

Microscopic description of ${}^6\text{He}$ elastic scattering

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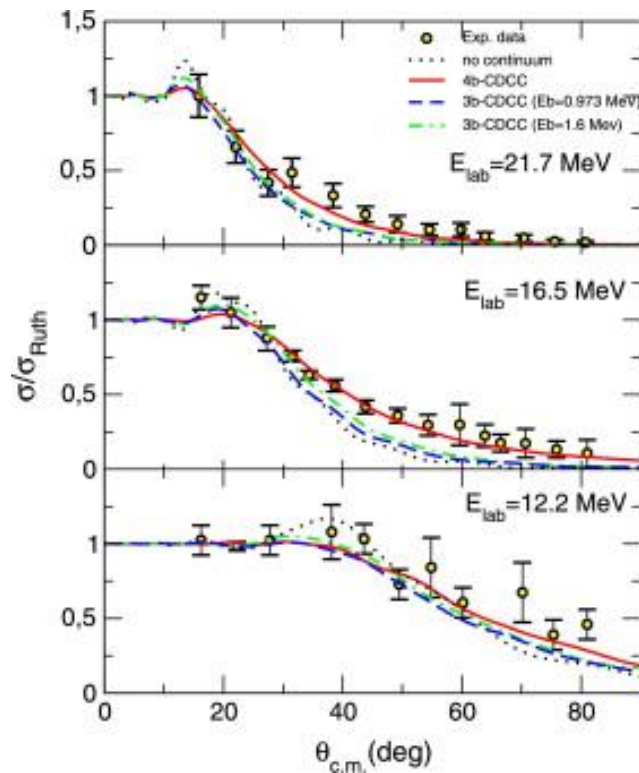
1. Introduction

Presentation of the ${}^6\text{He}$ nucleus

- Simplest halo 3-body nucleus
- Well described by an $\alpha+n+n$ structure
- Many theoretical calculations: rms radius, energy spectrum, E1 strength, etc.
- Challenge: describing reactions involving ${}^6\text{He}$

Many data

- Essentially elastic scattering around the Coulomb barrier: ${}^{208}\text{Pb}$, ${}^{120}\text{Sn}$, ${}^{64}\text{Ni}$, ${}^{209}\text{Bi}$, etc.



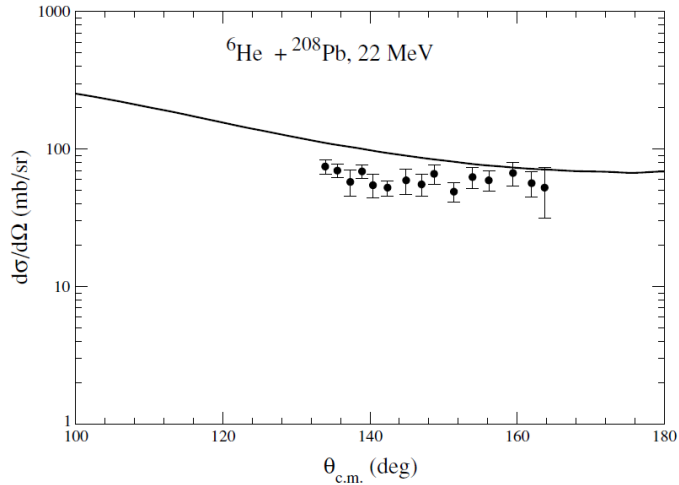
${}^6\text{He}+{}^{58}\text{Ni}$

V. Morcelle et al., PLB 732 (2014) 228

1. Introduction

Other cross sections (non elastic)

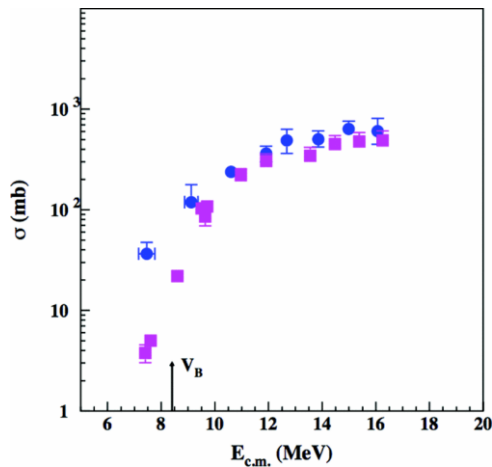
Example: **Alpha production**



${}^6\text{He} + {}^{208}\text{Pb}$

A. Sanchez Benitez et al., JPG31
(2005) S1953

Fusion: more difficult (must include transfer)



${}^6\text{He} + {}^{64}\text{Zn}$

V. Scuderi et al., PRC 84 (2011) 064604

1. Introduction

Current status of reactions with ${}^6\text{He}$

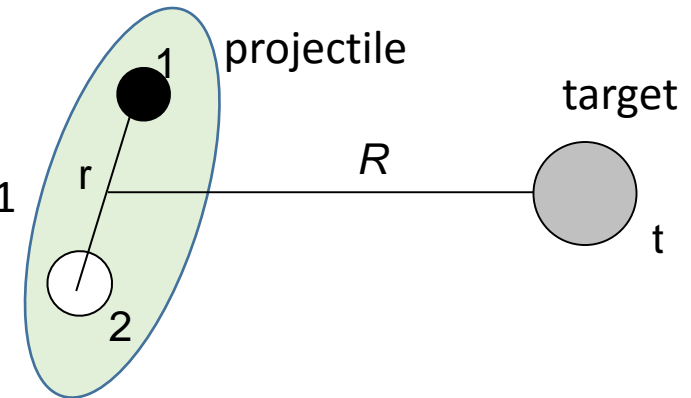
- Optical model: several parameters \rightarrow predictive power? (used to fit data)
- CDCC (Continuum Discretized Coupled Channels)

Solution of 3-body (4-body) scattering problem

G. Rawitscher, Phys. Rev. C 9, 2210 (1974)

M. Kamimura et al, Prog. Theor. Phys. Suppl. 89 (1986) 1

N. Austern et al., Phys. Rep. 154 (1987) 126

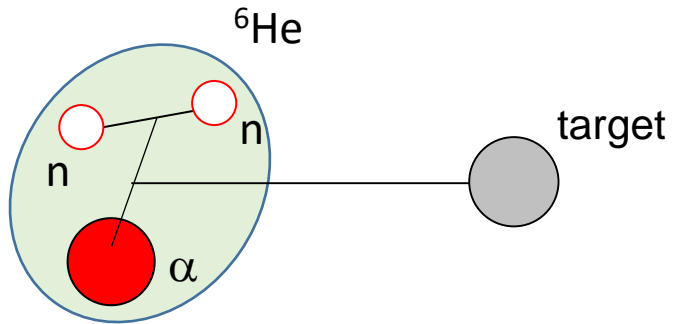


- Projectile initially assumed to be described by two clusters
- Introduced for deuteron-induced reactions (breakup important)
 \rightarrow well adapted to exotic nuclei
- Various cross sections: elastic, breakup, fusion, etc. (**Energy near the Coulomb barrier**)
- Recently extended to 3-body projectiles (${}^6\text{He}=\alpha+n+n$, ${}^9\text{Be}=\alpha+\alpha+n$)

1. Introduction

Current status of reactions with ${}^6\text{He}$

- Non microscopic CDCC (Continuum Discretized Coupled Channel)



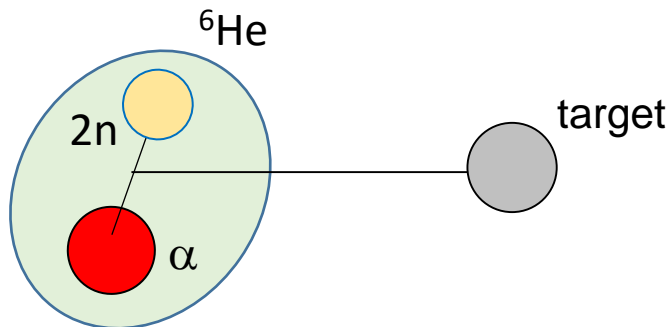
$\alpha+n+n$ description of ${}^6\text{He}$

${}^6\text{He}+{}^{209}\text{Bi}$

T. Matsumoto et al., PRC 73 (2006) 051602(R)

${}^6\text{He}+{}^{58}\text{Ni}$

V. Morcelle et al., PLB 732 (2014) 228



$\alpha+2n$ description of ${}^6\text{He}$

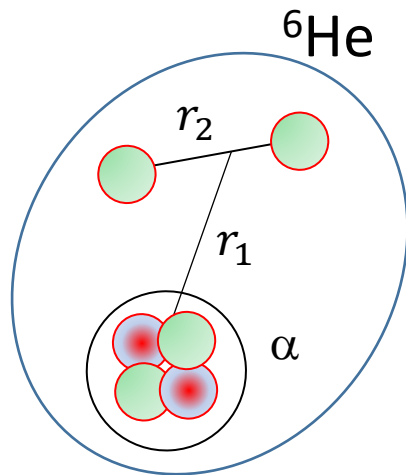
${}^6\text{He}+{}^{58}\text{Ni}$

V. Morcelle et al., PLB 732 (2014) 228

➔ Need for n +Target, α +Target, $2n$ +Target optical potentials

1. Introduction

Here: 6-nucleon description of ${}^6\text{He}$ + cluster approximation



$$\text{RGM: } \psi_{0,k}^{j\pi} = \mathcal{A} \varphi_{\alpha} \varphi_n \varphi_n g_k^{j\pi}(r_1, r_2)$$

j =spin of the ${}^6\text{He}$ system (here: 0^+ to 3^-)

k =excitation level

φ_{α} = $0s$ shell-model wave function



Scattering

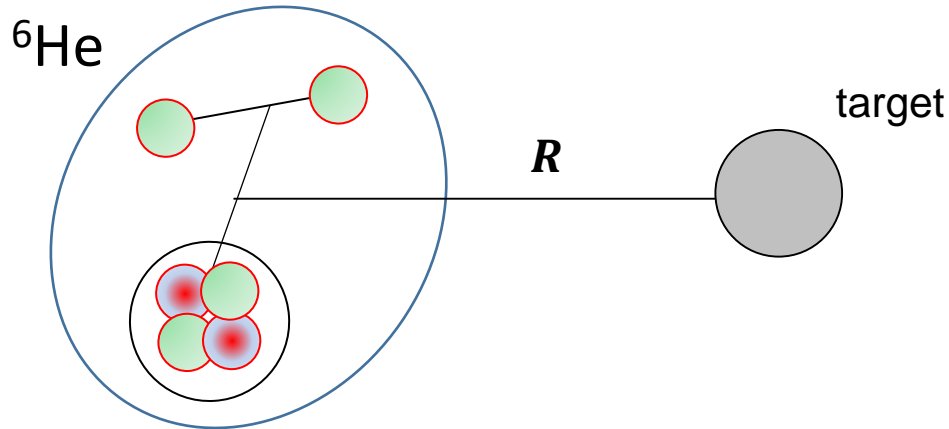


Microscopic CDCC

P.D., M. Hussein, PRL 111 (2013) 082701
(${}^7\text{Li}=\alpha+t$)

- Antisymmetrization included in ${}^6\text{He}$
- Only nucleon-target potential
- *Breakup included*

1. Introduction



Microscopic CDCC

Total Hamiltonian: $H = H_0 + T_R + \sum_{i=1}^6 v_{i-T}(r_i - R)$

With $H_0 = \sum_i t_i + \sum_{i<j} v_{ij}$ = microscopic Hamiltonian of ${}^6\text{He}$

$v_{i-T}(r)$ =optical potential neutron/proton + target (includes Coulomb)

