Magnetic properties of light nuclei from lattice QCD

William Detmold, MIT

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Magnetic properties of light nuclei from lattice QCD

I. Nuclei in LQCD

II. Magnetic moments & polarisabilities
[NPLQCD PRL 113, 252001 (2014), 1506.05518]

III. Thermal neutron capture cross-section: $np \to d\gamma$
[NPLQCD PRL 115, 132001 (2015)]

IV. Nuclear physics at the Intensity Frontier

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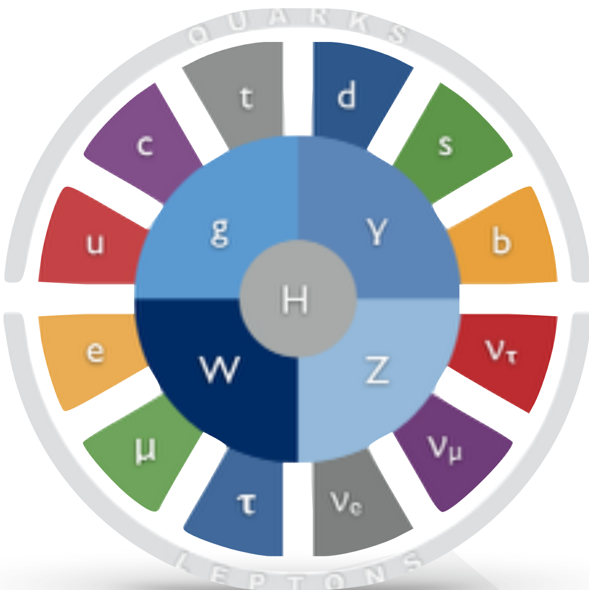
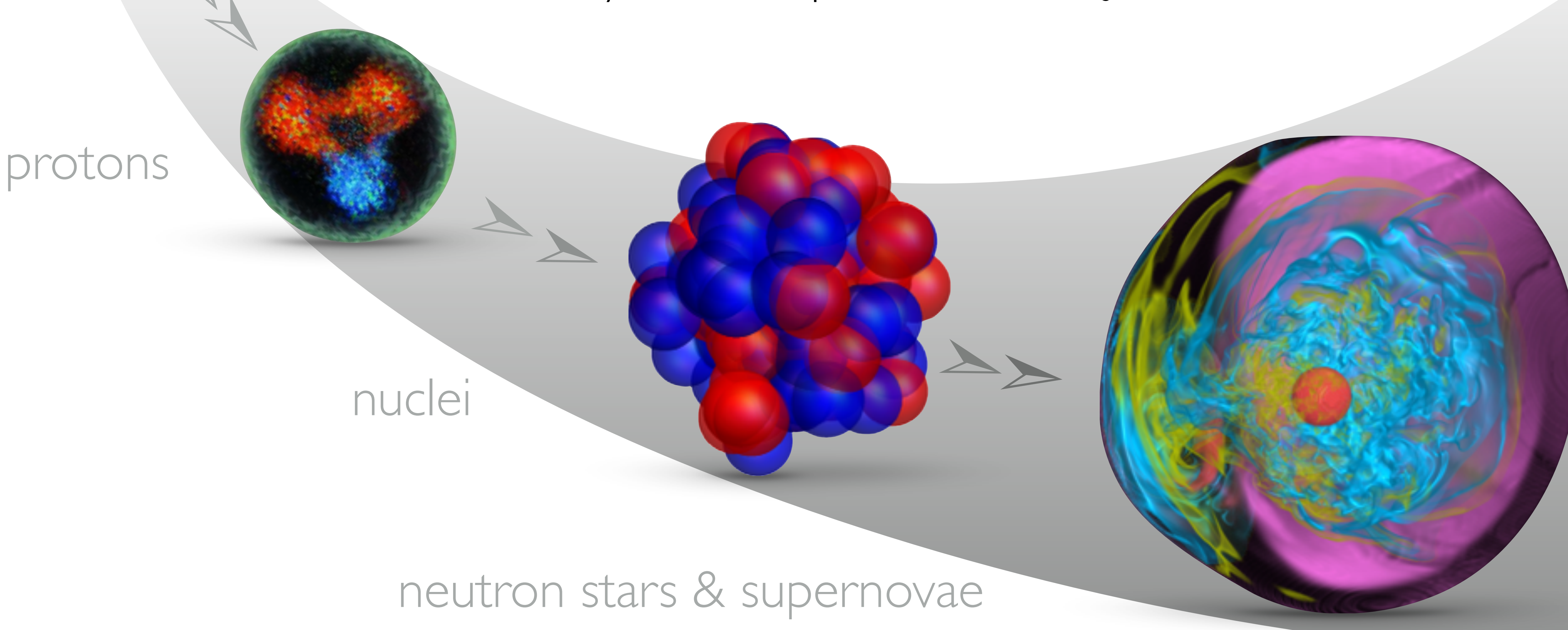
Nuclear physics from Lattice QCD



- Complexity of nuclear physics emerges from the Standard Model
  - Same underlying physics at vastly different scales
  - EM, weak and strong (QCD) interactions
  - Only relevant parameters: Λ_{QCD}, $m_{u,d,s}$, α

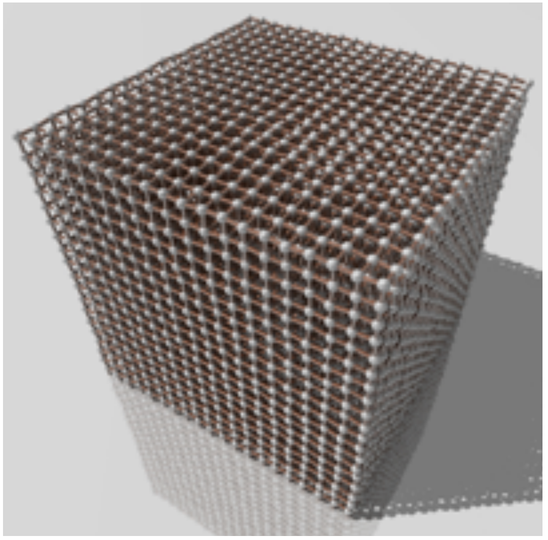


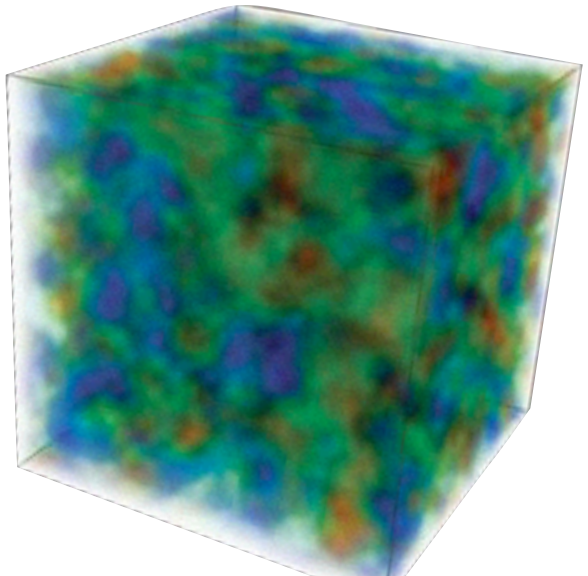
Quantum Chromodynamics

- Lattice QCD: tool to deal with quarks and gluons
  - Formulate problem as functional integral over quark and gluon d.o.f. on R_4

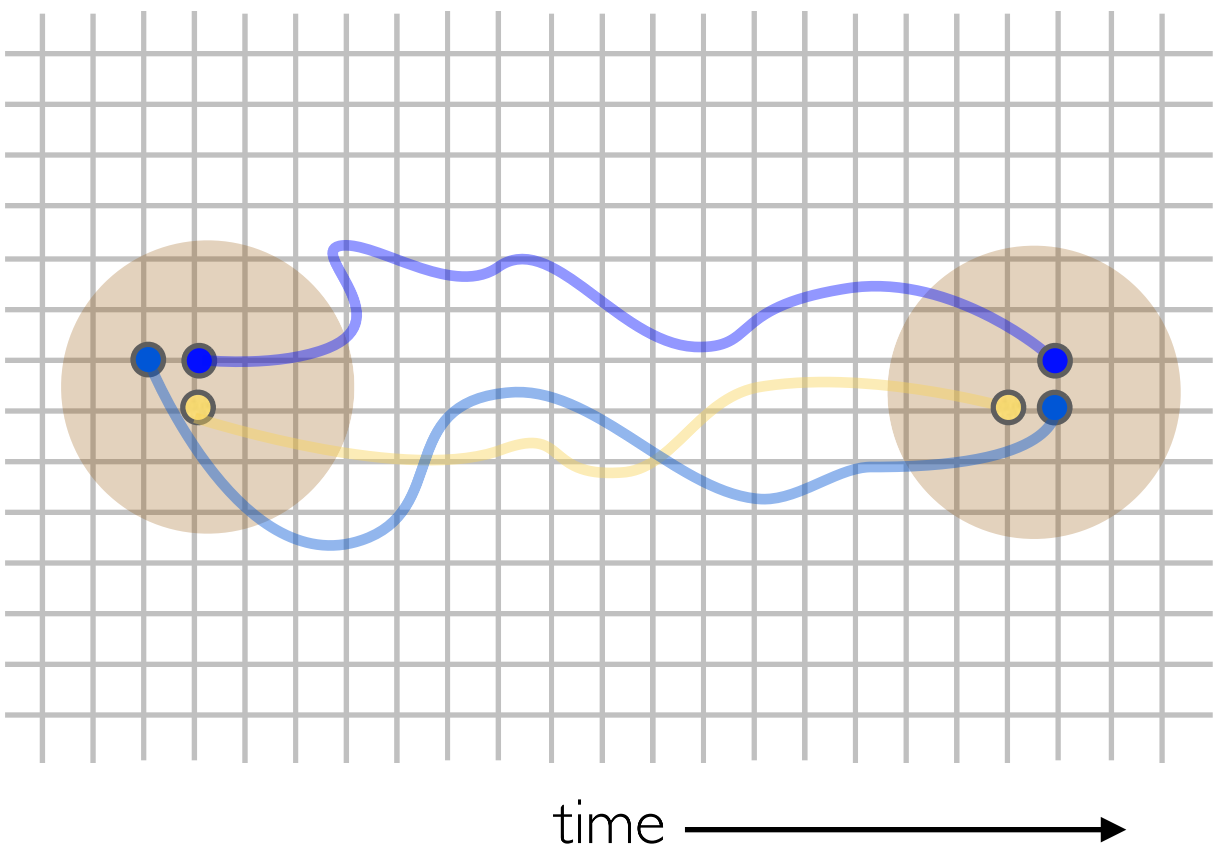
$$\langle \mathcal{O} \rangle = \int dA_\mu dq d\bar{q}\, \mathcal{O}[q,\bar{q},A] e^{-S_{QCD}[q,\bar{q},A]}$$

  - Discretise and compactify system
    - Finite but large number of d.o.f $(\sim 10^{10})$
  - Integrate via importance sampling (average over important configurations)
  - Undo the harm done in previous steps





- How do we calculate the proton mass?
- Create three quarks at a source: and annihilate the three quarks at sink far from source
- QCD adds all the quark anti-quark pairs and gluons automatically: only eigenstates with correct q#'s propagate



- Correlation decays exponentially with distance

$$C(t) = \sum_n Z_n \exp(-E_n t)$$

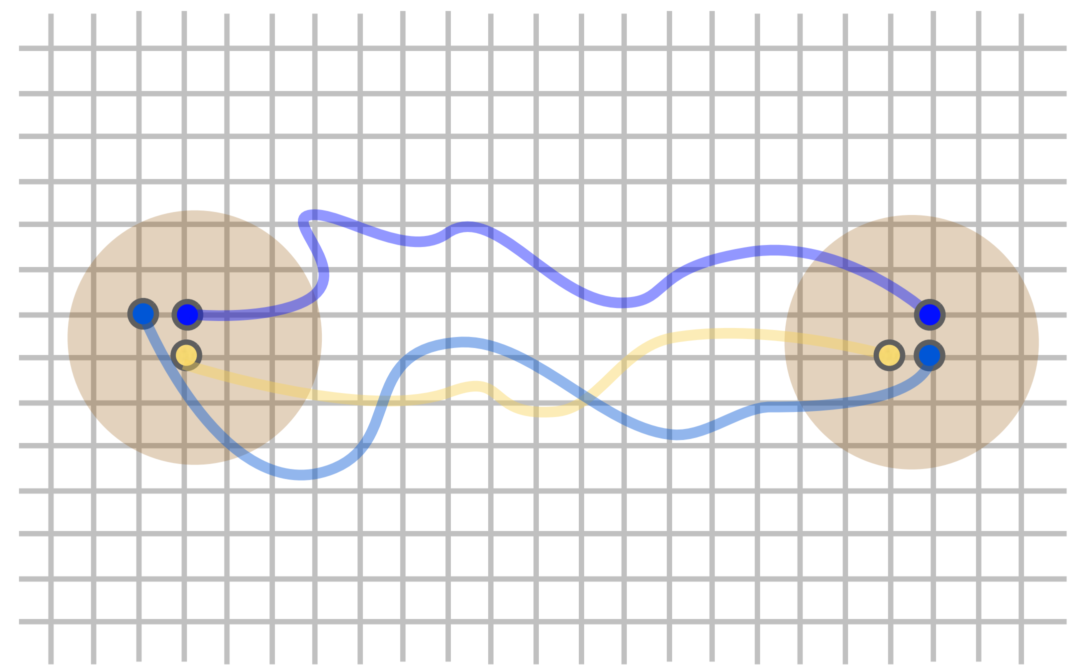
n ← all eigenstates with q#'s of proton

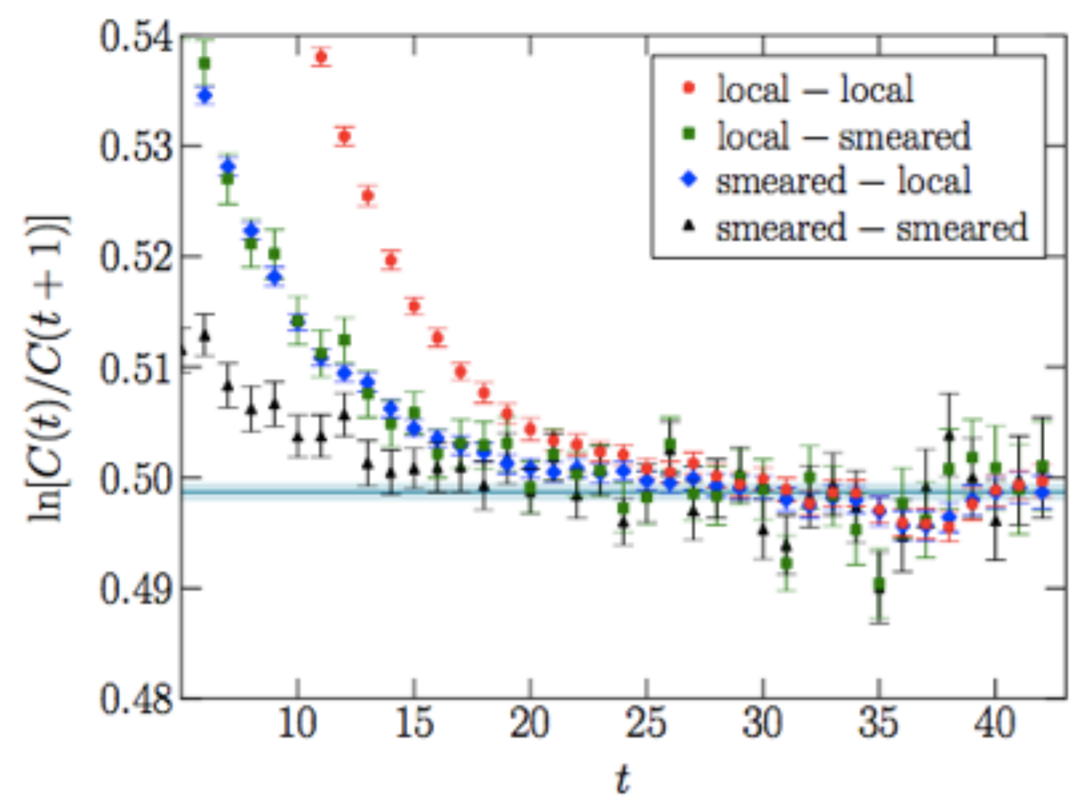
at late times

$$\rightarrow Z_0 \exp(-E_0 t)$$

- Ground state energy revealed through "effective mass plot"

$$M(t) = \ln\left[\frac{C(t)}{C(t+1)}\right] \xrightarrow{t\to\infty} E_0$$





21st century LQCD

- LQCD is an mature field: 30+ years since first calculations
  - ~2000: QCD (no "quenched" mutilation)
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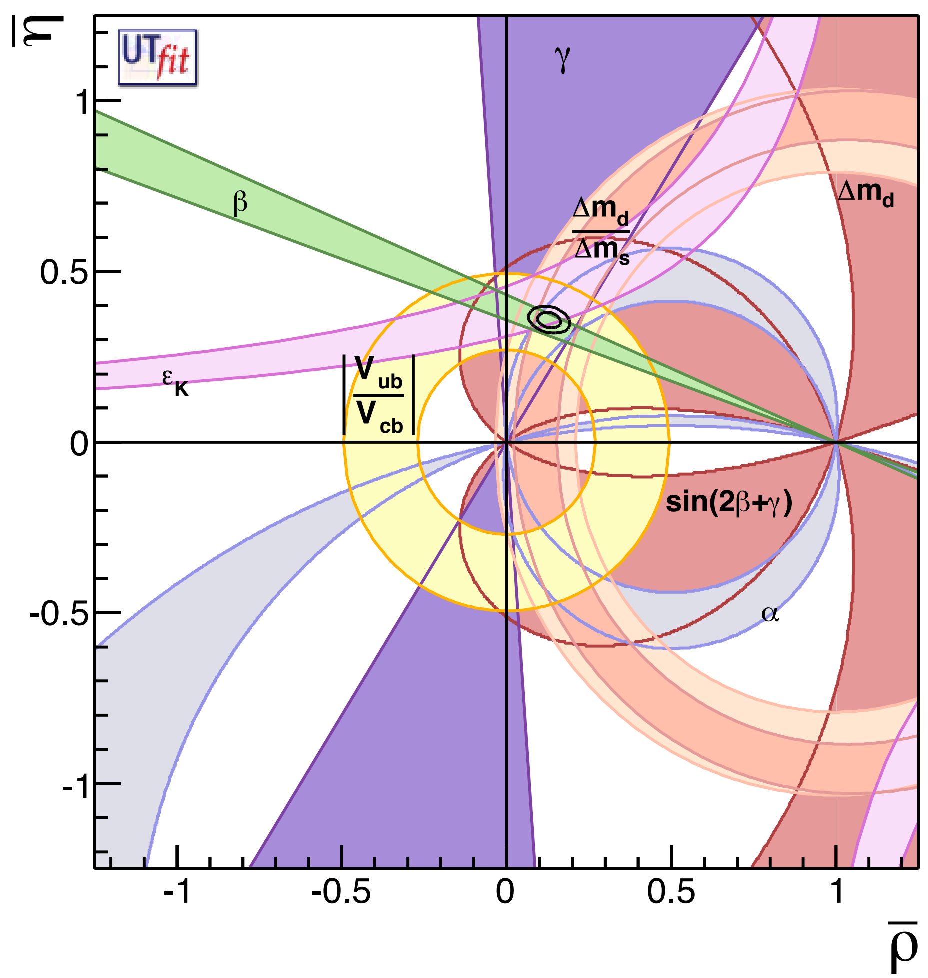
  - Combine with experiment to determine SM parameters



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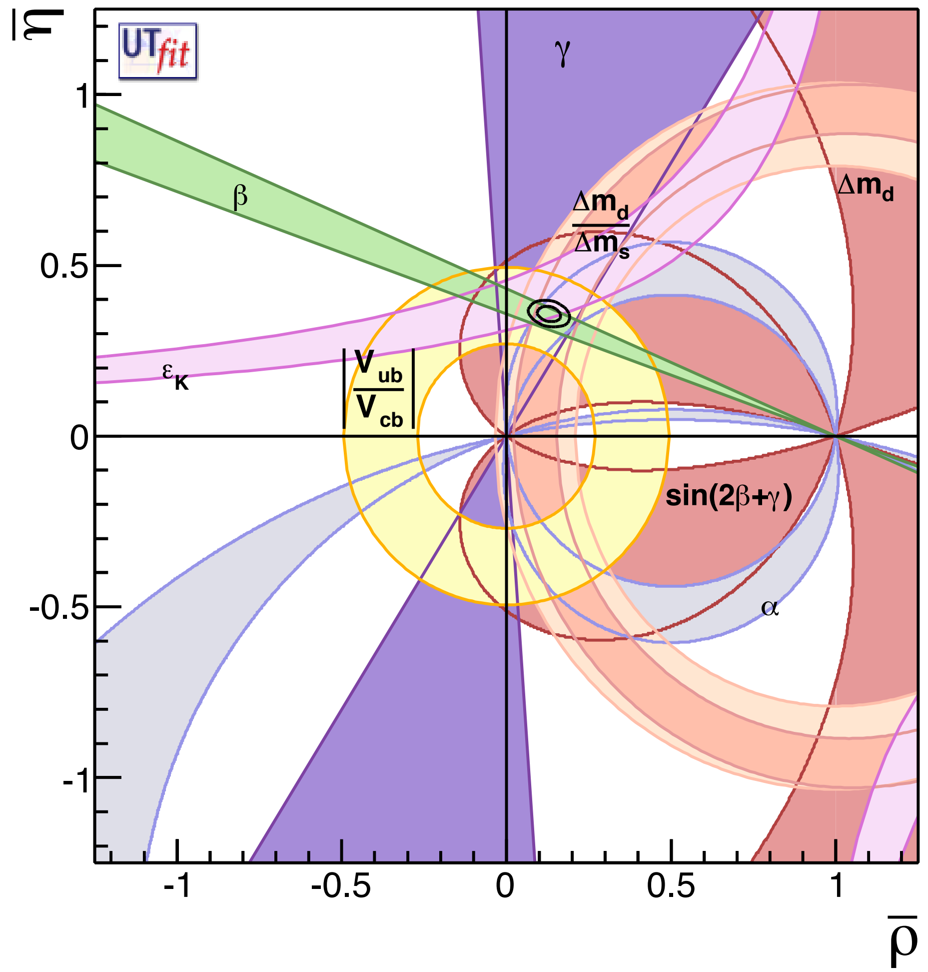
    - Verify CKM paradigm



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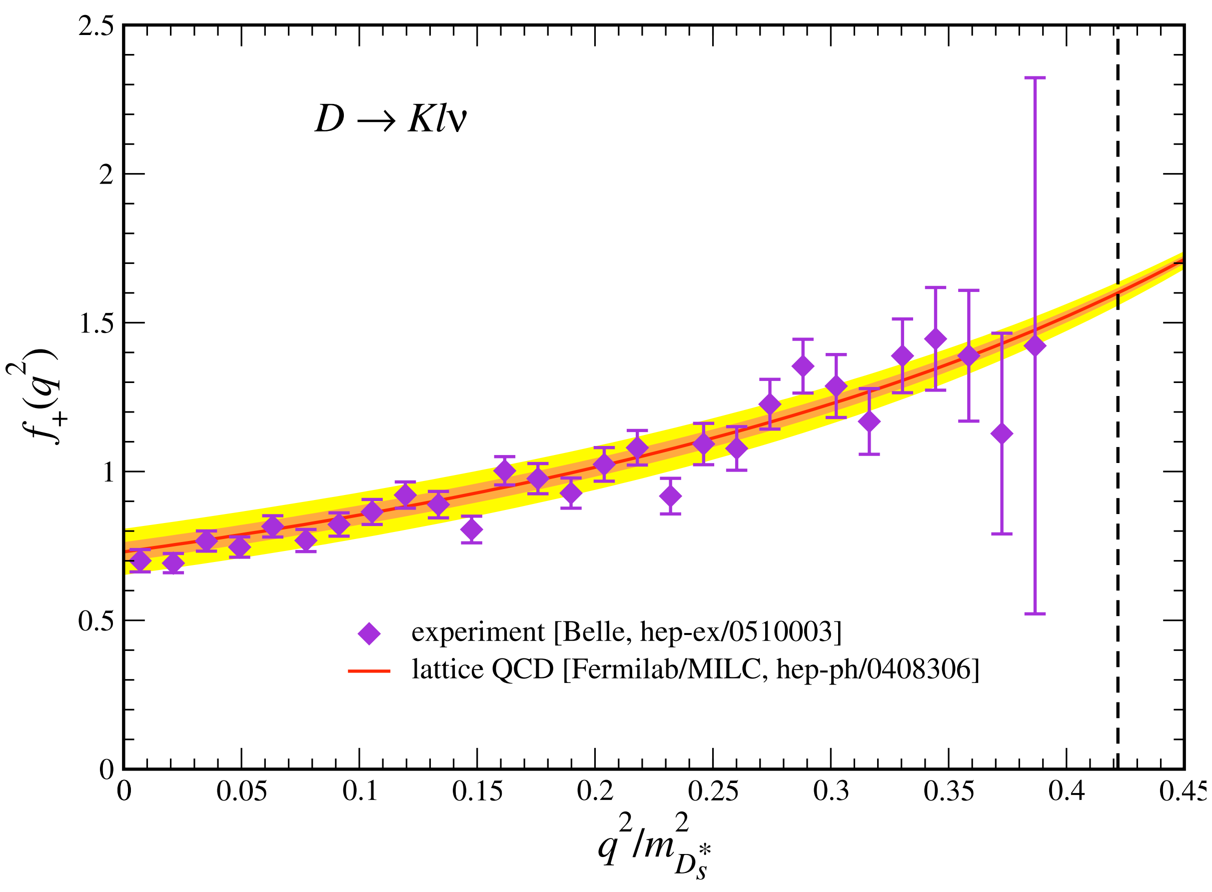
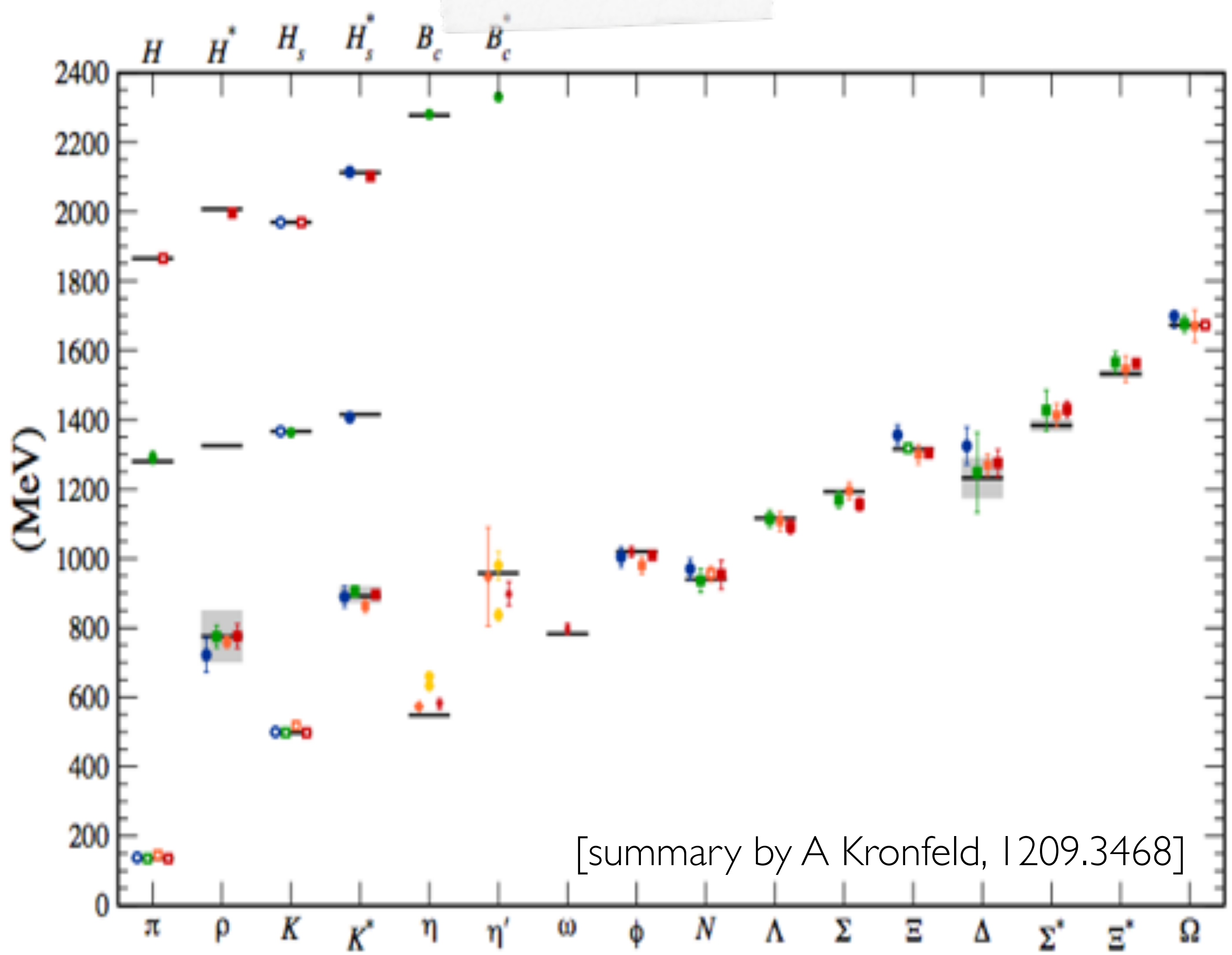
    - Verify CKM paradigm
  - SM predictions with reliable uncertainty quantification



QCD Spectrum

- After 30 years of developments

😍 Ground state hadron spectrum reproduced



- After 30 years of developments
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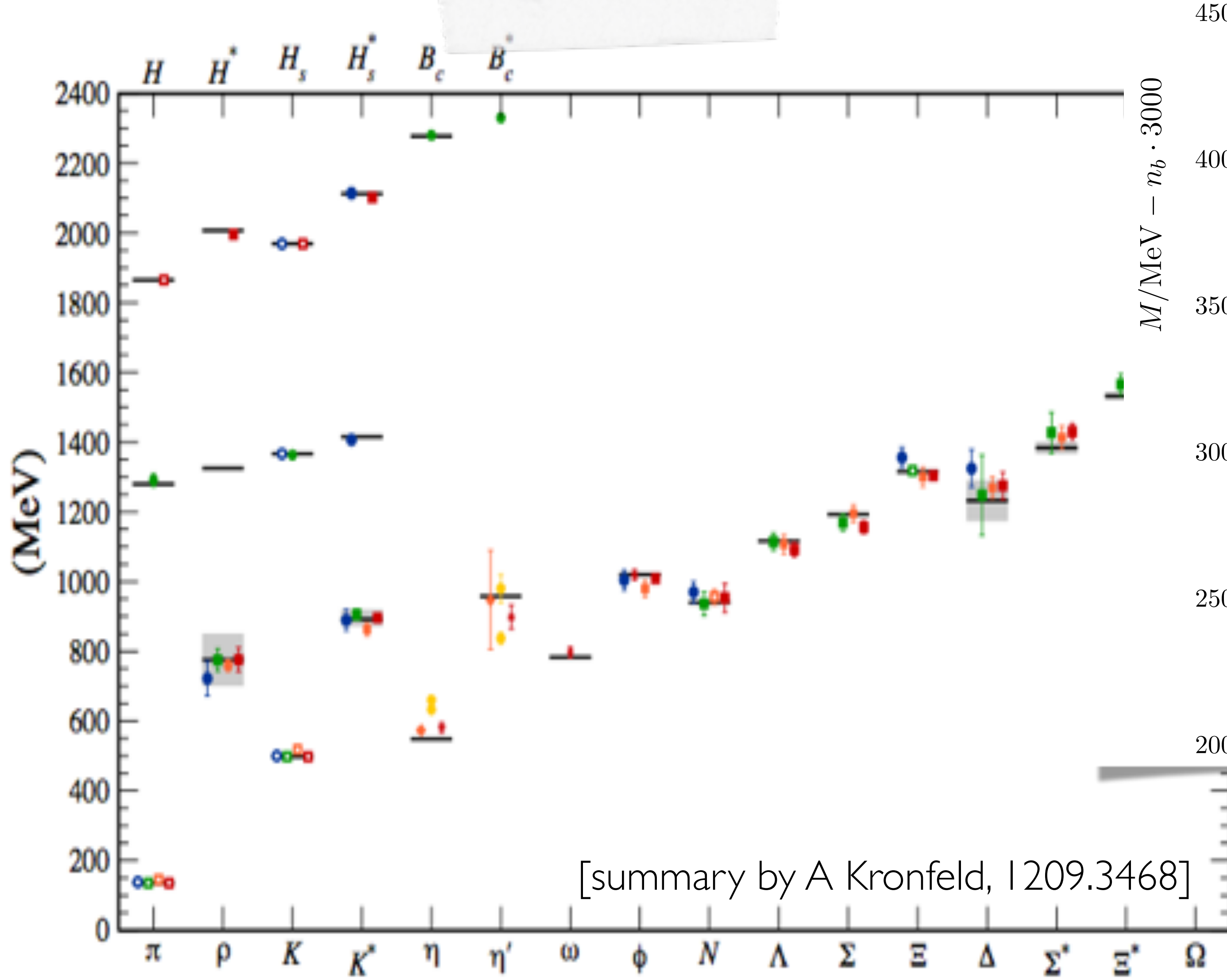
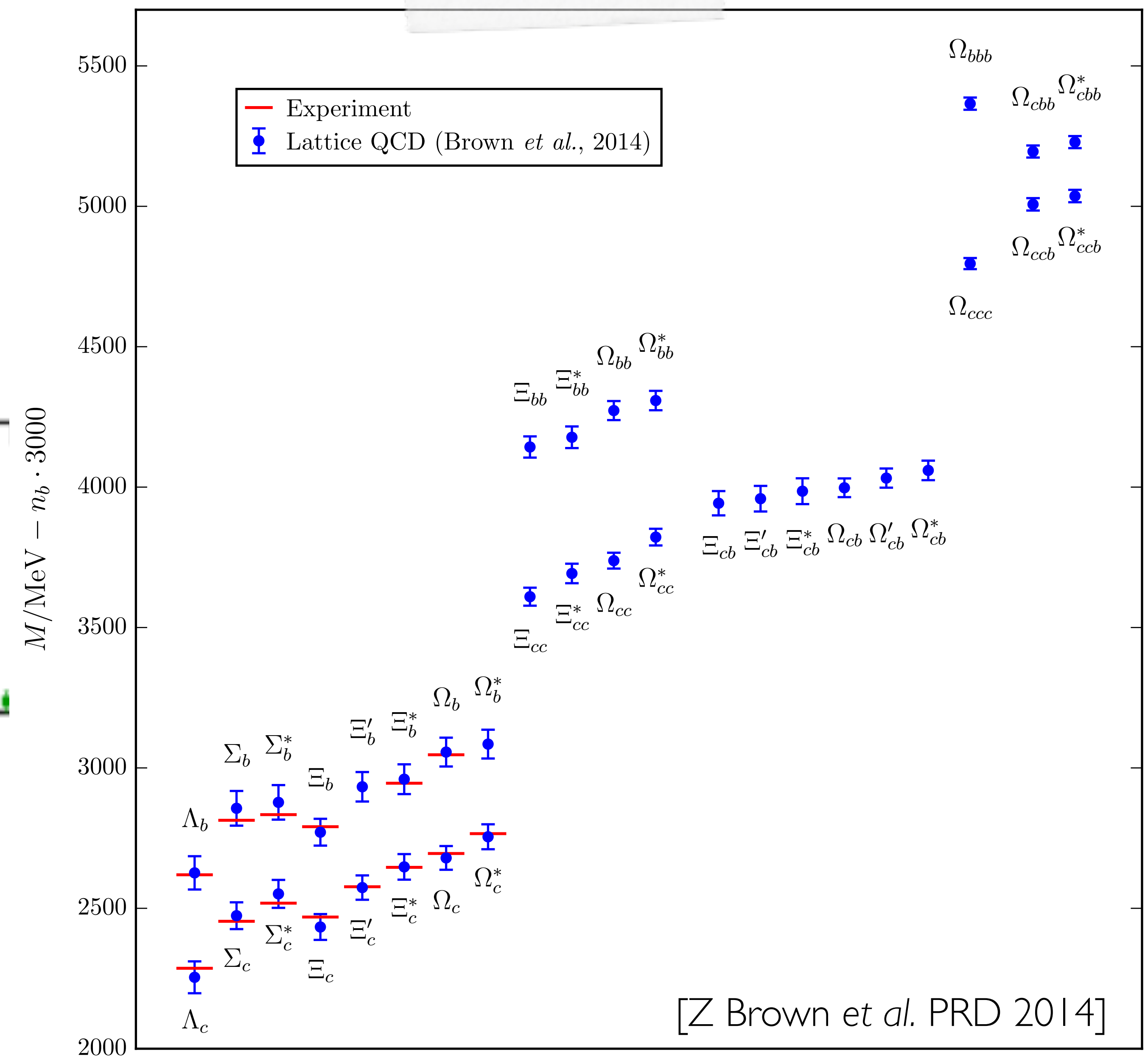
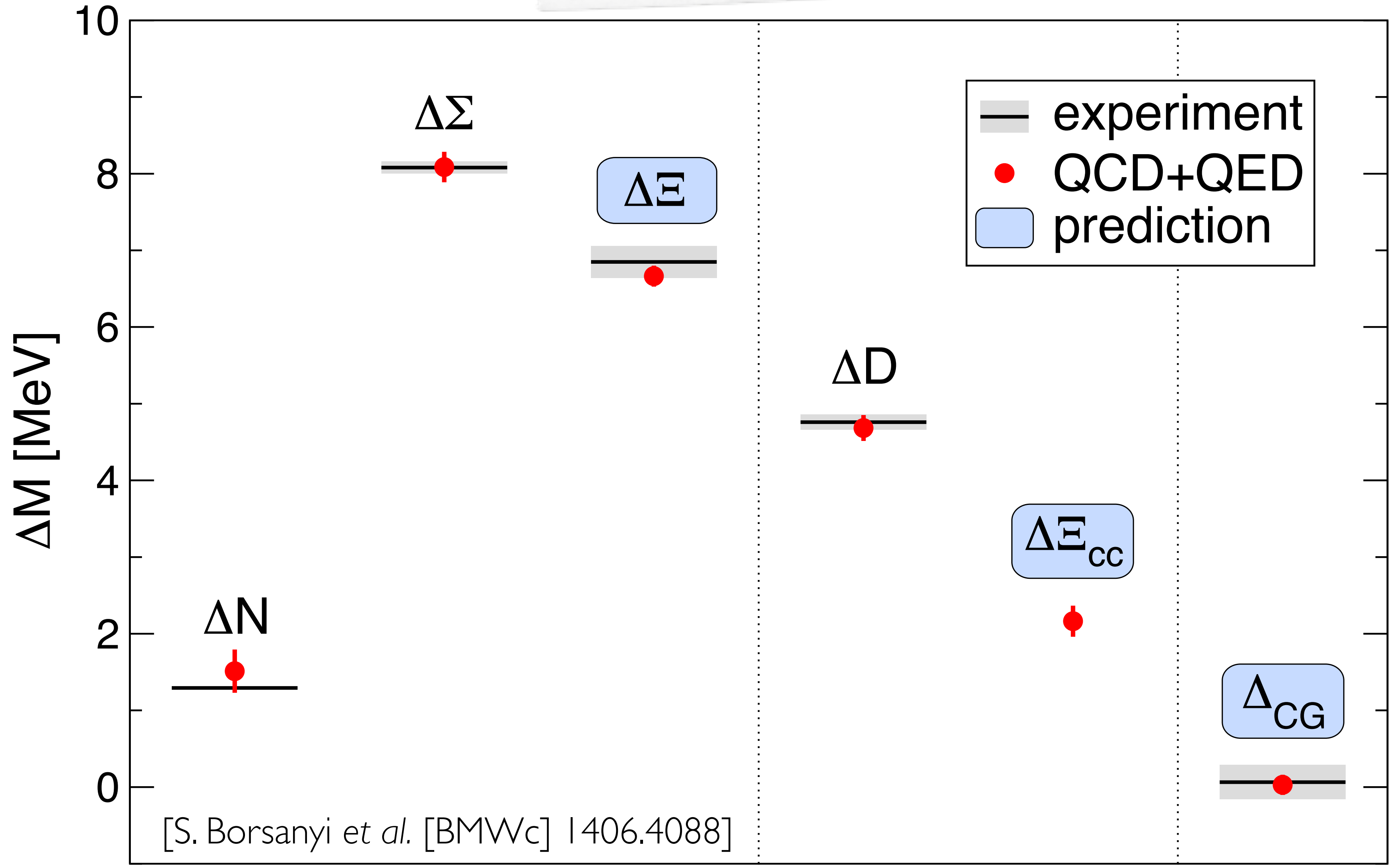




Standard Model Spectrum

😍 Precise isospin mass splittings in QCD+QED



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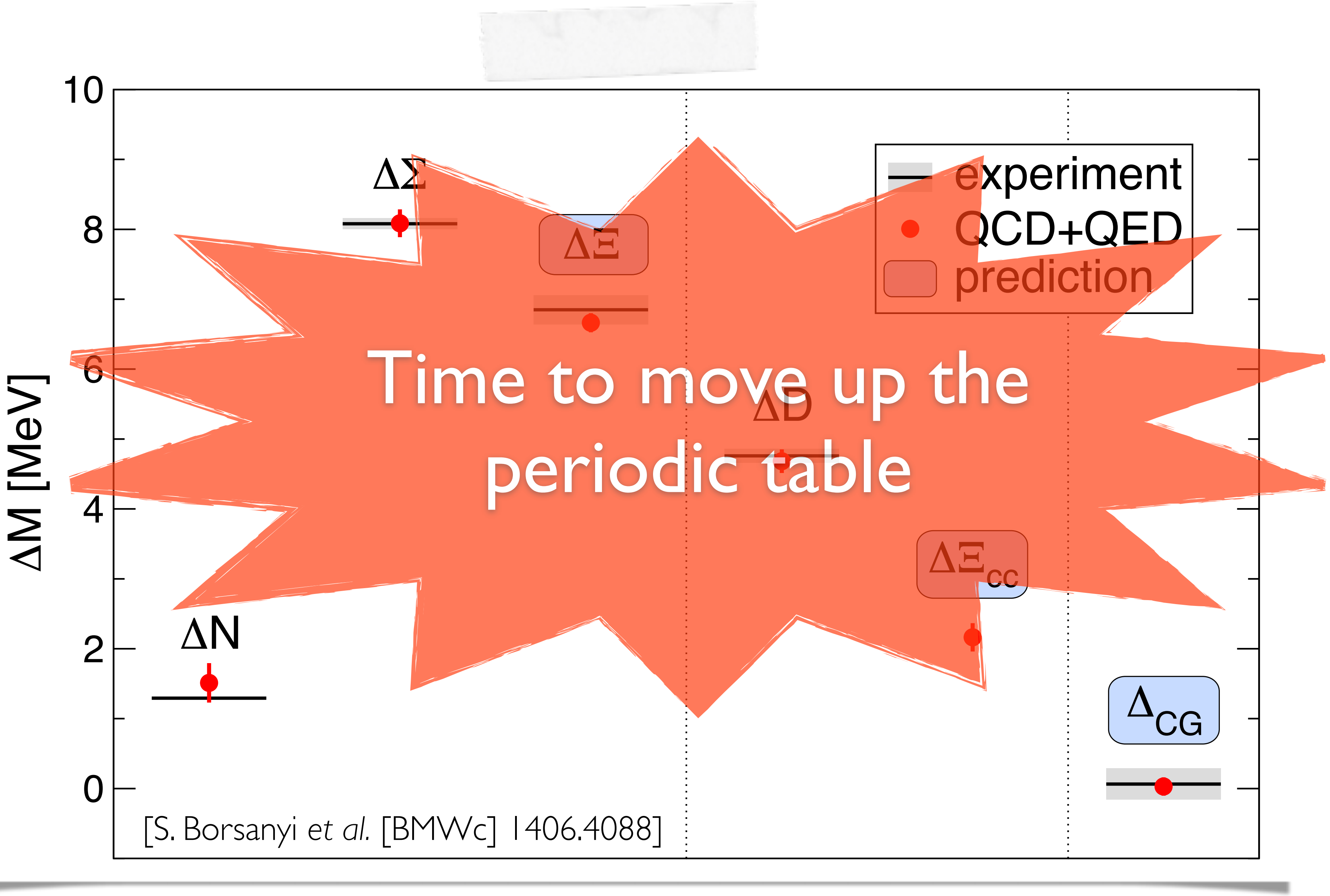
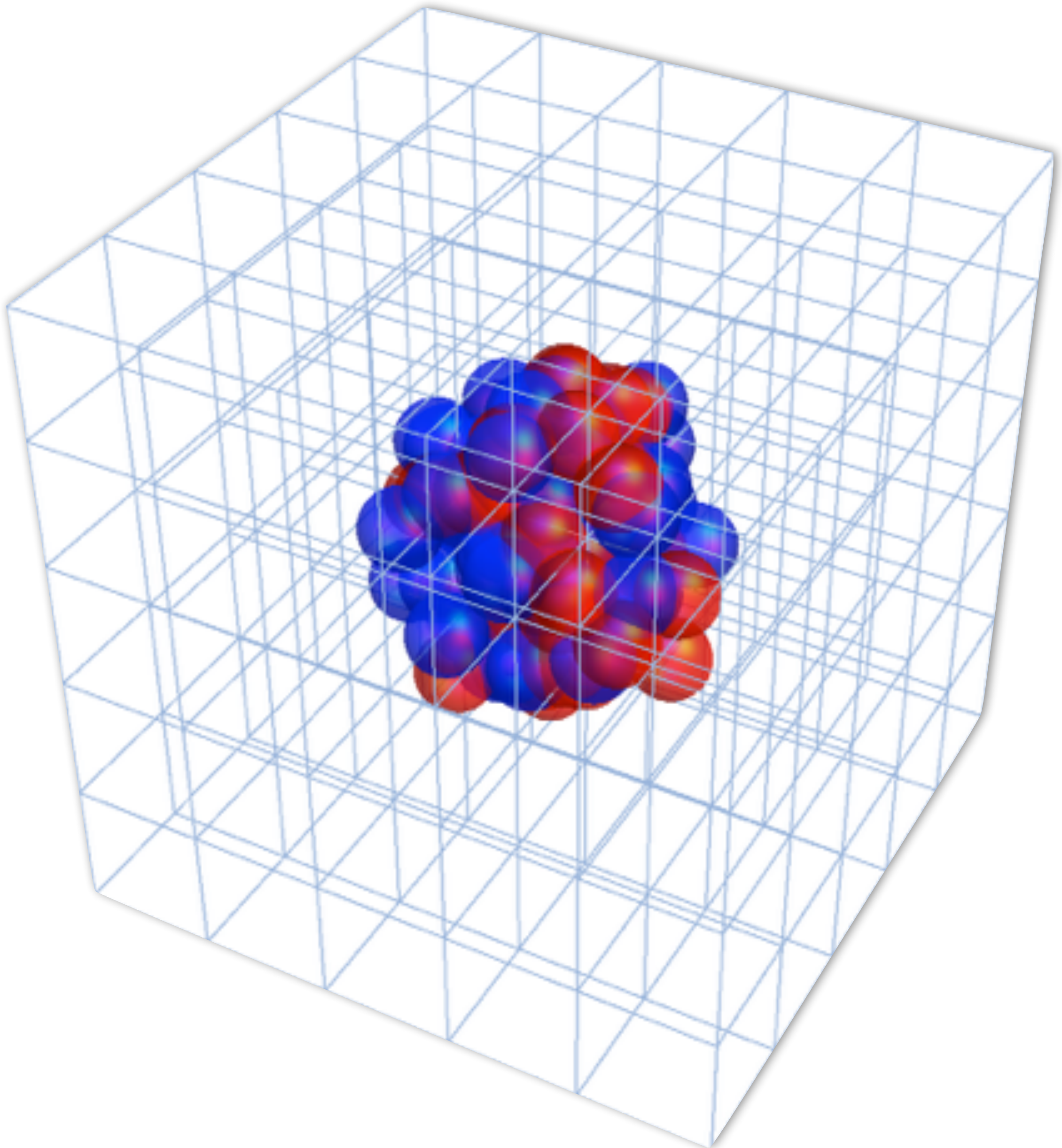


QCD for Nuclear Physics

- Nuclear physics is Standard Model physics
    - QCD (+ electroweak)
    - Can compute the mass of lead nucleus ... in principle
- In practice: a hard problem
    - QCD in non-perturbative domain
    - Physics at multiple scales



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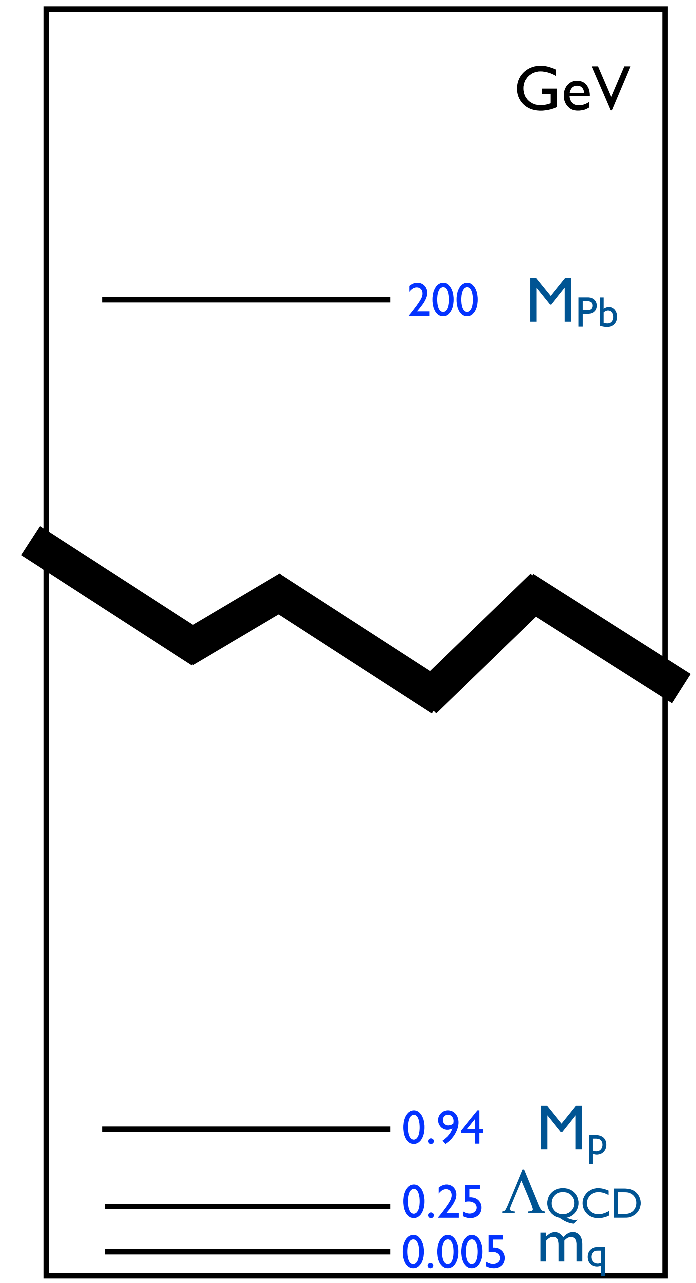

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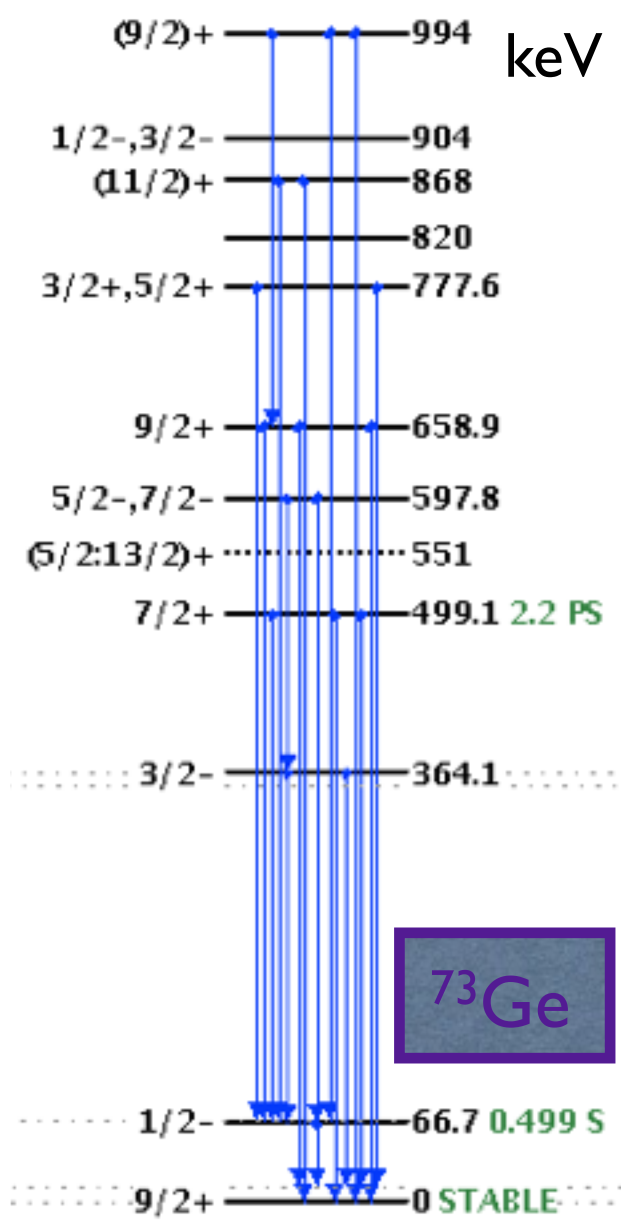

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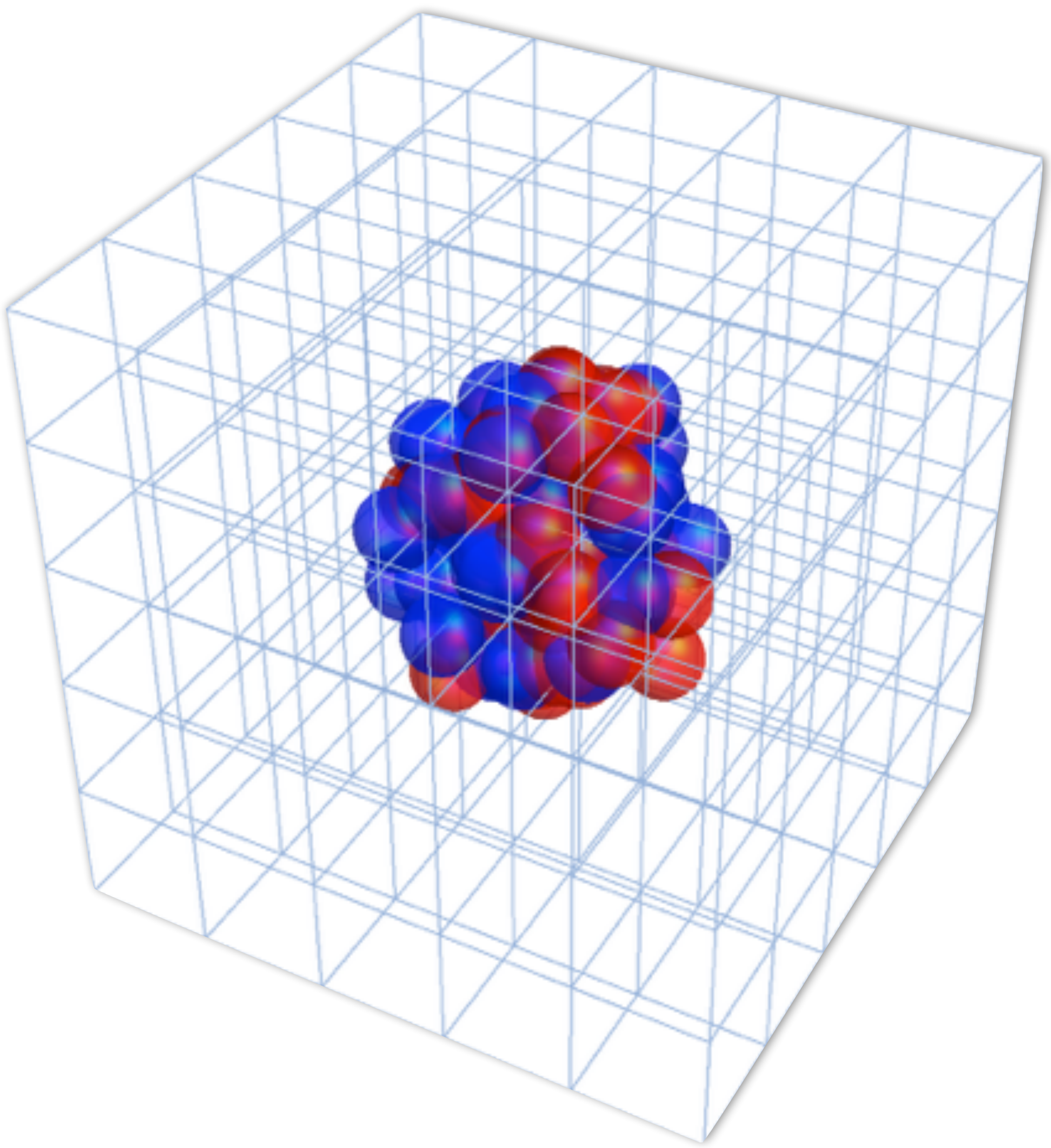
- At least two exponentially difficult challenges
  - Noise: statistical uncertainty grows exponentially with A
  - Contraction complexity grows factorially



