Continuum shell model for nuclear structure and reactions

Marek Ploszajczak
GANIL, Caen

Content:

- Introduction and recent advances in nuclear theory
- Continuum Shell Model and Gamow Shell Model
- Interplay between Hermitian and anti-Hermitian couplings
  - segregation of time scales
  - configuration mixing in weakly bound/unbound states
- One-nucleon spectroscopic factors
- Gamow Shell Model in Coupled Channel representation:
  - (p,p’), (p/n,g)
- Outlook and future challenges

Collaborators

W. Nazarewicz - NSCL-FRIB MSU, East Lansing and Univ. of Warsaw
K. Fossez, Y. Jaganathen, J. Rotureau - NSCL-FRIB MSU, East Lansing
J. Okolowicz - Institute of Nuclear Physics, Krakow
G. Dong, A. Mercenne, N. Michel - GANIL
Y.H. Lam – Institute of Modern Physics, Lanzhou
G. Papadimitriou - LLNL
R.M. Id Betan – Physics Institute of Rosario
B. Barrett – Univ. of Arizona, Tucson

Evolution of paradigms

- In medium nucleon-nucleon interaction from basic principles (EFT);
  3-body interactions

- Nuclear shell model for open quantum systems (GSM);
  structure and reactions in the low-energy continuum

- *Ab initio* many-body theories of structure and reactions
  (GFMC, NCSM/RGM, NCGSM, CC,...)

… the recent progress in many-body methods surpass
the progress in effective interactions

Challenges for the theory

- How to reconcile shell model with reaction models?

- Weakly bound systems; role of continuum…
  How to deal with non-localities due to the coupling to the decay channels and antisymmetrization

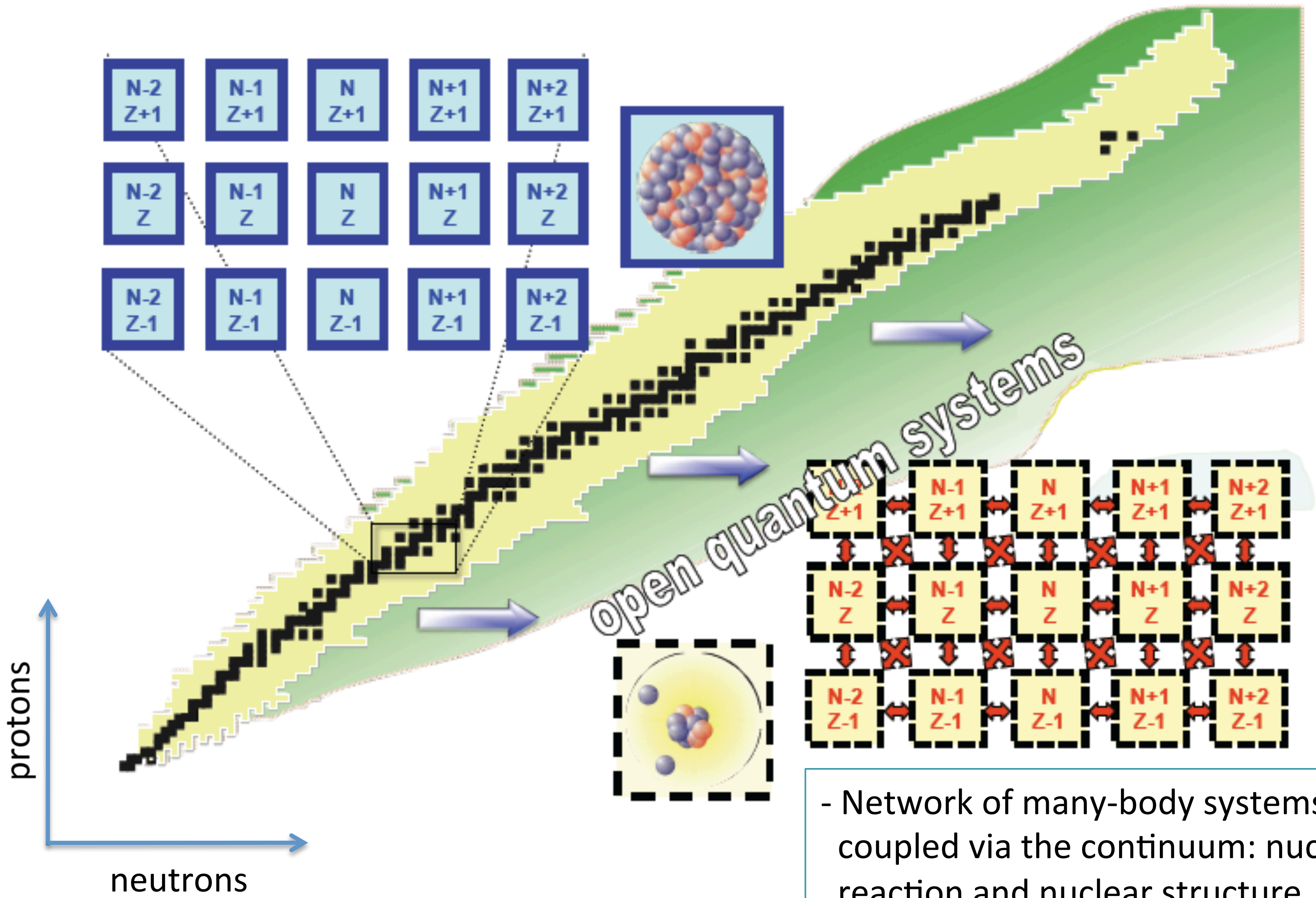
- Handling multi-configuration effects in reaction theory; How to reduce many-body to a few-body problem?

- Having useful microscopic input:
  Understanding (optical) potentials from microscopic interactions

Shell model for open quantum systems

Continuum Shell Model
Hilbert space formulation

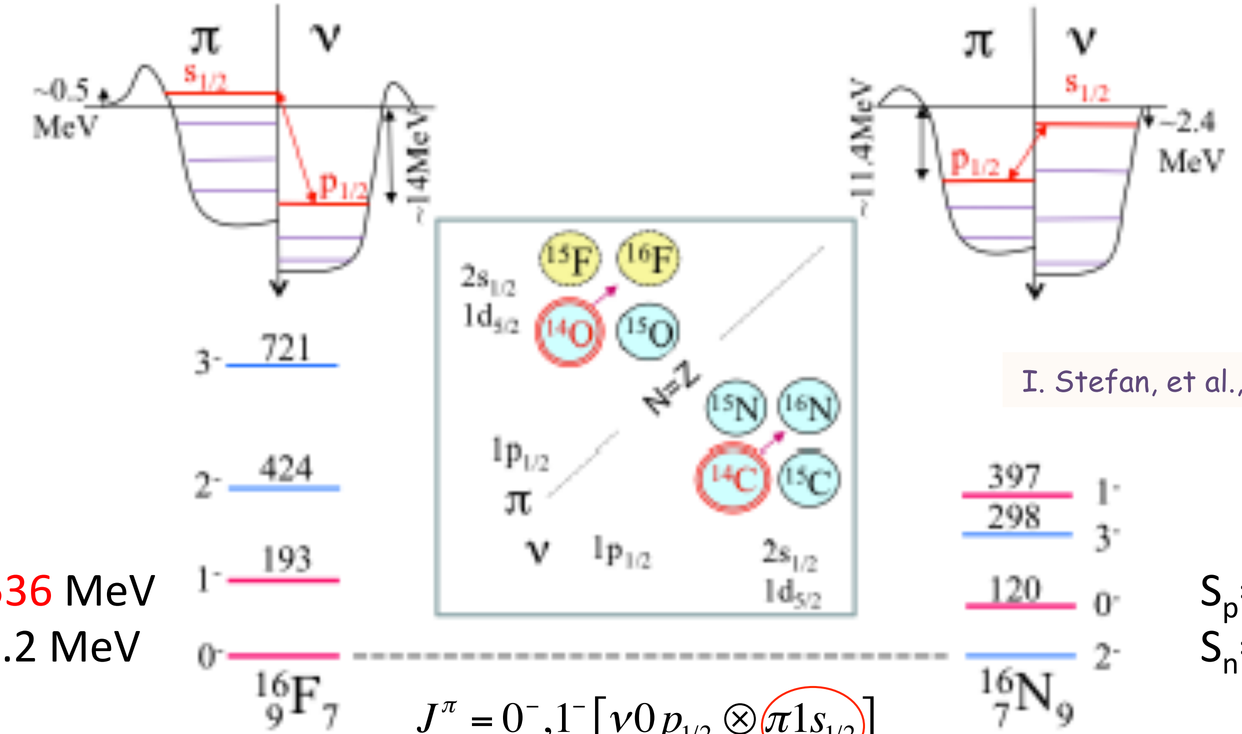
Gamow Shell Model
Rigged-Hilbert space formulation



- Network of many-body systems coupled via the continuum: nuclear reaction and nuclear structure approaches meet together
- Emergence of new scale(s) related to the threshold(s) energy

Mapping of continuum coupling effects in the SM analysis

I. Stefan, et al., PRC 90, 014307 (2014)



S_p=-0.536 MeV
S_n=+14.2 MeV

S_p=+11.48 MeV
S_n=+2.49 MeV

$$J^\pi = 0^-, 1^- \left[\nu 0 p_{1/2} \otimes \pi 1 s_{1/2}\right]$$

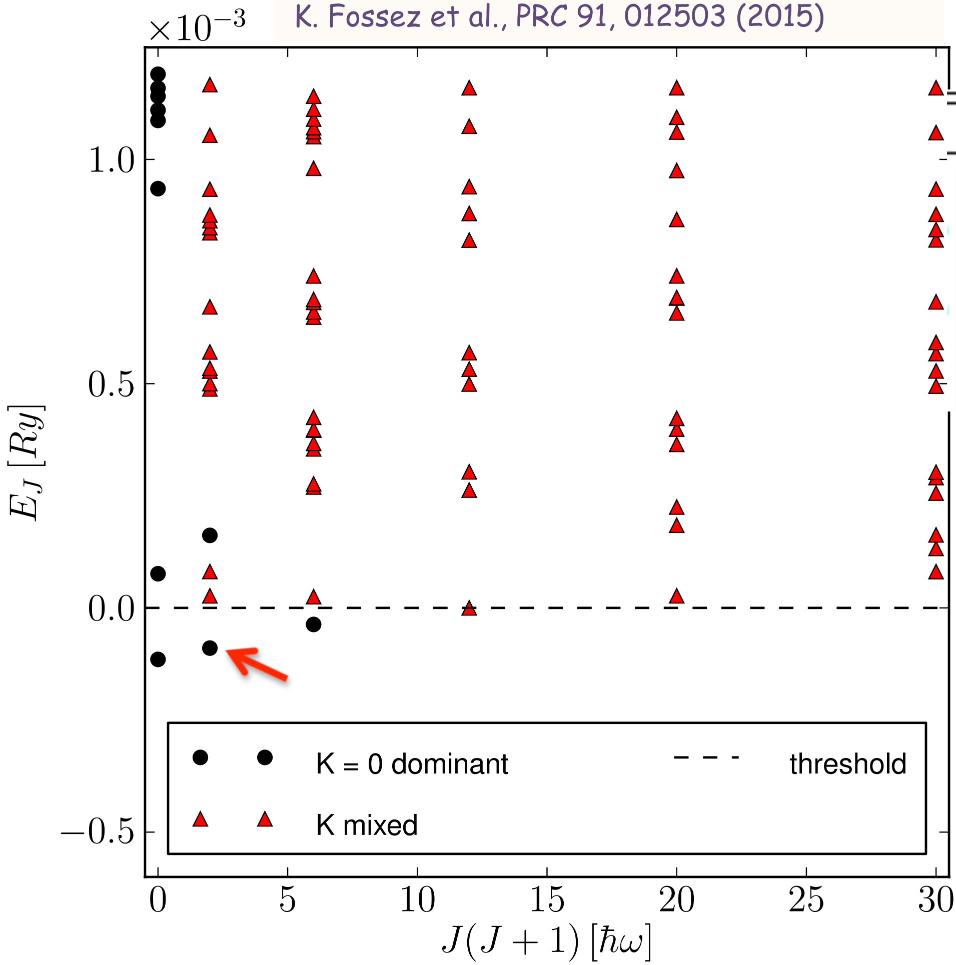
$$J^\pi = 2^-, 3^- \left[\nu 0 p_{1/2} \otimes \pi 0 d_{5/2}\right]$$

$$\boxed{\begin{array}{ll} \upsilon_{np}^{^{16}F}\left(0^-\right) = -0.775 MeV & -1.151 MeV = \upsilon_{np}^{^{16}N}\left(0^-\right) \\ \upsilon_{np}^{^{16}F}\left(1^-\right) = -0.577 MeV & -0.874 MeV = \upsilon_{np}^{^{16}N}\left(1^-\right) \end{array}} \quad \approx 40\%$$

$$\boxed{\begin{array}{ll} \upsilon_{np}^{^{16}F}\left(2^-\right) = -1.829 MeV & -2.011 MeV = \upsilon_{np}^{^{16}N}\left(2^-\right) \\ \upsilon_{np}^{^{16}F}\left(3^-\right) = -1.523 MeV & -1.713 MeV = \upsilon_{np}^{^{16}N}\left(3^-\right) \end{array}} \quad \approx 10\%$$

Breaking of K quantum number in the low-energy continuum of HCN-

K. Fossez et al., PRC 91, 012503 (2015)



Bands of isolated resonant states

$E(2^+)$	$E(3^-)$	$E(4^+)$
-3.69(-5)	3.89(-8) -i 1.06(-8)	2.70(-5) -i 5.55(-9)
2.51(-5) -i 9.68(-6)	2.63(-4) -i 1.88(-6)	1.84(-4) -i 2.02(-6)
2.69(-4) -i 3.45(-10)	3.03(-4) -i 9.25(-6)	2.25(-4) -i 2.47(-5)
2.77(-4) -i 3.58(-9)	4.99(-4) -i 1.28(-6)	3.65(-4) -i 1.40(-6)
3.55(-4) -i 7.20(-7)	5.32(-4) -i 1.01(-6)	3.99(-4) -i 1.43(-6)
3.67(-4) -i 1.21(-6)	5.69(-4) -i 1.25(-4)	4.23(-4) -i 1.26(-4)

Dipolar potential, finite-size effects, repulsive core, polarization effects, quadrupolar interaction described by a pseudo-potential V

$$H_{tot} = \frac{\boldsymbol{p}_e^2}{2m_e} + \frac{\boldsymbol{j}^2}{2I} + V$$

W. R. Garrett, J. Chem. Phys. 77, 3666 (1982)

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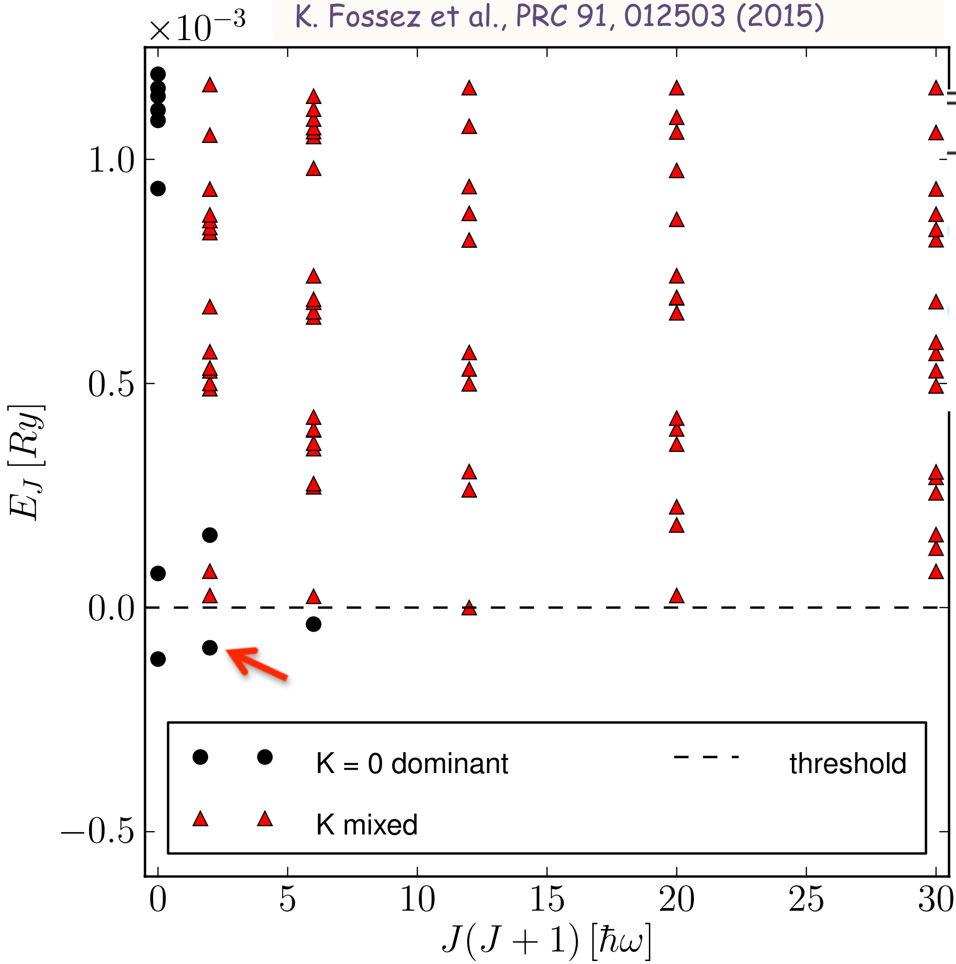

