Many-body resonances and continua in light unstable nuclei using the complex scaling method

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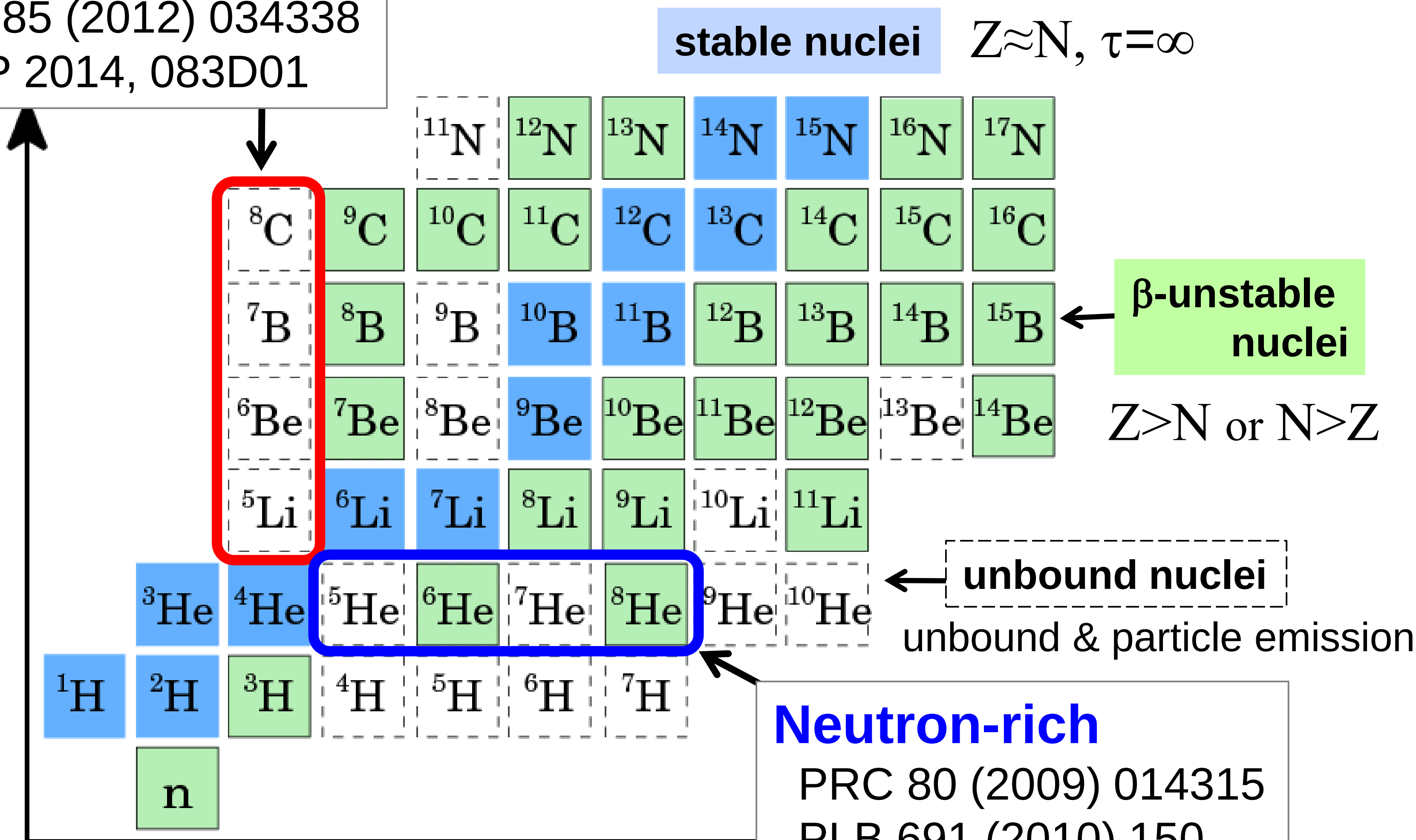
YIPQS Long-term workshop on Computational Advances in Nuclear and Hadron Physics (CANHP 2015), Kyoto, 2015.10

Outline

- Structure of Light Unstable Nuclei
  - He isotopes (neutron-rich)
  - mirror nuclei (proton-rich)
- Cluster Orbital Shell Model (**COSM**)
  - core nuclei + valence protons / neutrons
- Complex Scaling Method (**CSM**)
  - many-body resonances & continuum states
  - continuum level density, Green's function
  - strength functions, breakup reactions

Nuclear Chart

Proton-rich
PRC 84 (2011) 064306
PRC 85 (2012) 034338
PTEP 2014, 083D01

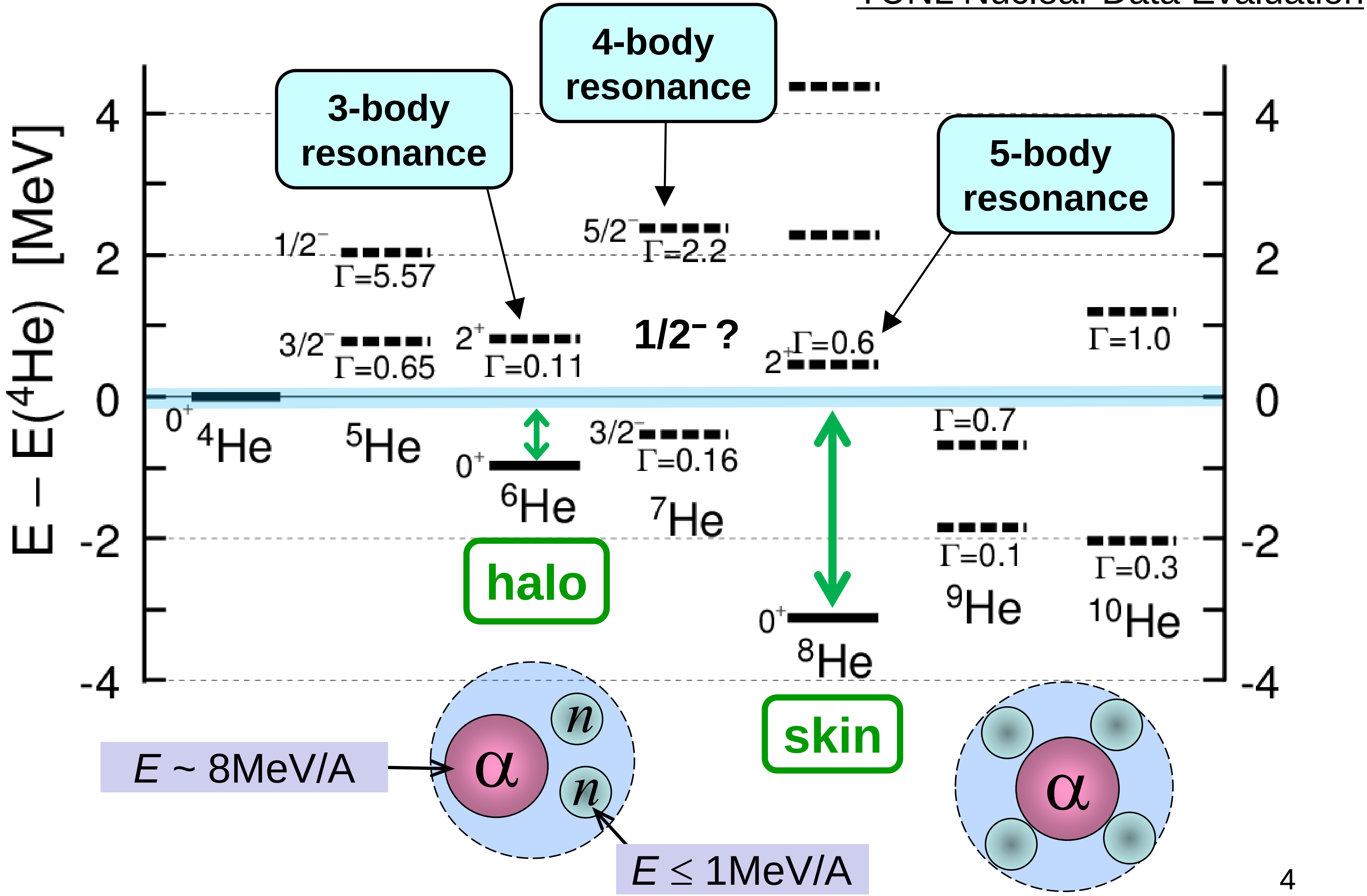


Neutron-rich
PRC 80 (2009) 014315
PLB 691 (2010) 150

Mirror symmetry between **proton-rich** & **neuron-rich**
(with Coulomb)

Neutron-rich He isotopes : experiment

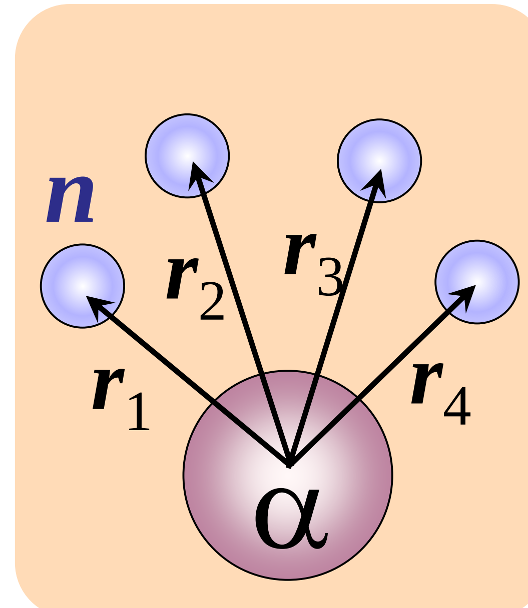
TUNL Nuclear Data Evaluation



Method

- Cluster Orbital Shell Model (**COSM**)
  - Include open channel effects.

    ^{8}He : ^{7}He+n, ^{6}He+n+n, ^{5}He+n+n+n, ...



- Complex Scaling Method

$$\mathbf{r} \to \mathbf{r}e^{i\theta}, \quad \mathbf{k} \to \mathbf{k}e^{-i\theta}$$

  - Obtain resonance w.f. with correct boundary condition as **Gamow states** $E=E_r - i\Gamma/2$
  - Give the continuum level density, ΔE
    - resonance+continuum, Green's function
    - strength function, Lippmann-Schwinger Eq., T-matrix

A.T. Kruppa, R.G. Lovas, B. Gyarmati, PRC37(1988) 383 (**^{8}Be as 2α**)
S. Aoyama, TM, K. Kato, K. Ikeda, PTP116(2006) 1 (**CSM review**)
C. Kurokawa , K. Kato, PRC71 (2005) 021301 (**^{12}C as 3α**)

Kikuchi (**LS eq.**)
Matsumoto (**CDCC**)

Cluster Orbital Shell Model (n-rich)

- System is obtained based on RGM equation

$$H(^A\mathrm{He}) = H(^4\mathrm{He}) + H_{\mathrm{rel}}(N_V n) \qquad \Phi(^A\mathrm{He}) = \mathcal{A}\left\{\psi(^4\mathrm{He}) \cdot \sum_{i=1}^{N} C_i \cdot \chi_i(N_V n)\right\}$$

valence neutron number

i : configuration

$\psi(^4\mathrm{He})$: $(0\mathrm{s})^4$ ← No explicit tensor correlation

$\chi_i(N_V n) = \mathcal{A}\{\varphi_{i1}\varphi_{i2}\varphi_{i3}\cdots\}$ $\qquad \varphi_i : L \le 2$ few-body method with Gaussian expansion

- Orthogonarity Condition Model (OCM) is applied.

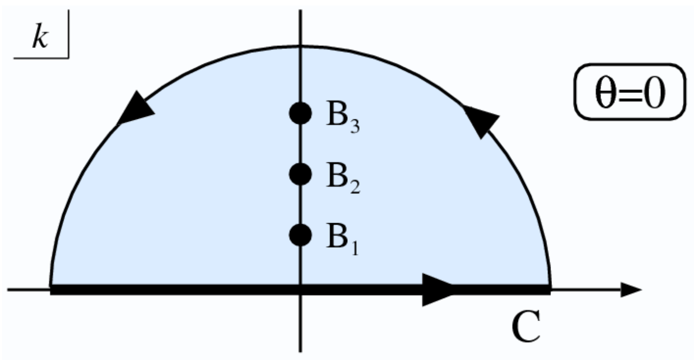
$$\sum_{i=1}^{N} \left\langle \chi_j \left| \sum_{k=1}^{N_V} \left(T_k + V_k^{cn} \right) + \sum_{k<l}^{N_V} \left(V_{kl}^{nn} + \frac{\vec{p}_i \cdot \vec{p}_j}{A_c m} \right) \right| \chi_i \right\rangle C_i = (E - E_{4\mathrm{He}})\, C_j$$

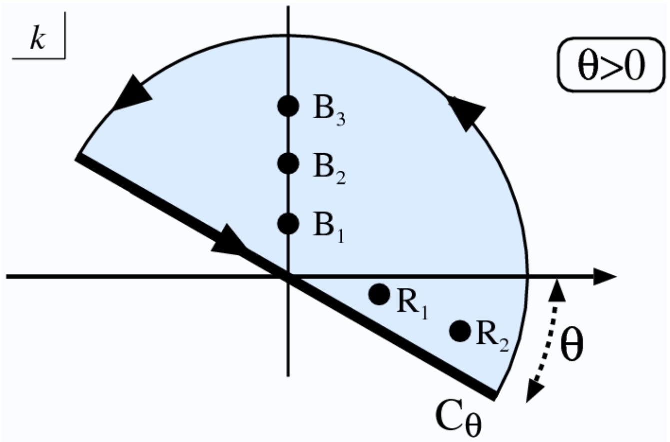
$\langle \varphi_i | \phi_{\mathrm{PF}} \rangle = 0$ Remove Pauli Forbidden states (PF)

Y. Suzuki, K. Ikeda, PRC38(1988)410, H. Masui, K. Kato, K. Ikeda, PRC73(2006)034318

Complex Scaling for 2-body case

$$U(\theta)\ :\ \mathbf{r} \to \mathbf{r}\cdot\exp(i\theta)\ ,\quad \mathbf{k} \to \mathbf{k}\cdot\exp(-i\theta)\ ,\quad \theta \in \mathbb{R}$$





Completeness relation

$$1 = \sum_B |\varphi_B\rangle\langle\tilde{\varphi}_B| + \int_C dk\, |\varphi_k\rangle\langle\tilde{\varphi}_k|$$

T. Berggren, NPA109('68)265.

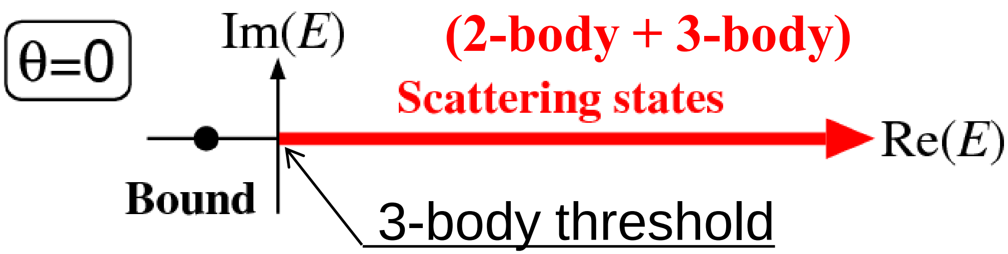
$$1 = \sum_B |\varphi_B\rangle\langle\tilde{\varphi}_B| + \sum_R |\varphi_R\rangle\langle\tilde{\varphi}_R| + \int_{C_\theta} dk_\theta\, |\varphi_{k_\theta}\rangle\langle\tilde{\varphi}_{k_\theta}|$$

J.Aguilar and J.M.Combes, Commun. Math. Phys.,22('71)269.
E.Balslev and J.M.Combes, Commun. Math. Phys.,22('71)280.

