From EFTs to Nuclei

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CANHP 2015

Research partly funded by the US Department of Energy

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Energy scales and relevant degrees of freedom

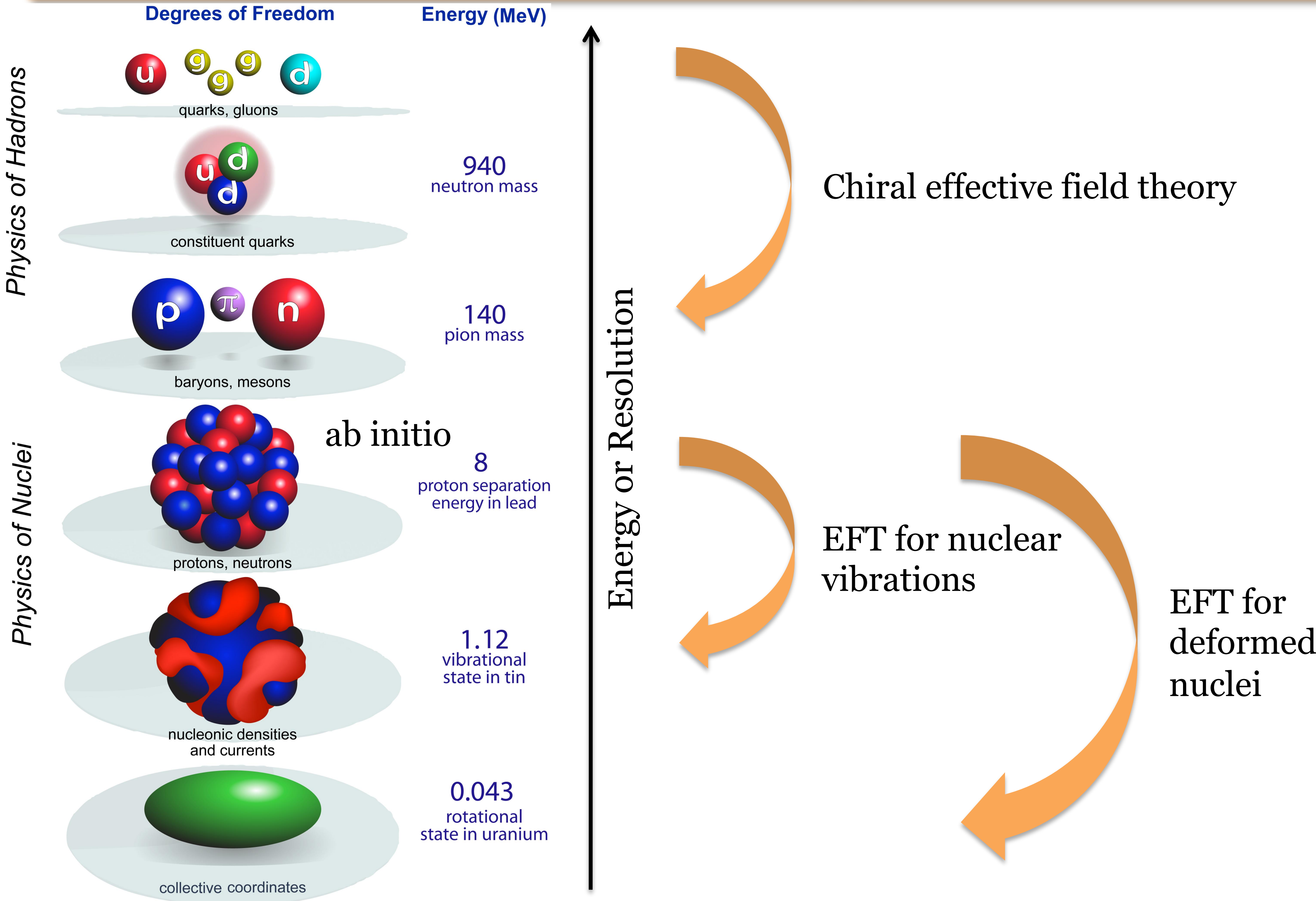
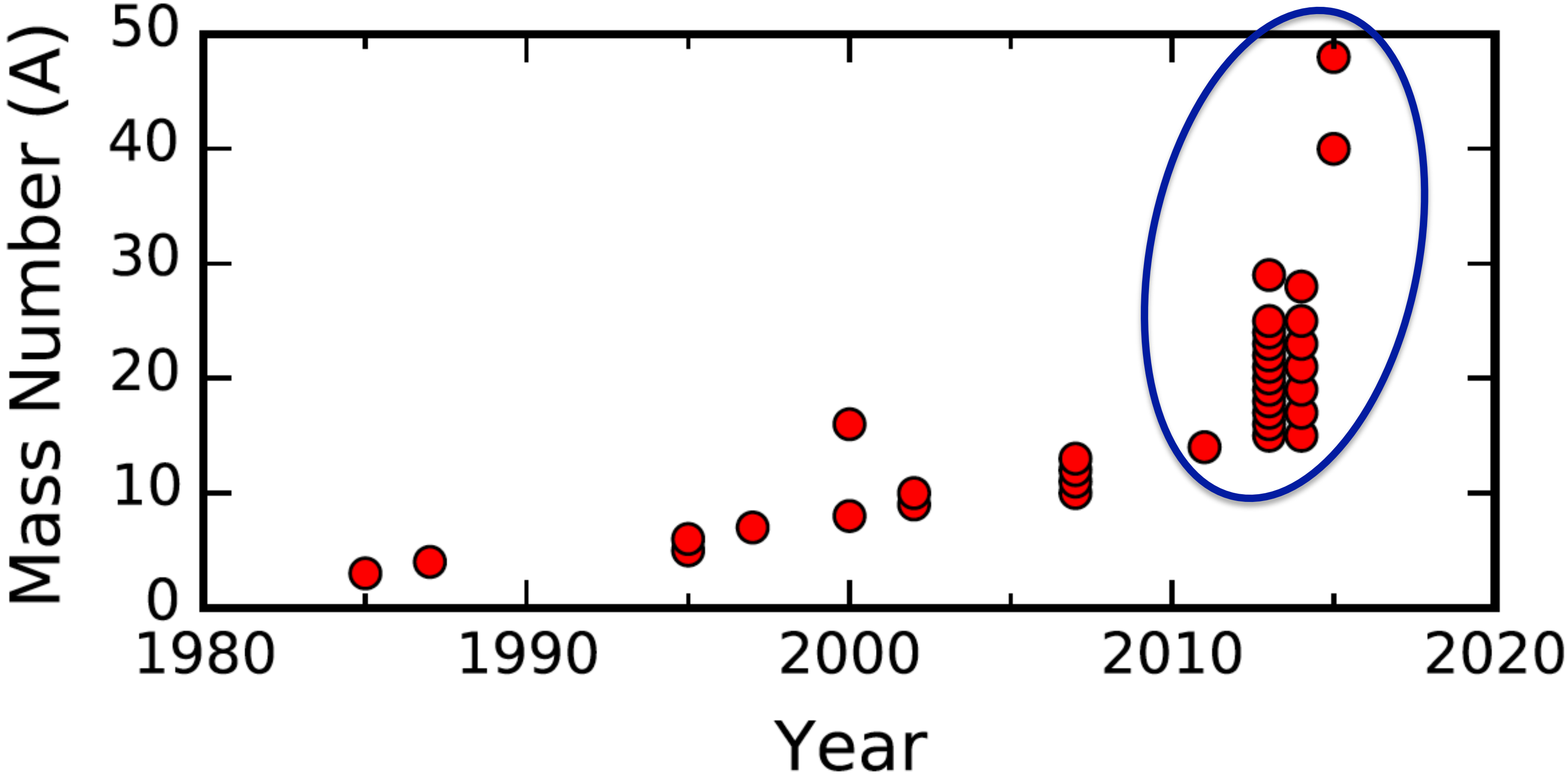


Fig.: Bertsch, Dean, Nazarewicz, SciDAC review (2007)

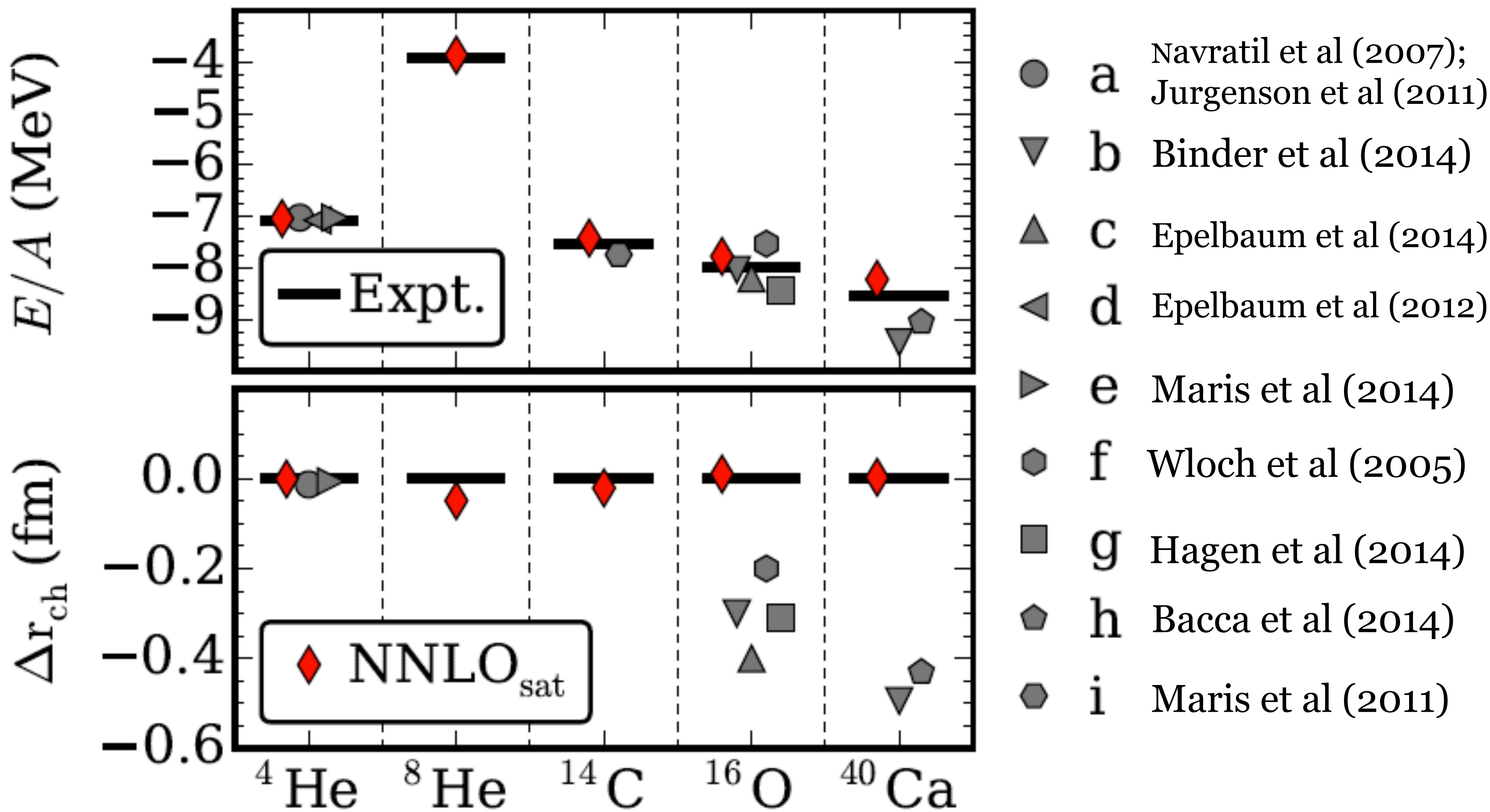
Trend in realistic *ab initio* calculations

