Light nuclei and nucleon form factors in $N_f = 2 + 1$ lattice QCD

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for PACS Collaboration

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1. Light nuclei

in collaboration with

K.-I. Ishikawa, Y. Kuramashi, and A. Ukawa for PACS Collaboration

Refs: PRD81:111504(R)(2010); PRD84:054506(2011); PRD86:074514(2012)
PRD92:014501(2015)

2. Nucleon form factors

in collaboration with

K.-I. Ishikawa, Y. Kuramashi, S. Sasaki and A. Ukawa
for PACS Collaboration

Outline

- Introduction
- Calculation method of nuclei in lattice QCD
- Simulation parameters
- Results of light nuclei
  - ^{4}He and ^{3}He channels
  - NN channels
- Preliminary result at $m_\pi \sim 0.145$ GeV
  - Light nuclei binding energy
  - nucleon form factors
- Summary and future work

Introduction

$$\text{Binding force} \begin{cases} \text{protons and neutrons} & \rightarrow & \text{nuclei} \\ \text{quarks and gluons} & \rightarrow & \text{protons and neutrons} \end{cases}$$

both from fundamental strong interaction of quark and gluon

well known, but hard to prove

quark and gluon $\rightarrow$ proton and neutron$\rightarrow$ nucleus

Introduction

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Spectrum of proton and neutron (nucleons)

success of non-perturbative calculation of QCD

degrees of freedom of quarks and gluons

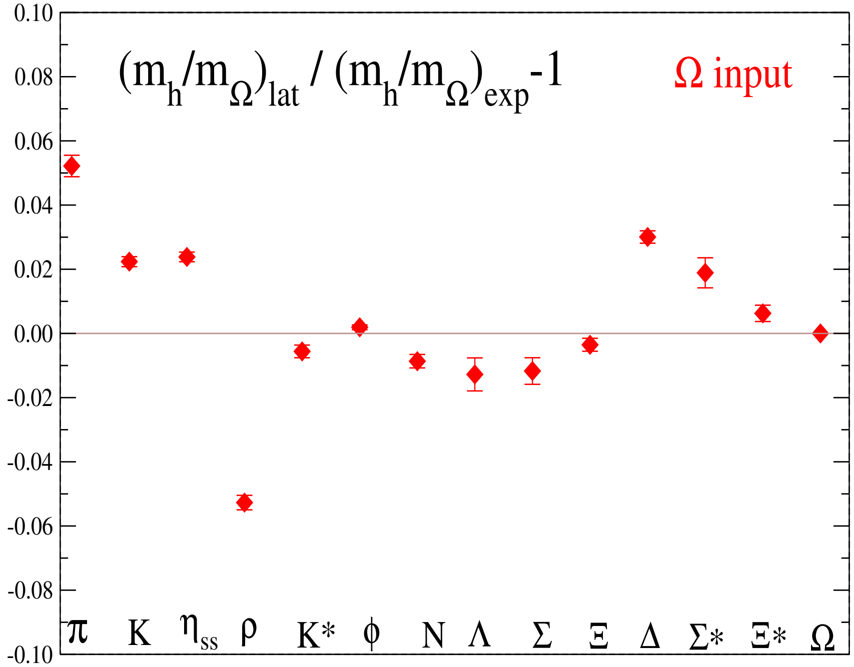
$\overbrace{\text{quark and gluon} \rightarrow \text{proton and neutron}} \rightarrow \text{nucleus}$

Hadron spectrum in $N_f = 2+1$ QCD

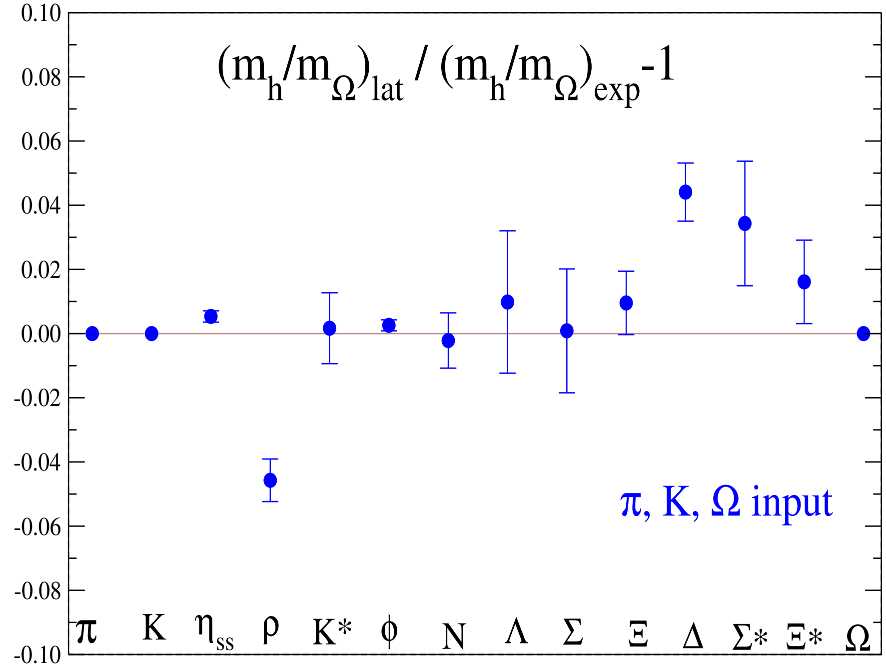
Lattice 2015, Ukita for PACS Collaboration

$m_\pi \sim$ 0.145 GeV on $L \sim 8$ fm (K computer, SPIRE Field 5)
using reweighting $m_{u,d}, m_s$ + extrapolation $\to$ physical m_π and m_K

$m_\pi \sim$ 0.145 GeV



physical point



Stable hadron mass: well reproduced from lattice QCD

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$\underbrace{\overbrace{\text{quark and gluon} \rightarrow \text{proton and neutron}} \rightarrow \text{nucleus}}$

goal: quantitatively understand property of nucleus from QCD

So far not many studies for multi-baryon bound states

$\rightarrow$ Can we reproduce binding energy of known light nuclei?

Introduction

$$\text{Binding force} \begin{cases} \text{protons and neutrons} & \rightarrow \quad \text{nuclei} \\ \text{quarks and gluons} & \rightarrow \quad \text{protons and neutrons} \end{cases}$$

both from fundamental strong interaction of quark and gluon

well known, but hard to prove

Spectrum of proton and neutron (nucleons)

success of non-perturbative calculation of QCD

degrees of freedom of quarks and gluons

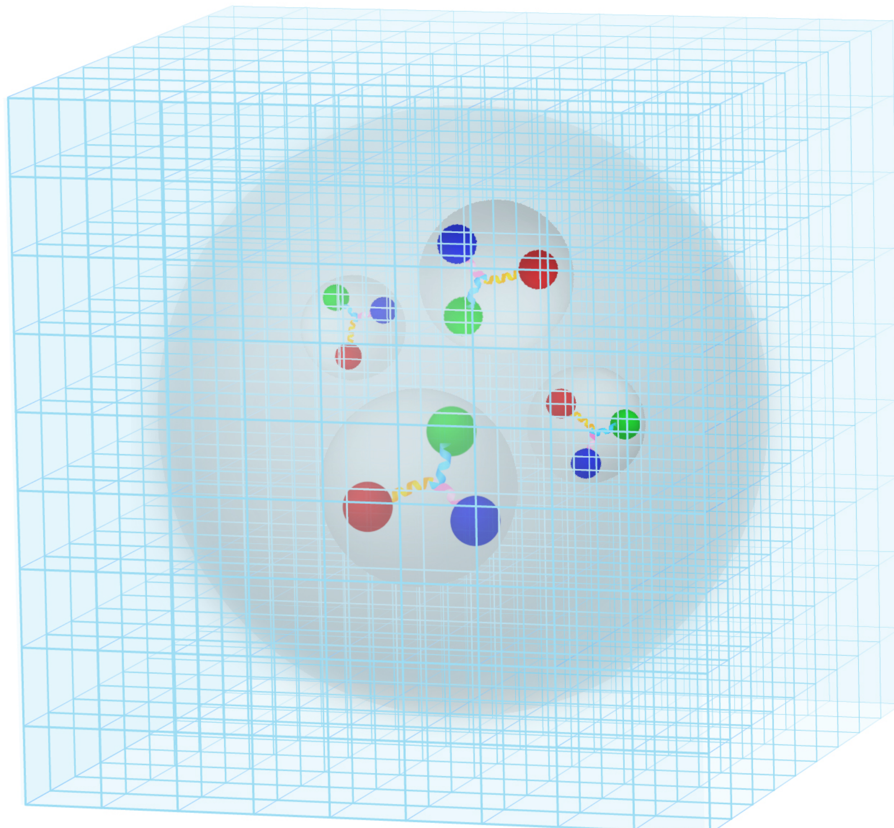
2nd motivation: Nucleon form factors not well understood

$\rightarrow$ 2nd part of talk

$$\underbrace{\overbrace{\text{quark and gluon} \rightarrow \text{proton and neutron}} \rightarrow \text{nucleus}}$$

goal: quantitatively understand property of nucleus from QCD

Ultimate goal of lattice QCD



http://www.jicfus.jp/jp/promotion/pr/mj/2014-1/

quantitatively understand property of nuclei from QCD

Multi-baryon system from lattice QCD at '09

1. $\Lambda\Lambda$ system (Quenched QCD)

'85 Mackenzie & Thacker
'88 Iwasaki *et al.*
'99 Pochinsky *et al.*
'00 Wetzorke *et al.*
'02 Wetzorke & Karsch
'09 NPLQCD ($N_f = 2 + 1$)

H dibaryon: unbound except Iwasaki *et al.*

2. NN system $^3S_1$ and $^1S_0$

'95 Fukugita *et al.* : Quenched QCD

'06 NPLQCD : $N_f = 2 + 1$ QCD

'08 Ishii *et al.* : Quenched and $N_f = 2 + 1$ QCD

'09 NPLQCD : $N_f = 2 + 1$ QCD

Deuteron: unbound due to $m_\pi \gtrsim 0.3$ GeV

3. NNN system

'09 NPLQCD : $N_f = 2 + 1$ QCD

Triton: likely unbound

Multi-baryon bound state from lattice QCD

Not observed before '09 (except H-dibaryon '88 Iwasaki *et al.*)

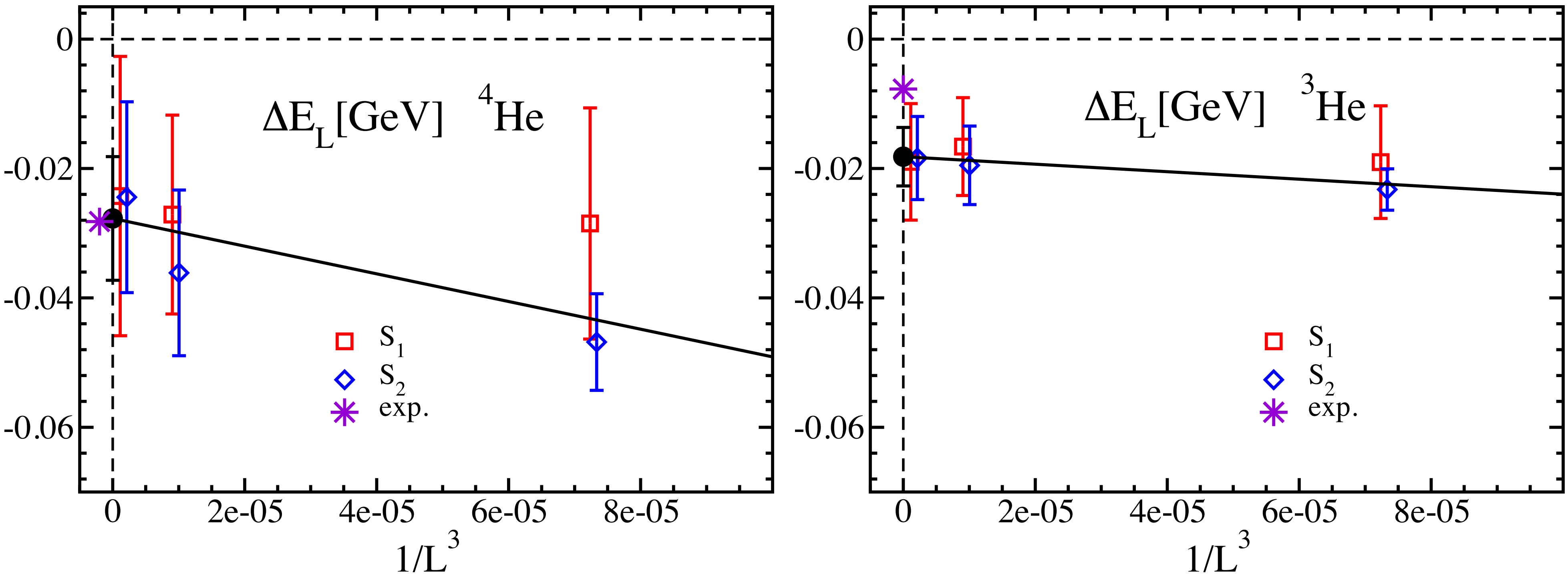
1. ^{4}He and ^{3}He

   '10 PACS-CS $N_f = 0$ $m_\pi = 0.8$ GeV PRD81:111504(R)(2010)