Wave function: $\Psi^{JM\pi}(\mathbf{R}, \mathbf{r}_i) = \sum_{jLk} \underbrace{u_{jLk}^{J\pi}(R)}_{\text{To be determined}} \left[\underbrace{\psi_{0,k}^{j\pi}(\mathbf{r}_i)}_{\text{{}^6He wave functions}} \otimes \underbrace{Y_L(\Omega_R)}_{\text{Angular functions}} \right]^{JM}$

To be determined
→ scattering matrices

Angular functions

${}^6\text{He}$ wave functions

1. Introduction

Several steps:

1. Solve the ${}^6\text{He}$ problem: $H_0 \Psi_{0,k}^{j\pi} = E_k^{j\pi} \Psi_{0,k}^{j\pi}$
→ Generator Coordinate Method (GCM)
2. Compute the coupling potentials $V_{k,k'}^{j\pi,j'\pi'}(\mathbf{R}) = \langle \psi_{0,k}^{j\pi} | \sum_i v_{i-T}(\mathbf{r}_i - \mathbf{R}) | \psi_{0,k'}^{j'\pi'} \rangle$
→ densities + folding method
→ multipole expansion
3. Solve the coupled-channel system for each $j\pi$
→ R-matrix method (provides the scattering matrices)
4. Compute the cross sections from the scattering matrices (standard formula)

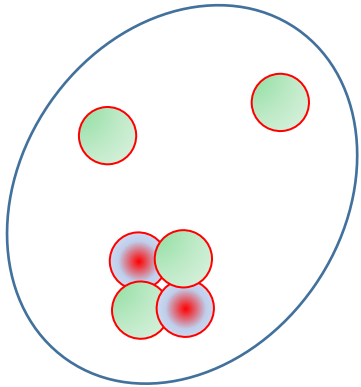
First step:

Microscopic description of ${}^6\text{He}$

2. Microscopic description of ${}^6\text{He}$

References

- Bound states: S. Korenov and P.D., *Nucl. Phys. A740* (2004) 249
M. Theeten, D. Baye and P. D. *Phys. Rev. C 74* (2006) 044304
- Scattering states: A. Damman and P. D., *Phys. Rev. C80* (2009) 044310



6-body Hamiltonian: $H_0 = \sum_i t_i + \sum_{i<j}(v_{ij}^N + v_{ij}^C)$

v_{ij}^C =Coulomb interaction (exact)

v_{ij}^N =Minnesota (parameter u)+ zero-range spin-orbit (parameter S_0)

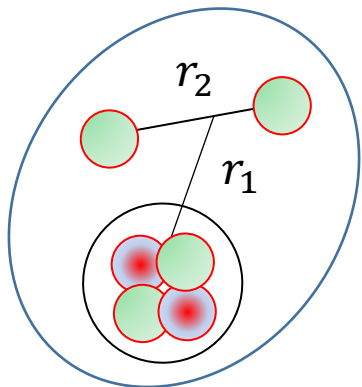


RGM approximation (clusters) : $\psi_{0,k}^{j\pi} = \mathcal{A}\varphi_\alpha\varphi_n\varphi_n g_k^{j\pi}(r_1, r_2)$

GCM: $\psi_{0,k}^{j\pi} = \sum_{i,i'} f_k^{j\pi}(R_{1i}, R_{2i'})\Phi^{jm\pi}(R_{1i}, R_{2i'})$

$R_{1i}, R_{2i'}$ = generator coordinates associated with r_1, r_2

$\Phi^{jm\pi}(R_{1i}, R_{2i'})$ =projected Slater determinant



2. Microscopic description of ${}^6\text{He}$

Hyperspherical coordinates

$$X = \frac{R_2}{\sqrt{\mu_{12}}}$$
$$Y = \frac{R_1}{\sqrt{\mu_{(12)3}}}$$

- Hyperradius $\rho = \sqrt{X^2 + Y^2}$
- A single generator coordinate, associated with the hyperradius ρ
- The ${}^6\text{He}$ wave function is expanded as

$$\psi_{0,k}^{jm\pi} = \sum_{K=0}^{\infty} \sum_{l_x, l_y, L, S} \sum_{i=1}^N f_{\gamma K}^{j\pi}(\rho_i) \Phi_{\gamma K}^{jm\pi}(\rho_i)$$

K =hypermoment (truncated at K_{max})

$$\gamma = (l_x, l_y, L, S)$$

with $K = l_x + l_y + 2n$ ($n > 0$)

the number of γ values depends on K_{max}

ρ_i : values of the generator coordinate ($N \approx 10 - 15$)

2. Microscopic description of ${}^6\text{He}$

$$\psi_{0,k}^{jm\pi} = \sum_{K=0}^{\infty} \sum_{l_x, l_y, L, S} \sum_{i=1}^N f_{\gamma K, k}^{j\pi}(\rho_i) \Phi_{\gamma K}^{jm\pi}(\rho_i)$$

$\Phi_{\gamma K}^{jm\pi}(\rho_i)$ = projected Slater determinant

$f_{\gamma K}^{j\pi}(\rho_i)$ = generator function

Standard variational problem

$$\sum_{\gamma, K, i} f_{\gamma K, k}^{j\pi}(\rho_i) \left(H_{\gamma K, \gamma' K'}^{j\pi}(\rho_i, \rho_{i'}) - E_{0,k}^{j\pi} H_{\gamma K, \gamma' K'}^{j\pi}(\rho_i, \rho_{i'}) \right) = 0$$

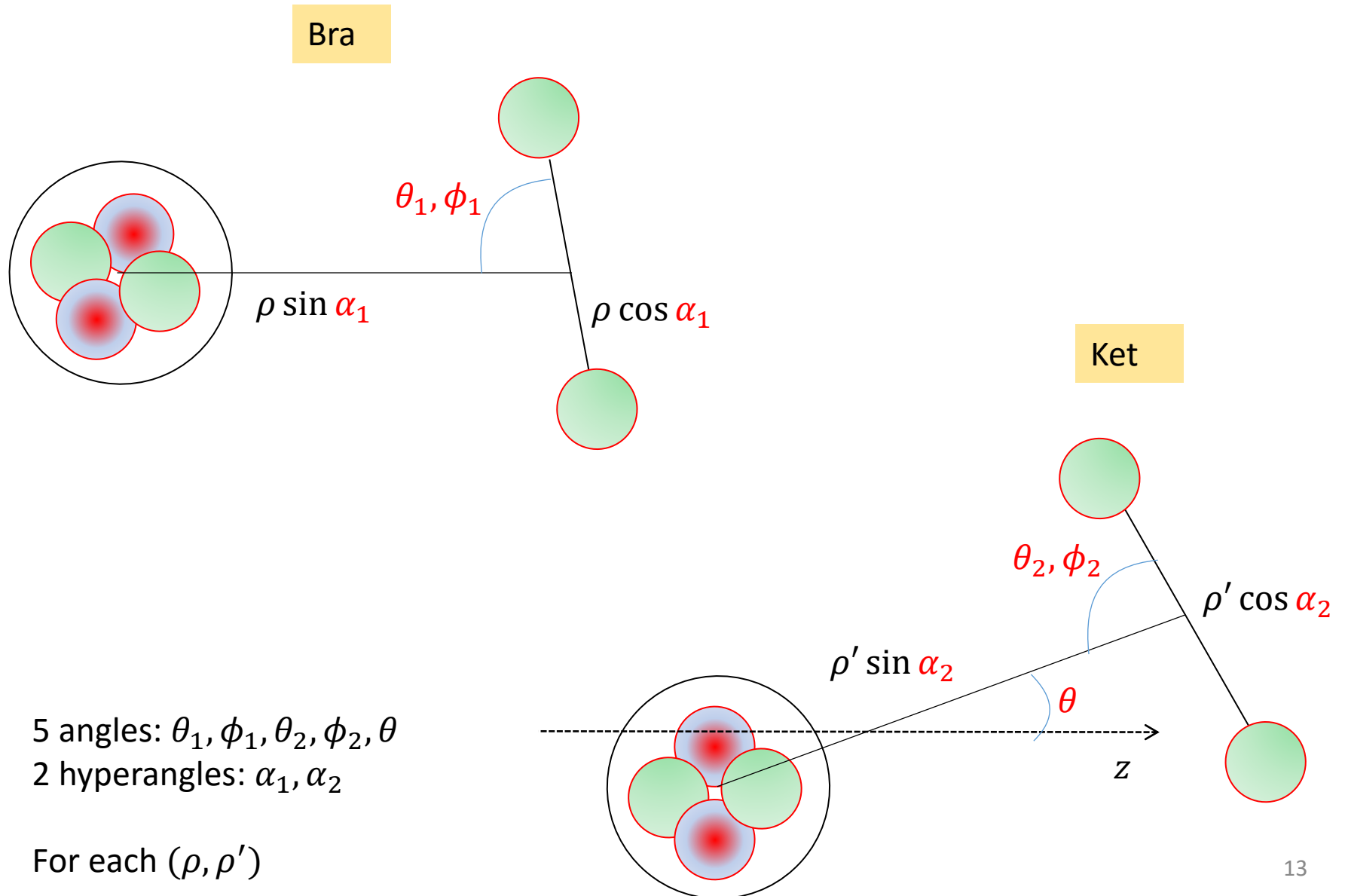
With

$$\left. \begin{aligned} H_{\gamma K, \gamma' K'}^{j\pi}(\rho_i, \rho_{i'}) &= \langle \Phi_{\gamma K}^{j\pi}(\rho_i) | H | \Phi_{\gamma' K'}^{j\pi}(\rho_{i'}) \rangle \\ N_{\gamma K, \gamma' K'}^{j\pi}(\rho_i, \rho_{i'}) &= \langle \Phi_{\gamma K}^{j\pi}(\rho_i) | 1 | \Phi_{\gamma' K'}^{j\pi}(\rho_{i'}) \rangle \end{aligned} \right\} \text{7-dimension integrals}$$

Also needed: densities $\langle \Phi_{\gamma K}^{j\pi}(\rho_i) | \sum_n \delta(\mathbf{r} - \mathbf{r}_n) | \Phi_{\gamma' K'}^{j\pi}(\rho_{i'}) \rangle$ (expanded in multipoles)