Explosion of many-body methods
(Coupled clusters, Green's function Monte Carlo, In-Medium SRG, Lattice EFT, No-Core Shell Model, Self-Consistent Green's Function, UMOA, …)



Computational capabilities exceed accuracy of available interactions
[Binder *et al*, Phys. Lett. B 736 (2014) 119]

Chiral interactions fail to saturate accurately: too much binding and too small radii



A. Ekström, G. Jansen, K. Wendt et al, Phys. Rev. C91, 051301(R) (2015)

Chiral EFT interaction $\mathrm{NNLO}_{\mathrm{sat}}$

New approach:

1. Simultaneous optimization of NN and 3NF at NNLO

2. Input:
• NN scattering below 35 MeV, deuteron properties
• Ground-state energies and radii of ^{3}H, ^{3}He, ^{4}He, ^{14}C, ^{16}O
• Ground-state energies of ^{22,24,25}O

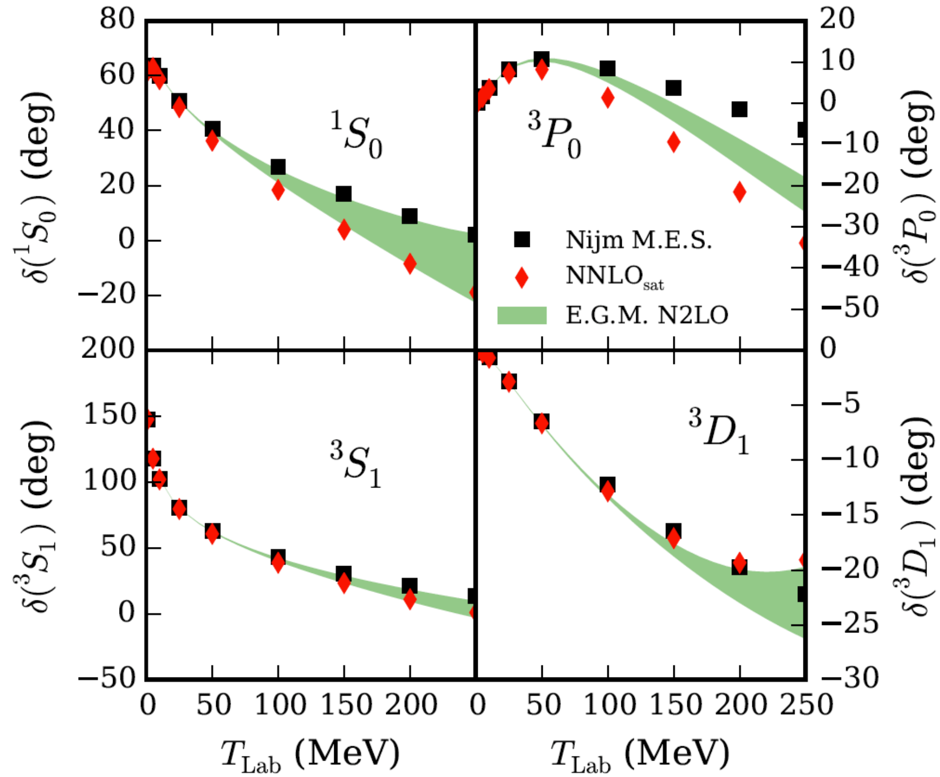
Rationale:

- Nuclear saturation an emergent phenomenon
- 3NF in T=3/2 not constrained in A≤4 nuclei
- No interaction yet achieves saturation from fit to A≤4 nuclei alone
- Binding energies and radii are low-energy data
- ... give up (for the moment) on predicting saturation.

A. Ekström, G. Jansen, K. Wendt et al, Phys. Rev. C91, 051301(R) (2015)

Results in *NN* sector

	NNLO$_{\text{sat}}$	N^{3}LO$_{\text{EM}}$	Exp.
a^C_{pp}	-7.8258	-7.8188	$-7.8196(26)$
r^C_{pp}	2.855	2.795	2.790(14)
a_{nn}	-18.929	-18.900	$-18.9(4)$
r_{nn}	2.911	2.838	2.75(11)
a_{np}	-23.728	-23.732	$-23.740(20)$
r_{np}	2.798	2.725	2.77(5)
E_D	2.22457	2.22458	2.224566
r_D	1.978	1.975	1.97535(85)
Q_D	0.270	0.275	0.2859(3)



Deviations above 35 MeV probably at limit of one would expect at NNLO.

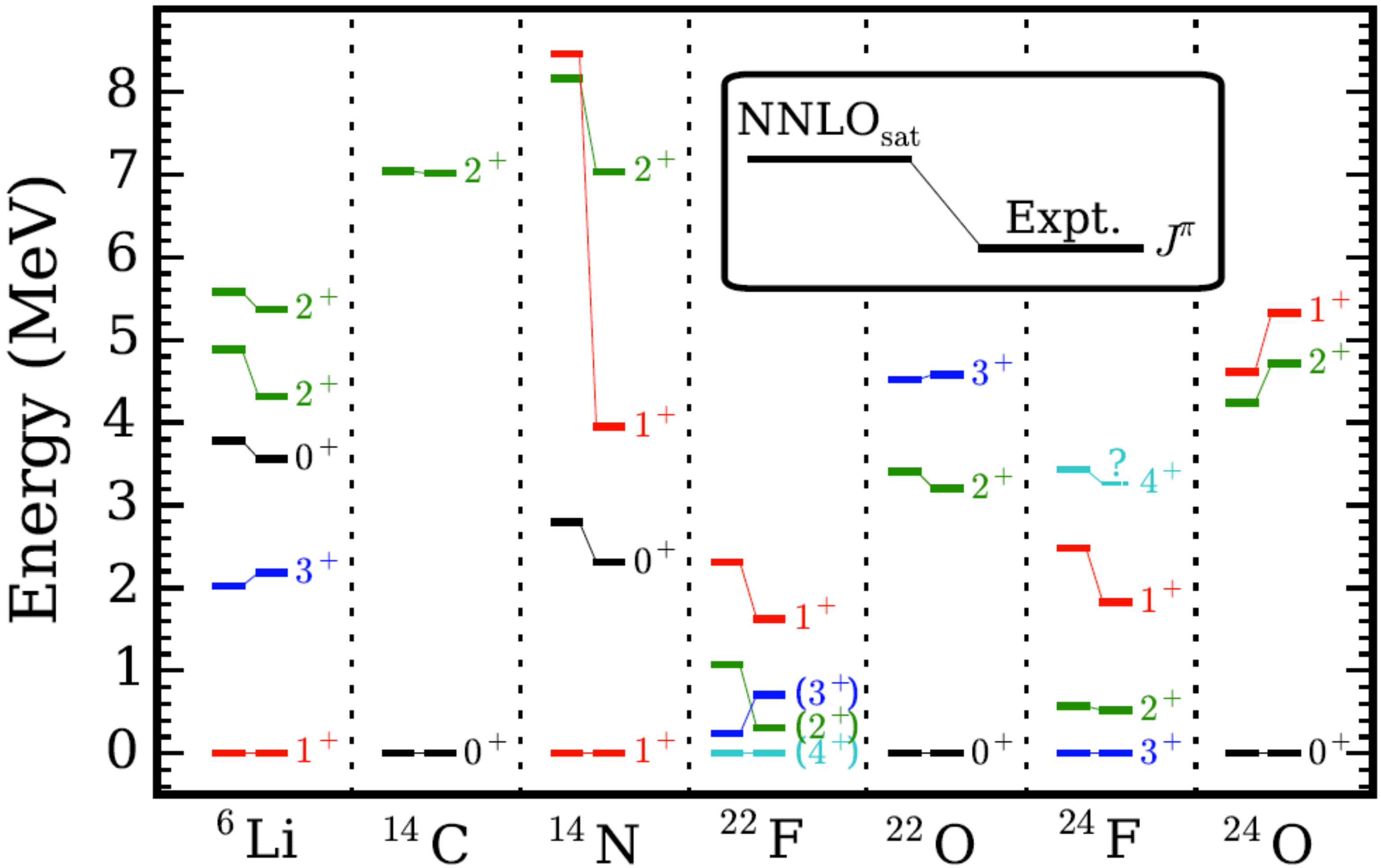
A. Ekström, G. Jansen, K. Wendt et al, Phys. Rev. C91, 051301(R) (2015)