Exploratory study of three- and four-nucleon systems

PACS-CS Collaboration, PRD81:111504(R)(2010)

Identification of bound state from volume dependence of ΔE



$\Delta E_{^4\mathrm{He}} = 27.7(7.8)(5.5)$ MeV $\quad \Delta E_{^3\mathrm{He}} = 18.2(3.5)(2.9)$ MeV

1. Observe bound state in both channels
2. Same order of ΔE to experiment

Several systematic errors included, e.g., $N_f = 0$, $m_\pi = 0.8$ GeV

Multi-baryon bound state from lattice QCD

1. ^{4}He and ^{3}He
   - '10 PACS-CS $N_f = 0$ $m_\pi = 0.8$ GeV PRD81:111504(R)(2010)
   - '12 HALQCD $N_f = 3$ $m_\pi = 0.47$ GeV, $m_\pi > 1$ GeV ^{4}He
   - '12 NPLQCD $N_f = 3$ $m_\pi = 0.81$ GeV
   - '12 TY *et al.* $N_f = 2 + 1$ $m_\pi = 0.51$ GeV PRD86:074514(2012)
   - '15 TY *et al.* $N_f = 2 + 1$ $m_\pi = 0.30$ GeV PRD92:014501(2015)
2. H dibaryon in ΛΛ channel (S=−2, I=0)
   - '11, '12 NPLQCD $N_f = 2 + 1$ $m_\pi = 0.39$ GeV, $N_f = 3$ $m_\pi = 0.81$ GeV
   - '11, '12 HALQCD $N_f = 3$ $m_\pi = 0.47$–1.02 GeV
   - '11 Luo et al. $N_f = 0$ $m_\pi = 0.5$–1.3 GeV
   - '14 Mainz $N_f = 2$ $m_\pi = 0.45, 1.0$ GeV
3. NN
   - '11 PACS-CS $N_f = 0$ $m_\pi = 0.8$ GeV PRD84:054506(2011)
   - '12 NPLQCD $N_f = 2 + 1$ $m_\pi = 0.39$ GeV (Possibility)
   - '12 NPLQCD, '15 CalLat $N_f = 3$ $m_\pi = 0.81$ GeV
   - '12 TY *et al.* $N_f = 2 + 1$ $m_\pi = 0.51$ GeV PRD86:074514(2012)
   - '15 TY *et al.* $N_f = 2 + 1$ $m_\pi = 0.30$ GeV PRD92:014501(2015)
   - '15 NPLQCD $N_f = 2 + 1$ $m_\pi = 0.45$ GeV

Other states: ΞΞ, '12 NPLQCD; spin-2 $N\Omega$, ^{16}O and ^{40}Ca, '14 HALQCD, $\cdots$

Calculation method

Calculation method of multi-nucleon bound state

Traditional method for example ^{4}He channel

$$\langle 0|O_{^4\text{He}}(t)O^\dagger_{^4\text{He}}(0)|0\rangle = \sum_n \langle 0|O_{^4\text{He}}|n\rangle\langle n|O^\dagger_{^4\text{He}}|0\rangle e^{-E_n t} \xrightarrow[t\gg 1]{} A_0\, e^{-E_0 t}$$

Difficulties for multi-nucleon calculation

1. Statistical error

Statistical error $\propto$ exp $\left(N_N\left[m_N - \frac{3}{2}m_\pi\right]t\right)$

2. Calculation cost

Wick contraction for ^{4}He $= p^2n^2 = (udu)^2(dud)^2$: 518400

proton $= p = (udu)$: 2

3. Identification of bound state on finite volume

Finite volume effect of attractive scattering state

$$\Delta E_L = E_0 - N_N m_N = O(L^{-3}) < 0 \leftrightarrow \text{binding energy}$$

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Most severe problem before '09: (every t)$\times N_{\text{meas}} \sim O(10^6)$

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Statistical error $\propto \exp\left(N_N\left[m_N - \frac{3}{2}m_\pi\right]t\right)$

$\rightarrow$ heavy quark $m_\pi = 0.8$–0.3 GeV + large # of measurements

2. Calculation cost PACS-CS PRD81:111504(R)(2010)

Wick contraction for $^4\mathrm{He} = p^2n^2 = (udu)^2(dud)^2$: 518400 $\rightarrow$ 1107

$\rightarrow$ reduction using $p(n) \leftrightarrow p(n)$ $p \leftrightarrow n$, $u(d) \leftrightarrow u(d)$ in $p(n)$

+ block of 3 quark props(parallel) and contraction(workstation)

'12 Doi and Endres; Detmold and Orginos; '13 Günther et al.; '15 Nemura

3. Identification of bound state on finite volume

attractive scattering state $\Delta E_L = E_0 - N_N m_N = O(L^{-3}) < 0$

'86,'91 Lüscher, '07 Beane *et al.*

$\rightarrow$ Volume dependence of $\Delta E_L \rightarrow \Delta E_\infty \neq 0 \rightarrow$ bound state

Spectral weight: '04 Mathur *et al.*, Anti-PBC '05 Ishii *et al.*

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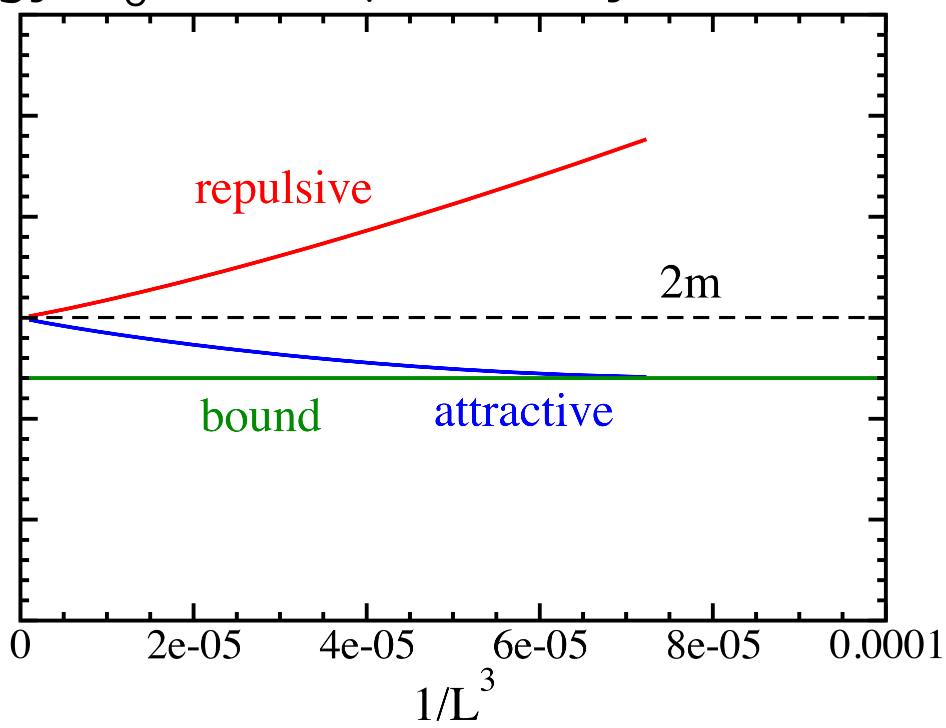
Ground state energy E_0 in two particle system on finite volume



Hard to distinguish attractive scattering from bound

from only 1 (small) volume calculation

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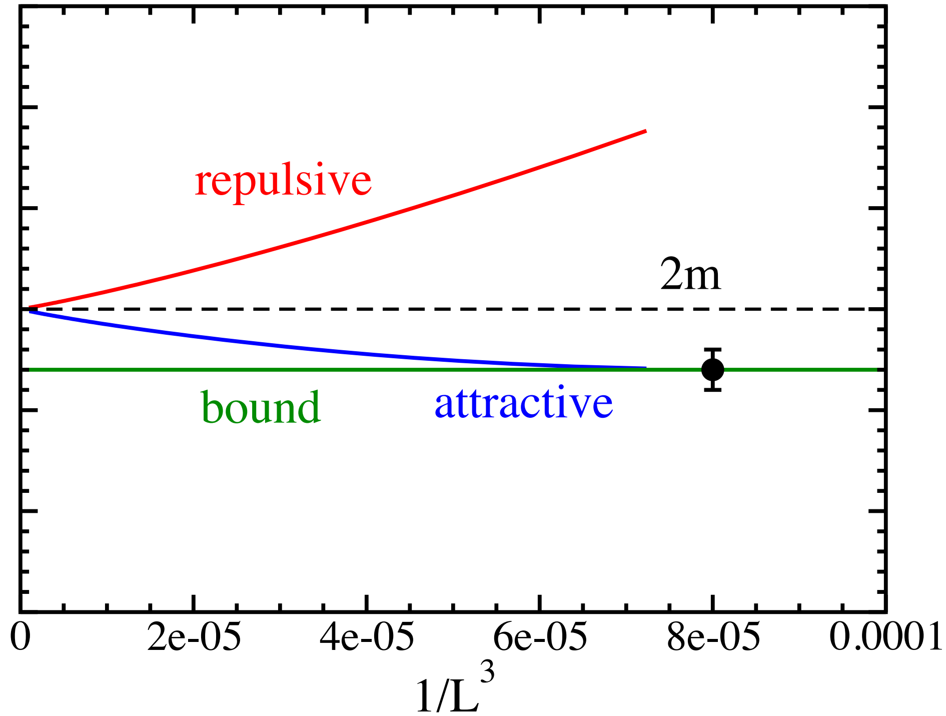

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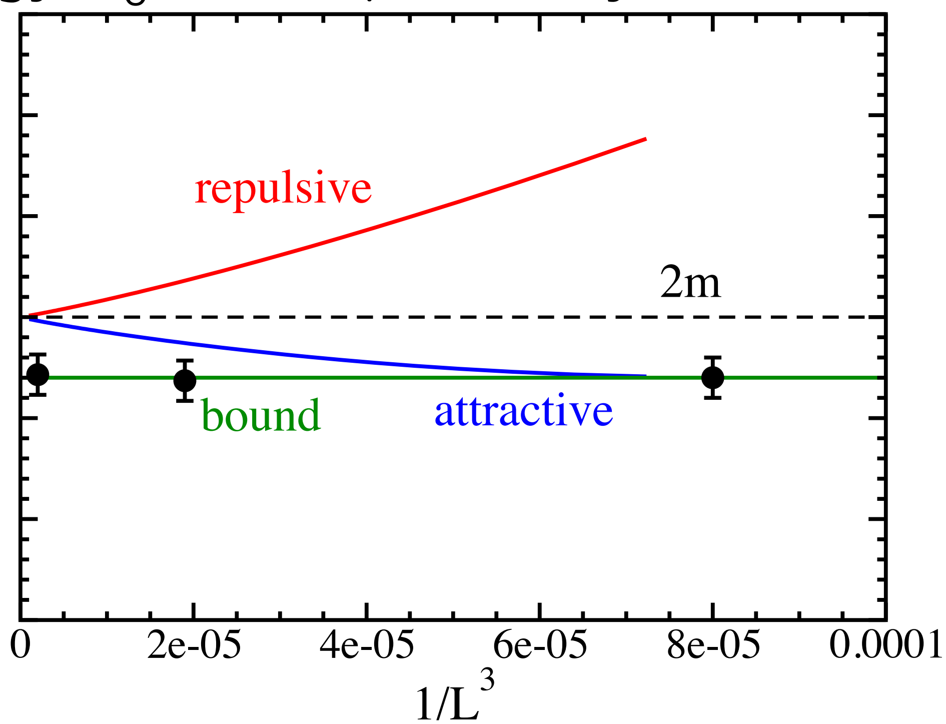

Simplest way: extrapolation of $E_0(\Delta E_L)$ to infinite volume limit

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Difficulties for multi-nucleon calculation

1. Statistical error

Statistical error $\propto \exp\left(N_N\left[m_N - \frac{3}{2}m_\pi\right]t\right)$

Most severe problem at present

2. Calculation cost PACS-CS PRD81:111504(R)(2010)

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proton $= p = (udu)$: 2

Used to be most severe problem

3. Identification of bound state on finite volume

Finite volume effect of attractive scattering state

$$\Delta E_L = E_0 - N_N m_N = O(L^{-3}) < 0 \leftrightarrow \text{binding energy}$$

Simulation parameters

$N_f = 2+1$ QCD

Iwasaki gauge action at $\beta = 1.90$

$a^{-1} = 2.194$ GeV with $m_\Omega = 1.6725$ GeV '10 PACS-CS

non-perturbative $O(a)$-improved Wilson fermion action

$m_\pi = 0.51$ GeV and $m_N = 1.32$ GeV PRD86:074514(2012)

$m_\pi = 0.30$ GeV and $m_N = 1.05$ GeV PRD92:014501(2015)

$m_s \sim$ physical strange quark mass

^{4}He, ^{3}He, NN($^3S_1$ and $^1S_0$)

		$m_\pi = 0.5$ GeV		$m_\pi = 0.3$ GeV		R
L	L [fm]	$N_{\rm conf}$	$N_{\rm meas}$	$N_{\rm conf}$	$N_{\rm meas}$	
32	2.9	200	192			
40	3.6	200	192			
48	4.3	200	192	400	1152	12
64	5.8	190	256	160	1536	5

