Constraining the Relativistic Nuclear Energy Density Functional

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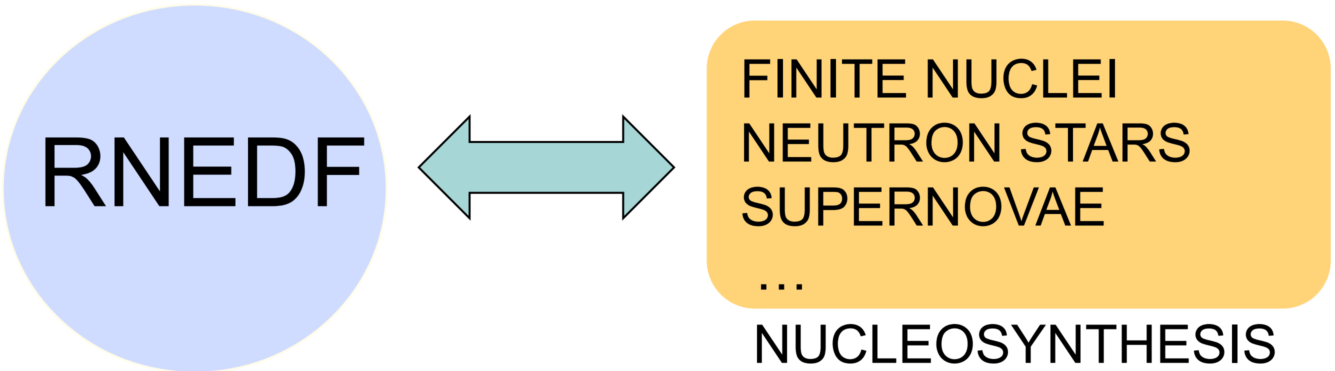
WORKING HYPOTHESIS

Final understanding of how supernova explosions and nucleosynthesis work, with self-consistent microscopic description of all relevant nuclear physics included, has not been achieved yet.

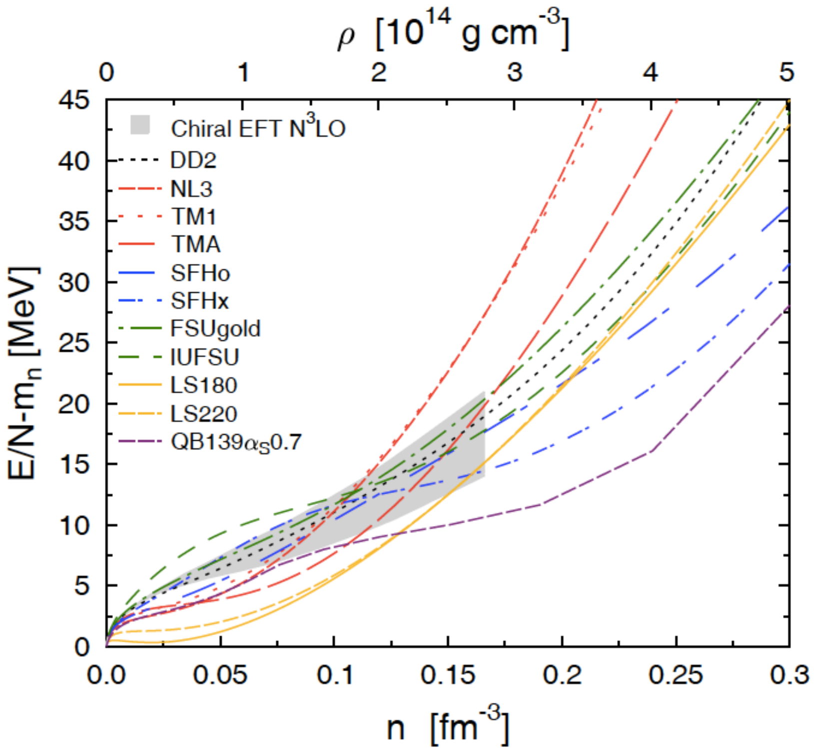
OUR GOAL: Universal relativistic nuclear energy density functional (RNEDF) for

- properties of finite nuclei (nuclear masses, radii, excitations,…)
- nuclear equation of state (EOS) ➔ supernova equation of state (with M. Hempel)
- neutron star properties (mass/radius, …)
- electron capture in presupernova collapse
- neutrino-nucleus reactions and beta decays of relevance for the nucleosynthesis
- other astrophysically related phenomena…



A FEW EXAMPLES

Neutron matter energy per particle



T. Fischer et al. EPJ A 50, 46 (2014).

Stellar electron capture rates

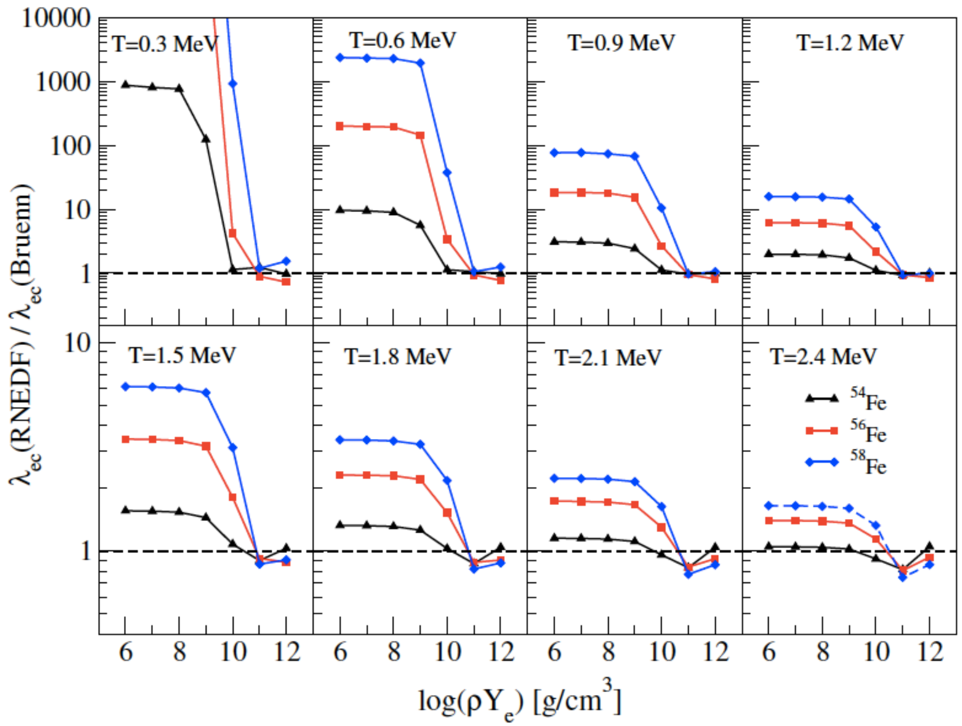


FIG. 7: The ratio between the RNEDF and Bruenn [5] electron capture rates for ^{54,56,58}Fe, shown as a function of ρY_e for the range of temperatures T=0.3,0.6,...,2.4 MeV.

N. P. et al. (2015).
S. W. Bruenn, A.J. Supp. 58, 771 (1985).

THEORY FRAMEWORK

T. Niksic, et al., Comp. Phys. Comm. 185, 1808 (2014).

- The implementation of relativistic nuclear energy density functional

- The basis is an effective Lagrangian with four-fermion (contact) interaction terms; isoscalar-scalar, isoscalar-vector, isovector-vector, derivative term

$$
\begin{aligned}
\mathcal{L} &= \bar{\psi}(i\gamma \cdot \partial - m)\psi \\
&- \frac{1}{2}\alpha_S(\hat{\rho})(\bar{\psi}\psi)(\bar{\psi}\psi) - \frac{1}{2}\alpha_V(\hat{\rho})(\bar{\psi}\gamma^\mu\psi)(\bar{\psi}\gamma_\mu\psi) \\
&- \frac{1}{2}\alpha_{TV}(\hat{\rho})(\bar{\psi}\vec{\tau}\gamma^\mu\psi)(\bar{\psi}\vec{\tau}\gamma_\mu\psi) \\
&- \frac{1}{2}\delta_S(\partial_\nu\bar{\psi}\psi)(\partial^\nu\bar{\psi}\psi) - e\bar{\psi}\gamma \cdot A\frac{(1-\tau_3)}{2}\psi
\end{aligned}
$$

- many-body correlations encoded in density-dependent coupling functions that are motivated by microscopic calculations but parameterized in a phenomenological way

- In addition: pairing correlations in finite nuclei
  - Relativistic Hartree-Bogoliubov model
  (with separable form of the pairing interaction Y. Tian, Z. Y. Ma, P. Ring, PLB 676, 44 (2009).)

- Relativistic Q(RPA)

THEORY FRAMEWORK

- Density dependence of the couplings

$$\alpha_i(\rho) = a_i + (b_i + c_i x)e^{-d_i x} \qquad (i \equiv S, V, TV)$$

$$x = \rho/\rho_{sat}$$

- 12 model parameters:

a_s, b_s, c_s, d_s	isoscalar-scalar
a_v, b_v, d_v	isoscalar-vector
b_{TV}, d_{TV}	isovector-vector
δ_s	derivative term
g_n, g_p	pairing correlations (strength parameters)

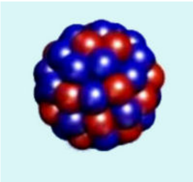
CONSTRAINING THE FUNCTIONAL

- The model parameters $\mathbf{p} = (p_1, ..., p_n)$ are constrained directly by many-body observables using χ^2 minimization

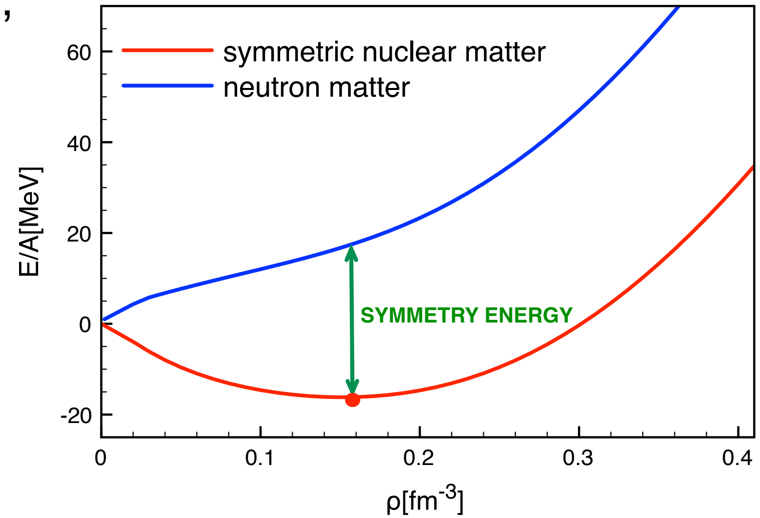
$$\chi^2(\boldsymbol{p}) = \sum_{\imath=1}^{m} \left(\frac{\mathcal{O}_\imath^{\text{theo.}}(\boldsymbol{p}) - \mathcal{O}_\imath^{\text{ref.}}}{\Delta \mathcal{O}_\imath^{\text{ref.}}} \right)^2$$

- Calculated values can be compared to experimental, observational, and pseudo-data, e.g.