Results for nuclei employed in optimization

	E_{gs}	Exp.	r_{ch}	Exp.
^{3}H	8.52	8.482	1.78	1.7591(363)
^{3}He	7.76	7.718	1.99	1.9661(30)
^{4}He	28.43	28.296	1.70	1.6755(28)
^{14}C	103.6	105.285	2.48	2.5025(87)
^{16}O	124.4	127.619	2.71	2.6991(52)
^{22}O	160.5	162.028(57)		
^{24}O	167.8	168.96(12)		
^{25}O	167.1	168.18(10)		

A. Ekström, G. Jansen, K. Wendt et al, Phys. Rev. C91, 051301(R) (2015)

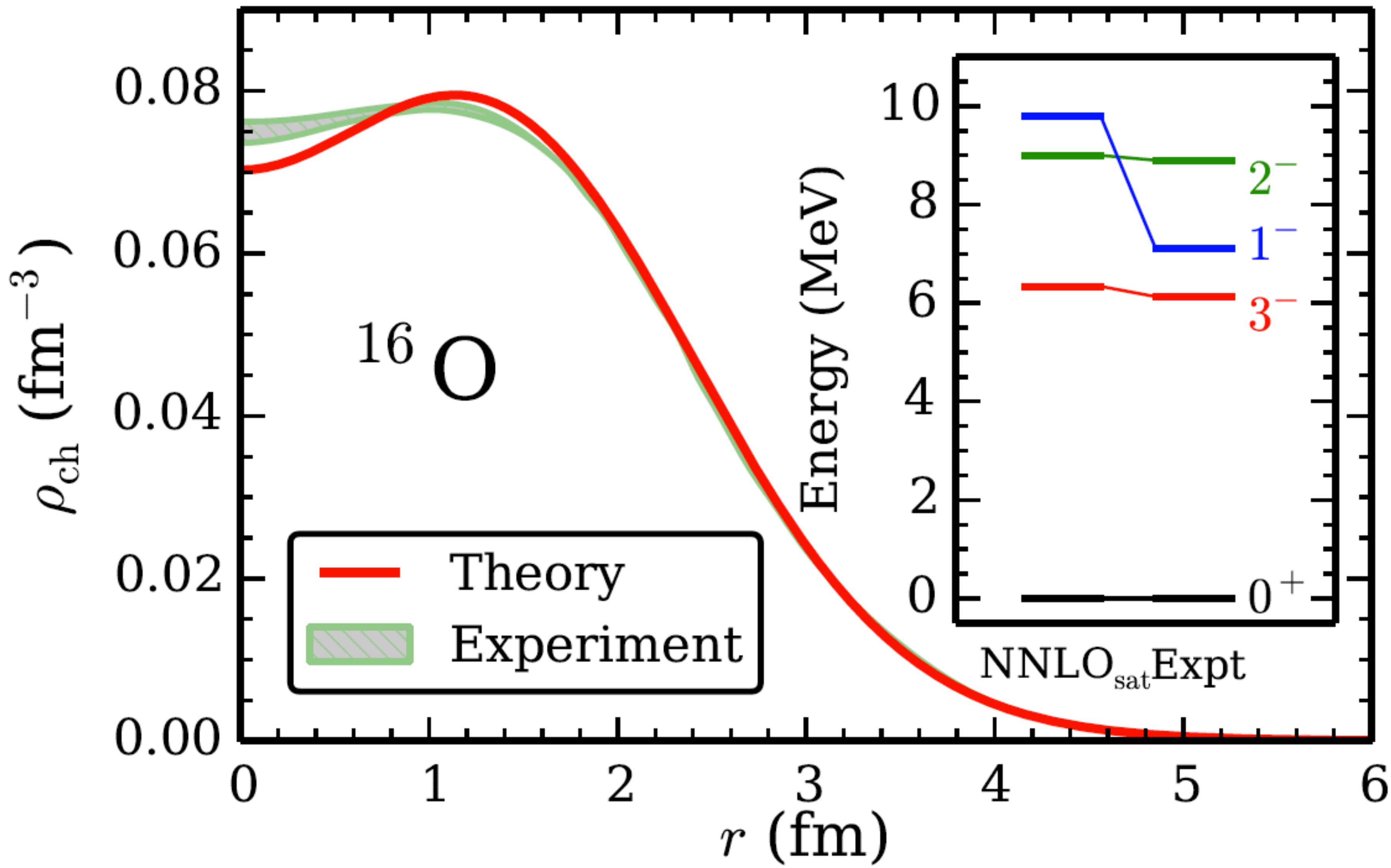
NNLO$_{sat}$ spectra



- Other deficiencies: ^{17,18}O ($s_{1/2}$ and $d_{3/2}$ too high, 2^+ too low)
- Overall NNLO$_{sat}$ spectra comparable to other chiral interactions

A. Ekström, G. Jansen, K. Wendt et al, Phys. Rev. C91, 051301(R) (2015)

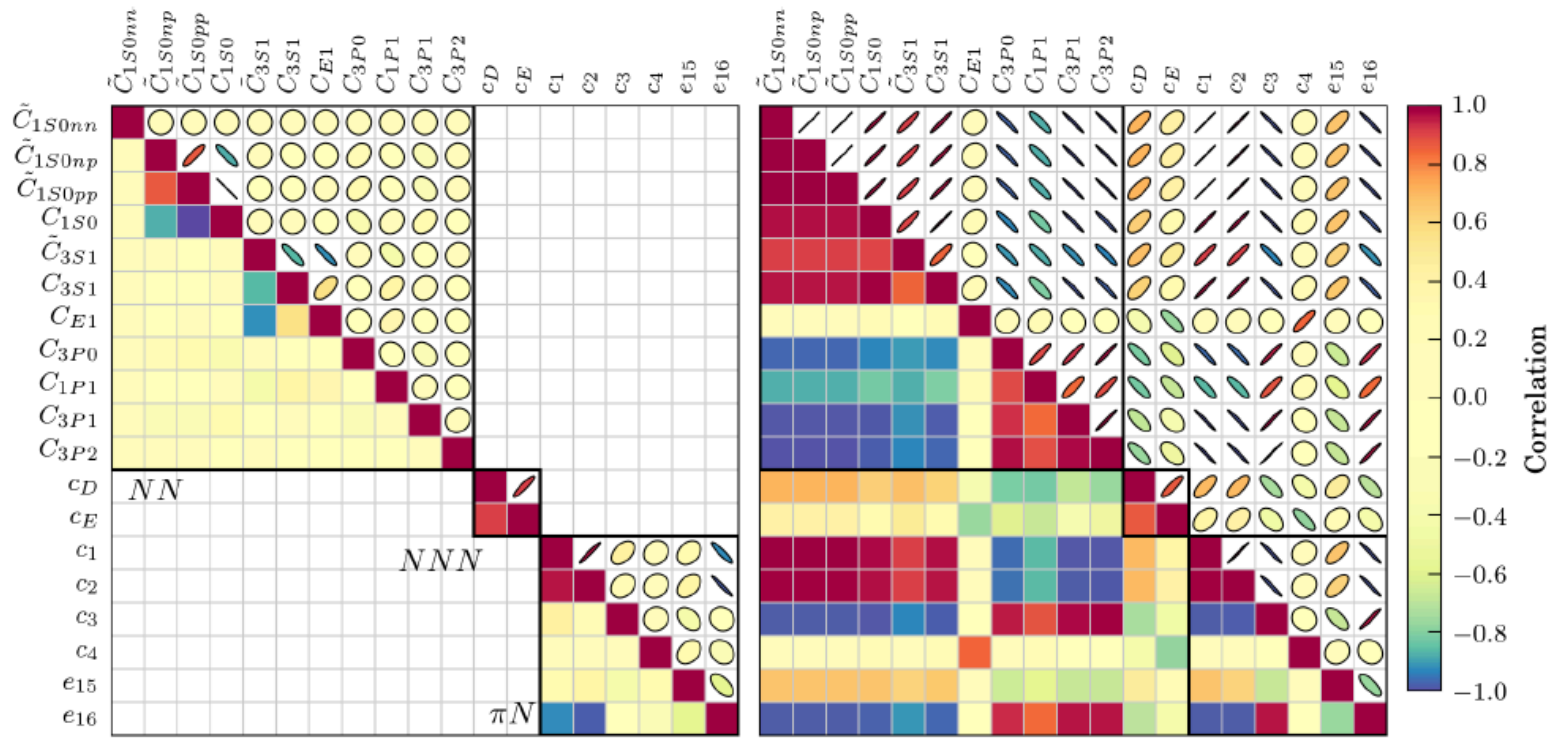
Charge density and excitations of ^{16}O



A. Ekström, G. Jansen, K. Wendt et al, Phys. Rev. C91, 051301(R) (2015)

Simultaneous (NN, NNN, πN) vs. separate optimization: Correlations in LECs and uncertainties