2. Microscopic description of ${}^6\text{He}$

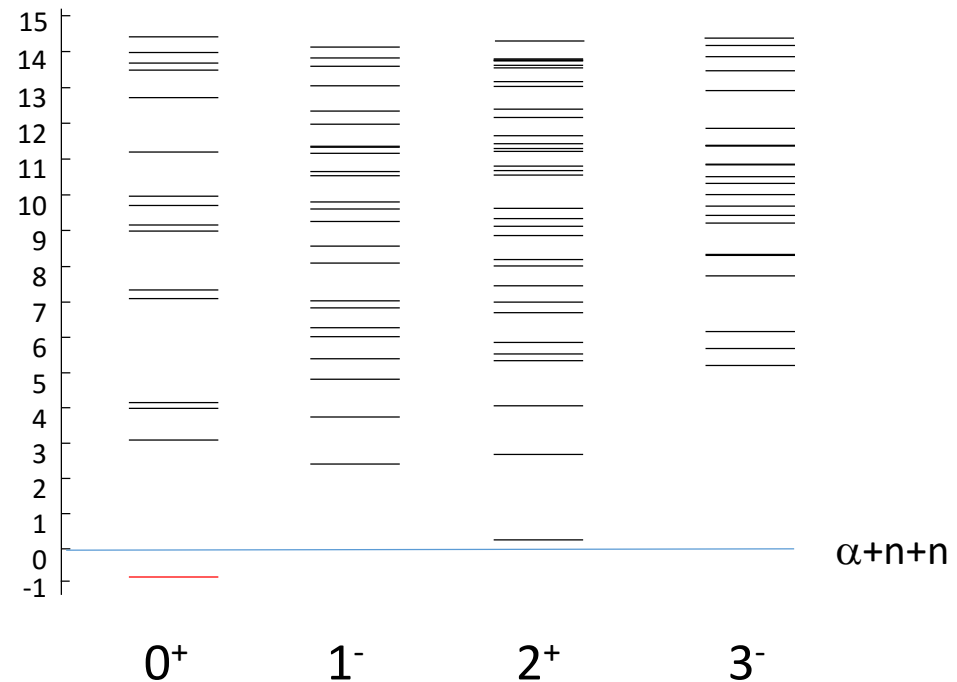
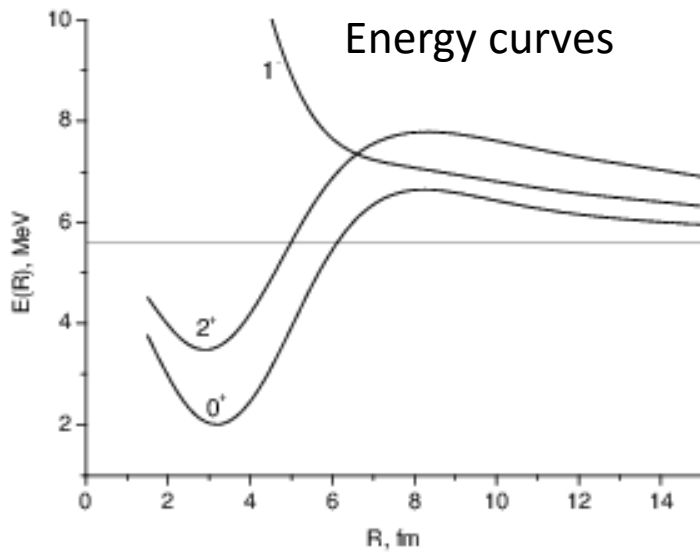
Projected matrix elements: 7-dimension integrals



2. Microscopic description of ${}^6\text{He}$

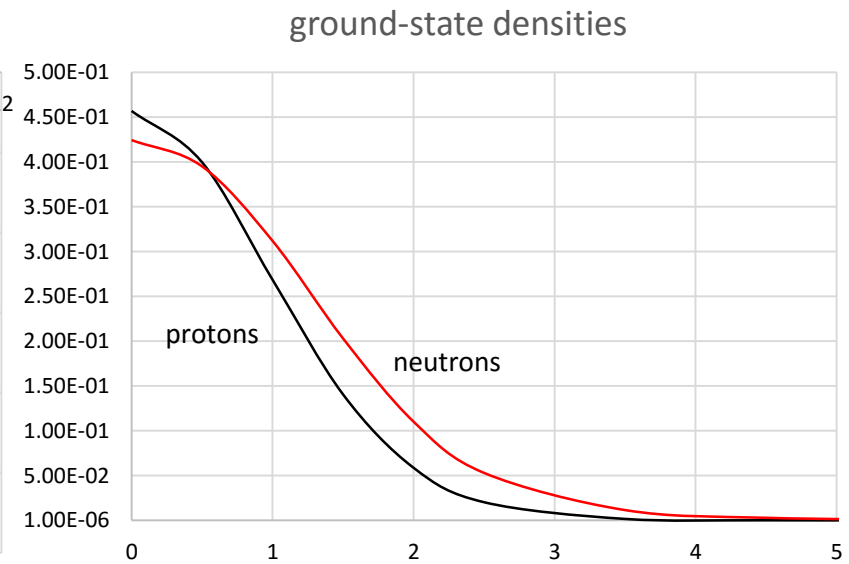
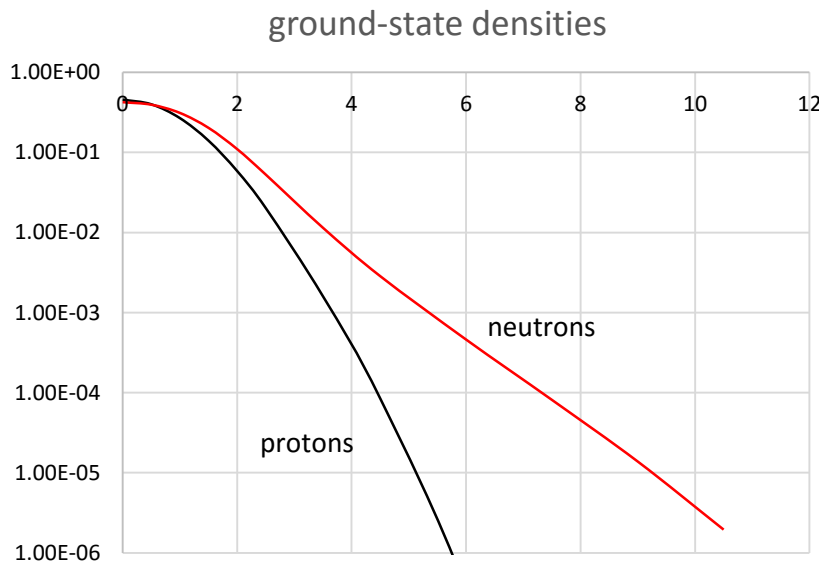
Conditions of the calculations

- NN interaction: Minnesota with $u=1.050$, $S_0=30 \rightarrow$ reproduce ${}^6\text{He}$ binding energy, and $\alpha+n$ phase shifts
- $j=0^+, 1^-, 2^+, 3^-$
- $K_{\text{max}}=18$
- ρ_i : from 1.5 fm to 12 fm (by step of 1.5 fm)



2. Microscopic description of ${}^6\text{He}$

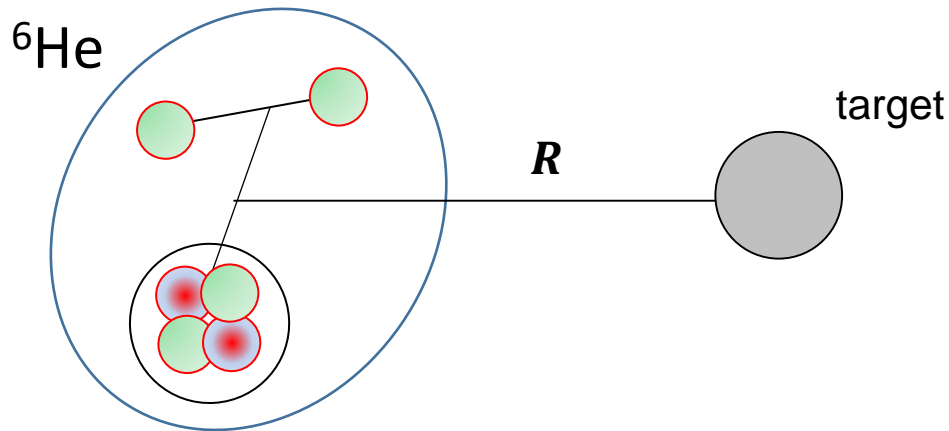
	GCM	exp
$\sqrt{\langle r^2 \rangle_p}$	1.34	
$\sqrt{\langle r^2 \rangle_n}$	2.77	
$\sqrt{\langle r^2 \rangle}$	2.39	2.33 ± 0.04



Second step:

Calculation of the coupling potentials

3. Coupling potentials



$$\text{Coupling potentials } V_{k,k'}^{jm,j'm'}(\mathbf{R}) = \langle \psi_{0,k}^{jm\pi} | \sum_i v_{i-T}(\mathbf{r}_i - \mathbf{R}) | \psi_{0,k'}^{j'm'\pi'} \rangle$$

- $\psi_{0,k}^{jm\pi}$ = combination of projected Slater determinants
- $v_{i-T}(\mathbf{r}_i - \mathbf{R})$ = nucleon-target interaction (including Coulomb)

→ Standard one-body matrix element (such as kinetic energy, rms radius...)

→ Must be expanded in multipoles:

$$V_{k,k'}^{jm,j'm'}(\mathbf{R}) = \sum_{\lambda} \langle jm \lambda m' - m | j'm' \rangle V_{k,k'}^{j,j'}(\lambda, R) Y_{\lambda}^{m'-m}(\Omega_R)$$

3. Coupling potentials

→ Two calculation methods:

1) Brink's formula for Slater determinants

One-body matrix elements (kinetic energy, rms radius, densities, etc.)

- Matrix elements between individual orbitals φ_i : $M_{ij} = \langle \varphi_i | v(\mathbf{r} - \mathbf{R}) | \varphi_j \rangle$
- Overlap matrix $B_{ij} = \langle \varphi_i | \varphi_j \rangle$
- Angular momentum projection

2) Folding procedure

$$\begin{aligned} V_{k,k'}^{jm,j'm'}(\mathbf{R}) &= \langle \psi_{0,k}^{jm\pi} \left| \sum_i v(\mathbf{r}_i - \mathbf{R}) \right| \psi_{0,k'}^{j'm'\pi'} \rangle \\ &= \int d\mathbf{S} v(\mathbf{S} - \mathbf{R}) \langle \psi_{0,k}^{jm\pi} \left| \sum_i \delta(\mathbf{r}_i - \mathbf{S}) \right| \psi_{0,k'}^{j'm'\pi'} \rangle \\ &= \int d\mathbf{S} v(\mathbf{S} - \mathbf{R}) \rho_{kk'}^{jm,j'm'}(\mathbf{S}) \end{aligned}$$

With $\rho_{kk'}^{jm,j'm'}(\mathbf{S}) = \langle \psi_{0,k}^{jm\pi} \left| \sum_i \delta(\mathbf{r}_i - \mathbf{S}) \right| \psi_{0,k'}^{j'm'\pi'} \rangle$ = nuclear densities

expanded in multipoles as $\rho_{kk'}^{jm,j'm'}(\mathbf{S}) = \sum_{\lambda} \langle jm \lambda m' - m | j'm' \rangle \rho_{k,k'}^{jj'\lambda}(\mathbf{S}) Y_{\lambda}^{m'-m}(\Omega_{\mathbf{S}})$

→ Test of the calculation

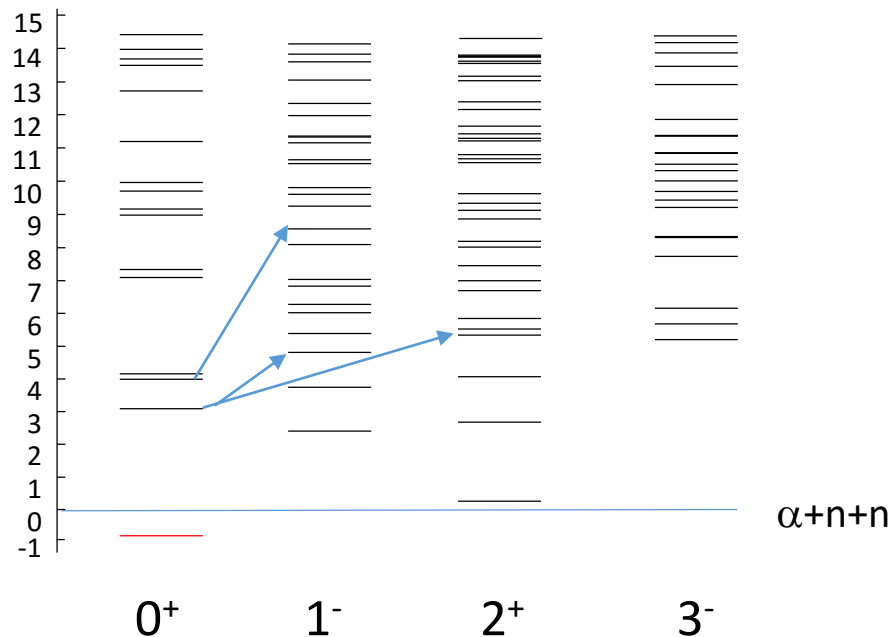
→ 2nd method more efficient since changing the potential is a minor work

