Taking the ratio between shear viscosity and electric conductivity of the QGP

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Computational Physics for High-Energy Heavy-Ion Collisions
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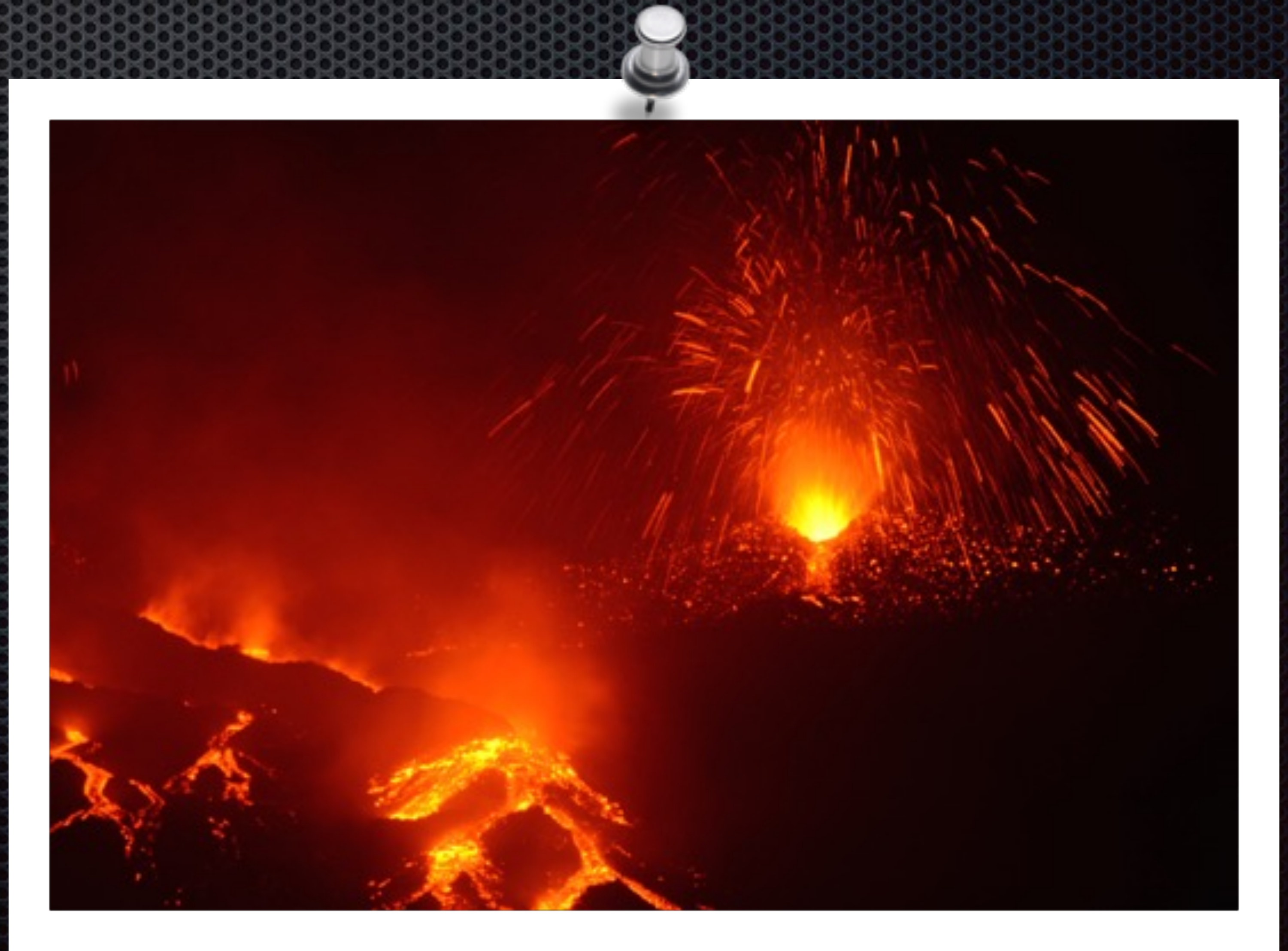
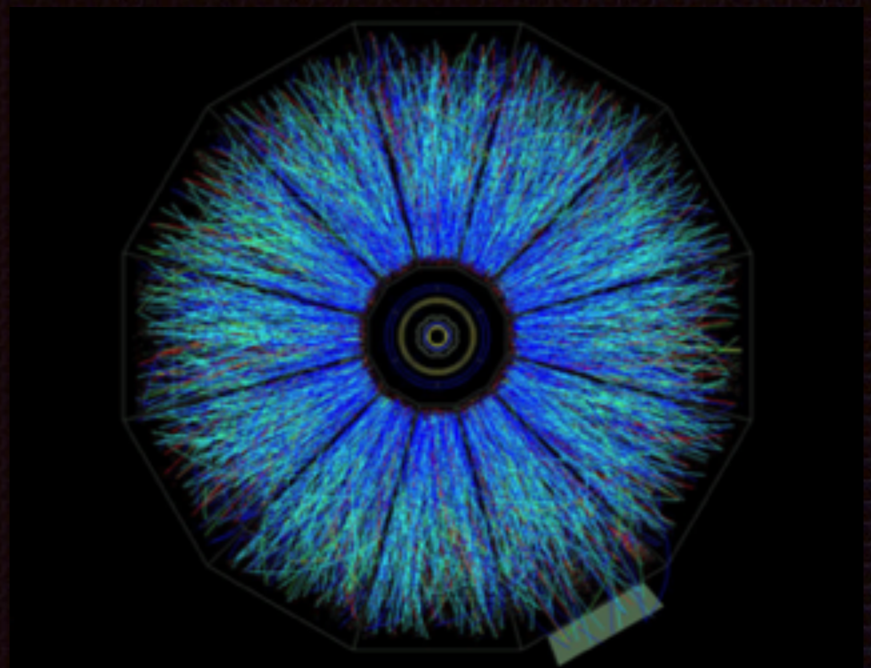
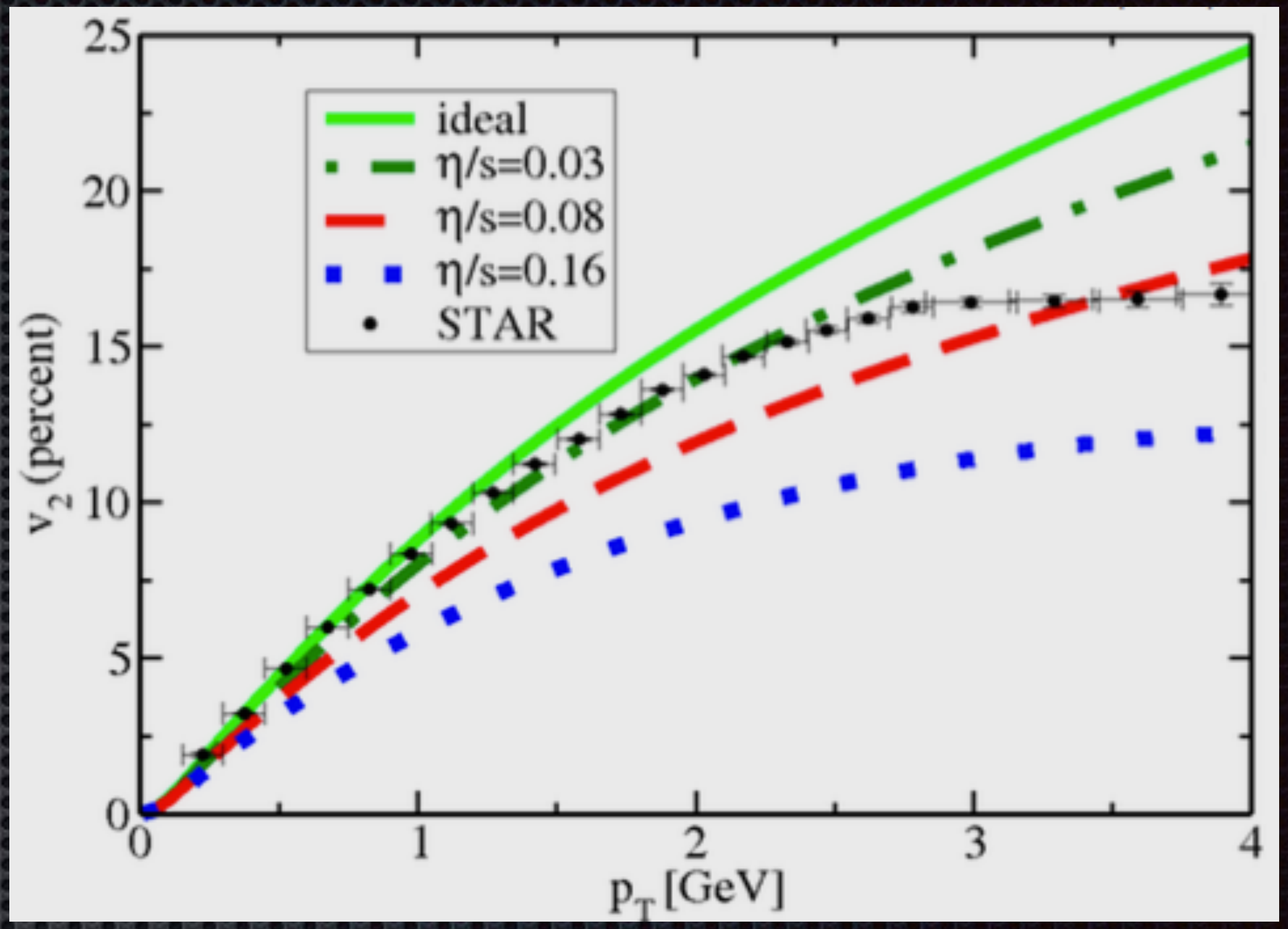