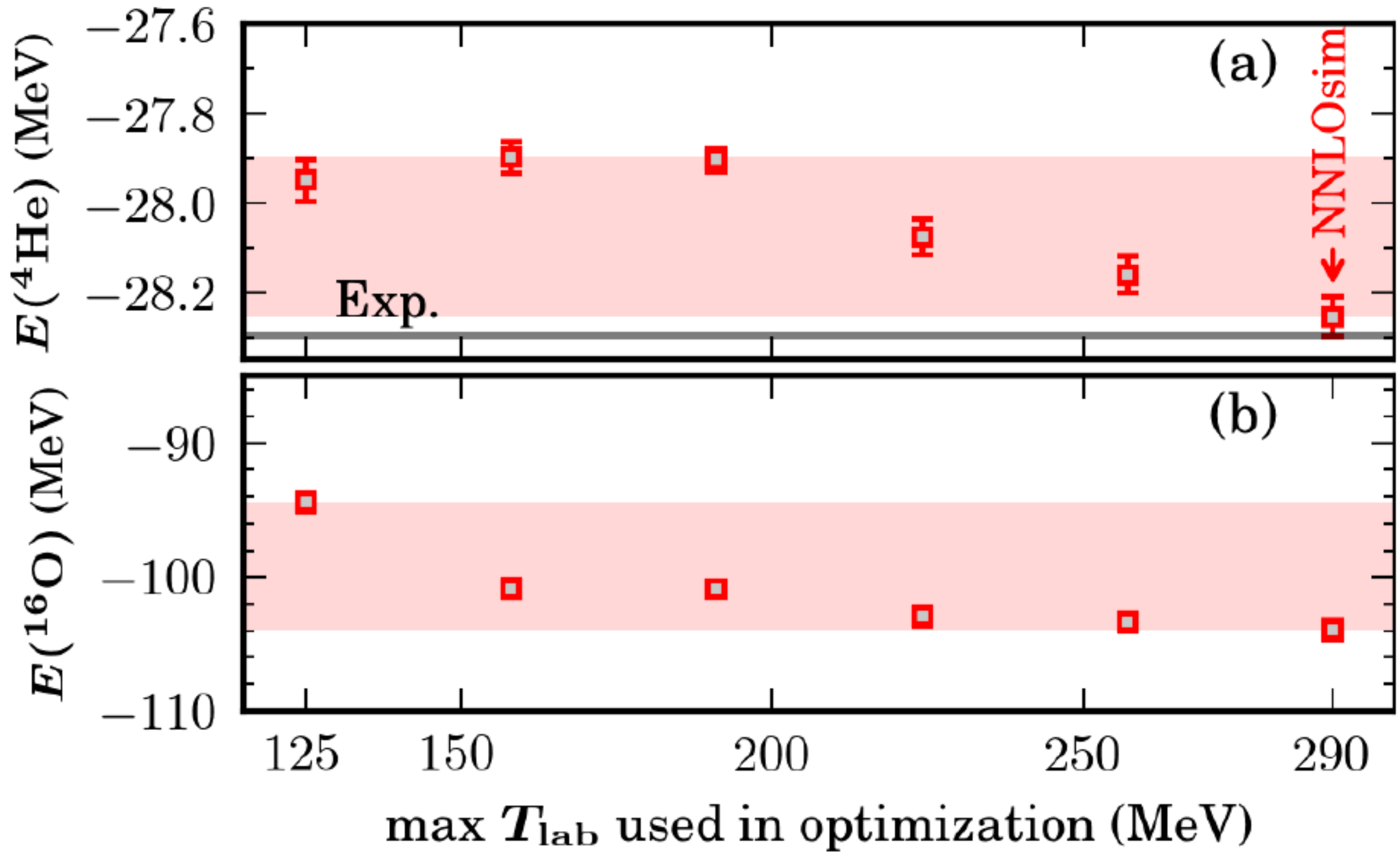
NNLO$_{\text{sep}}$: separate optimization   NNLO$_{\text{sim}}$: simultaneous optimization



	LOsep	NLOsep	NNLOsep	LOsim	NLOsim	NNLOsim	Exp.
$E(^2\mathrm{H})$	$-2.211(15)$	$-2.163^{(+9)}_{(-16)}$	$-2.2^{(+12)}_{(-25)}$	-2.225	$-2.225^{(+1)}_{(-6)}$	$-2.225^{(+0)}_{(-1)}$	-2.225
$E(^3\mathrm{H})$	$-11.40(4)$	$-8.220^{(+27)}_{(-43)}$	$-8.5^{(+31)}_{(-64)}$	-11.44	$-8.268^{(+27)}_{(-38)}$	$-8.482^{(+2)}_{(-5)}$	$-8.482(3)$
$E(^3\mathrm{He})$	$-10.39(4)$	$-7.474^{(+29)}_{(-45)}$	$-7.7^{(+30)}_{(-62)}$	-10.44	$-7.528^{(+20)}_{(-31)}$	$-7.718^{(+2)}_{(-6)}$	$-7.718(4)$
$E(^4\mathrm{He})$	$-40.27(13)$	$-27.56^{(+14)}_{(-18)}$	$-28^{(+8)}_{(-18)}$	$-40.39(1)$	$-27.44^{(+13)}_{(-15)}$	$-28.26^{(+4)}_{(-5)}$	$-28.30(1)$

B. Carlsson, A. Ekström, C. Forssén *et al*, arXiv:1506.02466

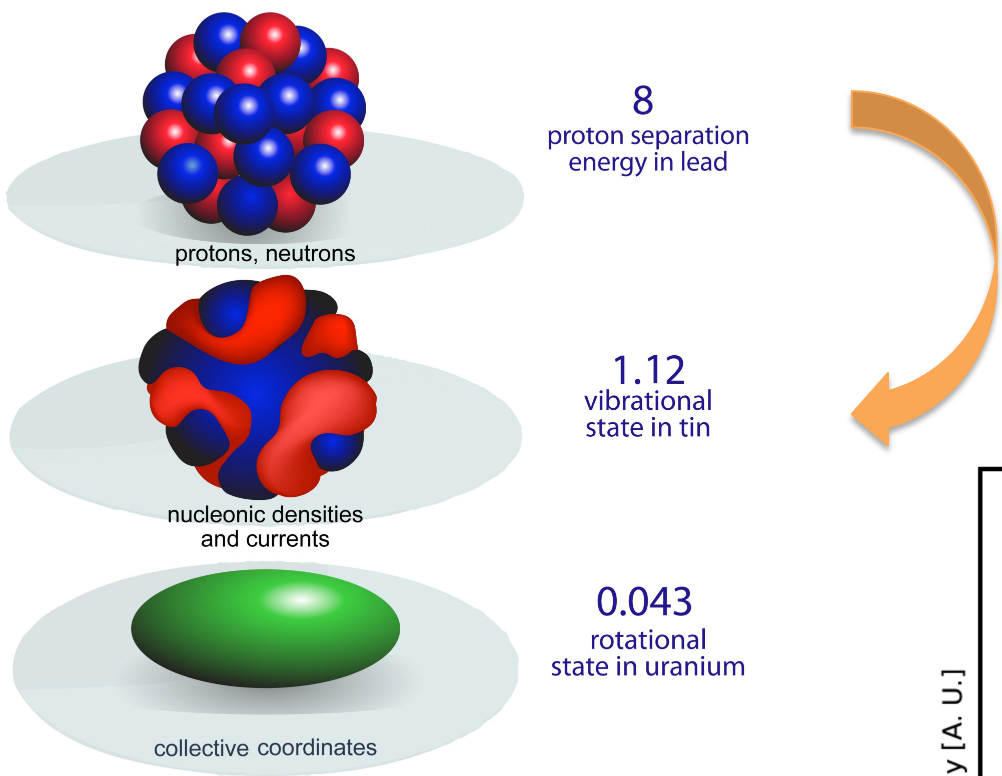
Sensitivity analysis for $NNLO_{sim}$



A 1% change in the binding energy of ^4He yields a 10% change in ^{16}O

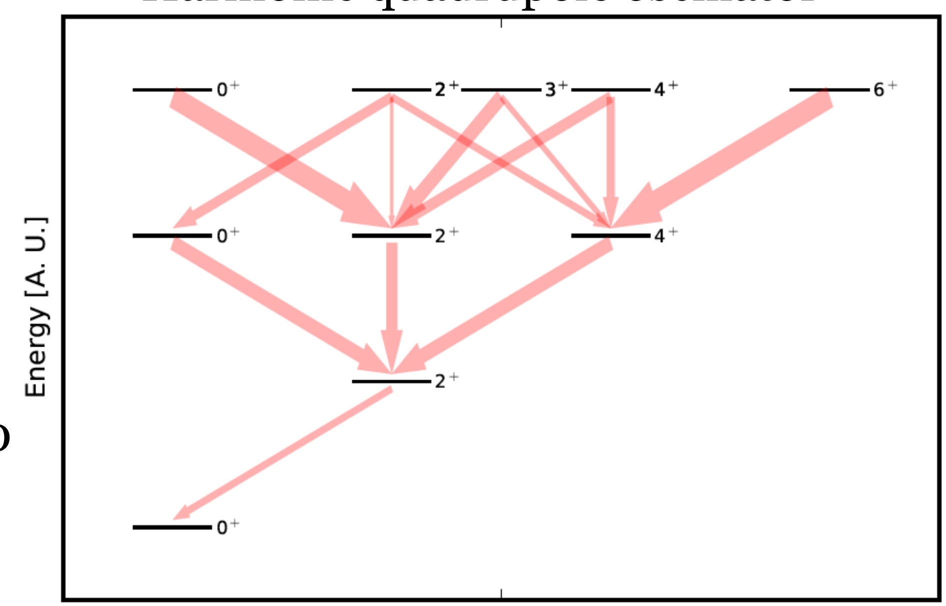
B. Carlsson, A. Ekström, C. Forssén *et al*, arXiv:1506.02466

EFT for nuclear vibrations
[with E. A. Coello Peréz, arXiv:1510.02401]



EFT for nuclear vibrations

Harmonic quadrupole oscillator

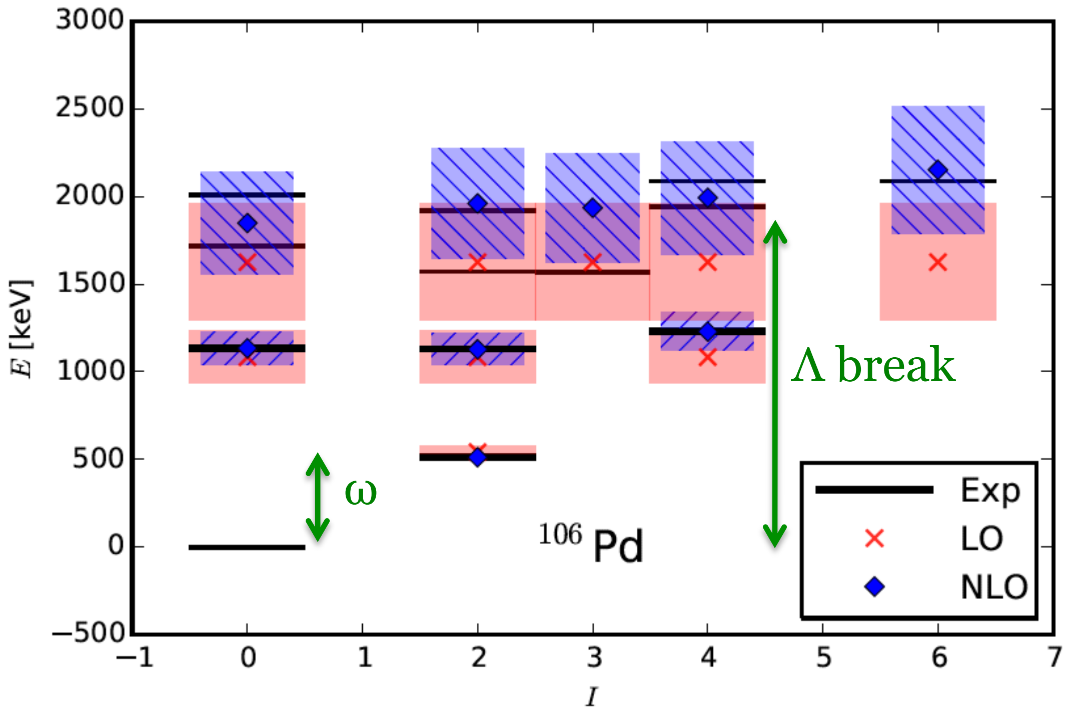


Spectrum and B(E2) transitions

While spectra of certain nuclei appear to be harmonic, B(E2) transitions do not.

