Isoscalar spin-triplet pairing and tensor correlations on Spin-Isospin response

-- Kyoto CANHP2015 Workshop 5th week "Energy density functionals" ---
Oct. 19, 2015

Hiroyuki Sagawa RIKEN/University of Aizu

- Skyrme tensor interactions
  - Spin-isospin excitations and sum rules
- Pairing correlations T=1,S=0 and T=0,S=1 pairs
  - Energy spectra of N=Z odd-odd nuclei.
  - Gamow-Teller excitations in N=Z+2 *pf*- shell nuclei
    - interplay between T=0 pairing and tensor correlations

Tensor correlations on Nuclear Structure

Binding energies
Deformations
EOS

Shell evolution by tensor correlations
（spin-orbit splitting)

Spin-Isospin excitations

a) Gamow-Teller excitations
b) Spin-Dipole (SD) excitations

Quenching of sum rules within RPA model
Multipole dependence of SD excitations

A self-consistent framework to describe nuclei in the whole mass region

Skyrme-type tensor interactions

$$
\begin{aligned}
V^T = & \frac{T}{2}\{[(\sigma_1\cdot\mathbf{k}')(\sigma_2\cdot\mathbf{k}') - \frac{1}{3}(\sigma_1\cdot\sigma_2)\,\mathbf{k}'^2]\delta(\mathbf{r}_1-\mathbf{r}_2) \\
& +\delta(\mathbf{r}_1-\mathbf{r}_2)\left[(\sigma_1\cdot\mathbf{k})(\sigma_2\cdot\mathbf{k}) - \tfrac{1}{3}(\sigma_1\cdot\sigma_2)\,\mathbf{k}^2\right]\} \\
& +\frac{U}{2}\{(\sigma_1\cdot\mathbf{k}')\,\delta(\mathbf{r}_1-\mathbf{r}_2)\,(\sigma_2\cdot\mathbf{k}) + (\sigma_2\cdot\mathbf{k}')\,\delta(\mathbf{r}_1-\mathbf{r}_2)(\sigma_1\cdot\mathbf{k}) \\
& -\tfrac{2}{3}\left[(\sigma_1\cdot\sigma_2)\mathbf{k}'\cdot\delta(\mathbf{r}_1-\mathbf{r}_2)\mathbf{k}\right]\}
\end{aligned}
$$

:Triplet-even

:Triplet-odd

Two Advantages

1.A simple formula for spin-orbit splitting

2.Analytic formulas for multipole expansion for spin-dependent excitations

T.H.R. Skyrme, Nucl.Phys. 9,615(1959).
F.L. Stancu, D. M. Brink and H. Flocard, PLB68,108 (1977).

T.Lesinski, M. Bender, K. Bennaceur, T. Duguet, J. Meyer, Phys. Rev.C76, 014312(2007).
G.Colo, H. Sagawa, S. Fracasso, P.F. Bortignon, Phys. Lett. B 646 (2007) 227.
B.A.Brown, T. Duguet,T. Otsuka, D. Abe and T. Suzuki, Phys. Rev. C74(2006) 061303(R)

Mean field ----Spin-orbit splitting----

$$\delta H = \frac{1}{2}\alpha(J_n^2 + J_p^2) + \beta J_n J_p.$$

$$U_{s.o.}^{(q)} = \frac{W_0}{2r}\left(2\frac{d\rho_q}{dr} + \frac{d\rho_{q'}}{dr}\right) + \left(\alpha\frac{J_q}{r} + \beta\frac{J_{q'}}{r}\right),$$

$q \ (n = 0, p = 1)$ $\qquad q' = 1 - q$

$$J_q(r) = \frac{1}{4\pi r^3}\sum_i v_i^2(2j_i + 1)\left[j_i(j_i + 1) - l_i(l_i + 1) - \frac{3}{4}\right] R_i^2(r).$$

$$\alpha_C = \frac{1}{8}(t_1 - t_2) - \frac{1}{8}(t_1 x_1 + t_2 x_2) = 80.7\text{MeV.fm}^5$$

$$\beta_C = -\frac{1}{8}(t_1 x_1 + t_2 x_2) = -48.9\text{MeV.fm}^5$$

$$\alpha_T = \frac{5}{12}U = -170\text{MeV.fm}^5$$

$$\beta_T = \frac{5}{24}(\text{T} + \text{U}) = 100\text{MeV.fm}^5$$

TIJ family

$$\alpha = \alpha_C + \alpha_T = 60(J - 2)\text{MeV.fm}^5$$

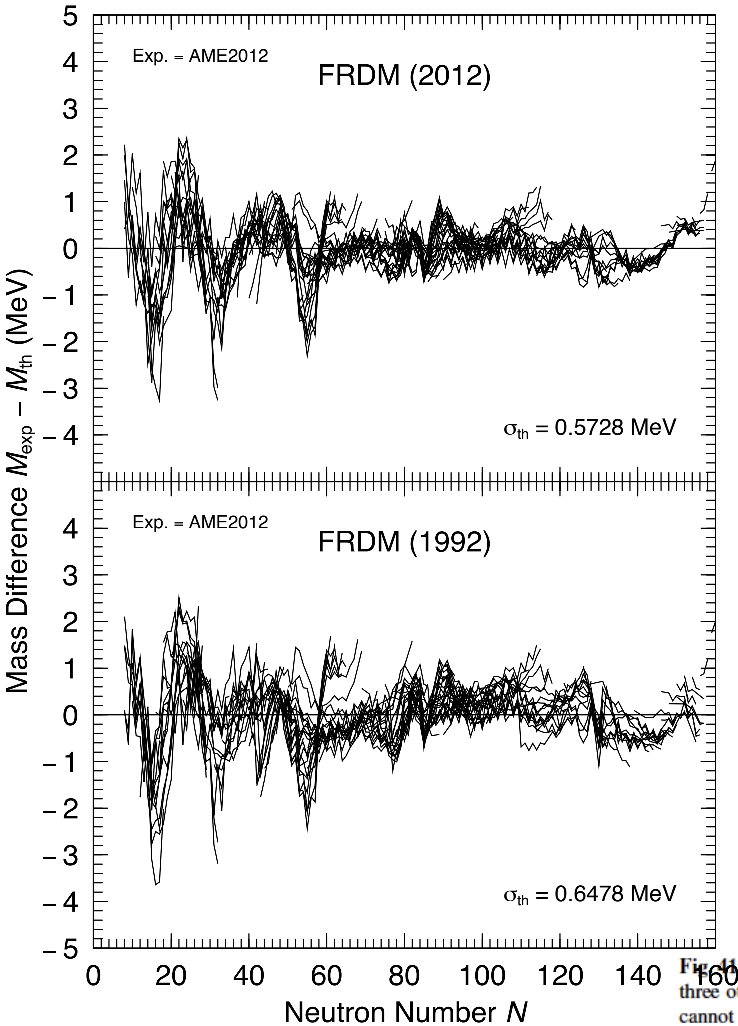
$$\beta = \beta_C + \beta_T = 60(I - 2)\text{MeV.fm}^5$$

	sign	spin-orbit splitting
α, β	negative	larger
	positive	smaller

Nuclear ground-state masses and deformations: FRDM(2012)

P. Möller[a,*], A. J. Sierk[a], T. Ichikawa[b], H. Sagawa[c,d]

ADNDT(2015) in press.



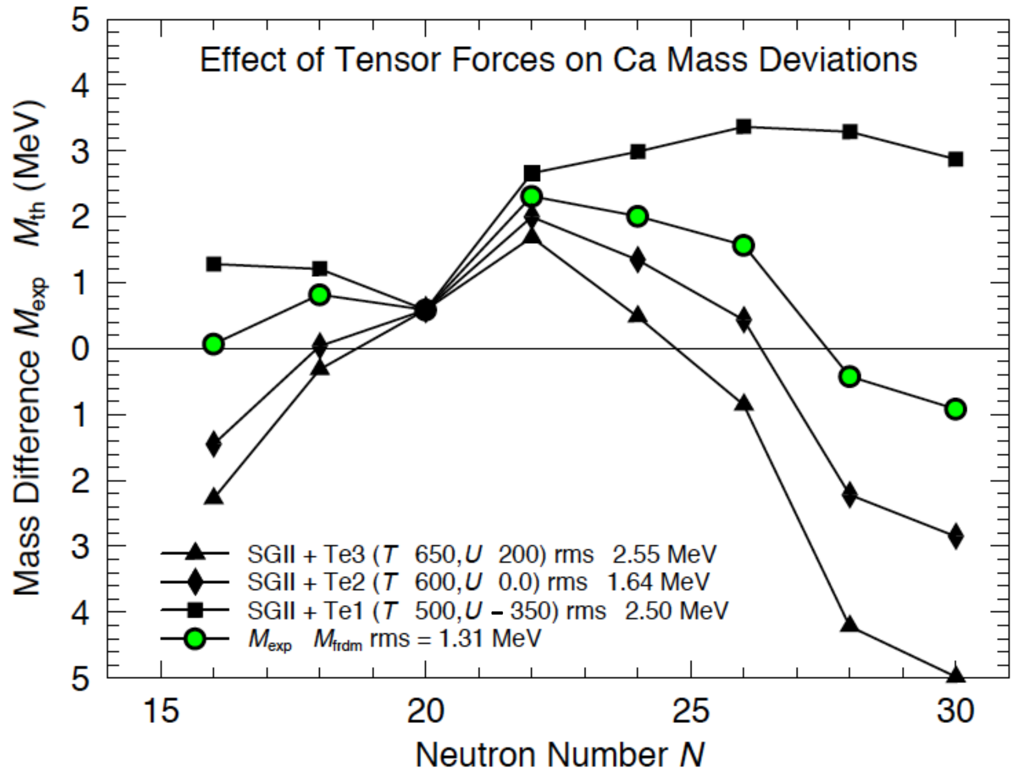
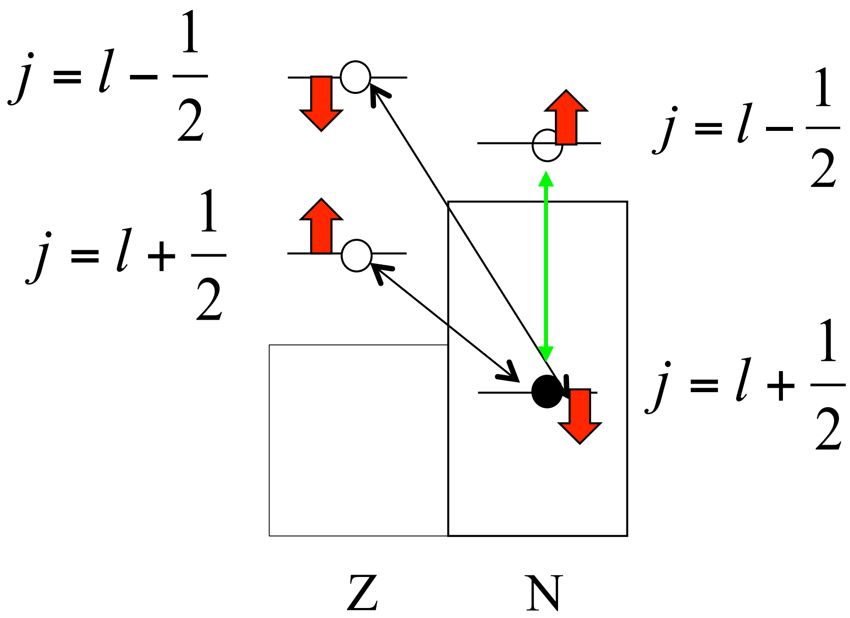


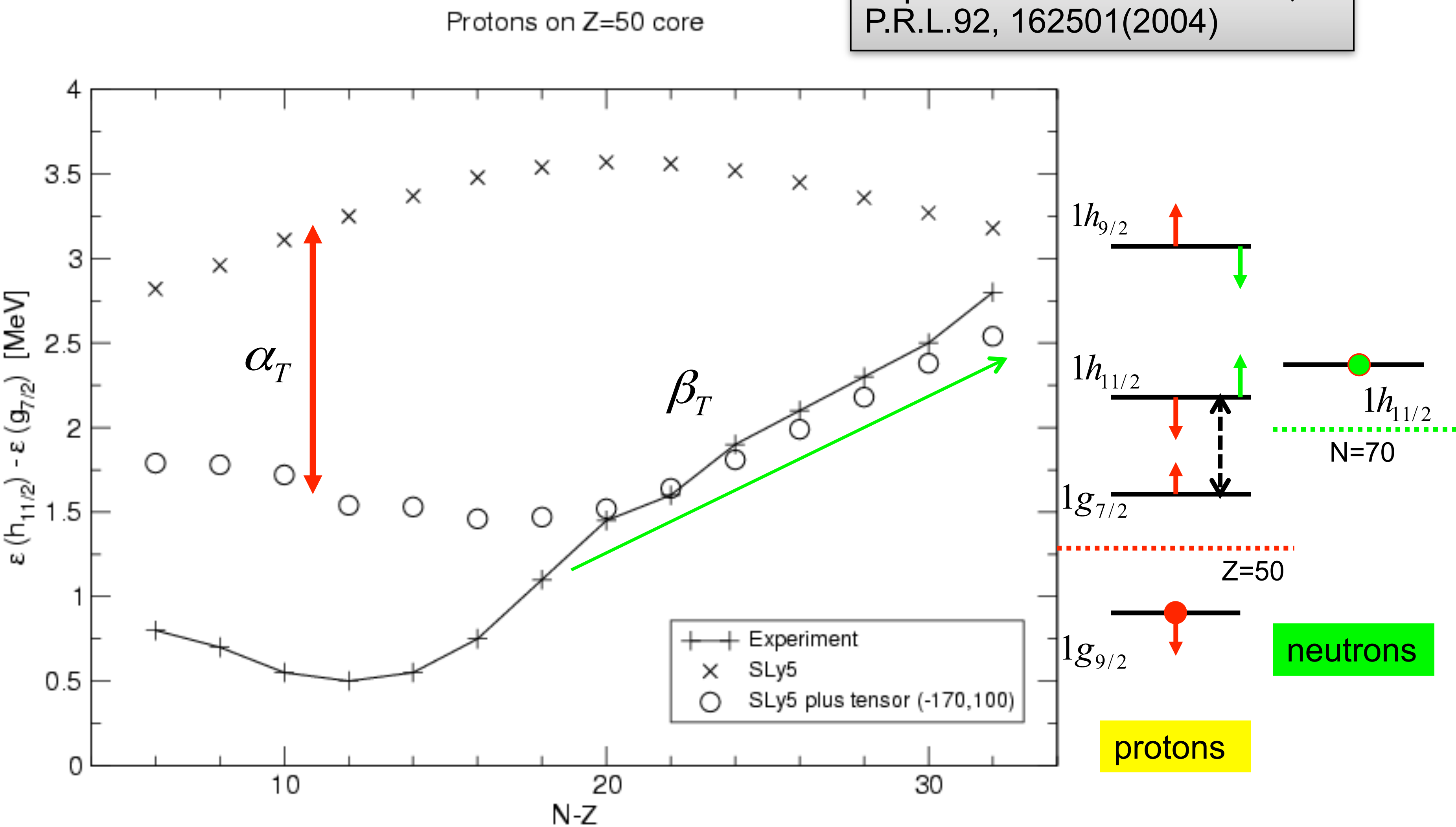
Fig. 11: Effect of tensor force on Ca-isotope mass deviations. The large filled circles show the mass deviations in the FRDM(2012) mass model. Th three other curves show how these are modified due to the effect of different tensor forces. It seems that the oscillatory behavior of the deviation cannot be eliminated by these tensor forces.

Effect of tensor interaction on spin-orbit splitting

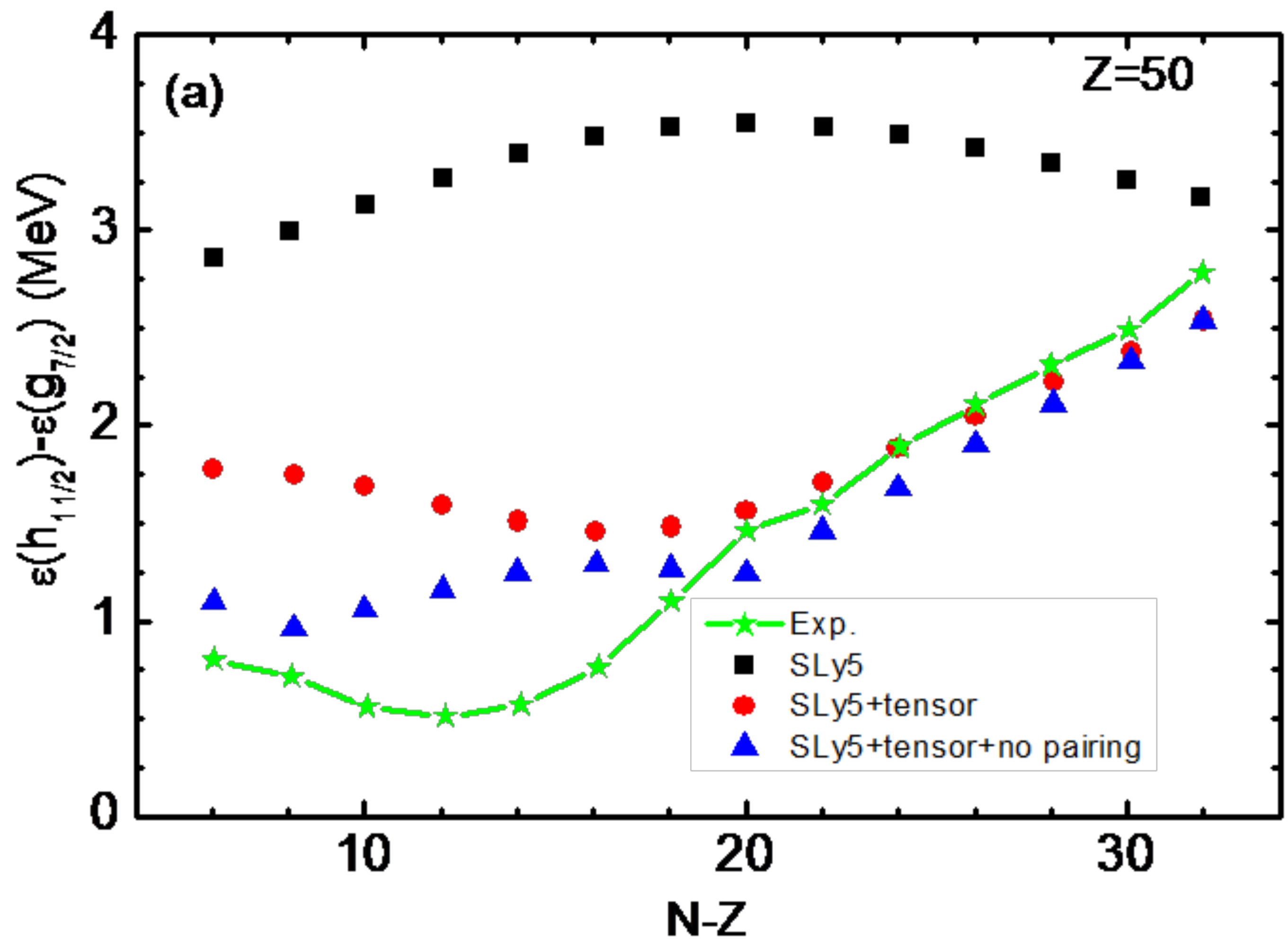


α:n-n or p-p larger with $j_>$
β : n-p smaller with $j_>$

Exp. Data : J.P.Schiffer et al., P.R.L.92, 162501(2004)



G.Colo, H. Sagawa, S. Fracasso, P.F. Bortignon, Phys. Lett. B 646 (2007) 227.