Shear Viscosity

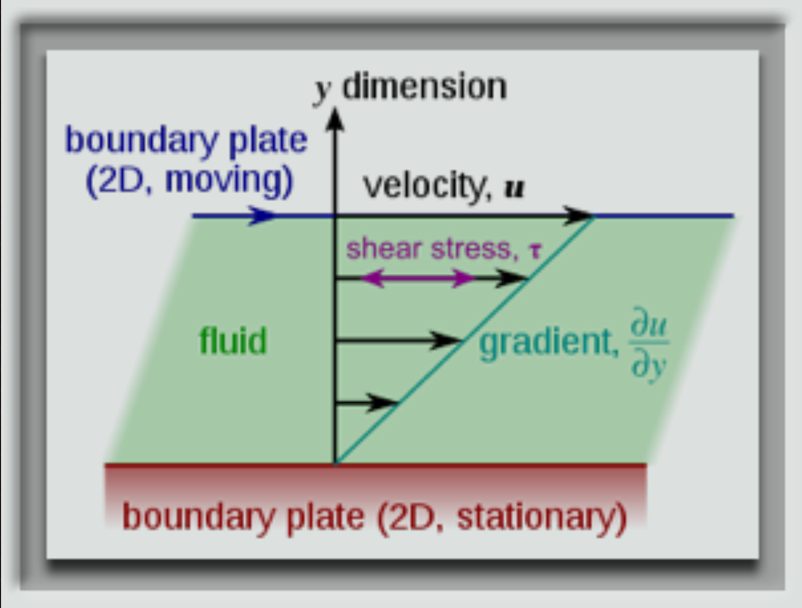


First Au-Au @ 100 GeV, RHIC, STAR

$$\frac{d^3N}{p_T dp_T dy d\phi} = \frac{1}{2\pi p_T}\frac{d^2N}{dp_T dy}\left(1 + 2\sum_{n=1}^{\infty} v_n(p_T, y)\cos[n(\phi - \psi_R)]\right)$$



elliptic flow $v_2(p_T, b) = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$



shear stress

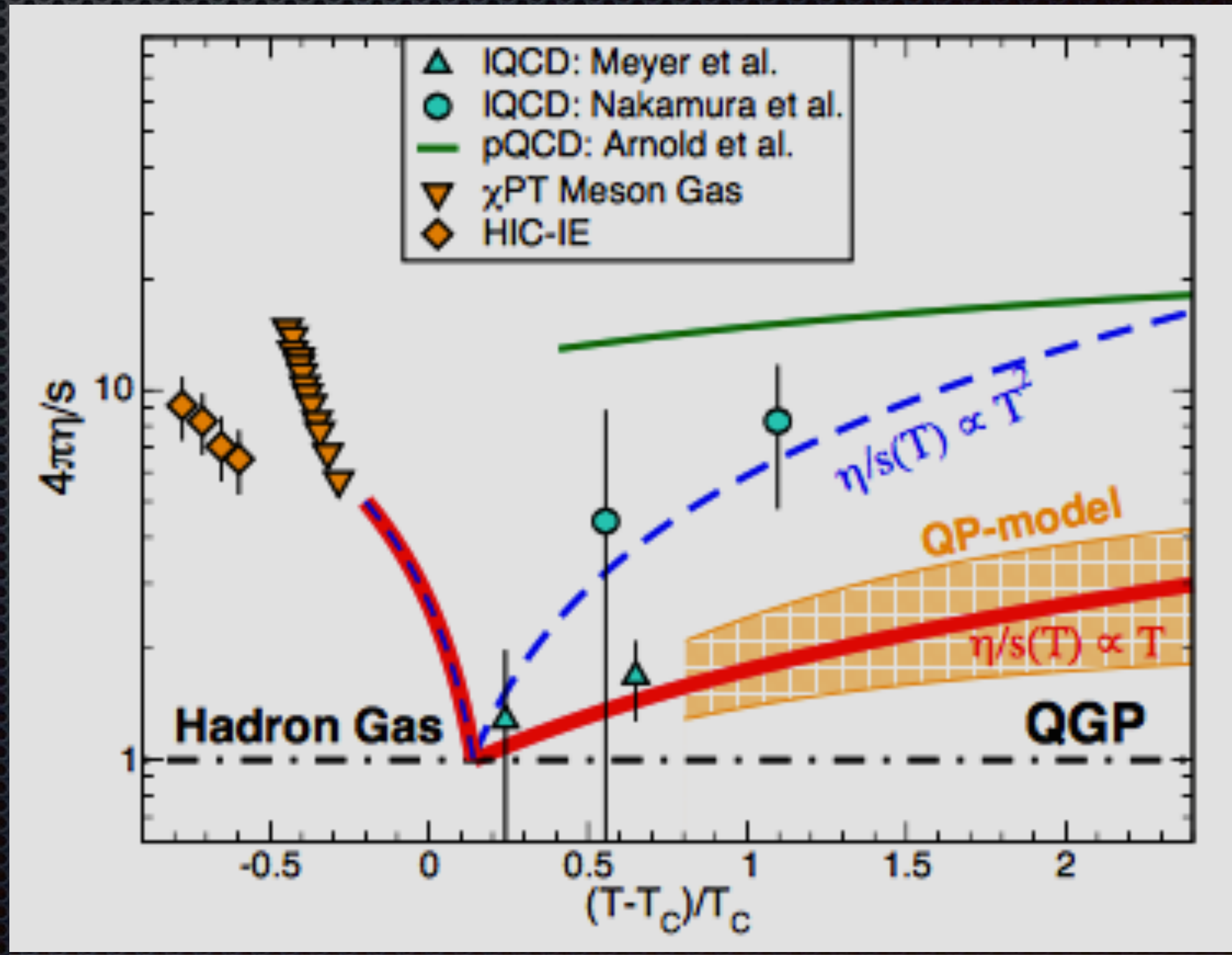
$$\tau = \frac{F_{xy}}{A_{yz}} = -\eta\frac{\partial u_x}{\partial y}$$

fluid	P [Pa]	T [K]	η [Pa · s]	η/s [$\hbar/k_B$]
H_2O	$0.1 \cdot 10^6$	370	$2.9 \cdot 10^{-4}$	8.2
^4He	$0.1 \cdot 10^6$	2.0	$1.2 \cdot 10^{-6}$	1.9
H_2O	$22.6 \cdot 10^6$	650	$6.0 \cdot 10^{-5}$	2.0
^4He	$0.22 \cdot 10^6$	5.1	$1.7 \cdot 10^{-6}$	0.7
QGP	$88 \cdot 10^{33}$	$2 \cdot 10^{12}$	$\lesssim 5 \cdot 10^{11}$	$\lesssim 0.4$

T.Schäfer, D. Teaney, Rep. Prog. Phys 72 (2009) 126001

pitch drop experiment

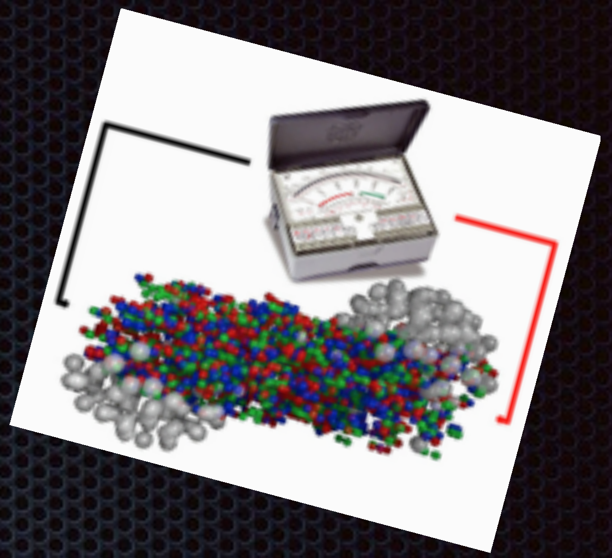
- started in 1927
- 8th drop in November 2000
- $\eta_{pitch} = 2 \times 10^{11} \eta_{water}$
- $\eta_{pitch} \ll \eta_{QGP}$