B.G.Giraud, K.Kato, A.Ohnishi
J. Phys. A **37** ('04)11575

Complex Scaling for 3-body case

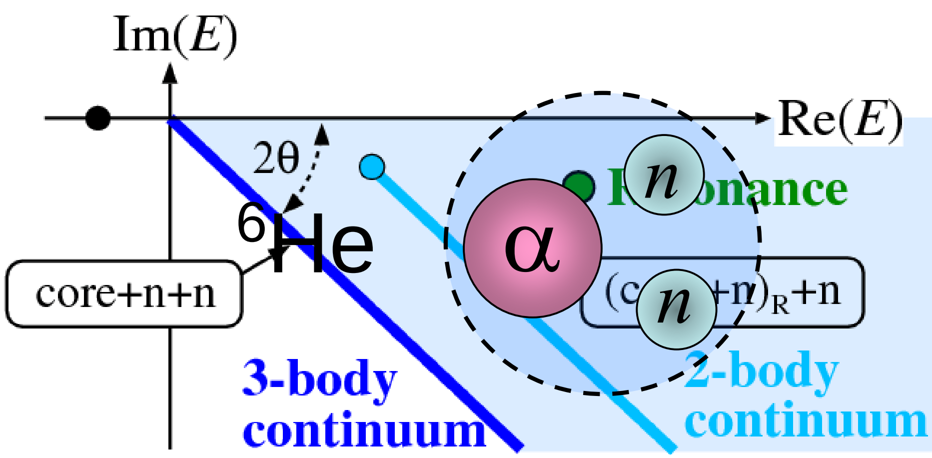
$$U(\theta)\ :\ \mathbf{r} \to \mathbf{r}\cdot\exp(i\theta)\ ,\qquad \mathbf{k} \to \mathbf{k}\cdot\exp(-i\theta)\ ,\qquad \theta \in \mathbb{R}$$



Completeness relation

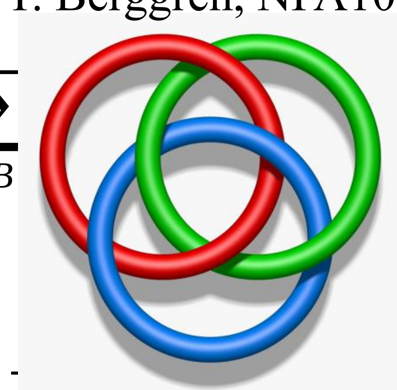
$$1 = \sum_B |\varphi_B\rangle\langle\tilde{\varphi}_B| + \int_C dE\, |\varphi_E\rangle\langle\tilde{\varphi}_E|$$

T. Berggren, NPA109('68)265.



$$1 = \sum_B \cdots$$

$$+ \cdots \rangle\langle\tilde{\varphi}_{E_\theta}|$$

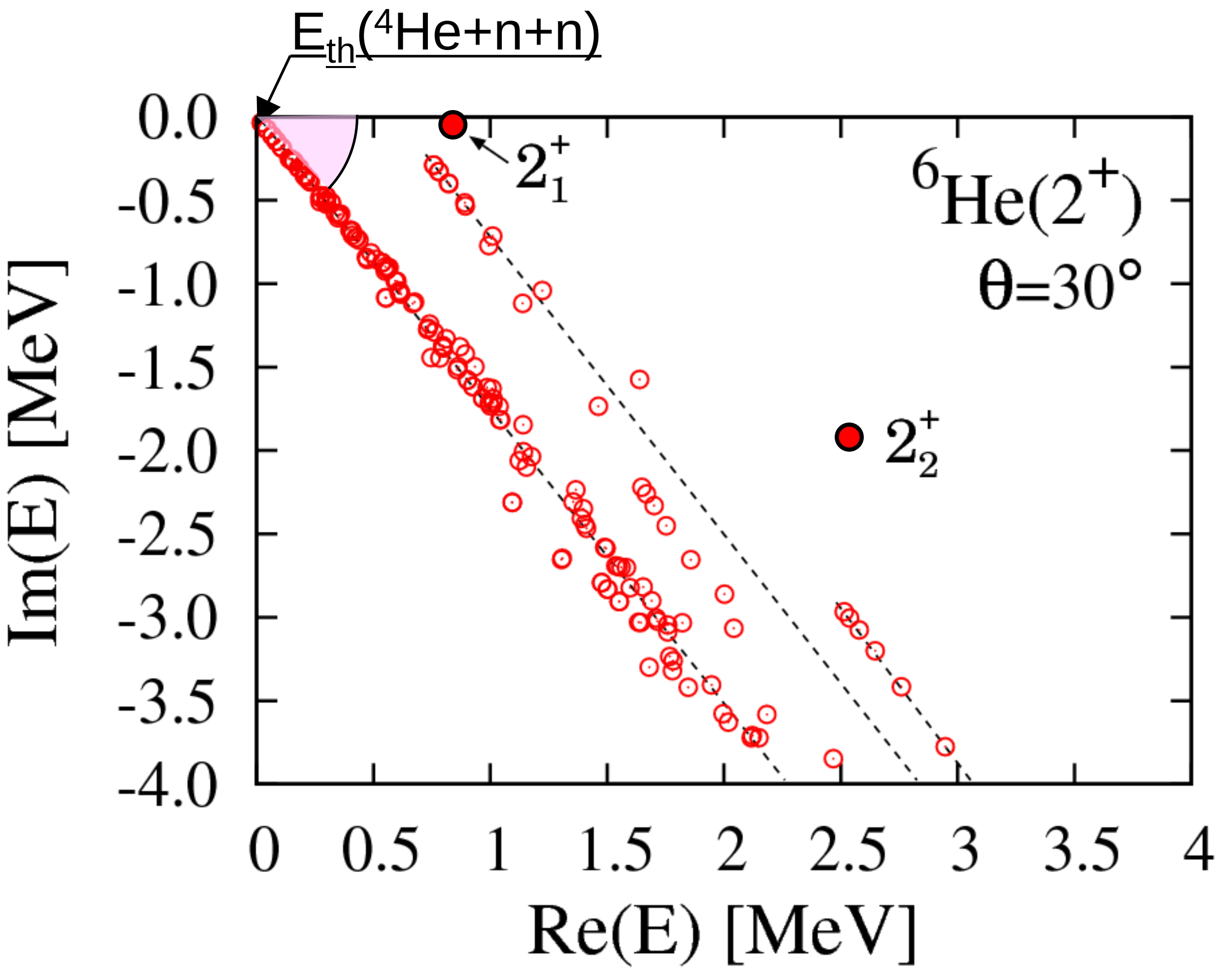


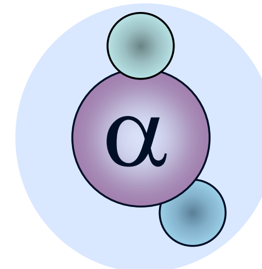
Borromean rings

Halo nuclei : "core+n+n" with Borromean condition

^{6}He=^{4}He+n+n, ^{11}Li=^{9}Li+n+n, ^{14}Be=^{12}Be+n+n, …

Spectrum of ^{6}He with ^{4}He+n+n model





^{6}He$^{(*)}$
^{5}He+n
^{4}He+n+n

Continuum states are discretized using **Gaussian basis functions** (Kamimura)

$$\phi_\ell(\mathbf{r}) = \sum_n C_n \cdot r^\ell e^{-\left(r/b_n\right)^2} Y_\ell(\hat{\mathbf{r}})$$

A. Csoto, PRC49 ('94) 3035,
S. Aoyama et al. PTP94('95)343, T. Myo et al. PRC63('01)054313

Progress in Particle and Nuclear Physics 79 (2014) 1–56

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journal homepage: www.elsevier.com/locate/ppnp

Review

Recent development of complex scaling method for many-body resonances and continua in light nuclei

Takayuki Myo [a,b,*], Yuma Kikuchi [c], Hiroshi Masui [d], Kiyoshi Katō [e]

A B S T R A C T

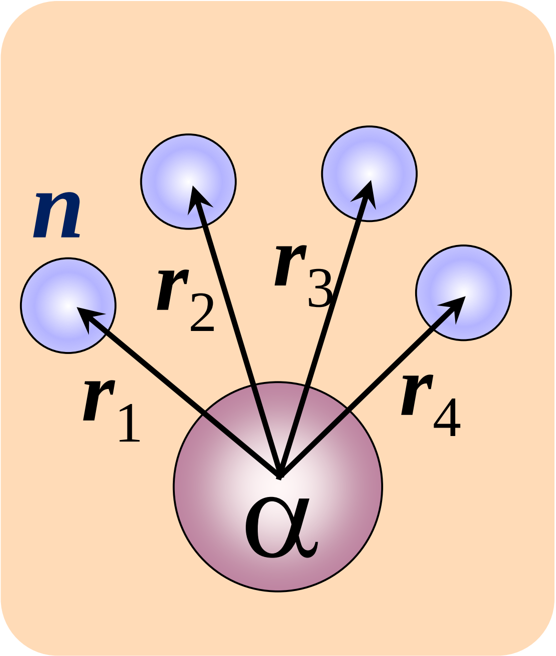
The complex scaling method (CSM) is a useful similarity transformation of the Schrödinger equation, in which bound-state spectra are not changed but continuum spectra are separated into resonant and non-resonant continuum ones. Because the asymptotic wave functions of the separated resonant states are regularized by the CSM, many-body resonances can be obtained by solving an eigenvalue problem with the L^2 basis functions. Applying this method to a system consisting of a core and valence nucleons, we investigate many-body resonant states in weakly bound nuclei very far from the stability lines. Non-resonant continuum states are also obtained with the discretized eigenvalues on the rotated branch cuts. Using these complex eigenvalues and eigenstates in CSM, we construct the extended completeness relations and Green's functions to calculate strength functions and breakup cross sections. Various kinds of theoretical calculations and comparisons with experimental data are presented.

Hamiltonian

- $V_{\alpha\text{-}n}$: microscopic KKNN potential
  - s,p,d,f-waves of α-n scattering
- V_{nn} : Minnesota potential with slightly strengthened (+ Coulomb for *p*-rich nuclei)

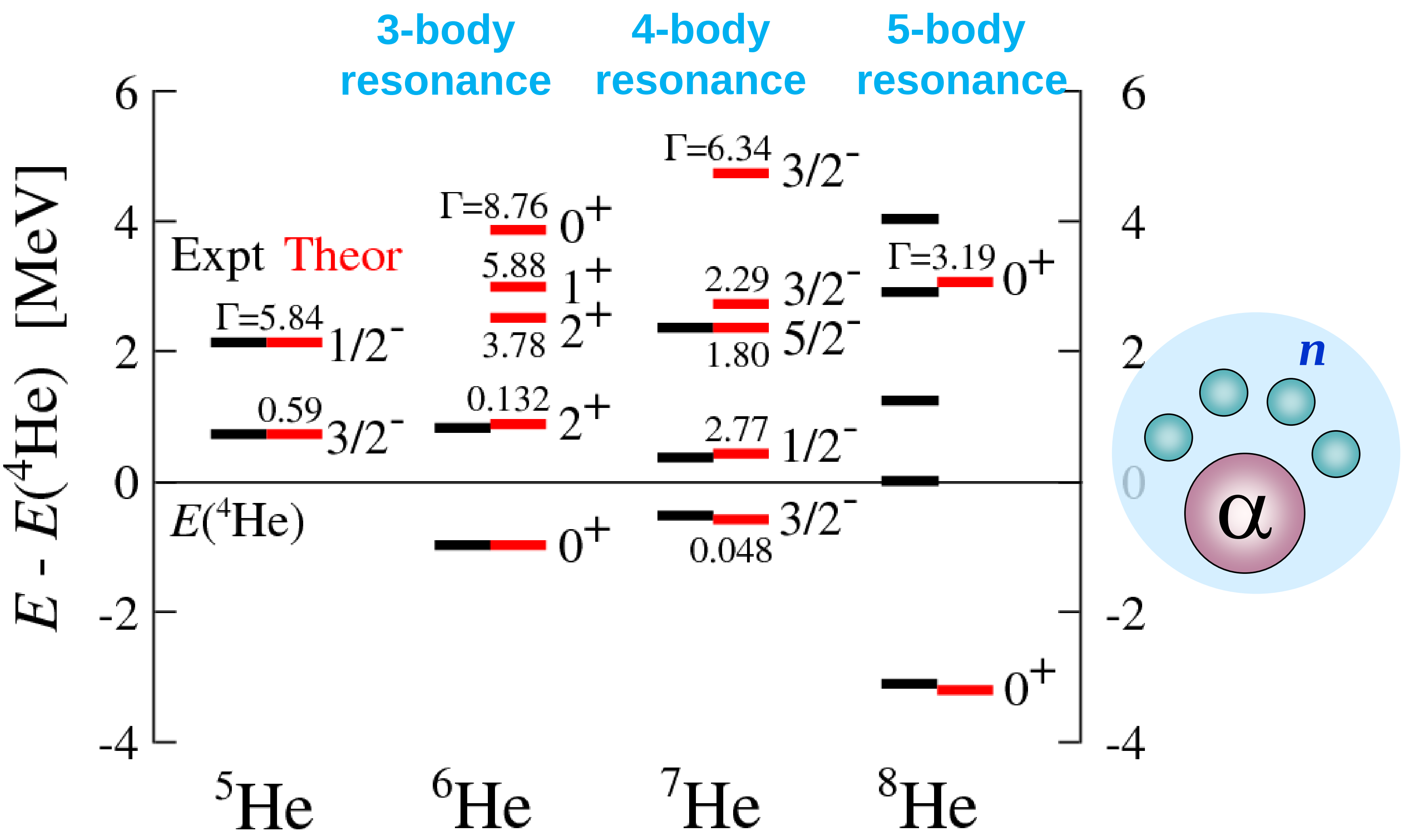
Fit energy of ^{6}He(0^+)

COSM



A. Csoto, PRC48(1993)165.
K. Arai, Y. Suzuki and R.G. Lovas, PRC59(1999)1432.
TM, S. Aoyama, K. Kato, K. Ikeda, PRC63(2001)054313.
TM et al. PTP113(2005)763.

