Existence of rotational states in the continuum

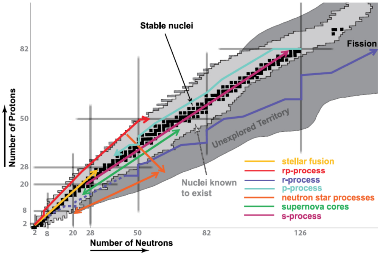
Kévin FOSSEZ

NSCL/MSU

October 29, 2015

Introduction

Next generation of RIB facilities:



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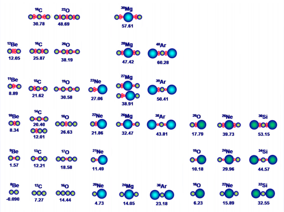

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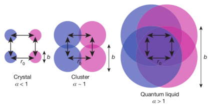

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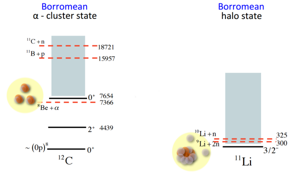

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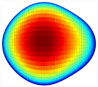

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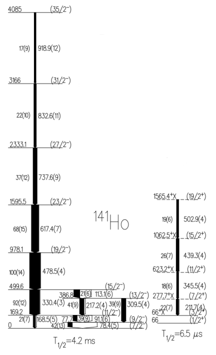

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Introduction

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FRIB

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GANIL spiral2

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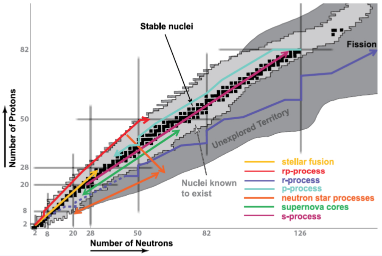

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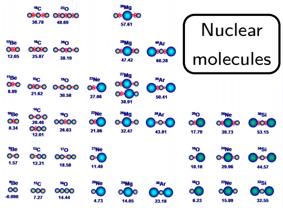
Nuclear molecules



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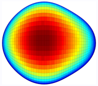

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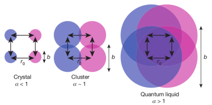

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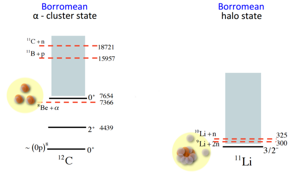

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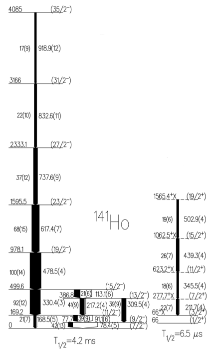

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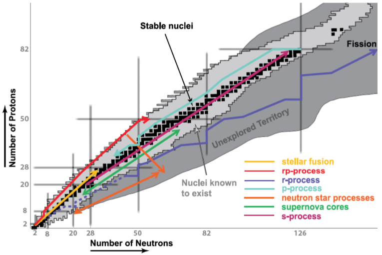

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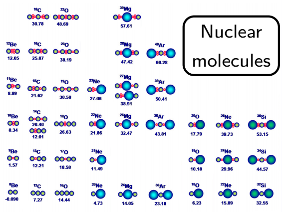

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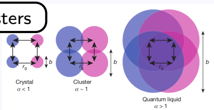

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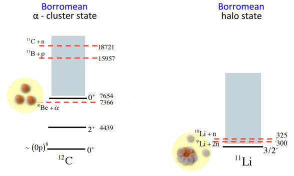

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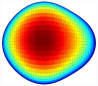

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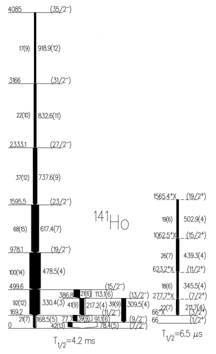

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INTRODUCTION

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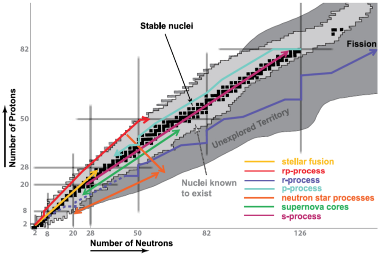

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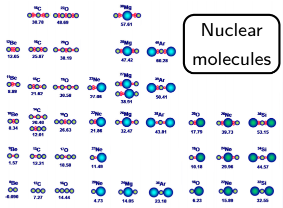

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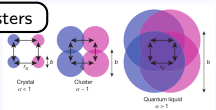

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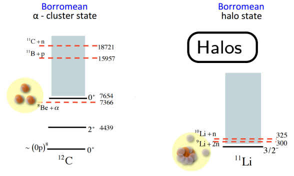
Halos



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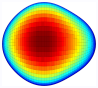

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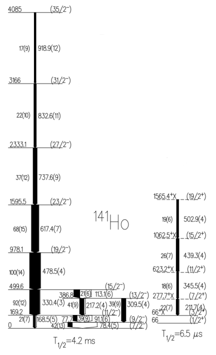

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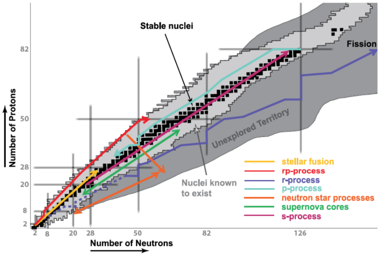

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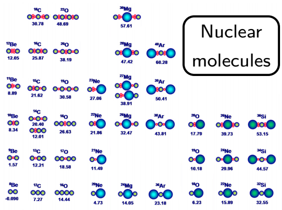

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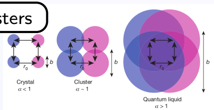

