CANHP2015 (week 6), 25-30 Oct. 2015, Kyoto, Japan

Cluster-orbital shell model approach for unstable nuclei and the developments

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Outline of my talk

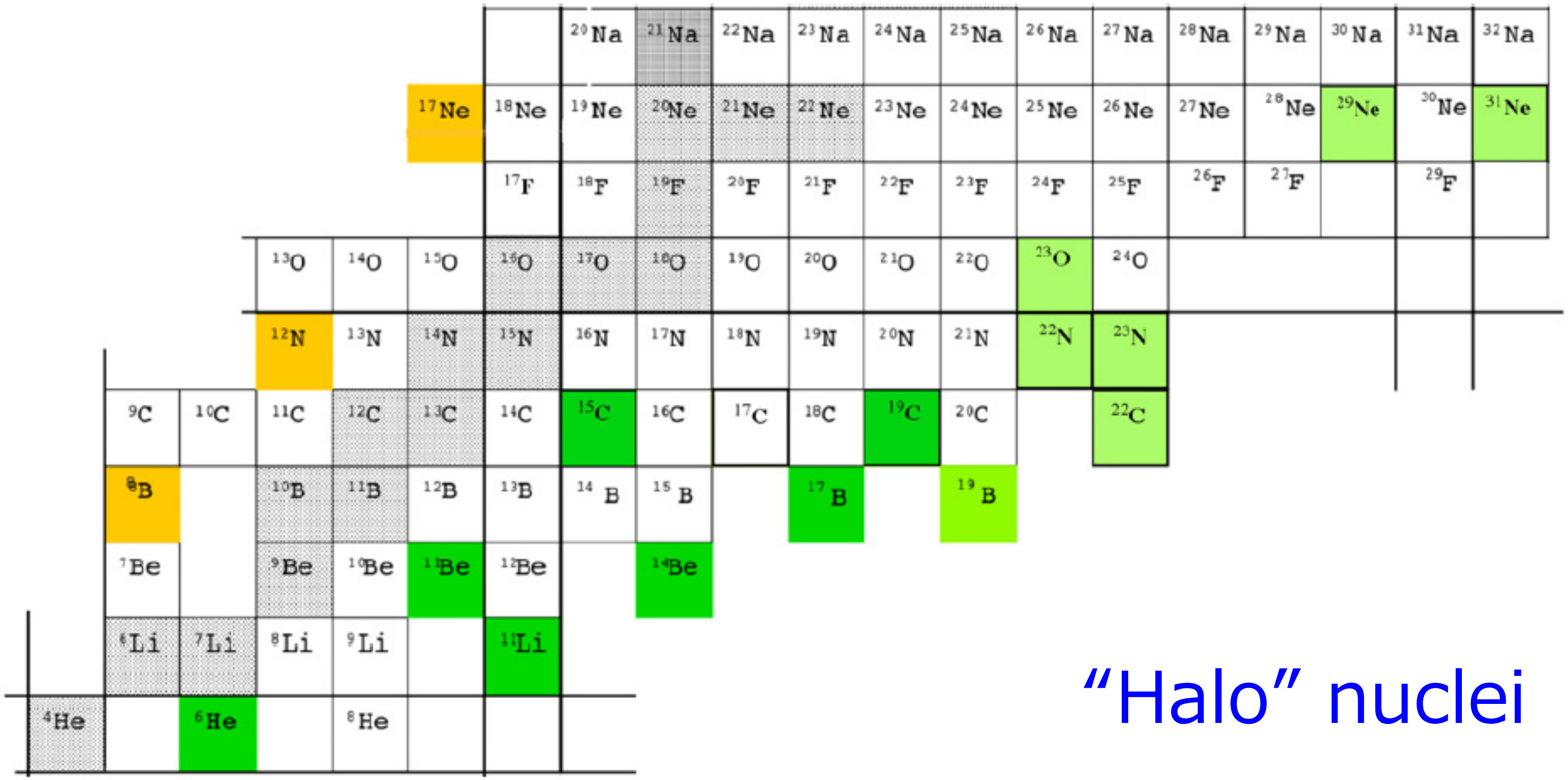
1. Cluster-orbital shell model approach

2. Comparison with Gamow shell model

3. Radius of oxygen isotopes

0. Basic motivation

Nuclear structure and the size

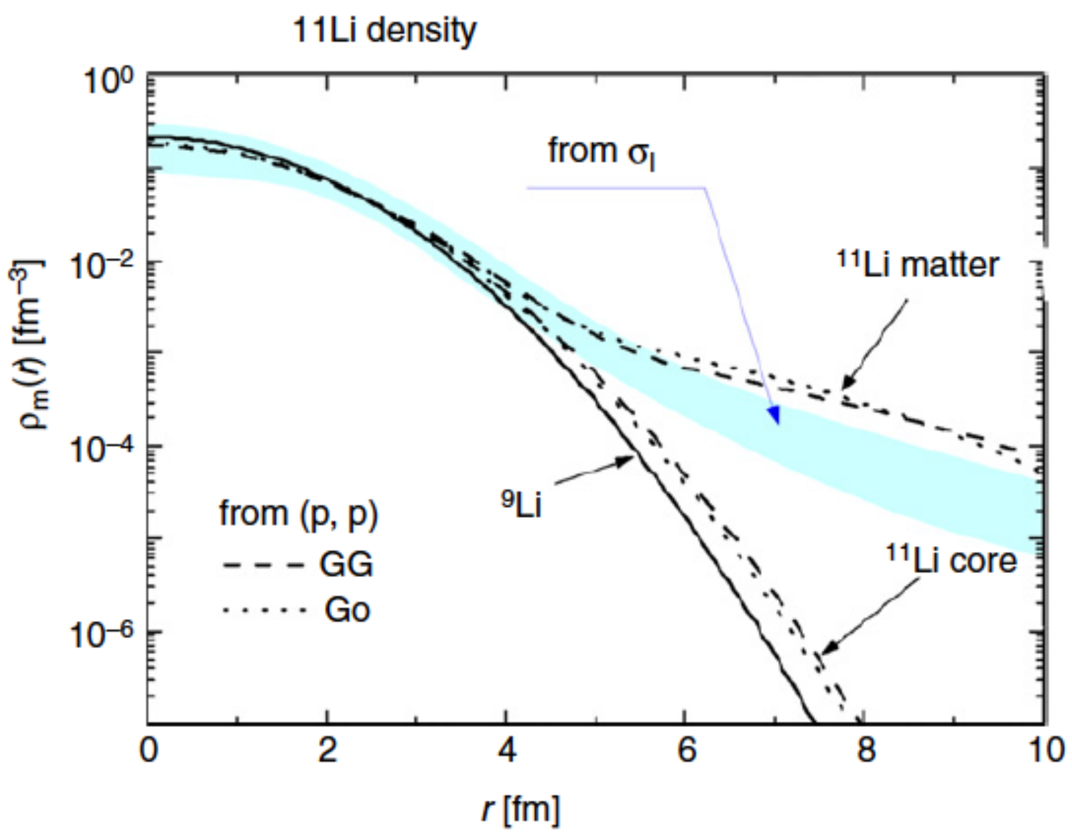


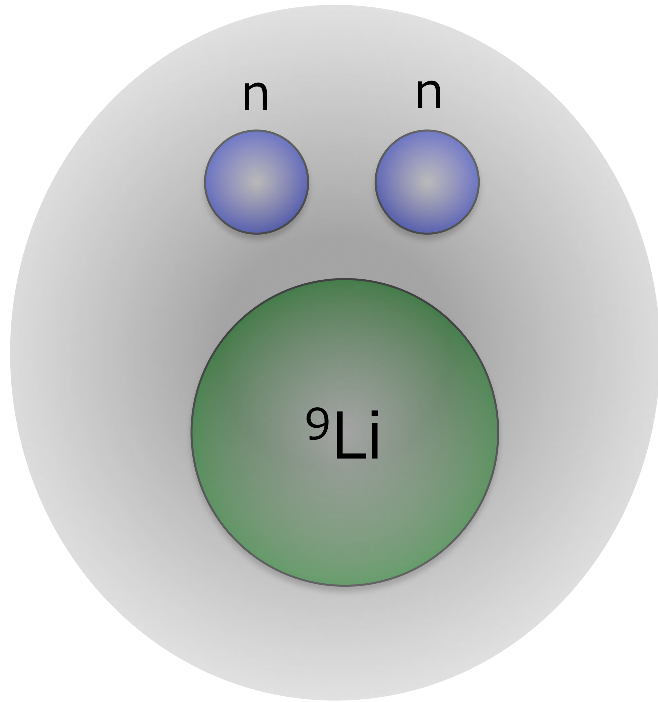
"Halo" nuclei

I. Tanihata, H. Savajols, R. Kanungo, PPNP68 (2013)

The typical halo nuclei: ^{11}Li (^{9}Li+2n)

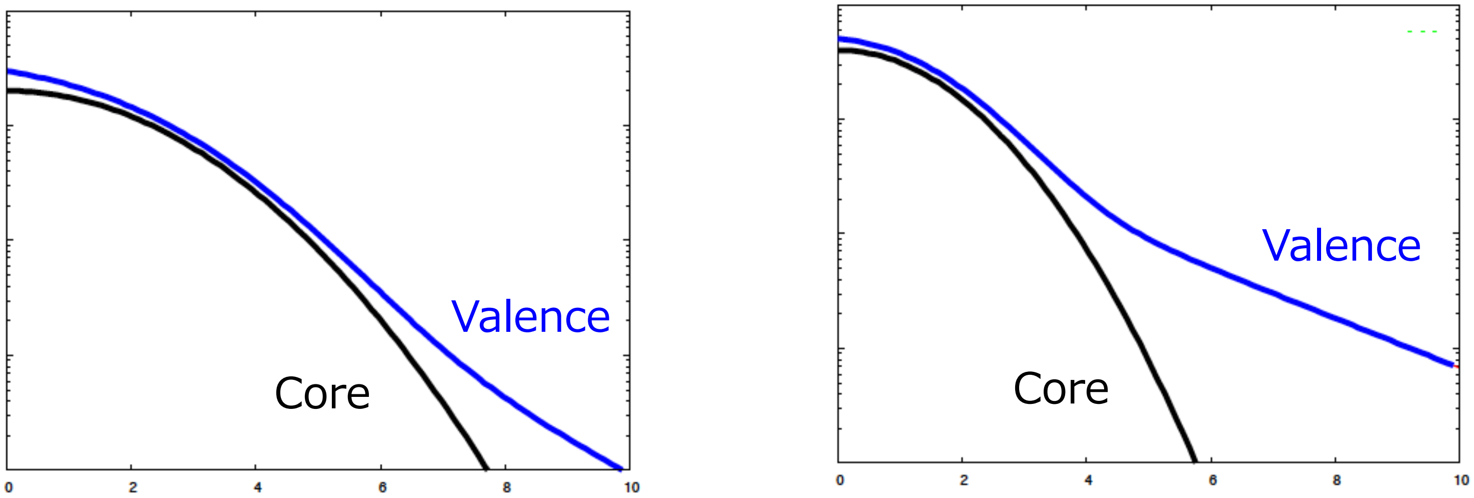
Loosely bound valence-nucleon above the core nucleus





I. Tanihata, H. Savajols, R. Kanungo, PPNP68 (2013)

We want to study these nuclei on the same footing



Cluster-orbital shell model (COSM)

Y. Suzuki and K. Ikeda, PRC38 (1988)

1. Cluster-orbital shell model

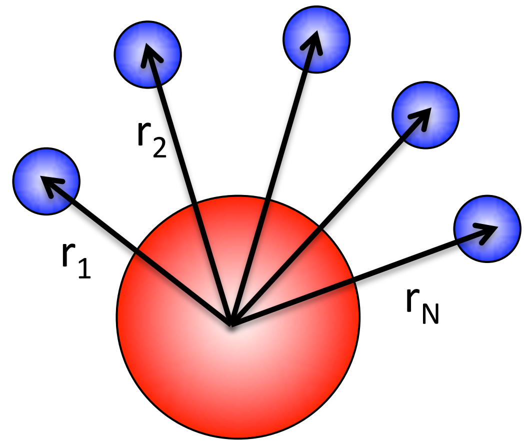
Cluster-orbital shell model approach

<u>Hamiltonian</u>   CoM motion is removed from the Hamiltonian

$$\hat{H} = \sum_i \hat{h}_i + \sum_{i<j} \left(\frac{p_i \cdot p_j}{A_C} + v_{ij} \right)$$

Core-N (1-body)   N-N (2-body)

Y. Suzuki and K. Ikeda, PRC38 (1988)



Core nucleus

- Anti-symmetrization with the nucleons in the core

→ Orthogonality Condition Model (OCM)

$$\boxed{\hat{\Lambda} \equiv \lambda |F.S.\rangle\langle F.S.|} \qquad \lambda \to \infty \qquad \text{(F.S. : Pauli-forbidden states)}$$

Basis function

$$\hat{H}\Psi_k = E_k\Psi_k$$

(For N-valence nucleon system)

Eigenfunction: $\Psi_k = \sum_{\alpha_1\alpha_2\cdots\alpha_N} C^{(k)}_{\alpha_1\alpha_2\cdots\alpha_N} \underline{\Phi_{\alpha_1\alpha_2\cdots\alpha_N}}$

$$\boxed{\Phi_{\alpha_1\alpha_2\cdots\alpha_N;\mathrm{core}} \equiv \mathcal{A}\left\{\Psi_{\mathrm{core}} \cdot [[\phi_{\alpha_1} \otimes \phi_{\alpha_2}] \otimes \cdots \otimes \phi_{\alpha_N}]^{JM}_{TM_T}\right\}}$$

Gaussian expansion (GEM) $\phi_{\alpha_i}(\boldsymbol{r}_i) = N_i r_i^{\ell_i} \exp(-\frac{1}{2}a_i r_i^2)\,[\ell_i \otimes s_i]_{j_i m_i}\,\chi^{\tau}_{m_{t_i}}$

Not eigenfunctions of the core+N sub-system

Width parameter: Geometrical progression

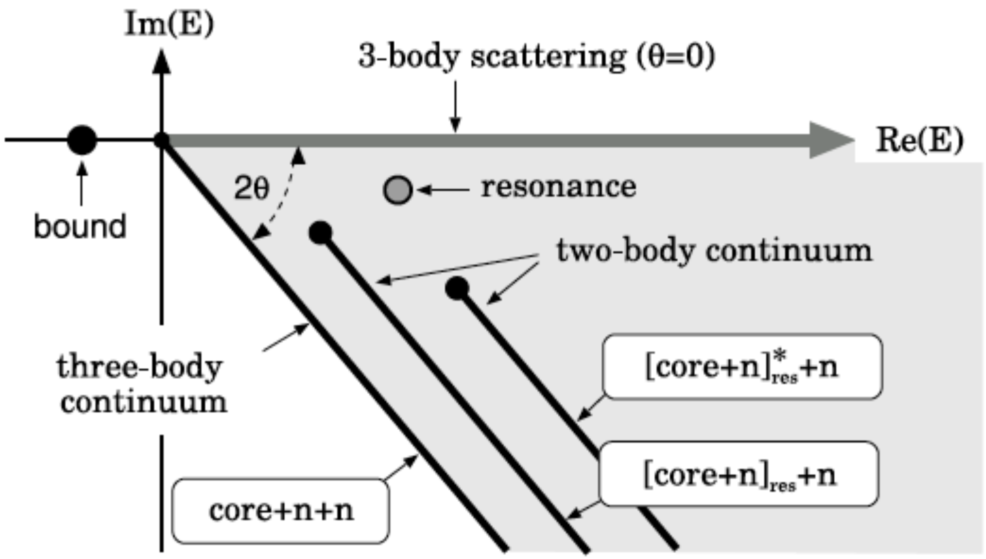
$$a_i = b_0\gamma^{q-1} \quad (q = 1, 2, \cdots, q_{\max})$$

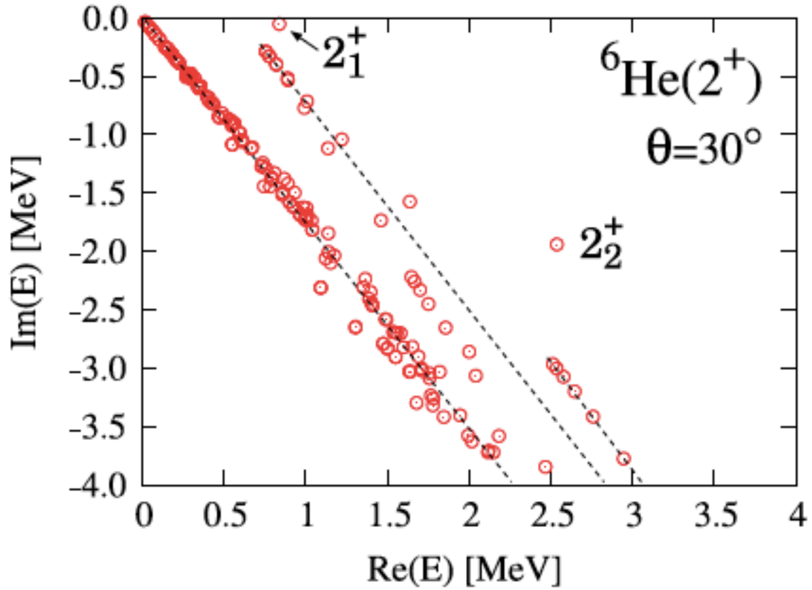
E. Hiyama, Y. Kino, and M. Kamimura, Prog. Part. Nucl. Phys. 51, 223 (2003).
S. Aoyama, T. Myo, K. Kato, and K. Ikeda, Prog. Theor. Phys. 116, 1 (2006).
T. Myo, Y. Kikuchi, H. M and K. Kato, Prog. Part. Nucl. Phys. 79, 1 (2014).