3. Coupling potentials

In practice: folding potentials are computed with Fourier transforms

$$\text{if } V(\mathbf{r}) = \int v(\mathbf{r} - \mathbf{S})\rho(\mathbf{S})d\mathbf{S} \rightarrow \tilde{V}(q) = \tilde{v}(q)F(q)$$

- $\tilde{v}(q)$ =Fourier transform of the nucleon-target interaction
- $F(q)$ =form factor (=Fourier transform of the density)
densities expanded in Gaussians
- Must be done for protons and neutrons
- For pseudostates: densities extend to large distances \rightarrow numerical problems
tests with the Coulomb interaction: analytical calculations possible

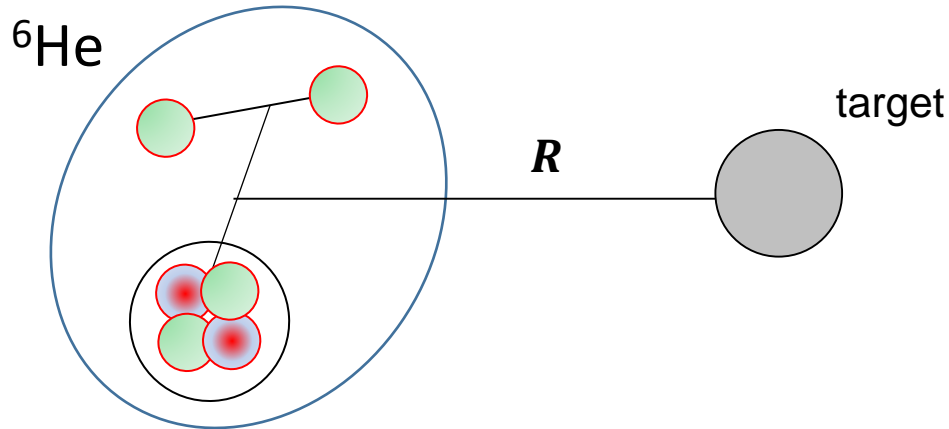


All couplings are needed
 \rightarrow many possibilities

Third step:

Solving the coupled-channel equations

4. Solving the coupled-channel equations



Hamiltonian: $H = H_0 + T_R + \sum_i v_{i-T}(r_i - R)$

Wave function: $\Psi^{JM\pi}(\mathbf{R}, \mathbf{r}_i) = \sum_{jLk} u_{jLk}^{J\pi}(R) \left[\psi_{0,k}^{j\pi}(\mathbf{r}_i) \otimes Y_L(\Omega_R) \right]^{JM}$

→ Set of coupled equations

$$\left[-\frac{\hbar^2}{2\mu} \left(\frac{d^2}{dR^2} - \frac{L(L+1)}{R^2} \right) + E_c - E \right] u_c^{J\pi}(R) + \sum_{c'} V_{cc'}^{J\pi}(R) u_{c'}^{J\pi}(R) = 0$$

$V_{cc'}^{J\pi}(R)$ obtained from $V_{k,k'}^{jm,j'm'}(\mathbf{R})$ with additional angular momentum couplings

Channel $c=j$: projectile quantum numbers

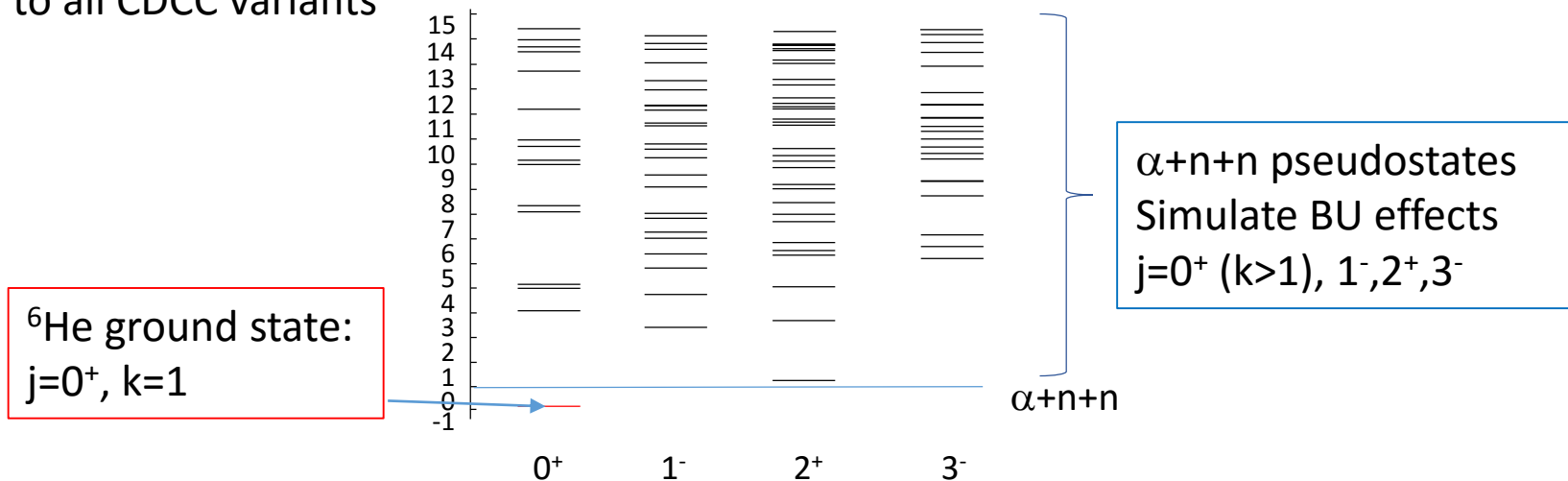
k : excitation level of the projectile [physical state ($E < 0$) or pseudostate ($E > 0$)]

L : orbital angular momentum between projectile and target

4. Solving the coupled-channel equations

$$\left[-\frac{\hbar^2}{2\mu} \left(\frac{d^2}{dR^2} - \frac{L(L+1)}{R^2} \right) + E_c - E \right] u_c^{J\pi}(R) + \sum_{c'} V_{cc'}^{J\pi}(R) u_{c'}^{J\pi}(R) = 0$$

- Common to all CDCC variants



- Solved with: Numerov algorithm / **R-matrix method**

At large distances

- Nuclear potential negligible, only Coulomb remains
- Wave function $u_c^{J\pi}(R) \rightarrow I_c(k_c R) \delta_{\omega c} - U_{\omega c}^{J\pi} O_c(k_c R)$
with ω =entrance channel

$I_c(x), O_c(x)$ = incoming and outgoing Coulomb functions

$U_{\omega c}^{J\pi}$ = scattering matrix \rightarrow various cross sections (elastic, breakup, etc)

4. Solving the coupled-channel equations

Solving the coupled-channel system

R-matrix theory: based on 2 regions (channel radius a)

A.M. Lane and R.G. Thomas, Rev. Mod. Phys. 30 (1958) 257

P.D. and D. Baye, Rep. Prog. Phys. 73 (2010) 036301

P.D., Computer Physics Communications (in press) <http://arxiv.org/abs/1510.03540>

Internal region: $R \leq a$

$R = a$

External region: $R \geq a$

R

Full Hamiltonian

$u_c^{J\pi}(R)$ expanded over a basis
(N functions)

$$u_{c,int}^{J\pi}(R) = \sum_{i=1}^N c_i \phi_i(R)$$

Only Coulomb

$$V_{cc'}^{J\pi}(R) = \frac{Z_p Z_t e^2}{R} \delta_{cc'}$$

$u_c^{J\pi}(R)$ has its asymptotic form

$$u_{c,ext}^{J\pi}(R) = I_c(kR) \delta_{c\omega} - O_c(kR) U_{c\omega}^{J\pi}$$

matching at $R=a$ provides: scattering matrices $U^{J\pi} \rightarrow$ cross sections

4. Solving the coupled-channel equations

Choice of the basis: **the Lagrange-mesh method** (*D. Baye, Phys. Rep. 565 (2015) 1-107*)

- **Gauss approximation:** $\int_0^a g(x)dx \approx \sum_{k=1}^N \lambda_k g(x_k)$
 - N= order of the Gauss approximation
 - x_k =roots of an orthogonal polynomial $P_N(x)$, l_k =weights
 - If interval [0,a]: Legendre polynomials
[0,∞]: Laguerre polynomials
- **Lagrange functions** for [0,1]: $f_i(x) \sim \frac{P_N(2x-1)}{(x-x_i)}$
 - x_i are roots of $P_N(2x - 1) = 0$
 - with the Lagrange property: $f_i(x_j) = \lambda_i^{-1/2}$
- **Matrix elements** with Lagrange functions: Gauss approximation is used
$$\langle f_i | f_j \rangle = \int f_i(x) f_j(x) dx \approx \delta_{ij}$$
$$\langle f_i | T | f_j \rangle \text{ analytical}$$
$$\langle f_i | V | f_j \rangle = \int f_i(x) V(x) f_j(x) dx \approx V(x_i) \delta_{ij} \Rightarrow \text{no integral needed}$$

Computer time: 2 main parts

- **Matrix elements**: very fast with Lagrange functions
- **Inversion of a complex matrix C** → R-matrix (long times for large matrices)

For reactions involving halo nuclei:

- Long range of the potentials (Coulomb)

$$\frac{Z_1 Z_t e^2}{R + \frac{A_2 r}{A_p}} + \frac{Z_2 Z_t e^2}{R - \frac{A_1 r}{A_p}} = \sum_{\lambda} V_{\lambda}(r, R) P_{\lambda}(\cos \theta_{Rr})$$

$$V_{cc'}(R) \approx \frac{Z_p Z_t e^2}{R} + \frac{Z_t Q_p}{R^3} + \dots$$

Can be large (large quadrupole moments of PS)

- Radius a must be large
- Many basis functions (N large)
- Even stronger for dipole terms ($\sim 1/R^2$)



- **Distorted Coulomb functions (FRESCO)**
- **Propagation techniques in the R-matrix** (well known in atomic physics)

