \langle u(2)\bar{u}(2')\rangle & \langle u(2)\bar{u}(3')\rangle \end{vmatrix} \langle d(4)\bar{d}(5')\rangle, \tag{52}$$

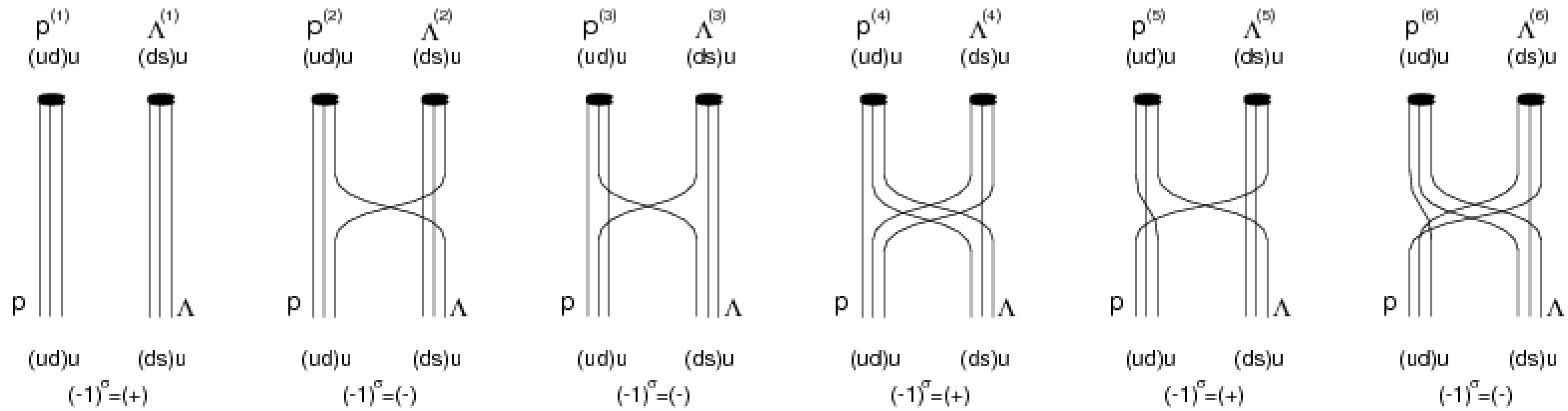
$$[\Lambda^{(6)}_{\beta}](\vec{y}; c'_2, c'_6, \alpha'_6) \tag{53}$$

$$= \varepsilon(5,6,3) \{(C\gamma_5)(5,6)\delta(\beta,3) + (C\gamma_5)(6,3)\delta(\beta,5) - 2(C\gamma_5)(3,5)\delta(\beta,6)\} \tag{54}$$

$$\times \varepsilon(1',4',2')(C\gamma_5)(1',4') \tag{55}$$

$$\times \langle u(3)\bar{u}(1')\rangle \langle d(5)\bar{d}(4')\rangle \langle s(6)\bar{s}(6')\rangle. \tag{56}$$

are exchanged between $[p^{(6)}_{\alpha;\alpha'\beta'}]$ and $[\Lambda^{(6)}_{\beta}]$ while $\delta(\alpha',2')$ is kept in $[p^{(6)}_{\alpha\alpha'\beta'}]$. The number of explicit summations of indices reduces to two colors and one spinor which makes the reduction factor $(N_c!N_\alpha)^2 \times 2^{B-N_\Lambda-N_{\Sigma^0}}/(N_c^2 N_\alpha) = 32$.



Performed these manipulations based on the diagrammatic classification, most of the summations can be carried out prior to evaluating the FFT so that the number of iterations significantly reduces; The numbers of iteration are $\{1, 9, 144, 144, 36, 36\}$ for the baryon blocks $\{([p_\alpha^{(i)}] \times [\Lambda_\beta^{(i)}]); i = 1, \cdots, 6\}$. Therefore only 370 iterations should be explicitly performed to obtain the four-point correlation function of the $p\Lambda$ system when we take the operator $\overline{X}_u$ in $\overline{\Lambda}_{\beta'}$ in the source. For the sake of completeness, the total number of iterations does not change when we take the operator $\overline{X}_s$ in $\overline{\Lambda}_{\beta'}$ in the source whereas the numbers of iteration are $\{1, 36, 36, 144, 144, 36\}$ when we consider the contribution from the operator $\overline{X}_d$ in $\overline{\Lambda}_{\beta'}$ in the source which slightly differ from the former cases and the total number of iterations is 397.

Generalization to the various baryon-baryon channels strangeness S=0 to -4 systems

$$\langle pn\overline{pn}\rangle, \tag{4.1}$$

$$\begin{aligned}
&\langle p\Lambda\overline{p\Lambda}\rangle, \quad \langle p\Lambda\overline{\Sigma^+ n}\rangle, \quad \langle p\Lambda\overline{\Sigma^0 p}\rangle, \\
&\langle \Sigma^+ n\overline{p\Lambda}\rangle, \langle \Sigma^+ n\overline{\Sigma^+ n}\rangle, \langle \Sigma^+ n\overline{\Sigma^0 p}\rangle, \\
&\langle \Sigma^0 p\overline{p\Lambda}\rangle, \langle \Sigma^0 p\overline{\Sigma^+ n}\rangle, \langle \Sigma^0 p\overline{\Sigma^0 p}\rangle,
\end{aligned} \tag{4.2}$$

$$\begin{aligned}
&\langle\Lambda\Lambda\overline{\Lambda\Lambda}\rangle, \quad \langle\Lambda\Lambda\overline{p\Xi^-}\rangle, \quad \langle\Lambda\Lambda\overline{n\Xi^0}\rangle, \quad \langle\Lambda\Lambda\overline{\Sigma^+\Sigma^-}\rangle, \quad \langle\Lambda\Lambda\overline{\Sigma^0\Sigma^0}\rangle, \\
&\langle p\Xi^-\overline{\Lambda\Lambda}\rangle, \quad \langle p\Xi^-\overline{p\Xi^-}\rangle, \quad \langle p\Xi^-\overline{n\Xi^0}\rangle, \quad \langle p\Xi^-\overline{\Sigma^+\Sigma^-}\rangle, \quad \langle p\Xi^-\overline{\Sigma^0\Sigma^0}\rangle, \quad \langle p\Xi^-\overline{\Sigma^0\Lambda}\rangle, \\
&\langle n\Xi^0\overline{\Lambda\Lambda}\rangle, \quad \langle n\Xi^0\overline{p\Xi^-}\rangle, \quad \langle n\Xi^0\overline{n\Xi^0}\rangle, \quad \langle n\Xi^0\overline{\Sigma^+\Sigma^-}\rangle, \quad \langle n\Xi^0\overline{\Sigma^0\Sigma^0}\rangle, \quad \langle n\Xi^0\overline{\Sigma^0\Lambda}\rangle, \\
&\langle\Sigma^+\Sigma^-\overline{\Lambda\Lambda}\rangle, \langle\Sigma^+\Sigma^-\overline{p\Xi^-}\rangle, \langle\Sigma^+\Sigma^-\overline{n\Xi^0}\rangle, \langle\Sigma^+\Sigma^-\overline{\Sigma^+\Sigma^-}\rangle, \langle\Sigma^+\Sigma^-\overline{\Sigma^0\Sigma^0}\rangle, \langle\Sigma^+\Sigma^-\overline{\Sigma^0\Lambda}\rangle, \\
&\langle\Sigma^0\Sigma^0\overline{\Lambda\Lambda}\rangle, \quad \langle\Sigma^0\Sigma^0\overline{p\Xi^-}\rangle, \quad \langle\Sigma^0\Sigma^0\overline{n\Xi^0}\rangle, \quad \langle\Sigma^0\Sigma^0\overline{\Sigma^+\Sigma^-}\rangle, \quad \langle\Sigma^0\Sigma^0\overline{\Sigma^0\Sigma^0}\rangle, \\
&\qquad \langle\Sigma^0\Lambda\overline{p\Xi^-}\rangle, \quad \langle\Sigma^0\Lambda\overline{n\Xi^0}\rangle, \quad \langle\Sigma^0\Lambda\overline{\Sigma^+\Sigma^-}\rangle, \qquad \langle\Sigma^0\Lambda\overline{\Sigma^0\Lambda}\rangle,
\end{aligned} \tag{4.3}$$

$$\begin{aligned}
&\langle\Xi^-\Lambda\overline{\Xi^-\Lambda}\rangle, \quad \langle\Xi^-\Lambda\overline{\Sigma^-\Xi^0}\rangle, \quad \langle\Xi^-\Lambda\overline{\Sigma^0\Xi^-}\rangle, \\
&\langle\Sigma^-\Xi^0\overline{\Xi^-\Lambda}\rangle, \langle\Sigma^-\Xi^0\overline{\Sigma^-\Xi^0}\rangle, \langle\Sigma^-\Xi^0\overline{\Sigma^0\Xi^-}\rangle, \\
&\langle\Sigma^0\Xi^-\overline{\Xi^-\Lambda}\rangle, \langle\Sigma^0\Xi^-\overline{\Sigma^-\Xi^0}\rangle, \langle\Sigma^0\Xi^-\overline{\Sigma^0\Xi^-}\rangle,
\end{aligned} \tag{4.4}$$

$$\langle\Xi^-\Xi^0\overline{\Xi^-\Xi^0}\rangle. \tag{4.5}$$

Make better use of the computing resources!

Generalization to the various baryon-baryon channels strangeness S=0 and -1 systems

channel	# of diagrams	$\{(\#\text{ of iterations})^{\text{sign}}\}$	# of total iterations
$\langle pn\overline{pn}\rangle$	9	$\{1^+, 36^-, 144^-, 36^+, 36^+, 144^-, 144^+, 9^-, 36^+\}$	586
$\langle p\Lambda\overline{p\Lambda_{X_{u,s}}}\rangle$	6	$\{1^+, 9^-, 144^-, 144^+, 36^+, 36^-\}$	370
$\langle p\Lambda\overline{p\Lambda_{X_d}}\rangle$	6	$\{1^+, 36^-, 36^-, 144^+, 144^+, 36^-\}$	397
$\langle p\Lambda\overline{\Sigma^+ n}\rangle$	6	$\{144^-, 36^+, 36^+, 144^-, 9^-, 36^+\}$	405
$\langle p\Lambda\overline{\Sigma^0_{X_u} p}\rangle$	6	$\{144^+, 36^-, 9^-, 36^+, 144^+, 1^-\}$	370
$\langle p\Lambda\overline{\Sigma^0_{X_d} p}\rangle$	6	$\{144^-, 36^+, 36^+, 144^-, 36^-, 1^+\}$	397
$\langle \Sigma^+ n\overline{p\Lambda_{X_u}}\rangle$	3	$\{144^-, 144^+, 36^-\}$	324
$\langle \Sigma^+ n\overline{p\Lambda_{X_d}}\rangle$	3	$\{144^-, 36^+, 9^-\}$	189
$\langle \Sigma^+ n\overline{p\Lambda_{X_s}}\rangle$	3	$\{36^-, 144^+, 36^-\}$	216
$\langle \Sigma^+ n\overline{\Sigma^+ n}\rangle$	3	$\{1^+, 36^-, 144^+\}$	181
$\langle \Sigma^+ n\overline{\Sigma^0_{X_u} p}\rangle$	3	$\{144^-, 36^+, 144^-\}$	324
$\langle \Sigma^+ n\overline{\Sigma^0_{X_d} p}\rangle$	3	$\{36^+, 9^-, 144^+\}$	189
$\langle \Sigma^0 p\overline{p\Lambda_{X_{u,s}}}\rangle$	6	$\{36^+, 144^-, 144^+, 36^-, 9^+, 1^-\}$	370
$\langle \Sigma^0 p\overline{p\Lambda_{X_d}}\rangle$	6	$\{36^+, 144^-, 36^+, 144^-, 36^+, 1^-\}$	397
$\langle \Sigma^0 p\overline{\Sigma^+ n}\rangle$	6	$\{36^-, 144^+, 36^-, 9^+, 36^-, 144^+\}$	405
$\langle \Sigma^0 p\overline{\Sigma^0_{X_u} p}\rangle$	6	$\{1^+, 36^-, 9^+, 144^-, 36^+, 144^-\}$	370
$\langle \Sigma^0 p\overline{\Sigma^0_{X_d} p}\rangle$	6	$\{1^-, 144^+, 36^-, 36^+, 36^-, 144^+\}$	397