$R = (N_{\rm conf} \cdot N_{\rm meas})_{0.3{\rm GeV}} / (N_{\rm conf} \cdot N_{\rm meas})_{0.5{\rm GeV}}$

Smear source and point sink (N with $p = 0$) operators

$\rightarrow$ after some tests in $N_f = 0$, consider large overlap to ground state

Results at $m_\pi = 0.5$ and 0.3 GeV

Computational resources

PACS-CS, T2K-Tsukuba, HA-PACS, COMA at Univ. of Tsukuba

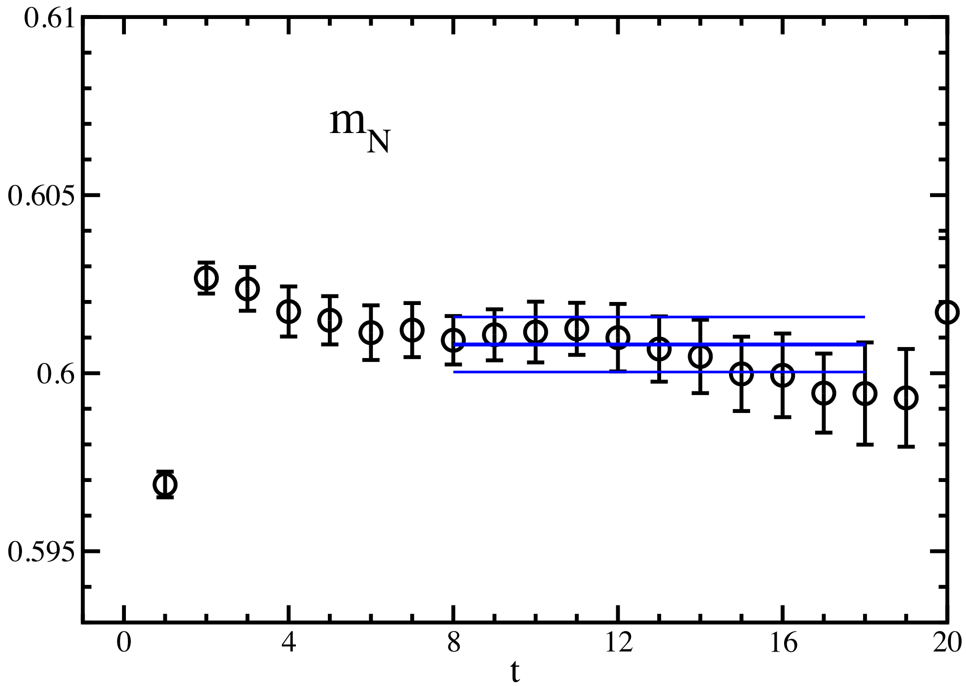
T2K-Tokyo and FX10 at Univ. of Tokyo, and K at AICS

Results

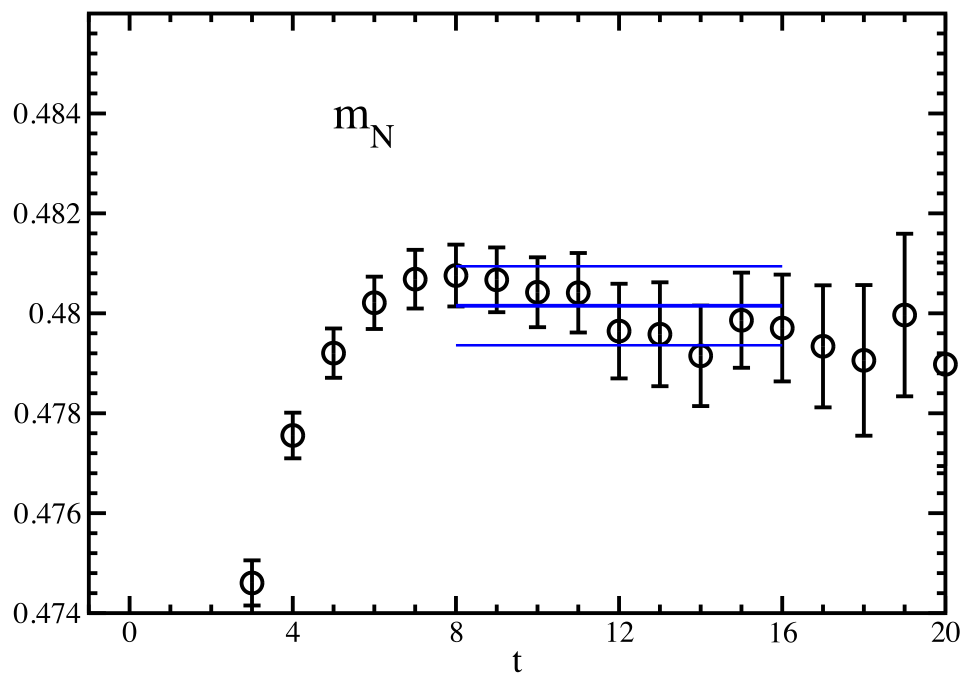
Effective mass of nucleon on $L = 5.8$ fm

$$\text{Effective } m_N = \log\left(\frac{C_N(t)}{C_N(t+1)}\right)$$

$m_\pi = 0.5$ GeV



$m_\pi = 0.3$ GeV



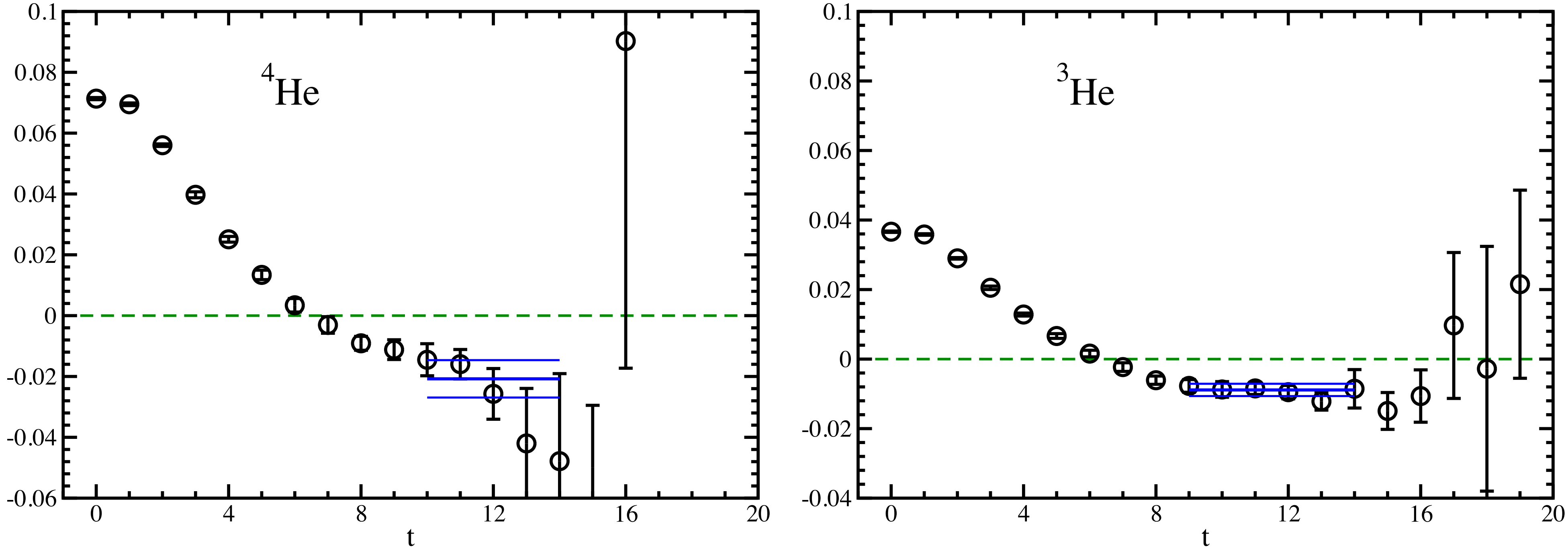
- Good plateau $t \gtrsim 7$
- Statistical error $< 0.2\%$

$\Delta E_L = E_0 - N_N m_N$ in ^{4}He and ^{3}He channels

at $m_\pi = 0.5$ GeV on $L = 5.8$ fm

TY *et al.*, PRD86:074514(2012)

$$\Delta E_L = \log\left(\frac{R_{^4\mathrm{He}}(t)}{R_{^4\mathrm{He}}(t+1)}\right) \text{ with } R_{^4\mathrm{He}}(t) = \frac{C_{^4\mathrm{He}}(t)}{(C_N(t))^4}$$

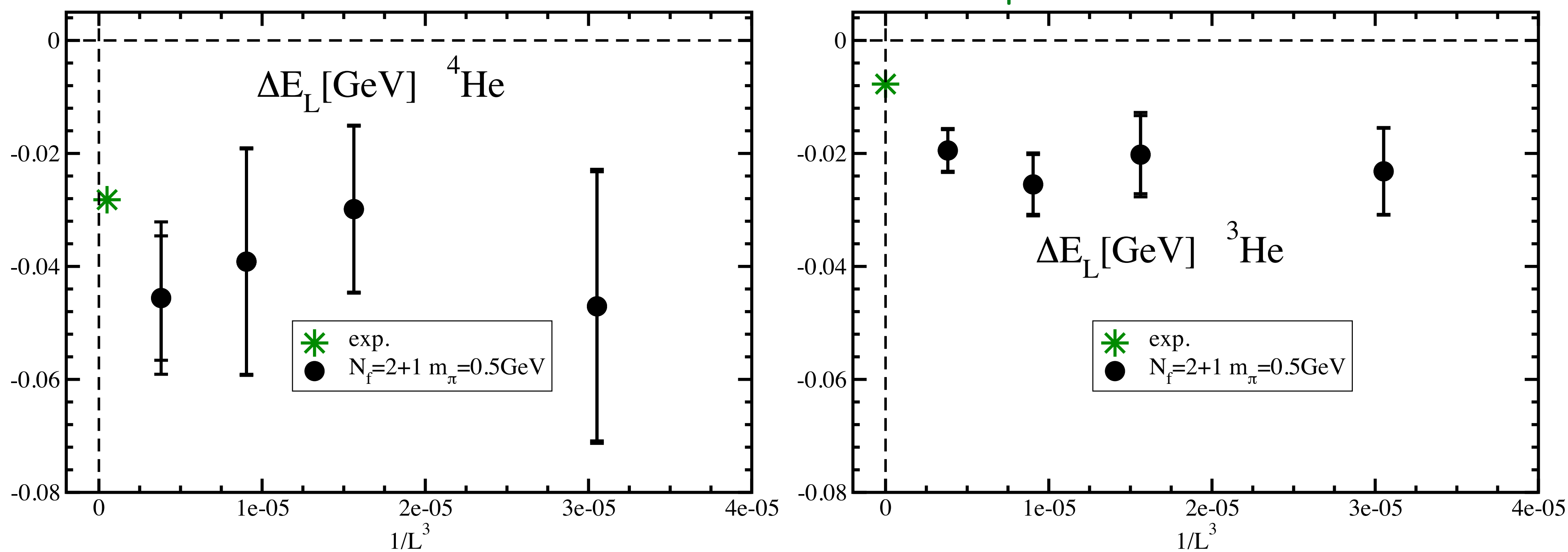


- Statistical error under control in $t < 12$
- Negative ΔE_L in both channels
- Plateau region $\sim$ plateau region of m_N

^{4}He and ^{3}He channels $\Delta E_L = E_0 - N_N m_N$ at $m_\pi = 0.5$ GeV

TY *et al.*, PRD86:074514(2012)

Identification of bound state from volume dependence of ΔE



- $\Delta E_L < 0$ and mild volume dependence
- Infinite volume extrapolation with $\Delta E_L = -\Delta E_{\text{bind}} + C/L^3$