Not … upling
effe … st to look
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1. Experiments : Spectroscopic factors
2. Theory : Consistent Skyrme spin-orbit interaction parameters including tensor interactions

Spin-isospin physics: Gamow-Teller responses

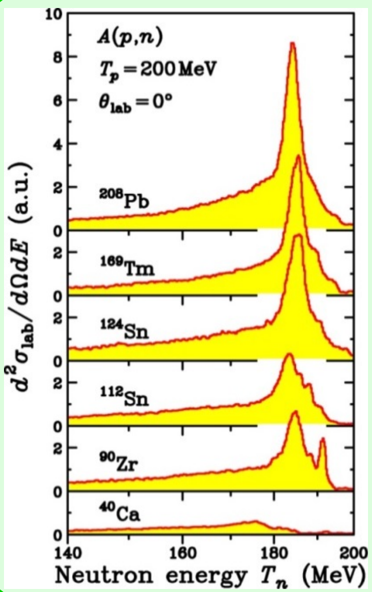
Progress in Last century

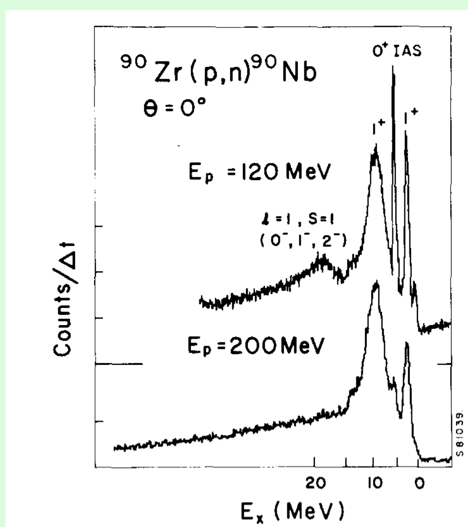
Courtesy of H. Sakai

- 1963 GT giant resonance predicted, GT(Ikeda) sum rule 3(N-Z)
  collectivity?
- ~1980 GT giant resonances established
- Strength quenched/missing: 50-60% of 3(N-Z) due to Δ-h or 2p-2h ?
- 1997 ~90% of 3(N-Z) found (2p-2h dominance)

- Charge-exchange reactions on stable target nuclei
- CHEX reactions: (p,n)/(n,p) and (^{3}He,t)/(t,^{3}He) reactions at intermediate energy

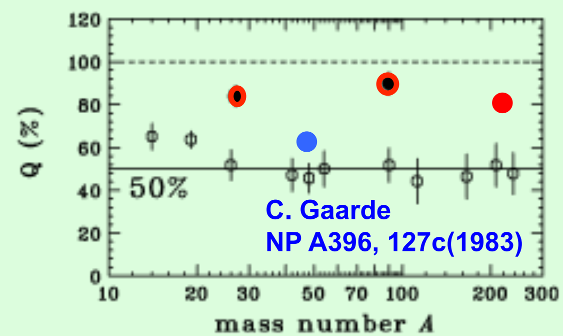
- C. Garrde, NPA396(1982)127c.





- Wakasa et al., PR C55, 2909 (1997)

GT strength quenching problem



- **Wakasa et al.**, PR C **55**, 2909 (1997)
- Yako et al.,

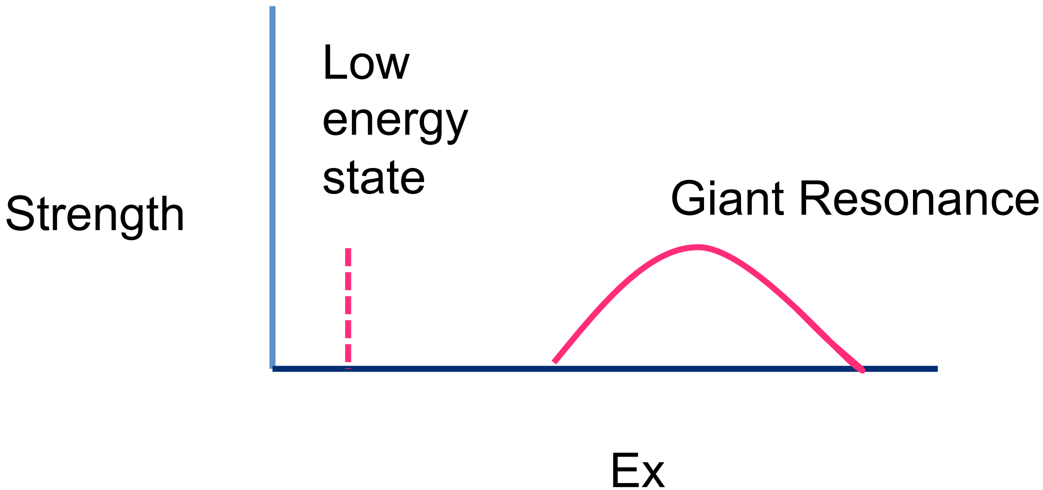
Energy-weighted (EW) and NEW sum rules

$$m(k) = \sum_i E^k{}_i \left| \langle i | \hat{O}_\lambda | 0 \rangle \right|^2$$

$$m(1) = \frac{1}{2} \langle 0 | [\hat{O}_\lambda, [H, \hat{O}_\lambda]] | 0 \rangle$$

$$\begin{aligned} m_-(0) - m_+(0) &= \sum_\nu (|\langle \nu | O_- | 0 \rangle|^2 - |\langle \nu | O_+ | 0 \rangle|^2) \\ &= \langle 0 | [O_-, O_+] | 0 \rangle, \end{aligned}$$

Gamow-Teller
Ikeda sum rule



Model-independent sum rule : GT(Ikeda) sum rule

$$
\begin{aligned}
S_{\beta-} - S_{\beta+} &= \frac{1}{2J_i+1}\sum_f |\langle f\| \sum_{i=1}^{A} t_-(i)\boldsymbol{\sigma}_i \|i\rangle|^2 \\
&\quad - \frac{1}{2J_i+1}\sum_f |\langle f\| \sum_{i=1}^{A} t_+(i)\boldsymbol{\sigma}_i \|i\rangle|^2 \\
&= \langle i| \sum_{i,j=1}^{A} (t_+(j)t_-(i) - t_-(i)t_+(j))\, \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\, |i\rangle
\end{aligned}
$$

$$
[t_+(j), t_-(i)] = \delta_{ij}\, 2t_z(i), \qquad \sum_{i=1}^{A} 2t_z(i) = 2T_z \qquad \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_i = 3
$$

$$
\boxed{S_{\beta-} - S_{\beta+} = \langle i|\, 2T_z\cdot 3\, |i\rangle = 3(N-Z)}
$$

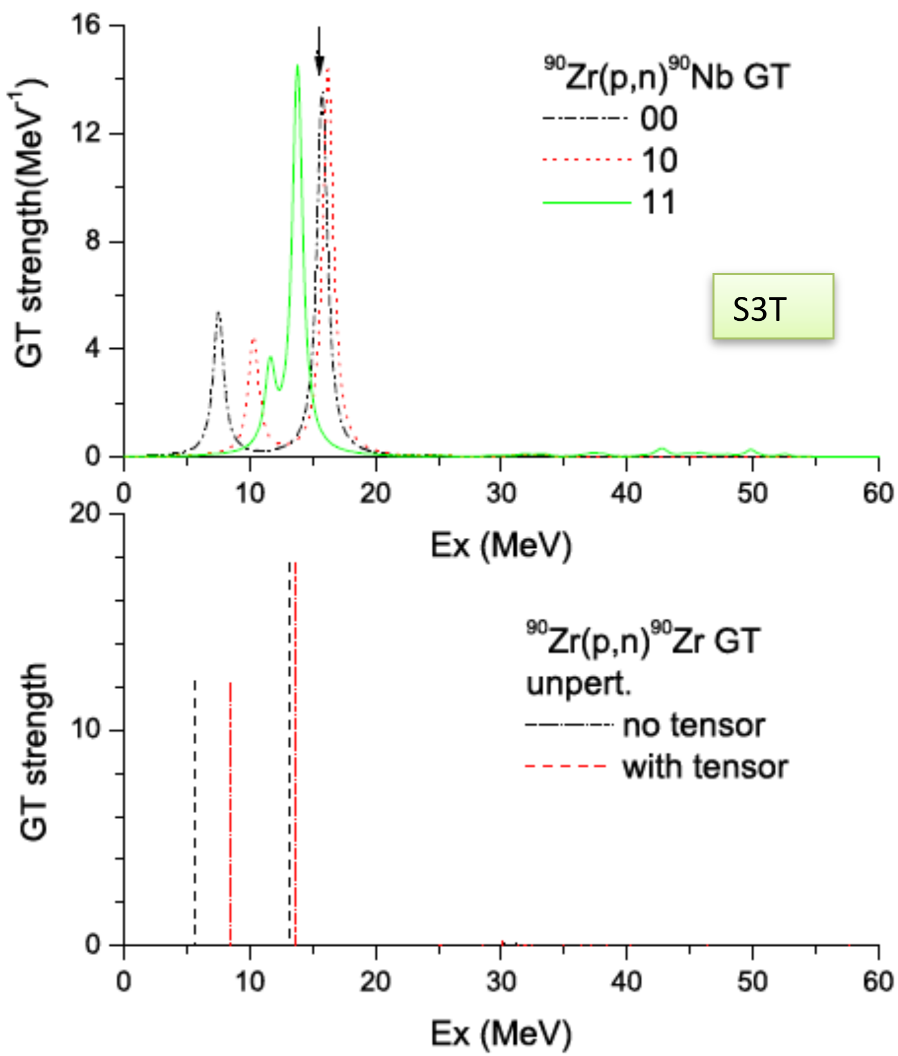
=30 for ^{90}Zr

=132 for ^{208}Pb

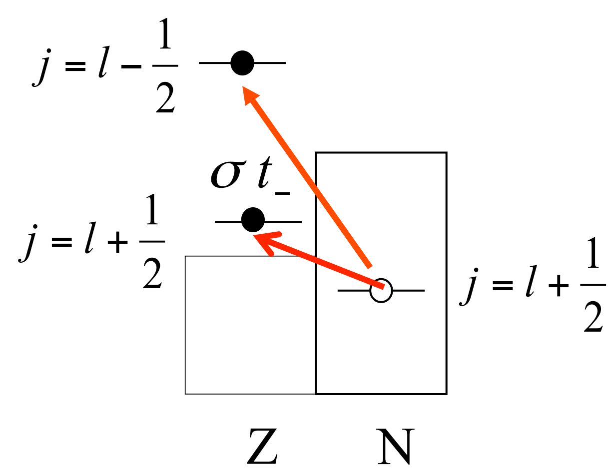
cf: Fermi transition

$$
S_{\mathrm{F}-} - S_{\mathrm{F}+} = \langle i|\, 2T_z\, |i\rangle = N - Z
$$

The tensor force and charge-exchange excitations



Gamow-Teller $\lambda^{\pi} = 1^{+}$



The main peak is moved downward by the tensor force but the centroid is moved upwards !

C.L.Bai, HS, H.Q.Zhang, X.Z.Zhang, G.Colo and F.R.Xu, P.L.B675,28 (2009).
C.L.Bai, H.Q. Zhang, X.Z.Zhang, F,R,Xu, HS and G.Colo, PRC79, 041301(R) (2009).

Tensor correlations on Spin-Isospin mode

Effect of Tensor Correlations on Gamow-Teller States in ^{90}Zr and ^{208}Pb

C.L. Bai[1,2], H. Sagawa[3], H.Q. Zhang[1,2], X.Z. Zhang [2], G. Colò[4] and F.R. Xu[1]

$$O_- = \sigma t_-$$
$$O_+ = \sigma t_+$$

$$\begin{aligned} V^T = & \frac{T}{2}\{[(\sigma_1\cdot\mathbf{k}')(\sigma_2\cdot\mathbf{k}') - \frac{1}{3}(\sigma_1\cdot\sigma_2)\,\mathbf{k}'^2]\delta(\mathbf{r}_1-\mathbf{r}_2) \\ & +\delta(\mathbf{r}_1-\mathbf{r}_2)\left[(\sigma_1\cdot\mathbf{k})(\sigma_2\cdot\mathbf{k}) - \tfrac{1}{3}(\sigma_1\cdot\sigma_2)\,\mathbf{k}^2\right]\} \\ & +\frac{U}{2}\{(\sigma_1\cdot\mathbf{k}')\,\delta(\mathbf{r}_1-\mathbf{r}_2)\,(\sigma_2\cdot\mathbf{k}) + (\sigma_2\cdot\mathbf{k}')\,\delta(\mathbf{r}_1-\mathbf{r}_2)(\sigma_1\cdot\mathbf{k}) \\ & -\tfrac{2}{3}\left[(\sigma_1\cdot\sigma_2)\mathbf{k}'\cdot\delta(\mathbf{r}_1-\mathbf{r}_2)\mathbf{k}\right]\} \end{aligned}$$

$$\begin{aligned} m_-(0) - m_+(0) &= \sum_\nu \left(|\langle\nu|O_-|0\rangle|^2 - |\langle\nu|O_+|0\rangle|^2\right) \\ &= \langle 0|\,[O_-,O_+]\,|0\rangle, \quad =3(N-Z) \end{aligned}$$

$$\begin{aligned} m_-(1) + m_+(1) &= \sum_\nu \left(|\langle\nu|O_-|0\rangle| + |\langle\nu|O_+|0\rangle|^2\right) E_\nu \\ &= \langle 0|\,[O_+,[H,O_-]]\,|0\rangle, \end{aligned}$$

$$\boxed{\begin{aligned} \Delta E_{GT} &= \frac{m_-(1)}{m_-(0)} \\ &\sim \frac{m_-(1)+m_+(1)}{m_-(0)-m_+(0)} \\ &= \frac{4\pi}{3(N-Z)}\int drr^2[-(\tfrac{5}{2}U+\tfrac{5}{2}T)J_nJ_p - \tfrac{5}{3}U(J_n^2+J_p^2)] \end{aligned}}$$

S3+Tensor

	$m_-(1;\text{no tensor})$	$m_-(1;\text{with tensor})$	δE_{RPA}	δE_{DC}
	MeV	MeV	MeV	MeV
^{90}Zr	271.45	338.68	2.241	2.276
^{208}Pb	1854.12	2000.76	1.111	1.118

	type of calculation	$m_-(0)$ 0-30MeV	$m_-(0)$ 30-60MeV	$m_-(1)$ 0-30 MeV	$m_-(1)$ 30-60 MeV	$m_-(1)$ total	$m_+(1)$ total
^{90}Zr	00	29.16	0.71	395	26.2	421.8	10.1
	10	29.16	0.79	444	22	466	11.1
	11	27.00	2.89	366.9	122	493.2	10.3
^{208}Pb	00	127.54	3.43	2080	124.5	2212.8	18.8
	10	127.38	3.68	2176	93	2269	21
	11	114.10	16.58	1658	694	2370	19.3

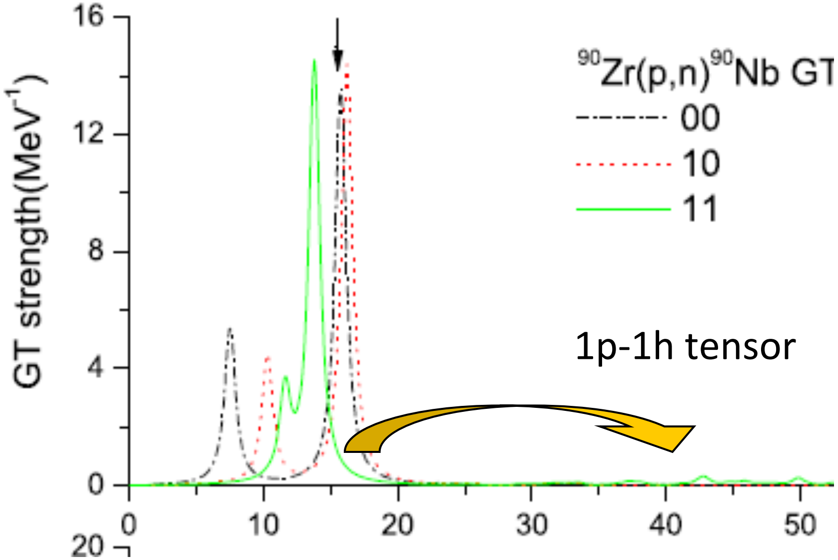
Energy-weighted sum rules

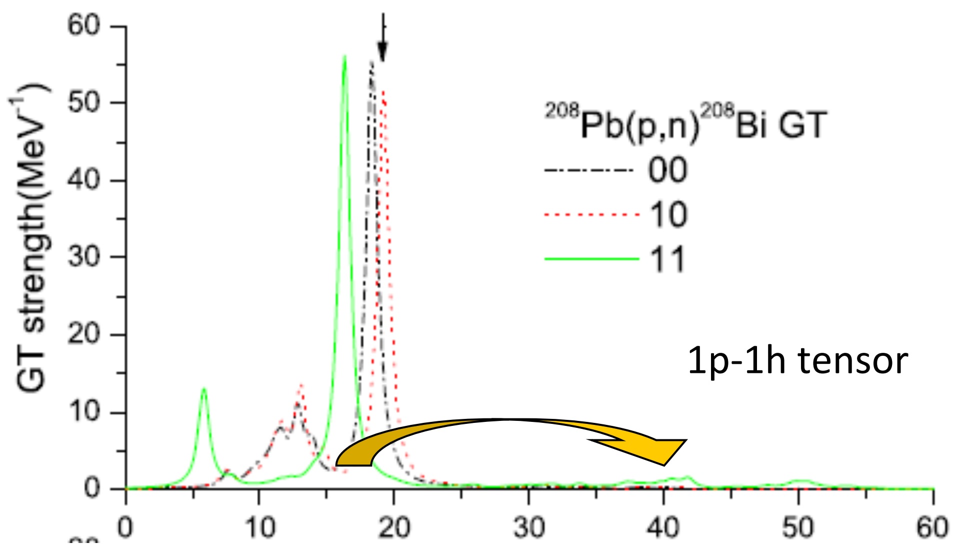
$$m(k) = \sum_i E^k{}_i \left|\langle i|\hat{O}_\lambda|0\rangle\right|^2$$

$$m(1) = \frac{1}{2}\langle 0|\left[\hat{O}_\lambda,\left[H,\hat{O}_\lambda\right]\right]|0\rangle$$

$$\boxed{m_-(0) - m_+(0) = 3(N-Z) = \begin{cases} 30 \text{ for } {}^{90}Zr \\ 132 \text{ for } {}^{208}Pb \end{cases}}$$

About 10% of strength is moved by the tensor correlations to the energy region above 30 MeV.
Relevance for the GT quenching problem.