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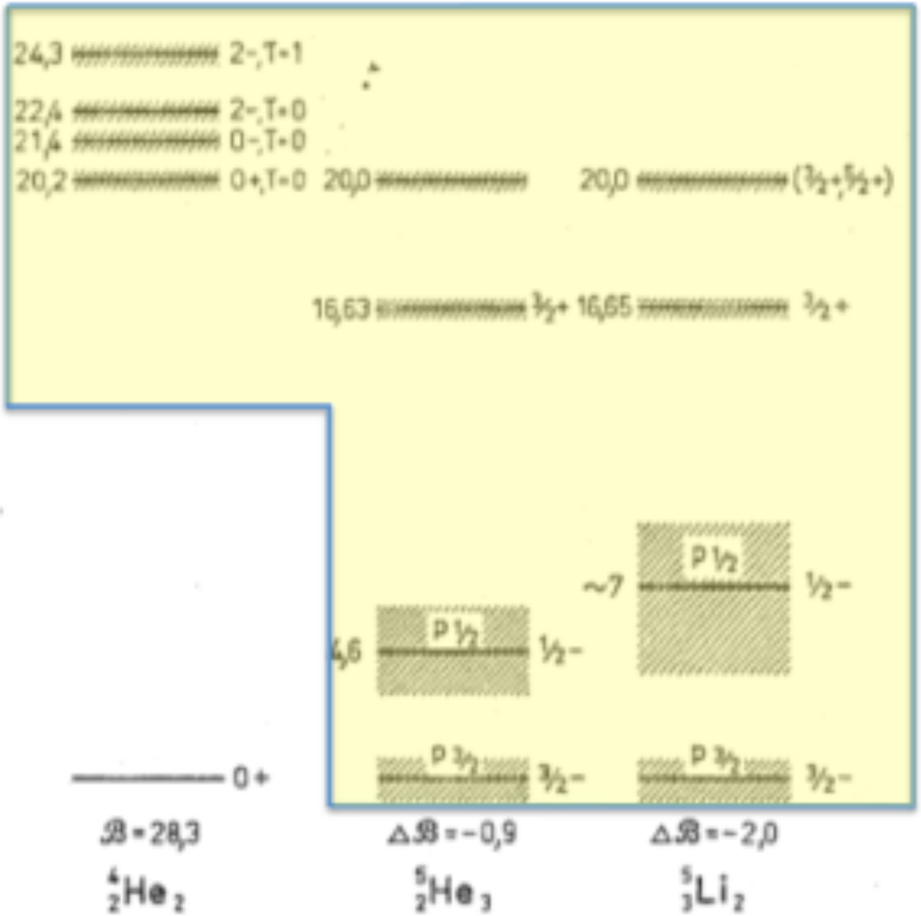
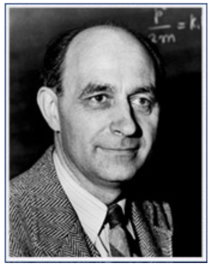
Challenge: How SM wave functions (gamma transition probabilities, selection rules, conserved quantum numbers, band structure) are changed by the coupling to the continuum?

See the talk by Kevin Fossez

Shell model describes atomic nuclei as the closed quantum system
i.e. coupling to decay channels are neglected





Enrico Fermi

Maria Goeppert-Mayer

J. Hans D. Jensen

Role of boundary conditions in universal properties of reaction cross-sections at the threshold

E.P. Wigner (1948)

To what extent the change in boundary conditions at the nuclear surface due to Coulomb wave function distortion in the external region can explain relative displacement of states in mirror nuclei?

J.B. Ehrman (1950)

The exact coincidence of the energies of different configurations makes the ordinary perturbation theory inadequate, so that special procedures are required…

U. Fano (1961)

Eugene P. Wigner

U. Fano

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U. Fano (1961)

Hilbert space includes real-energy bound and scattering states
Resonances are discarded as unphysical!

The resolution of this paradox required:

- New mathematical concepts: Rigged Hilbert Space (≥1964),…
- Generalized completeness relation including s.p. bound states, resonances, and scattering states (~1968)
- Gamow Shell Model (~2002)

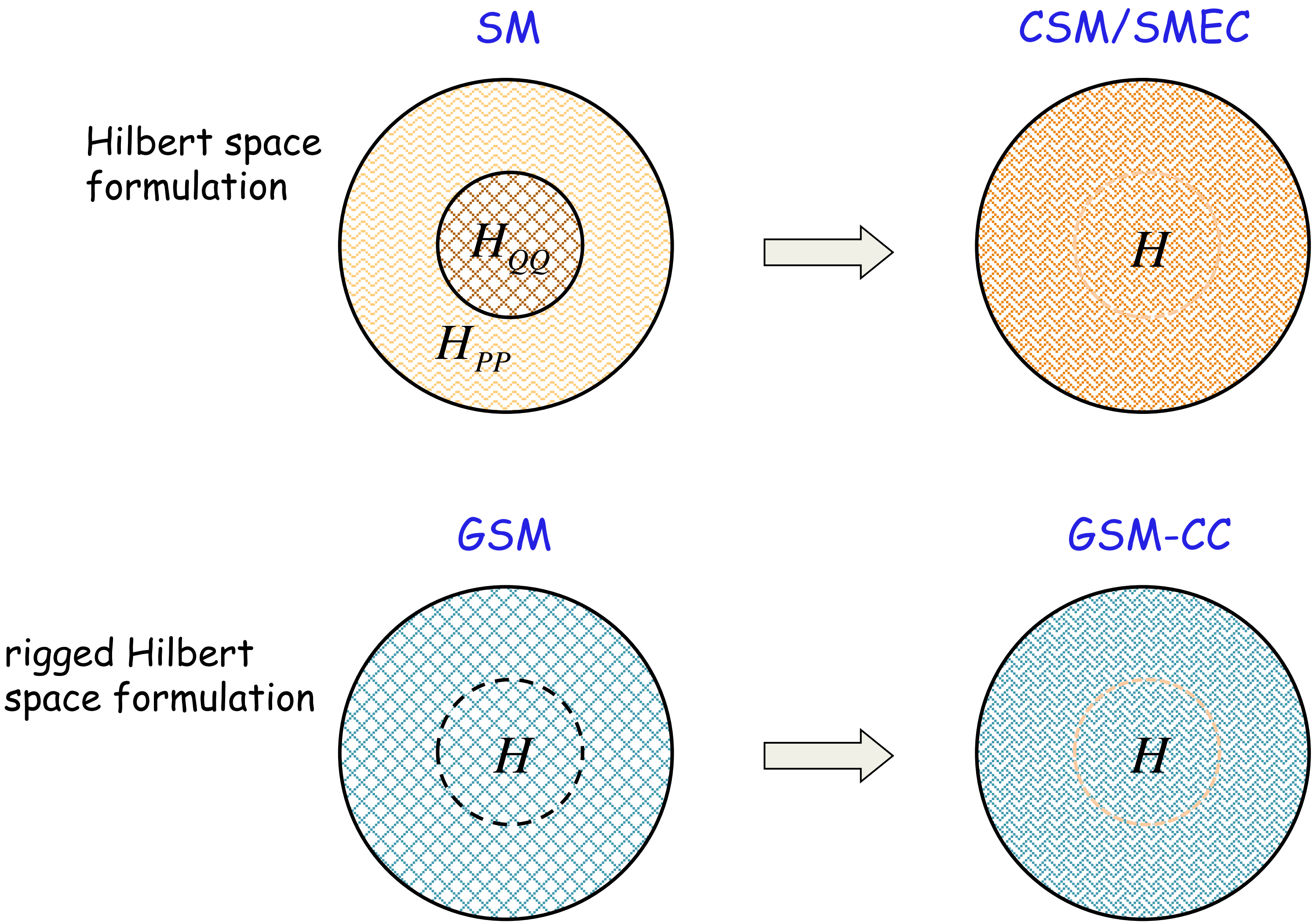
Eugene P. Wigner

U. Fano

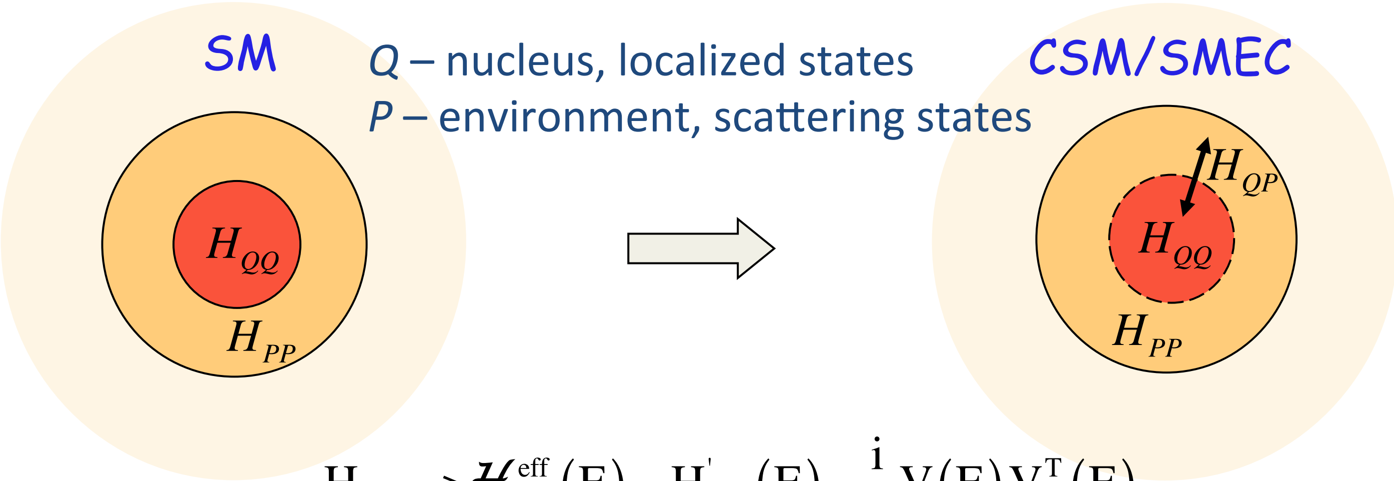
I.M. Gelfand

T. Berggren

Two strategies to formulate the continuum shell model



Continuum shell model: Shell model with the real-energy continuum



$$\mathrm{H}_{\mathrm{QQ}} \rightarrow \mathcal{H}_{\mathrm{QQ}}^{\mathrm{eff}}(\mathrm{E}) = \mathrm{H}'_{\mathrm{QQ}}(\mathrm{E}) - \frac{\mathrm{i}}{2}\mathrm{V}(\mathrm{E})\mathrm{V}^{\mathrm{T}}(\mathrm{E})$$

closed quantum system — hermitian — anti-hermitian — open quantum system

Open QS solution in Q : $\Psi_\alpha = \sum_i b_{\alpha i}\Phi_i^{(\mathrm{SM})}$

For bound states: $\mathcal{E}_\alpha(E)$ is real and $\mathcal{E}_\alpha(E) = E$
For unbound states: physical resonances = poles of S-matrix

C. Mahaux, H.A. Weidenmüller, *Shell Model Approach to Nuclear Reactions (1969)*
H.W.Bartz et al, Nucl. Phys. A275 (1977) 111
R.J. Philpott, Nucl. Phys. A289 (1977) 109
K. Bennaceur et al, Nucl. Phys. A651 (1999) 289
J. Rotureau et al, Nucl. Phys. A767 (2006) 13

Coupling of 'internal' (in Q) and 'external' (in P) states induces effective A-particle correlations and determines the structure of many-body states

Quasi-stationary extension of the Shell Model in the complex k-plane

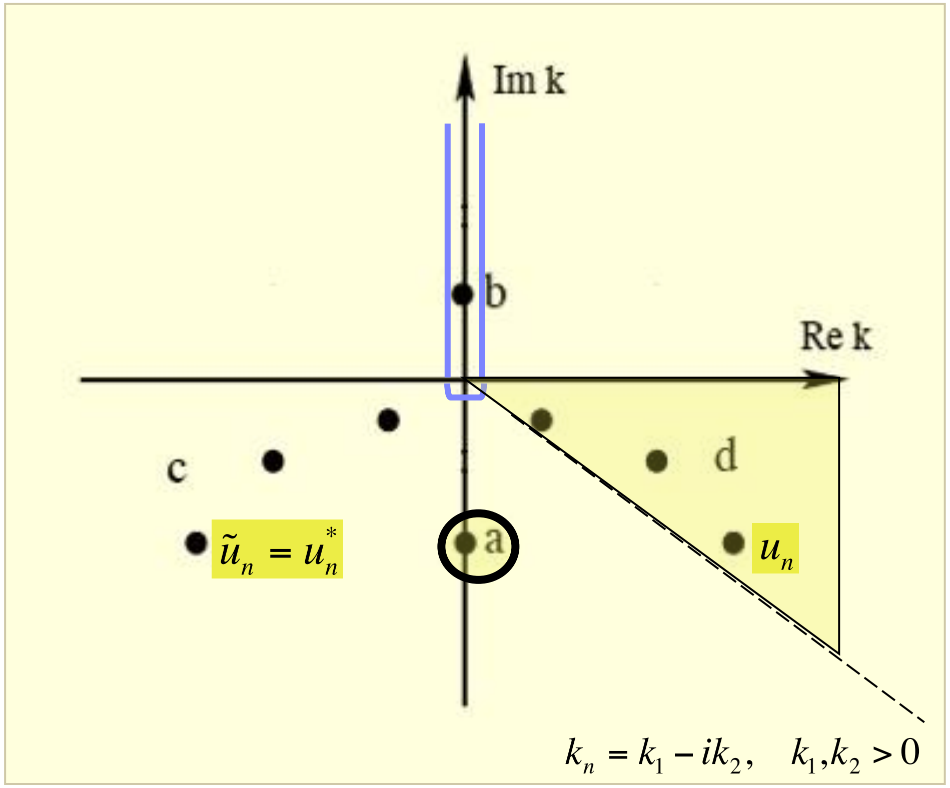
$$i\hbar\frac{\partial}{\partial t}\Phi(r,t) = \hat{H}\Phi(r,t) \qquad \Phi(r,t) = \tau(t)\Psi(r)$$

$$\hat{H}\Psi = \left(e - i\frac{\Gamma}{2}\right)\Psi \;\rightarrow\; \tau(t) = \exp\left(-i\left(e - i\frac{\Gamma}{2}\right)\right)$$

$$\Psi(0,k) = 0\ ,\ \begin{cases} \Psi(\vec{r},k) \underset{r\to\infty}{\longrightarrow} O_l(kr) \\ \Psi(\vec{r},k) \underset{r\to\infty}{\longrightarrow} I_l(kr) + O_l(kr) \end{cases}$$

$$k_n = \sqrt{\frac{2m}{\hbar^2}\left(e_n - i\frac{\Gamma_n}{2}\right)} \qquad \text{(poles of the S-matrix)}$$

Bound states	$k_n = i\kappa_n$
Antibound states	$k_n = -i\kappa_n$
Resonances	$k_n = \pm\gamma_n - i\kappa_n$



Only bound states are integrable!