Combined with the Complex scaling method (CSM)

$$r_i \to r_i e^{i\theta} \quad (i : 1, 2, \cdots N)$$

Complex-scaled eigenvalues on the physical Riemann sheet

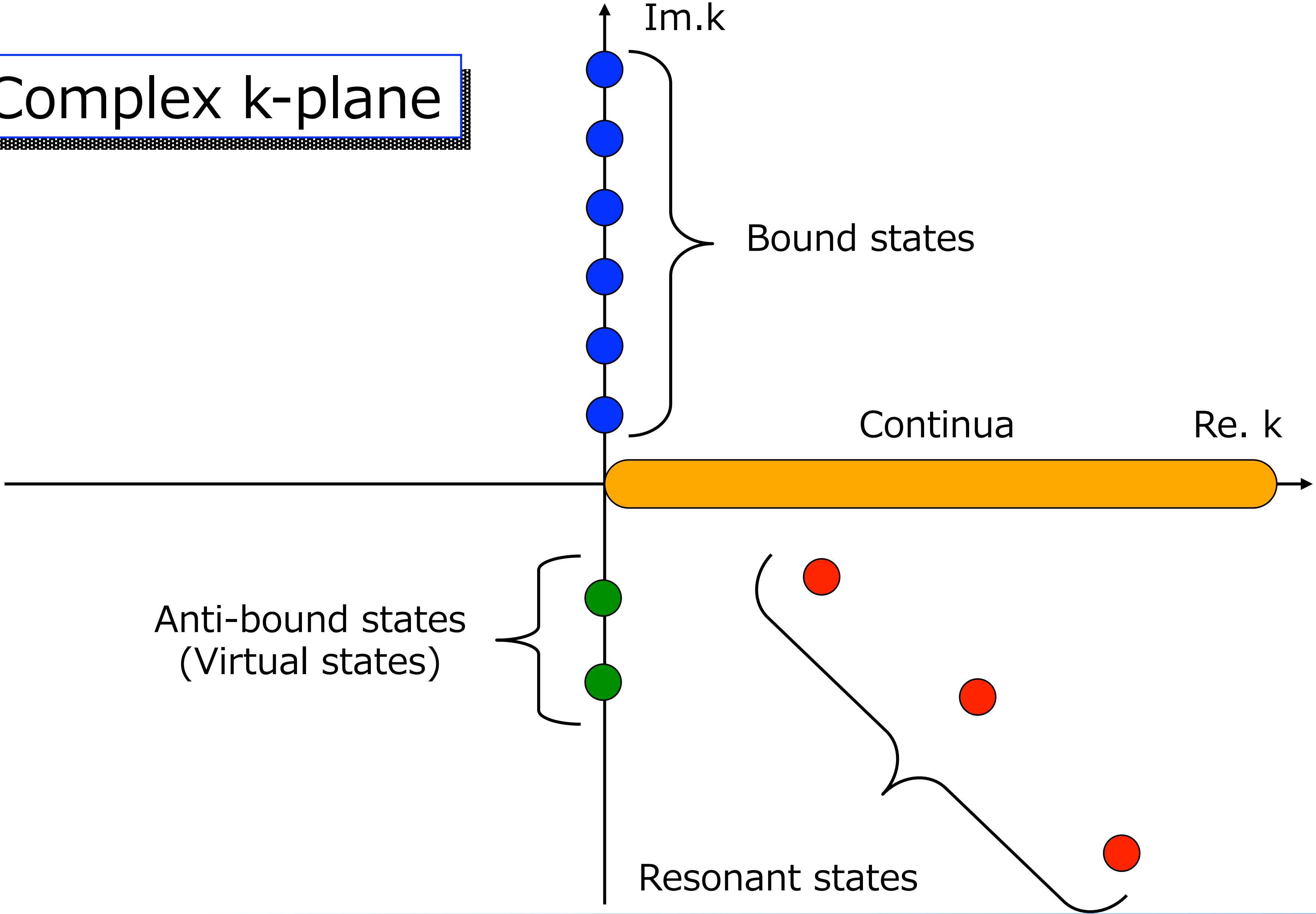




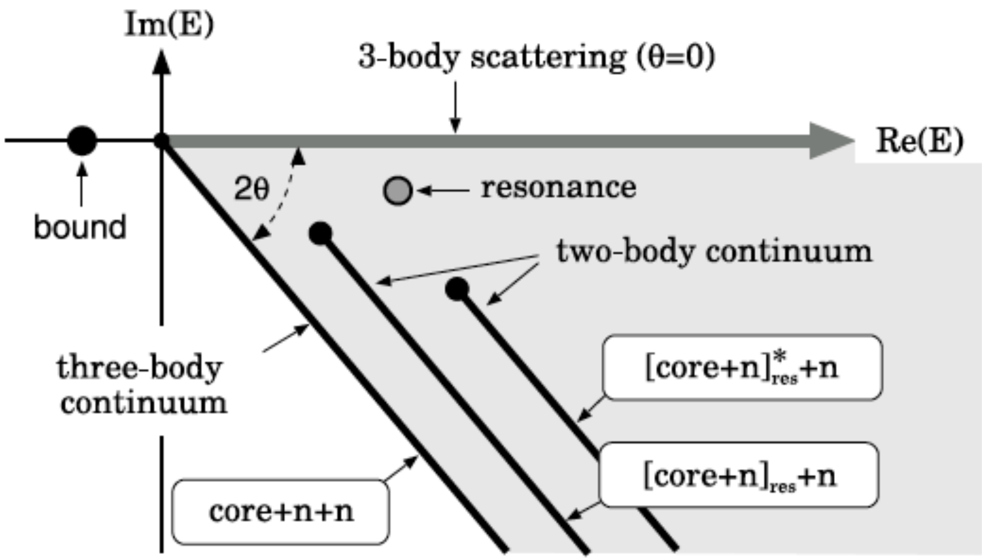
T. Myo, Y. Kikuchi, H. M and K. Kato, Prog. Part. Nucl. Phys. 79, 1 (2014).

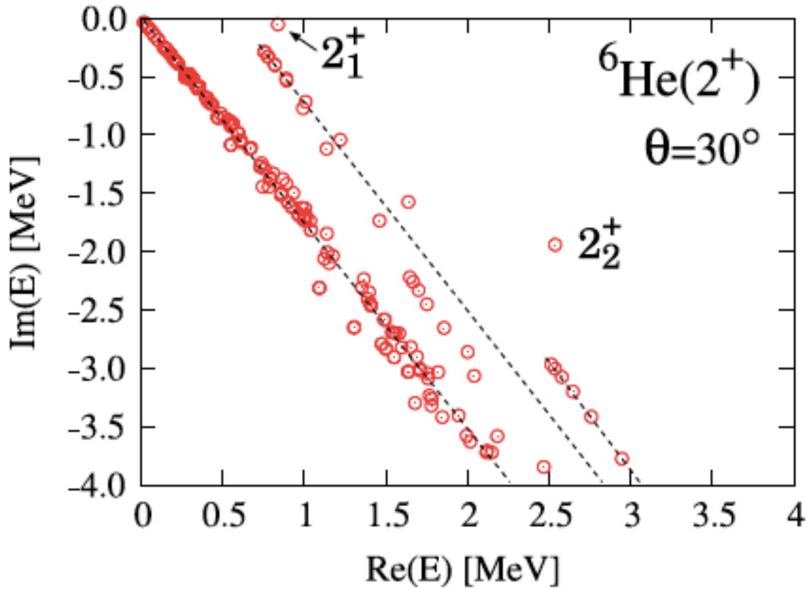
2. Comparison with Gamow shell model

Complex k-plane



Complex scaling method





"Resolution of the identity"

$$\mathbf{1} = \sum_b^{n_b} |\Psi_b^\theta\rangle\langle\tilde{\Psi}_b^\theta| + \sum_r^{n_r^\theta} |\Psi_r^\theta\rangle\langle\tilde{\Psi}_r^\theta| + \int_{L_\theta^E} dE|\Psi_E^\theta\rangle\langle\tilde{\Psi}_E^\theta|$$

$$= \sum_b^{n_b} |\Psi_b^\theta\rangle\langle\tilde{\Psi}_b^\theta| + \sum_r^{n_r^\theta} |\Psi_r^\theta\rangle\langle\tilde{\Psi}_r^\theta| + \int_{L_\theta^k} dk|\Psi_k^\theta\rangle\langle\tilde{\Psi}_k^\theta|$$

Gamow shell model approach

N. Michel, W. Nazarewicz, M. Płoszajczak, and K. Bennaceur,
Phys. Rev. Lett. 89, 042502 (2002)

R. Id Betan, R. J. Liotta, N. Sandulescu, and T. Vertse,
Phys. Rev. Lett. 89, 042501 (2002)

N. Michel, W. Nazarewicz, M. Płoszajczak, and J. Okołowicz,
Phys. Rev. C 67, 054311 (2003)

N. Michel, W. Nazarewicz, and M. Płoszajczak,
Phys. Rev. C70, 064313 (2004).

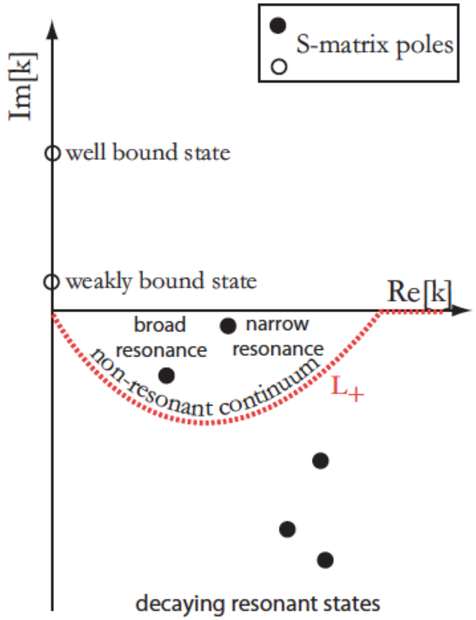
K. Fossez, N. Michel, M. Płoszajczak, Y. Jaganathen, and R. M. Id Betan
Phys. Rev. C 91, 034609 (2015)

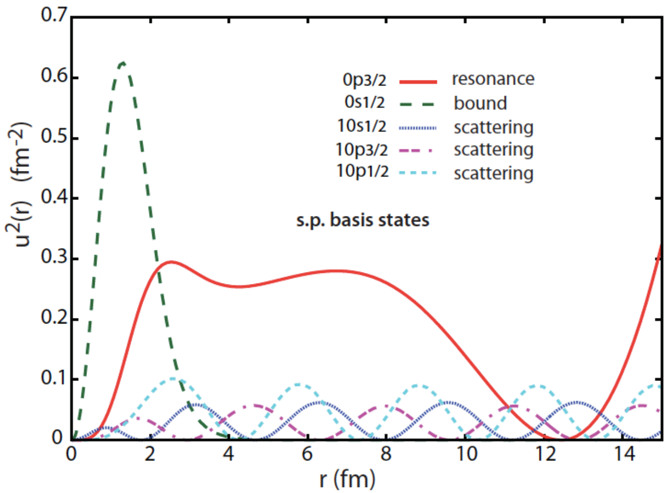
Gamow shell model

$$\Phi_{\mu_1\mu_2\cdots\mu_N;\,\mathrm{core}} \equiv \mathcal{A}\left\{\Psi_{\mathrm{core}}\cdot[[\psi_{\mu_1}\otimes\psi_{\mu_2}]\otimes\cdots\otimes\psi_{\mu_N}]^{JM}_{TM_T}\right\}$$

Single-particle states $\hat{h}\psi_{\mu_i} = \epsilon_{\mu_i}\psi_{\mu_i}$ $\Rightarrow$ $\sum_n \underline{|u_n\rangle\langle\tilde{u}_n|} + \int_{L_+} |u_k\rangle\langle\tilde{u}_k|dk = 1$

"Berggren basis"





G. Papadimitriou, J. Rotureau, N. Michel, M. Płoszajczak,and B. R. Barrett, Phys. Rev. C 88, 044318 (2013).

Basis function GEM with CS and GSM

GEM with CS

$$\boxed{\Phi_{\alpha_1\alpha_2\cdots\alpha_N;\,\mathrm{core}} \quad \equiv \quad \mathcal{A}\left\{\Psi_{\mathrm{core}} \cdot [[\phi_{\alpha_1} \otimes \phi_{\alpha_2}] \otimes \cdots \otimes \phi_{\alpha_N}]^{JM}_{TM_T}\right\}}$$

$$\phi_{\alpha_i}(\boldsymbol{r}_i) = N_i r_i^{\ell_i} \exp(-\frac{1}{2}a_i r_i^2)\,[\ell_i \otimes s_i]_{j_i m_i}\,\chi^{\tau}_{m_{t_i}}$$

GSM

$$\boxed{\Phi_{\mu_1\mu_2\cdots\mu_N;\,\mathrm{core}} \quad \equiv \quad \mathcal{A}\left\{\Psi_{\mathrm{core}} \cdot [[\psi_{\mu_1} \otimes \psi_{\mu_2}] \otimes \cdots \otimes \psi_{\mu_N}]^{JM}_{TM_T}\right\}}$$

$$\hat{h}\psi_{\mu_i} = \epsilon_{\mu_i}\psi_{\mu_i} \qquad\qquad \sum_n |u_n\rangle\langle\widetilde{u_n}| + \int_{L_+} |u_k\rangle\langle\widetilde{u_k}|dk = 1$$

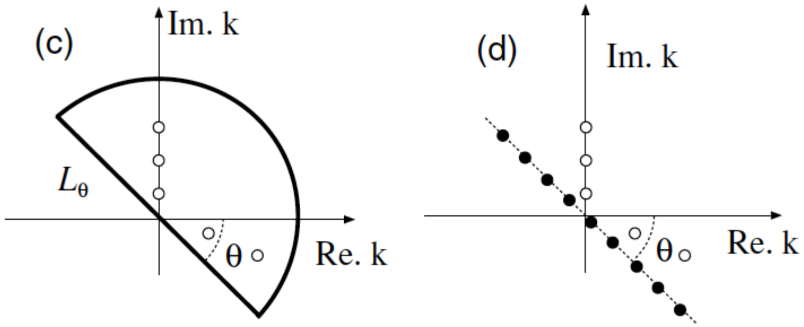
2-Steps for the comparison with GSM

H.M, K. Kato, K. Ikeda, PRC75(2007)034316

1. Prepare the single-particle complete set with the complex scaling

$$\mathbf{1}_i = \sum_{m=b,r} \left|\phi_{\theta,i}^{(m)}\right\rangle\left\langle\tilde{\phi}_{\theta,i}^{(m)}\right| + \oint_{L_\theta} dk|\phi_{\theta,i}(k)\rangle\langle\tilde{\phi}_{\theta,i}(k)| \equiv \sum_{\nu} |\phi_{\theta,i}[\nu]\rangle\langle\tilde{\phi}_{\theta,i}[\nu]|.$$

$$\phi_\theta(r) = \sum_{m'} d^{(m')}u^{(m')}(re^{i\theta}) = \sum_{m'} d^{(m')}g^{(m')}(re^{i\theta})\cdot\{[l\otimes s]_j\cdot\chi^\tau\}^{(m')}$$



2. Expand the many-body state with the complex-scaled single-particle states

$$\begin{aligned}|\Psi_V\rangle &= \mathbf{1}_1\otimes\ldots\otimes\mathbf{1}_N\sum_m c^{(m)}\mathcal{A}\left\{F^{(m)}\left|JMTM_T^{(m)}\right\rangle\right\} \\ &= C_1|(\phi_{\theta,1}[\nu_1]\ldots\phi_{\theta,N}[\nu_N])_1\rangle \\ &\quad + C_2|(\phi_{\theta,1}[\nu_1']\ldots\phi_{\theta,N}[\nu_N'])_2\rangle + \ldots \\ &= \sum_k C_k|(\phi_{\theta,1}[\nu_1]\ldots\phi_{\theta,N}[\nu_N])_k\rangle.\end{aligned}$$

$$C_k \equiv \langle(\tilde{\phi}_{\theta,1}[\nu_1]\ldots\tilde{\phi}_{\theta,N}[\nu_N])_k|\Psi_V\rangle$$

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Comparison with the Gamow shell model

H.M, K. Kato, K. Ikeda, PRC75(2007)034316

Two examples:

^{18}O (^{16}O+2n) : Stable nucleus

Bound, narrow resonant states, continua

^{6}He (^{4}He+2n) : “Halo” nucleus

Resonant states, continua

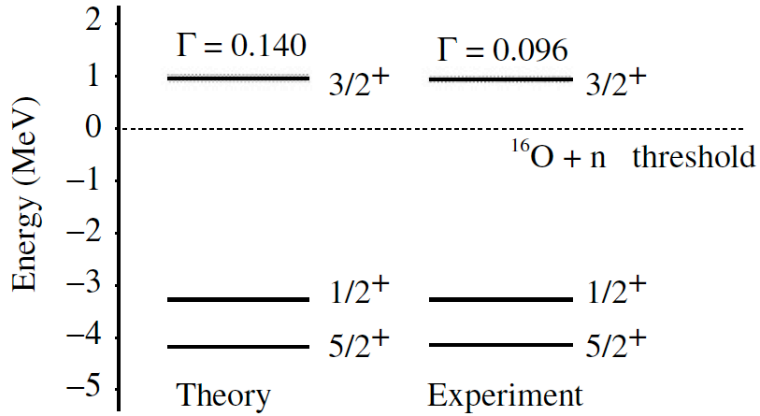
Oxygen systems

Interaction

- Core – N: Semi-microscopic potential [3]

$$\hat{h}_i[b] = \hat{t}_i + \boxed{\hat{V}_i^d[b] + \hat{V}_i^{\mathrm{ex}}[b]} + \hat{V}_{ls}[b] + \lambda\hat{\Lambda}_i[b].$$

Adjusted to ^{17}O ($5/2^+$, $1/2^+$, $3/2^+$)



- N-N : Volkov No.2 [4]
  (M=0.58, B=H=0.07)

Adjusted to ^{18}O (0^+)

Size parameter of the core

^{16}O core (h.o.): fixed size as $R_{rms}(^{16}O)$ = 2.54 fm (b = 1.723 fm)

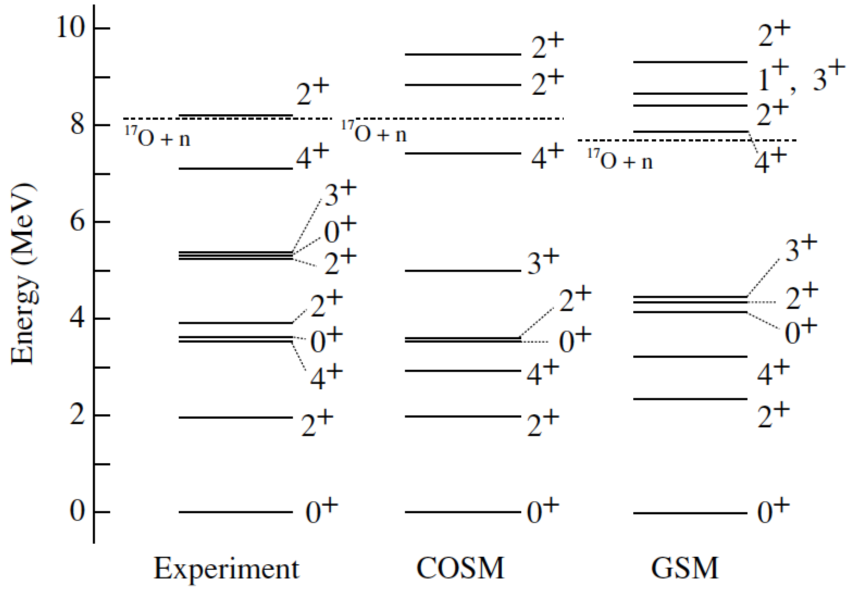
[3] T. Kaneko, M. LeMere, and Y. C. Tang, PRC44 (1991)
[4] A. B. Volkov, NPA74 (1965).

Results for the oxygen isotopes

Energy and R_{rms} of $^{17\text{-}20}O$

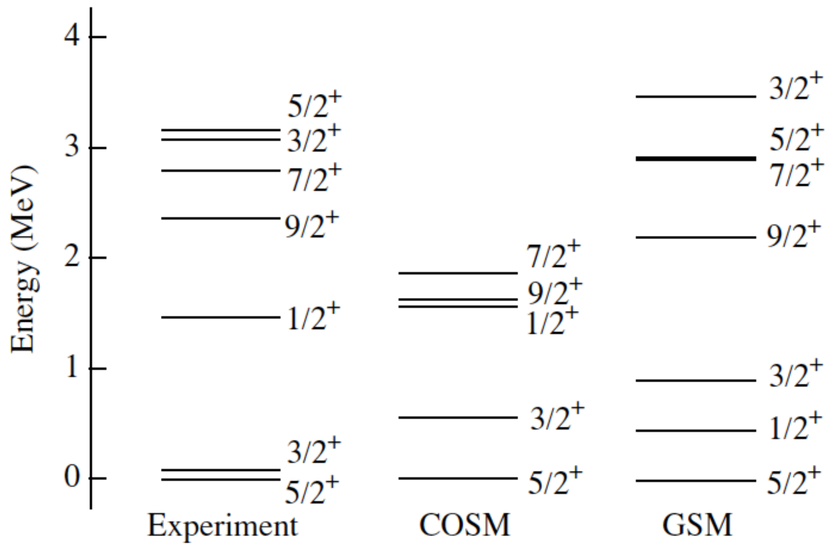
	^{17}O	^{18}O	^{19}O	^{20}O
S_n (MeV)				
Calc.	4.143	8.048	4.204	7.331
Exp. [38,39]	4.143	8.044	3.957	7.607
R_{rms} (fm)				
Calc.	2.59	2.64	2.67	2.71
Exp. [1]	2.59 ± 0.05	2.61 ± 0.08	2.68 ± 0.03	2.69 ± 0.03

Energy levels of ^{18}O

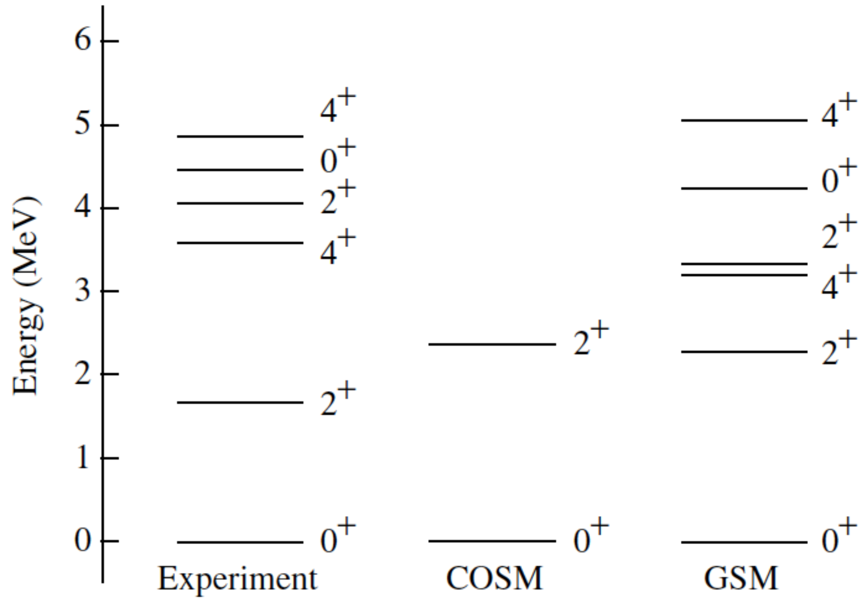


GSM: N. Michel, W. Nazarewicz, M. Płoszajczak, and J. Okołowicz, Phys. Rev. C 67, 054311 (2003)

Energy levels of ^{19}O



Energy levels of ^{20}O



GSM: N. Michel, W. Nazarewicz, M. Płoszajczak, and J. Okołowicz,
Phys. Rev. C 67, 054311 (2003)

Expansion with the single-particle states for ^{18}O

$(C_k)^2$	COSM	GSM [14]
$(0d_{5/2})^2$	$0.830 - i\epsilon$	$0.872 + i\epsilon$
$(1s_{1/2})^2$	$0.096 - i\epsilon$	$0.044 - i\epsilon$
$(0d_{3/2})^2$	$0.028 - i0.005$	$0.028 - i0.007$
$S1$	$0.020 + i0.004$	$0.042 + i0.005$
$S2$	$0.026 + i0.001$	$0.015 + i0.002$

Although the NN-int. are different, COSM and GSM give almost the same result.