The trouble with baryons

[Lepage '89]

- Importance sampling of QCD functional integrals
  ➤ correlators determined stochastically

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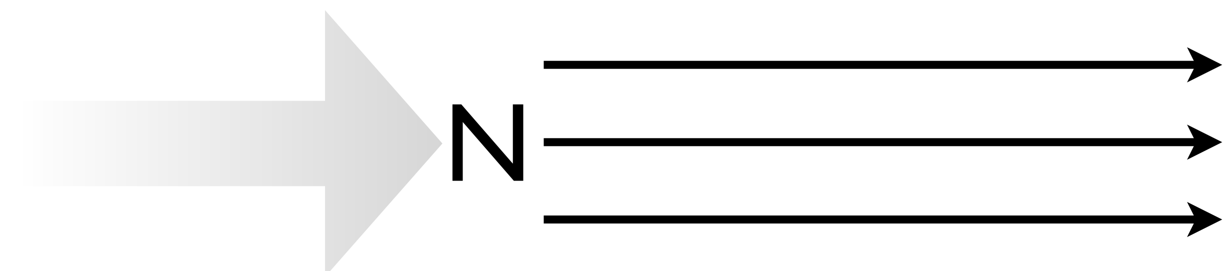
- Proton

  $$\text{signal} \sim \langle C \rangle \sim \exp[-M_N t]$$



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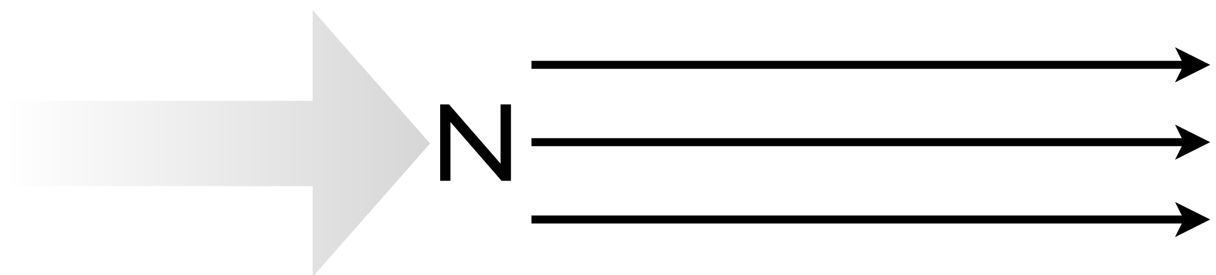

  - Variance determined by

$$\sigma^2(C) = \langle C C^\dagger \rangle - |\langle C \rangle|^2$$

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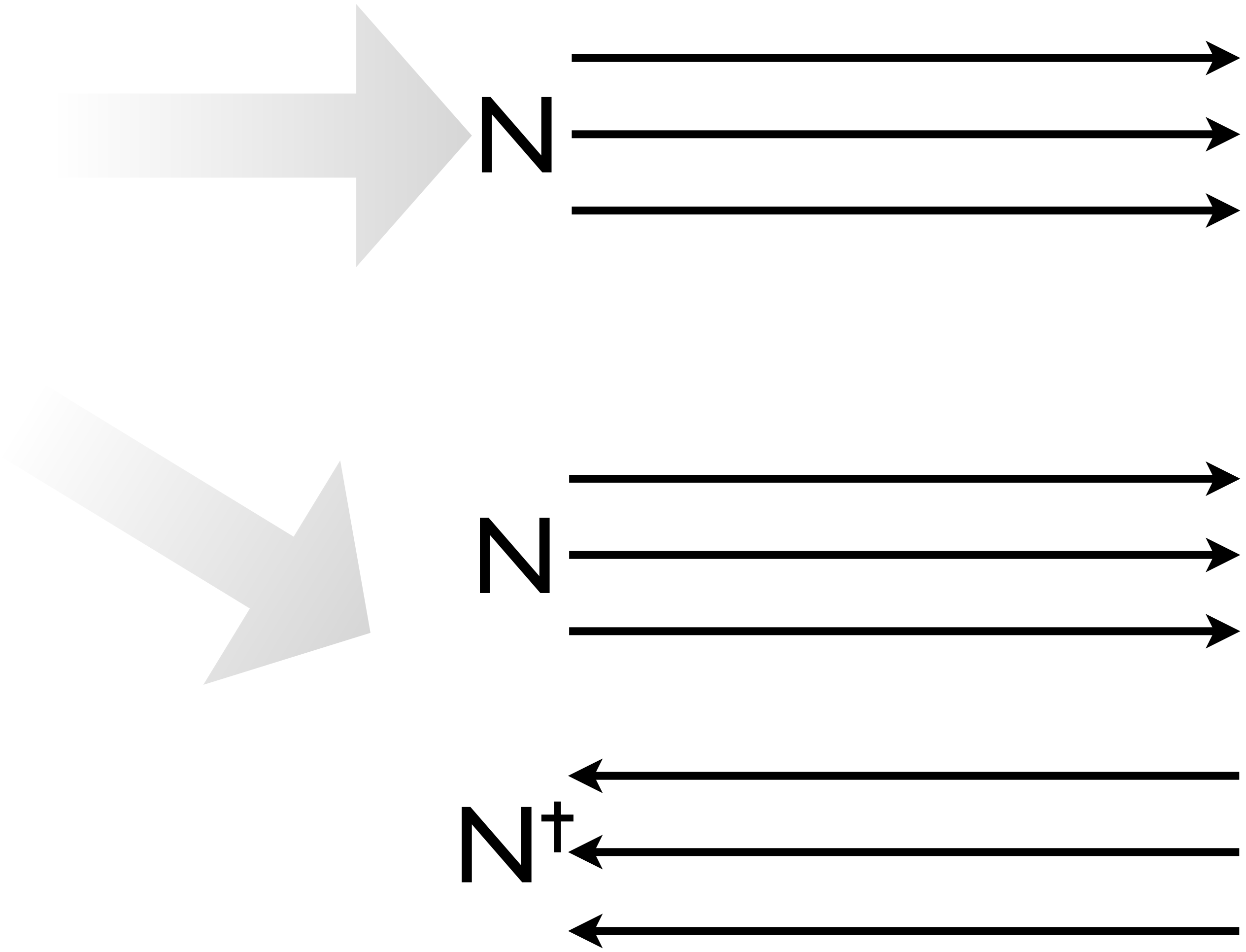

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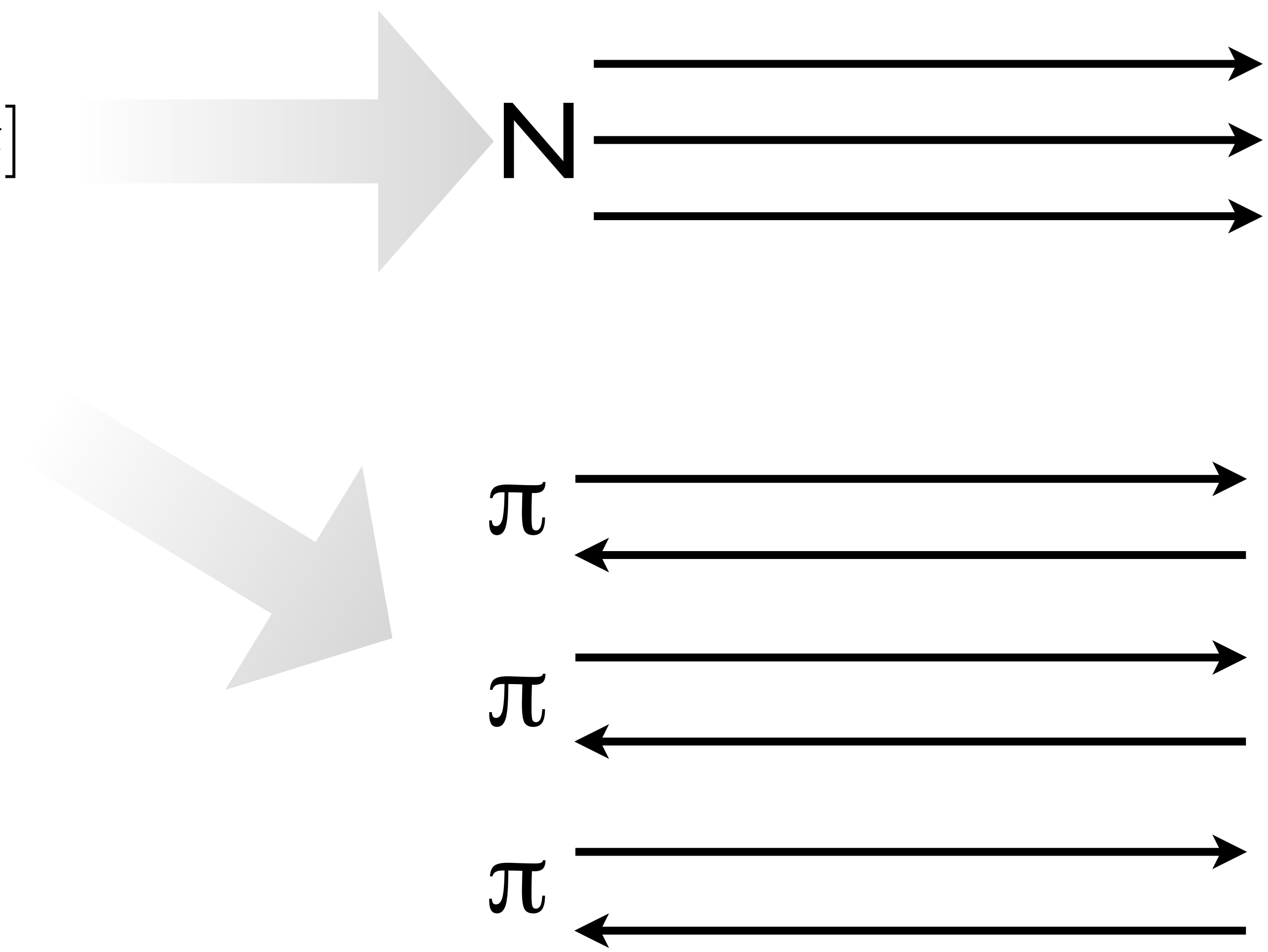

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$$\sigma^2(C) = \langle CC^\dagger \rangle - |\langle C \rangle|^2$$

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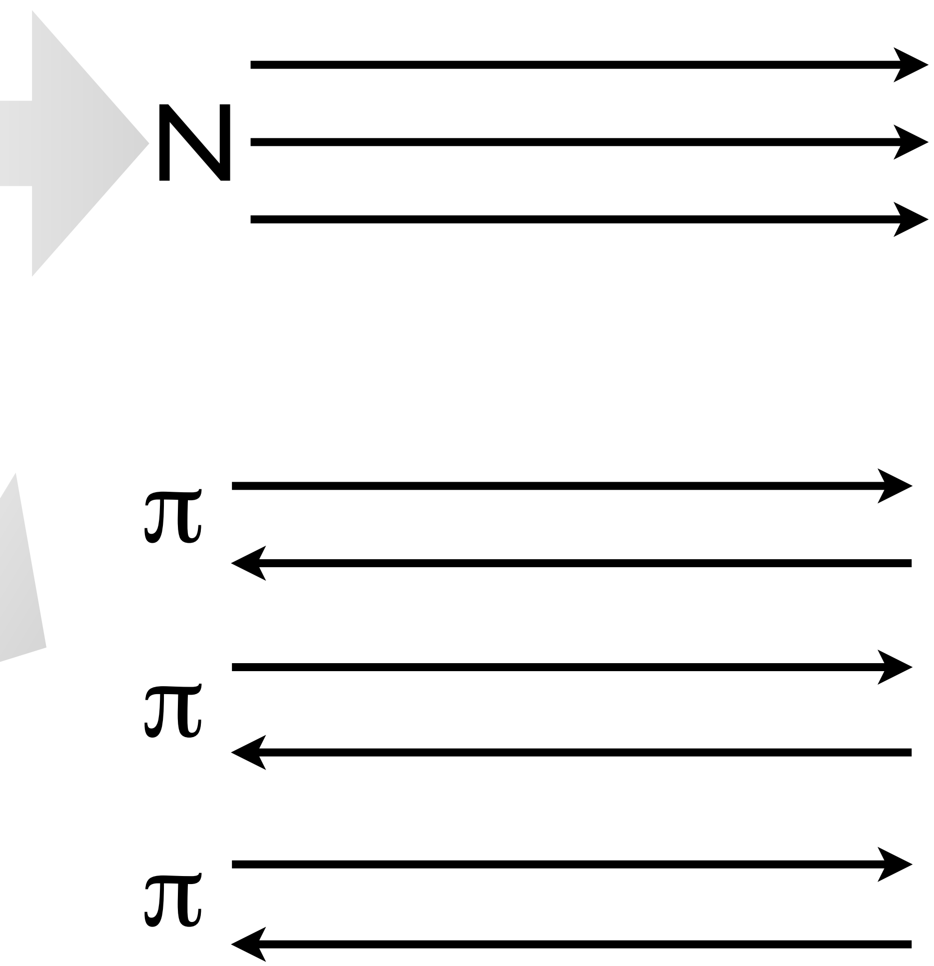

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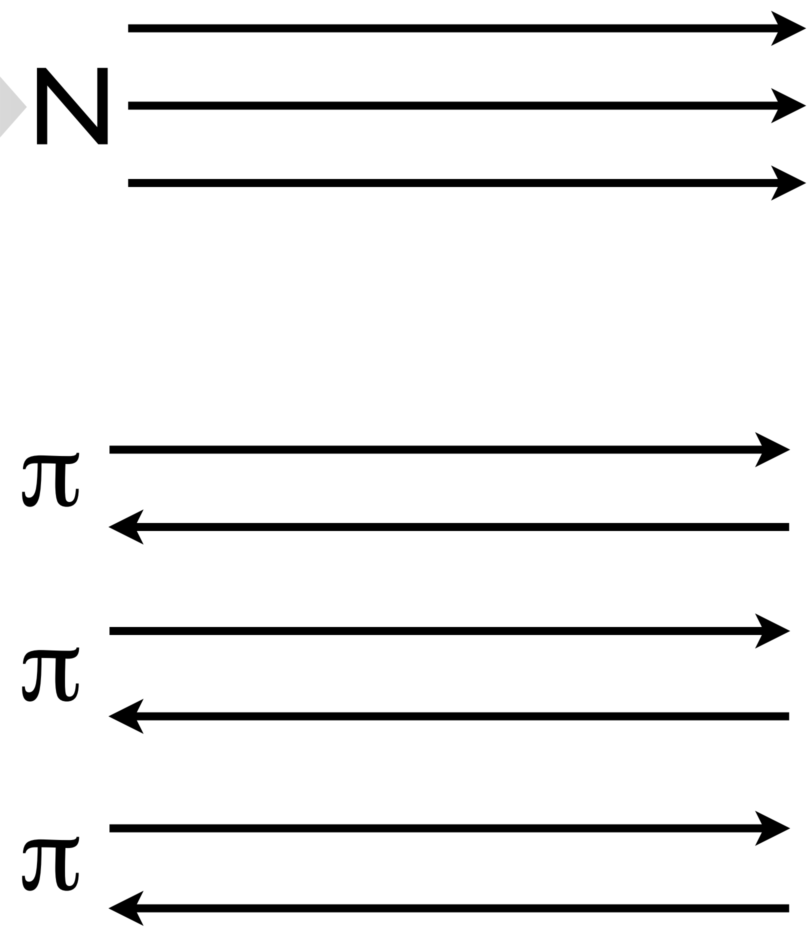

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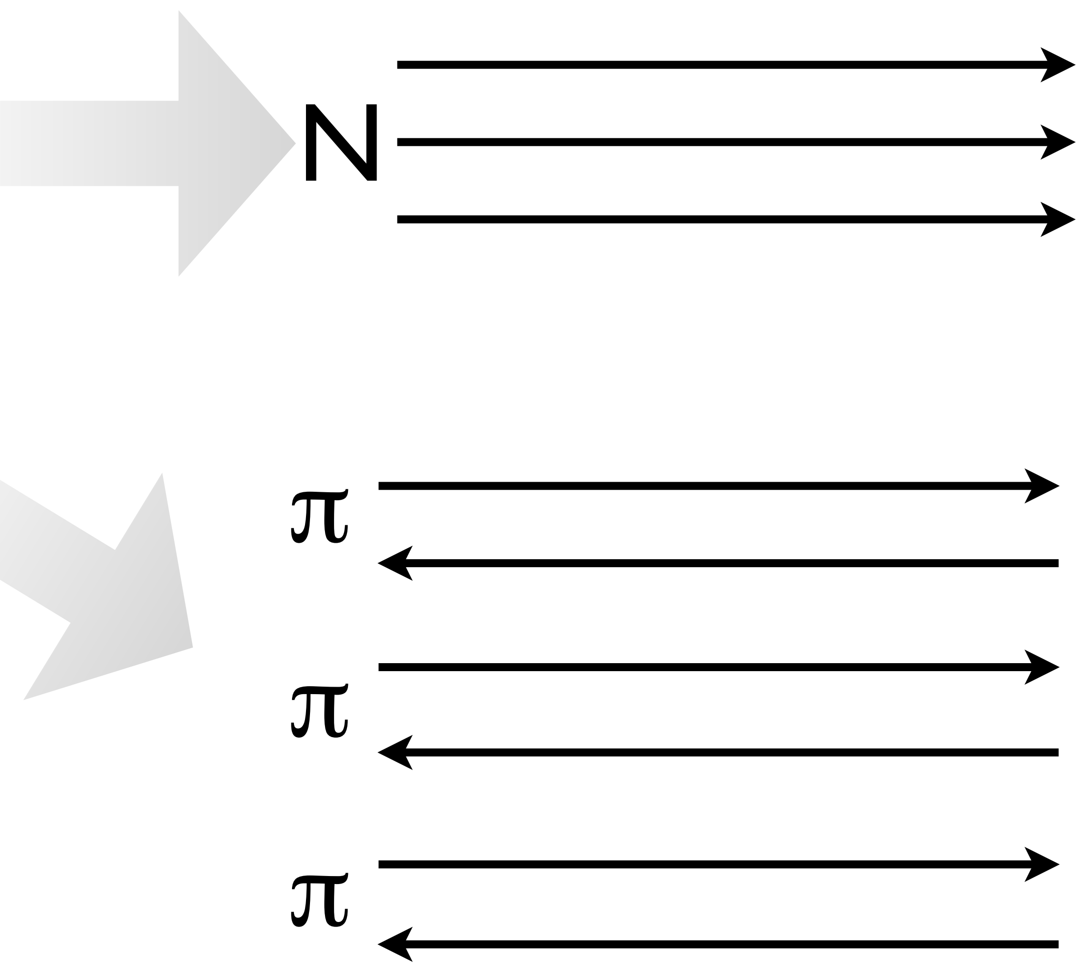


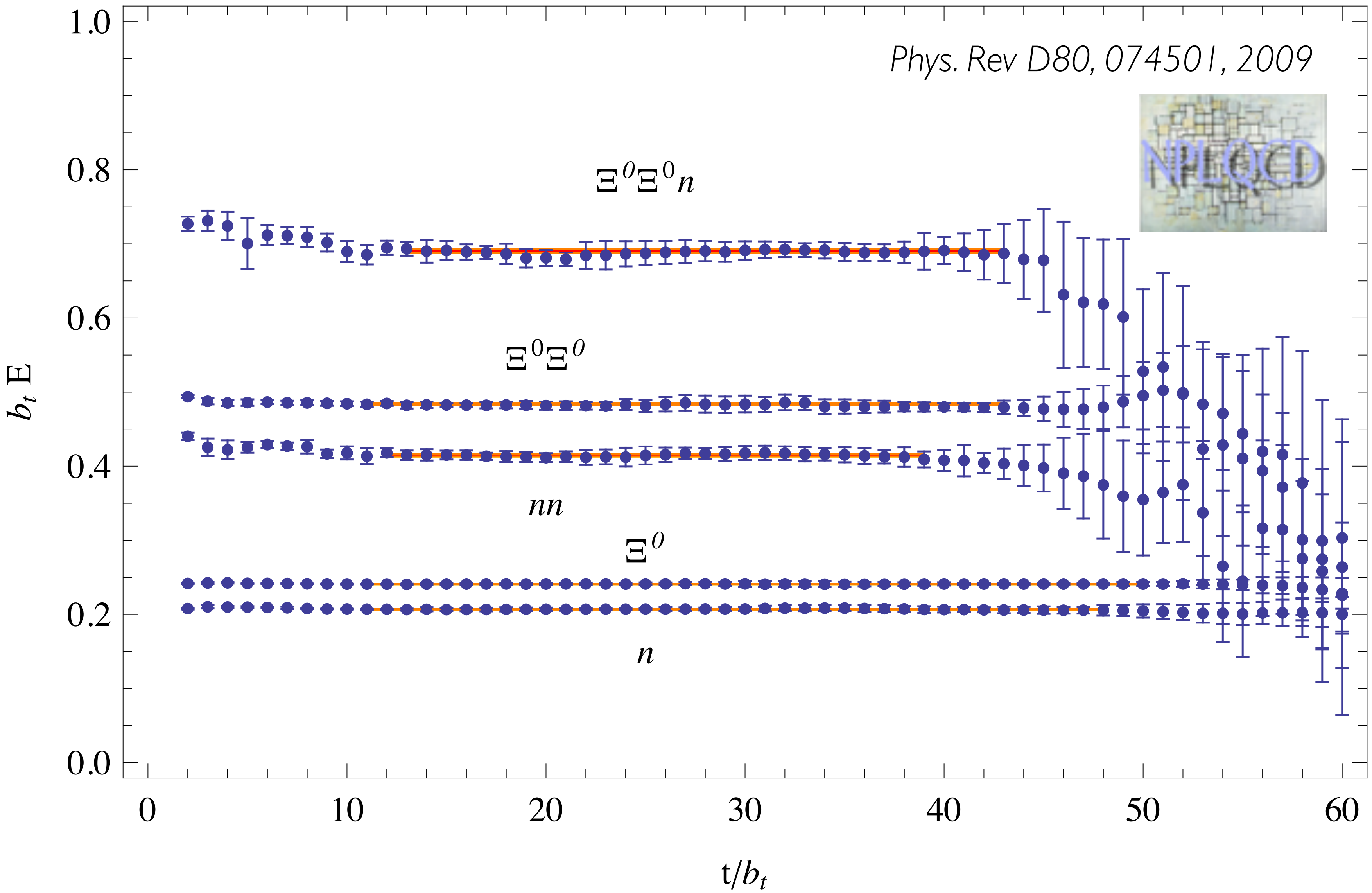
- For nucleus A:

$$\frac{\text{signal}}{\text{noise}} \sim \exp\left[-A(M_N - 3/2 m_\pi) t\right]$$

[Lepage '89]

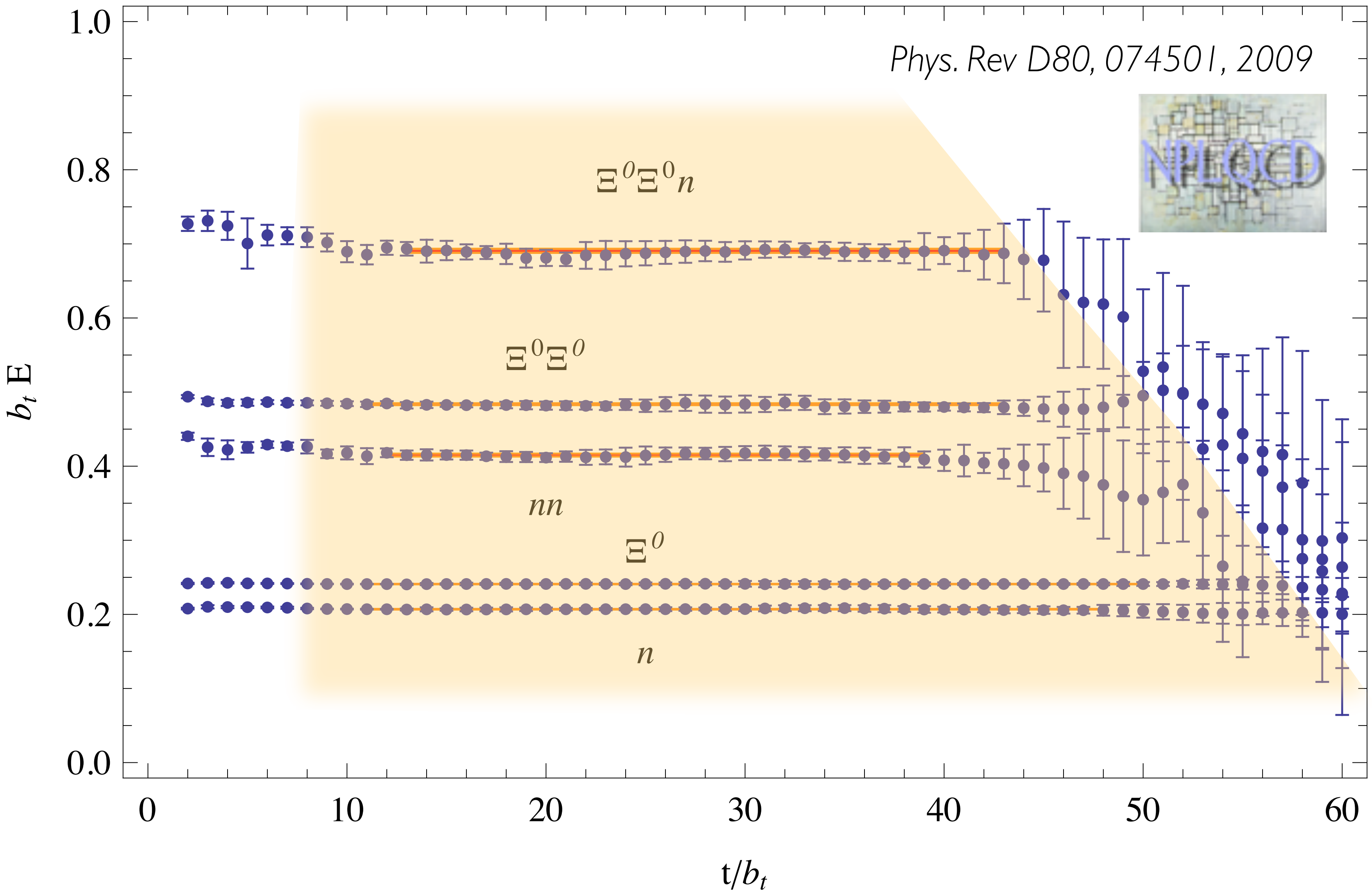
The trouble with baryons

High statistics study using anisotropic lattices (fine temporal resolution)



@ m_π = 390 MeV

High statistics study using anisotropic lattices (fine temporal resolution)



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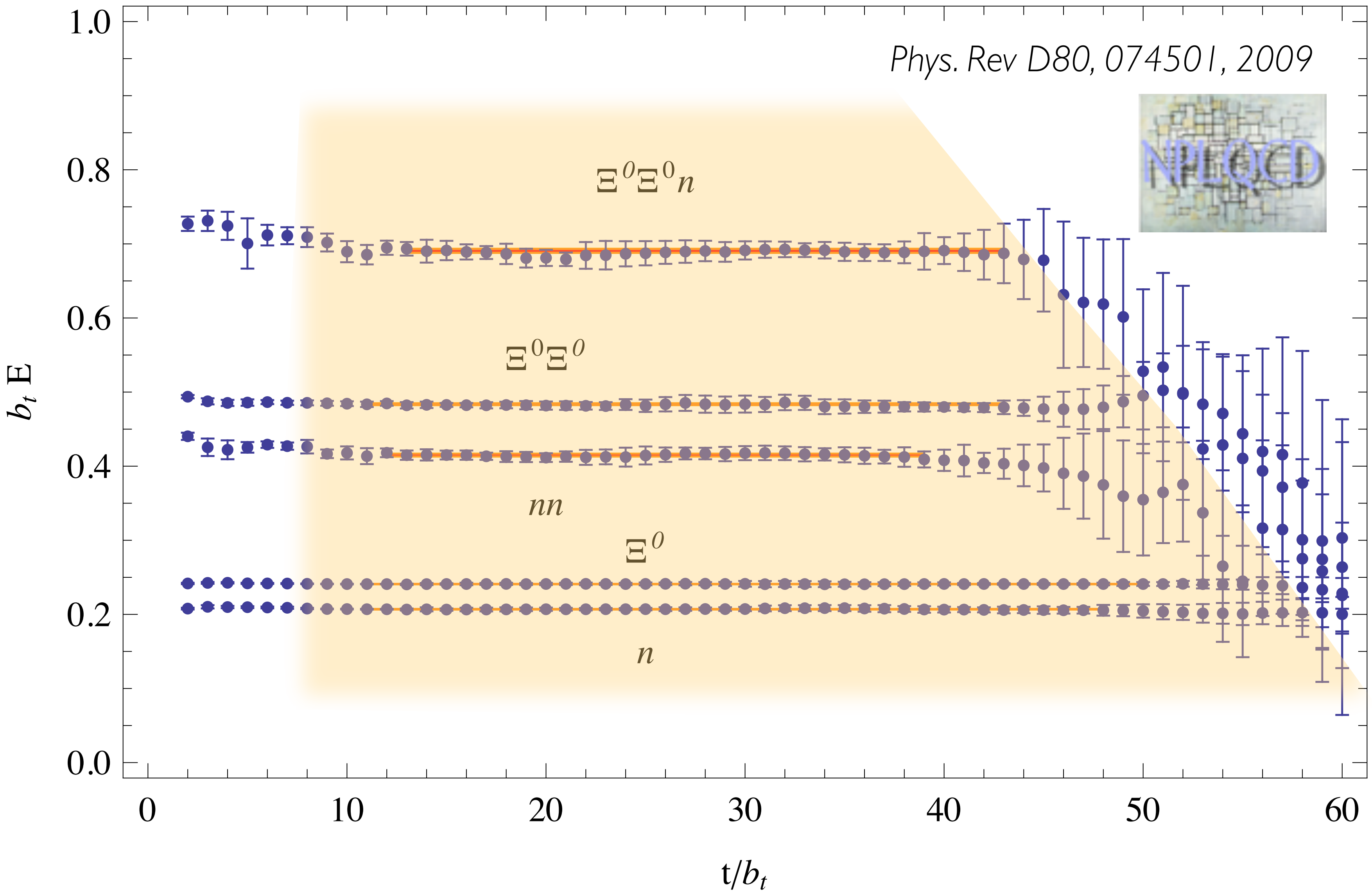
Golden window of time-slices where signal/noise const

High statistics study using anisotropic lattices (fine temporal resolution)



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No? trouble with baryons

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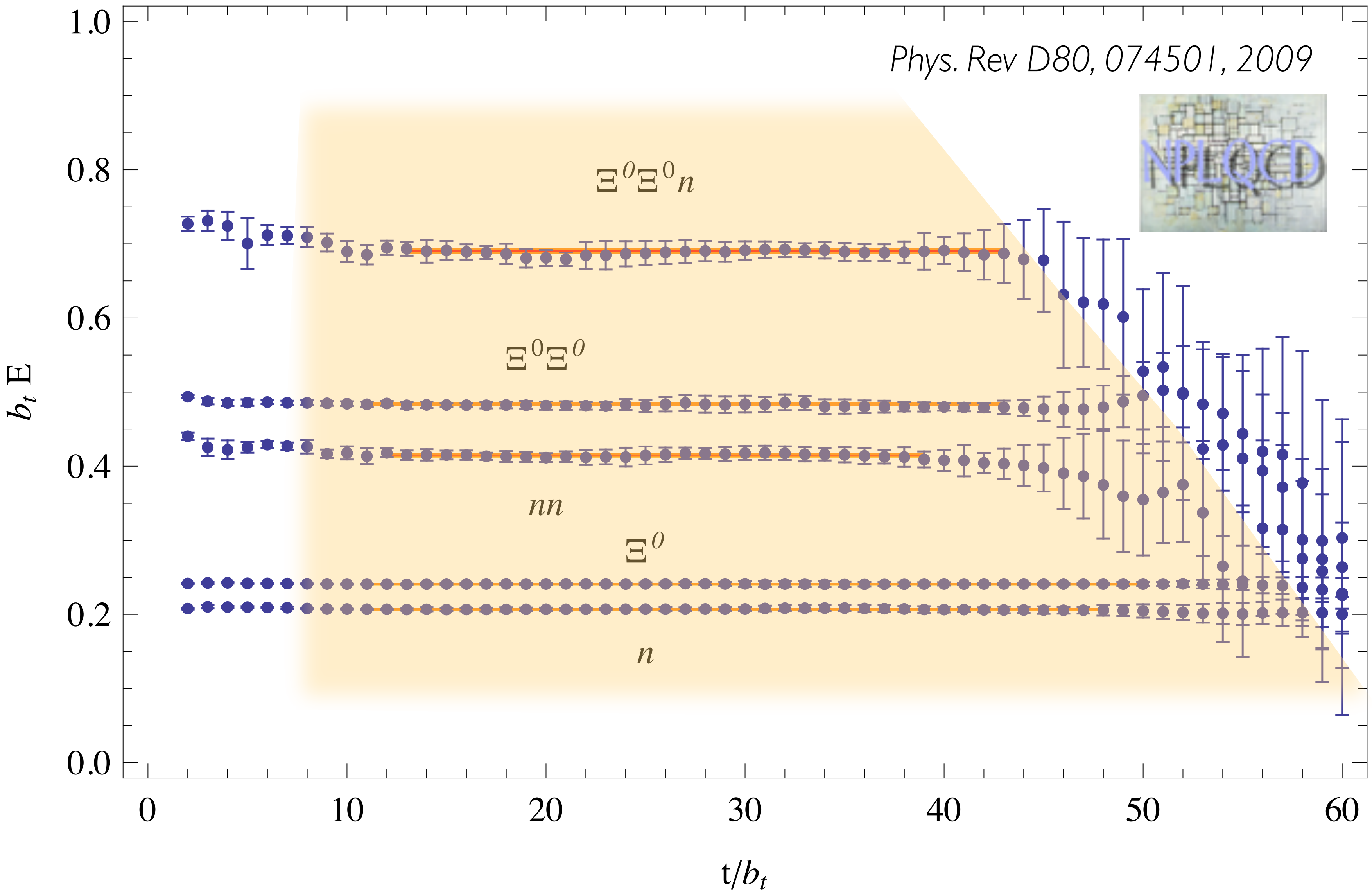

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Golden window of time-slices where signal/noise const

Interpolator choice can be used to suppress noise

Signal-noise optimisation

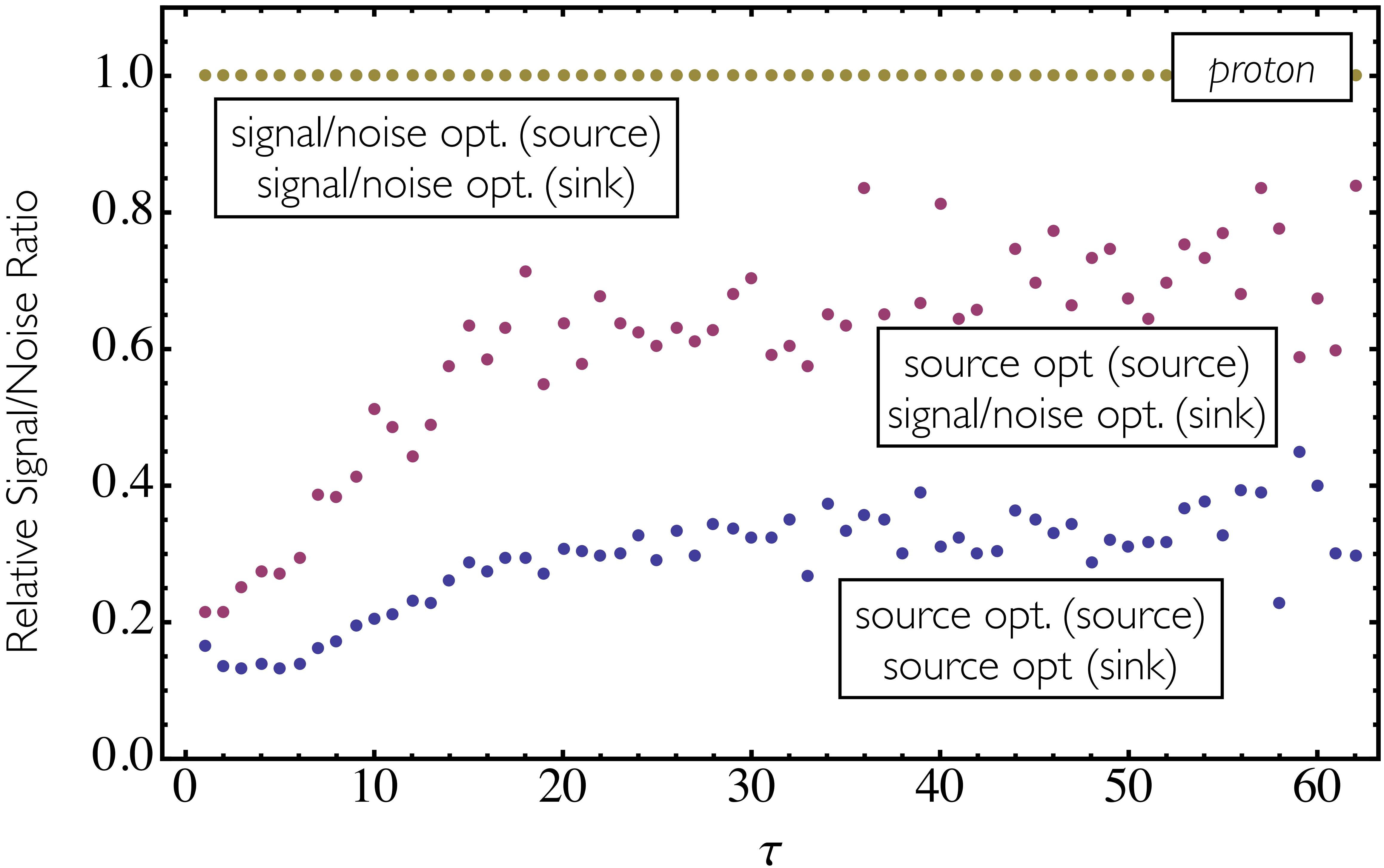
- Matrix of correlation functions

$$C_{ij}(\tau) = \langle\Omega|\hat{\mathcal{O}}'_i e^{-\hat{H}\tau}\hat{\mathcal{O}}_j^\dagger|\Omega\rangle$$
$$= \sum_n Z'_{in} Z^*_{jn} e^{-E_n\tau}$$

- Solve generalised eigenvalue problem to optimise overlap onto eigenstates: variational method of Michael/Lüscher&Wolff
- Solve optimisation problem to maximize signal-noise ratio [WD & M Endres, PRD 2014]
  - Very large enhancements theoretically possible
- Combination of both

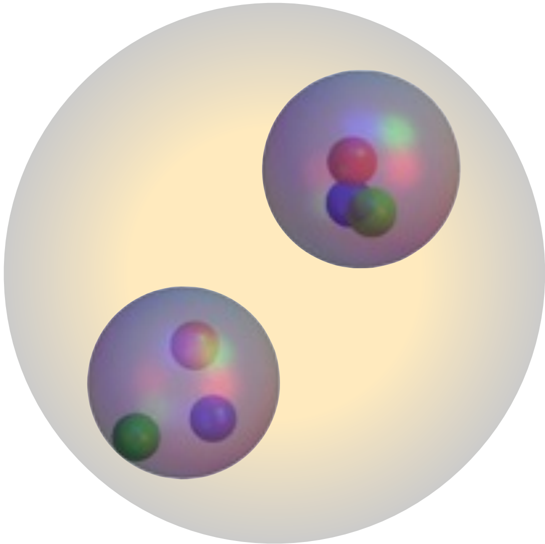
Signal-noise optimisation

5x5 correlation matrix



[WD & M Endres, PRD 2014]

Bound states at finite volume



- Focus on bound states
- Two particle scattering amplitude in infinite volume

$$\mathcal{A}(p) = \frac{8\pi}{M}\frac{1}{p\cot\delta(p) - ip}$$

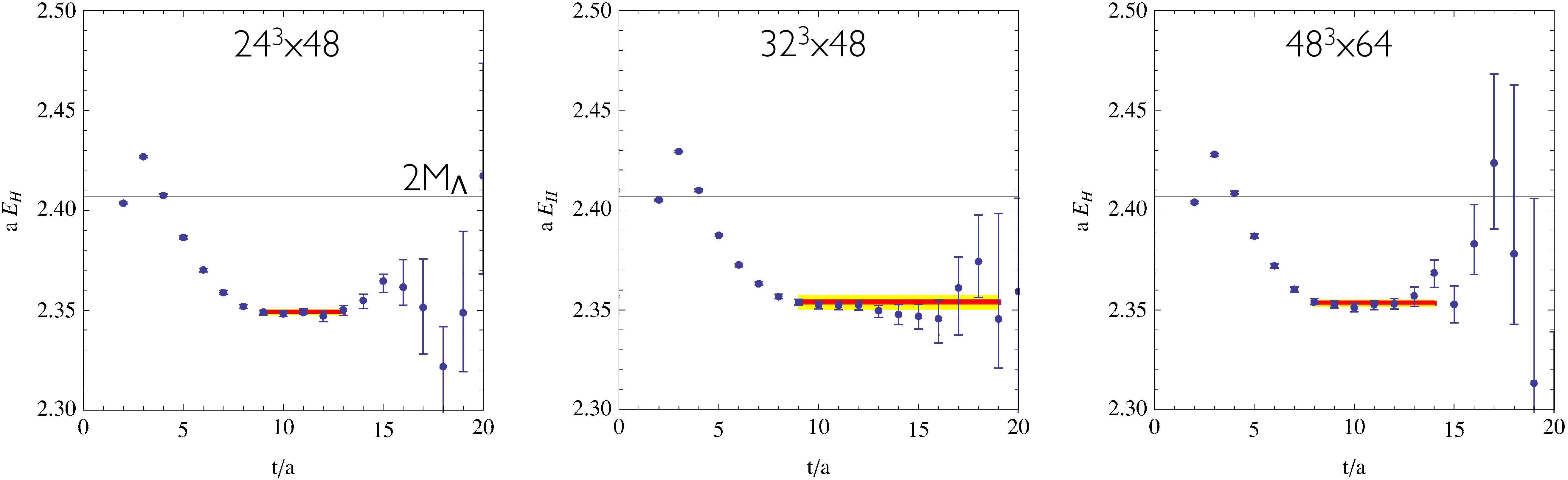
bound state at $p^2 = -\gamma^2$ when $\cot\delta(i\gamma) = i$

- Scattering amplitude in finite volume (Lüscher method)

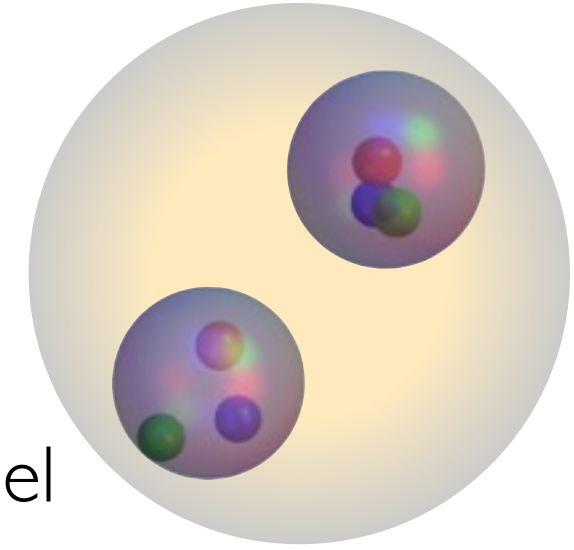
$$\cot\delta(i\kappa) = i - i\sum_{\vec{m}\neq 0}\frac{e^{-|\vec{m}|\kappa L}}{|\vec{m}|\kappa L} \qquad \kappa \xrightarrow{L\to\infty} \gamma$$

  - Need multiple volumes
- More complicated for n>2 body bound states

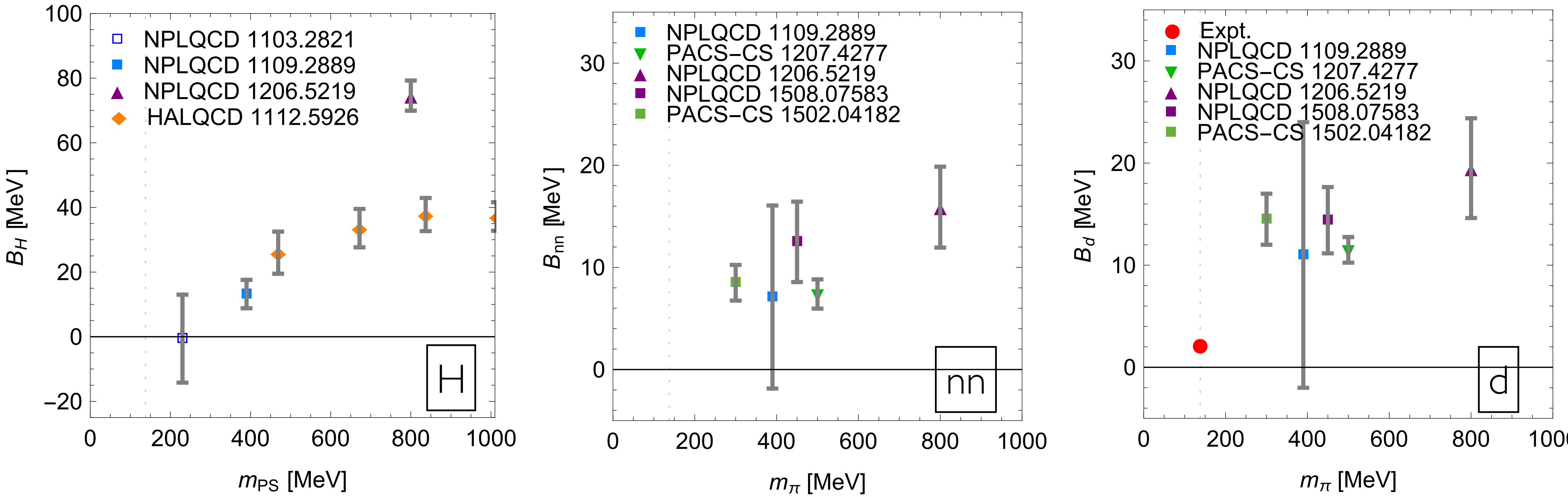
Ex: H dibaryon ($\Lambda\Lambda$)



- Effective mass plots of energies
- Multiple volumes needed to disentangle bound state from attractive scattering state
- Bayes factor: test bound state vs scattering state model

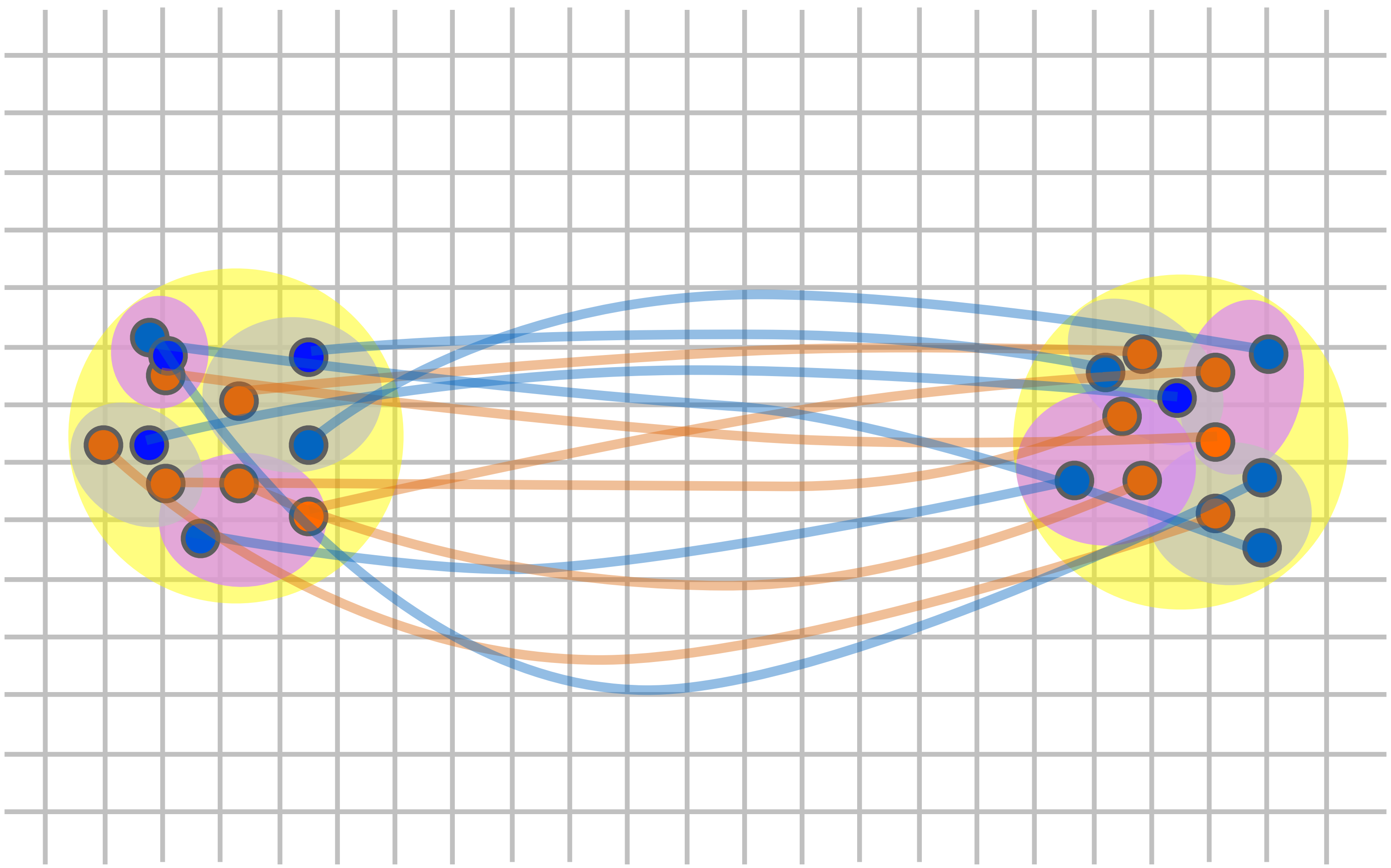


Dibaryons

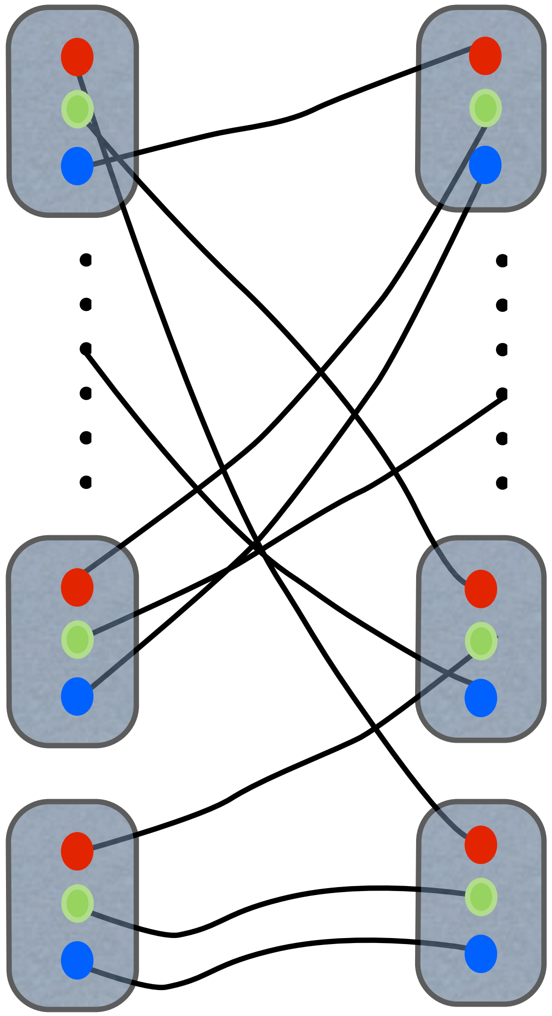


- H dibaryon, di-neutron and deuteron
- More exotic channels also considered ($\Xi\Xi$, nΩ and $\Omega\Omega$)
- Clearly more work needed at lighter masses

- Quarks need to be tied together in all possible ways
  - $N_{\text{contractions}} = N_u! N_d! N_s!$



- Managed using algorithmic trickery [WD & Savage; Doi & Endres; WD & Orginos; Günther et al.]
  - Study up to N=72 pion systems, A=5 nuclei



- Many baryon correlator construction is messy
- Interpolating fields – express weighted sums

color/spin/flavour/spatial indices

$$\bar{\mathcal{N}}^h = \sum_{k=1}^{N_w} \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \sum_{\mathbf{i}} \epsilon^{i_1,i_2,\cdots,i_{n_q}} \bar{q}(a_{i_1})\bar{q}(a_{i_2})\cdots\bar{q}(a_{i_{n_q}})$$

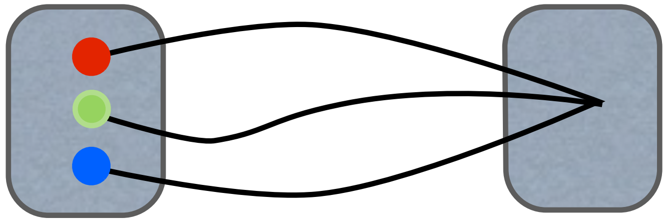
  - Generation of weights can be automated (symbolic c++ code) for given quantum numbers
    - Specify final quantum numbers (spin, isospin, strangeness etc)
    - Build up from states of smaller quantum numbers using rules of eg angular momentum addition
  - Contraction just reads in weights and can be implemented independent of the particular process being considered

[WD, K Orginos, 1207.1452]

Many baryon systems

- Given a complex many baryon system to perform contractions for, always possible to group colour singlets at one end (sink)
- Contractions can be written in terms of baryon blocks (objects that are contracted at sink)
- A particular set of quantum numbers b for the block is select by a weighted sum of components of quark propagators

$$\mathcal{B}_b^{a_1,a_2,a_3}(\mathbf{p},t;x_0) = \sum_{\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}} \sum_{k=1}^{N_{B(b)}} \tilde{w}_b^{(c_1,c_2,c_3),k} \sum_{\mathbf{i}} \epsilon^{i_1,i_2,i_3} \times S(c_{i_1},x;a_1,x_0) S(c_{i_2},x;a_2,x_0) S(c_{i_3},x;a_3,x_0)$$



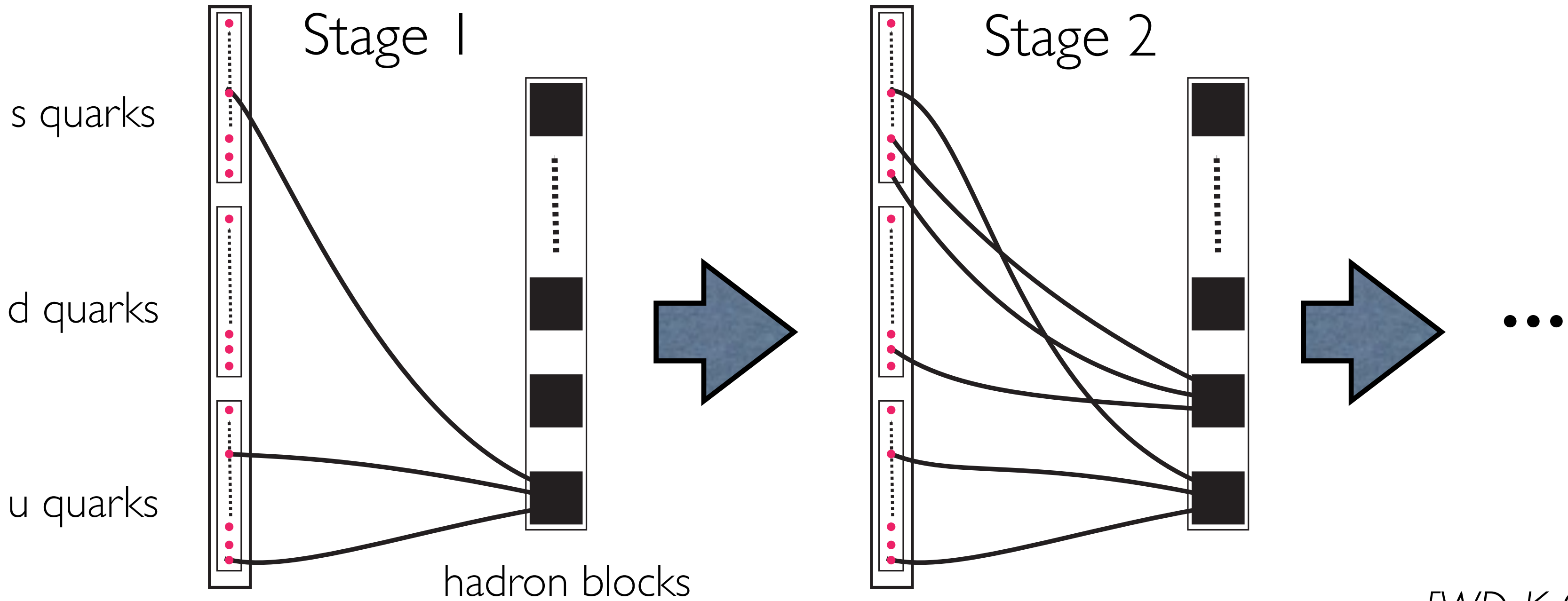
- Can be generalised to multi-baryon blocks if desired although storage requirements rapidly increase