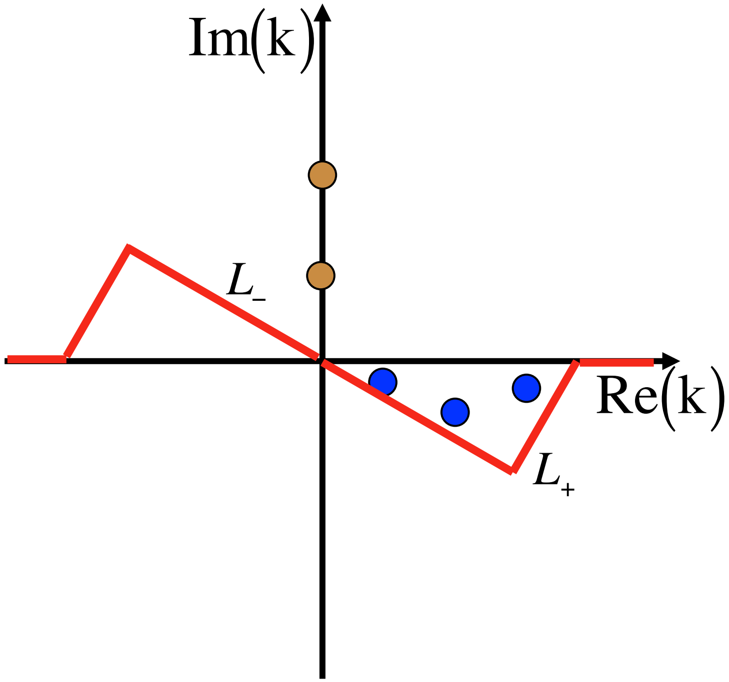
Euclidean inner product

$$\langle u_n | u_n \rangle = \int_0^\infty dr u_n^*(r) u_n(r) \xrightarrow[r\to\infty]{} e^{2k_2 r}$$

RHS inner product

$$\langle \tilde{u}_n | u_n \rangle = \int_0^\infty dr \tilde{u}_n^*(r) u_n(r) \xrightarrow[r\to\infty]{} e^{2ir(k_1 - ik_2)}$$

Gamow Shell Model: Shell model in the complex k-plane



$$H \rightarrow [H]_{ij} = [H]_{ji}$$

Complex-symmetric eigenvalue problem for hermitian Hamiltonian

$$\sum_n |u_n\rangle\langle\tilde{u}_n| + \int_{L_+} |u_k\rangle\langle\tilde{u}_k|\,dk = 1\,;\ \langle u_i|\tilde{u}_j\rangle = \delta_{ij}$$

T. Berggren, Nucl. Phys. A109, 265 (1968)
K. Maurin, Generalized Eigenfunction Expansion, Polish Scientific Publishers, Warsaw (1968)

bound states resonances

non-resonant continuum

$$|SD_i\rangle = |u_{i_1} \ldots u_{i_A}\rangle \rightarrow \sum_k |SD_k\rangle\langle\widetilde{SD}_k| \cong 1$$

Gamow Shell Model

N. Michel et al, PRL 89 (2002) 042502
R. Id Betan et al, PRL 89 (2002) 042501
N. Michel et al, PRC 70 (2004) 064311

Ab initio no-core GSM studies of resonances in A=4,5 systems

See the talk of George Papadimitriou

Interplay between Hermitian and anti-Hermitian couplings is a source of new phenomena

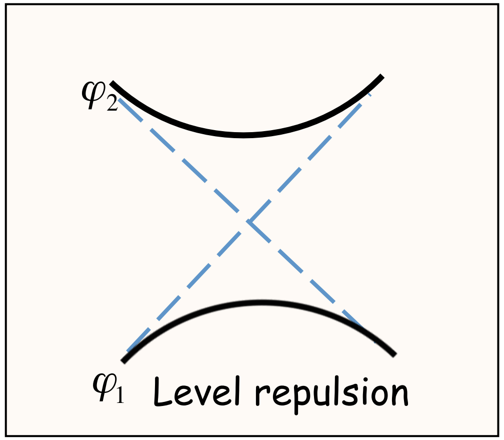
- Coalescence of eigenfunctions/eigenvalues (exceptional points); entangled states
  Example: doublet of 3+ resonances in ^{8}Be: 19.07(0.27) MeV and 19.23(0.23) MeV

- Violation of the orthogonal invariance; significant modification of the Porter-Thomas distribution, …
  Example: Reduced neutron width distribution — P.E. Koehler et al., PRL 105, 072502 (2010)
  – disagreement with the prediction of the random matrix theory

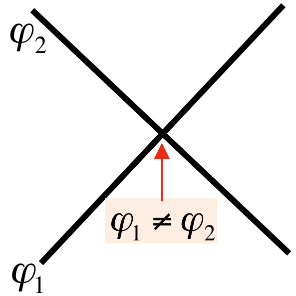
- Segregation of time scales: resonance trapping vs super-radiance phenomenon

- Collective phenomena:
  - Multichannel effects in reaction cross-section and shell occupancies
  - Instability of SM eigenstates at the channel threshold; near-threshold clustering
  - …

Coalescence of eigenfunctions



Bound states (Hermitian problem)



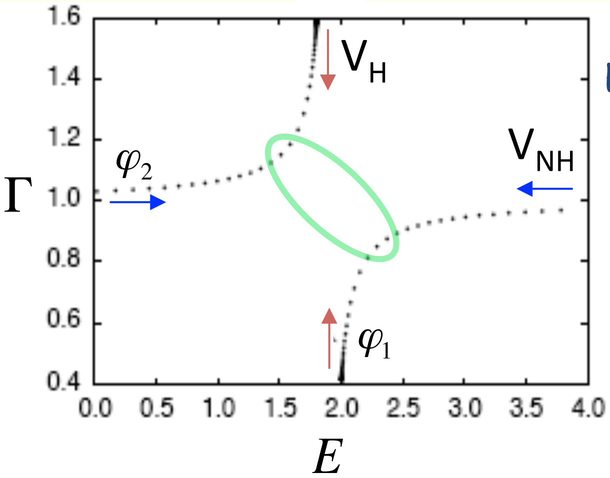
Resonances (non-Hermitian problem)

V_H | V_{NH}

Level repulsion
Width clustering

vs

Level clustering
Width repulsion

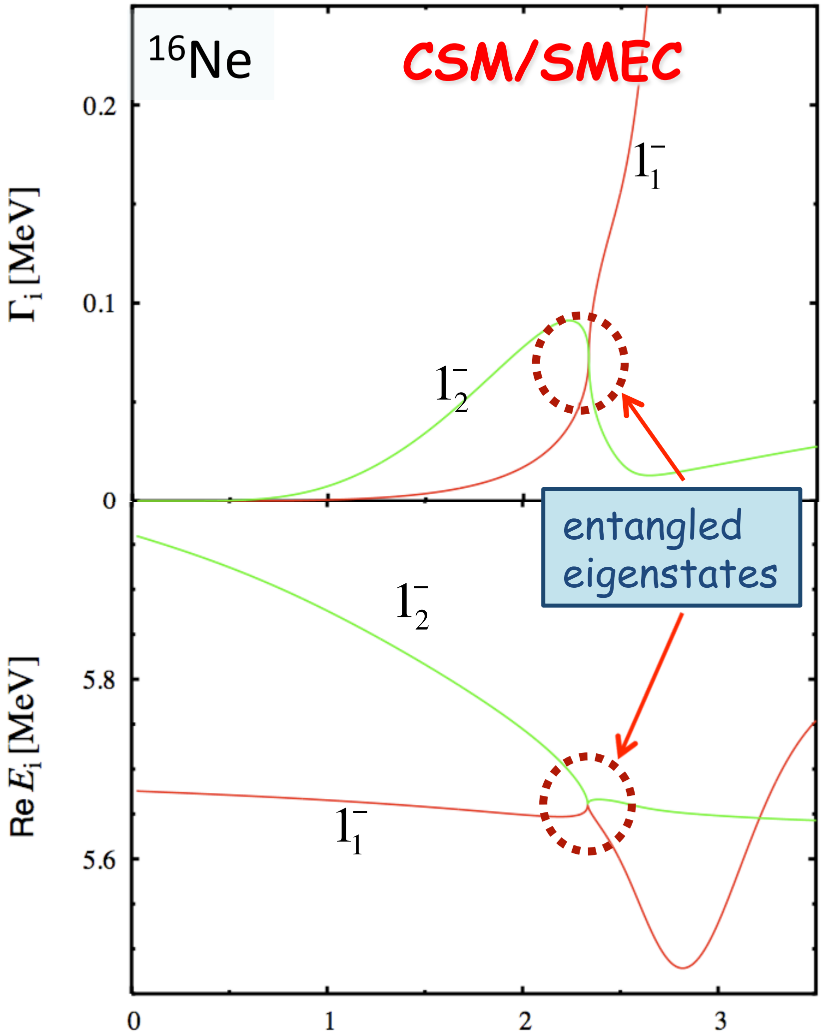


Exceptional point: $\varphi_1 = \varphi_2$, $\varphi_1 = \varphi_1^*$

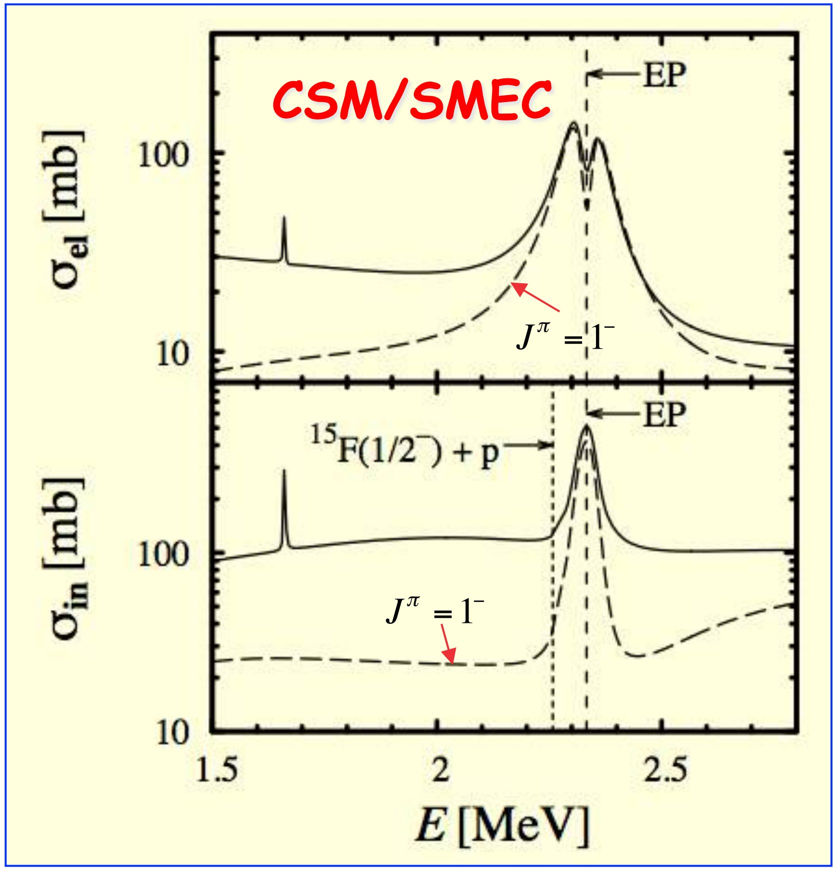
$\omega = v_{in} + i w_{ex}$

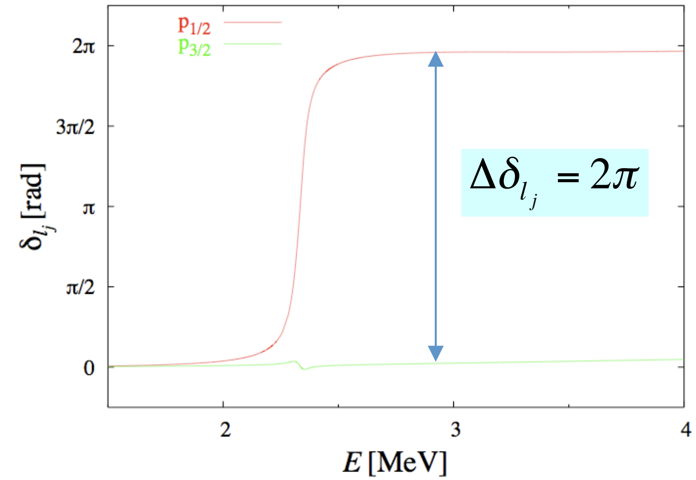
$$\mathscr{H} = \begin{pmatrix} \epsilon_1 & \omega \\ \omega & \epsilon_2 \end{pmatrix} \equiv \begin{pmatrix} e_1 - \frac{i}{2}\gamma_1 & 0 \\ 0 & e_2 - \frac{i}{2}\gamma_2 \end{pmatrix} + \begin{pmatrix} 0 & \omega \\ \omega & 0 \end{pmatrix}$$

M.R. Zirnbauer et al., Nucl. Phys. A411 (1983) 161
J. Okolowicz et al., Phys. Reports 374 (2003) 271



Experimental signatures





Channels: $\left[{}^{15}\mathrm{F}\left(1/2_1^+\right) \oplus \mathrm{p}\left(\ell_j\right)\right]^{J^\pi}$

$\left[{}^{15}\mathrm{F}\left(5/2_1^+\right) \oplus \mathrm{p}\left(\ell_j\right)\right]^{J^\pi}$ ZBM + WB interaction

$\left[{}^{15}\mathrm{F}\left(1/2_1^-\right) \oplus \mathrm{p}\left(\ell_j\right)\right]^{J^\pi}$ $V_0 = -1617.4 \mathrm{MeV} \bullet \mathrm{fm}^3$

J. Okolowicz, et al, PRC 80 (2009) 034619

Statistical aspects of nuclear coupling to continuum

$$\mathcal{H}_{QQ}^{\text{eff}}(E) = H_{QQ} + H_{QP}G_P^{(+)}(E)H_{PQ} \equiv H_{QQ} + \left(W^R + W^I\right)$$

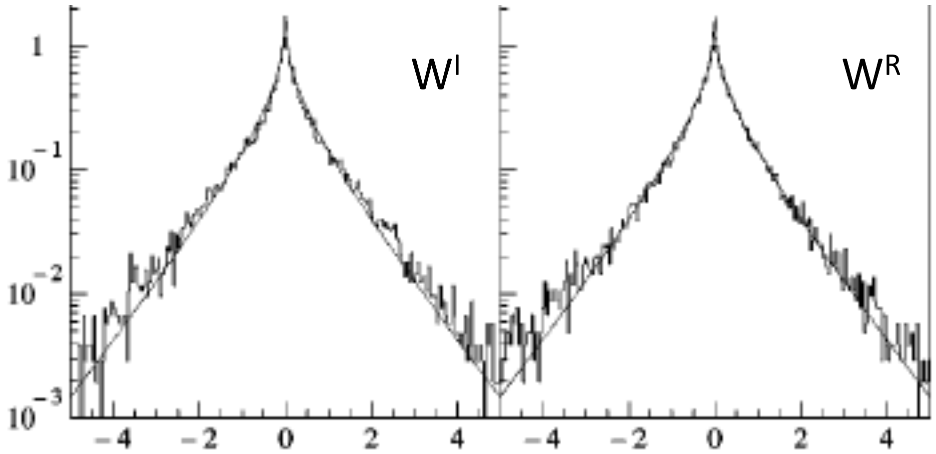
$$W_{ij}(E) = \sum_{c=1}^{\Lambda} \int_{\epsilon_c}^{\infty} dE' \frac{\langle\Phi_j|H_{QP}|\xi_E^c\rangle\langle\xi_E^c|H_{PQ}|\Phi_i\rangle}{E-E'}$$

$$-i\pi \sum_{c=1}^{\Lambda} \underbrace{\langle\Phi_j|H_{QP}|\xi_E^c\rangle\langle\xi_E^c|H_{PQ}|\Phi_i\rangle}$$

$$W^I = -\frac{i}{2}VV^T$$

CSM/SMEC

J=0$^+$, T=0 states in ^{24}Mg, 1 channel



$$s = \frac{W_{ij}^X - \langle W_{ij}^X\rangle}{\sigma_X}$$

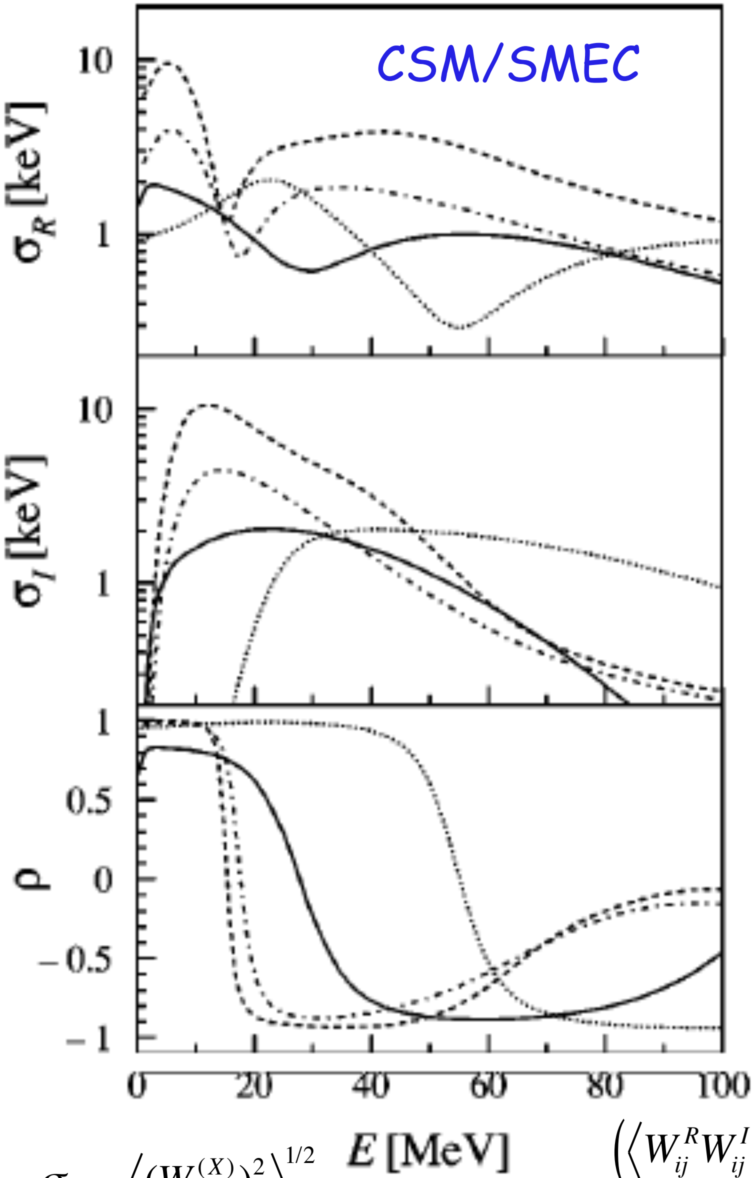
Assuming internal dynamics is governed by the GOE and different channels are equivalent one finds:

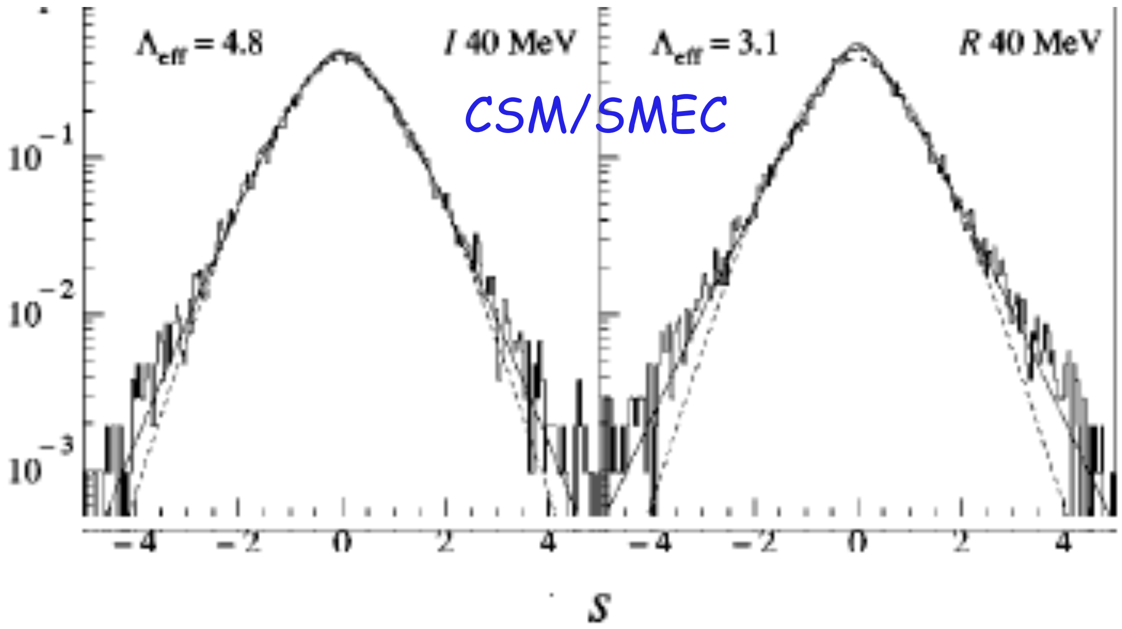
$$P_\Lambda\left(W_{ij}^I\right) = \frac{\left|W_{ij}^I\right|^{(\Lambda-1)/2} K_{(\Lambda-1)/2}\left(\left|W_{ij}^I\right|\right)}{\Gamma(\Lambda/2)\sqrt{\pi}\,2^{(\Lambda-1)/2}}$$

The assumption of Gaussian distribution of V_i^c is justified in a generic case of a single channel. The distribution for both W^I and W^R is the same.

J=0$^+$, T=0 states in ^{24}Mg, 10 channels

$$P_\Lambda\left(W_{ij}^I\right)=\frac{\left|W_{ij}^I\right|^{(\Lambda-1)/2}K_{(\Lambda-1)/2}\left(\left|W_{ij}^I\right|\right)}{\Gamma(\Lambda/2)\sqrt{\pi}\,2^{(\Lambda-1)/2}}$$





- W^R and W^I obey the same statistical ensembles
- M.E. of W^R and W^I are energy dependent and (often) strongly correlated
- Different decay channels are not equivalent
- Coupling to continuum breaks the statistical model assumption of orthogonal invariance and channel equivalence for several open channels

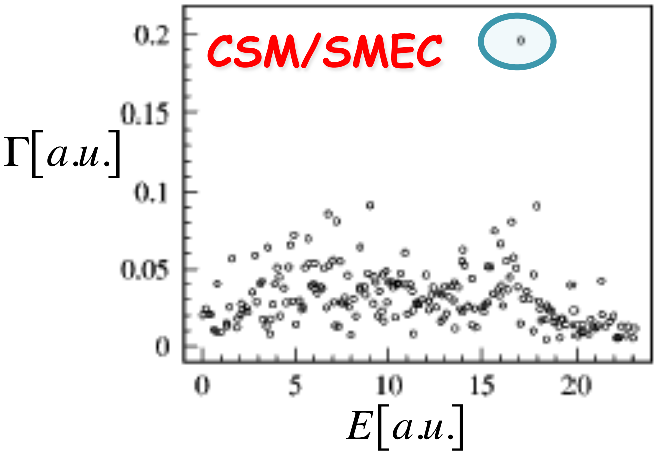
$$\sigma_X=\left\langle (W_{ij}^{(X)})^2\right\rangle^{1/2}$$

$$\rho=\frac{\left(\left\langle W_{ij}^R W_{ij}^I\right\rangle-\left\langle W_{ij}^R\right\rangle\left\langle W_{ij}^I\right\rangle\right)}{\sigma_R\sigma_I}$$

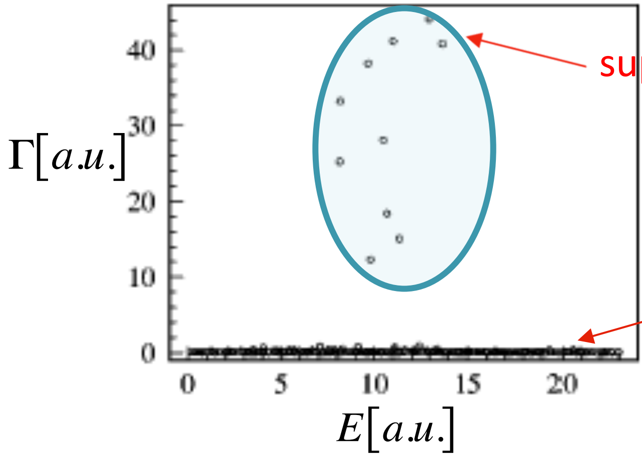
Segregation of time scales

J=0$^+$, T=0 states in ^{24}Mg, 10 channels

S. Drozdz et al, PRC 62, 24313 (2000)



intermediate coupling

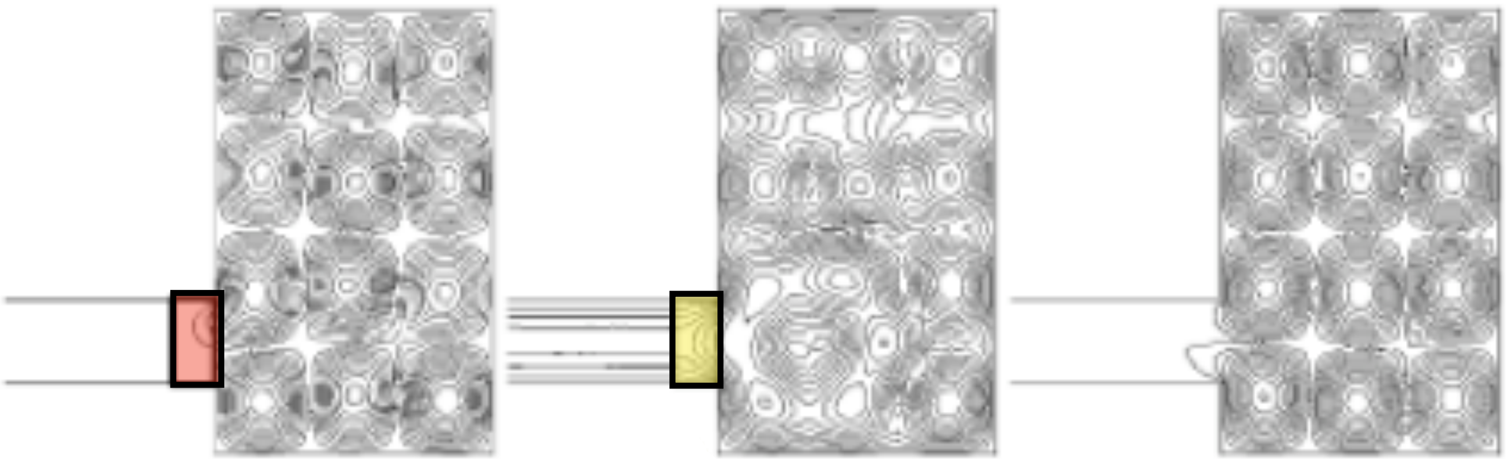


strong coupling

super-radiant states

trapped states

Trapped resonances are narrower than uncoupled resonances



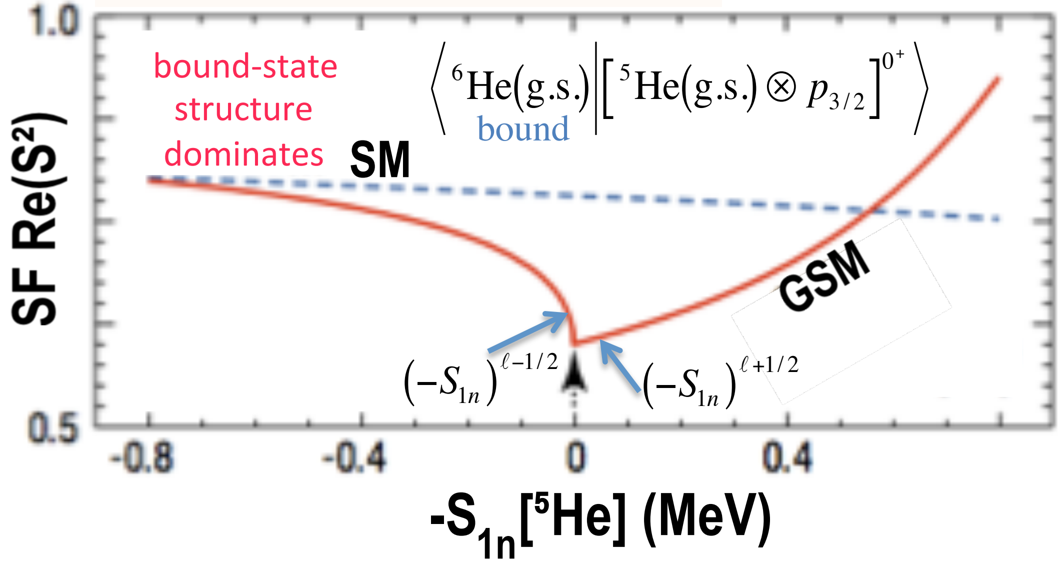
closed cavity

open cavity

'Quasi-bound' states in the continuum

Configuration mixing in weakly bound/unbound states

N. Michel et al. PRC 75, 0311301(R) (2007)



Analogy with the Wigner threshold phenomenon for reaction cross-sections

E.P. Wigner, PR 73, 1002 (1948)

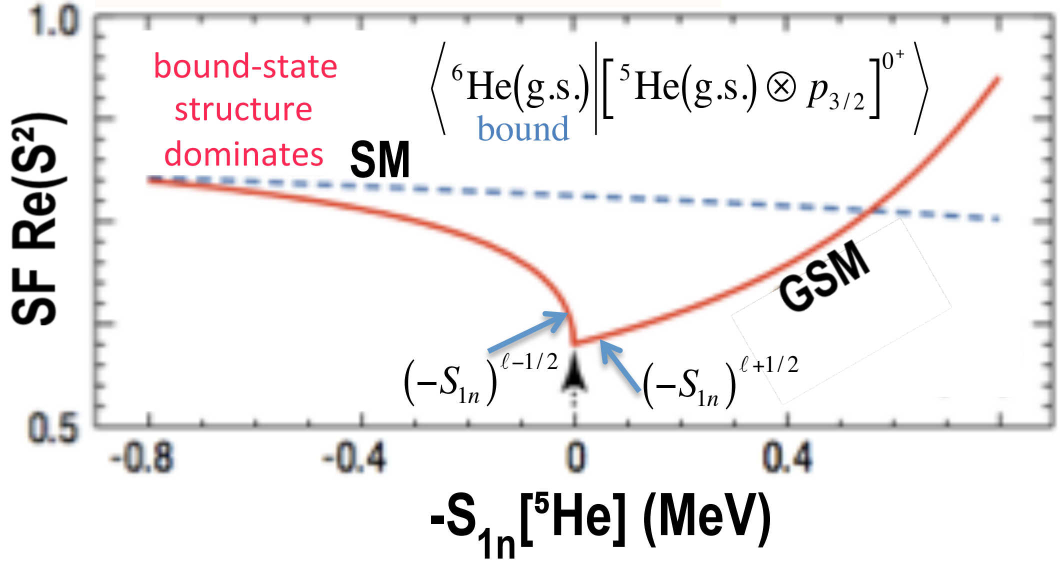
$$Y(b,a)X : \sigma_\ell \sim k^{2\ell-1} \qquad (-S_n)^{\ell-1/2} \quad \text{for } S_n < 0$$

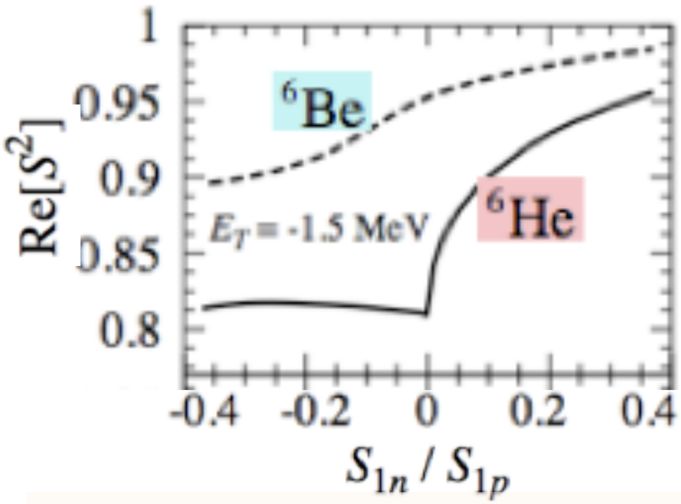
$$X(a,b)Y : \sigma_\ell \sim k^{2\ell+1} \qquad (-S_n)^{\ell+1/2} \quad \text{for } S_n > 0$$

Unification of discrete (nuclear structure) and continuum (nuclear reactions) aspects of the nuclear many-body problem

Configuration mixing in weakly bound/unbound states

N. Michel et al. PRC 75, 0311301(R) (2007)





N. Michel, W. Nazarewicz, M.P., PRC 82, 044315 (2010)

Analogy with the Wigner threshold phenomenon for reaction cross-sections

E.P. Wigner, PR 73, 1002 (1948)

$$Y(b,a)X : \sigma_\ell \sim k^{2\ell-1} \quad \Longleftrightarrow \quad (-S_n)^{\ell-1/2} \quad \text{for } S_n < 0$$