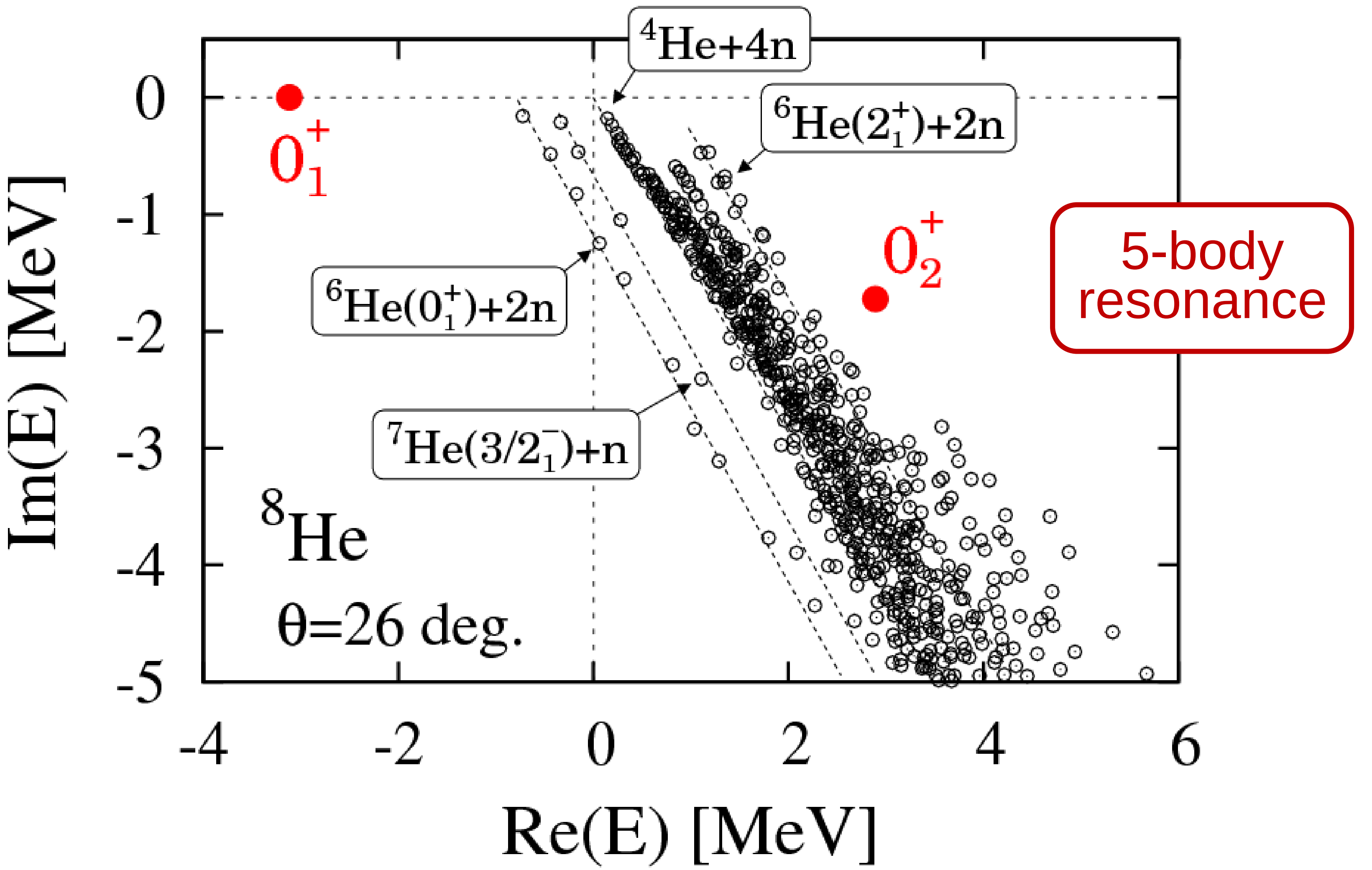
He isotopes : Expt vs. Complex Scaling



TM, K.Kato, K.Ikeda PRC76('07)054309
TM, R.Ando, K.Kato PRC80('09)014315

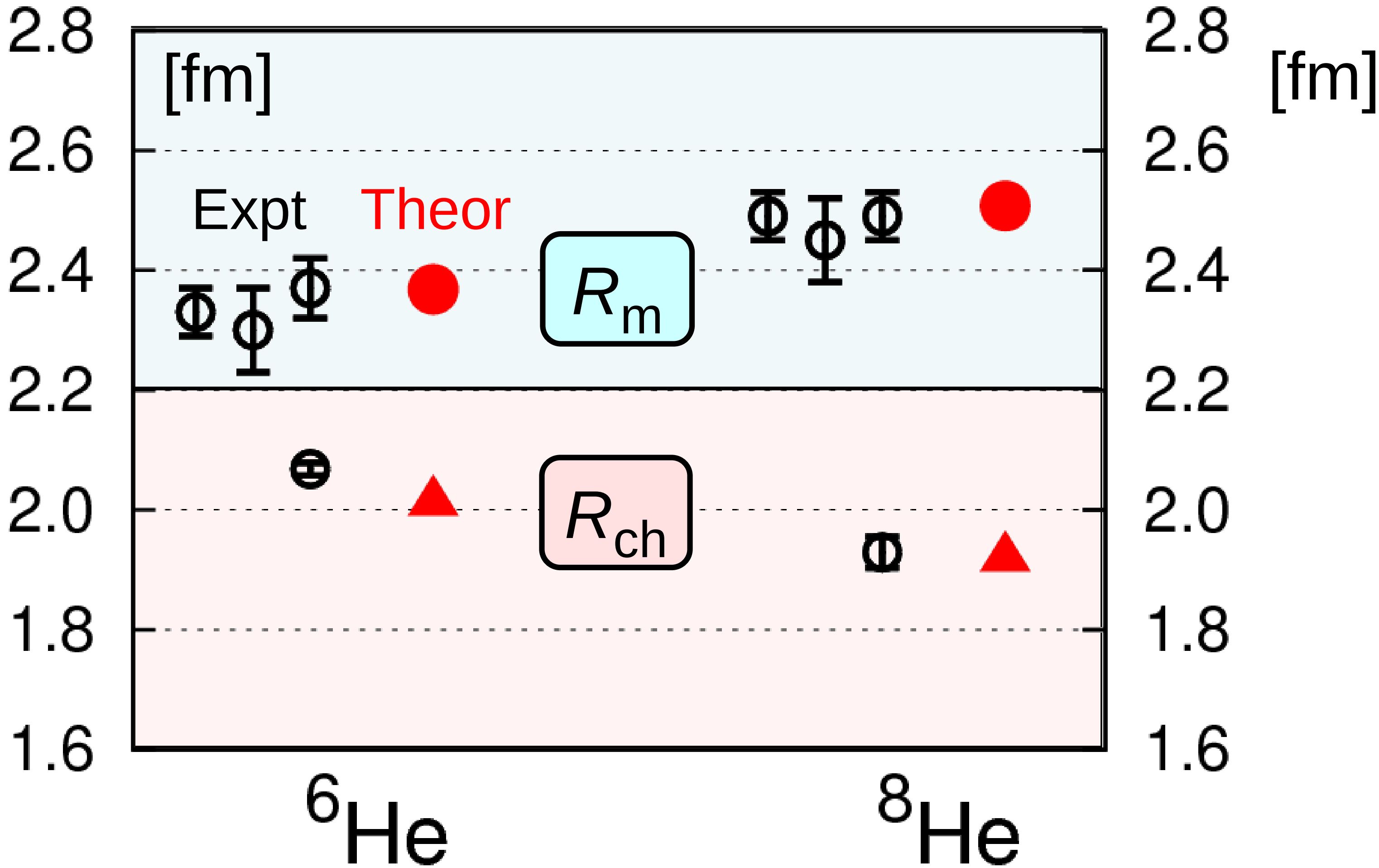
TM, R.Ando, K.Kato, PLB691('10)150
TUNL Nuclear Data Evaluation

Energy of ^{8}He with complex scaling



Eigenvalue problem with 32,000 dim.
Full diagonalization of complex matrix @ SX8R of NEC

Matter & Charge radii of ^{6,8}He



I. Tanihata et al., PLB289('92)261
O. A. Kiselev et al., EPJA 25, Suppl. 1('05)215.
G. D. Alkhazov et al., PRL78('97)2313
P. Mueller et al., PRL99(2007)252501

$p(^8He,t)$@SPIRAL
PLB718(2012)441

^6He

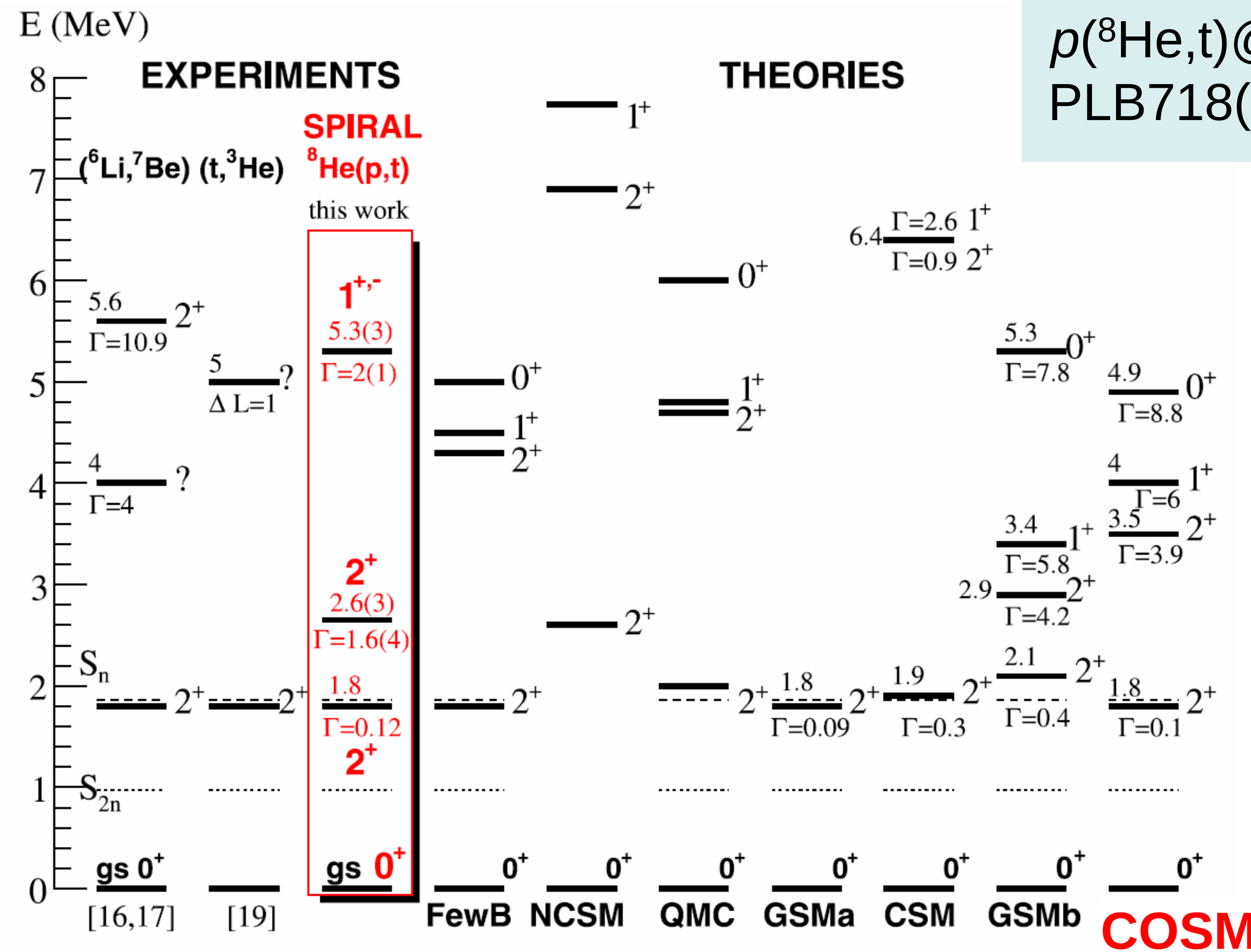
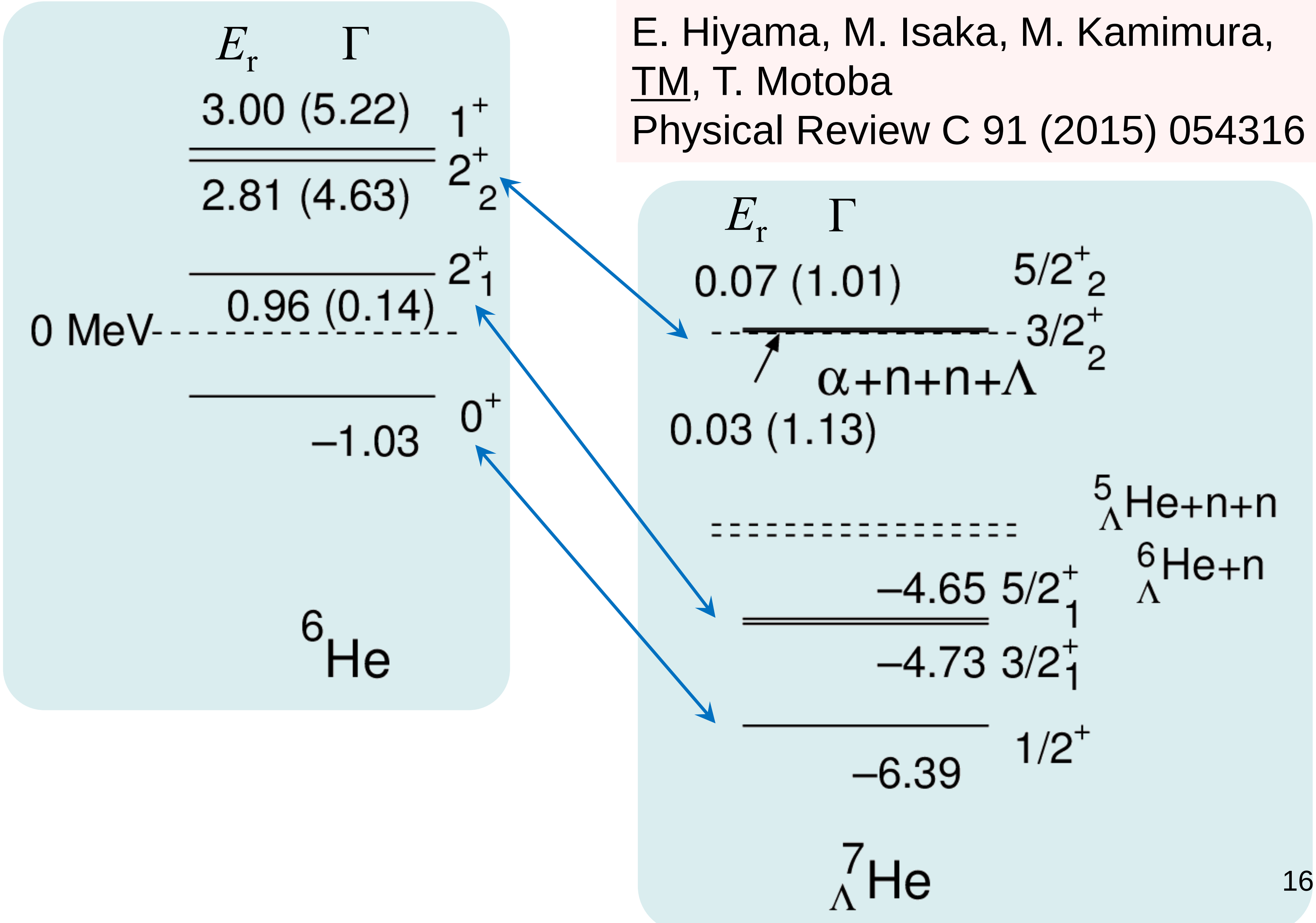


Fig. 6. Spectroscopy of ^6He: comparison between our new results with the previous experiments and with several theories, few-body model (FewB) [14], QMC [4], NCSM [15], CSM [10], GSMa [9], GSMb [8], and the COSM [11].