^{4}He and ^{3}He channels $\Delta E_L = E_0 - N_N m_N$ at $m_\pi = 0.5$ GeV

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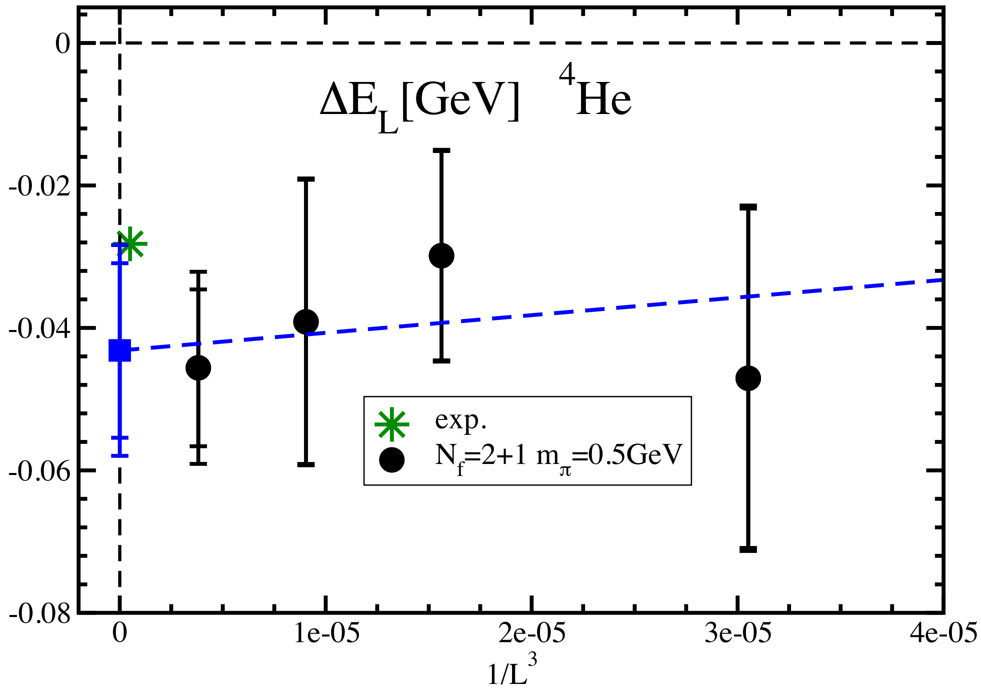
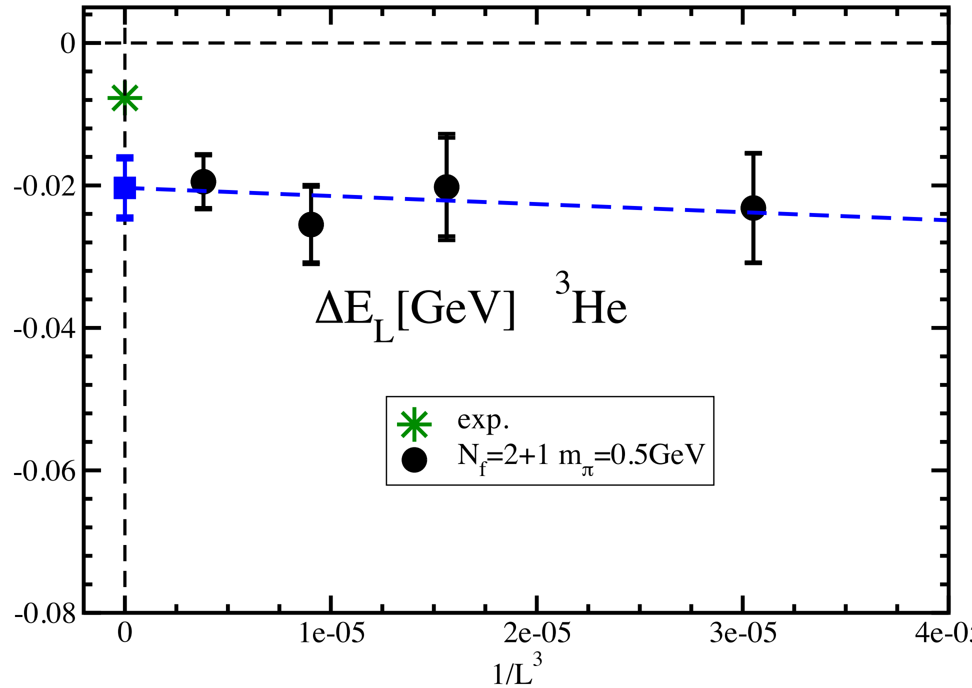




$\Delta E_{^4\text{He}} = 43(12)(8)$ MeV

$\Delta E_{^3\text{He}} = 20.3(4.0)(2.0)$ MeV

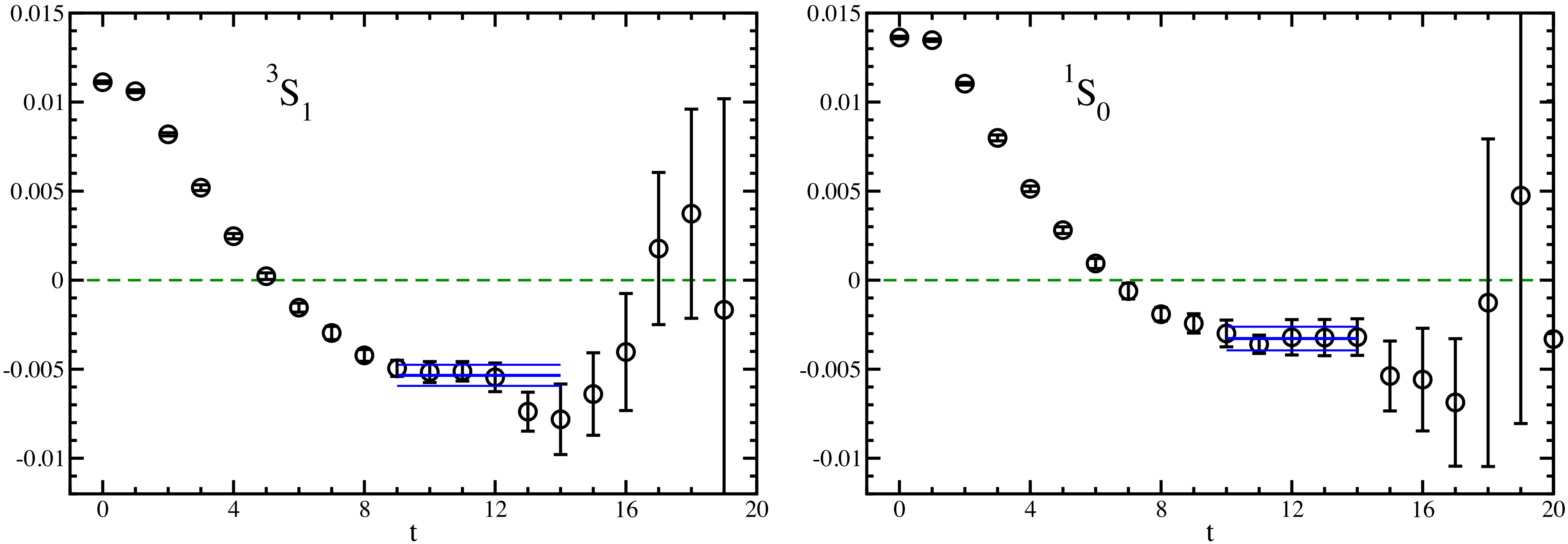
Observe bound state in both channels

ΔE: small difference from $\exp(-cL)$ fit due to large error

ΔE_L in 2-nucleon channels at $m_\pi = 0.5$ GeV on $L = 5.8$ fm

TY *et al.*, PRD86:074514(2012)

$$\Delta E_L = \log\left(\frac{R_{NN}(t)}{R_{NN}(t+1)}\right) \text{ with } R_{NN}(t) = \frac{C_{NN}(t)}{(C_N(t))^2}$$



- Statistical error under control in $t \leq 12$
- Smaller error than ^{4}He and ^{3}He channels
- Negative ΔE_L in both channels
- Plateau region $\sim$ plateau region of m_N

NN (^{3}S$_1$ and ^{1}S$_0$) channels $\Delta E_L = E_0 - 2m_N$ at $m_\pi = 0.5$ GeV

TY *et al.*, PRD86:074514(2012)

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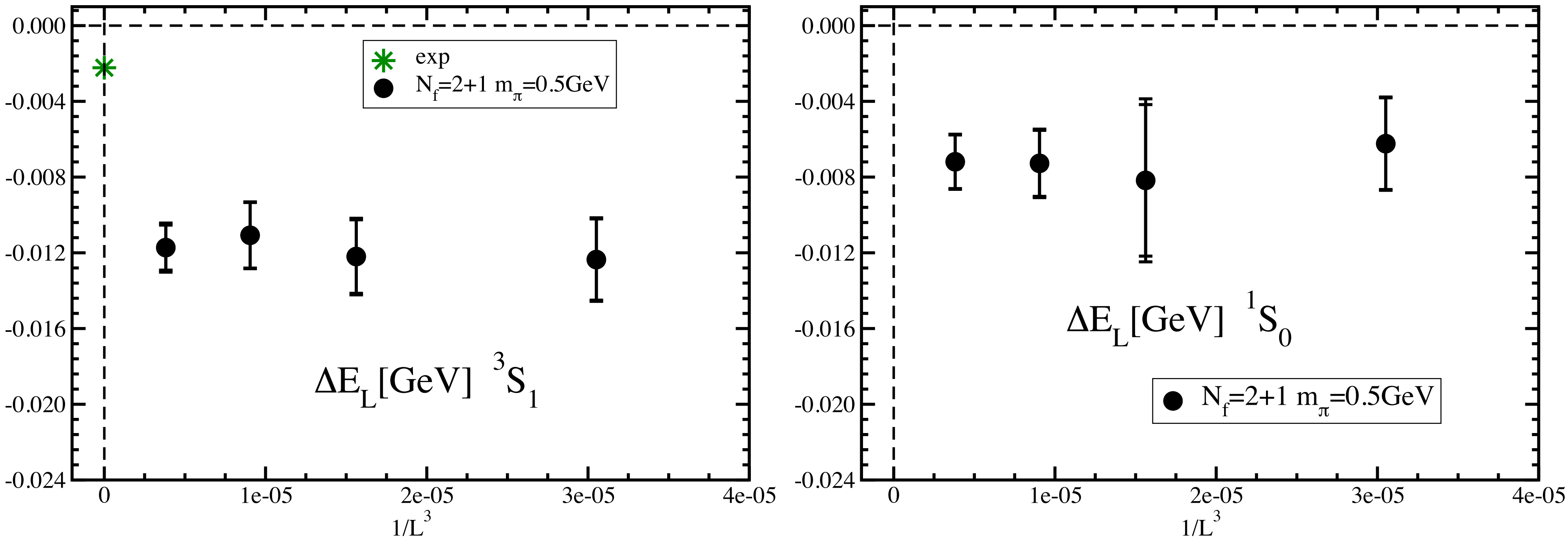

- Negative ΔE_L
- Infinite volume extrapolation of ΔE_L '04 Beane *et al.*, '06 Sasaki & TY

$$\Delta E_L = -\frac{\gamma^2}{m_N}\left\{1 + \frac{C_\gamma}{\gamma L}\sum_{\vec{n}}{}' \frac{\exp(-\gamma L\sqrt{\vec{n}^2})}{\sqrt{\vec{n}^2}}\right\}, \quad \Delta E_{\mathrm{bind}} = \frac{\gamma^2}{m_N}$$

based on Lüscher's finite volume formula

NN ($^3S_1$ and $^1S_0$) channels $\Delta E_L = E_0 - 2m_N$ at $m_\pi = 0.5$ GeV

TY *et al.*, PRD86:074514(2012)

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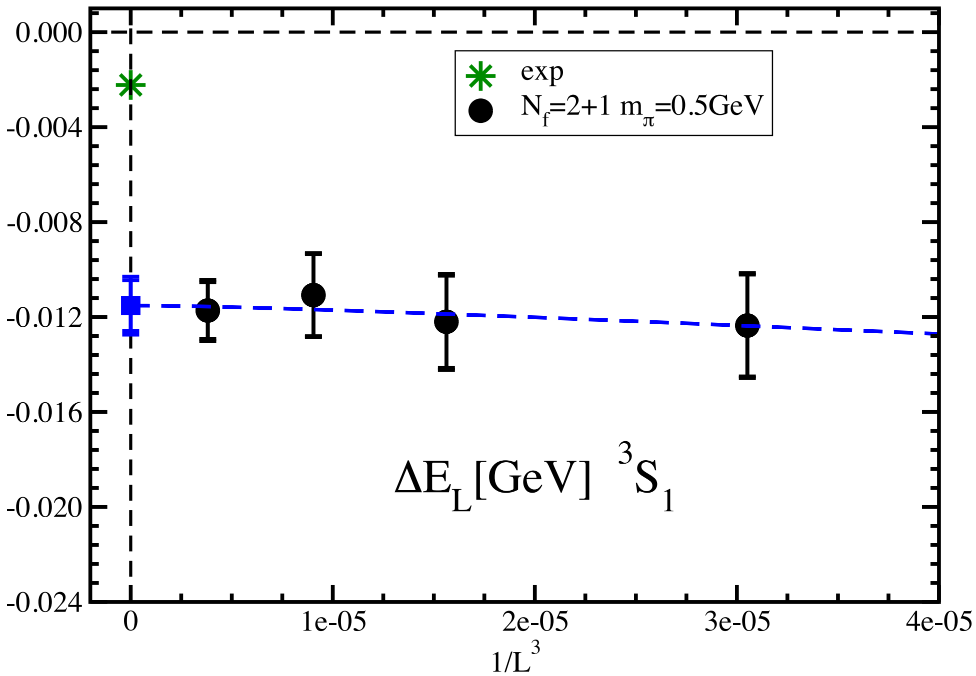
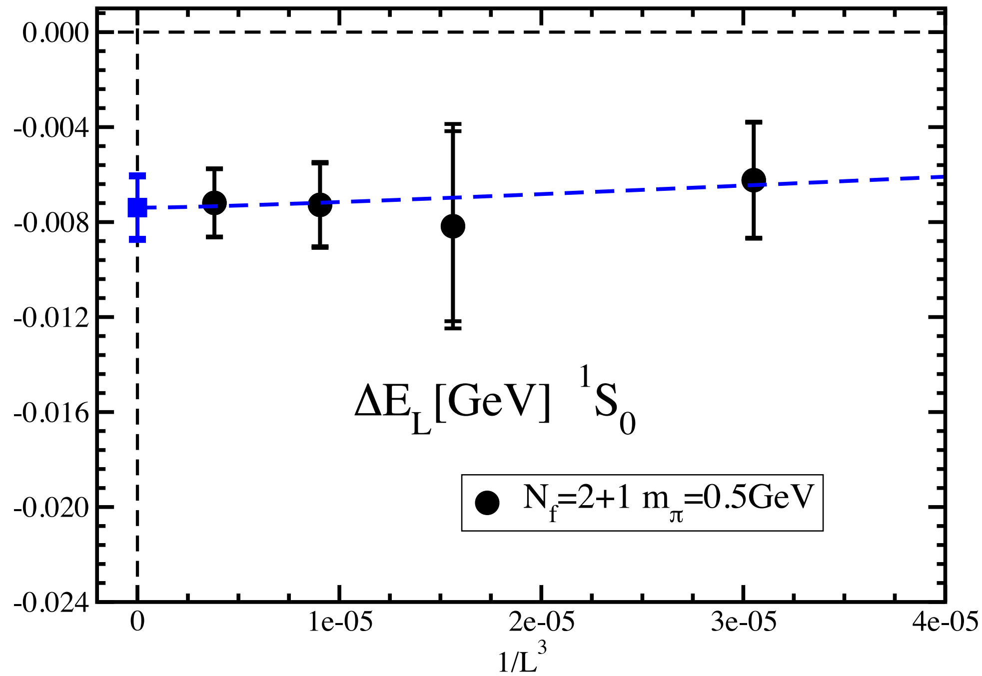