Ref.: Baluja et al. Comp. Phys. Comm. 27 (1982) 299

Well adapted to Lagrange-mesh calculations

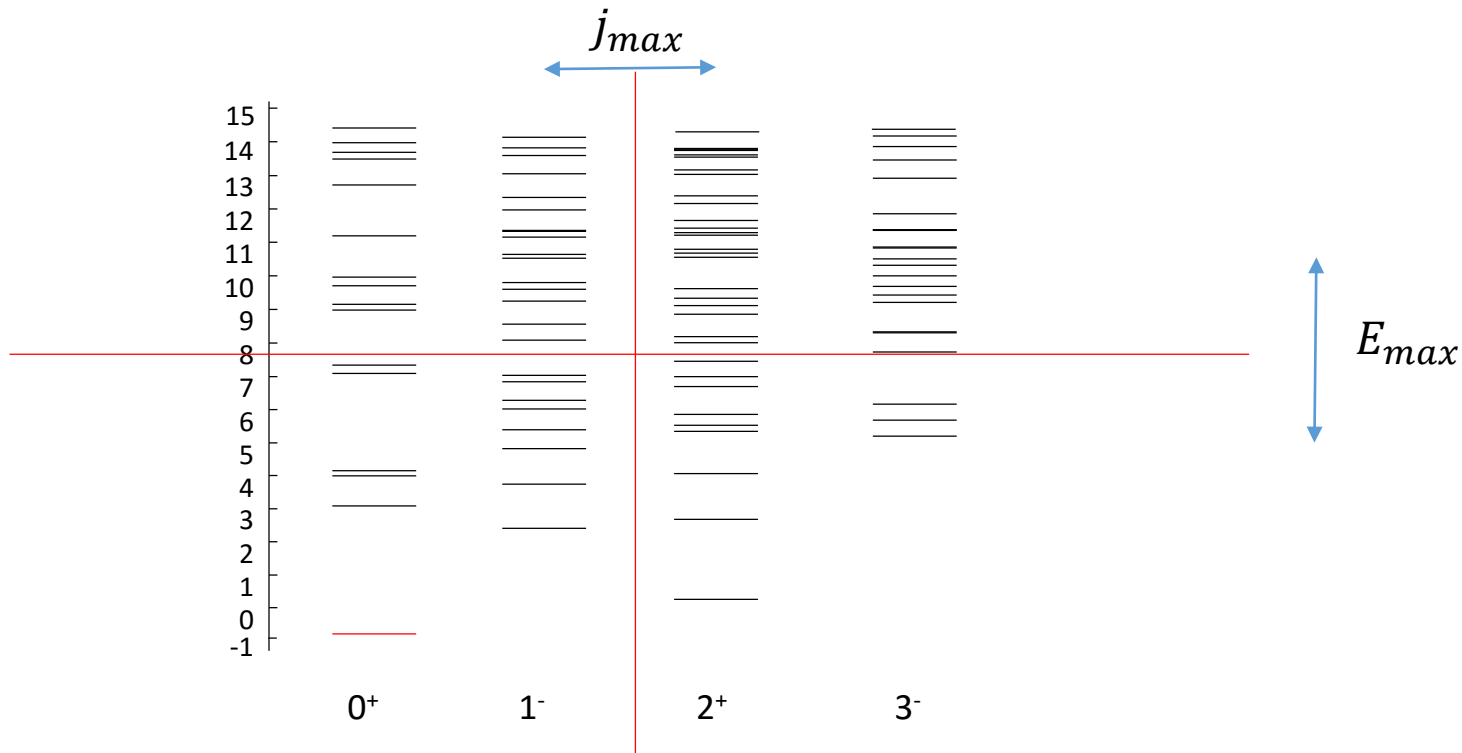
Applications:

${}^6\text{He}$ scattering on ${}^{58}\text{Ni}$, ${}^{120}\text{Sn}$, ${}^{208}\text{Pb}$

5. Applications

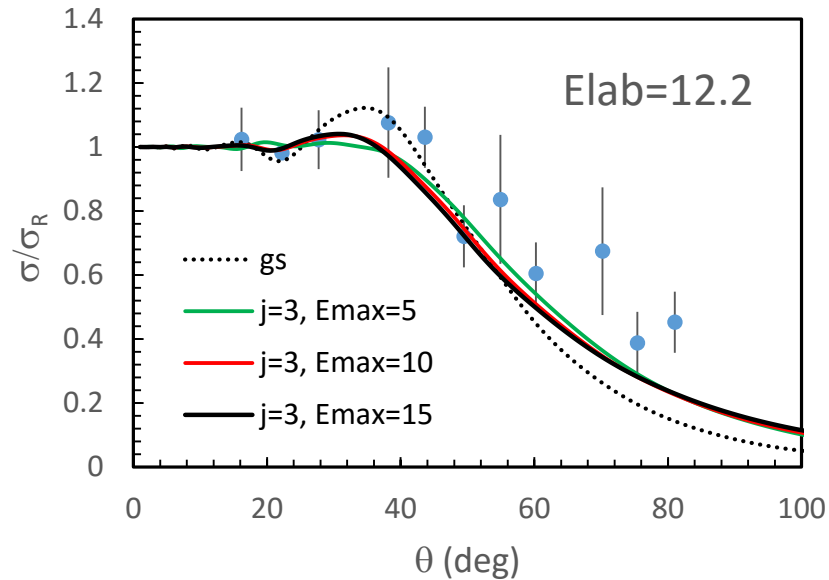
${}^6\text{He}$ scattering on ${}^{58}\text{Ni}$, ${}^{120}\text{Sn}$, ${}^{208}\text{Pb}$

- Optical potentials n-T, p-T: taken from Koning and Delaroche, NPA 713 (2003) 231
 - Complex potentials (simulate the excitation of the target)
 - ${}^{58}\text{Ni}$, ${}^{120}\text{Sn}$, ${}^{208}\text{Pb}$: “local” potentials (specific fits)
- Convergence problems: truncation on E_{max}, j_{max}

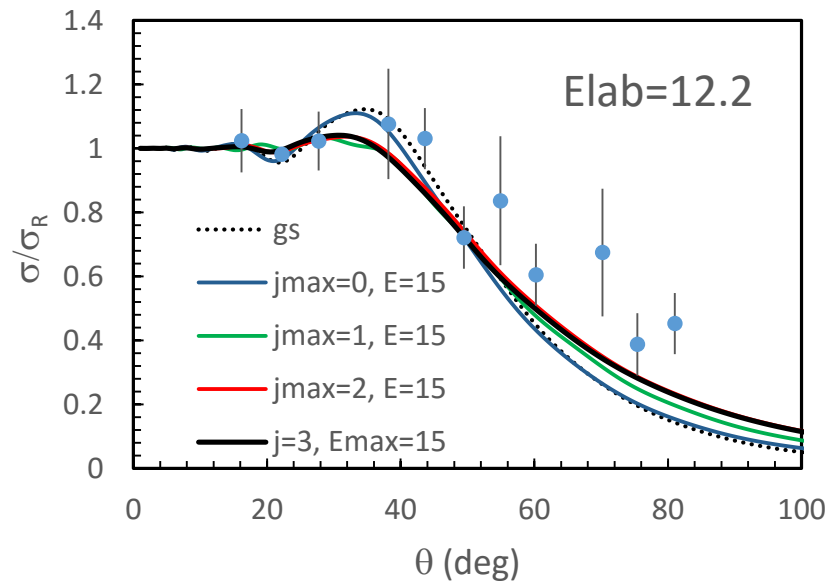


5. Applications

${}^6\text{He} + {}^{58}\text{Ni}$: V. Morcelle et al., PLB 732 (2014) 228
Coulomb barrier $V_B \sim 7.3$ MeV

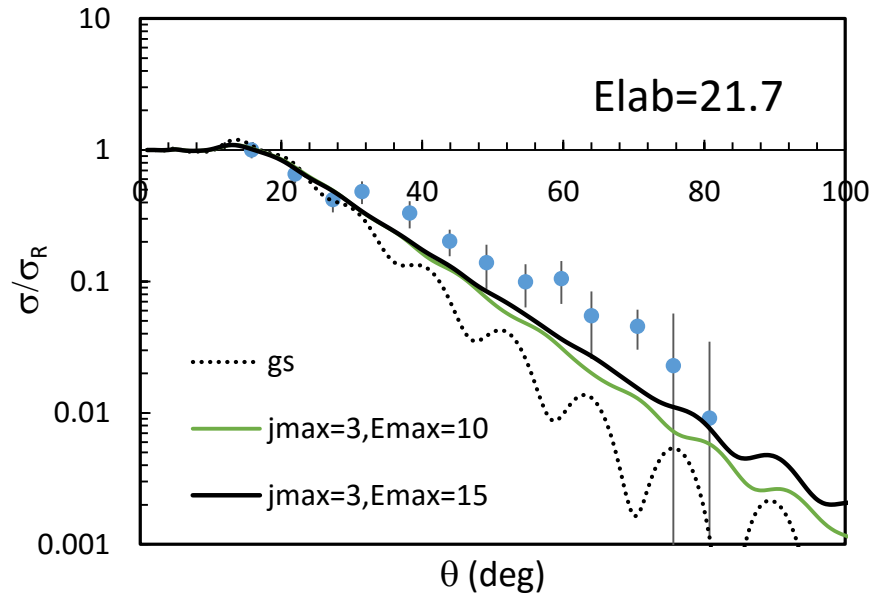


Convergence with E_{max}
 $j_{max} = 3$



Convergence with j_{max}
 $E_{max} = 15$ MeV
 \rightarrow 1- important

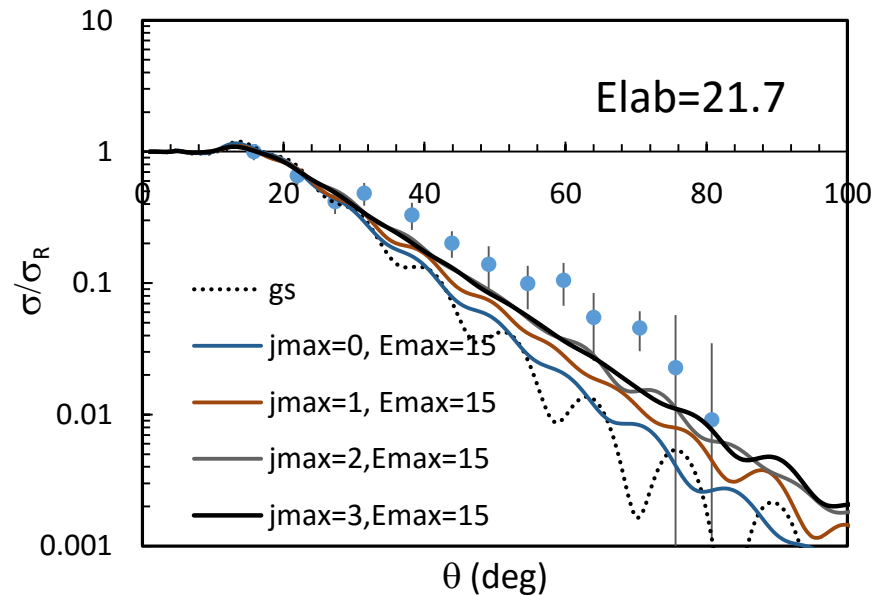
5. Applications



${}^6\text{He} + {}^{58}\text{Ni}$

Convergence with E_{max}
 $j_{max} = 3$

Convergence with j_{max}
 $E_{max} = 15$ MeV

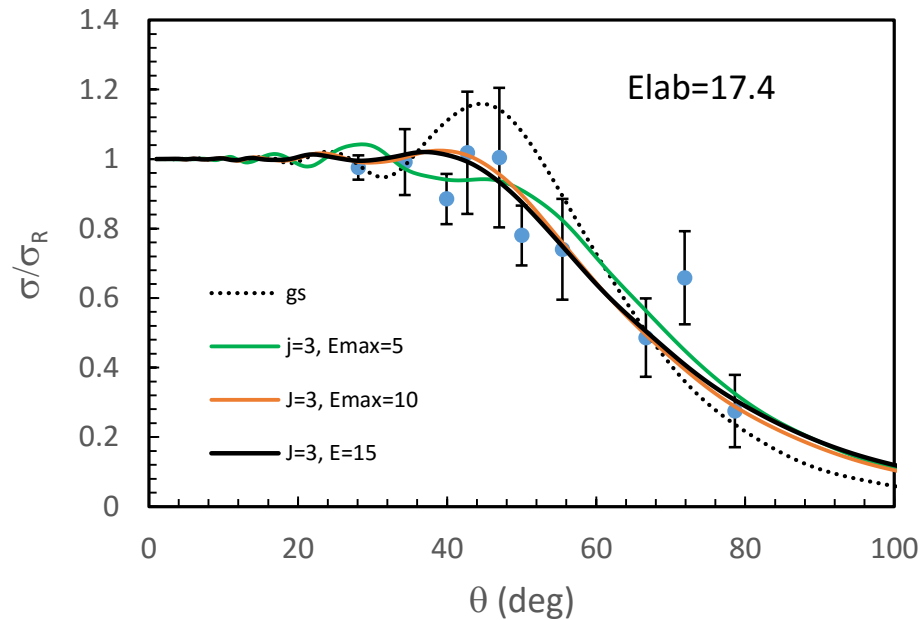
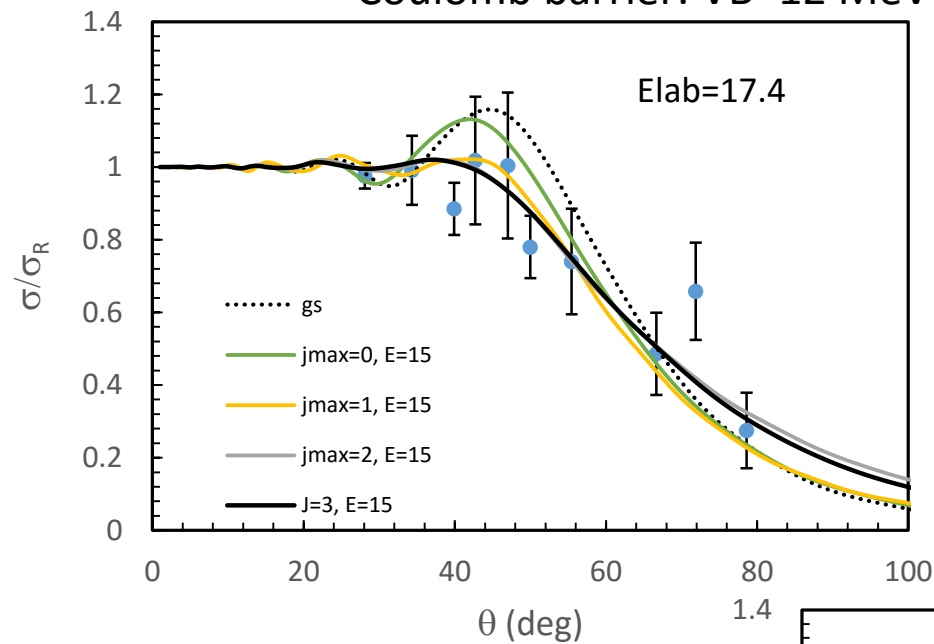


