



# effects of clustering in dipole and monopole transitions

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## 1. Introduction

## 2. Monopole excitations in $^{16}\text{O}$ and $^{12}\text{C}$

Y. K-E. PRC89, 024302 (2014)

## 3. Dipole excitations in $^{9,10}\text{Be}$

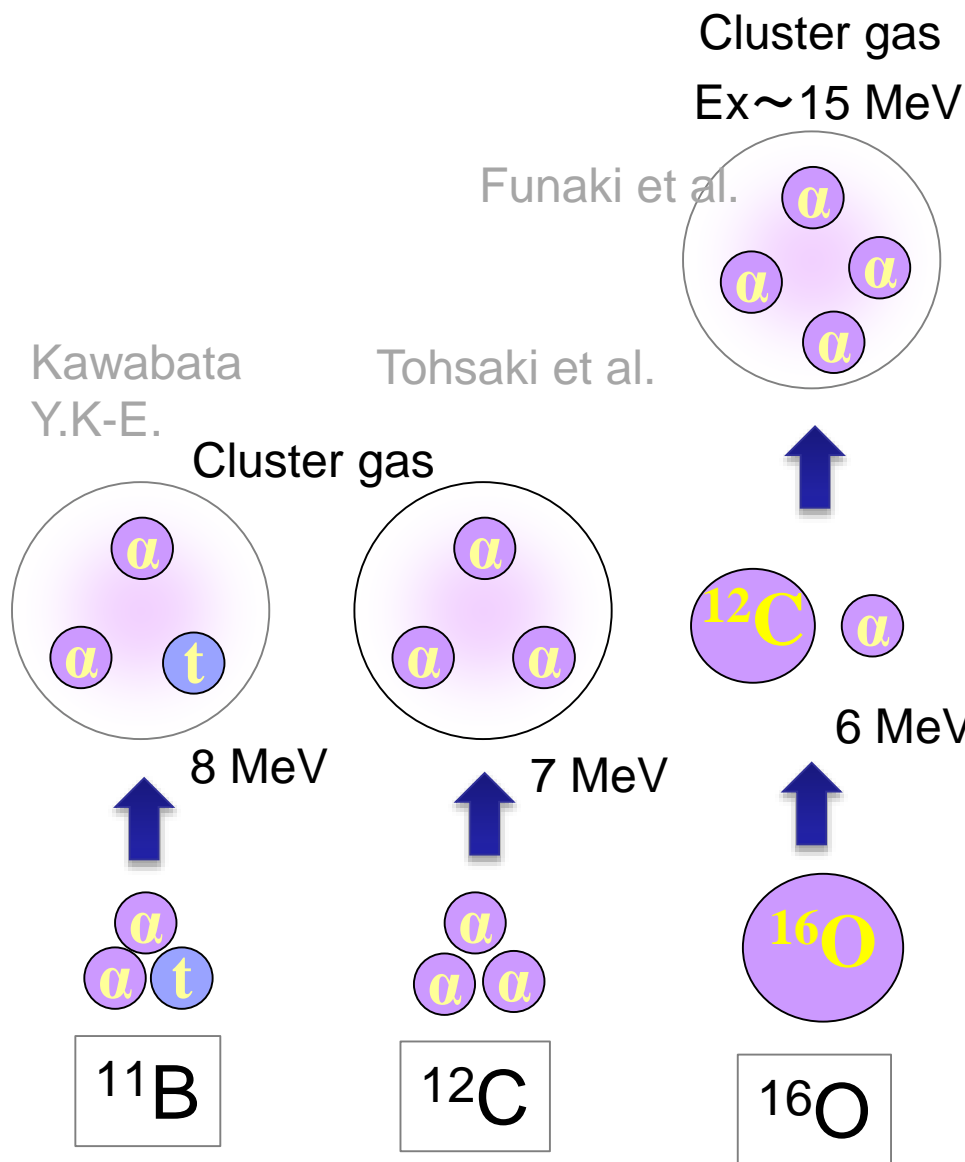
## 4. Summary



# **1. Introduction**

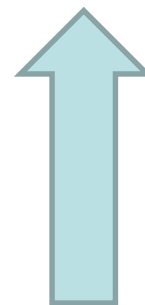
**Cluster excitations and Giant resonances**

# Cluster states in $Ex < \sim 15$ MeV



For cluster states in 7~10 MeV energy, strong ISO trans. because ISO operator excites radial motion of a cluster -> ISO is a good probe

1 $\alpha$  excitation



Strong ISO

cluster correlations in g.s. & excited states.

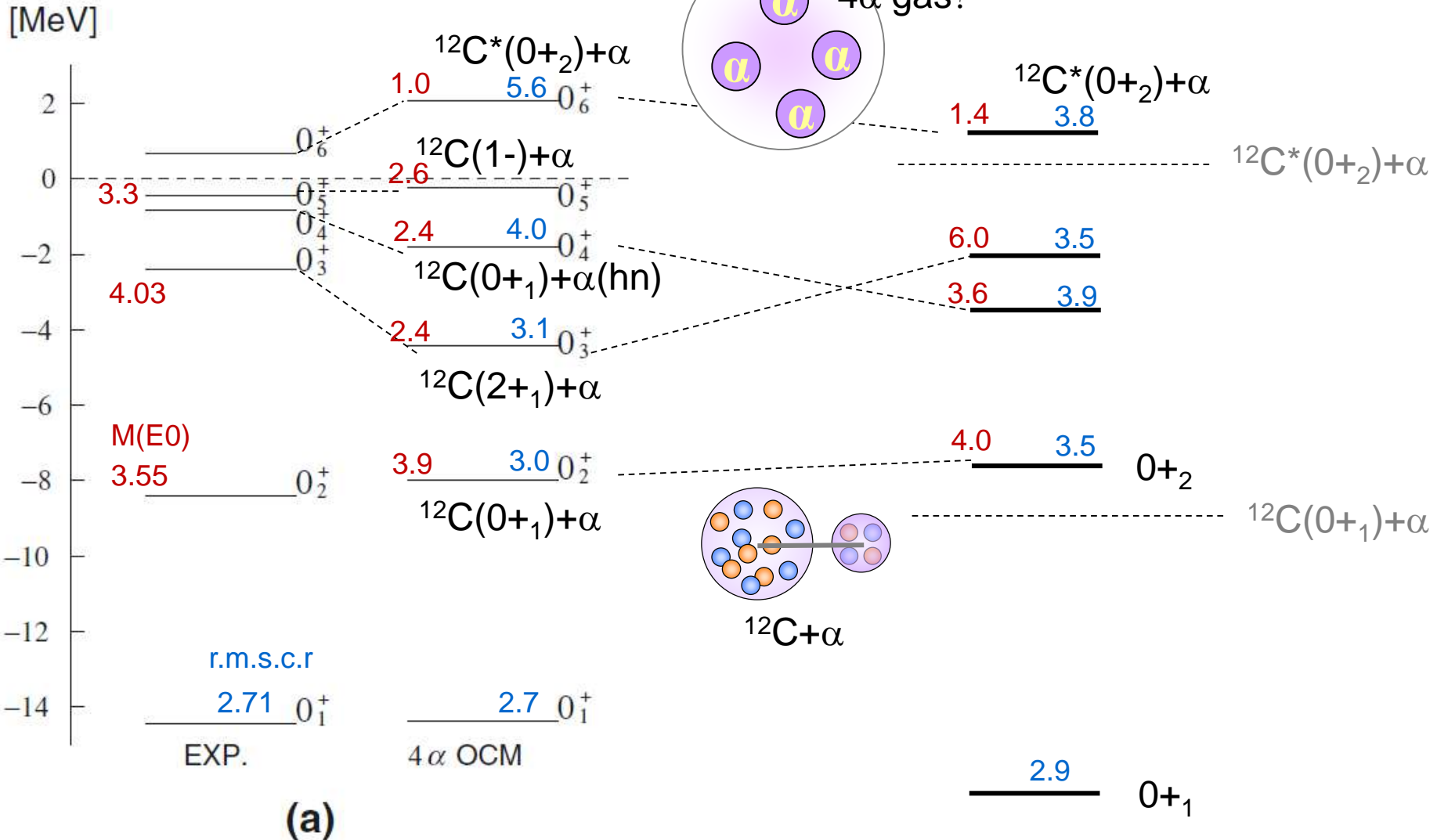
# Cluster excited states in $^{16}\text{O}$

4 $\alpha$ -OCM

$^{12}\text{C}(\text{AMD})+\alpha$ GCM

Funaki et al. PRL101, 082502 (2008),  
PRC82, 024312 (2010)

Y. K-E. PRC89, 024302 (2014)



# Isoscalar monopole (IS0) strengths in $^{16}\text{O}$

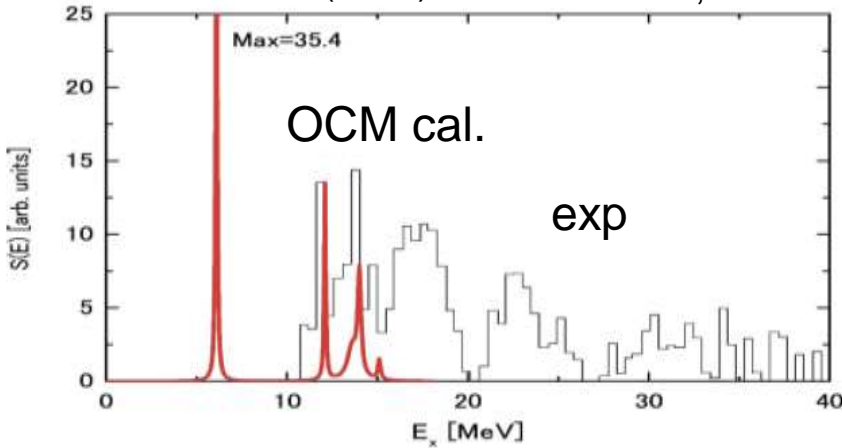
Strong IS0 for cluster states

IS0 operator excites radial motion of cluster.

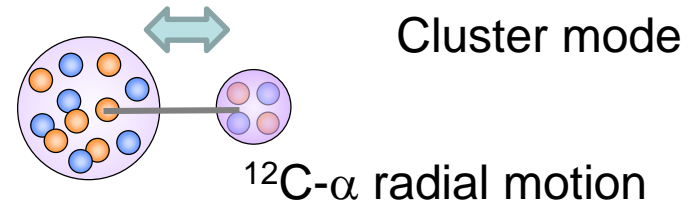
Cluster mode decouples from collective vibration mode.  
GR feeds low-energy strengths.

4-alpha OCM: Yamada et al.  
PRC85, 034315 (2012)

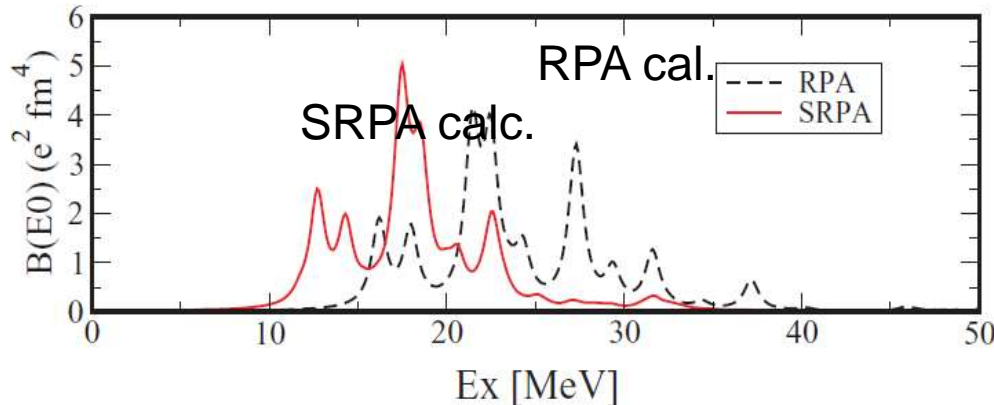
exp: Y.-W. Lui, et al.  
PRC64, 064308 (2001).