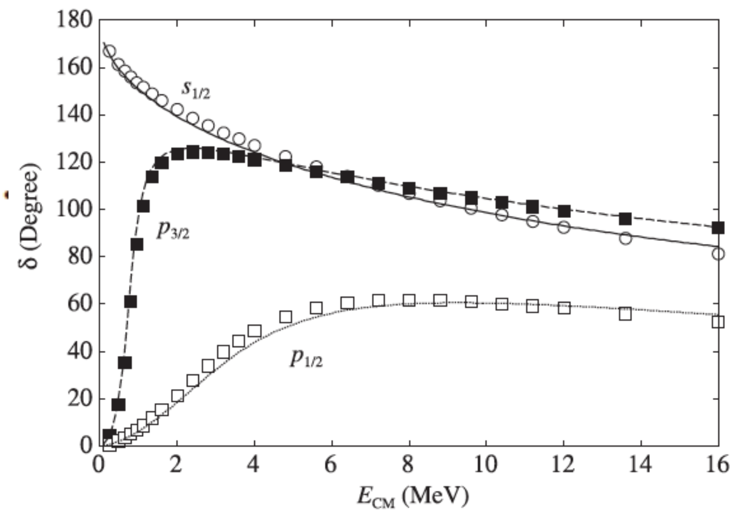
Helium systems

Interaction

- Core – N: Semi-microscopic potential [5]

$$\hat{h}_i = \hat{t}'_i + \boxed{\hat{V}^d_i + \hat{V}^{\mathrm{ls}}_i} + \lambda\hat{\Lambda}_i.$$

Reproduce the phase-shift of ^{4}He+n



- N-N : Minnesota [6]
  (u=1.0)

- Effective 3-body force[7]

Adjusted to ^{6}He ground state

[5] H. Kanada, T. Kaneko, S. Nagata, andM. Nomoto, Prog. Theor. Phys. 61, 1327 (1979).

[6] D. R. Thompson, M. LeMere, and Y. C. Tang, Nucl. Phys. A286, 53 (1977).

[7] T. Myo, K. Kato, S. Aoyama, and K. Ikeda, Phys. Rev. C 63, 054313 (2001).

Results for the Helium isotopes

Energy of Helium isotopes

		^{5}He	^{6}He	^{7}He	^{8}He
E (MeV)	GSM [14]	0.75	−0.98	0.18	−1.60
	GSM [19]	0.75	−0.98	−0.14	–
	COSM	0.74	−0.79	−0.89	−2.94
	Exp. [42]	0.89	−0.98	−0.53	−3.11

GSM[14]: N. Michel, W. Nazarewicz, M. Płoszajczak, and J. Okołowicz, Phys. Rev. C 67, 054311 (2003)

GSM[19]: G. Hagen, M. Hjorth-Jensen, and J. S. Vaagen, Phys. Rev. C 71, 044314 (2005).

R_{rms} of Helium isotopes

	Calc.	exp.: Ref. [43]	Ref. [44]	Ref. [45]
^{6}He	2.48	2.48 ± 0.03	2.33 ± 0.04	2.30 ± 0.07
^{8}He	2.66	2.52 ± 0.03	2.49 ± 0.04	2.45 ± 0.07

[42] F. Ajzenberg-Selove, NPA490, 1 (1988).
[43] I. Tanihata, NPA478, 795c (1998).
[44] I. Tanihata et al., PLB289, 261 (1992).
[45] G. D. Alkhazov et al., PRL78, 2313 (1997).

Expansion with the single-particle states for ^{6}He

$(C_k)^2$	COSM	GSM [14]	GSM [19]
$(0p_{3/2})^2$	$1.211 - i0.666$	$0.891 - i0.811$	$1.105 - i0.832$
$(0p_{1/2})^2$	$1.447 + i0.007$	$0.004 - i0.079$	$0.226 - i0.161$
$S1$	$-2.909 + i0.650$	$0.255 + i0.861$	$-0.259 + i1.106$
$S2$	$1.251 + i0.009$	$-0.150 + i0.029$	$-0.072 - i0.113$

$(p_{3/2})$-contribution for $^{6\text{-}8}$He

	COSM	GSM [14]	GSM [19]
^{6}He	$1.211 - i0.666$	$0.891 - i0.811$	$1.105 - i0.832$
^{7}He	$1.429 - i1.017$	$1.110 - i0.879$	$1.296 - i0.987$
^{8}He	$0.252 - i1.597$	$0.296 - i1.323$	–

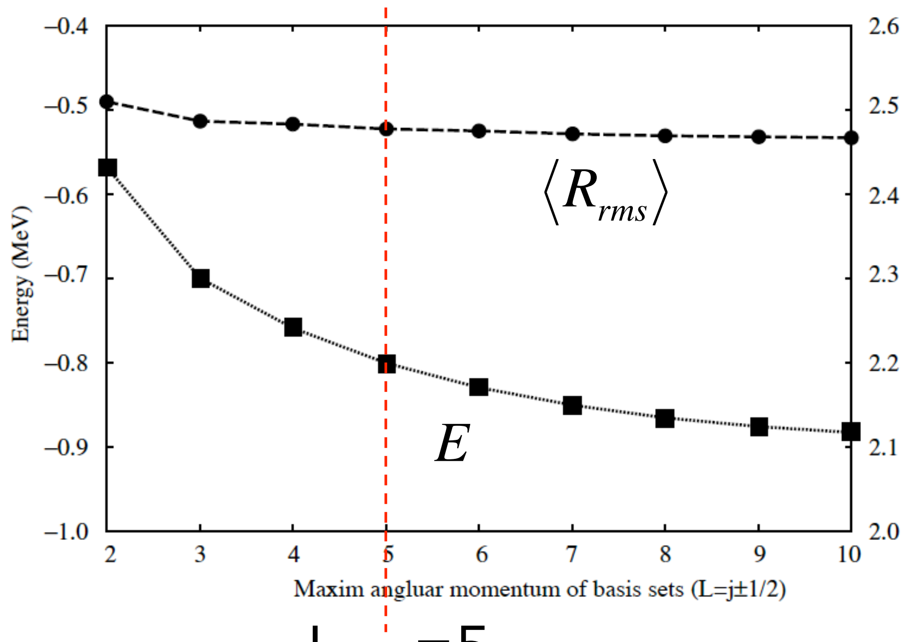
GSM[14]: N. Michel, W. Nazarewicz, M. Płoszajczak, and J. Okołowicz, Phys. Rev. C 67, 054311 (2003)
GSM[19]: G. Hagen, M. Hjorth-Jensen, and J. S. Vaagen, Phys. Rev. C 71, 044314 (2005).

$(p_{3/2})$- and $(p_{1/2})$-contributions for ^{6}He

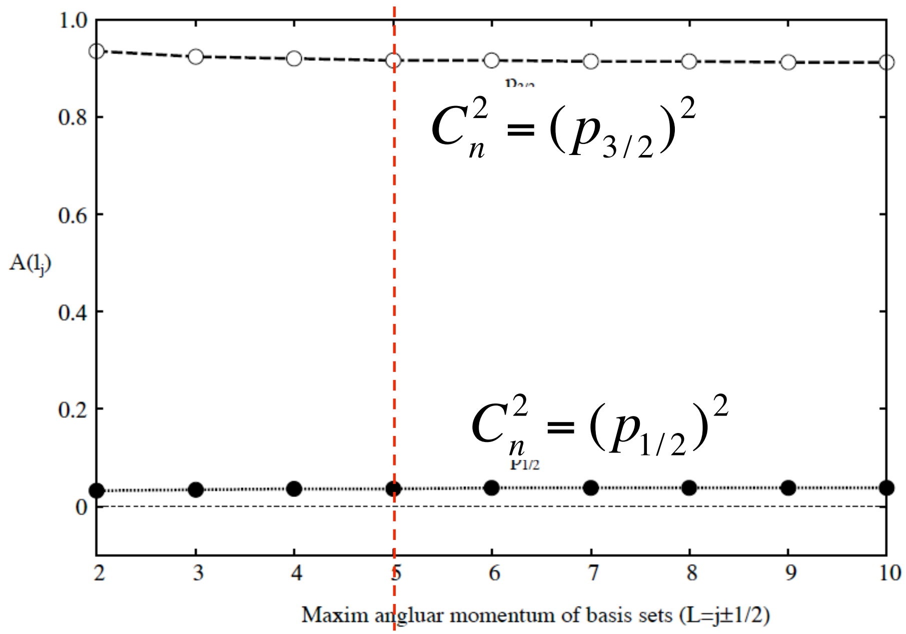
$(C_k)^2$	COSM	COSM ($l = 1$)	GSM [19]
$(0p_{3/2})^2$	$1.211 - i0.666$	$1.139 - i0.742$	$1.105 - i0.832$
$(S1)_{p3/2}$	$-0.252 + i0.692$	$-0.119 + i0.773$	$-0.060 + i0.881$
$(S2)_{p3/2}$	$-0.042 - i0.026$	$-0.060 - i0.031$	$-0.097 - i0.050$
sum	0.917	0.960	0.948
$(0p_{1/2})^2$	$1.447 + i0.007$	$0.353 - i0.077$	$0.226 - i0.161$
$(S1)_{p1/2}$	$-2.658 - i0.042$	$-0.534 + i0.065$	$-0.198 + i0.224$
$(S2)_{p1/2}$	$1.249 + i0.034$	$0.221 + i0.012$	$0.025 - i0.063$
sum	0.038	0.040	0.053
	(L_{max}=5)	(L=1)	(L=1)

When the model space is the same, results becomes similar

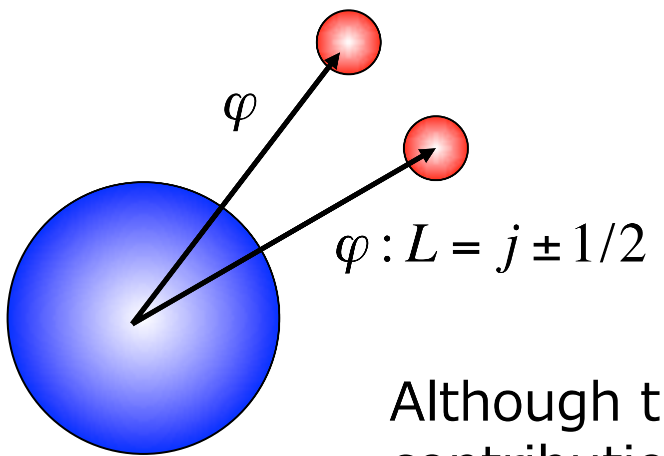
GSM[19]: G. Hagen, M. Hjorth-Jensen, and J. S. Vaagen, Phys. Rev. C 71, 044314 (2005).



$L_{max}=5$



$L_{max}=5$

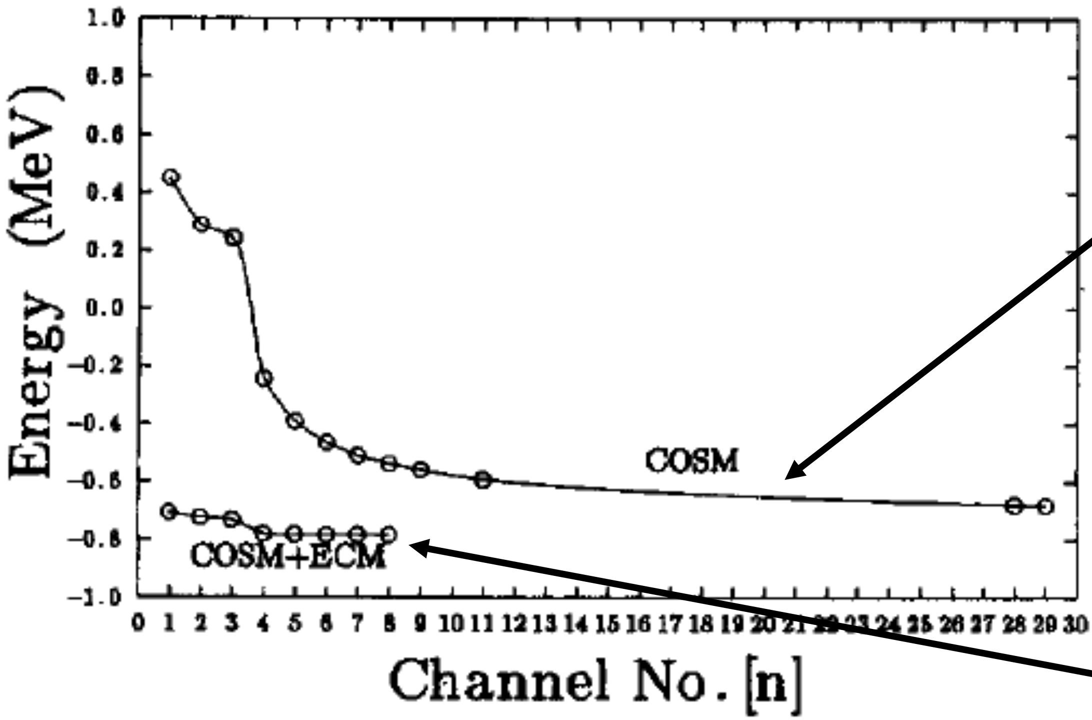


$$\left|\Psi\right\rangle = \sum C_n \left|\Phi_n\right\rangle$$

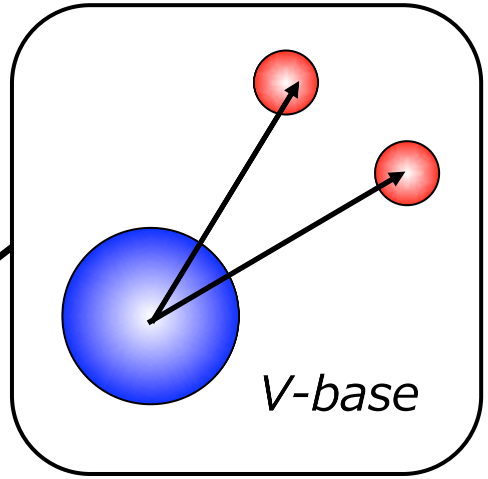
Although the number of channel increases, contributions of $(p_{3/2})^2$ and $(p_{1/2})^2$ are almost the same

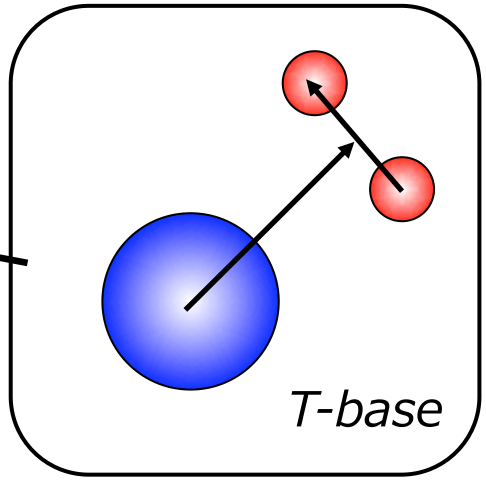
Correlation of valence nucleons

Energy of ^{6}He



S. Aoyama et al. PTP93 (1995)

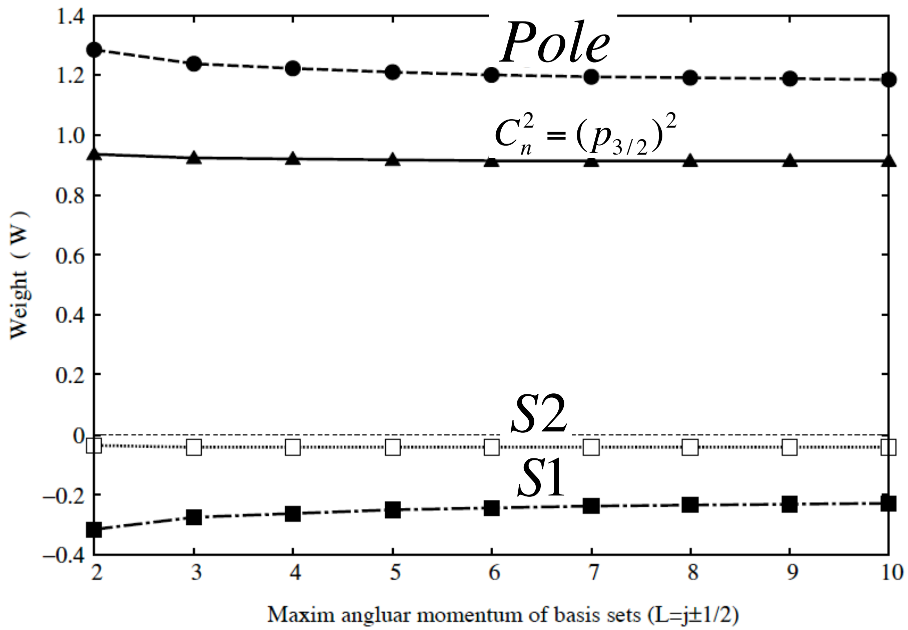




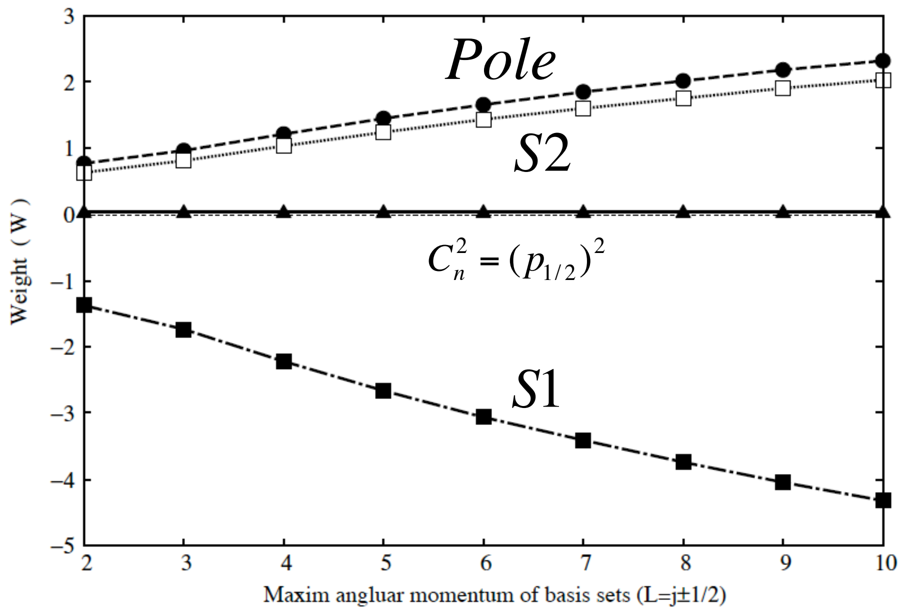
Details of poles and continua

$$C_n^2 = Pole + S1 + S2$$

$(p_{3/2})^2$



$(p_{1/2})^2$



Precise comparison between GEM with CS and GSM

PHYSICAL REVIEW C **89**, 044317 (2014)

Precise comparison of the Gaussian expansion method and the Gamow shell model

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PHYSICAL REVIEW C **89**, 014330 (2014)

Nuclear three-body problem in the complex energy plane: Complex-scaling Slater method

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(Received 28 October 2013; published 31 January 2014)