${}^{7}_{\Lambda}$He spectrum with α+n+n+Λ

E. Hiyama, M. Isaka, M. Kamimura, TM, T. Motoba
Physical Review C 91 (2015) 054316

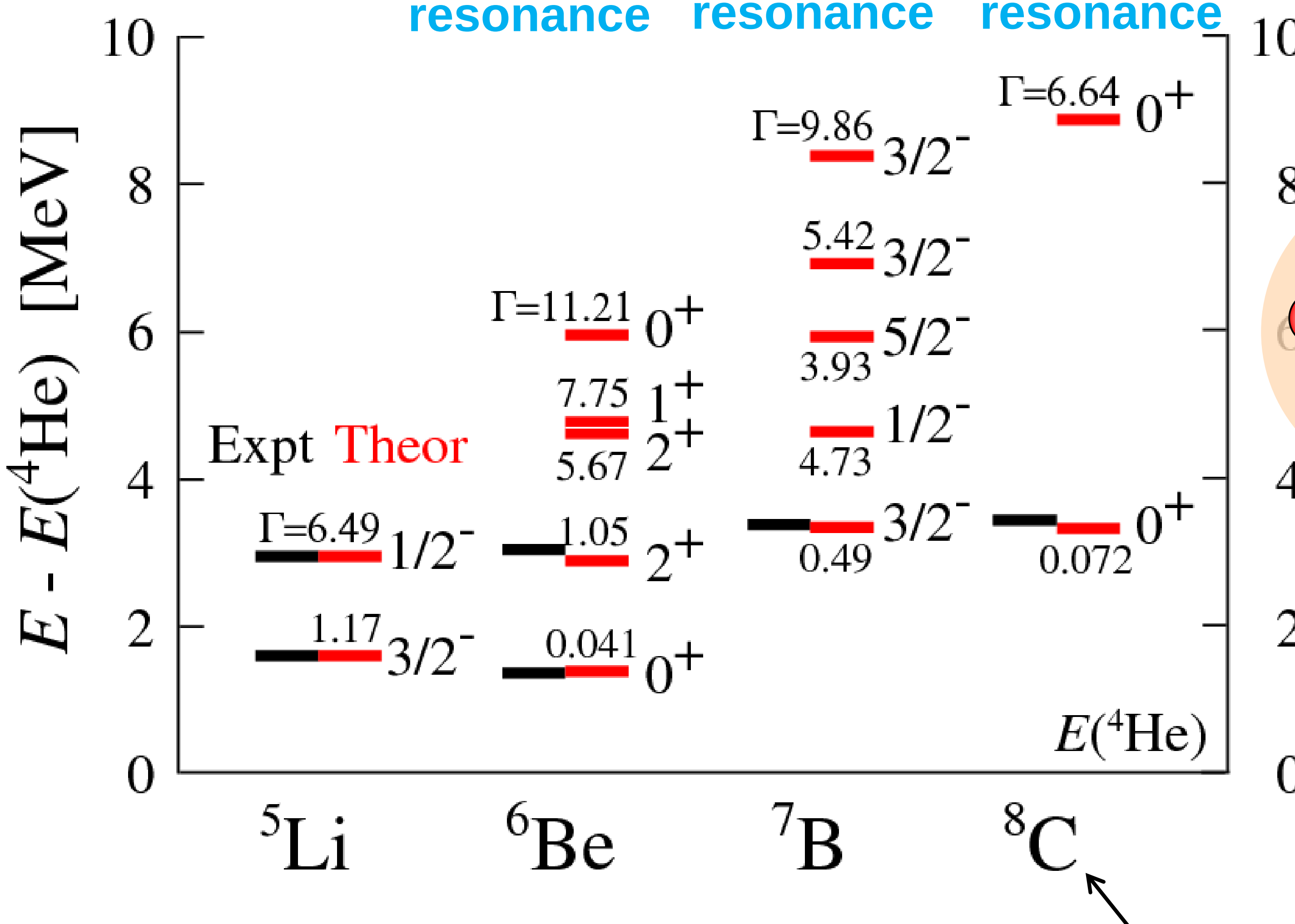


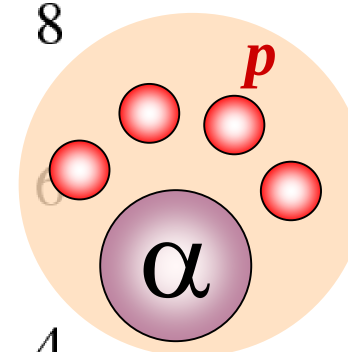
Proton-rich ^{7}B & ^{8}C

- Proton-rich unbound nucleus
  - ^{4}He-^{5}Li-^{6}Be-**^{7}B**-**^{8}C**, decay into $\alpha+p+p+p(+p)$ systems
- Experiments
  - Only the ground states are observed.
    - **^{7}B:** L. R. McGrath & J. Cerny, Phys. Rev. Lett. **19**, 1442 (1967).
    - **^{8}C:** R. G. H. Robertson, S. Martin, W. R. Falk, D. Ingham, A. Djaloeis, Phys. Rev. Lett. **32**, 1207 (1974). **^{8}C & ^{20}Mg**
  - R. J. Charity et al., Phys. Rev. C 84, 014320 (2011).
    ^{9}C beam: ^{7}B, $^{8}B^{*}$, ^{8}C, ... @MSU
- Mirror symmetry of ***p***-rich & ***n***-rich unstable nuclei
  - ^{7}B-^{7}He, ^{8}C-^{8}He : energies levels, configurations

Proton-rich side : ^{4}He+4p

3-body resonance **4-body resonance** **5-body resonance**

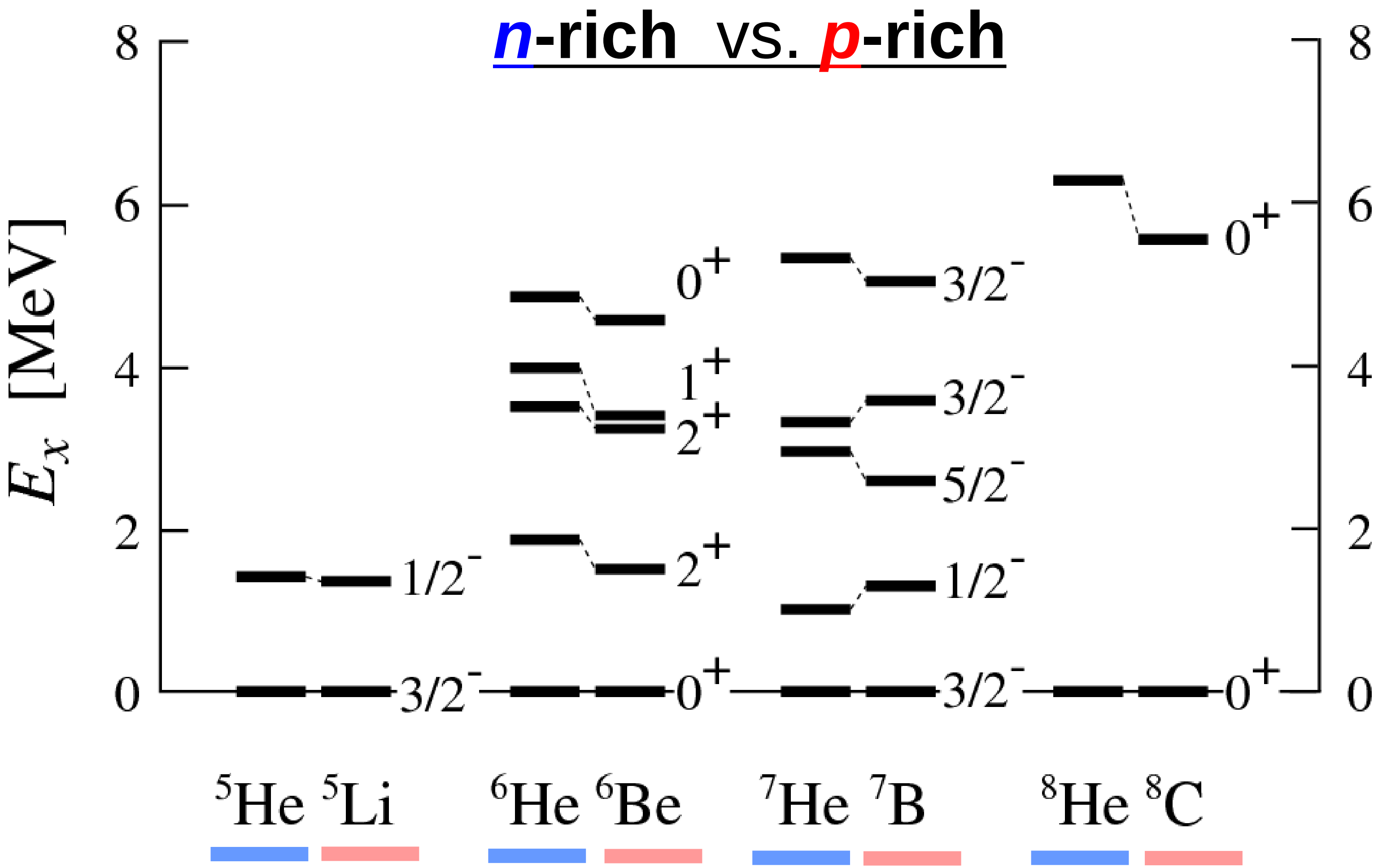




Γ(exp) ~ 0.130(50) MeV

Charity et al. PRC84(2011)014320.
TUNL Nuclear Data Evaluation

Mirror symmetry in resonances



TM, Kikuchi, Kato PRC84 (2011) 064306
PRC85 (2012) 034338

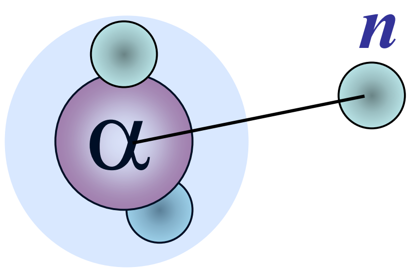
- Good symmetry

S-factors of ^{7}He & ^{7}B

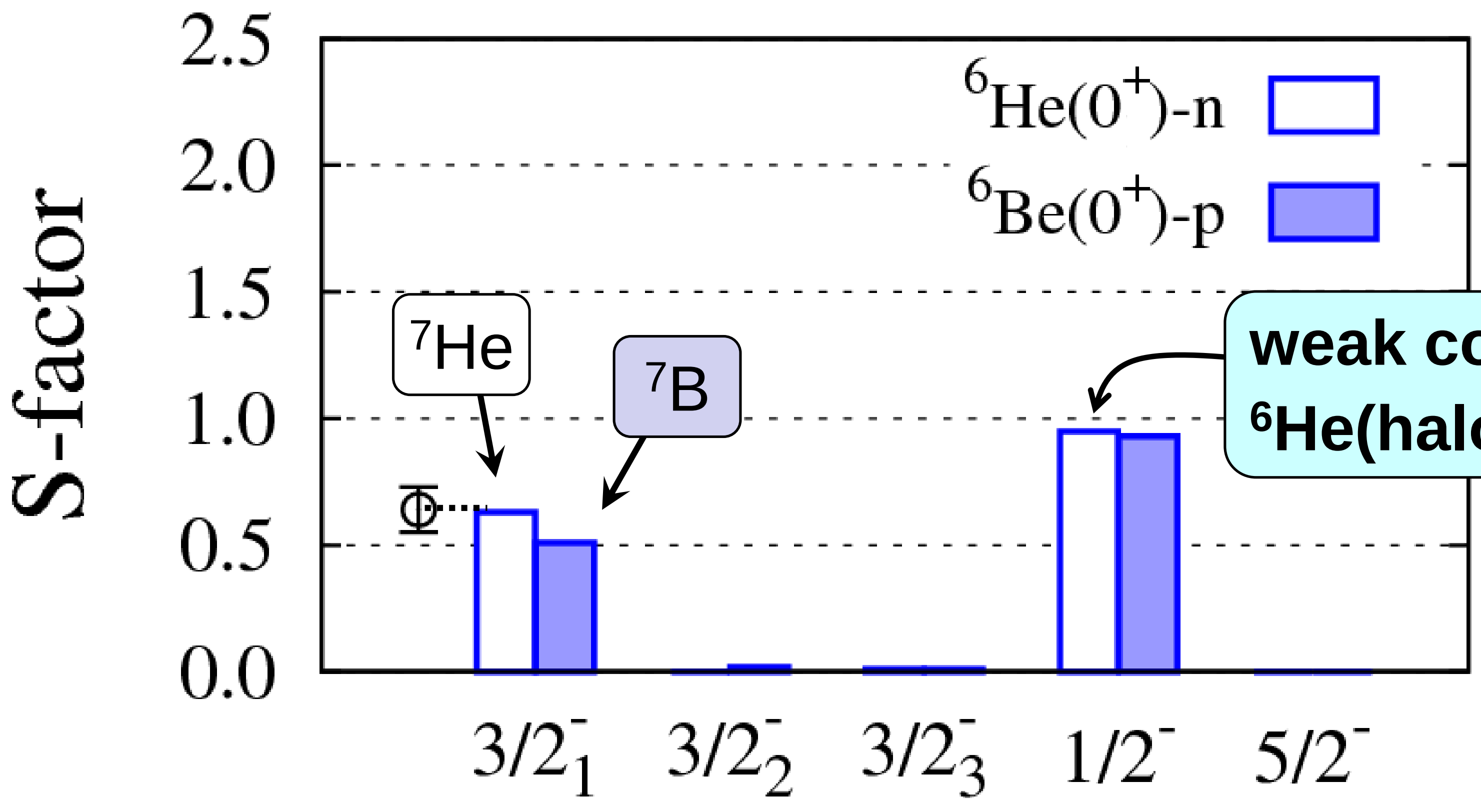
neutron removal

$$S_{J',J} = \sum_{nlj} \left\langle {}^6\mathrm{He}(0^+) \middle| a_{nlj}(n) \middle| {}^7\mathrm{He}(J^\pi) \right\rangle^2$$

$$S_{J',J} = \sum_{nlj} \left\langle {}^6\mathrm{Be}(0^+) \middle| a_{nlj}(p) \middle| {}^7\mathrm{B}(J^\pi) \right\rangle^2$$



^{6}He (halo)



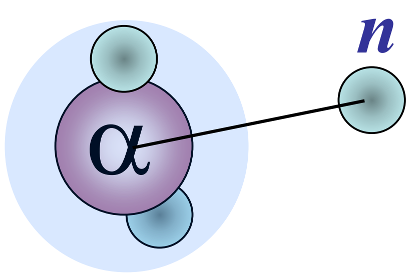
weak coupling of ^{6}He(halo) +n($p_{1/2}$)

Expt. of ^{7}He : F. Beck et al., Phys. Lett. B 645 (2007) 128

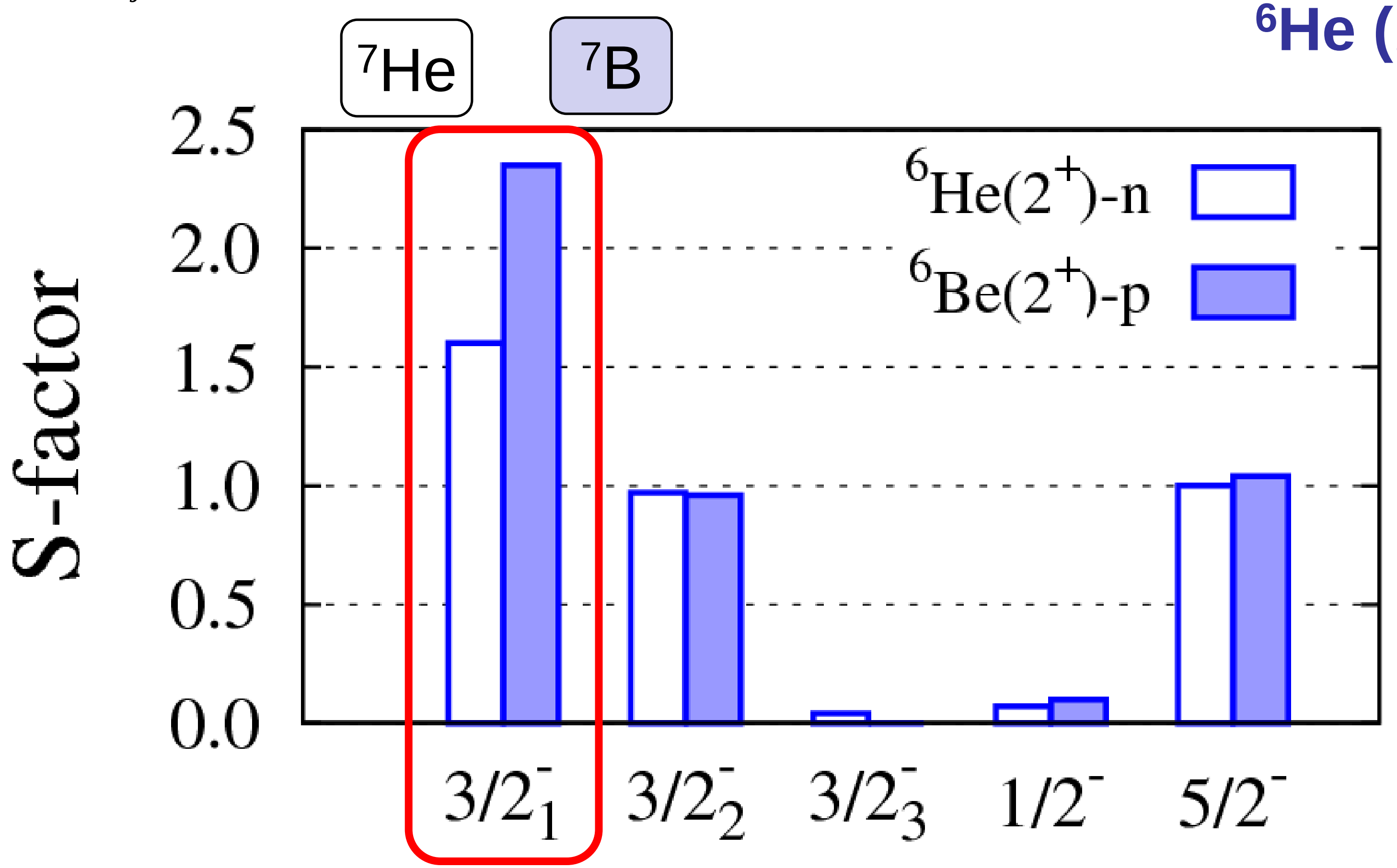
S-factors of ^{7}He & ^{7}B

$$S_{J',J} = \sum_{nlj} \left\langle {}^6\mathrm{He}(2^+) \middle| a_{nlj}(n) \middle| {}^7\mathrm{He}(J^\pi) \right\rangle^2$$