Multipole Expansion of Tensor Interactions

$$
\begin{aligned}
V^T = {} & \frac{T}{2}\{[(\sigma_1\cdot\mathbf{k}')(\sigma_2\cdot\mathbf{k}')-\frac{1}{3}(\sigma_1\cdot\sigma_2)\mathbf{k}'^2]\delta(\mathbf{r}_1-\mathbf{r}_2) \\
& +\delta(\mathbf{r}_1-\mathbf{r}_2)\left[(\sigma_1\cdot\mathbf{k})(\sigma_2\cdot\mathbf{k})-\tfrac{1}{3}(\sigma_1\cdot\sigma_2)\mathbf{k}^2\right]\} \\
& +\frac{U}{2}\{(\sigma_1\cdot\mathbf{k}')\,\delta(\mathbf{r}_1-\mathbf{r}_2)\,(\sigma_2\cdot\mathbf{k})+(\sigma_2\cdot\mathbf{k}')\,\delta(\mathbf{r}_1-\mathbf{r}_2)(\sigma_1\cdot\mathbf{k}) \\
& -\tfrac{2}{3}\left[(\sigma_1\cdot\sigma_2)\mathbf{k}'\cdot\delta(\mathbf{r}_1-\mathbf{r}_2)\mathbf{k}\right]\}
\end{aligned}
$$

$$\delta(\vec{r}_1-\vec{r}_2)=\sum_{lm} Y_{lm}(\hat{r}_1)Y^*_{lm}(\hat{r}_2)\frac{\delta(r_1-r_2)}{r_1 r_2}$$

$$V^T \propto T_{(\lambda,\kappa)}\{[\sigma_1\times[\nabla_1\times Y_{l=1}(\hat{r}_1)]^{(\lambda)}\}^{(\kappa)}[\sigma_2\times[\nabla_2\times Y_{l=1}(\hat{r}_2)]^{(\lambda')}\}^{(\kappa)}\}^{(0)}\delta(r_1-r_2)$$

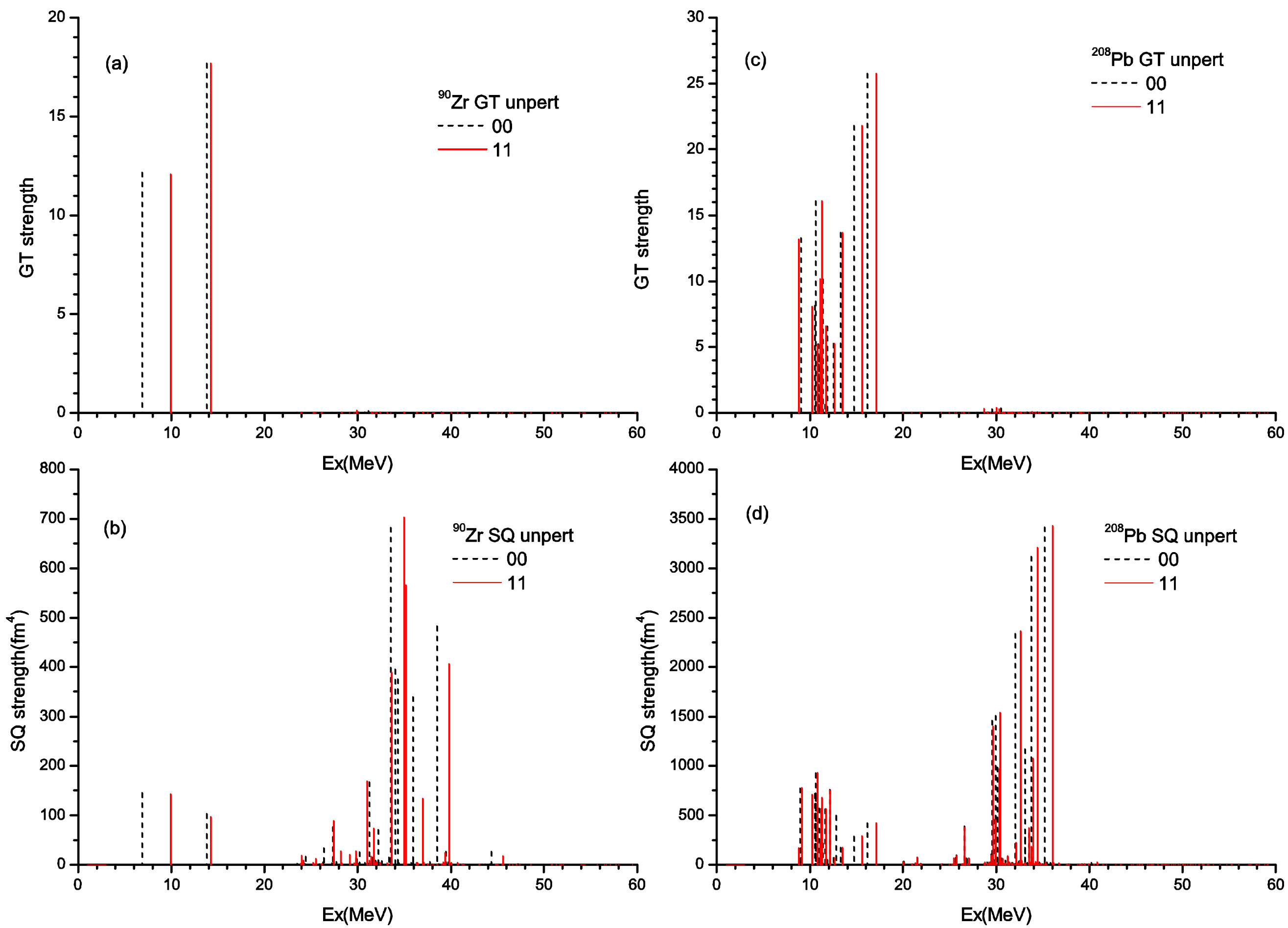
$1^+ \quad T_{(\lambda=\lambda'=2,\kappa=1)} \Rightarrow$ *repulsive*

$1^+ \quad T_{(\lambda=2,\lambda'=0,\kappa=1)} \Rightarrow$ strong mixing between Gamow-Teller and spin-quadrupole excitations!

Why does Tensor interaction decrease GT strength in peak region?

$$1^{+}: \text{ Gamow-Teller excitation } \sigma\cdot\tau$$

$$\text{Spin-Quadrupole excitation } \; r^{2}[\sigma\times \mathrm{Y}_{2}]^{(\lambda)} \;\; \lambda = 1^{+}, 2^{+}, 3^{+}$$



type of calculation		$m_-(0)$ 0-30MeV	$m_-(0)$ 30-60MeV	$m_-(1)$ 0-30 MeV	$m_-(1)$ 30-60 MeV	$m_-(1)$ total	$m_+(1)$ total
^{90}Zr	00	29.16	0.71	395	26.2	421.8	10.1
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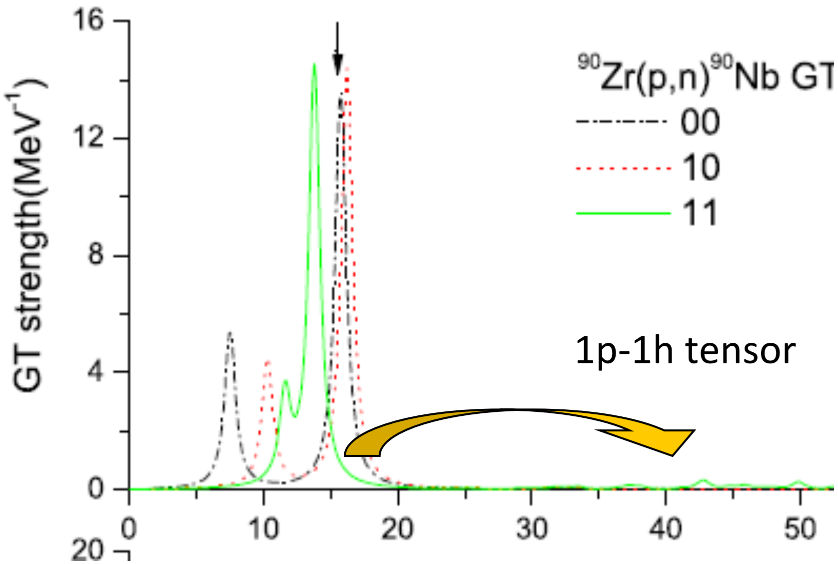
Energy-weighted sum rules

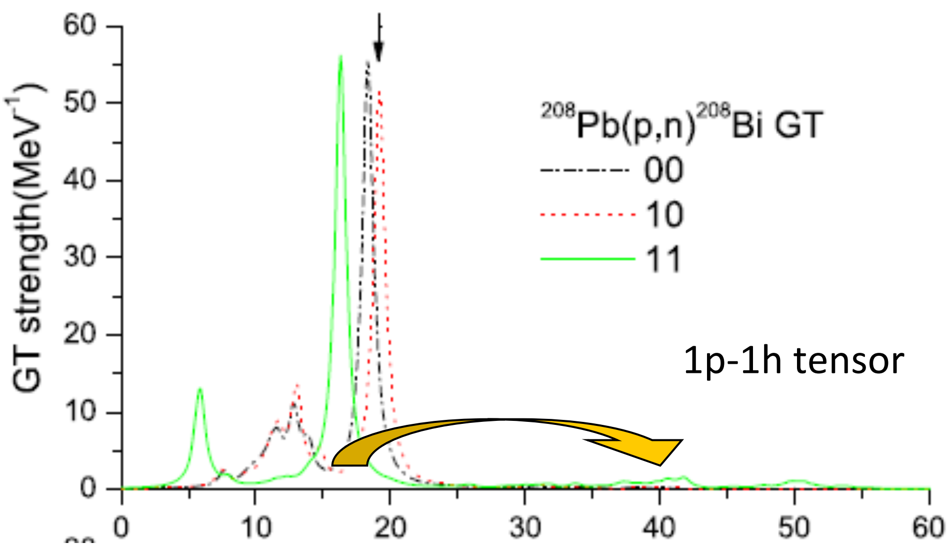
$$m(k) = \sum_i E^k{}_i \left| \langle i | \hat{O}_\lambda | 0 \rangle \right|^2$$

$$m(1) = \frac{1}{2} \langle 0 | [\hat{O}_\lambda, [H, \hat{O}_\lambda]] | 0 \rangle$$

$$m_-(0) - m_+(0) = 3(N-Z) = \begin{cases} 30 \text{ for } {}^{90}Zr \\ 132 \text{ for } {}^{208}Pb \end{cases}$$

About 10% of strength is moved by the tensor correlations to the energy region above 30 MeV.
Relevance for the GT quenching problem.

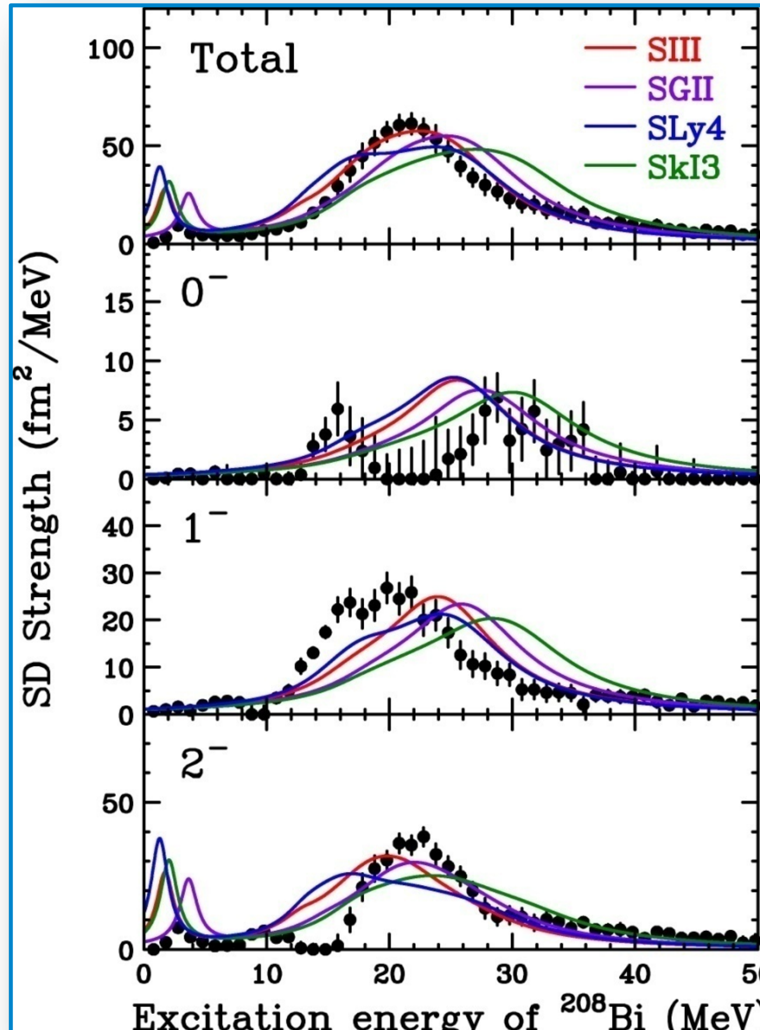




Puzzle in SD Strength Distributions（Wakasa, SIR2010,18-21 Feb.,2010)

H. Sagawa et al., PRC 76, 024301 (2007).

- Total strength
  - Asymmetric single bump
    - Extend up to ～50 MeV
    - Same as ^{90}Zr(p,n)results
  - SIII provides better description
- 0^- strength
  - Quenched
    - Seems to be fragmented
- 1^- strength
  - Softened compared with theory
    - Peak shift to lower E_x
- 2^- strength
  - Hardened compared with theory
    - Peak shift to higher E_x



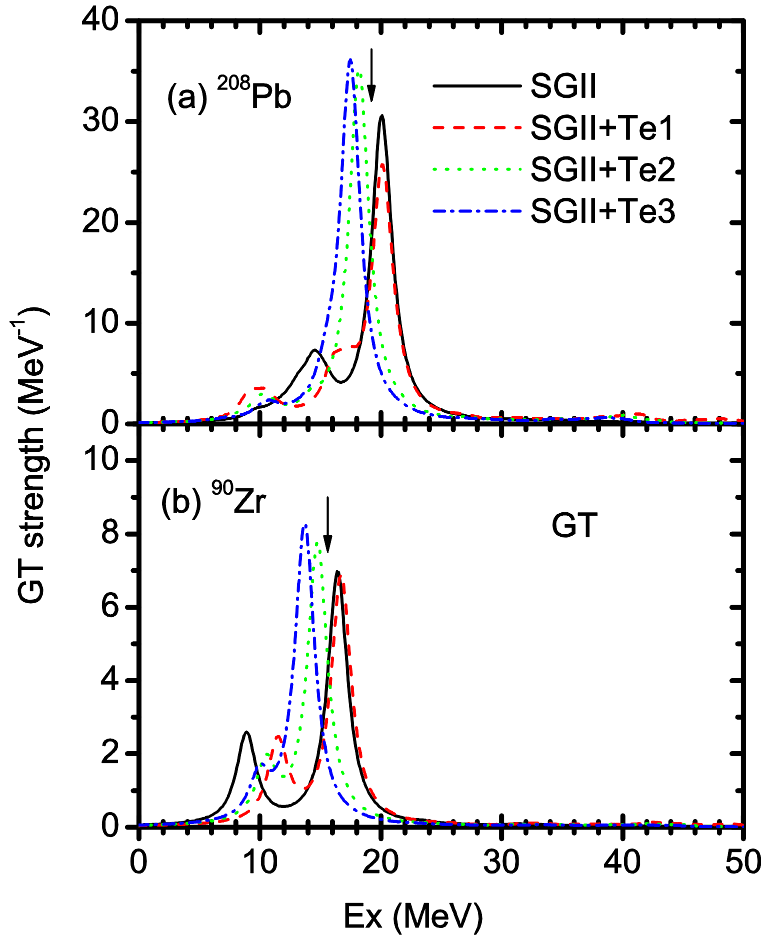
- No Skyrme int. which reproduces both total and separated strengths
- ΔJ^{π}-dependent correlation ? → Require further investigations

A systematic study of tensor interactions on Spin-Isospin excitations

PHYSICAL REVIEW C 83, 054316 (2011)

Spin-isospin excitations as quantitative constraints for the tensor force

C. L. Bai,[1,2] H. Q. Zhang,[2] H. Sagawa,[3] X. Z. Zhang,[2] G. Colò,[4] and F. R. Xu[5]



	(T , U)
SG2+Te1	(500,-350)
Te2	(600, 0)
Te3	(650, 200)
Sly5+Tw	(888,-408)

TABLE I. The calculated peak energies of the SD and GT strengths in ^{90}Zr and ^{208}Pb obtained by using the four interactions that reproduce the experimental data [14,18,19] within an accuracy of 2.5 MeV. See the text for a discussion.

	^{90}Zr					^{208}Pb				
	0^-	1^-	2^-	total SD	GT	0^-	1^-	2^-	total SD	GT
T21	39.3	23.3	25.3	23.5	15.9	40.8	24.1	25.0	23.3	18.0
T32	39.0	23.8	25.4	24.3	15.9	39.4	23.4	25.3	23.3	17.4
T43	38.6	24.3	25.3	24.9	16.2	37.7	24.0	25.4	23.6	17.2
T54	38.3	24.5	25.4	25.2	16.2	37.1	23.8	25.4	23.5	16.7
exp	...	...	...	26.0	15.6	34.5	22.8	25.8	25.2	19.2