Each number of iterations is less than 600

Generalization to the various baryon-baryon channels strangeness S=-2 systems

channel	# of diagrams	$\{(\text{\# of iterations})^{\text{sign}}\}$	# of total iterations
$\langle \Lambda\Lambda\overline{\Lambda_{X_q}\Lambda_{X_{q'}}}\rangle$ $(q=q')$	8	$\{1^+,9^-,144^-,144^+,144^-,144^+,9^+,1^-\}$	596
$\langle \Lambda\Lambda\overline{\Lambda_{X_q}\Lambda_{X_{q'}}}\rangle$ $(q\neq q')$	8	$\{1^+,36^-,144^-,36^+,36^-,144^+,36^+,1^-\}$	434
$\langle \Lambda\Lambda\overline{p\Xi^-}\rangle$	8	$\{36^+,144^-,9^-,36^+,36^-,9^+,144^+,36^-\}$	450
$\langle \Lambda\Lambda\overline{n\Xi^0}\rangle$	8	$\{36^+,36^-,9^-,144^+,144^-,9^+,36^+,36^-\}$	450
$\langle \Lambda\Lambda\overline{\Sigma^+\Sigma^-}\rangle$	8	$\{36^-,144^+,36^+,9^-,9^+,36^-,144^-,36^+\}$	450
$\langle \Lambda\Lambda\overline{\Sigma^0_{X_q}\Sigma^0_{X_{q'}}}\rangle$ $(q=q')$	8	$\{1^+,9^-,144^-,144^+,144^-,144^+,9^+,1^-\}$	596
$\langle \Lambda\Lambda\overline{\Sigma^0_{X_q}\Sigma^0_{X_{q'}}}\rangle$ $(q\neq q')$	8	$\{1^-,36^+,144^+,36^-,36^+,144^-,36^-,1^+\}$	434
$\langle p\Xi^-\overline{\Lambda_{X_q}\Lambda_{X_q}}\rangle$ $(q=u,s)$	2	$\{36^+,36^-\}$	72
$\langle p\Xi^-\overline{\Lambda_{X_q}\Lambda_{X_{q'}}}\rangle$ $((q,q')=(d,u),(u,d),(s,d),(d,s))$	2	$\{36^+,144^-\}$	180
$\langle p\Xi^-\overline{\Lambda_{X_q}\Lambda_{X_{q'}}}\rangle$ $((q,q')=(s,u),(u,s))$	2	$\{9^+,144^-\}$	153
$\langle p\Xi^-\overline{\Lambda_{X_d}\Lambda_{X_d}}\rangle$	2	$\{144^+,144^-\}$	288
$\langle p\Xi^-\overline{p\Xi^-}\rangle$	2	$\{1^+,144^-\}$	145
$\langle p\Xi^-\overline{n\Xi^0}\rangle$	2	$\{36^+,144^-\}$	180
$\langle p\Xi^-\overline{\Sigma^+\Sigma^-}\rangle$	2	$\{144^-,36^+\}$	180
$\langle p\Xi^-\overline{\Sigma^0_{X_u}\Sigma^0_{X_u}}\rangle$	2	$\{36^+,36^-\}$	72
$\langle p\Xi^-\overline{\Sigma^0_{X_q}\Sigma^0_{X_{q'}}}\rangle$ $(q\neq q')$	2	$\{36^-,144^+\}$	180
$\langle p\Xi^-\overline{\Sigma^0_{X_d}\Sigma^0_{X_d}}\rangle$	2	$\{144^+,144^-\}$	288
$\langle p\Xi^-\overline{\Sigma^0_{X_u}\Lambda_{X_u}}\rangle$	2	$\{36^+,36^-\}$	72
$\langle p\Xi^-\overline{\Sigma^0_{X_q}\Lambda_{X_{q'}}}\rangle$ $((q,q')=(d,u),(u,d),(d,s))$	2	$\{36^-,144^+\}$	180
$\langle p\Xi^-\overline{\Sigma^0_{X_d}\Lambda_{X_d}}\rangle$	2	$\{144^-,144^+\}$	288
$\langle p\Xi^-\overline{\Sigma^0_{X_u}\Lambda_{X_s}}\rangle$	2	$\{144^+,9^-\}$	153

Each number of iterations is less than 600

Generalization to the various baryon-baryon channels strangeness S=-2 systems (cont' d)

$\langle p\Xi^- \overline{\Sigma^0_{X_u}\Sigma^0_{X_u}}\rangle$	2	$\{36^+,36^-\}$	72
$\langle p\Xi^- \overline{\Sigma^0_{X_q}\Sigma^0_{X_{q'}}}\rangle$ $(q \neq q')$	2	$\{36^-,144^+\}$	180
$\langle p\Xi^- \overline{\Sigma^0_{X_d}\Sigma^0_{X_d}}\rangle$	2	$\{144^+,144^-\}$	288
$\langle p\Xi^- \overline{\Sigma^0_{X_u}\Lambda_{X_u}}\rangle$	2	$\{36^+,36^-\}$	72
$\langle p\Xi^- \overline{\Sigma^0_{X_q}\Lambda_{X_{q'}}}\rangle$ $((q,q')=(d,u),(u,d),(d,s))$	2	$\{36^-,144^+\}$	180
$\langle p\Xi^- \overline{\Sigma^0_{X_d}\Lambda_{X_d}}\rangle$	2	$\{144^-,144^+\}$	288
$\langle p\Xi^- \overline{\Sigma^0_{X_u}\Lambda_{X_s}}\rangle$	2	$\{144^+,9^-\}$	153
$\langle n\Xi^0 \overline{\Lambda_{X_u}\Lambda_{X_u}}\rangle$	2	$\{144^+,144^-\}$	288
$\langle n\Xi^0 \overline{\Lambda_{X_q}\Lambda_{X_{q'}}}\rangle$ $((q,q')=(d,u),(u,d),(s,u),(u,s))$	2	$\{144^+,36^-\}$	180
$\langle n\Xi^0 \overline{\Lambda_{X_q}\Lambda_{X_q}}\rangle$ $(q=d,s)$	2	$\{36^+,36^-\}$	72
$\langle n\Xi^0 \overline{\Lambda_{X_q}\Lambda_{X_{q'}}}\rangle$ $((q,q')=(s,d),(d,s))$	2	$\{9^+,144^-\}$	153
$\langle n\Xi^0 \overline{p\Xi^-}\rangle$	2	$\{36^+,144^-\}$	180
$\langle n\Xi^0 \overline{n\Xi^0}\rangle$	2	$\{1^+,144^-\}$	145
$\langle n\Xi^0 \overline{\Sigma^+\Sigma^-}\rangle$	2	$\{144^-,36^+\}$	180
$\langle n\Xi^0 \overline{\Sigma^0_{X_u}\Sigma^0_{X_u}}\rangle$	2	$\{144^+,144^-\}$	288
$\langle n\Xi^0 \overline{\Sigma^0_{X_q}\Sigma^0_{X_{q'}}}\rangle$ $(q \neq q')$	2	$\{144^-,36^+\}$	180
$\langle n\Xi^0 \overline{\Sigma^0_{X_d}\Sigma^0_{X_d}}\rangle$	2	$\{36^+,36^-\}$	72
$\langle n\Xi^0 \overline{\Sigma^0_{X_u}\Lambda_{X_u}}\rangle$	2	$\{144^+,144^-\}$	288
$\langle n\Xi^0 \overline{\Sigma^0_{X_q}\Lambda_{X_{q'}}}\rangle$ $((q,q')=(d,u),(u,d),(u,s))$	2	$\{144^-,36^+\}$	180
$\langle n\Xi^0 \overline{\Sigma^0_{X_d}\Lambda_{X_d}}\rangle$	2	$\{36^-,36^+\}$	72
$\langle n\Xi^0 \overline{\Sigma^0_{X_d}\Lambda_{X_s}}\rangle$	2	$\{144^-,9^+\}$	153

Each number of iterations is less than 600

Generalization to the various baryon-baryon channels strangeness S=−2 systems (cont' d)