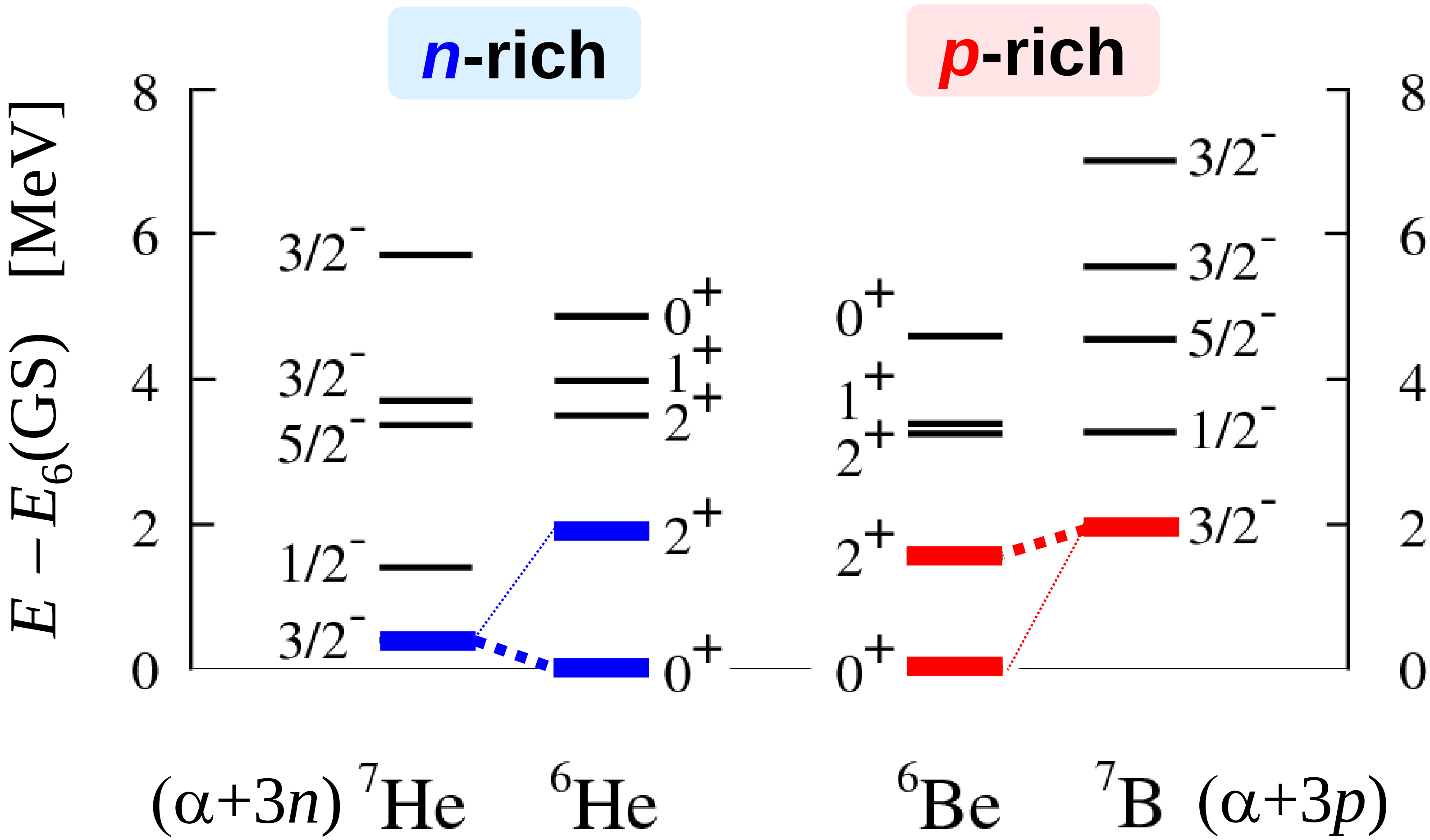
$$S_{J',J} = \sum_{nlj} \left\langle {}^6\mathrm{Be}(2^+) \middle| a_{nlj}(p) \middle| {}^7\mathrm{B}(J^\pi) \right\rangle^2$$



^{6}He (2_1^+)

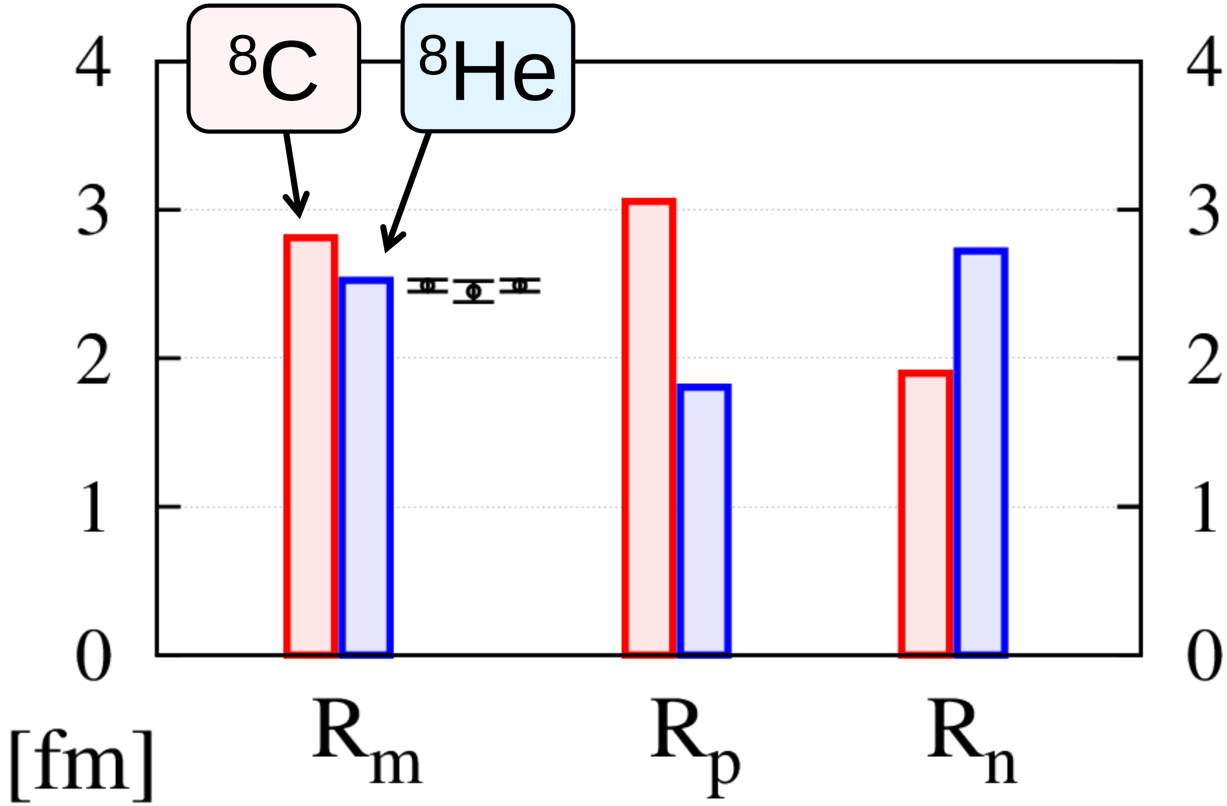


Thresholds of [A=6]+N system



Mirror symmetry breaking due to the channel coupling effect caused by Coulomb force

Radial properties of ^{8}C, ^{8}He – **G.S.** –

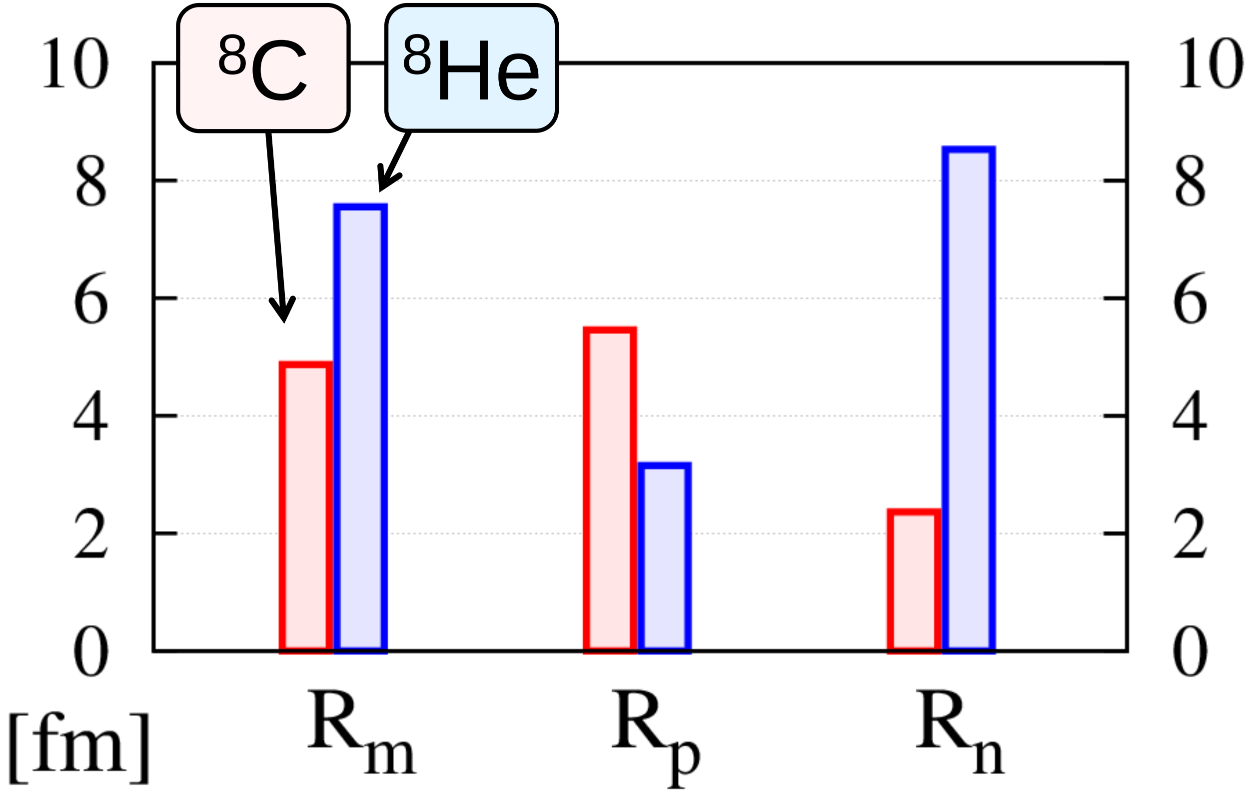


<u>10%-15% increase</u> due to Coulomb repulsion

cf. ^{6}Be-^{6}He, 20% increase
(2p) (2n)

I. Tanihata et al., PLB289('92)261
G. D. Alkhazov et al., PRL78('97)2313
O. A. Kiselev et al., EPJA 25, Suppl. 1('05)215

Radial properties of ^{8}C, ^{8}He – $\mathbf{0^+_2}$ –



TM, Kato, PTEP2014, 083D01

30% decrease due to Coulomb barrier

$\mathbf{0^+_2}$

- ^{8}C $(E_r, \Gamma) = (8.9, 6.4)$ (MeV)
- ^{8}He $(E_r, \Gamma) = (3.1, 3.2)$ comparable

Continuum Level Density (CLD) in CSM

$$\Delta E = -\frac{1}{\pi}\mathrm{Im}\left[\mathrm{Tr}\left[G(E)-G_0(E)\right]\right], \qquad G_{(0)} = \frac{1}{E-H_{(0)}},$$

$$\Delta E = \frac{1}{2i\pi}\mathrm{Tr}\left[S(E)^\dagger \frac{d}{dE}S(E)\right] \quad \rightarrow \quad \frac{1}{\pi}\frac{d\delta_\ell}{dE} \ \text{(single channel case)}$$