Garrett & Wood (2010): "Where are the qudrupole vibrations in atomic nuclei?"

EFT for nuclear vibrations



EFT ingredients:

- quadrupole degrees of freedom
- breakdown scale around three-phonon levels
- "small" expansion parameter: ratio of vibrational energy to breakdown scale: $\omega/\Lambda \approx 1/3$

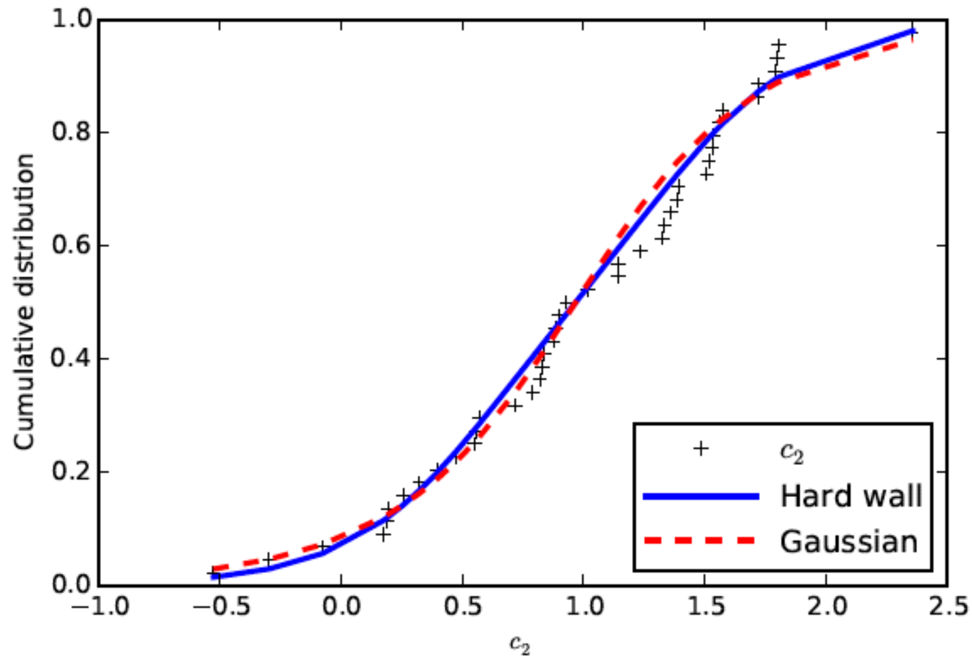
- Uncertainties show 68% DOB intervals from Bayesian analysis of EFT truncation effects, following [Cacciari & Houdeau (2011); Bagnaschi et al (2015); Furnstahl, Klco, Phillips & Wesolowski (2015)]
  - Expand observables according to power counting
  - Employ "naturalness" assumptions as log-normal priors in Bayes' theorem
  - Compute distribution function of uncertainties due to EFT truncation
  - Compute degree-of-believe (DOB) intervals.

Uncertainty quantification

$$E_{\mathrm{NLO}} = \omega N + g_\omega N + g_N N^2 + g_v v(v+3) + g_I I(I+1)$$

$$\begin{aligned} c_2 &\equiv c_2(N, v, I) \\ &= \frac{g_\omega N + g_N N^2 + g_v v(v+3) + g_I I(I+1)}{\varepsilon^2 \omega} \end{aligned}$$

$$X = X_0 \sum_{n=0}^{\infty} c_n \varepsilon^n$$



Linear combinations of LECs enter observables. LECs are random, but with EFT expectations, i.e. log-normal distributed. Making assumptions about these distributions then allows one to quantify uncertainties. The assumptions can be tested.

$$\Delta_k^{(M)} = \sum_{n=k+1}^{k+M} c_n \varepsilon^n$$

$$p_M(\Delta | c_0, \ldots, c_k) = \frac{\int_0^\infty \mathrm{d}c \; \mathrm{pr}(c) p_M(\Delta|c) \prod_{m=0}^{k} \mathrm{pr}(c_m|c)}{\int_0^\infty \mathrm{d}c \; \mathrm{pr}(c) \prod_{m=0}^{k} \mathrm{pr}(c_m|c)}$$

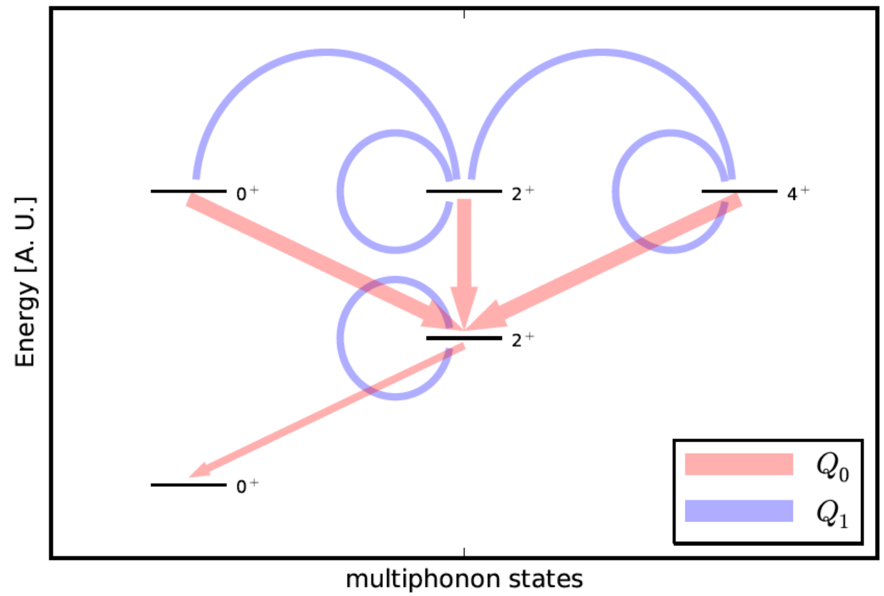
New EFT result: sizeable quadrupole matrix elements

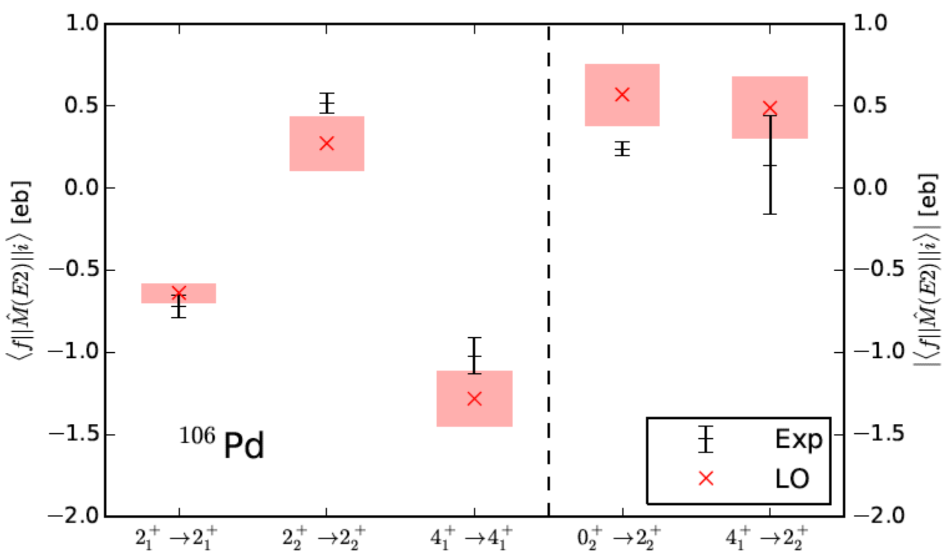
In the EFT, the quadrupole operator is also expanded:

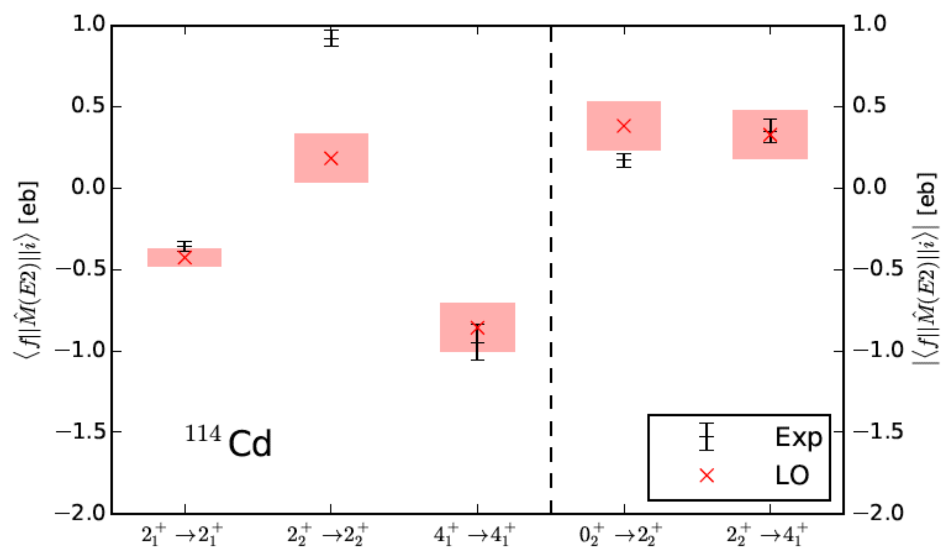
$$\hat{Q}_\mu = Q_0 \left(d_\mu^\dagger + \tilde{d}_\mu \right) + Q_1 \left(d^\dagger \times d^\dagger + \tilde{d} \times \tilde{d} + 2 d^\dagger \times \tilde{d} \right)_\mu^{(2)}$$