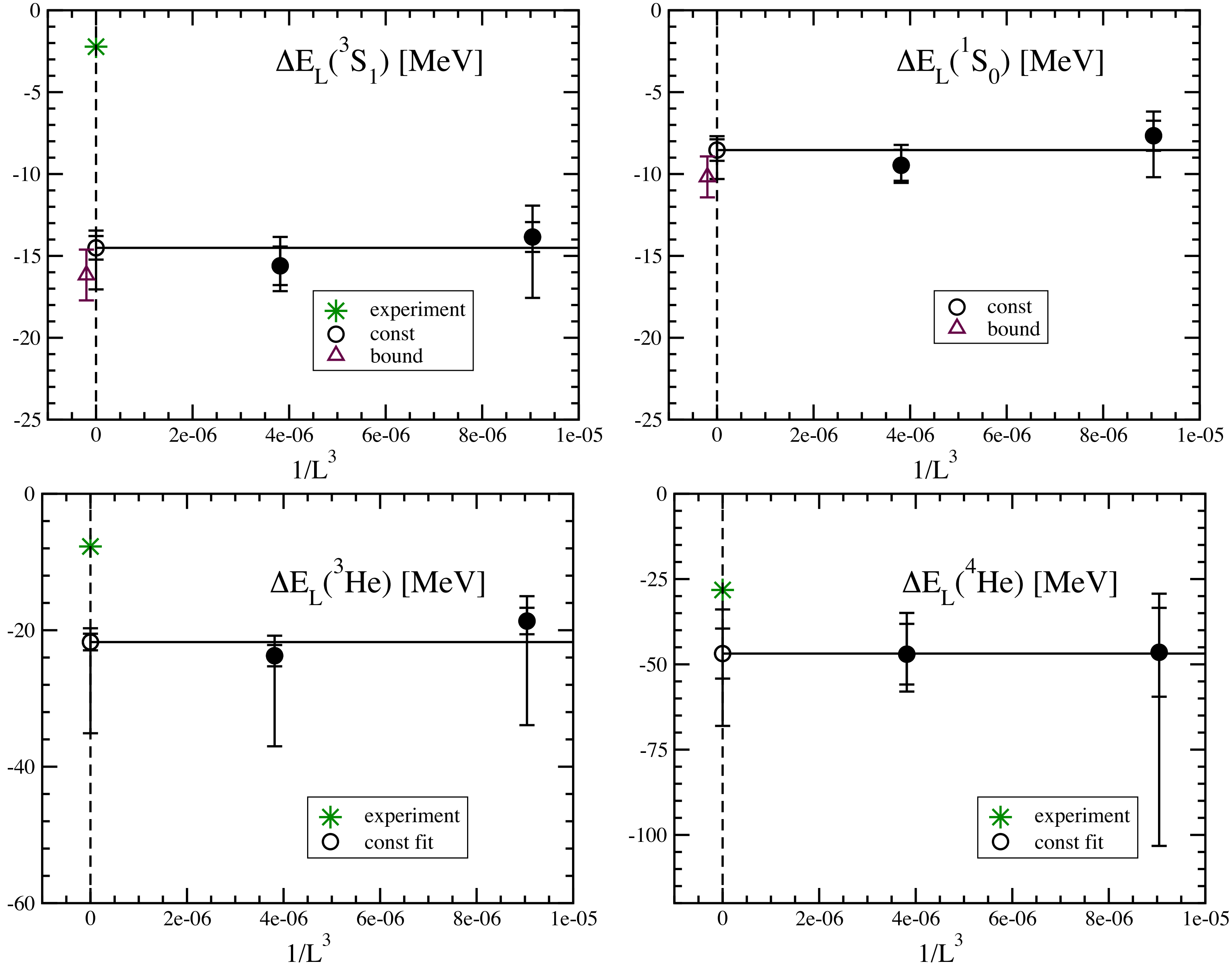


Bound state in both channels ← different from experiment

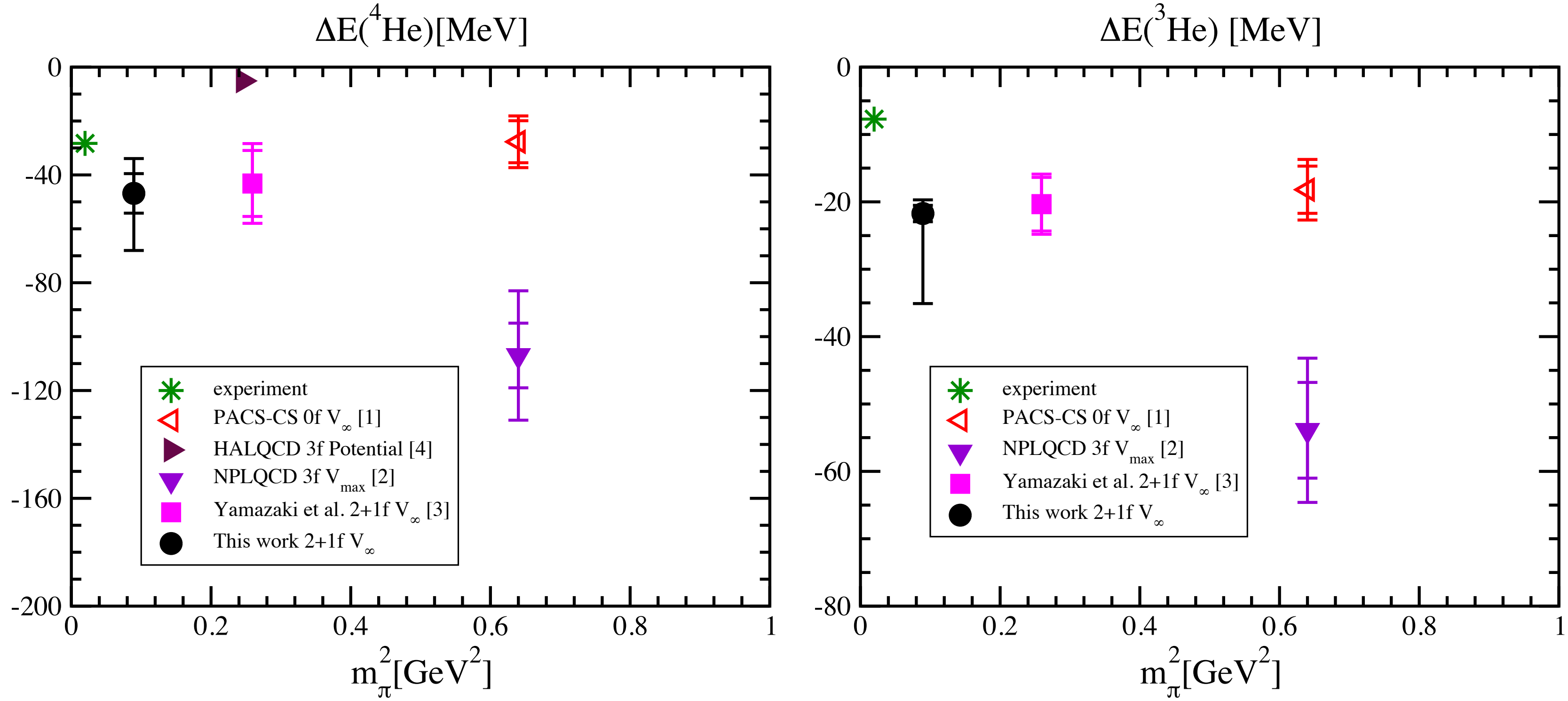
$\Delta E_{^3S_1} = 11.5(1.1)(0.6)$ MeV $\qquad \Delta E_{^1S_0} = 7.4(1.3)(0.6)$ MeV

Results at $m_\pi = 0.3$ GeV

TY *et al.*, PRD92:014501(2015)



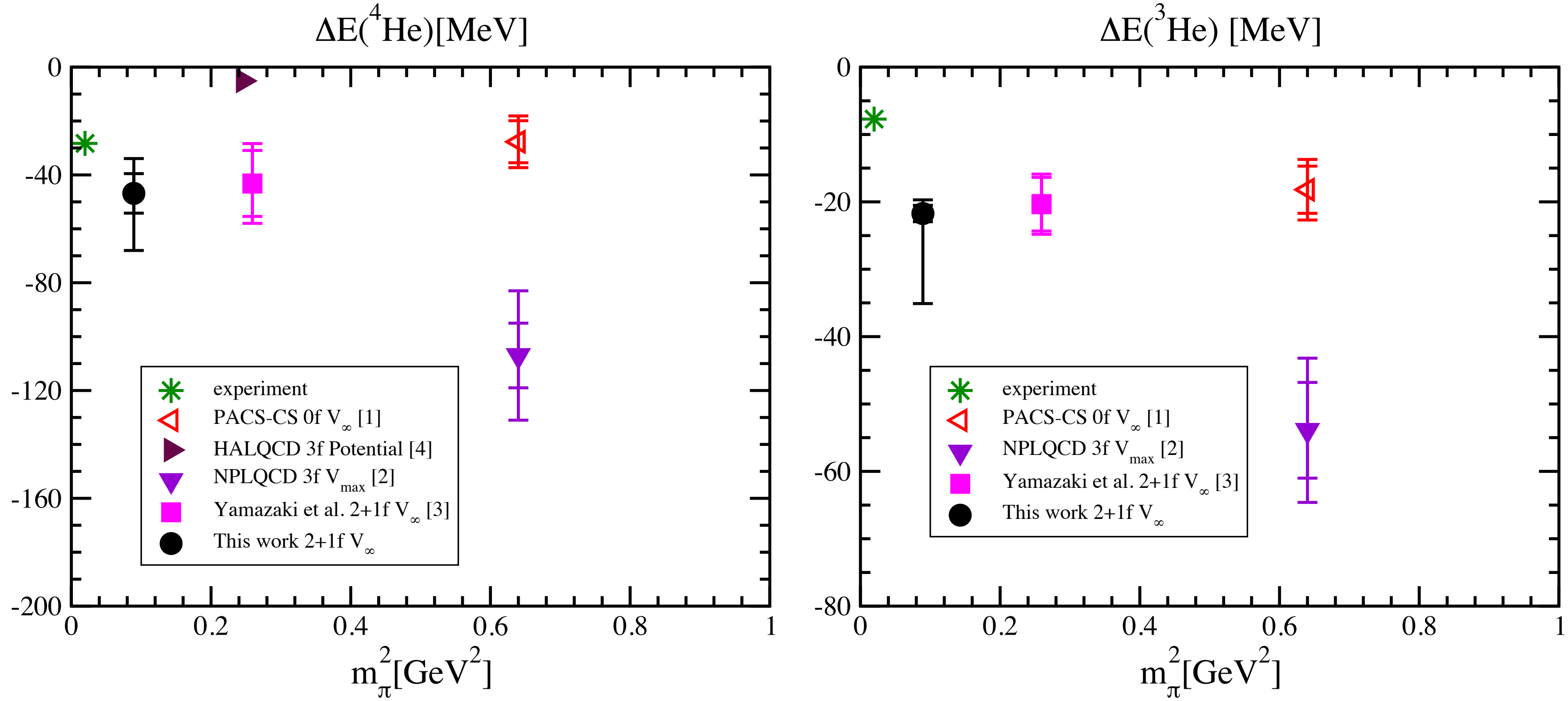
Comparison of ^{4}He and ^{3}He channels