Precise comparison between two criteria

[Gaussian expansion/Slater basis] with complex scaling

$$\Phi_{\alpha_1\alpha_2\cdots\alpha_N;\mathrm{core}} \quad \equiv \quad \mathcal{A}\left\{\Psi_{\mathrm{core}}\cdot[[\phi_{\alpha_1}\otimes\phi_{\alpha_2}]\otimes\cdots\otimes\phi_{\alpha_N}]^{JM}_{TM_T}\right\}$$

Gamow shell model

$$\Phi_{\mu_1\mu_2\cdots\mu_N;\mathrm{core}} \quad \equiv \quad \mathcal{A}\left\{\Psi_{\mathrm{core}}\cdot[[\psi_{\mu_1}\otimes\psi_{\mu_2}]\otimes\cdots\otimes\psi_{\mu_N}]^{JM}_{TM_T}\right\}$$

Hamiltonian

$$\hat{H} = \sum_{i=1}^{2}\left(\hat{t}_i + \hat{V}_i^{(C)}\right) + \left(\hat{T}_{12} + \hat{v}_{12} + \hat{V}_{12}^{(C)}\right)$$

One-body part

$$\hat{V}_i^{(C)} = \hat{V}_i^{\alpha n} + \hat{V}_i^{\mathrm{Coul}} + \lambda\,\hat{\Lambda}_i$$

$$\hat{V}_i^{\alpha n}(r_i) = V_0^{\alpha n}(r_i) + 2V_{LS}^{\alpha n}(r_i)\,\boldsymbol{L}\cdot\boldsymbol{S},$$

KKNN-potential

$$\hat{V}_i^{\mathrm{Coul}}(r_i) = \frac{2e^2}{r_i}\,\mathrm{Erf}(\alpha\, r_i)$$

$$\hat{\Lambda}_i \equiv \left|F.S.\right\rangle\left\langle F.S.\right|_i$$

Two-body part

$$\hat{v}_{12}(\boldsymbol{r}_{12}) = \sum_{k=1}^{3} V_k^0 \left(W_k^{(u)} - M_k^{(u)} P^\sigma P^\tau + B_k^{(u)} P^\sigma - H_k^{(u)} P^\tau\right) \times \exp(-\rho_k\, \boldsymbol{r}_{12}^2)\ . \qquad (13)$$

$$\hat{T}_{12} = -\frac{\hbar^2}{M^{(C)}}\nabla_1\cdot\nabla_2$$

$$\hat{V}_{12}^{(C)}(r_1,r_2) = V_{\alpha nn}^0\,\exp\left(-\rho_{\alpha nn}\left(r_1^2 + r_2^2\right)\right)$$

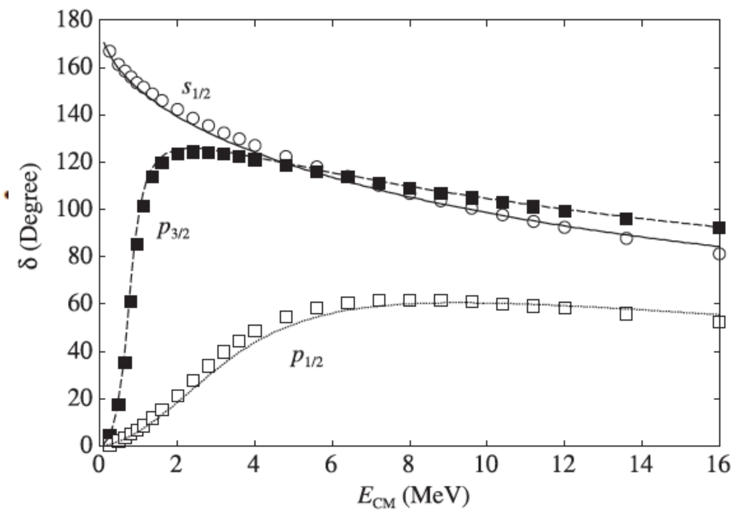
^{4}He+pp/nn systems

Interaction

- Core – N: Semi-microscopic potential [5]

$$\hat{h}_i = \hat{t}'_i + \boxed{\hat{V}^d_i + \hat{V}^{\mathrm{ls}}_i} + \lambda\hat{\Lambda}_i.$$

Reproduce the phase-shift of ^{4}He+n



- N-N : Minnesota [6]
  (u=1.0)

- Effective 3-body force[7]

Adjusted to ^{6}He ground state

- Core-N Coulomb pot.

$$\hat{V}^{\mathrm{Coul}}_i(r_i) = \frac{2e^2}{r_i}\,\mathrm{Erf}(\alpha\, r_i).$$

[5] H. Kanada, T. Kaneko, S. Nagata, andM. Nomoto, Prog. Theor. Phys. 61, 1327 (1979).

[6] D. R. Thompson, M. LeMere, and Y. C. Tang, Nucl. Phys. A286, 53 (1977).

[7] T. Myo, K. Kato, S. Aoyama, and K. Ikeda, Phys. Rev. C 63, 054313 (2001).

TBMEs

Gaussian/Slater basis (basis functions are analytic function)

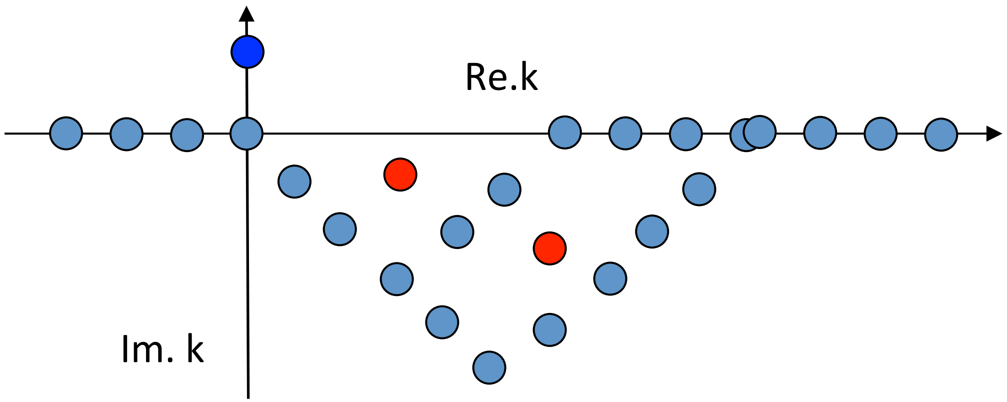
TBMEs can be calculated analytically

Gamow shell model (sinle-particle states are solved numerically)

Numerically obtained w.f. : Expanded with analytical func.

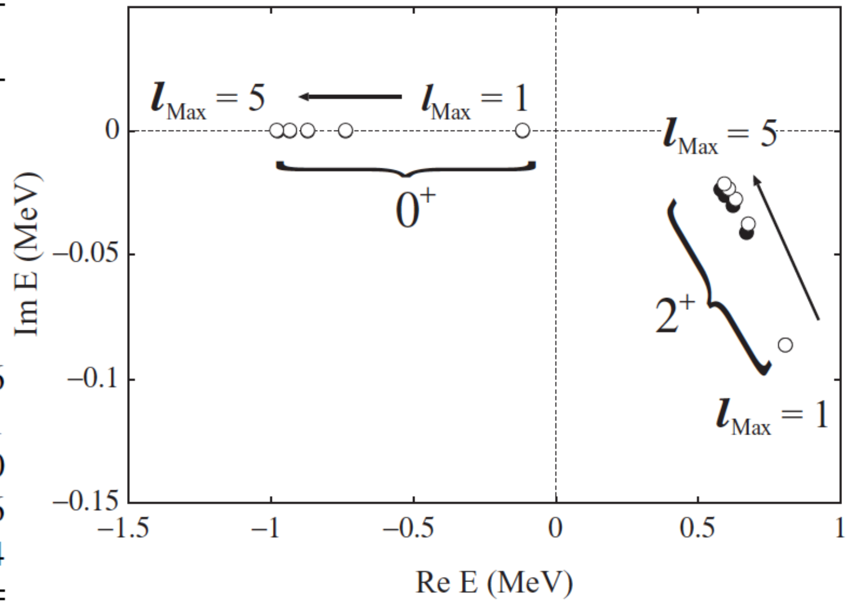
$$\langle \Phi_{\rm GSM}^{(i)} | \hat{O}_{12} | \Phi_{\rm GSM}^{(j)} \rangle$$
$$= \sum_{\alpha,\beta} \langle \Phi_{\rm GSM}^{(i)} | \Phi_{\rm HO}^{(\alpha)} \rangle \langle \Phi_{\rm HO}^{(\alpha)} | \hat{O}_{12} | \Phi_{\rm HO}^{(\beta)} \rangle \langle \Phi_{\rm HO}^{(\beta)} | \Phi_{\rm GSM}^{(j)} \rangle$$
$$= \sum_{\alpha,\beta} d_{i,\alpha}^{*} \, d_{j,\beta} \langle \Phi_{\rm HO}^{(\alpha)} | \hat{O}_{12} | \Phi_{\rm HO}^{(\beta)} \rangle,$$

$$d_{i,\alpha} \equiv \langle \Phi_{\rm HO}^{(\beta)} | \Phi_{\rm GSM}^{(i)} \rangle$$



Energy of ^{6}He (0^+ and 2^+)

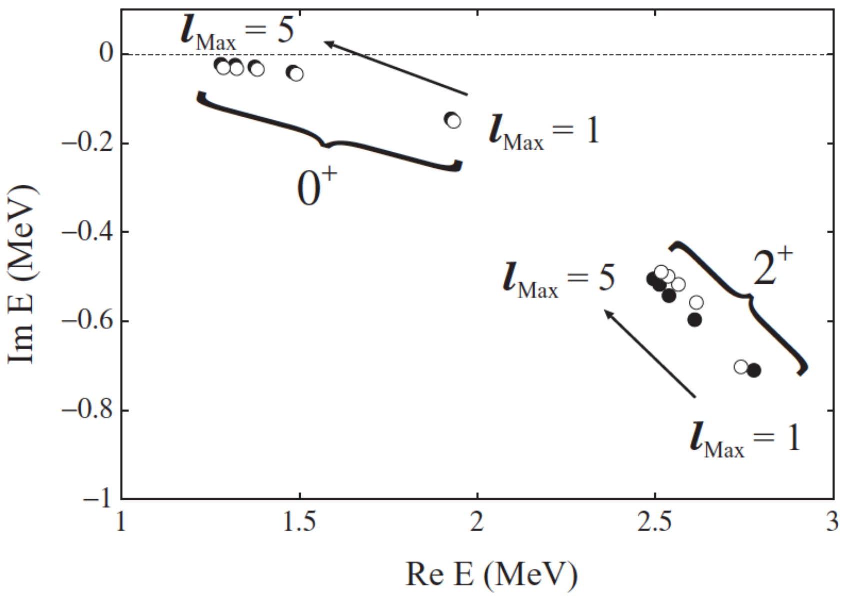
	ℓ_{max}	GEM+CS	GSM
$E(0_1^+)$	1	−0.117	−0.116
	2	−0.737	−0.737
	3	−0.870	−0.870
	4	−0.933	−0.932
	5	−0.978	−0.977
	ℓ_{max}	GEM+CS	GSM
$E(2_1^+)$	1	$0.805 - i0.086$	$0.804 - i0.086$
	2	$0.675 - i0.038$	$0.669 - i0.041$
	3	$0.628 - i0.027$	$0.619 - i0.030$
	4	$0.605 - i0.023$	$0.595 - i0.026$
	5	$0.589 - i0.021$	$0.577 - i0.024$



H. M, K. Kato, N. Michel, M. Płoszajczak, Phys. Rev. C 89 (2014) 044317.

Energy of ^{6}Be (0^+ and 2^+)

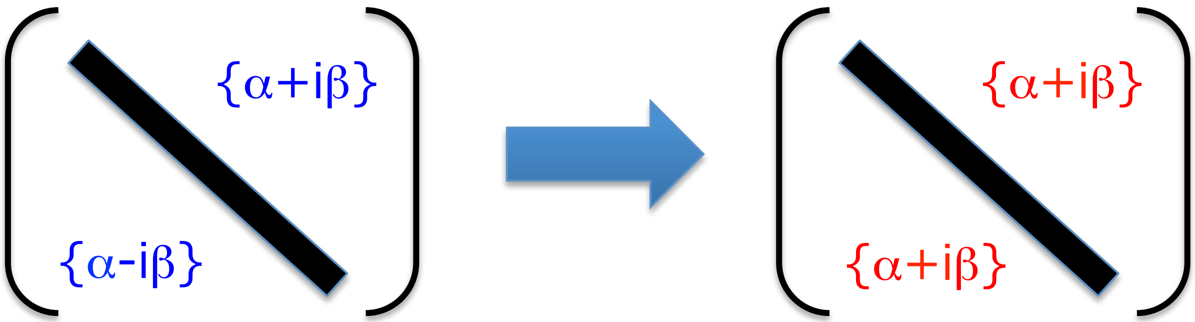
	ℓ_{max}	GEM+CS	GSM
	1	$1.932 - i0.152$	$1.926 - i0.146$
	2	$1.490 - i0.046$	$1.482 - i0.041$
$E(0_1^+)$	3	$1.380 - i0.036$	$1.374 - i0.030$
	4	$1.324 - i0.033$	$1.318 - i0.026$
	5	$1.285 - i0.031$	$1.279 - i0.024$
	ℓ_{max}	GEM+CS	GSM
	1	$2.741 - i0.703$	$2.776 - i0.711$
	2	$2.614 - i0.559$	$2.610 - i0.596$
$E(2_1^+)$	3	$2.565 - i0.518$	$2.538 - i0.543$
	4	$2.537 - i0.500$	$2.512 - i0.518$
	5	$2.517 - i0.491$	$2.495 - i0.505$



H. M, K. Kato, N. Michel, M. Płoszajczak, Phys. Rev. C 89 (2014) 044317.

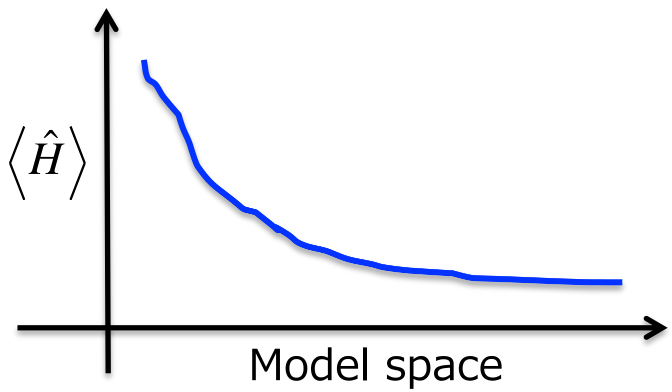
Convergence of the calculation and the related parameters

Complex rotation/Berggren basis function

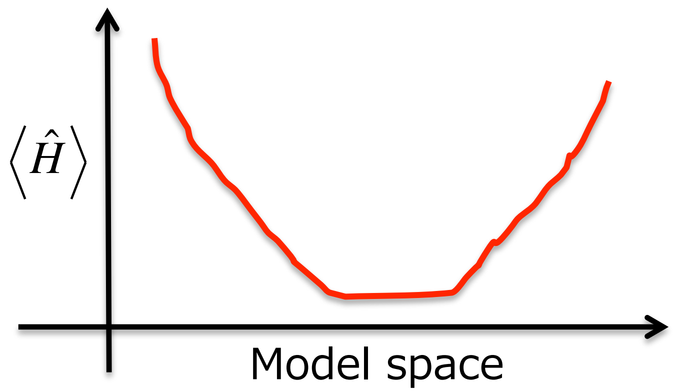


Non-Hermitian (complex symmetric Hamiltonian matrix elements)

variational problem



"Generalized variational problem"



Variational parameters

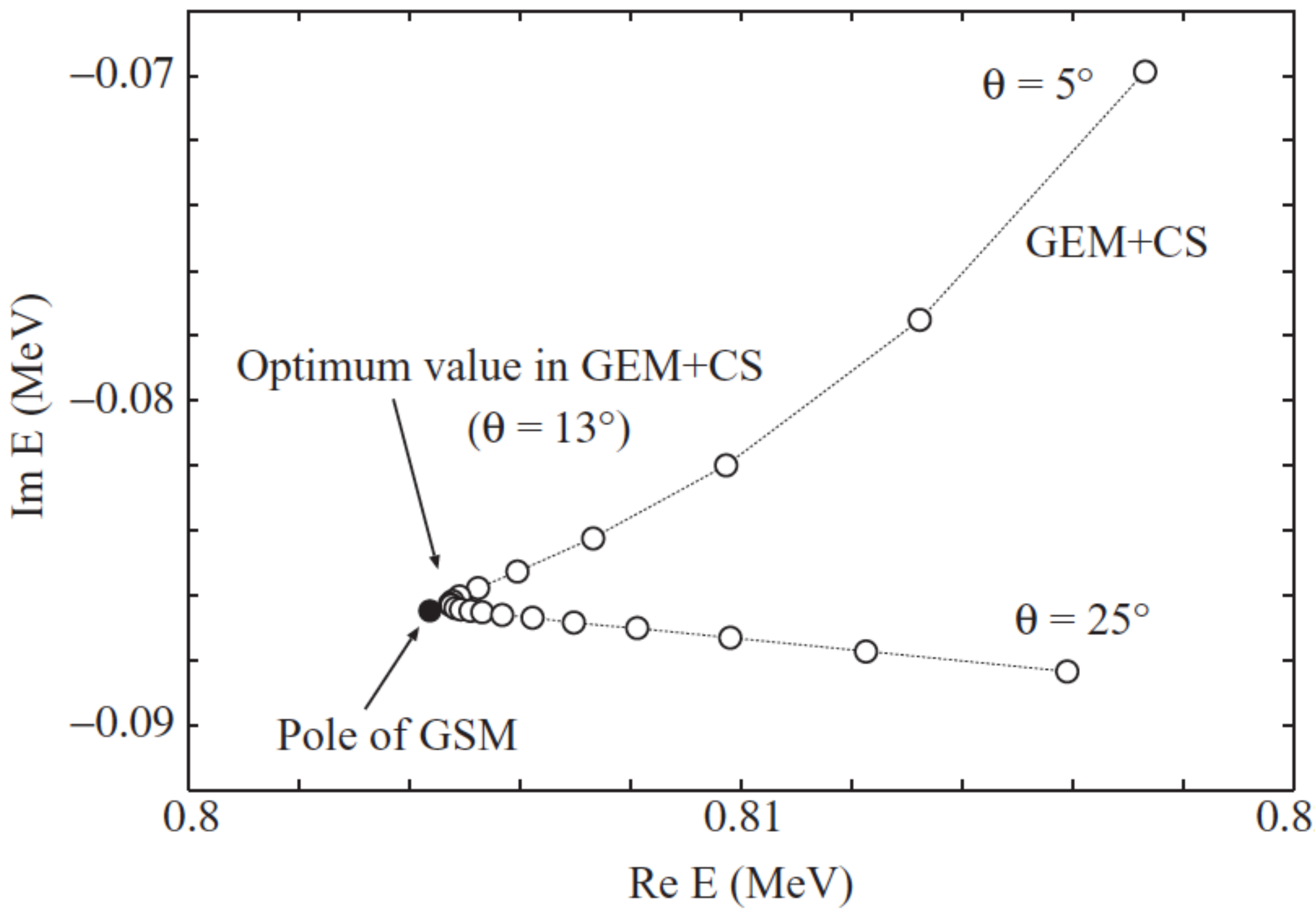
$\boxed{\text{GEM + CS}}$

- Gaussian width parameters: b, N_{max}
- Complex rotation angle: θ

$\boxed{\text{GSM}}$

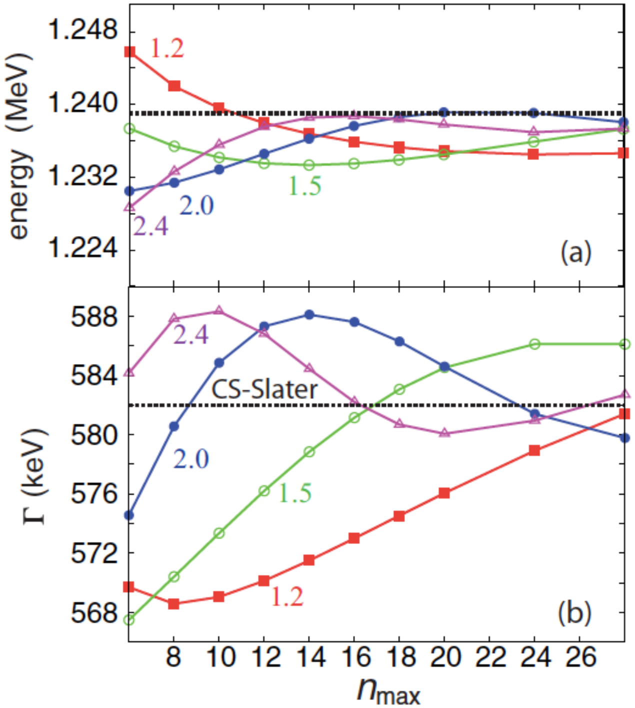
- Discretization of continuum and the contour
- Parameters in H.O. expansion: $h\omega$, N_{max}