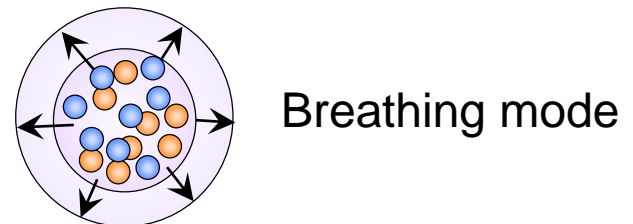
## ◆ Low-lying IS0 strengths



PRA cal. D. Gambacurta, et al. PRC81, 054312 (2010).



## ◆ High-lying IS0 (GMR): Collective vibration coherent 1p-1h motion



# LE strengths decoupling from GR = Decoupling of Scale

The universal feature

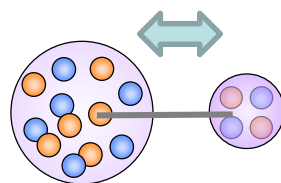
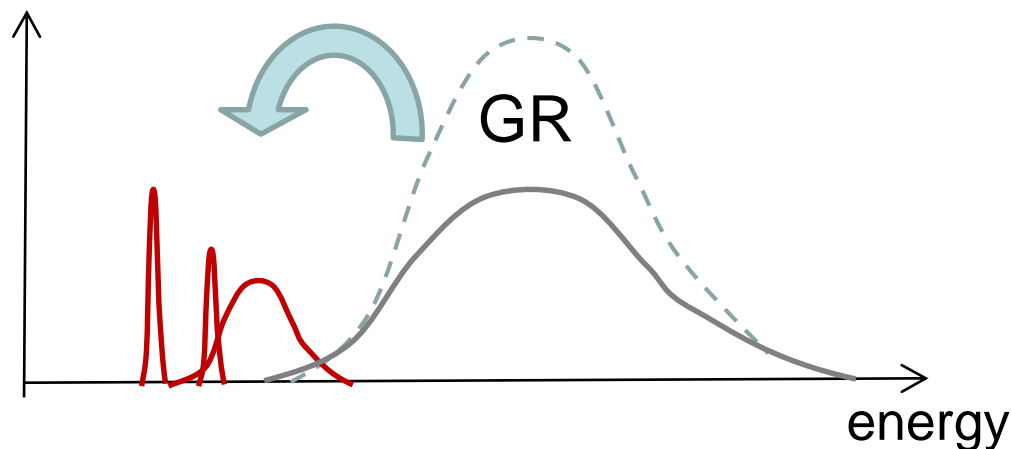
**Decoupling of scale (energy)**



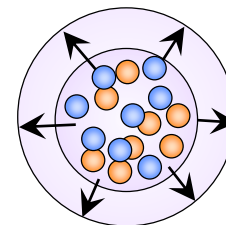
**New excitation modes**

transition strength

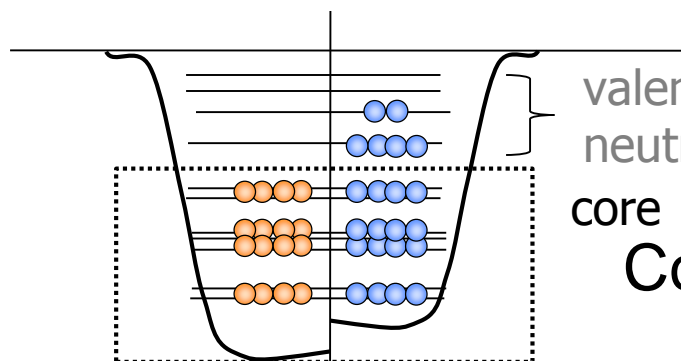
GMR, GDR



Cluster

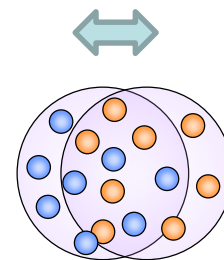
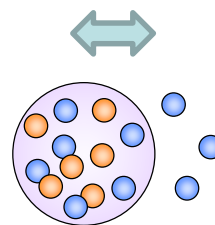


GMR



valence neutrons  
core

Core+Xn



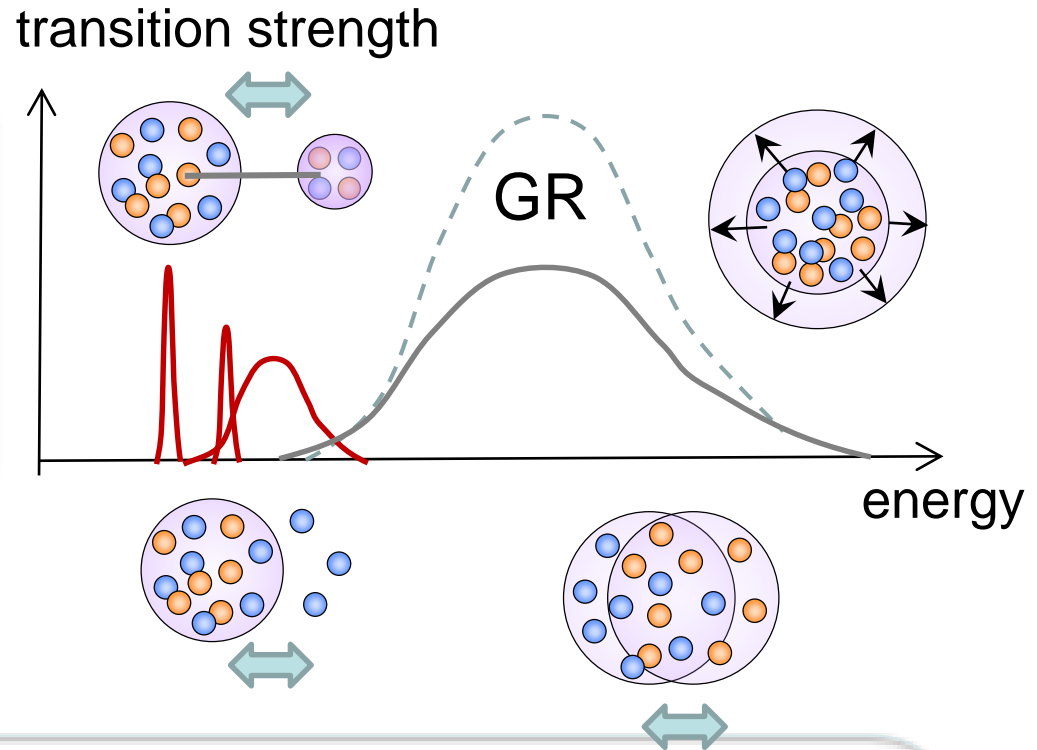
GDR

# LE strengths decoupling from GR=Decoupling of Scale

**Decoupling of  
scale (energy)**



**New excitation modes**



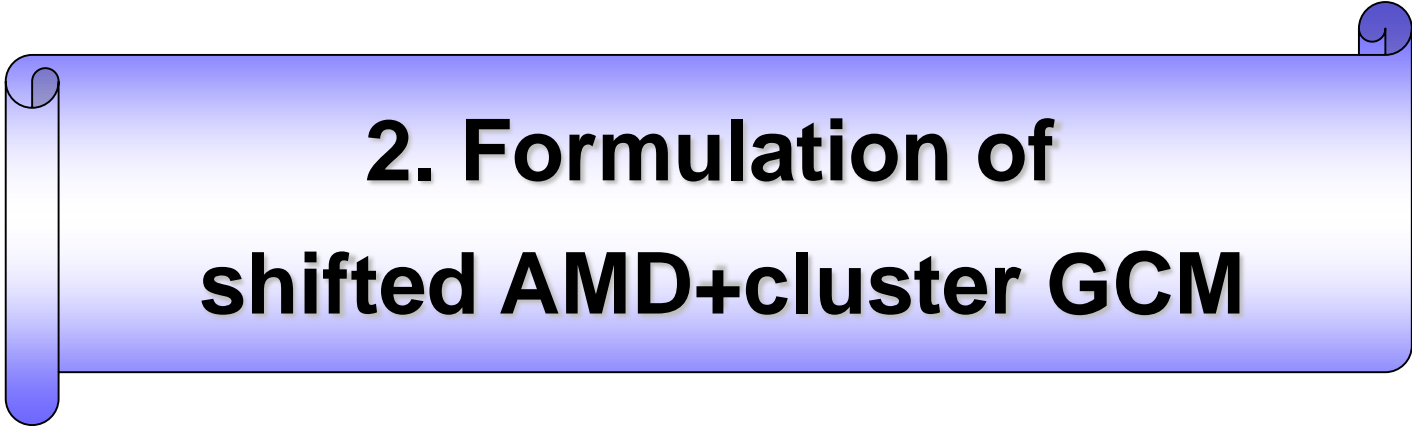
Aim: to study LE and GR strengths

Theoretical model describes both 1p-1h excitation  
(collective vibration) and cluster excitations

Present method: shifted AMD+cluster GCM

small amp. (1p1h)

large amp. cluster mode



## **2. Formulation of shifted AMD+cluster GCM**



# Formulation of AMD

A approach to study coexistence of cluster and mean-field natures

AMD wave function

$$\Phi = c\Phi_{\text{AMD}} + c'\Phi'_{\text{AMD}} + c''\Phi''_{\text{AMD}} + \dots$$

$$\Phi_{\text{AMD}} = \det\{\varphi_1, \varphi_2, \dots, \varphi_A\}$$

Slater det.

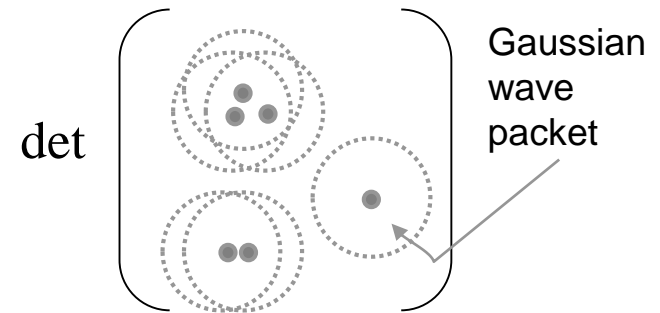
Gaussian

$$\varphi_i = \phi_{\mathbf{Z}_i} \chi_i \begin{cases} \text{spatial} \\ \phi_{\mathbf{Z}_i}(\mathbf{r}_j) \propto \exp\left[-\nu\left(\mathbf{r} - \frac{\mathbf{Z}_i}{\sqrt{\nu}}\right)^2\right] \\ \chi_i = \begin{pmatrix} \frac{1}{2} + \xi_i \\ \frac{1}{2} - \xi_i \end{pmatrix} \times \begin{matrix} p \text{ or } n \\ \text{spin} \end{matrix} \end{matrix} \text{ isospin}$$

$$\Phi_{\text{AMD}}(\mathbf{Z})$$

variational parameters

$$\mathbf{Z} = \{ \mathbf{Z}_1, \mathbf{Z}_2, \dots, \mathbf{Z}_A, \xi_1, \dots, \xi_A \}$$



Variation, projection, super position

$$\delta \frac{\langle \Phi | H | \Phi \rangle}{\langle \Phi | \Phi \rangle} = 0$$

VAP: Variation after spin-parity projection

GCM:  $\beta$ -GCM,  $\alpha$ -cluster GCM ( $\alpha$ GCM)

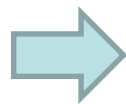
Various cluster configurations are contained in the AMD model space

# sAMD (1p1h excitations)

AMD wave fn.  
for g.s.