S. Shlomo, NPA539('92)17

K. Arai and A. Kruppa, PRC60('99)064315

R. Suzuki, T. Myo and K. Kato, PTP113('05)1273.

CLD in CSM

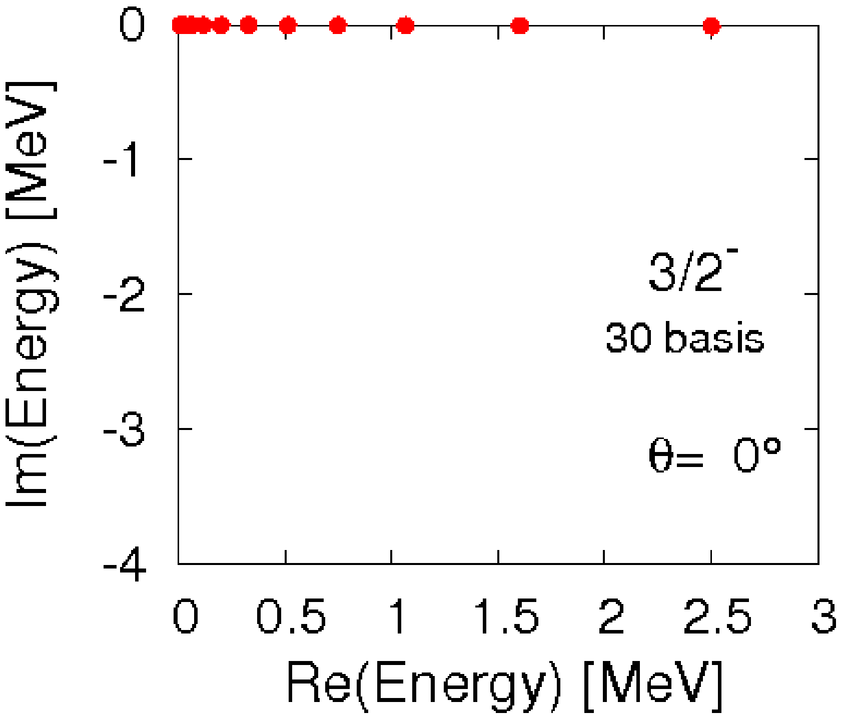
$$\Delta E = -\frac{1}{\pi}\mathrm{Im}\left[\mathrm{Tr}\left[G^\theta(E)-G_0{}^\theta(E)\right]\right]$$

$$G = \frac{1}{E-H^\theta}$$

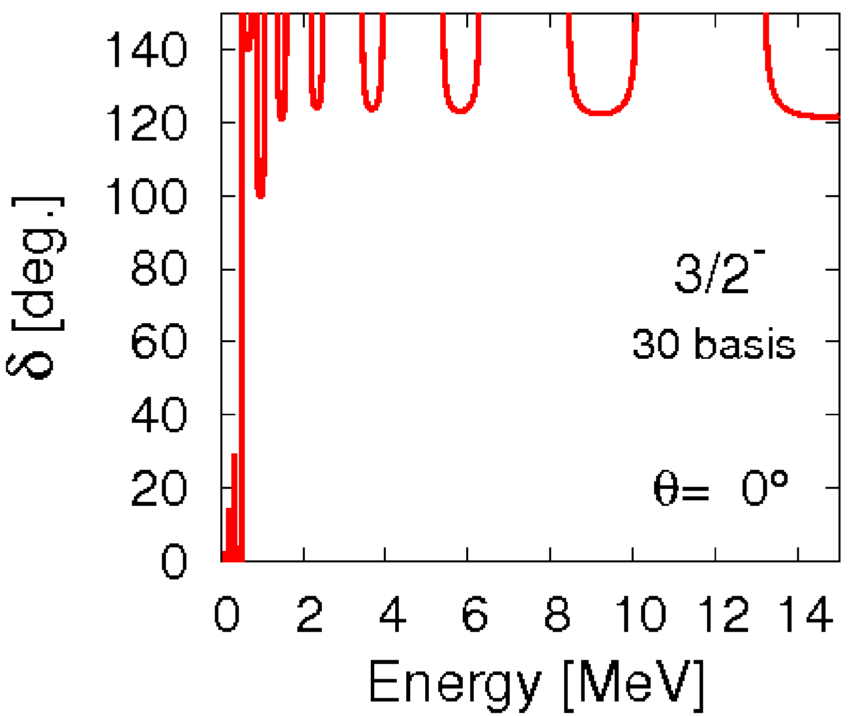
$$G_0 = \frac{1}{E-H_0^\theta} \ \text{(asymptotic)}$$

α+n scattering with complex scaling using discretized continuum states

energy eigenvalues



$P_{3/2}$ scattering phase shift



30 Gaussian basis functions

Strength function $S(E)$ in CSM

- Strength function and response function

Bi-orthogonal relation

$$S(E) = \sum_i \langle \tilde{\Phi}_0 | \hat{O}^\dagger | \varphi_i \rangle \langle \tilde{\varphi}_i | \hat{O} | \Phi_0 \rangle \cdot \delta(E - E_i)$$

initial state

$$= -\frac{1}{\pi} \mathrm{Im}\left[R(E) \right]$$

$$R(E) = \sum_i \frac{\langle \tilde{\Phi}_0 | \hat{O}^\dagger \boxed{| \varphi_i \rangle \langle \tilde{\varphi}_i |} \hat{O} | \Phi_0 \rangle}{\boxed{E - E_i}}$$

Response function

- Complex-scaled Green's function

complete set in CSM

$$G^\theta(E) = \frac{1}{E - H_\theta} = \sum_i \frac{| \varphi_i^\theta \rangle \langle \tilde{\varphi}_i^\theta |}{E - E_i^\theta}$$

Bound+Resonance+Continuum

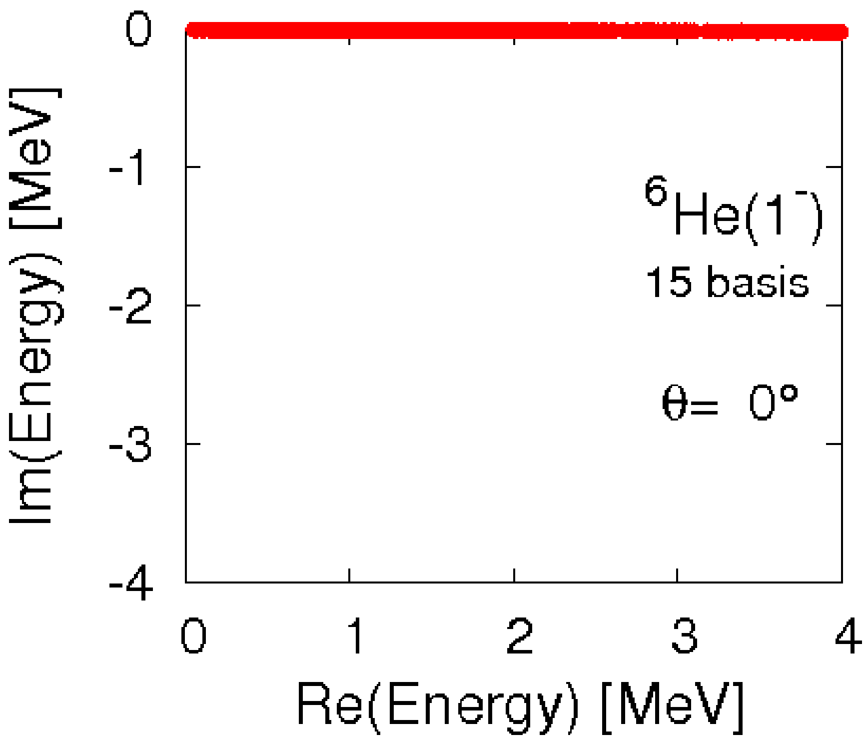
Reaction theory

- LS-eq. (Kikuchi)
- CDCC (Matsumoto)
- Scatt. Amp. (Kruppa, Dote($K^{bar}N$))

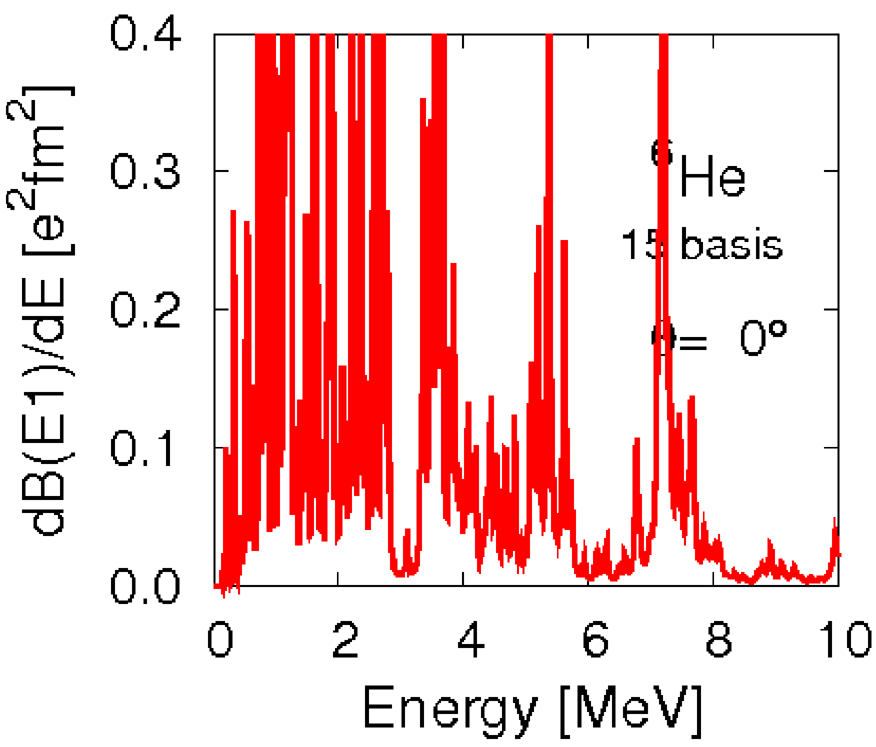
T. Berggren, NPA109('68)265, TM, A. Ohnishi and K. Kato, PTP99('98)801

^{6}He $\alpha+n+n$ scattering states with complex scaling

energy eigenvalues

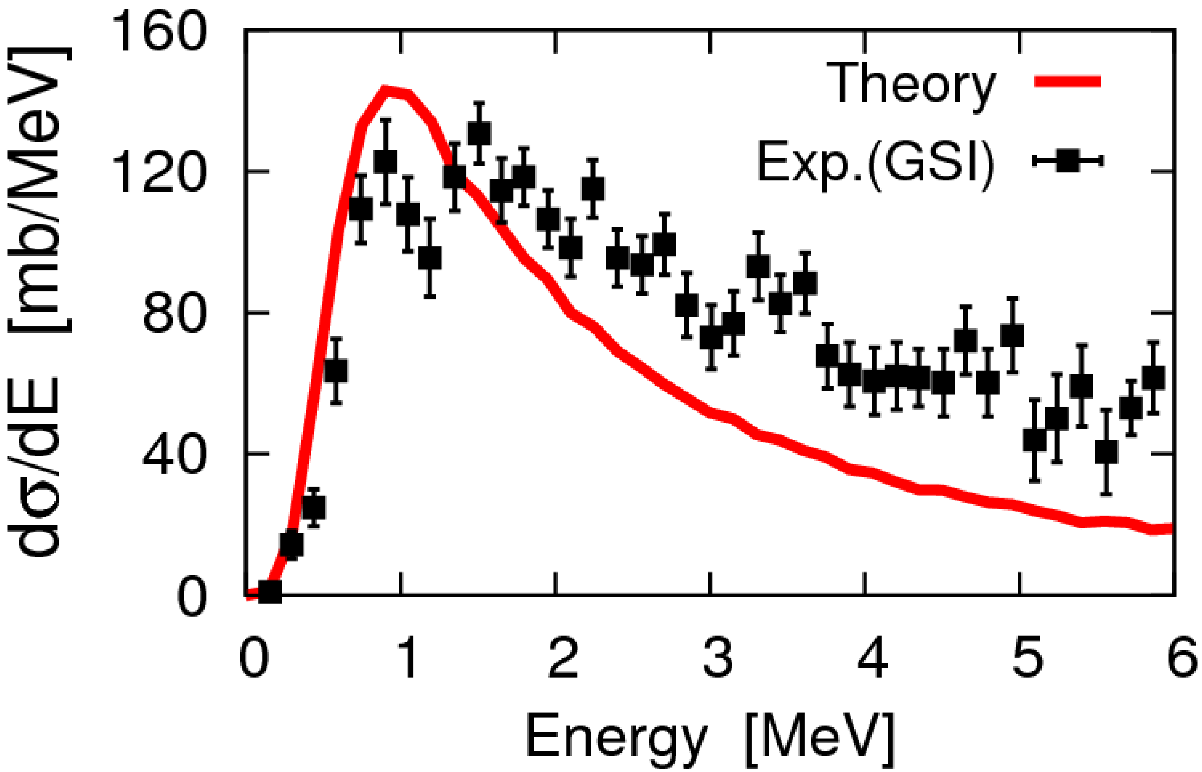


E1 strength



Coulomb breakup strength of ^{6}He

$$^{6}\mathrm{He}\,(\mathrm{G.S.}) \rightarrow {}^{4}\mathrm{He} + n + n$$



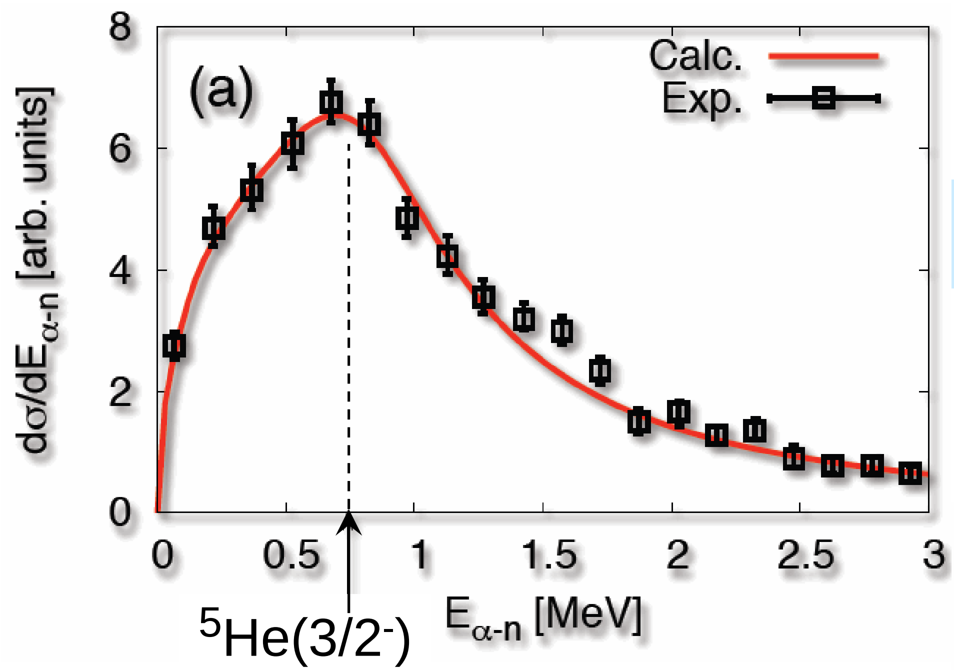
E1+E2 (complex scaling)
Equivalent photon method

TM, K. Kato, S. Aoyama and K. Ikeda PRC63(2001)054313.

Kikuchi, TM, Takashina, Kato, Ikeda
PTP122(2009)499
PRC81(2010)044308.
(invariant mass of α-n & n-n)

^{6}He : 240MeV/A, Pb Target (T. Aumann et.al, PRC59(1999)1252)

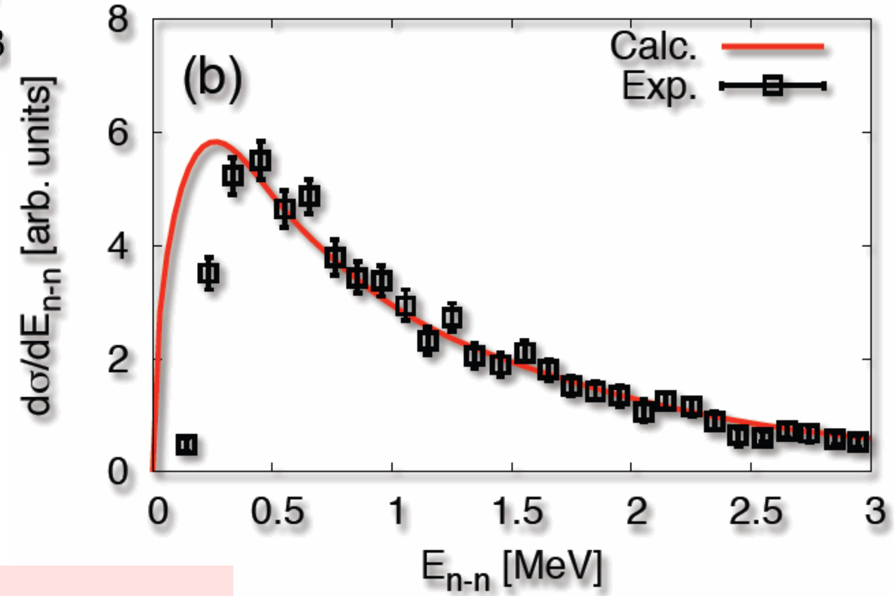
Invariant mass spectra of ^{6}He breakup



$$^6\text{He (G.S.)} \rightarrow {}^4\text{He} + n + n$$

α-n system

n-n system



Complex Scaling +
Lippmann-Schwinger Eq.