5. Applications

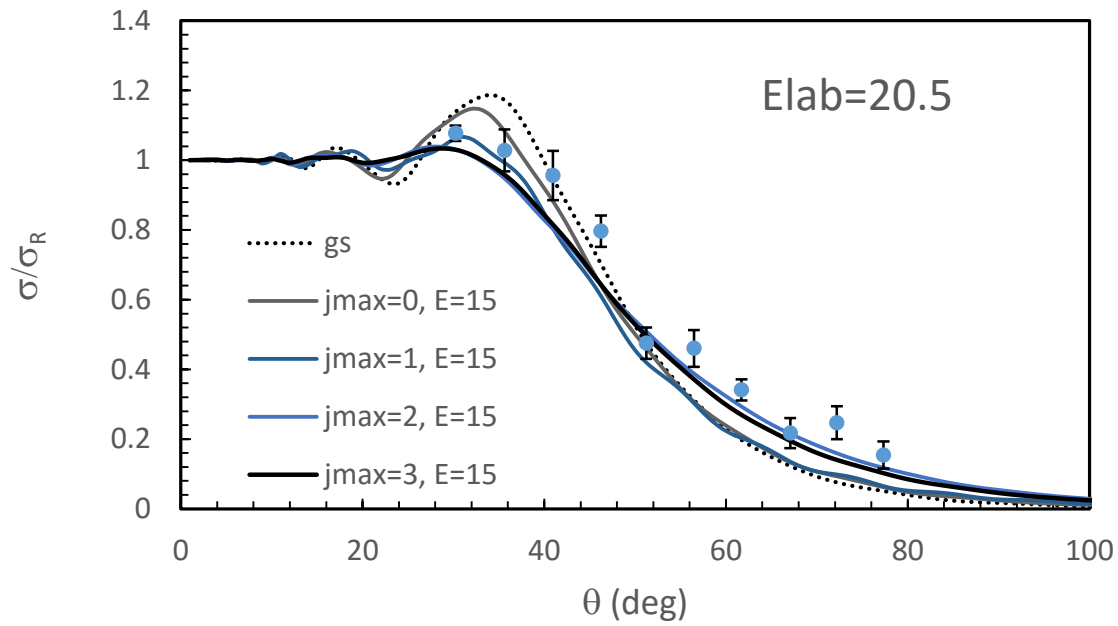
${}^6\text{He} + {}^{120}\text{Sn}$:

Ref: P. N. de Faria et al., PRC 81 (2010) 044605

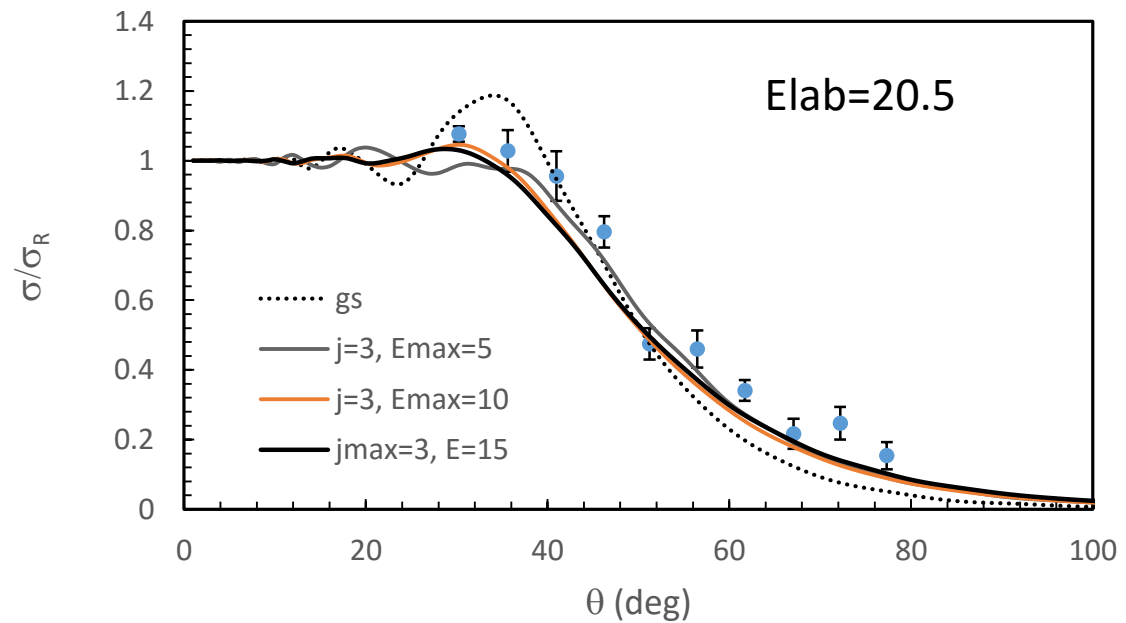
Coulomb barrier: $V_B \sim 12$ MeV



5. Applications

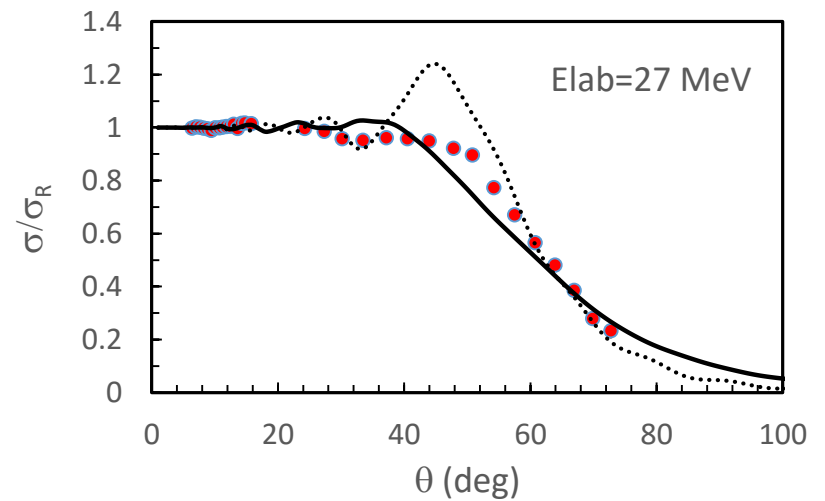
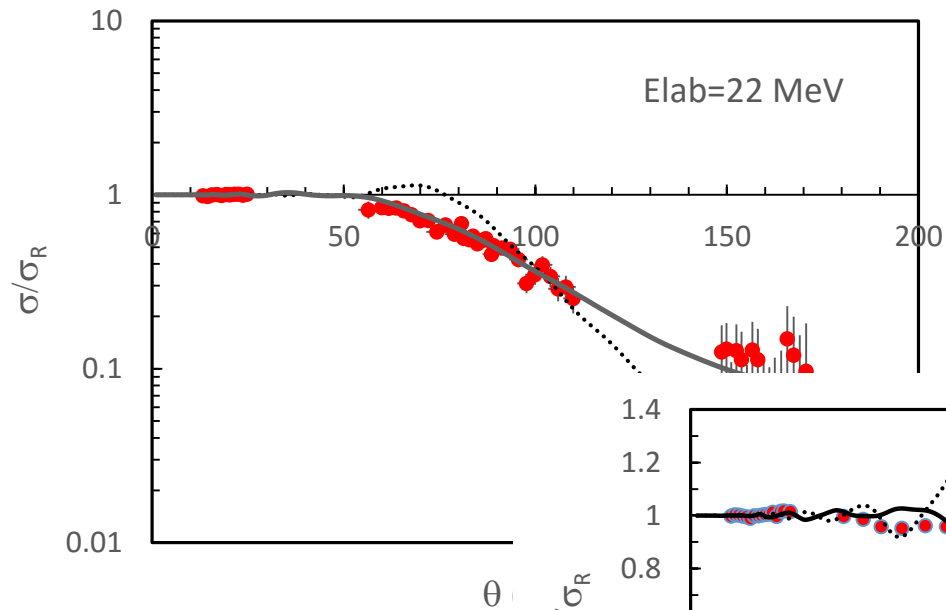
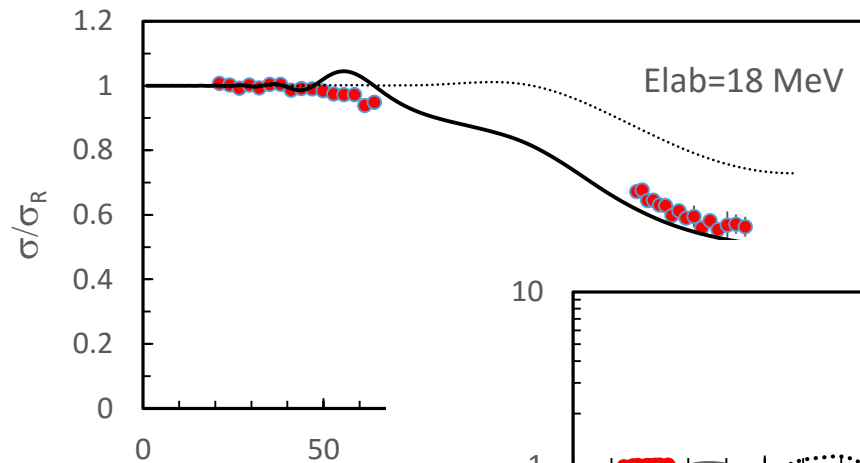