Many baryon systems

- Contractions

$$[\mathcal{N}_1^h(t)\bar{\mathcal{N}}_2^h(0)]_U = \int \mathcal{D}q\mathcal{D}\bar{q}\, e^{-S_{QCD}[U]} \sum_{k'=1}^{N'_w}\sum_{k=1}^{N_w} \tilde{w}_h^{\prime(a'_1,a'_2\cdots a'_{n_q}),k'} \; \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \times$$
$$\sum_{\mathbf{j}}\sum_{\mathbf{i}} \epsilon^{j_1,j_2,\cdots,j_{n_q}} \epsilon^{i_1,i_2,\cdots,i_{n_q}} q(a'_{j_{n_q}})\cdots q(a'_{j_2})q(a'_{j_1}) \times \bar{q}(a_{i_1})\bar{q}(a_{i_2})\cdots\bar{q}(a_{i_{n_q}})$$
$$= e^{-\mathcal{S}_{eff}[U]} \sum_{k'=1}^{N'_w}\sum_{k=1}^{N_w} \tilde{w}_h^{\prime(a'_1,a'_2\cdots a'_{n_q}),k'} \; \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \times$$
$$\sum_{\mathbf{j}}\sum_{\mathbf{i}} \epsilon^{j_1,j_2,\cdots,j_{n_q}} \epsilon^{i_1,i_2,\cdots,i_{n_q}} S(a'_{j_1};a_{i_1})S(a'_{j_2};a_{i_2})\cdots S(a'_{j_{n_q}};a_{i_{n_q}})$$

- Make a particular choice of correlation function (momentum projection at sink) and express in terms of blocks (quark-hadron level contraction)



[WD, K Orginos, 1207.1452;]

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- Contractions

$$\left[\mathcal{N}_1^h(t)\bar{\mathcal{N}}_2^h(0)\right]_U = \int \mathcal{D}q\mathcal{D}\bar{q}\; e^{-S_{QCD}[U]} \sum_{k'=1}^{N'_w}\sum_{k=1}^{N_w} \tilde{w}_h^{\prime(a'_1,a'_2\cdots a'_{n_q}),k'}\;\tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \times$$

$$\sum_{\mathbf{j}}\sum_{\mathbf{i}} \epsilon^{j_1,j_2,\cdots,j_{n_q}}\epsilon^{i_1,i_2,\cdots,i_{n_q}} q(a'_{j_{n_q}})\cdots q(a'_{j_2})q(a'_{j_1}) \times \bar{q}(a_{i_1})\bar{q}(a_{i_2})\cdots\bar{q}(a_{i_{n_q}})$$

$$= e^{-\mathcal{S}_{eff}[U]} \sum_{k'=1}^{N'_w}\sum_{k=1}^{N_w} \tilde{w}_h^{\prime(a'_1,a'_2\cdots a'_{n_q}),k'}\;\tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \times$$

$$\sum_{\mathbf{j}}\sum_{\mathbf{i}} \epsilon^{j_1,j_2,\cdots,j_{n_q}}\epsilon^{i_1,i_2,\cdots,i_{n_q}} S(a'_{j_1};a_{i_1})S(a'_{j_2};a_{i_2})\cdots S(a'_{j_{n_q}};a_{i_{n_q}})$$

- Or write as determinant (quark-quark level contraction)

$$\langle\mathcal{N}_1^h(t)\bar{\mathcal{N}}_2^h(0)\rangle = \frac{1}{\mathcal{Z}}\int \mathcal{D}\mathcal{U}\; e^{-\mathcal{S}_{eff}} \sum_{k'=1}^{N'_w}\sum_{k=1}^{N_w} \tilde{w}_h^{\prime(a'_1,a'_2\cdots a'_{n_q}),k'}\;\tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \times \det G(\mathbf{a}';\mathbf{a})$$

where
$$G(\mathbf{a}';\mathbf{a})_{j,i} = \begin{cases} S(a'_j;a_i) & a'_j \in \mathbf{a}' \text{ and } a_i \in \mathbf{a} \\ \delta_{a'_j,a_i} & \text{otherwise} \end{cases}$$

- Determinant can be evaluated in polynomial number of operations (LU decomposition)

[WD, K Orginos, 1207.1452;]

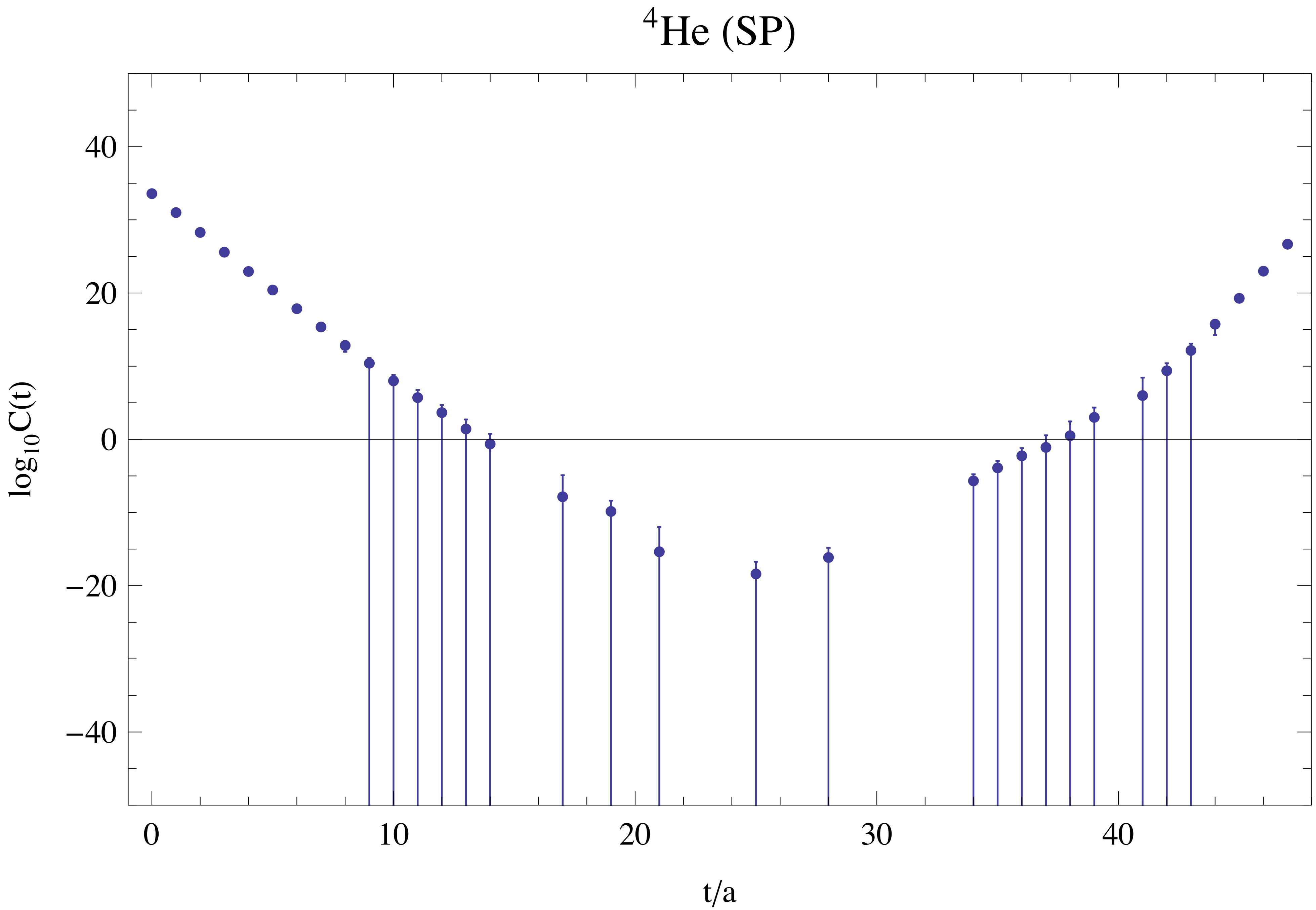
Quark-quark determinant based contraction method

- Many baryon correlators using determinant-based method

(low statistics, single volume)

WD, Kostas Orginos, Phys.Rev. D87 (2013) 114512

Quark-quark determinant based contraction method



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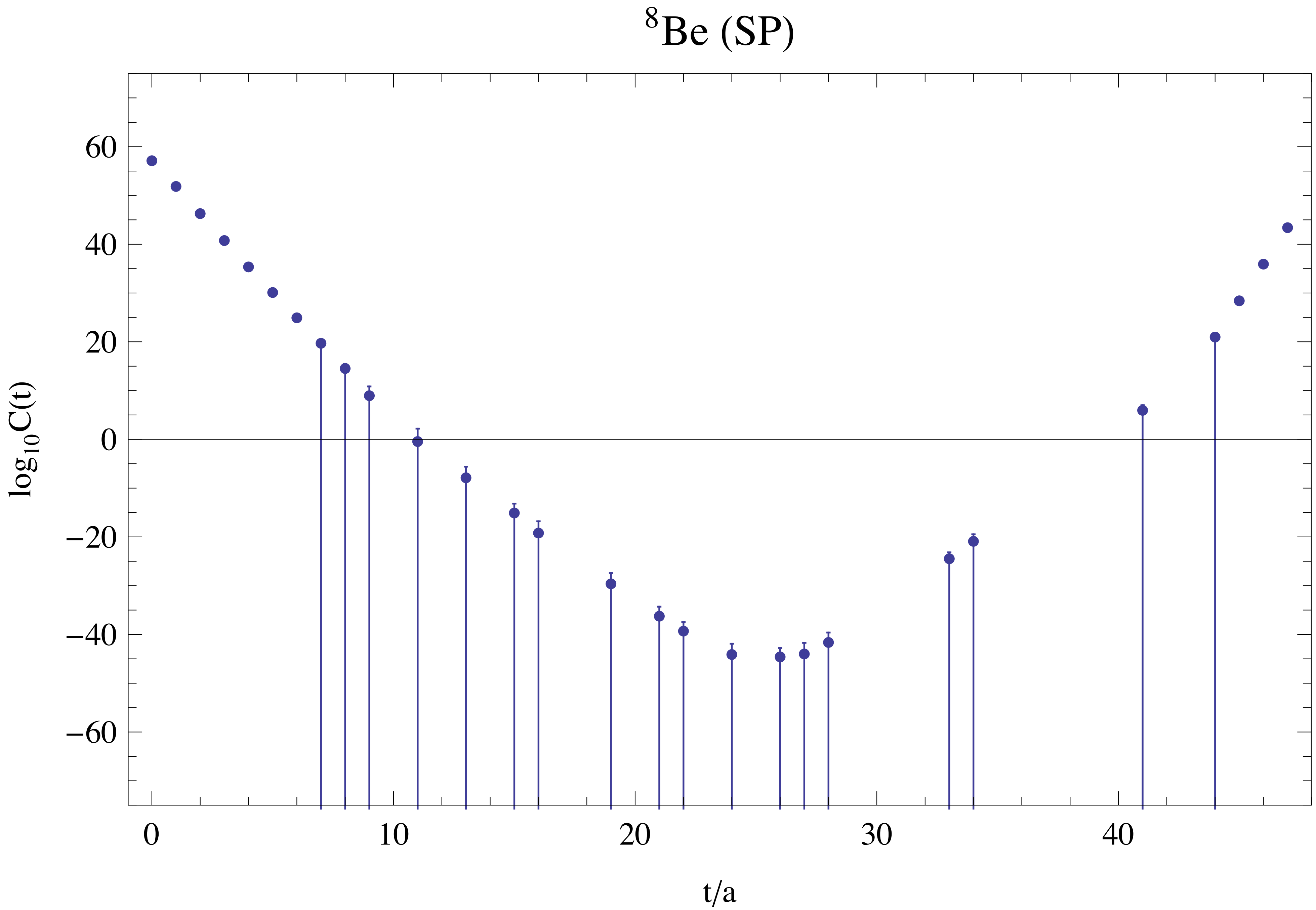

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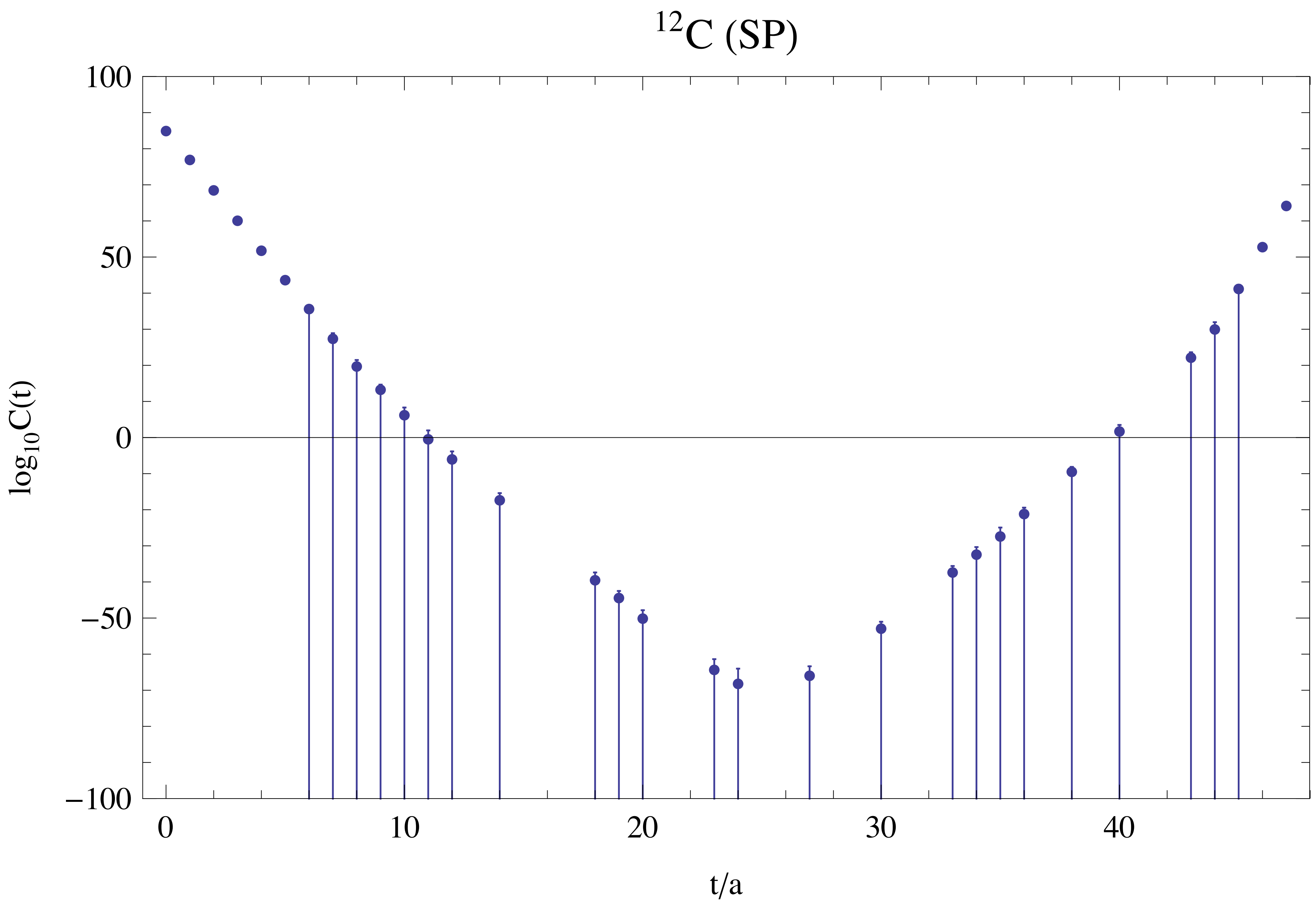

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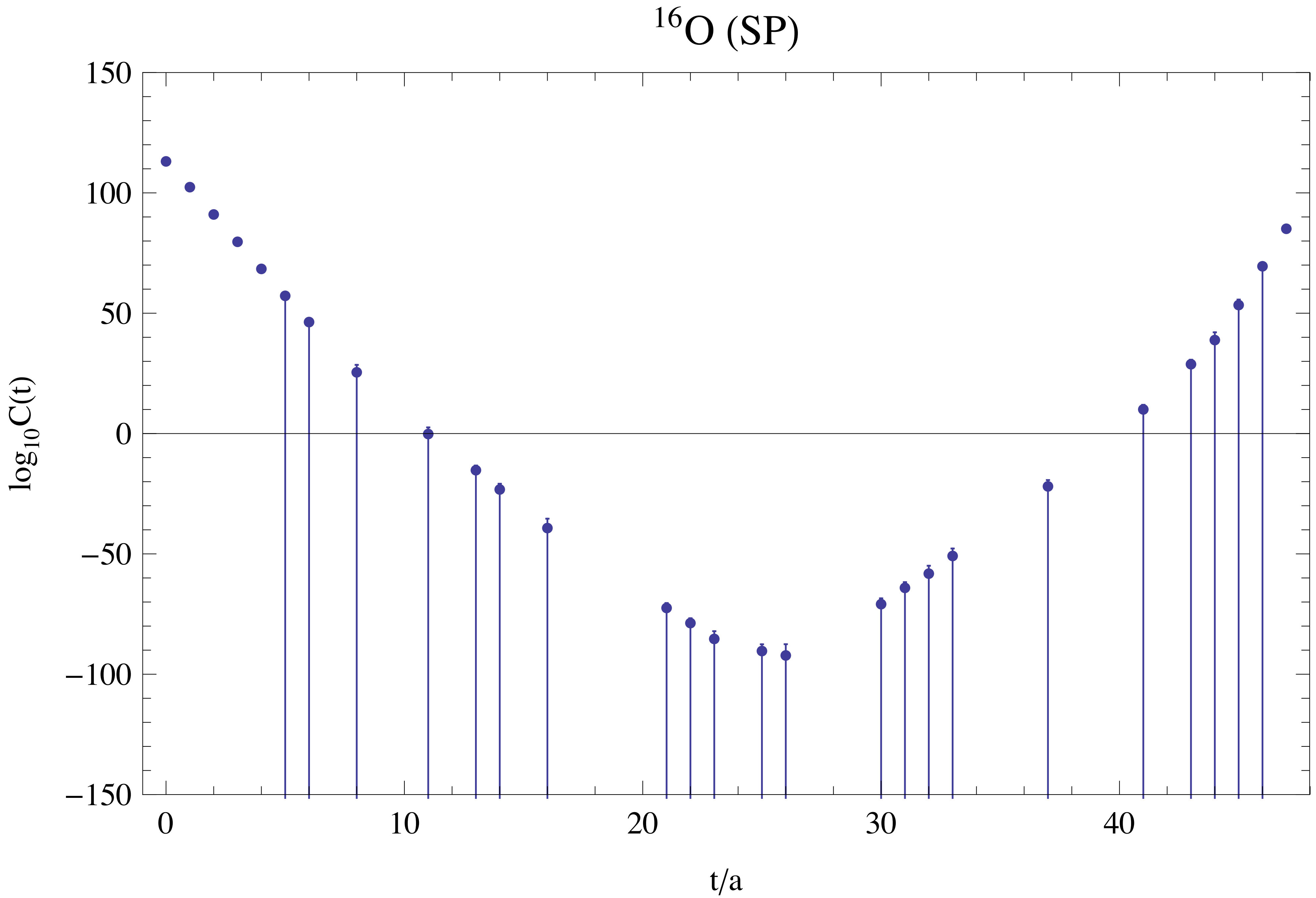

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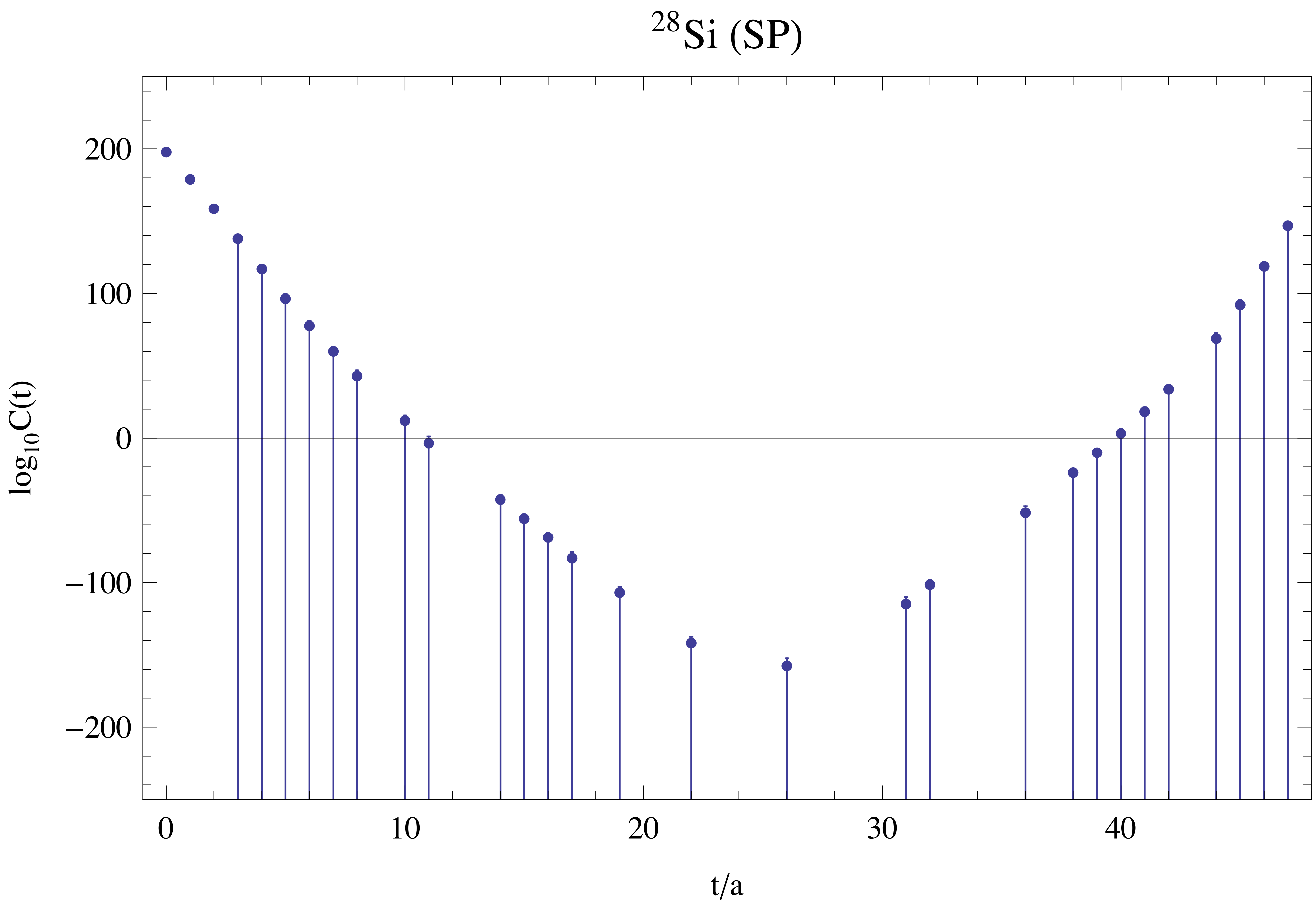


(low statistics, single volume)

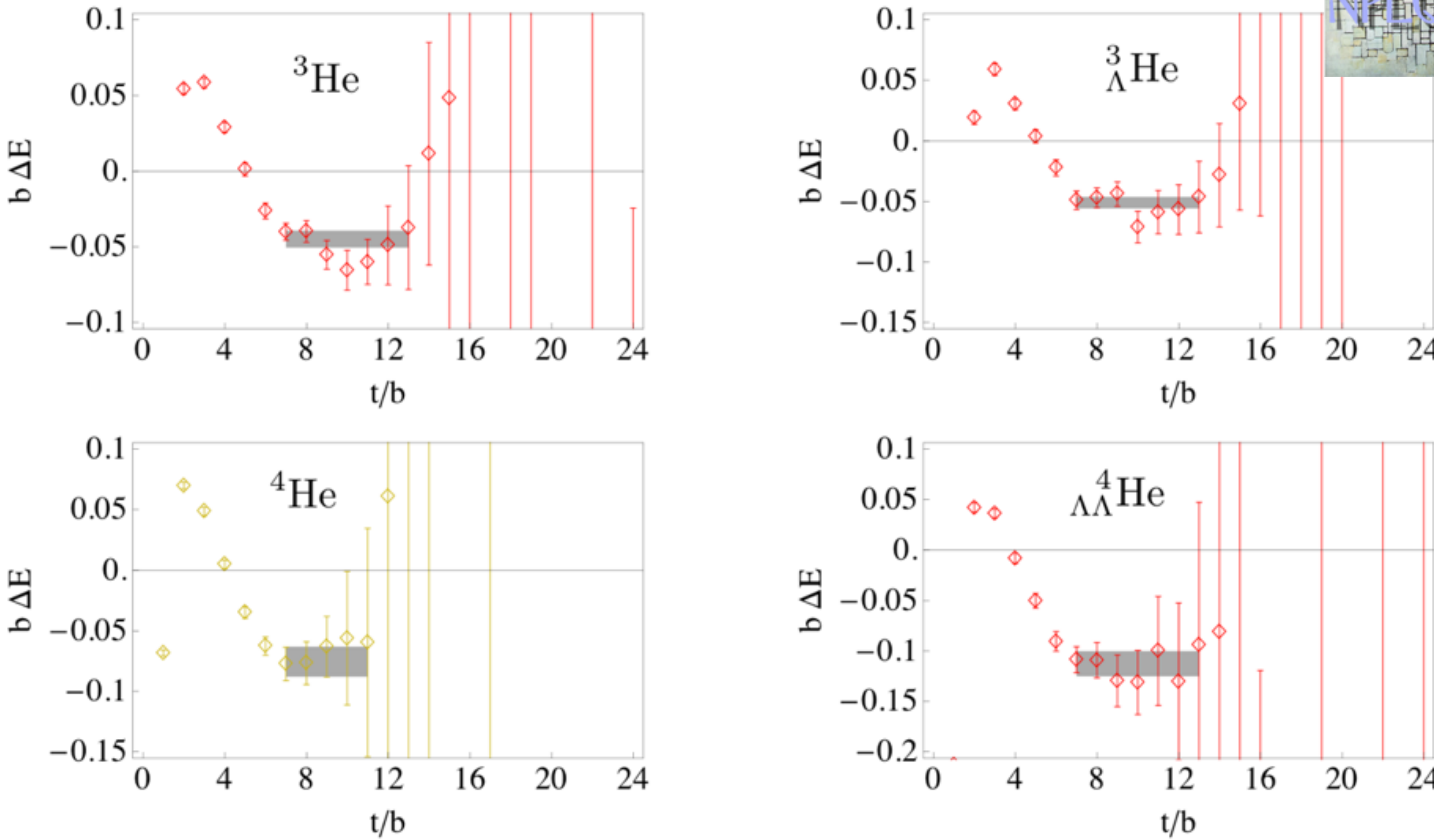
WD, Kostas Orginos, Phys.Rev. D87 (2013) 114512

[NPLQCD Phys.Rev. D87 (2013), 034506]

- NPLQCD study at SU(3) point (physical m_s)
  - Isotropic clover lattices
  - Single lattice spacing
  - Multiple volumes: 3.4, 4.5, 6.7 fm
  - High statistics

Label	L/b	T/b	β	$b\ m_q$	b [fm]	L [fm]	T [fm]	m_π [MeV]	$m_\pi\ L$	$m_\pi\ T$	N_{cfg}	N_{src}
A	24	48	6.1	-0.2450	0.145	3.4	6.7	806.5(0.3)(0)(8.9)	14.3	28.5	3822	48
B	32	48	6.1	-0.2450	0.145	4.5	6.7	806.9(0.3)(0.5)(8.9)	19.0	28.5	3050	24
C	48	64	6.1	-0.2450	0.145	6.7	9.0	806.7(0.3)(0)(8.9)	28.5	38.0	1212	32

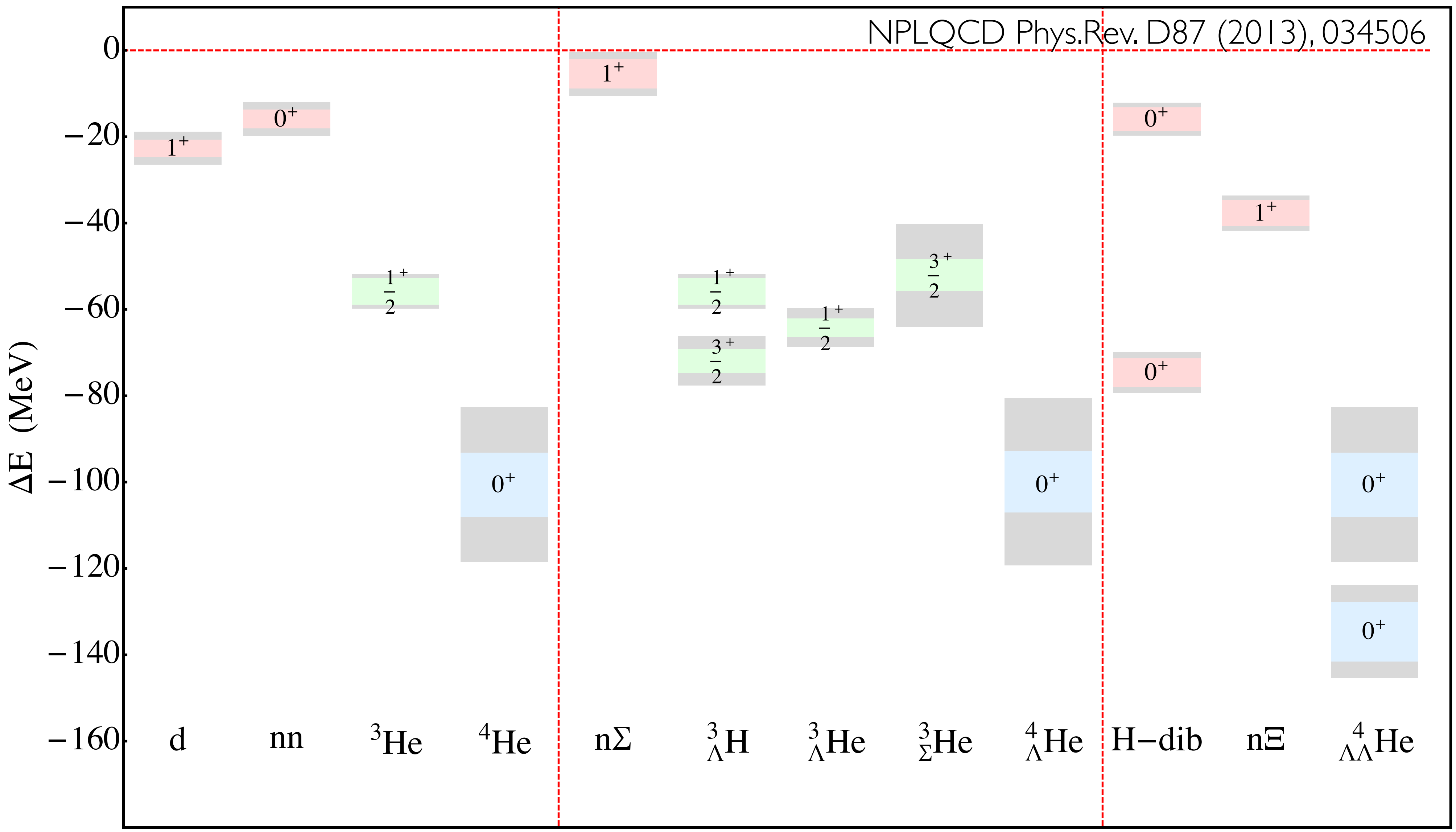
Nuclei (A=3,4)



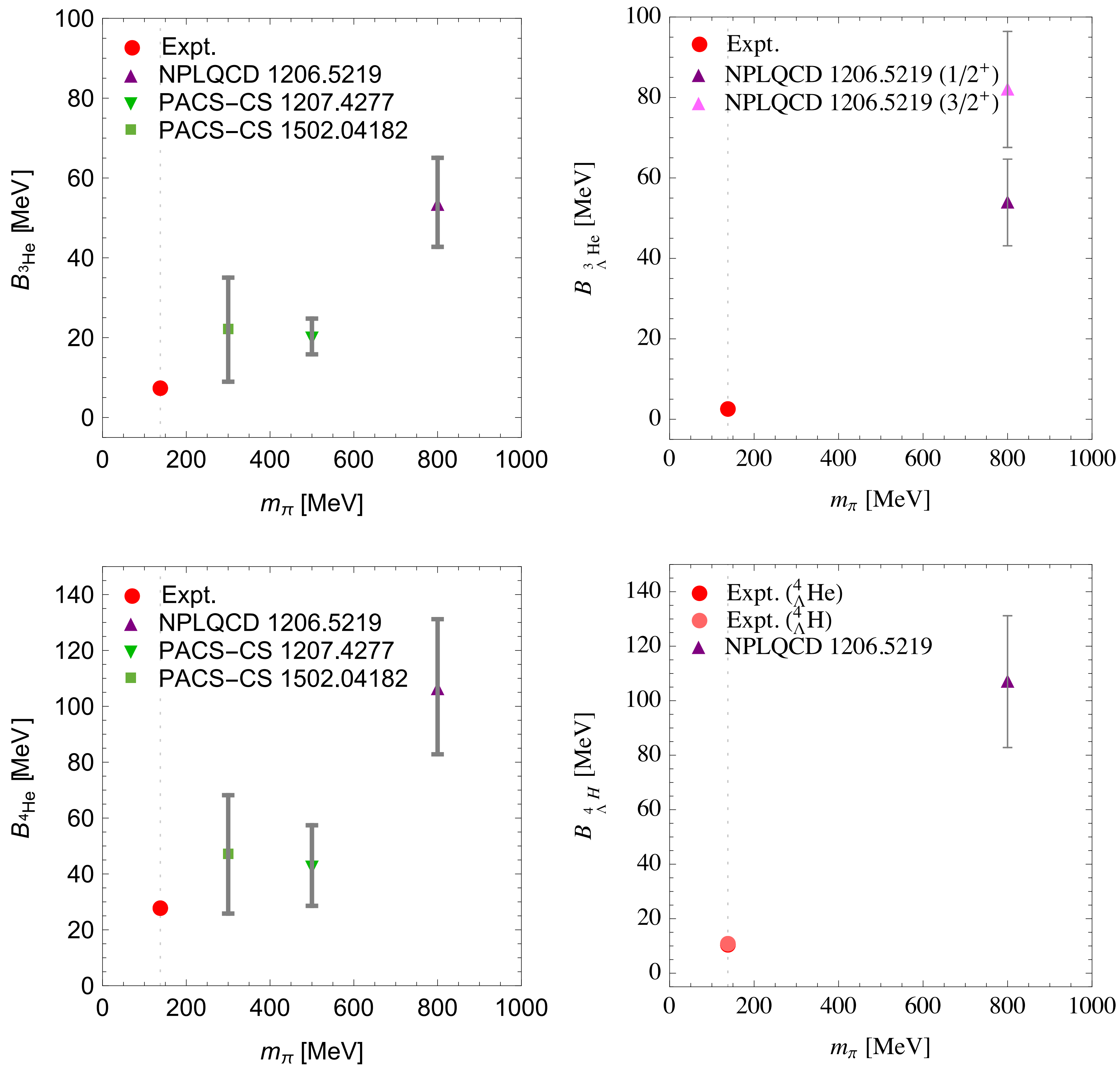
[NPLQCD Phys.Rev. D87 (2013), 034506]

Light nuclei and hypernuclei

- Light hypernuclear binding energies @ m_π=800 MeV



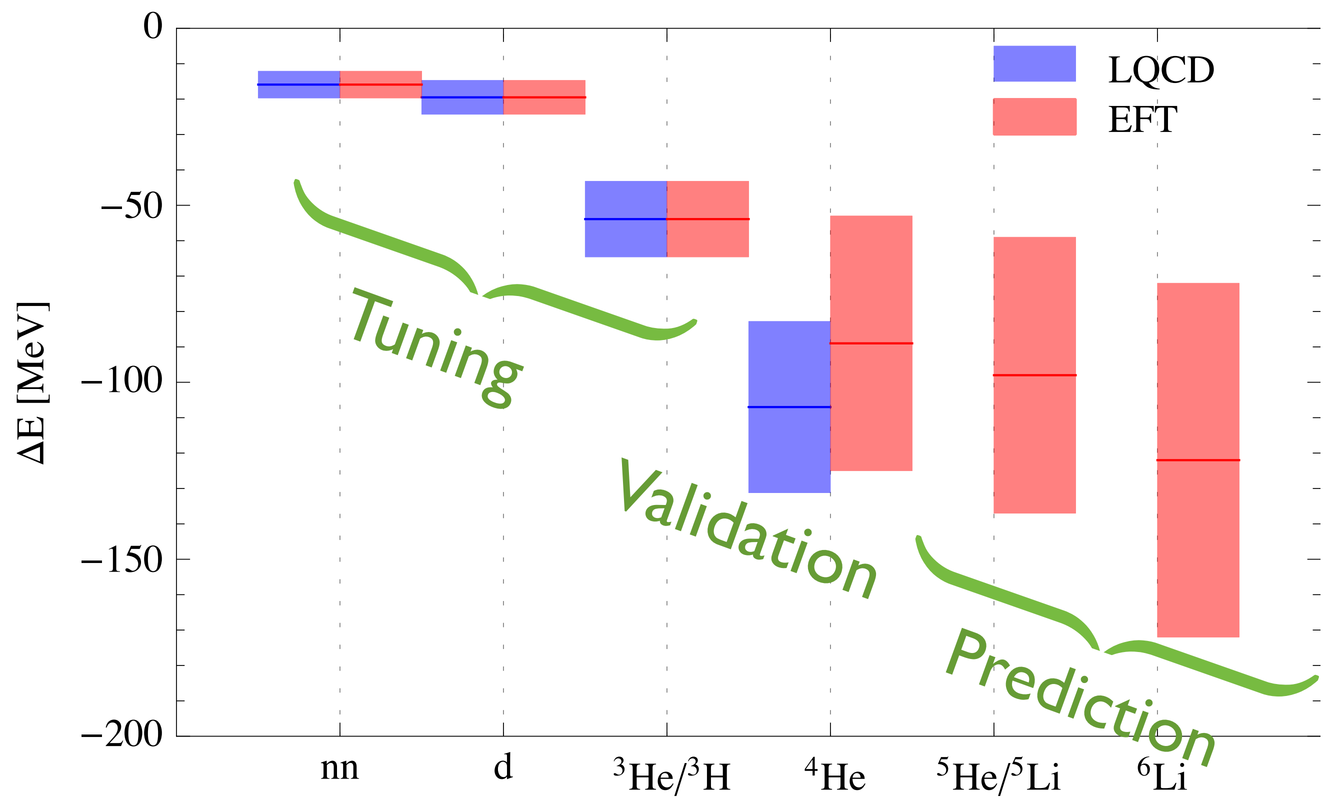
QCD Nuclei (s=0,-1)



Heavy quark universe

[Barnea et al. PRL 2014; see also Kirscher et al. 1506.09048]

- Combine LQCD and nuclear EFT (pionless EFT)
- EFT matching to LQCD determines NN, NNN interactions: allows predictions for larger nuclei

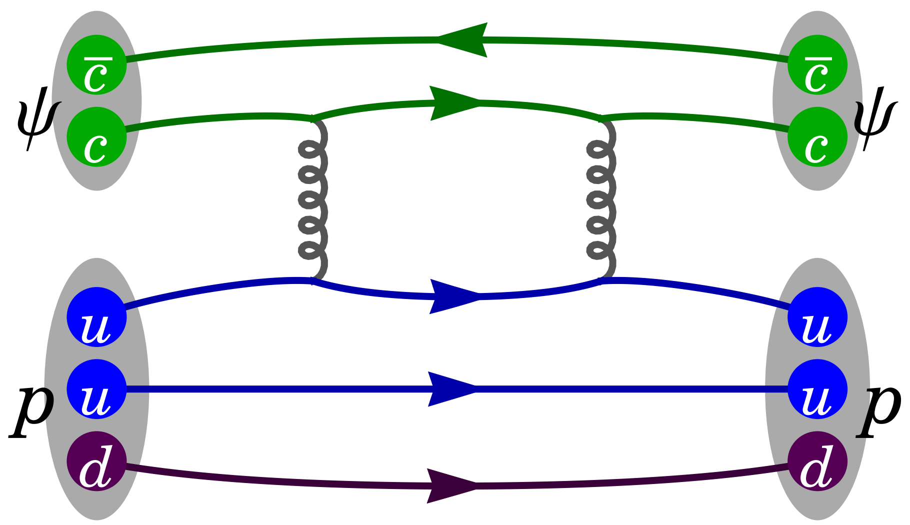


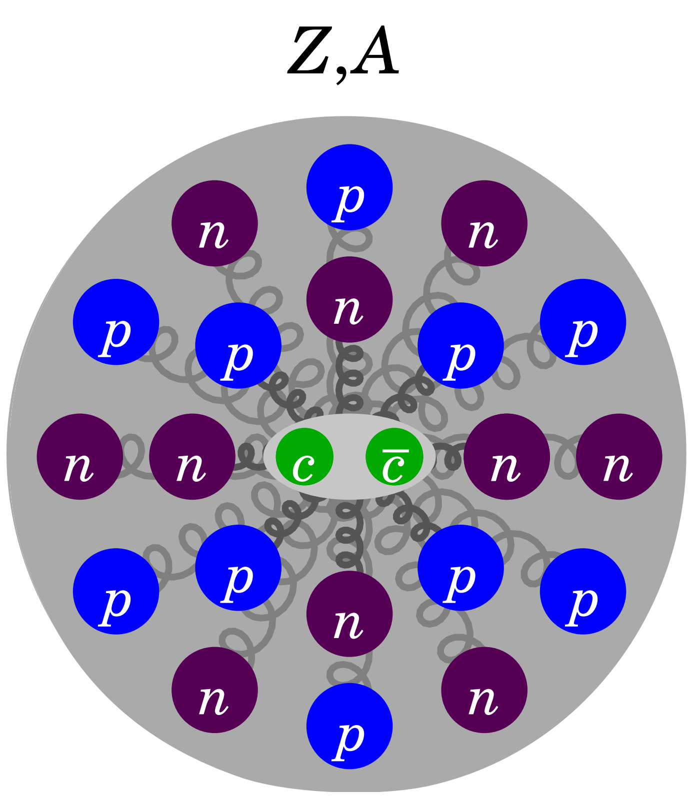
In a world @ m_π = 800 MeV

- Other many-body methods significantly extend reach

Onium-nucleus binding

- Quarkonium interactions with light quark systems via colour van der Waals
- Colour stark effect: quarkonium induces dipoles in nucleons that attract
  - Brodsky et al. [PRL64,1011 (1990)] suggested large binding: ^{9}Be–η_c ~ 400 MeV
  - Nuclei not point-like: gluons screened Typical model estimates now: J/Ψ–A ~ 10 MeV
- Eta-mesic nuclei possibly seen at COSY
- ATHENNA experiment at JLab12GeV will look for charmonium nuclei





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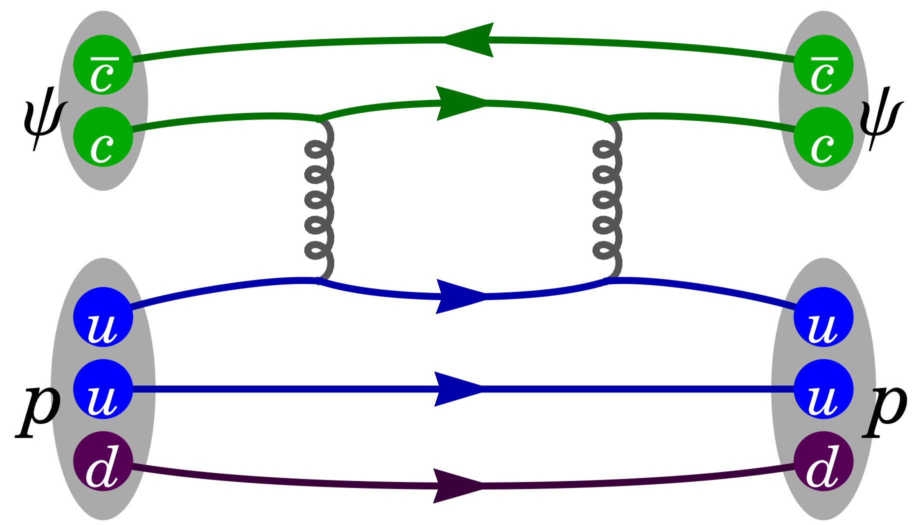
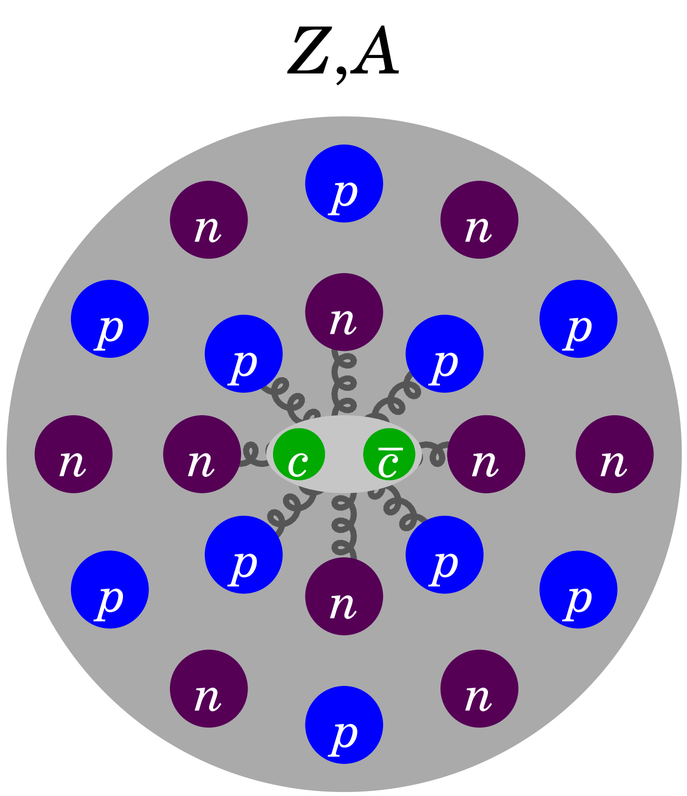
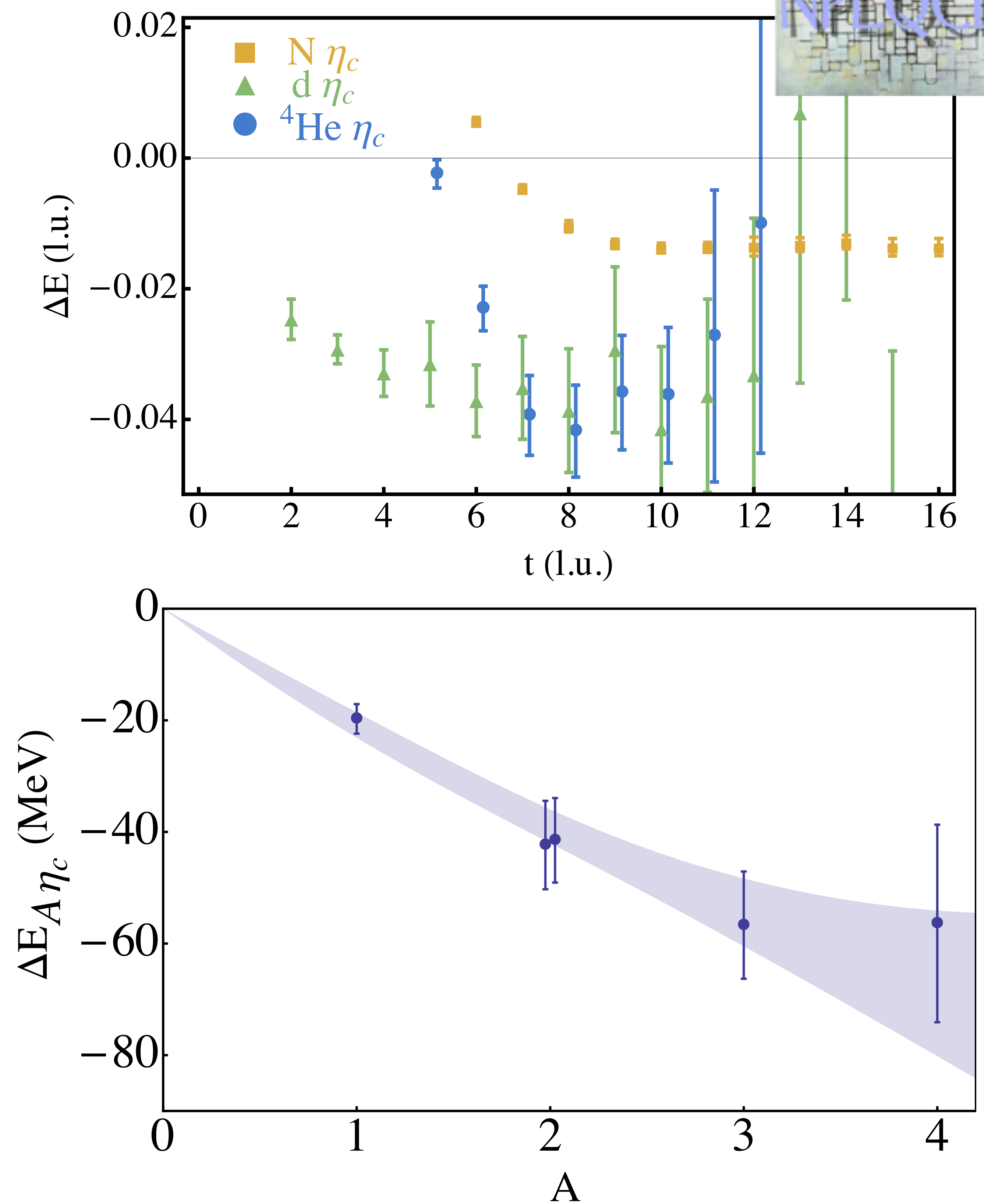




Onium-nucleus binding

[NPLQCD Phys.Rev. D91 (2015), 114503]

- Straightforward LQCD calculation
- Study at m_π~800 MeV using strangonium and charmonium
- Energy shift from $M_\eta+M_A$
- Multiple volumes to extract infinite volume binding energy
- Binding energy vs A
  - Very strong binding!
- Quantum numbers of pentaquark!