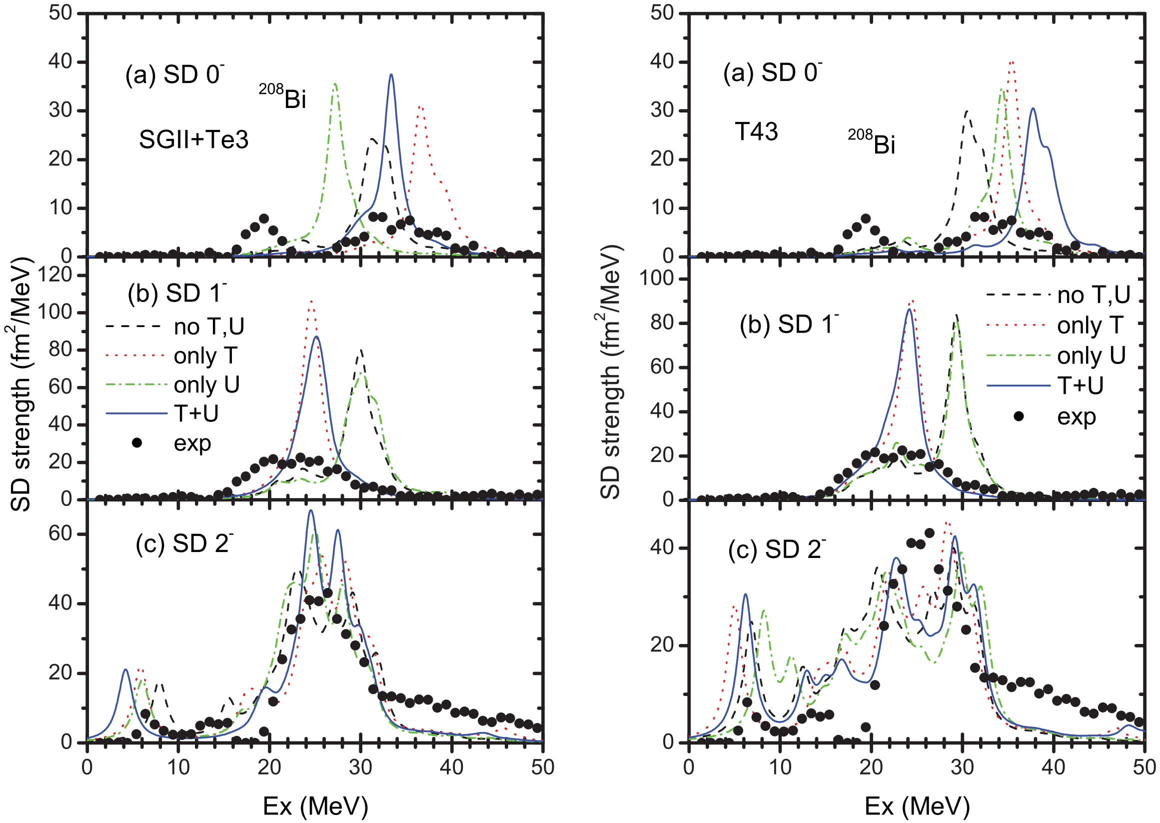
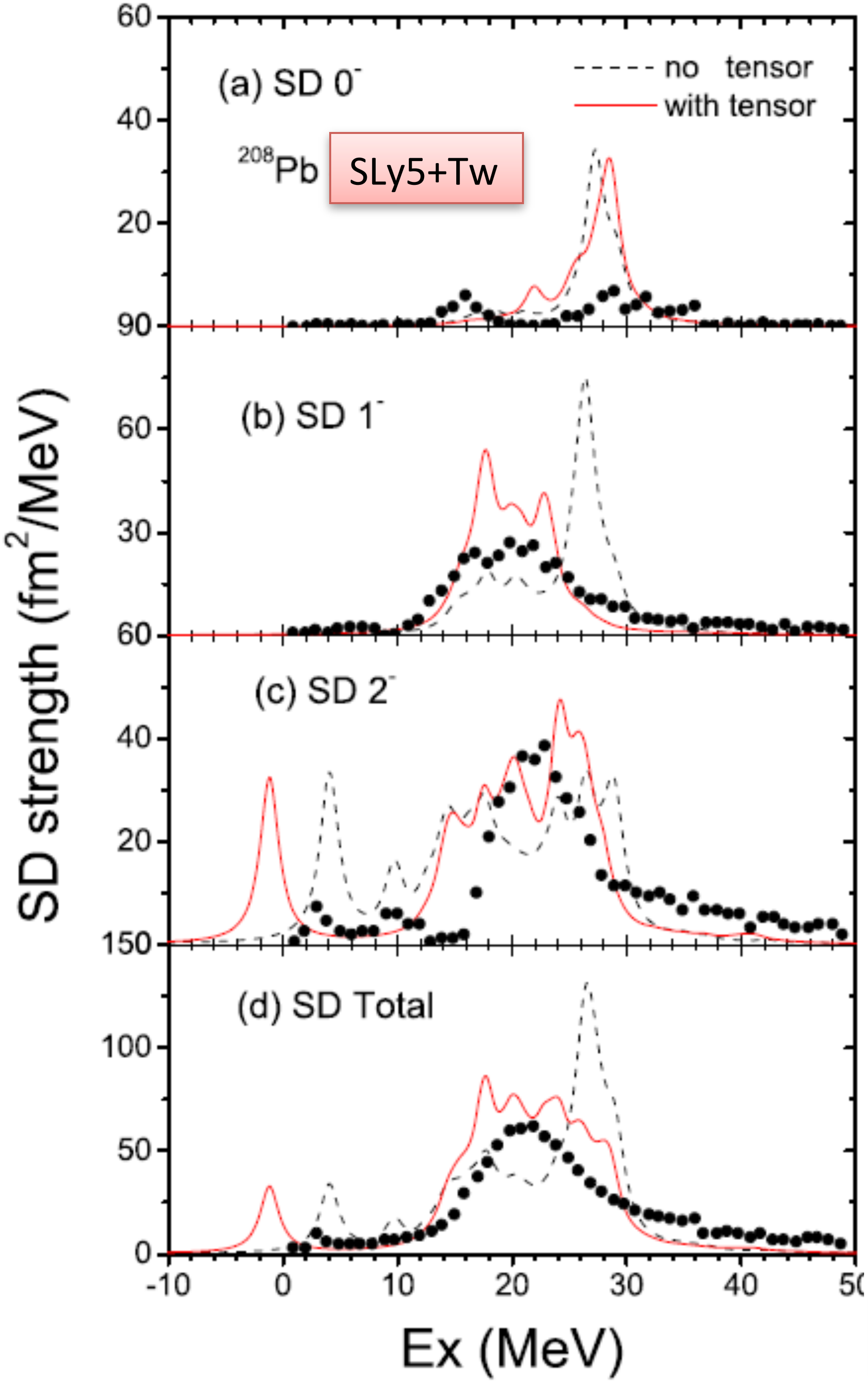
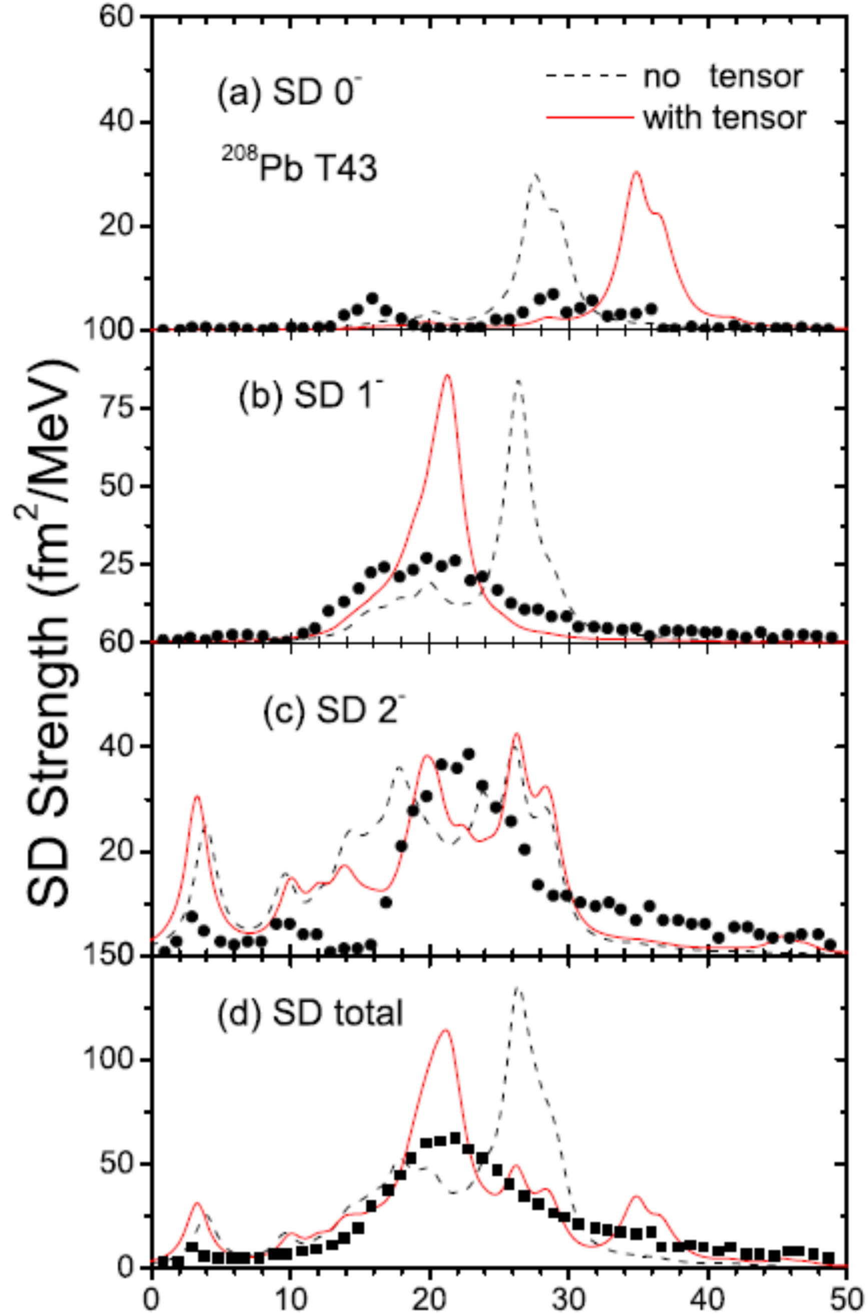


FIG. 3. (Color online) The charge-exchange SD strength distributions of ^{208}Bi, calculated with the T43 interaction. The dashed, dotted, and dashed-dotted curves show the results without tensor interaction, with the only triplet-even and only triplet-odd tensor interactions, respectively, while the solid curve shows the results with the full tensor interaction. The discrete RPA results have been smoothed by using Lorentzian functions having a width of 2 MeV. See the text for more details.

TABLE IV. Energy differences between the main 0^- and 1^- peaks, $\delta E_p \equiv E(0^-) - E(1^-)$, in ^{16}F and ^{208}Bi. The four RPA results correspond to the case without tensor interactions, with the triplet-even T term, with the triplet-odd U term and with both T and U terms, respectively.

		W/o tensor	With T	With U	With T and U
^{16}F	T43	0.7	3.1	1.5	3.9
	SGII+Te3	0.9	4.0	−0.6	2.5
^{208}Bi	T43	1.2	11.0	4.9	13.6
	SGII+Te3	1.9	12.0	−2.9	8.2

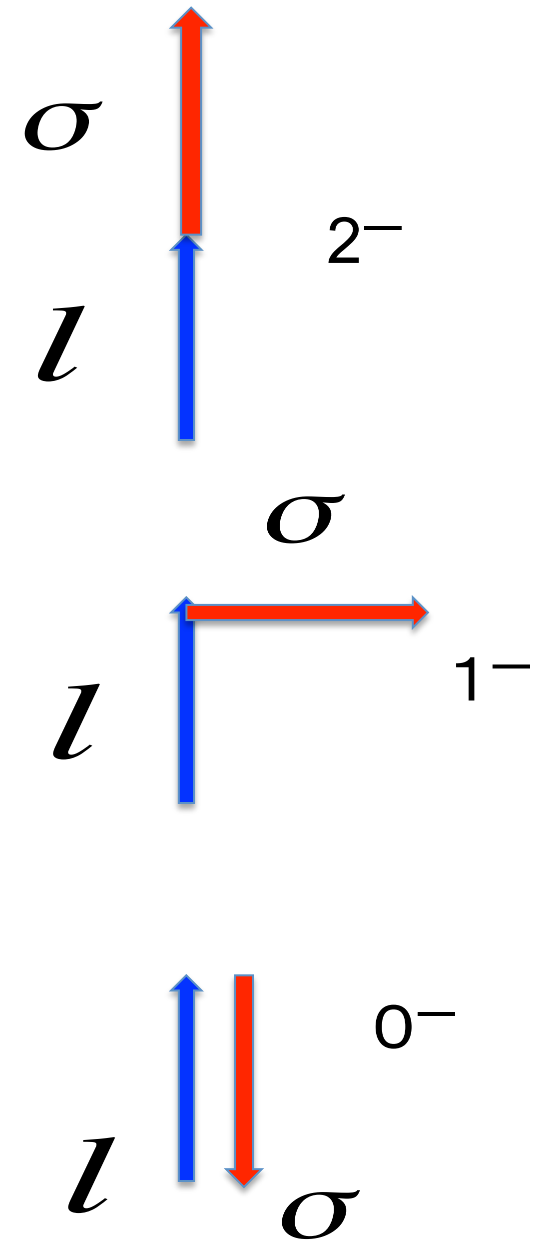




C.L.Bai et al., PRL 105, 072501 (2010)

$$V^{(\lambda)}{}_{\mathrm{TE}} = \frac{-5}{12} T \begin{Bmatrix} 1 \\ -1/6 \\ 1/50 \end{Bmatrix} \left| \langle p \| O_{1,\lambda} \| h \rangle \right|^2 \quad \text{for } \lambda = \begin{Bmatrix} 0^- \\ 1^- \\ 2^- \end{Bmatrix} \quad = aT$$

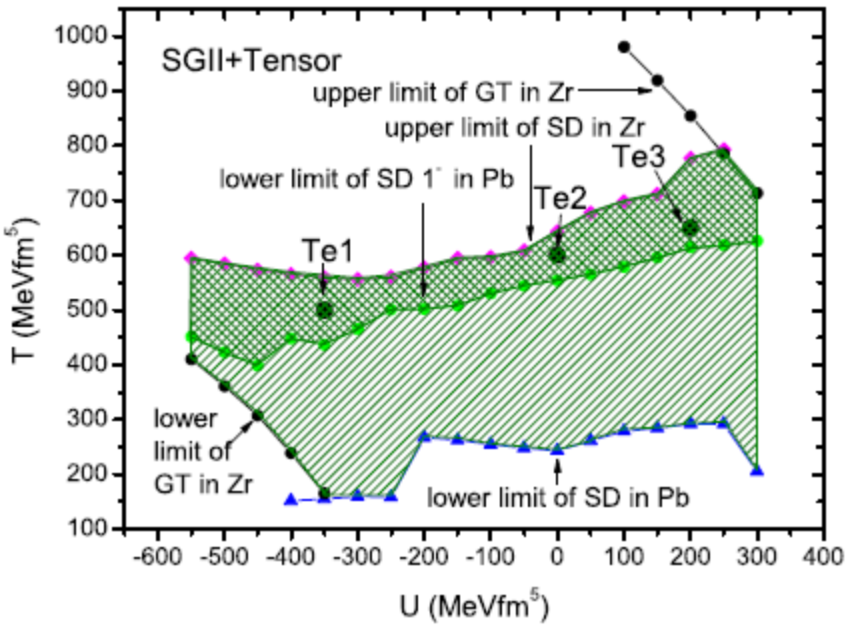
$$V^{(\lambda)}{}_{\mathrm{TO}} = \frac{5}{12} U \begin{Bmatrix} 1 \\ -1/6 \\ 1/50 \end{Bmatrix} \left| \langle p \| O_{1,\lambda} \| h \rangle \right|^2 \quad \text{for } \lambda = \begin{Bmatrix} 0^- \\ 1^- \\ 2^- \end{Bmatrix} \quad = bU$$

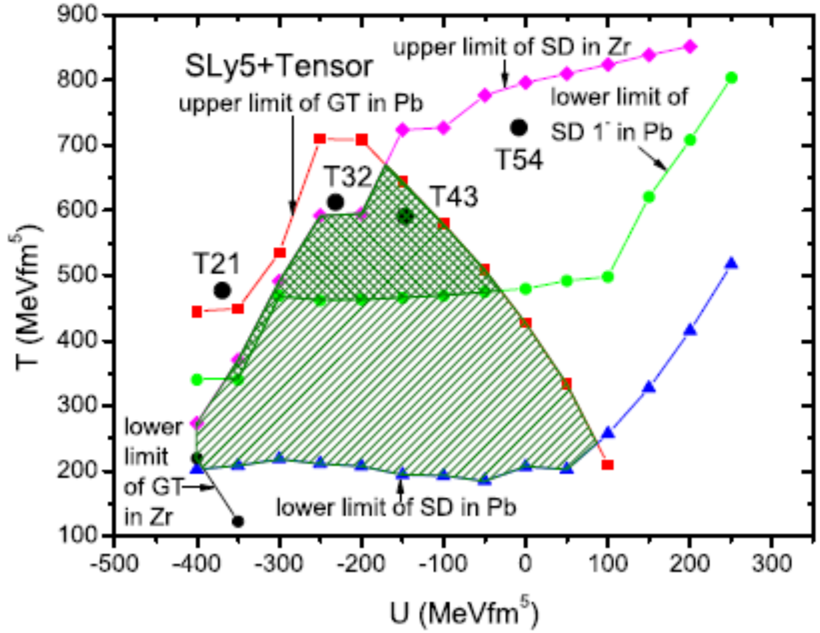


antisymmtric matrix $$V^{(\lambda)}{}_{\mathrm{T,AS}} = \left[-\frac{1}{2} a_\lambda T + \frac{1}{2} b_\lambda U \right] \langle \tau . \tau \rangle$$

$$\begin{Bmatrix} \text{repulsive} \\ \text{attractive} \\ \text{repulsive} \end{Bmatrix} \quad \text{for } \lambda = \begin{Bmatrix} 0^- \\ 1^- \\ 2^- \end{Bmatrix}$$

A systematic study of tensor interactions on Spin-Isospin excitations by HF+RPA





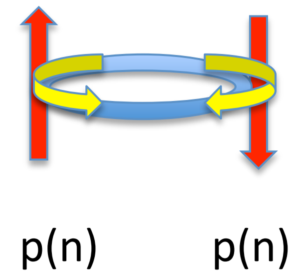
T(triplet-even tensor) is well constrained by spin-isopin excitations irrespective of central part of Skyrme forces. T=500+/-100MeVfm^(5)

U(triplet-odd) is not well constrained by existing sets of experimental data.

T=1 S=0 pairing and T=0 S=1 pairing interactions

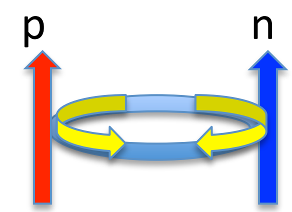
T=1, S=0 pair

$$\left|(L = S = 0)J = 0, T = 1\right\rangle \Rightarrow$$



T=0, S=1 pair

$$\left|(L = 0, S = 1)J = 1, T = 0\right\rangle \Rightarrow$$



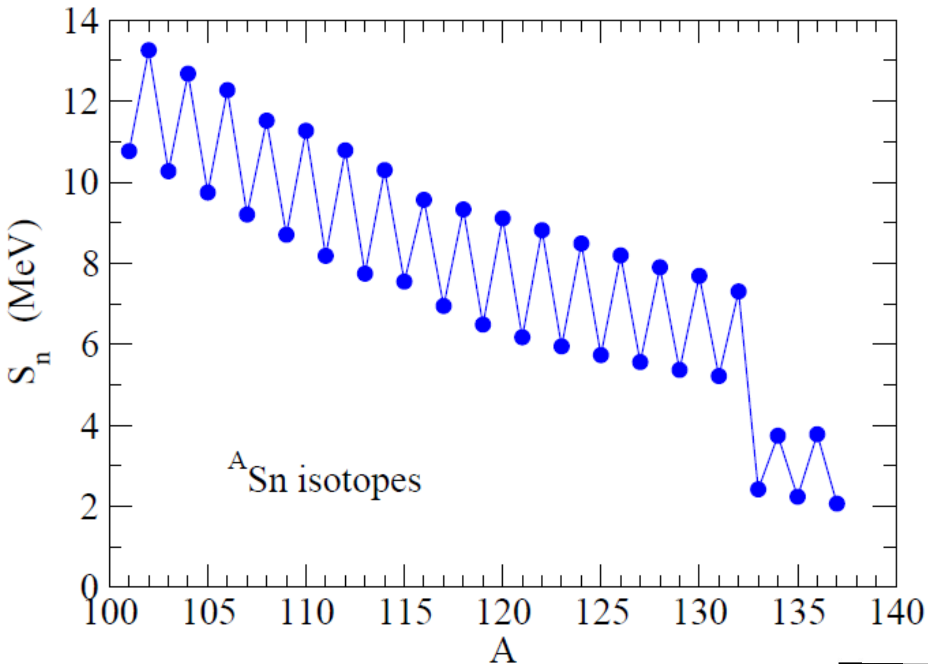
How we can disentangle in quantum many-body systems.
➔ two kinds of superfluidity?