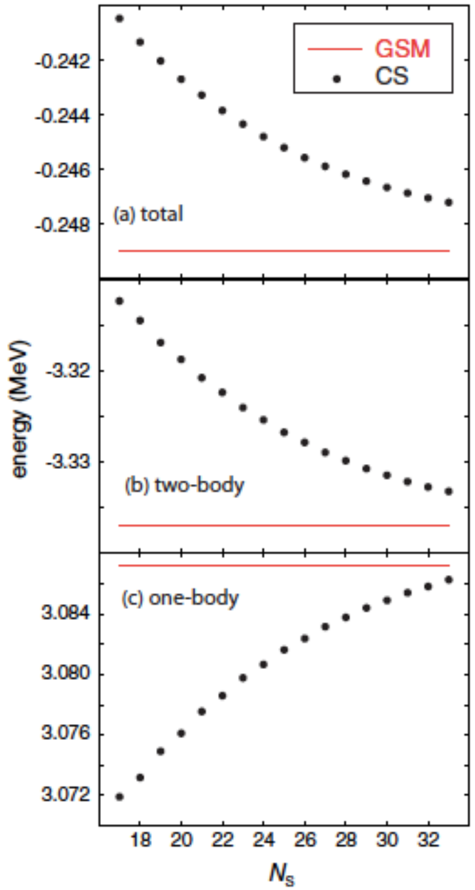
Convergence of resonant poles of ^{6}He (2^+)

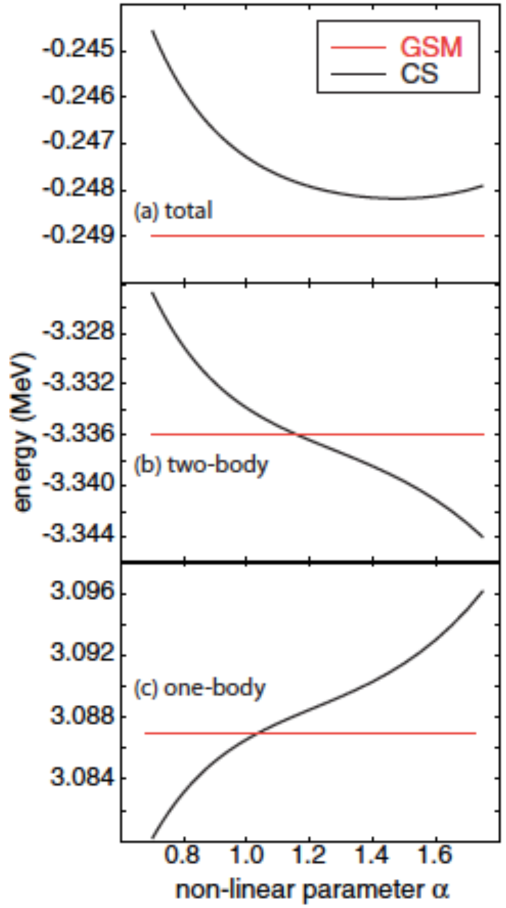


“Nuclear three-body problem in the complex energy plane: Complex-scaling Slater method”

A. T. Kruppa, G. Papadimitriou, W. Nazarewicz, and N. Michel, Phys. Rev. C 89, 014330 (2014).



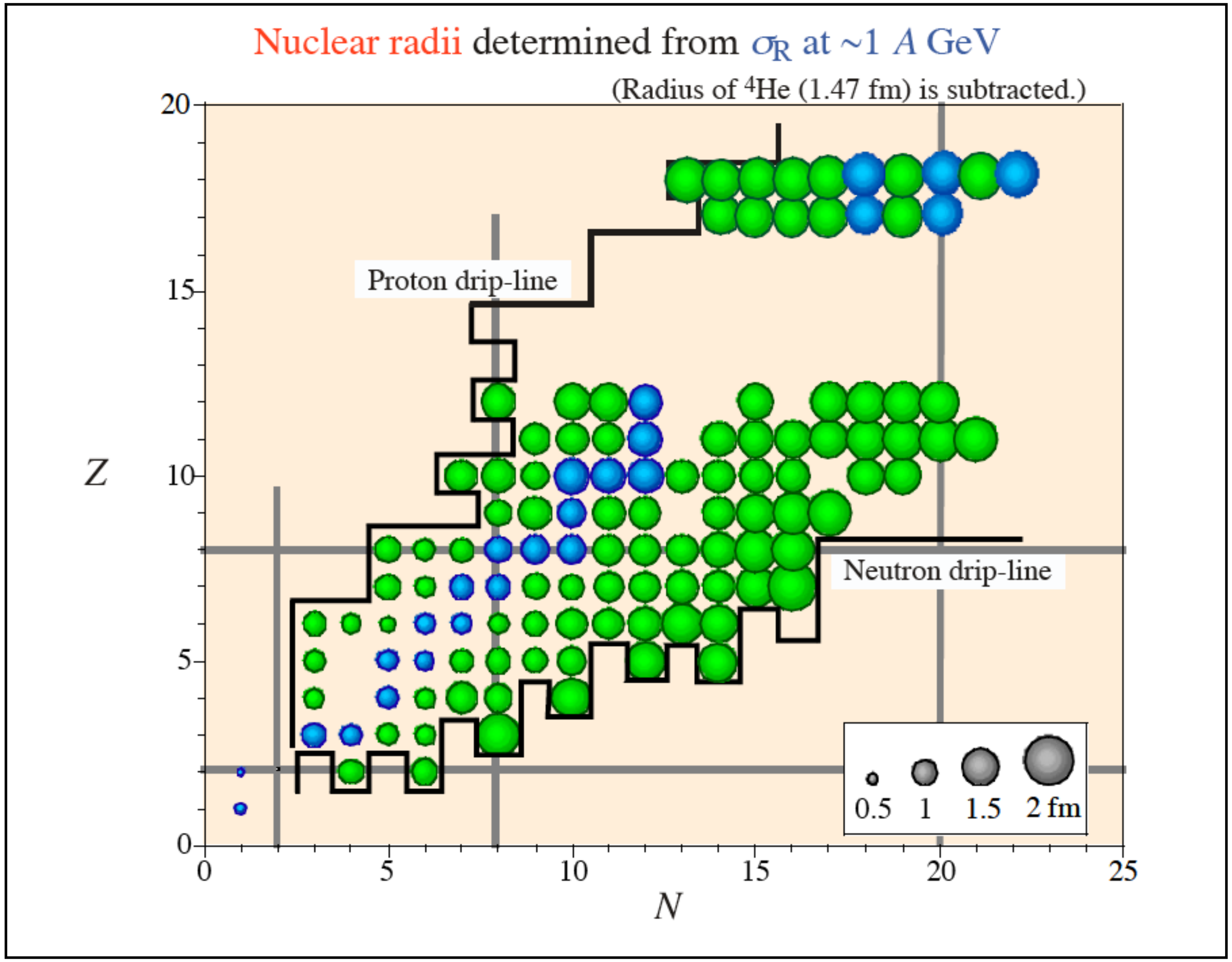




$$
\begin{aligned}
&\langle \Phi^{(i)}_{\mathrm{GSM}} | \hat{O}_{12} | \Phi^{(j)}_{\mathrm{GSM}} \rangle \\
&\quad = \sum_{\alpha,\beta} \langle \Phi^{(i)}_{\mathrm{GSM}} | \Phi^{(\alpha)}_{\mathrm{HO}} \rangle \langle \Phi^{(\alpha)}_{\mathrm{HO}} | \hat{O}_{12} | \Phi^{(\beta)}_{\mathrm{HO}} \rangle \langle \Phi^{(\beta)}_{\mathrm{HO}} | \Phi^{(j)}_{\mathrm{GSM}} \rangle \\
&\quad = \sum_{\alpha,\beta} d^*_{i,\alpha}\, d_{j,\beta} \langle \Phi^{(\alpha)}_{\mathrm{HO}} | \hat{O}_{12} | \Phi^{(\beta)}_{\mathrm{HO}} \rangle,
\end{aligned}
$$

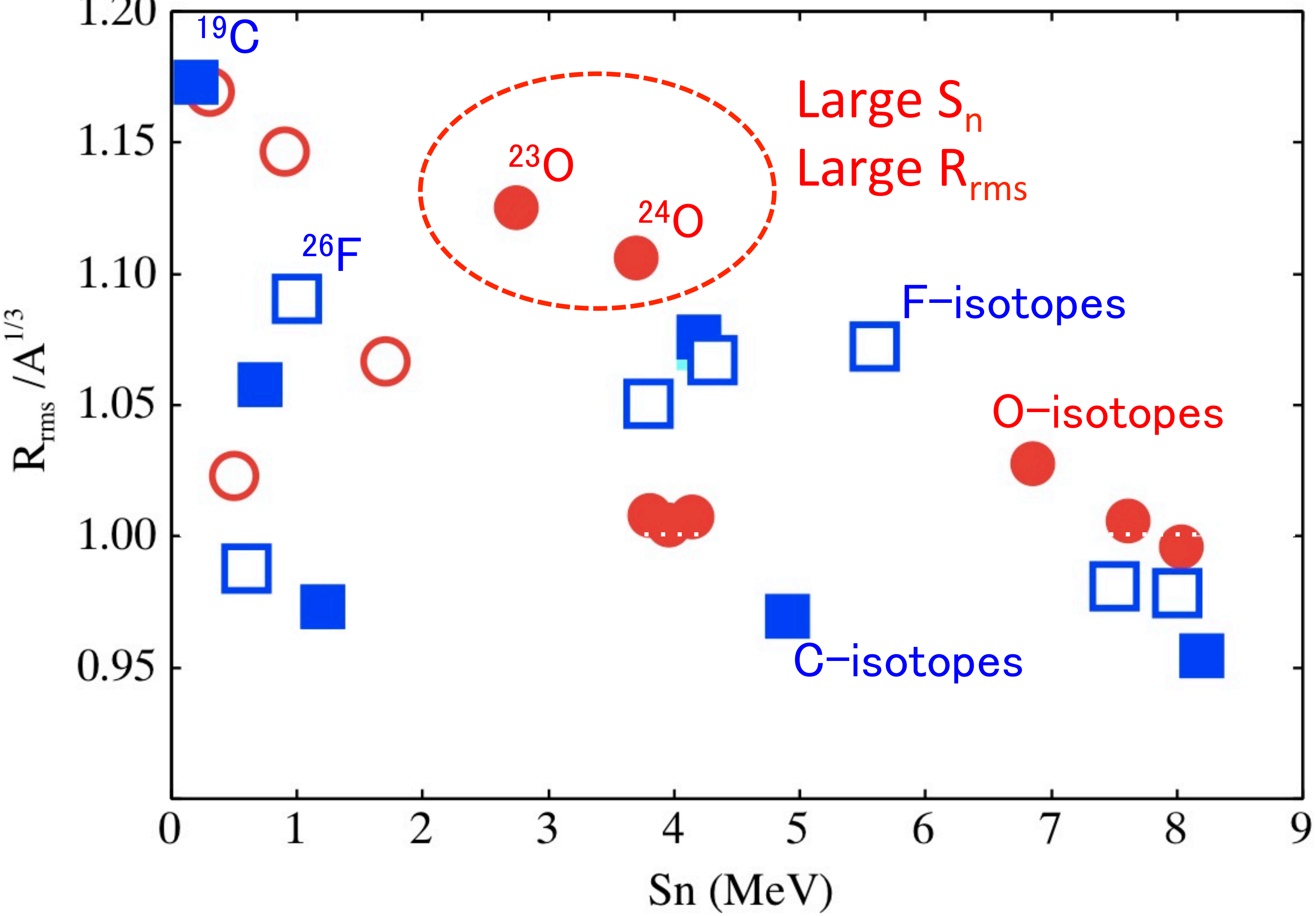
3. Radius of oxygen isotopes

Matter radius of nuclei near the drip-lines

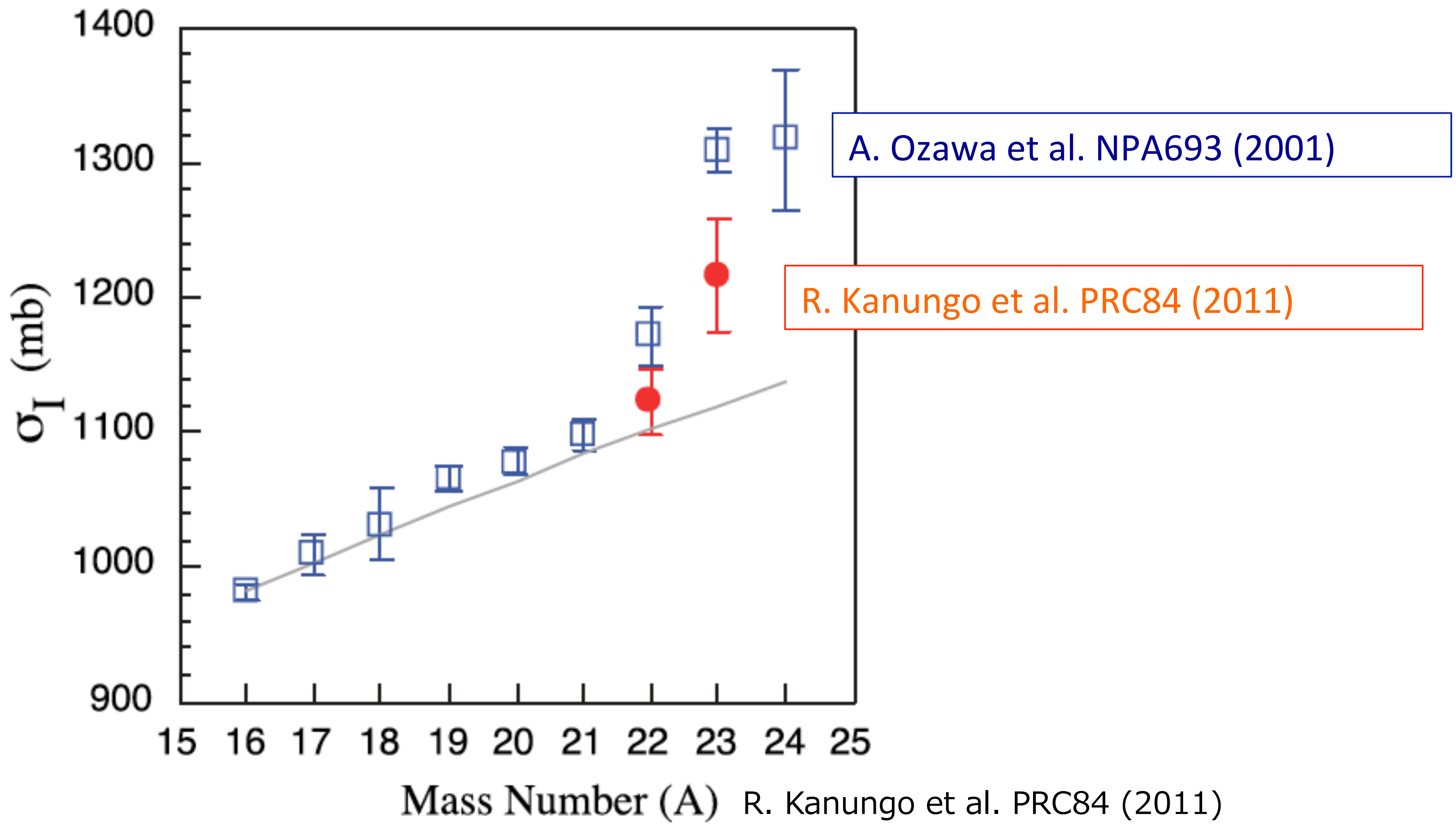


A. Ozawa (2001)

Relation for the S_n and $(R_{rms}/A^{1/3})$



Reaction cross-section of O-isotopes



R. Kanungo et al. PRC84 (2011)

^{16}O+XN systems

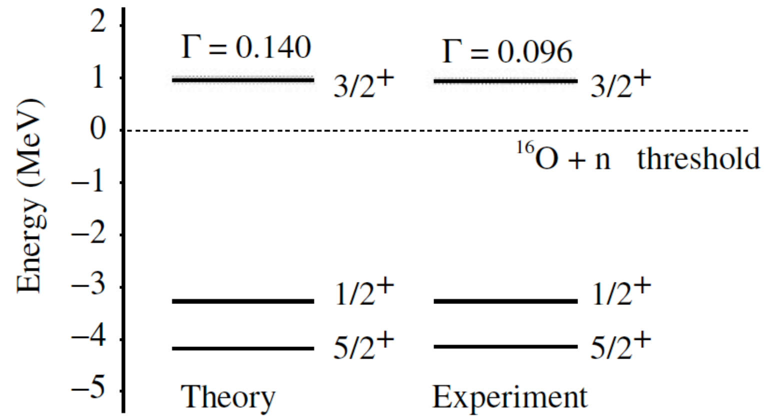
Interaction

H. M, K. Kato and K. Ikeda, PRC73, (2006)

- Core – N: Semi-microscopic potential [3]

$$\hat{h}_i[b] = \hat{t}_i + \boxed{\hat{V}_i^{d}[b] + \hat{V}_i^{\mathrm{ex}}[b]} + \hat{V}_{ls}[b] + \lambda \hat{\Lambda}_i[b].$$

Adjusted to ^{17}O ($5/2^+$, $1/2^+$, $3/2^+$)



- N-N : Volkov No.2 [4]
  (M=0.58, B=H=0.07)

Adjusted to ^{18}O (0^+)

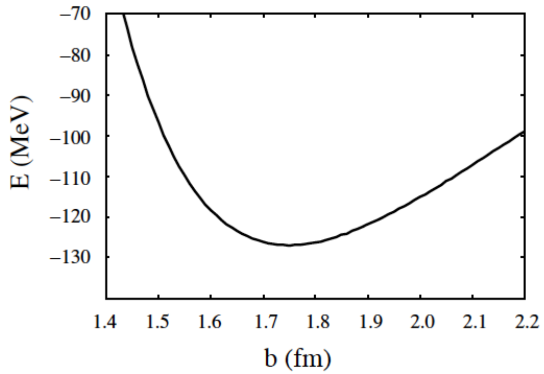
Size parameter of the core

^{16}O core (h.o.): fixed size as $R_{rms}(^{16}\mathrm{O}) = 2.54$ fm ($b = 1.723$ fm)

[3] T. Kaneko, M. LeMere, and Y. C. Tang, PRC44 (1991)
[4] A. B. Volkov, NPA74 (1965).