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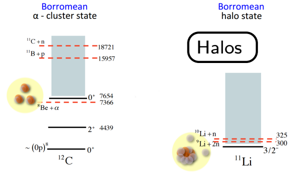

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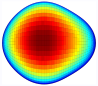
Deformations



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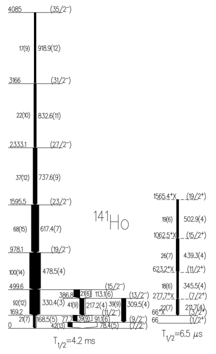

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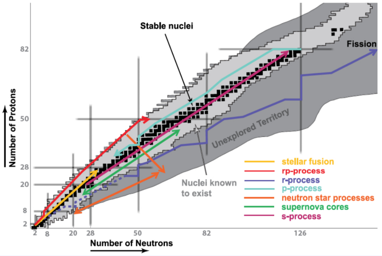

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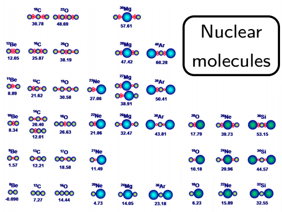

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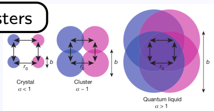

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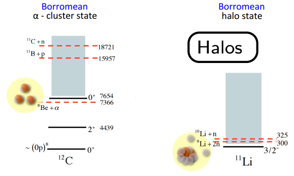

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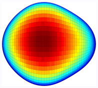

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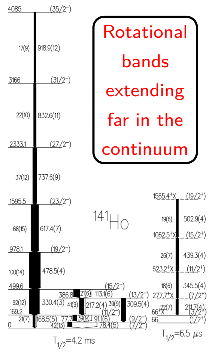
Rotational bands extending far in the continuum



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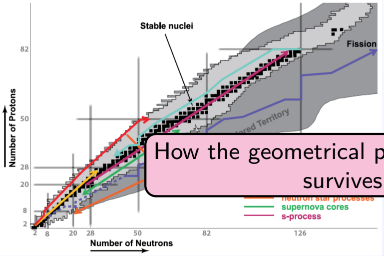

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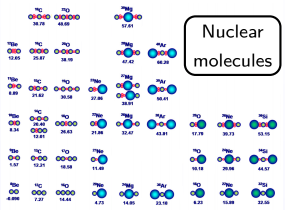

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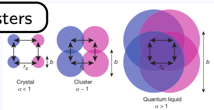

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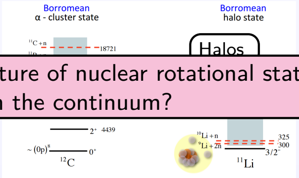

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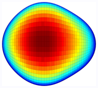

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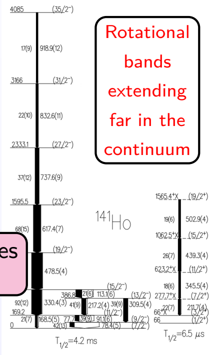

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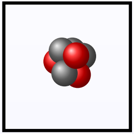
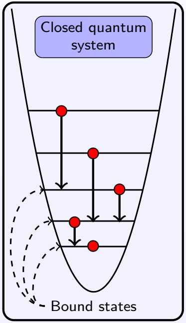
How the geometrical picture of nuclear rotational states survives in the continuum?

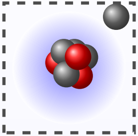
Open quantum systems (OQSs): What are they?

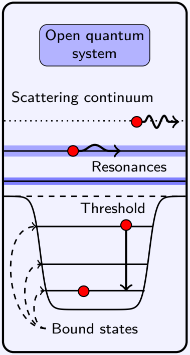
→ **Quantum systems coupled to the environment of scattering states and decay channels.**





- Examples of OQSs in many domains of physics: hadrons, nuclei, atoms, molecules, quantum dots, microwave cavities.
- Exotic phenomena in OQSs: superradiance phenomena, spontaneous two-proton radioactivity, near-threshold clustering phenomena...
- General properties of OQSs (resonances, halos, exceptional points) are common to all mesoscopic systems.





Rotational states

→ **What "makes" rotational bands?**

- Experimental identification: Energy spectrum regularities, large B($E2$).
- Theoretical interpretation:

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"A clue for understanding the deviations in the nuclear coupling scheme from that of the single-particle model was provided by the fact that many nuclei have quadrupole moments that are more than an order of magnitude larger than could be attributed to a single particle. This finding directly implied a sharing of angular momentum with many particles, and might seem to imply a break-down of the one-particle model. However, essential features of the single-particle model could be retained by assuming that the average nuclear field in which a nucleon moves deviates from spherical symmetry. This picture leads to a nuclear model resembling that of a molecule, in which the nuclear core possesses vibrational and rotational degrees of freedom." *Rotational motion in nuclei, Nobel Lecture, December 11, 1975 Aage Bohr*

Rotational states

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- Experimental identification: Energy spectrum regularities, large B($E2$).
- Theoretical interpretation:
  - ⋆ Deviation from spherical symmetry.
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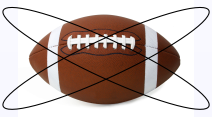
  - ⋆ Single particle properties.
  - ⋆ Intrinsic state.



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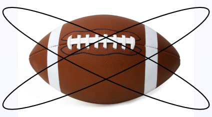

- Additional conditions:
  - ⋆ A nuclear mean-field: $T_{1/2} > T_{\text{s.p.}}$.
  - ⋆ No salient cluster degrees of freedom.
  - ⋆ The ^{8}Be example: $T_{1/2} \approx T_{\text{s.p.}}$.