obtained by VAP

$$\delta \frac{\langle P_{MK}^{J\pm} \Phi_{\text{AMD}} | H | P_{MK}^{J\pm} \Phi_{\text{AMD}} \rangle}{\langle P_{MK}^{J\pm} \Phi_{\text{AMD}} | P_{MK}^{J\pm} \Phi_{\text{AMD}} \rangle} = 0$$



$$\Phi_{\text{AMD}}(\mathbf{Z}) = \det\{\varphi_1, \varphi_2, \dots, \varphi_A\}$$

$$\varphi_i = \phi_{\mathbf{Z}_i} \chi_i$$

$$\phi_{\mathbf{Z}_i}(\mathbf{r}_j) = \exp\left[-\nu\left(\mathbf{r} - \frac{\mathbf{Z}_i}{\sqrt{\nu}}\right)^2\right]$$

optimized parameters

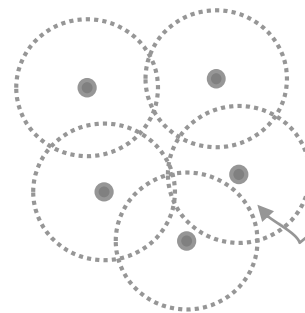
$$\mathbf{Z} = \{ \mathbf{Z}_1, \mathbf{Z}_2, \dots, \mathbf{Z}_A, \xi_1, \dots, \xi_A \}$$

sAMD

small variation of single-particle wave functions

$$\Phi = \det\{\varphi_1, \varphi_2, \dots, \varphi_A\} + \sum_i c_i \det\{\varphi_1, \dots, \varphi_i + \delta\varphi_i, \dots, \varphi_A\}$$

$$\varphi_i + \delta\varphi_i = \phi_{\mathbf{Z}_i + \delta\mathbf{Z}_i} \chi_i$$



Gaussian center

$$\mathbf{Z}_i \rightarrow \mathbf{Z}_i + \delta\mathbf{Z}_i$$

small shift of spatial part

Linear combination of 1p1h excitations from the g.s. configuration  
(small amplitude oscillation within shifts of Gaussians)

describes monopole, dipole excitations

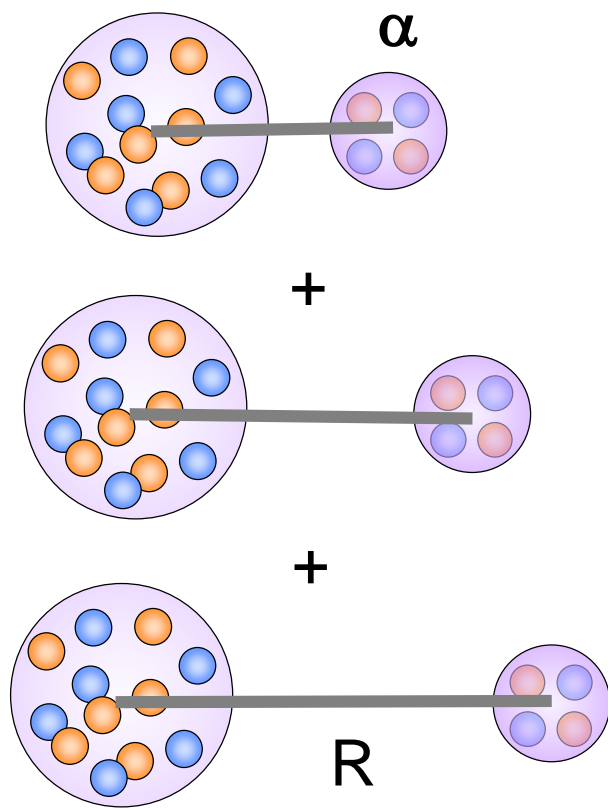
(applicable to GT, M1, SD etc.)

# AMD+ $\alpha$ GCM

**AMD+ $\alpha$ GCM**

Core(AMD)+ $\alpha$  = **AMD+ $\alpha$ GCM**

Core(AMD)



Generator coordinate: R

Large amplitude cluster mode

# sAMD+GCM(cluster)

sAMD+GCM      superposing spin-parity projected states  
of all wave functions (shifted AMD, and cluster wave functions.)

- ground state: VAP describes g.s.cluster correlations
- sAMD describes small amplitude oscillation (1p1h)
- GCM describes large amplitude cluster mode
- Parity and angular momentum projections  
describe coupling of intrinsic modes with the rotation

RPA: only small amplitude oscillation (->higher: SRPA)  
no projections

QRPA: pairing correlation but no cluster correlation  
no projections

Time-dependent AMD: large amplitude oscillation  
but not quantized  
no projections

# Monopole and dipole excitations

Isoscalar monopole (IS0):

$$M(IS0) = \sum_i r_i^2 Y_{00}(\hat{\mathbf{r}}_i)$$

Breathing mode  
coupling with radial excitation  
and cluster mode

Isovector dipole (E1):

$$M(E1; \mu) = \sum_{i=proton} r_i Y_{1\mu}(\hat{\mathbf{r}}_i)$$

Translational mode

Isoscalar dipole (ISD):

$$M(ISD; \mu) = \sum_i r_i^3 Y_{1\mu}(\hat{\mathbf{r}}_i)$$

Compressive dipole mode  
coupling with radial excitation  
and cluster mode

## 3. Results

1.  $^{16}\text{O}$ : sAMD+ GCM( $^{12}\text{C}+\alpha$ ) for B(IS0)
2.  $^{12}\text{C}$ : sAMD+ GCM( $3\alpha$ ) for B(IS0) & B(ISD)
3.  $^{9,10}\text{Be}$ : sAMD+GCM( $^{5,6}\text{He}+\alpha$ ) for B(E1) & B(ISD)

# Effective interactions

Phenomenological effective two-body and three-body interactions

Central force

Modified Volkov force: finite range two-body + zero-range three-body

Spin-orbit force

finite range two-body

Coulomb

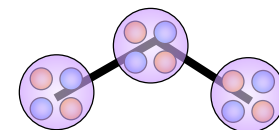
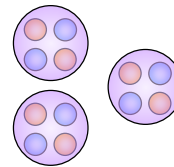
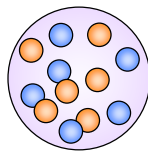
Present parameter set reproduce energy spectra of  $^{12}\text{C}$ .

# 3-1. B(IS0) in $^{16}\text{O}$

Y. K-E. PRC89, 024302 (2014)

$$^{16}\text{O}(\text{sAMD}) + \text{GCM}(^{12}\text{C}(\text{AMD}) + \alpha)$$

$$^{12}\text{C}(\text{AMD}) = \left| \Phi_{AMD}(^{12}\text{C}:0_1^+) \right\rangle, \left| \Phi_{AMD}(^{12}\text{C}:0_2^+) \right\rangle, \left| \Phi_{AMD}(^{12}\text{C}:1_1^-) \right\rangle$$

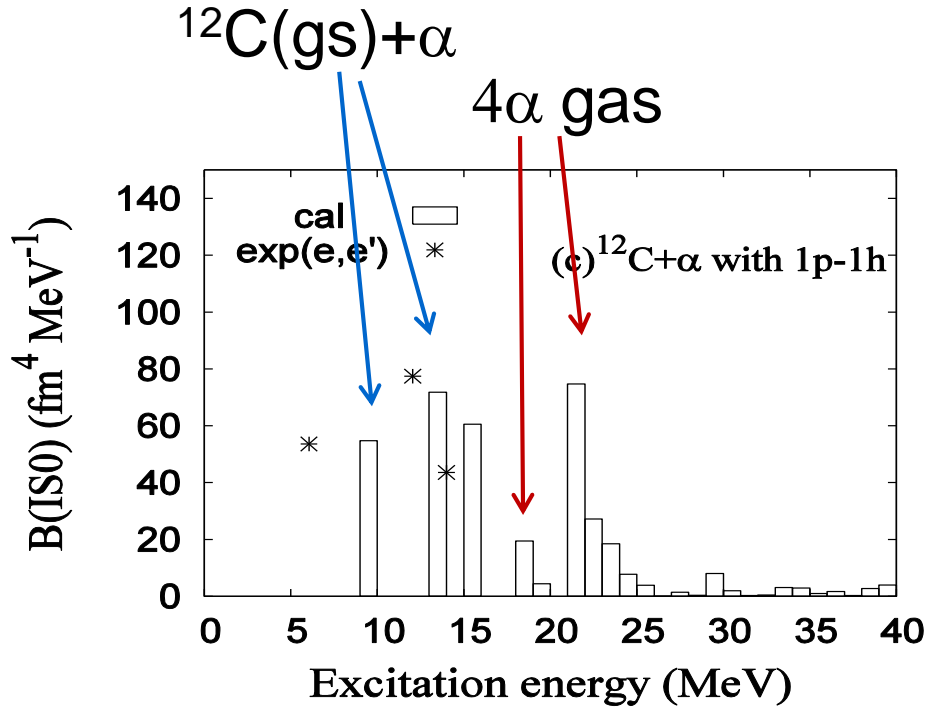


Three states with AMD+VAP



# B(IS0) in $^{16}\text{O}$

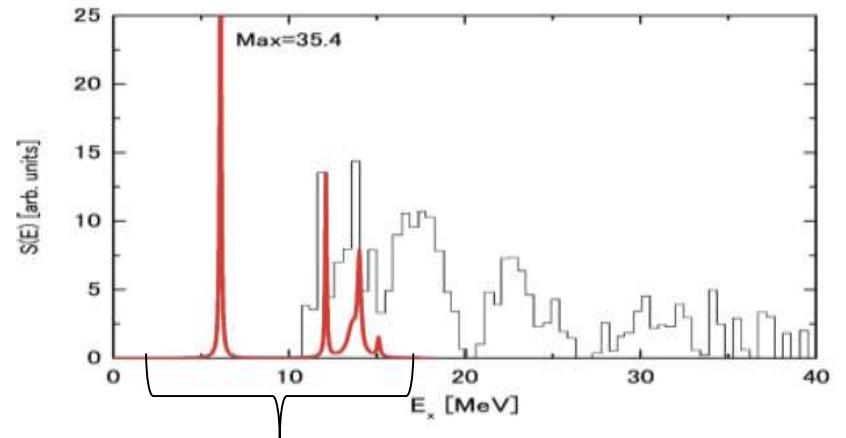
sAMD+ $\alpha$ GCM



LE cluster mode

GMR

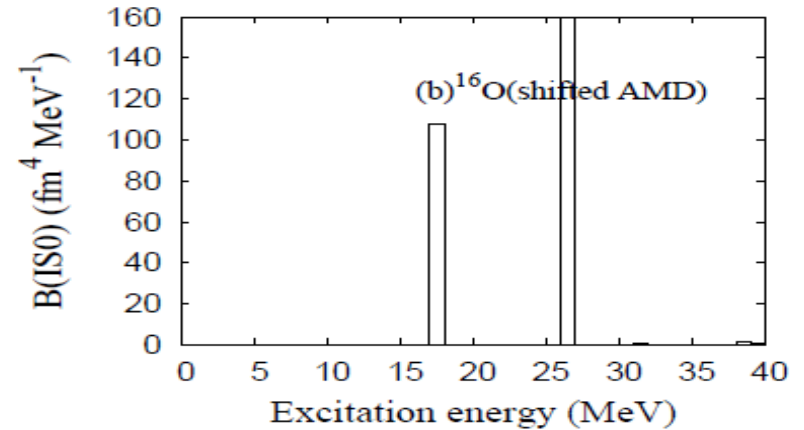
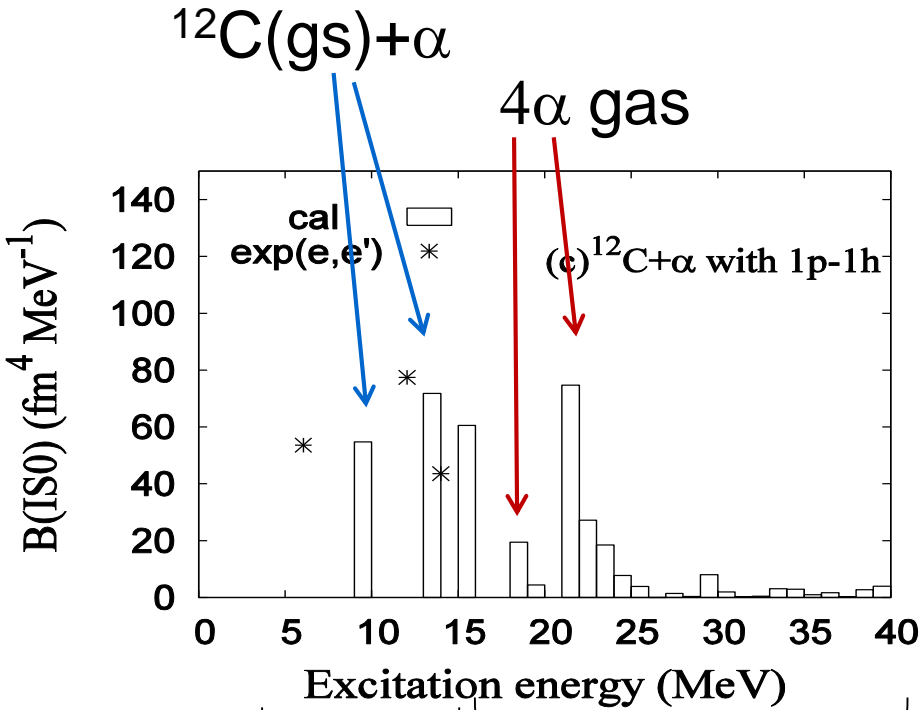
Yamada et al.