Light nuclei likely formed in 0.3 GeV $\leq m_\pi \leq$ 0.8 GeV

Same order of ΔE to experiments

Comparison of ^{4}He and ^{3}He channels



Light nuclei likely formed in 0.3 GeV $\leq m_\pi \leq$ 0.8 GeV

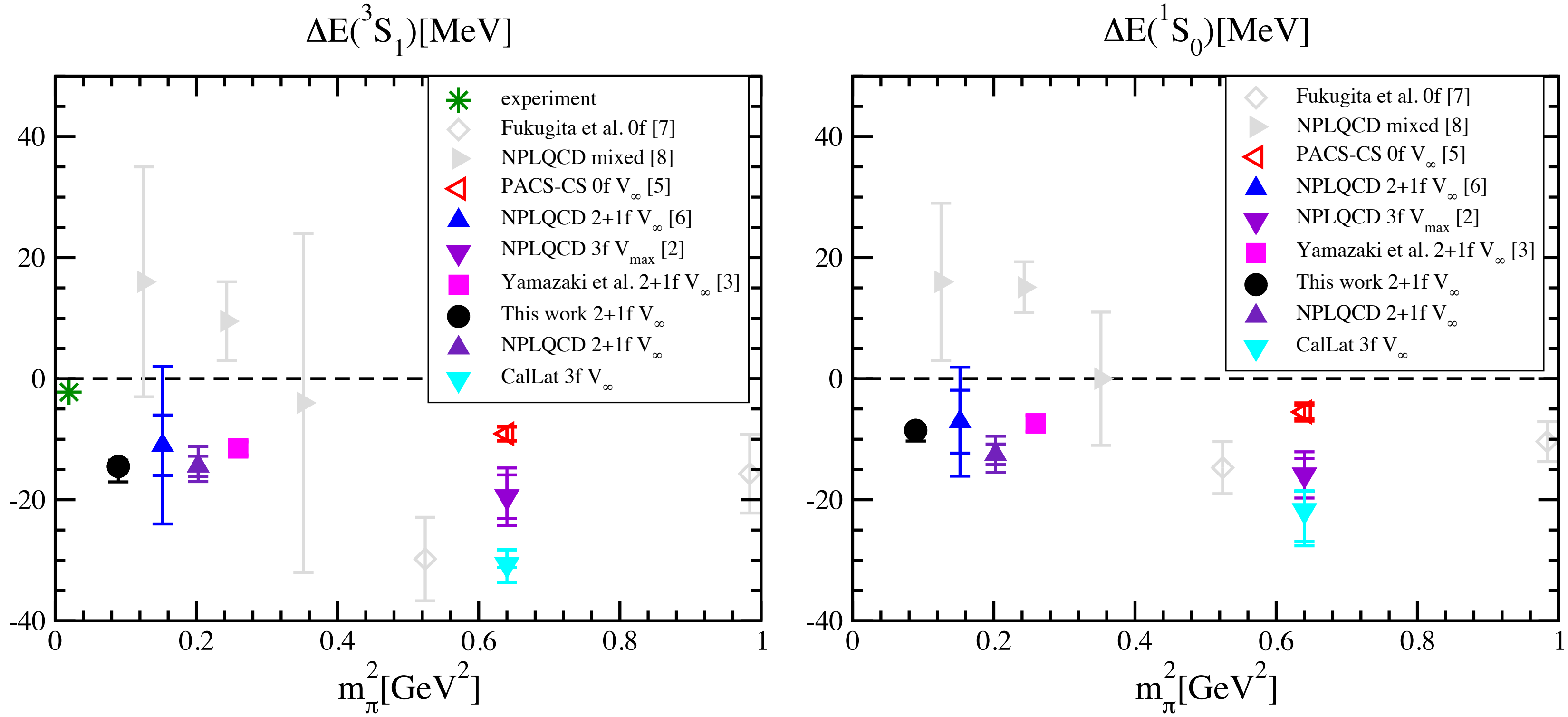
Same order of ΔE to experiments $\rightarrow$ relatively easier than NN

large $|\Delta E|$ makes less V dependence at physical m_π

touchstone of quantitative understanding of nuclei from lattice QCD

Investigations of m_π dependence $\rightarrow m_\pi \sim 0.145$ GeV on $L \sim 8$ fm

Comparison of NN channels



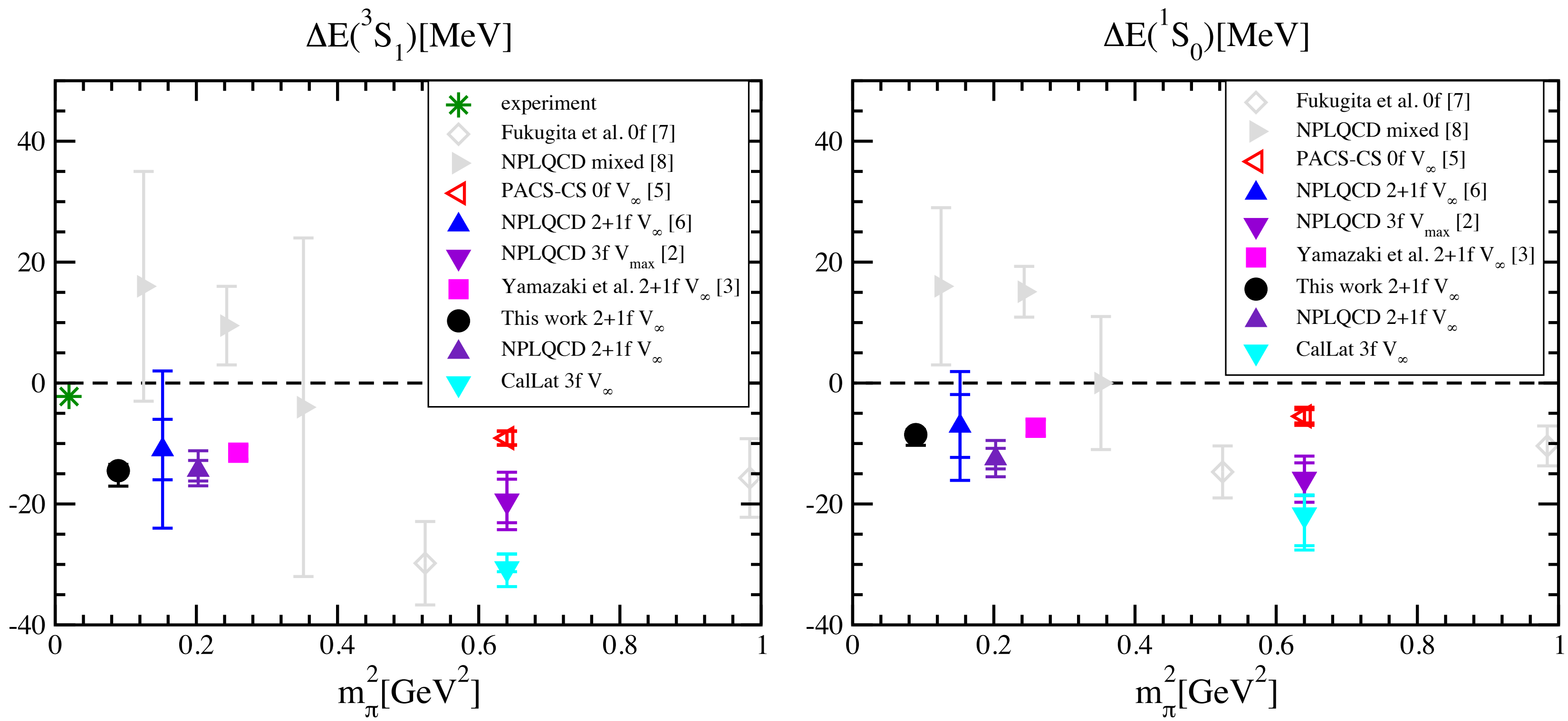
gray data: single volume calculation

$L^3 \to \infty$ data: existence of bound states in ^{3}S$_1$ and ^{1}S$_0$

inconsistent with experiment due to larger m_π(?)

Investigation of m_π dependence $\to m_\pi \sim 0.145$ GeV on $L \sim 8$ fm

Comparison of NN channels



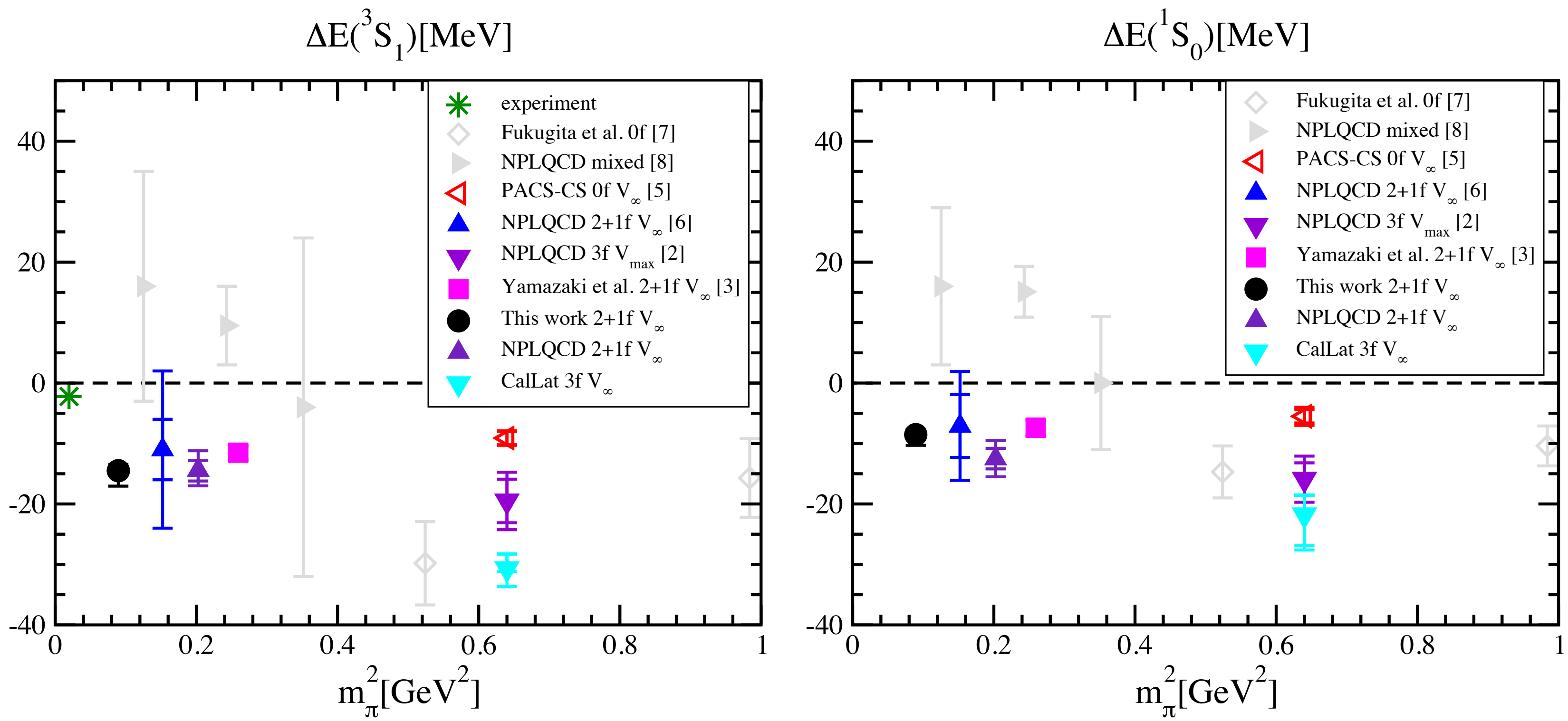
gray data: single volume calculation

Investigations of m_π dependence $\rightarrow$ $m_\pi \sim 0.145$ GeV on $L \sim 8$ fm

Large finite volume effect expected even on $L \sim 8$ fm

'86 Lüscher, '04 Beane *et al.*, '14 Briceño *et al.*

Comparison of NN channels



gray data: single volume calculation

Investigations of m_π dependence $\rightarrow m_\pi \sim 0.145$ GeV on $L \sim 8$ fm