E. Garrido *et al.*, Phys. Rev. C **88**, 024001 (2013)

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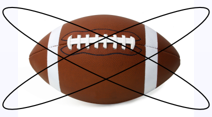

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MeV

$\alpha - \alpha?(4^+)$ — 11.7 (3.5 MeV)

^{8}Be(2^+) — 3.0 (1.5 MeV)

^{8}Be(0^+) — 0.0 (5.5 eV)

Formalism

→ **Quasi-stationary formalism and Rigged Hilbert space.**

- **Gamow states:** discrete solutions of the quasi-stationary Schrödinger equation that are regular at the origin and with outgoing boundary conditions.

  G. Gamow, Z. Physik **51**, 204 (1928); A. F. J. Siegert, Phys. Rev. **56**, 750 (1939)

- Eigenenergies: $E_n = e_n - i\Gamma_n/2$.
- Lifetime: $T_{1/2} = \hbar \ln(2)/\Gamma$

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  Gel'fand, Vilenkin *et al.* (1964), Maurin (1968)
- Rigorous framework for quantum mechanics
  Böhm (1964), Roberts (1966), Antoine (1969) and Melsheimer (1974)
- RHS inner product:

$$\langle \tilde{u}_n | u_n \rangle = \int_0^\infty dr\, \tilde{u}_n^*(r) u_n(r).$$

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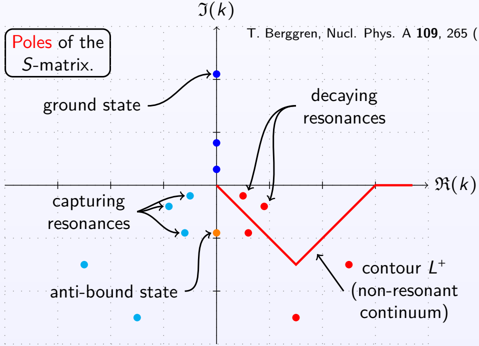
→ **Berggren completeness relation:**

$$\boxed{\sum_{n\in(b,d)} |u_l(k_n)\rangle\langle\tilde{u}_l(k_n)| + \int_{L^+_{l,j}} dk\, |u_l(k)\rangle\langle\tilde{u}_l(k)| = \hat{\mathbb{1}}.}$$



T. Berggren, Nucl. Phys. A **109**, 265 (1968)

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  $$\langle\tilde{u}_n|u_n\rangle = \int_0^\infty dr\, \tilde{u}_n^*(r) u_n(r).$$

Formalism

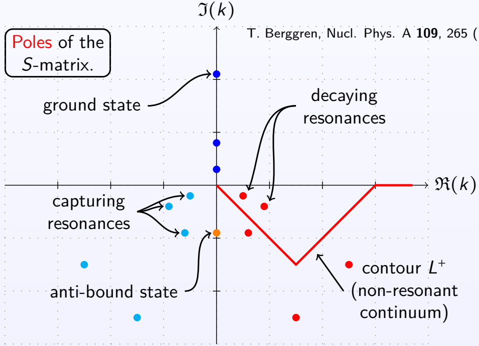
→ **Quasi-stationary formalism and Rigged Hilbert space.**

- **Gamow states:** discrete solutions of the quasi-stationary Schrödinger equation that are regular at the origin and with outgoing boundary conditions.
  G. Gamow, Z. Physik **51**, 204 (1928); A. F. J. Siegert, Phys. Rev. **56**, 750 (1939)

- Eigenenergies:
  $E_n = e_n - i\Gamma_n/2.$
- Lifetime:
  $T_{1/2} = \hbar \ln(2)/\Gamma$

→ **Berggren completeness relation:**

$$\boxed{\sum_{n\in(b,d)} |u_l(k_n)\rangle \langle \tilde{u}_l(k_n)| + \int_{L^+_{l,j}} dk \, |u_l(k)\rangle \langle \tilde{u}_l(k)| = \hat{\mathbb{1}}.}$$



T. Berggren, Nucl. Phys. A **109**, 265 (1968)

- **RHS:** Extension of HS to distribution.
  Gel'fand, Vilenkin *et al.* (1964), Maurin (1968)
- Rigorous framework for quantum mechanics
  Böhm (1964), Roberts (1966), Antoine (1969) and Melsheimer (1974)
- RHS inner product:

$$\langle \tilde{u}_n | u_n \rangle = \int_0^\infty dr \, \tilde{u}_n^*(r) u_n(r).$$

- **In practice:**

→ Exterior complex-scaling.
$\hat{U}_a(\theta)\chi(r) = \chi(r_a + |r - r_a|e^{i\theta})$
if $|r| > r_a$.

A. M. Dykhne *et al.*, Sov. Phys. JETP **13**, 1002 (1961)
B. Gyarmati, Nucl. Phys. A **160**, 523 (1971)
B. Simon, Phys. Lett. A **71**, 211 (1979)

→ Contour discretization.

Coupled channel formalism

→ **Coupled channel formalism:**

- A truncation scheme:

$l_{\max} = 1$

$l_{\max} = 0$

$|l - s| \leq j \leq l + s$

$|J - j| \leq j_r \leq J + j$

Example:

J fixed,

$s = 1/2$,

$\hat{\vec{J}} = \hat{\vec{j}} + \hat{\vec{j}}_r$,

$\hat{\vec{j}} = \hat{\vec{l}} + \hat{\vec{s}}$.

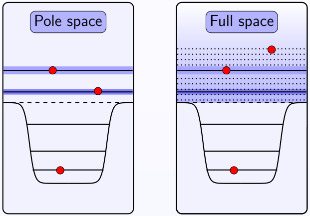
$$|\Psi\rangle = \sum_c |\Psi_c\rangle, \qquad c = (l, j, j_r).$$

- A clear physical interpretation:

$$\sum_c \left(H_{c',c}(r) - E\right) \frac{u_c(r)}{r} = 0.$$

- Expansion of the $u_c(r)$ in the Berggren basis, generated by the diagonal part of the potential $V_{cc}(r)$.

- Overlap method to identify resonances.