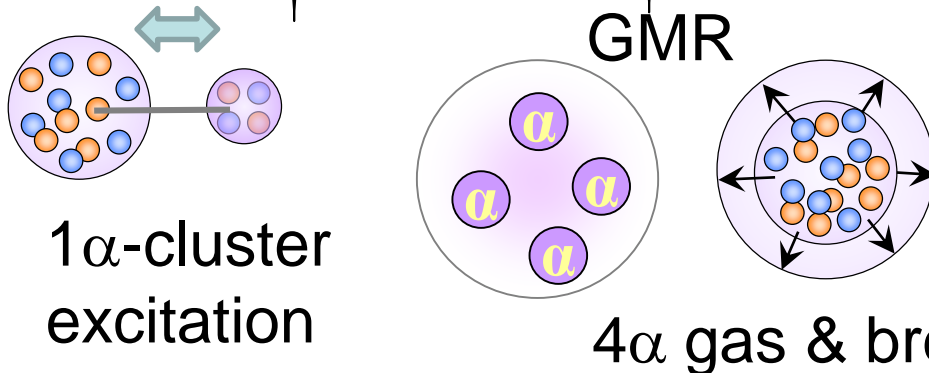
4a-OCM: EWSR( $E < 16$ )  $\sim 20\%$   
 exp. EWSR( $E < 40$ ) = 50%

# sAMD+ $\alpha$ GCM

# sAMD (only 1p1h)



- ✓  $^{12}\text{C}-\alpha$  mode decouple from GMR: low-energy IS0 strength.
- ✓  $4\alpha$  cluster gas does not decouple From GMR, contributes to GMR. Isotropic radial excitation coupling with collective breathing mode.

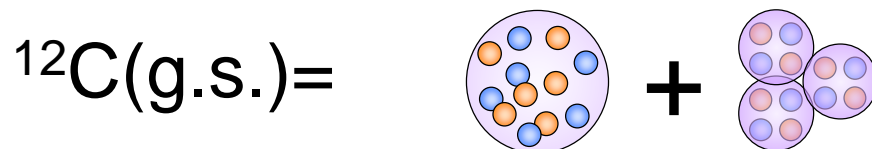


## 3-2. B(ISO) & B(ISD) in $^{12}\text{C}$

$^{12}\text{C}(\text{sAMD})+\text{GCM}(3\alpha)$

# sAMD+ $\alpha$ GCM for $^{12}\text{C}$

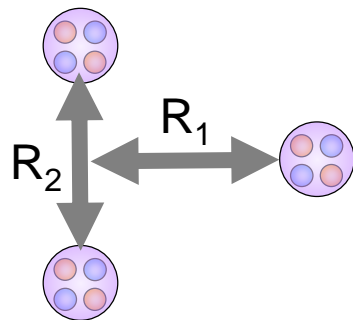
$^{12}\text{C}(\text{sAMD})+\text{GCM}(3\alpha)$



VAP result contains  
the g.s. cluster correlation

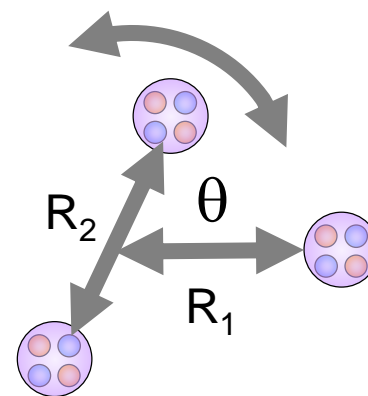
$^{12}\text{C}(\text{sAMD})$ : 1p1h excitations on g.s.

GCM( $3\alpha$ )



(A) radial mode

$^{12}\text{C}(0^+_2)$

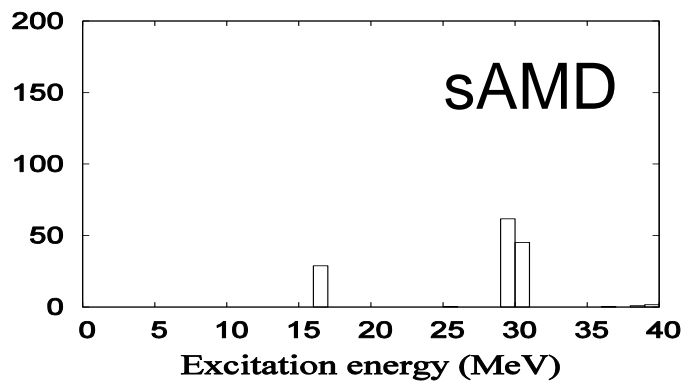


(B) radial & angular mode

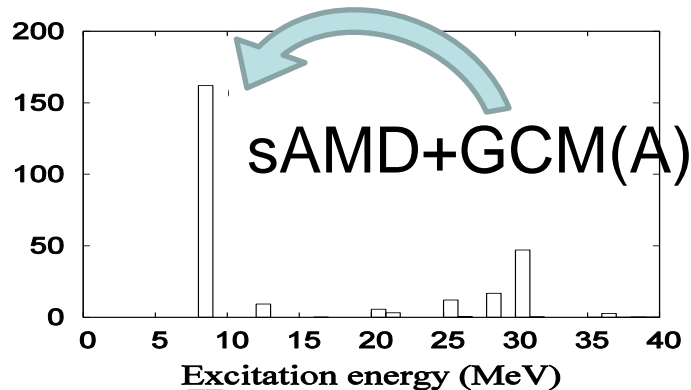
$^{12}\text{C}(0^+_3)$

Details of  $3\alpha$  mode are investigated by Y. Yoshida.

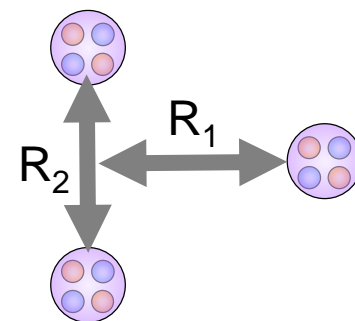
# B(IS0) of $^{12}\text{C}$



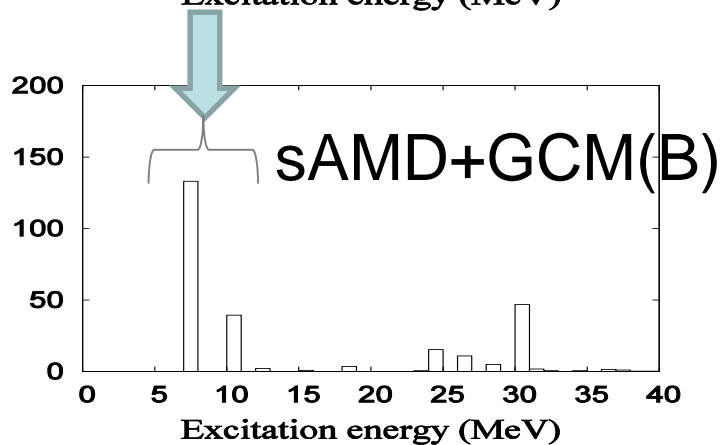
sAMD contains g.s. cluster corr.  
LE cluster mode & GMR



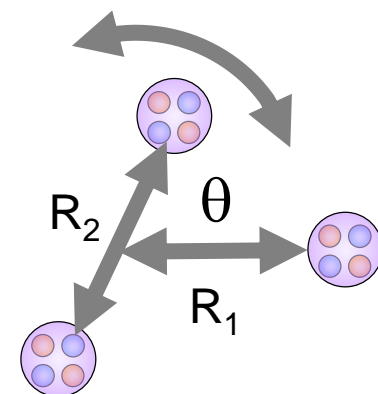
(A)  $3\alpha$  radial mode



GMR feeds  
strength of LE cluster mode

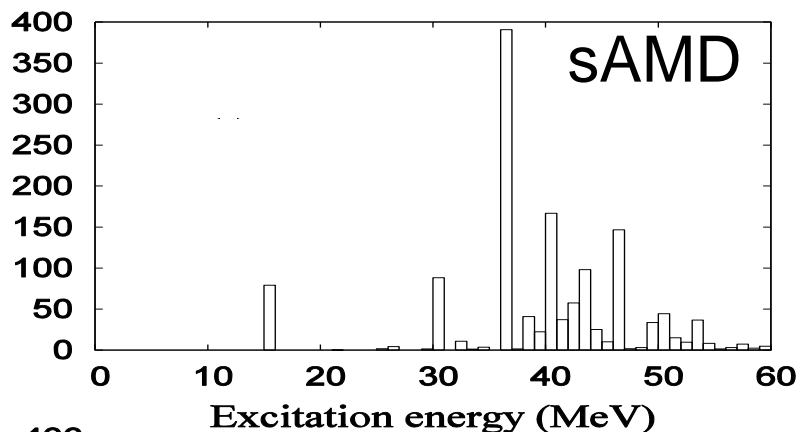


(B)  $3\alpha$  angular mode

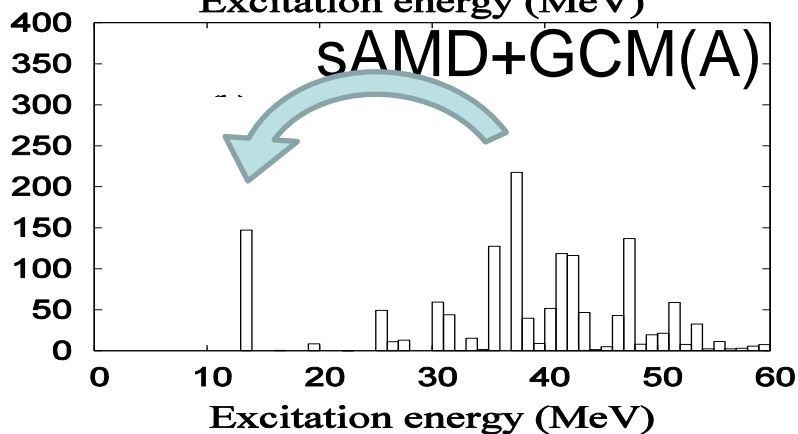


Split IS0 of  
LE cluster mode

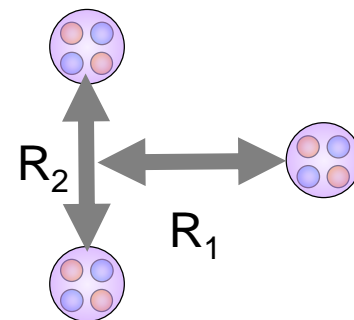
# B(ISD) of $^{12}\text{C}$



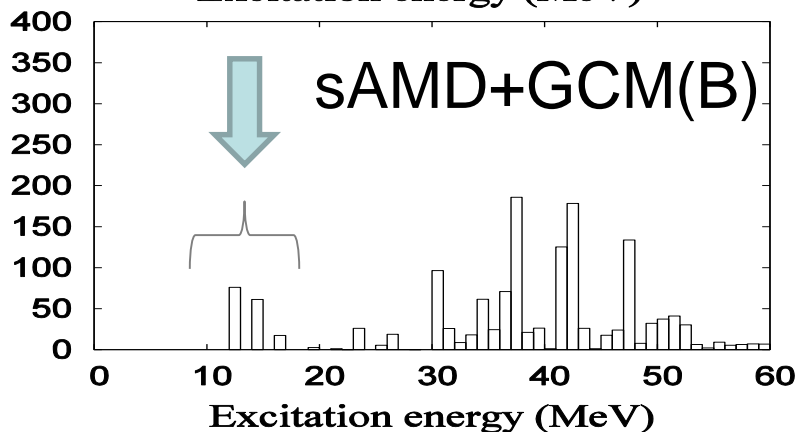
sAMD contains g.s. cluster corr.  
LE cluster mode & GR