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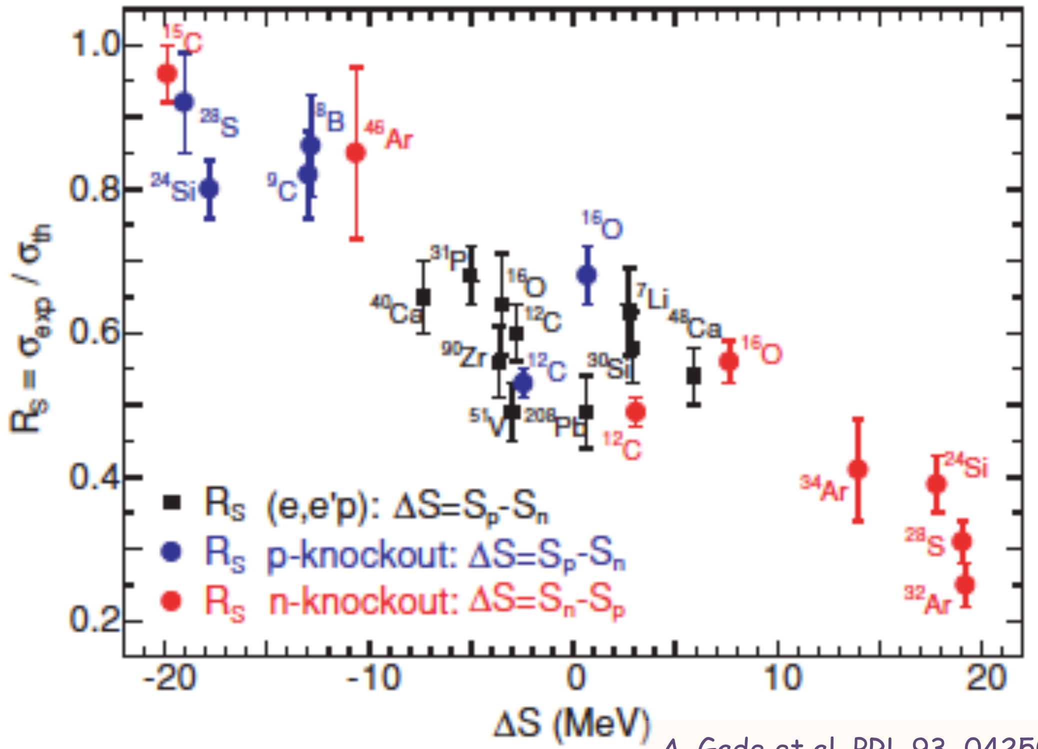
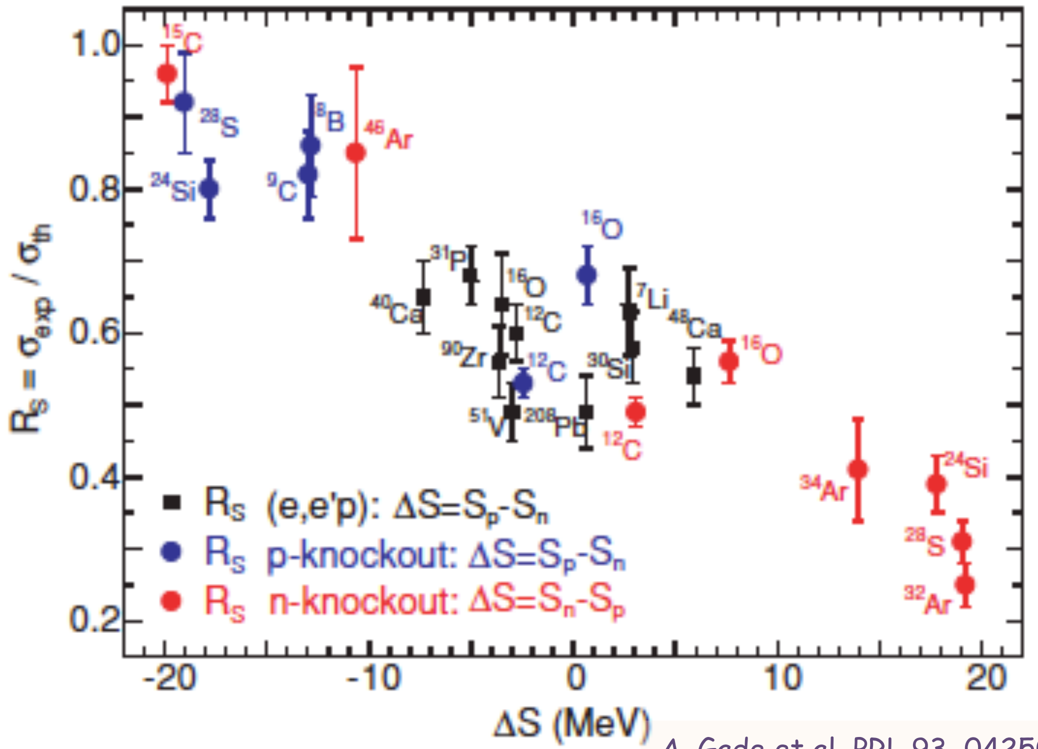
Unification of discrete (nuclear structure) and continuum (nuclear reactions) aspects of the nuclear many-body problem

One-nucleon spectroscopic factors involving weakly- and strongly-bound nucleons

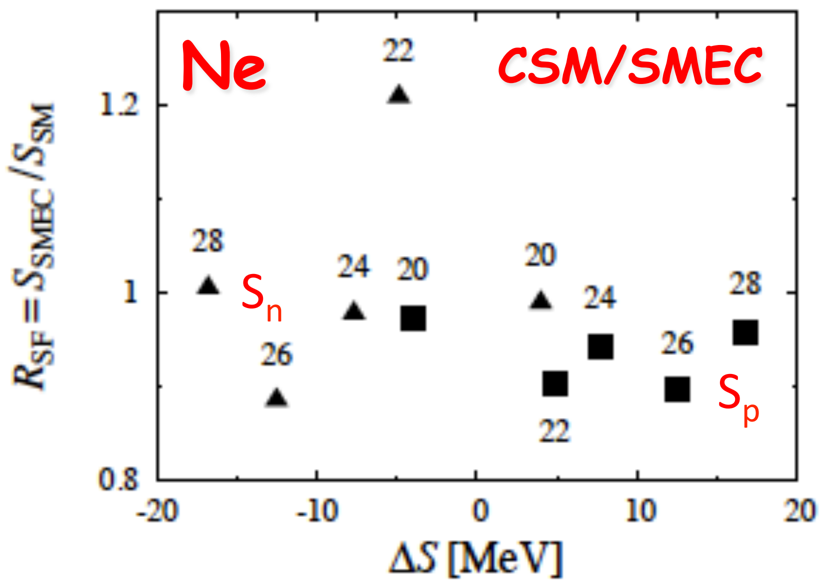


A. Gade et al, PRL 93, 042501

One-nucleon spectroscopic factors involving weakly- and strongly-bound nucleons

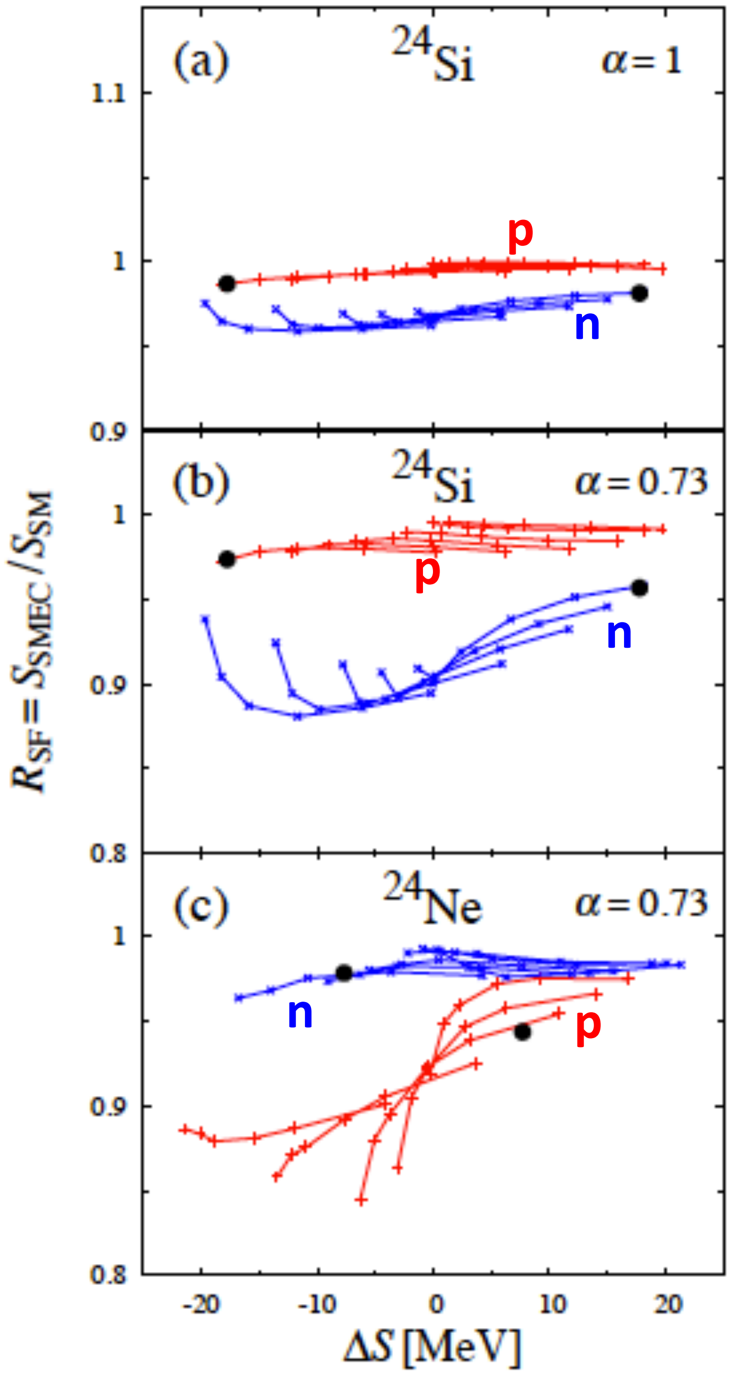


A. Gade et al, PRL 93, 042501



J. Okolowicz et al, (2015)

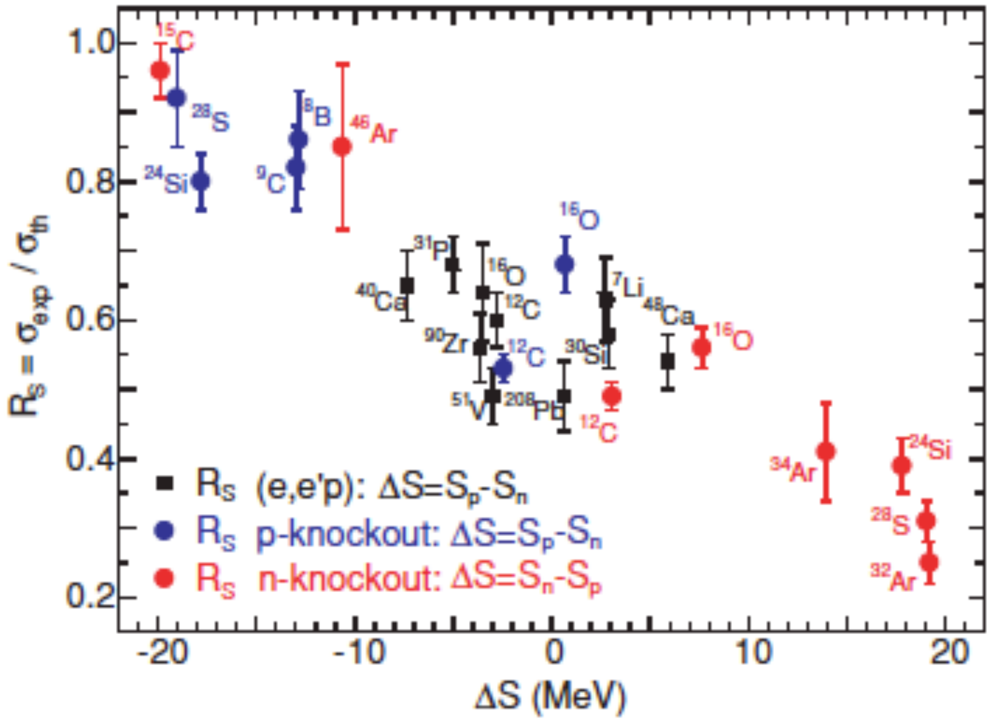
- Less than 15% of the ground state spectroscopic strength shifted to higher excitations
- One-nucleon spectroscopic factors are not correlated with the asymmetry of S_n and S_p separation energies



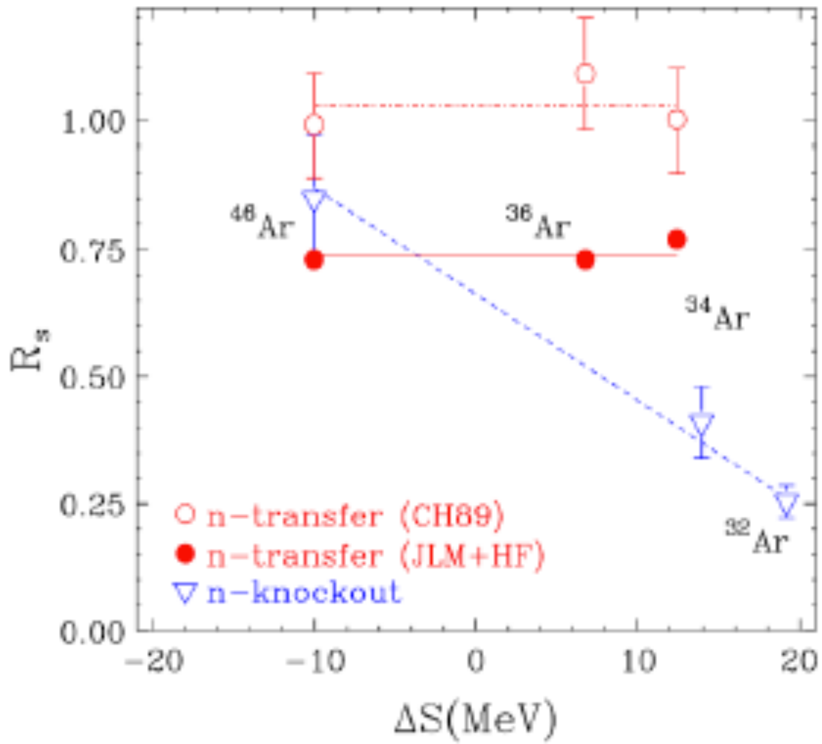
CSM/SMEC

- Usually stronger changes of R_{SF} for $S_{n,p}$ close to 0 (threshold effect)
- R_{SF} correlates better with $S_{n/p}$

Transfer vs knockout?



Gade et al, PRL 93, 042501



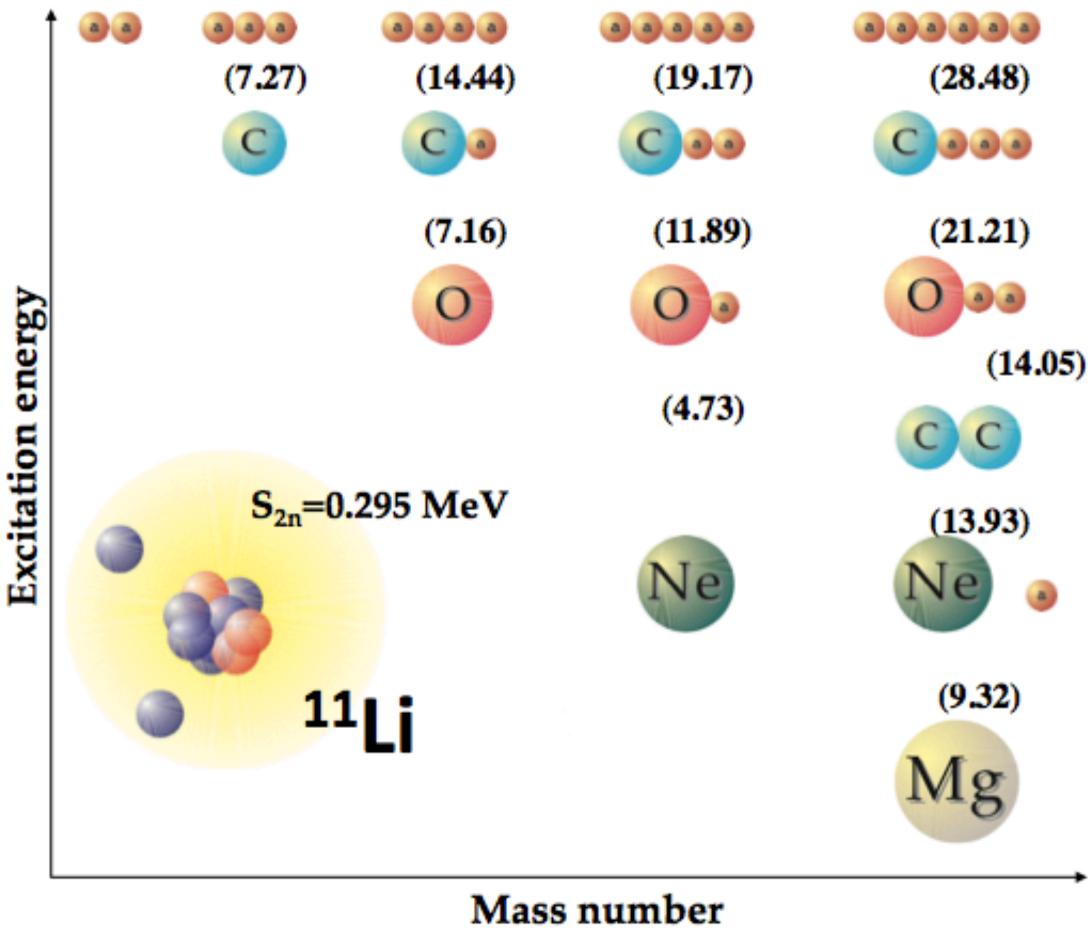
Jenny Lee et al., PRL 104, 112701 (2010)

Studies of exotic nuclei require accurate reaction models

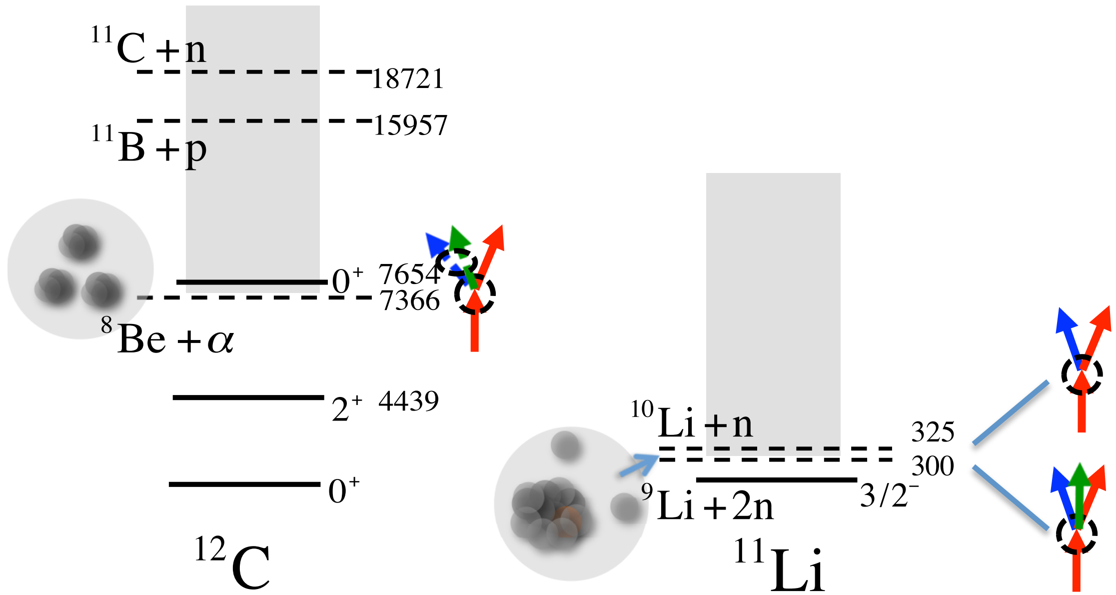
The origin of near-threshold clustering

α-cluster states can be found in the proximity of α-particle decay threshold

K. Ikeda, N. Takigawa, H. Horiuchi (1968)



Why cluster states appear in the proximity of cluster thresholds?