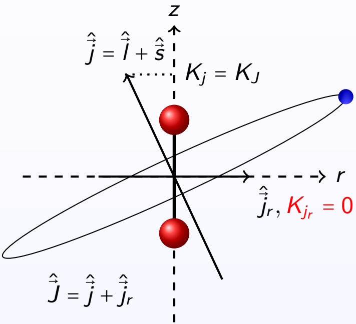
$$\langle \Psi_n^{(\text{pole})} | \Psi_n^{(\text{full})} \rangle \approx 70\% - 90\%.$$

- Hermitian Hamiltonian but complex-symmetric matrix $A = A^{\mathrm{T}}$.

- Davidson diagonalization.

Model

→ **System: particle-plus-rotor.**



- Just the minimum:

$$\hat{H} = \frac{\hat{\vec{p}}^2}{2m} + \frac{\hat{\vec{j}}_r^2}{2I} + \hat{V}.$$

- Adiabatic limit: $I \to \infty$.

  ⇒ Same intrinsic state.

- How to study the rotational structure?

  ⋆ Density in the rotor frame:

$$\hat{R}(\Omega)\,|\Psi\rangle = \sum_{K_J} |\Psi_{K_J}\rangle_\Omega,$$

$$\rho_{J,K_J}(\vec{r}) = \int d\Omega_\Omega \, \langle \Psi_{K_J} | \hat{\rho}_{\vec{r}} | \Psi_{K_J} \rangle_\Omega,$$

$$\rho_J(\vec{r}) = \sum_{K_J} \rho_{J,K_J}(\vec{r}).$$

  ⋆ Weights of K_J components:

$$n_{J,K_J} = \int d^3\vec{r}\, \rho_{J,K_J}(\vec{r}),$$

  ⋆ Test in the adiabatic limit: $I \to \infty$

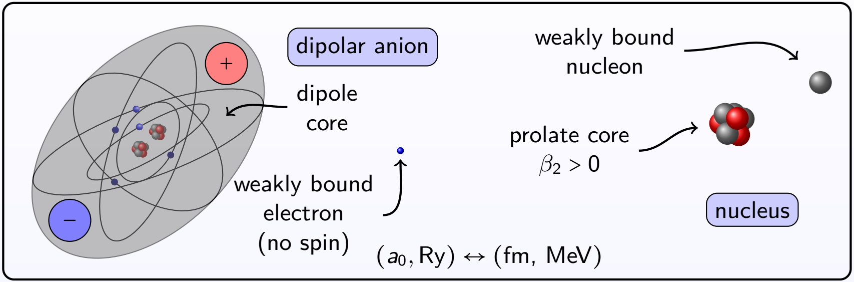
$$E_J, E_{J+1}, E_{J+2}, \cdots \to E^{(I\to\infty)}.$$

$$\rho_J, \rho_{J+1}, \rho_{J+2}, \cdots \to \rho^{(I\to\infty)}.$$

1. Introduction
2. Formalism
3. **Applications**
4. Conclusion
5. Outlook

Rotational degrees of freedom and continuum

→ **Digression: A scary insight from a molecular open quantum system.**



- A unique quantum system with few bound states.
- Very limited literature for resonances.
- At large distances: no analytical asymptotic solution for finite l with $V_{cc'} \propto -1/r^2$.
- Effective potential $V(r, \theta)$.
- Realistic case: HCN^-.

W. R. Garrett, *J. Chem. Phys.* **133**, 224103 (2010)

→ The worst case scenario for rotational states in the continuum.

Rotational degrees of freedom and continuum

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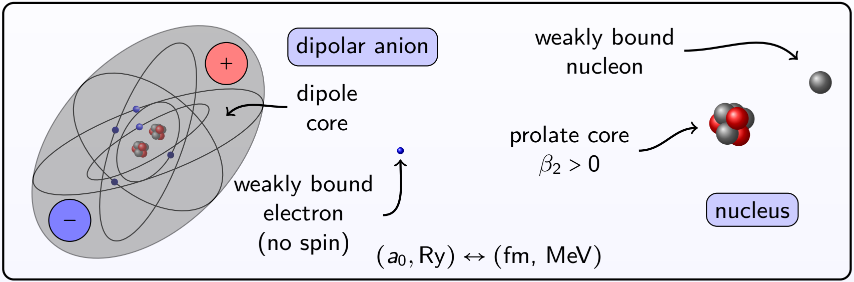

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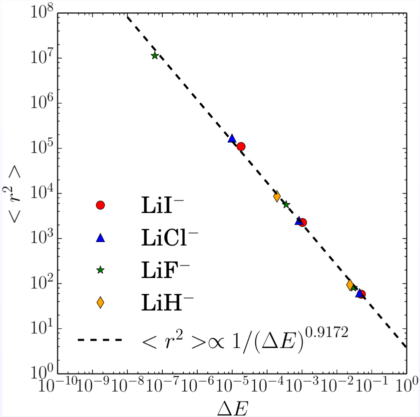
- *Phys.* **133**, 224103 (2010)

$E_{rot} \approx E_{s.p.} \Rightarrow$ Strong nonadiabatic couplings.
→ Competition between threshold effects and rotations.

→ The worst case scenario for rotational states in the continuum.

Dipolar anions

→ **An extreme halo system.**



- For a relative angular momentum:
  - ⋆ $l = 0 :< r^2 >$ diverges as $1/\Delta E$.
  - ⋆ $l = 1 :< r^2 >$ diverges as $1/\sqrt{\Delta E}$.
  - ⋆ $l > 1 :< r^2 >=$ constant.

K. Riisager *et al.*, Nucl. Phys. A **548**, 393 (1992)

K. Riisager *et al.*, *Europhys. Lett.* **49**, 547 (2000)

- Scaling properties of two-body systems (bound states).