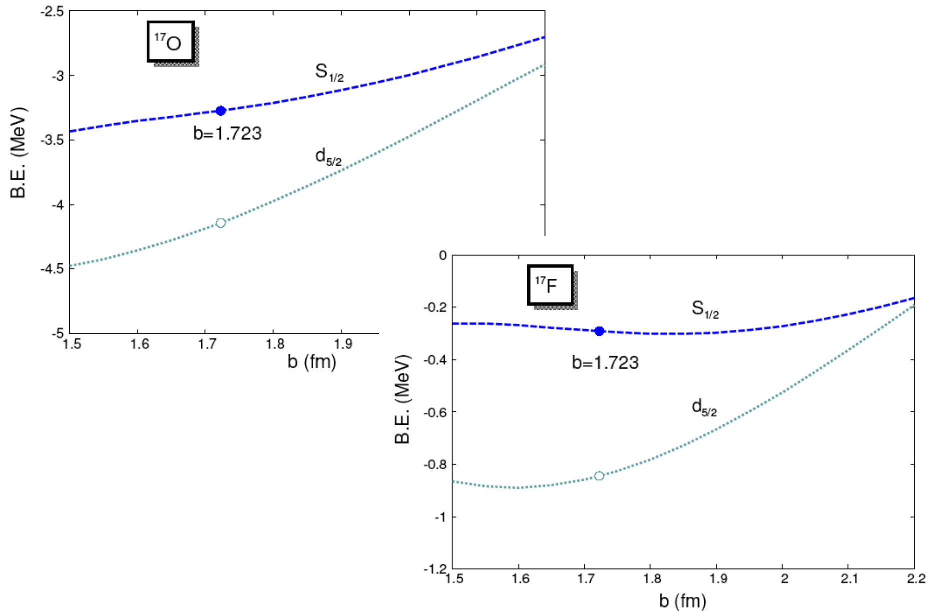
Core-size dependence

^{16}O-core



Core configuration:
h.o. wave function

Core-N



$$\boxed{\langle E(b)\rangle = \langle E(b)\rangle_{Core} + \langle E(b)\rangle_{Valence}}$$

Optimum *b* might be different in isotopes/isotones

^{16}O-core part

Energy of ^{16}O-core with effective NN-int. [5]

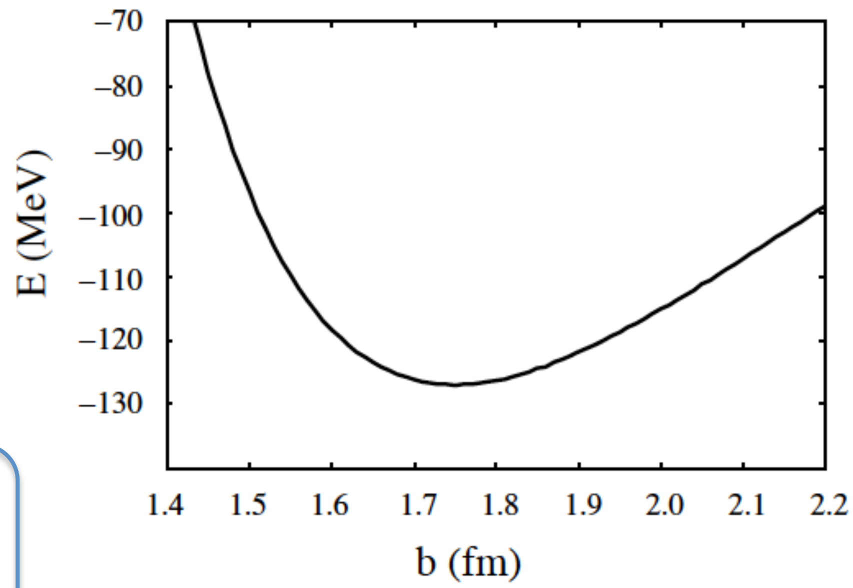
$$E(b) = \frac{69}{4}\hbar\omega + \sum_k v_k \beta_k^{3/2}\left[\left(\frac{31}{4} - \frac{3}{2}\beta_k + \frac{15}{4}\beta_k^2\right)(X_k^d + X_k^{ex}) + 6\beta_k(X_k^d - X_k^{ex})\right]$$

$$+ e^2\frac{83}{4}\left(\frac{2}{\pi b^2}\right)^{1/2} + 16\cdot\frac{29}{9}\left(\frac{1}{3\pi b^4}\right)^{3/2} v^{(3)} ,$$

(additional 3-body force)

$$V_{ijk}^{(3)} = v^{(3)}\,\delta(\boldsymbol{r}_i - \boldsymbol{r}_j)\,\delta(\boldsymbol{r}_j - \boldsymbol{r}_k)$$

Energy of ^{16}O



Volkov No.2 (M=0.58)

[5] T. Ando, K. Ikeda, and A. Tohsaki-Suzuki, PTP64 (1980).

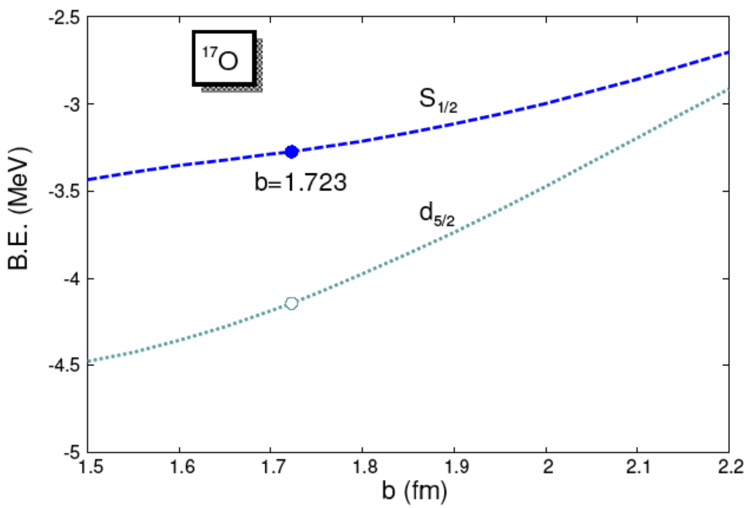
The core-N part

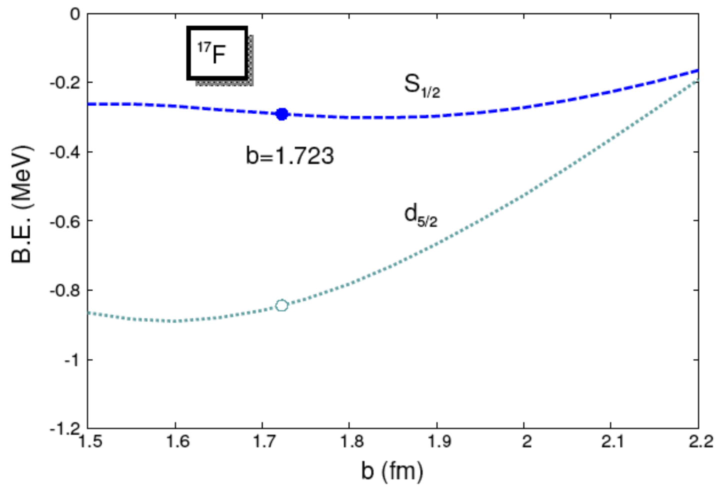
Core-N Hamiltonian:

$$\hat{h}_i[b] = \hat{t}_i + \boxed{\hat{V}_i^d[b] + \hat{V}_i^{\mathrm{ex}}[b] + \hat{V}_{ls}[b] + \lambda\hat{\Lambda}_i[b].}$$

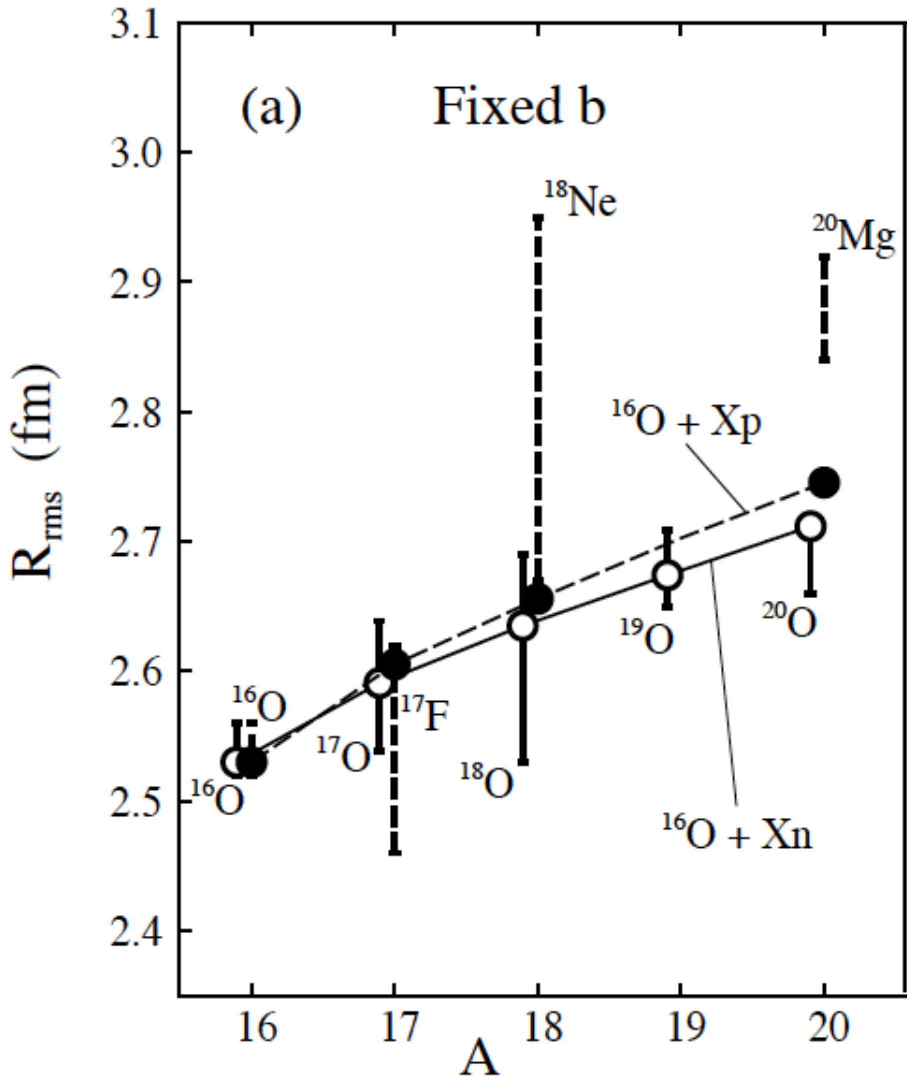
Change of b-parapeter:

Strength of the potential





Inclusion of the dynamics of the core: R_{rms} are improved



H. M, K. Kato and K. Ikeda, PRC73, (2006)

Our approaches

I) Role of many valence neutrons

^{16}O+Xn model

m-scheme COSM + Gaussian basis

II) Role of last one- or two-neutrons

"Core" + n or "Core"+2n model

Coupled-channel model for the core

^{16}O+XN systems

Interaction

H. M, K. Kato and K. Ikeda, EPJA42 (2009)

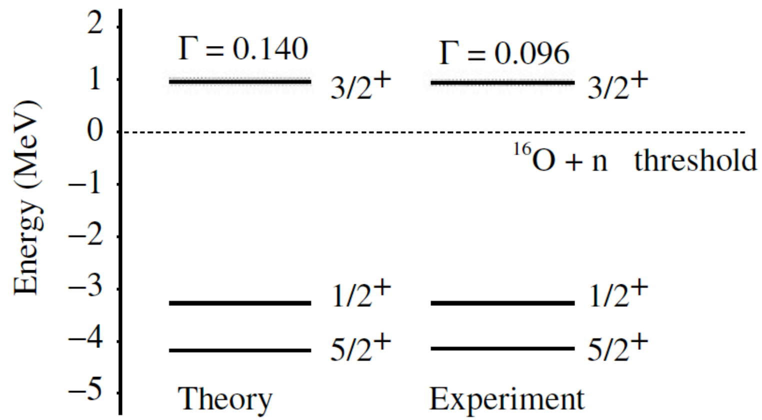
- Core – N: Semi-microscopic

$$\hat{h}_i[b] = \hat{t}_i + \boxed{\hat{V}_i^d[b] + \hat{V}_i^{\mathrm{ex}}[b]} + \hat{V}_{ls}[b] + \lambda\hat{\Lambda}_i[b].$$

Adjusted to ^{17}O ($5/2^+$, $1/2^+$, $3/2^+$)

- N-N : Volkov No.2
  (M=0.58, B=H=0.07, 0.25)

Adjusted to ^{18}O (0^+), drip-line at ^{25}O



Size parameter of the core

^{16}O core (h.o.): fixed size, $A^{1/6}$

Wave function

$$\Phi = \sum_m c_m F_m(r_1, r_2, \ldots, r_N) \left| (M\, M_T)_m \right\rangle$$

•Radial part

$$F_m(r_1, r_2, \ldots, r_N) = \left(g(r_1)\, g(r_2) \cdots g(r_N) \right)_m$$

Product of Gaussian with polynomial

•Spin-isospin part

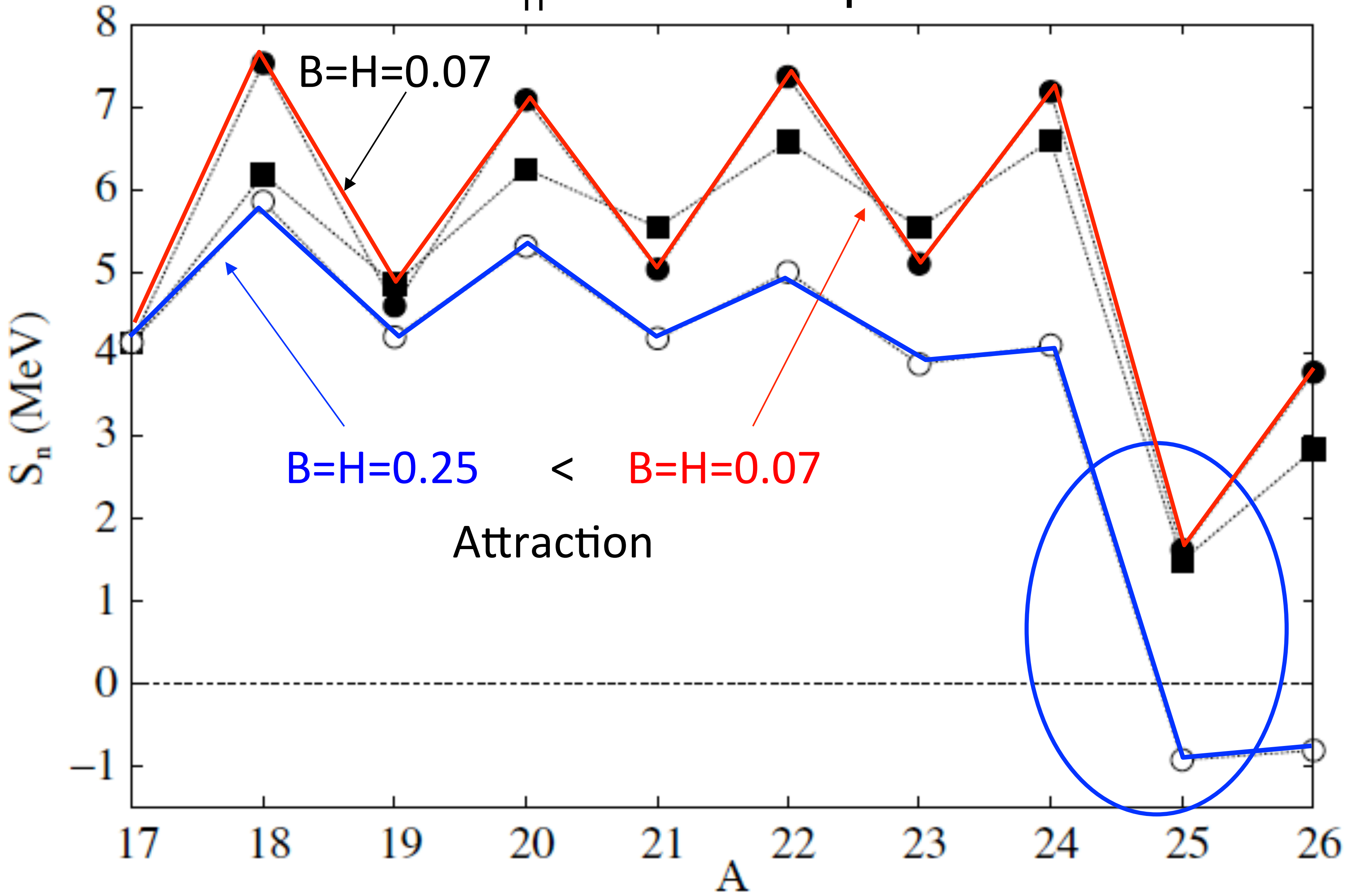
$$\left| (M\, M_T)_m \right\rangle$$

Total M and M_T are fixed

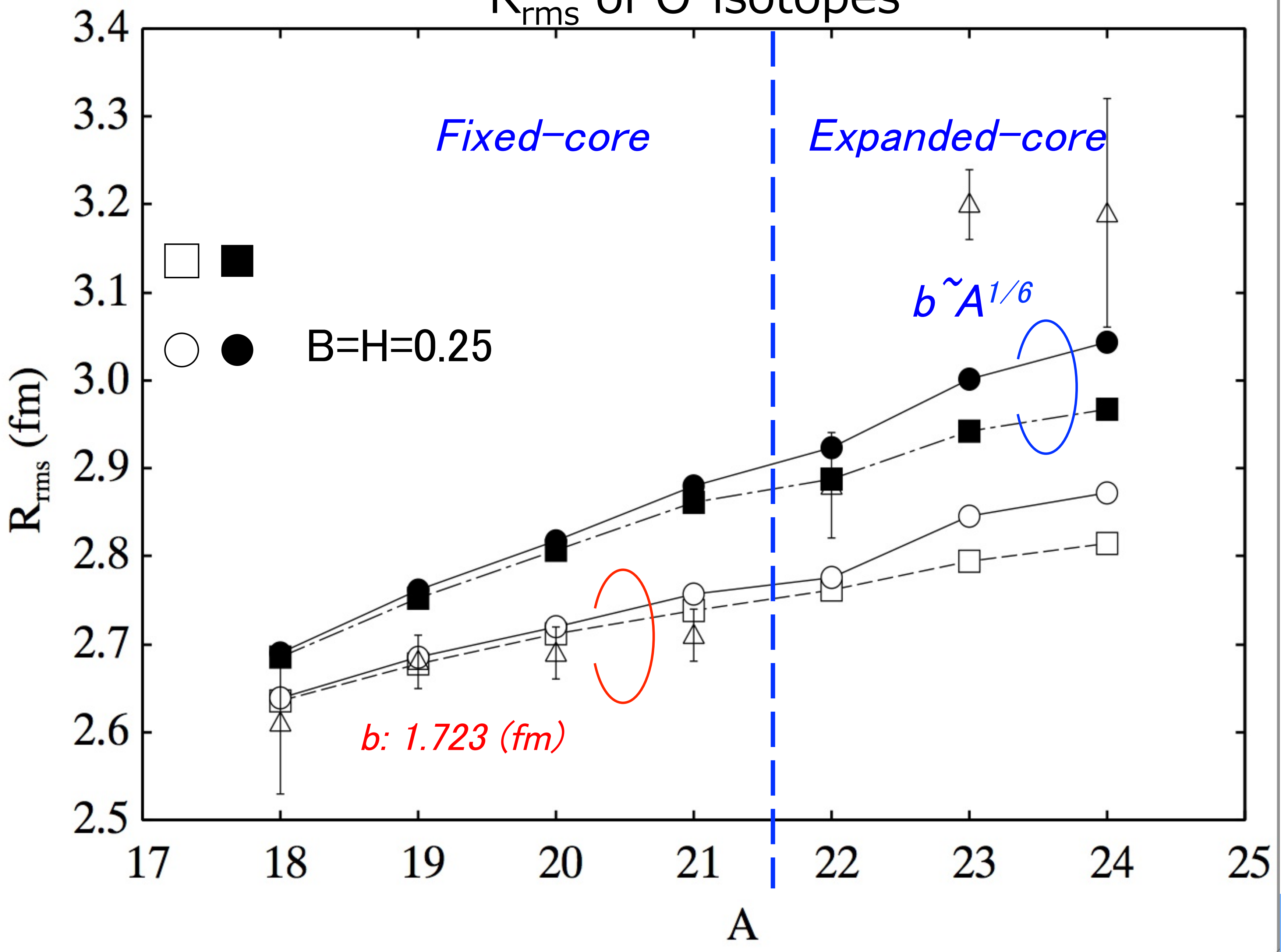
$$\begin{cases} M = m_1 + m_2 + \cdots + m_N \\ M_T = m_{T1} + m_{T2} + \cdots + m_{TN} \end{cases}$$