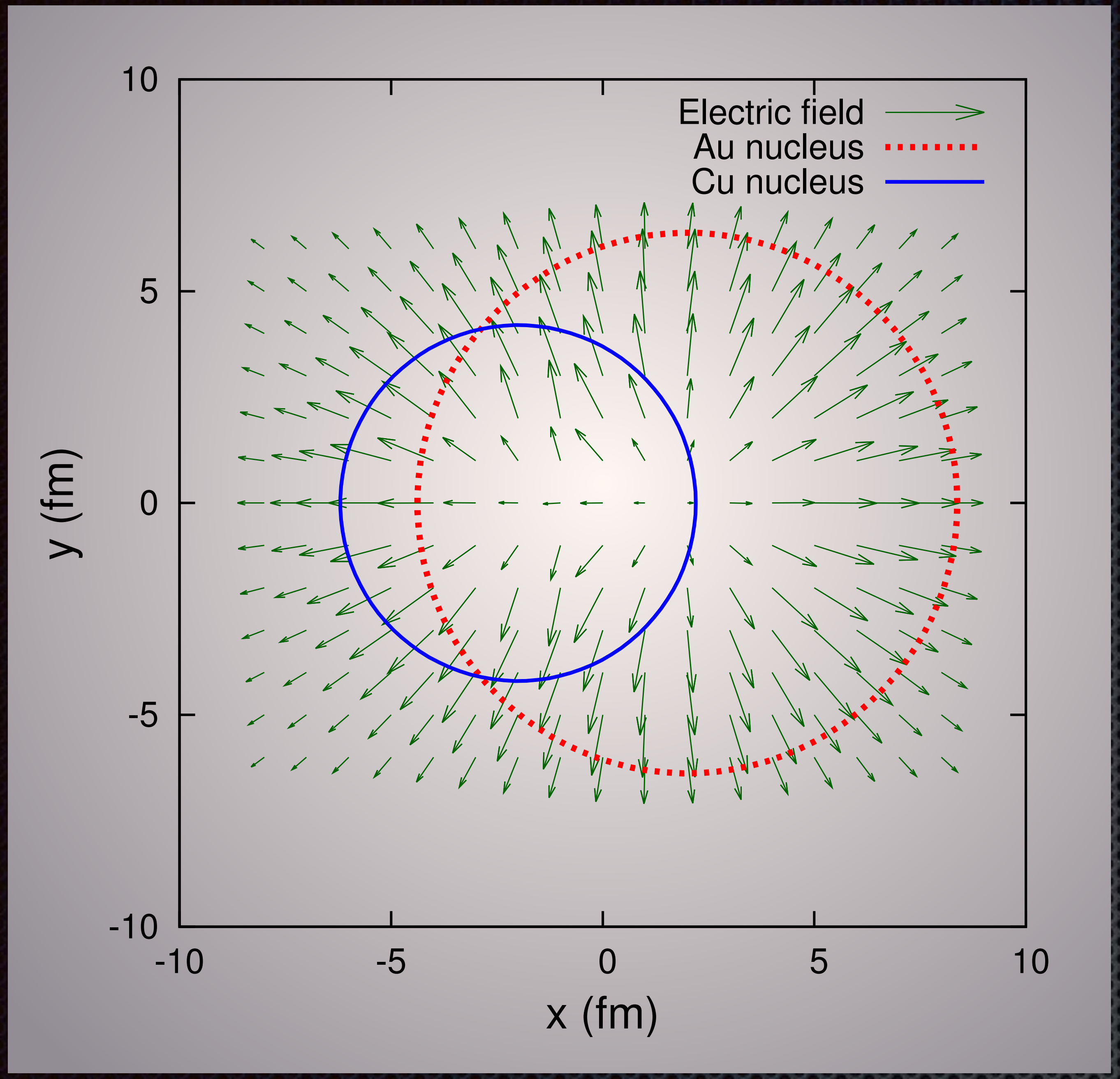
Electric Conductivity

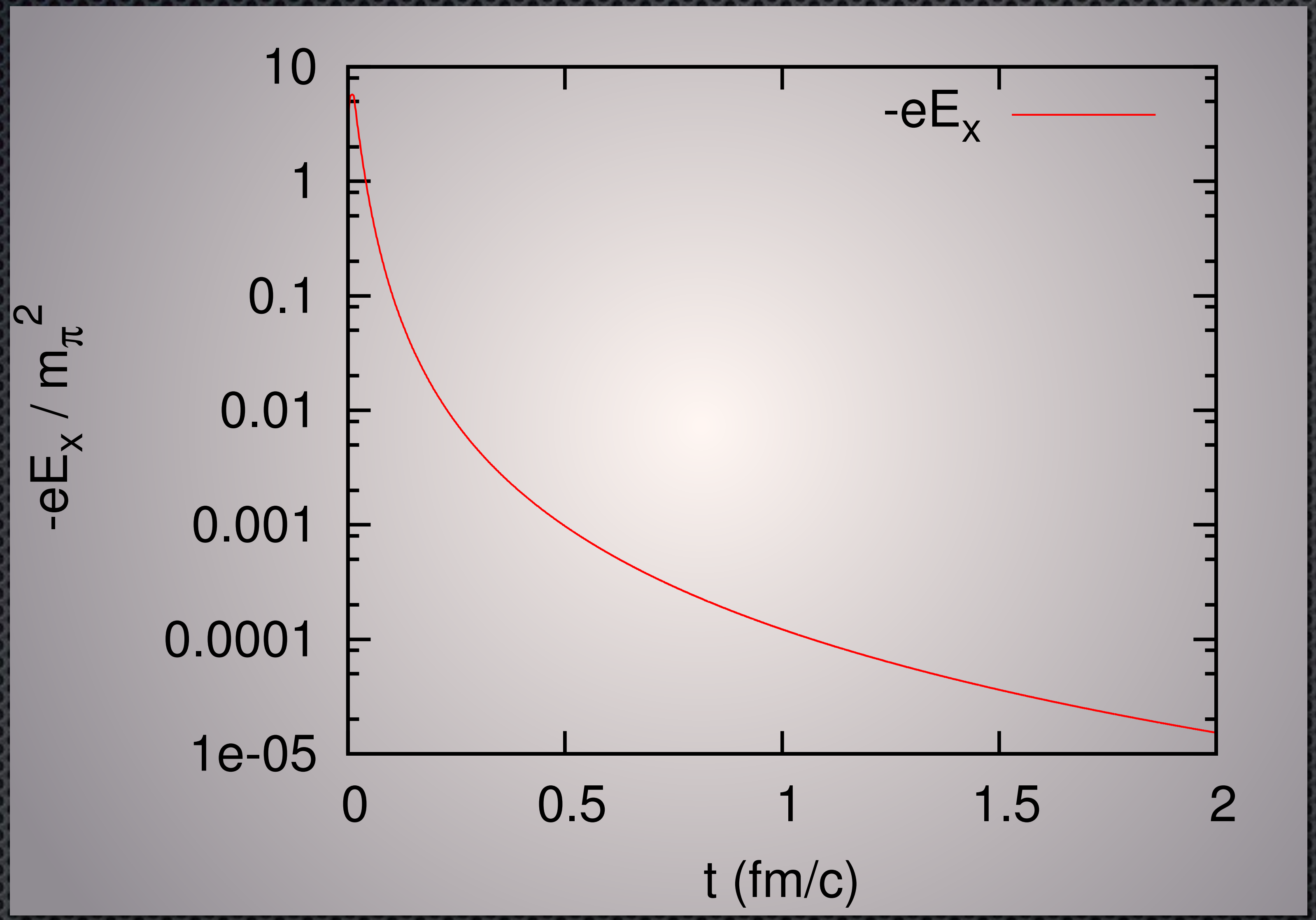
Mho

$$j = \sigma_{el} E$$

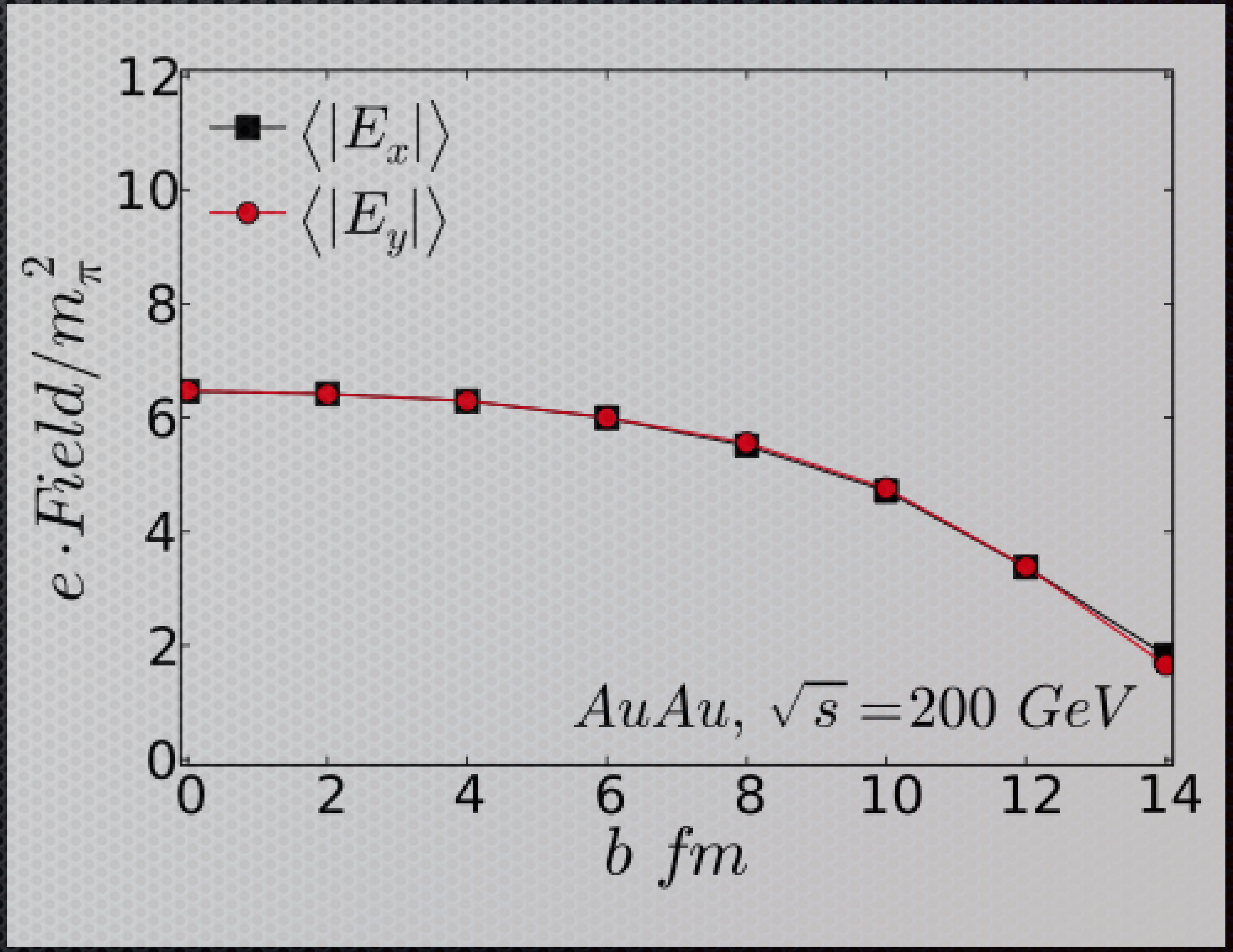


$$eE \simeq (m_\pi^2) \rightarrow E \sim 10^{21} V/cm$$





Y. Hirono, M. Hongo, T. Hirano, arXiv:1211.1114 (2012)



K. Tuchin, Adv.High Energy Phys. 2013 (2013) 490495

Direct flow

$$v_1 = \left\langle \frac{p_x}{p_T} \right\rangle$$

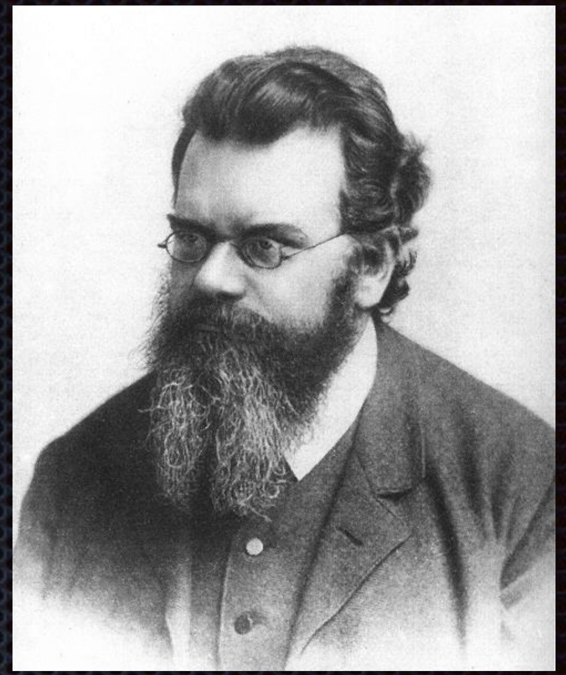
Photon Rate

$$\omega \frac{d\Gamma_\gamma}{d^3p} = \frac{\alpha_{EM}}{\pi^2 e^2} \frac{\sigma_{el}\omega}{e^{\omega/T} - 1}$$

Lattice QCD

$$G_{\mu\nu}(\tau, T) = \int d^3x \langle J_\mu(\tau, \mathbf{x}) J_\nu(0, \mathbf{0})^\dagger \rangle$$

Relativistic Transport Equation



$$p^\mu \partial_\mu f(x,p) = C[f]$$

$$\mathcal{C}_{22} = \frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int \frac{d^3p_1'}{(2\pi)^3 2E_1'}\frac{d^3p_2'}{(2\pi)^3 2E_2'} f_1' f_2' \times |\mathcal{M}_{1'2'\to 12}|^2 (2\pi)^4 \delta^{(4)}(p_1' + p_2' - p_1 - p_2)$$

$$-\frac{1}{2E_1}\times\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int \frac{d^3p_1'}{(2\pi)^3 2E_1'}\frac{d^3p_2'}{(2\pi)^3 2E_2'} f_1 f_2 \times |\mathcal{M}_{12\to 1'2'}|^2 (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_1' - p_2')$$

Test-particle method

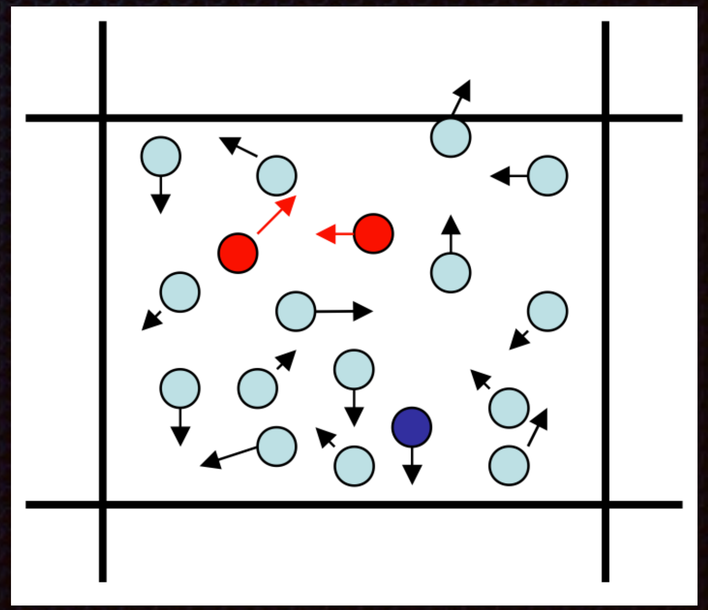
$$f(x,p) = \sum_{i=1}^{N} \delta^4(x_i(t) - x)\delta^4(p_i(t) - p)$$

$$N = N_{real} \times N_{test} \quad , \quad \sigma \to \sigma / N_{test}$$

Stochastic method

$$P_{22} = \frac{\Delta N_{coll}^{2\to 2}}{\Delta N_1 \Delta N_2} = v_{rel}\frac{\sigma_{22}}{N_{test}}\frac{\Delta t}{\Delta^3 x}$$