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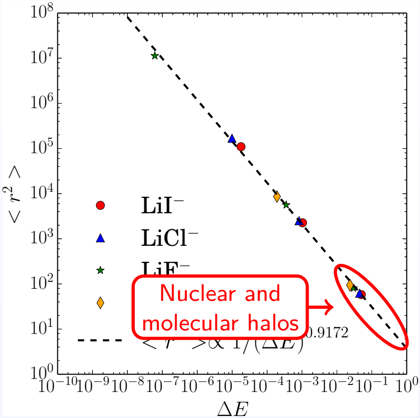

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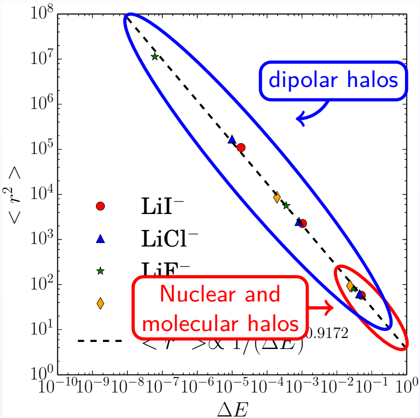

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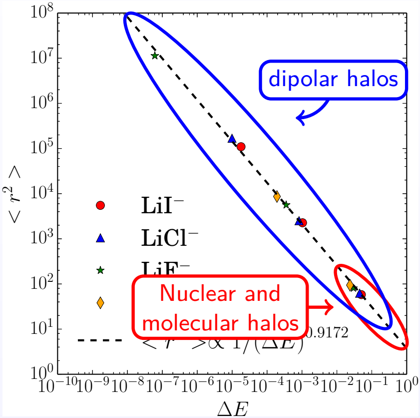

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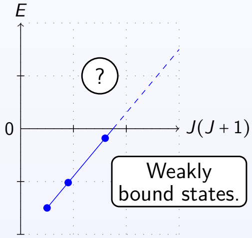

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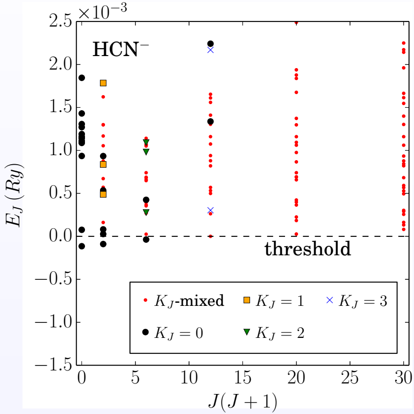
Dipolar anions

→ **Rotational states.**



$$\rho_J(\vec{r}) = \sum_{K_J} \rho_{J,K_J}(\vec{r}).$$

- Intrinsic density: all K_J-components except one vanish.
- No rotational states above the threshold.
- What is happening in the continuum?
  - ⋆ Bound states: low-ℓ channels (0,1).
  - ⋆ Resonances: high-ℓ channels (6-8).
  - ⋆ Groups of resonances in the complex-energy plane.
  - ⋆ In each group, same dominant ℓ, but... $j_r = 0, 2, 4, 6, 8, \ldots$

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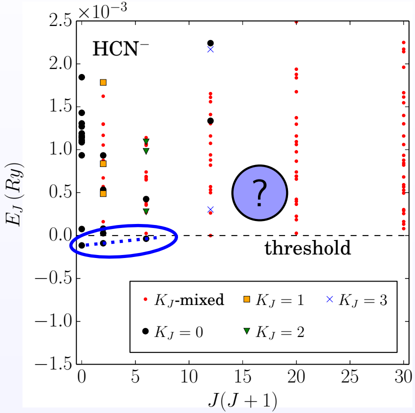

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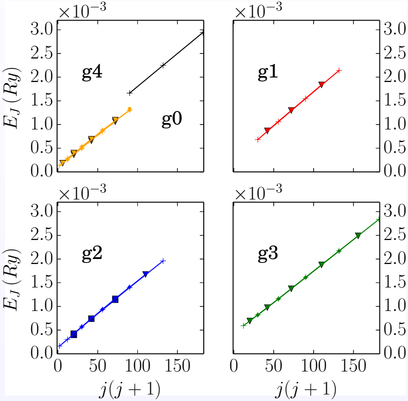
Dipolar anions

→ **Competition between threshold effects and rotation.**



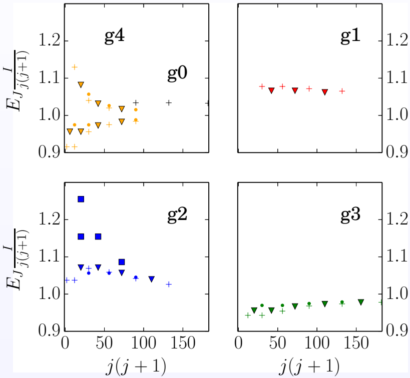
- Collective bands.

- Above the threshold: weak coupling of the rotational motion of the dipole and the valence electron.

(Decoupled motions)

Dipolar anions

→ **Competition between threshold effects and rotation.**



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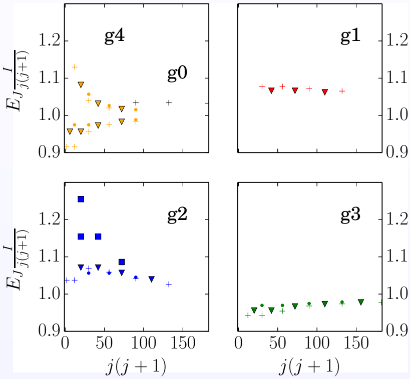

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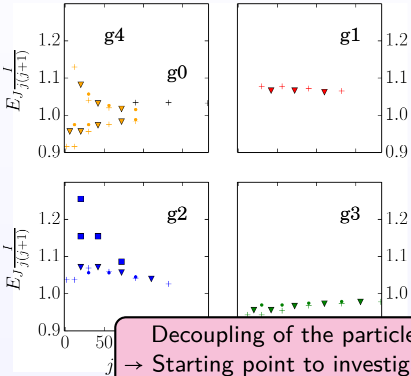

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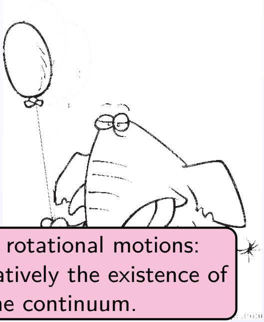


Decoupling of the particle and core rotational motions:
→ Starting point to investigate qualitatively the existence of nuclear rotational states in the continuum.