We check the expectation value of the total J as <J²>

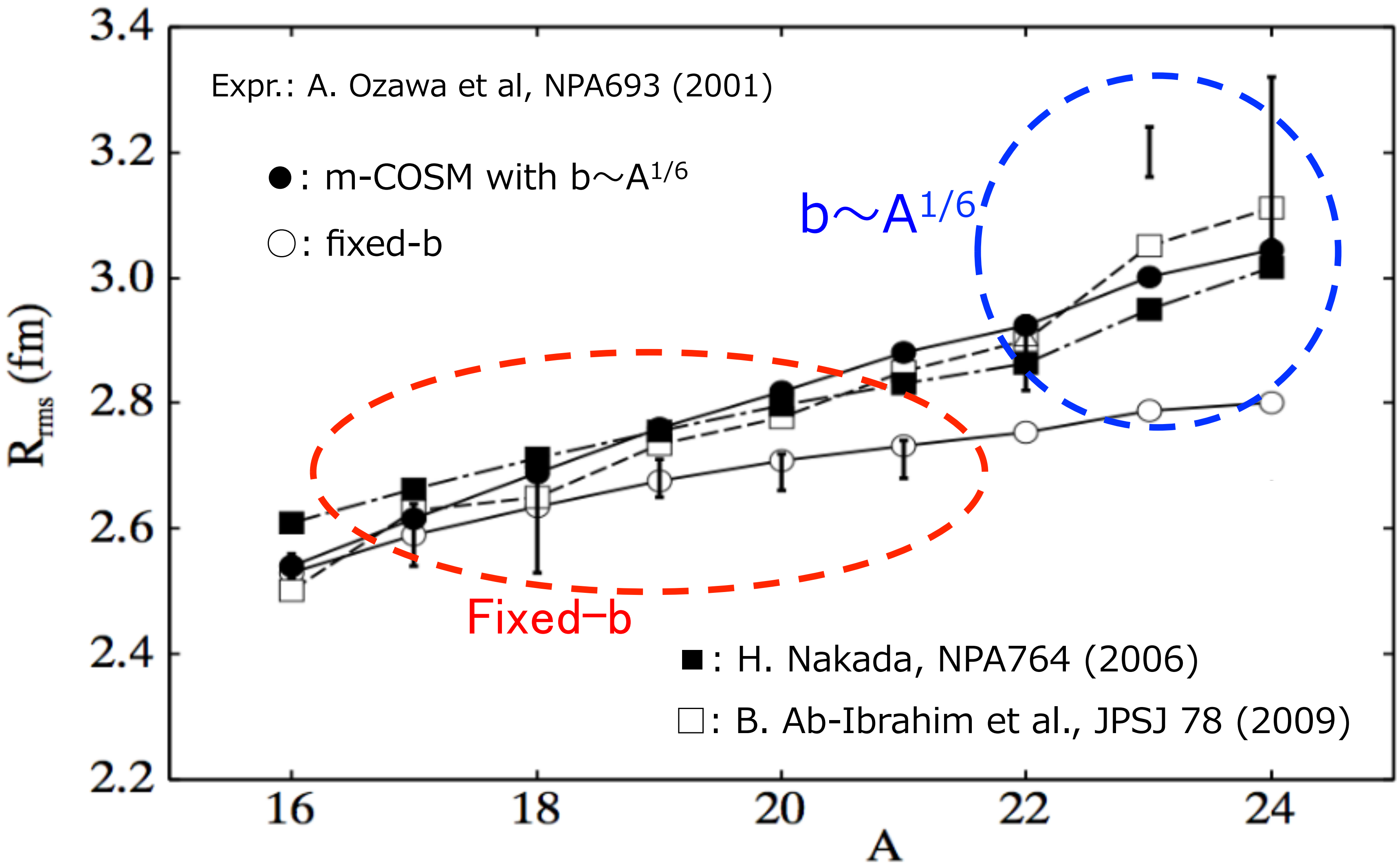
S_n of O-isotopes

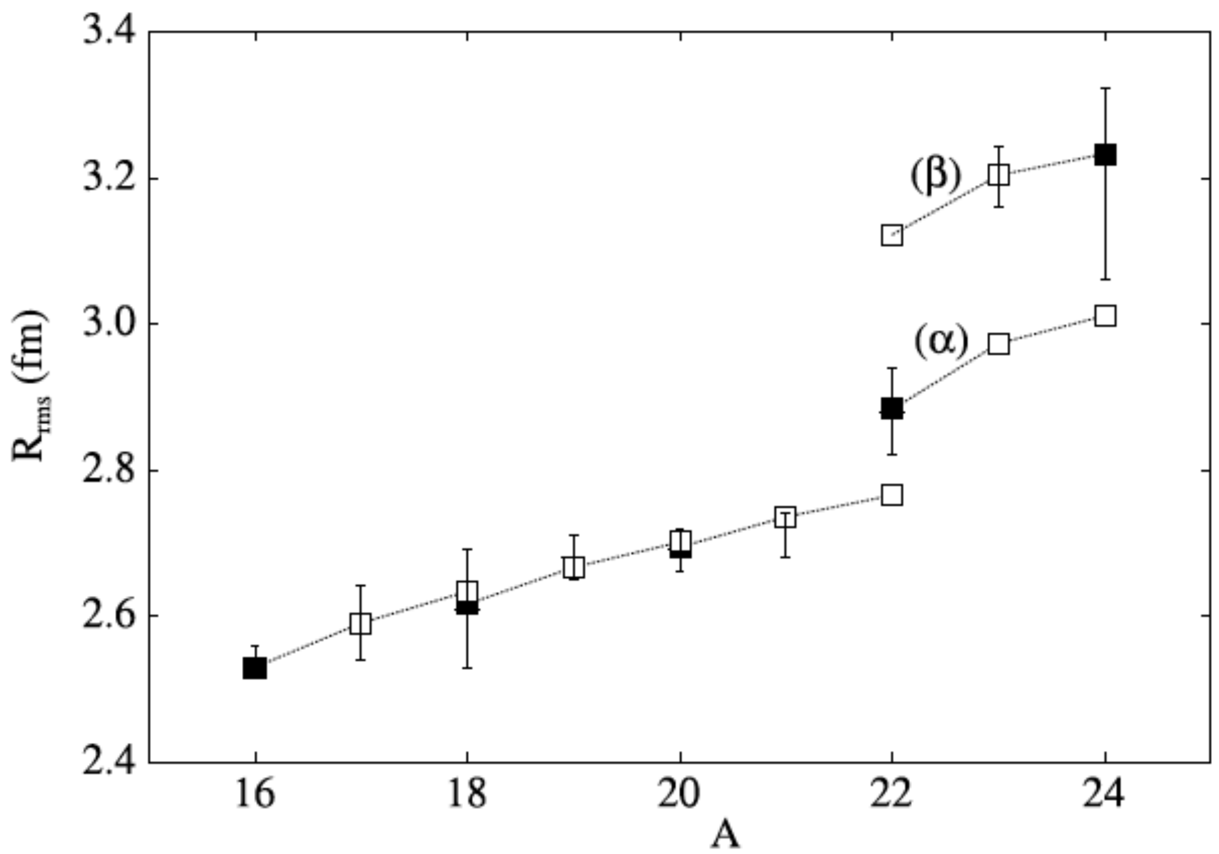


R_{rms} of O-isotopes

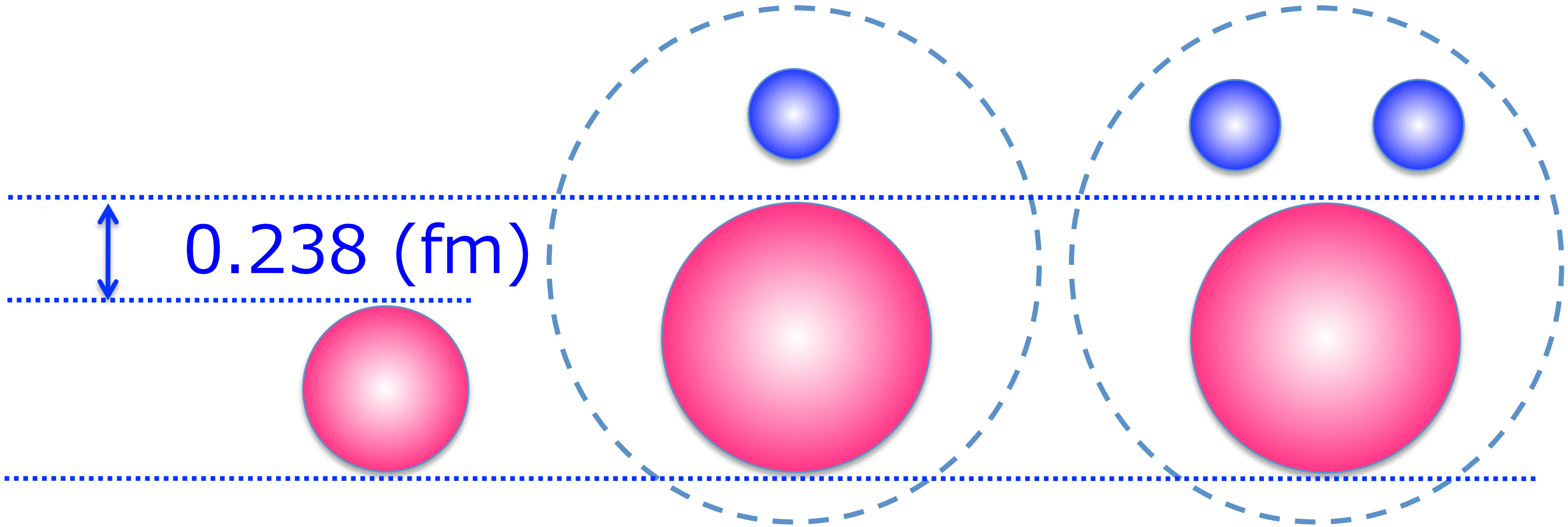


Comparison with other approaches




3.4
3.2
3.0
2.8
2.6
2.4
R_rms (fm)
(β)
(α)
16
18
20
22
24
A

A schematic figure to illustrate the change of the radius of ^{22}O



	^{22}O	^{23}O	^{24}O
R_{rms} [1]	2.88±0.06	3.20±0.04	3.19±0.13

[1] A. Ozawa et al, NPA693 (2001)

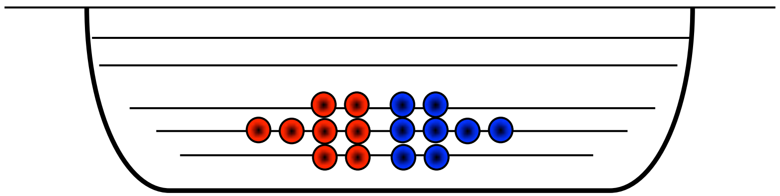
Inclusion of the core excitation

a coupled–channel picture for ^{16}O

H.M, K. Kato, K. Ikeda, NPA895 (2012)

0p-0h

$b \propto A^{1/6}$



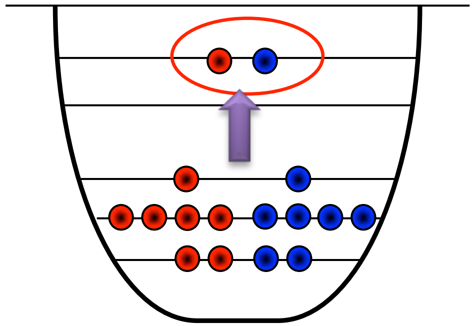
Radius : large

$(0s)^4(0p)^{12}$

"Mean-field-like" core

2p-2h ex.

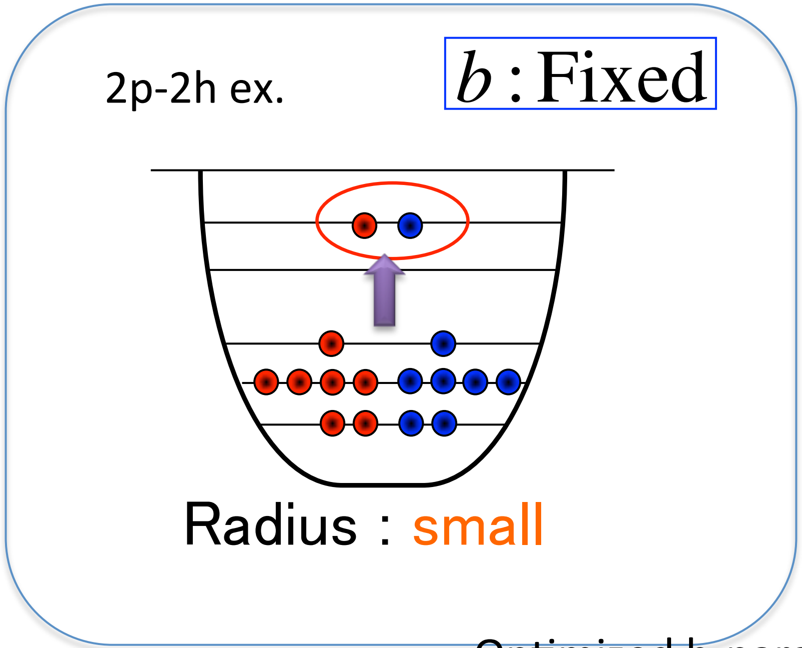
b : Fixed



Radius : small

$[(1s_{1/2})_\pi(1s_{1/2})_\nu]_{S=1,T=0}$

$[(0p_{1/2})_\pi(0p_{1/2})_\nu]_{S=1,T=0}$



"TOSM" 4P-1 (Myo)

Tensor-optimized shell model

T. Myo et al.,PTP117 (2007)

Coupling to the higher orbits

High-momentum components

Small b-parameter

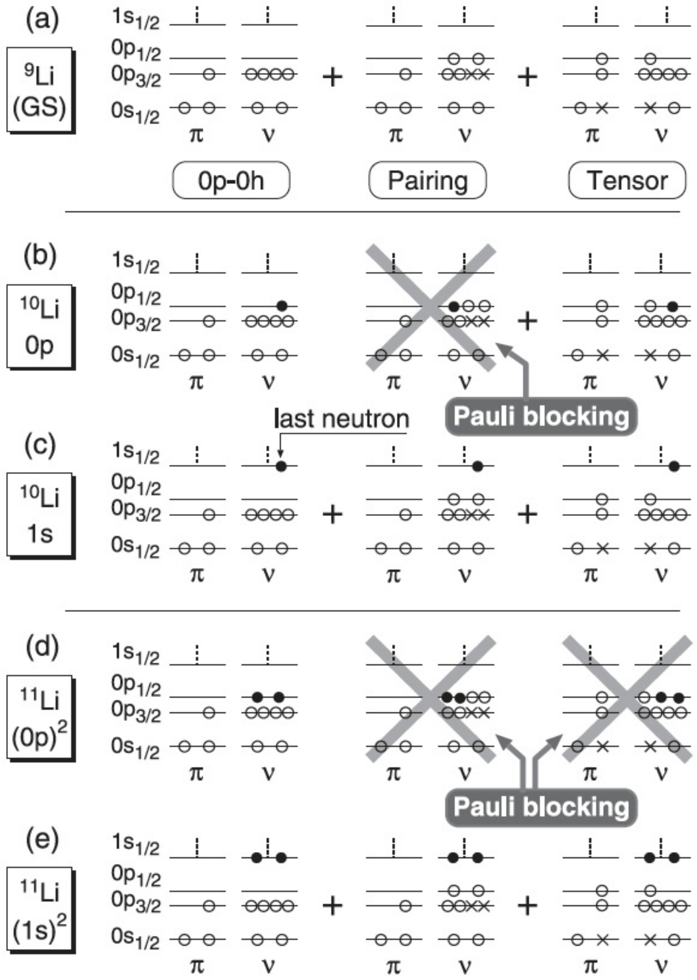
Optimized b-parameter for ^{4}He

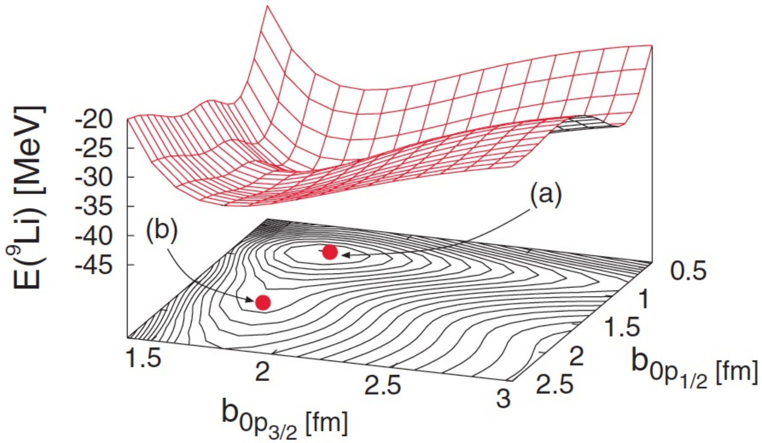
$l_{\max}$	$0s_{1/2}$	$0p_{1/2}$	$0p_{3/2}$	$1s_{1/2}$	$0d_{3/2}$	$0d_{5/2}$	$0f_{5/2}$	$0f_{7/2}$
1	1.26	0.75	0.69	—	—	—	—	—
2	1.19	0.78	0.75	0.76	0.69	0.62	—	—
3	1.16	0.74	0.66	0.73	0.67	0.62	0.77	0.66
4	1.16	0.75	0.67	0.73	0.67	0.61	0.77	0.67
5	1.16	0.76	0.67	0.73	0.67	0.61	0.77	0.64
6	1.16	0.76	0.67	0.73	0.67	0.61	0.77	0.64

Inclusion of the core excitation

TOSM in ^{9}Li

T. Myo, K. Kato, H. Toki and K. Ikeda, PRC76(2007)

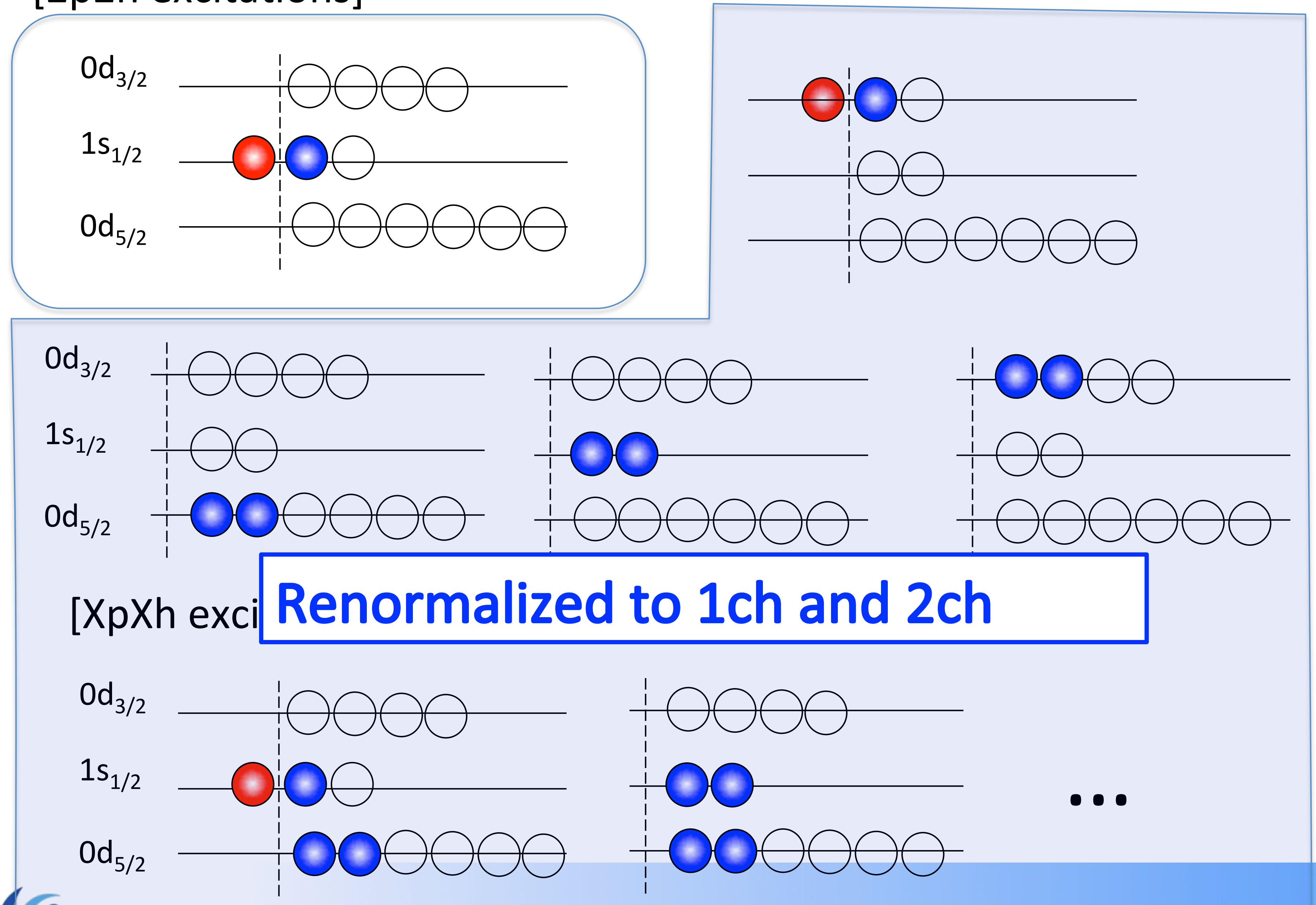




1. Different size for each orbit

2. Specific configurations are suppressed due to the Pauli-blocking effect

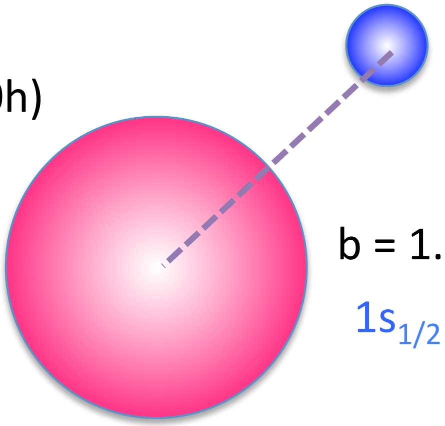
[2p2h excitations]