  - properties of finite nuclei – e.g., binding energies, charge radii, diffraction radii, surface thicknesses, pairing gaps, etc.,…



  - nuclear matter properties – equation of state, binding energy and density at the saturation, symmetry energy J & L, incompressibility…



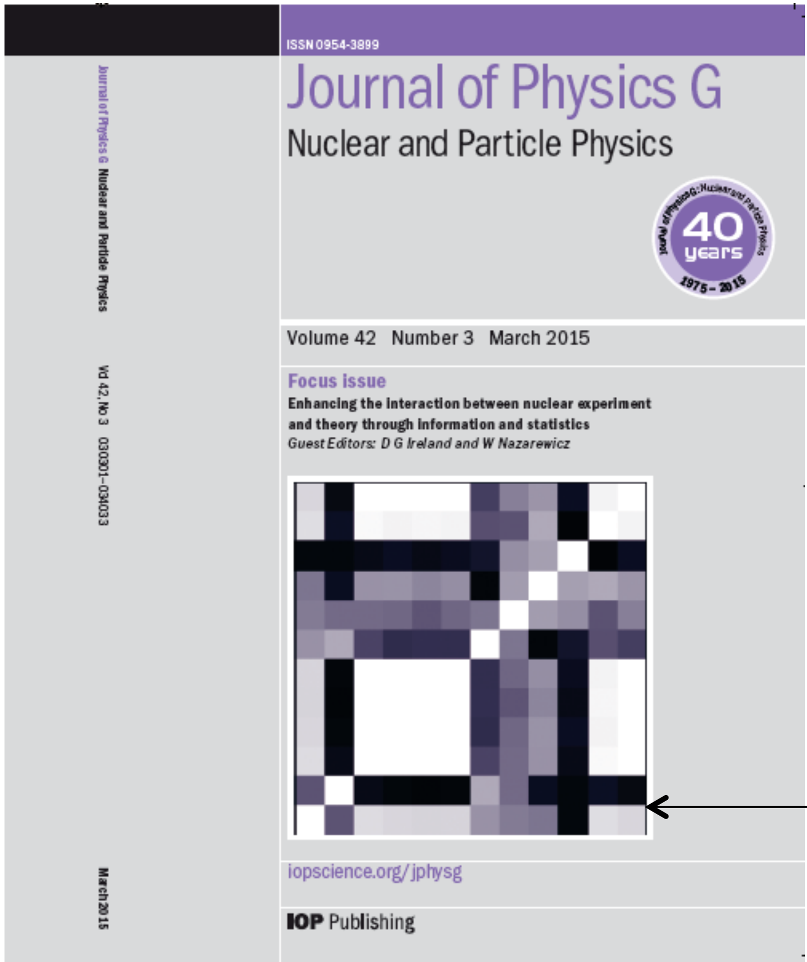
- Isovector channel of the EDF is weakly constrained by exp. data such as binding energies and charge radii. Possible observables for the isovector properties: *neutron radii, neutron skins, dipole polarizability, pygmy dipole strength, neutron star radii*

COVARIANCE ANALYSIS IN THE FRAMEWORK OF EDFs

The quality of X^2 minimization to exp. data is an indicator of the statistical uncertainty

- Curvature matrix:

$$\mathcal{M}_{ij} = \frac{1}{2}\partial_{p_i}\partial_{p_j}\chi^2|_{\mathbf{p}_0}$$



Covariance between two quantities A & B:

$$\overline{\Delta A\,\Delta B} = \sum_{ij}\partial_{p_i}A(\hat{\mathcal{M}}^{-1})_{ij}\partial_{p_j}B$$

1) variance $\overline{\Delta^2 A}$ defines statistical uncertainty of calculated quantity

2) correlations between quantities A & B:

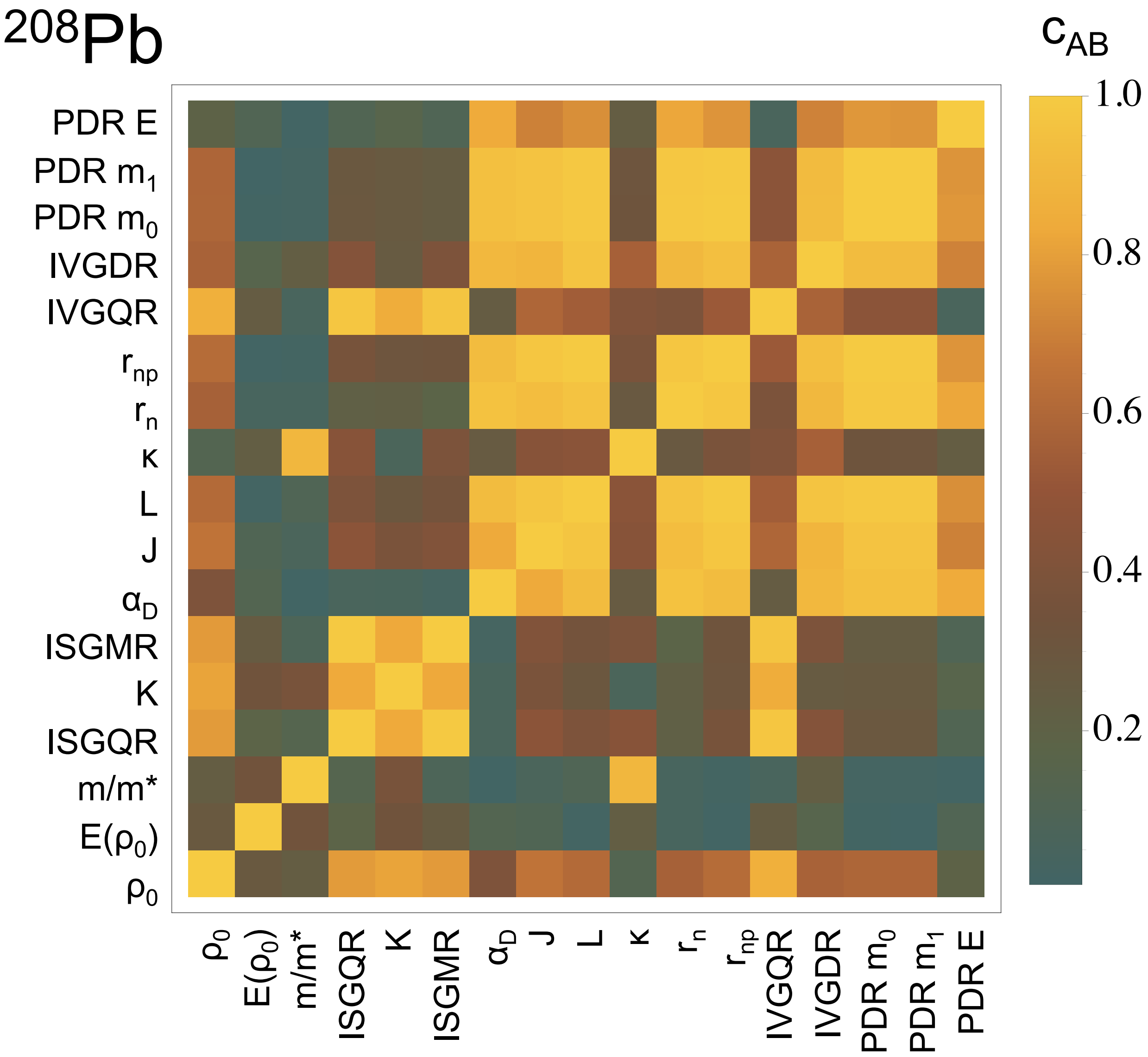
$$c_{AB} = \frac{|\overline{\Delta A\,\Delta B}|}{\sqrt{\overline{\Delta A^2}\;\overline{\Delta B^2}}}$$

- see: J. Dobaczewski, W. Nazarewicz, P.-G. Reinhard, JPG 41, 074001 (2014)

- X. Roca Maza et al., JPG 42, 034033 (2015)
- T. Niksic et al., JPG 42, 034008 (2015)

- Correlation matrix shows the correlations between various quantities.

CORRELATIONS: NUCLEAR MATTER vs. PROPERTIES OF NUCLEI



Correlation matrix between nuclear matter properties and several quantities in ^{208}Pb (DDME-min1)

- neutron skin thickness, properties of giant resonances, pygmy strength

c_{AB}=1: A & B strongly correlated

c_{AB}=0: A & B uncorrelated

CONSTRAINTS ON THE NUCLEAR MATTER INCOMPRESSIBILITY

- The compression modulus of nuclear matter K_{nm} can be obtained from the energies of isoscalar giant monopole resonance (ISGMR)

- ISGMR energies extracted from inelastic scattering of alpha particles

$$E_{ISGMR} = \hbar\sqrt{\frac{K_A}{m\langle r^2\rangle}}$$

- Strategy to reach K_{nm}:

1) Build the energy density functional (EDF), each parameterization corresponds to a K_{nm}

2) Calculate ISGMR excitation energy using the same EDF (e.g., RPA)

3) The K_{nm} value associated with the EDF that best describes the experimental ISGMR energy is considered as the “correct” one.