Large finite volume effect expected even on $L \sim 8$ fm

'86 Lüscher, '04 Beane *et al.*, '14 Briceño *et al.*

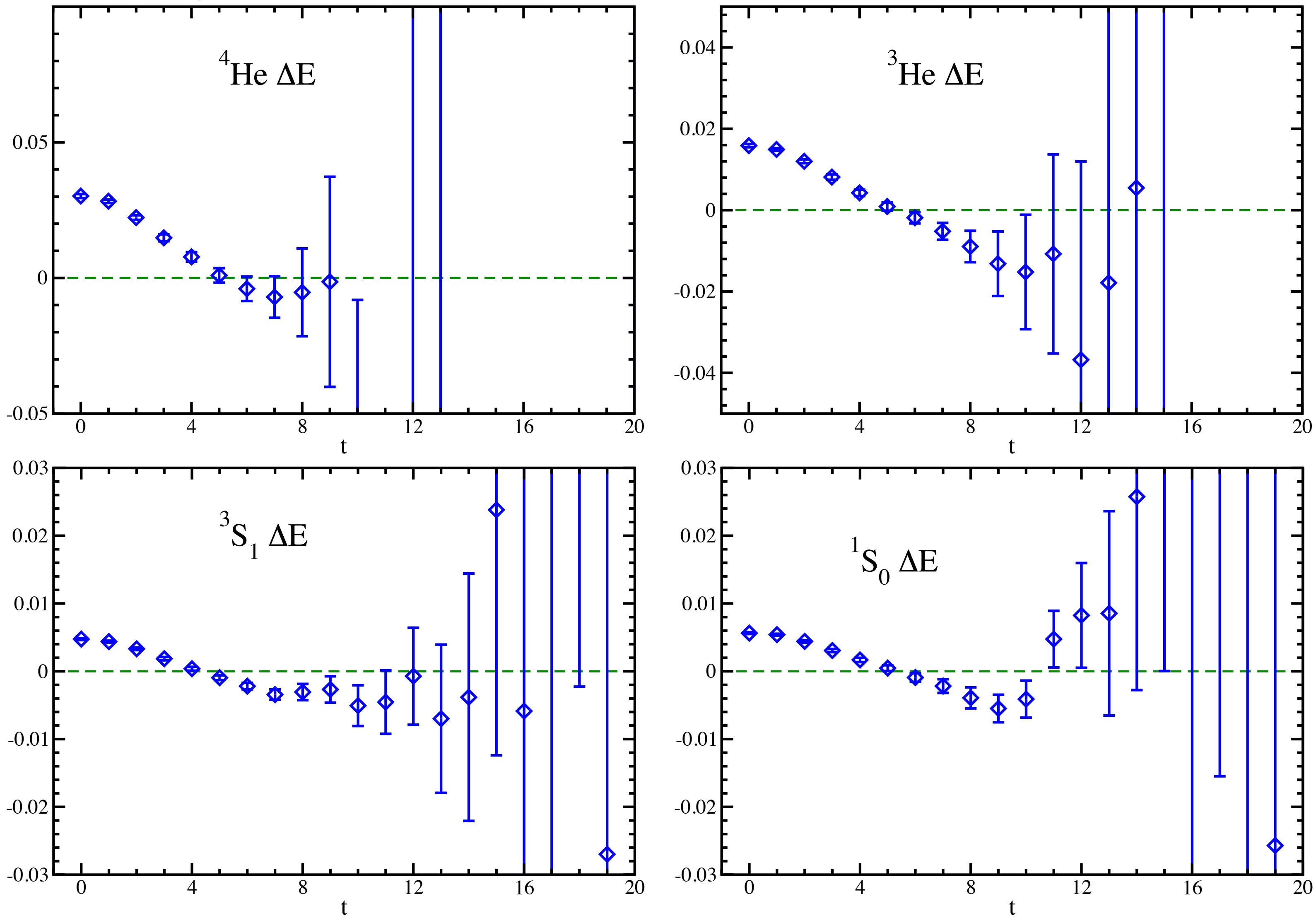
$^3S_1$: $\Delta E_{\text{exp}} = 2.2$ MeV

$$\Delta E_L = -\left(\Delta E_{\text{exp}} + \mathcal{O}(\exp(-L\sqrt{m_N \Delta E_{\text{exp}}}))\right) \lesssim -4 \text{ MeV}$$

$^1S_0$: $a_0^{\text{exp}} = 23.7$ fm

$$\Delta E_L = -\frac{4\pi a_0^{\text{exp}}}{m_N L^3} + \mathcal{O}(1/L^4) \lesssim -2 \text{ MeV}$$

Very preliminary results of ΔE at $m_\pi \sim 0.145$ GeV on $L \sim 8$ fm



Computational resources

HA-PACS, COMA @Univ. of Tsukuba, K @AICS, FX100 @RIKEN

Nucleon form factors at almost physical m_π

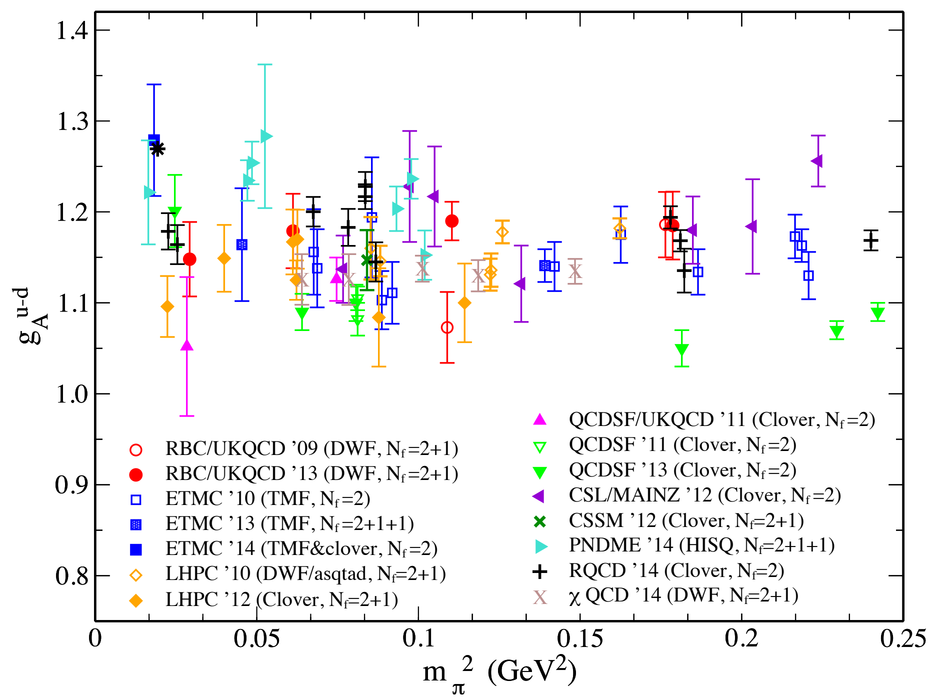
in collaboration with
K.-I. Ishikawa, Y. Kuramashi, S. Sasaki, and A. Ukawa
for PACS Collaboration

Computational resources (the HPCI System Research Project: hp140155, hp150135)
COMA @Univ. of Tsukuba, FX10 @Univ. of Tokyo,
FX100 @RIKEN, System E @Kyoto Univ., FX100 @Nagoya Univ.

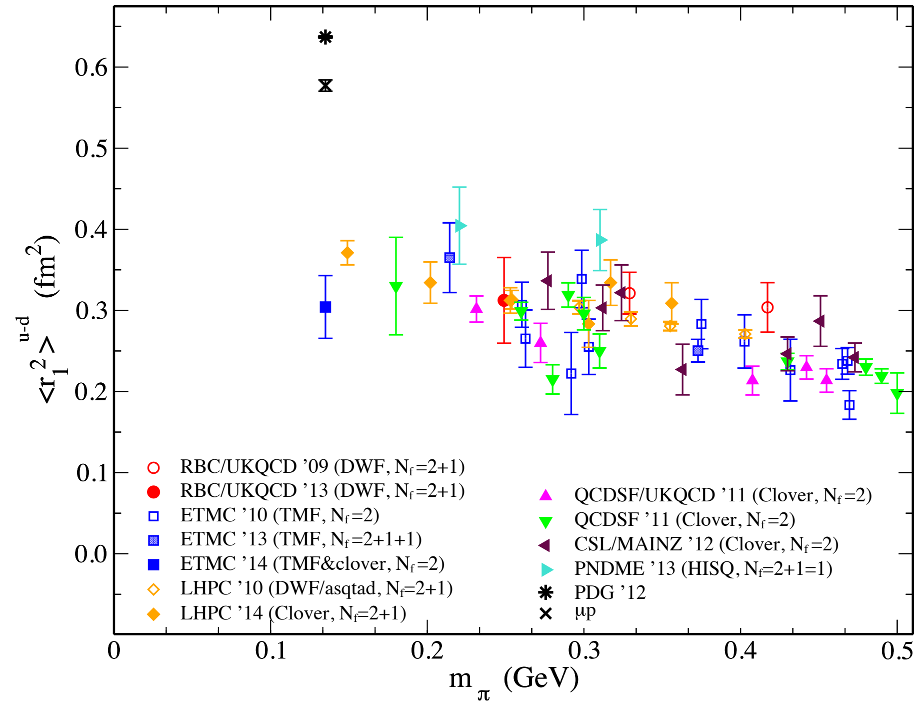
Example of large quark mass dependence near $m_\pi \to 0$
Isovector radii from form factors F_1 and F_2

Constantinou, Lat14 plenary

axial charge



charge radius $\langle r_1^2 \rangle$



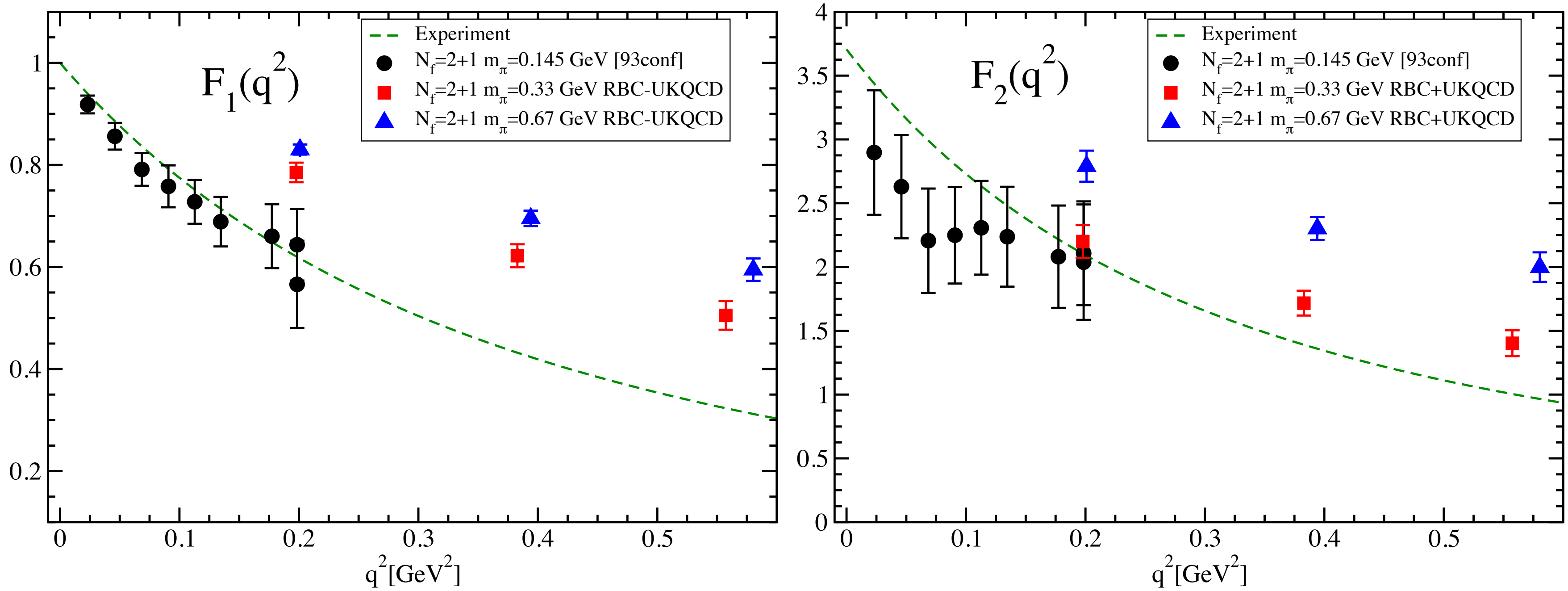
c.f.) '14 LHP, '15 Capitani *et al.*,'15 ETM, see also James's Lat15 plenary

important for understanding of nucleus property

Can we reproduce experiment at physical m_π?