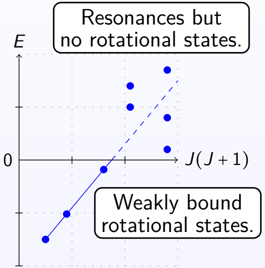
- Deviation to the rigid rotor reference.

(Decoupled motions)

From molecular to nuclear physics

→ **Competition between threshold effects and rotation.**

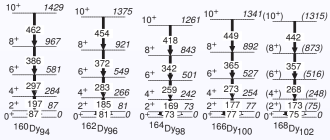
- In the worst case scenario, sharp transition between two coupling regimes at the threshold.



- Feature not observed in nuclear systems.

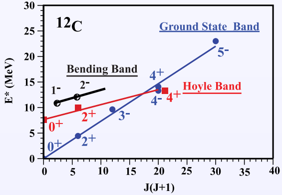
- On the one hand...

P. -A. Söderström *et al.*, Phys. Rev. C **81**, 034310 (2010)



- On the other hand...

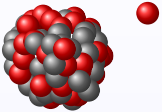
D. J. Marín-Lámbarri *et al.*, Phys. Rev. Lett. **113**, 012502 (2014)

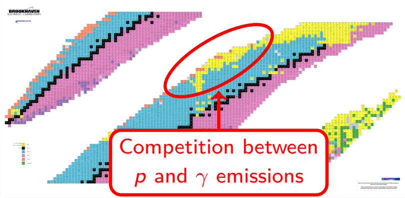


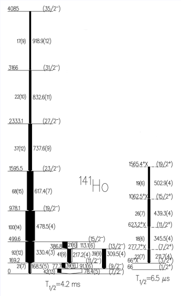
From proton- to neutron-rich systems

→ **What about nuclear systems?**

^{141}Ho (^{140}Dy + p)

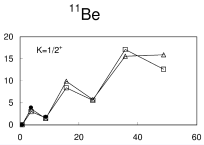






- Non-adiabatic couplings like in dipolar anions.
- Spin-orbite interaction cannot be neglected.
- Short-range interaction (∼ Woods-Saxon potential).

- Coulomb barrier in p-rich nuclei.
- Neutron resonances more intriguing.
- Example: ^{11}Be = ^{10}Be + n (halo).

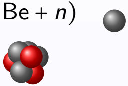


P. Descouvemont, Nucl. Phys. A **699**, 463 (2002)

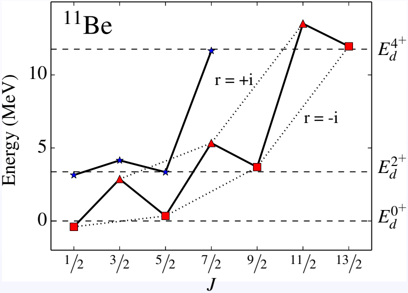
The ^{11}Be and ^{141}Ho cases

→ **Nuclear rotational bands:** (preliminary)

^{11}Be (^{10}Be + n)



- A qualitative study, $K_J = 1/2$.



- Moment of inertia adjusted to exp. data when available.
- Partial waves: $\ell = 0, 2, 4$ and 6.
- Fit using POUNDerS. (http://www.mcs.anl.gov/tao)

	d (fm)	R_0 (fm)	β_2	V_0 (MeV)	V_{so} (MeV)
^{11}Be	0.7721	2.548	0.5184	-52.95	12.70

- Yrast and yrare bands.
- Eigenenergies collapse to the same value when $I \to \infty$.

- Two tools:

$\star$ $n_{J,K_J} = \int d^3\vec{r}\, \rho_{J,K_J}(\vec{r})$.

$\star$ $n_{l,j,j_r} = \int_0^\infty dr\, u^2_{l,j,j_r}(r)$.

Rotational structure

→ **Rotational structure.**

- Similar and dominant $K_J = 1/2$ densities in the favored band ($e^{-i\pi J} = -i$).

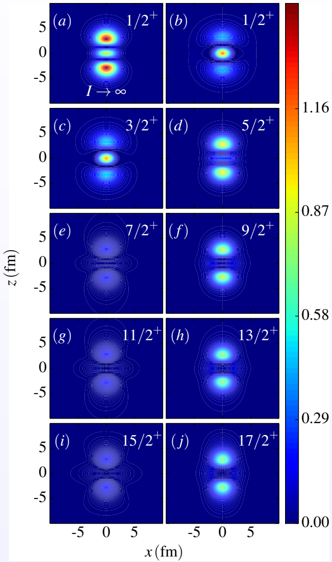
- Same densities when $I \to \infty$ (100% $K_J = 1/2$).

J^π, K_J	1/2	3/2	5/2
$1/2^+$	100		
$3/2^+$	82	18	
$5/2^+$	39	27	34
$7/2^+$	7	36	57
$9/2^+$	48	34	18
$11/2^+$	8	43	48
$13/2^+$	49	33	14
$15/2^+$	9	46	43
$17/2^+$	50	33	13

- $n_{J,K_J>5/2} < 3\%$

- K_J mixing in most cases.

- Dominant $K_J = 5/2$ and 3/2 in the $11/2^+$ and $15/2^+$ states, respectively.