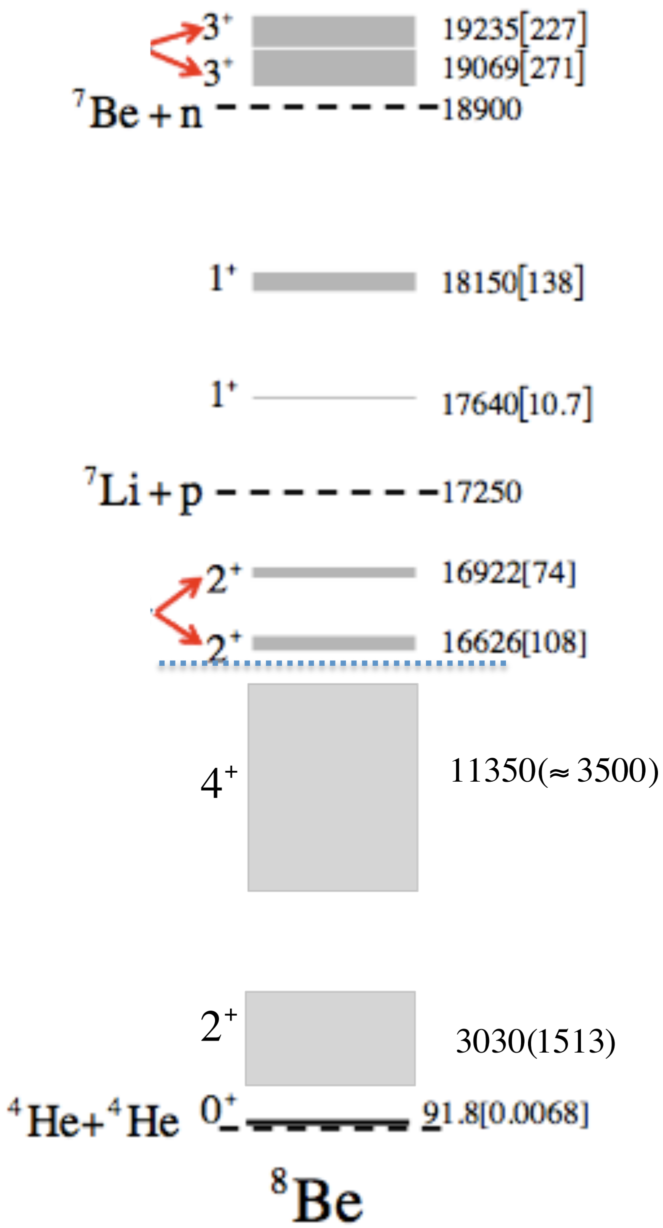


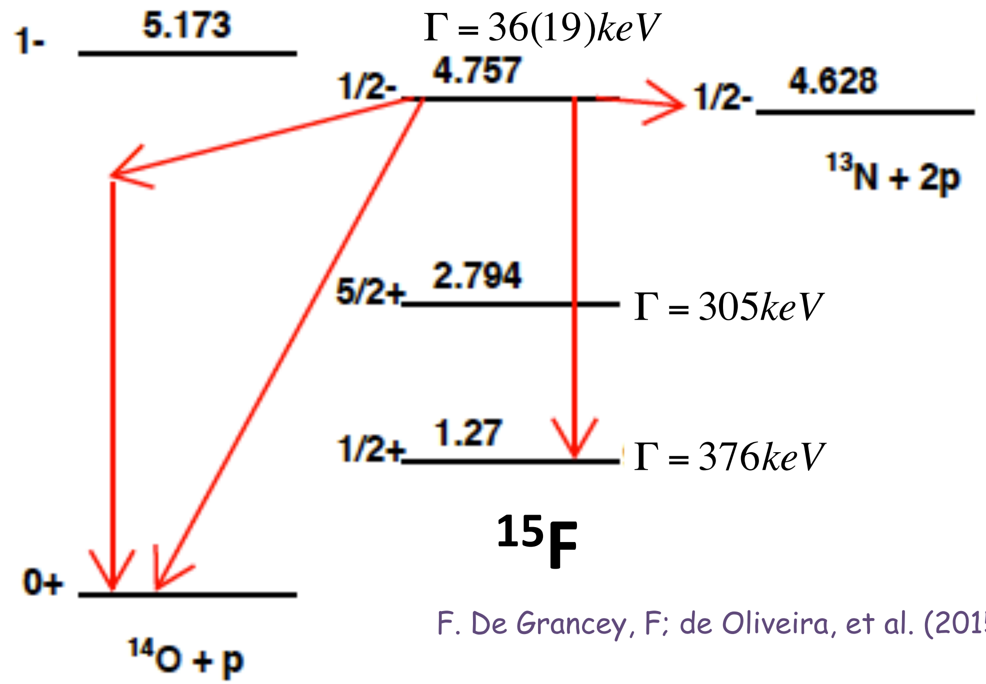
Correlations in the near-threshold states depend on the topology of the nearest branching point

Near-threshold clustering/correlations result from the interplay between internal mixing by interactions and external mixing via the decay channel(s) ➔ emergence of new energy scale in the problem!

J. Okolowicz, M.P., W. Nazarewicz, Prog. Theor. Phys. Suppl. 196 (2012) 230; Fortschr. Phys. 61 (2013) 66

Narrow near-threshold states in the continuum





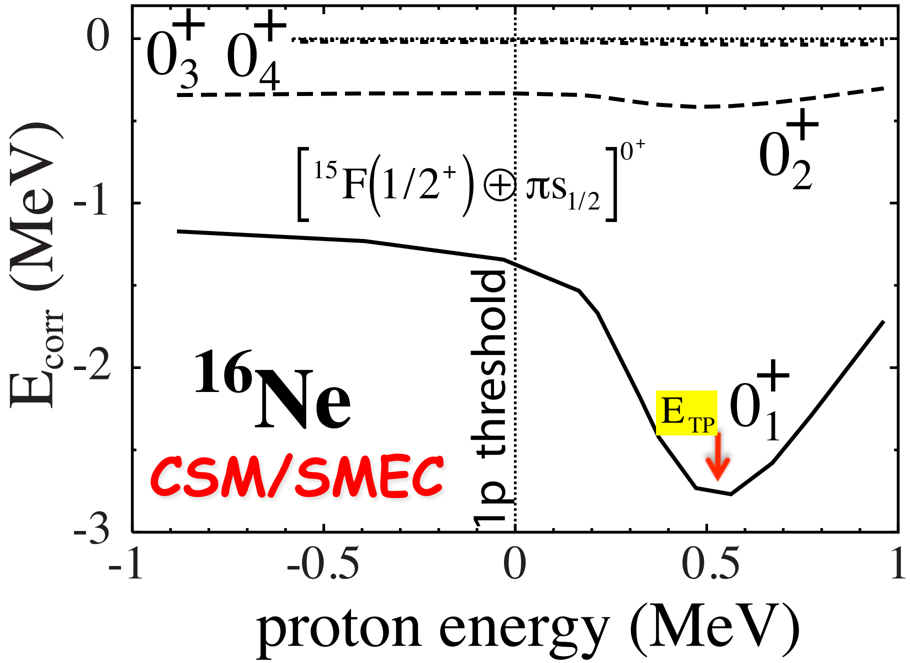
F. De Grancey, F; de Oliveira, et al. (2015)

Conjecture:
The coupling to a nearby decay channel induces correlations in the shell model wave functions which are the imprint of this channel.

Instability of SM eigenstates at the channel threshold: Appearance of the 'aligned' state

Continuum coupling correction to SM eigenstates

$$E^{(\ell)}_{\mathrm{corr};i}(E) \simeq V_0^2 \langle \psi_i | h(E) | \psi_i \rangle$$

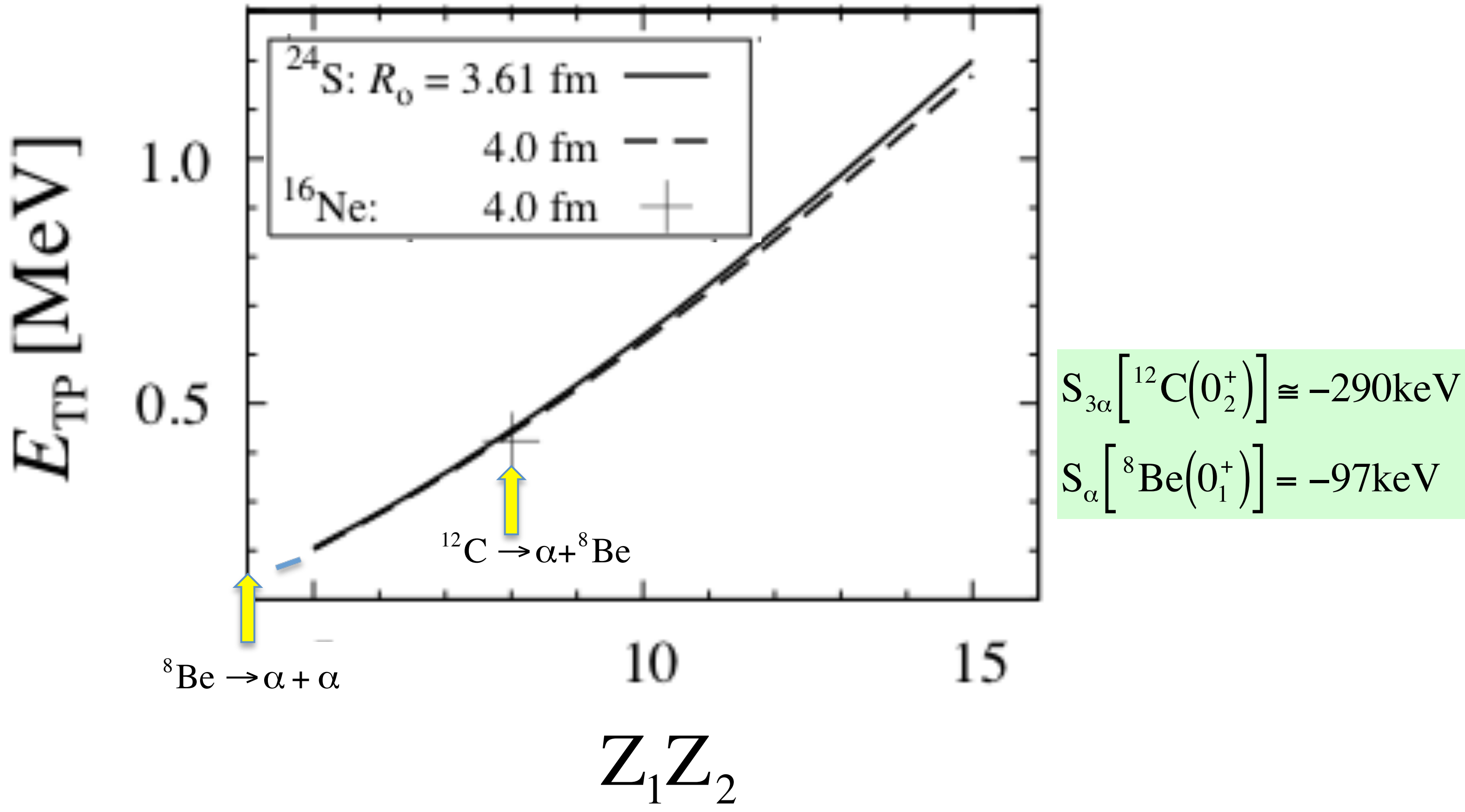


- Interaction through the continuum leads to a formation of the collective eigenstate which couples strongly to the decay channel and carries many of its characteristics

- The maximum continuum coupling point at E_{TP} is determined by an interplay between the Coulomb/centrifugal interactions, and the continuum coupling

- The centroid of the energy window E_{TP} is independent of the continuum coupling strength

J. Okolowicz et al., Prog. Theor. Phys. Suppl. 196, 230 (2012)
Fortschr. Phys. 61, 66 (2013)



For a given Z_1Z_2 , the centroid of the 'opportunity window' at E_{TP} depends weakly on the charged particle decay channel and parameters of the potential.

The continuum-coupling correlation energy and collectivity of the aligned state are reduced with increasing the Coulomb barrier.

Conclusion:

The mixing of SM eigenstates via the continuum has universal features which explain:

(i) the emergence of near-threshold clustering/correlations and optimal energies for its appearance,

(ii) a gradual disappearance of charged-particle clustering in heavier nuclei

Conclusion:

The mixing of SM eigenstates via the continuum has universal features which explain:

(i) the emergence of near-threshold clustering/correlations and optimal energies for its appearance,

(ii) a gradual disappearance of charged-particle clustering in heavier nuclei

What is specific and what is generic in this emerging phenomenon?

Specific	Generic
Energetic order of emission thresholds and absence of stable cluster entirely composed of like nucleons	Correlations in the near-threshold states depend on the nature of the nearby decay threshold

- Experimental verification of the predicted by this mechanism exotic clustering(s)/correlation(s) (e.g. ^3H, ^3He, …) is needed!

What about the multineutron clustering?