Isovector F_1 and F_2 form factors

Preliminary results at $m_\pi \sim 0.145$ GeV on $L \sim 8$ fm



Need much more statistics

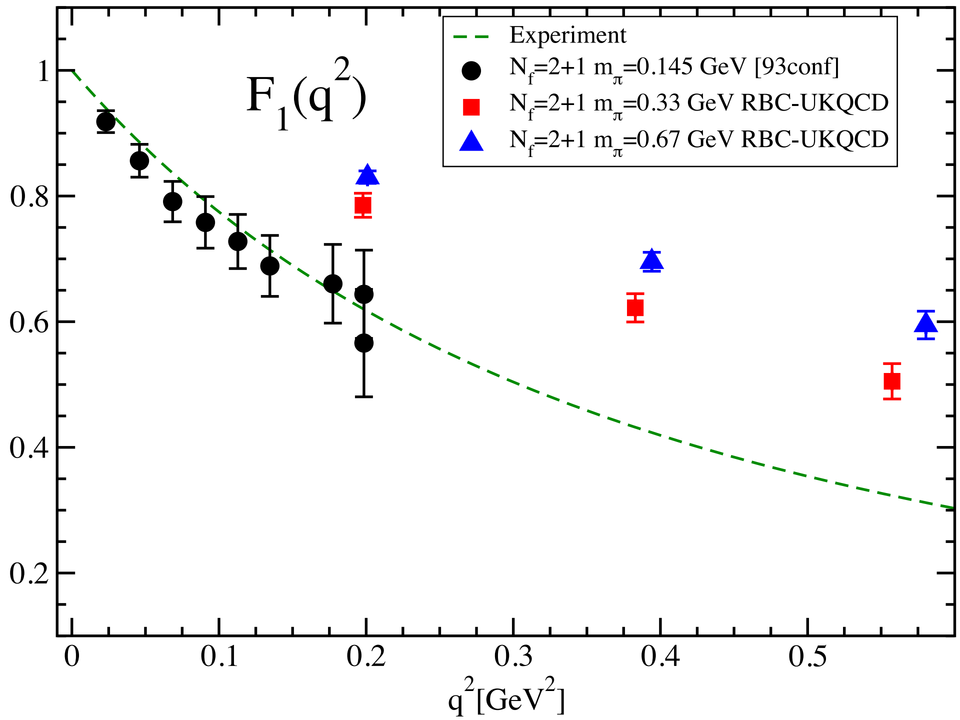
but encouraging signal in G_E

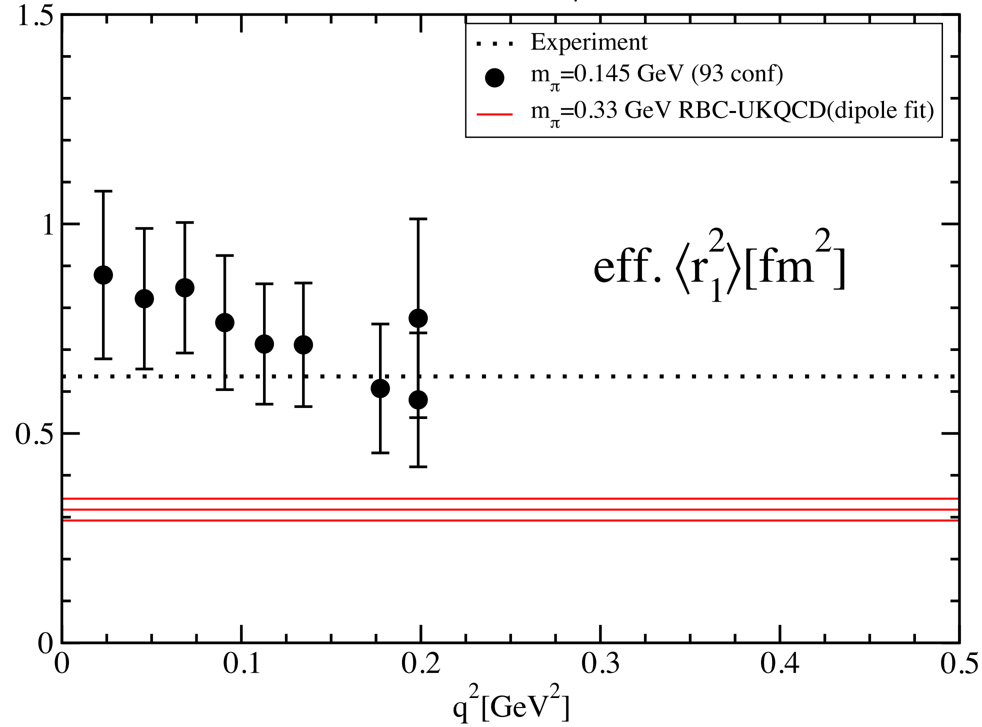
Charge radius $\langle r_1^2 \rangle$

Preliminary results at $m_\pi \sim 0.145$ GeV on $L \sim 8$ fm

Dipole form $F_1(q^2) = \left(1 + \frac{q^2}{12}\langle r_1^2 \rangle\right)^{-2}$

Eff. $\langle r_1^2 \rangle = \frac{12}{q^2}\left(\sqrt{\frac{1}{F_1(q^2)}} - 1\right)$





Statistical error is large

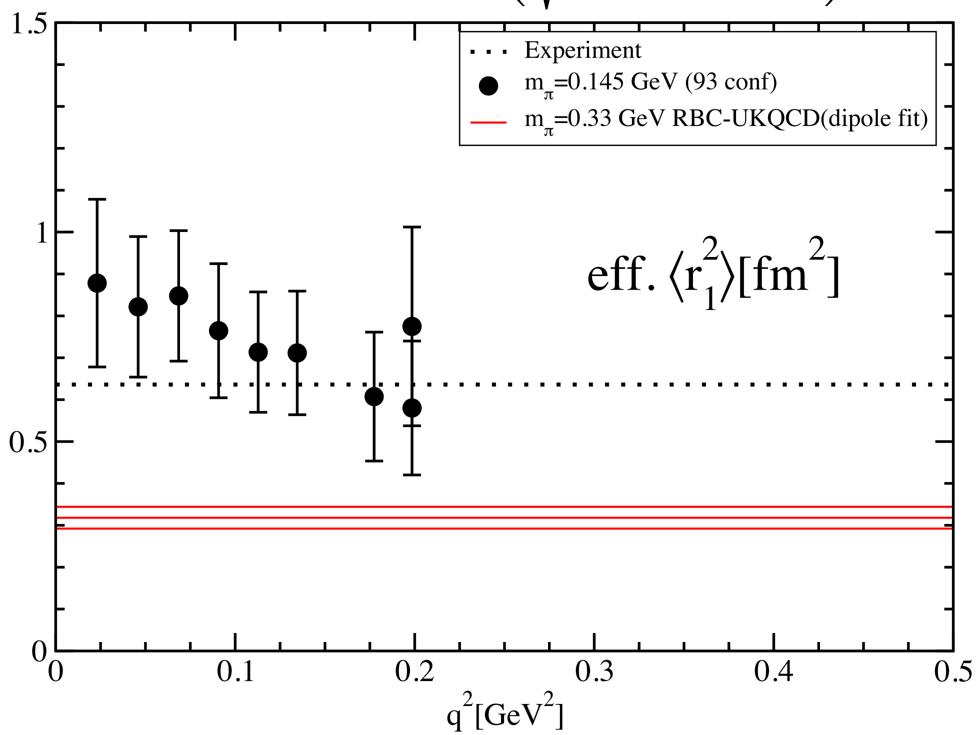
Charge radius $\langle r_1^2 \rangle$

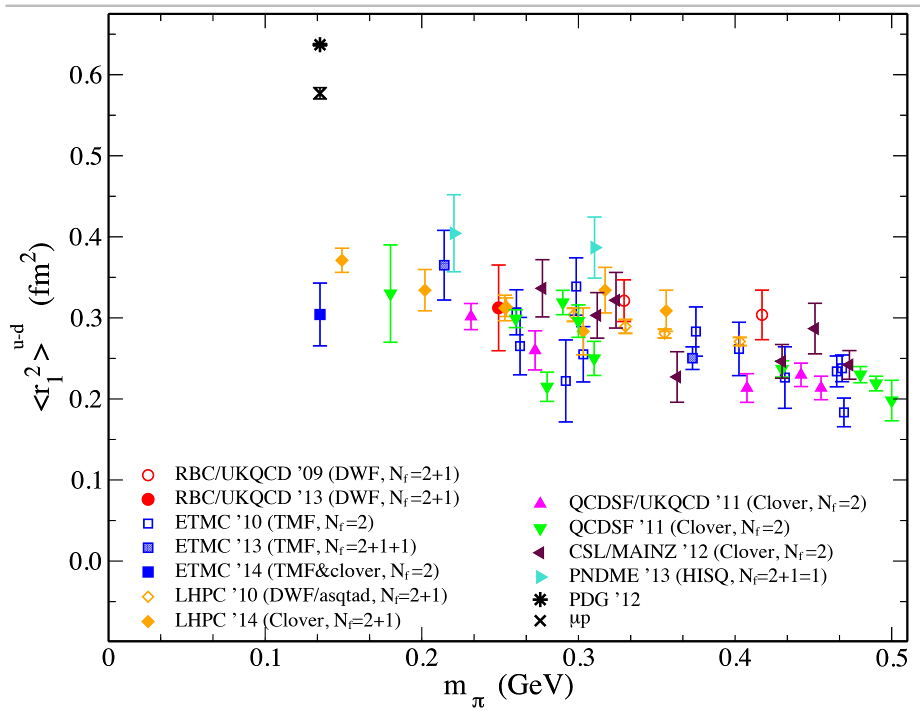
Preliminary results at $m_\pi \sim 0.145$ GeV on $L \sim 8$ fm

$$\text{Dipole form } F_1(q^2) = \left(1 + \frac{q^2}{12}\langle r_1^2 \rangle\right)^{-2}$$

$$\text{Eff. } \langle r_1^2 \rangle = \frac{12}{q^2}\left(\sqrt{\frac{1}{F_1(q^2)}} - 1\right)$$

Constantinou, Lat14 plenary





Need much more statistics
but encouraging signal

Axial charge $g_A = Z_A g_A^{\text{bare}}$

Preliminary results at $m_\pi \sim 0.145$ GeV on $L \sim 8$ fm

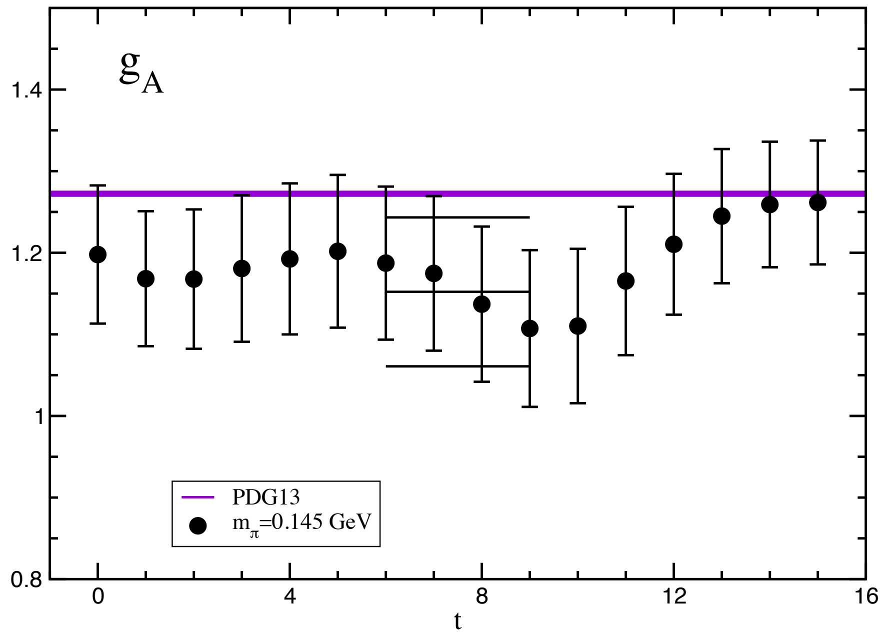
Z_A from SF scheme (Lattice 2015, Ishikawa for PACS Collaboration)

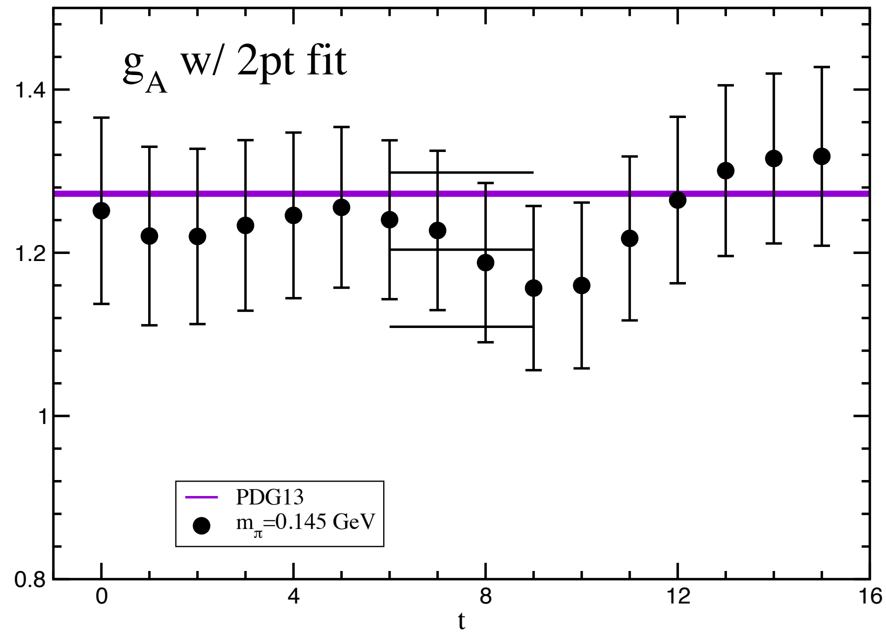
consistent with $Z_V = 1/G_E(0)$ within 1–2%

$g_A^{\text{bare}} = C_{A_3}(t)/C_N(t_{\text{sink}})$

$g_A^{\text{bare}} = C_{A_3}(t)/(Z_N^2 \exp(-M_N t_{\text{sink}}))$

Z_N and M_N from fit of $C_N(t)$





Discrepancy of two results → systematic error

roughly consistent with experiment,

but need much more statistics for stringent test

Summary

$N_f = 2+1$ lattice QCD at $m_\pi = 0.5$ and 0.3 GeV

- Volume dependence of ΔE

  $\Delta E \neq 0$ of 0th state in infinite volume limit
  $\rightarrow$ bound state in ^{4}He, ^{3}He, ^{3}S$_1$ and ^{1}S$_0$
  at $m_\pi = 0.5$ and 0.3 GeV

- ΔE larger than experiment and small m_π dependence
- Bound state in ^{1}S$_0$ not observed in experiment

  $N_f = 3$ at $m_\pi = 0.8$ GeV by NPLQCD
  and CalLat with sophisticated sources
  $N_f = 2+1$ $m_\pi = 0.45$ GeV by NPLQCD

  No bound state in HALQCD method
  variational method could give hint to solve the difference

Need further investigations

e.g. systematic error from large m_π and finite lattice spacing

$N_f = 2+1$ $m_\pi \sim 0.145$ GeV on $L \sim 8$ fm
ΔE for nuclei and Isovector form factors of nucleon