Subleading corrections are sizable:

$$Q_1 \sim \left(\frac{\omega}{\Lambda} \right)^{1/2} Q_0$$







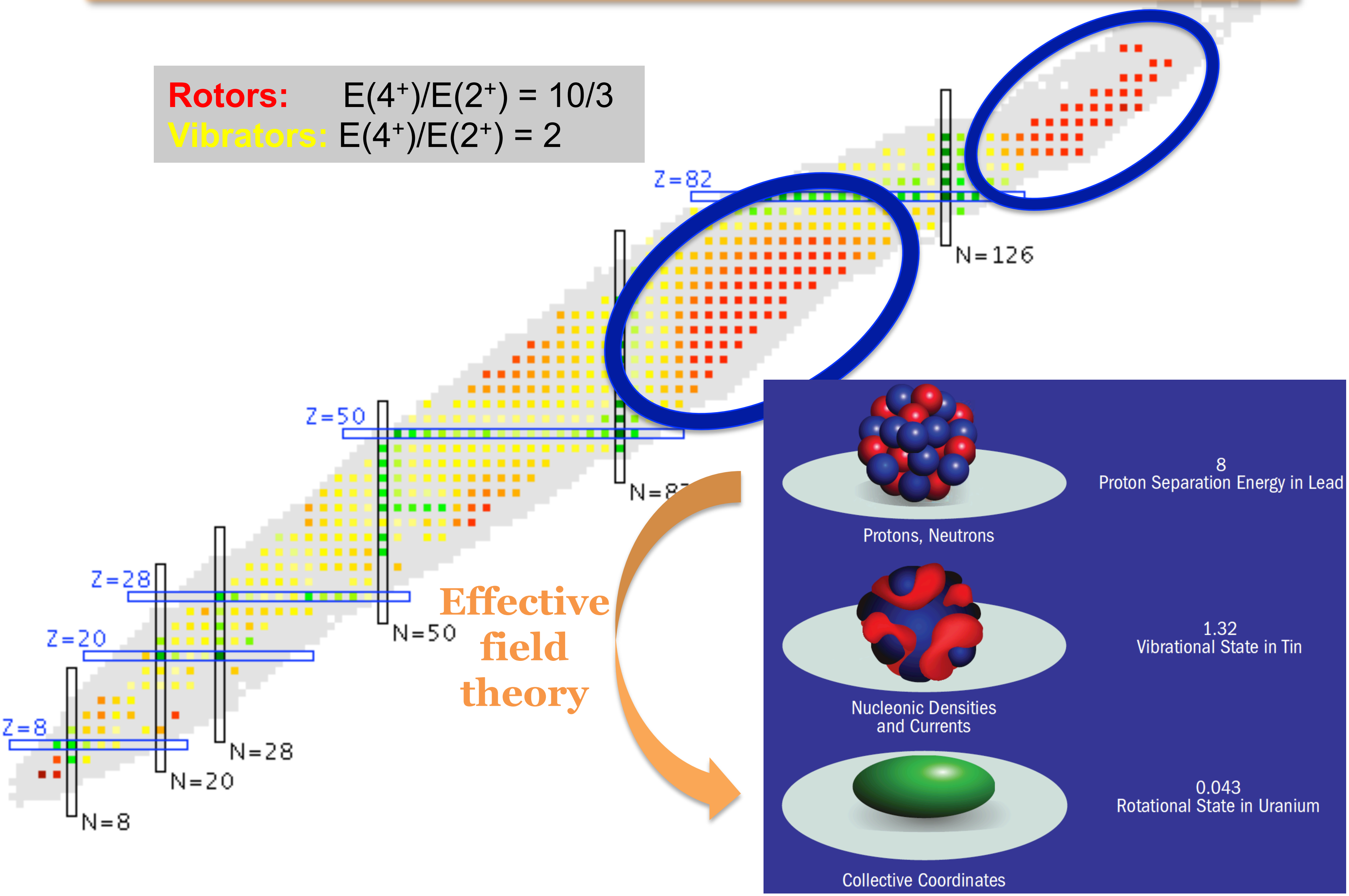
B(E2) transitions in vibrational nuclei

Nucleus	$2_1^+ \to 0_1^+$	EFT	$0_2^+ \to 2_1^+$	$2_2^+ \to 2_1^+$	$4_1^+ \to 2_1^+$	EFT
^{62}Ni	12.1(4)	11(4)	42(23)	14.9(42)	21(6)	21(7)
^{98}Ru	31(1)	28(9)		47(5)	57.6(40)	56(19)
^{100}Ru	35.6(4)	24(8)	35(5)	30.9(4)	51(4)	47(16)
^{106}Pd	44.3(15)	30(10)	35(8)	44(4)	76(11)	61(20)
^{108}Pd	49.5(13)	37(12)	52(5)	71(5)	73(8)	74(25)
^{110}Cd	27.0(8)	21(7)		30(5)	42(9)	42(14)
^{112}Cd	30.2(3)	23(8)	51(14)	15(3)	61(6)	46(15)
^{114}Cd	31.1(19)	22(7)	27.4(17)	22(6)	62(4)	43(15)
^{120}Te	31 (6)	31(10)				62(21)
^{122}Te	36.9(3)	41(14)		100(30)		81(27)

B(E2) transition strengths consistent with EFT expectations within the theoretical uncertainties (68% DOB intervals).

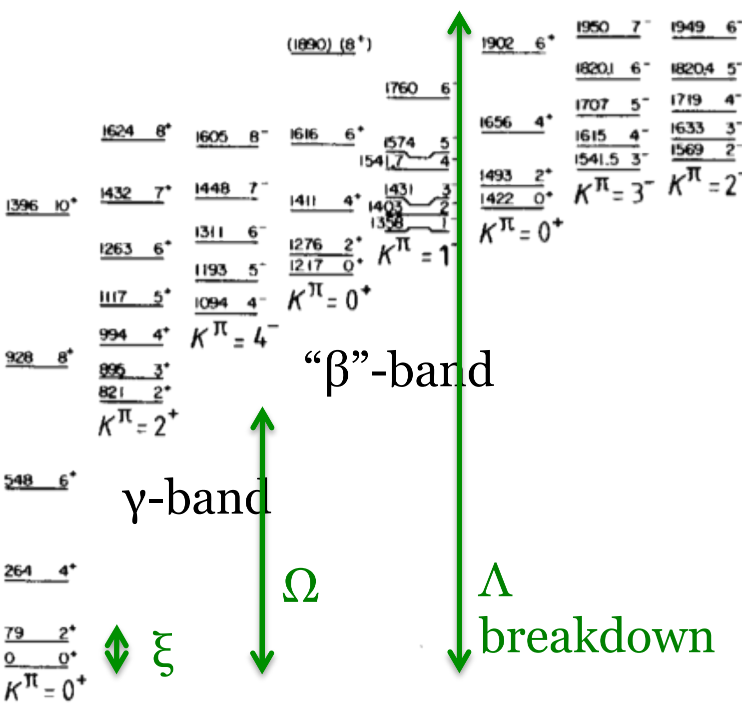
EFT for deformed nuclei

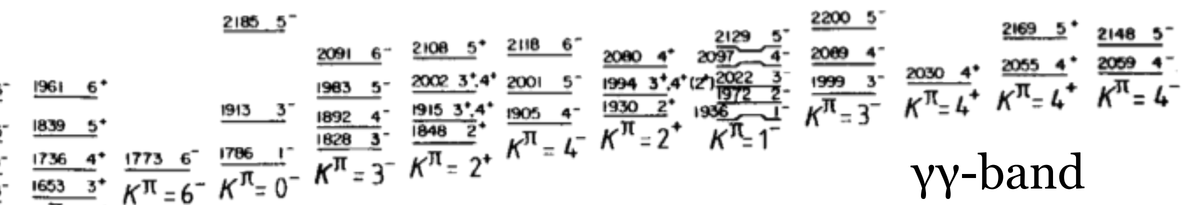
Rotors: $E(4^+)/E(2^+) = 10/3$
Vibrators: $E(4^+)/E(2^+) = 2$



Electromagnetic transitions in deformed nuclei

“Complete” spectrum of ^{168}Er [Davidson *et al.*, J. Phys. G 7, 455 (1981)]





Separation of scale: $\xi << \Omega << \Lambda$

Key features of geometric collective model

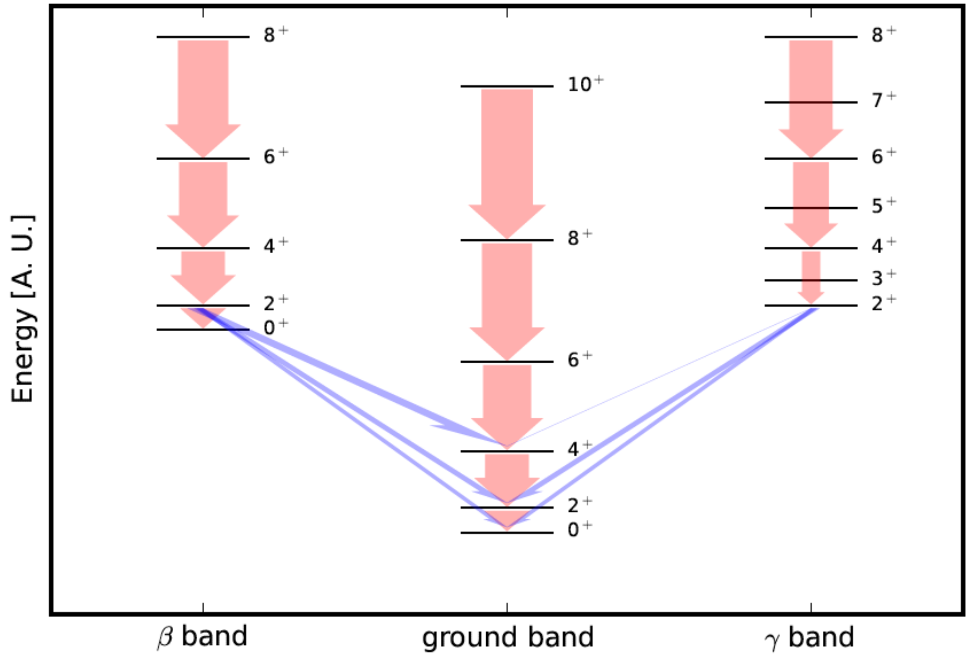
✓ Rotational bands on top of vibrational band heads
✓ Strong in-band E2 transitions
✓ Weaker inter-band E2 transitions
✓ Spectra are reproduced rather well

✗ Inter-band transitions are factors 2-10 too strong [Garrett, J. Phys. G 27 (2001) R1; Rowe & Wood “Fundamentals of Nuclear Models” (2010)]

Consistent coupling of EM fields addresses this problem

Spectra and transitions in deformed nuclei

Problem: Traditional collective models overpredict faint inter-band transitions by factors 2—10.