CONSTRAINING THE SYMMETRY ENERGY

Nuclear matter equation of state:

$$E(\rho, x) = E_{SNM}(\rho) + E_{sym}(\rho)(1-2x)^2 + ...$$

$$\rho = \rho_n + \rho_p \ , x = \rho_p/\rho$$

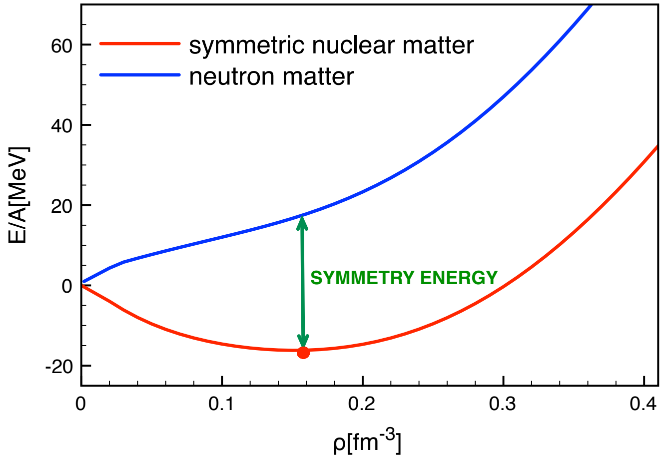
Symmetry energy term:

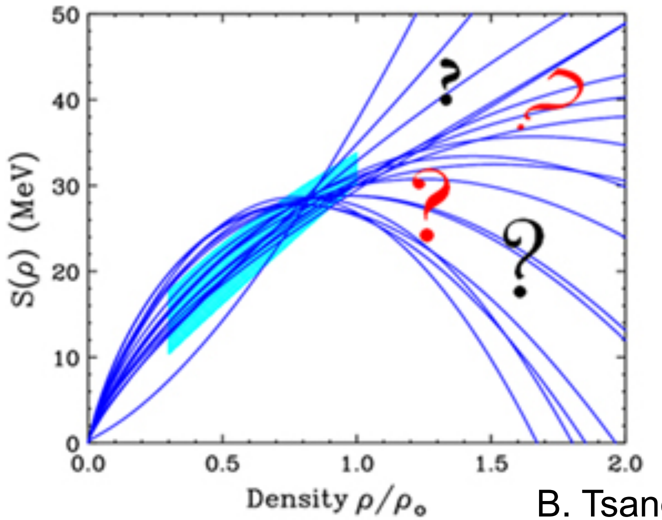
$$E_{sym}(\rho) \equiv S_2(\rho) = J - L\epsilon + \dots$$

$$\epsilon = (\rho_0 - \rho)/(3\rho_0)$$

$$L = 3\rho_0 \frac{dS_2(\rho)}{dr}|_{\rho_0}$$

J – symmetry energy at saturation density
L – slope of the symmetry energy (related to the pressure of neutron matter)



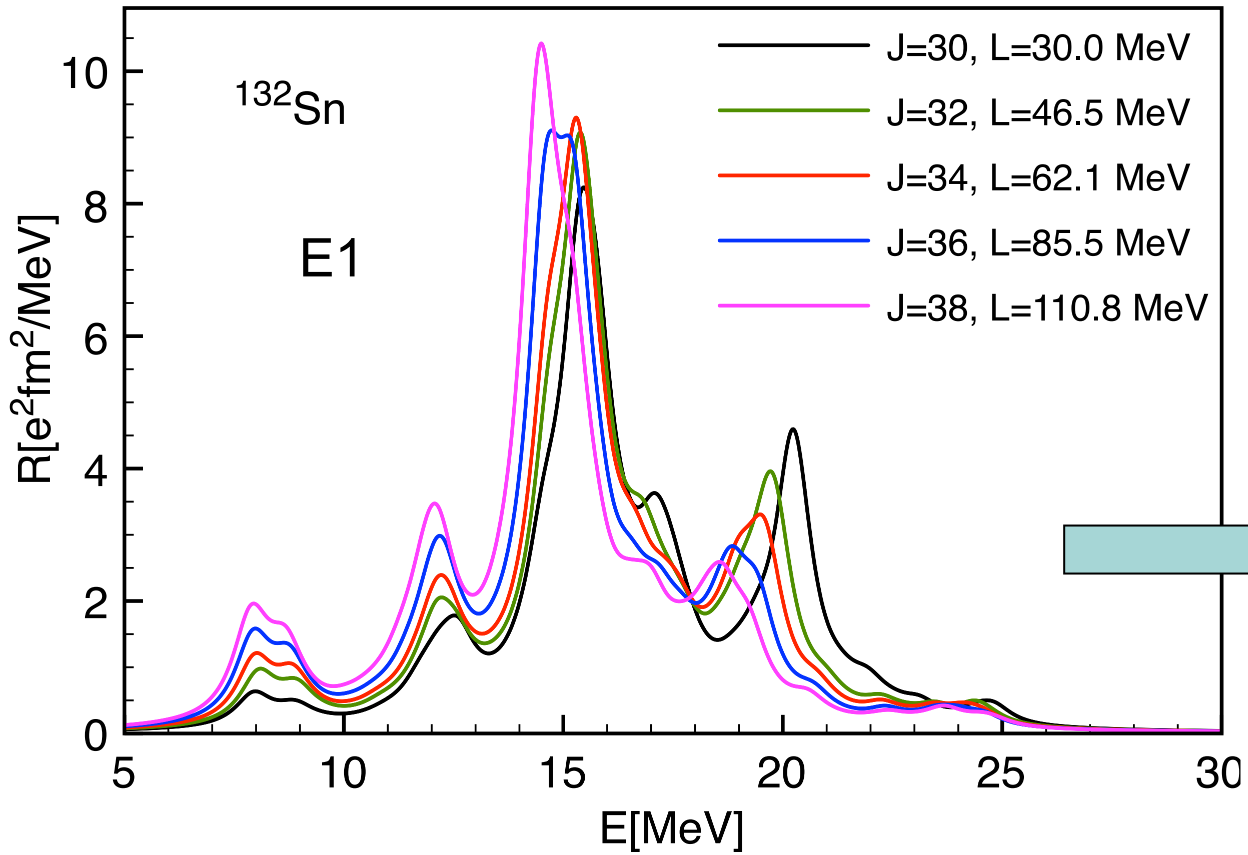


B. Tsang, NSCL

CONSTRAINING THE SYMMETRY ENERGY

• Isovector dipole transition strength is sensitive to the symmetry energy at saturation density (J) / slope of the symmetry energy (L).

• Set of effective DD-ME interactions constrained on the same data, but with different constraint on J (30,32,…,38 MeV).

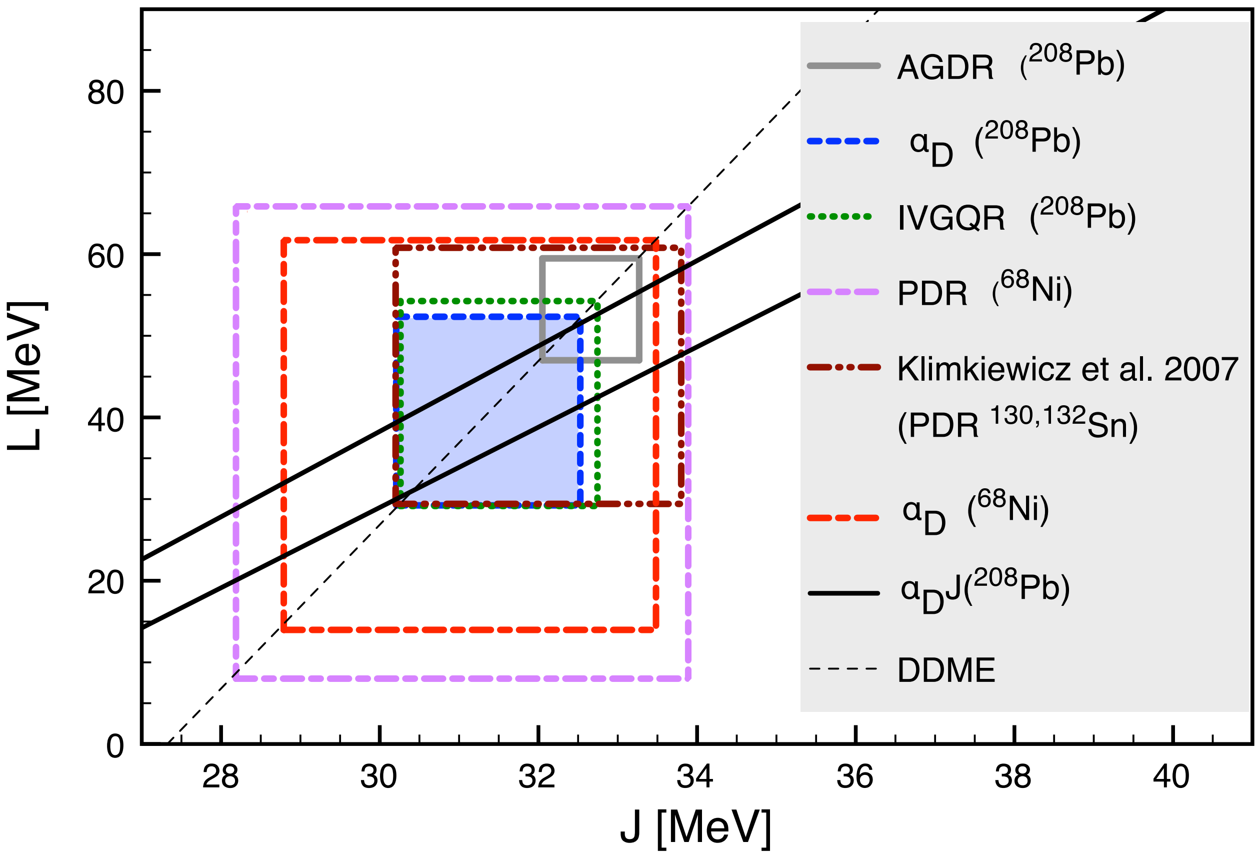


Dipole polarizability

$$\alpha_D = \frac{8\pi}{9} e^2 m_{-1}$$

:

CONSTRAINING THE SYMMETRY ENERGY



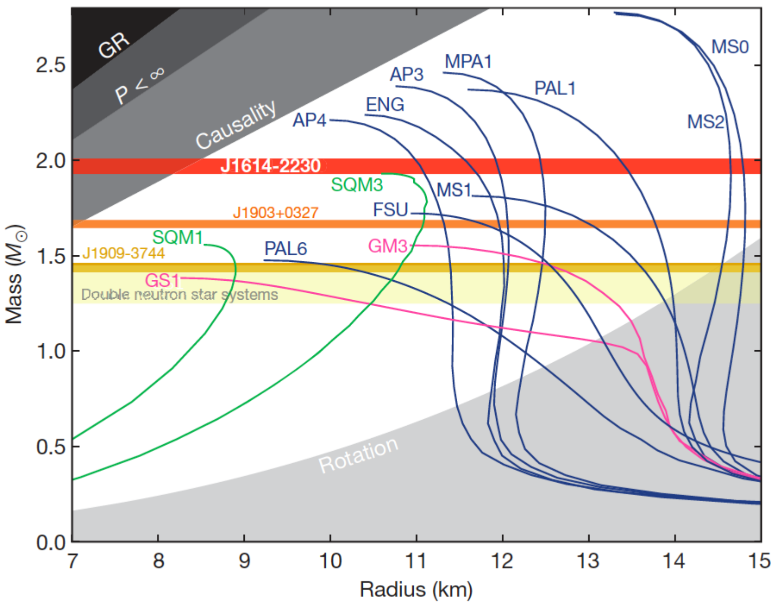
- The same set of DD-ME interactions used in the analysis based of various giant resonances and pygmy strengths (consistent theory !)
- Excellent agreement, except for the AGDR – new measurements are needed for the AGDR

Exp. data for various excitations:

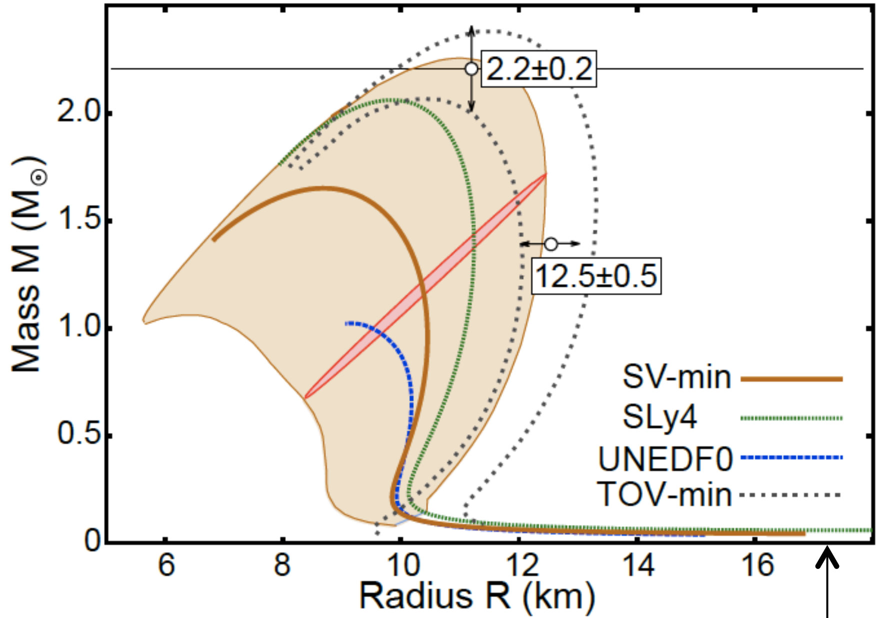
- α_D (^{208}Pb) ➔ A. Tamii et al., PRL 107, 062502 (2011) – update A. Tamii et al. (2015) (no q-deuteron).
- α_D (^{68}Ni) ➔ K. Boretzky, D. Rossi, T. Aumann, et al., (2015).
- PDR (^{68}Ni) ➔ O. Wieland, A. Bracco, F. Camera et al., PRL 102, 092502 (2009).
- (^{130,132}Sn) A. Klimkiewicz et al., PRC 76, 051603(R) (2007).
- IVGQR (^{208}Pb) ➔ S. S. Henshaw, M. W. Ahmed G. Feldman et al, PRL 107, 222501 (2011).
- AGDR (^{208}Pb) ➔ A. Krasznahorkay et al., arXiv:1311.1456 (2013)

NEUTRON STAR PROPERTIES

- Mass-radius relations of cold neutron stars for different EOS – observational constraints on the neutron star mass rule out many models for EOS.



P. B. Demorest et al., Nature 467, 1081 (2010)



- Building EDFs for finite nuclei and neutron stars
  Wei-Chia Chen and J. Piekarewicz, PRC 90, 044305 (2014).
  J. Erler. C.J. Horowitz, W. Nazarewicz et al., PRC 87, 044320 (2013).

- Constraints on the maximal neutron star mass from observation:
  J. Antoniadis, P. C. C. Freire, N. Wex et al. Science 340, 448 (2013) ➔ 2.01(4) Msun
  P. B. Demorest et al., Nature 467, 1081 (2010) ➔1.97(4) Msun

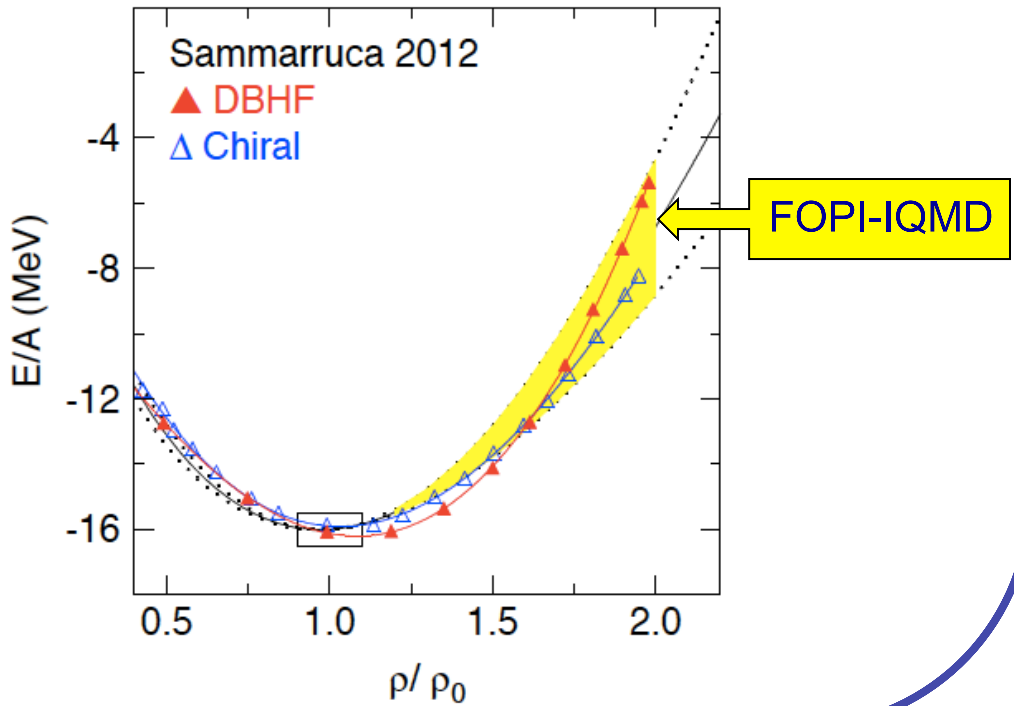
CONSTRAINTS ON THE NUCLEAR EOS BEYOND SATURATION

- The knowledge on the nuclear matter equation of state (EOS) beyond the saturation density ρ_0 is limited

- Some constraints on the EOS are possible from heavy ion collisions

- The FOPI (GSI) detector data on elliptic flow in Au+Au collisions between 0.4 and 1.5A GeV were used to establish empirical constraints on the nuclear EOS
  A. Le Fèvre, Y. Leifels, W. Reisdorf, J. Aichelin, Ch. Hartnack, arXiv:1501.05246 (2015).

- FOPI-IQMD (transport code) provides limits to the symmetric nuclear matter EOS up to $2\rho_0$



Towards a universal relativistic nuclear energy density functional for astrophysical applications – RNEDF1 (N.P., M. Hempel et al. 2015)

The strategy to constrain the functional (relativistic point coupling model)

- Adjust the properties of 72 spherical nuclei to exp. data (binding energies (Δ=1 MeV), charge radii (0.02 fm), diffraction radii (0.05 fm), surface thickness (0.05 fm))

- Improve description of open-shell nuclei by adjusting the pairing strength parameters to empirical paring gaps (n,p) (0.14 MeV)

- constrain the symmetry energy $S_2(\rho_0)=J$ (2%) from exp. data on dipole polarizability (^{208}Pb) A. Tamii et al., PRL 107, 062502 (2011) + update (2015).

- constrain the nuclear matter incompressibility K_{nm} (2%) from exp. data on ISGMR modes (^{208}Pb); D. Patel et al., PLB 726, 178 (2013).