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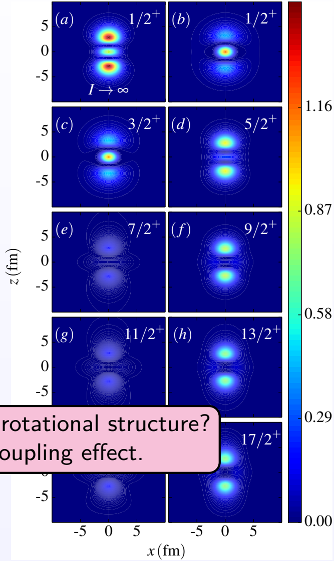
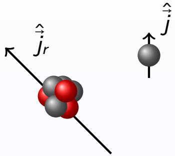


- Domina … and $15/2^+$ states, respectively.

How to explain these observations on the rotational structure?
→ Connection with the Coriolis decoupling effect.

The Coriolis decoupling

→ **Dominant channel analysis and Coriolis decoupling.**



- Coriolis effect through non-adiabatic couplings.
- Channel decomposition:

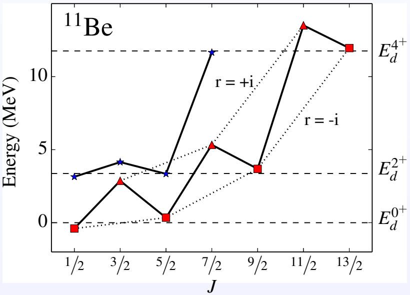
$$|\Psi\rangle = \sum_c |\Psi_c\rangle\,, \qquad c = (l, j, j_r).$$

- The Coriolis decoupling favors the alignment of $\vec{\hat{j}}$ and $\vec{\hat{j}}_r$.

- Effect on widths:

J^π	Γ (MeV)
$1/2_1^+$	0.000
$3/2_1^+$	0.095
$5/2_1^+$	0.009
$7/2_1^+$	0.587
$9/2_1^+$	0.009
$11/2_1^+$	0.458
$13/2_1^+$	0.002
$15/2_1^+$	0.394
$17/2_1^+$	0.000

- Greatly reduced widths in the favored band.
- $l = 0$ channels cannot contribute to decay widths.



- Decay widths dominated by $l = 2$ channels.

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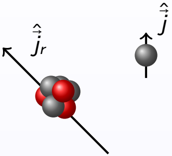

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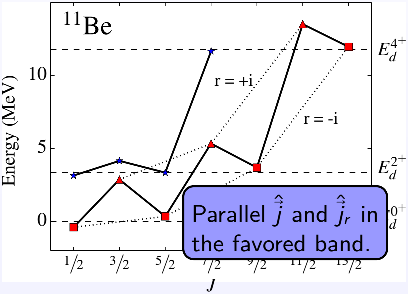
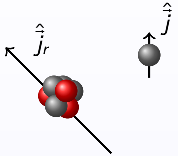


Parallel $\vec{\hat{j}}$ and $\vec{\hat{j}}_r$ in the favored band.

- Decay widths dominated by $l = 2$ channels.

The Coriolis decoupling

→ **Dominant channel analysis and Coriolis decoupling.**



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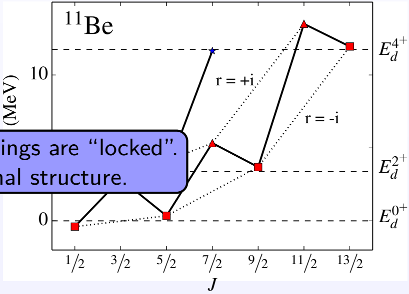
$15/2_1^+$	0.394
$17/2_1^+$	0.000

- Greatly reduced widths in the favored band.
- $l = 0$ channels cannot contribute

- The Coriolis decoupling favors the alignment of $\hat{\vec{j}}$ and $\hat{\vec{j}}_r$.

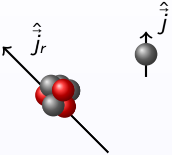


Angular momentum couplings are “locked”.
→ Affect the rotational structure.

- Decay widths dominated by $l = 2$ channels.

The Coriolis decoupling

→ **Dominant channel analysis and Coriolis decoupling.**



- Coriolis effect through non-adiabatic couplings.
- Channel decomposition:

$$|\Psi\rangle = \sum_c |\Psi_c\rangle \,, \qquad c = (l, j, j_r).$$

- Effect on widths:

J^π	Γ (MeV)
$1/2^+_1$	0.000
$3/2^+_1$	0.095
$5/2^+_1$	0.009
$7/2^+_1$	0.587
$9/2^+_1$	0.009
$11/2^+_1$	0.458
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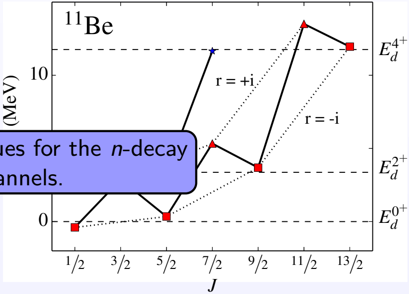

Consequence: small Q-values for the n-decay in the $l = 2$ channels.

- Decay widths dominated by $l = 2$ channels.

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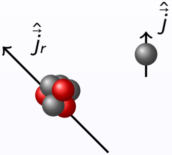

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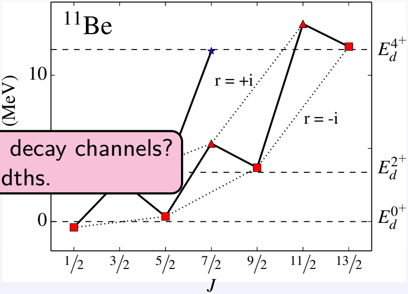


How to show the prefered decay channels?
→ Channel widths.

- Decay widths dominated by $l = 2$ channels.