T=1 S=0 pairing pairing interactions

T=1 pairing (n-n, p-p pairing correlations) ➔ isovector spin-singlet superfluidity

- mass (odd-even staggering)
- energy spectra (gap between the first excited state and the ground state in even-even nuclei)
- moment of inertia
- n-n or p-p Pair transfer reactions
- fission barrier (large amplitude collective motion)

➢binding energy

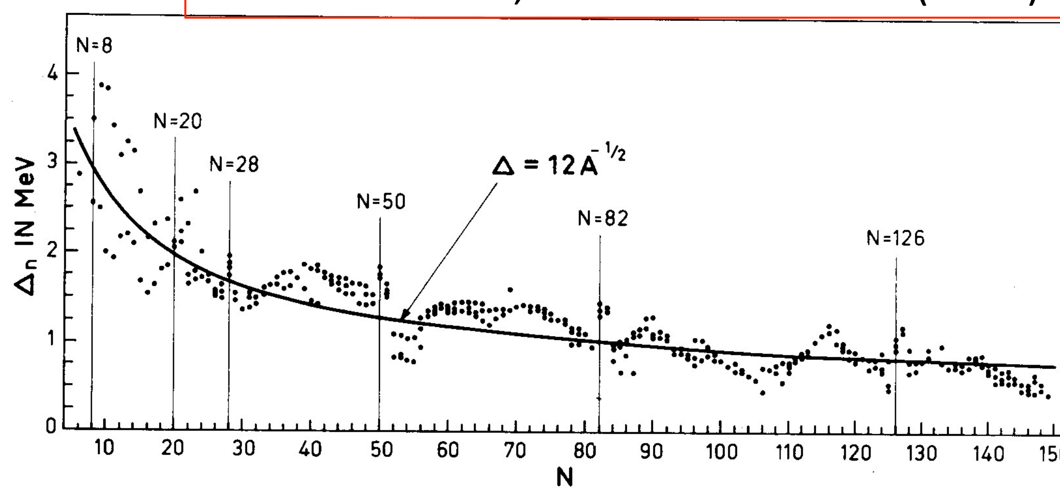


pairing gap parameter

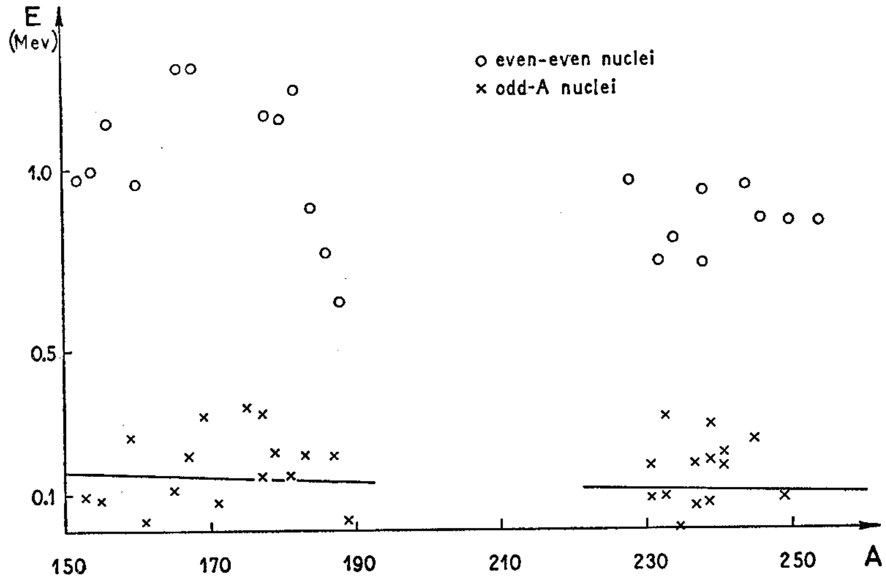
$$\Delta(N) = \frac{(-)^N}{2}(B(N-1) - 2B(N) + B(N+1)) = \frac{(-)^N}{2}(S_n(N-1) - S_n(N))$$

$S_n(N) = B(N) - B(N-1)$

Bohr-Mottelson, Nuclear Structure I (1969)



Quasi-particle excitation



Δ (pairing gap)

218 Pairing Correlations and Superfluid Nuclei

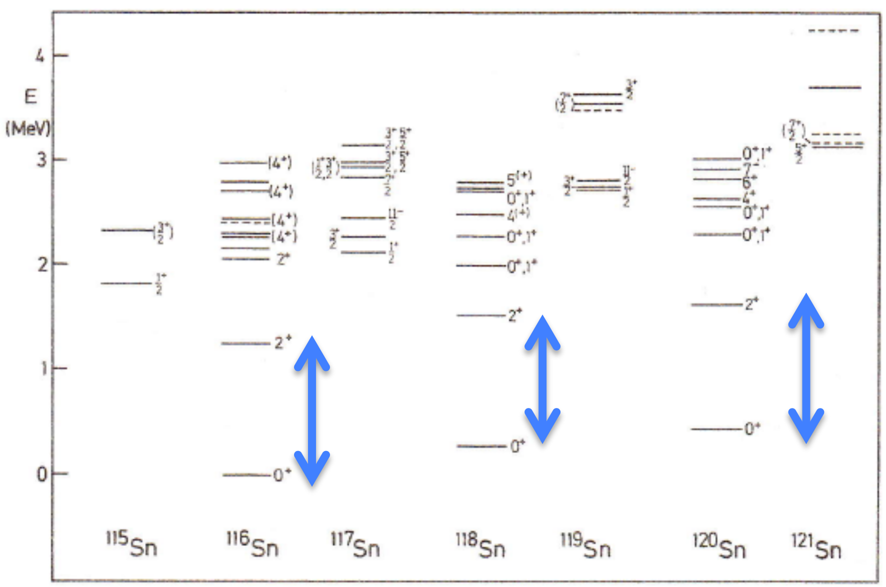


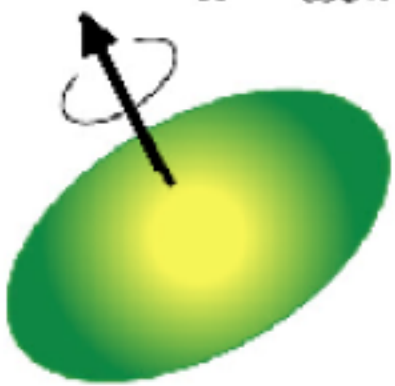
Figure 6.1. Excitation spectra of the $_{50}$Sn isotopes.

Sn isotopes: 2^+ states

Moment of Inertia

Bohr-Mottelson, Nuclear Structure II (1975)

$h - \omega J_x$



Rotation of deformed nucleus

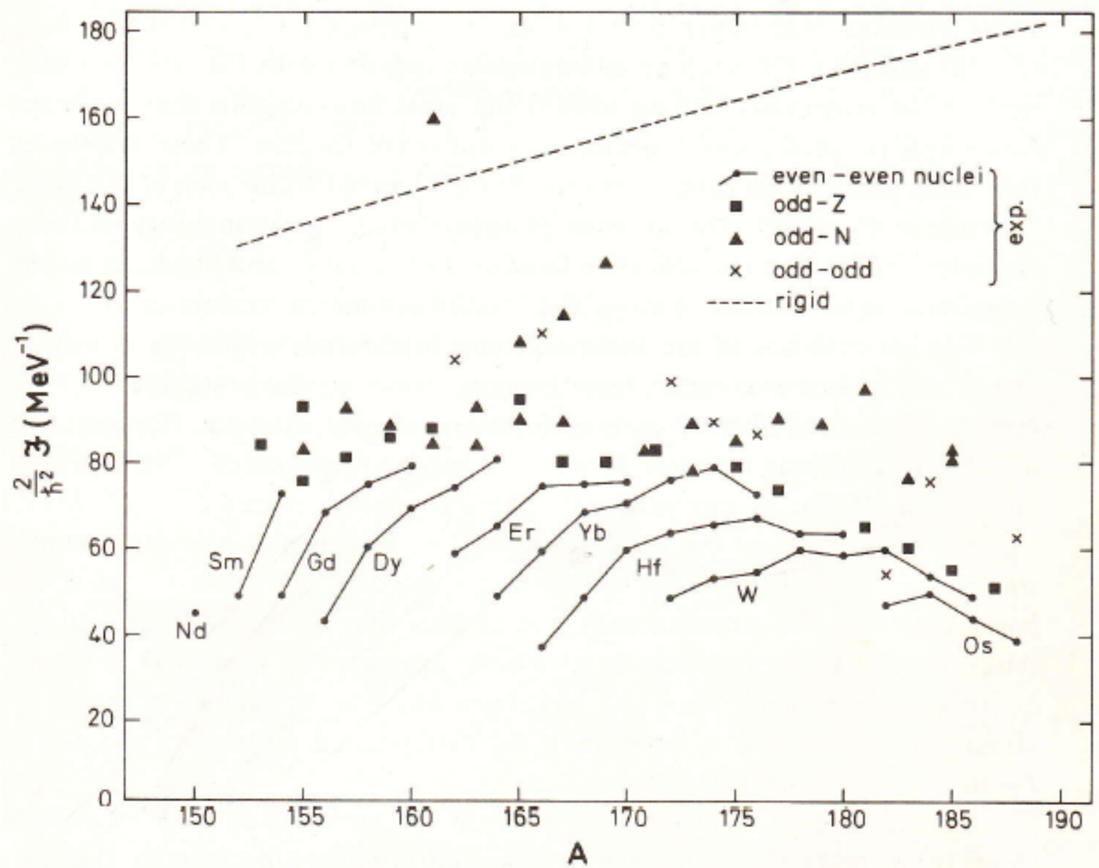


Figure 4-12 Systematics of moments of inertia for nuclei with $150 \leqslant A \leqslant 188$. The moments of inertia are obtained from the empirical energy levels in *Table of Isotopes* by Lederer *et al.*, 1967.

rigid rotor vs. superfluid rotor

$$I_{Inglis} = 2\sum_{i>j} \frac{\langle i | J_x | j \rangle^2}{\varepsilon_i - \varepsilon_j} = I_{rigid}$$

$$I_{pairing} = 2\sum_{i,j} \frac{\langle i | J_x | j \rangle^2 (u_i v_j - v_i u_j)^2}{E_i + E_j}$$

T=1 S=0 pairing and T=0 S=1 pairing interactions

T=1 pairing (n-n, p-p pairing correlations) ➔spin singlet superfluid

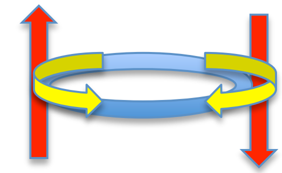
- mass (odd-even staggering)
- energy spectra (gap between the first excited state and the ground state in even-even nuclei
- moment of inertia
- n-n or p-p Pair transfer reactions
- fission barrier (large amplitude collective motion)

T=0 pairing (p-n pairing with S=1) ➔spin triplet superfluid ?

- N=Z Wigner energy (still controversial)
- Energy spectra in nuclei with N=Z (T=0 and $J=J_{max}$)
- n-p pair transfer reaction
- low-energy super-allowed Gamow-Teller transition in N=Z and N=Z+2 between SU(4) supermultiples (C.L. Bai et al.)

Two particle systems

T=1, S=0 pair

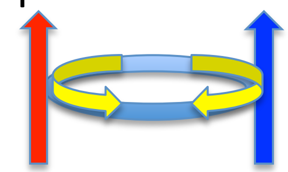


p(n) p(n)

$$\left|(L=S=0)J=0,T=1\right\rangle \Rightarrow \left|(j=j^{'})J=0,T=1\right\rangle$$

T=0, S=1 pair

p n



$$\left|(L=0,S=1)J=1,T=0\right\rangle \Rightarrow$$

$$a\left|(l=l^{'}j=j^{'})J=1,T=0\right\rangle + b\left|((l=l^{'})j,j^{'}=j\pm 1)J=1,T=0\right\rangle$$

If there is strong spin-orbit splitting, it is difficult to make (T=0,S=1)pair.

But, T=0 J= 1^+ state could be Gamow-Teller states in nuclei with N~Z
➔ strong GT states in N=Z+2 nuclei

SU(4) supermultiplet in spin-isospin space

Well-known in light p-shell nuclei (LS coupling dominance)

The spin-singlet T=1 pairing $V^{(T=1)}(\mathbf{r},\mathbf{r}') = -G^{(T=1)} \sum_{i,j} P^{(1,0)\dagger}_{i,i}(\mathbf{r},\mathbf{r}') P^{(1,0)}_{j,j}(\mathbf{r},\mathbf{r}')$

$$\begin{aligned}\langle (j_i j_i) T = 1, J = 0 | V^{(T=1)} | (j_j j_j) T = 1, J = 0 \rangle \\ = -\sqrt{(j_i + 1/2)(j_j + 1/2)}\, G^{(T=1)} I^2_{ij}\end{aligned} \qquad (5)$$

where I_{ij} is the overlap integral given by,

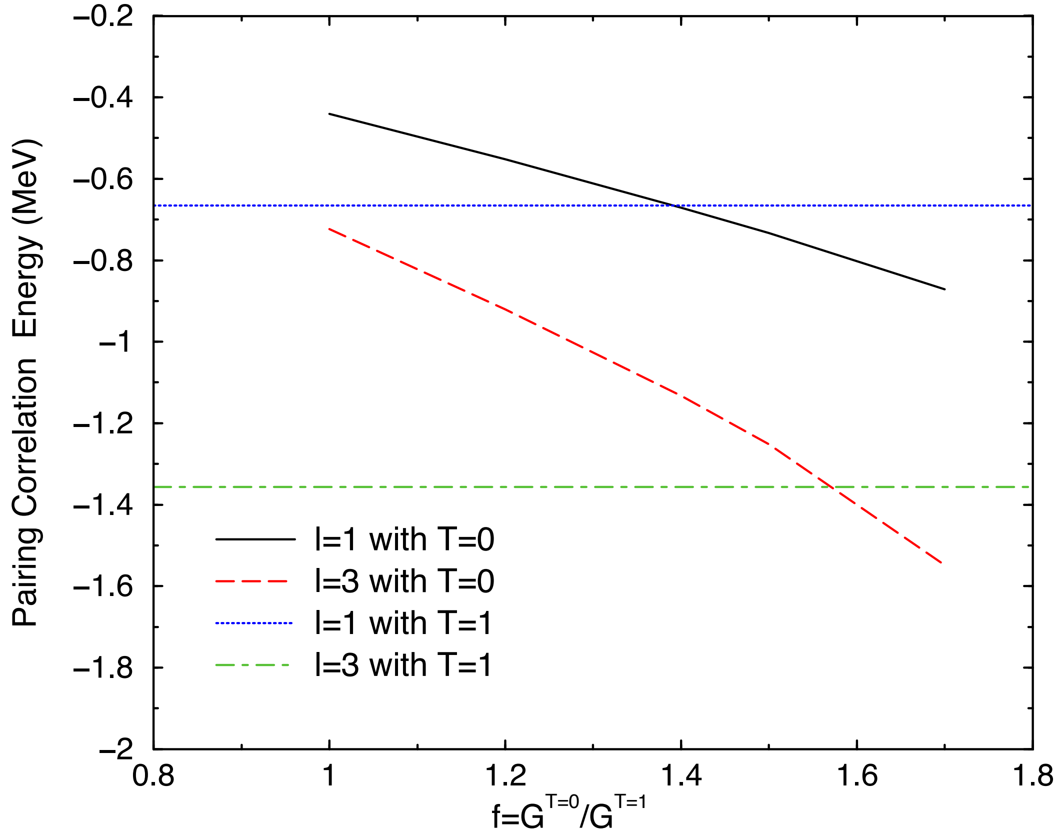
$$I_{ij} = \int \psi_i(\mathbf{r})^* \psi_j(\mathbf{r}) d\mathbf{r} \qquad (6)$$

(T=0, S=1) pairing $V^{(T=0)}(\mathbf{r},\mathbf{r}') = -fG^{(T=1)} \sum_{i \geqq i', j \geqq j'} P^{(0,1)\dagger}_{i,i'}(\mathbf{r},\mathbf{r}') P^{(0,1)}_{j,j'}(\mathbf{r},\mathbf{r}')$

$$\begin{aligned}&\langle (j_1 j_2) T = 0, J = 1 | V^{(T=0)} | (j_1' j_2') T = 0, J = 1 \rangle = \\ &- \left\langle \left[\left(l_1 \frac{1}{2} \right)^{j_1} \left(l_2 \frac{1}{2} \right)^{j_2} \right]^{J=1} \middle| \left[(l_1 l_2)^{L=0} \left(\frac{1}{2} \frac{1}{2} \right)^{S=1} \right]^{J=1} \right\rangle \\ &\times \left\langle \left[\left(l_1' \frac{1}{2} \right)^{j_1'} \left(l_2' \frac{1}{2} \right)^{j_2'} \right]^{J=1} \middle| \left[(l_1' l_2')^{L=0} \left(\frac{1}{2} \frac{1}{2} \right)^{S=1} \right]^{J=1} \right\rangle \\ &\times \frac{\sqrt{2l_1 + 1}\sqrt{2l_1' + 1}}{\sqrt{1 + \delta_{j_1,j_2}}\sqrt{1 + \delta_{j_1',j_2'}}} fG^{T=1} (I_{j_1 j_1'} I_{j_2 j_2'} + I_{j_1 j_2'} I_{j_1 j_2'}),\end{aligned}$$

TABLE I: The transformation coefficient R between the jj coupling and the LS coupling for the pair wave functions, $R = \langle [(l\frac{1}{2})^j (l\frac{1}{2})^{j'}]^{J=1} | [(ll)^{L=0} (\frac{1}{2}\frac{1}{2})^{S=1}]^{J=1} \rangle$. Ω is defined as $\Omega \equiv 3(2l+1)^2$.

j	j'	R	$l=1$	$l=3$
$l+1/2$	$l+1/2$	$\sqrt{\frac{(2l+2)(2l+3)}{2\Omega}}$	$\frac{1}{3}\sqrt{\frac{10}{3}}$	$\frac{2\sqrt{3}}{7}$
$l+1/2$	$l-1/2$	$-\sqrt{\frac{4l(l+1)}{\Omega}}$	$-\frac{2}{3}\sqrt{\frac{2}{3}}$	$-\frac{4}{7}$
$l-1/2$	$l-1/2$	$-\sqrt{\frac{2l(2l-1)}{2\Omega}}$	$-\frac{1}{3}\sqrt{\frac{1}{3}}$	$-\frac{\sqrt{5}}{7}$
$l-1/2$	$l+1/2$	$\sqrt{\frac{4l(l+1)}{\Omega}}$	$\frac{2}{3}\sqrt{\frac{2}{3}}$	$\frac{4}{7}$



Pairing correlation energy of (J,T)=(0,1) and (1,0) states in *pf* shell

Even with large spin-orbit splitting for *f* -orbits, the spin-triplet correlations will be larger than the spin-singlet one for f>1.5

HS, Y. Tanimura and K. Hagino, PRC87, 034310 (2013)

TABLE I. Strengths of triplet and singlet interactions from shell-model fits and their ratios. See text for details.