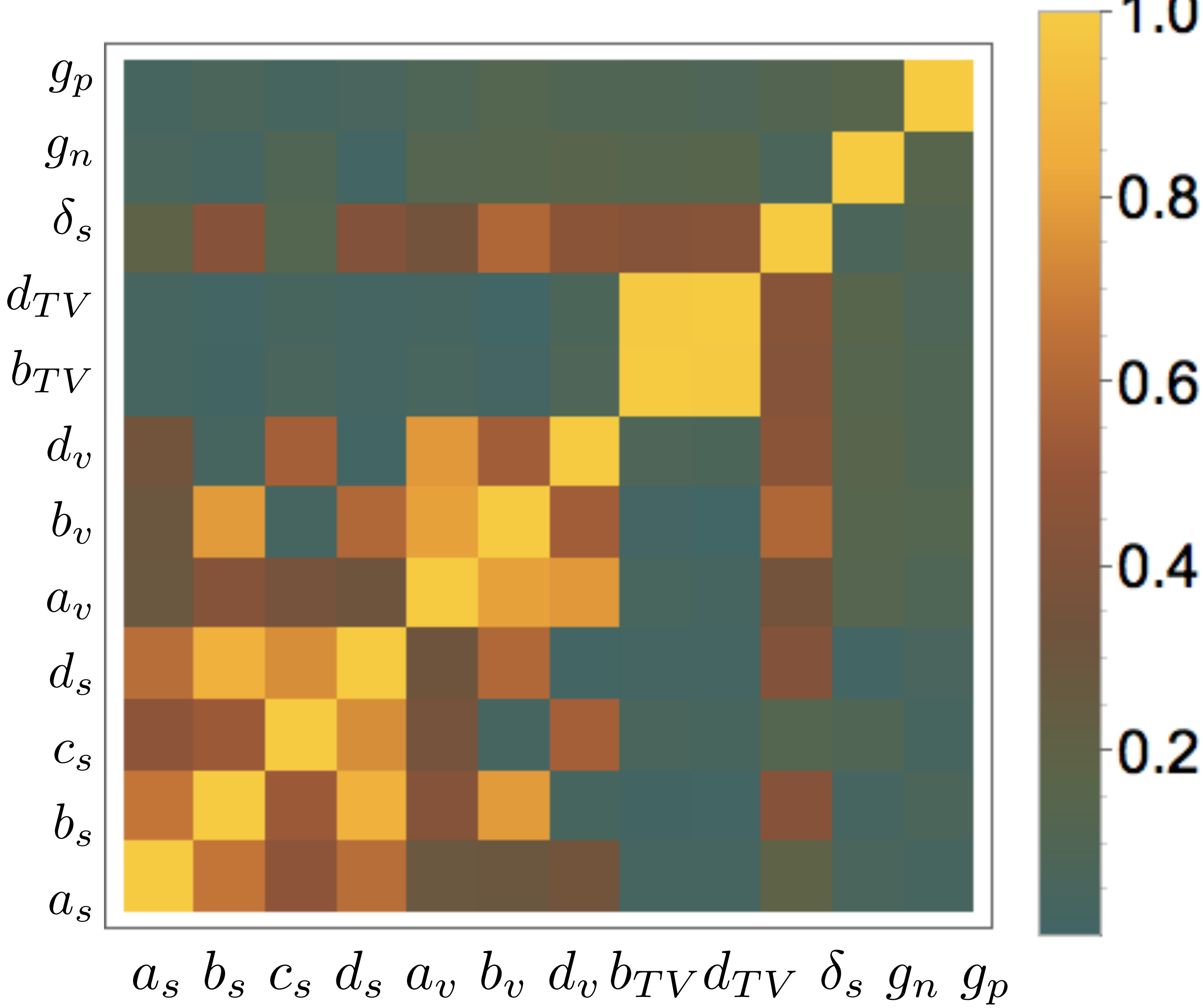
… the strategy to constrain the functional

- constrain the equation of state using the saturation point (ρ_0) and point at twice the saturation density ($2\rho_0$) from heavy ion collisions (FOPI-IQMD) (10%)
  A. Le Fevre et al., arXiv:1501.05246v1 (2015)

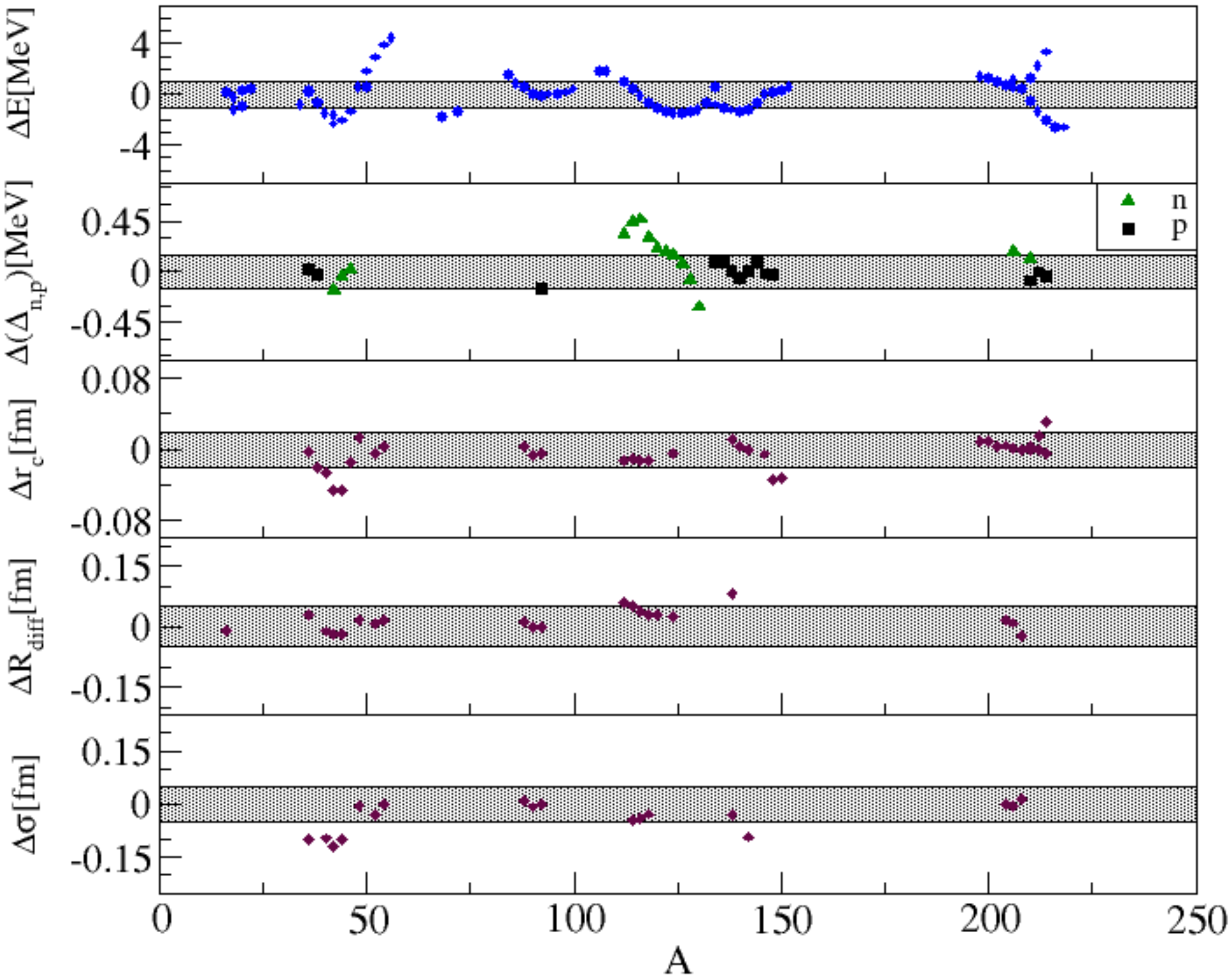
- constrain the maximal neutron star mass by solving the Tolman-Oppenheimer-Volkov (TOV) equations and using observational data (slightly larger value $M_{max}=2.2M_{\odot}$(5%); J. Erler. et al., PRC 87, 044320 (2013).)
  J. Antoniadis, et al. Science 340, 448 (2013); P. B. Demorest et al., Nature 467, 1081 (2010)

- the fitting protocol is supplemented by the covariance analysis
  - calculation of the curvature matrix, correlations, statistical uncertainties

RNEDF1: CORRELATIONS BETWEEN THE MODEL PARAMETERS



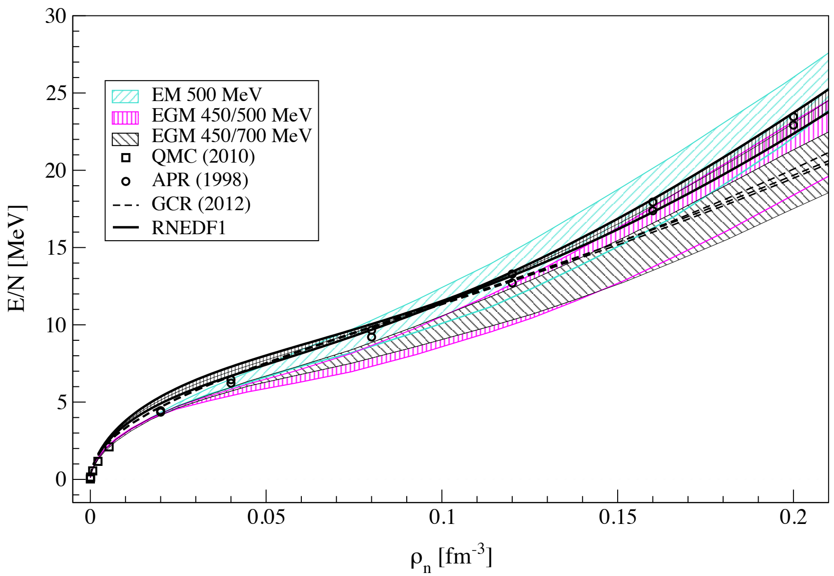
RNEDF1: DEVIATIONS FROM THE EXP. DATA

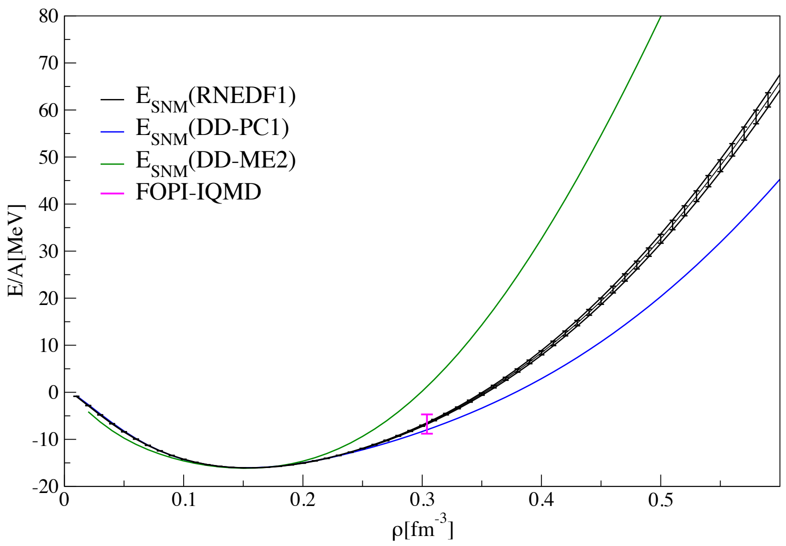


- binding energies
- pairing gaps
- charge radii
- diffraction radii
- surface thickness