- Correlated four neutrons reported in the disintegration of ^{14}Be

F.M. Marques et al, PRC 65, 044006 (2002)

- Bound tetraneutron incompatible with our understanding of nuclear forces

- Isospin structure of the nuclear force prevents that: $S_{4n} < S_{2n}/S_{1n}$

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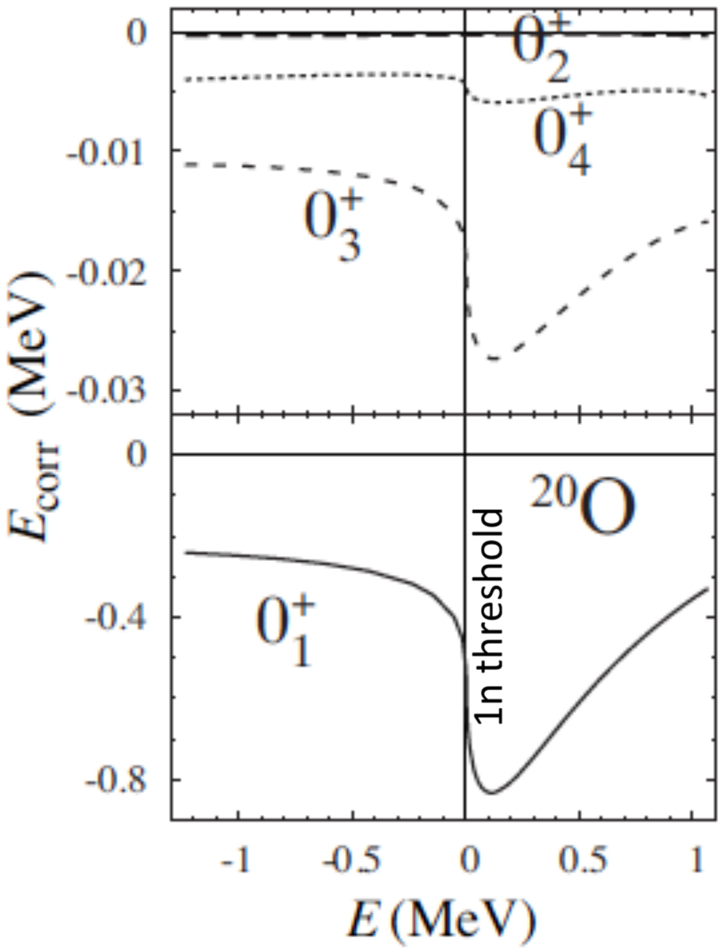
but

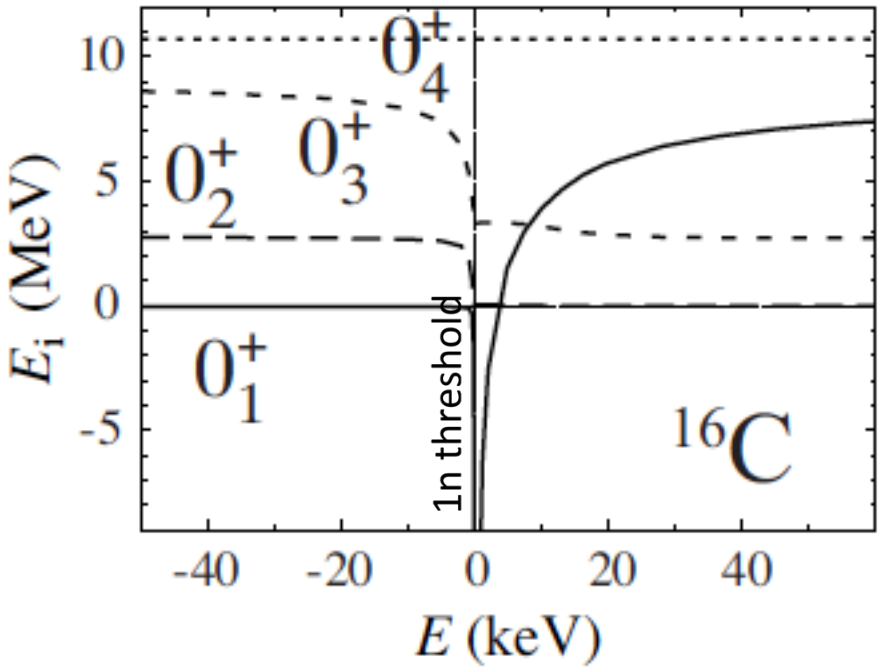
The nature of nuclear forces does not preclude the manifestation of tetraneutron correlations in the vicinity of the 4n-emission threshold as a consequence of the collective coupling of many-body resonances via the 4n-emission channel(s).

CSM/SMEC

$\ell = 2$



$\ell = 1$



J. Okolowicz, M.P., W. Nazarewicz, APP B45, 331 (2014)

Similarity between $\ell \geq 2$ neutral particle $\ell = 0$ clustering and charged particle clustering

Contrary to the charged-particle correlations, the multi-neutron correlations in channels with ℓ=2,3,… can appear both in light and heavy nuclei

Unified desciption of structure and reactions in the Gamow Shell Model

Direct reactions in Gamow Shell Model

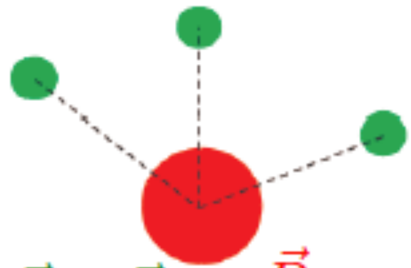
Y. Jaganathen, N. Michel, M.P., PRC 89 (2014) 034624

- Center of mass treatment: *Cluster Orbital Shell Model relative coordinates*

  Y. Suzuki, K. Ikeda, PRC 38, 410 (1988)

$$H = \sum_{i=1}^{A_v} \left(\frac{\mathbf{p}_i^2}{2\mu} + U_i \right) + \sum_{i<j}^{A_v} \left(V_{ij} + \frac{p_i p_j}{A_c} \right)$$

"Recoil" term coming from the expression of H in the COSM coordinates. No spurious states



$$\vec{r} = \vec{r}_{lab} - \vec{R}_{core}$$

- Emission channels identified in $|\Psi_{GSM}(A-p)\rangle$ and $|\Psi_{GSM}(A)\rangle$
- Scattering wave functions $|\Psi_{GSM}(A-p) \otimes \Phi_{proj}(p)\rangle$ are the many-body states
- Antisymmetry exactly handled
- Core arbitrary

- Resonating Group Method for A-body matrix elements:

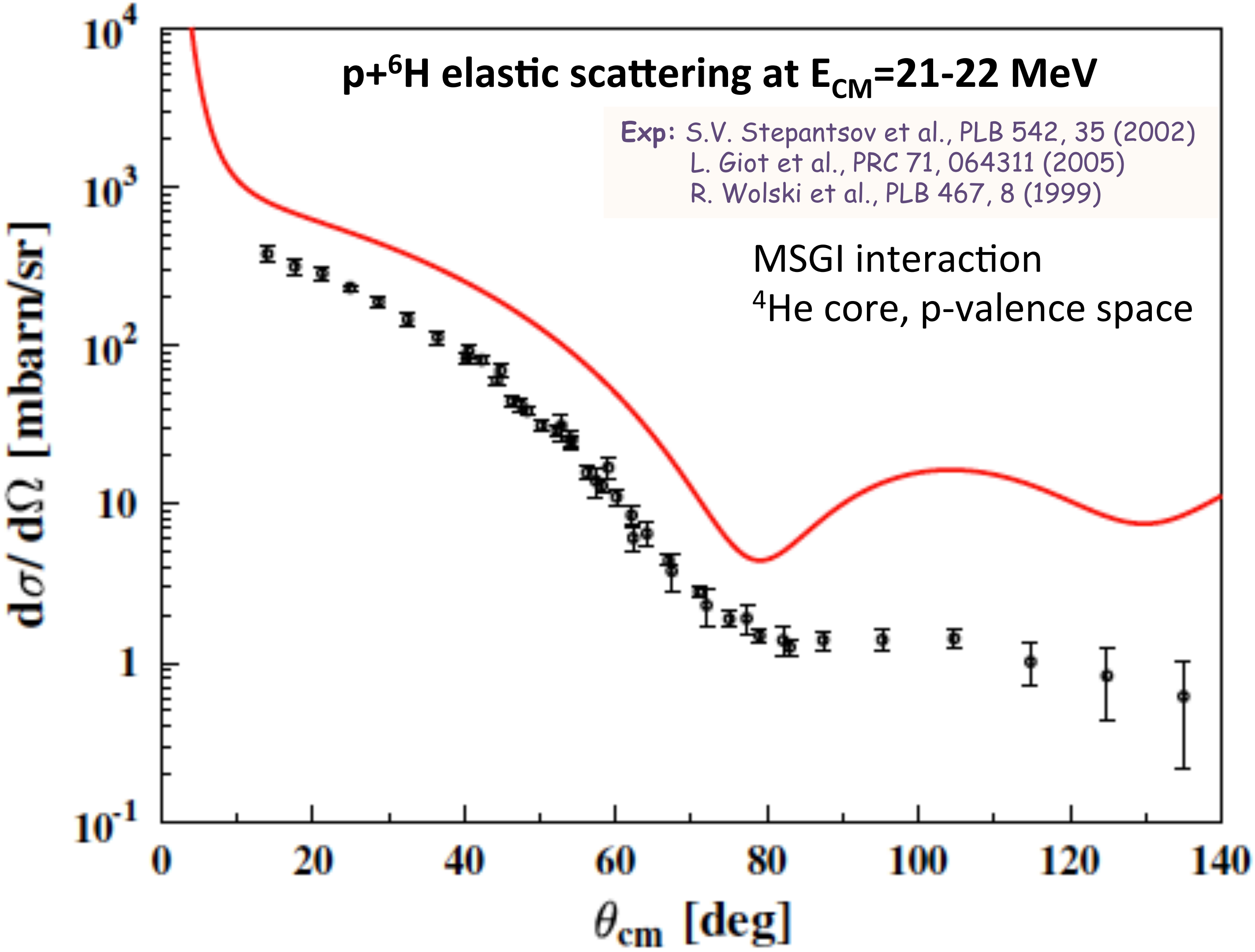
$$\left\langle \Psi_{\mathrm{GSM;f}}(A-p)\otimes\Phi_{\mathrm{proj;f}}(p)\right|H\left|\Psi_{\mathrm{GSM;i}}(A-p)\otimes\Phi_{\mathrm{proj;i}}(p)\right\rangle$$

leads to coupled-channel equations with microscopic potentials

- Differential elastic and inelastic scattering cross-sections obtained using effective interactions which reproduce spectroscopy/binding of (A) and (A-p) nuclei

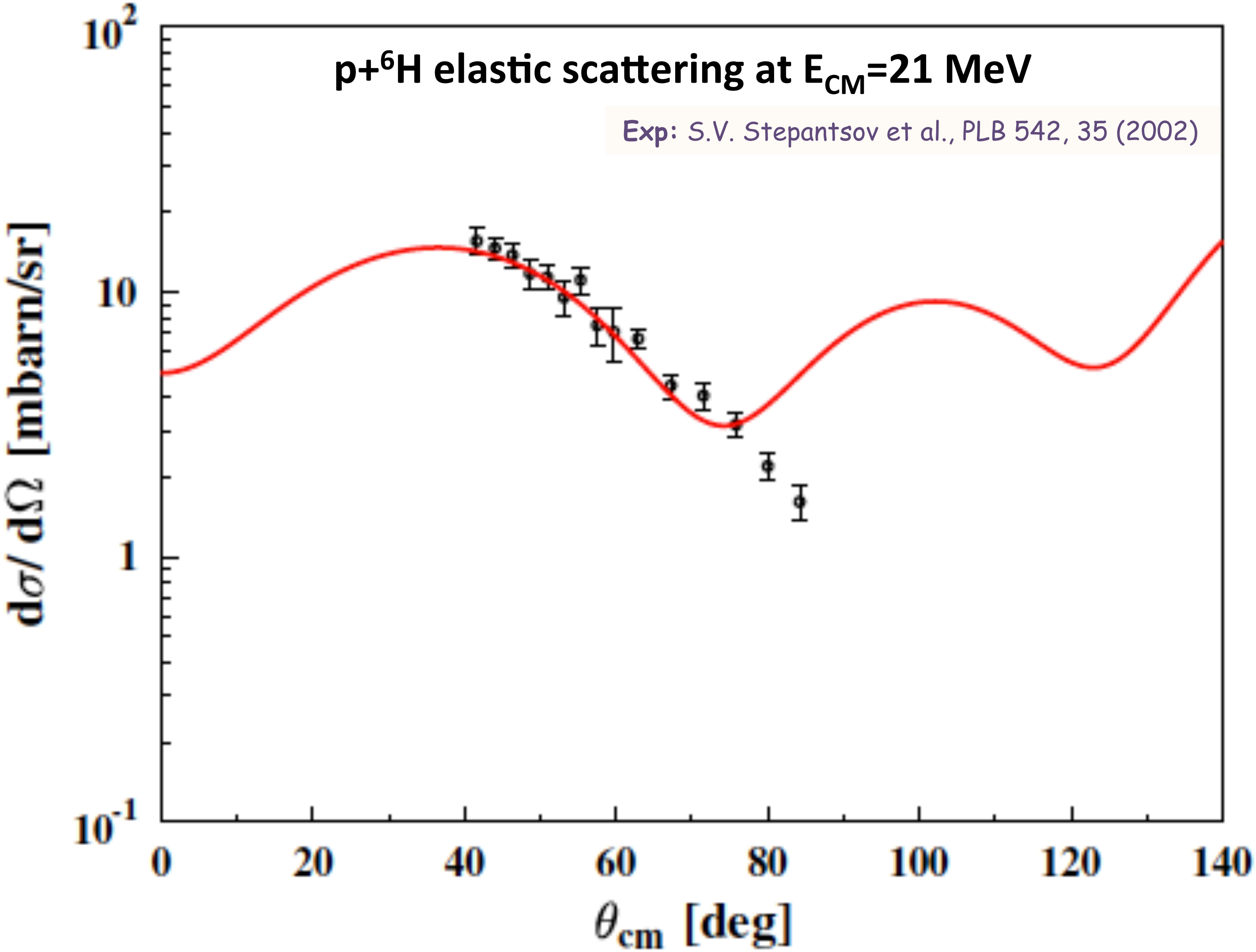
- Consistency test:

$E_{\mathrm{GSM;j}} \neq E_{\mathrm{GSM\text{-}CC;j}} \Rightarrow$ channels involving non-resonant scattering states of a target nucleus are essential in weakly-bound targets (S_n, S_p<3MeV)!



^{5}He, ^{6}He, ^{7}Li energies with respect to ^{4}He core are well reproduced

Channels constructed with discrete states of ^{6}He: 0^+,2^+



$E_{\mathrm{GSM;j}} \neq E_{\mathrm{GSM\text{-}CC;j}}$ → channels involving non-resonant scattering states of target nucleus are important!

p+^{18}Ne excitation function at different angles

	EXP	GSM	GSM-CC
18Ne			
0+	0.00	0.00	
2+	1.89	1.56	
19Na			
5/2+	0.32	0.28	0.29
3/2+	0.44	0.25	0.27
1/2+	1.07	1.08	1.13

S_p=3.921 MeV
S_n=19.237 MeV

S_p=-0.32 MeV
S_n=20.18 MeV

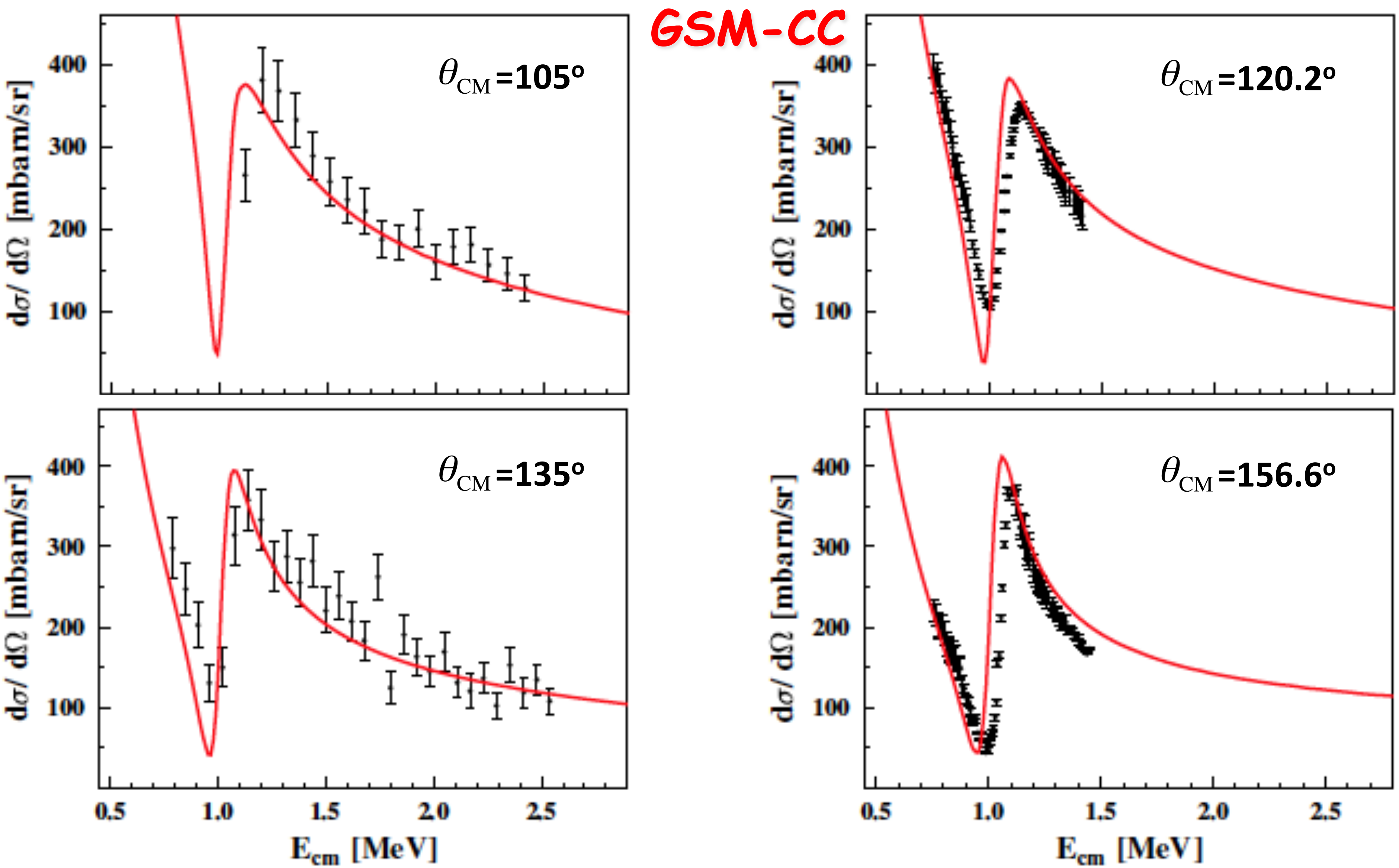
Model space: $0d_{5/2}$, $1s_{1/2}$, $0d_{3/2}$, $1p_{3/2}$, $1p_{1/2}$