Z. Xu and C. Greiner, Phys.Rev. C71 (2005) 064901



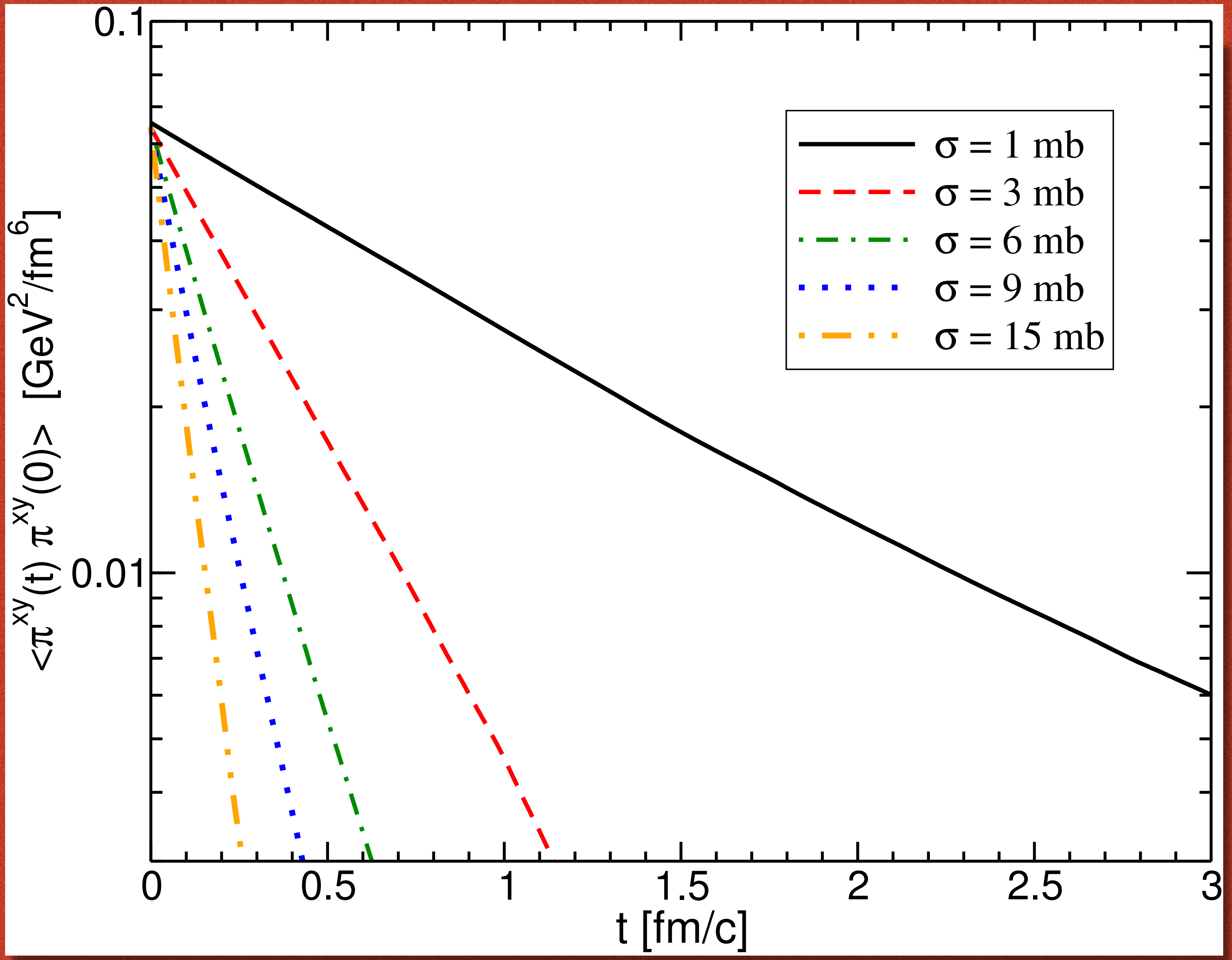
Green-Kubo Relations

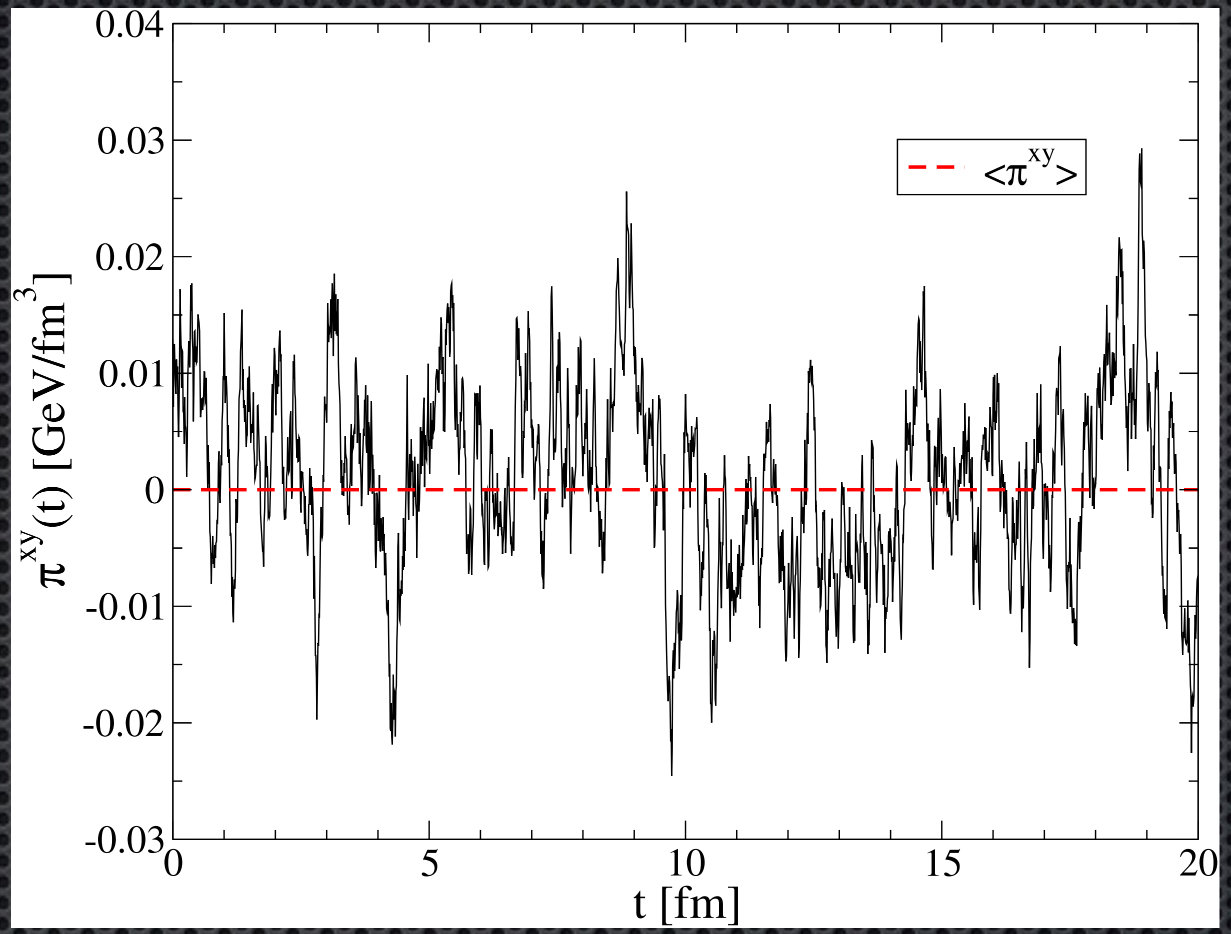
$$A = \frac{V}{T}\int_0^\infty dt\,\langle J(t)J(0)\rangle$$

$$\eta = \frac{V}{T}\int_0^\infty dt\,\langle \pi^{xy}(t)\pi^{xy}(0)\rangle = \frac{V}{T}\langle \pi^{xy}(0)^2\rangle\tau$$

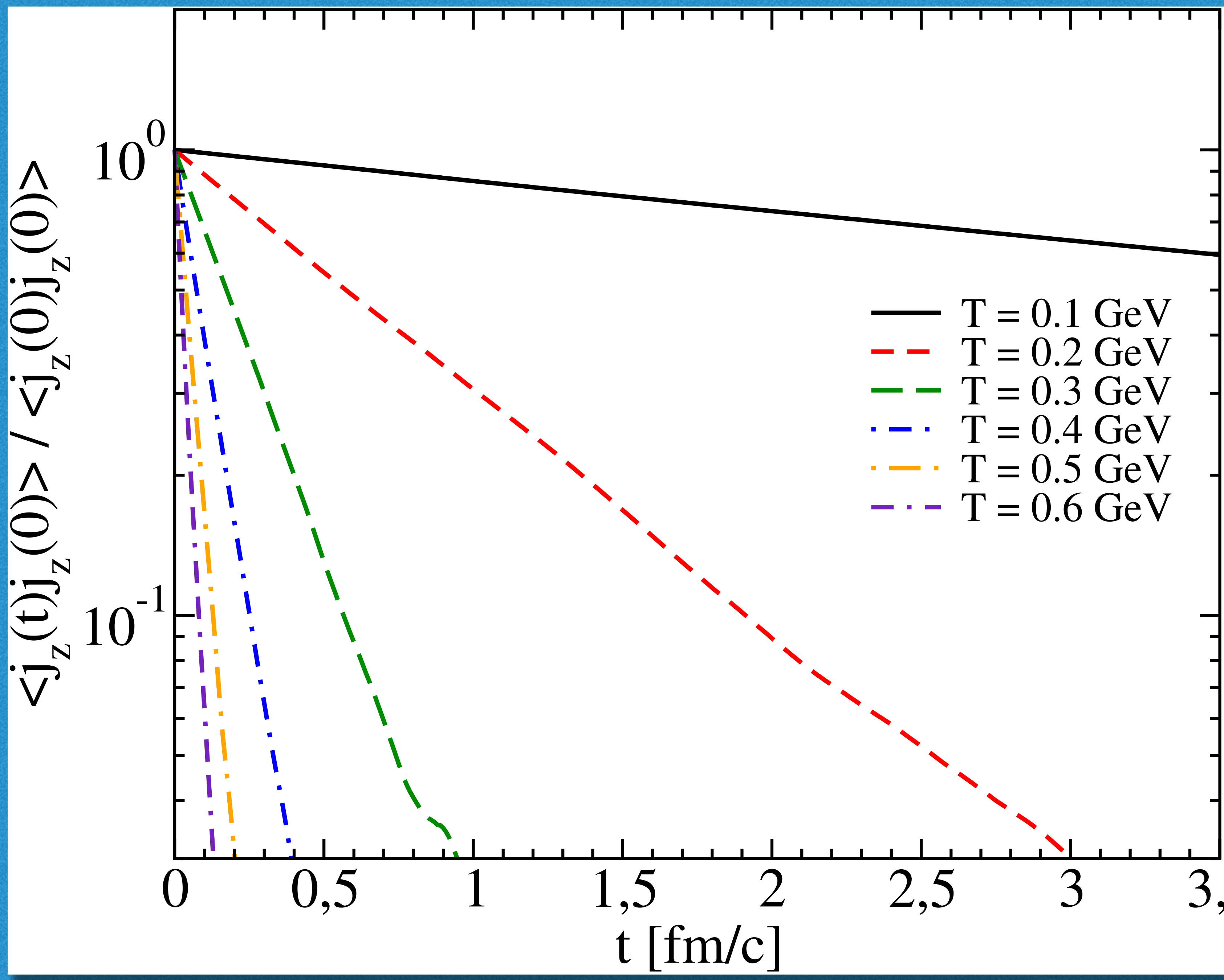
$$\sigma_{el} = \frac{V}{T}\int_0^\infty dt\,\langle J_z(t)J_z(0)\rangle = \frac{V}{T}\langle J_z(0)^2\rangle\tau$$

$$\eta \Rightarrow J = \pi^{xy} = -\eta\frac{\partial u_x}{\partial y}$$





$$\sigma_{el} \Rightarrow J = \sigma_{el}E$$



C. Wesp et al, Phys.Rev. C84 (2011) 054911

A. Puglisi et al., Phys.Rev. C86 (2012) 054902

A. Puglisi et al., PRD 90(2014)11,114009

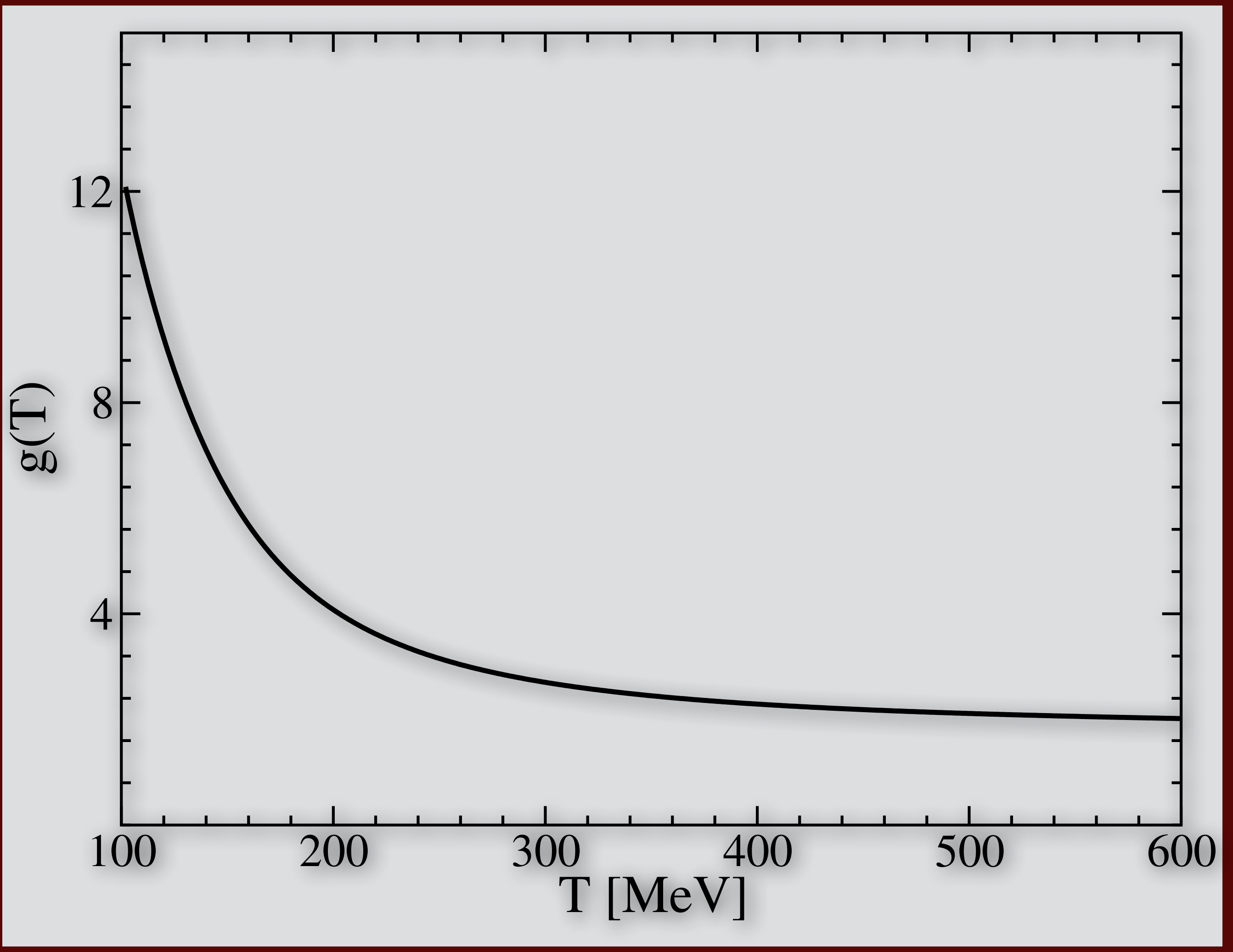
M. Greif et al., PRD 90(2014)9,094014

Fixing the Thermodynamics

$$\eta/s = \frac{1}{15\,T\,s}\left(\sum_q \int \frac{d^3p}{(2\pi)^3}\frac{p^4}{E^2}\tau_q f(p) + \int \frac{d^3p}{(2\pi)^3}\frac{p^4}{E^2}\tau_g f(p)\right) = \underbrace{\frac{1}{15Ts}\left\langle \frac{p^4}{E^2}\right\rangle}_{\text{Thermodynamics}} \overbrace{\left(\tau_q \rho_q^{\text{tot}} + \tau_g \rho_g\right)}^{\text{Dynamics}}$$

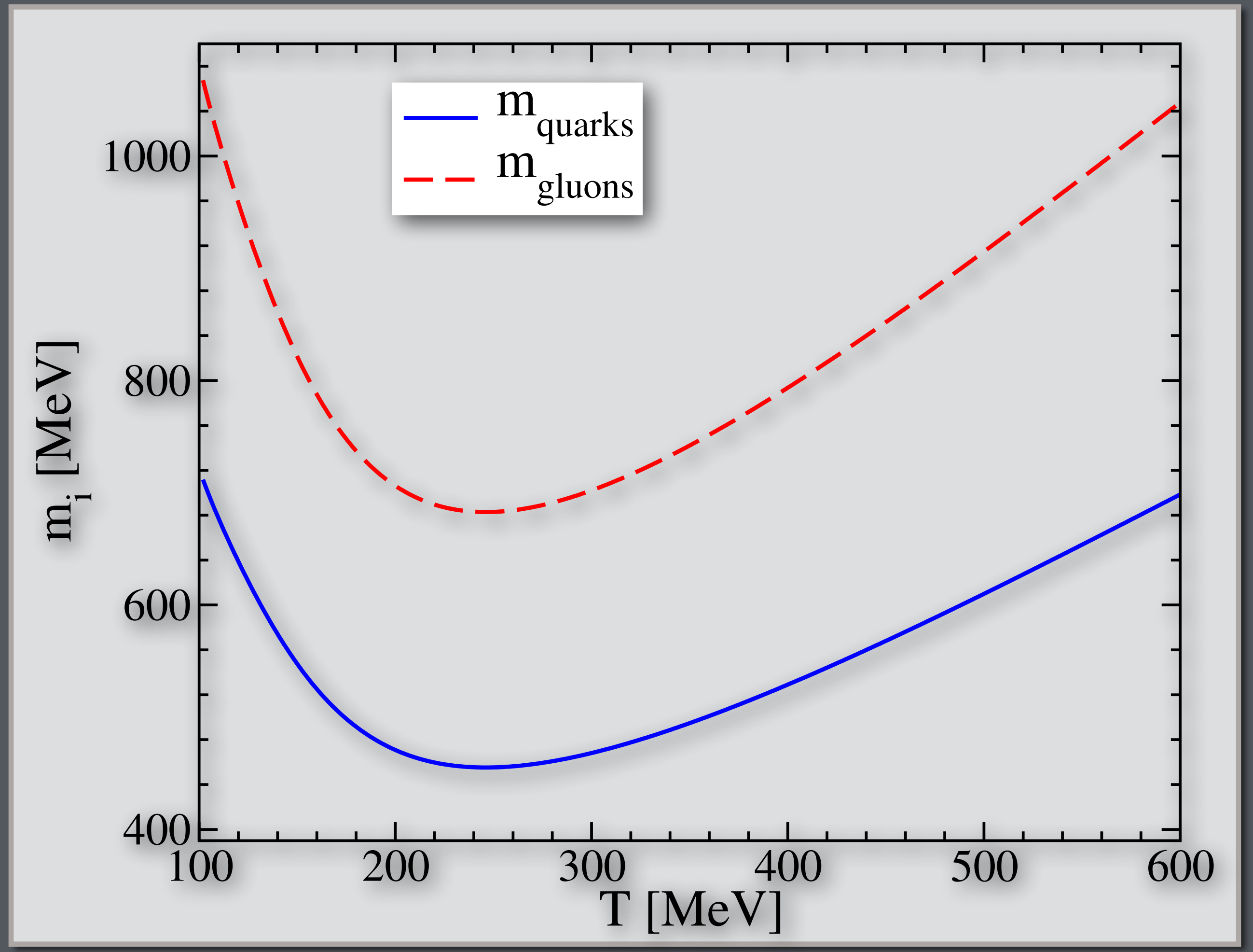
$$\frac{\sigma_{el}}{T} = \frac{e^2}{3\,T^2}\sum_{j=q,\bar{q}} q_j^2 \int \frac{d^3p}{(2\pi)^3}\frac{\vec{p}^2}{E^2}\tau_i f(p) = \underbrace{\frac{e_\star^2}{3T^2}\left\langle \frac{p^2}{E^2}\right\rangle}_{\text{Thermodynamics}} \overbrace{\tau_q}^{\text{Dynamics}} \rho_q$$

$$g^2(T) = \frac{48\pi^2}{(11N_c - 2N_f)\log\left[\lambda\left(\frac{T}{T_c} - \frac{T}{T_s}\right)\right]^2}$$



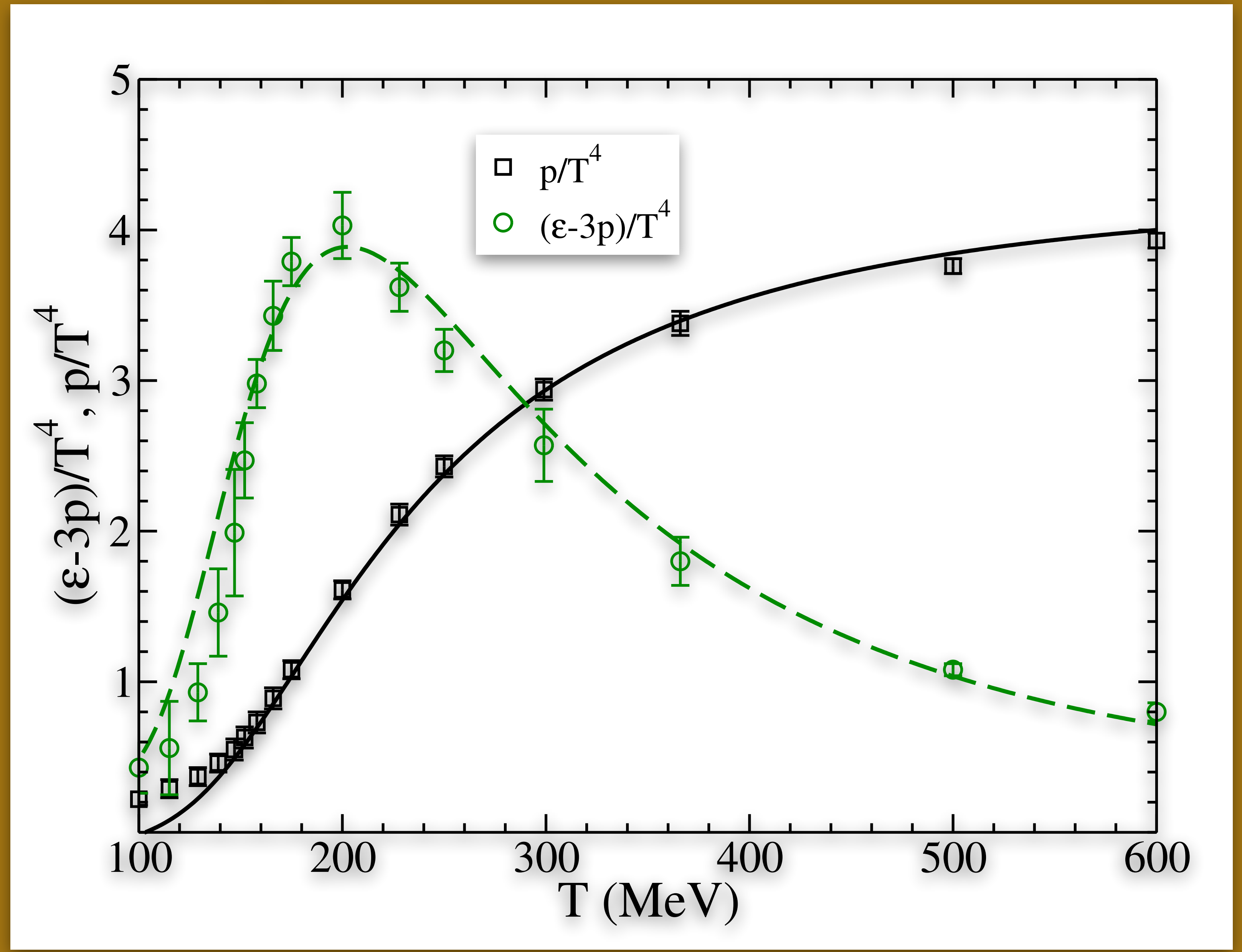
$$m_q^2(T) = \frac{N_c^2 - 1}{8N_c} g^2 T^2$$

$$m_g^2(T) = \frac{1}{6} g^2 (N_c + \frac{1}{2} N_f) T^2$$



$$P(T) = \sum_{i=q,g} d_i \int \frac{d^3p}{(2\pi)^3}\frac{\vec{p}^2}{3E_i(p)} f_i(p) - B(T)$$

$$\epsilon(T) = \sum_i d_i \int \frac{d^3p}{(2\pi)^3} E_i f(p) + B(m_i(T))$$



Thermodynamics from Lattice QCD, S. Plumari et al. PRD 84, 094004 (2011)

Fixing the dynamics: Shear Viscosity

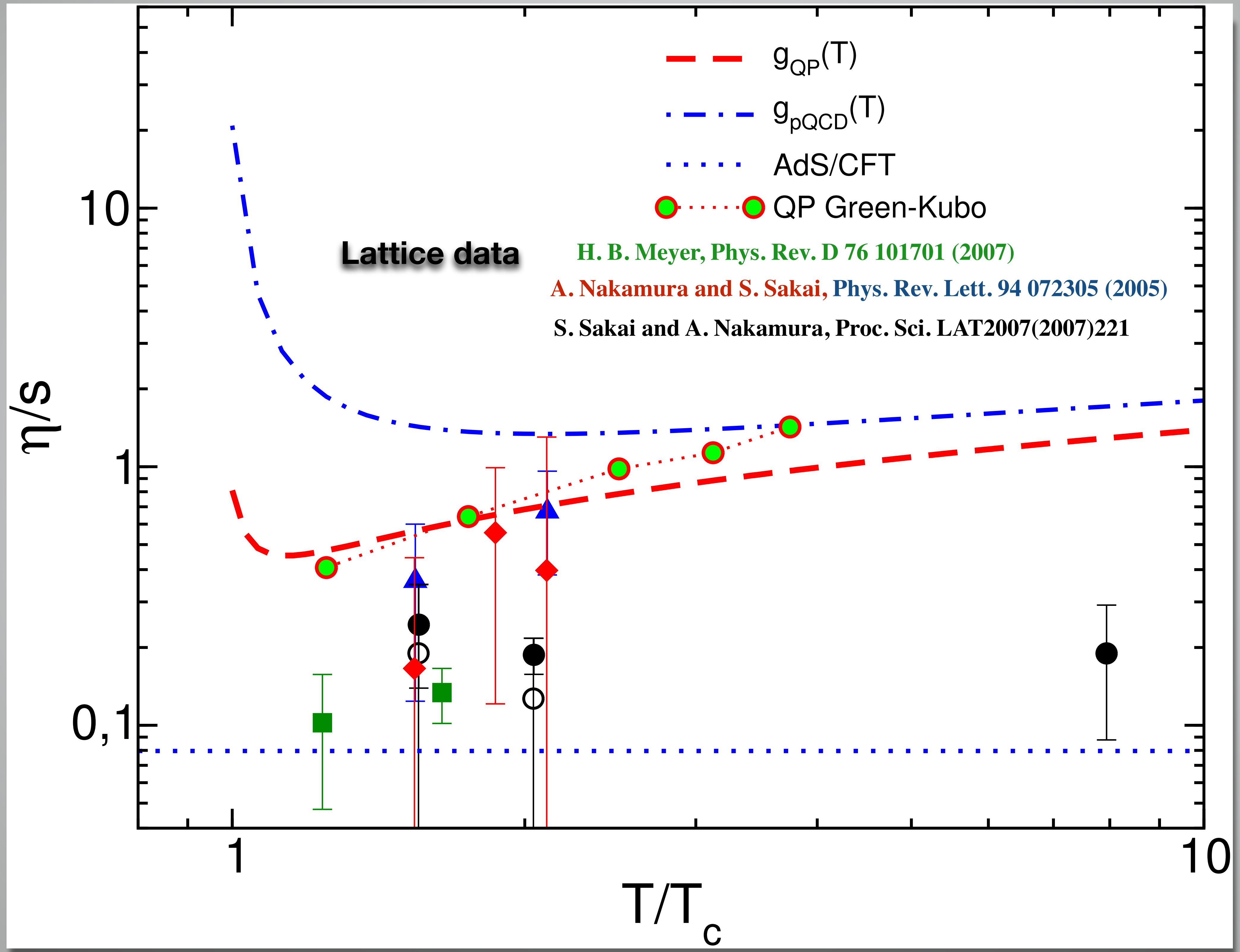
$$\tau_{q,tr}^{-1} = \langle \sigma(s)_{tr} v_{rel} \rangle \left(\rho_q \sum_{i=u,d,s}^{\bar{u},\bar{d},\bar{s}} \beta^{qi} + \rho_g \beta^{qg} \right)$$

$$\tau_{g,tr}^{-1} = \langle \sigma(s)_{tr} v_{rel} \rangle \left(\rho_q^{tot} \beta^{qg} + \rho_g \beta^{gg} \right)$$

$$\tau \sim \frac{1}{\rho\sigma}$$

$$\sigma_{tot}^{ij}(s) \sim \beta^{ij} \frac{\alpha_s^2}{m_D^2(T)} \frac{s}{(s + m_D^2(T))}$$

ij	β
$qq \to qq$	$2\frac{8\pi}{9}$
$q\bar{q} \to q\bar{q}$	$\frac{8\pi}{9}$
$qg \to qg$	2π
$gg \to gg$	9π



- ✦Green-Kubo vs. RTA
- ✦η/s predicted ≃5/4π (red dashed line)
- ✦symbols: Lattice QCD data [2]
- ✦upscaling the coupling by k-factor in order to reproduce η/s≃1/4π
- ✦What does happen to σ_{el} using the same relaxation times?

$$g_{QP}^2(T) = \frac{48\pi^2}{(11N_c - 2N_f)\log\left[\lambda\left(\frac{T}{T_c} - \frac{T}{T_s}\right)\right]^2}$$

$$g_{pQCD}^2 = \frac{16\pi^2}{\left(11 - \frac{2}{3}N_f\right)\log\left[\frac{2\pi T}{\Lambda_{QCD}}\right]^2}$$