EFT for deformed nuclei:

- Separation of scale between rotations, vibrations, and fermionic effects
- Emergent breaking of rotational symmetry SO(3)→ SO(2) requires rotational invariance to be realized nonlinearly. [TP 2011; TP & Weidenmüller 2014/2015]
- Quadrupole degrees of freedom

EFT for deformed nuclei

Spectrum of ground-state band

$$E(I) = \frac{I(I+1)}{2C_0} - \frac{C_2}{4C_0^4}[I(I+1)]^2$$

Strength of quadrupole transitions $I_i \rightarrow I_i - 2$ in ground-state band
(Clebsch-Gordan coefficient divided out)

$$Q_{if}^2 = Q_0^2\left[1 + \frac{b}{a} I_i(I_i - 1)\right]$$

No surprises here: the EFT reproduces well known results from phenomenological models (e.g. Variable Moment of Inertia, Mikhailov theory…)
EFT provides us with insight in scale of parameters in expansion of observables

E. A. Coello Pérez and TP, Phys. Rev. C 92, 014323 (2015)

EFT: expansion parameter & naturalness

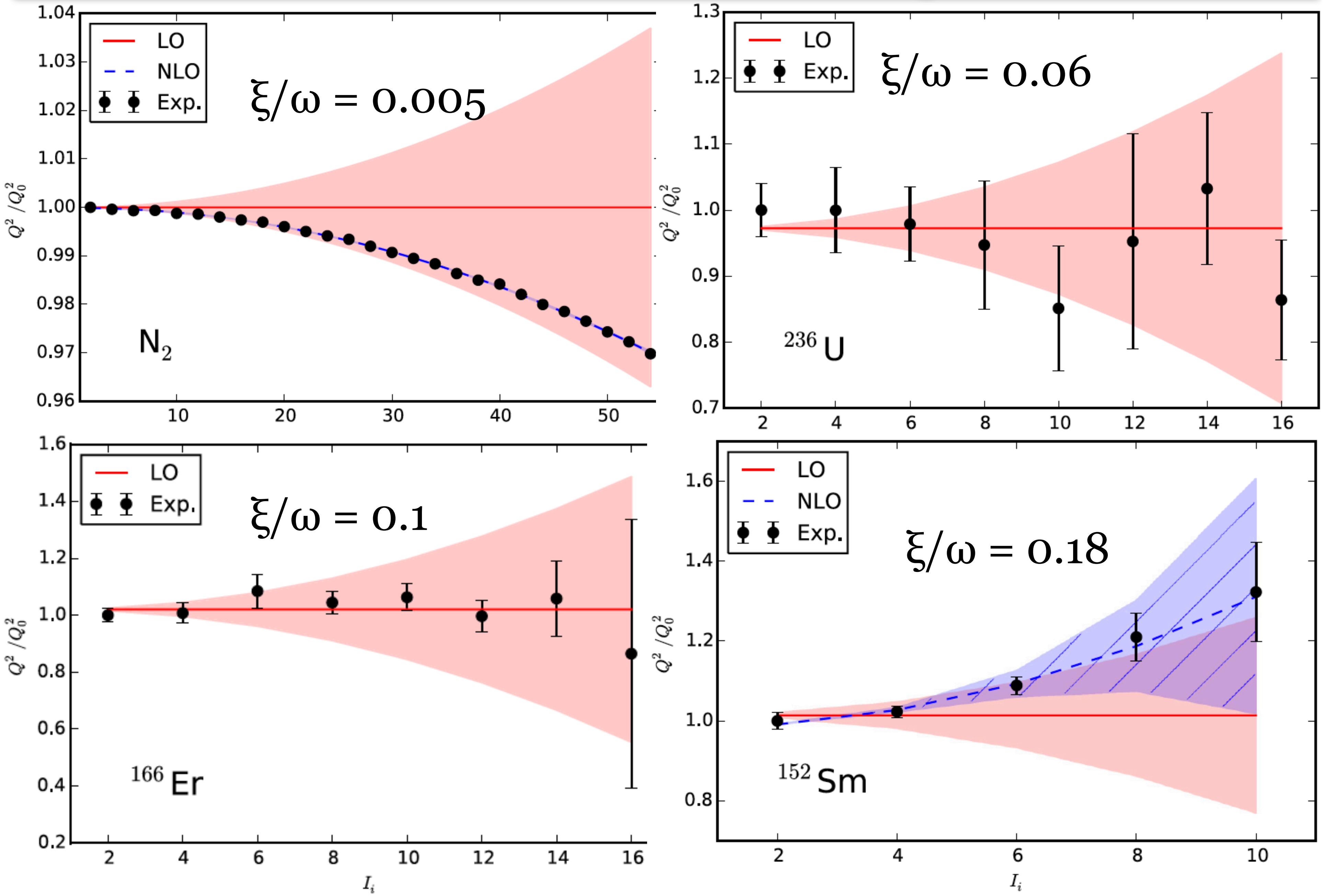
Natural sizes as expected!

	System	Expansion parameter: E_{rot} / E_{vib} $(\xi/\omega)^2$	Natural LECs: spectrum C_2/C_0^3	Natural LECs: transitions b/a
Molecules	N_2	0.000 026	0.000 006	-0.000 011
	H_2	0.0062	0.0015	0.0022
Rotational nuclei	^{236}U	0.0043	0.0011	—
	^{174}Yb	0.0026	0.0010	—
	^{168}Er	0.0094	0.0010	—
	^{166}Er	0.011	0.0020	—
	^{162}Dy	0.0083	0.0017	—
	^{154}Sm	0.0056	0.0033	—
Transitional nuclei	^{188}Os	0.06	0.012	0.008
	^{154}Gd	0.033	0.013	0.006
	^{152}Sm	0.032	0.013	0.003
	^{150}Nd	0.037	0.017	0.011

less rigid rotor

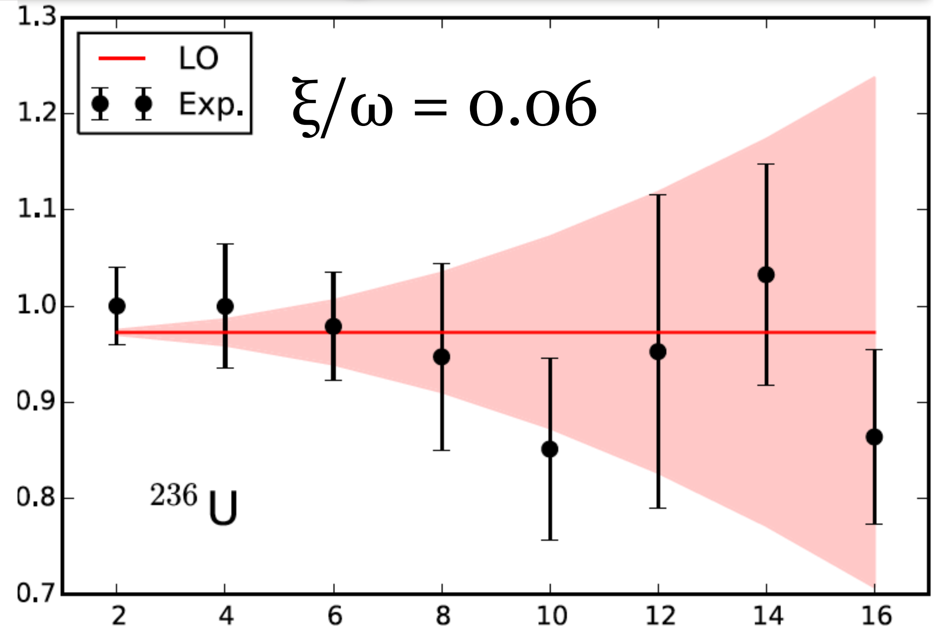
E. A. Coello Pérez and TP, Phys. Rev. C 92, 014323 (2015)

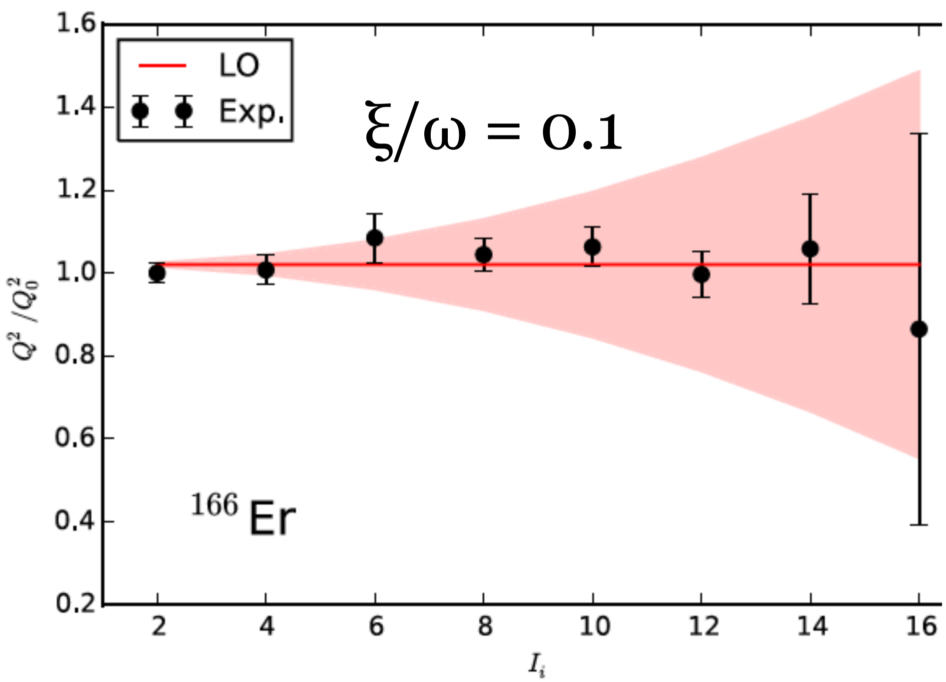
EFT works well for a wide range of rotors