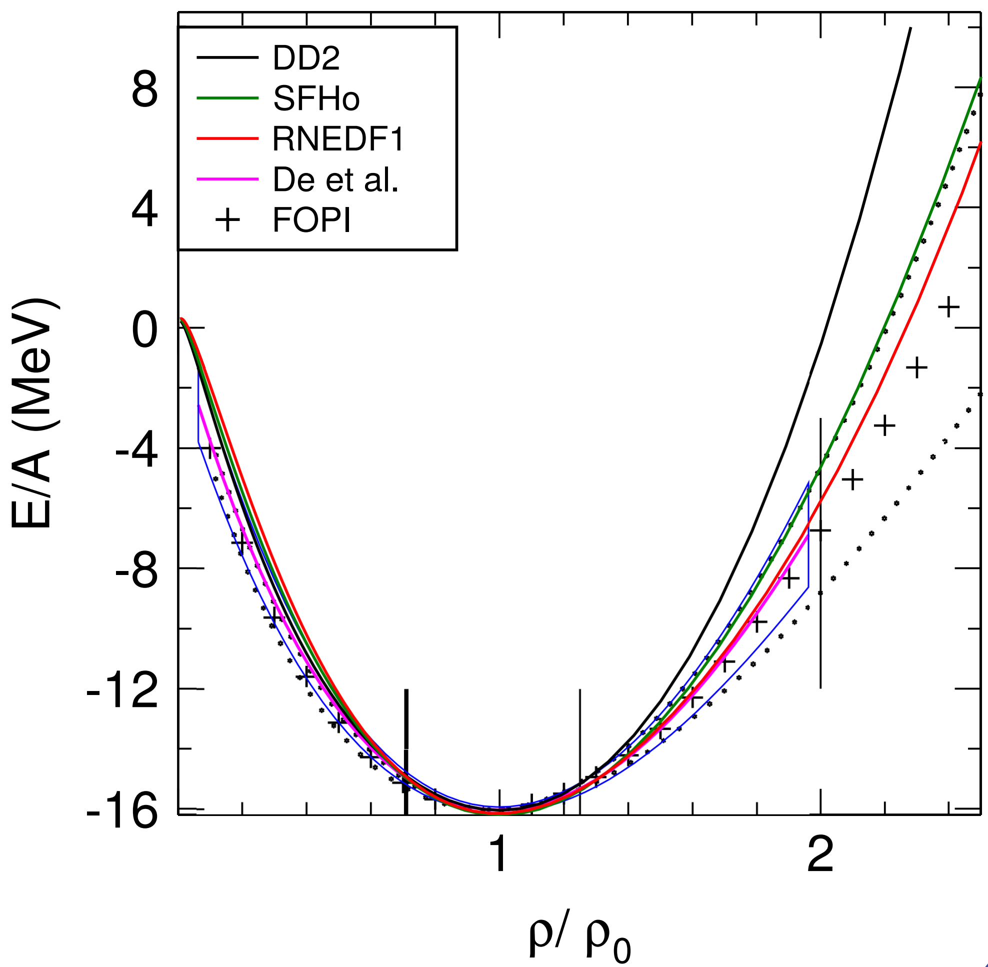
RNEDF1: NUCLEAR MATTER PROPERTIES

NEUTRON MATTER



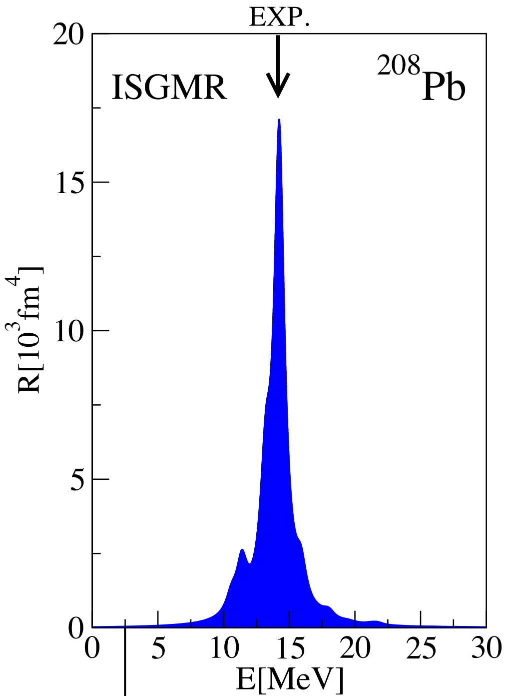


SYMMETRIC NUCLEAR MATTER

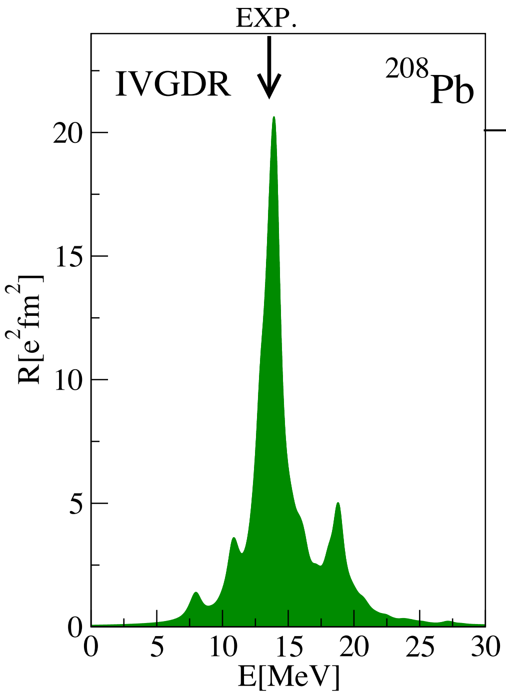


GIANT RESONANCES, COMPRESSIBILITY, SYMMETRY ENERGY

ISOSCALAR GIANT MONOPOLE RESONANCE



ISOVECTOR GIANT DIPOLE RESONANCE



- ISGMR energy determines the nuclear matter incompressibility:
K_{nm}=232.4 MeV

E (Exp.) = (13.91 ± 0.11) MeV (TAMU)
E (Exp.) = (13.7 ± 0.1) MeV (RCNP)

Dipole polarizability:
α_D= (19.68 ± 0.21) fm^3

Exp.
α_D= (19.6 ± 0.6) fm^3

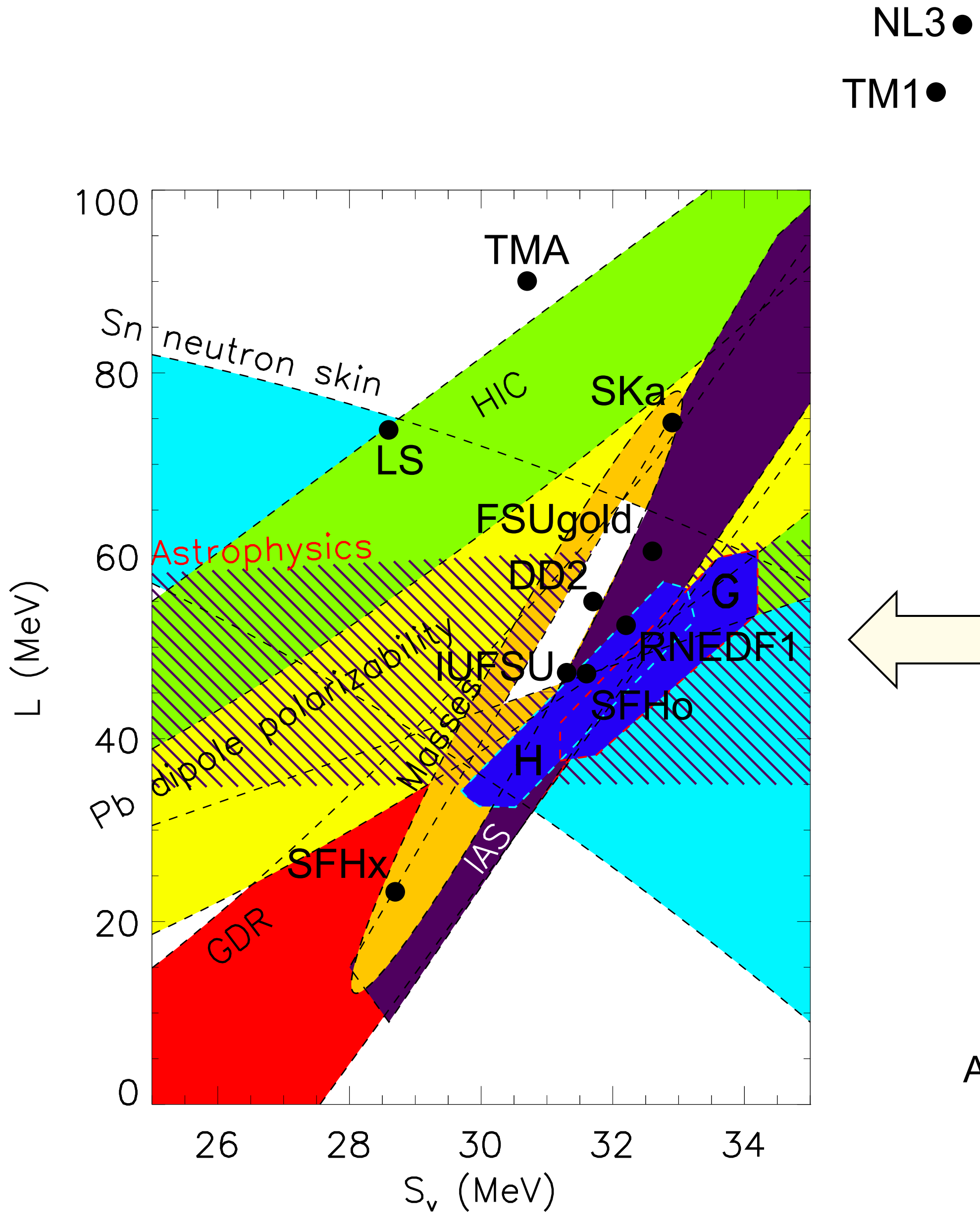
A.Tamii et al., PRL 107, 062502 (2011). + update (2015).