Interaction: FHT finite-range interaction: $V(ij)=V^C + V^{SO} + V^T + V^{Coul}$

H. Furutani, H. Horiuchi, R. Tamagaki, PTP 60 (1978) 307; 62 (1979) 981

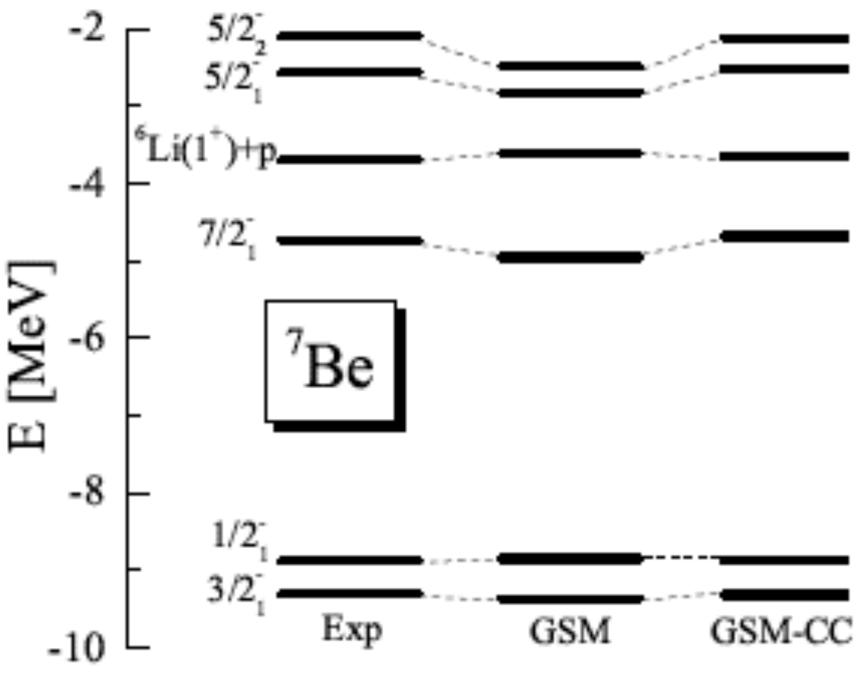
- Complete description of the scattering continuum in ^{19}Na

- Scattering continuum states $J=0^+,1^+,2^+$,... and higher lying (bound) states in ^{18}Ne are unimportant. GSM and GSM-CC results (almost) identical p+^{18}Ne excitation function at different angles

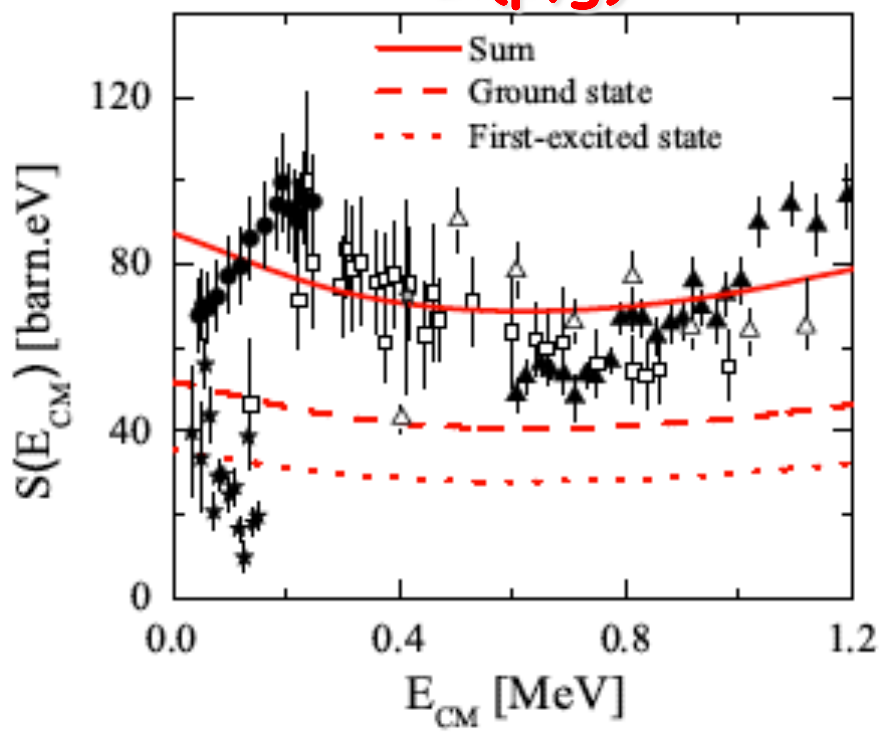


Exp: F. de Oliveira Santos et al., Eur. Phys. J. A24, 237 (2005)
B. Skorodumov et al., Phys. Atom. Nucl. 69, 1979 (2006)
C. Angulo et al., PRC 67, 014308 (2003)

^{6}Li(p,g) radiative capture cross sections



^{6}Li(p,g)



- Model space: $0p_{3/2}$, $0p_{1/2}$, $0s_{1/2}$, $0d_{5/2}$, $0d_{3/2}$
- Interaction: FHT finite-range interaction
- 1-3% renormalization of channel-channel coupling potentials to correct the spectra of ^{6}Li and ^{7}Be for missing channels involving non-resonant scattering states of ^{6}Li

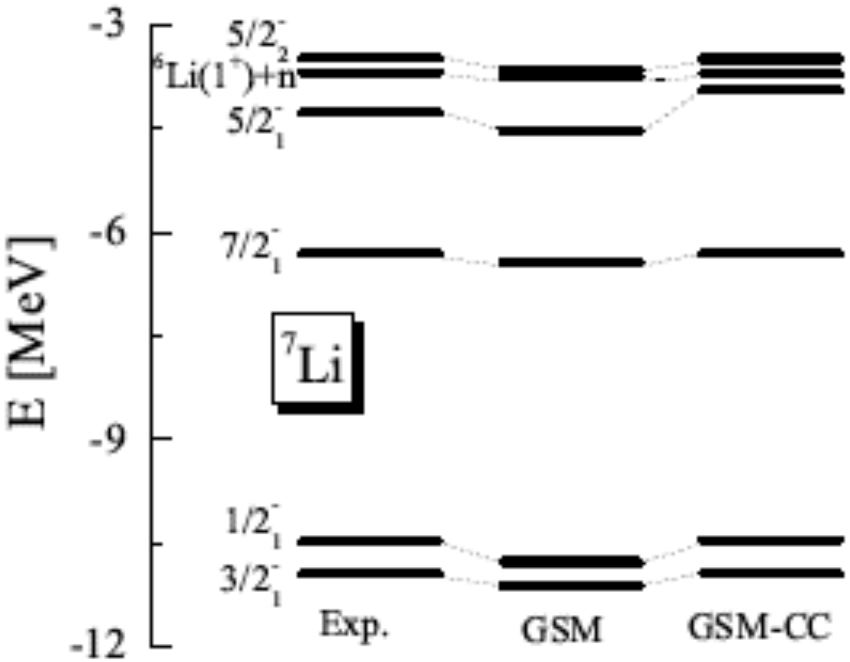
E1, E2, M1 components included

→ Effective interaction in p-shell:
See the talk of Yannen Jaganathen

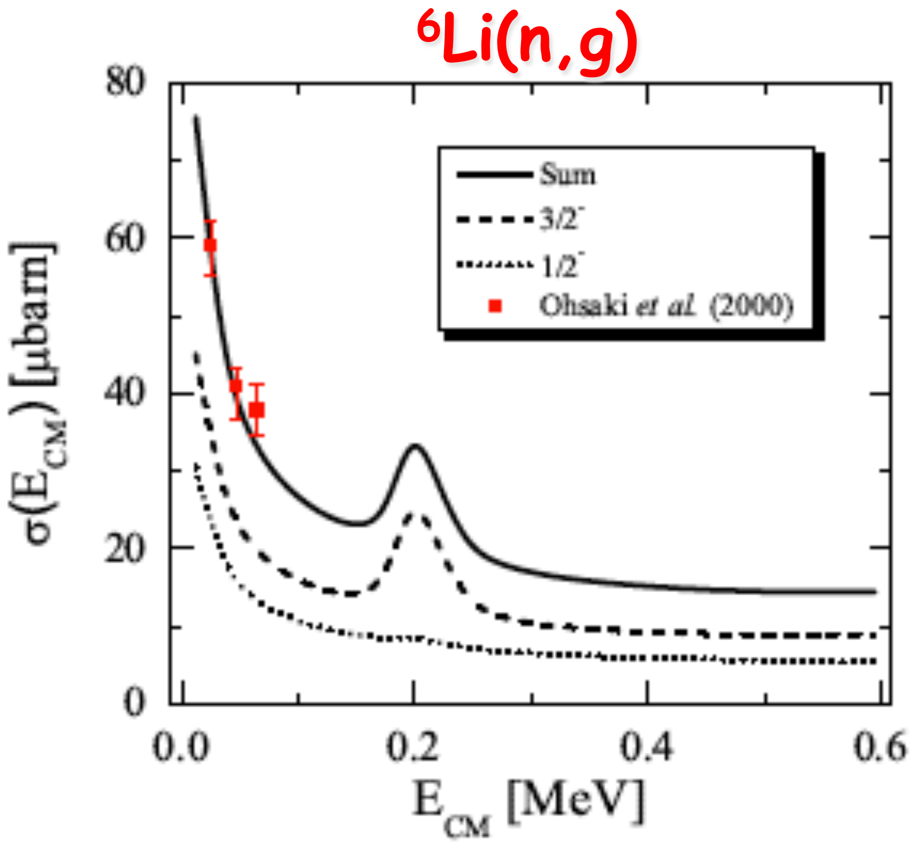
$S^{GSM\text{-}CC}(0)$=88.34 b eV
$S^{exp\text{-}acc}(0)$=79+/-18 b eV

N. Michel, G. Dong, et al. (2015)

Mirror reaction: ^{6}Li(n,g)



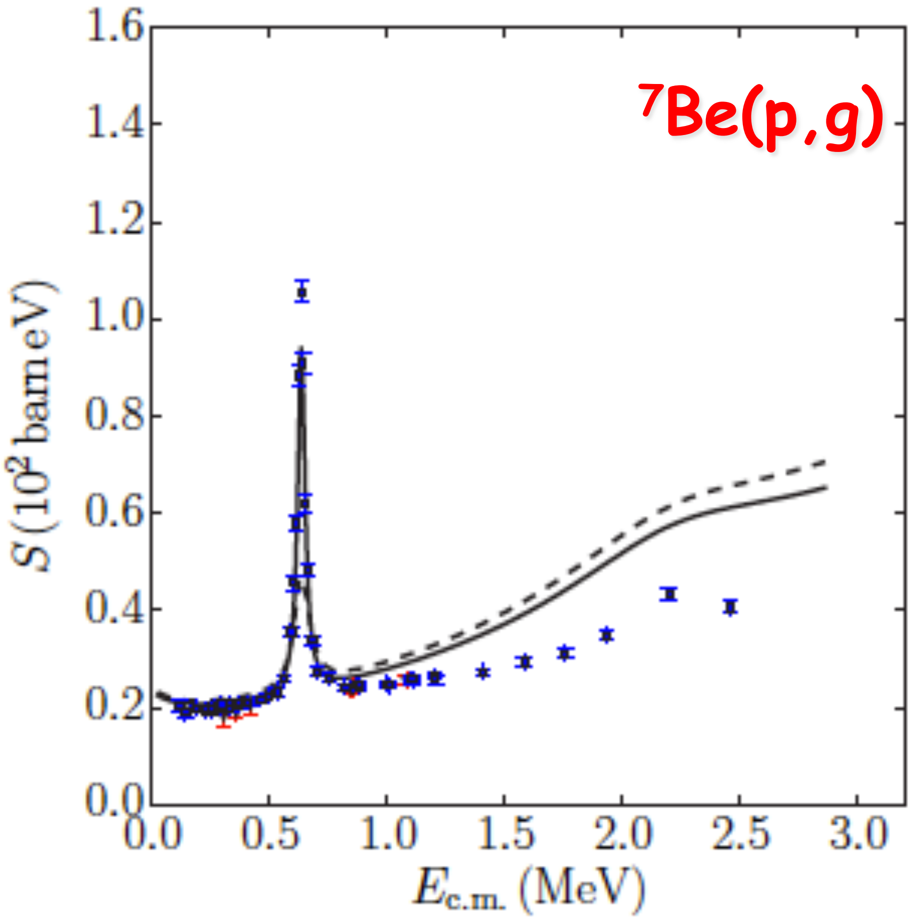
1-3% renormalization of channel-channel coupling potentials

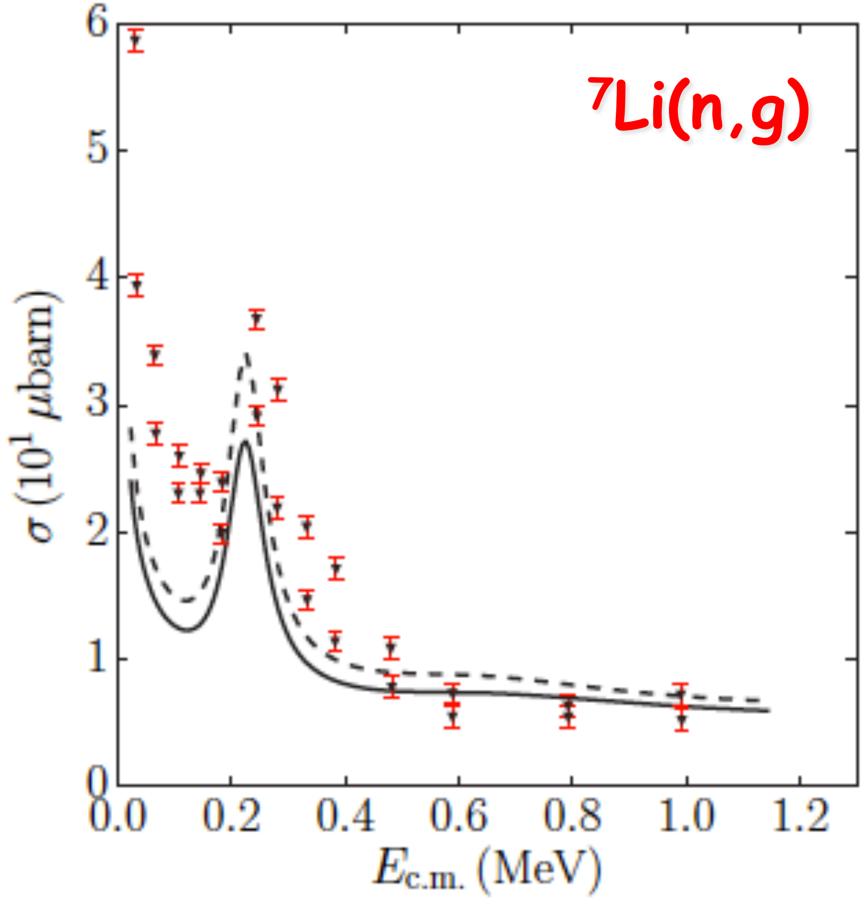


E1, E2, M1 components included

N. Michel, G. Dong, et al. (2015)

Mirror systems/reactions: ^{7}Be(p,g), ^{7}Li(n,g)





$S^{GSM\text{-}CC}(0)=23.21$ b eV

$S^{exp\text{-}acc}(0)=20.9+/-0.6$ b eV

E1, E2, M1 components included

K. Fossez, N. Michel, et al., PRC 91, 034609 (2015)

Outlook

1. Comprehensive and validated theory of nuclei on the horizon
2. Shell model treatment of weakly bound/unbound states ➔ unification of nuclear structure and reactions
3. Collectivization of nuclear wave functions can be the result of
   - internal mixing by interactions: low-energy collective vibrations, rotational states, giant resonances, ...
   - external mixing via the decay channel(s): coherent enhancement/suppression of radiation, multi-channel effects in spectroscopic factors and reaction cross-sections, ...
   - interplay of internal & external mixing: near-threshold cluster states, halos, modification of spectral fluctuations, breaking of the isospin symmetry, K quantum number non-conservation, Thomas-Ehrmann shift, coalescence of eigenvalues, ...

Future challenges

1. The assessments of the validity of random matrix theory is crucial for the statistical theory of nuclear reactions
   - Coupling to continuum leads to violation of the orthogonal invariance and deviations from PT distribution
   - Statistical theory neglects the energy dependence of the Hermitian part of the effective Hamiltonian and the strong correlation between matrix elements of Hermitian and anti-Hermitian parts (Ex.: segregation of time scales)

2. The connection of surrogate reactions with wanted reactions (ex. (d,p) with (n,g))

3. The resolution of transfer vs knockout reaction controversy

Thank You