The decay channels

→ **Partial wave contributions and channel widths.**

- Partial waves contributions:

$$n_{l,j} = \sum_{j_r} n_{l,j,j_r},$$

$$\sum_{l,j} n_{l,j} = 1.$$

- Channel widths:

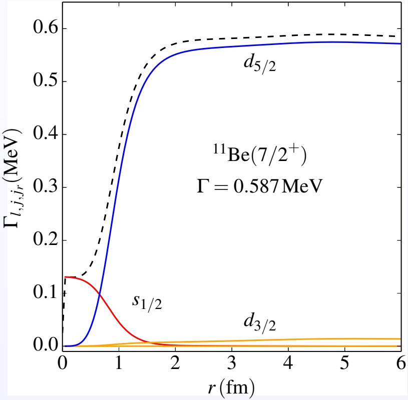
$$\Gamma_c(r) = -\frac{\hbar^2}{\mu} \frac{\mathrm{Im}[u_c'(r) u_c^*(r)]}{\sum_{c'} \int_0^r dr' \, |u_{c'}(r')|^2}$$

with $\Gamma = \sum_c \Gamma_c(r)$.

J. Humblet *et al.*, Nucl. Phys. **26**, 529 (1961)

B. Barmore *et al.*, Phys. Rev. C **62**, 054315 (2000)

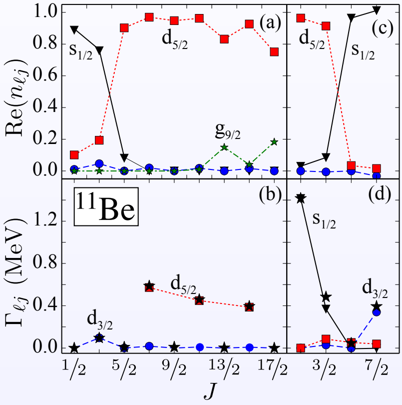
- Example:



The decay channels

→ **Partial wave contributions and channel widths.**

- Yrast states:
  - ⋆ Alignment parttern governed by a transition from $s_{1/2}$ to $d_{5/2}$ partial waves.
  - ⋆ Decay *via* $s_{1/2}$ partial waves is blocked.
  - ⋆ Small Q-value of n-decay *via* $d_{5/2}$ waves.
  - ⋆ Weak coupling for $J \leq 7/2$
  - ⋆ Contribution of $g_{9/2}$ for $J > 11/2$.
  - ⋆ Increased centrifugal barrier.
- Yrare states:
  - ⋆ Opposite situation.
  - ⋆ Width explodes for $J > 7/2$.



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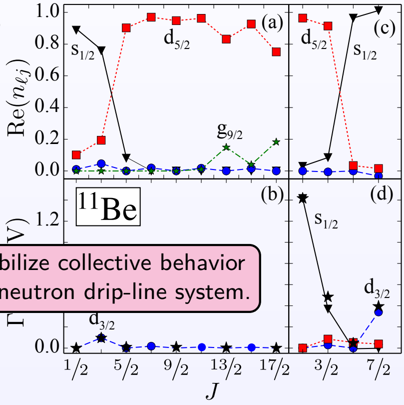

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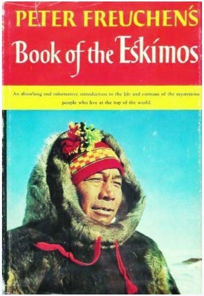
  - ⋆ Width expl

Angular momentum can stabilize collective behavior in highly excited states of a neutron drip-line system.

Cultural interlude

D. Graeber, *Debt: The First 5,000 Years*,
from the Danish writer Peter Freuchen's *Book of the Eskimo*:

Freuchen tells how one day, after coming home hungry from an unsuccessful walrus-hunting expedition, he found one of the successful hunters dropping off several hundred pounds of meat. He thanked him profusely. The man objected indignantly:



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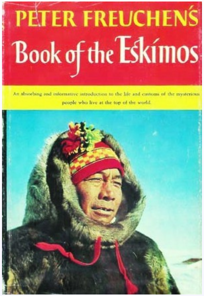

"*Up in our country we are human!*" said the hunter. "*And since we are human we help each other. We don't like to hear anybody say thanks for that. What I get today you may get tomorrow. Up here we say that by gifts one makes slaves and by whips one makes dogs.*"

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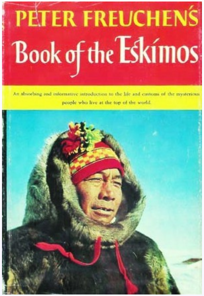

"*Up in our country we are human!*" said the hunter. "*And since we are human we help each* ... *anks* ... *row. Up here we say that by gifts one makes slaves and by whips one makes dogs.*"

You have about 5 min before the end of this talk.
→ Choose carefully if you still want to thank the speaker.

Rotational bands in the continuum

→ **Conclusion:**

- Coupled-channel formalism and the Berggren basis:
  1) The particle continuum is fully accounted for.
  2) The Coriolis effect appears naturally.
  3) Exact treatment of channel-channel couplings.
  4) Dominant channel analysis and clear interpretation.
  5) Density in the rotor frame.
  6) Test in the adiabatic limit.

- Limits:
  1) Pauli principle partially respected (deformation).
  2) Core width neglected.

- Results (^{11}Be):
  1) Strong Coriolis decoupling that align particle and core angular momenta.
  2) Increasing of the centrifugal barrier.
  3) Blocking of low-l channels.

Rotational bands in the continuum

→ **Conclusion:**

- The Coriolis decoupling and centrifugal forces act in concert to decrease decay widths of excited states.
- Narrow collective states can exist at high excitation energy in weakly bound neutron drip-line nuclei such as ^{11}Be.

→ Justifies the geometrical picture in such cases.
→ Support the applicability of bound state approaches.

- Open question:
  → Is a broad N-body nuclear resonance ($\Gamma \approx 3.5\,\mathrm{MeV}$) a N-body nucleus?

Thank you for your attention!