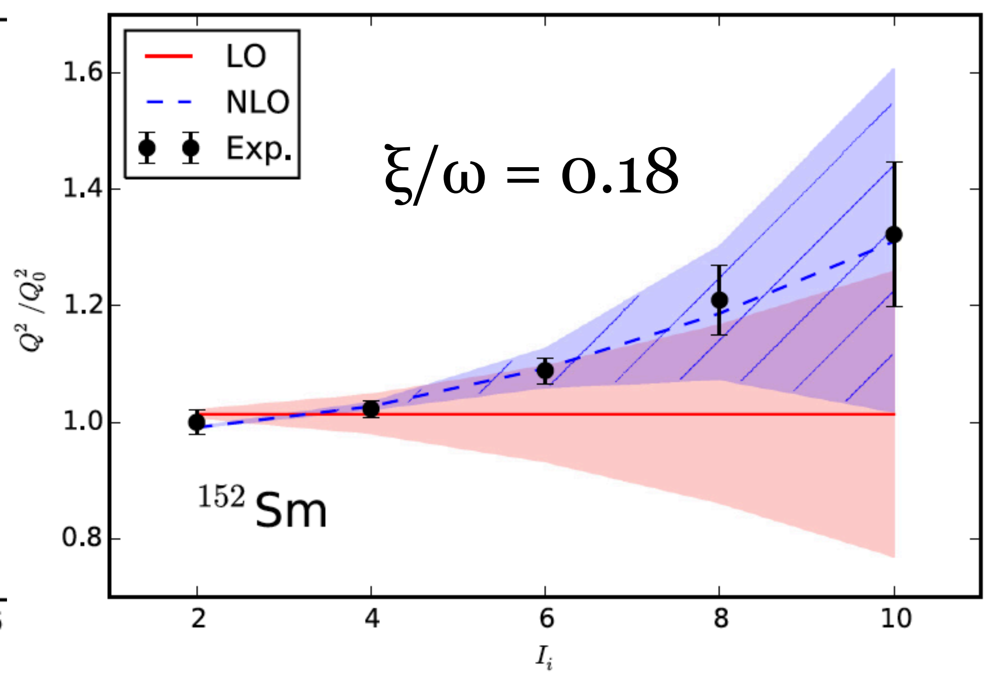


EFT works well for a wide range of rotors

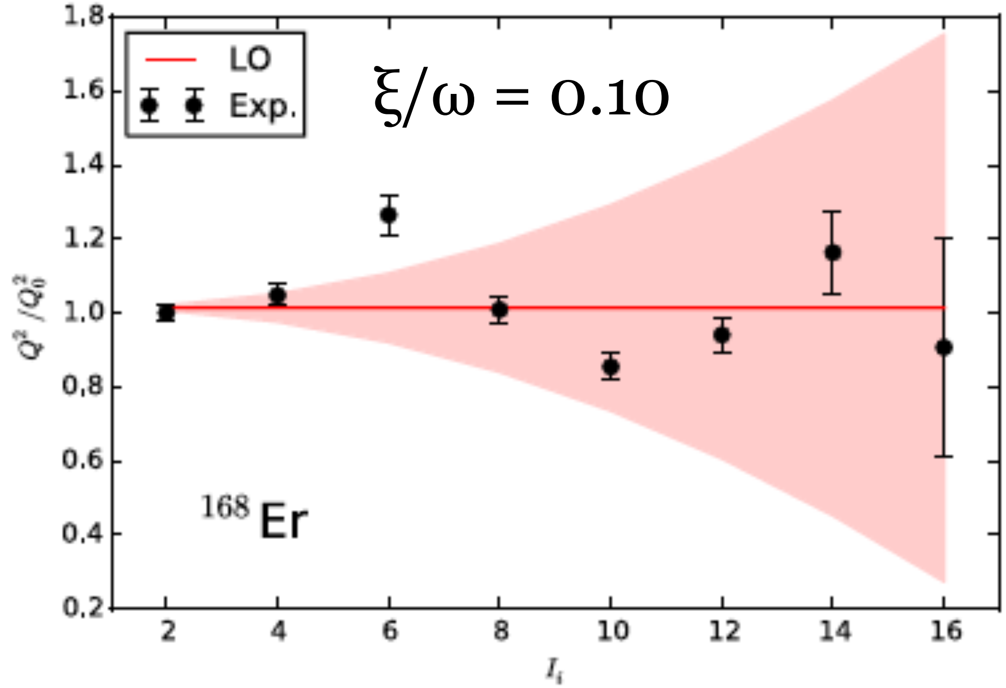
Bohr & Mottelson (1975):
"The accuracy of the present measurements of E2-matrix elements in the ground-state bands of even even nuclei is in most cases barely sufficient to detect deviations from the leading-order intensity relations."







EFT can not explain oscillatory patterns in supposedly “good” rotors ^{168}Er, ^{174}Yb

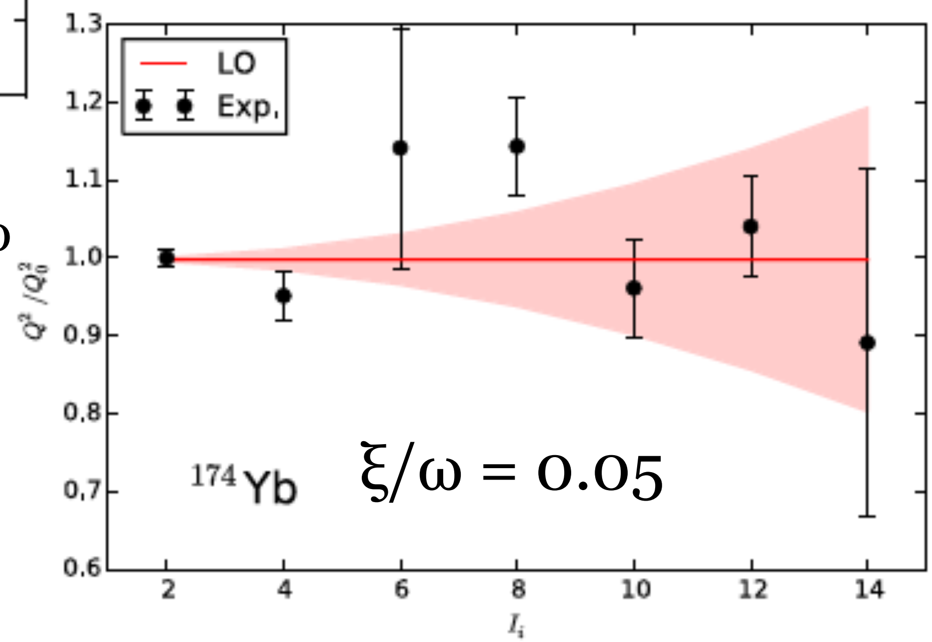


Based on results for molecules, well-deformed nuclei, and transitional nuclei, EFT suggests that a few transitions in text-book rotors could merit re-measurement.

^{168}Er: B(E2) for $6^+ \rightarrow 4^+$ very difficult to understand.

^{174}Yb: B(E2) for $8^+ \rightarrow 6^+$ difficult to reconcile with $4^+ \rightarrow 2^+$.

Theoretical uncertainty estimates relevant.



EFT and weak interband transitions (^{154}Sm)

$i \to f$	$B(E2)_{exp}$	$B(E2)_{ET}$	$B(E2)_{CBS}$	$B(E2)_{BH}$
$2_g^+ \to 0_g^+$	0.863 (5)	0.863[a]	0.853	0.863
$4_g^+ \to 2_g^+$	1.201 (29)	1.233 (9)	1.231	1.234
$6_g^+ \to 4_g^+$	1.417 (39)	1.358 (23)	1.378	1.355
$8_g^+ \to 6_g^+$	1.564 (83)	1.421 (43)	1.471	1.424
$2_\gamma^+ \to 0_g^+$	0.0093 (10)	0.0110 (28)		0.0492
$2_\gamma^+ \to 2_g^+$	0.0157 (15)	0.0157[a]		0.0703
$2_\gamma^+ \to 4_g^+$	0.0018 (2)	0.0008 (2)		0.0050
$2_\beta^+ \to 0_g^+$	0.0016 (2)	0.0025 (6)	0.0024	0.0319
$2_\beta^+ \to 2_g^+$	0.0035 (4)	0.0035[a]	0.0069	0.0456
$2_\beta^+ \to 4_g^+$	0.0065 (7)	0.0063 (16)	0.0348	0.0821

[a]Values employed to adjust the LECs of the effective theory.

In-band transitions [in e^2b^2] are LO, inter-band transitions are NLO. Effective theory is more complicated than Bohr Hamiltonian both in Hamiltonian and E2 transition operator. EFT correctly predicts strengths of inter-band transitions with natural LECs.

[E. A. Coello Pérez and TP, Phys. Rev. C 92, 014323 (2015)]

Summary

- **Exciting times in nuclear theory**
  - explosion of many-body solvers; capabilities not matched by interactions
  - many new developments regarding interactions
- **Optimization of chiral interaction NNLO$_{\mathbf{sat}}$**
  - spectra of p-shell and sd-shell nuclei comparable to other chiral interactions
  - considerably improved radii and binding energies
- **EFT for deformed nuclei & vibrational nuclei**
  - **vibrational nuclei**: consistent description of spectra and EM moments within uncertainties up to the 2-phonon limit; anharmonic quadrupole vibrators
  - **deformed nuclei**: description of strong in-band transitions with uncertainty estimates suggest re-measurements of some transitions; weak inter-band transitions correctly described within EFT.