${}^6\text{He} + {}^{120}\text{Sn}$: VB~12 MeV



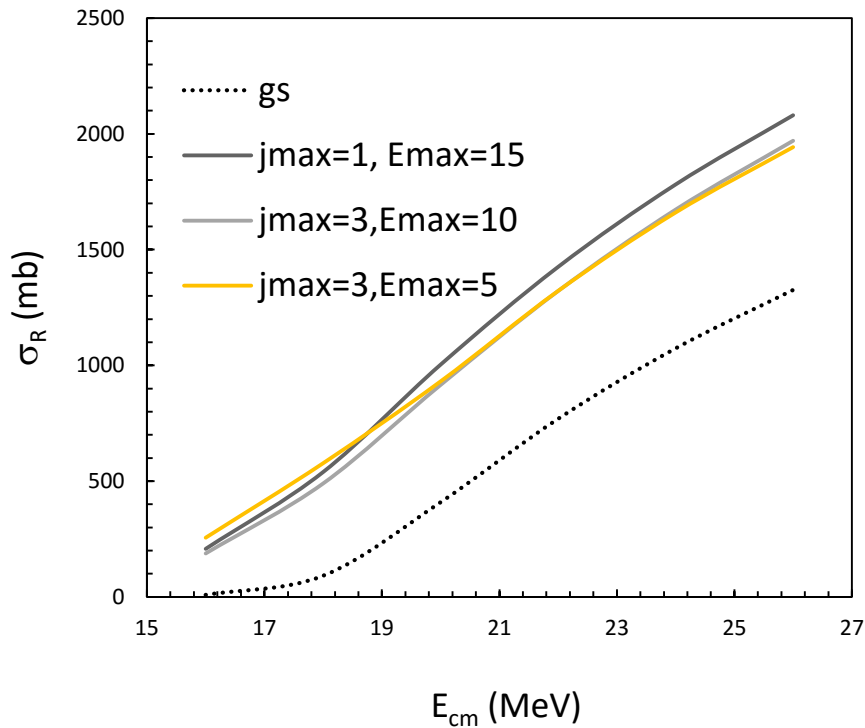
5. Applications

${}^6\text{He} + {}^{208}\text{Pb}$: L. Acosta et al., PRC 84 (2011) 044604
Coulomb barrier $V_B \sim 18.4$ MeV



5. Applications

${}^6\text{He} + {}^{208}\text{Pb}$: reaction cross sections



- BU channels yield a larger reaction cross section
- Stable with jmax and Emax

Discussion of the ${}^6\text{He}$ - ${}^{208}\text{Pb}$ potential

6. Discussion of the ${}^6\text{He}$ - ${}^{208}\text{Pb}$ potential

Goals:

- Sensitivity with respect to the p- ${}^{208}\text{Pb}$ and n- ${}^{208}\text{Pb}$ optical potentials
- Equivalent potentials (full CDCC \rightarrow single-channel)

1. Sensitivity

$$H = H_0 + T_R + \sum_{i=1}^6 \left(\frac{1}{2} - t_{iz} \right) \left[v_p(r_i - R) + v_C(r_i - R) \right] + \left(\frac{1}{2} + t_{iz} \right) v_n(r_i - R)$$

Proton-target
Optical potential

Neutron-target
Optical potential

v_p and v_n are taken from Koning-Delaroche, NPA 713 (2003) 231

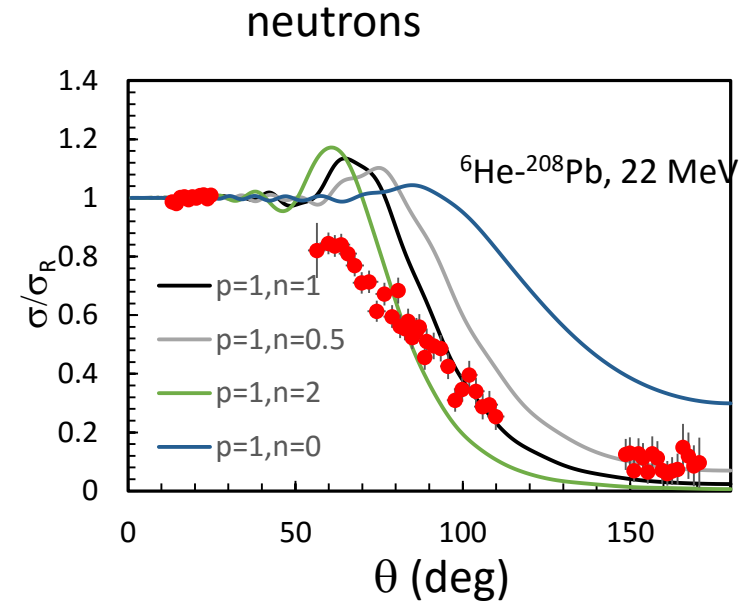
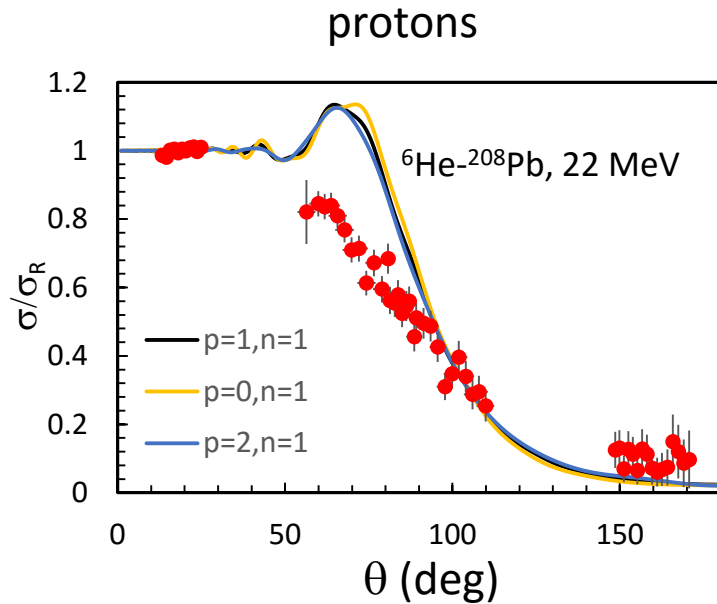
- Include real and imaginary parts
- Fit nucleon elastic scattering

\rightarrow How sensitive are the ${}^6\text{He}$ - ${}^{208}\text{Pb}$ cross sections?

\rightarrow Multiplicative factors F_p and F_n

6. Discussion of the ${}^6\text{He}$ - ${}^{208}\text{Pb}$ potential

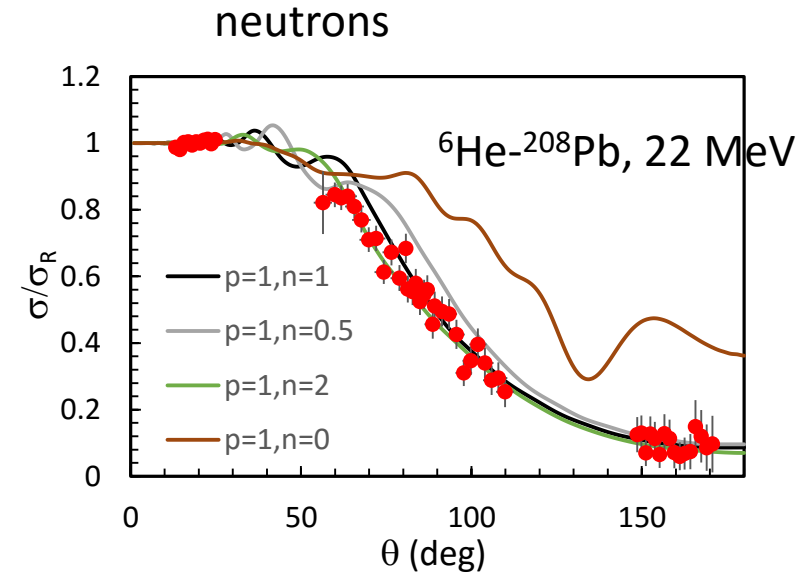
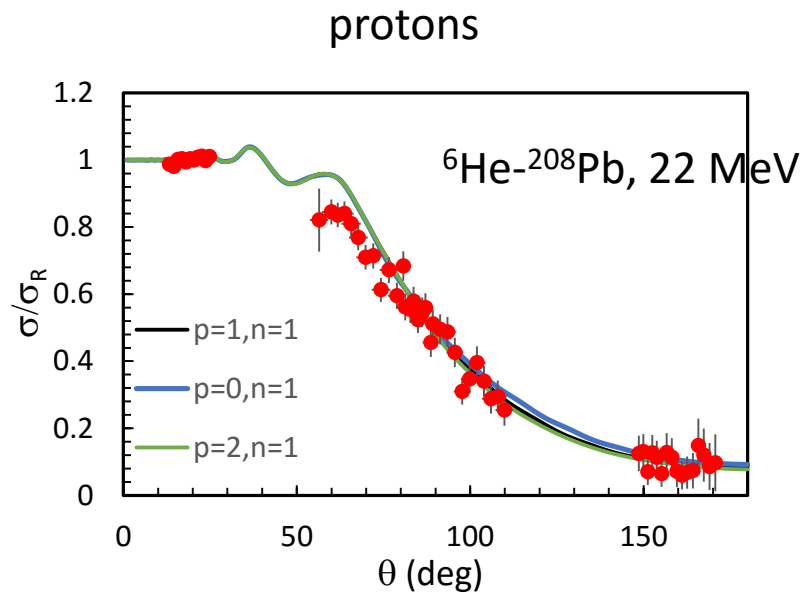
Single channel



- Negligible role of the p - ${}^{208}\text{Pb}$ optical potential
 $E_{\text{lab}}({}^6\text{He})=22$ MeV $\rightarrow E_{\text{lab}}(p)\sim 3.7$ MeV: much lower than the Coulomb barrier (~ 10 MeV)
- Role of n - ${}^{208}\text{Pb}$: not very strong
Consistent with R.C. Johnson et al., PRL 79 (1997) 2771
For halo nuclei: $\frac{d\sigma}{d\Omega} \approx |F(q)| \left(\frac{d\sigma}{d\Omega} \right)_{\text{core-target}}$
several conditions: adiabatic, core-target potential dominant, etc.

6. Discussion of the ${}^6\text{He}$ - ${}^{208}\text{Pb}$ potential

Multi channel



→ Similar conclusions for the p/n optical potentials

→ Importance of Coulomb couplings (difference between SC and MC)

6. Discussion of the ${}^6\text{He}$ - ${}^{208}\text{Pb}$ potential

2. Equivalent potentials

Question: can we find a single-channel equivalent potential?

a) J-dependent potential

For the elastic channel : $(T_R + V_{11}^J(R) - E)u_1^J(R) = -\sum_{c \neq 1} V_{1c}^J(R)u_c^J(R)$

Equivalent to $(T_R + V_{11}^J(R) + V_{pol}^J(R) - E)u_1^J(R) = 0$

$$\text{with } V_{pol}^J(R) = -\frac{\sum_{c \neq 1} V_{1c}^J(R)u_c^J(R)}{u_1^J(R)}$$

Problems: J dependent
contains singularities (nodes of the wave function)

→ Construction of a J-independent potential

6. Discussion of the ${}^6\text{He}$ - ${}^{208}\text{Pb}$ potential

b) J-independent potential

I.J. Thompson et al., Nucl. Phys. A 505 (1989) 84.