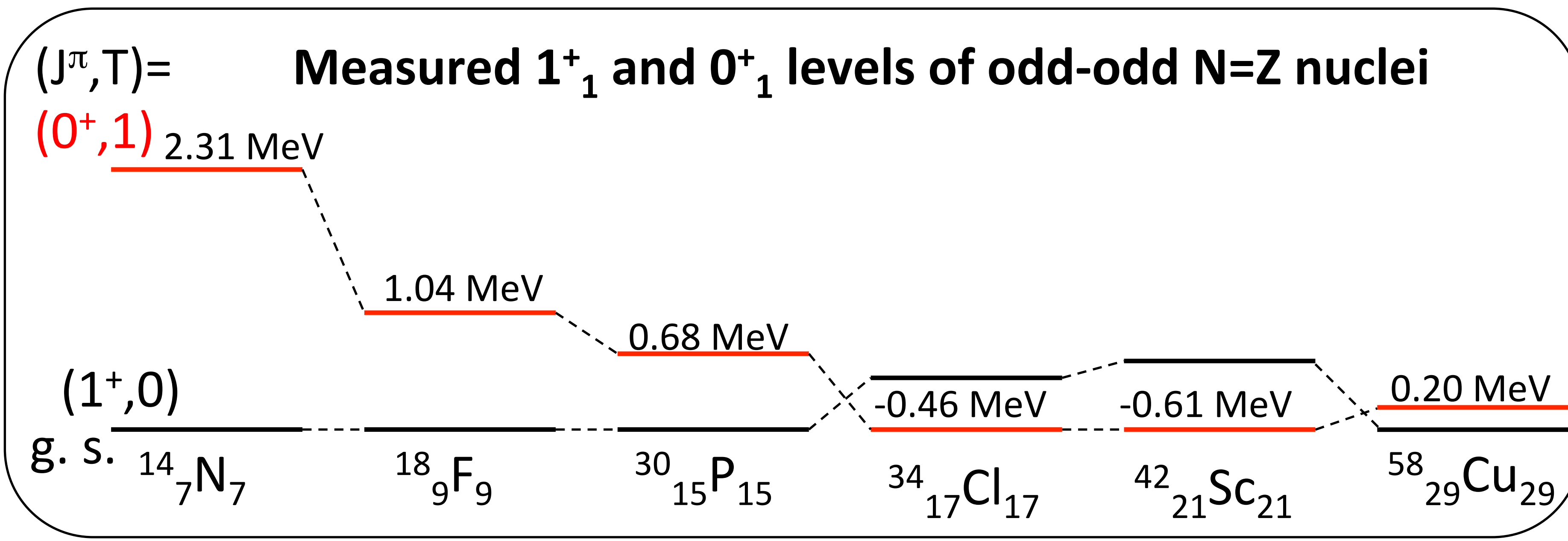
Source	v_s (MeV fm^3)	v_t (MeV fm^3)	Ratio
sd shell [8]	280	465	1.65
fp shell [9]	291	475	1.63

G.F. Bertsch and Y. Luo, PRC81, 064320 (2010)

Y. Tanimura, HS, K. Hagino, PTEP 053D02 (2014)

- n-p Pair correlations studied by 3-body model
  - ✓ T=0, 1 two channels
  - ✓ T=0, S=1 is attractive stronger than T=1, S=0 pair
    cf. dueteron, matrix elements in shell models
  - ✓ In finite nuclei N>Z , the strong spin-orbit coupling
    may quench or even kill T=0 pairing

when *l* is larger , the spin-orbit is larger and T=0 pair correlations decrease



Three-body Model

Total 3-body Hamiltonian

$$H = \frac{\boldsymbol{p}_p^2}{2m} + \frac{\boldsymbol{p}_n^2}{2m} + V_{pC}(\boldsymbol{r}_p) + V_{nC}(\boldsymbol{r}_n) + V_{pn}(\boldsymbol{r}_p, \boldsymbol{r}_n) + \frac{(\boldsymbol{p}_p + \boldsymbol{p}_n)^2}{2A_C m}$$

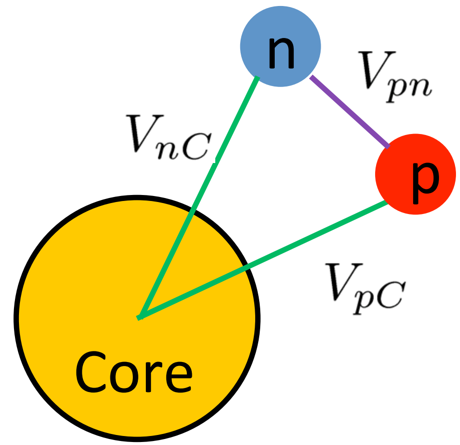
Core-N mean field

$$V_{(p/n)C}(r) = v_0 f(r) + v_{ls}\frac{1}{r}\frac{d}{dr}f(r)(\boldsymbol{l}\cdot\boldsymbol{s})(+Coulomb)$$

$$f(r) = \frac{1}{1+e^{(r-R)/a}}$$

p-n interaction

$$V_{pn} = \hat{P}_s v_s \delta(\boldsymbol{r}_p - \boldsymbol{r}_n)[1 + x_s(\frac{\rho(r)}{\rho_0})^\alpha] + \hat{P}_t v_t \delta(\boldsymbol{r}_p - \boldsymbol{r}_n)[1 + x_t(\frac{\rho(r)}{\rho_0})^\alpha]$$



Determination of parameters

v_0, v_{ls}: neutron separation energy

v_s, v_t: pn scattering length with E_{cut} (= 20 MeV)

v_s/v_t=1.7 (spin-triplet pairing is much stronger than spin-singlet)

x_S, x_t, α: 1^+, 3^+, 0^+ in ^{18}F energies are fitted

⇨ Diagonalization in a large model space

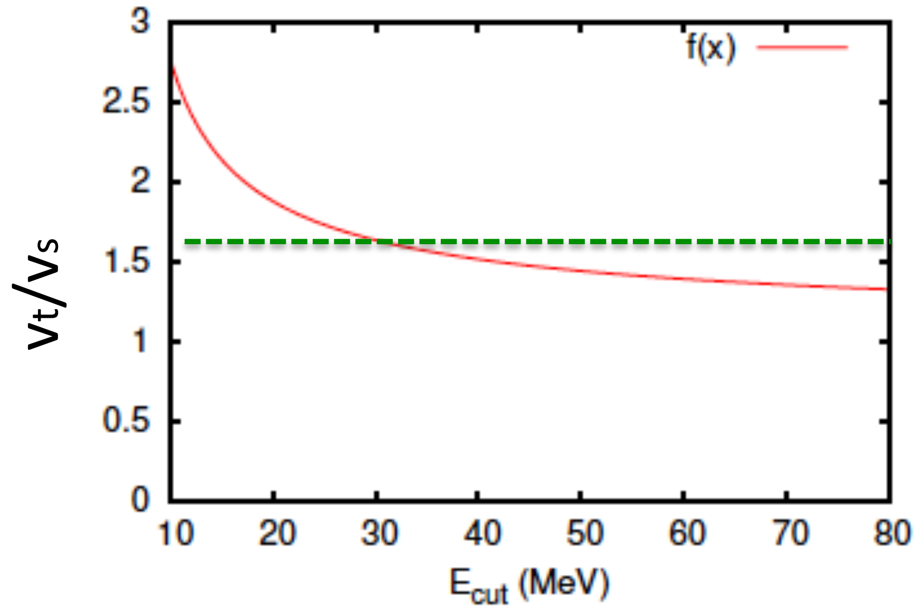
pn pairing interaction

$$V_{np}(\boldsymbol{r}_1,\boldsymbol{r}_2) = \hat{P}_s v_s \delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\left[1+x_s\left(\frac{\rho(r)}{\rho_0}\right)^{\alpha}\right] + \hat{P}_t v_t \delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\left[1+x_t\left(\frac{\rho(r)}{\rho_0}\right)^{\alpha}\right]$$

$$\hat{P}_s = \frac{1}{4}-\frac{1}{4}\boldsymbol{\sigma}_p\cdot\boldsymbol{\sigma}_n,\ \hat{P}_t = \frac{3}{4}+\frac{1}{4}\boldsymbol{\sigma}_p\cdot\boldsymbol{\sigma}_n.$$

$$v_s = \frac{2\pi^2\hbar^2}{m}\frac{2a_{pn}^{(s)}}{\pi-2a_{pn}^{(s)}k_{\rm cut}},$$

$$v_t = \frac{2\pi^2\hbar^2}{m}\frac{2a_{pn}^{(t)}}{\pi-2a_{pn}^{(t)}k_{\rm cut}},$$



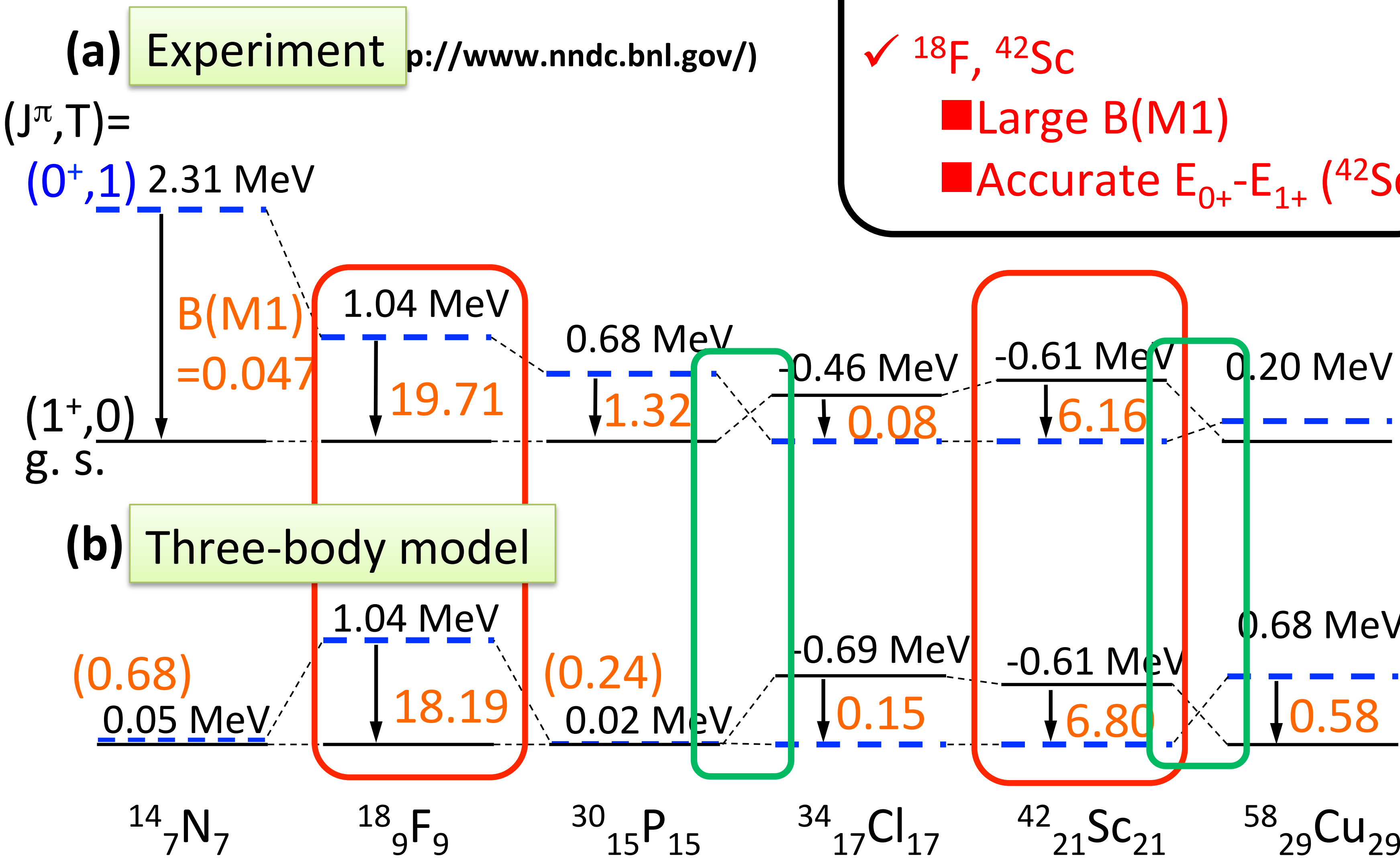
$$a_{pn}^{(s)} = -23.749 \text{ fm and } a_{pn}^{(t)} = 5.424 \text{ fm}$$

$E_{cut}=k_{cut}^2/2m$

Y. Tanimura, HS, K. Hagino, PTEP 053D02 (2014)

1. E_{0+}-E_{1+} and B(M1)

- ✓ Inversion of 1^+ and 0^+
- ✓ ^{18}F, ^{42}Sc
  - Large B(M1)
  - Accurate E_{0+}-E_{1+} (^{42}Sc)



The inversion of 1+ and 0+ shows a clear manifestation of the competition between spin-orbit and the spin-triplet pairing.

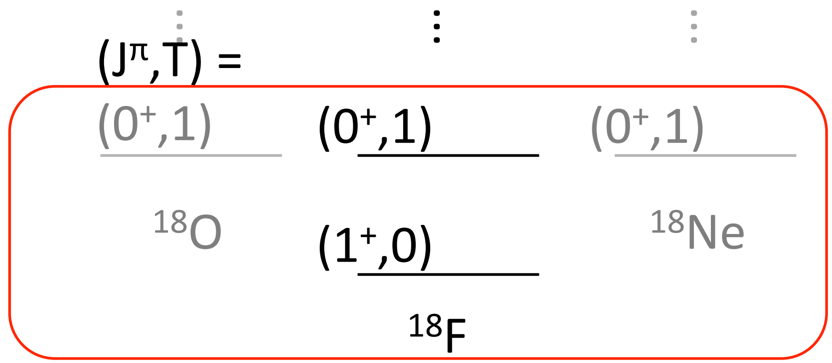
Results

Large B(M1) in ^{18}F and ^{42}Sc

^{18}F :

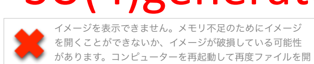
$$\left\{\begin{array}{l} 1^+ \rightarrow P(S=1) = 90.1\%,\ (1d)^2 \\ 0^+ \rightarrow P(S=0) = 82.2\%,\ (1d)^2 \end{array}\right.$$

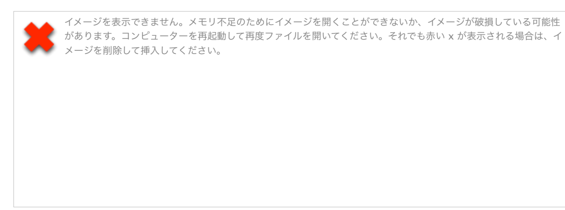
1⁺ and 0⁺ can be considered
as the states in the same SU(4) multiplets
(LST) = (0,1,0), (0,0,1)
The same as 42Sc in 1f-orbits



SU(4) multiplet

$$\begin{aligned} O(\mathrm{M1}) &\propto \sum_i [g_s(i)\boldsymbol{s}(i) + g_\ell(i)\boldsymbol{\ell}(i)] \\ &= \underbrace{g_s^{IV}}_{\text{Large}} \underbrace{\boxed{\sum_i \tau_3(i)\boldsymbol{s}(i)}}_{\text{SU(4)generator}} + \underbrace{g_s^{IS}}_{\text{(small)}} \sum_i \boldsymbol{s}(i) + \sum_i g_\ell(i)\boldsymbol{\ell}(i) \end{aligned}$$





results

^{18}F and ^{42}Sc: large B(M1)

Separate Contribution to <f||O(M1)||i> (μ_N)

	^{14}N	^{18}F	^{30}P	^{34}Cl	^{42}Sc	^{58}Cu
Valence orbital	p1/2	d5/2	s1/2	d3/2	f7/2	p3/2
orbital	1.09	**1.28**	0.21	2.28	**2.91**	0.09
$g_s^{IV}\sum_i \tau_3(i)\boldsymbol{s}(i)$	-2.78	<u>7.44</u>	-1.21	-3.65	<u>6.34</u>	<u>1.47</u>
$g_s^{IS}\sum_i \boldsymbol{s}(i)$	5x10^{-5}	3x10^{-3}	3x10^{-5}	-1x10^{-4}	2x10^{-3}	-2x10^{-3}
B(M1)↓ (μ_N^2) Exp.	0.047	19.71	1.32	0.08	6.16	---
Calc.	0.68	**18.19**	0.24	0.15	**6.80**	0.58

✓ (j=l-1/2)^2 spin and orbital are cancelled
(Lisetskiy et al., PRC60, 064310 ('99))

⇨ ^{14}N, ^{34}Cl B(M1) **small**

✓ (j=l+1/2)^2 spin and orbital coherent
(Lisetskiy et al., PRC60, 064310 ('99))

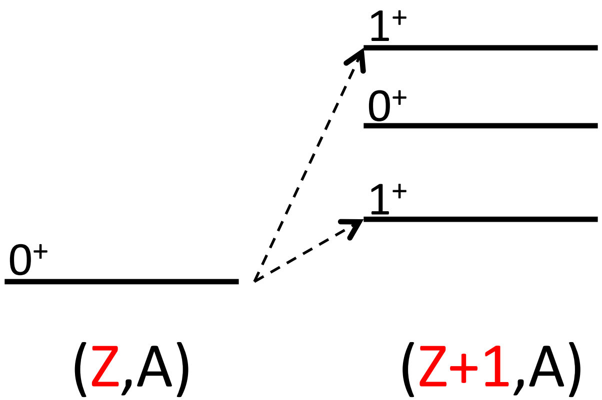
✓ **<u>good SU(4) symmetry</u>**

⇨ ^{18}F, ^{42}Sc B(M1)**large**

✓ even j=l+1/2 not good **<u>SU(4) symmetry</u>**

⇨ ^{58}Cuは(M1)**small**

2. Gamow-Teller B(GT)



$$O(\mathrm{GT}) \propto \sum_i \tau_-(i)\boldsymbol{\sigma}(i)$$

A=18, 42 : strong transition to 1^+_1
→good SU(4) symmetry

A=58 : a weak B(GT) to 1^+_1
→no SU(4) symmetry

※Halse and Barrett, Ann. Phys. (N. Y.) 192, 204 ('89). Consistent results

$^{18}\mathrm{O} \rightarrow {}^{18}\mathrm{F}$			
E_x (MeV)		$B(GT)$ $(g_A^2/4\pi)$	
cal.	(exp.)	cal.	(exp.)
0.0	(0.0)	2.48	(3.11± 0.03)
4.79	(—)	0.028	(—)
6.87	(—)	0.036	(—)
$^{42}\mathrm{Ca} \rightarrow {}^{42}\mathrm{Sc}$			
E_x (MeV)		$B(GT)$ $(g_A^2/4\pi)$	
cal.	(exp.)	cal.	(exp.)
0.61	(0.61)	1.80	(2.16 ± 0.05)
—	(1.89)	—	(0.09 ± 0.01)
3.71	(3.69)	0.346	(0.15 ± 0.02)
$^{58}\mathrm{Ni} \rightarrow {}^{58}\mathrm{Cu}$			
E_x (MeV)		$B(GT)$ $(g_A^2/4\pi)$	
cal.	(exp.)	cal.	(exp.)
0.0	(0.0)	0.097	(0.155 ± 0.01)
1.24	(1.05)	0.74	(0.32 ± 0.03)

D.R.Tilley et al, NPA595, 1 ('95)
T. Kurtukian et al, PRC80, 035502('09)
Y. Fujita et al, EPJ A 13, 411 ('02)
Y. Fujita, private communications

Cooperation of T=0 and T=1 pairing in Gamow-Teller states in N=Z nuclei

C. L. Bai, H.S., M.Sasano, T. Uesaka, K. Hagino, H.Q. Zhang, X.Z. Zhang,F.R.Xu

Phys. Lett. B719, pp. 116-121 (2013)

HFB+QRPA with T=1 and T=0 pairing
T=1 pairing in HFB
T=0 pairing in QRPA
How large is the spin-triplet T=0 pairing?