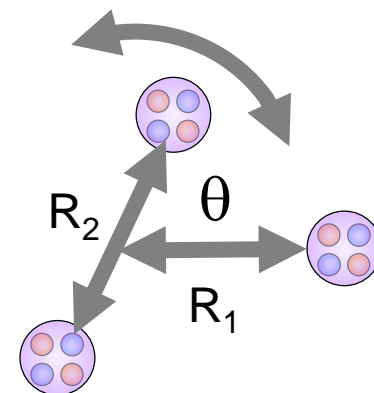
(A)  $3\alpha$  radial mode



GR feeds  
strength of LE cluster mode

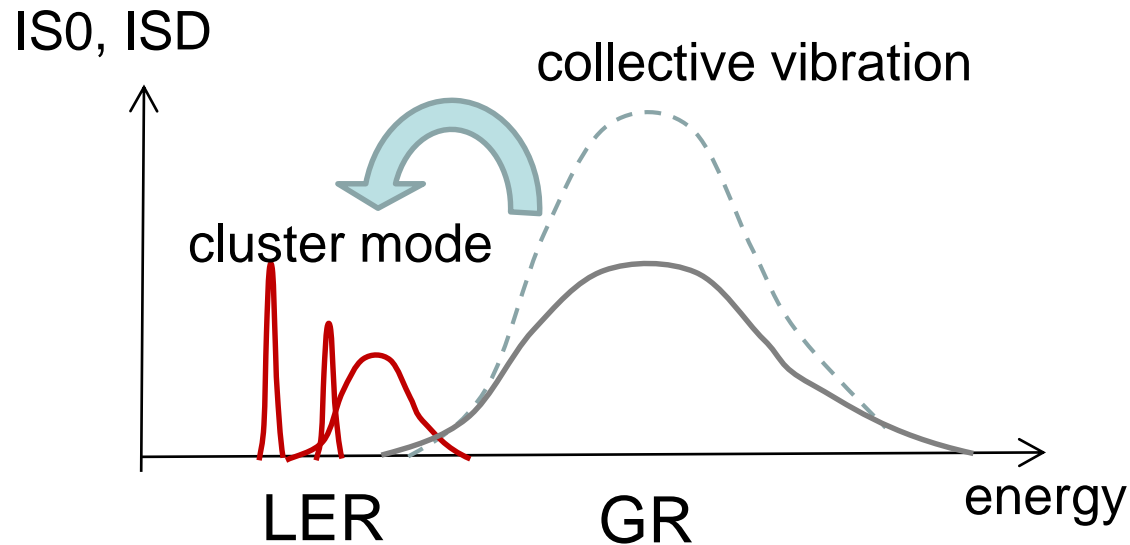


(B)  $3\alpha$  angular mode



Split IS0 of  
LE cluster mode

# general trend of ISO and ISD



- ✓ Cluster modes decouple from the collective vibration mode
- ✓ Low-energy strengths appears decoupling from the GR.

## 3-3. B(E1)&B(ISD) in ${}^9,{}^{10}\text{Be}$

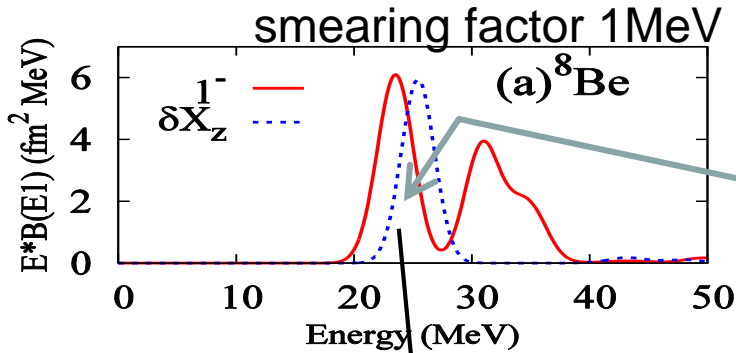
sAMD+ $\alpha$ GCM

$rY_{1\mu}$  E1: translational mode

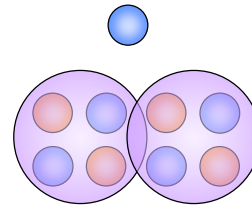
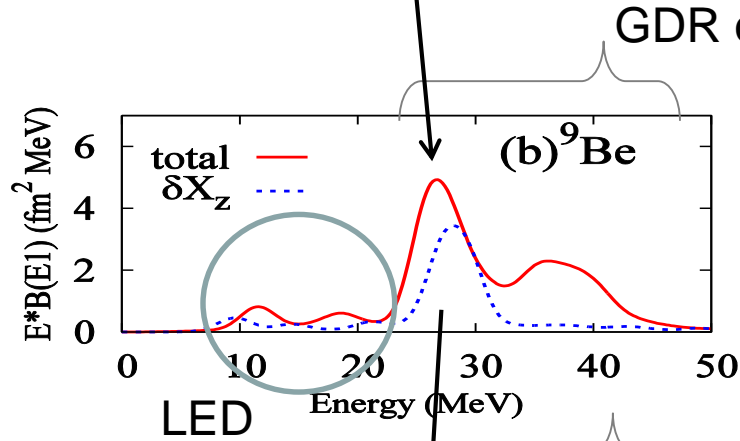
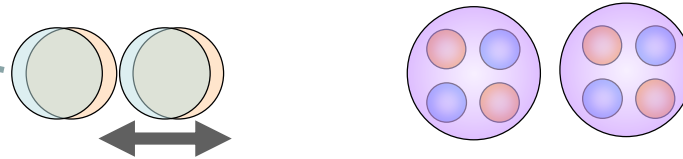
$r^3Y_{1\mu}$  ISD: compressive mode, radial excitation  
sensitive to coupling with cluster mode



# B(E1) of Be isotopes obtained by sAMD calc.

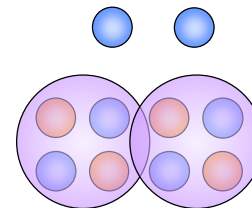
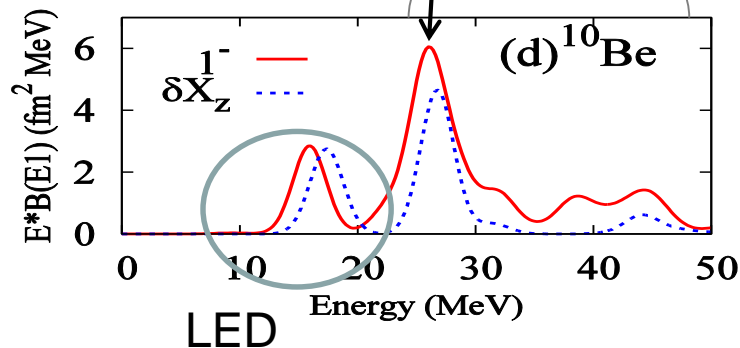


E1 on prolate deformed state



LE dipole strengths:  
valence neutrons

GDR:  
 $^8\text{Be}$  core

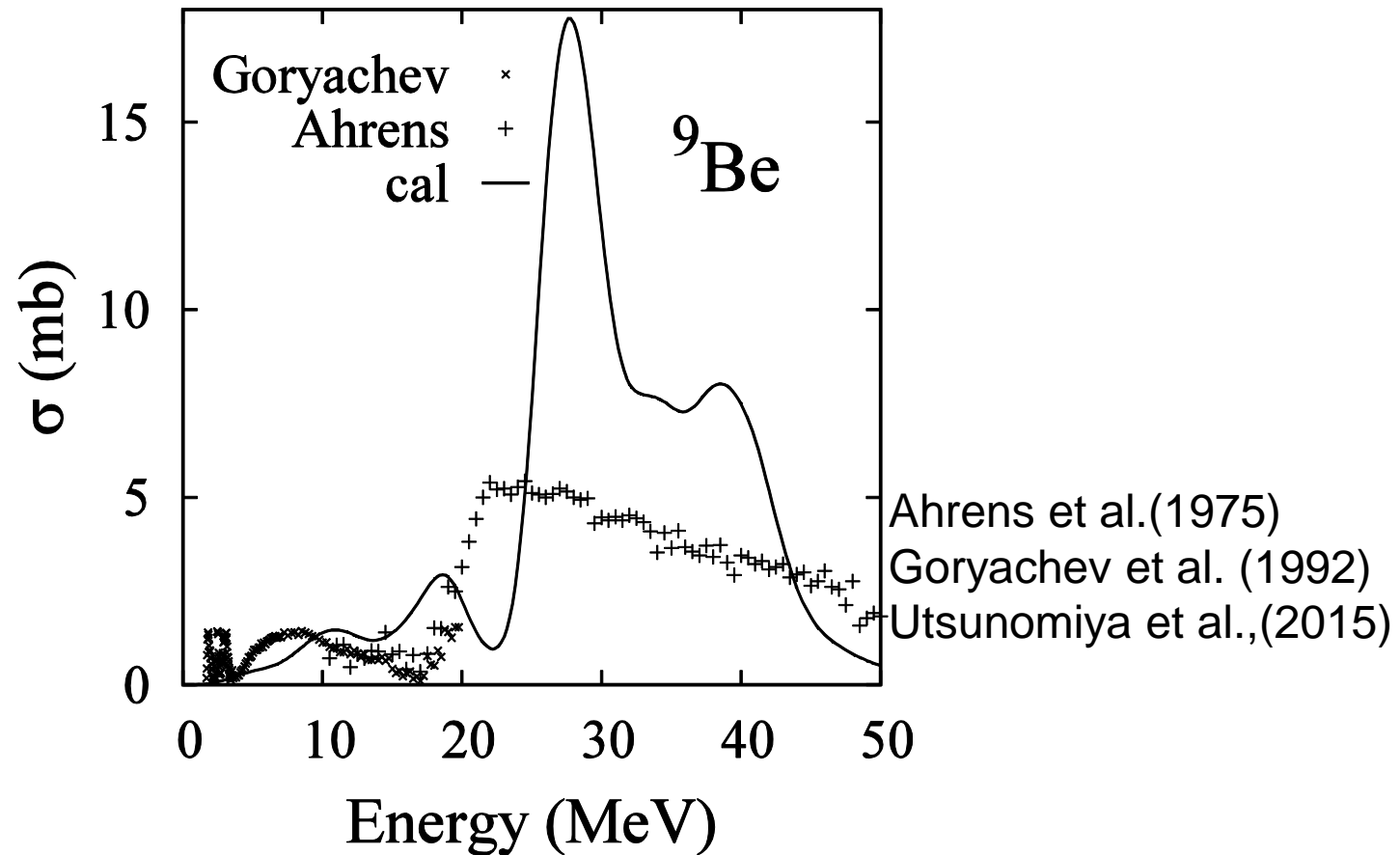


contribution of valence neutrons

- ✓ LE dipole strengths
- ✓ broadening higher peak of GDR

# B(E1) exp. of $^9\text{Be}$

Photonuclear cross section v.s. sAMD+ $\alpha$ GCM calc.