$$V_{pol}(R) = \frac{\sum_J V_{pol}^J(R) \omega^J(R)}{\sum_J \omega^J(R)}$$

With $\omega^J(R)$ =weight function

reduces the influence of the nodes

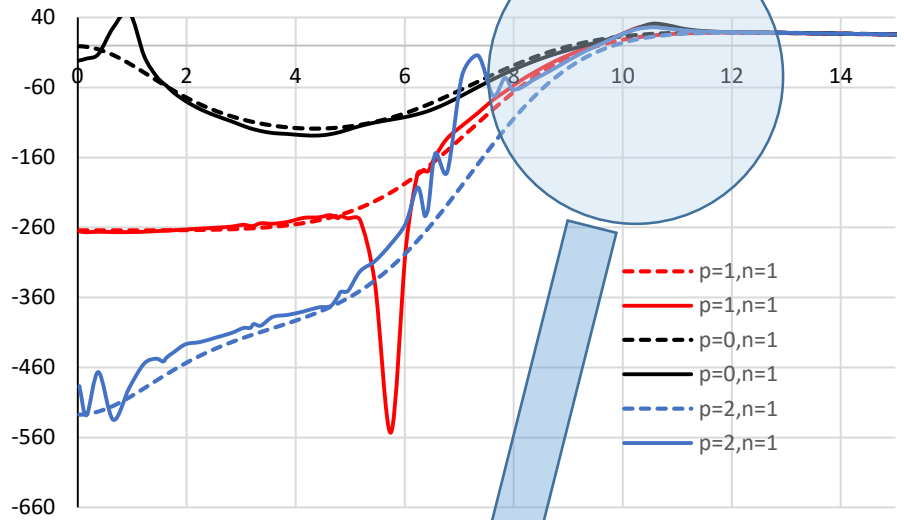
gives more weight to the dominant J-values

$$\omega^J(R) = (2J + 1) \left(1 - |U_{11}^J|^2\right) |u_1^J(R)|^2$$

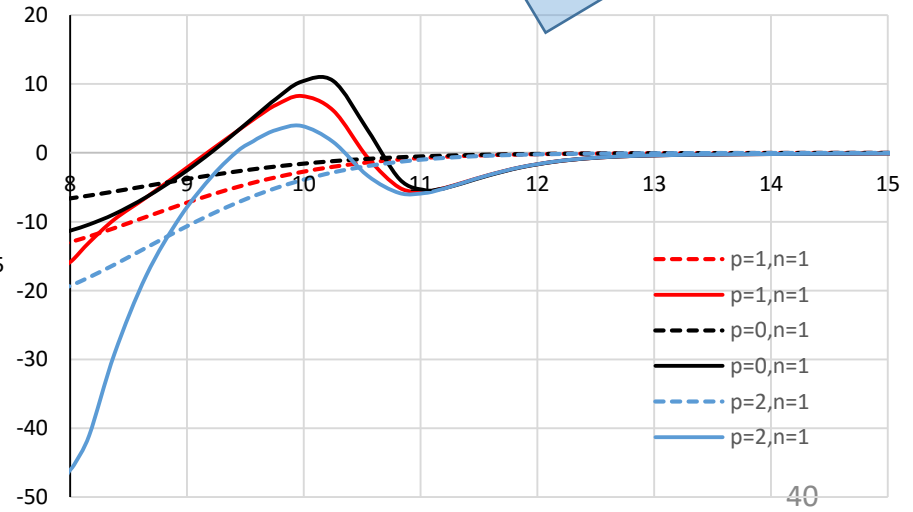
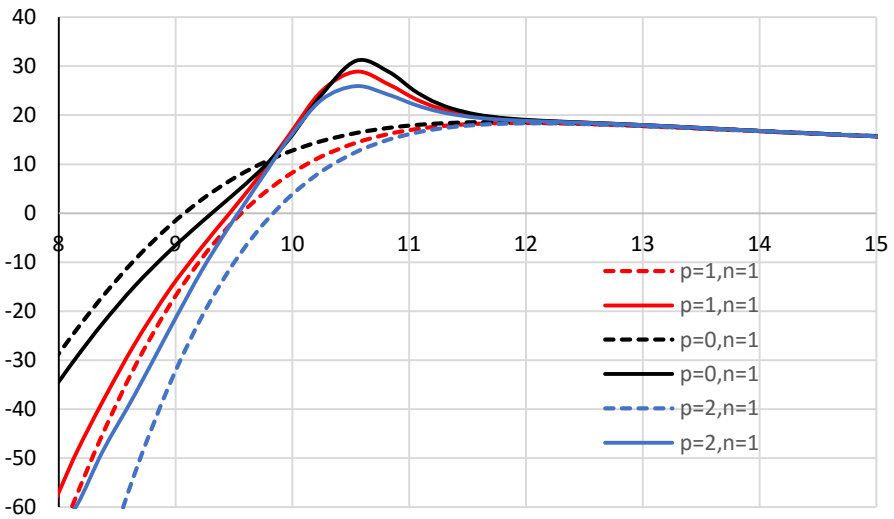
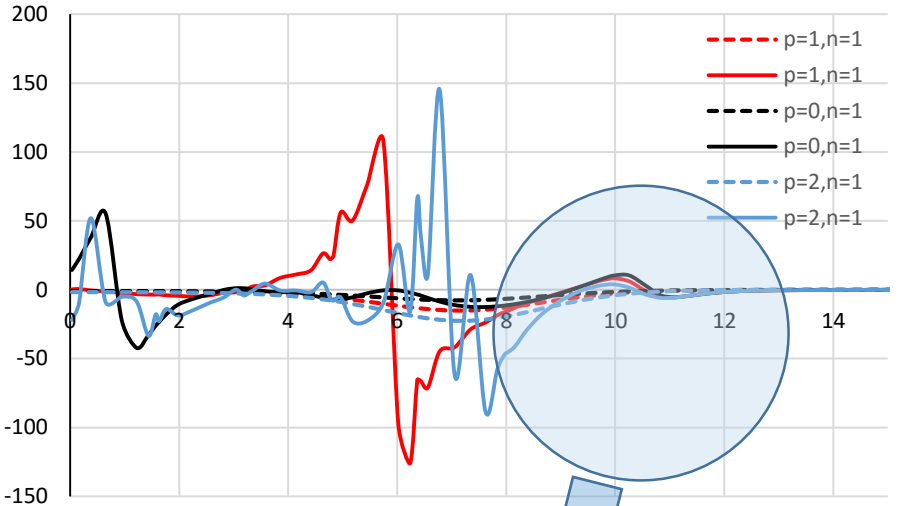
Test: verify that $V_{pol}(R)$ redroduces the full calculation

6. Discussion of the ${}^6\text{He}$ - ${}^{208}\text{Pb}$ potential

Real

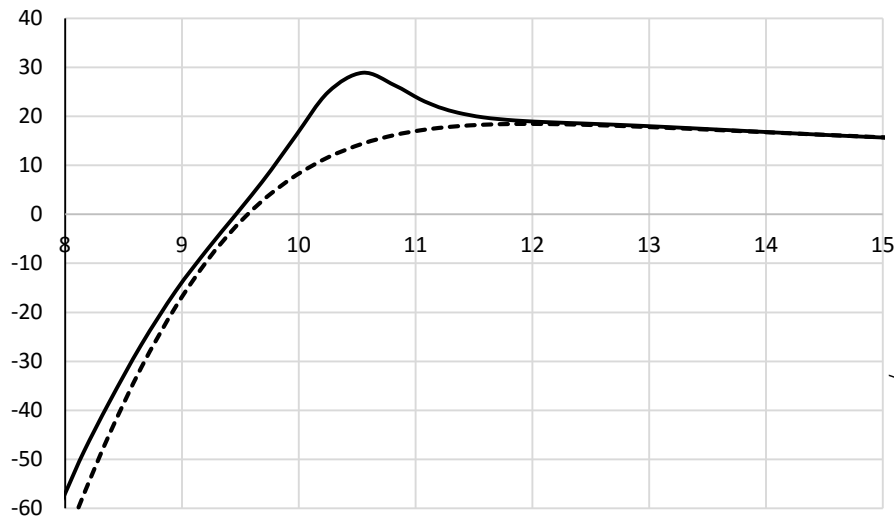


Imaginary

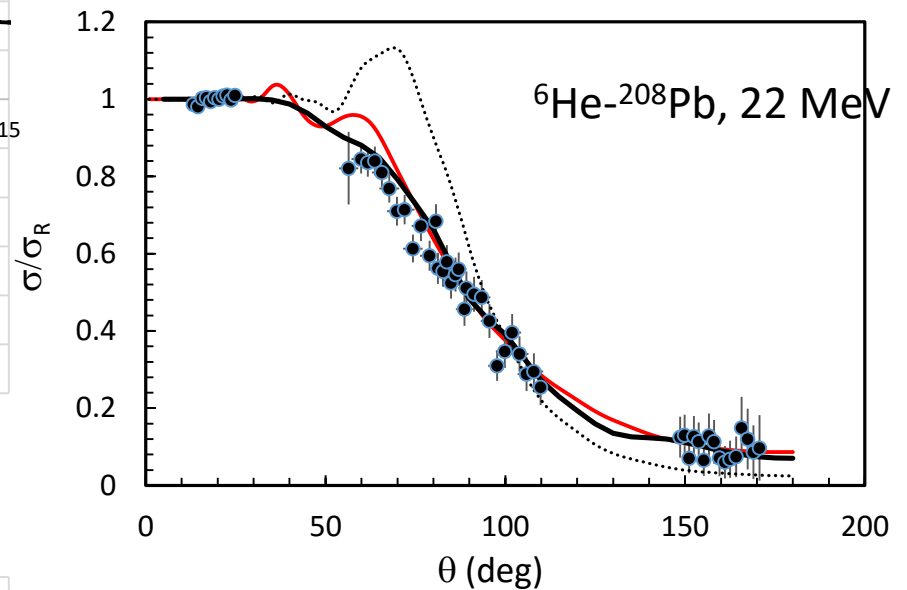
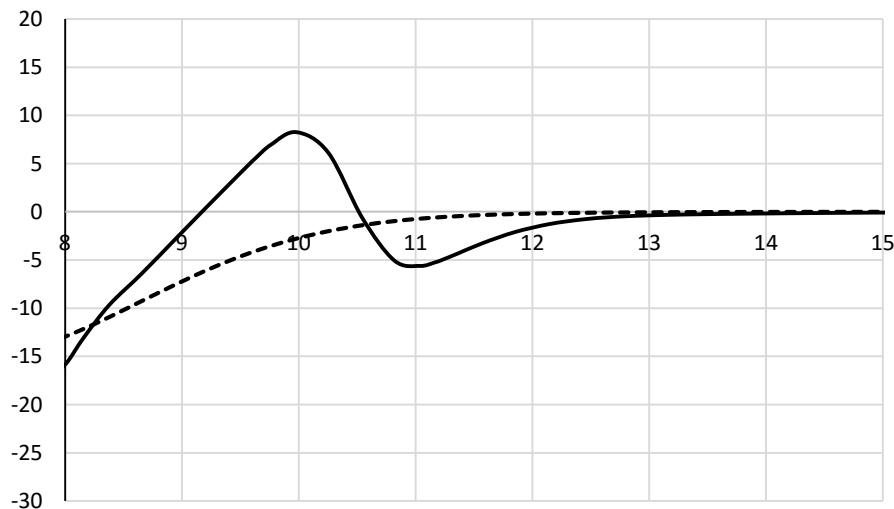


6. Discussion of the ${}^6\text{He}$ - ${}^{208}\text{Pb}$ potential

Real



Imaginary



- Long range of the polarization
- Real part repulsive
- Imaginary part negative
(consistent with the reaction cross section)

Conclusion

7. Conclusion

- **Microscopic CDCC:** Combination of CDCC and microscopic cluster model for the projectile
- Continuum simulated by pseudostates
- Extension to three-cluster projectiles (still in progress!)
- **Only a nucleon-target is necessary (no free parameter)**
- Application to ${}^6\text{He} + \text{target}$
 - ${}^6\text{He}$ is well described by a microscopic $\alpha+n+n$ structure
 - ${}^6\text{He}+{}^{58}\text{Ni}$, ${}^6\text{He}+{}^{120}\text{Sn}$, ${}^6\text{He}+{}^{208}\text{Pb}$ are investigated near the Coulomb barrier
 - Good agreement with experiment
 - Importance of break-up channels (poor agreement when they are neglected)
- ${}^6\text{He}+{}^{208}\text{Pb}$ potential
 - Equivalent potentials can be obtained
 - Break-up channels make the real part more repulsive
 - Imaginary part has a long range