- IVGDR – α_D constrain the symmetry energy of the interaction

J = 31.89 MeV
L = 51.48 MeV

- Lattimer & Lim, ApJ. 771, 51 (2013)

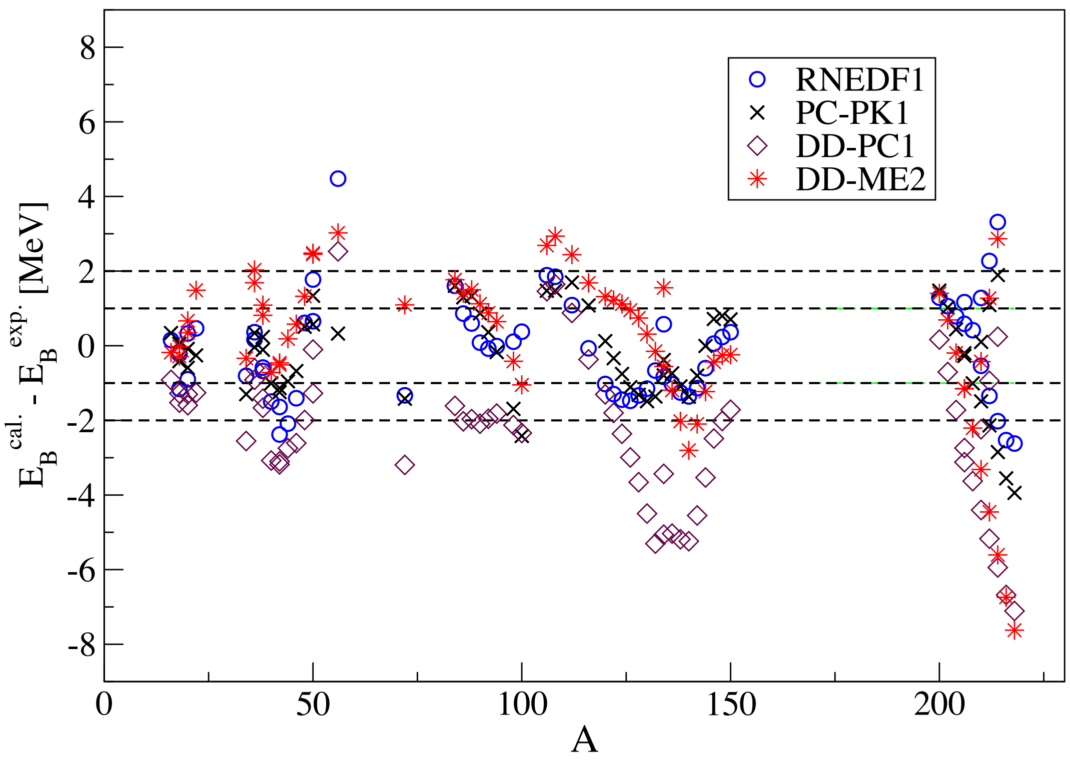
J = 29.0–32.7 MeV
L = 40.5–61.9 MeV



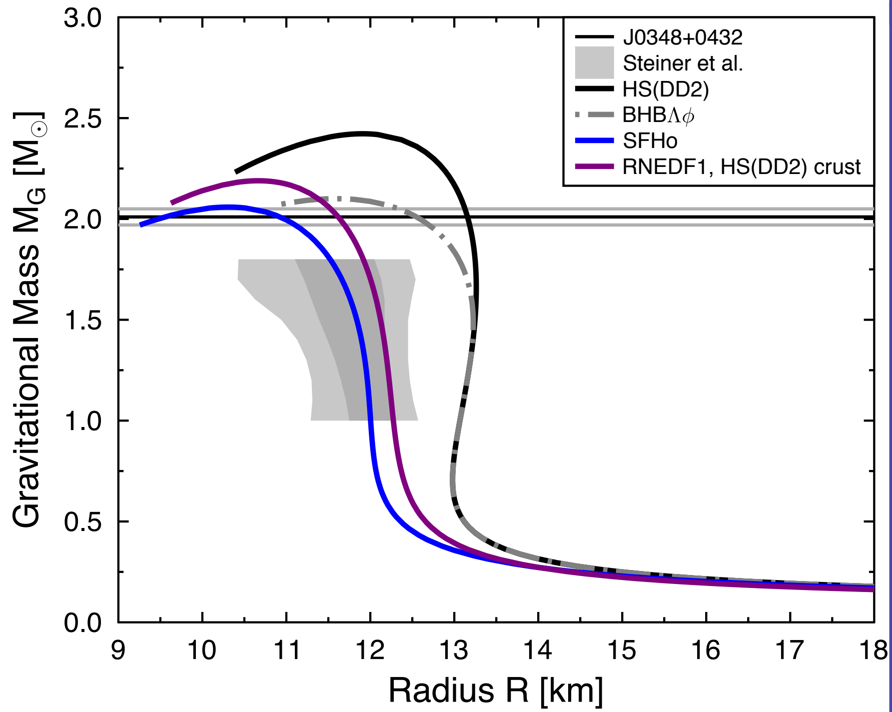
Also see: Lattimer & Lim, ApJ. 771, 51 (2013)

RNEDF1: FROM FINITE NUCLEI TOWARD THE NEUTRON STAR

Nuclear binding energies (calc. – exp.)