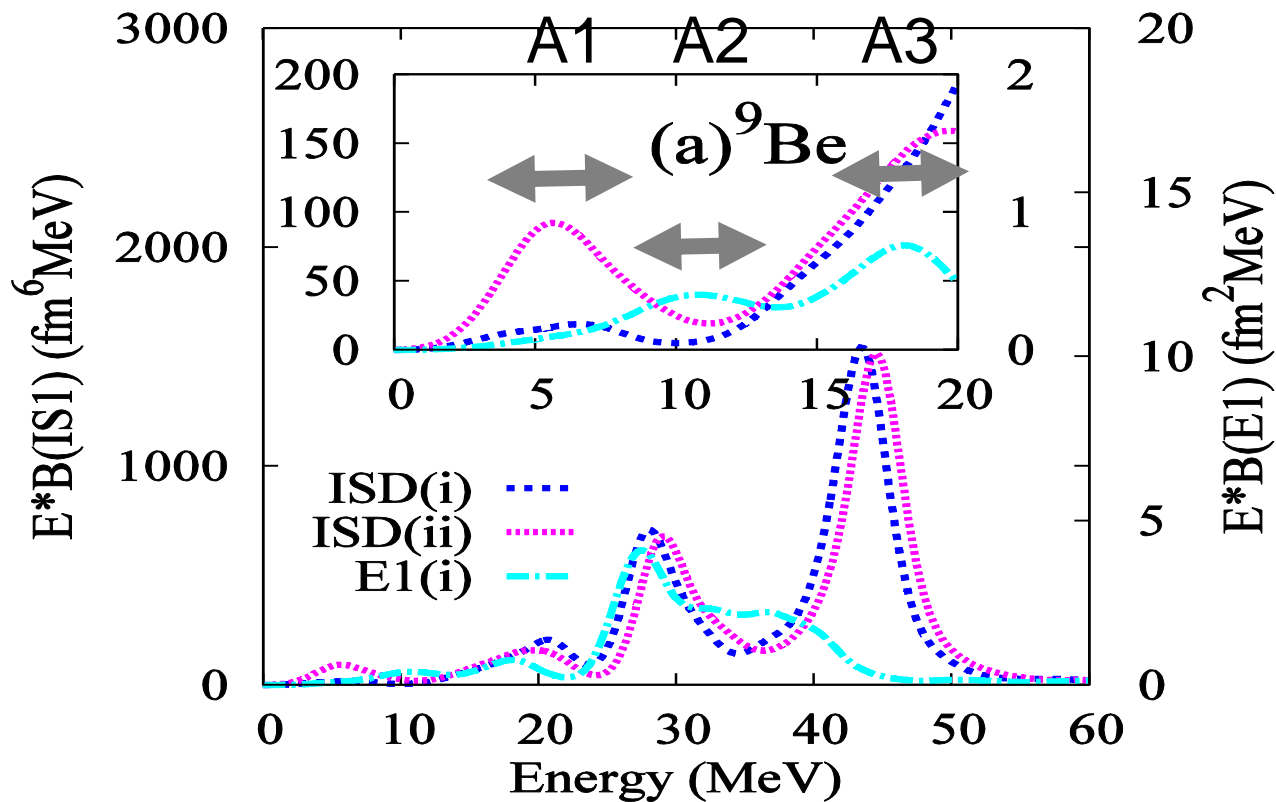


# B(E1) & B(ISD) of $^9\text{Be}$

sAMD+ $\alpha$ GCM

## $^9\text{Be}$

LE dipole excitations

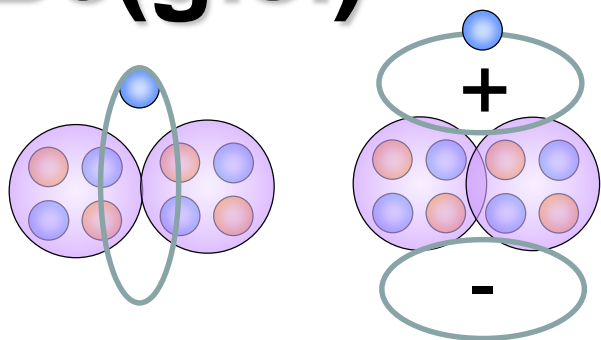


A1: ISD enhanced by  $\alpha$ -cluster mode

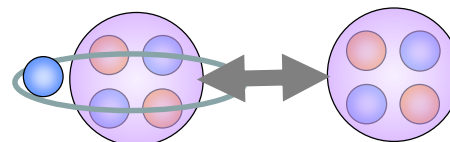
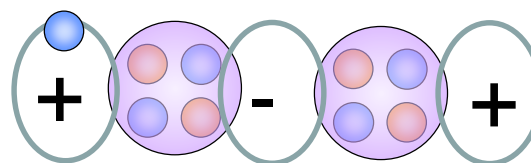
A2: weak ISD, weak coupling with cluster mode

# B(E1) & B(ISD) of $^9\text{Be}$

$^9\text{Be}(\text{g.s.})$

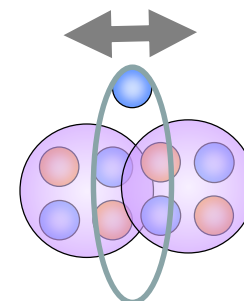
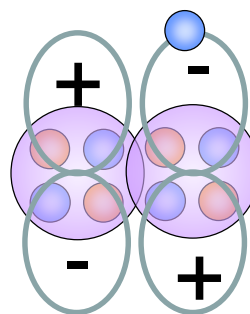


**A1** ISD enhanced by  $\alpha$ -cluster mode

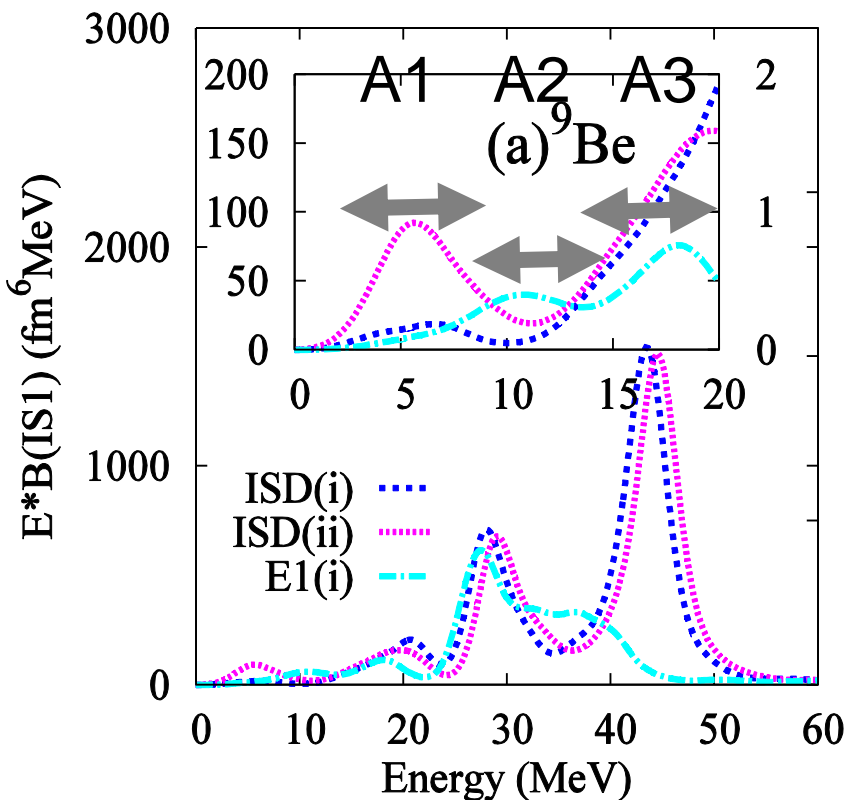


$^5\text{He} + \alpha$   
enhances ISD

**A2** weak ISD, weak coupling with cluster



not radial excitation



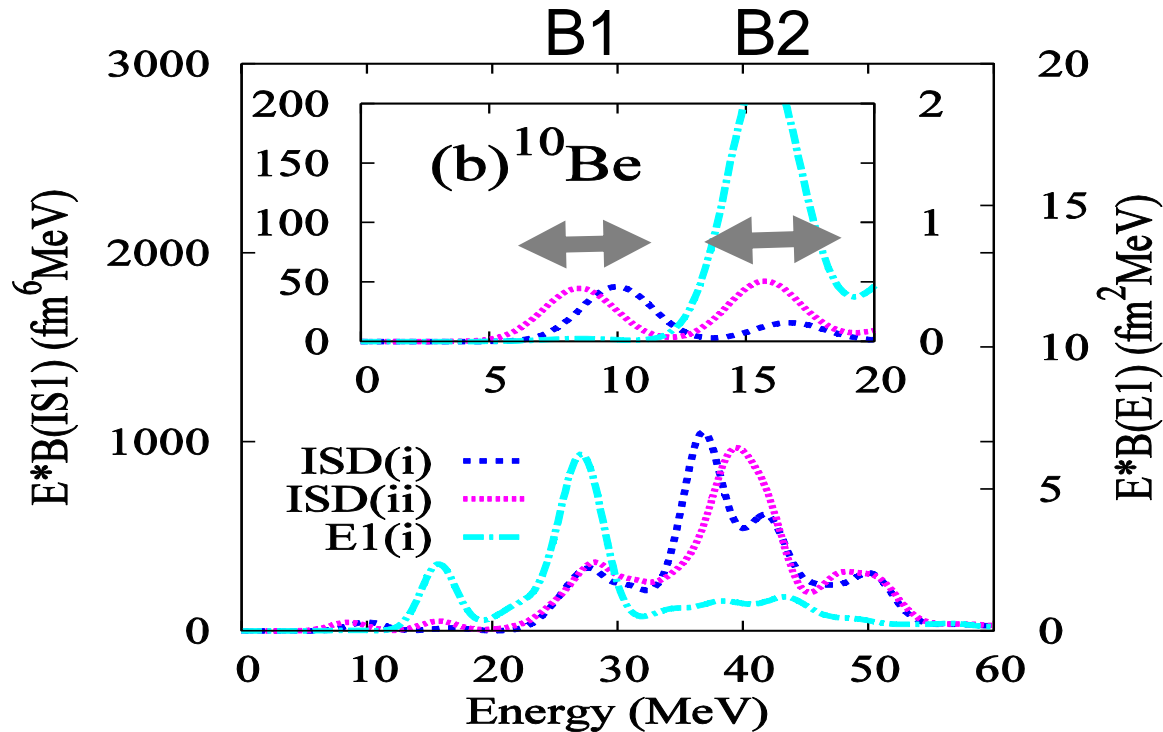
# B(E1) & B(ISD) of $^{10}\text{Be}$

sAMD+ $\alpha$ GCM

$^{10}\text{Be}$

sAMD

LE dipole excitations

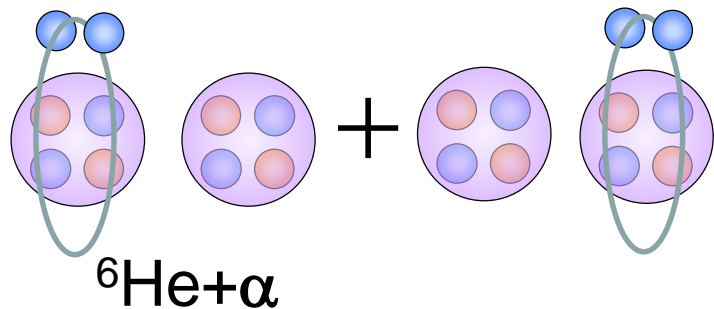


B1: weak E1, weak coupling with cluster mode

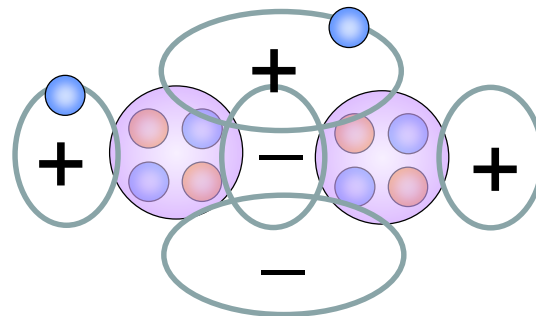
B2: significant E1, ISD enhanced by  $\alpha$ -cluster mode