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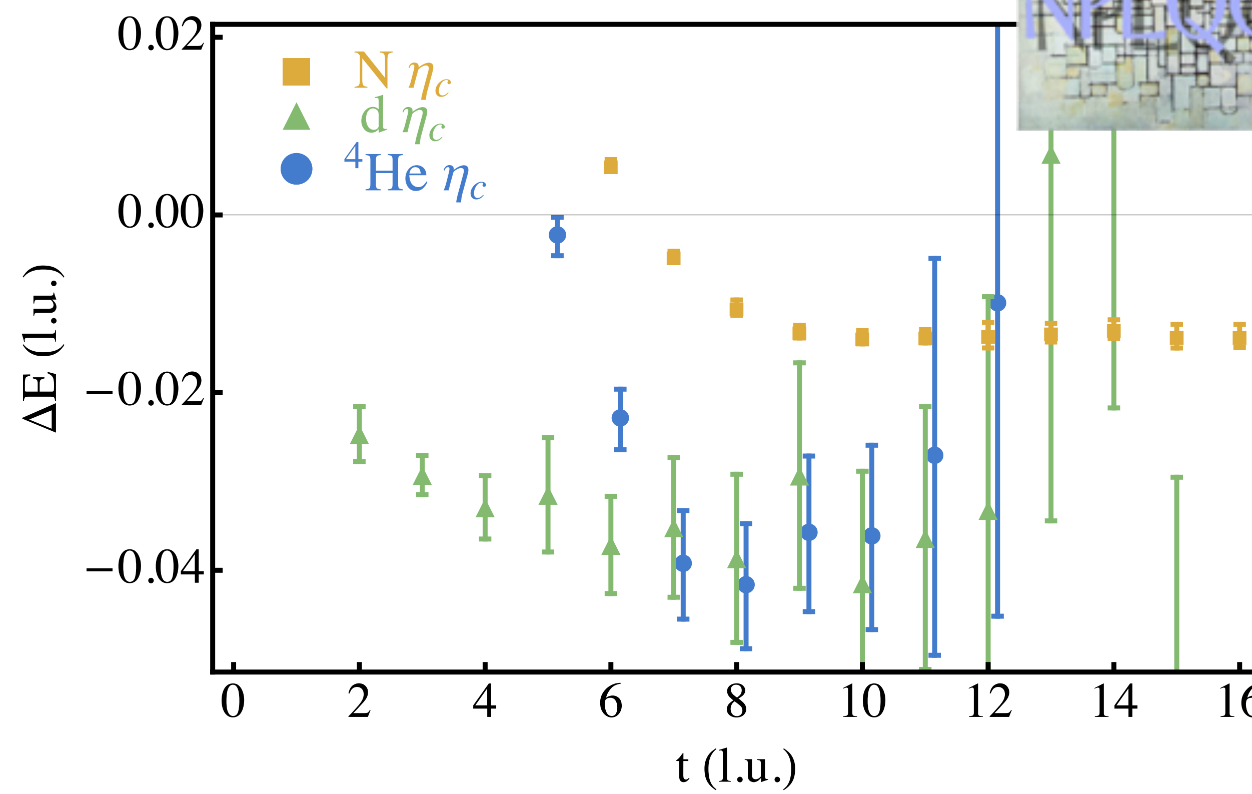

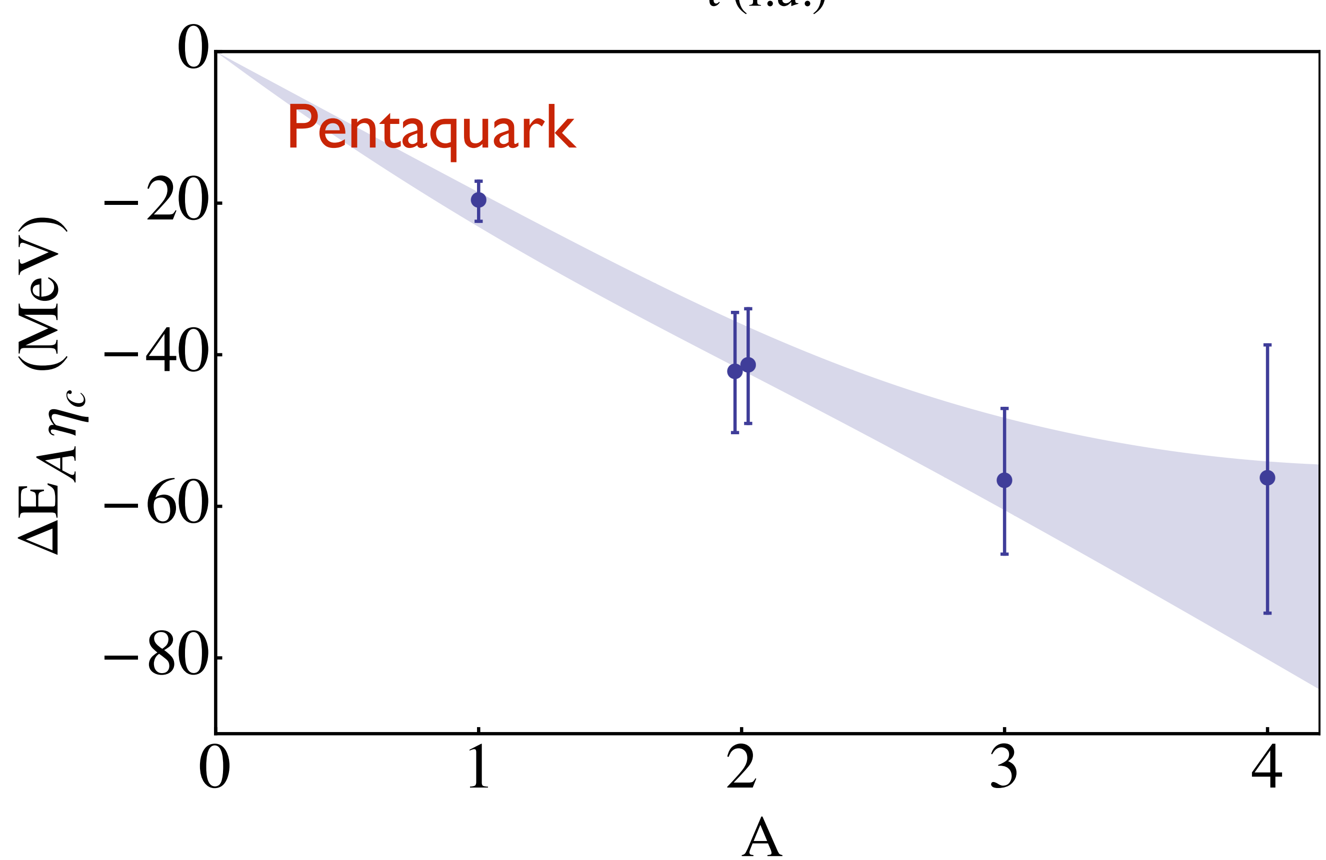

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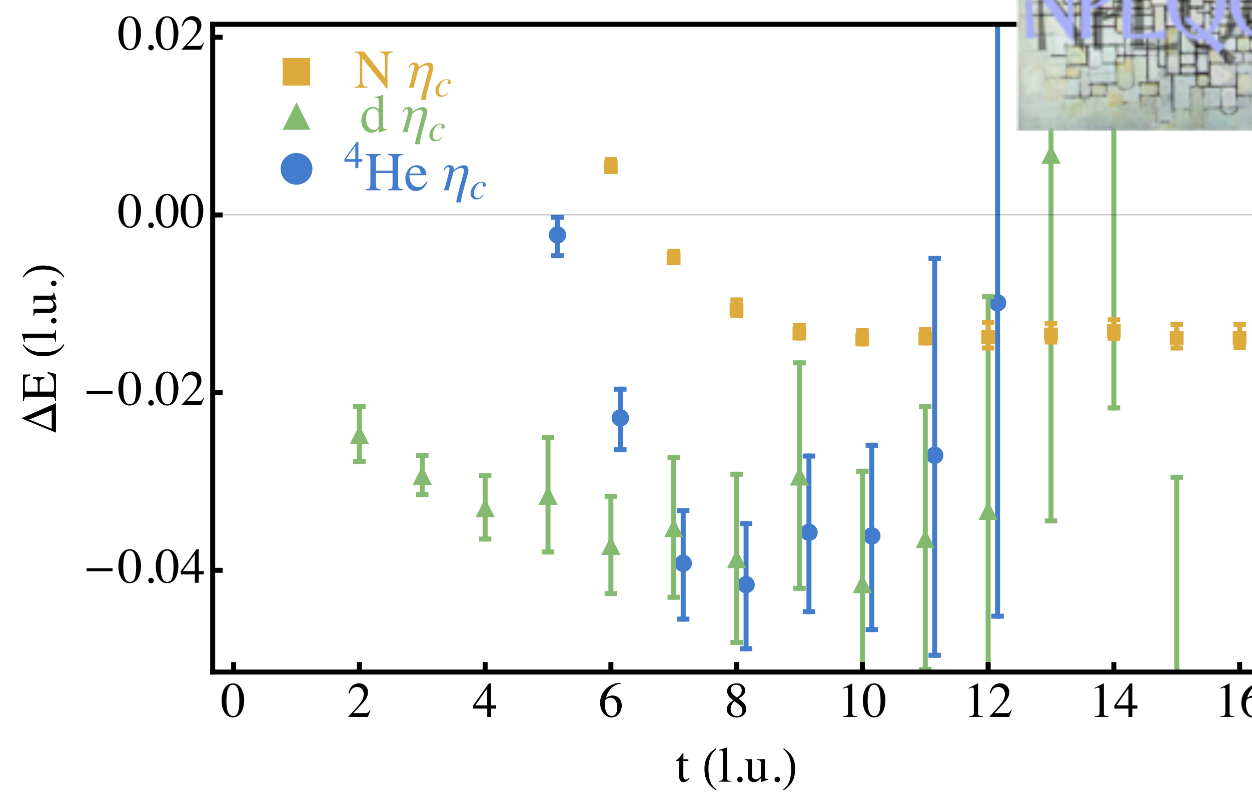
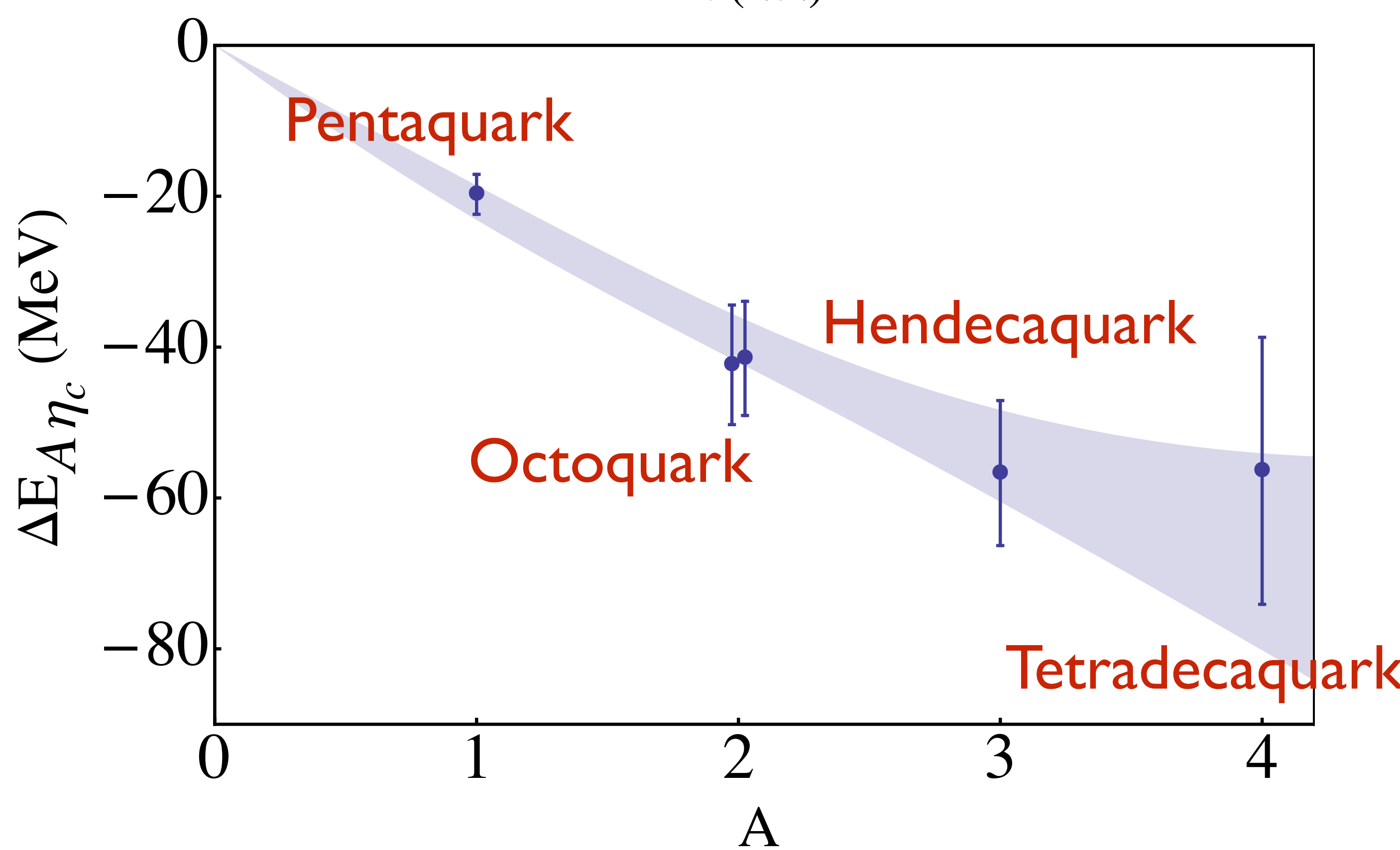
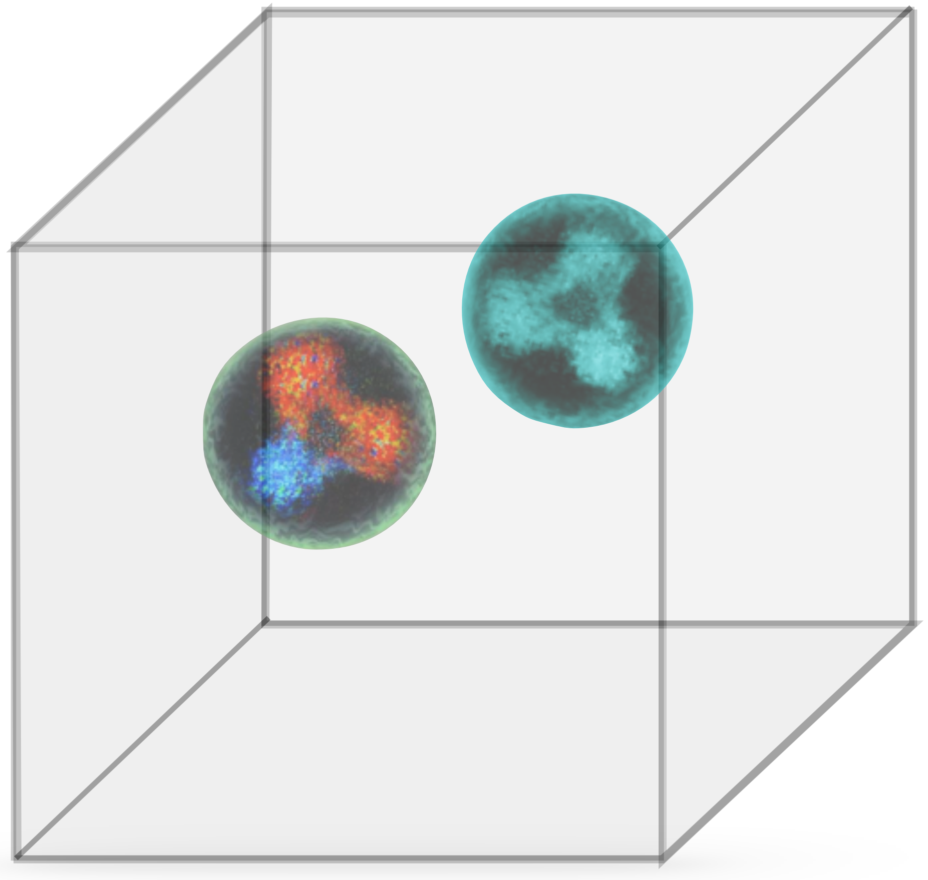




Hadron-hadron interactions

- Two particles in a box: eigen-energies depend on interactions



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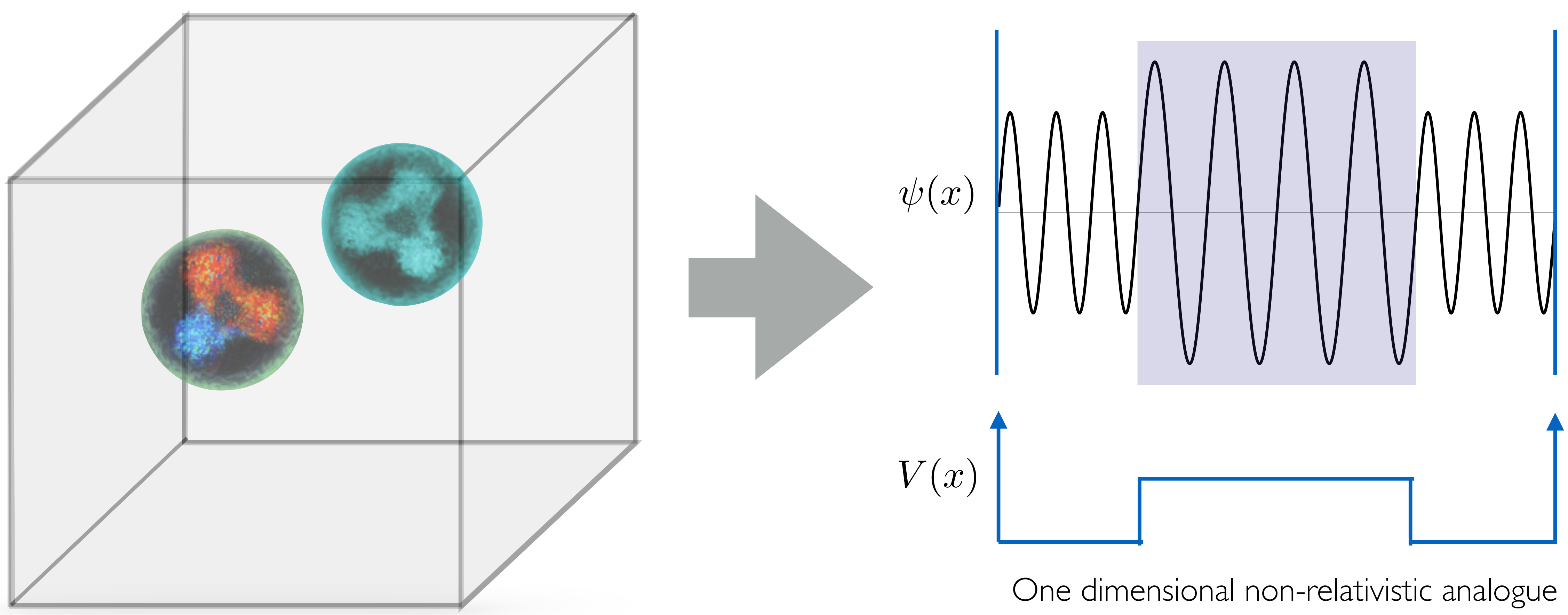
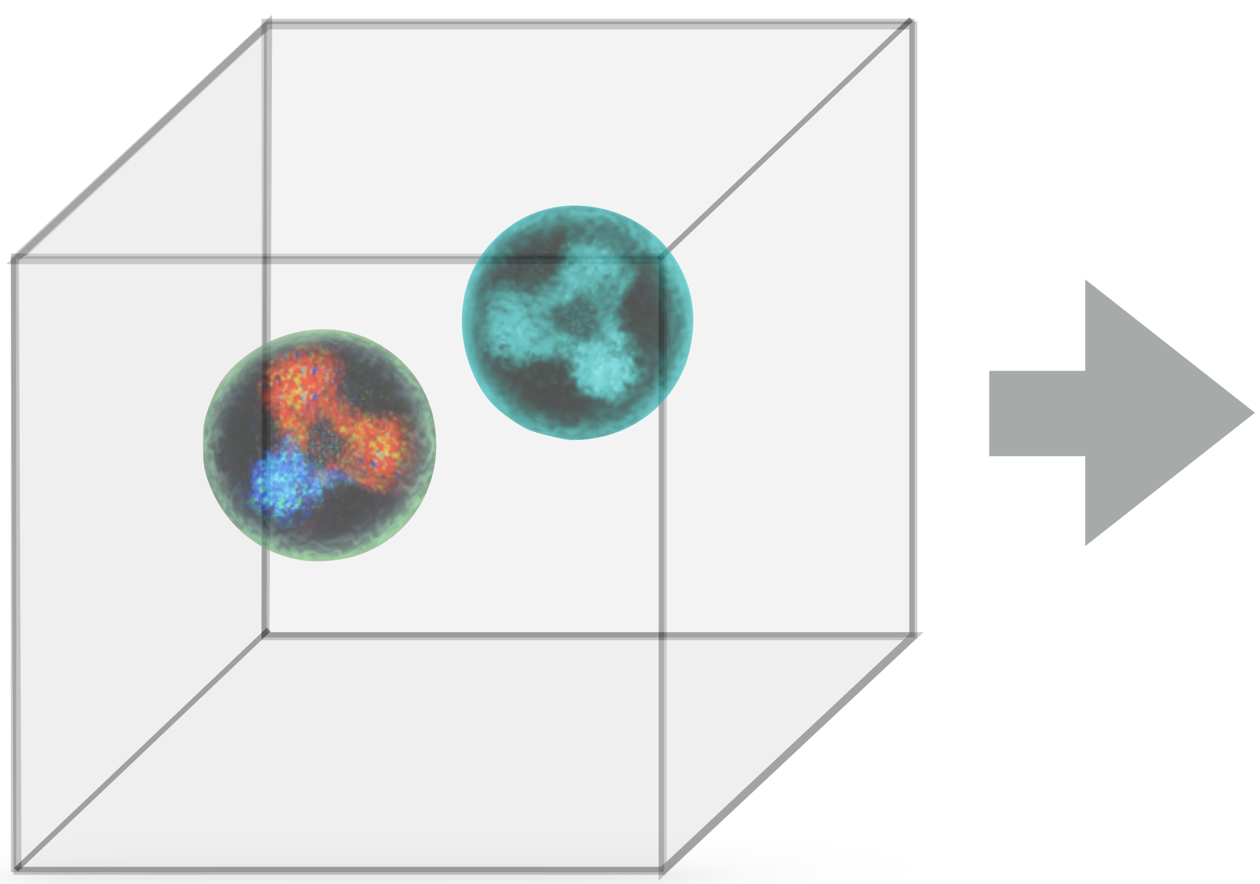
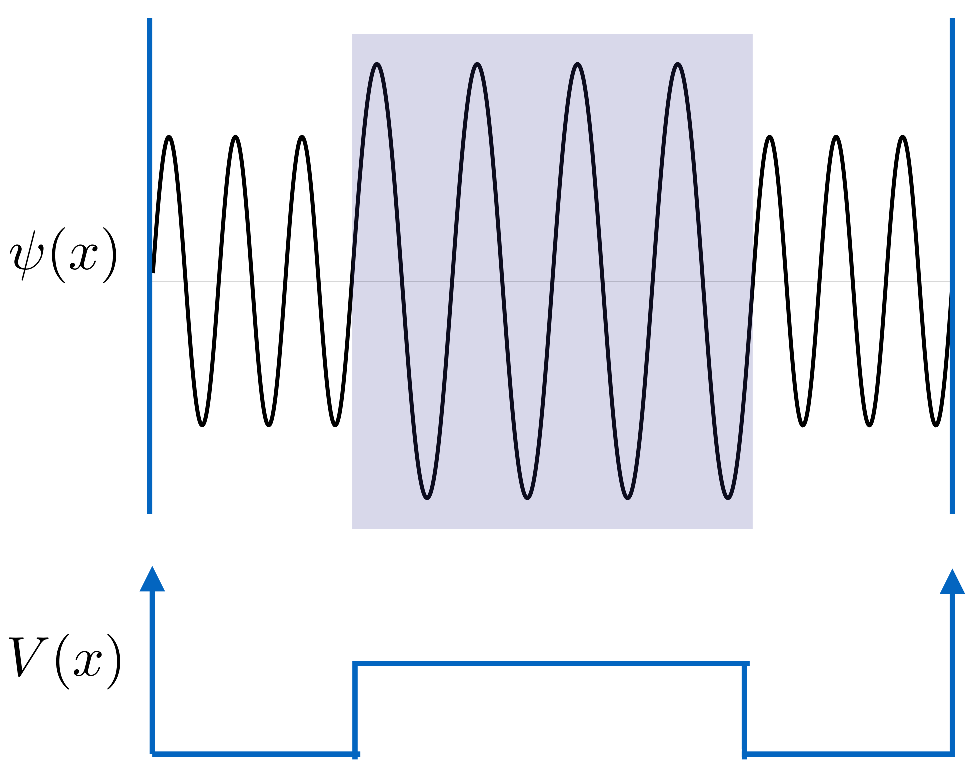


One dimensional non-relativistic analogue

- Two particles in a box: eigen-energies depend on interactions
- Calculate two particle energies to determine scattering phase shift

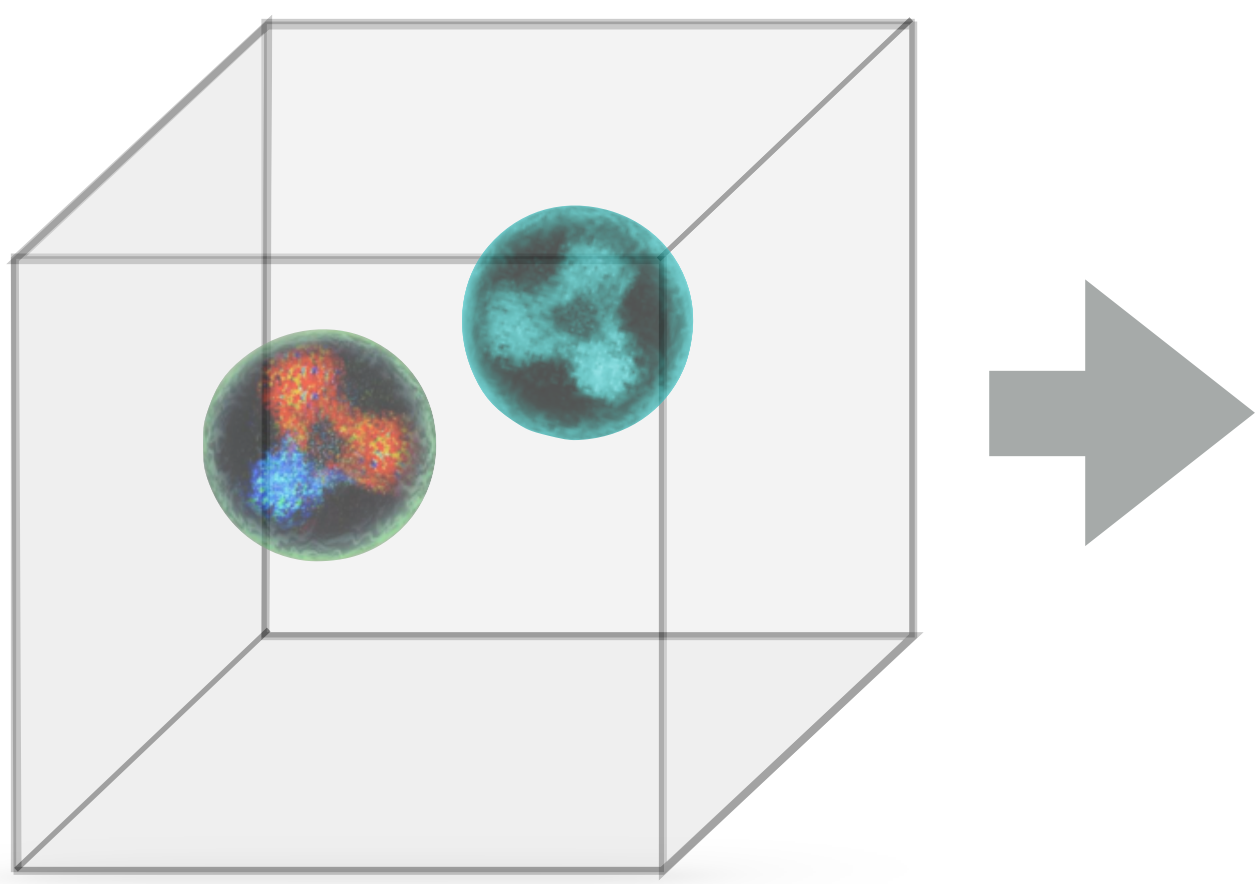


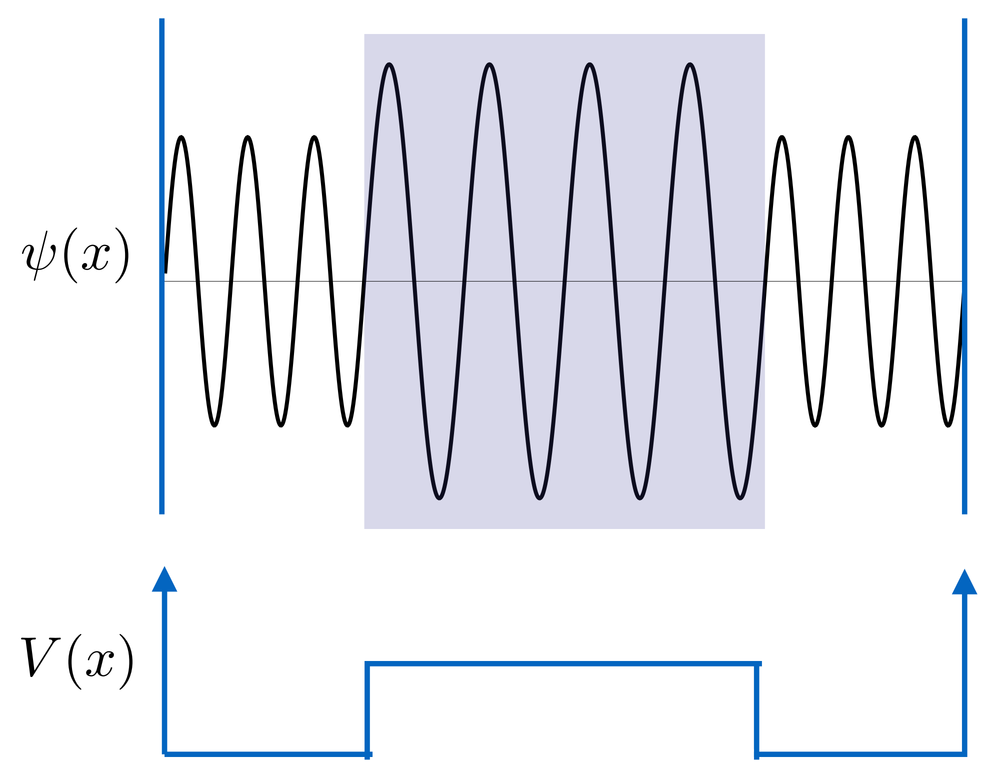


One dimensional non-relativistic analogue

Hadron-hadron interactions

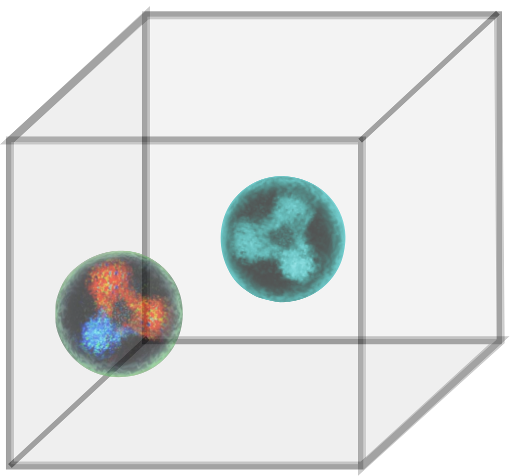
- Two particles in a box: eigen-energies depend on interactions
- Calculate two particle energies to determine scattering phase shift
- Requires multiple different volumes





One dimensional non-relativistic analogue

Hadron scattering



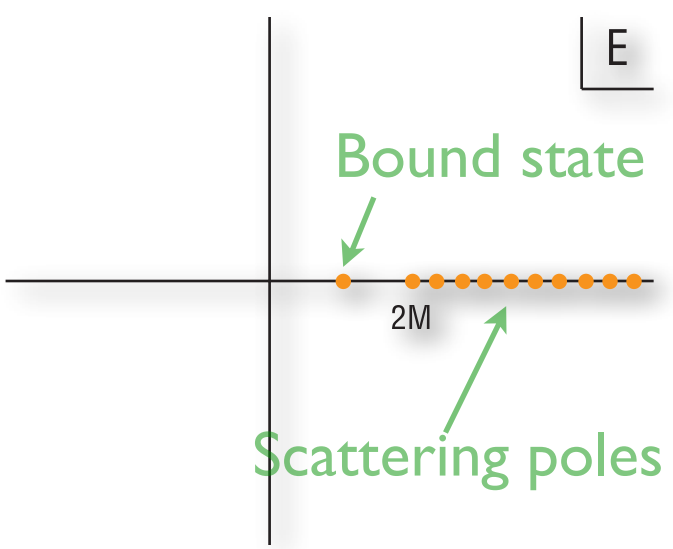
- Maiani-Testa: extracting multi-hadron S-matrix elements from Euclidean lattice calculations of Green functions in infinite volume is impossible

- Lüscher: volume dependence of two-particle energy levels
  ⇒ scattering phase-shift, δ(p), up to inelastic threshold

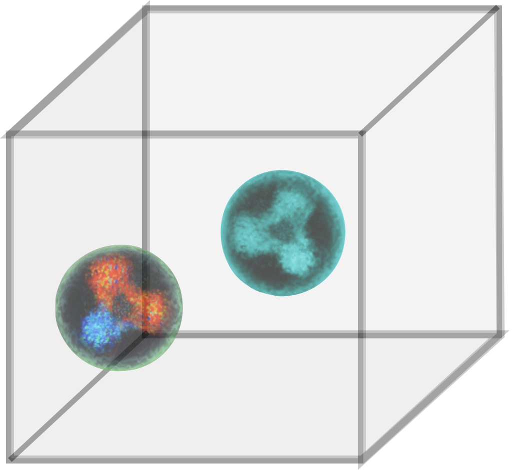
$$\Delta E_{(n)} = \sqrt{|\mathbf{q}_{(n)}|^2 + m_A^2} + \sqrt{|\mathbf{q}_{(n)}|^2 + m_B^2} - m_A - m_B$$

$$q_{(n)} \cot \delta(q_{(n)}) = \frac{1}{\pi L} S\left(\frac{q_{(n)} L}{2\pi}\right)$$

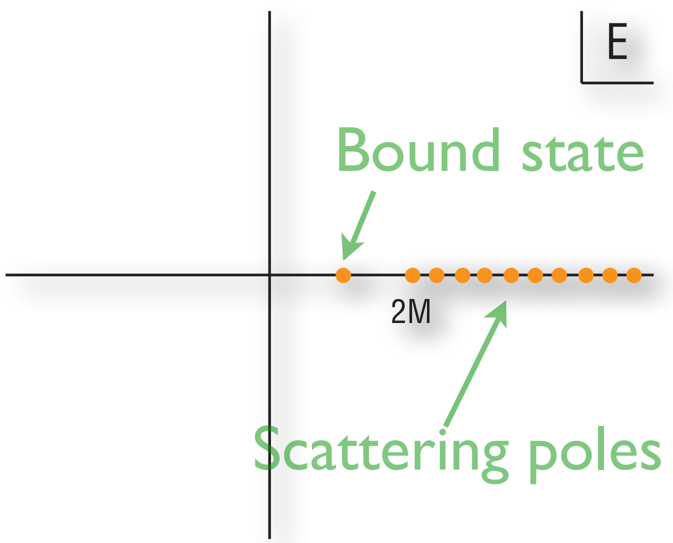
$$S(\eta) = \lim_{\Lambda\to\infty}\left[\sum_{\vec{n}}^{|\vec{n}|<\Lambda} \frac{1}{|\vec{n}|^2 - \eta^2} - 4\pi\Lambda\right]$$



Hadron scattering

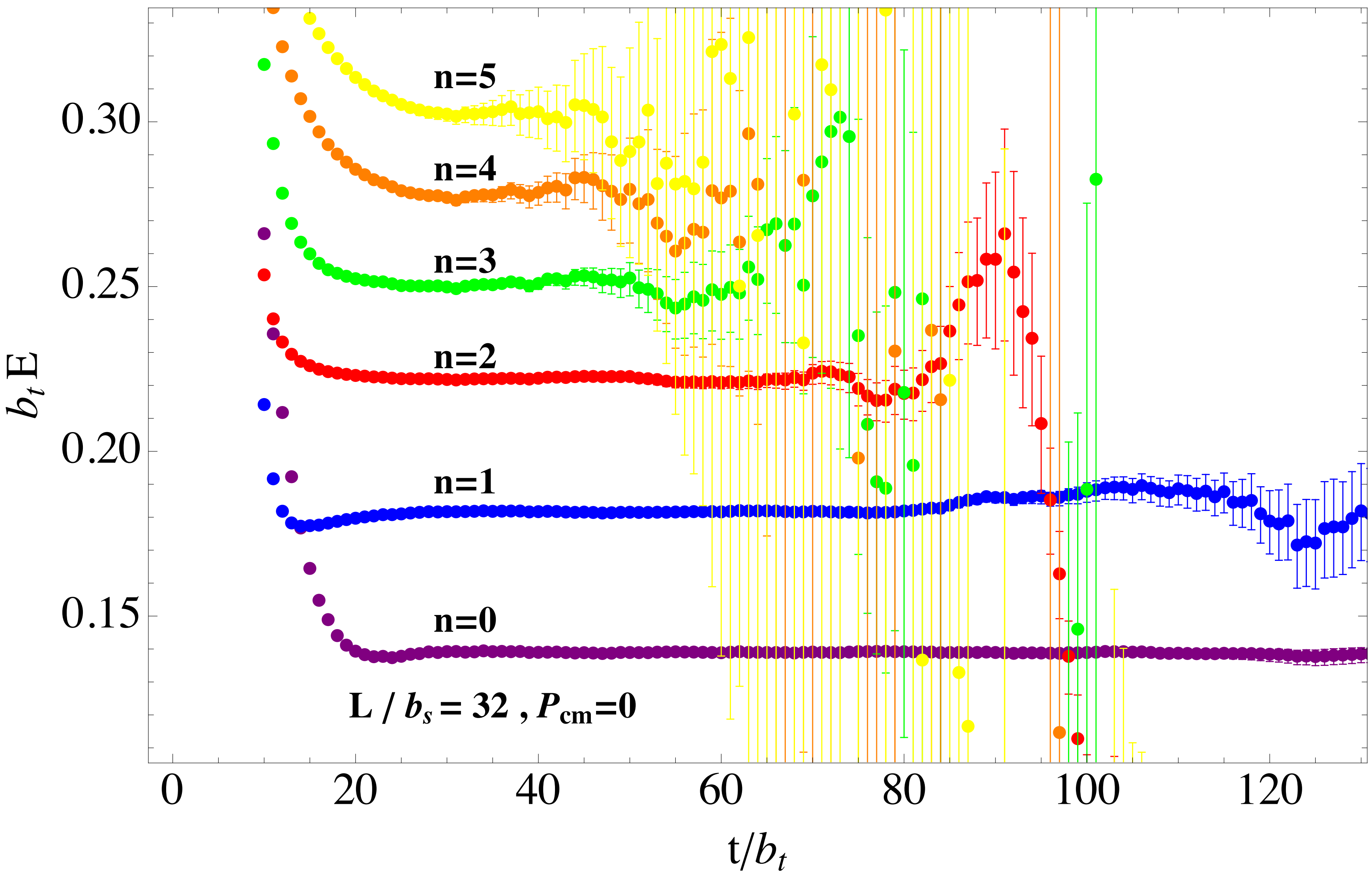


- Maiani-Testa: extracting multi-hadron S-matrix elements from Euclidean lattice calculations of Green functions in infinite volume is impossible
- Lüscher: volume dependence of two-particle energy levels
  $\Rightarrow$ scattering phase-shift, $\delta(p)$, up to inelastic threshold
- Exact relation provided r«L
- Used for $\pi\pi$, KK, ...
  - A precision science for stretched states
- Known for many years in QM, NP



Example: I=2 $\pi\pi$

- Study multiple energy levels of two pions in a box for multiple volumes and with multiple P_{CM}



@ m_π = 390 MeV

1107.5023 [prd]

Example: I=2 ππ

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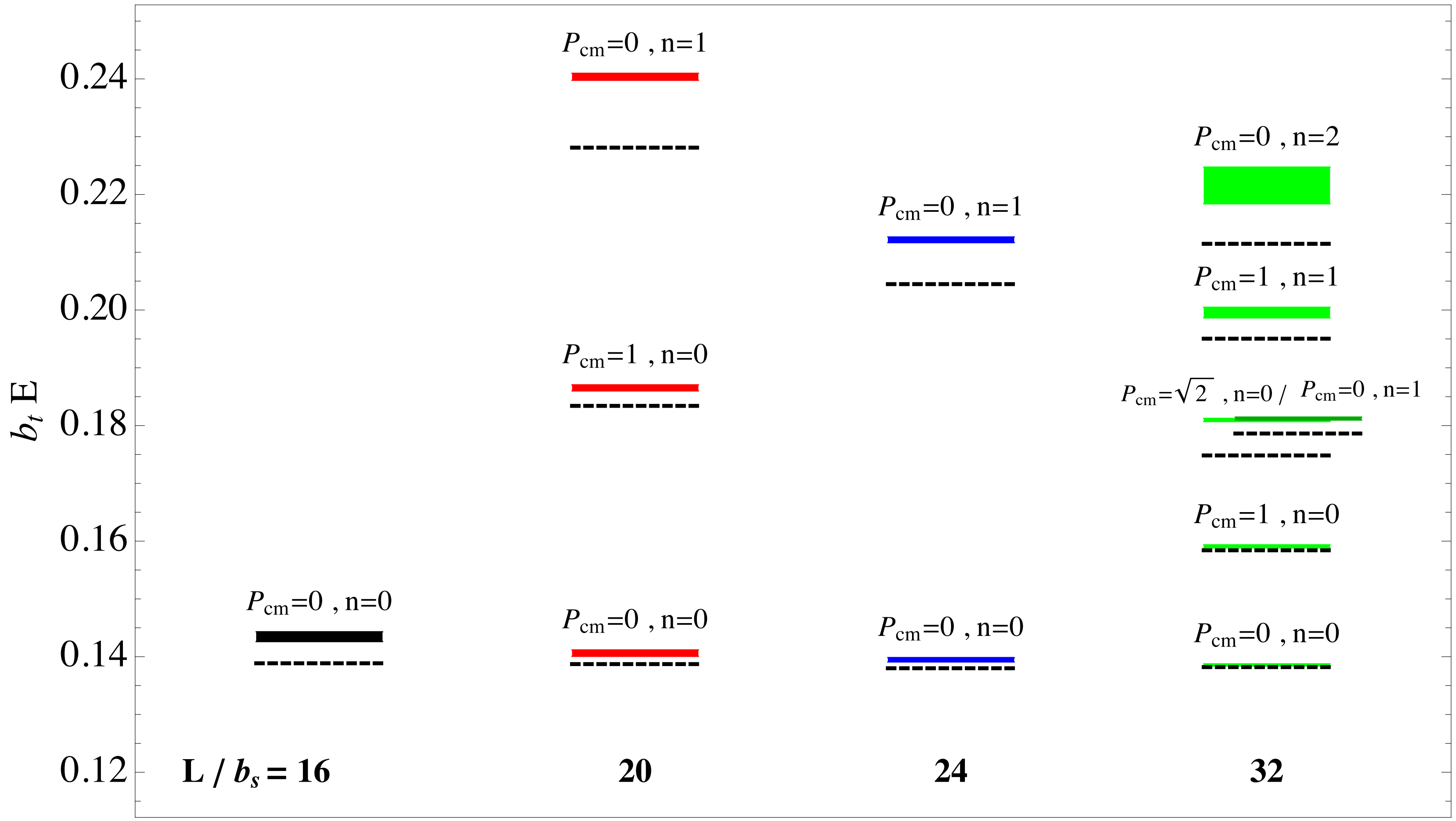


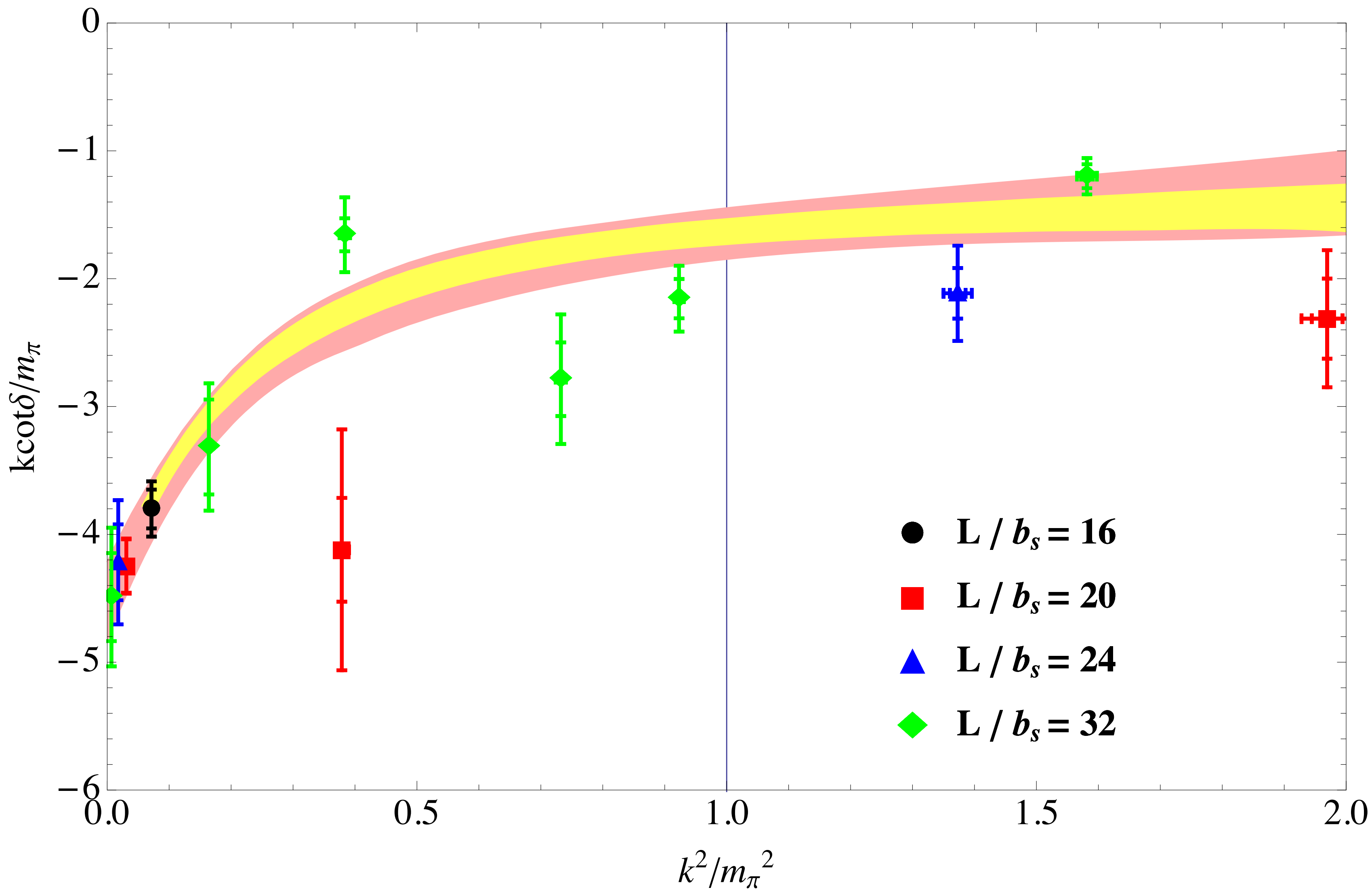
Dashed lines are non-interacting energy levels

@ m_π = 390 MeV

1107.5023 [prd]

Example: I=2 ππ

- Allows phase shift to be extracted at multiple energies

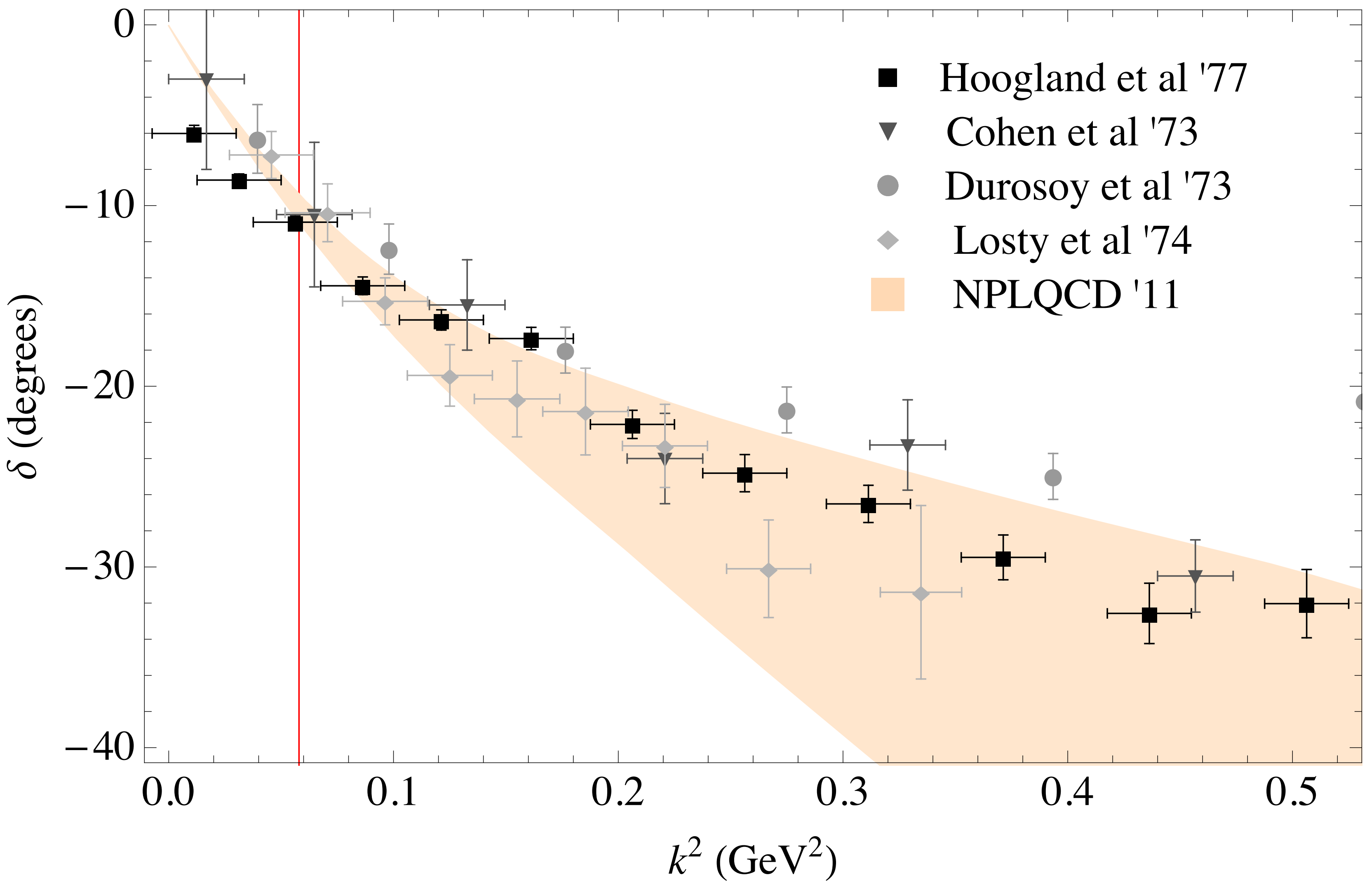


@ m_π = 390 MeV

1107.5023 [prd]

Example: I=2 $\pi\pi$

- Combine with chiral perturbation theory to interpolate to physical pion mass
- D wave phase shift also extracted [JLab]



1107.5023 [prd]

[NPLQCD 1301.5790]

@ m_π = 800 MeV

- Calculation of NN phase shifts at m_π=800 MeV

- Scattering length and effective range extracted with O(10%) precision

$$a^{(^3S_1)} = 1.82^{+0.14+0.17}_{-0.13-0.12}\ \text{fm} \qquad r^{(^3S_1)} = 0.906^{+0.068+0.068}_{-0.075-0.084}\ \text{fm}$$

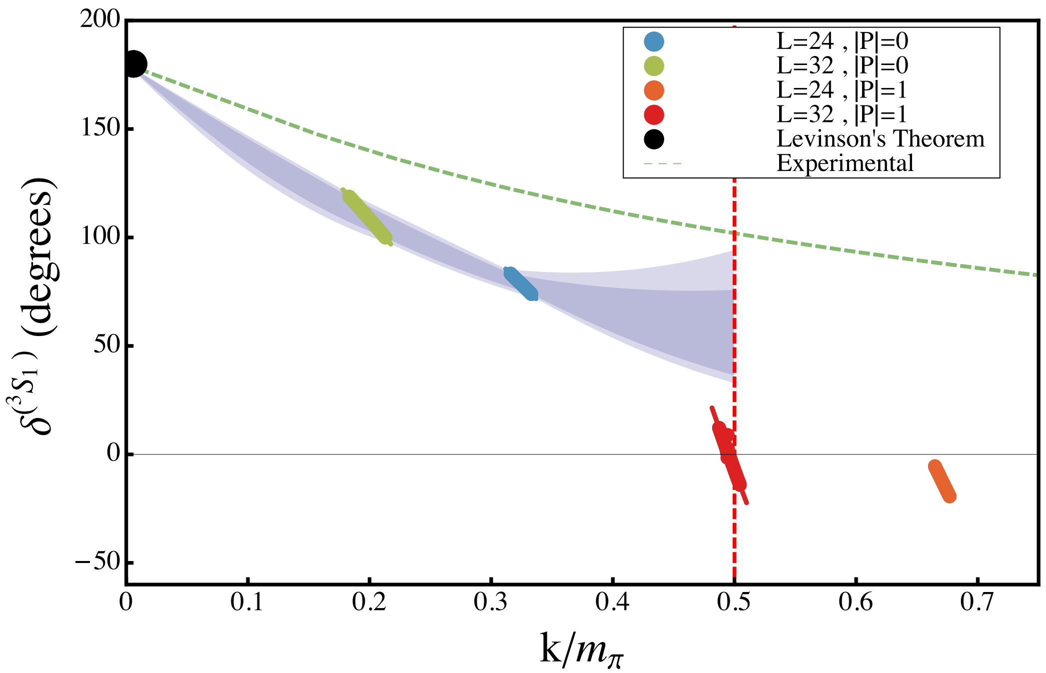
$$a^{(^1S_0)} = 2.33^{+0.19+0.27}_{-0.17-0.20}\ \text{fm} \qquad r^{(^1S_0)} = 1.130^{+0.071+0.059}_{-0.077-0.063}\ \text{fm}$$

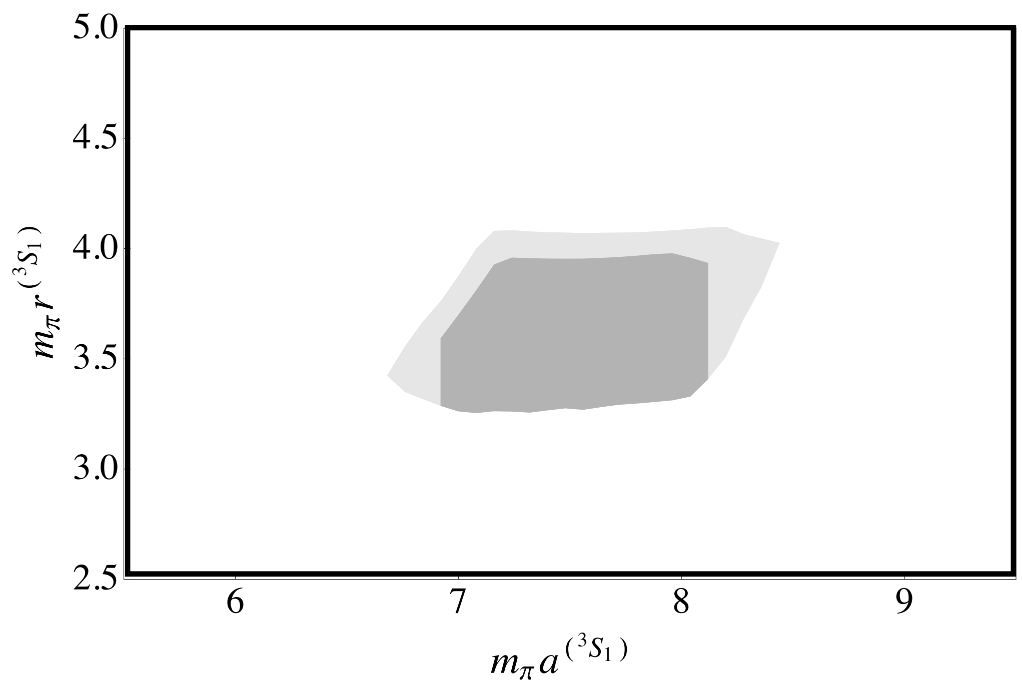
- Fine-tuning of NN at physical mass?

$$a^{(^3S_1)}/r^{(^3S_1)} = 2.06^{+0.22+0.25}_{-0.18-0.19}$$

$$a^{(^1S_0)}/r^{(^1S_0)} = 2.02^{+0.23+0.29}_{-0.19-0.18}$$

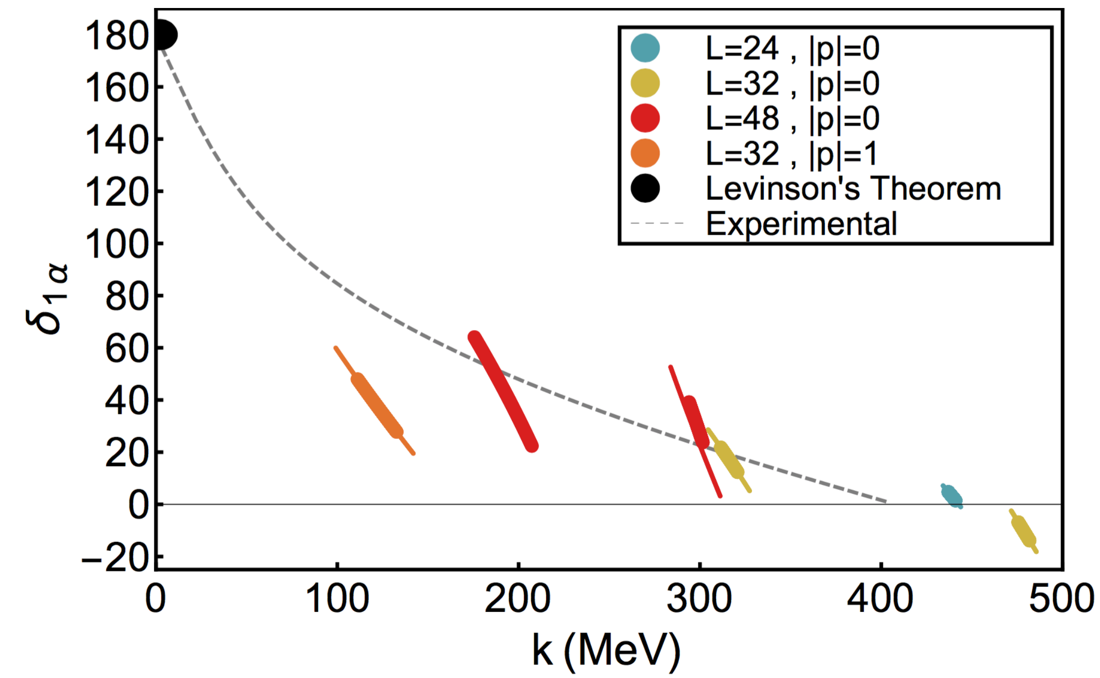
- Wigner SU(4) symmetry





[NPLQCD 1508.07583]

@ m_π = 450 MeV

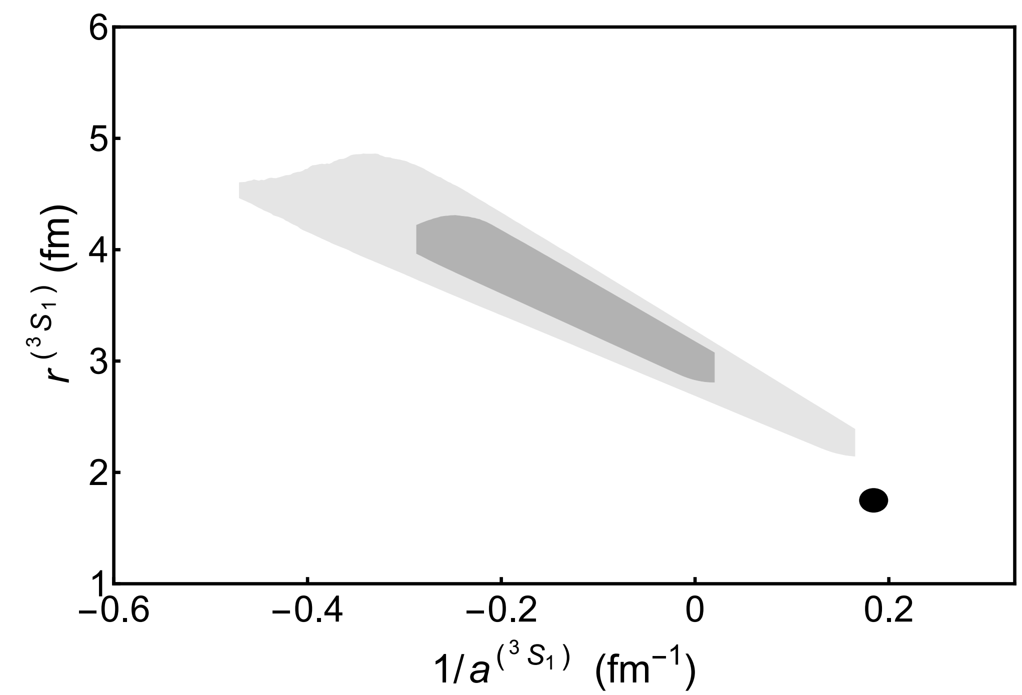


- Recent calculation of NN phase shifts at m_π=450 MeV
- Phase shifts closer to physical case
- Analyse in chiral EFT:

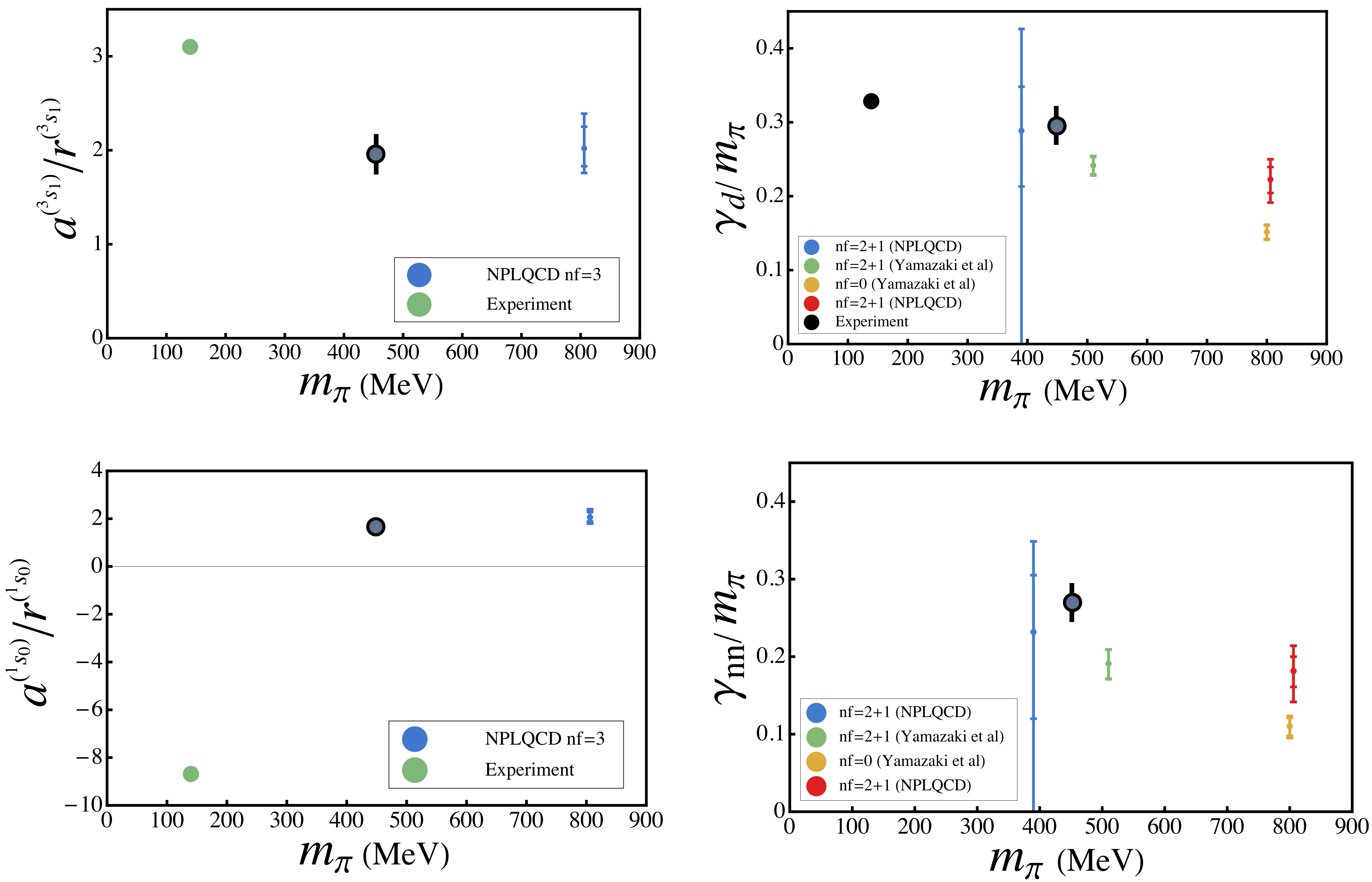
$$a^{(^1S_0)}_{\mathrm{NNLO}} = 2.99(07)(15)\ \mathrm{fm} \qquad r^{(^1S_0)}_{\mathrm{NNLO}} = 1.611(42)(83))\ \mathrm{fm}$$

$$a^{(^3S_1)}_{\mathrm{NLO}^*} = 2.72(22)(27)\ \mathrm{fm} \qquad r^{(^3S_1)}_{\mathrm{NLO}^*} = 1.43(12)(13))\ \mathrm{fm}$$

  - Enables extraction of $^3D_1$ phase and mixing
- Fine-tuning of NN at physical mass?