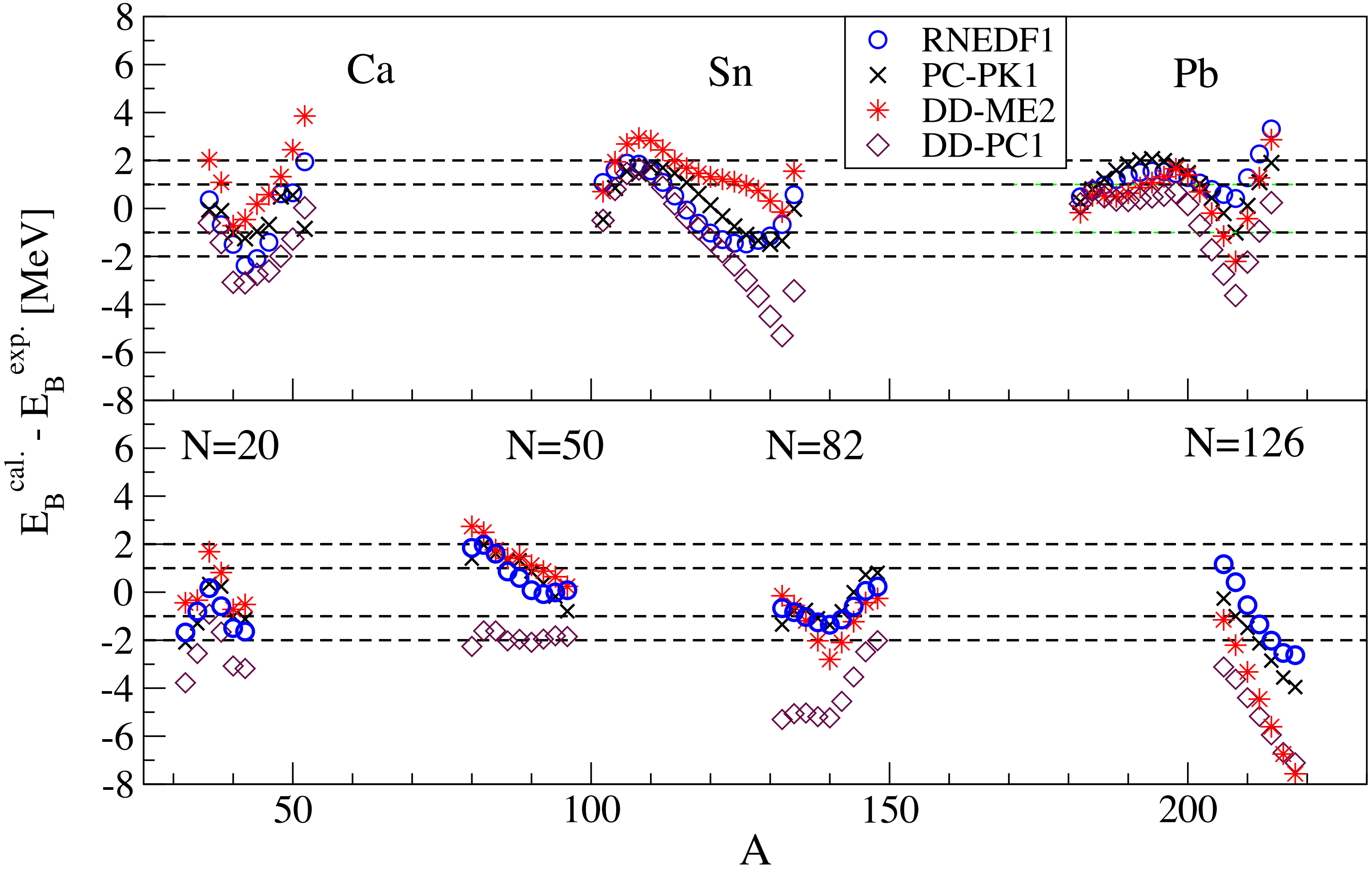


Neutron star mass-radius



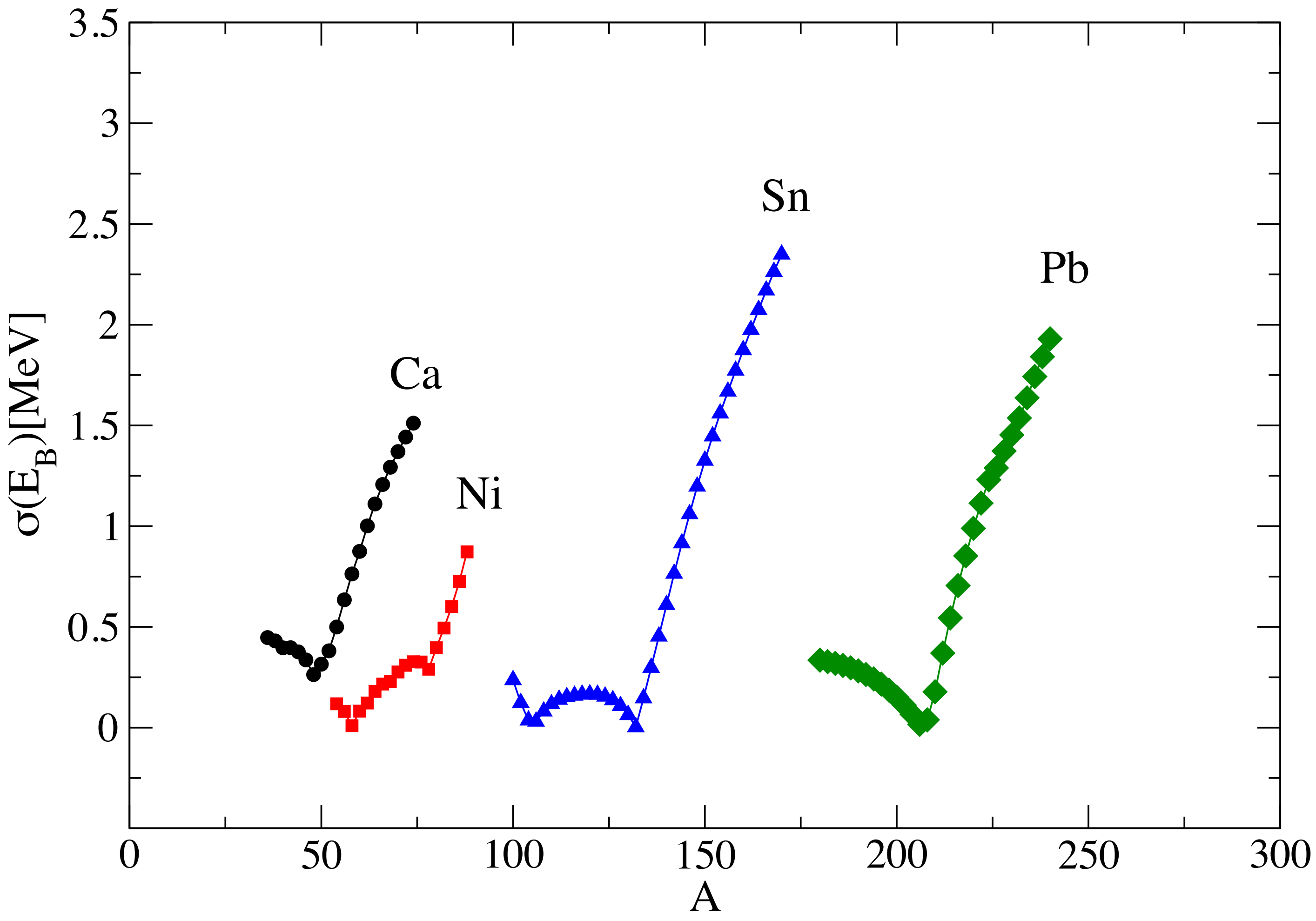
PC-PK1: P.W.Zhao, Z. P. Li, J. M. Yao, and J. Meng, PRC 82, 054319 (2010)
- (K=238 MeV, J=35.6, L = 113 MeV)

RNEDF1: ISOTOPE AND ISOTONE CHAINS

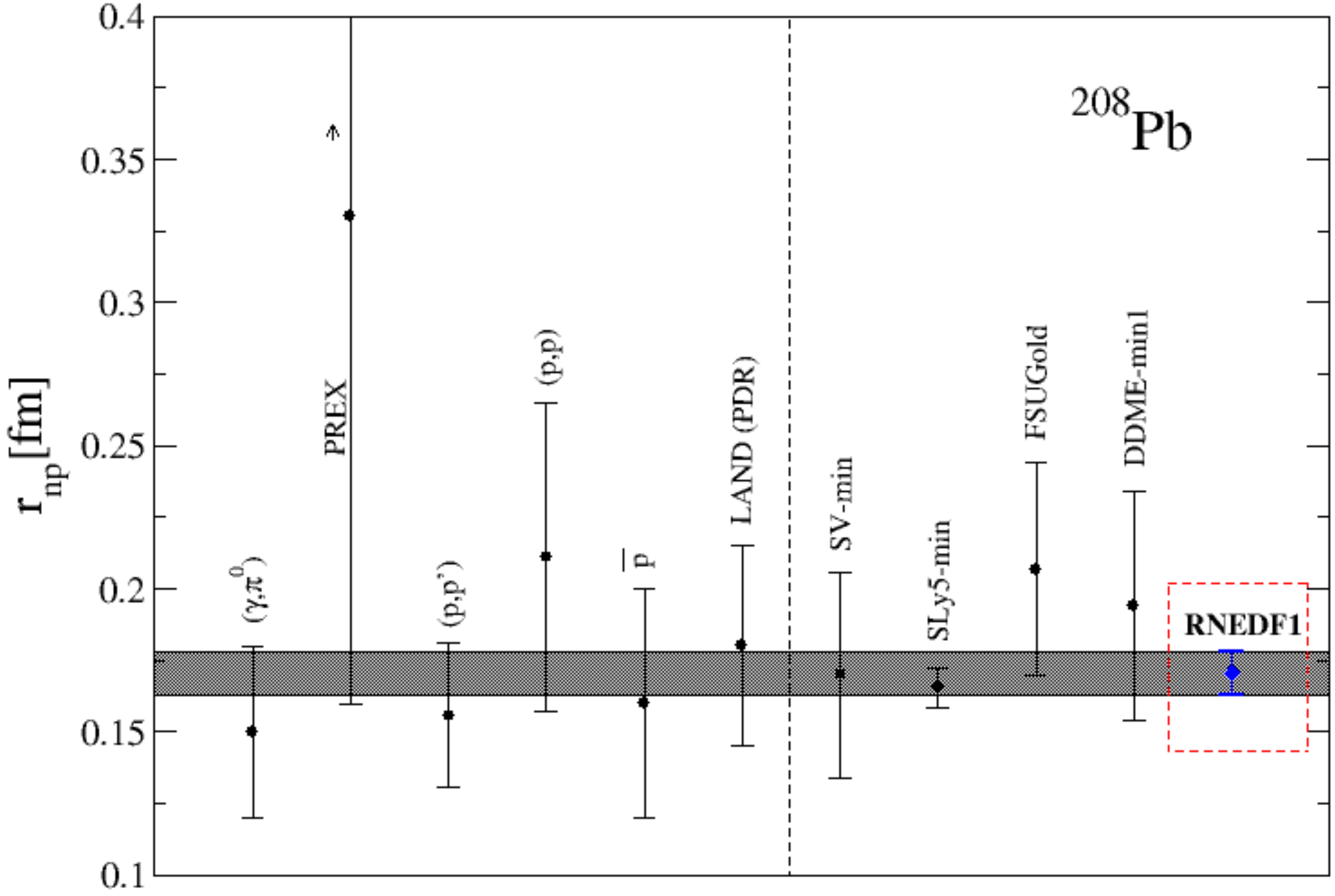


RNEDF1: ISOTOPE CHAINS

- The evolution of statistical uncertainties of the nuclear binding energies



RNEDF1: NEUTRON SKIN THICKNESS IN ^{208}Pb



(γ,π^0): C.M. Tarbert et al., PRL 112, 242502 (2014)
PREX: S. Abrahamyan et al., PRL. 108, 112502 (2012)
(p,p'): A. Tamii et al., PRL 107, 062502 (2011)
(p,p) J. Zenihiro et al., PRC 82, 044611 (2010)
Antipr. at.: B. Kłos et al., Phys. Rev. C 76, 014311 (2007).
LAND (PDR): A. Klimkiewicz et al., PRC 76, 051603 (2007).

SV-min: P.G. Reinhard et al.
SLy5-min: X. Roca-Maza, G. Colò et al.
FSUGold: J. Piekarewicz et al.
DDME-min1: N.P. et al.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to be continued ...

Strategy to determine the EDF parameters

"Unconstrained" fit
(only masses, radii, pairing gaps)

DD-PC (min) :
J=29.3 MeV
L=30.8 MeV
α_D= 18.18 fm^3

SV-min: α_D~ 20.4 MeV
(+ ls splitting in the fit)
T. Hashimoto et al. arXiv: 1503.08321 (2015).

"Constrained" fit
(masses, radii, pairing gaps and additional constraints to nuclear matter and/or isovector observables, etc.)

RNEDF1:
J = 31.9 MeV
L = 51.5 MeV
α_D= 19.68 fm^3

EXP. $\alpha_D(^{208}\mathrm{Pb}) = 19.6 \pm 0.6\ fm^3$

CONCLUDING REMARKS

- Toward self-consistent framework based on the relativistic nuclear energy density functional for astrophysical applications (RNEDF1): from finite nuclei toward neutron stars; new protocol used to constrain the functional

  - supplemented with the covariance analysis to determine correlations between various quantities and statistical uncertainties

Future work …

- supernova equation of state (with M. Hempel et al.)
- complete the mass table both for spherical and deformed nuclei (deformed nuclei necessitate additional rotational correction)
- implementation of charged-current quasiparticle RPA for the relativistic point coupling interaction (➔Haozhao Liang)
- systematic calculations of presupernova electron capture rates at finite temperature
- neutrino-nucleus cross sections, both for neutral-current and charged current reactions
- neutron star properties – mass/radius relationship, liquid-to-solid core-crust transition density and pressure
- …