# B(E1) & B(ISD) of $^{10}\text{Be}$

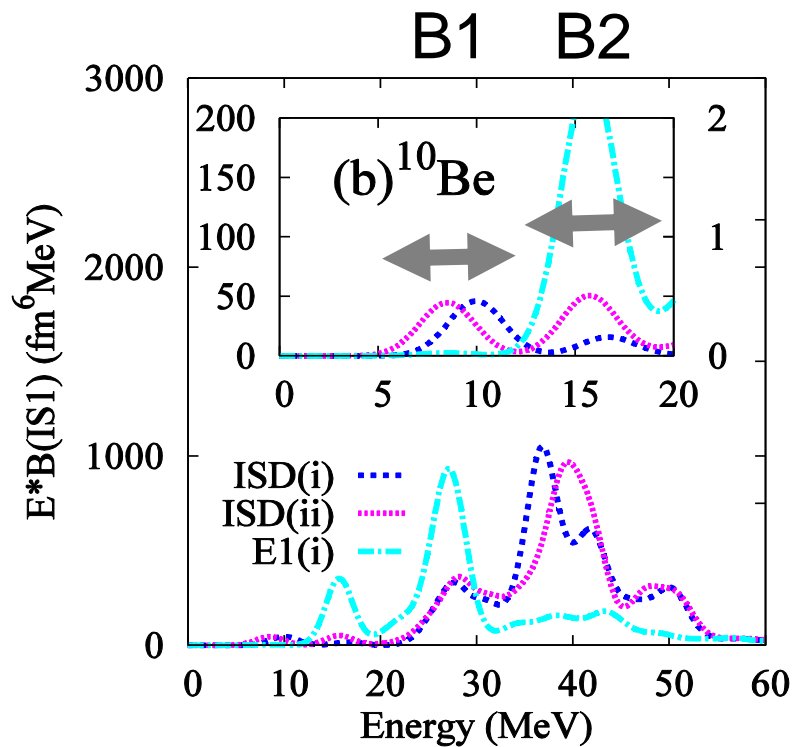
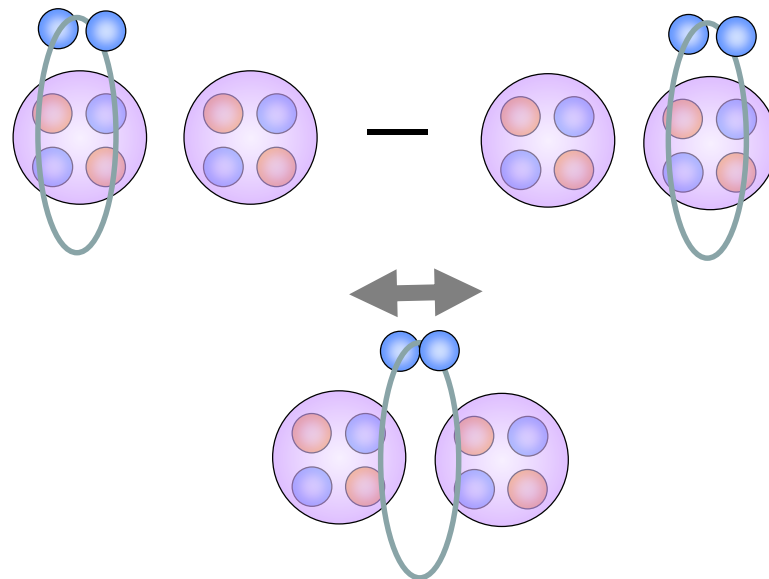
## $^{10}\text{Be}(\text{g.s.})$



**B1** weak E1,  
weak coupling with cluster mode



**B2** significant E1,  
ISD enhanced by  $\alpha$ -cluster mode





## **5. Summary**

# Summary

- sAMD+GCM

  - 1p1h+cluster excitations,  $J\pi$ -projections

- Monopole & Dipole excitations

- Monopole excitations in  $^{16}\text{O}$

- Monopole and ISD in  $^{12}\text{C}$

- Dipole excitations in Be isotopes

- IS0 & ISD enhanced by coupling with cluster modes

- Low-energy excitations decoupling from GR

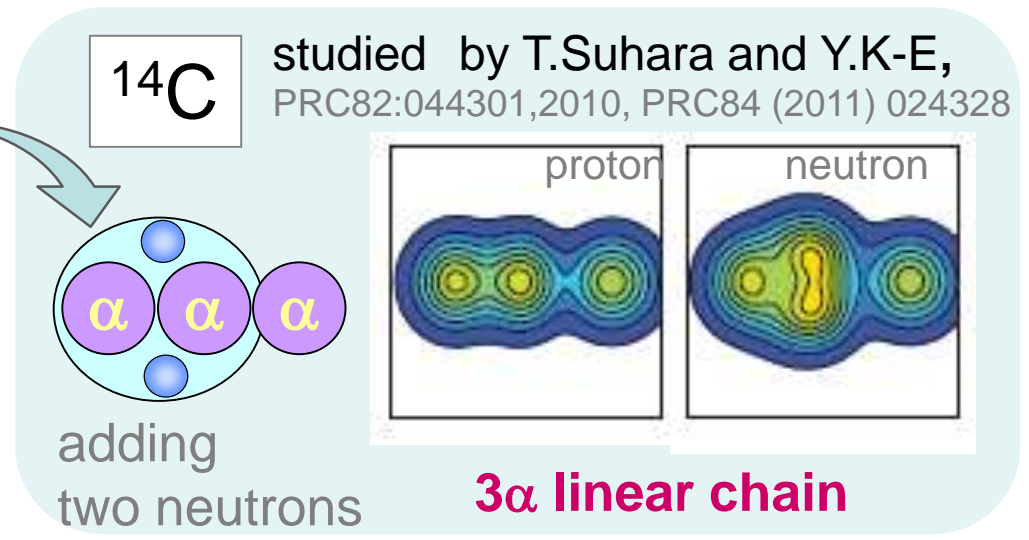
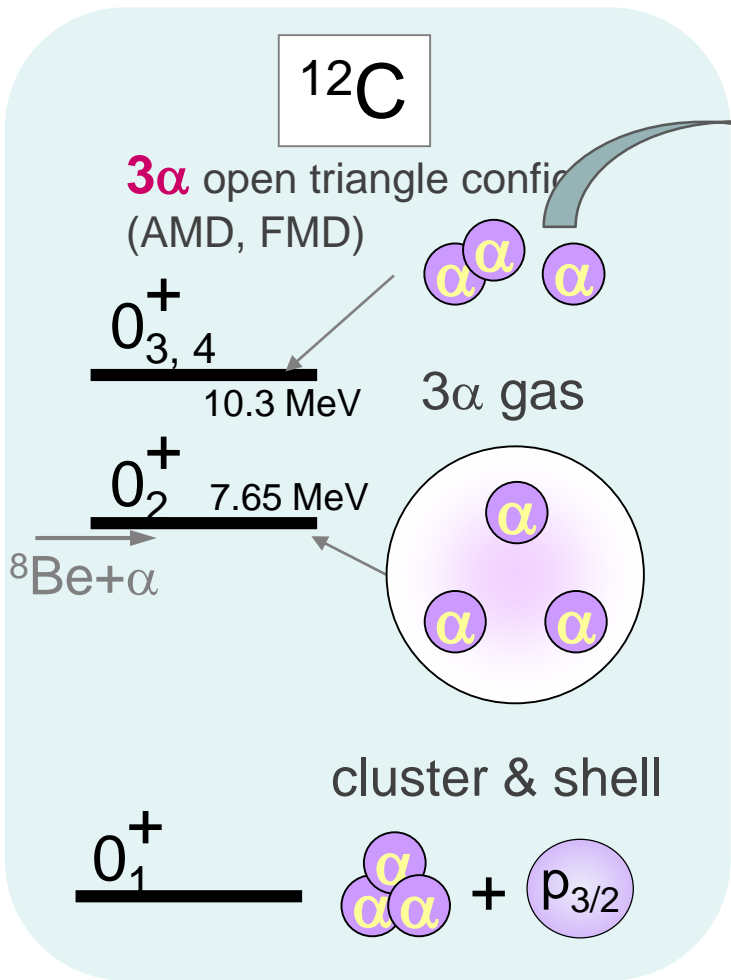
  - LE IS0 & ISD in  $^{12}\text{C}$ : cluster mode

  - LE E1 in  $^9,^{10}\text{Be}$ : valence neutron mode

  - LE ISD in  $^9,^{10}\text{Be}$ : cluster mode



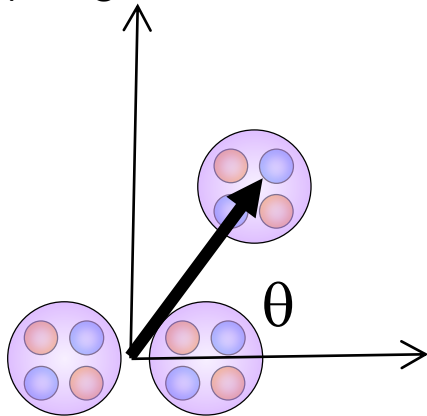
# 6. What is shape ? of bound and resonance states



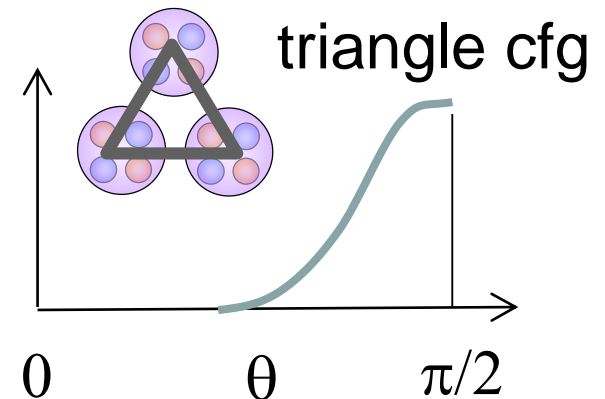
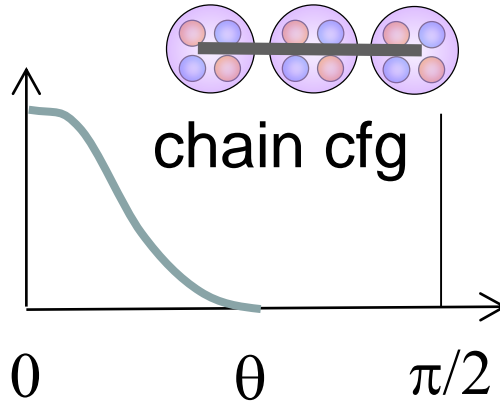
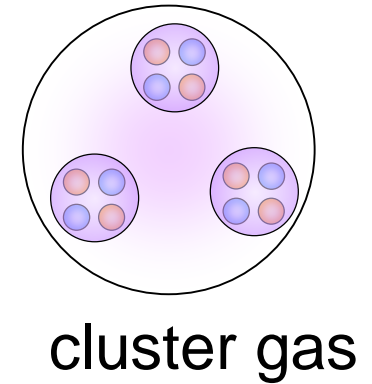
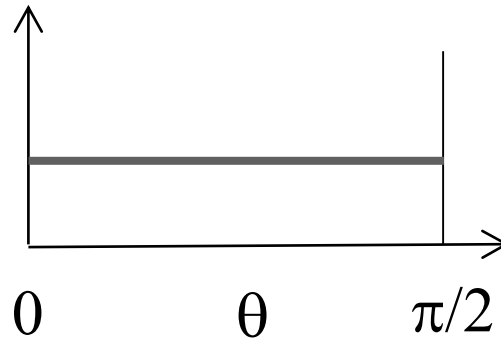
# Shape of quantum system

Localization in probability distribution  $\rightarrow$  shape  
A state dominantly contains a certain configuration (classical interpretation)