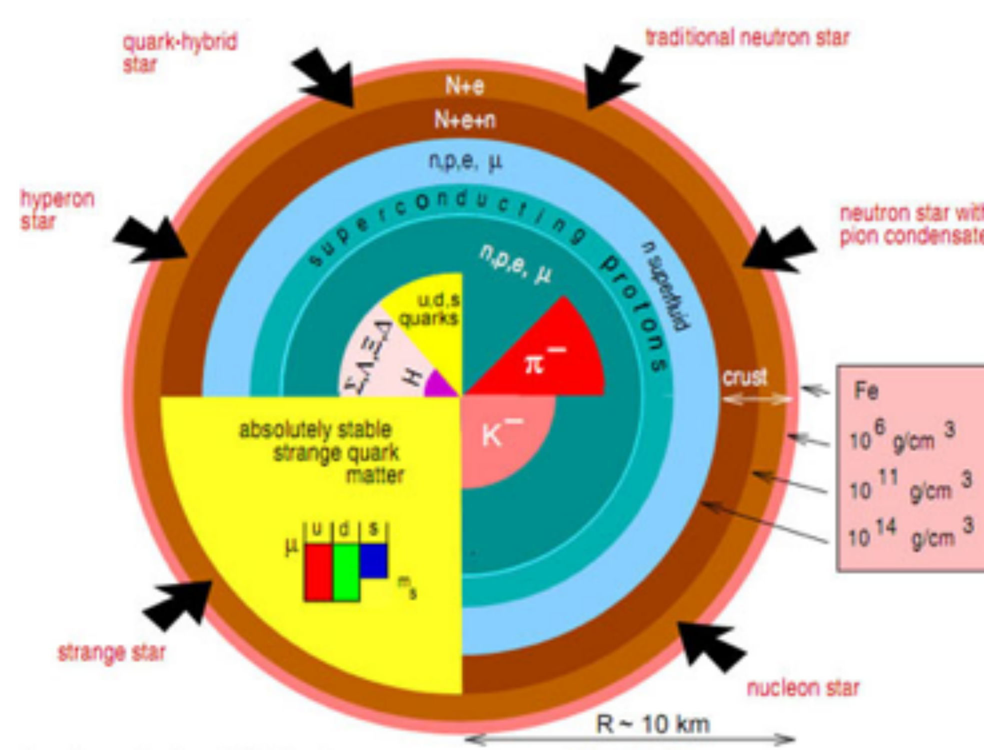


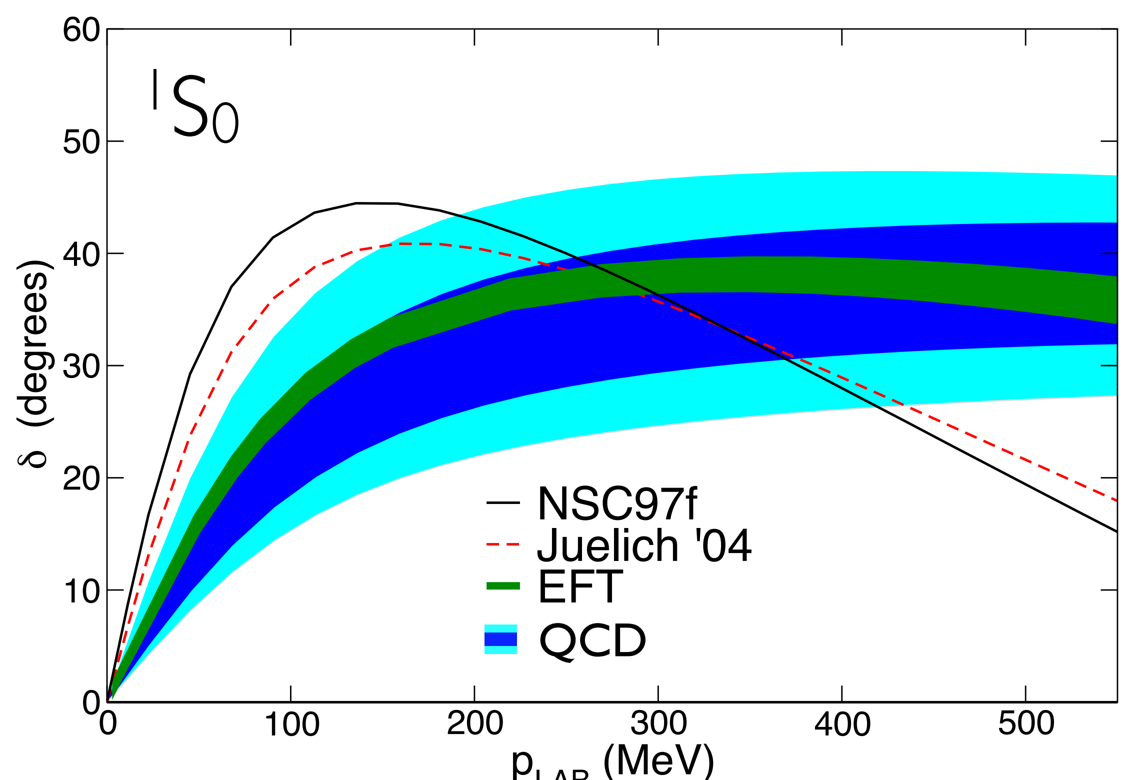
NN fine tuning

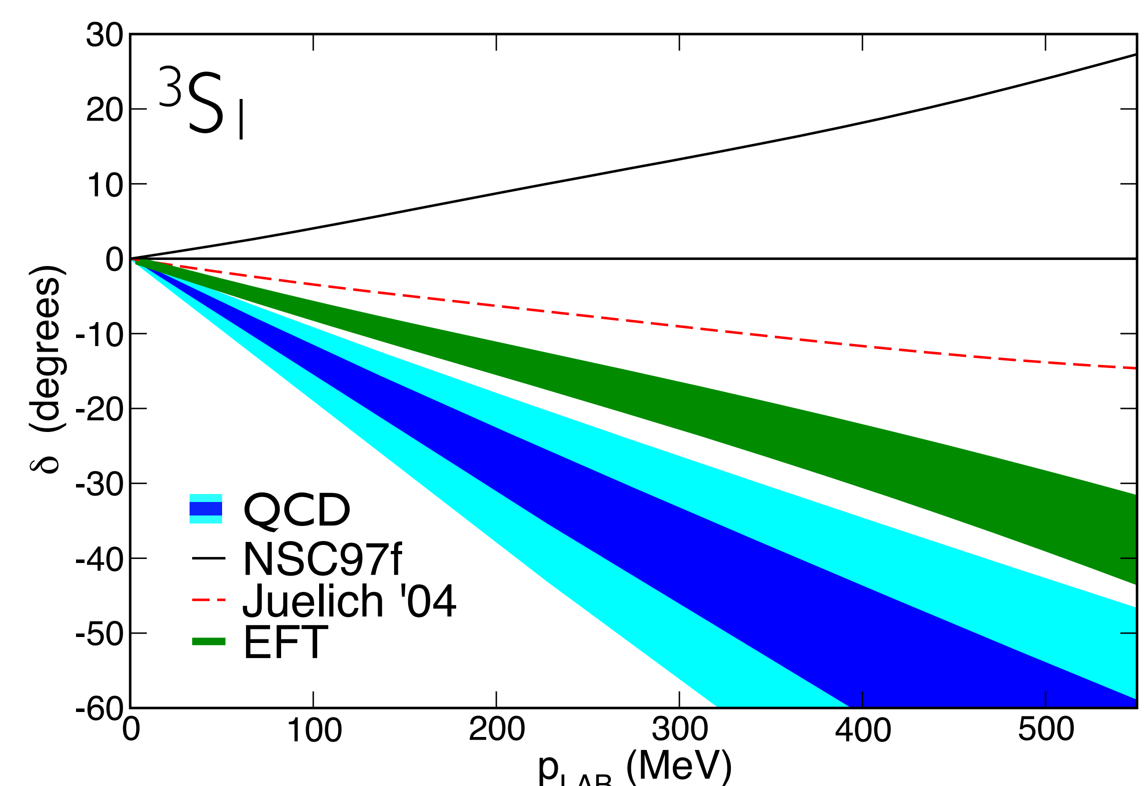


$\Sigma^{-}n$ (I=3/2) phase shifts

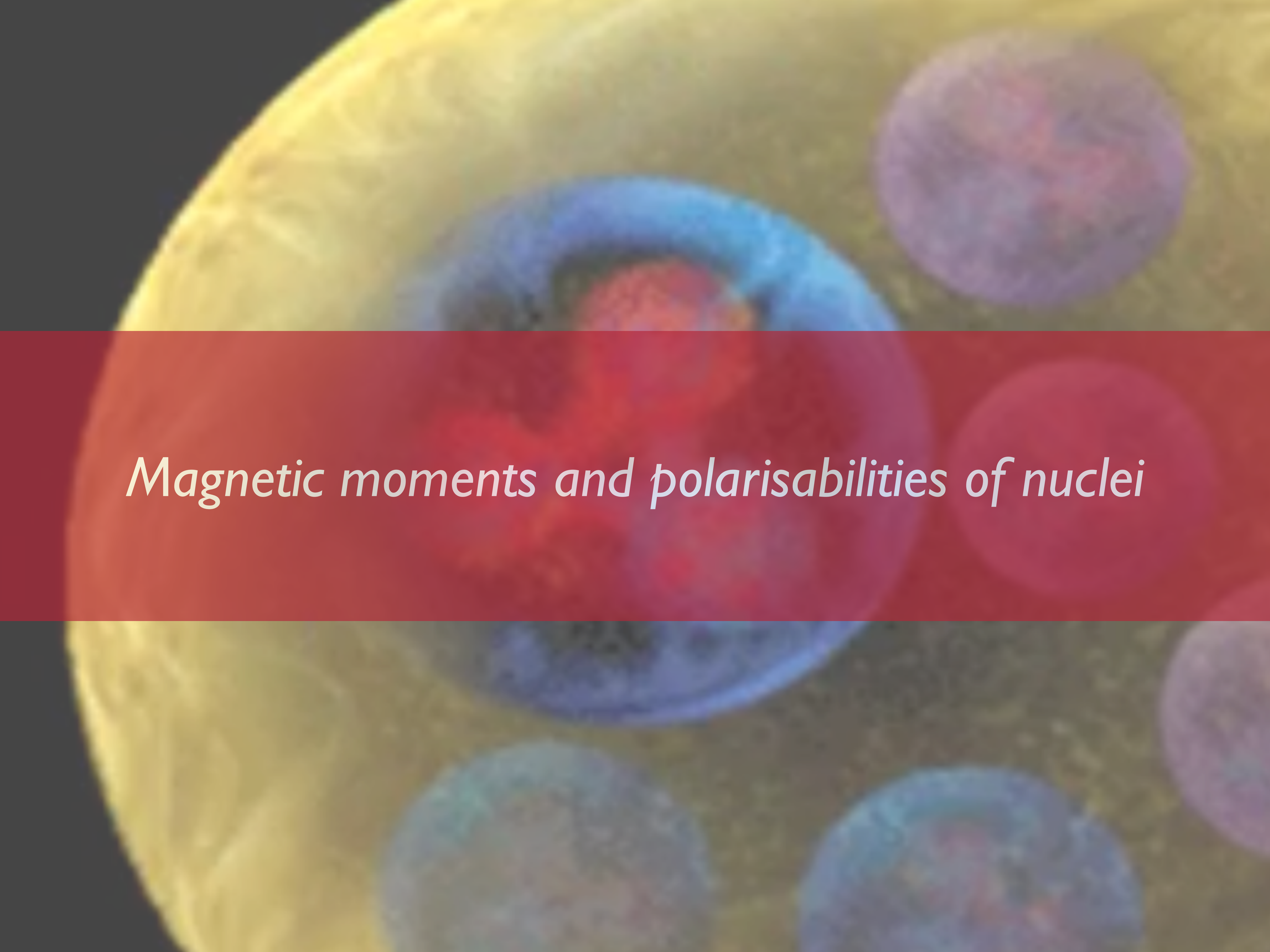
- Hyperon-nucleon phase shifts important EoS of neutron stars
- Determine at one quark mass
- Match to effective field theory to extract phase shift at physical mass
- Future calculations will inform EoS







[NPLQCD PRL109 (2012) 172001]



Magnetic moments and polarisabilities of nuclei

Unphysical Nuclei

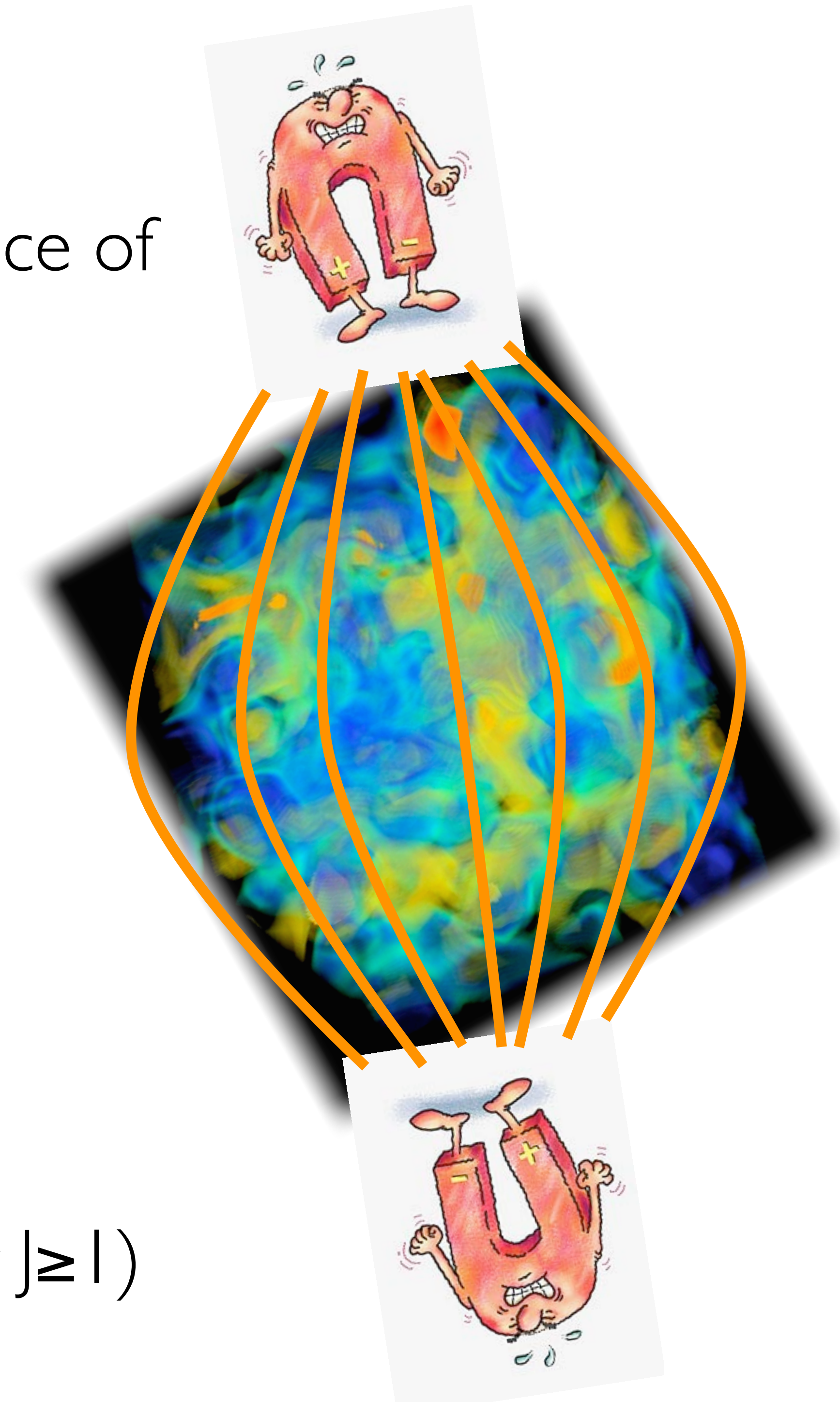
- Spectroscopy reveals nuclei at unphysical quark masses are quite different
  - Many improvements still needed
  - Much to learn at lighter quark masses (even lighter than physical)
- What is the structure of these nuclei?
  - Probe as in experiment
    - Magnetic moments, form factors, quadrupole moment,…

Background field methods

- Hadron/nuclear energies are modified by presence of fixed external fields
- Eg: fixed B field

$$E_{h;j_z}(\mathbf{B}) = \sqrt{M_h^2 + (2n+1)|Q_h eB|} - \boldsymbol{\mu}_h \cdot \mathbf{B} \\ - 2\pi\beta_h^{(M0)}|\mathbf{B}|^2 - 2\pi\beta_h^{(M2)}\langle \hat{T}_{ij} B_i B_j \rangle + \ldots$$

- QCD calculations with multiple fields enable extraction of coefficients of response
  - Magnetic moment
  - Magnetic polarisabilities (scalar and tensor for J≥1)



External field method

- Partially-quenched external fields easy to apply

$$U_\mu(x) \to U_\mu(x) \cdot U_\mu^{\text{ext}}(x) \qquad U_\mu = e^{i\,g\,A_\mu} \qquad U_\mu^{\text{ext}} = e^{i\,q\,A_\mu^{\text{ext}}}$$

existing SU(3) link fields

SU(3) gauge fields

U(1) gauge field

  - PQ is enough for non-singlet linear shifts
  - Full background field needs to be present during ensemble generation (or requires reweighting)

- E.g.: constant Euclidean magnetic field

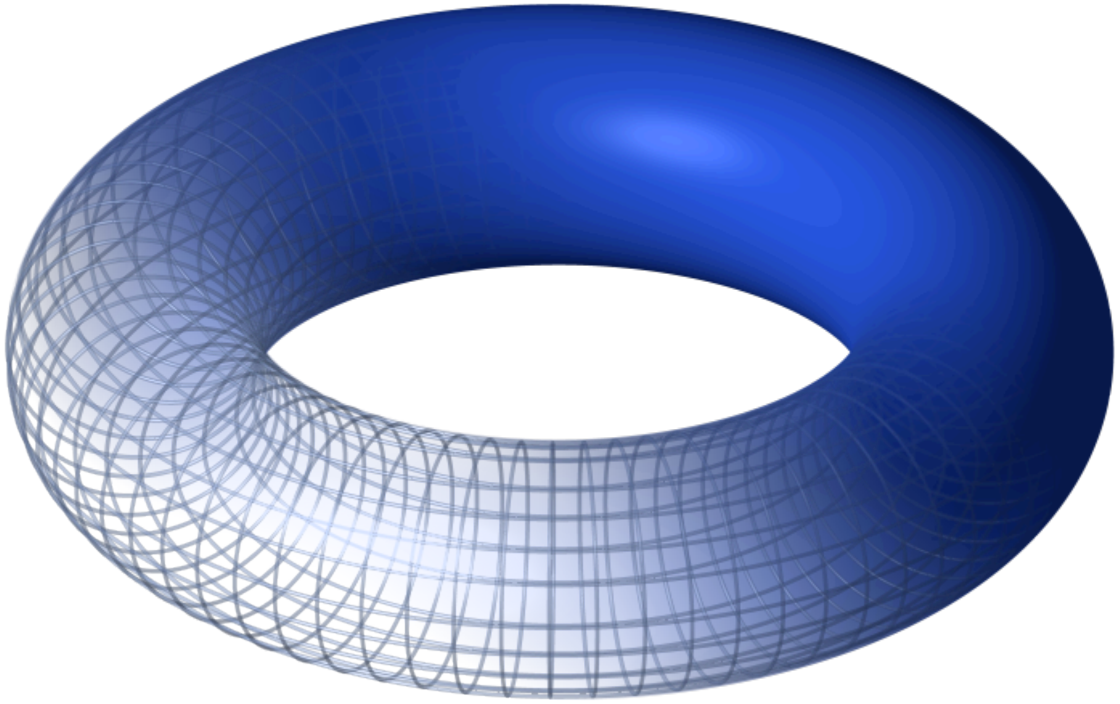
$$A_\mu^{\text{ext}} = \mathcal{B} x_1 \delta_{\mu,2} \qquad \Longrightarrow \qquad U_{1,3,4}^{\text{ext}} = 1, \; U_2^{\text{ext}} = e^{iq\mathcal{B}x_1}$$

- Look for shift in energy quadratic in $\mathcal{B}$ (spin 0)

$$C_2(t;\mathcal{B}) = \sum_{\mathbf{x}} \langle 0|\chi(\mathbf{x},t)\overline{\chi}\mathbf{0},0)|0\rangle_{\mathcal{B}} \xrightarrow{t\to\infty} Z(\mathcal{B})\exp\left(-[M + 2\pi\beta\mathcal{B}^2 + \mathcal{O}(\mathcal{B}^4)]t\right)$$

Magnetic polsarisability

Quantisation conditions



- On a torus, L^3xT not all values of external field are allowed
- Example: constant B field
- Periodicity of links requires

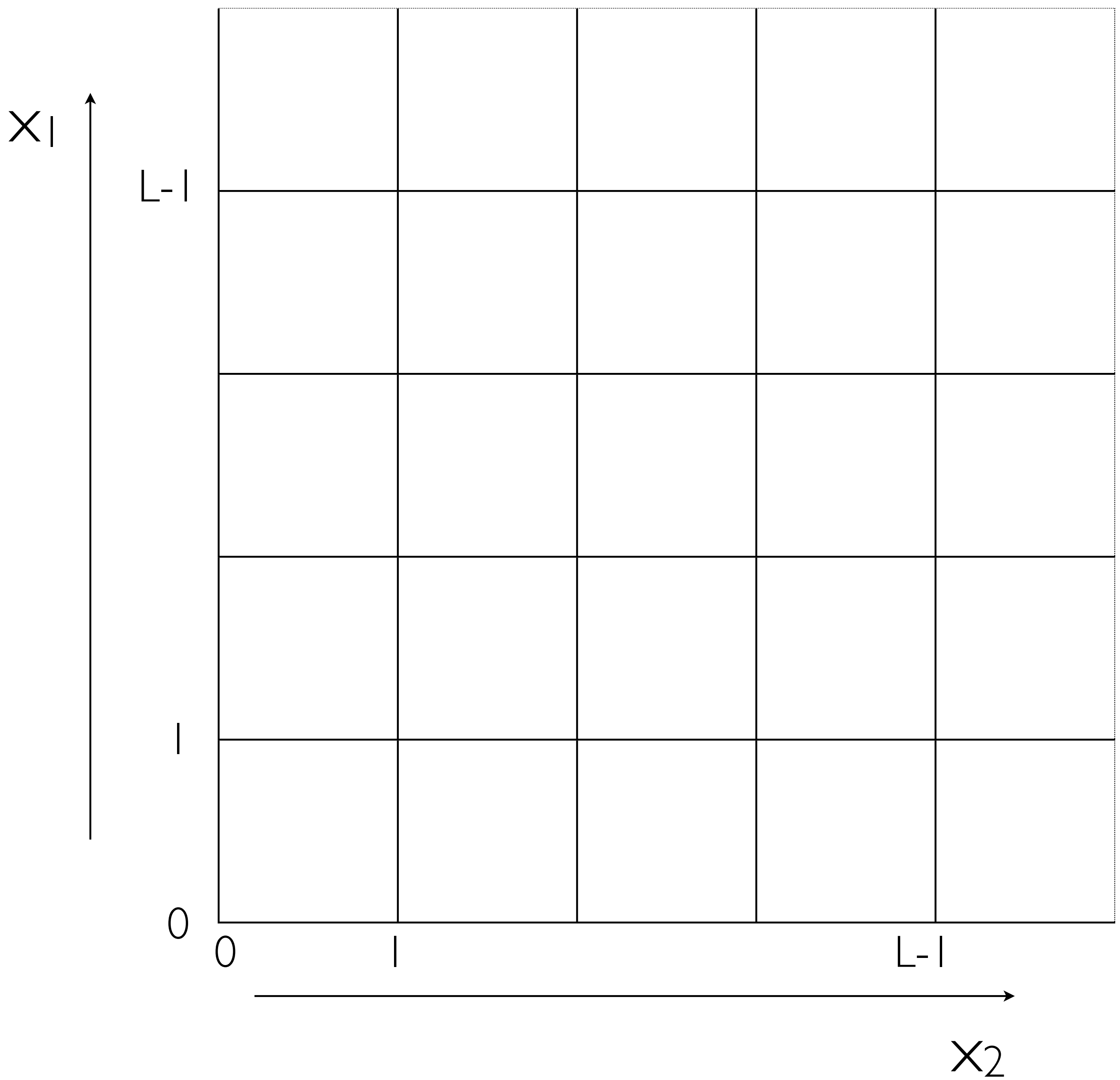
$$q\mathcal{B} = \frac{2\pi\, n}{L}, \qquad n \in \mathbb{Z}$$

- Periodicity up to a gauge transform is less restrictive ['t Hooft 79]
- Add in transverse links on the periodic boundary

$$A_\mu^{\rm ext} \longrightarrow A_\mu^{{\rm ext},\perp} = \mathcal{B} x_1 \delta_{\mu,2} - \mathcal{B} x_2 L \delta_{\mu,1} \delta_{x_1,L-1}$$

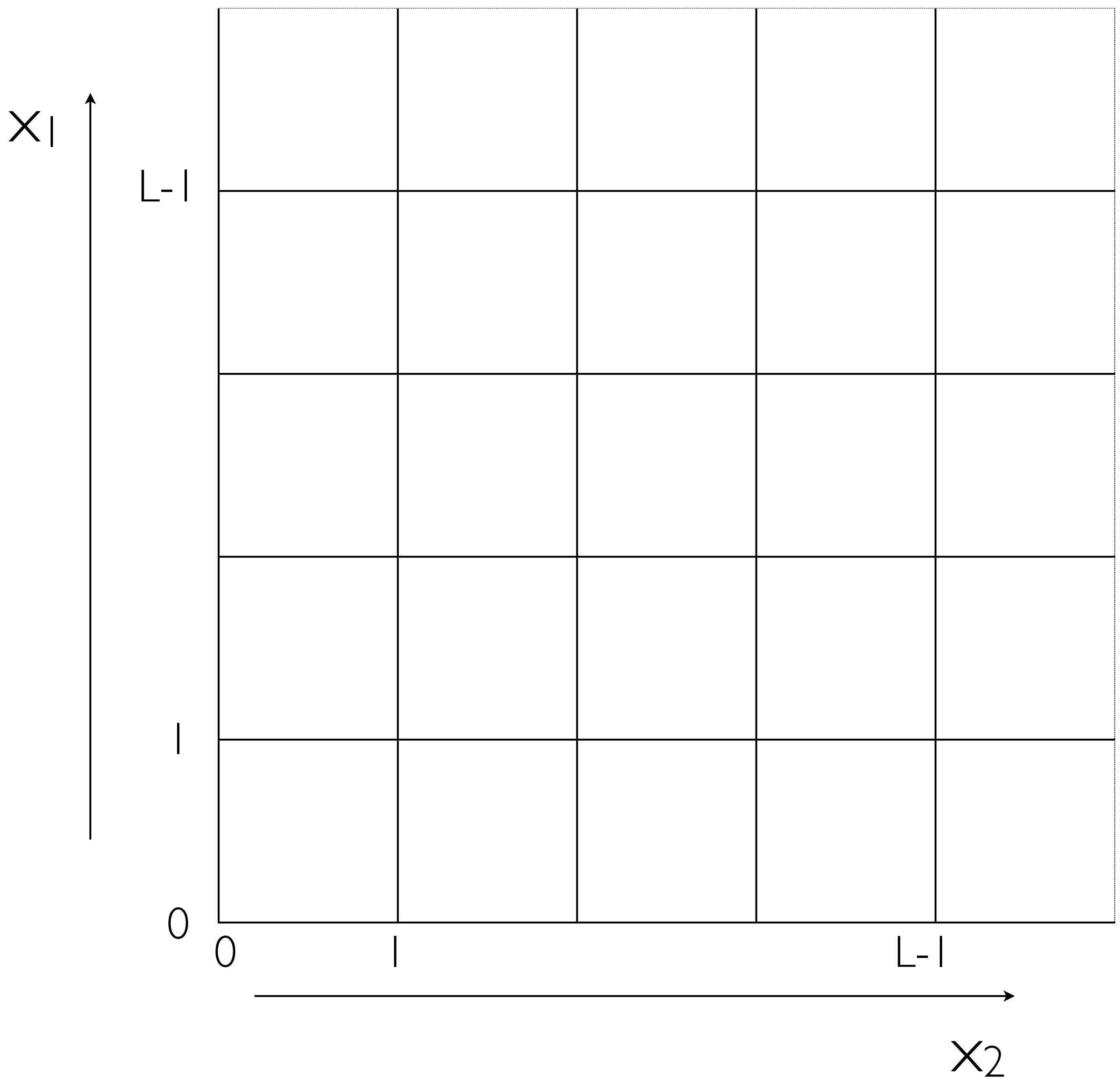
- Quantisatiation condition for electric field: $q\mathcal{B} = \dfrac{2\pi\, n}{L^2}, \qquad n \in \mathbb{Z}$

Plaquettes



$$\mathcal{P} = U_1(x)U_2(x+\hat{1})U_1^\dagger(x+\hat{1}+\hat{2})U_2^\dagger(x+\hat{2})$$

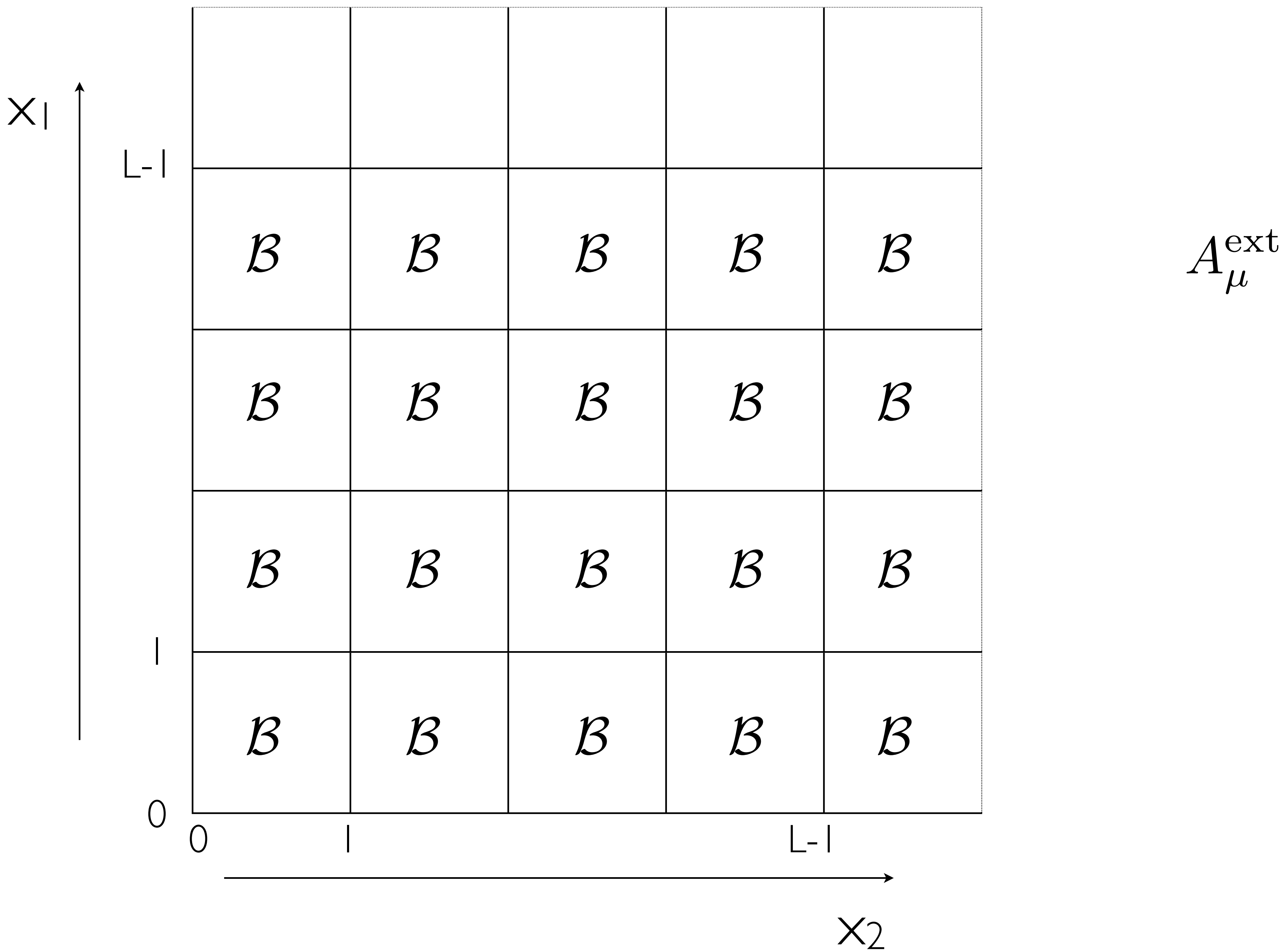
Plaquettes



$$A_\mu^{\text{ext}}$$

$$\mathcal{P} = U_1(x)U_2(x+\hat{1})U_1^\dagger(x+\hat{1}+\hat{2})U_2^\dagger(x+\hat{2})$$

Plaquettes



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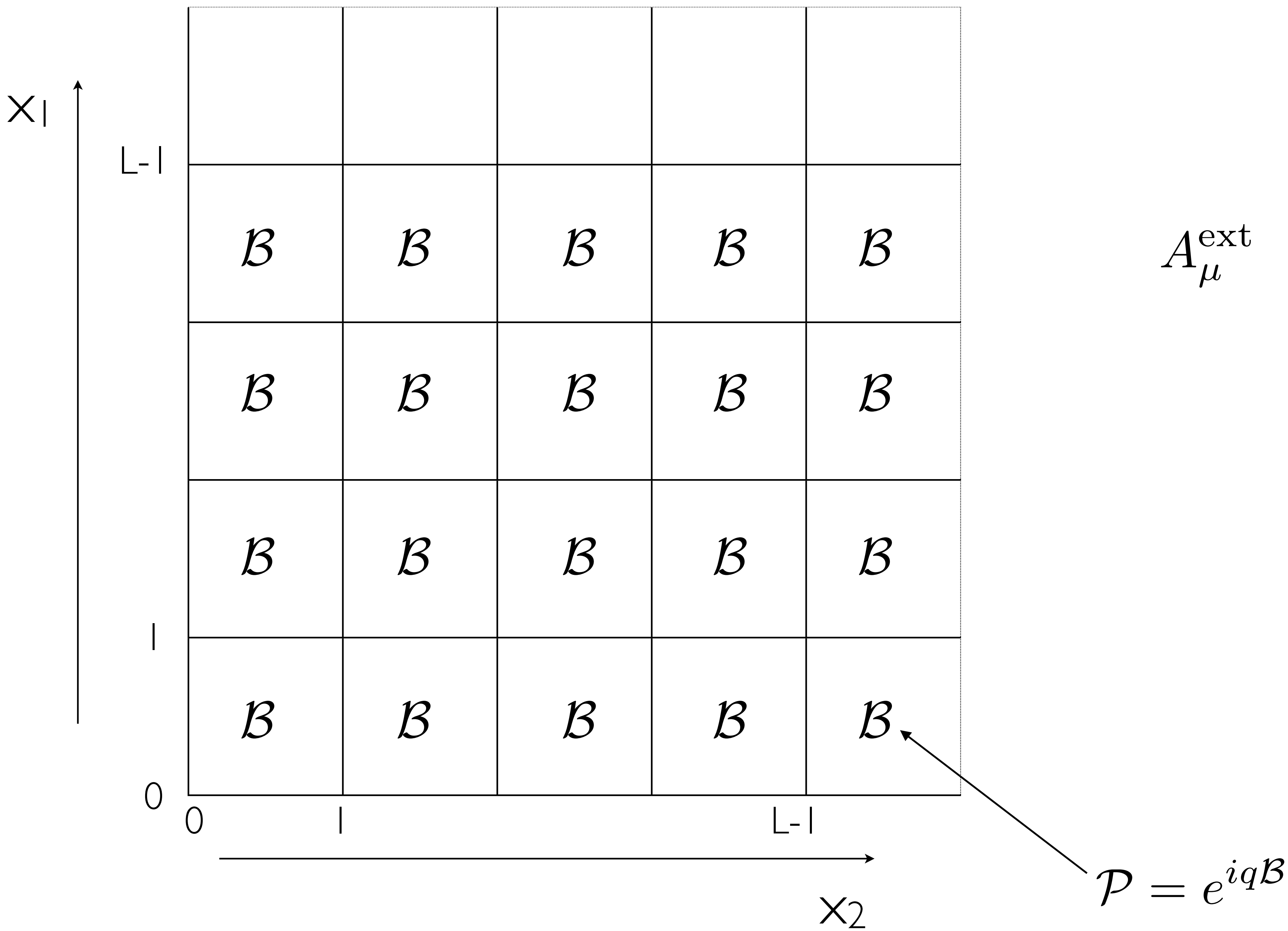

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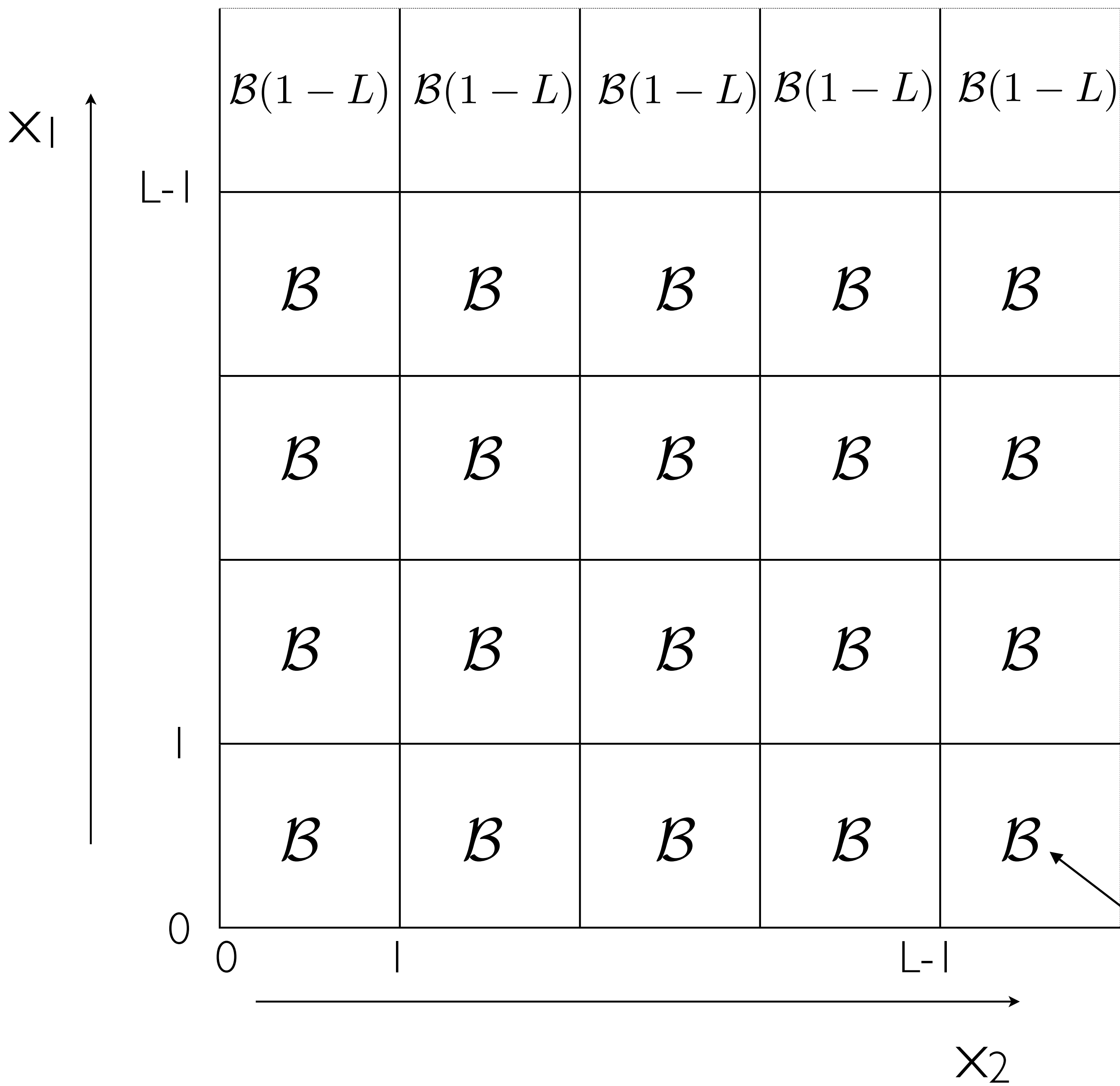
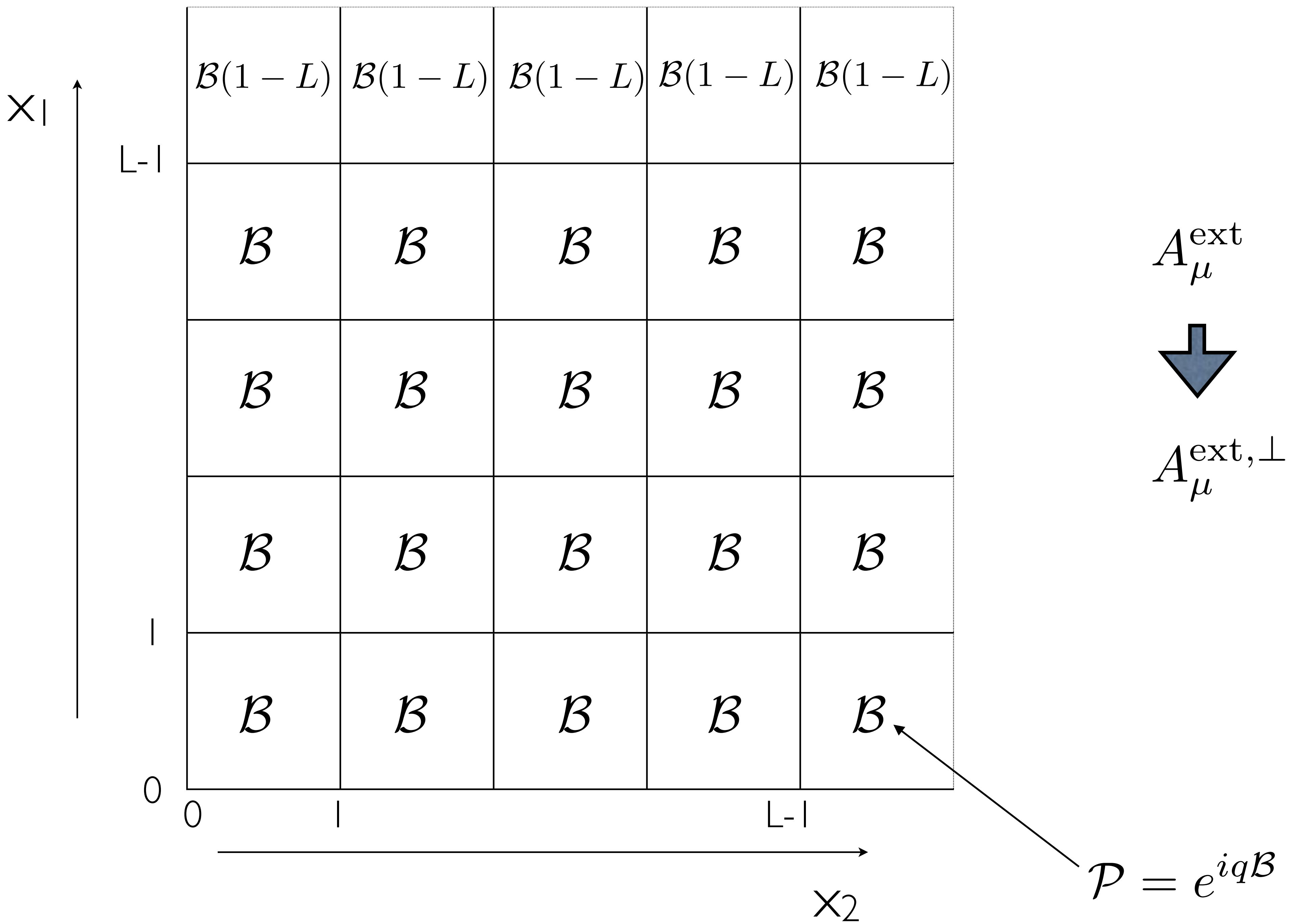
Plaquettes



A_μ^{ext}

$\mathcal{P} = e^{iq\mathcal{B}}$

$$\mathcal{P} = U_1(x)U_2(x+\hat{1})U_1^\dagger(x+\hat{1}+\hat{2})U_2^\dagger(x+\hat{2})$$



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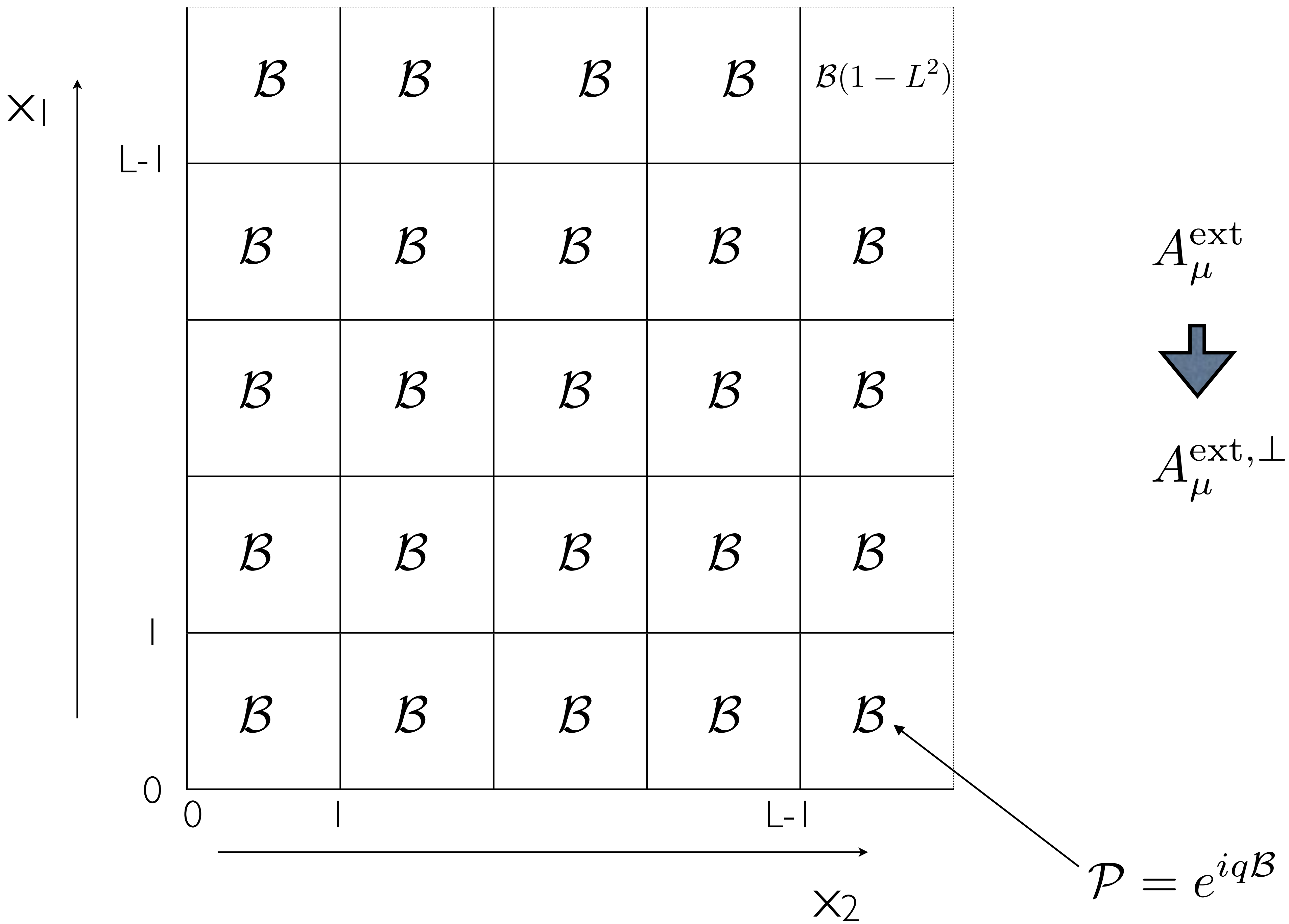
Plaquettes



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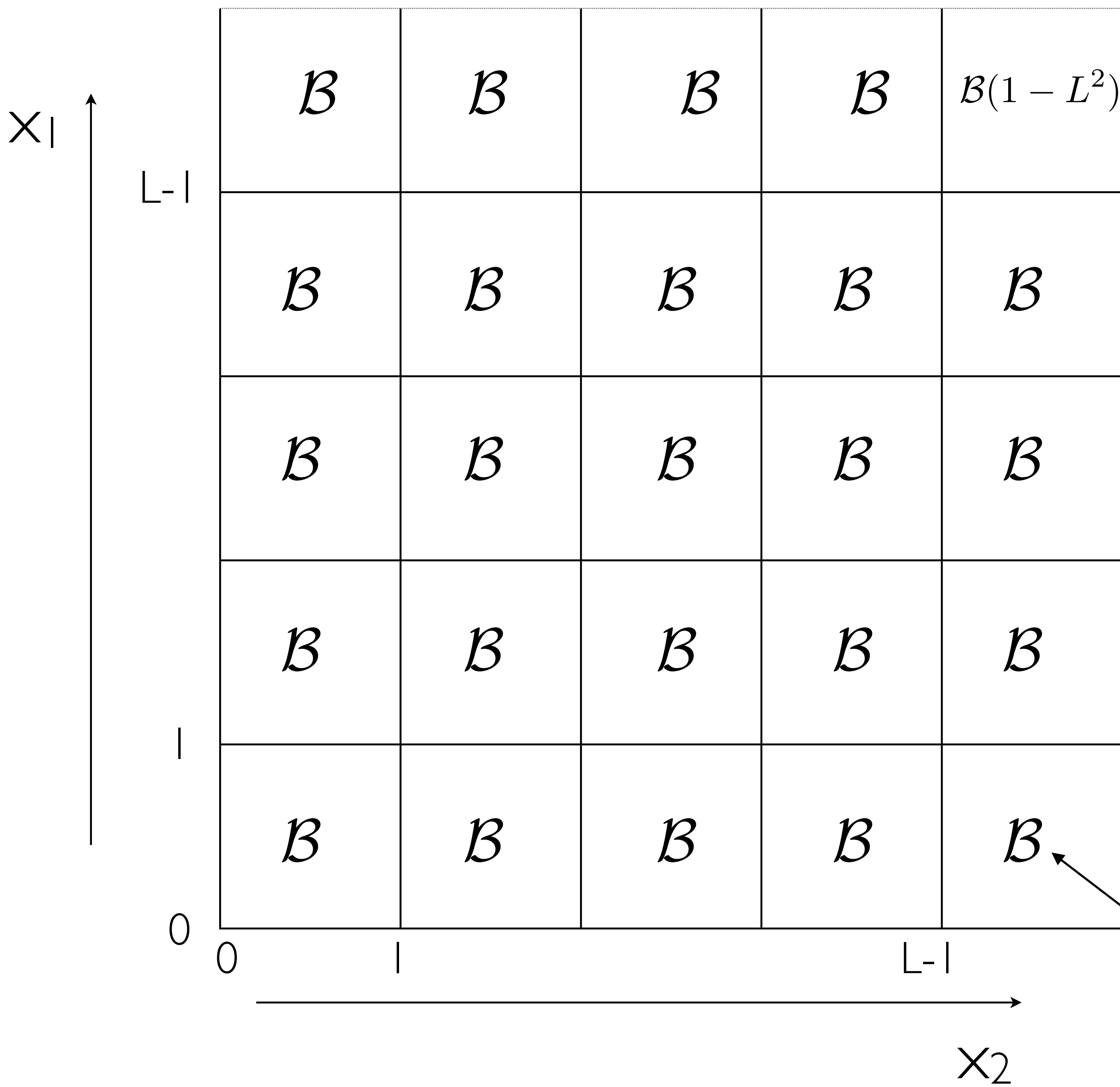

$$A_\mu^{\text{ext}}$$

$$A_\mu^{\text{ext},\perp}$$

$$q\mathcal{B} = \frac{2\pi\, n}{L^2}, \qquad n \in \mathbb{Z}$$

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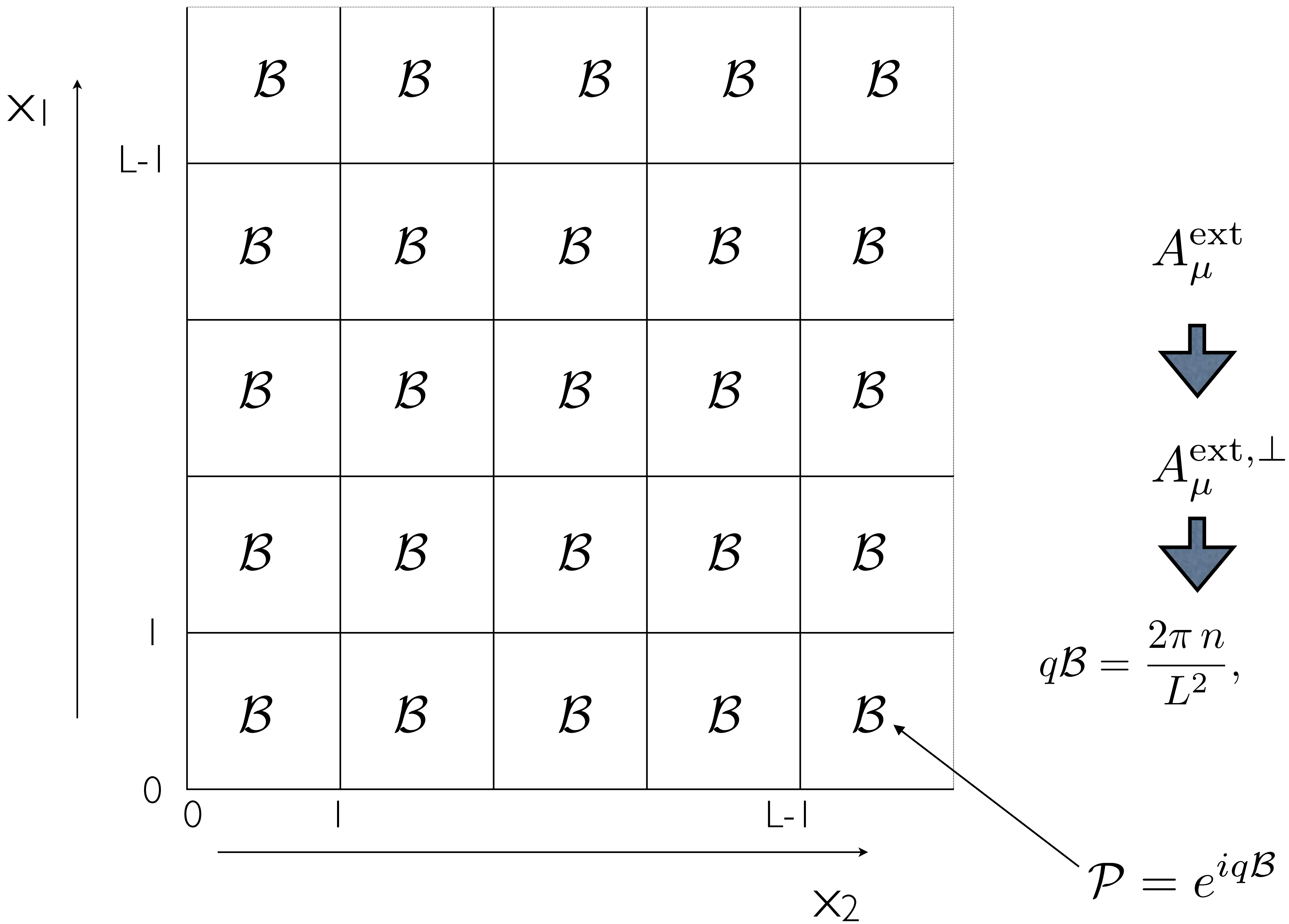
Plaquettes