channel	# of diagrams	$\{(\#\text{ of iterations})^{\text{sign}}\}$	# of total iterations
$\langle\Sigma^+\Sigma^-\overline{\Lambda_{X_q}\Lambda_{X_q}}\rangle$ $(q=u,d)$	2	$\{36^-, 36^+\}$	72
$\langle\Sigma^+\Sigma^-\overline{\Lambda_{X_q}\Lambda_{X_{q'}}}\rangle$ $((q,q')=(d,u),(u,d))$	2	$\{9^-, 144^+\}$	153
$\langle\Sigma^+\Sigma^-\overline{\Lambda_{X_q}\Lambda_{X_{q'}}}\rangle$ $((q,q')=(s,u),(s,d),(u,s),(d,s))$	2	$\{36^-, 144^+\}$	180
$\langle\Sigma^+\Sigma^-\overline{\Lambda_{X_s}\Lambda_{X_s}}\rangle$	2	$\{144^-, 144^+\}$	288
$\langle\Sigma^+\Sigma^-\overline{p\Xi^-}\rangle$	2	$\{144^-, 36^+\}$	180
$\langle\Sigma^+\Sigma^-\overline{n\Xi^0}\rangle$	2	$\{36^-, 144^+\}$	180
$\langle\Sigma^+\Sigma^-\overline{\Sigma^+\Sigma^-}\rangle$	2	$\{1^+, 144^-\}$	145
$\langle\Sigma^+\Sigma^-\overline{\Sigma^0_{X_q}\Sigma^0_{X_q}}\rangle$ $(q=u,d)$	2	$\{36^-, 36^+\}$	72
$\langle\Sigma^+\Sigma^-\overline{\Sigma^0_{X_q}\Sigma^0_{X_{q'}}}\rangle$ $(q\neq q')$	2	$\{9^+, 144^-\}$	153
$\langle\Sigma^+\Sigma^-\overline{\Sigma^0_{X_q}\Lambda_{X_q}}\rangle$ $(q=u,d)$	2	$\{36^-, 36^+\}$	72
$\langle\Sigma^+\Sigma^-\overline{\Sigma^0_{X_q}\Lambda_{X_{q'}}}\rangle$ $((q,q')=(d,u),(u,d))$	2	$\{9^+, 144^-\}$	153
$\langle\Sigma^+\Sigma^-\overline{\Sigma^0_{X_q}\Lambda_{X_s}}\rangle$ $(q=u,d)$	2	$\{144^-, 36^+\}$	180
$\langle\Sigma^0\Sigma^0\overline{\Lambda_{X_q}\Lambda_{X_{q'}}}\rangle$ $(q=q')$	8	$\{1^+, 9^-, 144^-, 144^+, 144^-, 144^+, 9^+, 1^-\}$	596
$\langle\Sigma^0\Sigma^0\overline{\Lambda_{X_q}\Lambda_{X_{q'}}}\rangle$ $(q\neq q')$	8	$\{1^+, 36^-, 144^-, 36^+, 36^-, 144^+, 36^+, 1^-\}$	434
$\langle\Sigma^0\Sigma^0\overline{p\Xi^-}\rangle$	8	$\{36^+, 144^-, 9^-, 36^+, 36^-, 9^+, 144^+, 36^-\}$	450
$\langle\Sigma^0\Sigma^0\overline{n\Xi^0}\rangle$	8	$\{36^+, 36^-, 9^-, 144^+, 144^-, 9^+, 36^+, 36^-\}$	450
$\langle\Sigma^0\Sigma^0\overline{\Sigma^+\Sigma^-}\rangle$	8	$\{36^-, 144^+, 36^+, 9^-, 9^+, 36^-, 144^-, 36^+\}$	450
$\langle\Sigma^0\Sigma^0\overline{\Sigma^0_{X_q}\Sigma^0_{X_{q'}}}\rangle$ $(q=q')$	8	$\{1^+, 9^-, 144^-, 144^+, 144^-, 144^+, 9^+, 1^-\}$	596
$\langle\Sigma^0\Sigma^0\overline{\Sigma^0_{X_q}\Sigma^0_{X_{q'}}}\rangle$ $(q\neq q')$	8	$\{1^-, 36^+, 144^+, 36^-, 36^+, 144^-, 36^-, 1^+\}$	434
$\langle\Sigma^0\Lambda\overline{p\Xi^-}\rangle$	8	$\{36^+, 144^-, 9^-, 36^+, 36^-, 9^+, 144^+, 36^-\}$	450
$\langle\Sigma^0\Lambda\overline{n\Xi^0}\rangle$	8	$\{36^+, 36^-, 9^-, 144^+, 144^-, 9^+, 36^+, 36^-\}$	450

Each number of iterations is less than 600

Generalization to the various baryon-baryon channels strangeness S=−2 systems (cont’ d)

$\langle \Sigma^+\Sigma^-\overline{\Lambda_{X_q}\Lambda_{X_{q'}}}\rangle$ $((q,q')=(s,u),(s,d),(u,s),(d,s))$	2	$\{36^-,144^+\}$	180
$\langle \Sigma^+\Sigma^-\overline{\Lambda_{X_s}\Lambda_{X_s}}\rangle$	2	$\{144^-,144^+\}$	288
$\langle \Sigma^+\Sigma^-\overline{p\Xi^-}\rangle$	2	$\{144^-,36^+\}$	180
$\langle \Sigma^+\Sigma^-\overline{n\Xi^0}\rangle$	2	$\{36^-,144^+\}$	180
$\langle \Sigma^+\Sigma^-\overline{\Sigma^+\Sigma^-}\rangle$	2	$\{1^+,144^-\}$	145
$\langle \Sigma^+\Sigma^-\overline{\Sigma^0_{X_q}\Sigma^0_{X_q}}\rangle$ $(q=u,d)$	2	$\{36^-,36^+\}$	72
$\langle \Sigma^+\Sigma^-\overline{\Sigma^0_{X_q}\Sigma^0_{X_{q'}}}\rangle$ $(q\neq q')$	2	$\{9^+,144^-\}$	153
$\langle \Sigma^+\Sigma^-\overline{\Sigma^0_{X_q}\Lambda_{X_q}}\rangle$ $(q=u,d)$	2	$\{36^-,36^+\}$	72
$\langle \Sigma^+\Sigma^-\overline{\Sigma^0_{X_q}\Lambda_{X_{q'}}}\rangle$ $((q,q')=(d,u),(u,d))$	2	$\{9^+,144^-\}$	153
$\langle \Sigma^+\Sigma^-\overline{\Sigma^0_{X_q}\Lambda_{X_s}}\rangle$ $(q=u,d)$	2	$\{144^-,36^+\}$	180
$\langle \Sigma^0\Sigma^0\overline{\Lambda_{X_q}\Lambda_{X_{q'}}}\rangle$ $(q=q')$	8	$\{1^+,9^-,144^-,144^+,144^-,144^+,9^+,1^-\}$	596
$\langle \Sigma^0\Sigma^0\overline{\Lambda_{X_q}\Lambda_{X_{q'}}}\rangle$ $(q\neq q')$	8	$\{1^+,36^-,144^-,36^+,36^-,144^+,36^+,1^-\}$	434
$\langle \Sigma^0\Sigma^0\overline{p\Xi^-}\rangle$	8	$\{36^+,144^-,9^-,36^+,36^-,9^+,144^+,36^-\}$	450
$\langle \Sigma^0\Sigma^0\overline{n\Xi^0}\rangle$	8	$\{36^+,36^-,9^-,144^+,144^-,9^+,36^+,36^-\}$	450
$\langle \Sigma^0\Sigma^0\overline{\Sigma^+\Sigma^-}\rangle$	8	$\{36^-,144^+,36^+,9^-,9^+,36^-,144^-,36^+\}$	450
$\langle \Sigma^0\Sigma^0\overline{\Sigma^0_{X_q}\Sigma^0_{X_{q'}}}\rangle$ $(q=q')$	8	$\{1^+,9^-,144^-,144^+,144^-,144^+,9^+,1^-\}$	596
$\langle \Sigma^0\Sigma^0\overline{\Sigma^0_{X_q}\Sigma^0_{X_{q'}}}\rangle$ $(q\neq q')$	8	$\{1^-,36^+,144^+,36^-,36^+,144^-,36^-,1^+\}$	434
$\langle \Sigma^0\Lambda\overline{p\Xi^-}\rangle$	8	$\{36^+,144^-,9^-,36^+,36^-,9^+,144^+,36^-\}$	450
$\langle \Sigma^0\Lambda\overline{n\Xi^0}\rangle$	8	$\{36^+,36^-,9^-,144^+,144^-,9^+,36^+,36^-\}$	450
$\langle \Sigma^0\Lambda\overline{\Sigma^+\Sigma^-}\rangle$	8	$\{36^-,144^+,36^+,9^-,9^+,36^-,144^-,36^+\}$	450
$\langle \Sigma^0\Lambda\overline{\Sigma^0_{X_q}\Lambda_{X_{q'}}}\rangle$ $(q=q')$	8	$\{1^+,9^-,144^-,144^+,144^-,144^+,9^+,1^-\}$	596
$\langle \Sigma^0\Lambda\overline{\Sigma^0_{X_q}\Lambda_{X_{q'}}}\rangle$ $(q\neq q')$	8	$\{1^-,36^+,144^+,36^-,36^+,144^-,36^-,1^+\}$	434

Each number of iterations is less than 600

Generalization to the various baryon-baryon channels strangeness S=-3 and -4 systems

channel	# of diagrams	$\{(\#\text{ of iterations})^{\text{sign}}\}$	# of total iterations
$\langle \Xi^- \Lambda \overline{\Xi^- \Lambda_{X_{u,s}}} \rangle$	6	$\{1^+, 36^-, 144^+, 144^-, 36^+, 9^-\}$	370
$\langle \Xi^- \Lambda \overline{\Xi^- \Lambda_{X_d}} \rangle$	6	$\{1^+, 36^-, 144^+, 36^-, 144^+, 36^-\}$	397
$\langle \Xi^- \Lambda \overline{\Sigma^- \Xi^0} \rangle$	6	$\{36^-, 9^+, 144^-, 144^+, 36^-, 36^+\}$	405
$\langle \Xi^- \Lambda \overline{\Sigma^0_{X_u} \Xi^-} \rangle$	6	$\{36^+, 9^-, 144^+, 36^-, 144^+, 1^-\}$	370
$\langle \Xi^- \Lambda \overline{\Sigma^0_{X_d} \Xi^-} \rangle$	6	$\{144^-, 36^+, 36^-, 36^+, 144^-, 1^+\}$	397
$\langle \Sigma^- \Xi^0 \overline{\Xi^- \Lambda_{X_u}} \rangle$	3	$\{36^-, 144^+, 36^-\}$	216
$\langle \Sigma^- \Xi^0 \overline{\Xi^- \Lambda_{X_d}} \rangle$	3	$\{9^-, 36^+, 144^-\}$	189
$\langle \Sigma^- \Xi^0 \overline{\Xi^- \Lambda_{X_s}} \rangle$	3	$\{36^-, 144^+, 144^-\}$	324
$\langle \Sigma^- \Xi^0 \overline{\Sigma^- \Xi^0} \rangle$	3	$\{1^+, 144^-, 36^+\}$	181
$\langle \Sigma^- \Xi^0 \overline{\Sigma^0_{X_u} \Xi^-} \rangle$	3	$\{36^-, 36^+, 144^-\}$	216
$\langle \Sigma^- \Xi^0 \overline{\Sigma^0_{X_d} \Xi^-} \rangle$	3	$\{144^+, 9^-, 36^+\}$	189
$\langle \Sigma^0 \Xi^- \overline{\Xi^- \Lambda_{X_{u,s}}} \rangle$	6	$\{9^+, 36^-, 144^+, 144^-, 36^+, 1^-\}$	370
$\langle \Sigma^0 \Xi^- \overline{\Xi^- \Lambda_{X_d}} \rangle$	6	$\{36^+, 144^-, 36^+, 144^-, 36^+, 1^-\}$	397
$\langle \Sigma^0 \Xi^- \overline{\Sigma^- \Xi^0} \rangle$	6	$\{36^-, 36^+, 144^-, 144^+, 9^-, 36^+\}$	405
$\langle \Sigma^0 \Xi^- \overline{\Sigma^0_{X_u} \Xi^-} \rangle$	6	$\{1^+, 144^-, 36^+, 144^-, 9^+, 36^-\}$	370
$\langle \Sigma^0 \Xi^- \overline{\Sigma^0_{X_d} \Xi^-} \rangle$	6	$\{1^-, 144^+, 36^-, 36^+, 36^-, 144^+\}$	397
$\langle \Xi^- \Xi^0 \overline{\Xi^- \Xi^0} \rangle$	6	$\{1^+, 36^-, 9^+, 144^+, 36^-, 144^+\}$	370

Each number of iterations is less than 600

Effective block algorithm to calculate the 52 channels of 4pt correlator

$$\langle pn\overline{pn}\rangle, \tag{4.1}$$

$$\begin{aligned}
&\langle p\Lambda\overline{p\Lambda}\rangle, \quad \langle p\Lambda\overline{\Sigma^+ n}\rangle, \quad \langle p\Lambda\overline{\Sigma^0 p}\rangle,\\
&\langle \Sigma^+ n\overline{p\Lambda}\rangle, \quad \langle \Sigma^+ n\overline{\Sigma^+ n}\rangle, \quad \langle \Sigma^+ n\overline{\Sigma^0 p}\rangle,\\
&\langle \Sigma^0 p\overline{p\Lambda}\rangle, \quad \langle \Sigma^0 p\overline{\Sigma^+ n}\rangle, \quad \langle \Sigma^0 p\overline{\Sigma^0 p}\rangle,
\end{aligned} \tag{4.2}$$

$$\begin{aligned}
&\langle \Lambda\Lambda\overline{\Lambda\Lambda}\rangle, && \langle \Lambda\Lambda\overline{p\Xi^-}\rangle, && \langle \Lambda\Lambda\overline{n\Xi^0}\rangle, && \langle \Lambda\Lambda\overline{\Sigma^+\Sigma^-}\rangle, && \langle \Lambda\Lambda\overline{\Sigma^0\Sigma^0}\rangle, && \\
&\langle p\Xi^-\overline{\Lambda\Lambda}\rangle, && \langle p\Xi^-\overline{p\Xi^-}\rangle, && \langle p\Xi^-\overline{n\Xi^0}\rangle, && \langle p\Xi^-\overline{\Sigma^+\Sigma^-}\rangle, && \langle p\Xi^-\overline{\Sigma^0\Sigma^0}\rangle, && \langle p\Xi^-\overline{\Sigma^0\Lambda}\rangle,\\
&\langle n\Xi^0\overline{\Lambda\Lambda}\rangle, && \langle n\Xi^0\overline{p\Xi^-}\rangle, && \langle n\Xi^0\overline{n\Xi^0}\rangle, && \langle n\Xi^0\overline{\Sigma^+\Sigma^-}\rangle, && \langle n\Xi^0\overline{\Sigma^0\Sigma^0}\rangle, && \langle n\Xi^0\overline{\Sigma^0\Lambda}\rangle,\\
&\langle \Sigma^+\Sigma^-\overline{\Lambda\Lambda}\rangle, && \langle \Sigma^+\Sigma^-\overline{p\Xi^-}\rangle, && \langle \Sigma^+\Sigma^-\overline{n\Xi^0}\rangle, && \langle \Sigma^+\Sigma^-\overline{\Sigma^+\Sigma^-}\rangle, && \langle \Sigma^+\Sigma^-\overline{\Sigma^0\Sigma^0}\rangle, && \langle \Sigma^+\Sigma^-\overline{\Sigma^0\Lambda}\rangle,\\
&\langle \Sigma^0\Sigma^0\overline{\Lambda\Lambda}\rangle, && \langle \Sigma^0\Sigma^0\overline{p\Xi^-}\rangle, && \langle \Sigma^0\Sigma^0\overline{n\Xi^0}\rangle, && \langle \Sigma^0\Sigma^0\overline{\Sigma^+\Sigma^-}\rangle, && \langle \Sigma^0\Sigma^0\overline{\Sigma^0\Sigma^0}\rangle, && \\
& && \langle \Sigma^0\Lambda\overline{p\Xi^-}\rangle, && \langle \Sigma^0\Lambda\overline{n\Xi^0}\rangle, && \langle \Sigma^0\Lambda\overline{\Sigma^+\Sigma^-}\rangle, && && \langle \Sigma^0\Lambda\overline{\Sigma^0\Lambda}\rangle,
\end{aligned} \tag{4.3}$$

$$\begin{aligned}
&\langle \Xi^-\Lambda\overline{\Xi^-\Lambda}\rangle, \quad \langle \Xi^-\Lambda\overline{\Sigma^-\Xi^0}\rangle, \quad \langle \Xi^-\Lambda\overline{\Sigma^0\Xi^-}\rangle,\\
&\langle \Sigma^-\Xi^0\overline{\Xi^-\Lambda}\rangle, \quad \langle \Sigma^-\Xi^0\overline{\Sigma^-\Xi^0}\rangle, \quad \langle \Sigma^-\Xi^0\overline{\Sigma^0\Xi^-}\rangle,\\
&\langle \Sigma^0\Xi^-\overline{\Xi^-\Lambda}\rangle, \quad \langle \Sigma^0\Xi^-\overline{\Sigma^-\Xi^0}\rangle, \quad \langle \Sigma^0\Xi^-\overline{\Sigma^0\Xi^-}\rangle,
\end{aligned} \tag{4.4}$$

$$\langle \Xi^-\Xi^0\overline{\Xi^-\Xi^0}\rangle. \tag{4.5}$$

- Elapse times to calculate the 52 matrix correlators (MPI+OpenMP)
- [tasks_per_node] x [OMP_NUM_THREADS]

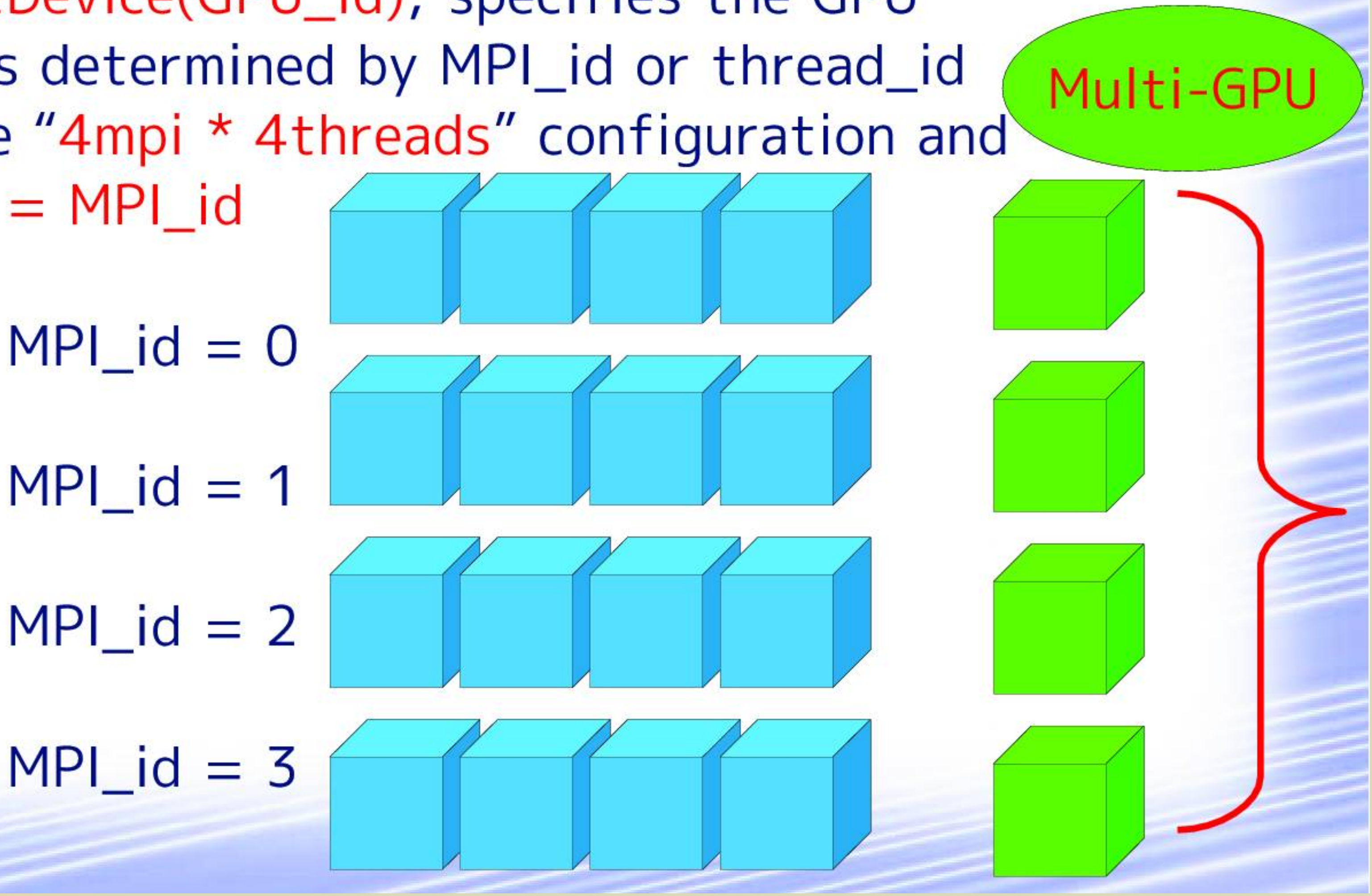
	64x1	32x2	16x4	8x4	4x8	2x16	1x32
Step-1	0:14	0:16	0:09	0:09	0:07	0:06	0:06
Step-2	0:10	0:11	0:12	0:12	0:12	0:13	0:14

HA-PACS

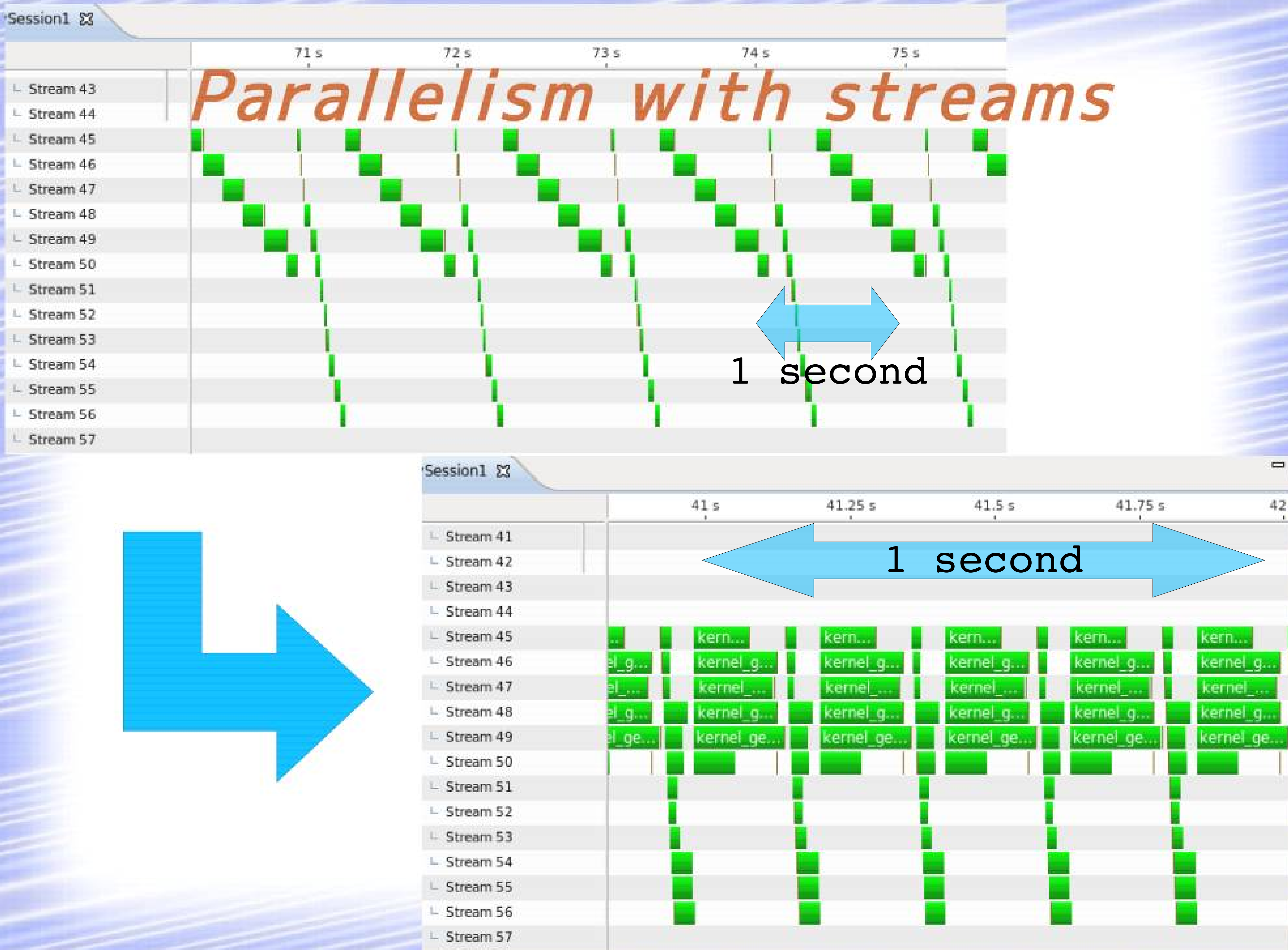
- **BASE**
  - Intel E5-2670 (16core) + NVIDIA M2090 (x4)
    332.8 GFlops   665 GFlops (x4)
    128 GBytes   6 GBytes (x4)
- **TCA**
  - Intel E5-2680v2 (20core) + NVIDIA K20X (x4)
    448 GFlops   1310 GFlops (x4)
    128 GBytes   6 GBytes (x4)

MPI+OpenMP + CUDA

- For HA-PACS, 1PE has 16 CPU cores and 4 GPUs:
  - cudaSetDevice(GPU_id); specifies the GPU
  - GPUid is determined by MPI_id or thread_id
  - We take "4mpi * 4threads" configuration and
  - GPU_id = MPI_id



Parallelism with streams



Results

- HA-PACS:

	M2090 (665 GFlops)	K20X (1310 GFlops)	
•	20 GFlops	27 GFlops	AllBaryons<<<...>>>(...)
•	1.6 GFlops	1.1 GFlops	A<<<...>>>(...)
•	5.4 GFlops	6.1 GFlops	B<<<...>>>(...)
•	4.7 GFlops	26 GFlops	B<<<...>>>(...) [using streams]

- Lattice size: 16^3 x 32

52 channel calculation with 16^3x32 lattice

- Without GPU, elapsed time is 2:22
- With GPU (M2090), 1:45

$$\langle pn\overline{pn}\rangle, \tag{4.1}$$

$$\begin{aligned}
&\langle p\Lambda\overline{p\Lambda}\rangle, \quad \langle p\Lambda\overline{\Sigma^+ n}\rangle, \quad \langle p\Lambda\overline{\Sigma^0 p}\rangle,\\
&\langle \Sigma^+ n\overline{p\Lambda}\rangle, \langle \Sigma^+ n\overline{\Sigma^+ n}\rangle, \langle \Sigma^+ n\overline{\Sigma^0 p}\rangle,\\
&\langle \Sigma^0 p\overline{p\Lambda}\rangle, \langle \Sigma^0 p\overline{\Sigma^+ n}\rangle, \langle \Sigma^0 p\overline{\Sigma^0 p}\rangle,
\end{aligned} \tag{4.2}$$

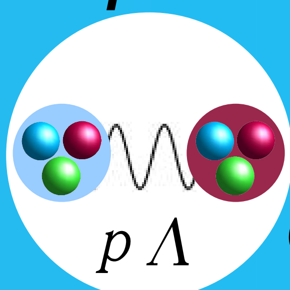
$$\begin{aligned}
&\langle \Lambda\Lambda\overline{\Lambda\Lambda}\rangle, \quad \langle \Lambda\Lambda\overline{p\Xi^-}\rangle, \quad \langle \Lambda\Lambda\overline{n\Xi^0}\rangle, \quad \langle \Lambda\Lambda\overline{\Sigma^+\Sigma^-}\rangle, \quad \langle \Lambda\Lambda\overline{\Sigma^0\Sigma^0}\rangle,\\
&\langle p\Xi^-\overline{\Lambda\Lambda}\rangle, \quad \langle p\Xi^-\overline{p\Xi^-}\rangle, \quad \langle p\Xi^-\overline{n\Xi^0}\rangle, \quad \langle p\Xi^-\overline{\Sigma^+\Sigma^-}\rangle, \quad \langle p\Xi^-\overline{\Sigma^0\Sigma^0}\rangle, \quad \langle p\Xi^-\overline{\Sigma^0\Lambda}\rangle,\\
&\langle n\Xi^0\overline{\Lambda\Lambda}\rangle, \quad \langle n\Xi^0\overline{p\Xi^-}\rangle, \quad \langle n\Xi^0\overline{n\Xi^0}\rangle, \quad \langle n\Xi^0\overline{\Sigma^+\Sigma^-}\rangle, \quad \langle n\Xi^0\overline{\Sigma^0\Sigma^0}\rangle, \quad \langle n\Xi^0\overline{\Sigma^0\Lambda}\rangle,\\
&\langle \Sigma^+\Sigma^-\overline{\Lambda\Lambda}\rangle, \langle \Sigma^+\Sigma^-\overline{p\Xi^-}\rangle, \langle \Sigma^+\Sigma^-\overline{n\Xi^0}\rangle, \langle \Sigma^+\Sigma^-\overline{\Sigma^+\Sigma^-}\rangle, \langle \Sigma^+\Sigma^-\overline{\Sigma^0\Sigma^0}\rangle, \langle \Sigma^+\Sigma^-\overline{\Sigma^0\Lambda}\rangle,\\
&\langle \Sigma^0\Sigma^0\overline{\Lambda\Lambda}\rangle, \quad \langle \Sigma^0\Sigma^0\overline{p\Xi^-}\rangle, \quad \langle \Sigma^0\Sigma^0\overline{n\Xi^0}\rangle, \quad \langle \Sigma^0\Sigma^0\overline{\Sigma^+\Sigma^-}\rangle, \quad \langle \Sigma^0\Sigma^0\overline{\Sigma^0\Sigma^0}\rangle,\\
&\qquad \langle \Sigma^0\Lambda\overline{p\Xi^-}\rangle, \quad \langle \Sigma^0\Lambda\overline{n\Xi^0}\rangle, \quad \langle \Sigma^0\Lambda\overline{\Sigma^+\Sigma^-}\rangle, \qquad \langle \Sigma^0\Lambda\overline{\Sigma^0\Lambda}\rangle,
\end{aligned} \tag{4.3}$$

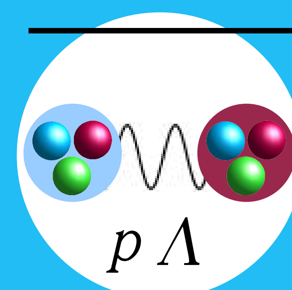
$$\begin{aligned}
&\langle \Xi^-\Lambda\overline{\Xi^-\Lambda}\rangle, \quad \langle \Xi^-\Lambda\overline{\Sigma^-\Xi^0}\rangle, \quad \langle \Xi^-\Lambda\overline{\Sigma^0\Xi^-}\rangle,\\
&\langle \Sigma^-\Xi^0\overline{\Xi^-\Lambda}\rangle, \langle \Sigma^-\Xi^0\overline{\Sigma^-\Xi^0}\rangle, \langle \Sigma^-\Xi^0\overline{\Sigma^0\Xi^-}\rangle,\\
&\langle \Sigma^0\Xi^-\overline{\Xi^-\Lambda}\rangle, \langle \Sigma^0\Xi^-\overline{\Sigma^-\Xi^0}\rangle, \langle \Sigma^0\Xi^-\overline{\Sigma^0\Xi^-}\rangle,
\end{aligned} \tag{4.4}$$

$$\langle \Xi^-\Xi^0\overline{\Xi^-\Xi^0}\rangle. \tag{4.5}$$

Benchmark

$$F_{\alpha\beta\alpha'\beta'}(\vec{r},\, t - t_0) \quad ,$$

$$\rightarrow \Big\langle$$


$$_{\alpha\beta}(\vec{r},\, t)$$


$$_{\alpha'\beta'}(t_0)\Big\rangle$$

Benchmark of the hybrid parallel C++ code implementation

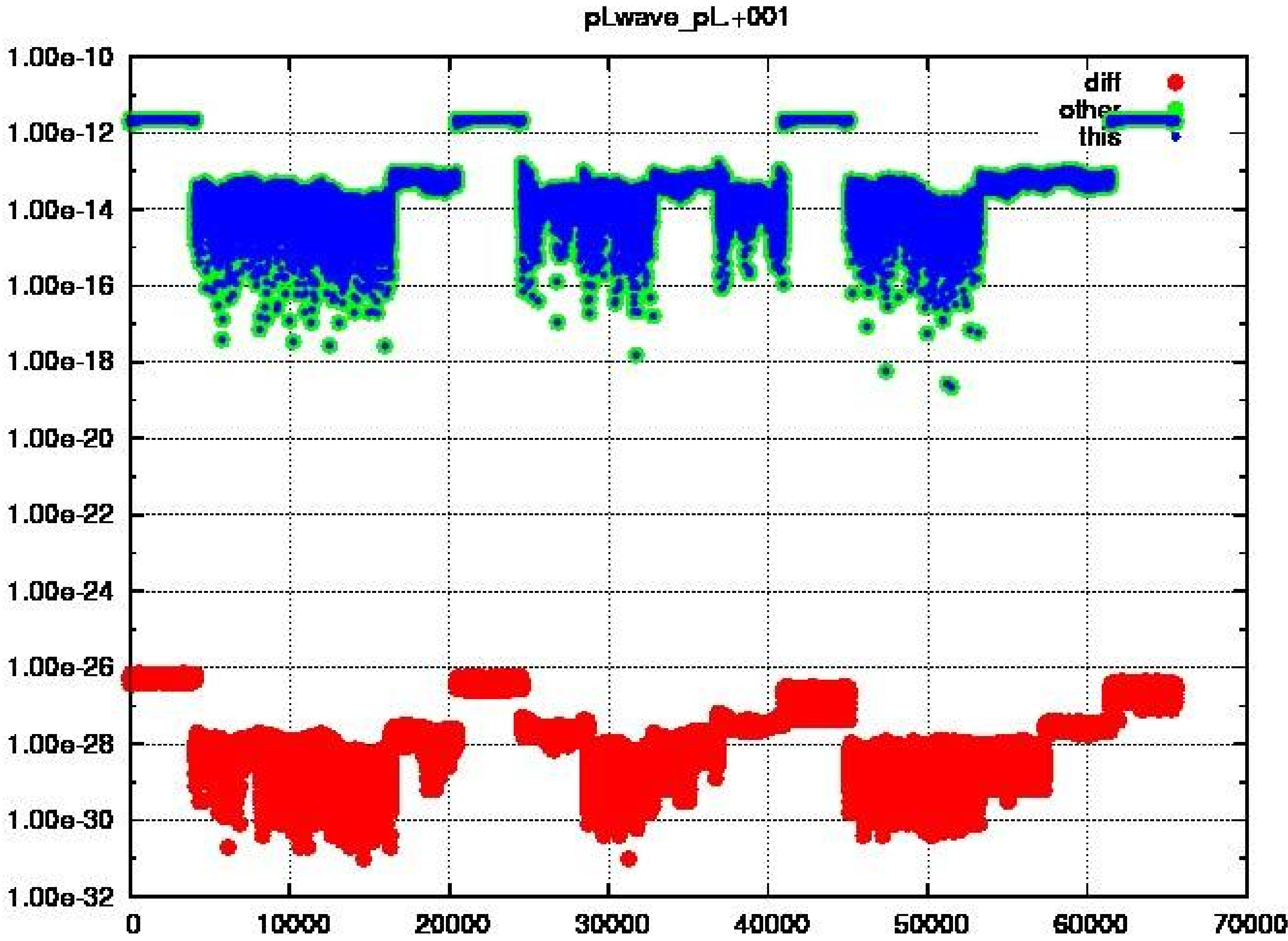
Comparisons have been made for all 52 channels over 31 time-slices, 16*16*16 points for spatial, and 2*2*2*2 points for the spin degrees of freedom.

There are 16*16*16*2*2*2*2 = 65536 points per time-slice per channel.

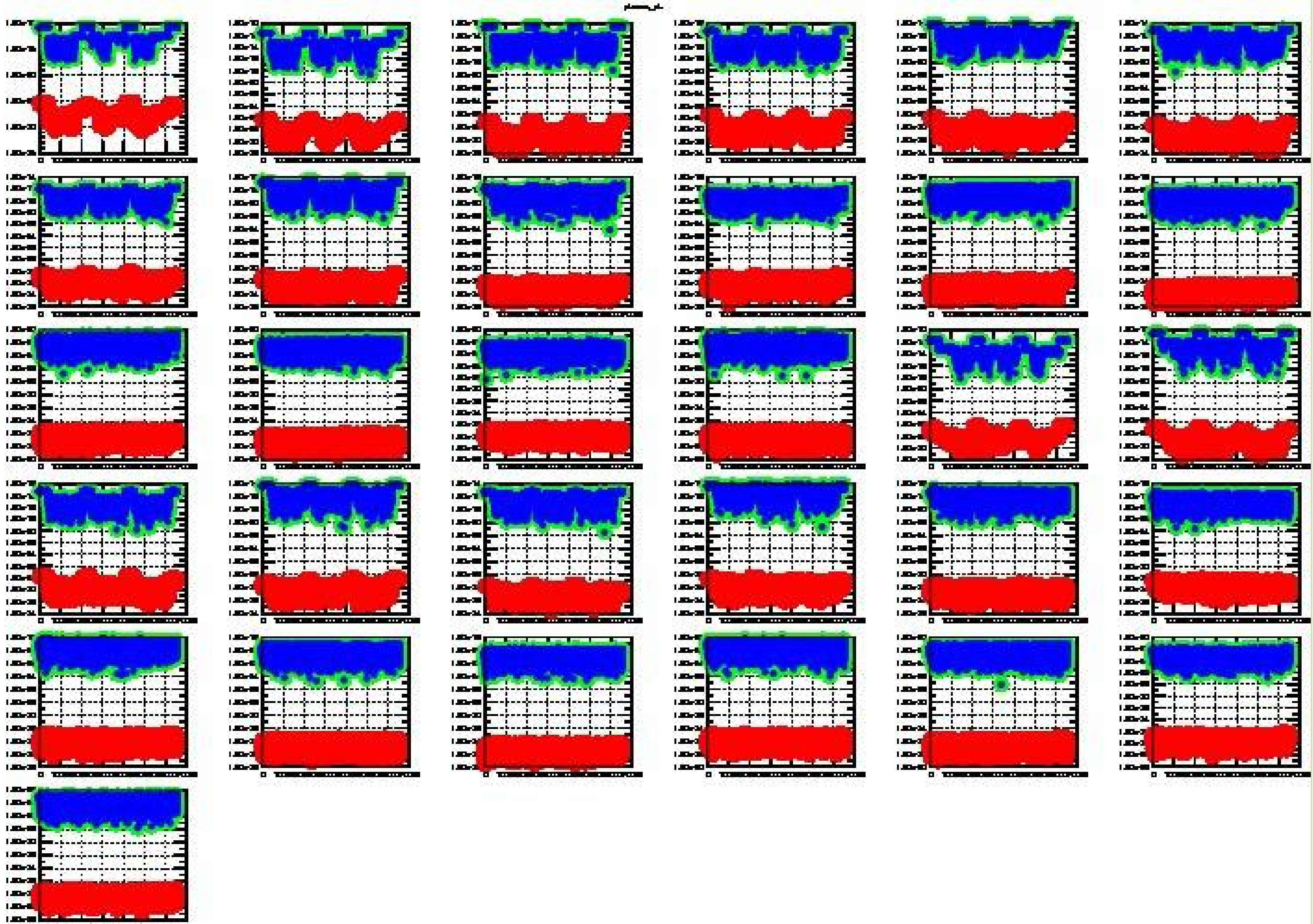
For example:

>>>	a	b	aPbP		x	y	z	ratio	From_other_work	From_this_work	diff (=From_other-From_this)
>>>	0	1	0	1	0	0	0	1.000000000000e+00	4.755520154185e-20	4.755520154185e-20	-8.907425992791e-34
>>>	0	1	0	1	1	0	0	1.000000000000e+00	1.064483789589e-19	1.064483789589e-19	-4.935195482492e-34
>>>	0	1	0	1	2	0	0	1.000000000000e+00	1.034021677632e-19	1.034021677632e-19	-8.185202263646e-34
>>>	0	1	0	1	3	0	0	1.000000000000e+00	9.903890629525e-20	9.903890629525e-20	-9.750020343460e-34
>>>	0	1	0	1	4	0	0	1.000000000000e+00	9.480310911656e-20	9.480310911656e-20	-1.095372655870e-33
>>>	0	1	0	1	5	0	0	1.000000000000e+00	1.030125191701e-19	1.030125191701e-19	-9.388908478888e-34
>>>	0	1	0	1	6	0	0	1.000000000000e+00	8.655174217981e-20	8.655174217981e-20	-6.500013562307e-34
>>>	0	1	0	1	7	0	0	1.000000000000e+00	2.229267218351e-20	2.229267218351e-20	-9.087981925077e-34
>>>	0	1	0	1	8	0	0	9.999999999996e-01	2.330235858756e-21	2.330235858757e-21	-8.911187574714e-34
>>>	0	1	0	1	9	0	0	1.000000000000e+00	2.034855576332e-20	2.034855576332e-20	-7.011588703785e-34
>>>	0	1	0	1	10	0	0	1.000000000000e+00	3.822483016345e-20	3.822483016345e-20	-6.981496048404e-34
>>>	0	1	0	1	11	0	0	1.000000000000e+00	6.506309796602e-20	6.506309796602e-20	-4.453712996395e-34
>>>	0	1	0	1	12	0	0	1.000000000000e+00	8.118512223003e-20	8.118512223003e-20	-6.981496048404e-34
>>>	0	1	0	1	13	0	0	1.000000000000e+00	5.202429085187e-20	5.202429085187e-20	-1.017131751880e-33
>>>	0	1	0	1	14	0	0	1.000000000000e+00	5.725220267276e-20	5.725220267276e-20	-7.342607912976e-34
>>>	0	1	0	1	15	0	0	1.000000000000e+00	4.331313772187e-20	4.331313772187e-20	-8.185202263646e-34

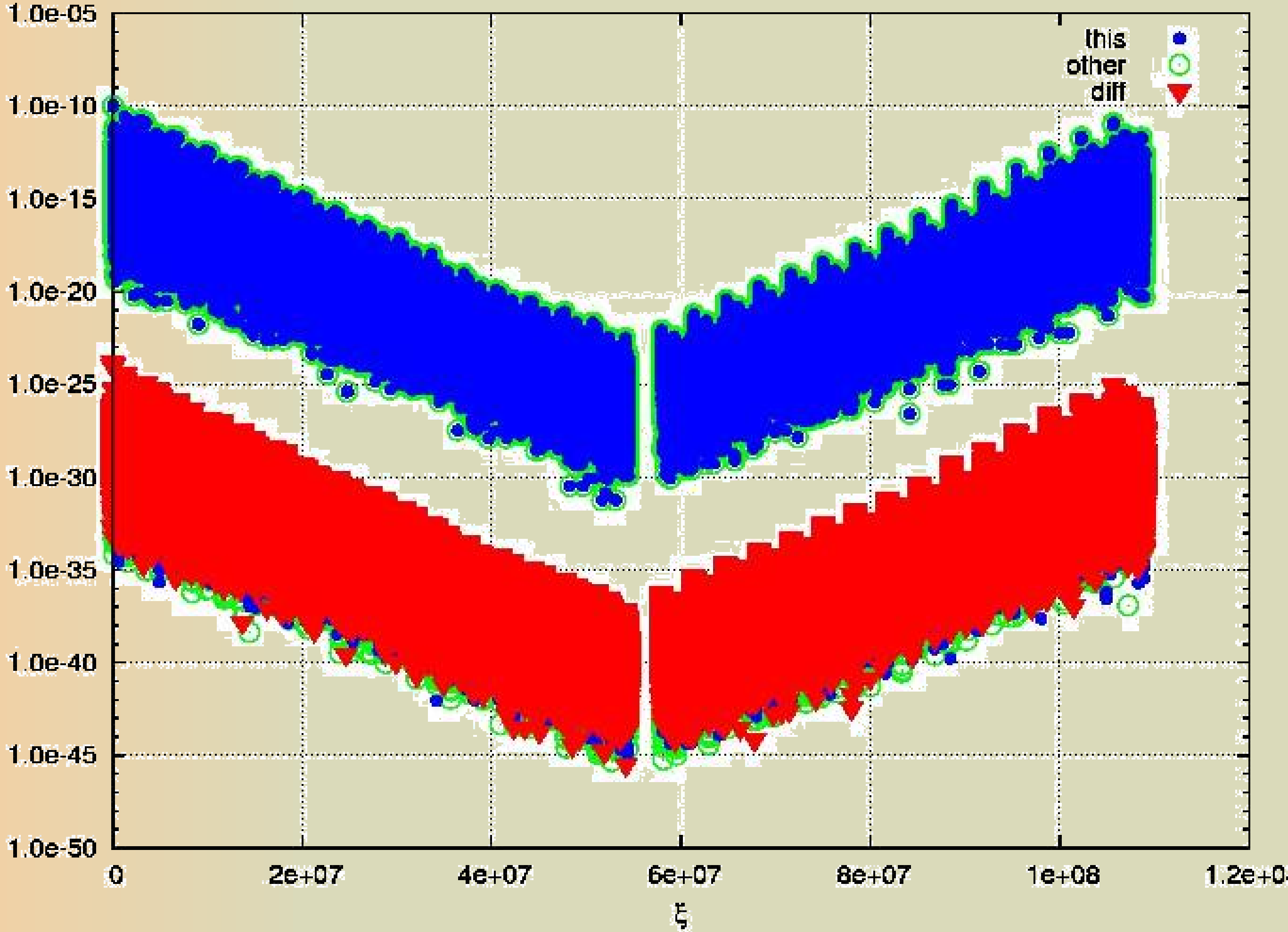
Benchmark (only LN-LN channel at $t-t_0$=+001)