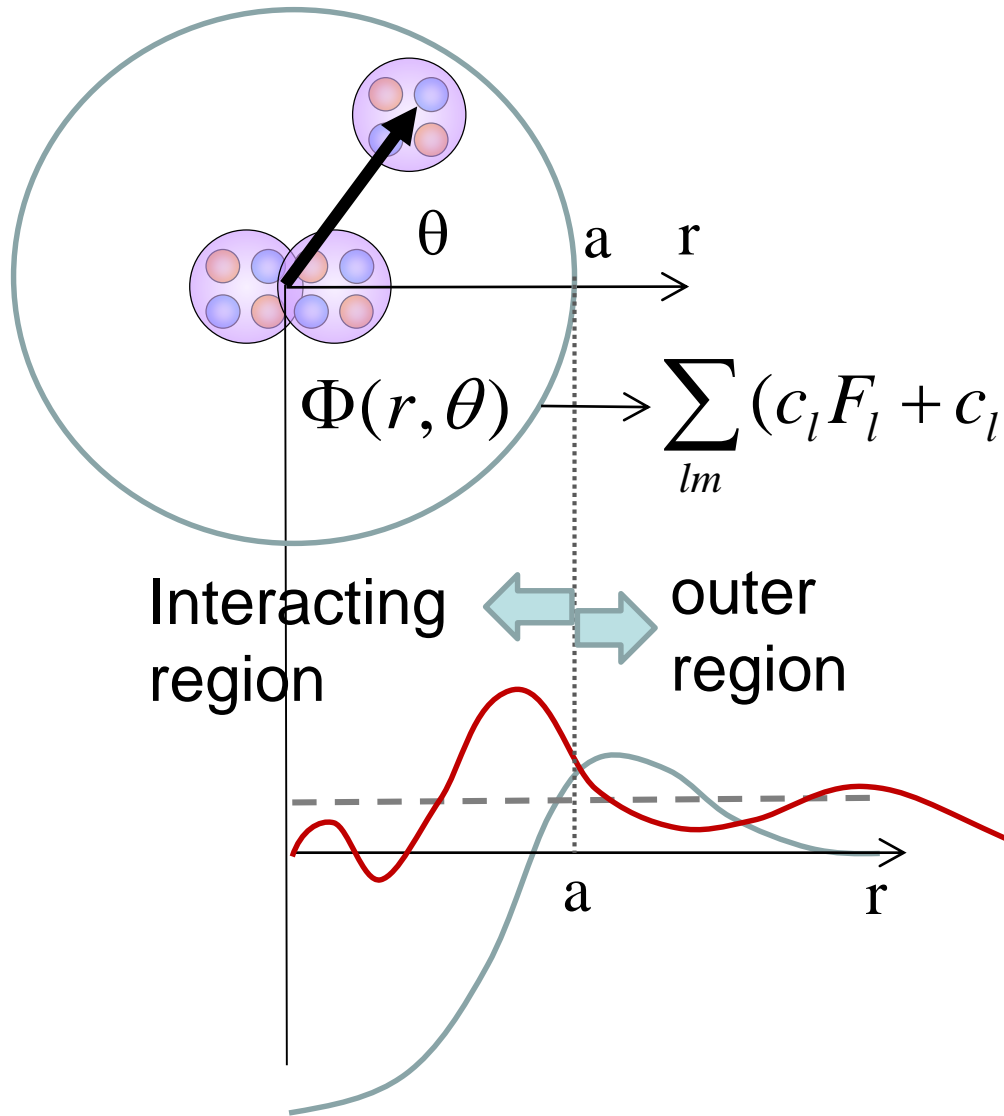
Ex) angular motion



probability



# Shape of resonance



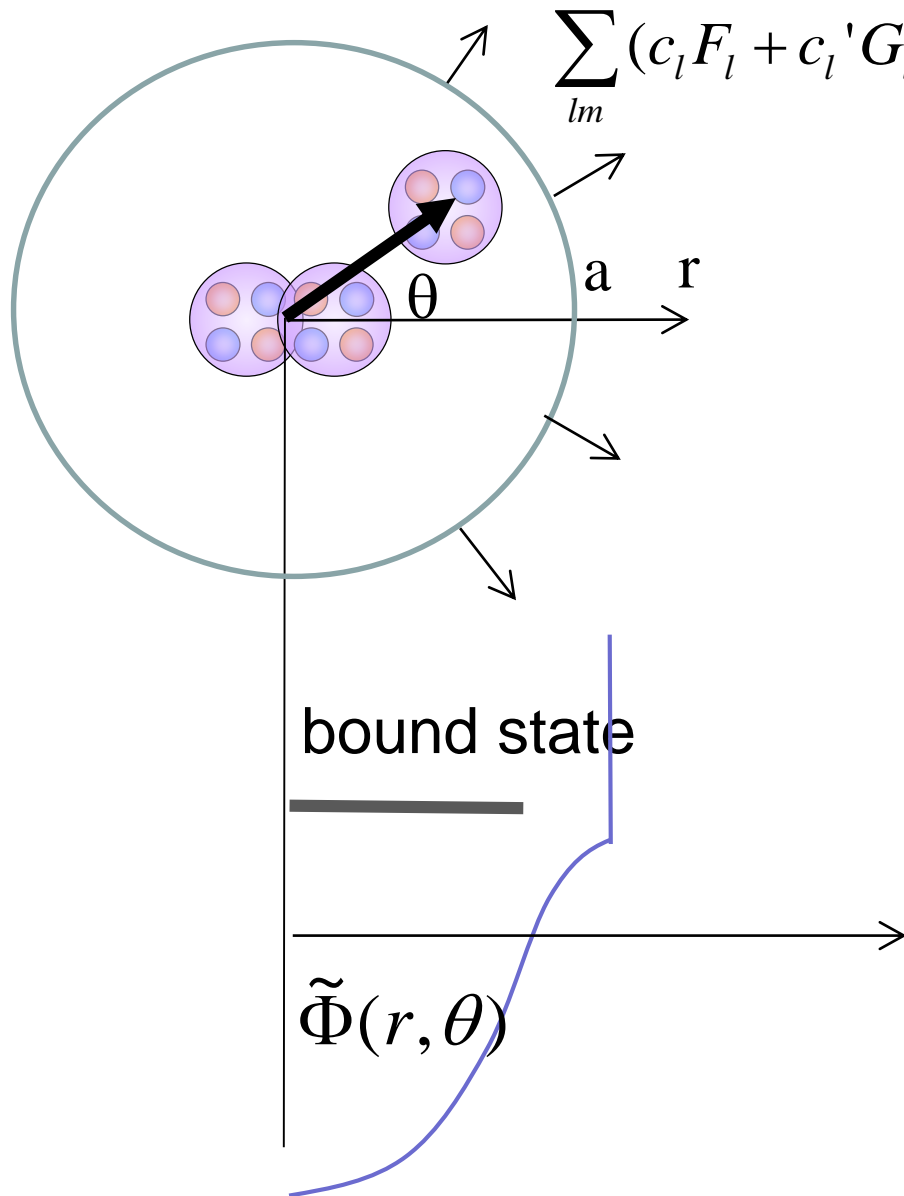
$$\Phi(r, \theta) \rightarrow \sum_{lm} (c_l F_l + c_l' G_l) Y_{lm} \quad \text{observable}$$

Inner: region of interest  
(3-body dynamics)

Outer: observable

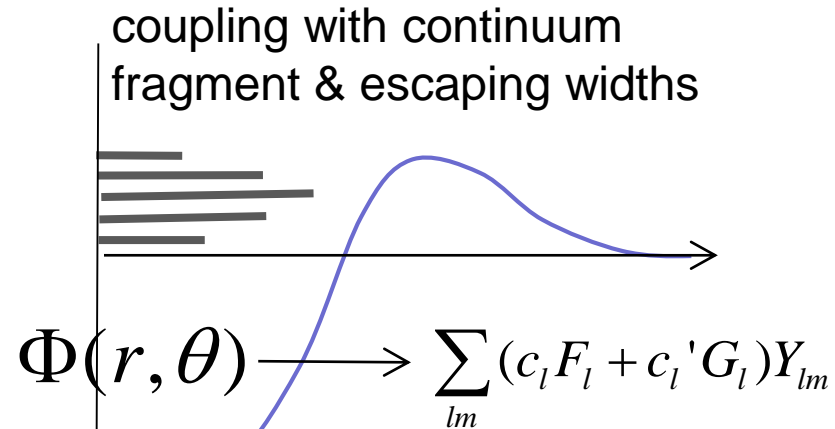
2b(3b) asymptotic  
reflects angular distribution  
in the inner region  $\Phi(r, \theta)$   
through coefficients  $c_l$

# Bound state approximation

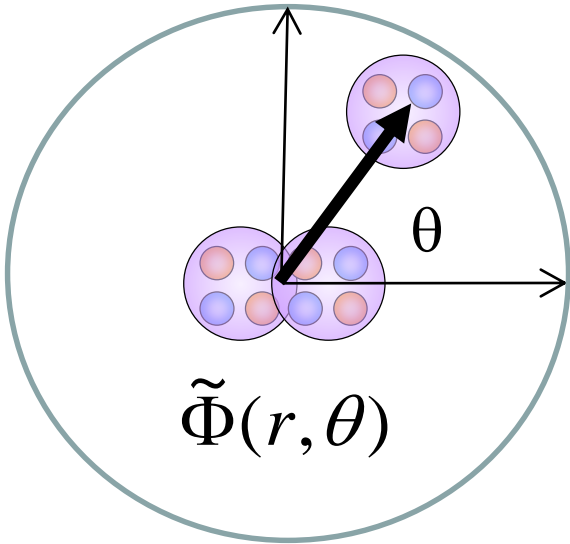


Case of linear chain state

- Resonance  $\tilde{\Phi} \approx \Phi$
- ✓ quasi bound state (complex  $E$ )  
linear chain state decaying to  $s+d+g\dots$  channels (mixed waves).
  - ✓ Scattering state (real  $E$ )  
head-on scattering



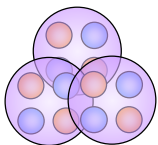
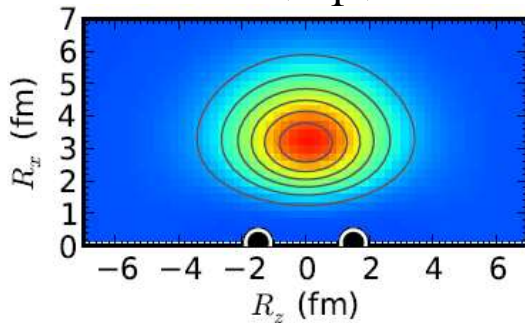
# 12C 0+ states (bound state approx.)



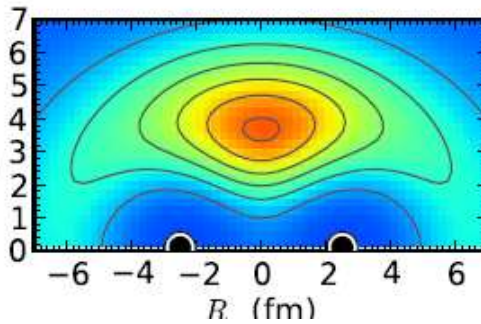
Linear chain state

- ✓ localization of amplitude
- large probability of linear chain cfg.
- ✓ head-on scattering
- large mixing of s,d,... waves

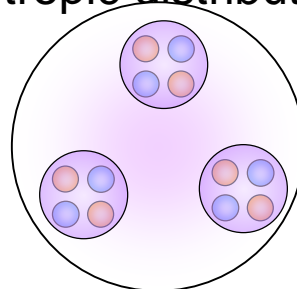
$^{12}\text{C}(0_1^+)$



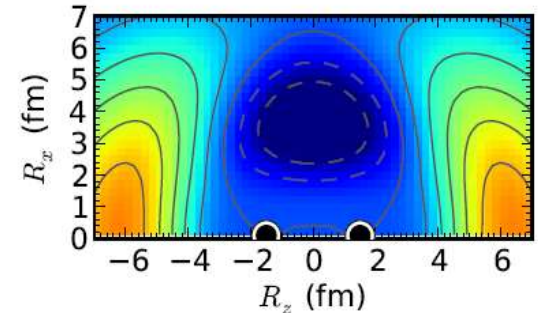
$^{12}\text{C}(0_2^+)$



Isotropic distribution



$^{12}\text{C}(0_3^+)$



Localized distribution  
dominant component:

