Computational Advances in Nuclear and Hadron Physics Workshop

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Relativistic Electro-Magneto-Fluid Dynamicsss

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Relativistic Electro-Magneto-Fluid Dynamics

REMFD in Astrophysics & Cosmology

- SKA Science – infer EoS and transport properties

- NICA & FAIR Science – measure the eos and transport properies

Pulsar Science Highlights

Key Science:

Strong-field Tests of Gravity

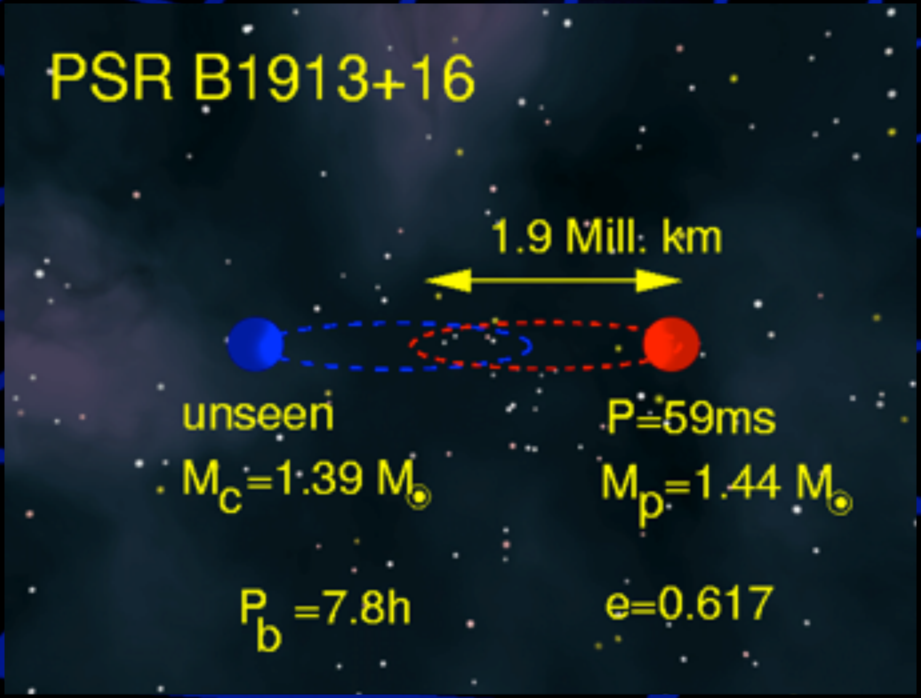
- Was Einstein Right?
- Cosmic Censorship, "No-Hair" Theorem
- Cosmic Gravitational Wave Background

Variety of Other Major Astrophysical Topics:

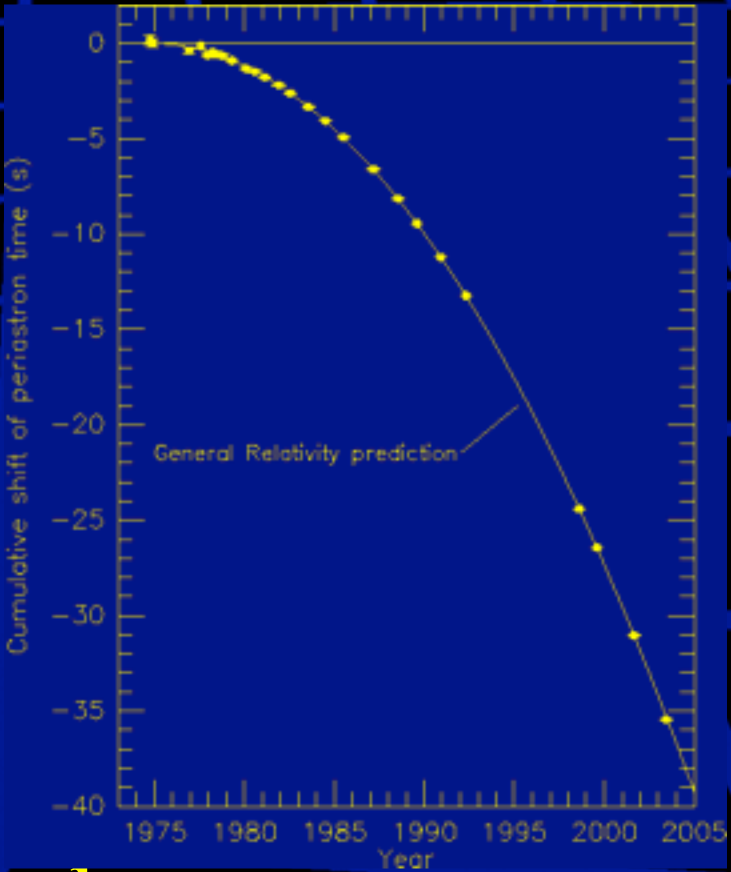
- Milky Way Structure, ISM
- Intergalactic Medium
- Relativistic Plasma Physics
- Extreme Densities

Noted GR Laboratories

Hulse & Taylor (1974)

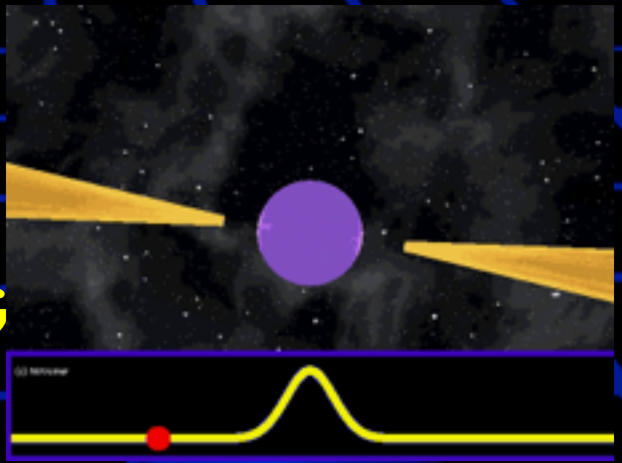


Weisberg & Taylor (priv. comm)



- Orbit shrinks every day by 1cm
- Confirmation of existence of gravitational waves

Pulsars…

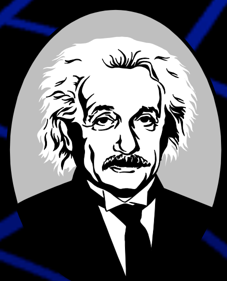


- embody physics of the EXTREME
  - surface speed ~0.1c
  - 10x nuclear density in centre
  - some have $B > B_q = 4.4 \times 10^{13}$ G
  - Voltage drops ~ 10^{12} volts
  - $F_{EM} = 10^9 F_g = 10^{11} F_{gEarth}$
  - Tsurf ~ million K
- …relativistic plasma physics in action
- …probes of turbulent and magnetized ISM
- …precision tools, e.g.
  - Period of B1937+21:

    P = 0.0015578064924327±0.0000000000000004 s
  - Orbital eccentricity of J1012+5307:

Was Einstein right?

General Relativity vs Alternative Theories



Strong Equivalence Principle

- Violation of Lorentz-Invariance
- Violation of Positional Invariance
- Violation of Conservation Laws etc.

Solar System tests provide constraints

but only in weak field!

$$\left(\frac{v}{c}\right)_{orbit} \approx 10^{-4}$$

$$\varepsilon_{Sun} \approx 10^{-6}$$

$$\varepsilon_{Earth} \approx 10^{-10}$$

$$\varepsilon_{Moon} \approx 10^{-11}$$

$$\left(\frac{v}{c}\right)_{orbit} \approx 10^{-3}$$

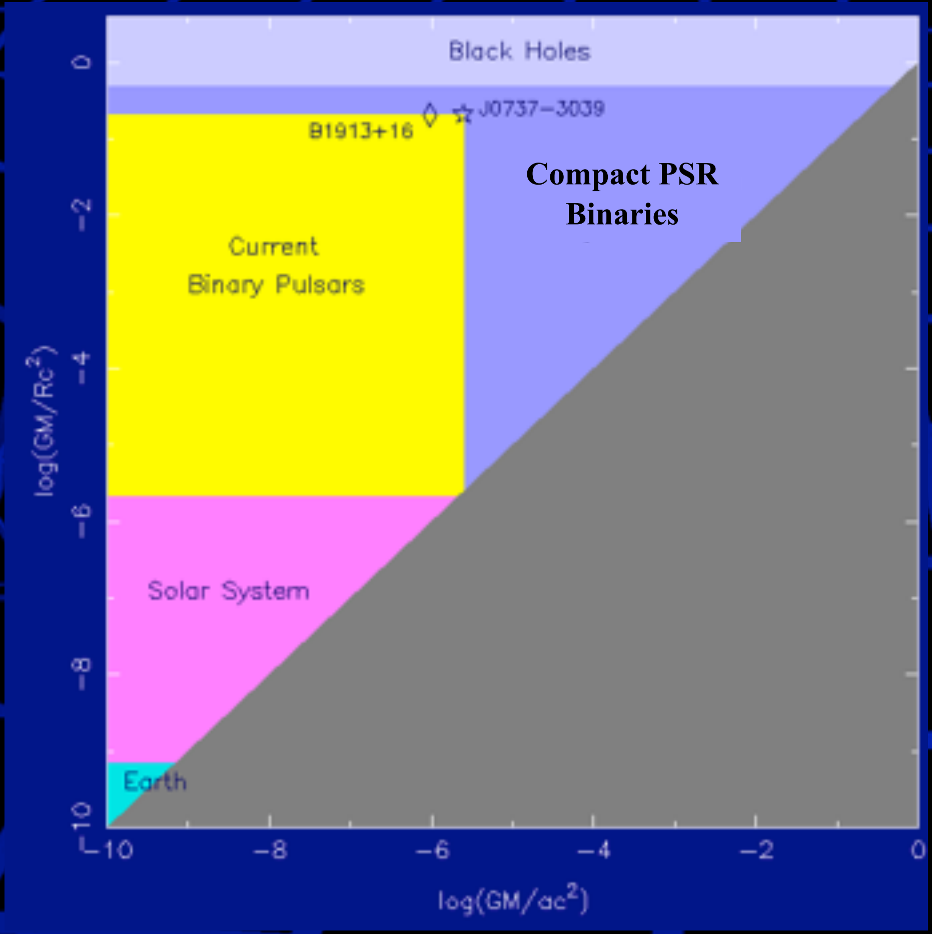
$$\varepsilon_{PSR} \approx 0.15$$

$$\varepsilon_{BH} \approx 0.5$$

No test of any theory of gravity is complete, if only done in solar system, i.e. strong field limit and radiative aspects need to be tested, too!

⇒ This is and will be done best with radio pulsars!

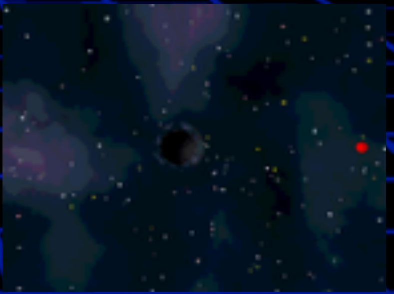
Exploration of Black Holes



We will probe BH properties with pulsars and SKA:

- precise measurements
- no assumptions about EoS or accretion physics
- test masses well separated, not deformed

Black Hole properties



spin and quadrupole moment:

- Astrophysical black holes are expected to rotate

$$\chi \equiv \frac{c}{G}\frac{S}{M^2} \qquad q \equiv \frac{c^4}{G^2}\frac{Q}{M^3}$$

S = angular momentum

Q = quadrupole moment

- Result is relativistic & classical spin-orbit coupling
- Visible as a precession of the orbit:

  Measure higher order derivatives of secular changes in semi-major axis and longitude of periastron (relativistic) or transient TOA perturbations (classical)

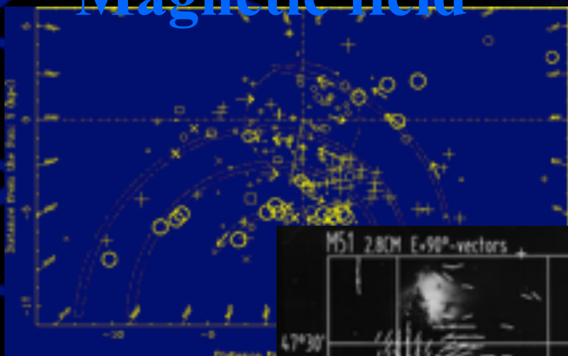
- Not easy! It is not possible today!
- Requires SKA sensitivity!

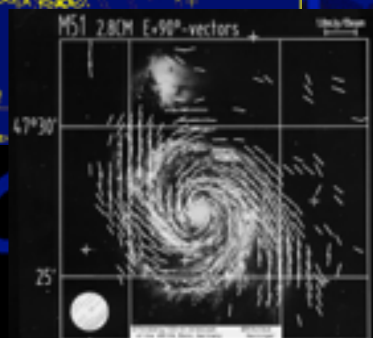
Pulsar Astrophysics with SKA

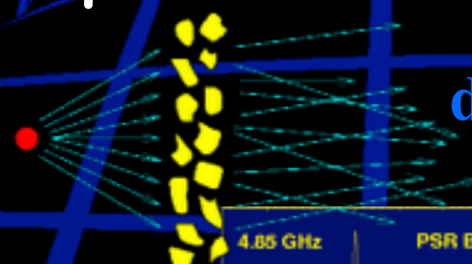
Wide range of applications:

- Galactic probes: Interstellar medium/magnetic field
  Star formation history
  Dynamics
  Population via distances (ISM, VLBI)

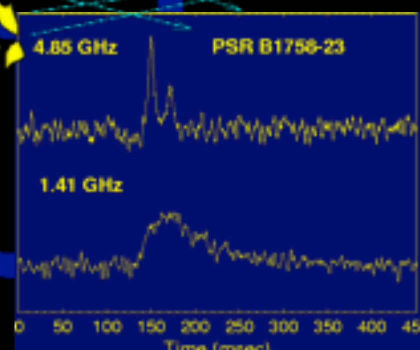
Magnetic field

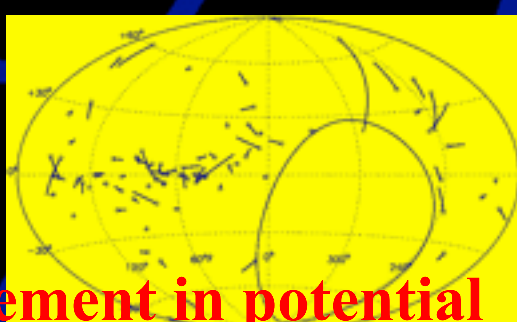




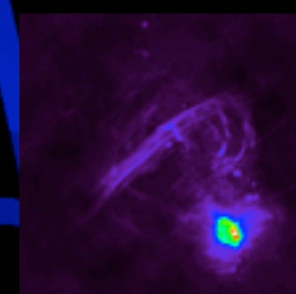


Electron distribution





Movement in potential



Galactic Centre

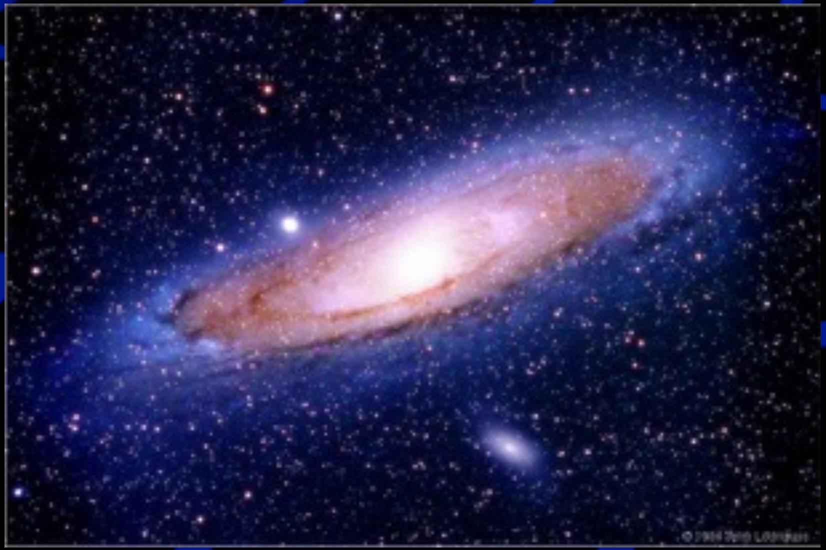
Pulsar Astrophysics with SKA

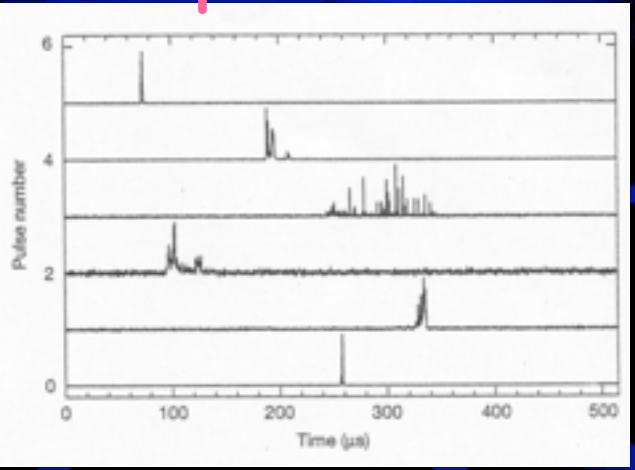
Wide range of applications:

- Galactic probes

Extragalactic pulsars: Missing Baryon Problem
Formation & Population
Turbulent magnetized IGM
Giant pulses

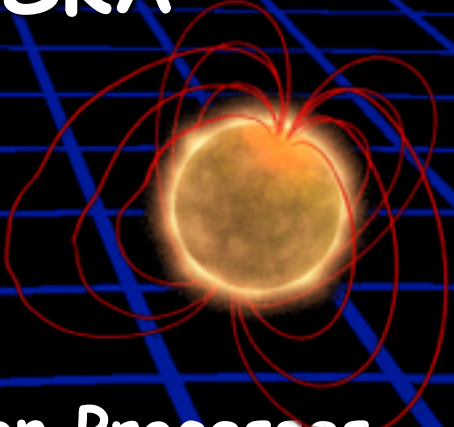
Search nearby galaxies!





Reach the local group!

Pulsar Astrophysics with SKA



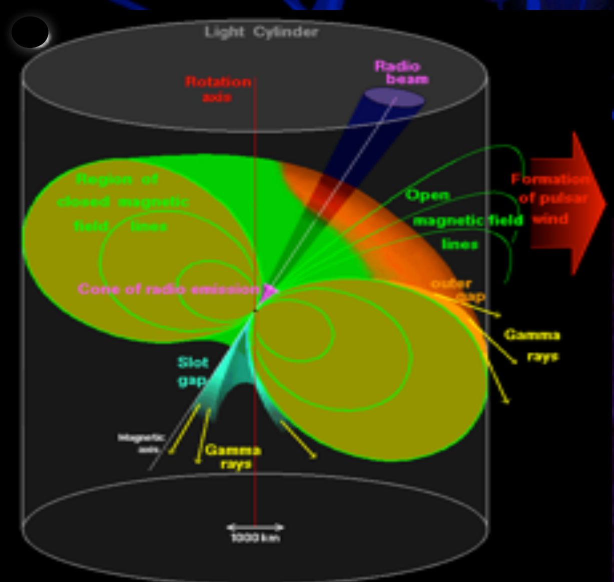
Wide range of applications:

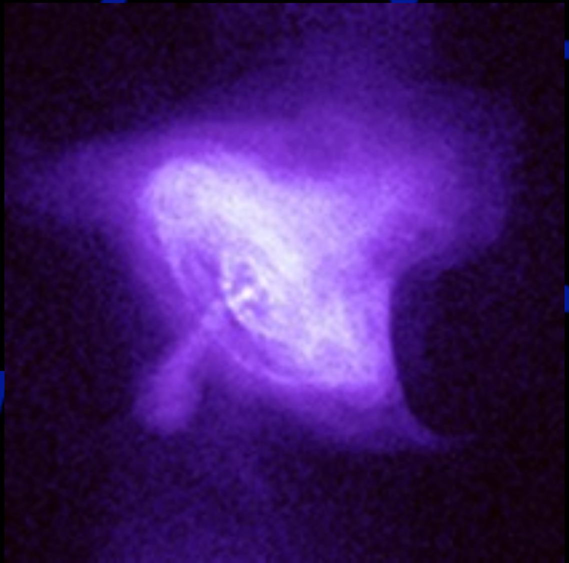
- Galactic probes
- Extragalactic pulsars

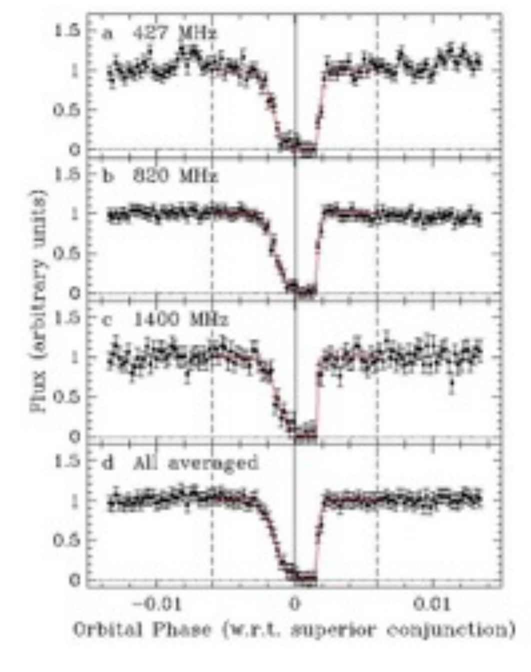
Relativistic plasma physics: Emission Processes

Pulsar Wind Nebulae

Magnetospheric Structure



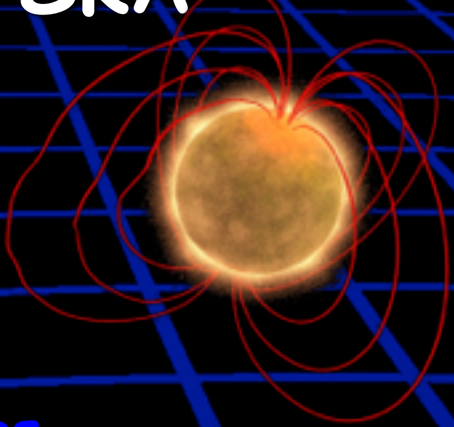




Pulsar Astrophysics with SKA

Wide range of applications:

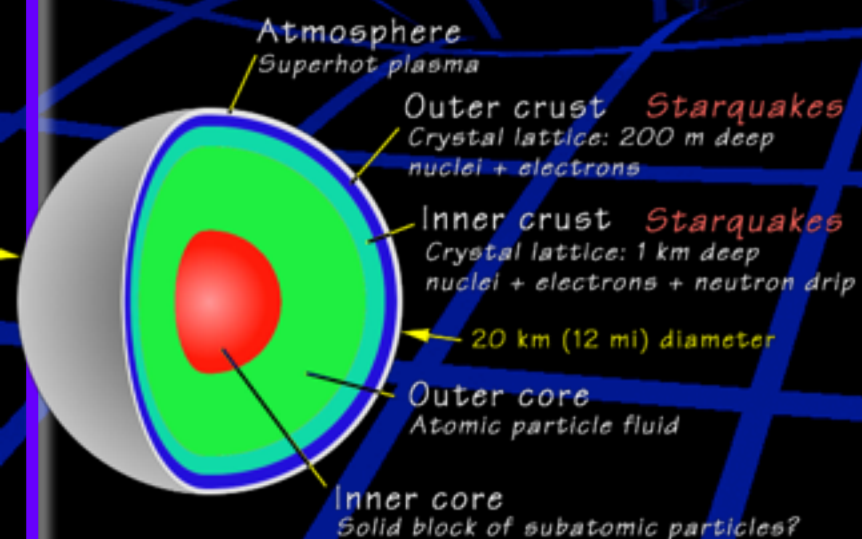
- Galactic probes
- Extragalactic pulsars
- Relativistic plasma physics

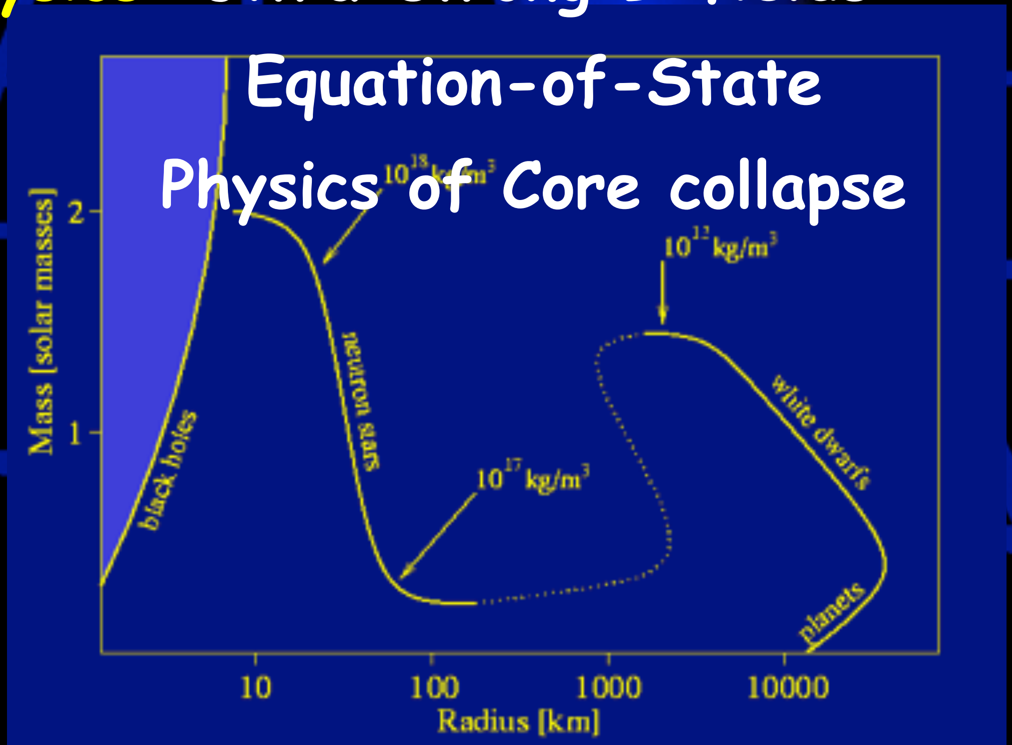


Matter Physics: Ultra-strong B-fields

Equation-of-State

Physics of Core collapse



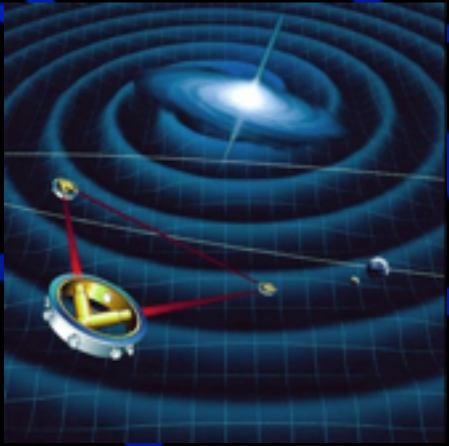


Pulsar Astrophysics with SKA

Wide range of applications:

- Galactic probes
- Extragalactic pulsar
- Relativistic plasma phy
- Extreme Dense Matter Physics
- Multi-wavelength studies: Photonic windows

Non-photonic windows

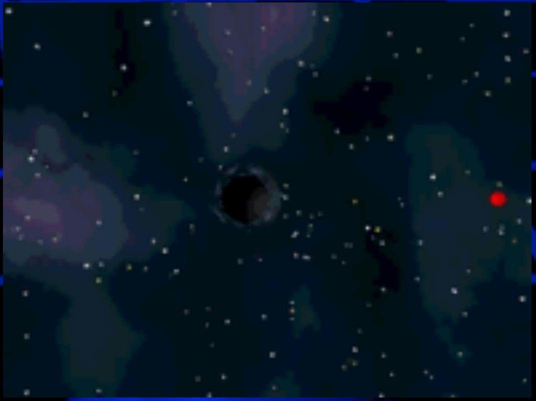


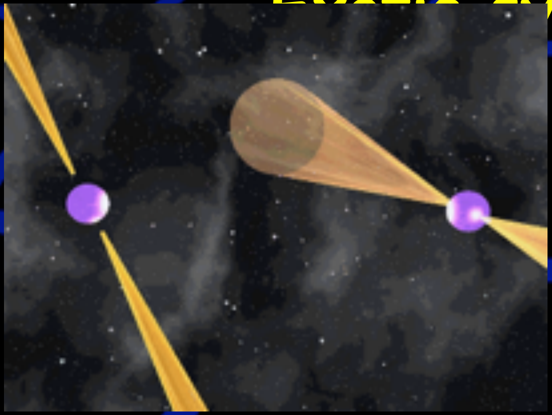
Pulsar Astrophysics with SKA

Wide range of applications:

- Galactic probes
- Extragalactic pulsars
- Relativistic plasma physics
- Extreme Dense Matter Physics
- Multi-wavelength studies
- Exotic systems: planets
  pulsar/MS binaries
  millisecond pulsars
  relativistic binaries
  double pulsars
  PSR-BH systems

Holy Grail: PSR-BH





Double Pulsars

Planets

Cosmological Gravitational Wave Background

- stochastic gravitational wave background expected on theoretical grounds

Possible Sources:

- Inflation
- String cosmology
- Cosmic strings
- phase transitions

} $h_0^2 \Omega_{GW}(f) \sim const.$

and also: merging massive BH binaries in early galaxy evolution } $h_0^2 \Omega_{GW}(f) \propto f^{2/3}$

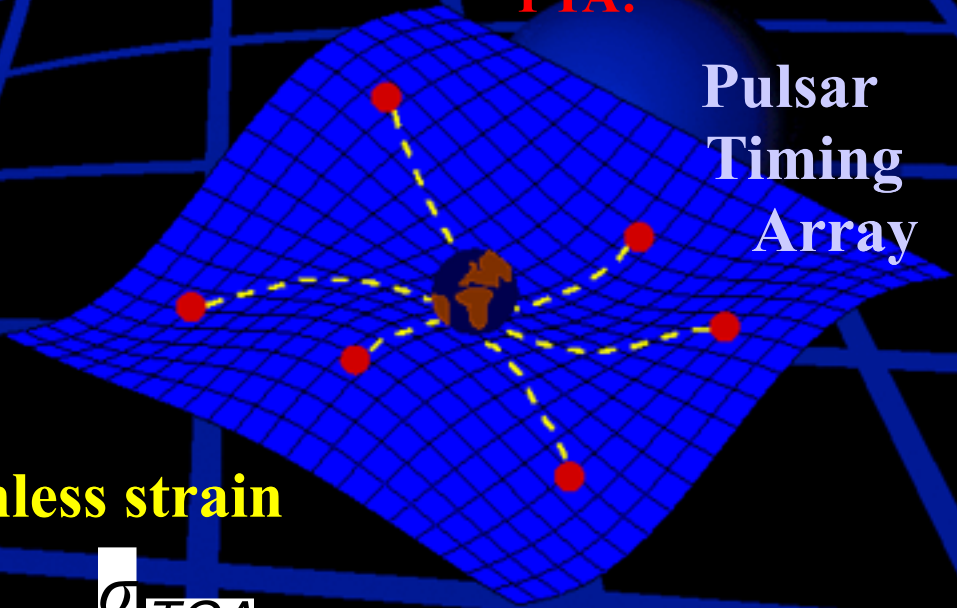
Cosmological Gravitational Wave Background

- Pulsars discovered in Galactic Census also provide network of arms of a huge cosmic gravitational wave detector

Perturbation in space-time can be detected in timing residuals

PTA:

Pulsar Timing Array



- Sensitivity: dimensionless strain

$$h_c(f) \sim \frac{\sigma_{TOA}}{T}$$

Relativistic Electro-Magneto-Fluid Dynamics

Combined EMFD

- Relativistic dissipative fluids relaxation equations
- Maxwell-Lorentz general equations
- Einstein GR equations
- Computational techniques

Thermodynamics of Polarized Media

The Maxwell-Lorentz field equations are

$$F_{\alpha\beta} = 2\partial_{[\alpha} A_{\beta]} \quad H^{\alpha\beta}{}_{|\beta} = 4\pi \vartheta^{\alpha} \quad H^{\alpha\beta} = \Phi^{\alpha\beta} - 4\pi M^{\alpha\beta}$$

The skew tensors may be decomposed into pairs of spatial vectors as

$$F_{\alpha\beta} = (B, E) \quad H^{\alpha\beta} = (H, \Delta) \quad M^{\alpha\beta} = (M, -\Pi)$$

so that, for example, $E_{\alpha} = \Phi_{\alpha\beta} \upsilon^{\beta}$

where u_{α} ις τηε 4-πελοχιτψ οφ τηε λοχολ οβσερπερ $(\upsilon_{\alpha}\upsilon^{\alpha} = -1)$

The energy-momentum tensor of the electromagnetic field is given by

$$4\pi T^{\lambda\mu}_{(\varepsilon.\mu)} = \Phi^{\lambda}{}_{\mu} H^{\mu\alpha} - \Pi^{\lambda}\upsilon^{\mu} - \Pi^{[\lambda} E^{\mu]} - M^{[\lambda} B^{\mu]} - \frac{1}{4}\gamma^{\lambda\mu}\Phi^{\alpha\beta}\Phi_{\alpha\beta}$$

with

$$\Pi^{\lambda} = 2u_{\mu} F^{[\lambda}{}_{\alpha} M^{\mu]\alpha} \quad \Pi = \mathbf{P} \times \mathbf{B} - \mathbf{M} \times \mathbf{E}$$

Thermodynamics of Polarized Media

The primary variables N^μ , $T^{\lambda\mu}{}_{(mat)}$ for an arbitrary state are decomposed in the form

$$N^\mu = \nu\upsilon^\mu$$

$$T^{\lambda\mu}{}_{(\mu\,\sigma)} = \rho\upsilon^\lambda\upsilon^\mu + 2\theta^{(\lambda}\upsilon^{\mu)} + \Pi\Delta^{\lambda\mu} + \pi^{\lambda\mu}$$

$$\upsilon_\lambda\Pi^\lambda = \upsilon_\lambda\theta^\lambda = \upsilon_\lambda\pi^{\lambda\mu} = \pi^{\lambda\mu}\upsilon_\mu = \pi^{\lambda\mu}\Delta_{\lambda\mu} = 0$$

Total energy-momentum tensor

The defination of the electromagnetic energy momentum tensor is purely formal.

The physically significant quantity is the total energy-momentum tensor , which couples to Einstein tensor in the gravitational field equations and satisfies

$$T^{[\lambda\mu]} = \mathrm{T}^{\lambda\mu}{}_{|\mu} = 0$$

Entropy production and transport equations

The fundamental reation between the primary variables for an arbitrary state close to equilibrium

$$S^{\mu} = \Pi_{(0)}\beta^{\mu} - \alpha N^{\mu} - \beta^{\lambda} T_{\lambda}^{(\mu\ldots)} - \Theta^{\mu}$$

From which we get the entropy production

$$TS^{\mu}_{|\mu} = \varphi^{\mu} E_{\mu} - \theta^{\mu}\left(\beta^{-1}\partial_{\mu}\beta - \dot{\upsilon}_{\mu}\right) + \pi^{\lambda\mu}_{(1)}\upsilon_{(\lambda|\mu)}$$

$$+ \Pi^{\mu}_{(1)}(\Lambda_{\mu} E_{\mu} - \upsilon_{(\lambda|\mu)} E^{\lambda}) + M^{\mu}_{(1)}(\Lambda_{\mu} B_{\mu} - \upsilon_{(\lambda|\mu)} B^{\lambda})$$

$$- T\Theta^{\mu}_{|\mu}$$

Relativistic equations for stellar structure

Static and sphericaly symmetric self-gravitating mass distribution

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = e^{2\Phi(r)} c^2 dt^2 - e^{2\lambda(r)} dr^2 - r^2 (d\theta^2 + \sin^2\theta d\phi^2)$$

$$e^{\lambda(r)} = \left[1 - \frac{2Gm(r)}{c^2 r}\right]^{-1/2}$$

for the present case the Einstein's field equations take the form called the

Tolman – Oppenheimer – Volkov equations (TOV)

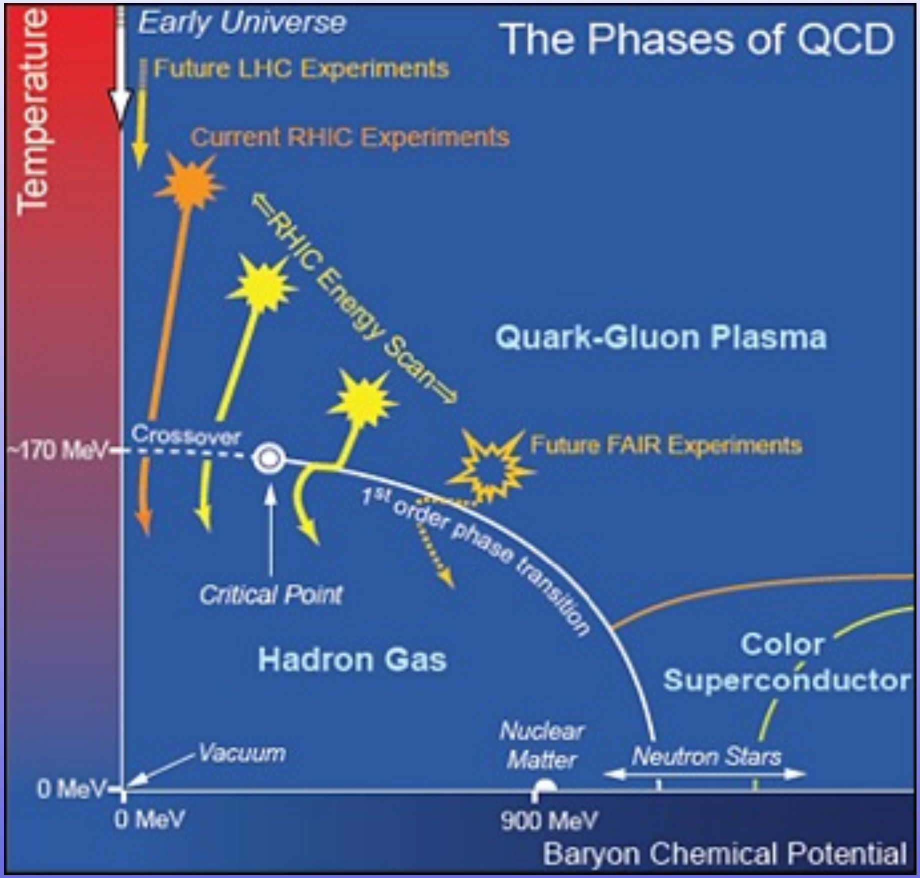
$$\frac{dP}{dr} = -G\frac{m(r)\rho(r)}{r^2}\left(1 + \frac{P(r)}{c^2\rho(r)}\right)\left(1 + 4\pi\frac{r^3 P(r)}{m(r)c^2}\right)\left[1 - \frac{2Gm(r)}{c^2 r}\right]^{-1}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho(r)$$

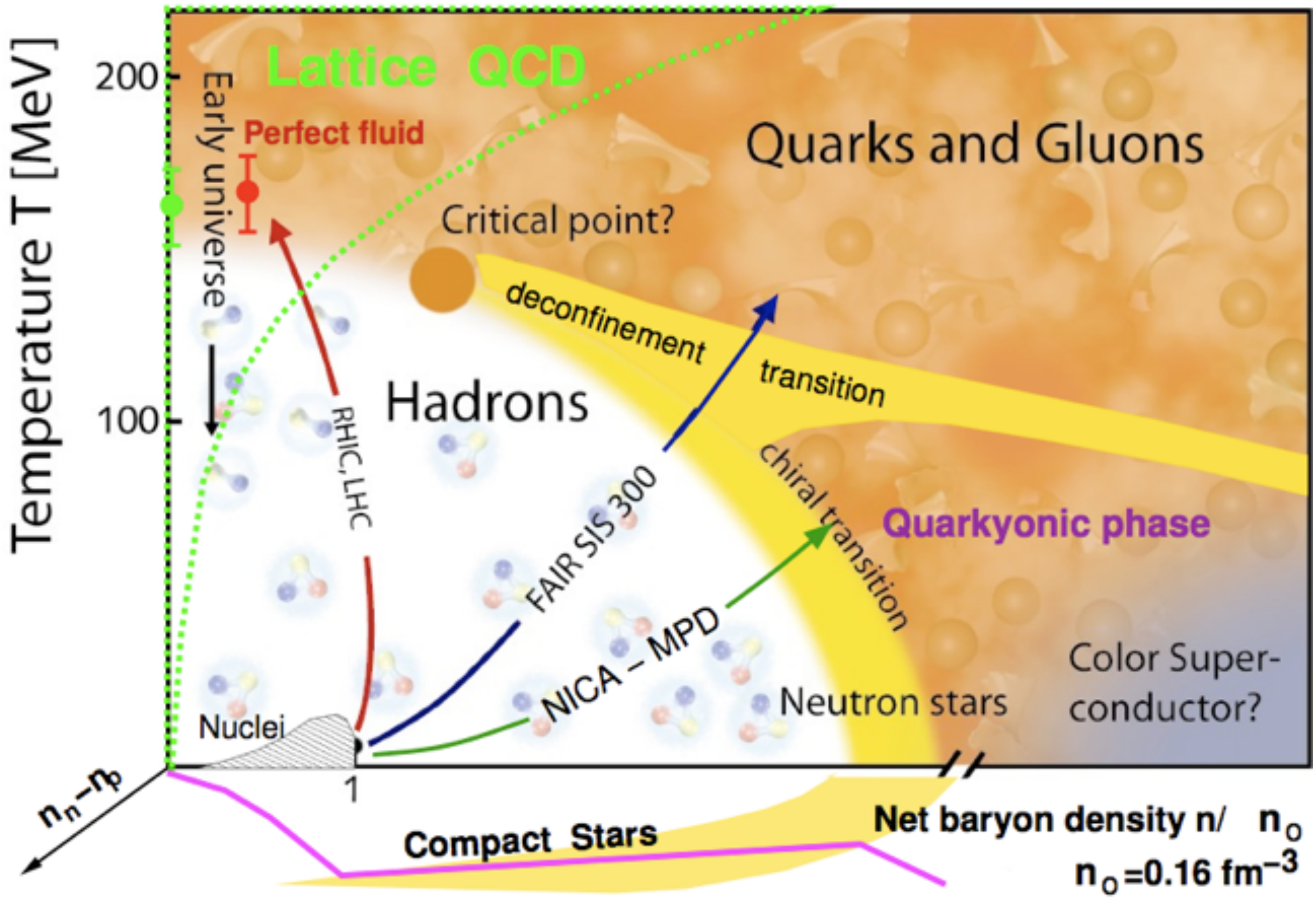
$$\frac{d\Phi}{dr} = -\frac{1}{\rho(r)c^2}\frac{dP}{dr}\left(1 + \frac{P(r)}{\rho(r)c^2}\right)^{-1}$$

One needs the equation of state (EOS) of dense matter, P = P(ρ), up to very high densities

Neutron Stars in the QCD phase diagram



Cristalline Color superconductor



Temperature T [MeV]
200
100
Lattice QCD
Early universe
Perfect fluid
Quarks and Gluons
Critical point?
deconfinement
transition
Hadrons
RHIC,LHC
FAIR SIS 300
chiral transition
Quarkyonic phase
NICA – MPD
Color Super-
conductor?
Neutron stars
Nuclei
1
$n_n - n_p$
Compact Stars
Net baryon density n/ n_0
n_0=0.16 fm^{-3}


27th Chris Engelbrecht Summer School 2016
Hot and Dense Matter
in Heavy Ion Collisions and Astrophysics
Pretoria, South Africa, 11 – 22 January 2016
Organizing Committee
A Muronga (UJ)
F.G. Scholtz (NITheP)
Topics
. Physics and astrophysics of quark gluon plasma
. Physics and astrophysics of compact stars
. Hydrodynamics and its application to heavy ion collisions
. Core collapse supernovae , neutron stars and the equation of state
. Finite temperature field theory
. Astroparticle physics