A_μ^{ext}

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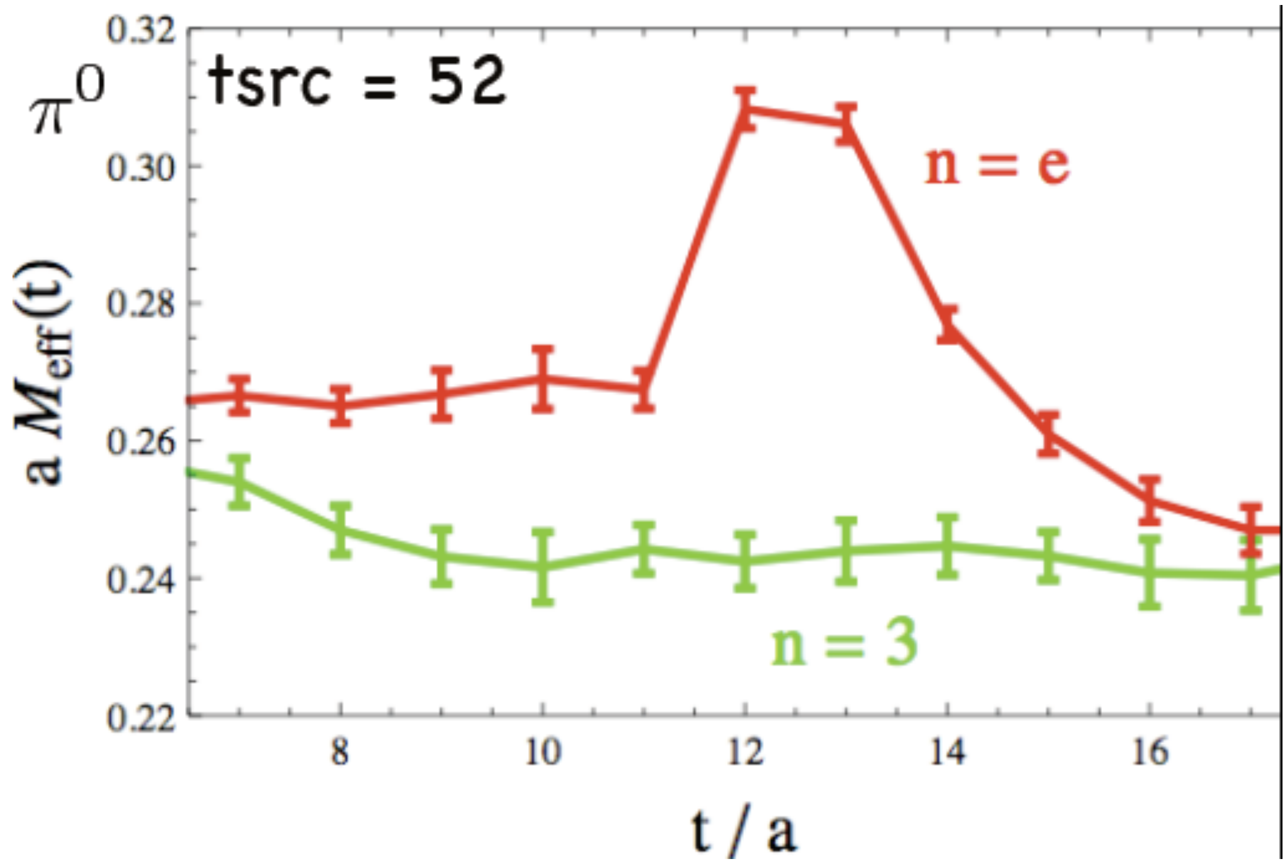
Non-quantised case

- Use non quantised field value n=e=2.71828 (Electric field)



  - Kinks in correlators
- Also problematic to use Dirichelet BCs
- Can go beyond constant field and preserve periodic nature
  [Z Davoudi, WD 2015]

Magnetic moments of nuclei

- Magnetic field in *z*-direction (quantised n)

$$U_\mu^{\rm QCD} \longrightarrow U_\mu^{\rm QCD} \cdot U_\mu^{(Q)}$$

$$U_\mu^{(Q)}(x) \;=\; e^{i\frac{6\pi Q_q \tilde{n}}{L^2} x_1 \delta_{\mu,2}} \times e^{-i\frac{6\pi Q_q \tilde{n}}{L} x_2 \delta_{\mu,1}\delta_{x_1,L-1}}$$

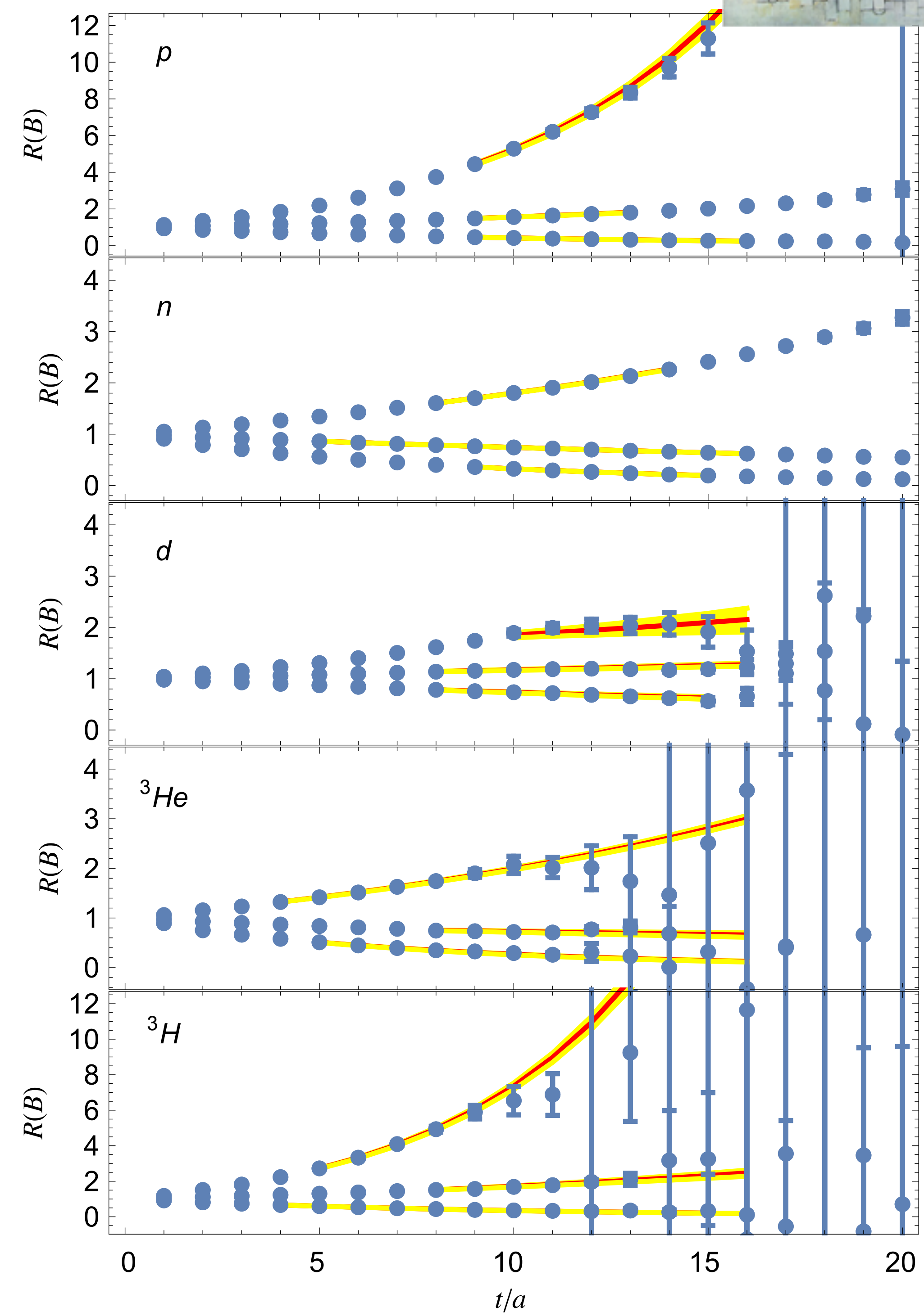
- Magnetic moments from spin splittings

$$\delta E^{(B)} \equiv E_{+j}^{(B)} - E_{-j}^{(B)} = -2\mu|\mathbf{B}| + \gamma|\mathbf{B}|^3 + \ldots$$

- Extract splittings from ratios of correlation functions

$$R(B) = \frac{C_j^{(B)}(t)\; C_{-j}^{(0)}(t)}{C_{-j}^{(B)}(t)\; C_j^{(0)}(t)} \;\xrightarrow{t\to\infty}\; Z e^{-\delta E^{(B)} t}$$

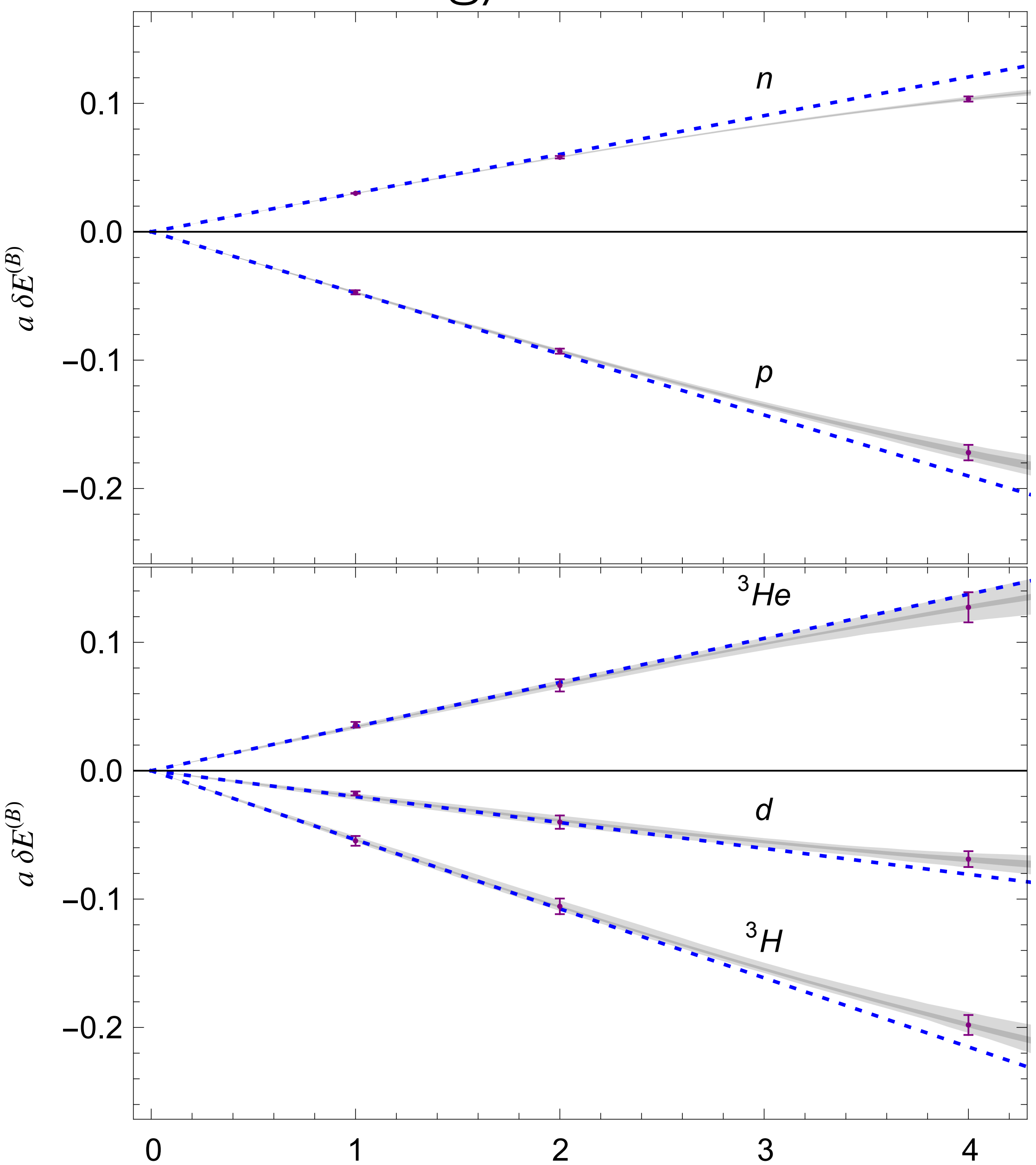
- Careful to be in single exponential region of each correlator



[NPLQCD 1409.3556]

Magnetic moments of nuclei

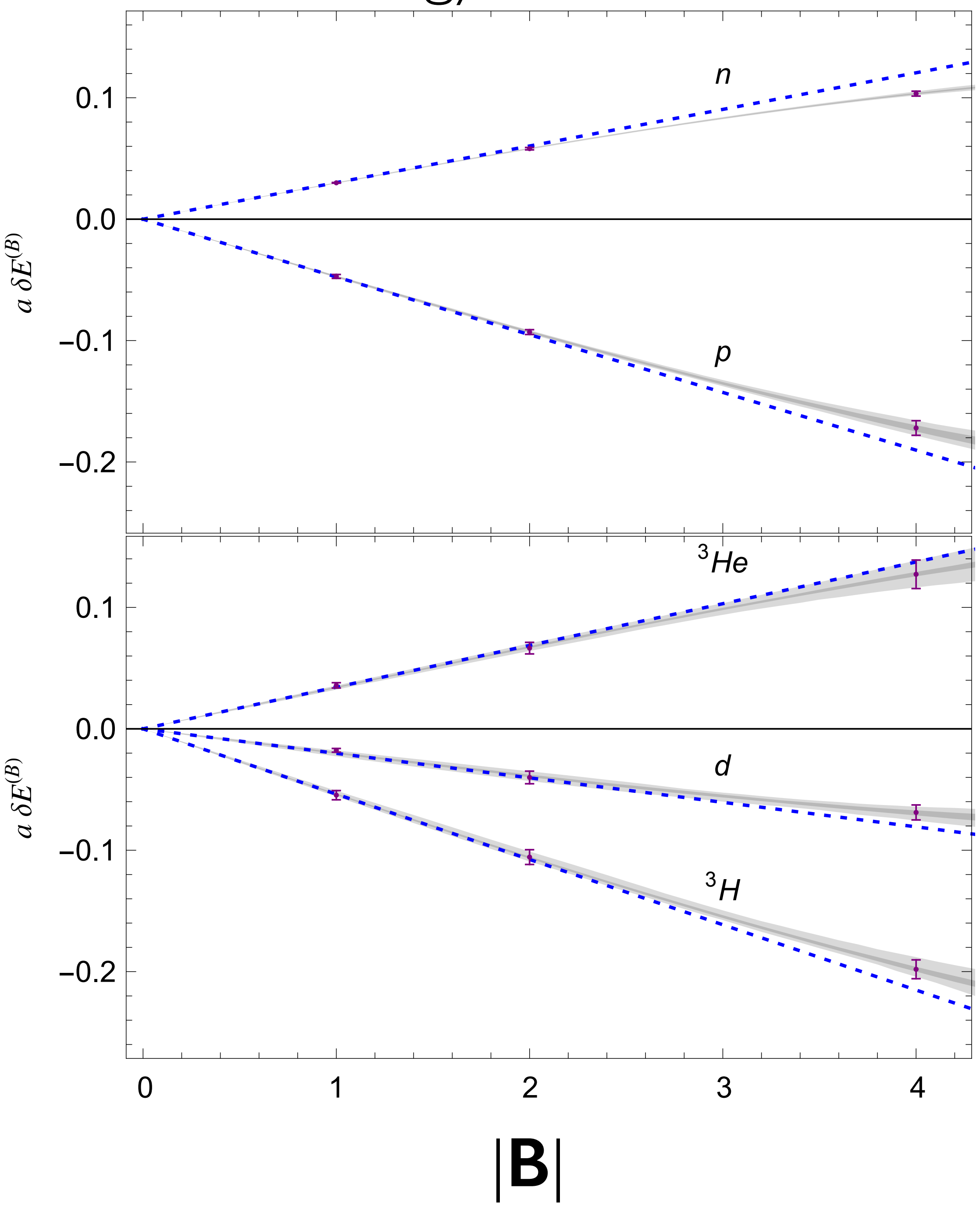
Energy shift vs B

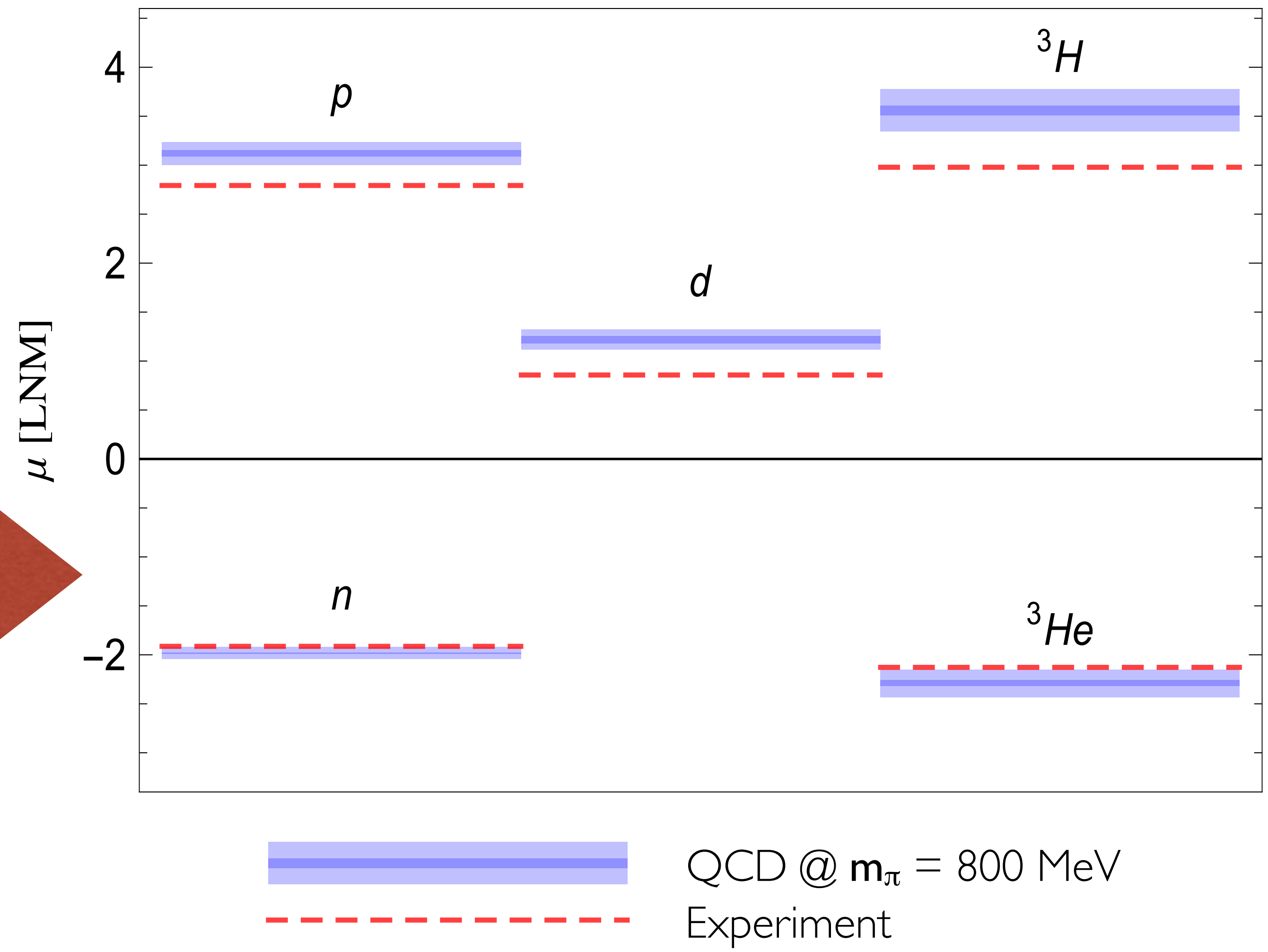


$|\mathbf{B}|$

[NPLQCD PRL 2014]

Energy shift vs B





	n	p	d	3	3
μ	-1.98(1)(2)	3.21(3)(6)	1.22(4)(9)	-2.29(3)(12)	3.56(5)(18)

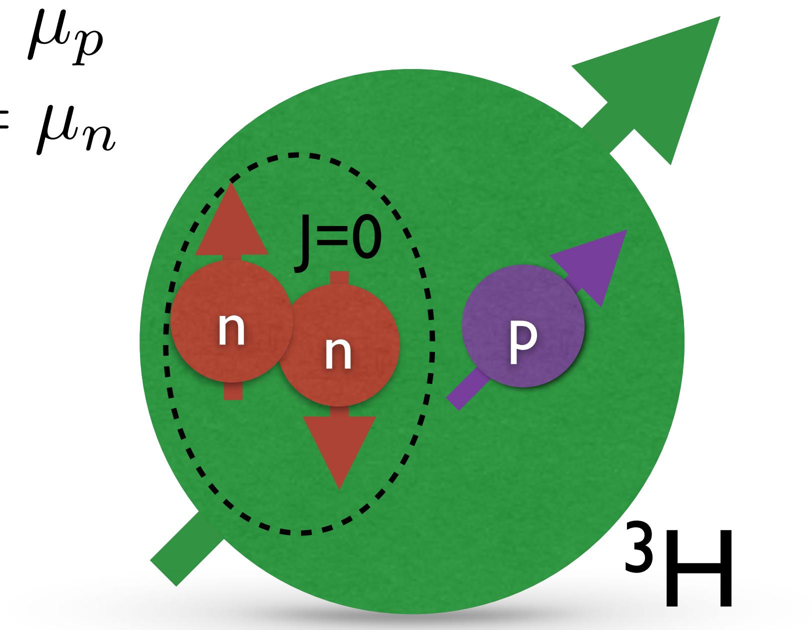
In units of appropriate nuclear magnetons (heavy M_N)

[NPLQCD PRL 2014]

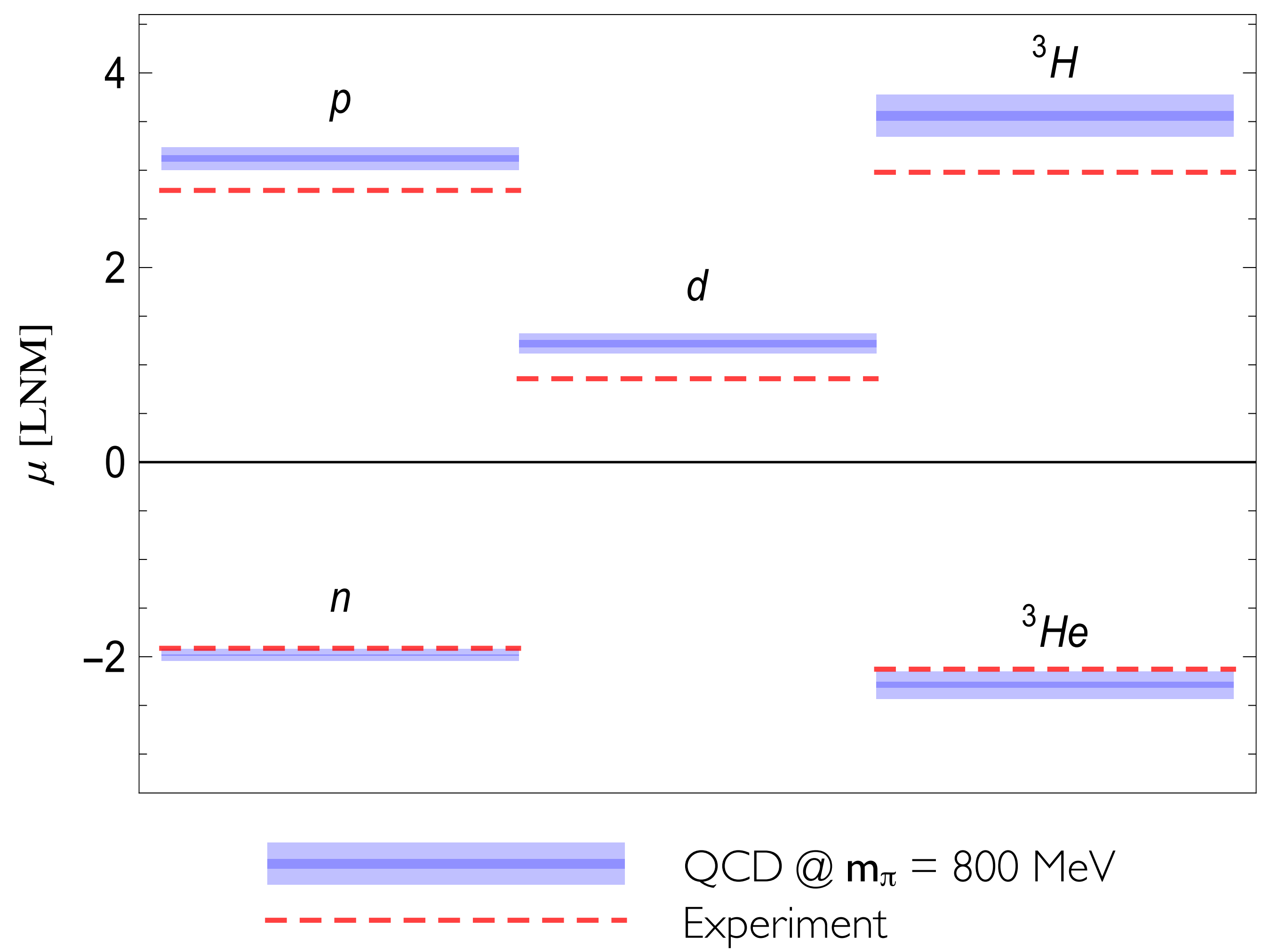
Magnetic moments of nuclei

- Numerical values are surprisingly interesting
- Shell model expectations

$$\mu_d = \mu_p + \mu_n$$
$$\mu_{^3\mathrm{H}} = \mu_p$$
$$\mu_{^3\mathrm{He}} = \mu_n$$



- Lattice results appear to suggest heavy quark nuclei are shell-model like!



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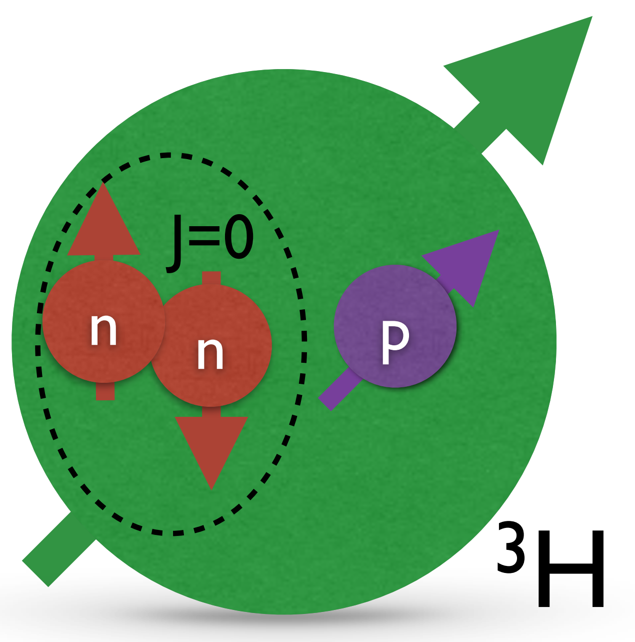

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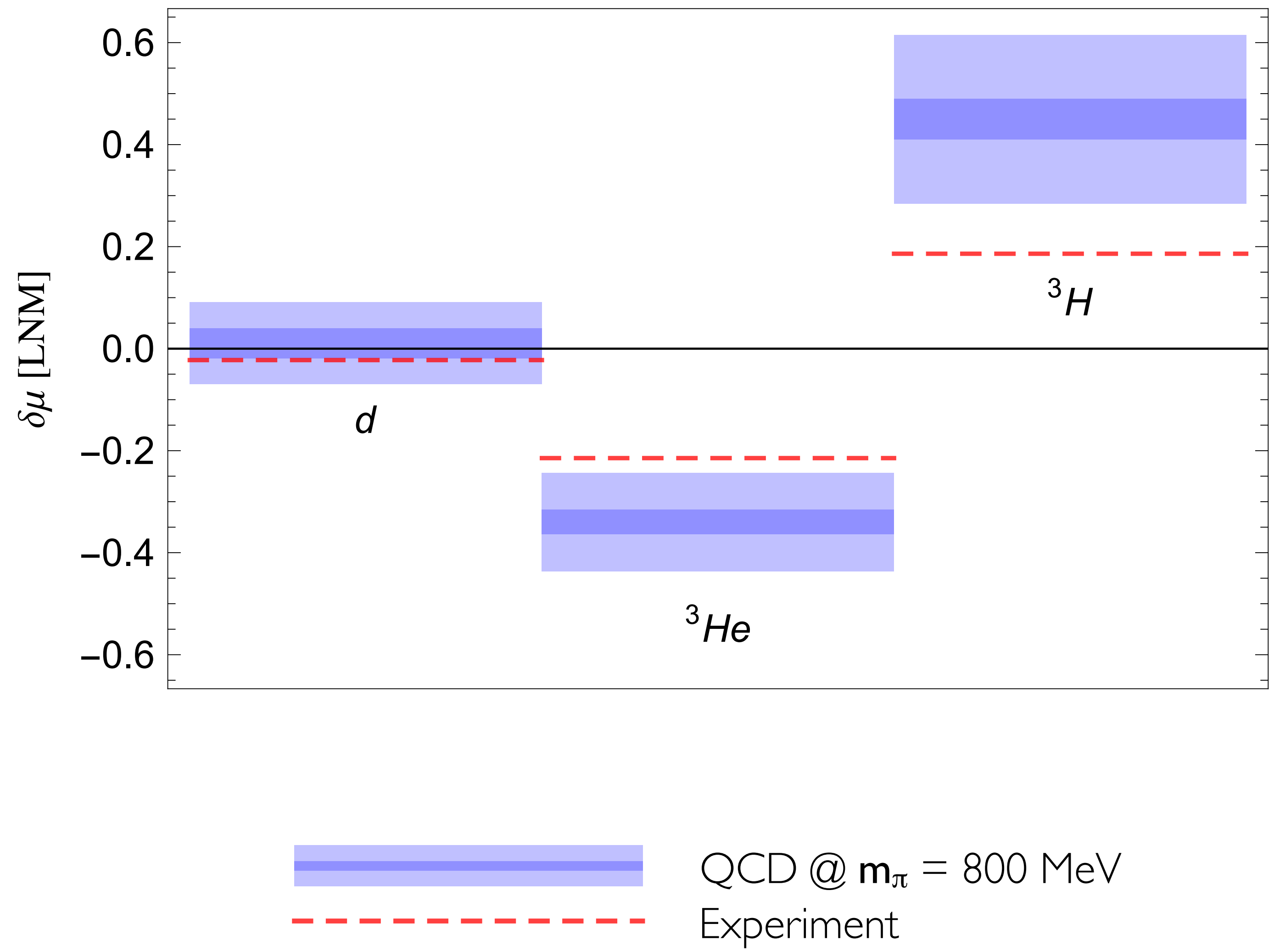


	d	3	3
δμ	0.01(3)(7)	-0.34(2)(9)	0.45(4)(16)

Difference from NSM expectation

[NPLQCD PRL 2014]

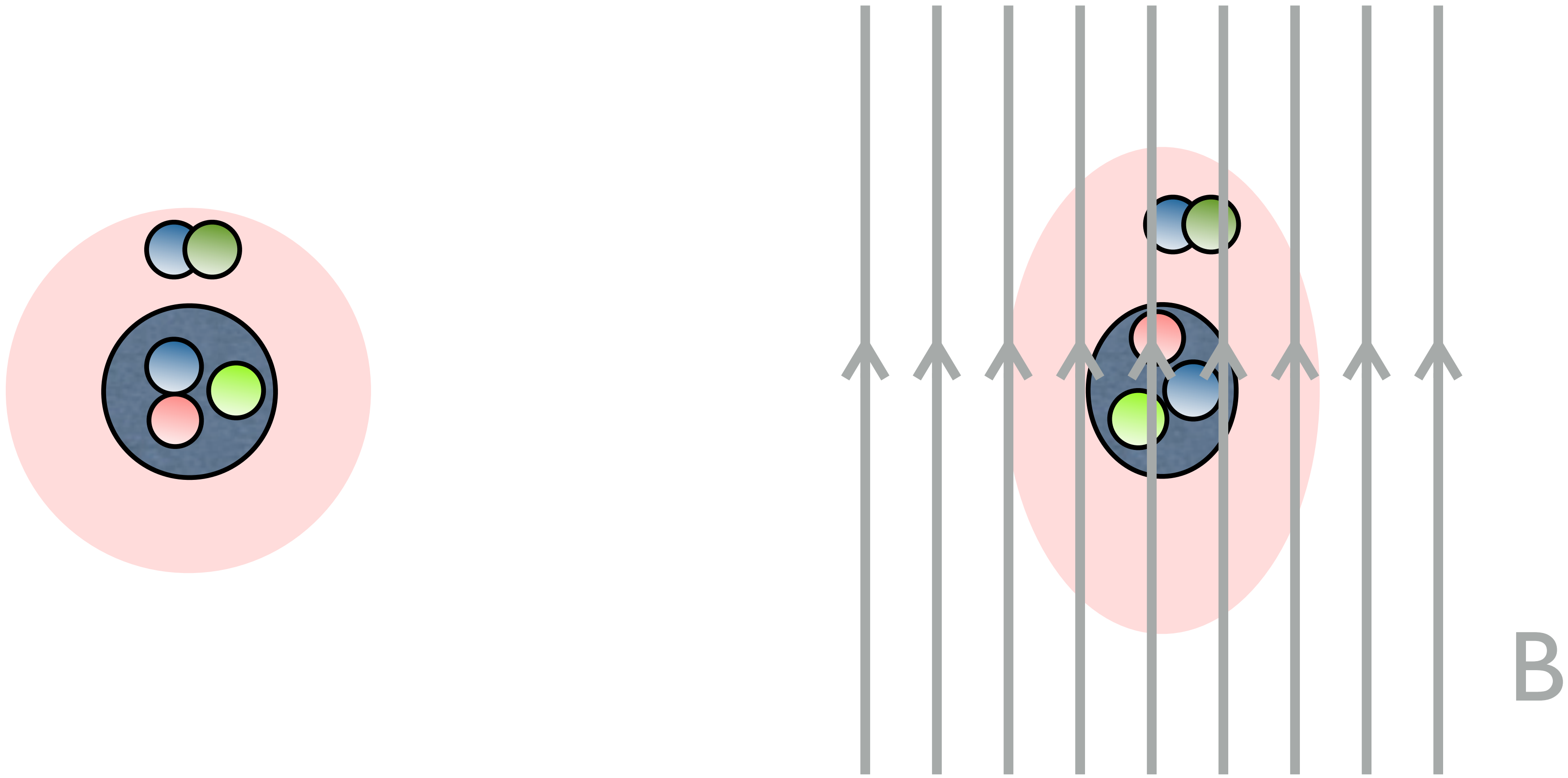
Magnetic Polarisabilities

[NPLQCD 1506.05518]

- Second order shifts determine polarisabilities

$$E_{h;j_z}(\mathbf{B}) \;=\; \sqrt{M_h^2 + (2n+1)|Q_h eB|} - \boldsymbol{\mu}_h \cdot \mathbf{B} - 2\pi\beta_h^{(M0)}|\mathbf{B}|^2 - 2\pi\beta_h^{(M2)}\langle \hat{T}_{ij} B_i B_j \rangle + \ldots$$

- Characterise deformation of state by constant magnetic field



Magnetic Polarisabilities

[NPLQCD 1506.05518]

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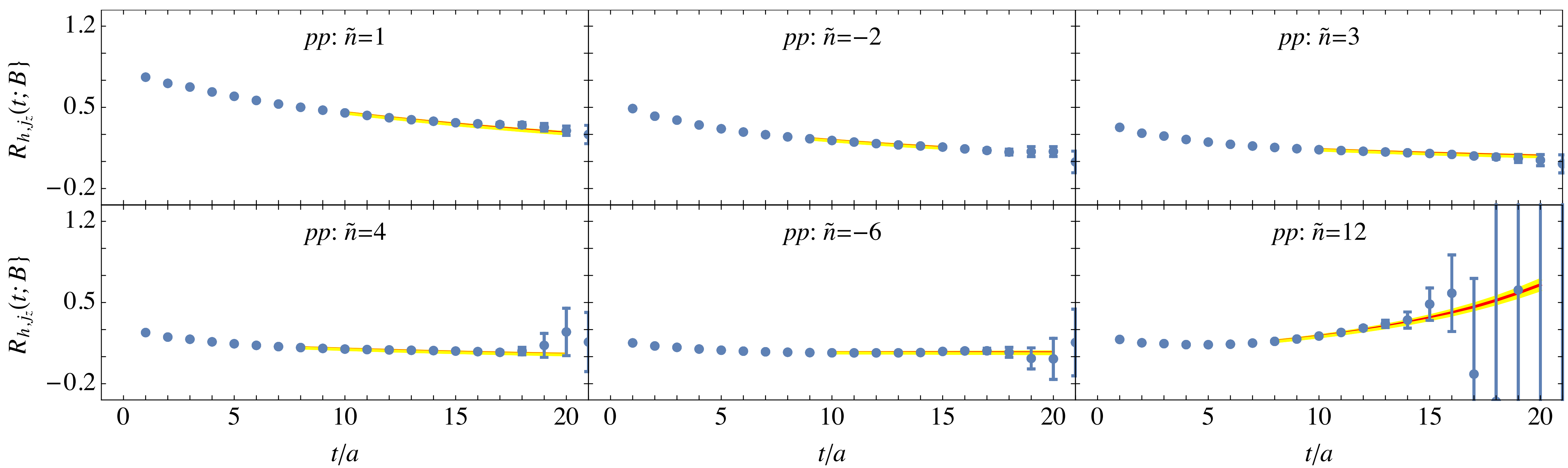
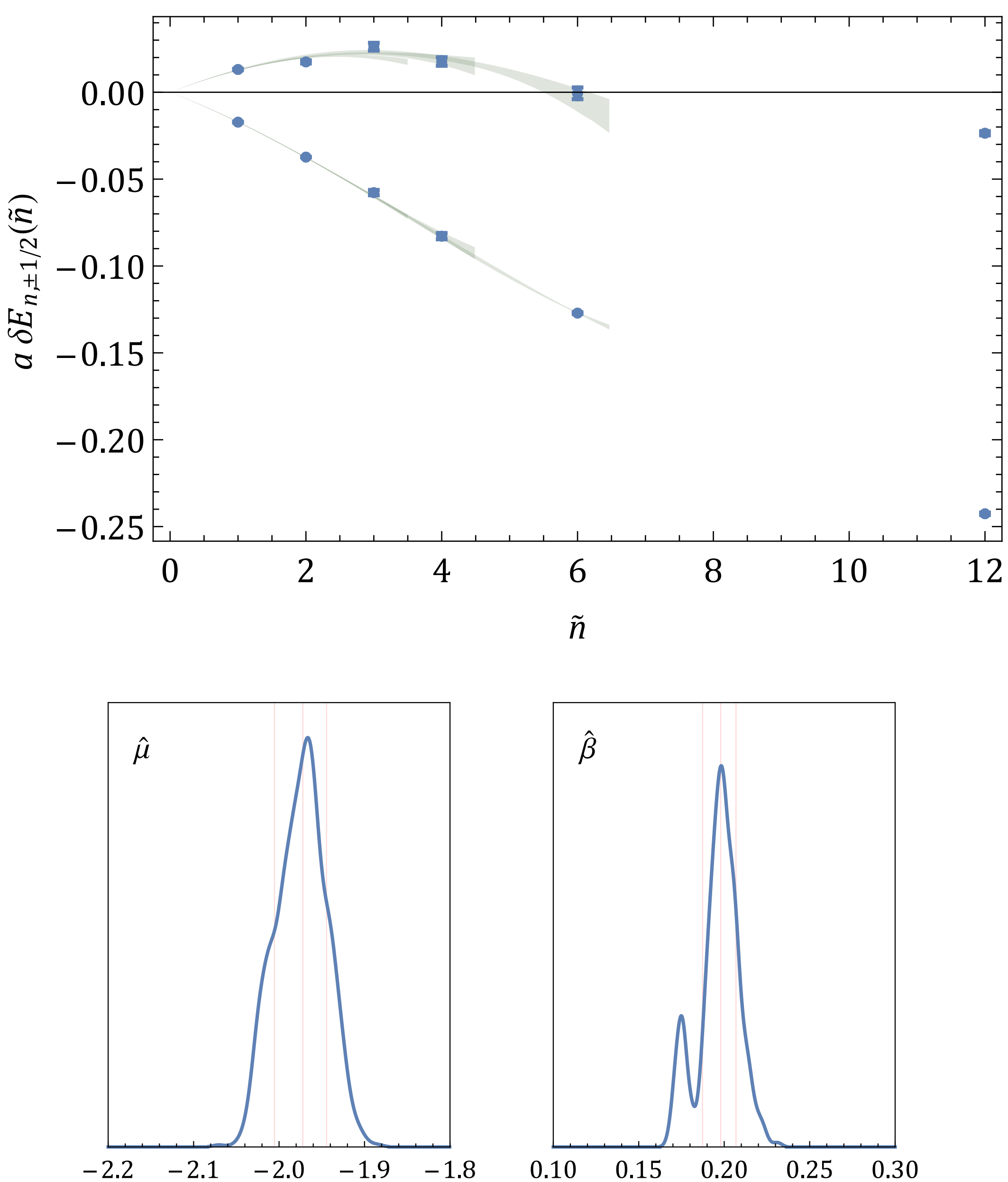
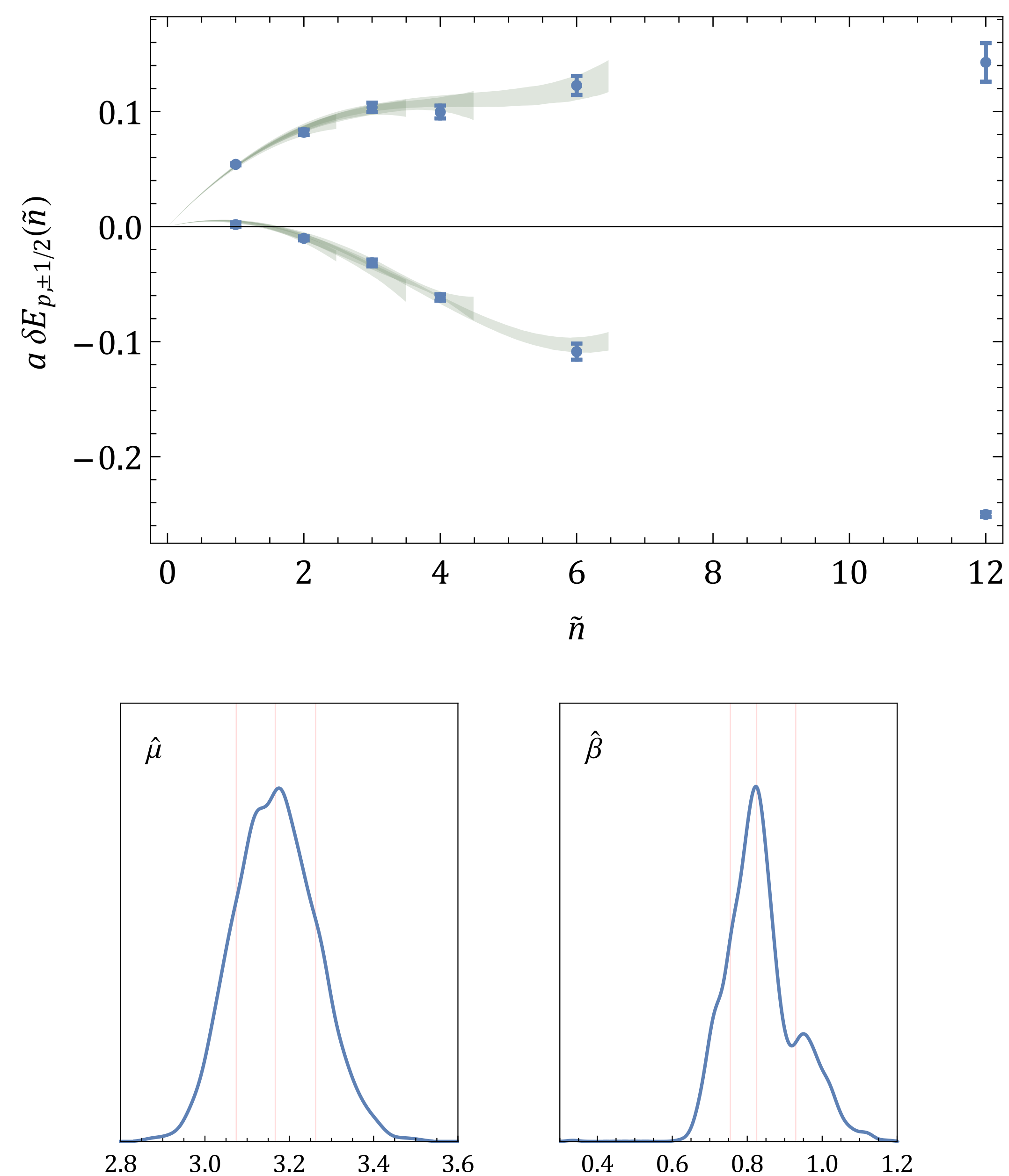
- Vary field strength to extract energy shifts
- Care required with Landau levels
  - Sources couple to strongly to n>0



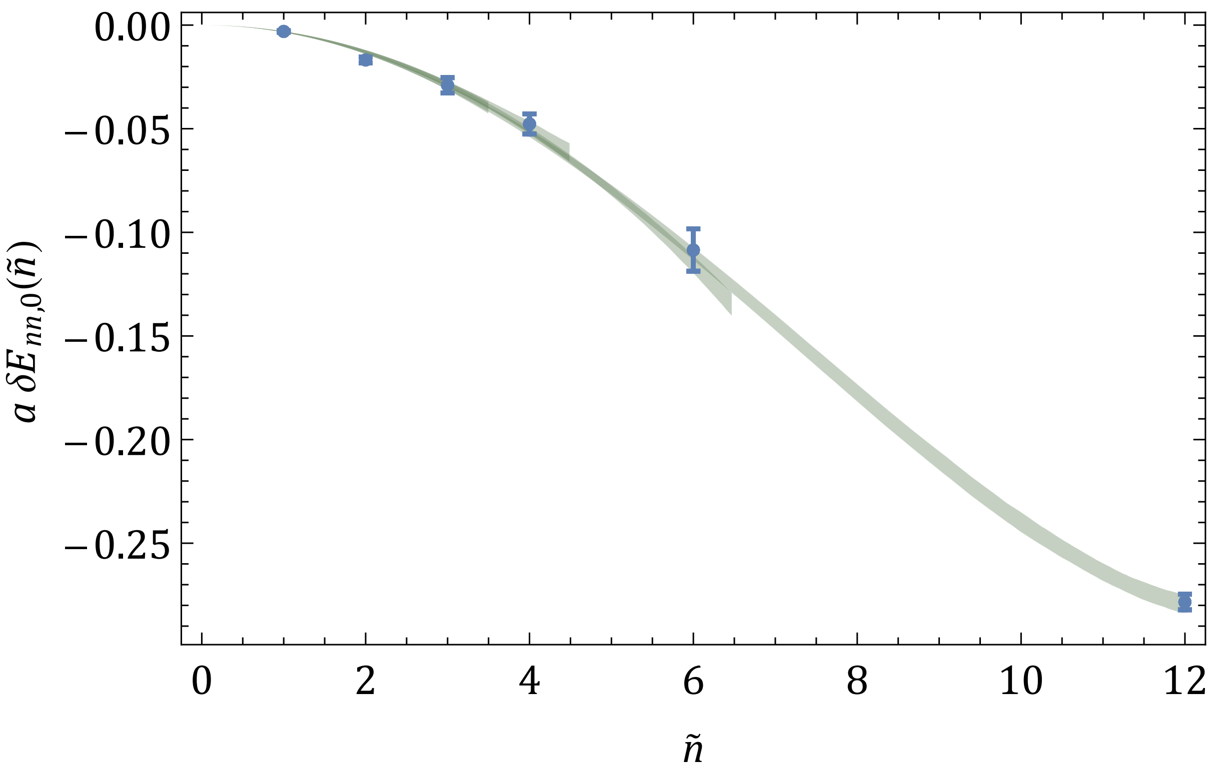
Neutron and proton

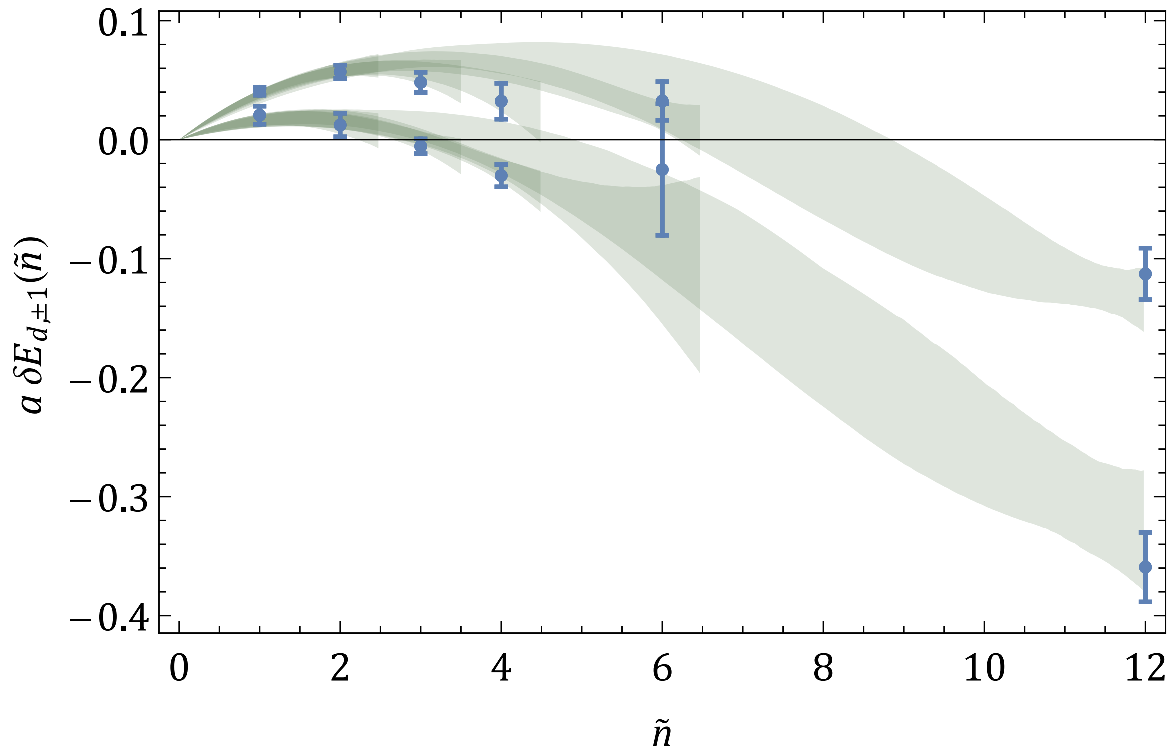
- Study how different spin states respond
- Estimate fit systematics based on multiple order polynomials