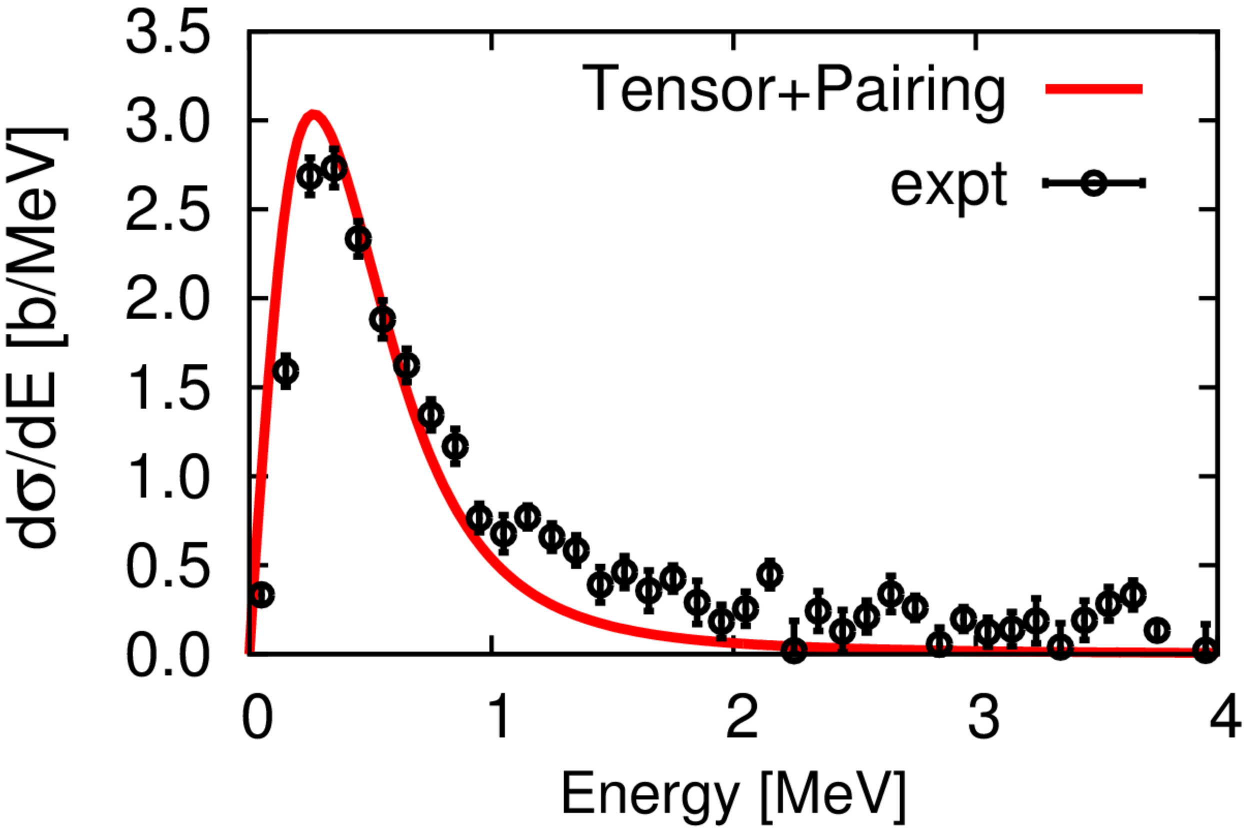
Kikuchi, TM, Takashina, Kato, Ikeda
PRC81(2010)044308.

$$|\Psi\rangle = |\Psi_0\rangle + \sum_i U^{-1}(\theta)|\phi_i\rangle \frac{1}{E - E_i^\theta} \langle\tilde{\phi}_i|U(\theta)V|\Psi_0\rangle$$

Coulomb breakup strength of ^{11}Li

T.Myo, K.Kato, H.Toki, K.Ikeda
PRC76(2007)024305

$^{11}Li \rightarrow {}^{9}Li+n+n$



No three-body resonance

E1 strength

+ **Complex scaling**

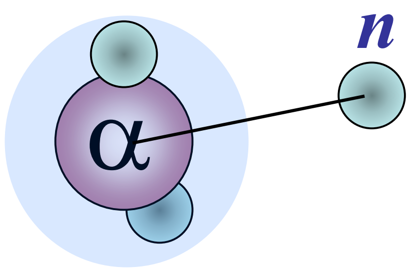
+Equivalent photon method

- Expt: T. Nakamura et al. , PRL96,252502(2006)
- Energy resolution with $\sqrt{E}$ =0.17 MeV.

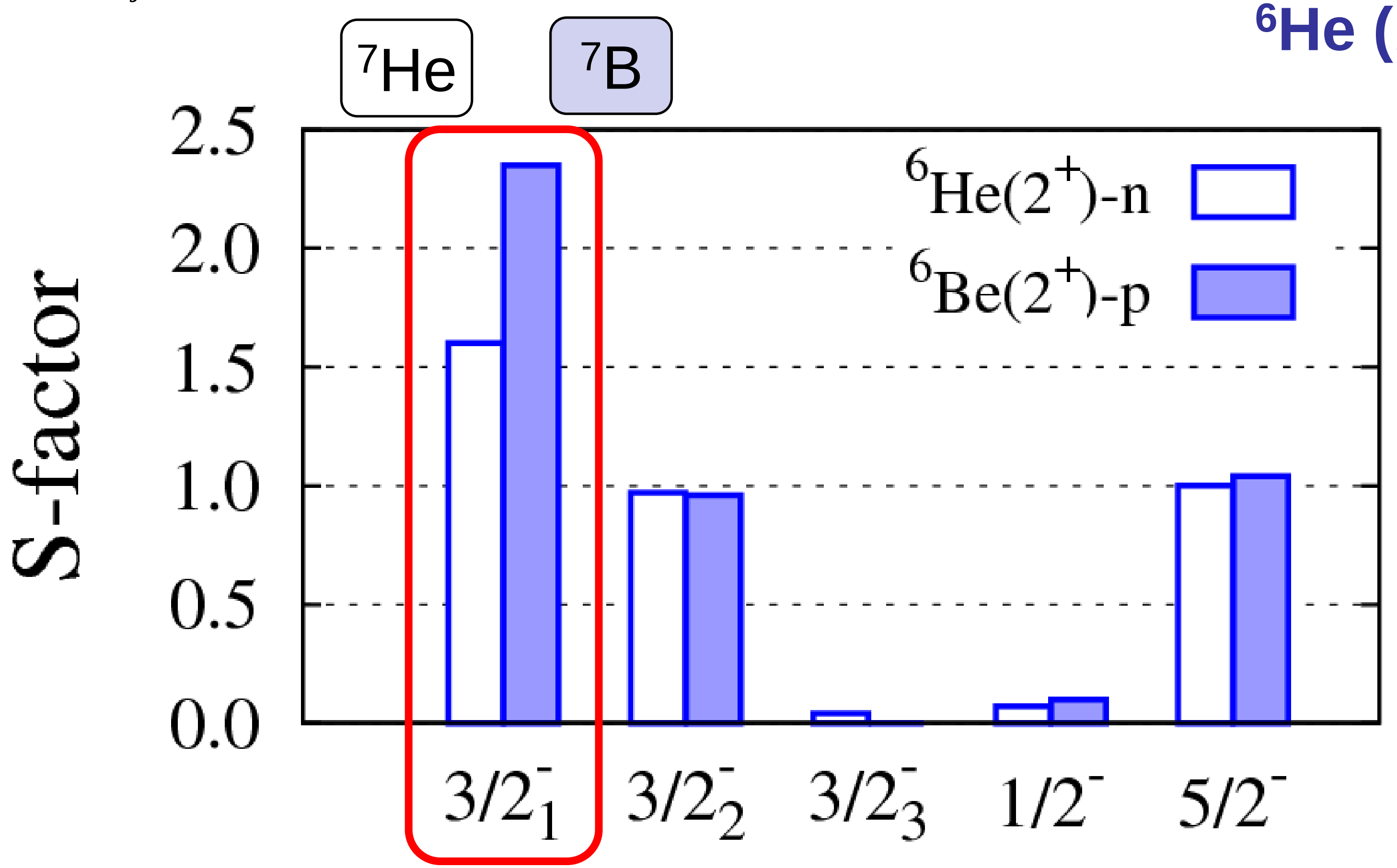
S-factors of ^{7}He & ^{7}B

$$S_{J',J} = \sum_{nlj} \left\langle {}^6\mathrm{He}(2^+) \middle| a_{nlj}(n) \middle| {}^7\mathrm{He}(J^\pi) \right\rangle^2$$

$$S_{J',J} = \sum_{nlj} \left\langle {}^6\mathrm{Be}(2^+) \middle| a_{nlj}(p) \middle| {}^7\mathrm{B}(J^\pi) \right\rangle^2$$



^{6}He (2_1^+)

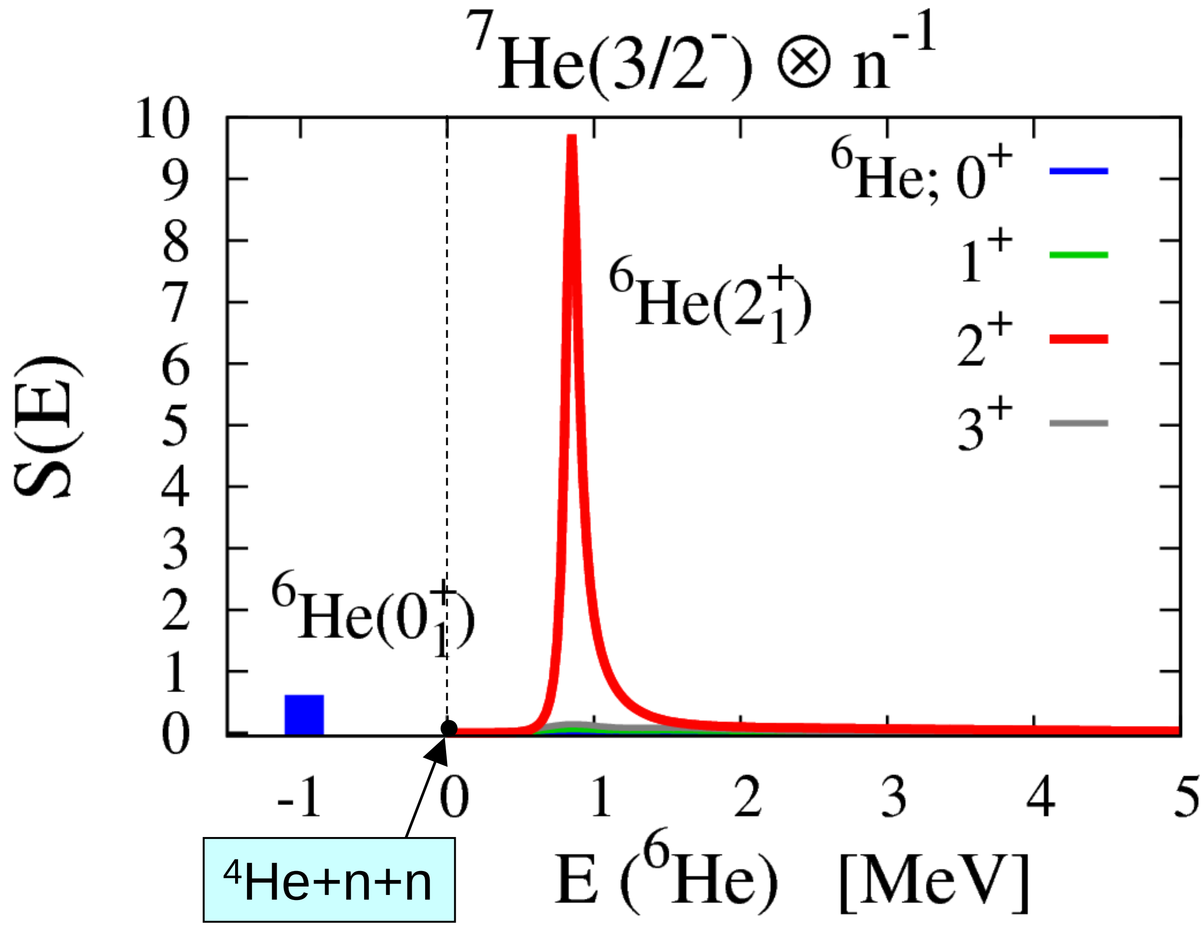


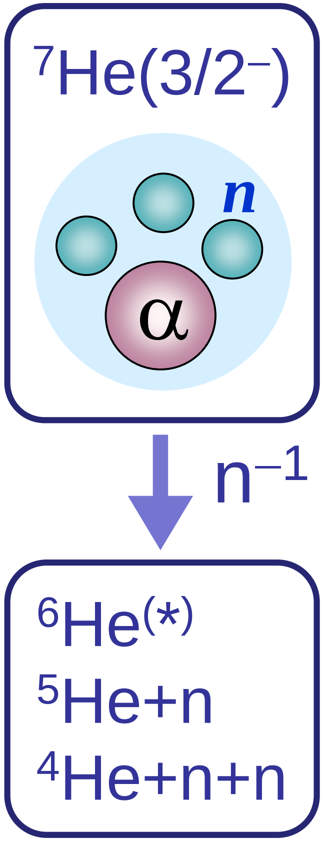
One-neutron removal strength of $^{7}He_{GS}$

$$S_{J',J}(E) = \sum_{nlj} \left| \left\langle {}^{6}\mathrm{He}^{J'}(E) \middle| a_{nlj}(n) \middle| {}^{7}\mathrm{He}^{J} \right\rangle \right|^2$$