Fixing the dynamics: Electric Conductivity

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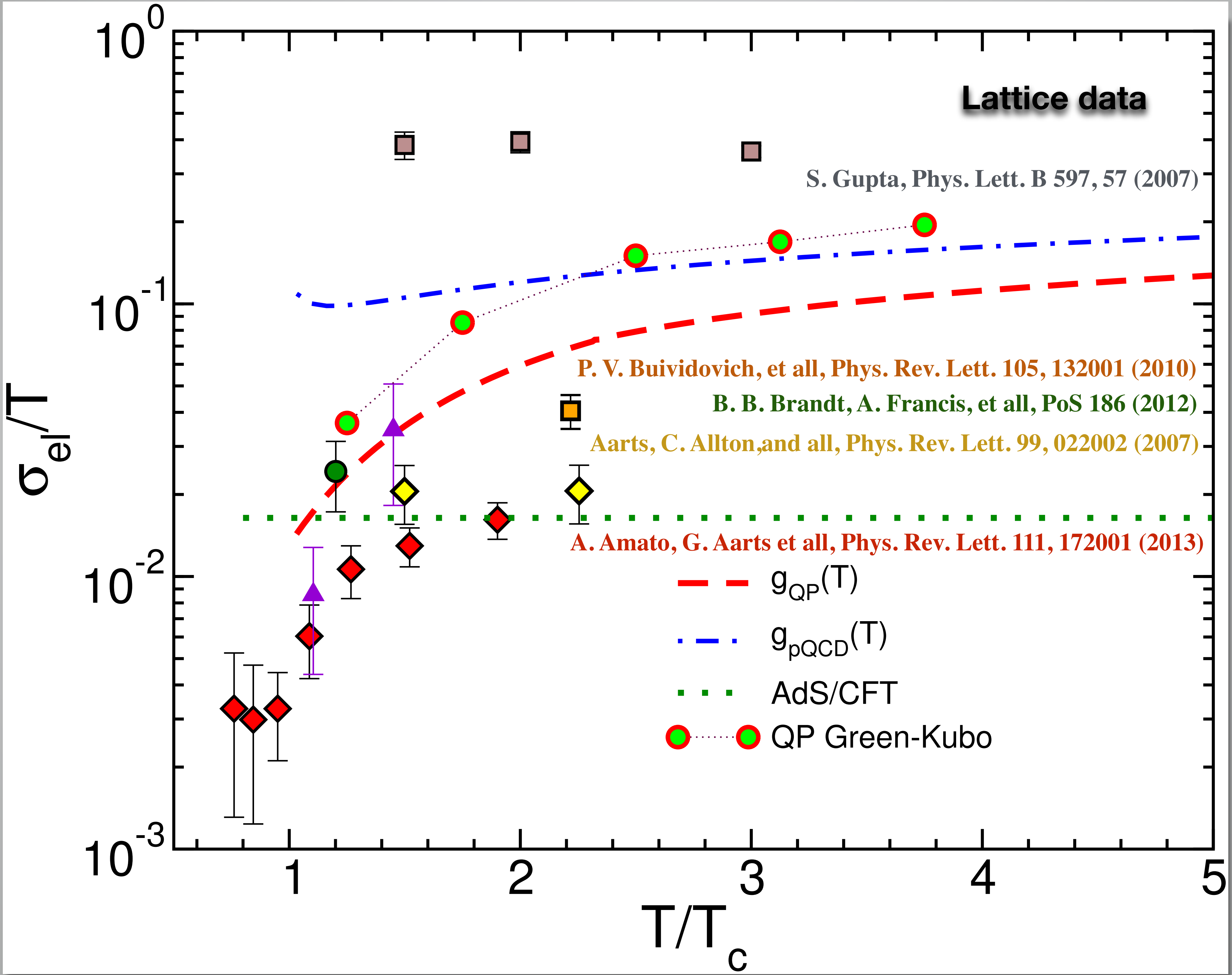

H.-T Ding, A. Francis, et all, PoS 185 (2012)

- extra T-dependence:

  $\sigma_{el}/T \simeq T/m(T)\, \eta/s$

- ε-3P>0 as the origin of extra T-dependence
- conformal theory: $\sigma_{el}/T \simeq \eta/s$

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Rescaling the dynamics

$$\sigma_{tot}^{ij}(s) \sim K \beta^{ij} \frac{\alpha_s^2}{m_D^2(T)} \frac{s}{(s + m_D^2(T))}$$

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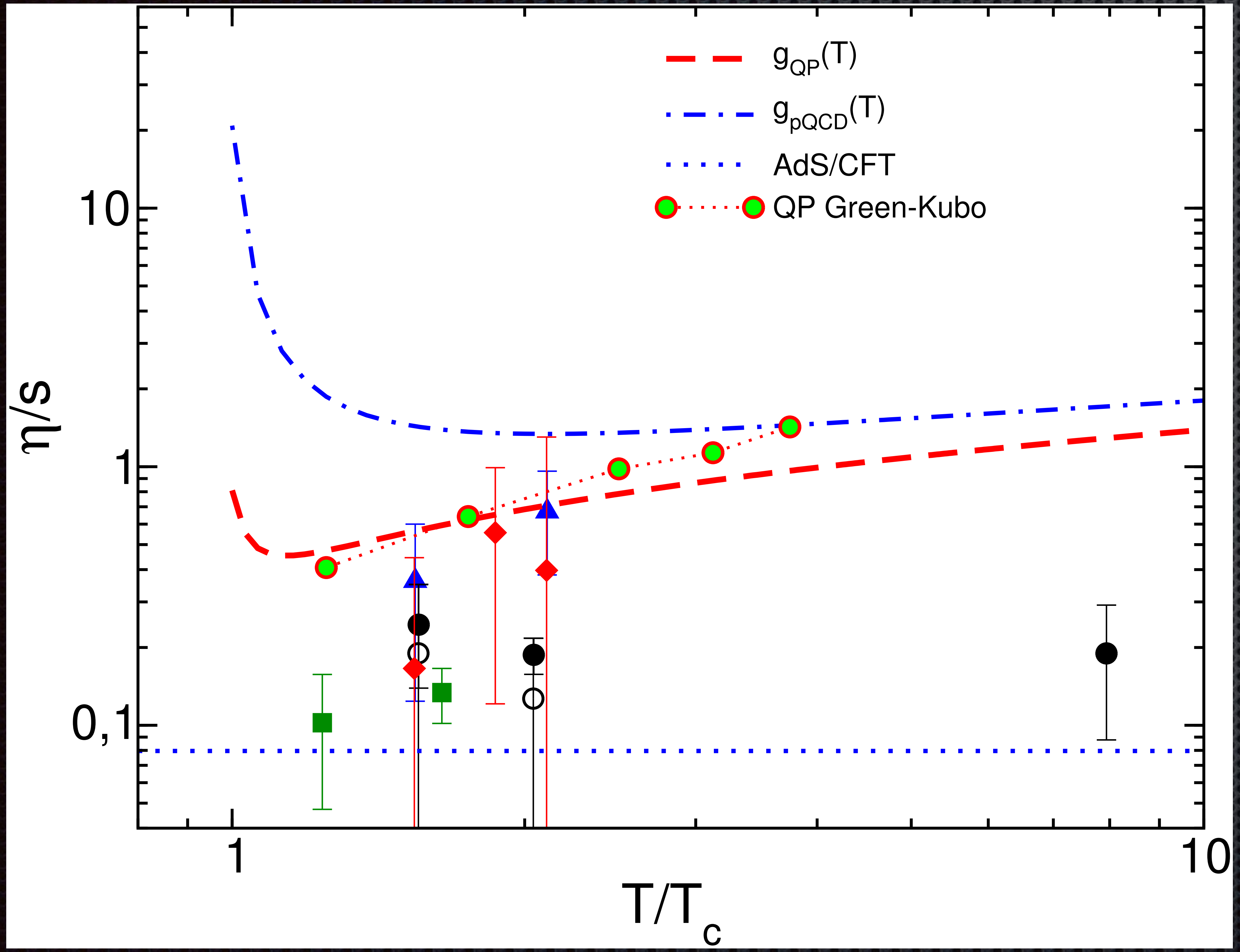