A coupled-channel approach

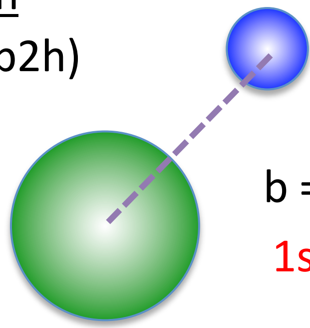
Hamiltonian:
$$\begin{pmatrix} T_1+V_1 & \Delta_{12} \\ \Delta_{21} & T_2+V_2+\Delta E \end{pmatrix}\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = E\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

<u>1ch</u>
(0p0h)



b = 1.1A$^{1/6}$ (fm)
1s$_{1/2}$ is allowed

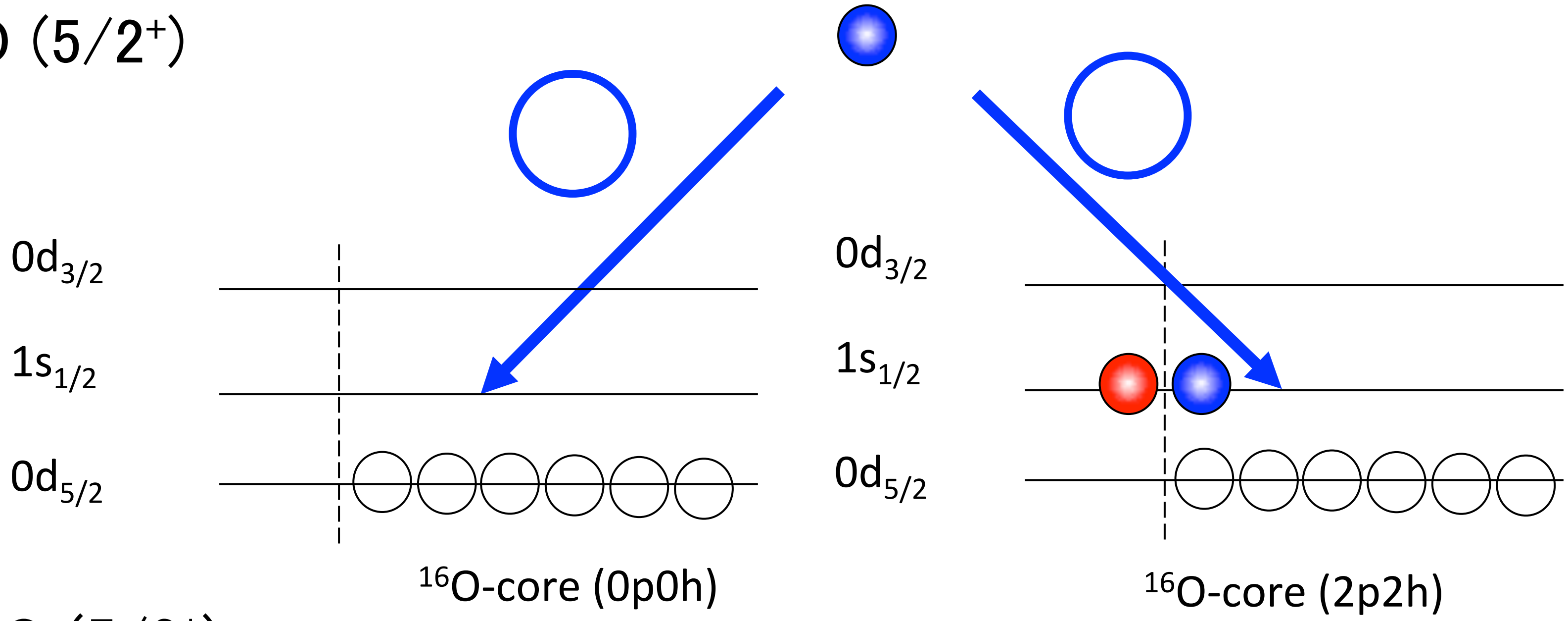
<u>2ch</u>
(2p2h)



b = 1.5(fm) : fixed
1s$_{1/2}$ is forbidden

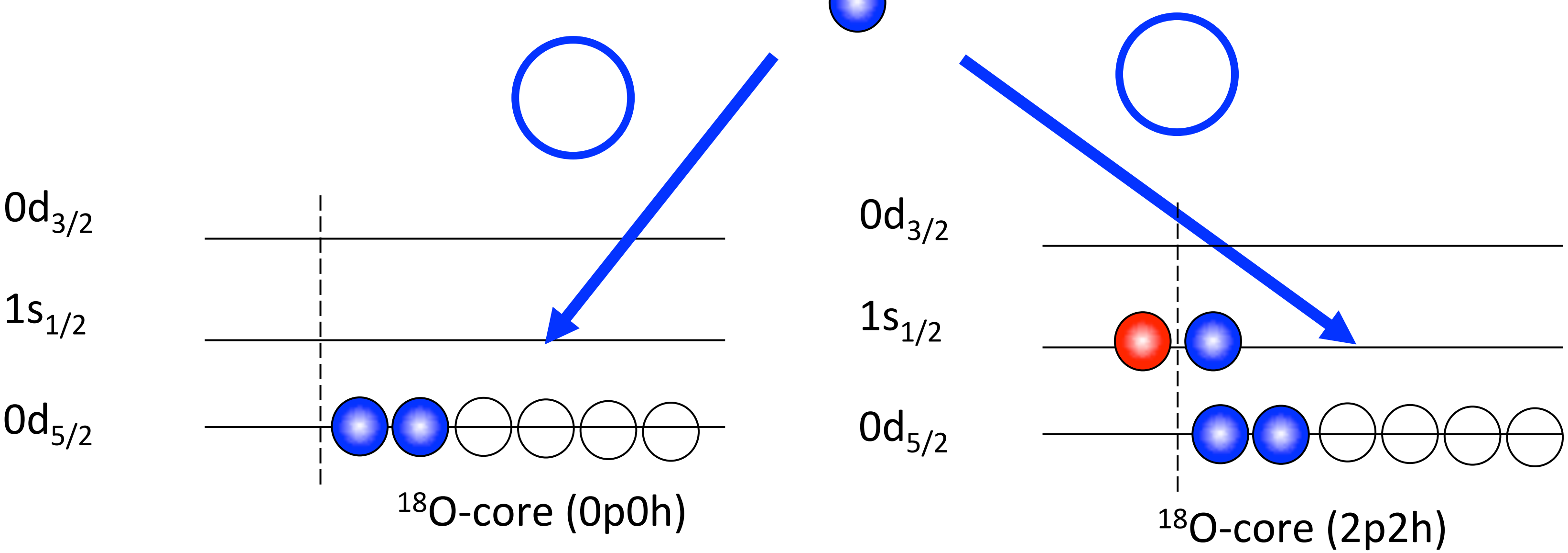
- Coupling term $\Delta_{12} = \Delta_{21} = -2$MeV
- Energy difference of the core $\Delta E = 2.5, 5, 7.5, 10$ (MeV)

$^{17}O\ (5/2^{+})$

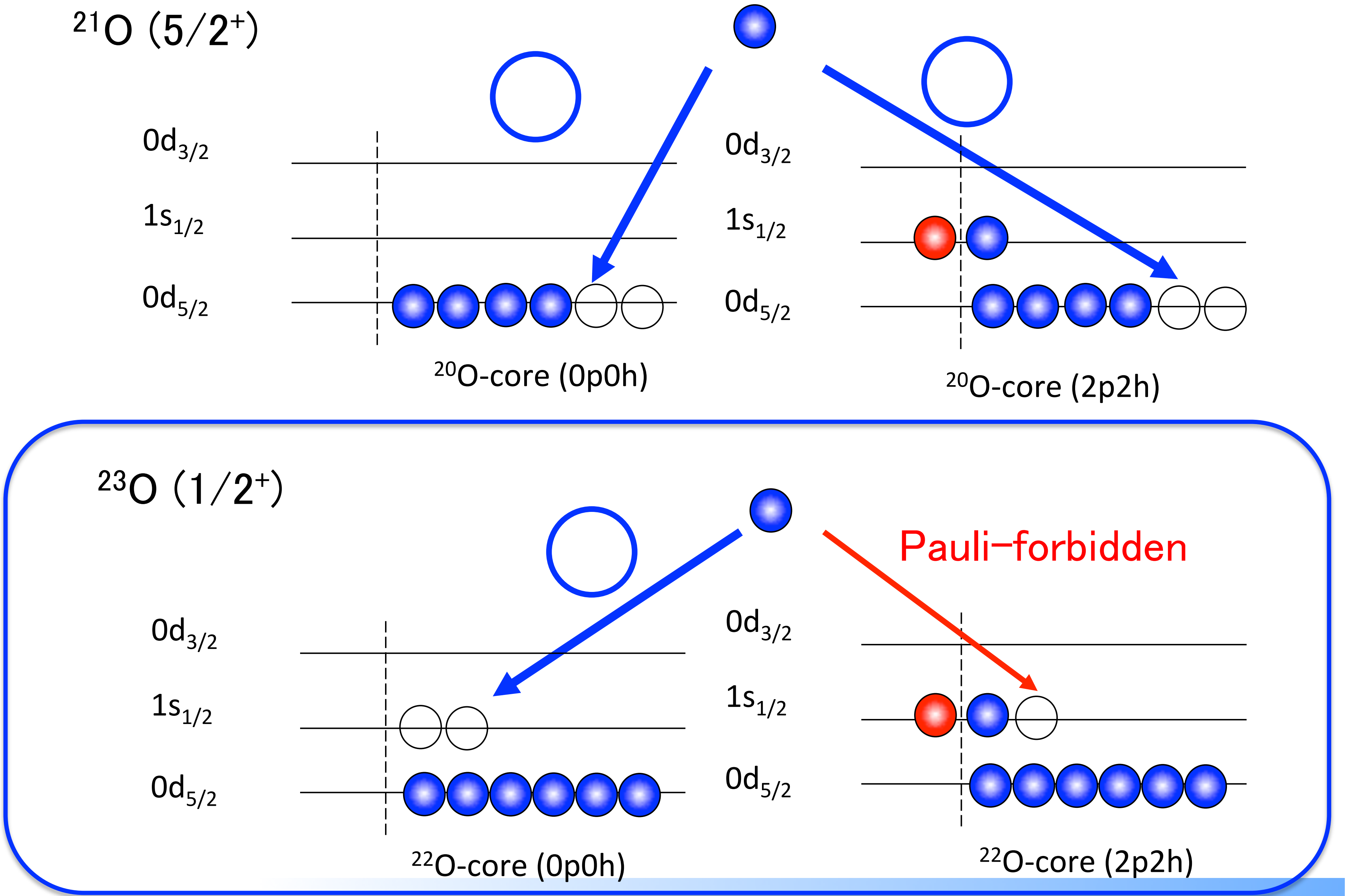


0d3/2
1s1/2
0d5/2
16O-core (0p0h)
0d3/2
1s1/2
0d5/2
16O-core (2p2h)

$^{19}O\ (5/2^{+})$



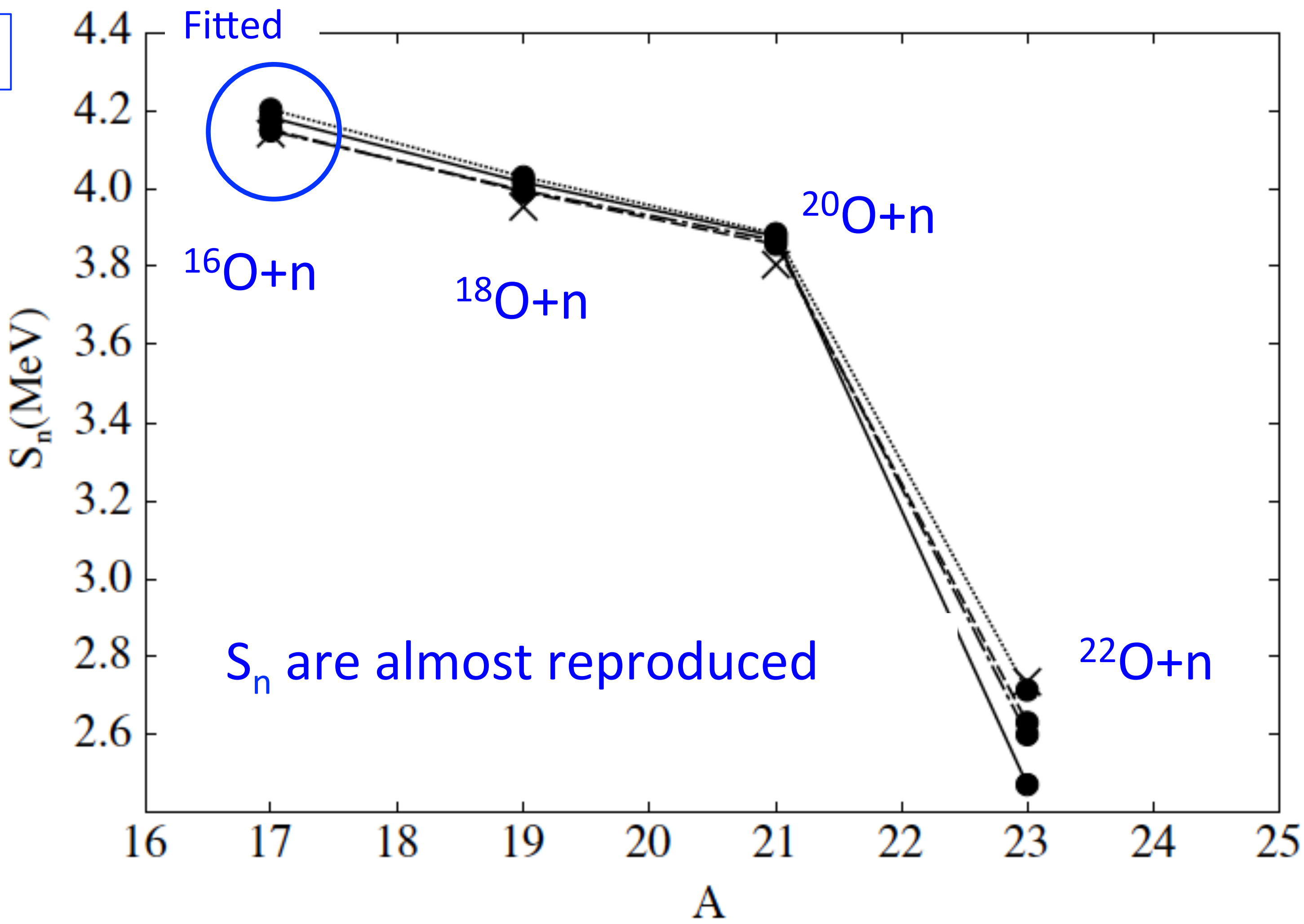
0d3/2
1s1/2
0d5/2
18O-core (0p0h)
0d3/2
1s1/2
0d5/2
18O-core (2p2h)



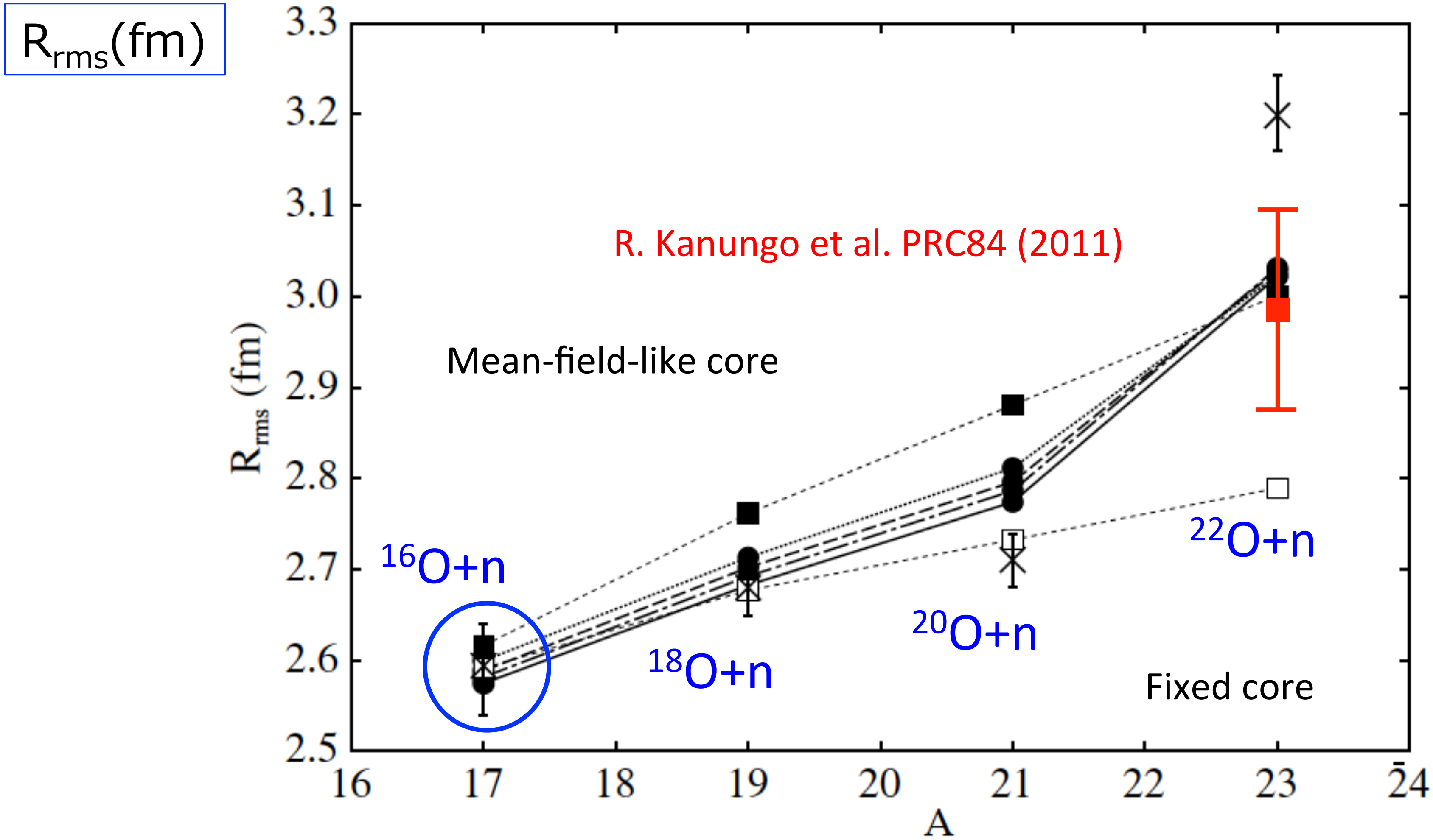
21O (5/2+)
0d3/2
1s1/2
0d5/2
20O-core (0p0h)
0d3/2
1s1/2
0d5/2
20O-core (2p2h)
23O (1/2+)
0d3/2
1s1/2
0d5/2
22O-core (0p0h)
Pauli−forbidden
0d3/2
1s1/2
0d5/2
22O-core (2p2h)

Results for a coupled-channel core+n model approach

S_n (MeV)



Results for a coupled-channel core+n model approach



Summary

- Comparison between GEM+CS and GSM

  Different procedures for preparing the "complete set" of the basis function

  They give numerically the same result (at least) for core+2N systems

  Generalized variational parameters should be optimized carefully

- Rrms ^{23}O and ^{24}O

  ^{16}O(fixed size) +Xn : failed to reproduce the experiment

  Modification of the core is important

  [An attempt to reproduce the Rrms]

  Coupled-channel picture

  (Shrunk core) + (Broad core) is a possible way