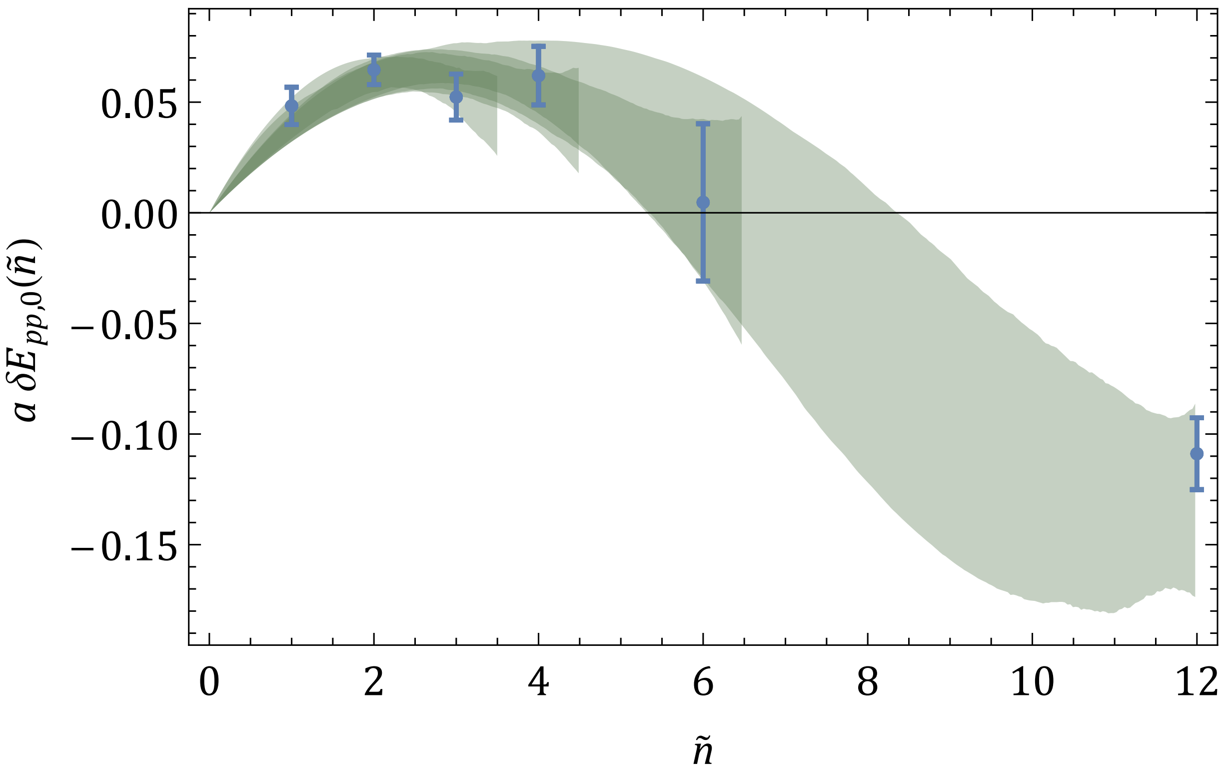




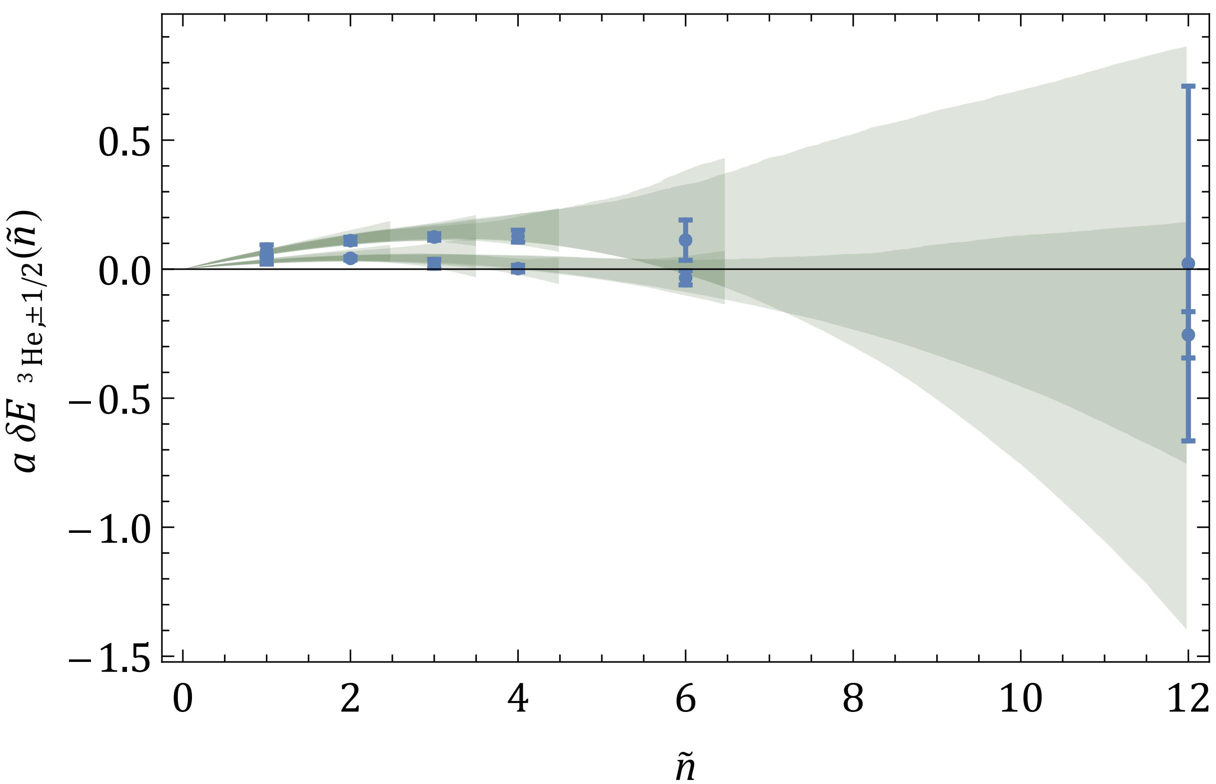
Two nucleon systems



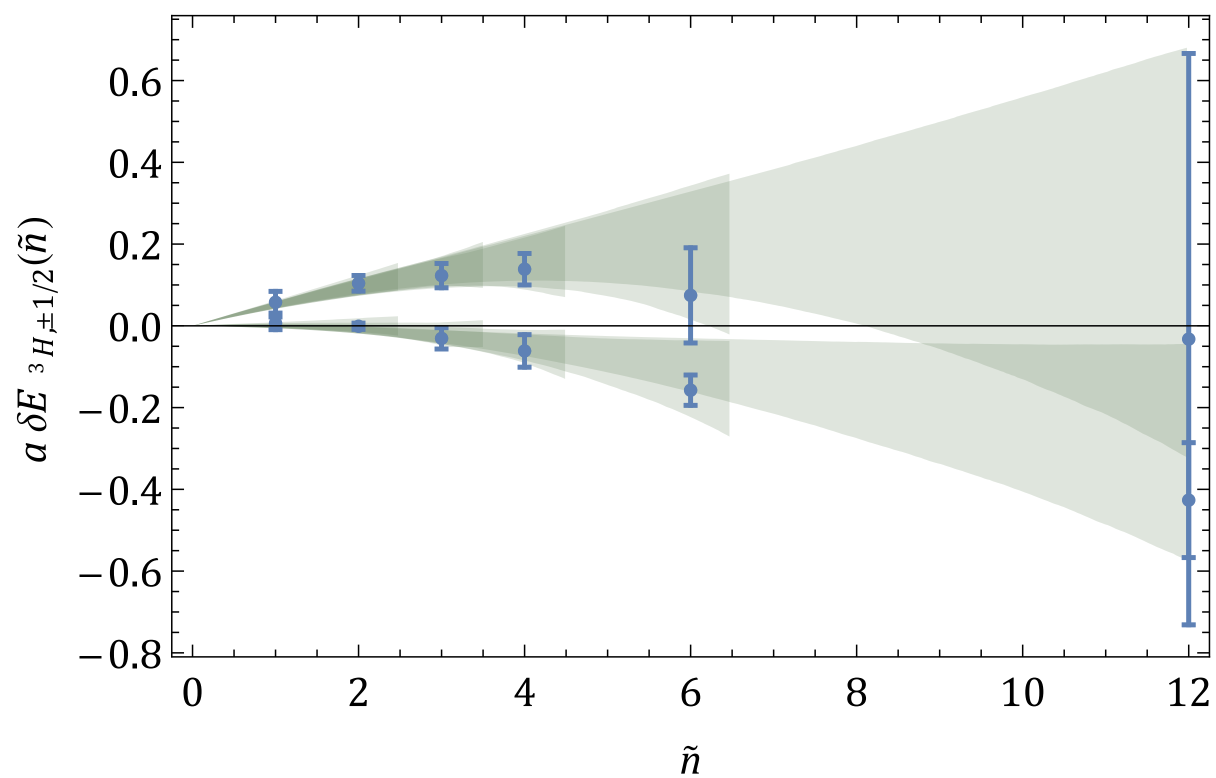




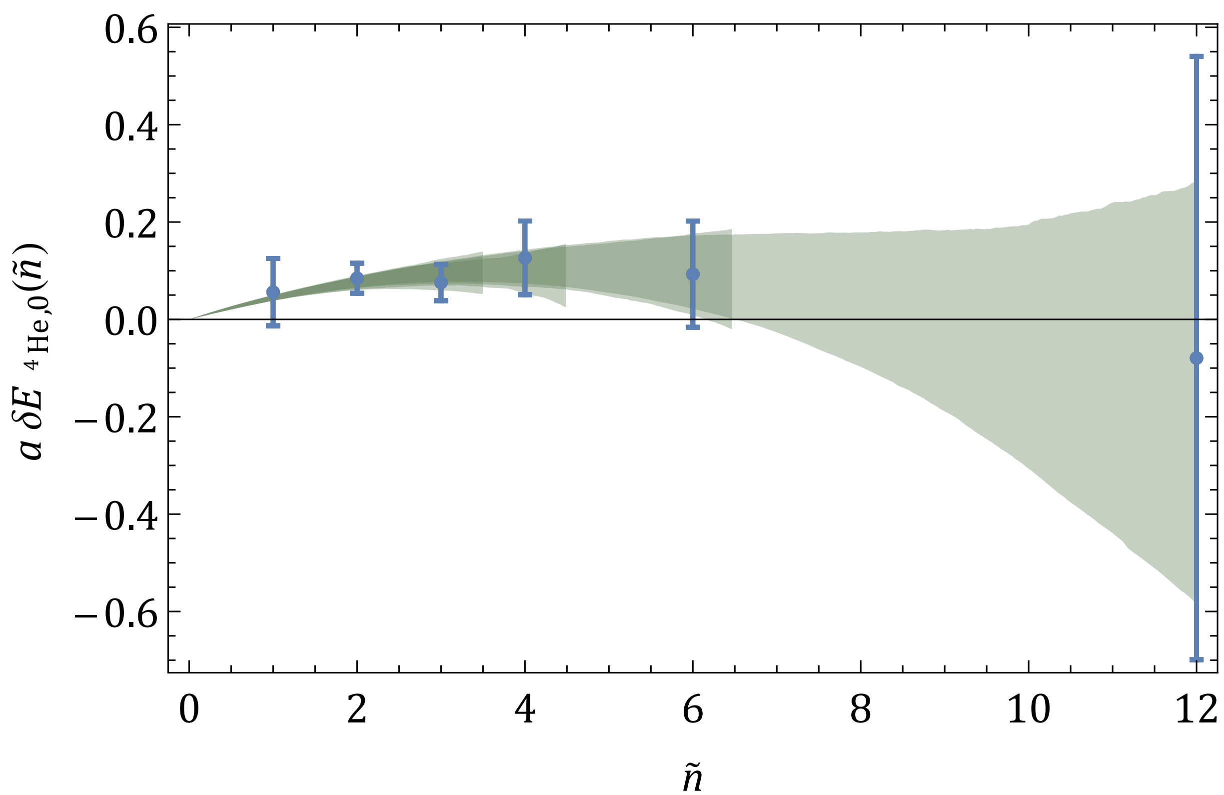
Nuclei: A=3,4



$a\,\delta E_{^3\mathrm{He},\pm 1/2}(\tilde{n})$
0.5
0.0
−0.5
−1.0
−1.5
0
2
4
6
8
10
12
$\tilde{n}$



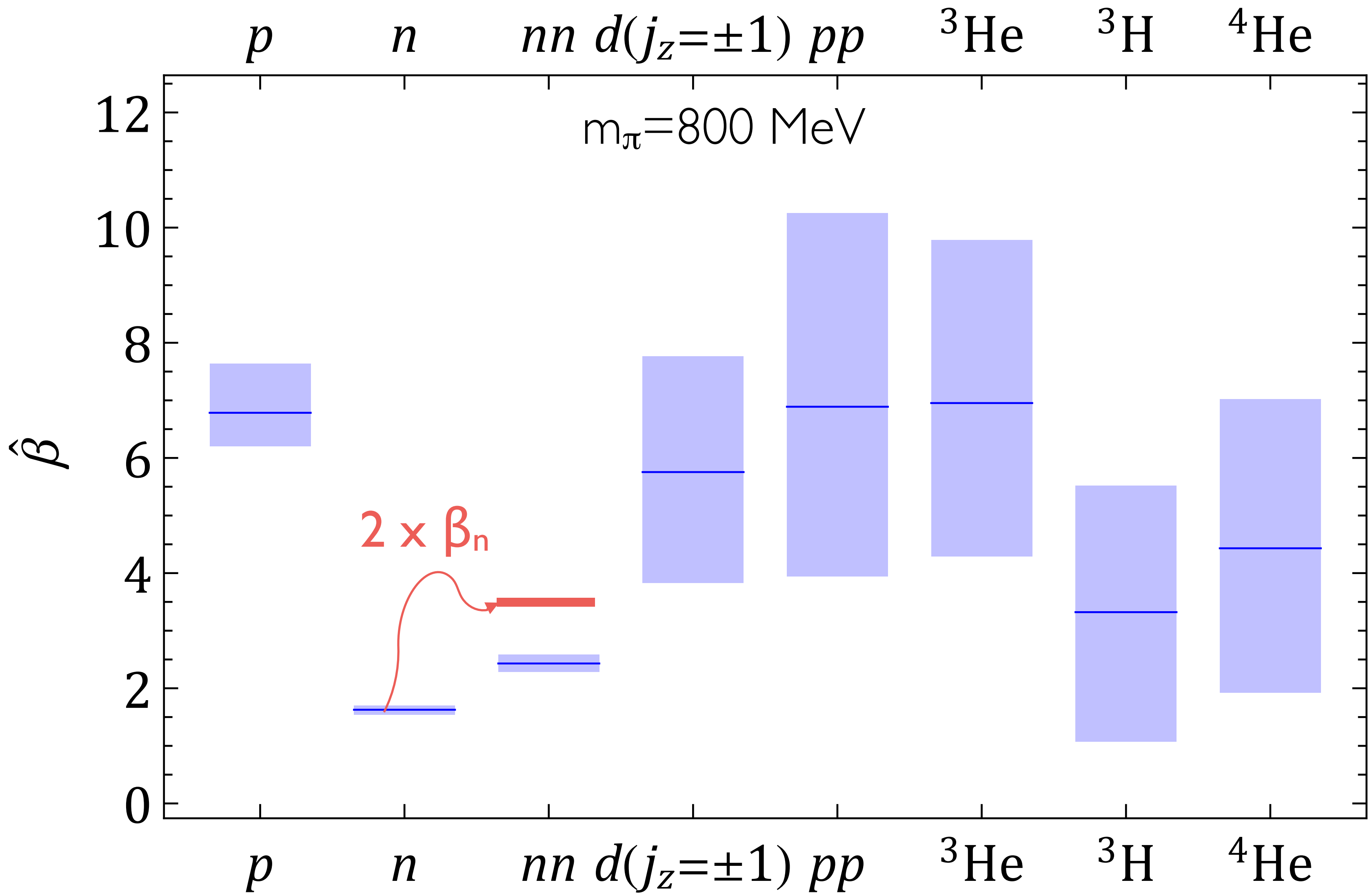
$a\,\delta E_{^3H,\pm 1/2}(\tilde{n})$
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
0
2
4
6
8
10
12
$\tilde{n}$



$a\,\delta E_{^4\mathrm{He},0}(\tilde{n})$
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0
2
4
6
8
10
12
$\tilde{n}$

Magnetic Polarisabilities

[NPLQCD 1506.05518]

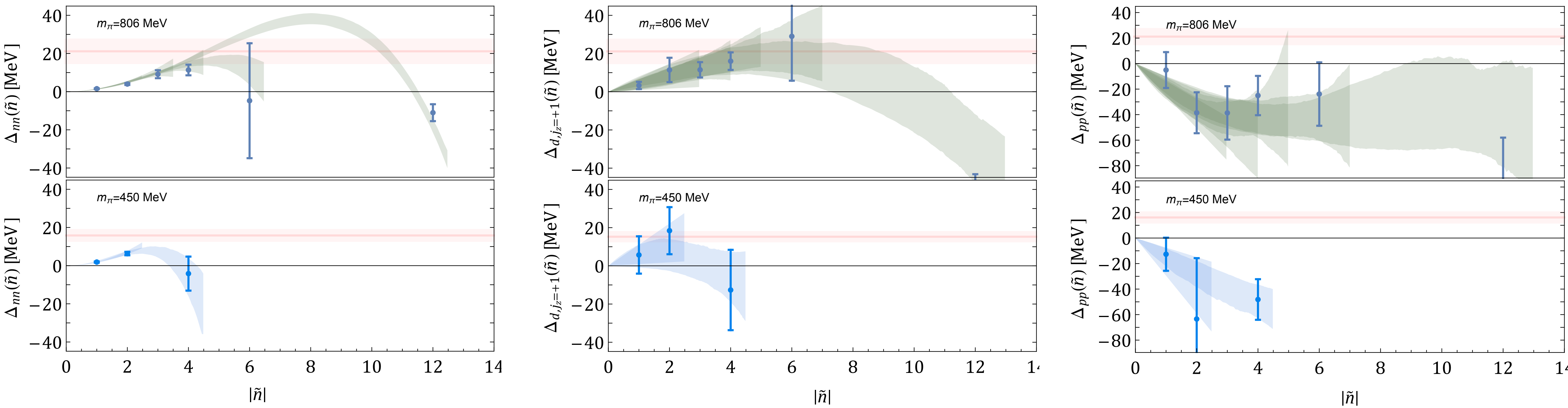


- Significant isovector nucleon polarisability
- Unambiguous two-body physics
- Nuclear polarisabilities similar to nucleon polarisabilities

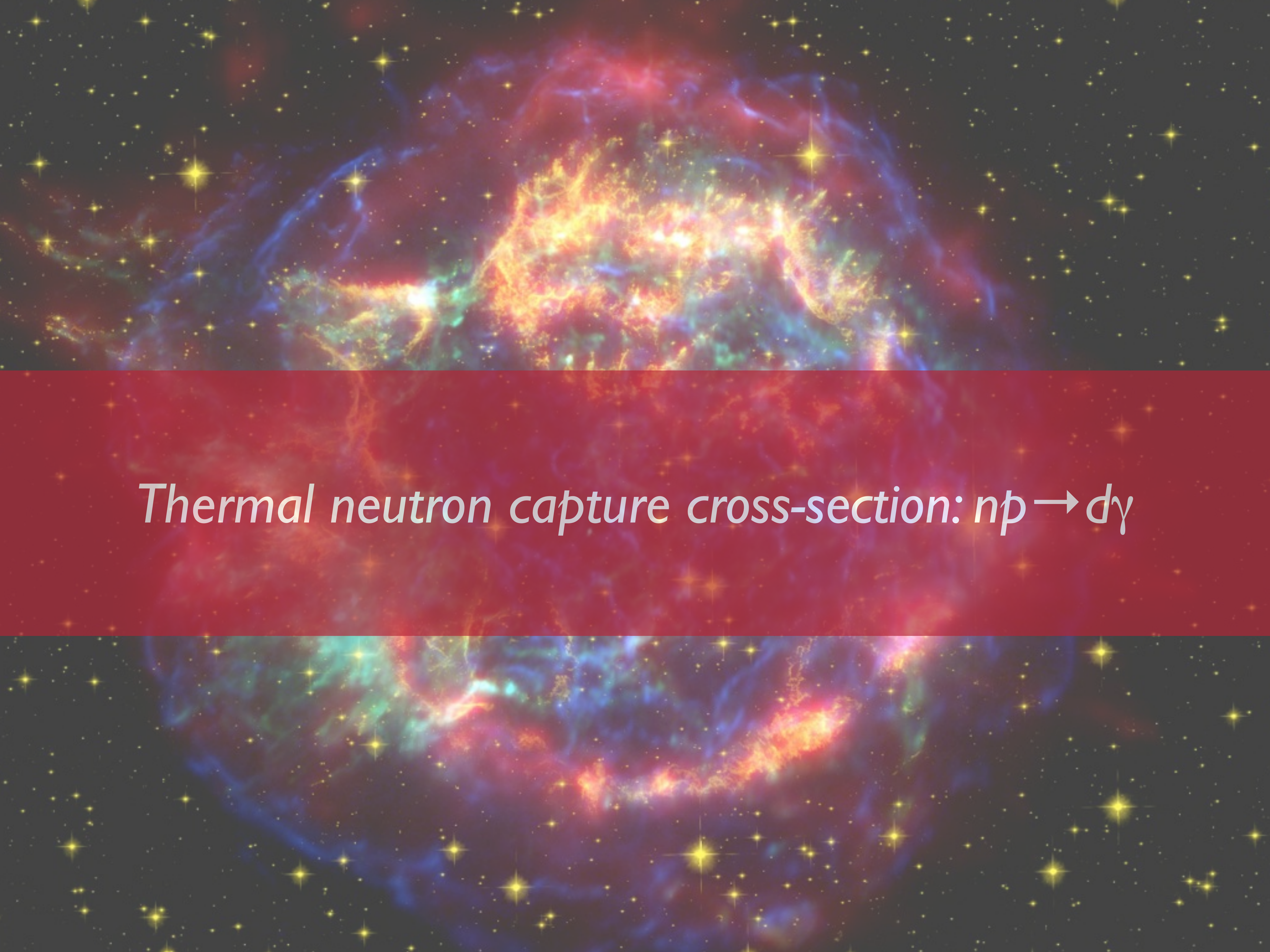
Feshbach resonances in nuclei?

[NPLQCD 1508.05884]

- Strong magnetic fields shift energies significantly compared to binding



- Appearance of Feshbach resonances where system binds/unbinds at large B (B~10^{19} Gauss)
  - Potentially persists at physical quark masses
  - Possible consequences in n-stars, ultra-peripheral HIC

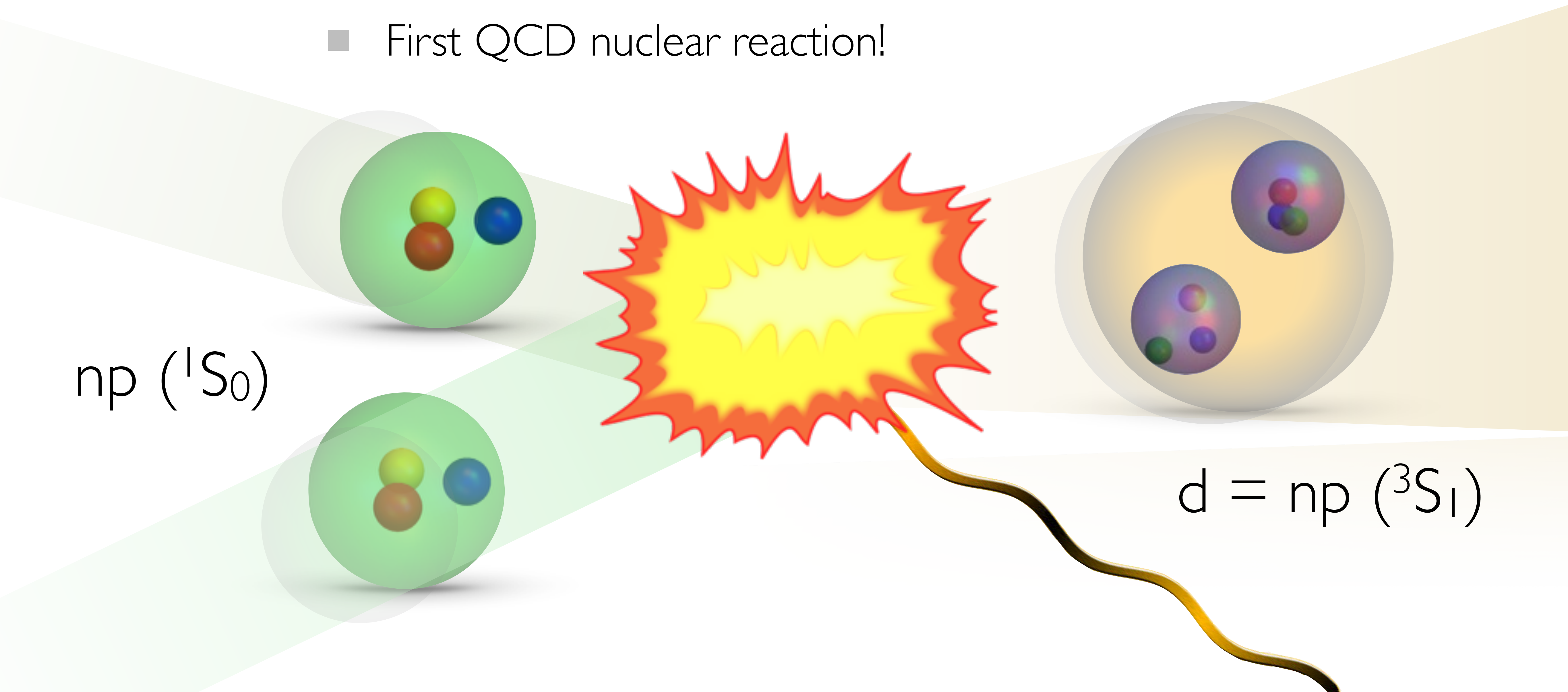


Thermal neutron capture cross-section: $np \to d\gamma$

Thermal Neutron Capture Cross-Section

[NPLQCD PRL 115, 132001 (2015)]

- Thermal neutron capture cross-section: $np \rightarrow d\gamma$
  - Critical process in Big Bang Nucleosynthesis
  - Historically important: 2-body contributions ~10%
  - First QCD nuclear reaction!



np→dγ in pionless EFT

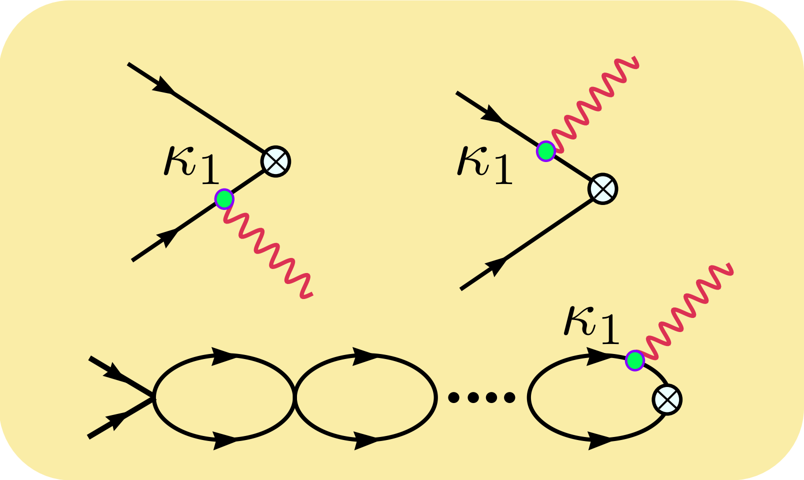
$$Z_d = 1/\sqrt{1-\gamma_0 r_3}$$

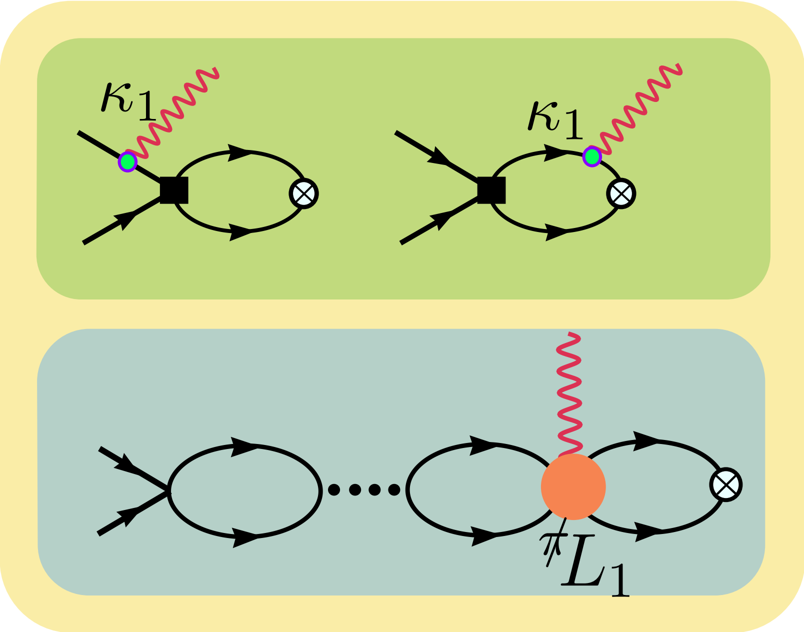
- Cross-section at threshold calculated in pionless EFT

$$\sigma(np \to d\gamma) = \frac{e^2(\gamma_0^2+|\mathbf{p}|^2)^3}{M^4\gamma_0^3|\mathbf{p}|}|\tilde{X}_{M1}|^2 + \ldots$$

- EFT expansion at LO given by mag. moments NLO contributions from short-distance two nucleon operators

$$\tilde{X}_{M1} = \frac{Z_d}{-\frac{1}{a_1}+\frac{1}{2}r_1|\mathbf{p}|^2 - i|\mathbf{p}|} \times \left[\frac{\kappa_1\gamma_0^2}{\gamma_0^2+|\mathbf{p}|^2}\left(\gamma_0 - \frac{1}{a_1}+\frac{1}{2}r_1|\mathbf{p}|^2\right)+\frac{\gamma_0^2}{2}l_1\right]$$



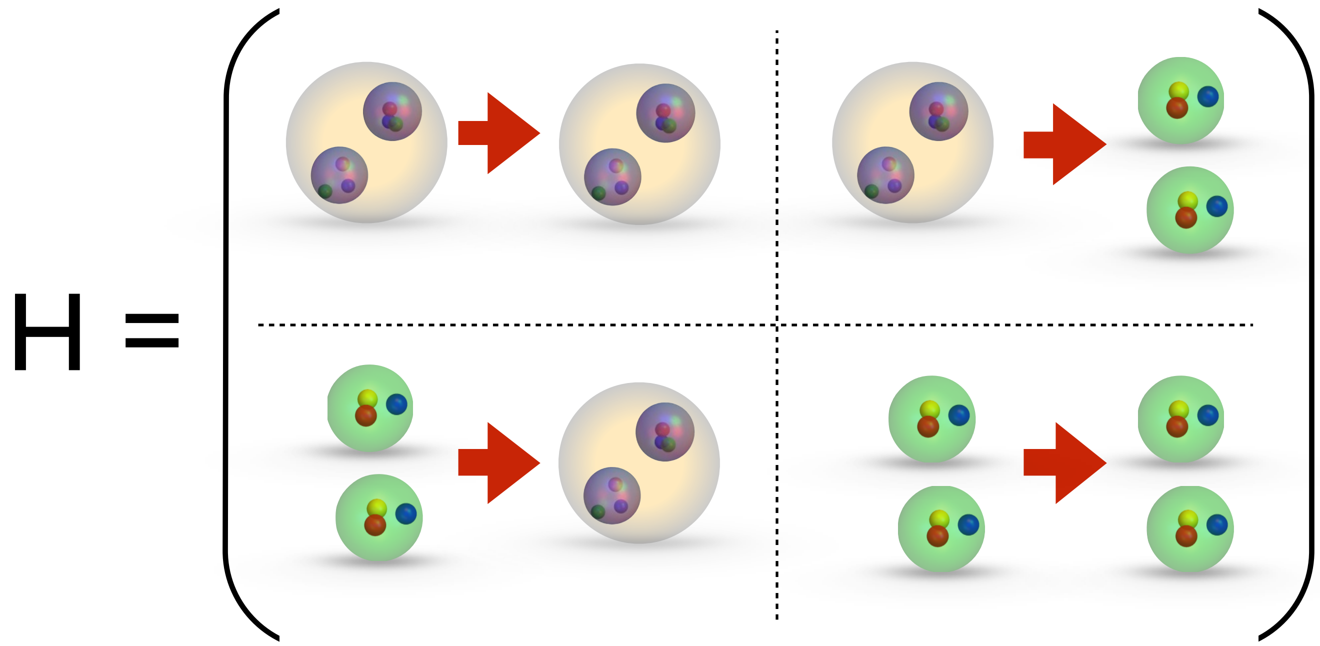


- Phenomenological description with 1% accuracy for E< 1MeV
  - Short distance (MEC) contributes ~10%

MECs: Riska, Phys.Lett. B38 (1972) 193
Hokert et al, Nucl.Phys. A217 (1973) 14
EFT: Chen et al.,Nucl.Phys. A653 (1999) 386
Chen et al, Phys.Lett. B464 (1999) 1
Rupak Nucl.Phys. A678 (2000) 405

[NPLQCD PRL 115, 132001 (2015)]

- Presence of magnetic field mixes $I_z=J_z=0$ $^3S_1$ and $^1S_0$ *np* systems



- Wigner SU(4) super-multiplet (spin-flavour) symmetry relates $^3S_1$ and $^1S_0$ states (diagonal elements approximately equal)
  - Shift of eigenvalues determined by transition amplitude

$$\Delta E_{^3S_1,^1S_0} = \mp \left(\kappa_1 + \overline{L}_1\right) \frac{eB}{M} + \ldots$$

- More generally eigenvalues depend on transition amplitude [WD, & M Savage 2004, H Meyer 2012]

np→dγ

[NPLQCD PRL 115, 132001 (2015)]

- Iz=Jz=0 correlation matrix

Lattice correlator with $^3S_1$ source and $^1S_0$ sink

$$\mathbf{C}(t;\mathbf{B}) \;=\; \begin{pmatrix} C_{^3S_1,^3S_1}(t;\mathbf{B}) & C_{^3S_1,^1S_0}(t;\mathbf{B}) \\ C_{^1S_0,^3S_1}(t;\mathbf{B}) & C_{^1S_0,^1S_0}(t;\mathbf{B}) \end{pmatrix}$$

- Generalised eigenvalue problem

$$[\mathbf{C}(t_0;\mathbf{B})]^{-1/2}\mathbf{C}(t;\mathbf{B})[\mathbf{C}(t_0;\mathbf{B})]^{-1/2}v = \lambda(t;\mathbf{B})v$$

- Ratio of correlator ratios to extract 2-body

$$R_{^3S_1,^1S_0}(t;\mathbf{B}) = \frac{\lambda_+(t;\mathbf{B})}{\lambda_-(t;\mathbf{B})} \xrightarrow{t\to\infty} \hat{Z}\exp\left[2\;\Delta E_{^3S_1,^1S_0}t\right]$$

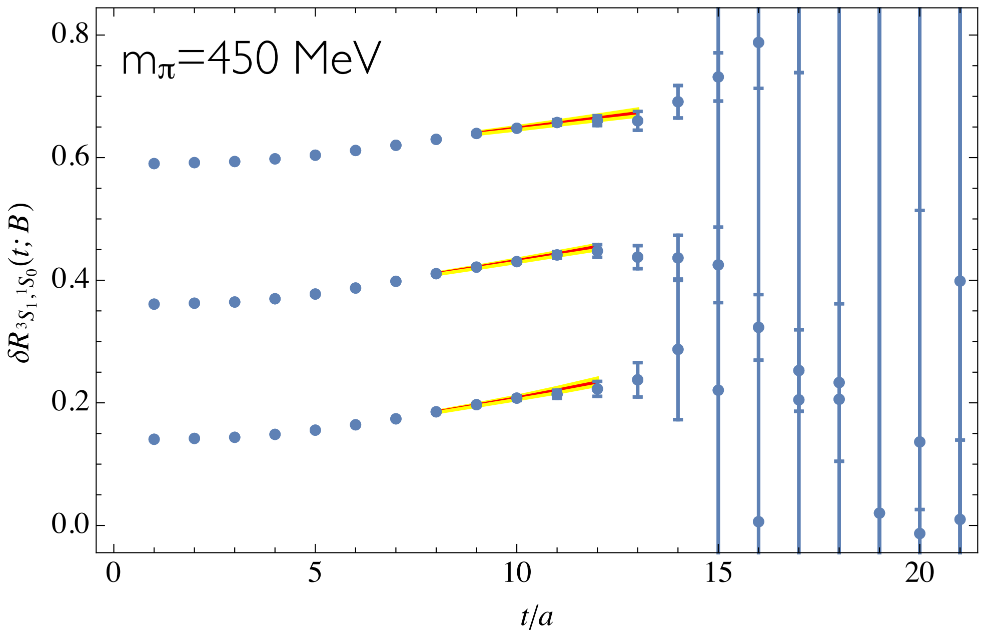
$$\delta R_{^3S_1,^1S_0}(t;\mathbf{B}) = \frac{R_{^3S_1,^1S_0}(t;\mathbf{B})}{\Delta R_p(t;\mathbf{B})/\Delta R_n(t;\mathbf{B})} \;\to A\; e^{-\delta E_{^3S_1,^1S_0}(\mathbf{B})t}$$

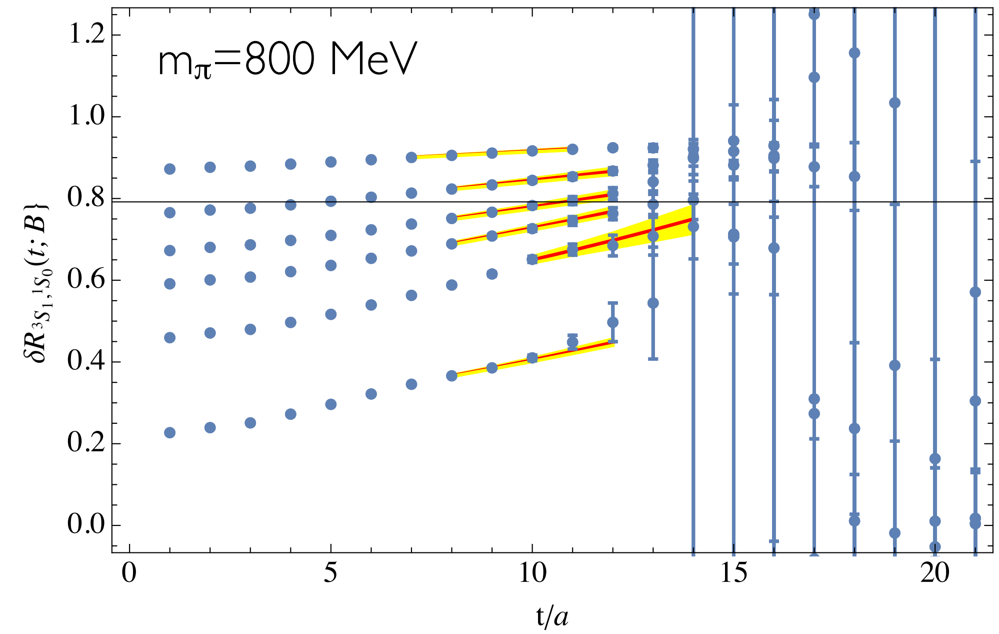
$$\begin{aligned}\delta E_{^3S_1,^1S_0} \;&\equiv\; \Delta E_{^3S_1,^1S_0} - [E_{p,\uparrow} - E_{p,\downarrow}] + [E_{n,\uparrow} - E_{n,\downarrow}] \\ &\to\; 2\overline{L}_1|e\mathbf{B}|/M + \mathcal{O}(\mathbf{B}^2)\end{aligned}$$

np→dγ

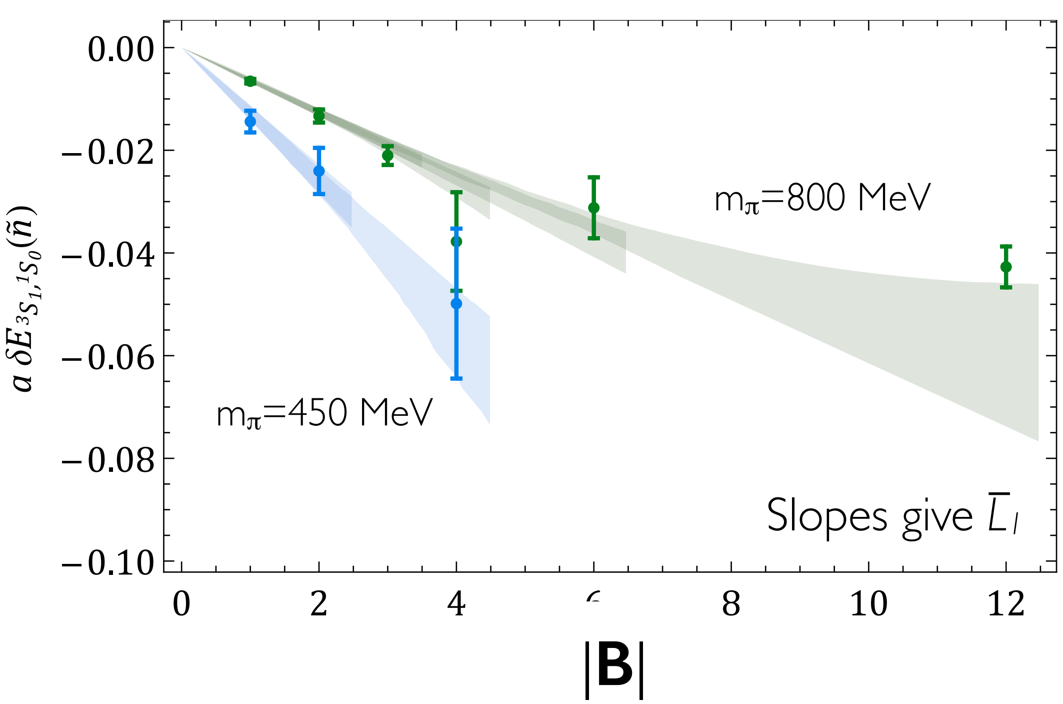
[NPLQCD PRL 115, 132001 (2015)]

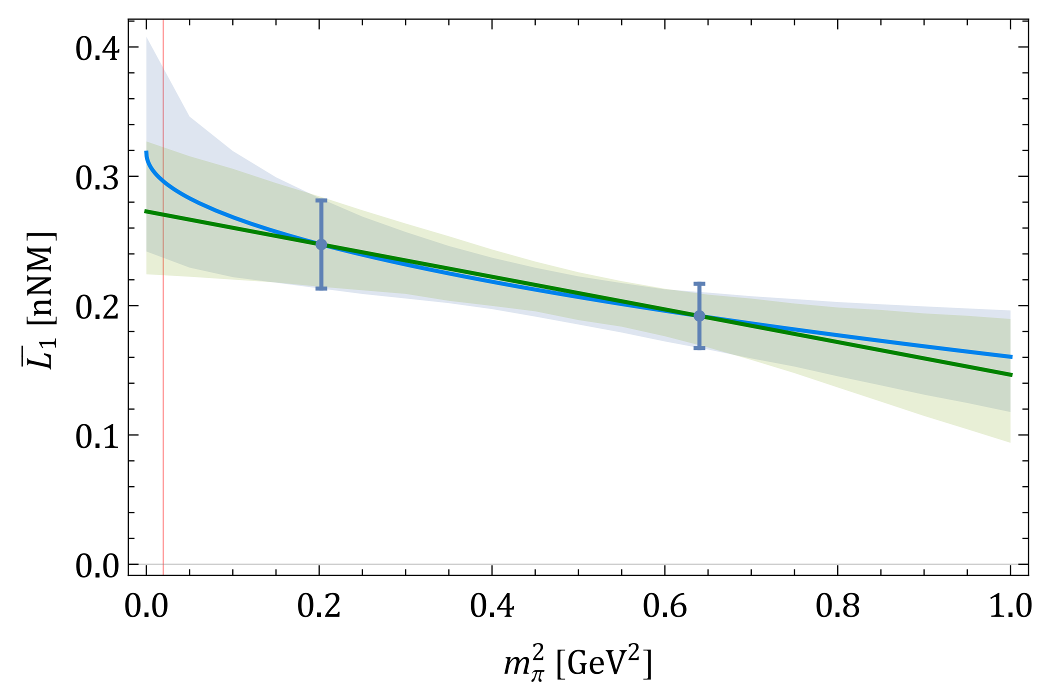
- Correlator ratios for different field strengths





- Field strength & mass dependence





$np \to d\gamma$

[NPLQCD PRL 115, 132001 (2015)]

- Extracted short-distance contribution at physical mass

$$\overline{L}_1^{\rm lqcd} = 0.285\left({}^{+63}_{-60} \right)\ {\rm nNM} \qquad\qquad l_1^{\rm lqcd} = -4.48\left({}^{+16}_{-15} \right)\ {\rm fm}$$

- Combine with phenomenological nucleon magnetic moment, scattering parameters at incident neutron velocity v=2,200 m/s

$$\sigma^{\rm lqcd}(np \to d\gamma) = 307.8(1 + 0.273\ \overline{L}_1^{\rm lqcd})\ {\rm mb}$$

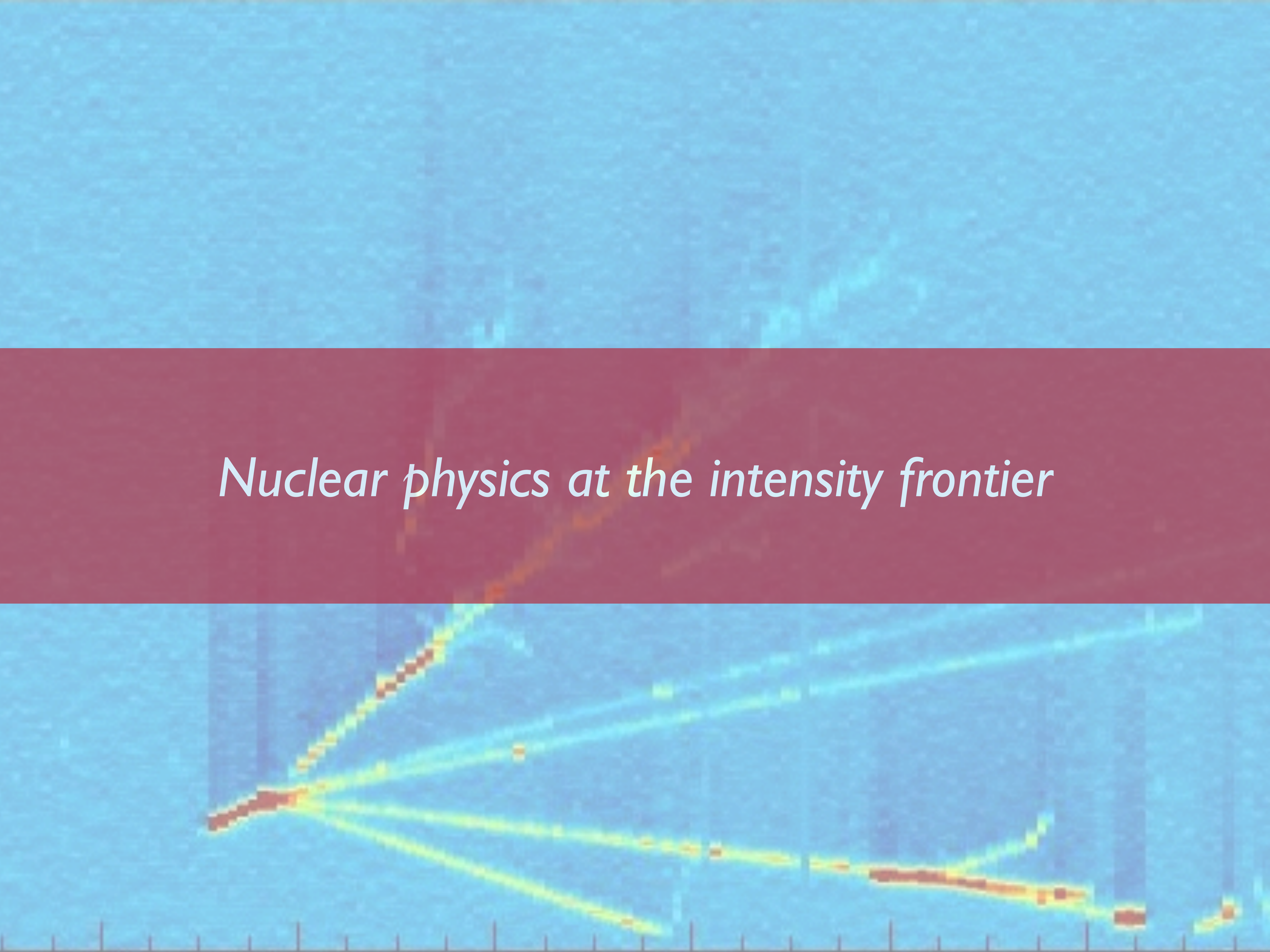
$$\sigma^{\rm lqcd}(np \to d\gamma) = 332.4\left({}^{+5.4}_{-4.7} \right)\ {\rm mb}$$

c.f. phenomenological value

$$\sigma^{\rm expt}(np \to d\gamma) = 334.2(0.5)\ {\rm mb}$$

- NB: at m_π=800 MeV, use LQCD for all inputs (ab initio)

$$\sigma^{800\ \rm MeV}(np \to d\gamma) \sim 10\ {\rm mb}$$



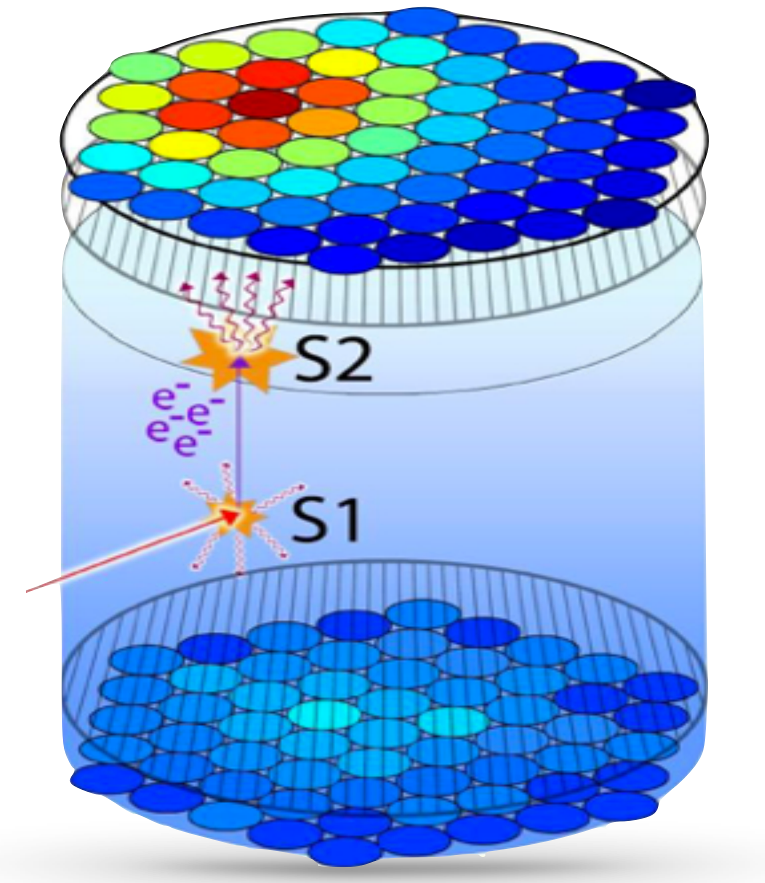
Nuclear physics at the intensity frontier

The intensity frontier

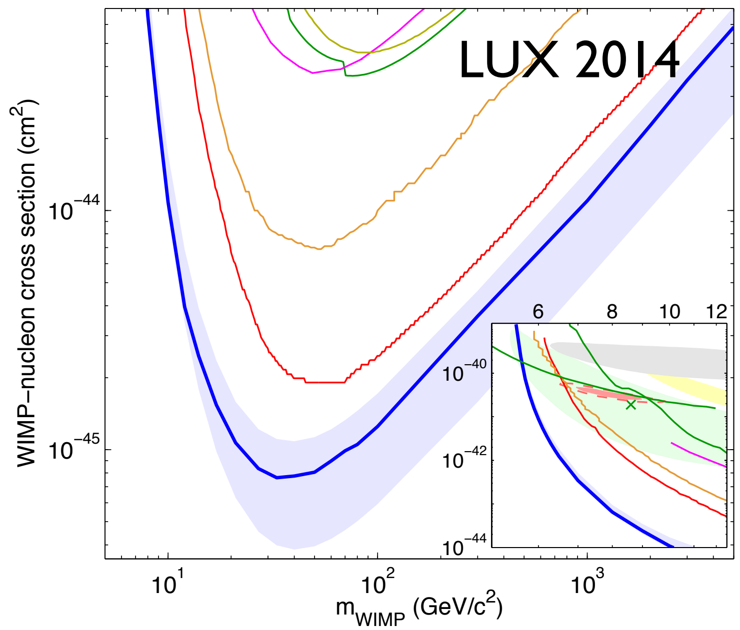
- Seek new physics through quantum effects
- Precise experiments
  - Sensitivity to probe the rarest interactions of the SM
  - Look for effects where there is no SM contribution
- Important focus of HEP(NP) experimental program
  - Dark matter direct detection
  - Neutrino physics
  - Charged lepton flavour violation, EDMs, proton decay, neutron-antineutron oscillations…
- ***Major component is nuclear targets***

The intensity frontier

- Dark matter direct detection: nuclear recoils in large bucket of nuclei as signal
  - Detection rate/bounds depends on dark matter properties/dynamics and x-sec on nucleus
- 😍 Positive signals would be unambiguous
- 😟 Post-detection: precise nuclear x-sec (with quantified uncertainties) to discern underlying dynamics
- Potentially understand seemingly conflicting positive and negative signals
- Inform experimental design and backgrounds

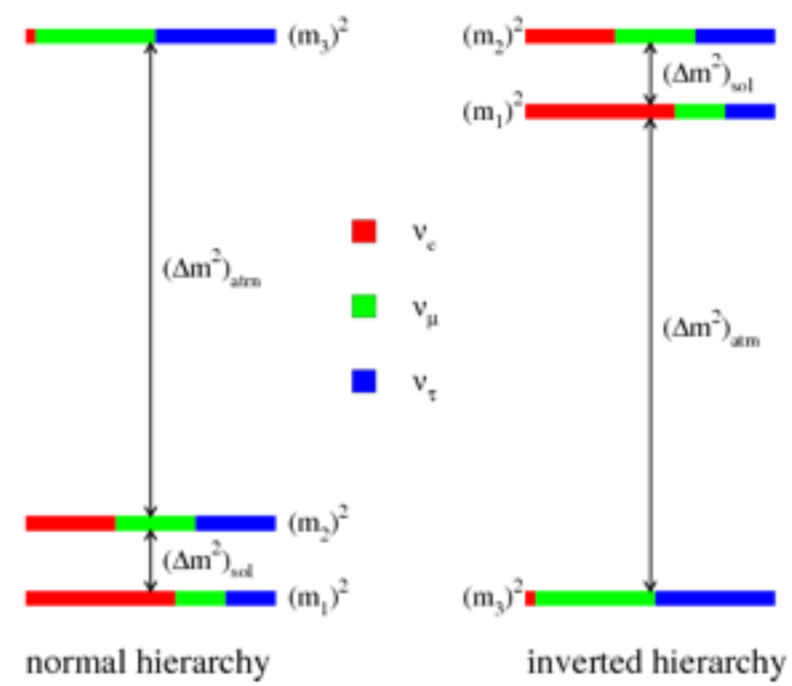


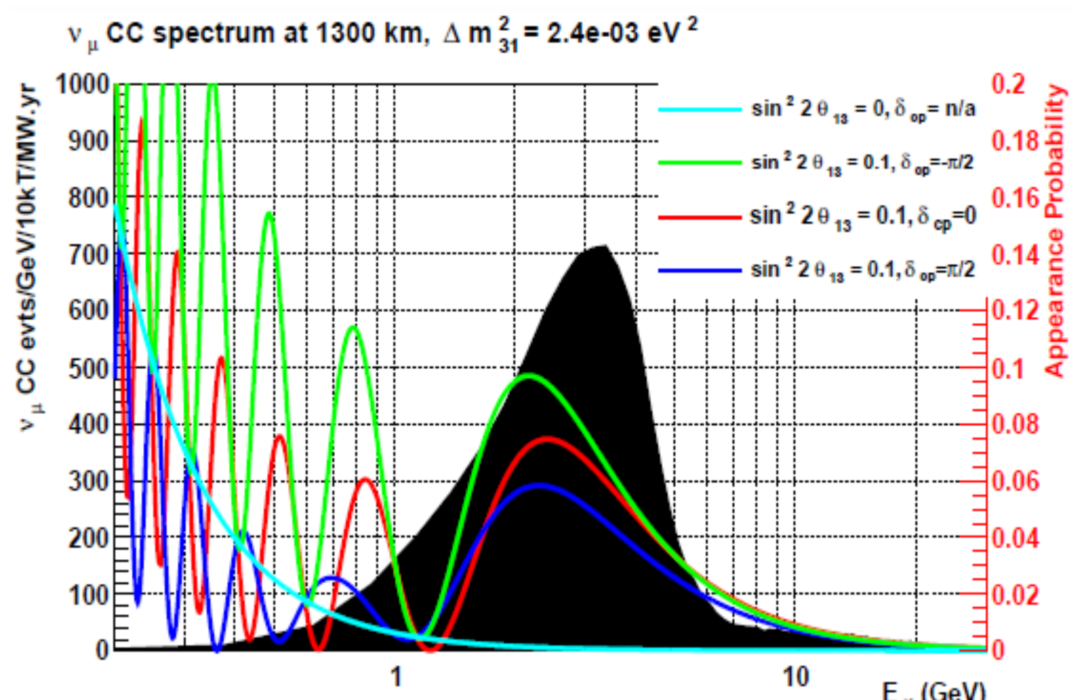
http://www.hep.ucl.ac.uk/darkMatter/



The intensity frontier

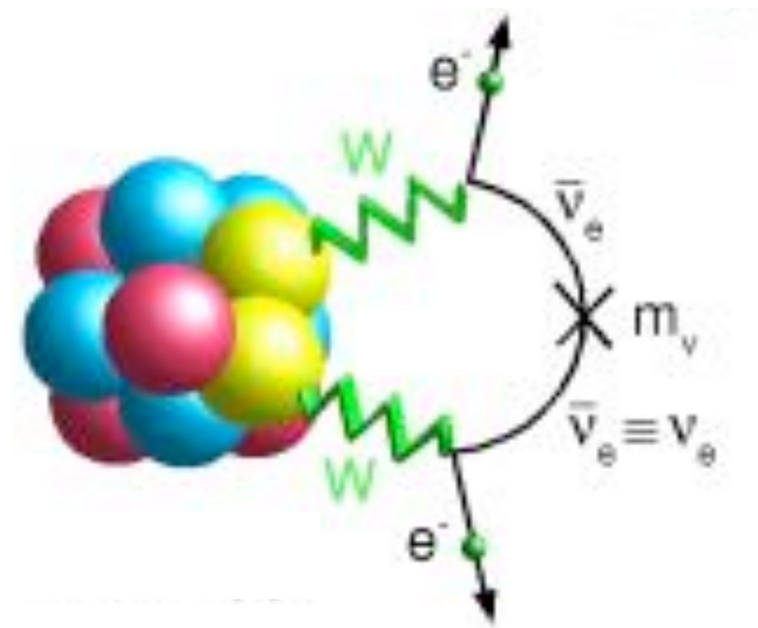
- Important goal of LBNF/DUNE: extraction of neutrino mass hierarchy and precise mixing parameters
- Neutrino scattering on argon target
- Requires knowing energies/fluxes to high accuracy
  - Nuclear axial & transition form factors
  - Resonances
  - Neutrino-nucleus DIS
- ~10% uncertainty on oscillation parameters [C Mariani, INT workshop 2013]



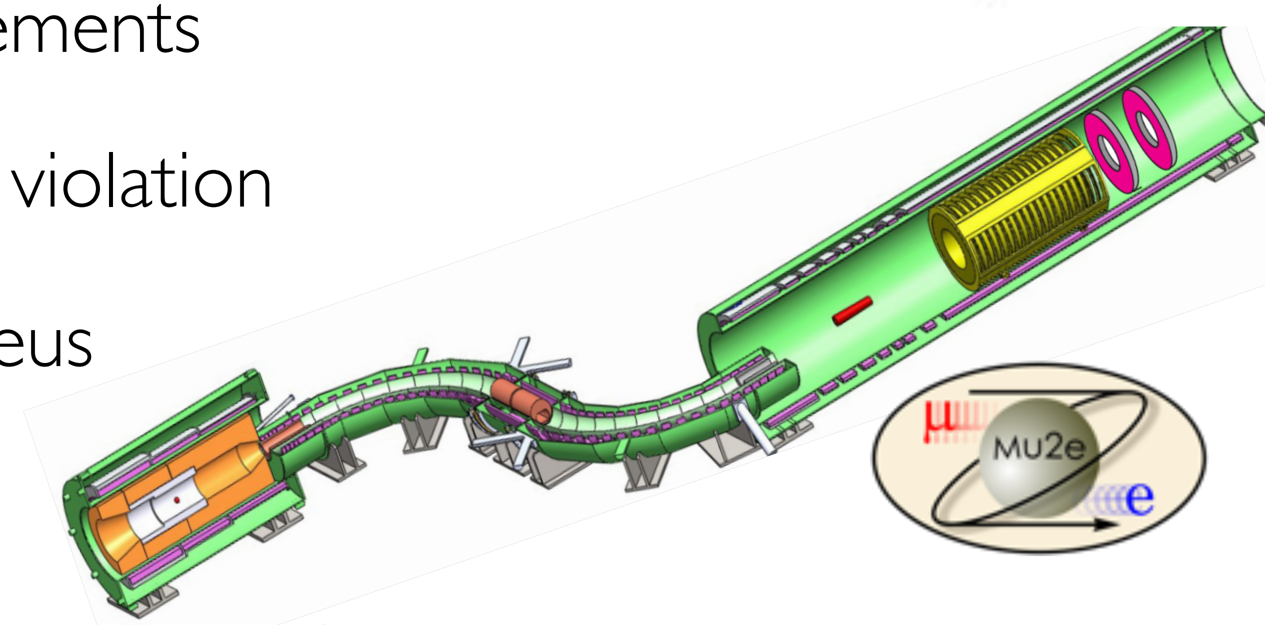


[U Mosel FSNu NP town meeting]

The intensity frontier



- EDMs: potential light nuclear EDM experiments offer complementary handles on CPV
- $0\nu\beta\beta$ decay: fundamental nature of neutrinos
  - Rates depend on nuclear matrix elements
- μ2e: search for charged lepton flavour violation
  - $\mu \rightarrow e$ conversion in field of Al nucleus



😍 Positive signals would be unambiguous

😟 Post-detection: precise nuclear matrix elements (with quantified uncertainties) to discern underlying dynamics

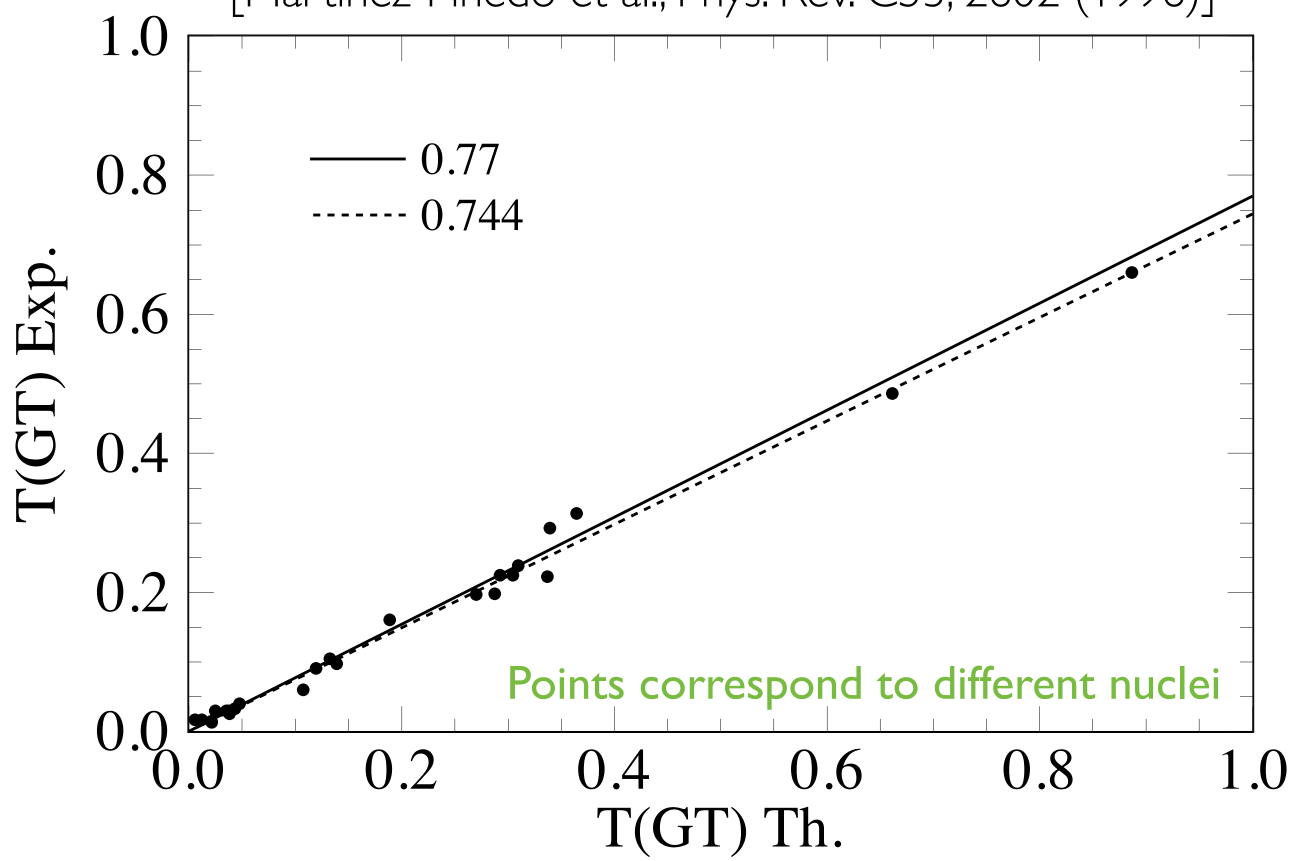
Nuclear uncertainties

- How well do we know nuclear matrix elements?