TM, Ando, Kato
PRC80(2009)014315

" ^{4}He+n+n" complete set using CSM



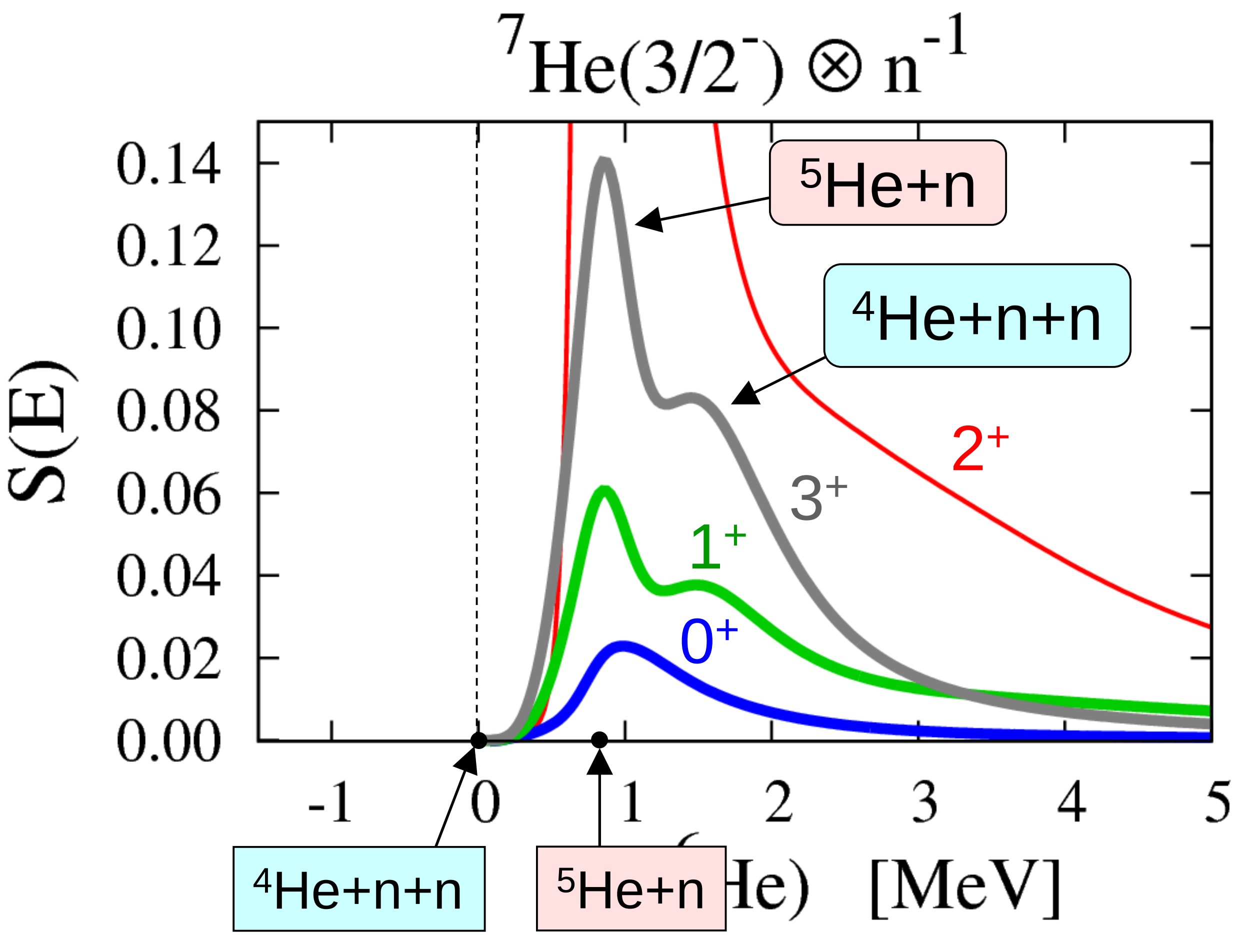


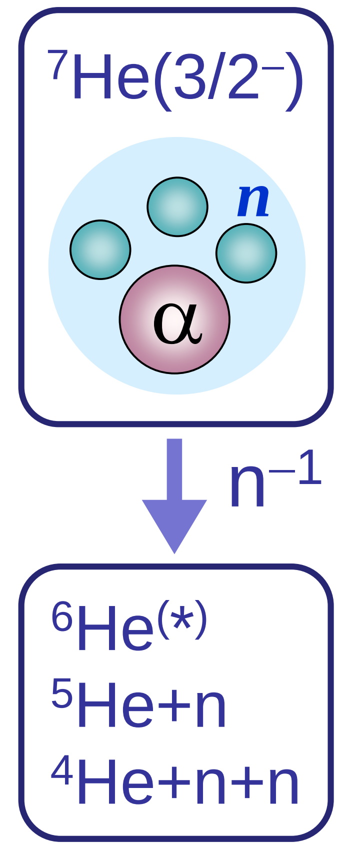
One-neutron removal strength of $^{7}\mathrm{He}_{\mathrm{GS}}$

$$S_{J',J}(E) = \sum_{nlj} \left| \left\langle {}^{6}\mathrm{He}^{J'}(E) \middle| a_{nlj}(n) \middle| {}^{7}\mathrm{He}^{J} \right\rangle \right|^{2}$$

TM, Ando, Kato
PRC80(2009)014315

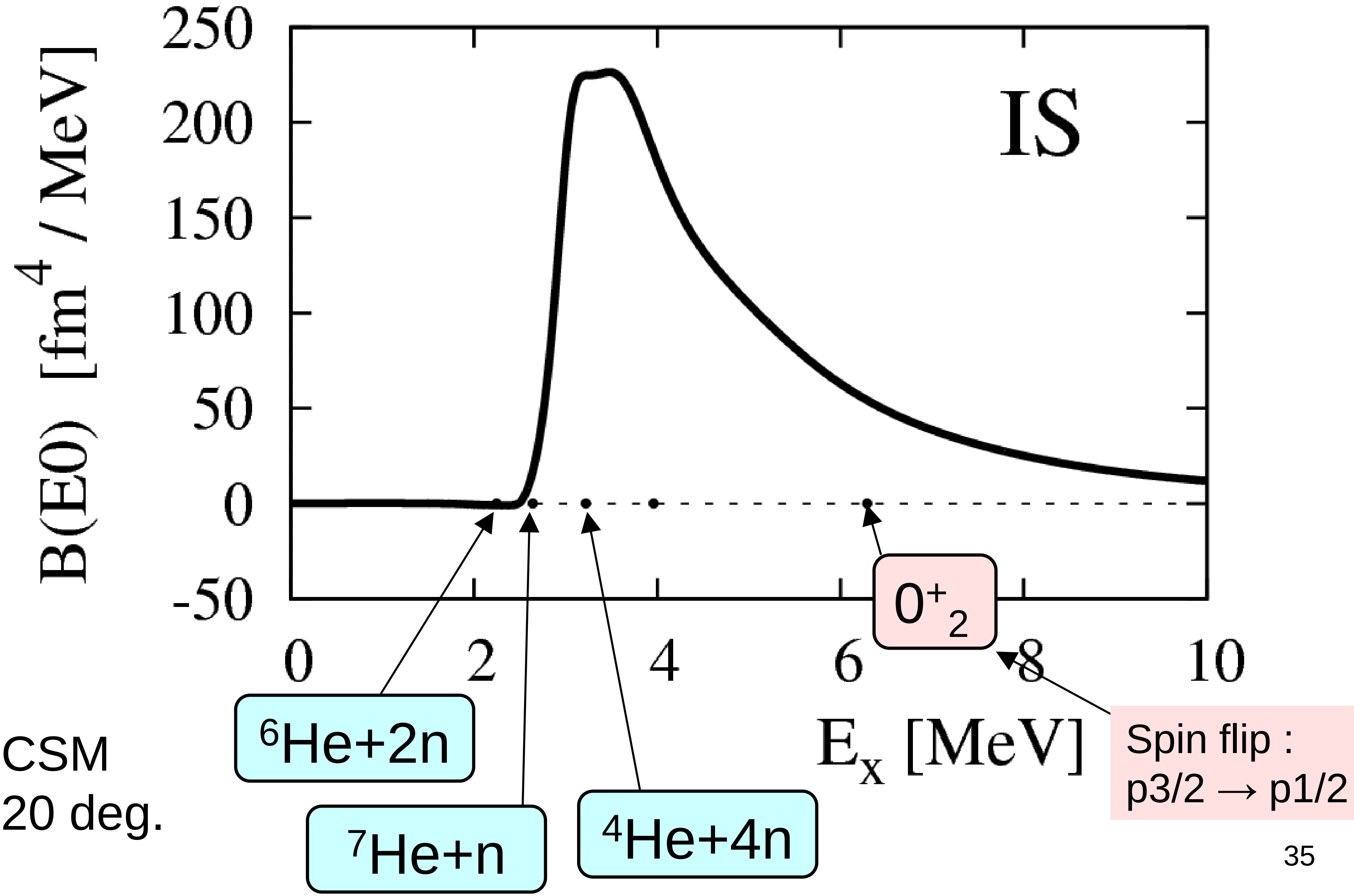
” ^{4}He+n+n” complete set using CSM





Monopole Strength of ^{8}He (Isoscalar)

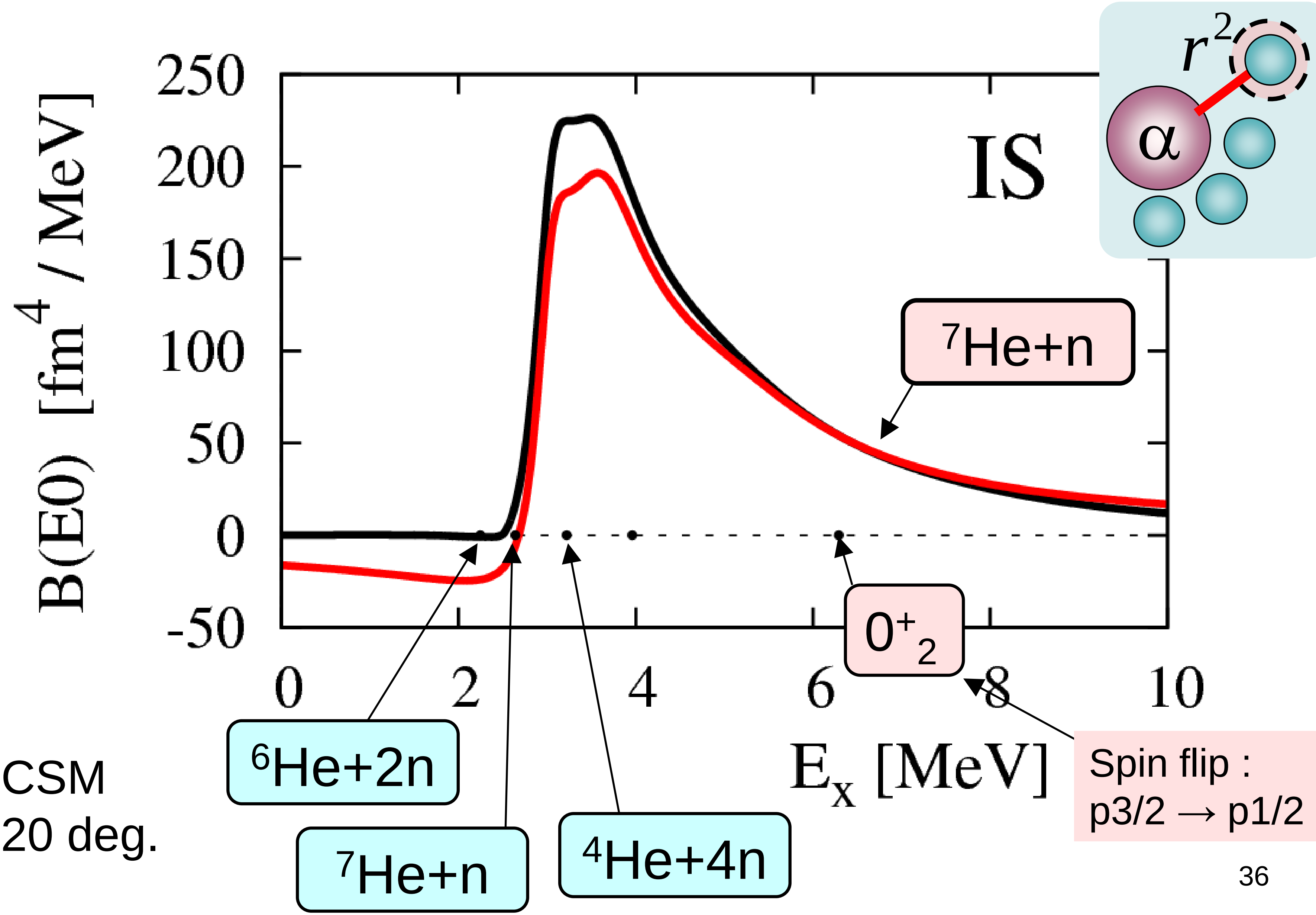
TM, Ando, Kato, PLB 691 (2010) 150



CSM

θ=20 deg.

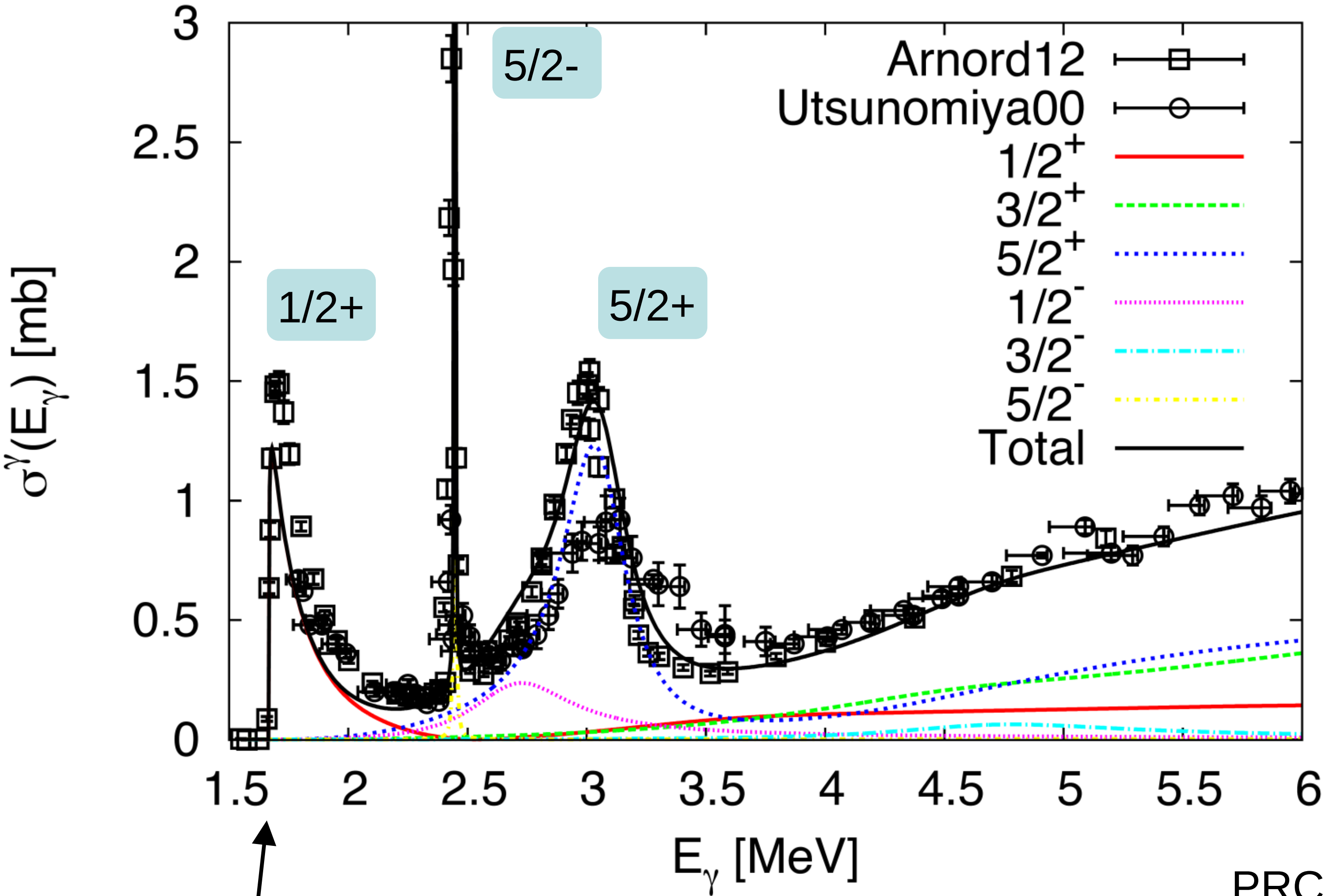
Monopole Strength of ^{8}He (Isoscalar)



Spin flip :
p3/2 → p1/2

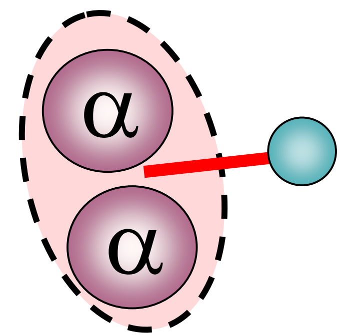
CSM
θ=20 deg.

^{9}Be Photodisintegration into $\alpha+\alpha+n$



Kikuchi, Kato, Odsuren, TM Aikawa

α+α+n OCM



PRC92 (2015) 014322

Low-lying 1/2+ enhancement suggests the ^{8}Be+n **s-wave virtual state** above the α+α+n threshold.

Summary

- **Light Unstable Nuclei**
  - He isotopes (***n*-rich**) & Mirror nuclei (***p*-rich**)
  - Mirror symmetry due to $\underline{V_{\mathrm{Coulomb}}}$
    - Channel coupling (threshold), Radius
- **Complex Scaling Method**
  - Many-body resonance spectroscopy
  - Continuum level density ΔE (resonance+continuum)
  - Strength functions using Green's function
    - Coulomb breakups, subsystem correlation, ...
    - Application to reaction theory (LS eq., CDCC, …)