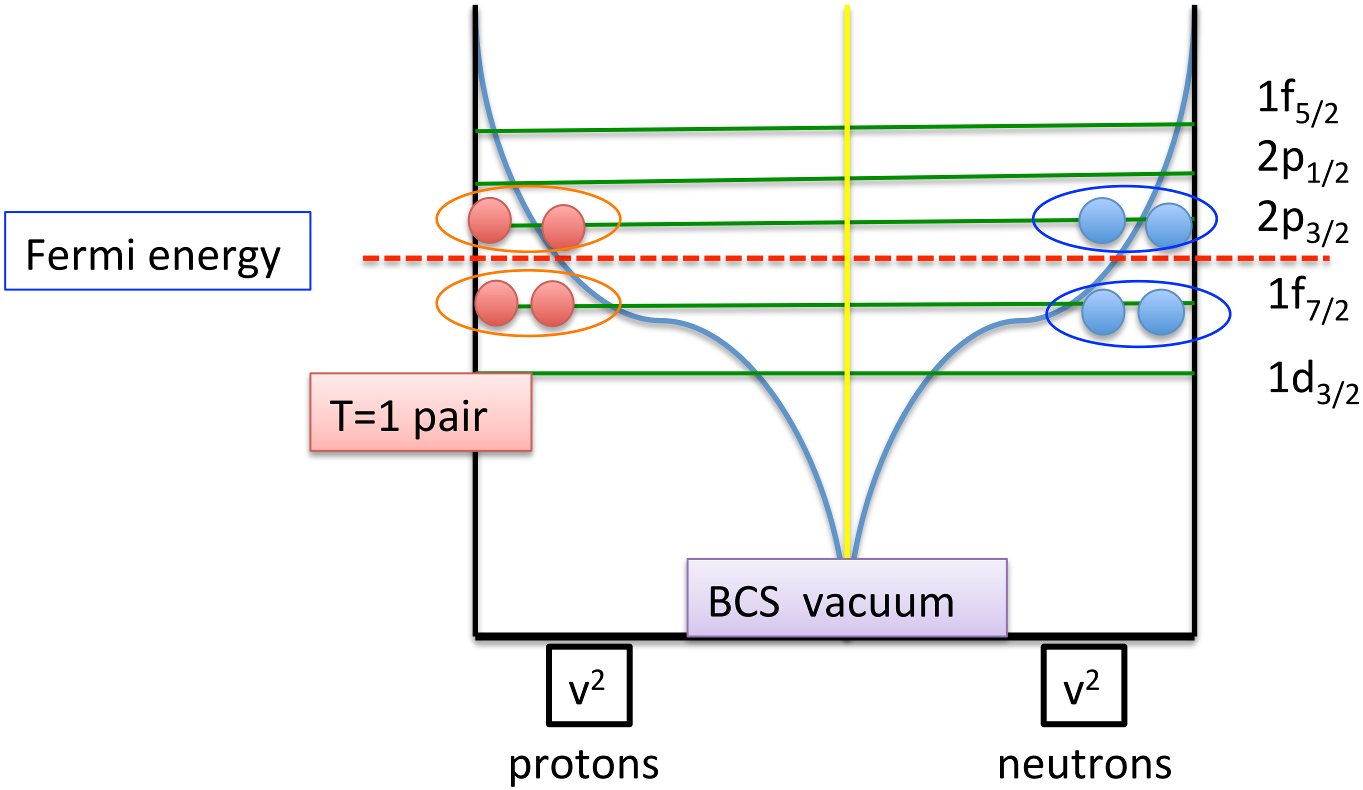
$$V_{T=1}(\mathbf{r}_1,\mathbf{r}_2) = V_0\frac{1-P_\sigma}{2}\left(1-\frac{\rho(\mathbf{r})}{\rho_o}\right)\delta(\mathbf{r}_1-\mathbf{r}_2), \quad (1)$$

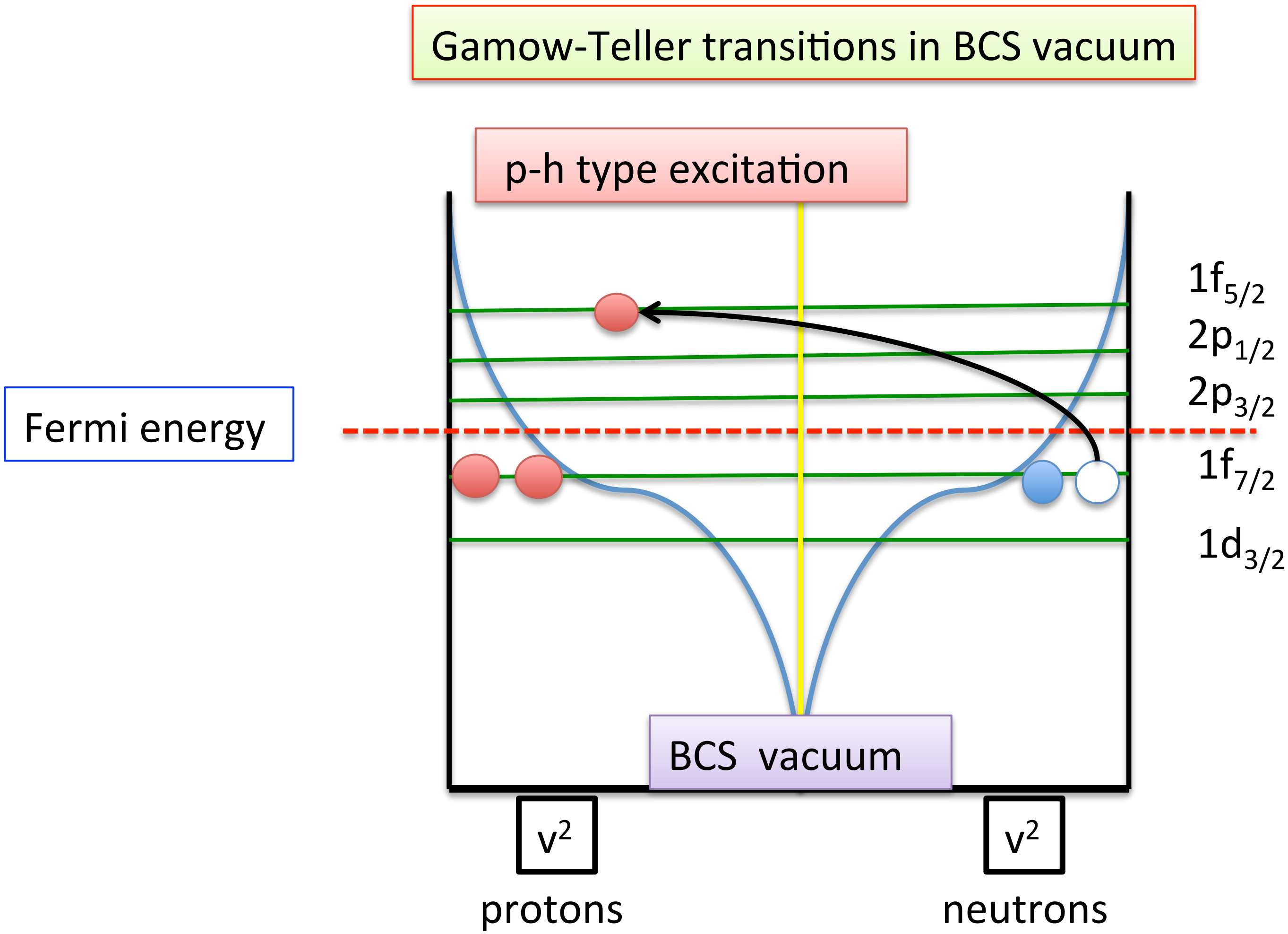
$$V_{T=0}(\mathbf{r}_1,\mathbf{r}_2) = fV_0\frac{1+P_\sigma}{2}\left(1-\frac{\rho(\mathbf{r})}{\rho_o}\right)\delta(\mathbf{r}_1-\mathbf{r}_2), \quad (2)$$

As a possible manifestation of T=0 S=1 pairing correlations in nuclei N=Z.

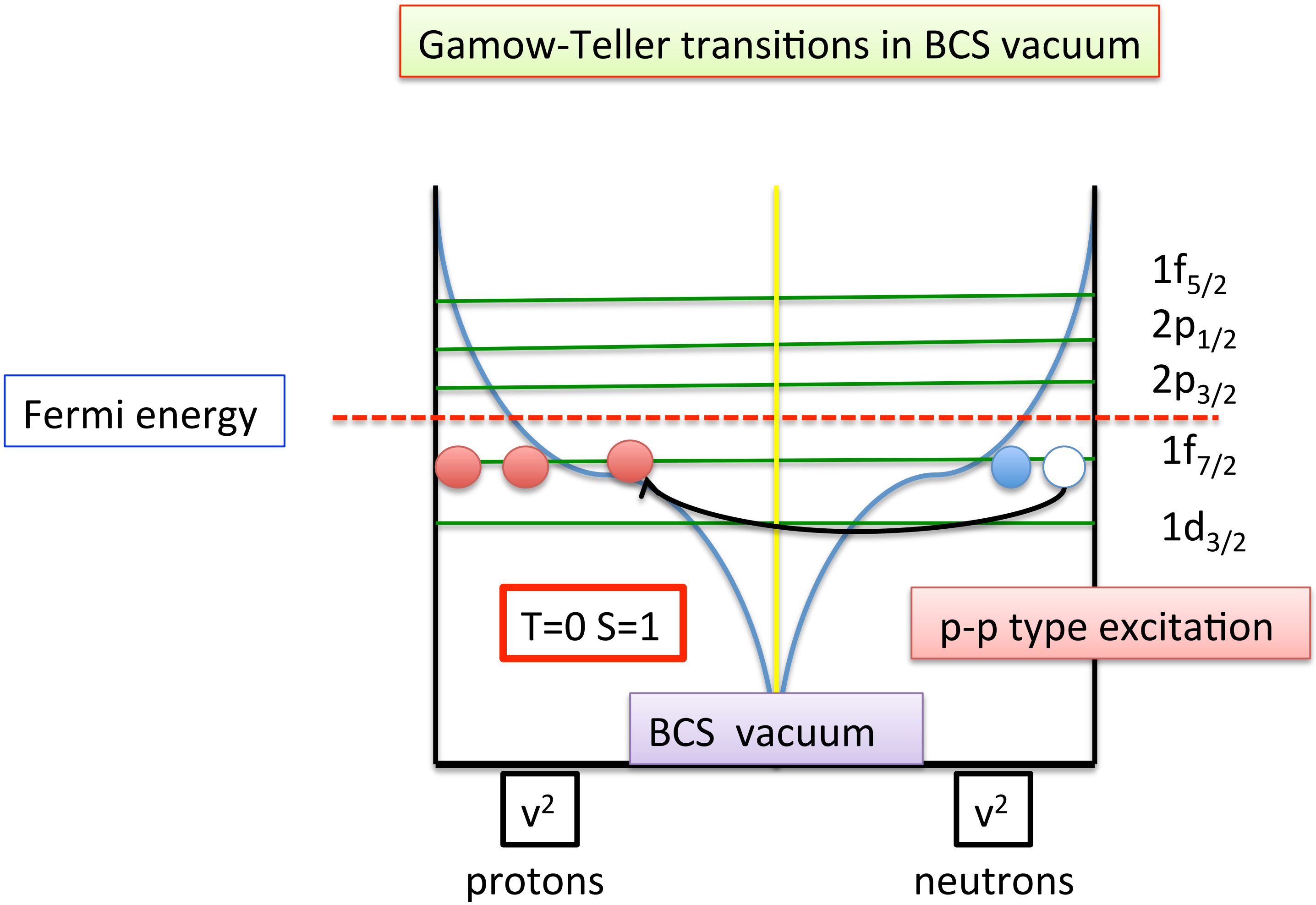
$$\hat{O}(GT) = \sigma\tau_{\pm}$$

Gamow-Teller transitions from BCS vacuum in N=Z nuclei

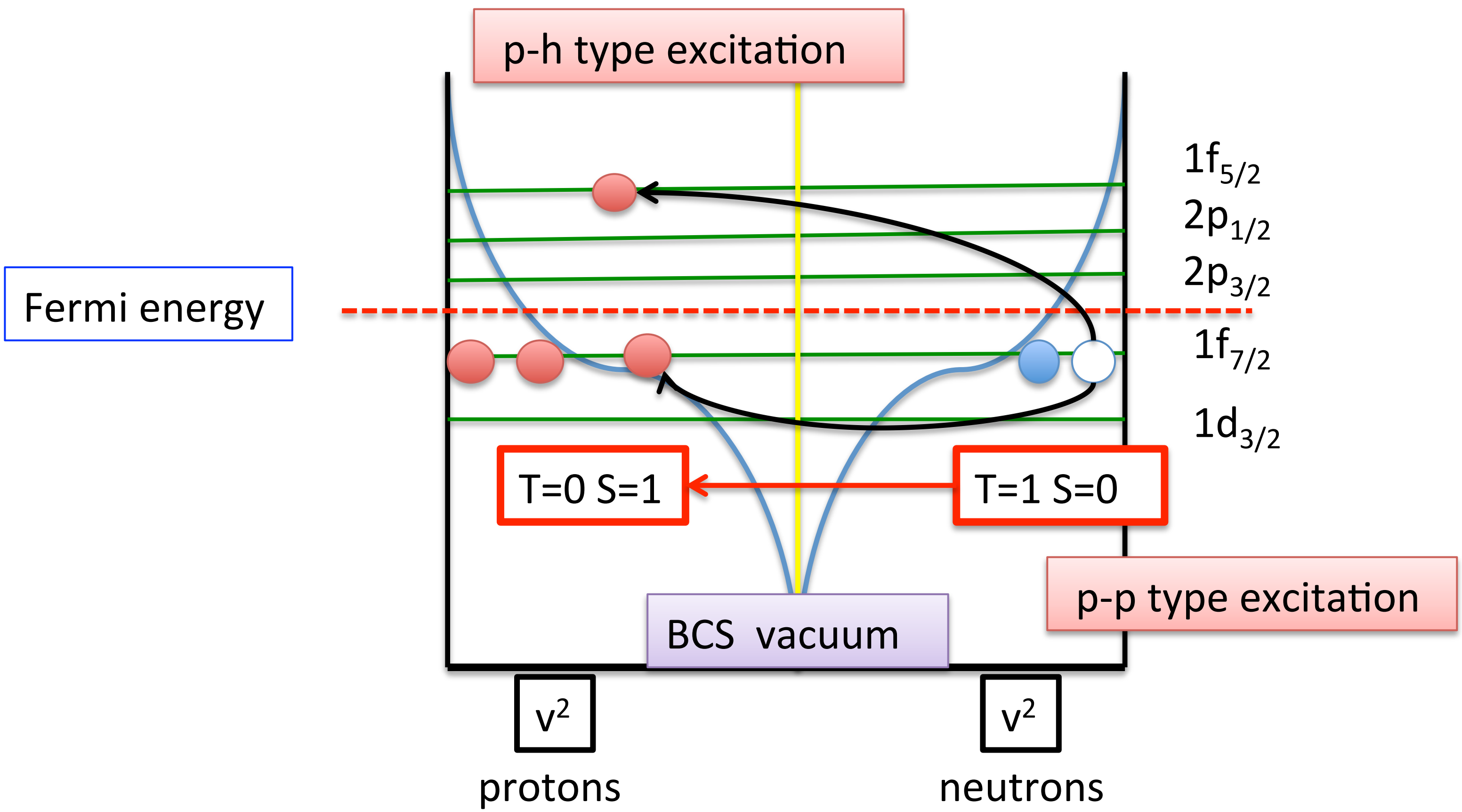




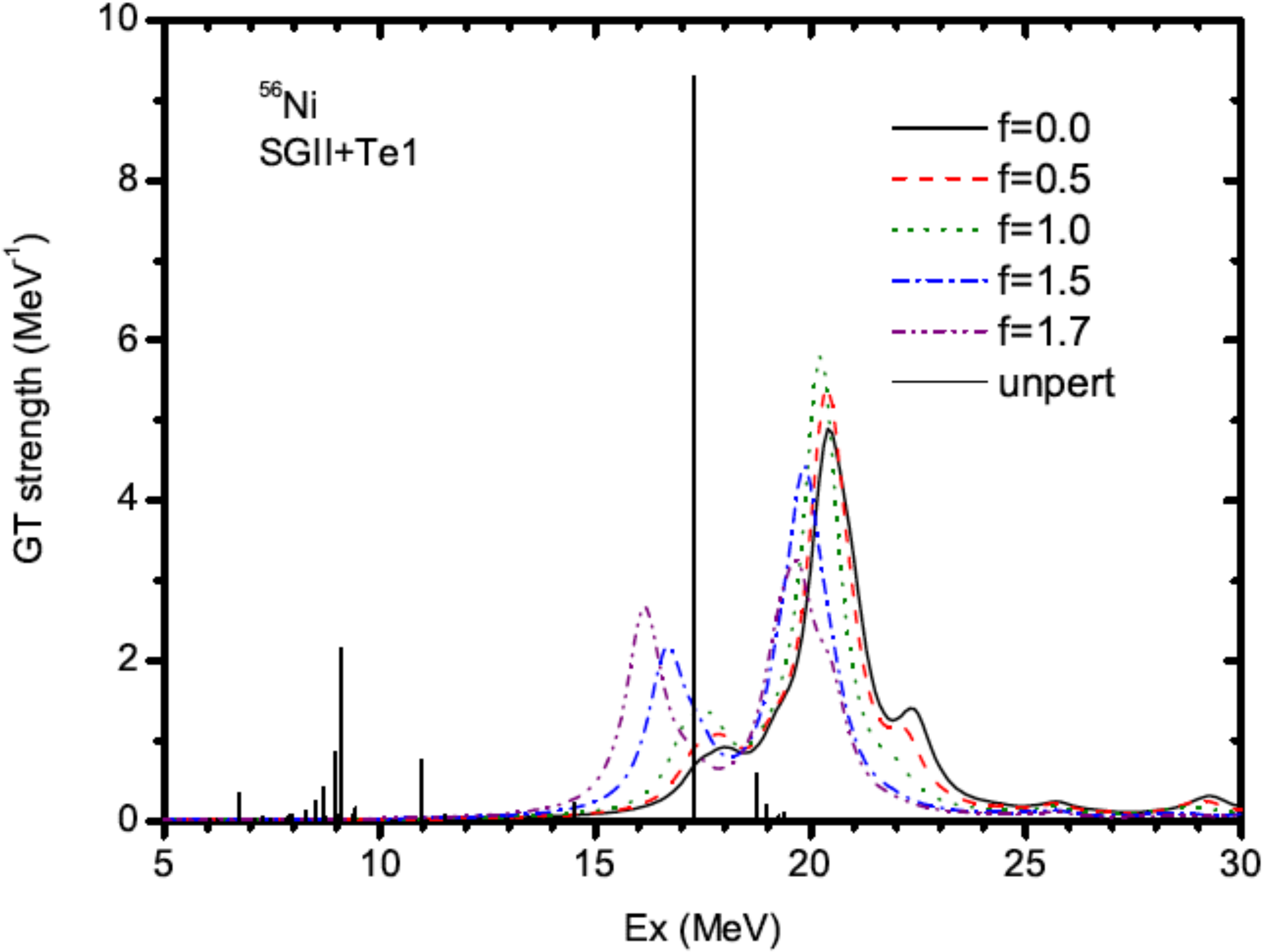
Gamow-Teller transitions in BCS vacuum
p-h type excitation
Fermi energy
1f5/2
2p1/2
2p3/2
1f7/2
1d3/2
BCS vacuum
v2
v2
protons
neutrons


Gamow-Teller transitions in BCS vacuum
Fermi energy
$1f_{5/2}$
$2p_{1/2}$
$2p_{3/2}$
$1f_{7/2}$
$1d_{3/2}$
T=0 S=1
p-p type excitation
BCS vacuum
v^2
v^2
protons
neutrons

Gamow-Teller transitions in BCS vacuum



A pair of SU(4) supermultiplet



^{56}Ni
SGII+Te1
f=0.0
f=0.5
f=1.0
f=1.5
f=1.7
unpert
GT strength (MeV^{-1})
Ex (MeV)
0
2
4
6
8
10
5
10
15
20
25
30

TABLE I: Amplitudes of main (np) particle-hole and particle-particle type configurations of GT states in ^{56}Ni. The QRPA calculations are performed without and with the T=0 pairing interaction in the cases of $f = 0$ and $f = 1.5$, respectively. The Skyrme interaction T21 is used for HF and p-h matrix calculations. The abbreviations B and C correspond to the GT reduced matrix element B=$(X u_\pi v_\nu - Y u_\nu v_\pi)\langle \pi || \hat{O}(\mathrm{GT}) || \nu \rangle$ and the normalization factor C=$X^2 - Y^2$, respectively, where X,Y are QRPA amplitudes and $\hat{O}$(GT) is GT transition operator in Eq. (3).