😢 Stark example of problems: Gamow-Teller transitions in nuclei

- Well measured for large range of nuclei (30<A<60)
- Many nuclear structure calcs (QRPA, shell-model,…) – spectrum well described
- Matrix elements systematically off by 20–30%
- "Correct" by "quenching" axial charge in nuclei ...

[Martinez-Pinedo et al., Phys. Rev. C53, 2602 (1996)]

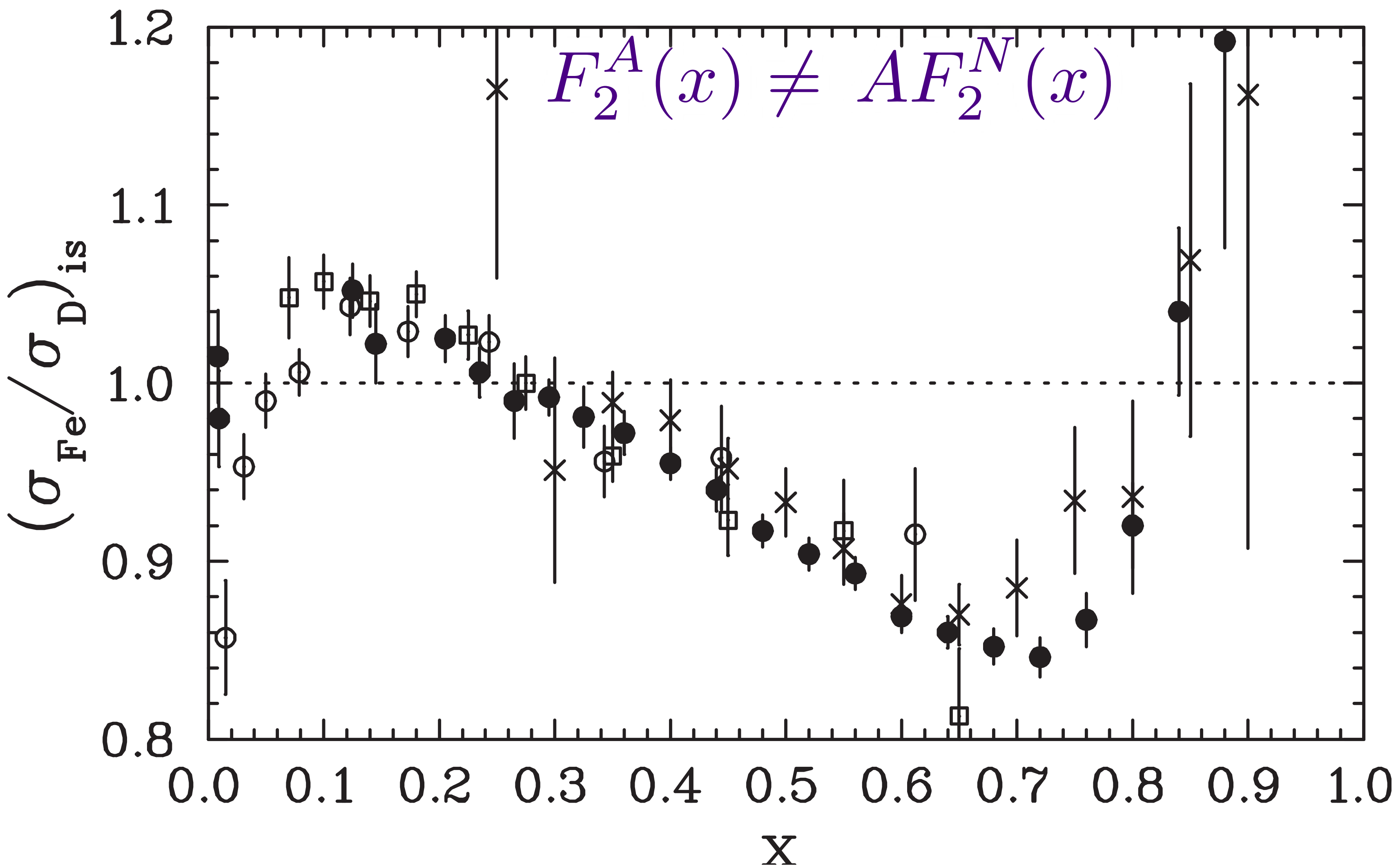


$$T(GT) \sim \sqrt{\sum_f \langle \boldsymbol{\sigma} \cdot \boldsymbol{\tau} \rangle_{i \to f}}$$

$$\langle \boldsymbol{\sigma\tau} \rangle = \frac{\langle f || \sum_k \boldsymbol{\sigma}^k \boldsymbol{t}^k_\pm || i \rangle}{\sqrt{2J_i + 1}}$$

The EMC effect

- Deep inelastic electron scattering on Fe target [EMC 1983]



- Proton structure modified in a nuclear environment
- Only "understood" in phenomenological models

Nuclear theory at the intensity frontier

- Definitive need for precision determinations of nuclear matrix elements
  - Must be based on the Standard Model (no hand-waving)
  - Must have fully quantified uncertainties
  - Timeframe and precision goals set by experiment
- Current state is far from this
- *Nuclear physics is the new flavour physics!*
  - Develop appropriate tools

Precision nuclear physics

- ***Goal: develop the tools for precision predictions***

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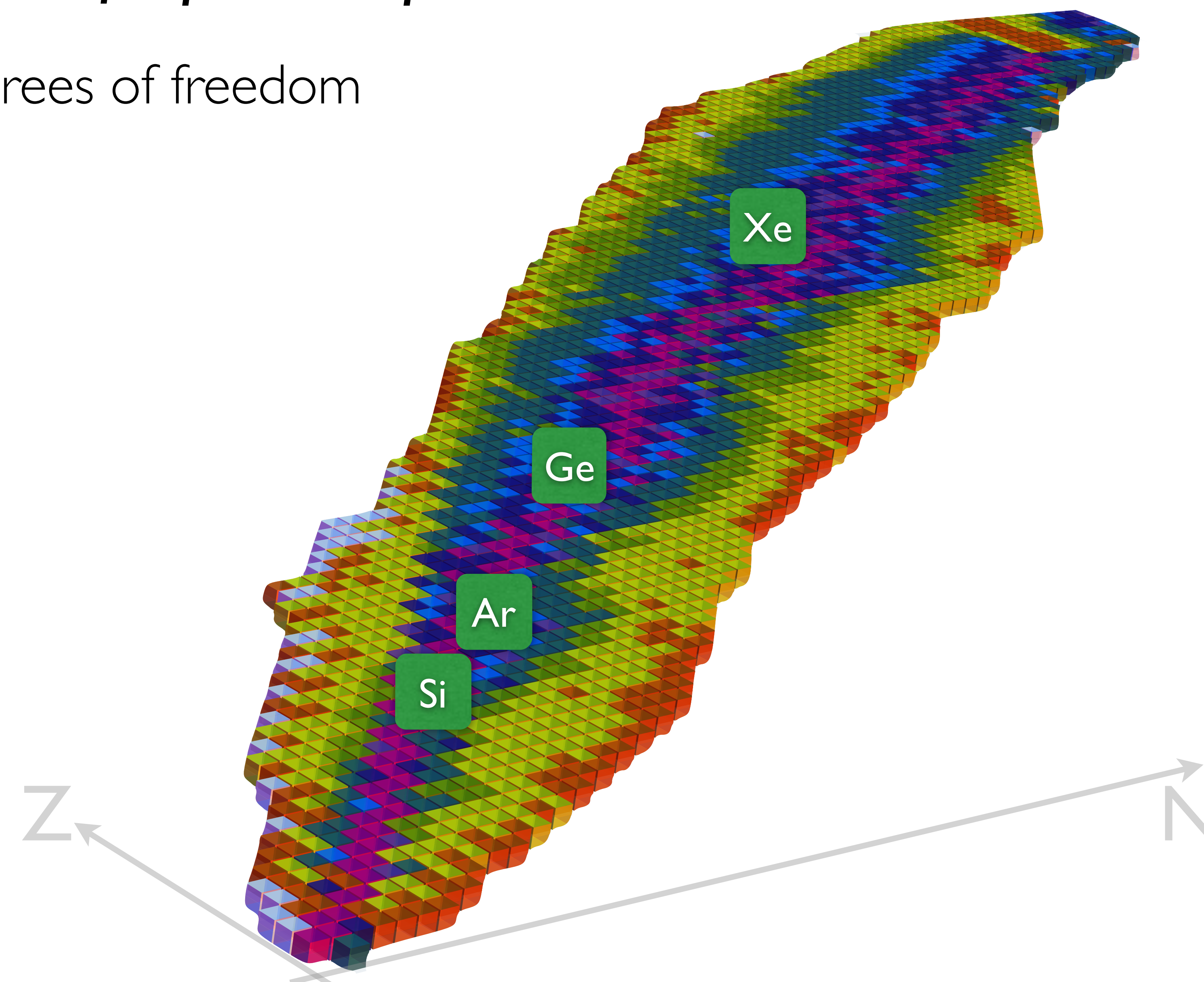
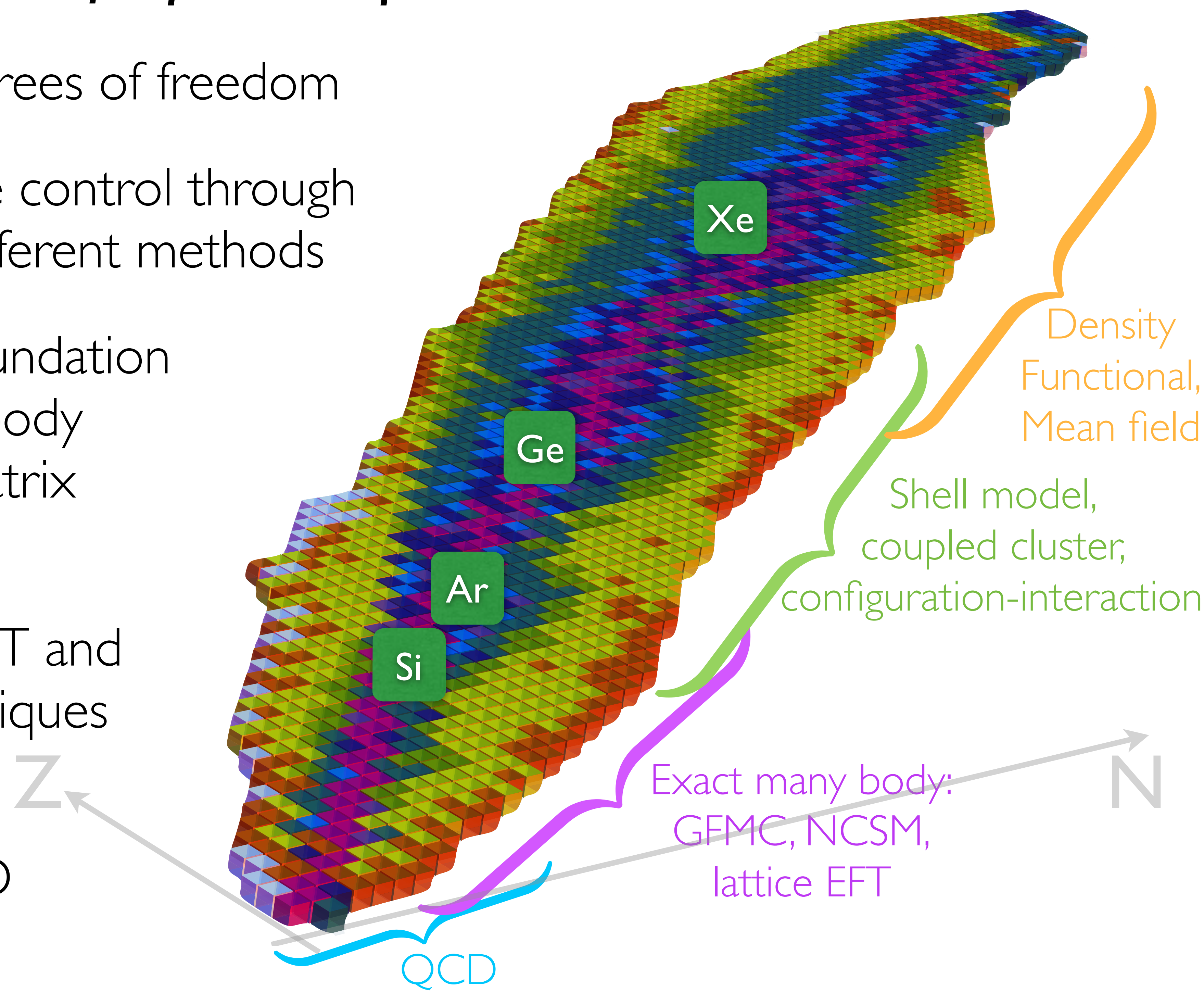
- Exploit effective degrees of freedom



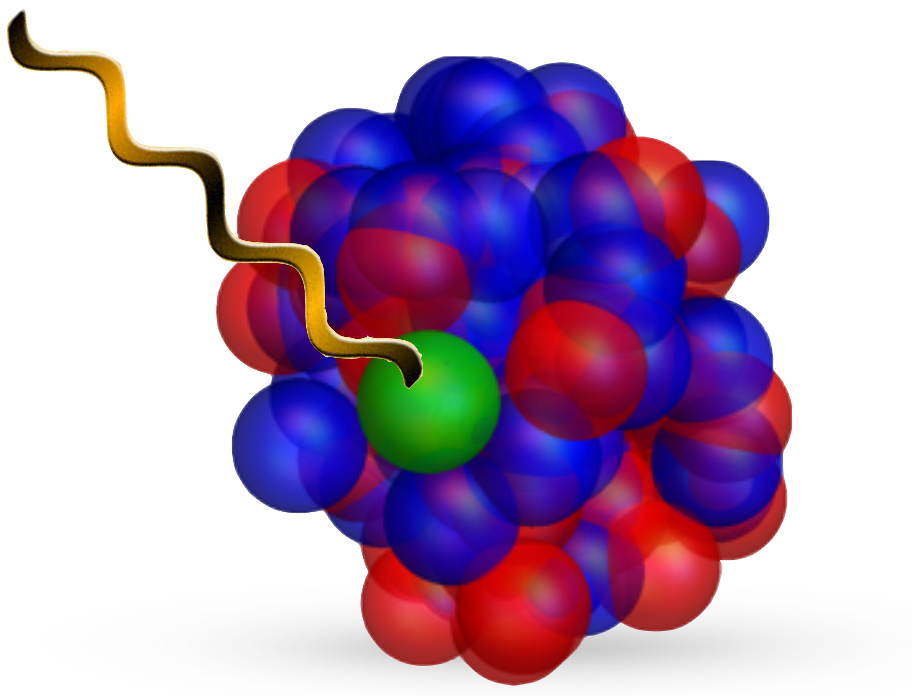
Precision nuclear physics

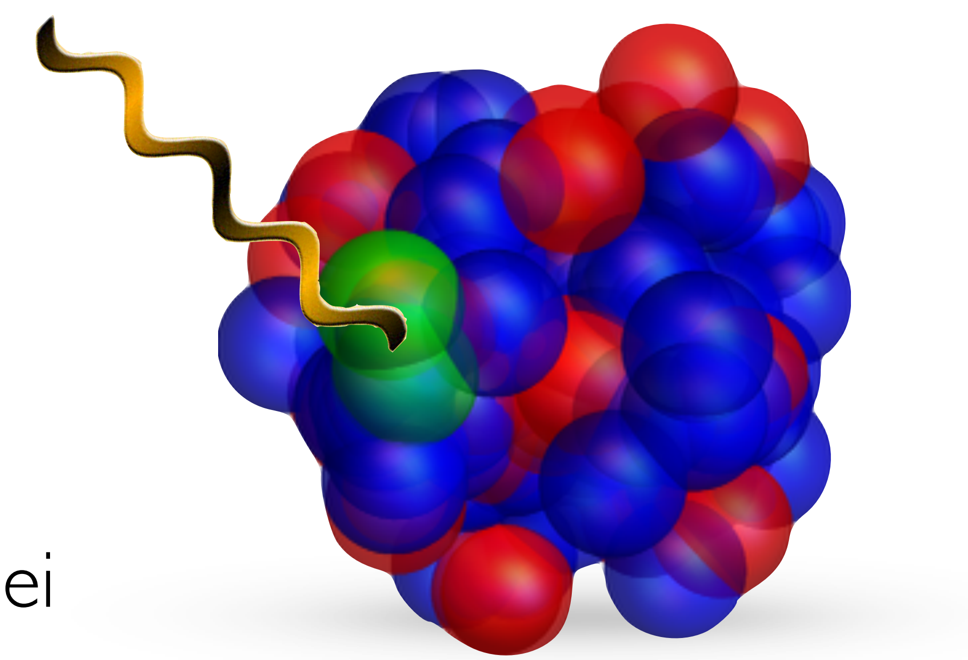
- ***Goal: develop the tools for precision predictions***
- Exploit effective degrees of freedom
- Establish quantitative control through linkages between different methods
  - QCD forms a foundation determines few body interactions & matrix elements
  - Match existing EFT and many body techniques onto QCD
- Now: focus on QCD



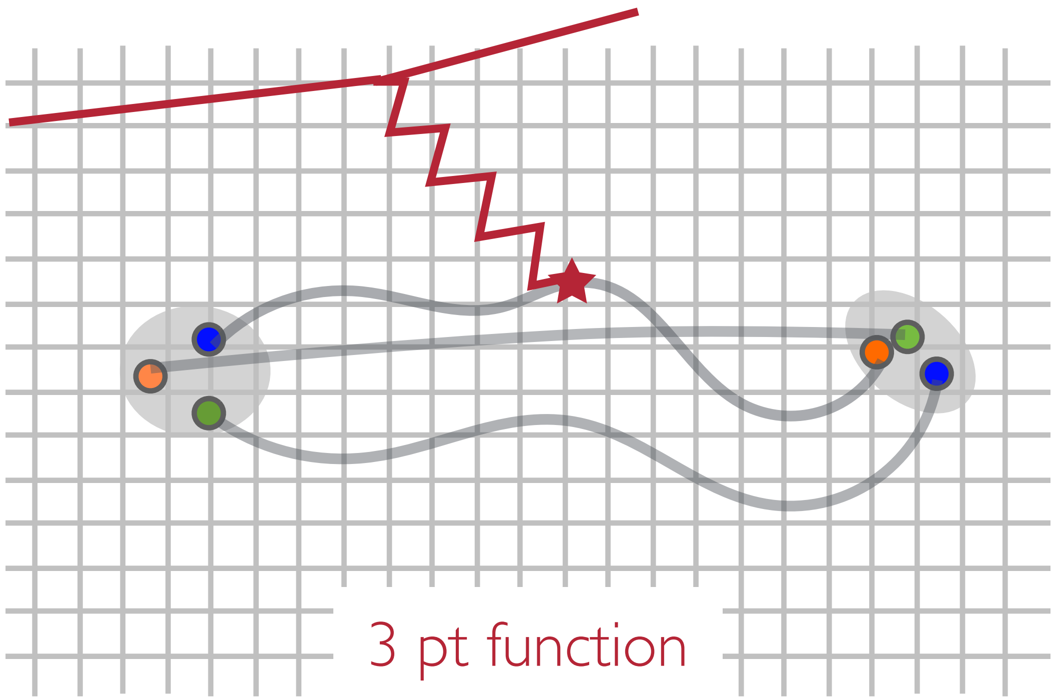
External currents and nuclei

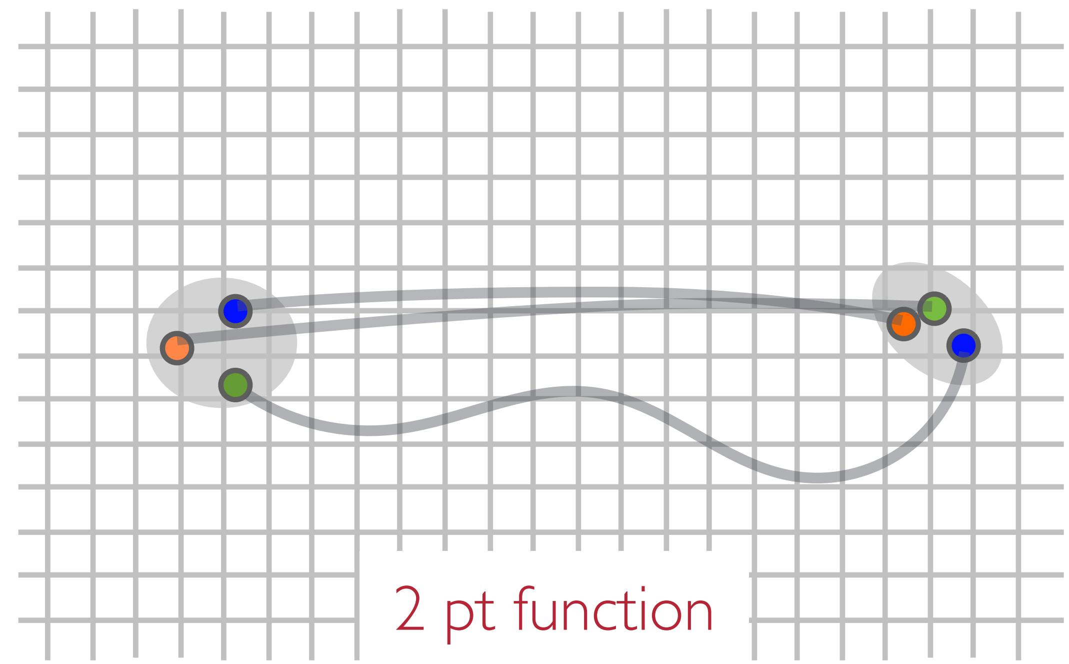
- Nuclear effective field theory:
  - 1-body currents are dominant
  - 2-body currents are sub-leading *but non-negligible*
- Determine one body contributions from single nucleon
- Determine few-body contributions from A=2,3,4...
- Match EFT and many body methods to LQCD to make predictions for larger nuclei





- LQCD FFs studied from ratios of 2- and 3-point correlators
  [Martinelli & Sachrajda1988]



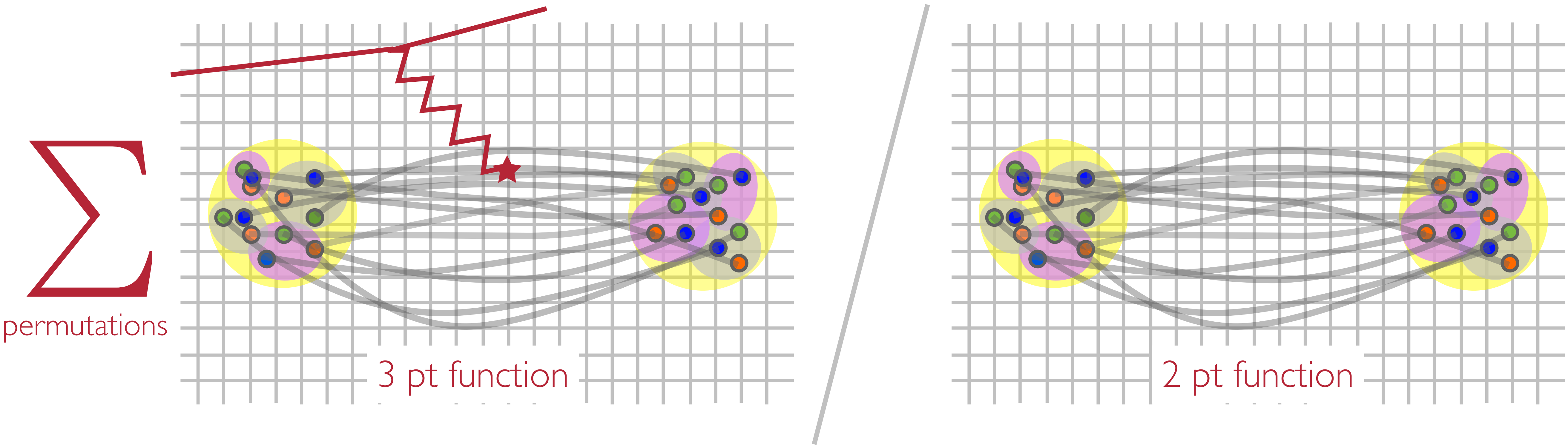


$$R(t,\tau;\mathbf{q}) \sim \sum_{\mathbf{y}} e^{i\mathbf{q}\cdot\mathbf{y}} \frac{\langle 0|\chi(0)\mathcal{O}(\mathbf{y},\tau)\chi^\dagger(t)|0\rangle}{\langle 0|\chi(0)\chi^\dagger(t)|0\rangle} \longrightarrow \langle N(p)|\mathcal{O}|N(p+q)\rangle$$

- Determines ground state FF at large source/operator/sink separation

Nuclear matrix elements

- For deeply bound nuclei, use the techniques as for single hadron matrix elements

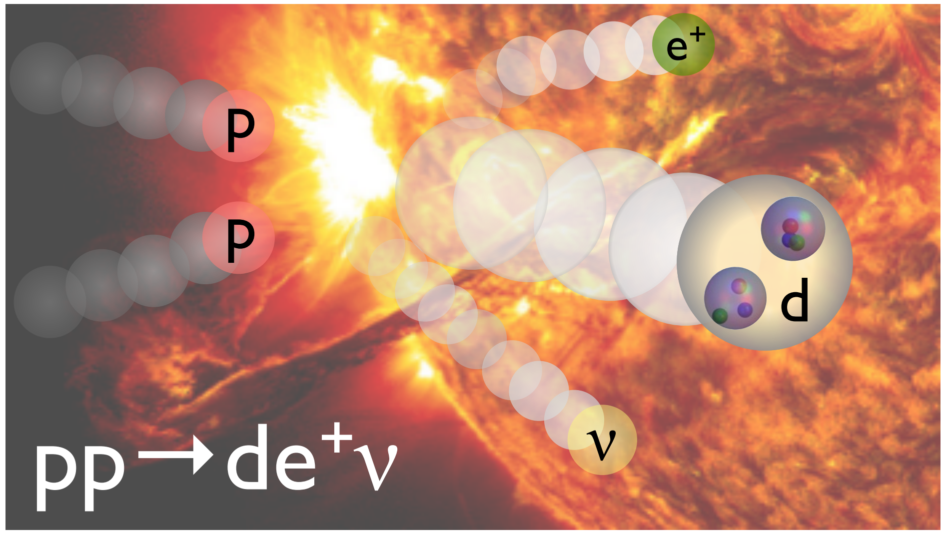


  - At large time separations gives matrix element of current
  - For near threshold states: care with volume effects

- ***Alternatively use background fields - works well for magnetic case***

Further matrix elements

- Axial coupling to NN system
  - pp fusion: "Calibrate the sun"
  - Muon capture: MuSun @ PSI
  - $d\,\nu \rightarrow n\,n\,e^+$: SNO



- Quadrupole moments: requires non-constant fields [Z Davoudi, WD 1507.01908]
- Axial form factors
- Scalar, … matrix elements for dark matter
- Twist-2 operators: EMC effect $\langle N, Z | \bar{q} \gamma_{\{\mu_1} D_{\mu_2} \dots D_{\mu_n\}} q | N, Z \rangle$

Nuclear sigma terms

- One possible DM interaction is through scalar exchange

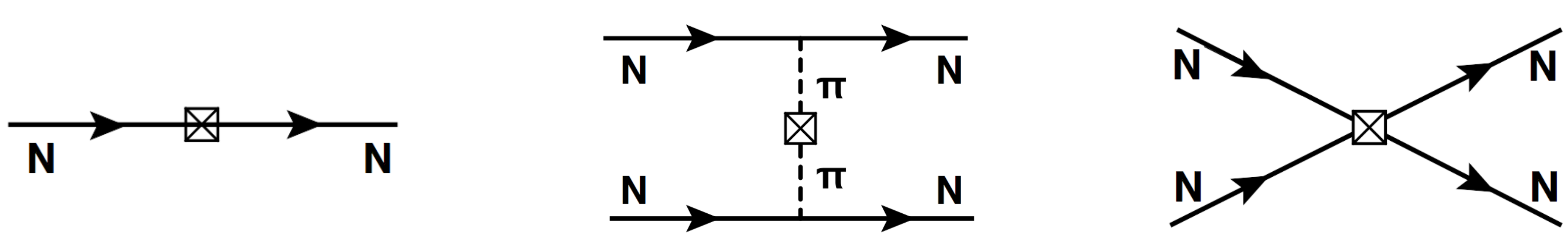
$$\mathcal{L} = \frac{G_F}{2} \sum_q a_S^{(q)} (\overline{\chi}\,\chi)(\overline{q}\,q)$$

- Accessible via Feynman-Hellman theorem

- At hadronic/nuclear level

$$\mathcal{L} \;\to\; G_F\;\overline{\chi}\chi\;\Big(\;\frac{1}{4}\langle 0|\overline{q}q|0\rangle\;\mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right]\;+\;\frac{1}{4}\langle N|\overline{q}q|N\rangle N^\dagger N \mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right]$$
$$-\;\frac{1}{4}\langle N|\overline{q}\tau^3 q|N\rangle\left(N^\dagger N\mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right]\;-\;4N^\dagger a_{S,\xi}N\right)\;+\;...\Big)$$

  - Contributions:

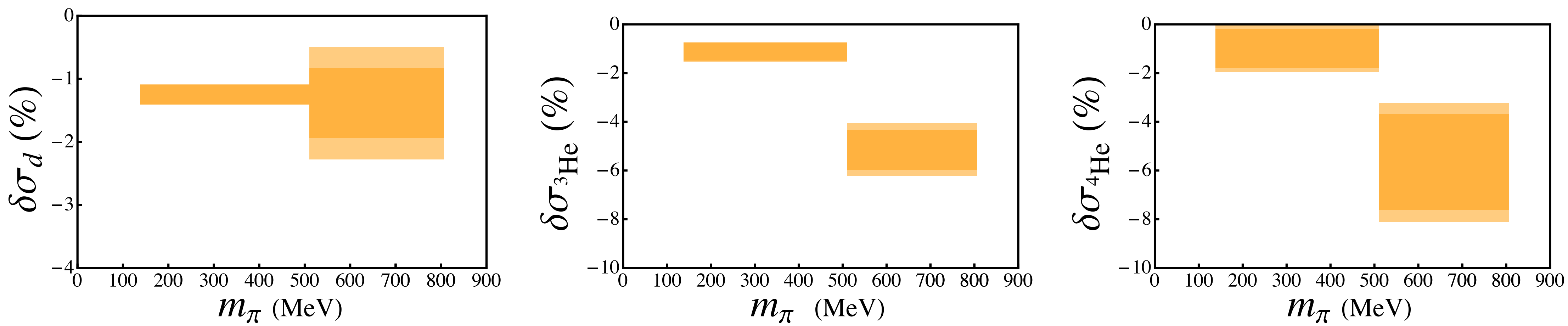


Nuclear sigma terms

- Previous work suggested scalar dark matter couplings to nuclei have O(50%) uncertainty arising from MECs [Prezeau et al 2003]
- Quark mass dependence of nuclear binding energies bounds such contributions

$$\delta\sigma_{Z,N} = \frac{\langle Z,N(\mathrm{gs})|\ \overline{u}u+\overline{d}d|Z,N(\mathrm{gs})\rangle}{A\ \langle N|\ \overline{u}u+\overline{d}d|N\rangle} - 1 = -\frac{1}{A\sigma_N}\frac{m_\pi}{2}\frac{d}{dm_\pi}B_{Z,N}$$

  - Lattice calculations + physical point suggest such contributions are O(10%) or less for light nuclei (A<4)



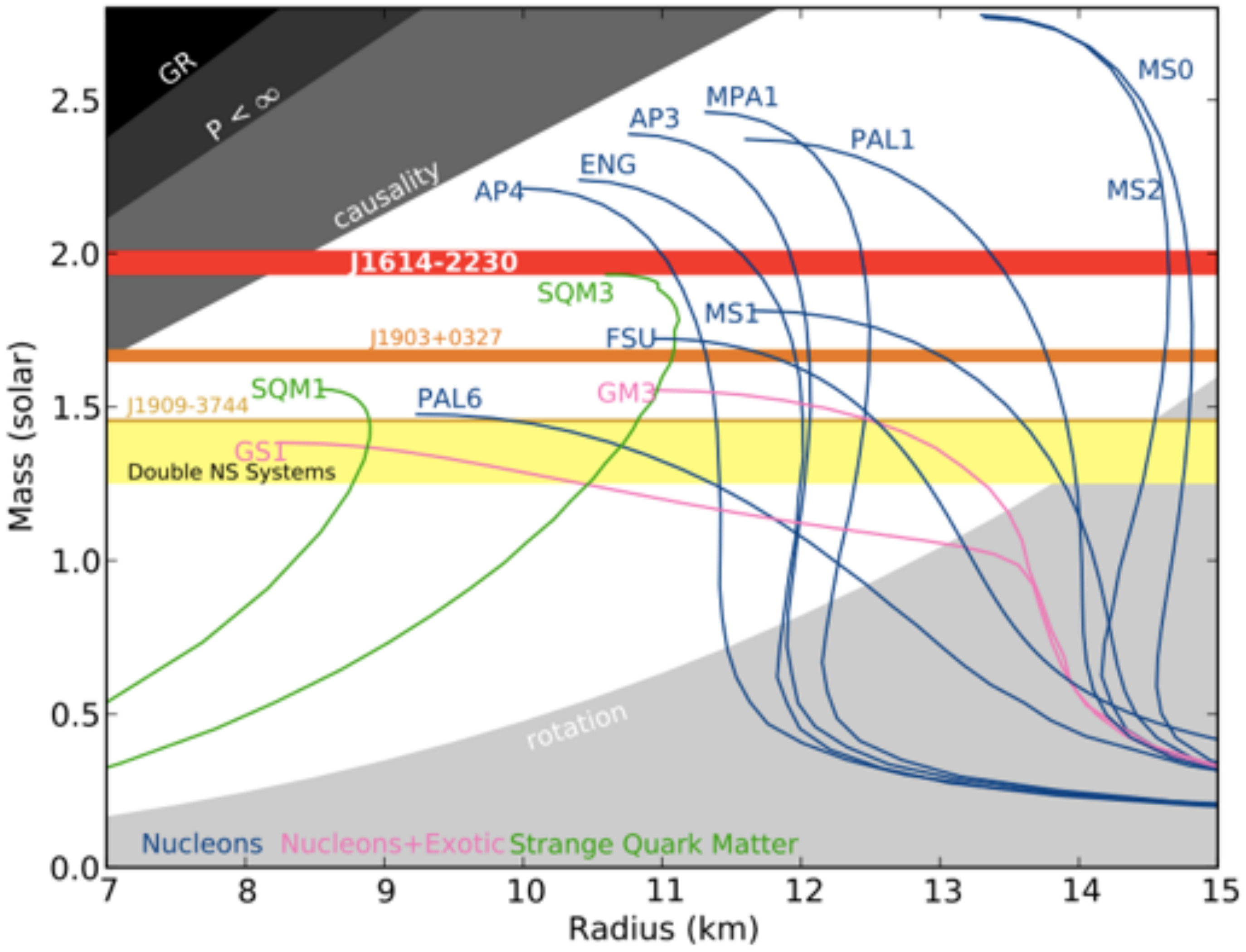
[NPLQCD PRD 2014]

QCD for nuclei

- Nuclei are under serious study directly from QCD
  - Spectroscopy of light nuclei and exotic nuclei (strange, charmed, …)
  - Structure: magnetic moments and polarisabilities
  - Electroweak interactions: thermal capture cross-section
- Prospect of a quantitative connection to QCD makes this a very exciting time for nuclear physics
  - Critical role in current and upcoming particle physics experimental program
  - Learn many interesting things about nuclear physics along the way

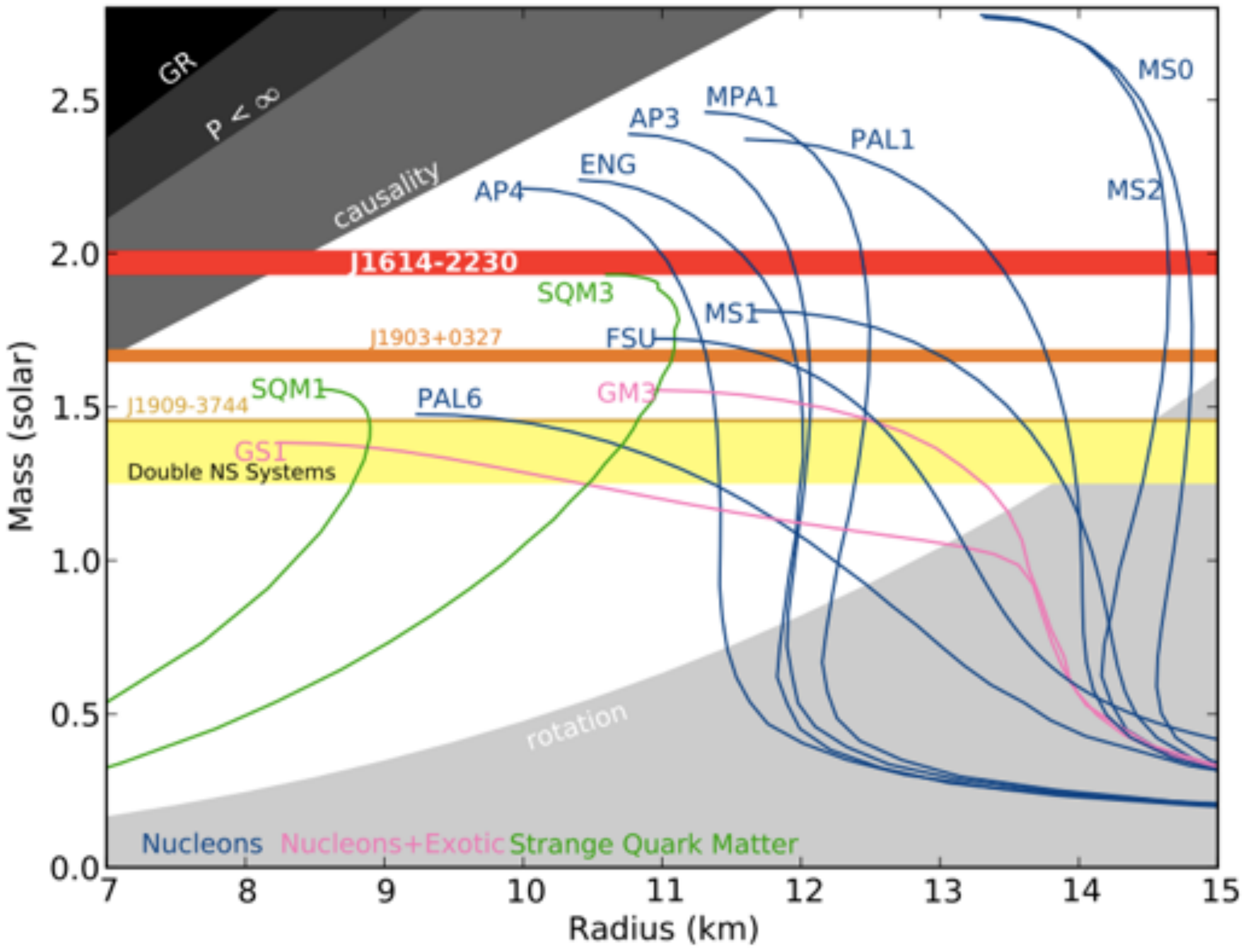
FIN

Hyperon-nucleon interactions



Hyperon-nucleon interactions

- Observation of 1.97 $M_{\odot}$ n-star [Demorest et al., Nature, 2010]
  "effectively rules out the presence of hyperons, bosons, or free quarks"



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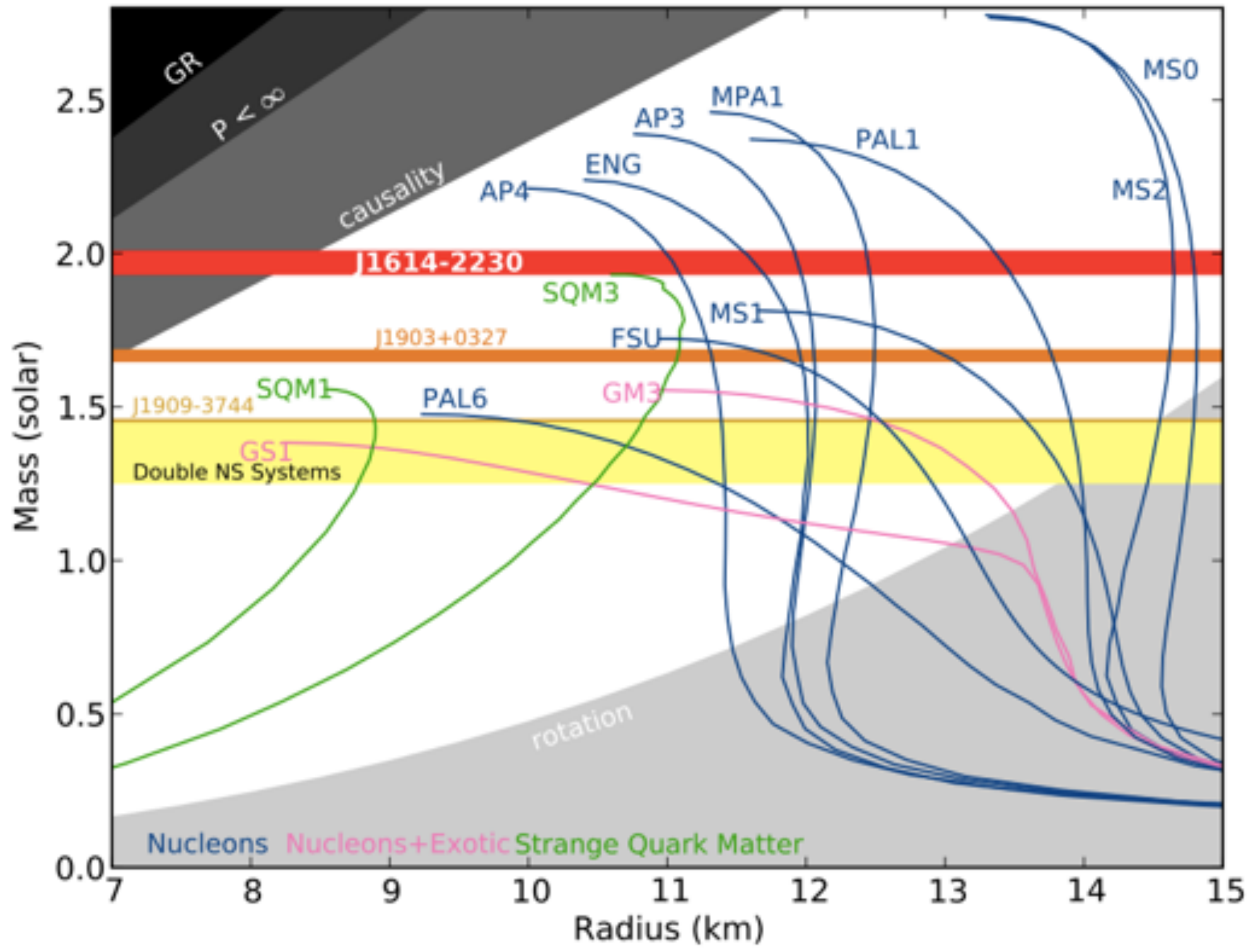
- Relies significantly on poorly known hadronic interactions at high density
  - Hyperon-nucleon
  - *nnn, ...*



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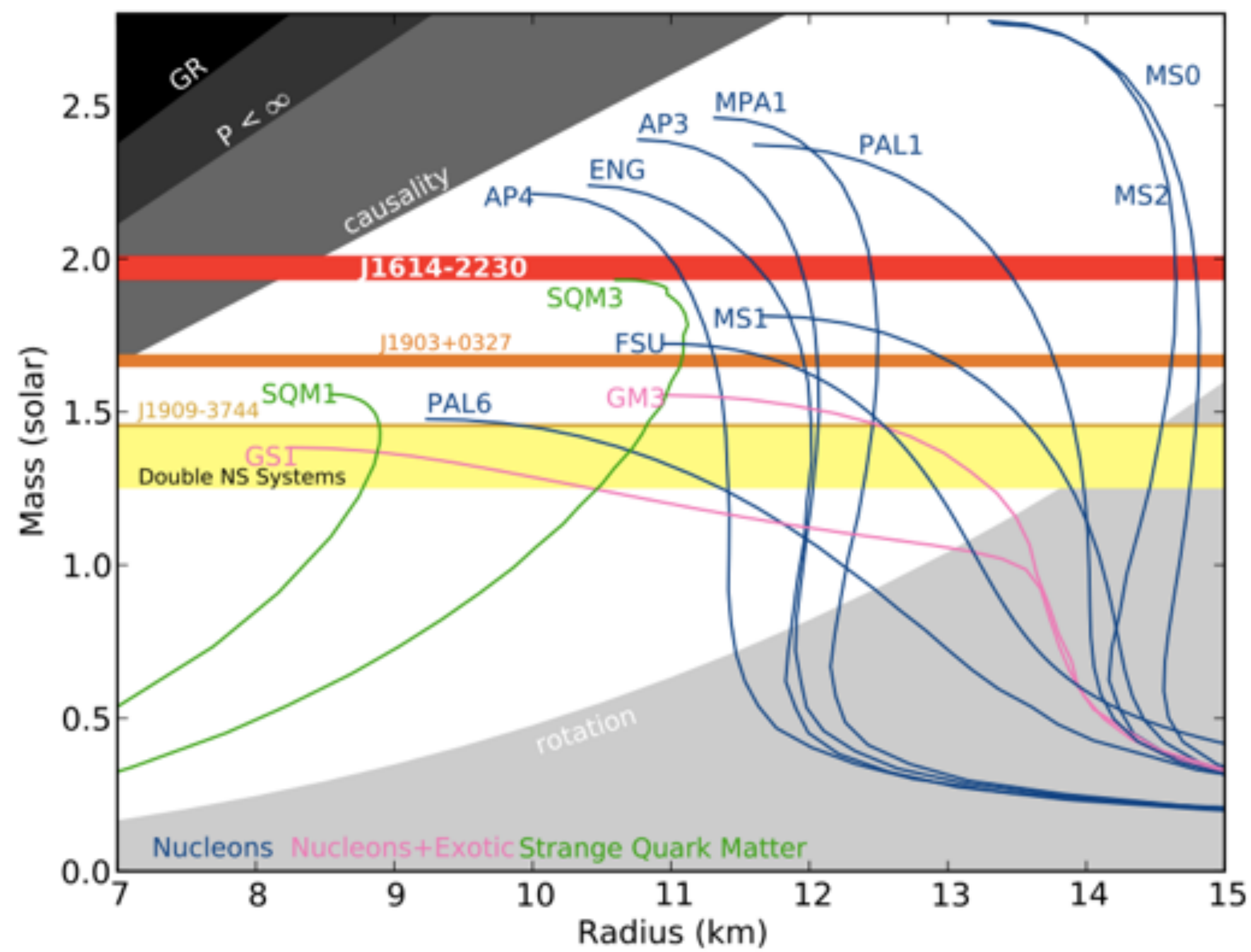
  "effectively rules out the presence of hyperons, bosons, or free quarks"
- Relies significantly on poorly known hadronic interactions at high density
  - Hyperon-nucleon
  - *nnn*, ...
- Calculable in QCD
  - 30% determinations would have impact
  - Happening for YN
    [NPLQCD PRL 109 (2012) 172001]



$\Sigma^- n$ (I=3/2) phase shifts

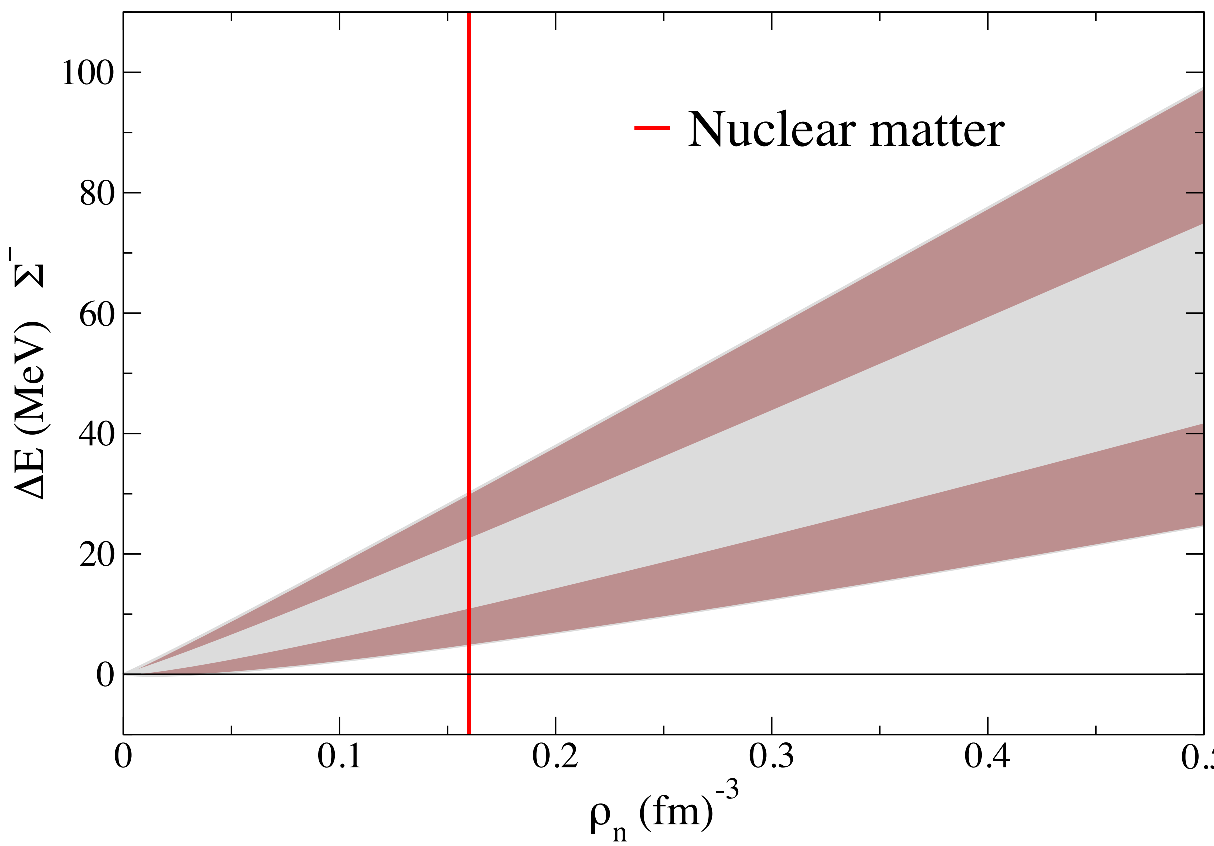
- Influence on EoS is complex
  - Crude approx: Fumi's theorem

$$\Delta E = -\frac{1}{\pi\mu}\int_0^{k_f} dk\, k \left[\frac{3}{2}\delta_{^3S_1}(k) + \frac{1}{2}\delta_{^1S_0}(k) \right]$$

- For $\rho_n \sim 0.4\ \mathrm{fm}^{-3}$,

  $\mu_n + \mu_{e^-} \sim 1290\ \mathrm{MeV}$

- If $\mu_{\Sigma^-} = M_\Sigma + \Delta E \lesssim 1290\ \mathrm{MeV}$ then **Σ**⁻s probably relevant to n-star structure



Phys. Rev. Lett. 109 (2012) 172001