Benchmark (at LN-LN channel)



Benchmark (from NN to XiXi channels)



numerical results of the correlators of entire 52 channels from NN to $\Xi\Xi$ systems given in Eqs. (32)–(36), over 31 time-slices, 16^3 points for spatial, and 2^4 points for the spin degrees of freedom, obtained by using this effective block algorithm (dot) and by using the unified contraction algorithm (open circle) as a function of one-dimensionally aligned data point $\xi = \tilde{\alpha} + 2(\tilde{\beta} + 2(\tilde{\alpha}' + 2(\tilde{\beta}' + 2(x + 16(y + 16(z + 16(c + 52((t - t_0 + T) \bmod T)))))))$, where $c = 0, \cdots, 51$ selects one of the 52 channels. The absolute value of their difference is also shown (triangle).

Summary on 1st part

(1) We present a fast algorithm to calculate the 4pt correlation function of Lambda-Nucleon system, which was used to study the hyperonic nuclear forces from lattice QCD.

(2) Generalize the target system to various baryon-baryon channels. (E.g., 52 channel NBS wave functions can be obtained at the same time from one computing job for the 2+1 lattice QCD.)

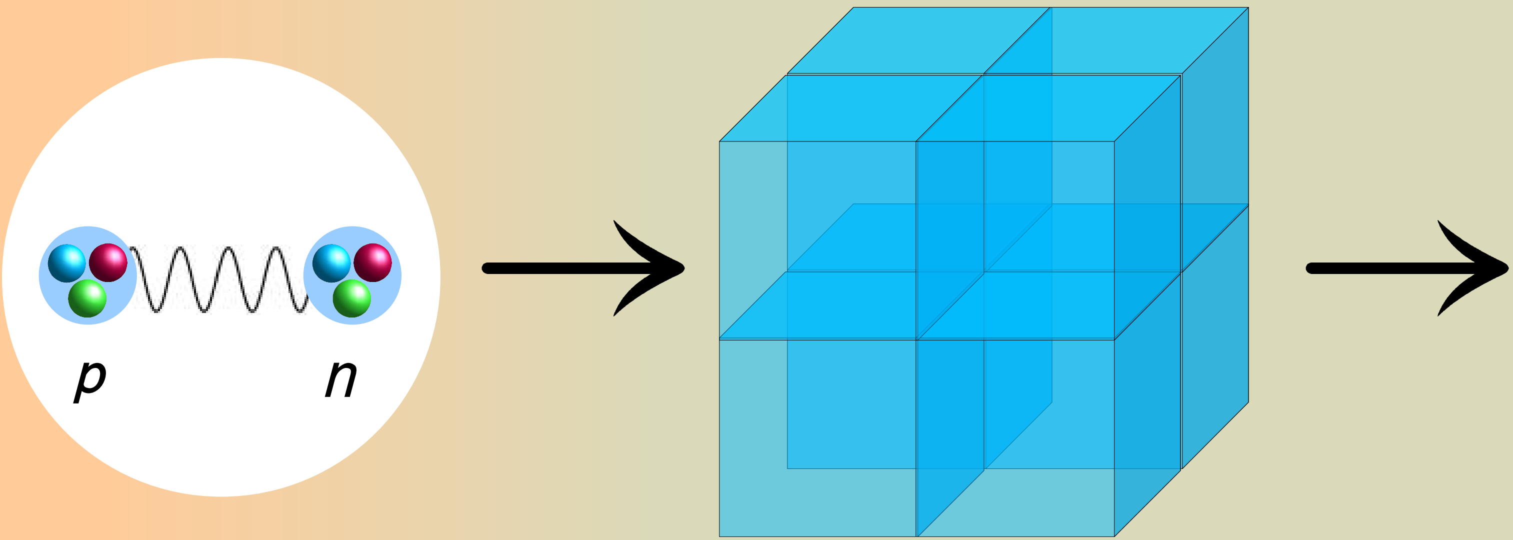
(3) In this approach, the number of iterations to obtain the four-point correlation function is remarkably smaller than the numbers given in the unified contraction algorithm[2].

(4) A hybrid parallel (C++) and multi-GPU (CUDA) program has been implemented with MPI and OpenMP, working on supercomputer (HA-PACS); Concurrent kernel executions with streams improve the computing performance for K20X (TCA part of HA-PACS).

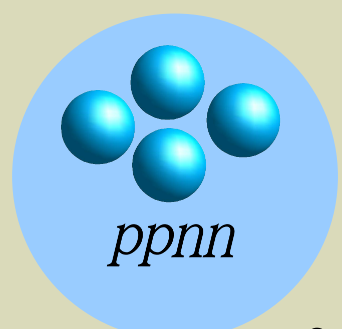
[1] H.N. PoS(LAT2013)426;(LAT2008)156;(LAT2009)152;(LAT2011)167; (LAT2013)426; arXiv:1510.00903(hep-lat).

[2] Doi and Endres, Comput. Phys. Commun. 184, 117 (2013).

Nuclear few-body problem



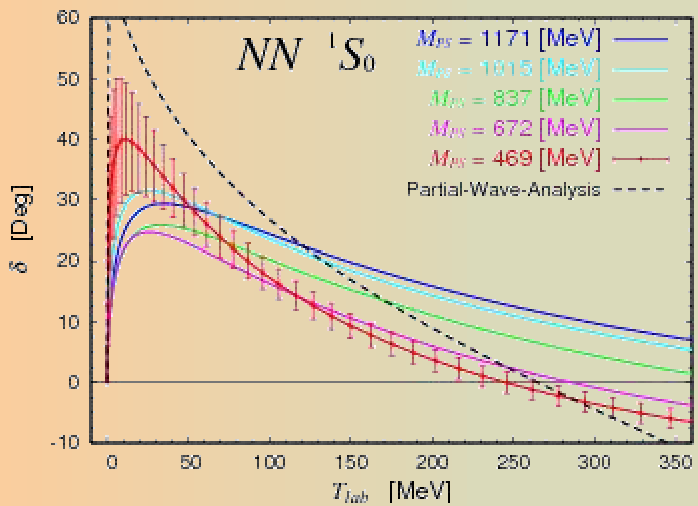
$$\rightarrow V_{NN}(r)$$

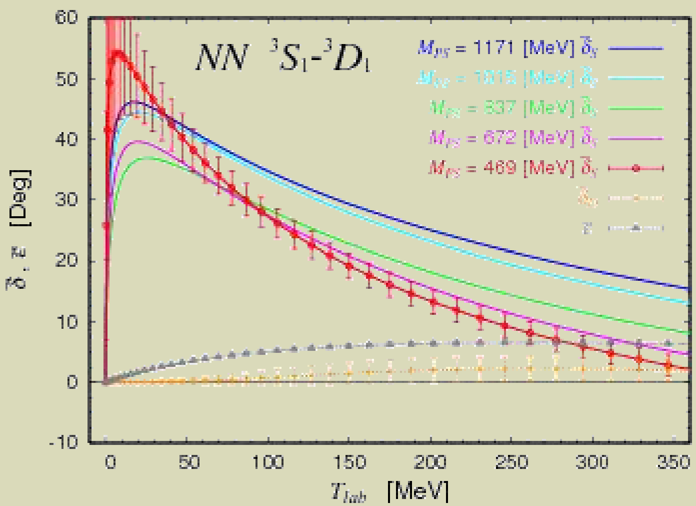


$$H = \sum_{i=1}^{A} \frac{\boldsymbol{p}_i^2}{2\mathrm{m}_N} - T_{c.m.} + \sum_{i<j}^{A} V_{NN}(r_{ij})$$

Stochastic variational calculation of 4He with using a lattice potential

- For NN potential, we use the SU(3) potential at the lightest quark mass(m_ps = 469 MeV), which has been reported to have a 4N bound state (about 5.1MeV) within a tensor-included effective central potential; NPA881, 28-43 (2011).





Stochastic variational calculation of 4He with using a lattice potential

The wave function of A-body system is described by a linear combination of basis functions as

$$\Psi = \sum_{k=1}^{K} c_k \varphi_k, \qquad \text{with} \qquad \varphi_k = \mathcal{A}\{G(\mathbf{x}; A_k)[\theta_{(LL')_k}(\mathbf{x}; (uu')_k), \chi_{S_k}]_{JM}\eta_{kIM_I}\}, \tag{11}$$

where c_k is the linear variational parameter determined by the variational principle, $\mathcal{A}$ is antisymmetrizer for identical particles. χ_{S_k} (η_{kIM_I}) is the spin (isospin) function of the system. $G(\mathbf{x}; A_k)$ is the correlated Gaussian function which is given by

$$G(\mathbf{x}; A_k) = \exp\left\{-\frac{1}{2}\sum_{i<j}^{A} \alpha_{kij}(\mathbf{r}_i - \mathbf{r}_j)^2\right\} = \exp\left\{-\frac{1}{2}\sum_{i,j=1}^{A-1} A_{kij}\mathbf{x}_i \cdot \mathbf{x}_j\right\}. \tag{12}$$

Stochastic variational calculation of 4He with using a lattice potential

A set of relative coordinates $\{\mathbf{x}_1, \cdots, \mathbf{x}_{A-1}\}$ and the center-of-mass coordinate $\mathbf{x}_A$ are given by a linear transformation of single particle coordinates $\{\mathbf{r}_1, \cdots, \mathbf{r}_A\}$ such as

$$\mathbf{x}_i = \sum_{j=1}^{A} U_{ij}\mathbf{r}_j, \qquad (i = 1, \cdots, A). \tag{13}$$

In order to obtain the accurate solution of the four-nucleon bound state with explicitly utilizing the the tensor potential, we consider nonzero orbital angular momentum states $(L, S)J^\pi = (1, 1)0^+$ and $(2, 2)0^+$ in addition to the $(0, 0)0^+$ configuration. We employ the global vector representation[11] for these nonzero orbital angular momentum states. Therefore, the angular part of the basis function is given by

$$\theta_{(LL')_k}(\mathbf{x}; (uu')_k) = v_k^{L_k} v_k'^{L'_k} [Y_{L_k}(\hat{\mathbf{v}}_k) \times Y_{L'_k}(\hat{\mathbf{v}}'_k)]_{L_k}, \qquad \begin{pmatrix} \mathbf{v} \\ \mathbf{v}' \end{pmatrix}_k = \sum_{i=1}^{A-1} \mathbf{x}_i \begin{pmatrix} u \\ u' \end{pmatrix}_{ki}. \tag{14}$$

The validity of the present choice of basis function is examined for several realistic NN potentials[11]. The A_{kij} and $(u, u')_{ki}$ are the nonlinear variational parameters which are determined by the stochastic variational method[12].