^{56}Ni		$f = 0$			
E_x (MeV)	B(GT)	$\nu(v_\nu^2)$	$\pi(v_\pi^2)$	B	C
17.5	1.41	$2p_{3/2}(0.20)$	$2p_{3/2}(0.21)$	0.358	0.127
		$1f_{7/2}(0.69)$	$1f_{5/2}(0.09)$	−0.229	0.006
		$1f_{7/2}(0.69)$	$1f_{7/2}(0.67)$	1.341	0.802
18.3	1.49	$2p_{1/2}(0.10)$	$2p_{3/2}(0.21)$	0.260	0.153
		$2p_{3/2}(0.20)$	$2p_{1/2}(0.11)$	0.846	0.740
21.3	7.88	$1f_{7/2}(0.69)$	$1f_{5/2}(0.09)$	2.48	0.742
S_-(GT)=18.28					
^{56}Ni		$f = 1.5$			
E_x (MeV)	B(GT)	$\nu(v_\nu^2)$	$\pi(v_\pi^2)$	B	C
16.6	4.82	$2p_{3/2}(0.20)$	$2p_{1/2}(0.11)$	−0.203	0.049
		$2p_{3/2}(0.20)$	$2p_{3/2}(0.21)$	−0.682	0.491
		$1f_{7/2}(0.69)$	$1f_{5/2}(0.09)$	−0.237	0.007
		$1f_{7/2}(0.69)$	$1f_{7/2}(0.67)$	−0.790	0.339
20.5	4.10	$1f_{7/2}(0.69)$	$1f_{5/2}(0.09)$	−1.900	0.437
S_-(GT)=14.75					

$$B = (Xu_\pi v_\nu - Yu_\nu v_\pi)\left\langle \pi \middle\| \hat{O}(GT) \middle\| \nu \right\rangle$$

$$C = X^2 - Y^2$$

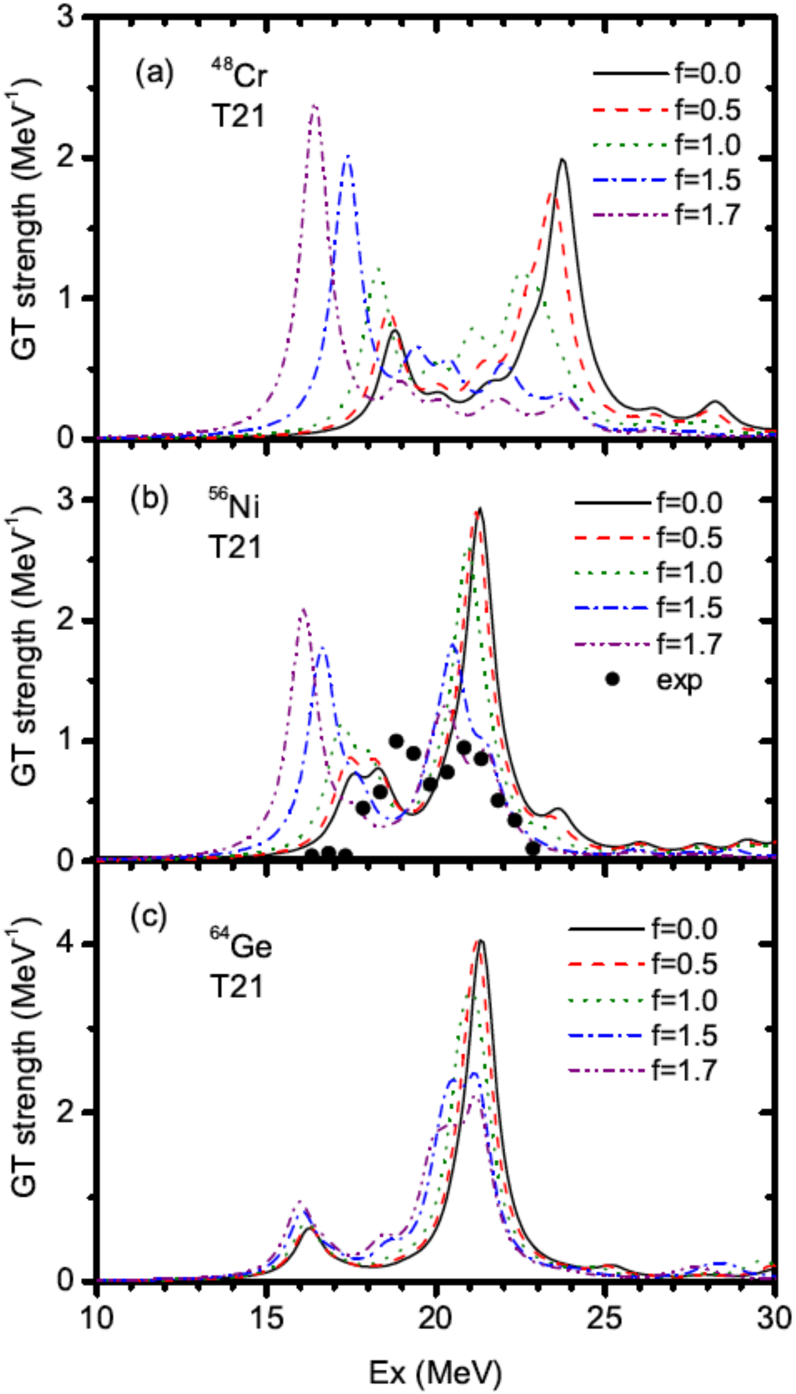
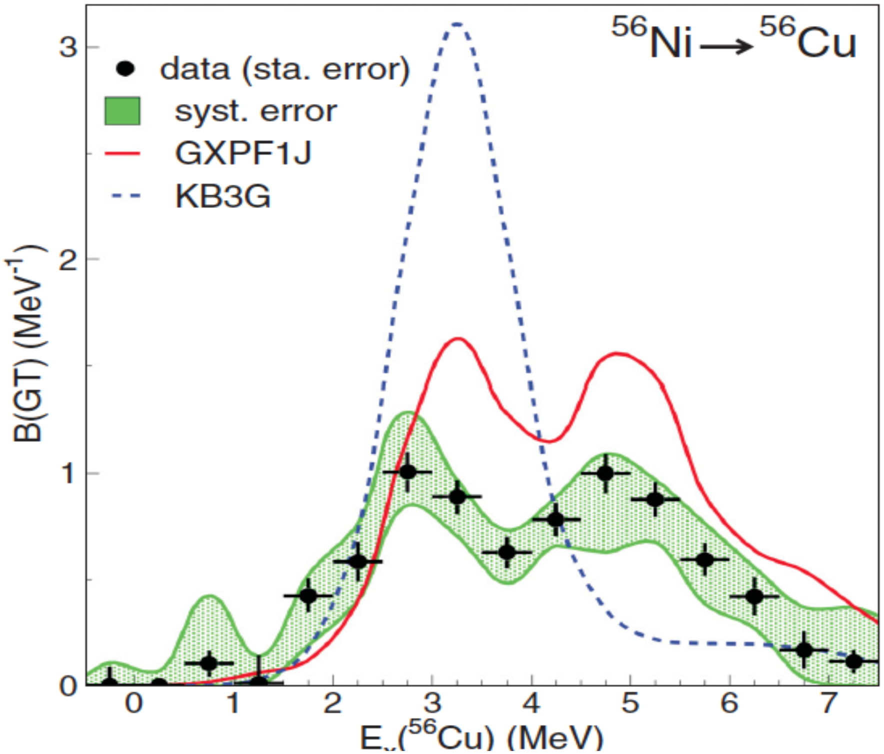
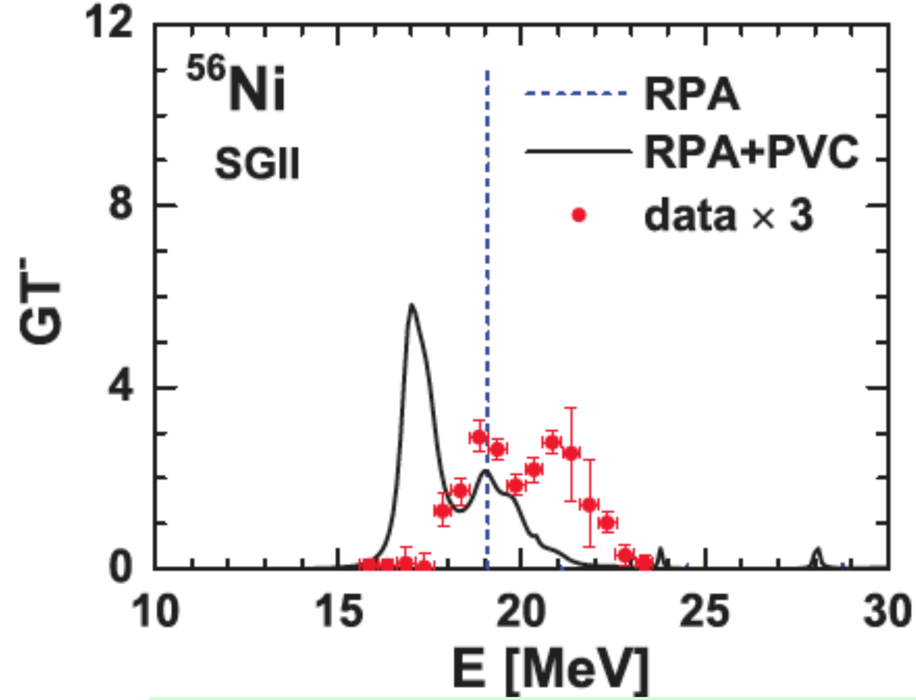


TABLE II: Same as Table I, but for ^{48}Cr and ^{64}Ge with the T=0 pairing interaction $f = 1.5$.

^{48}Cr		$f = 1.5$			
E_x (MeV)	B(GT)	$\nu(v_\nu^2)$	$\pi(v_\pi^2)$	B	C
17.4	5.68	$2p_{3/2}(0.12)$	$2p_{3/2}(0.12)$	0.186	0.062
		$1d_{3/2}(0.85)$	$1d_{3/2}(0.84)$	0.251	0.204
		$1f_{7/2}(0.33)$	$1f_{5/2}(0.07)$	0.268	0.021
		$1f_{7/2}(0.33)$	$1f_{7/2}(0.32)$	1.04	0.558
19.4	1.19	$2p_{3/2}(0.12)$	$2p_{1/2}(0.07)$	0.215	0.081
		$2p_{3/2}(0.12)$	$2p_{3/2}(0.12)$	0.559	0.461
		$1d_{3/2}(0.85)$	$1d_{3/2}(0.84)$	−0.217	0.142
		$1f_{7/2}(0.33)$	$1f_{5/2}(0.07)$	0.383	0.038
		$1f_{7/2}(0.33)$	$1f_{7/2}(0.32)$	−0.384	0.054
S_-(GT)=11.77					
^{64}Ge		$f = 1.5$			
E_x (MeV)	B(GT)	$\nu(v_\nu^2)$	$\pi(v_\pi^2)$	B	C
16.1	2.15	$2p_{3/2}(0.48)$	$2p_{1/2}(0.21)$	−1.316	0.889
21.2	5.21	$1f_{5/2}(0.14)$	$1f_{7/2}(0.90)$	−0.187	0.353
		$1f_{7/2}(0.92)$	$1f_{5/2}(0.15)$	2.392	0.561
S_-(GT)=17.26					

Fine adjustment of shell model int.





Beyond mean field effect:
Niu et al., PRC85, 034314(2012)

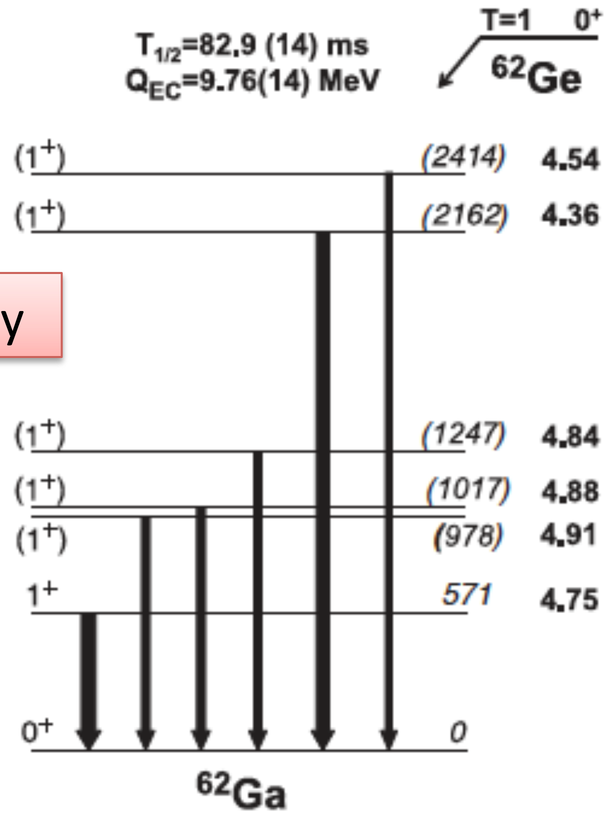
Sasano et al., PRC86, 034324(2012)

With $(0.74)^2$ quenching factor, still 30%missing strength

	E_x (MeV)		
^{56}Ni → ^{56}Cu	0–7	3.8 ± 0.2(stat.) ± 0.8(syst.)	5.5 (5.2)
^{55}Co → ^{55}Ni(g.s.)	0	0.267 (from Ref. [30])	0.209 (0.253)
^{55}Co → ^{55}Ni	0–15	5.3 ± 0.5(stat.)$^{+2.5}_{-1.5}$(syst.)	6.8 (6.2)

[a]The quoted error margins do not include the uncertainty in the value for the unit cross section (15%), which would change all strengths by a common scaling factor.
[b]A quenching factor of $(0.74)^2$ [44] has been applied to the shell-model summed strengths.

$^{62}Ge \rightarrow {}^{62}Ga$ β decay



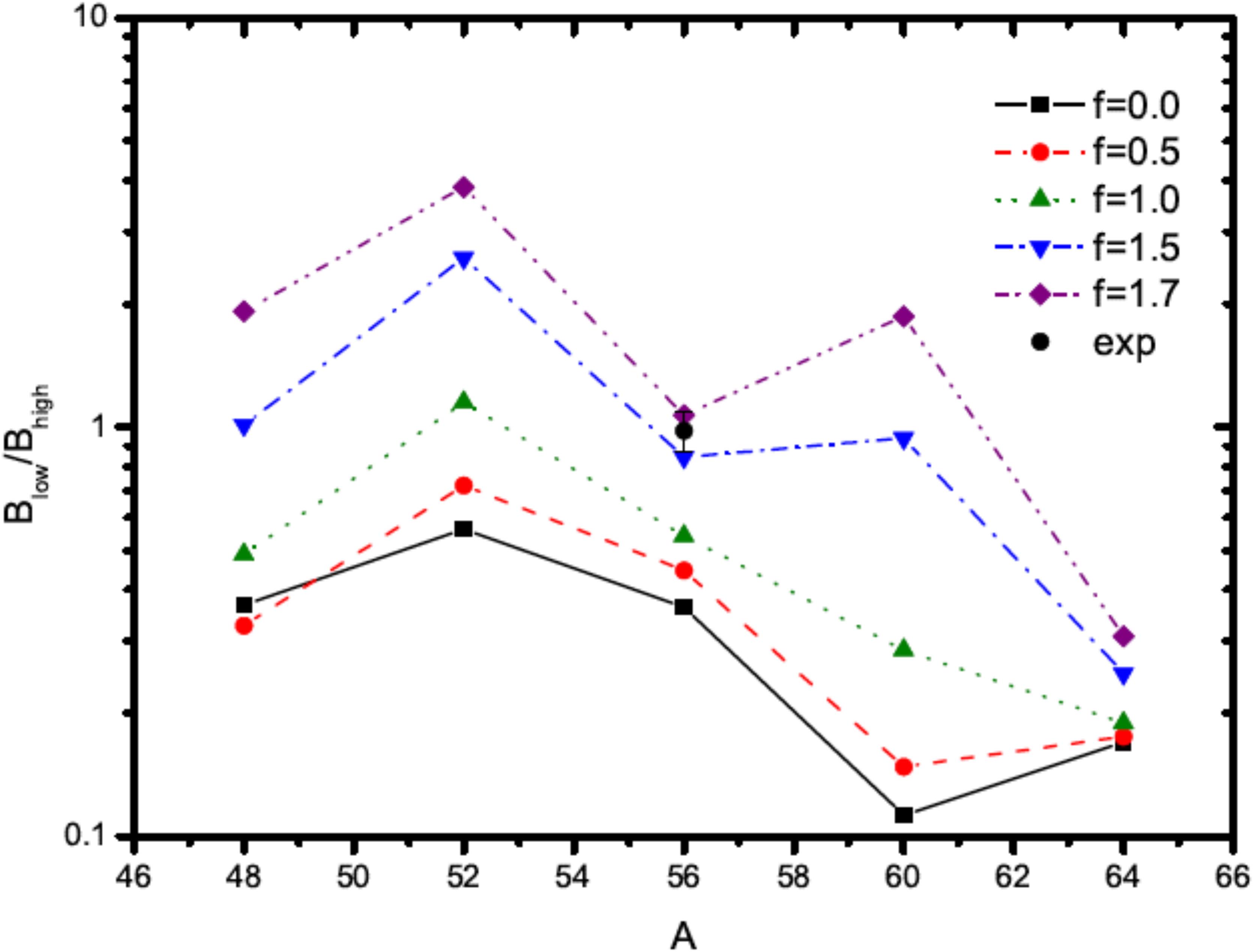
Log ft	B(GT)
4.75 ± 0.15	$0.070^{+0.017}_{-0.017}$
4.91 ± 0.15	$0.050^{+0.015}_{-0.017}$
4.88 ± 0.15	$0.054^{+0.013}_{-0.019}$
4.84 ± 0.13	$0.059^{+0.016}_{-0.022}$
4.36 ± 0.17	$0.17^{+0.05}_{-0.05}$
4.54 ± 0.17	$0.12^{+0.03}_{-0.05}$

Sum of B(GT)=0.52

FIG. 3. ^{62}Ga level scheme observed in the ^{62}Ge β decay built under the assumption that the populated (1^+) states will deexcite preferentially to the ground state. The excitation energies of the levels are in keV. The log ft values are indicated in the right side of the levels in bold characters.

GSI RISING experiment, E. Grodner et al., PRL113, 092501(2014)

No strong GT transitions in $^{62}_{32}Ge \rightarrow {}^{62}_{31}Ga$ β decay which is consistent with our picture of collectivity and np-pairing.



10
1
0.1
B_{low}/B_{high}
46
48
50
52
54
56
58
60
62
64
66
A
f=0.0
f=0.5
f=1.0
f=1.5
f=1.7
exp

Summary: N=Z nucleus

1. Inversion of 1+ and 0+ states in the energy spectra and strong M1 transitions in odd-odd N=Z nuclei is induced by a strong T=0 pairing correlations competing with T=1 pairing and spin-orbit force.

2. Cooperative role of T=0 and T=1 pairings is studied in Gamow-Teller transitions of N=Z nuclei
3. It is pointed out that the low energy peak appear due to the strong T=0 pairing correlations in the final states.
   Supermultiplets of T=1,S=0 and T=0 and S=1 pair
4. Energy difference of two peaks in ^{56}Ni➔ smaller spin-orbit splitting
5. Future perspective (experiment):
   New experiments in N=Z nuclei, ^{48}Cr, was approved by PAC in RIKEN .
   Further experiment in ^{64}Ge.

Theory

a. Further study of Particle-vibration coupling:
   Yifei Niu, Gianluca Colo
b. Fine fittings of energy density functions
   for RPA and QRPA
   (which was done already for Shell model interactions:
   Toshio Suzuki, Michio Honma