Benchmark test calculation of a four-nucleon bound state,
Phys. Rev. C64, 044001 (2001).

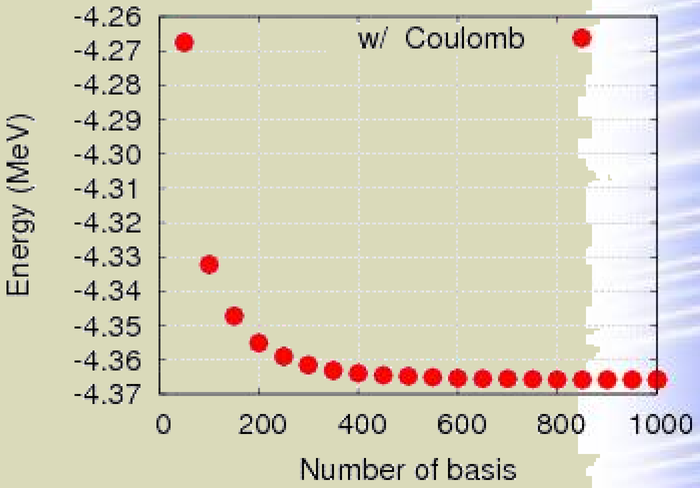
TABLE I. The expectation values $\langle T \rangle$ and $\langle V \rangle$ of kinetic and potential energies, the binding energies E_b in MeV, and the radius in fm.

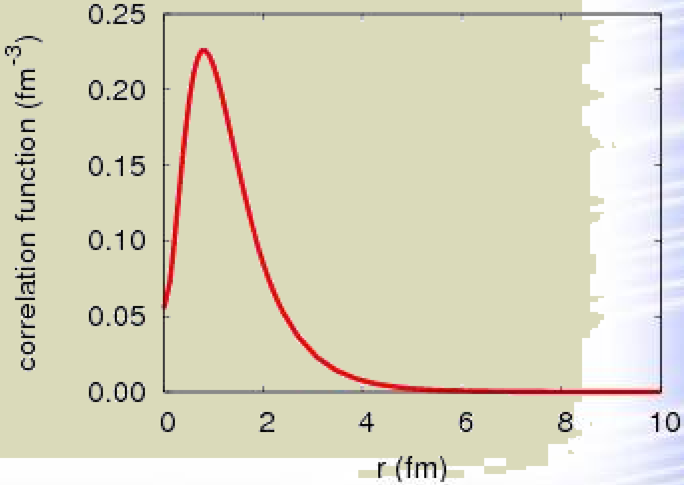
Method	$\langle T \rangle$	$\langle V \rangle$	E_b	$\sqrt{\langle r^2 \rangle}$
FY	102.39(5)	−128.33(10)	−25.94(5)	1.485(3)
CRCGV	102.30	−128.20	−25.90	1.482
SVM	102.35	−128.27	−25.92	1.486
HH	102.44	−128.34	−25.90(1)	1.483
GFMC	102.3(1.0)	−128.25(1.0)	−25.93(2)	1.490(5)
NCSM	103.35	−129.45	−25.80(20)	1.485
EIHH	100.8(9)	−126.7(9)	−25.944(10)	1.486

Results of few-body calculation

* Inputs:
  - m=1161.0 MeV,
  - hbar c = 197.3269602 MeV fm
  - hbar c/e^2 = 137.03599976
  - V_NN is treated as a Serber-type potential.

* Results:
  - B(4He)=4.37 MeV (w/ Coulomb)
    - Probabilities of (S, P, D) waves = (98.6%, 0.003%, 1.3%)
    - I also calculate the correlation function.





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Spin–orbit force from lattice QCD

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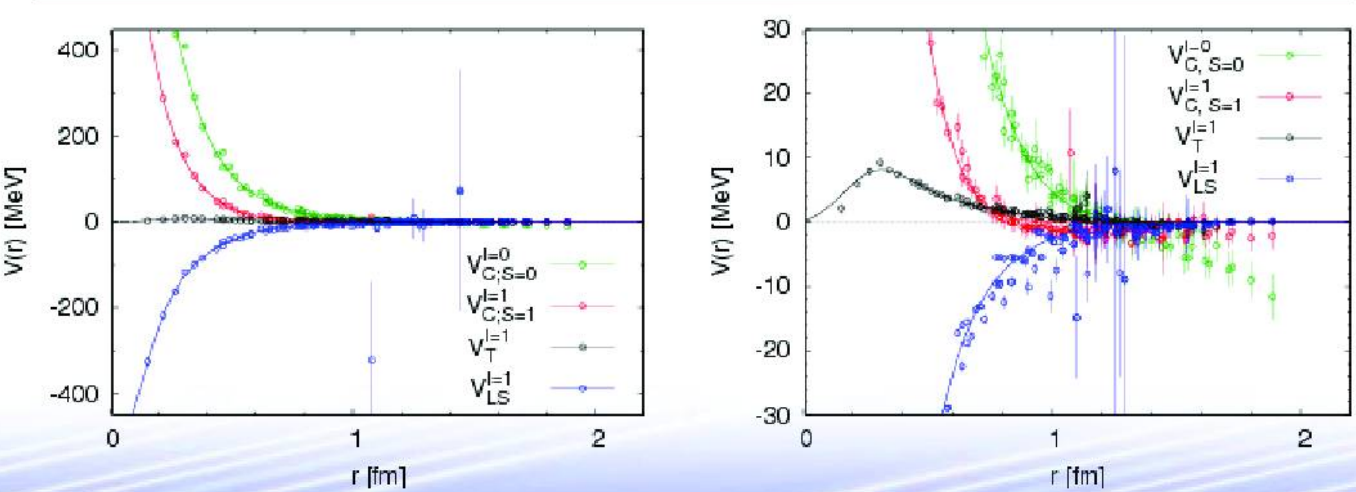
Scattering phase shift

ABSTRACT

We present a first attempt to determine nucleon–nucleon potentials in the parity-odd sector which appear in the $^1P_1$, $^3P_0$, $^3P_1$, $^3P_2$–$^3F_2$ channels, in $N_f = 2$ lattice QCD simulations. These potentials are constructed from the Nambu–Bethe–Salpeter wave functions for $J^P = 0^-$, 1^- and 2^-, which correspond to the A_1^-, T_1^- and $T_2^- \oplus E^-$ representation of the cubic group, respectively. We have found a large and attractive spin–orbit potential $V_{LS}(r)$ in the isospin-triplet channel, which is qualitatively consistent with the phenomenological determination from the experimental scattering phase shifts. The potentials obtained from lattice QCD are used to calculate the scattering phase shifts in the $^1P_1$, $^3P_0$, $^3P_1$ and $^3P_2$–$^3F_2$ channels. The strong attractive spin–orbit force and a weak repulsive central force in spin-triplet P-wave channels lead to an attraction in the $^3P_2$ channel, which is related to the P-wave neutron paring in neutron stars.



Inoue san's NN potential

- Central and spin-oribt potentials

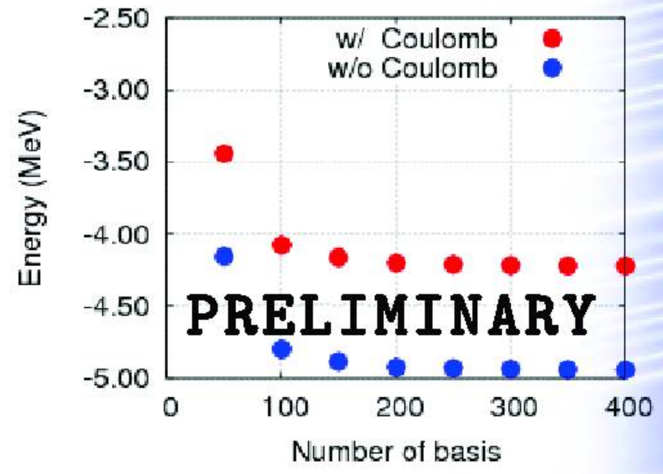
$$V_{C,LS}(r)=V_1\exp(-\alpha_1 r^2)+V_2\exp(-\alpha_2 r^2)$$
$$-V_3(1-\exp(-\alpha_3 r^2))^2(\exp(-\alpha_4 r)/r)^2$$

- Tensor potential

$$V_T(r)=V_1(1-\exp(-\alpha_1 r^2))^2\left(1+\frac{3}{\alpha_2 r}+\frac{3}{\alpha_2 r^2}\right)\frac{\exp(-\alpha_2 r)}{r}$$
$$+V_2(1-\exp(-\alpha_3 r^2))^2\left(1+\frac{3}{\alpha_4 r}+\frac{3}{\alpha_4 r^2}\right)\frac{\exp(-\alpha_4 r)}{r}$$

Results of few-body calculation

- Inputs:
  - m=1161.0 MeV,
  - hbar c = 197.3269602 MeV fm
  - hbar c/e^2 = 137.03599976
  - V_NN consists of AV8 type operators, determined from {1S0, 3S1, 3SD1, 1P1, 3P0, 3P1, 3PF2}.
    - V_0, V_σ, V_τ, $V_{\sigma\tau}$, V_T, $V_{T\tau}$, V_{LS}^{odd} are determined



- Preliminary results:
  - B(4He)=4.23 MeV (w/ Coulomb) (old: 4.37MeV)
    - Probabilities of (S, P, D) waves = (98.8%, 0.002%, 1.2%)
    - cf. roughly speaking (S,P,D)~(<90%, <0.1%, >10%) for a realistic NN force
  - B(4He)=4.95 MeV (w/o Coulomb) (old: 5.09MeV)
    - Probabilities of (S, P, D) waves = (98.8%, 0.002%, 1.2%)

Summary

(I-1) Lattice QCD calculation for hyperon potentials toward the physical point calculation. (Lambda-N, Sigma-N: central, tensor)

(I-2) Effective hadron block algorithm for the various baron-baryon interaction [arXiv:1510.00903(hep-lat)]

A hybrid parallel (C++) and multi-GPU (CUDA) program is implemented (MPI + OpenMP).

Reasonable performances at various hybrid parallel execution on the supercomputers (BlueGene/Q and HA-PACS)

(II-1) Four-nucleon bound state with a lattice NN potential (+Coulomb potential) has been solved by using the stochastic variational method

The lattice NN potential is represented by AV8-type operators.

The tensor potential is weaker than the phenomenological realistic NN potentials due to the heavier pion mass about 470MeV.

The D-state probability is only about 1.2%.