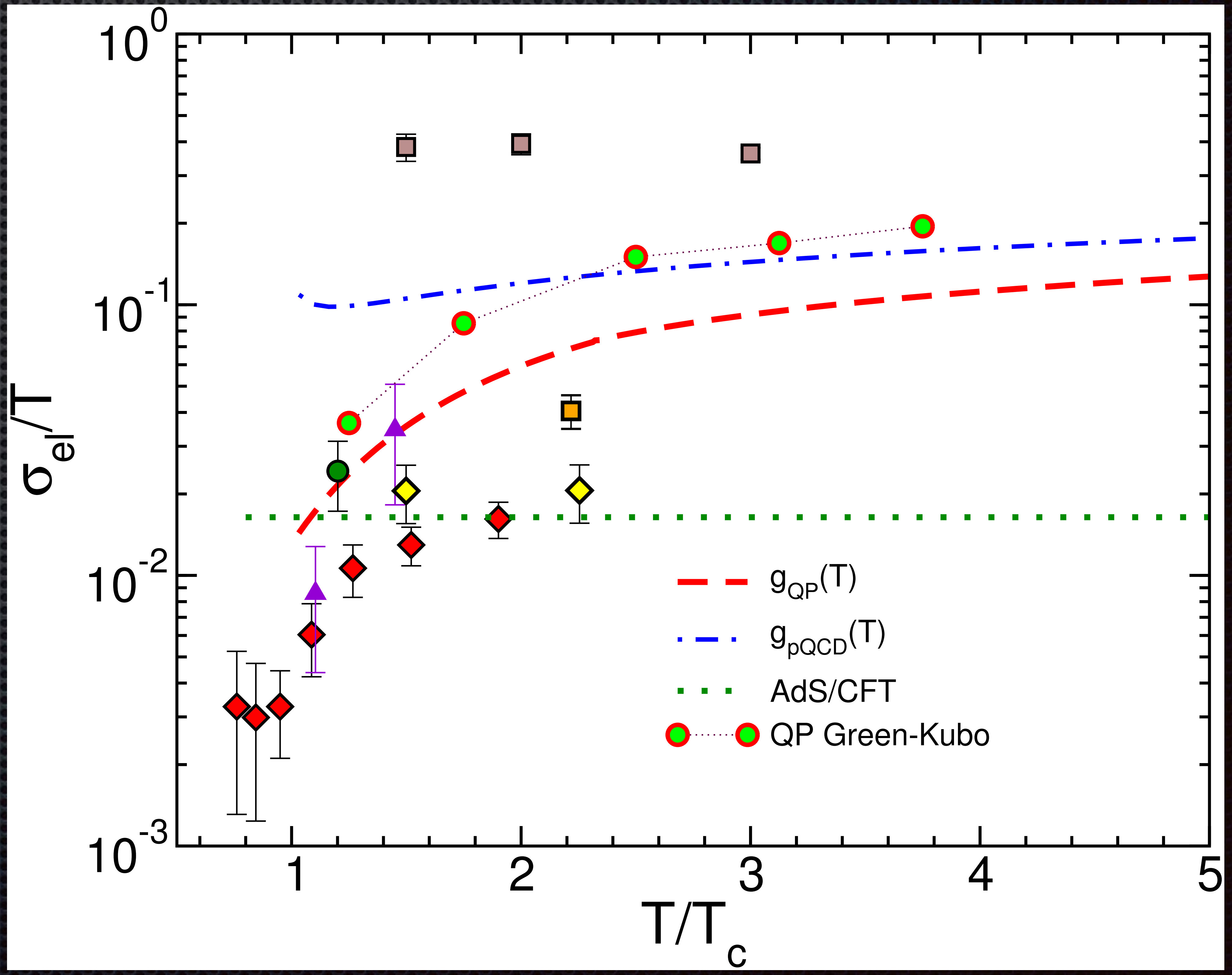

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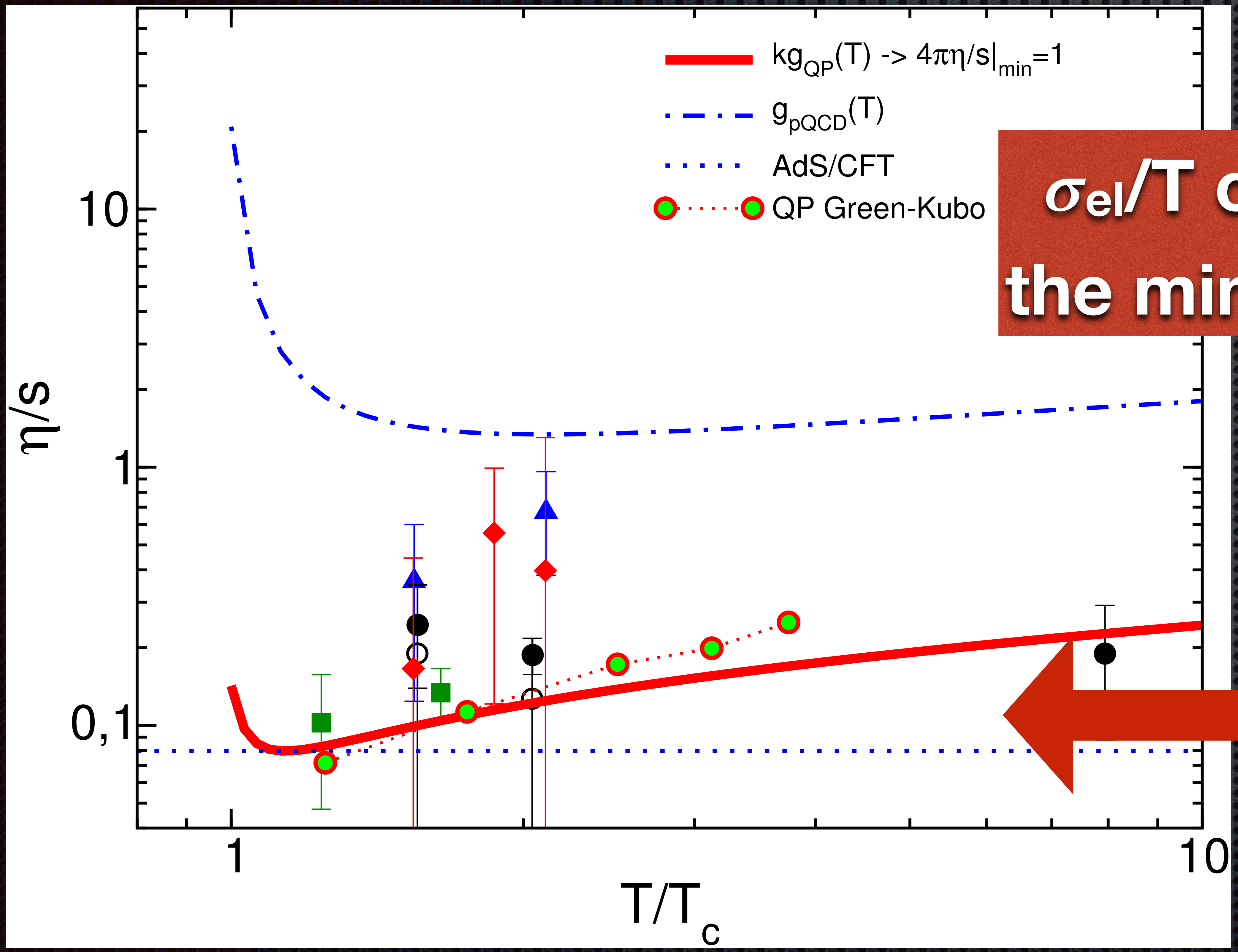
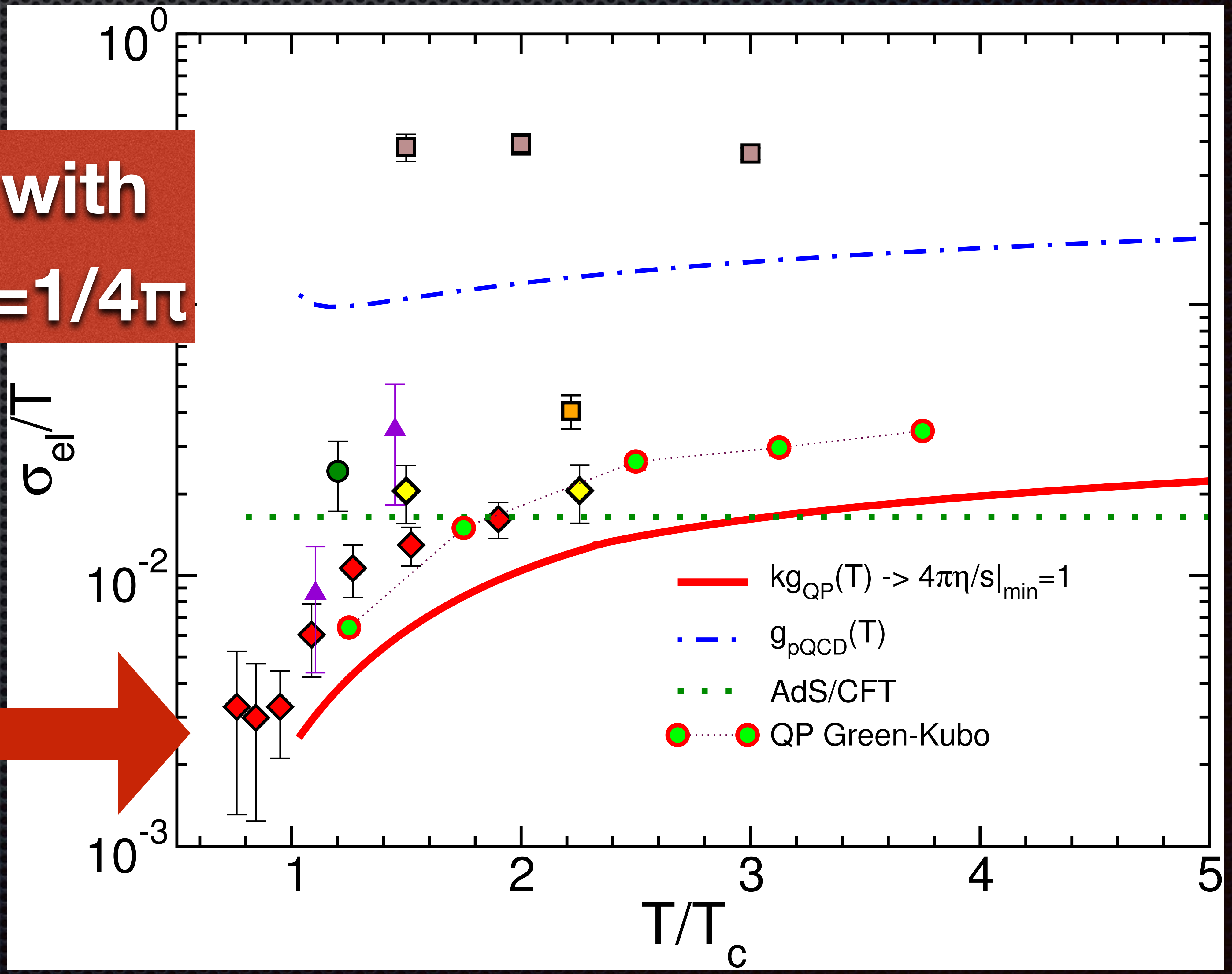


σ_{el}/T consistent with the minimum η/s=1/4π



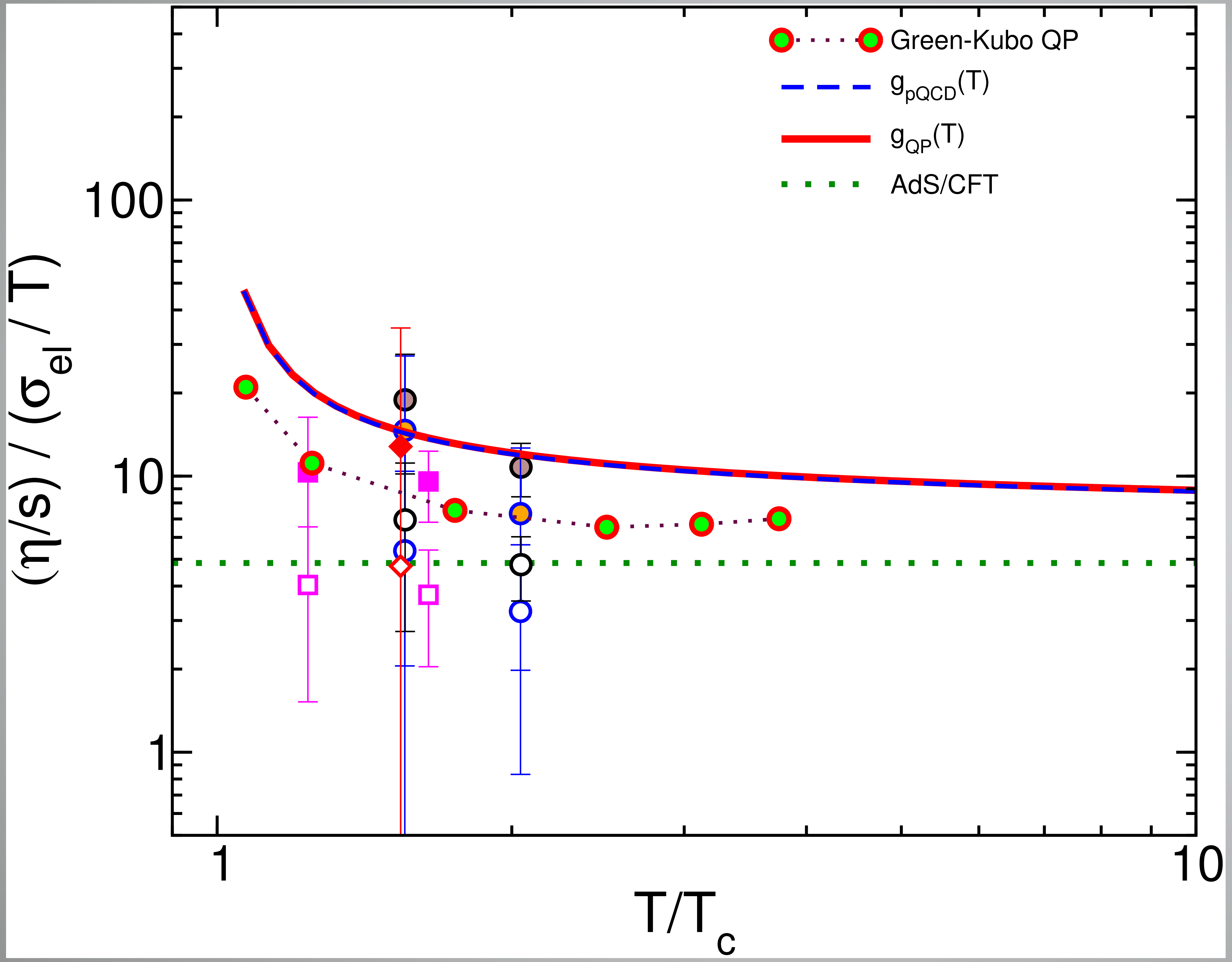
Taking the ratio

$$\frac{\eta/s}{\sigma_{el}/T} = \frac{6}{5}\frac{T\langle p^2/E^2\rangle^{-1}}{se_\star^2}\left\langle\frac{p^4}{E^2}\right\rangle\left(1+\frac{\tau_g}{\tau_q}\frac{\rho_g}{\rho_q^{tot}}\right), \quad \frac{\tau_g}{\tau_q} = \frac{C^q + \frac{\rho_g}{\rho_q}}{6 + C^g\frac{\rho_g}{\rho_q}}$$

$$C^q = (\beta^{qq} + \beta^{q\bar{q}} + 2\beta^{q\bar{q}'} + 2\beta^{qq'})/\beta^{qg} \qquad C^q|_{pQCD} = \frac{28}{9} \simeq 3.1$$

$$C^g = \beta^{gg}/\beta^{qg} \qquad C^g|_{pQCD} = \frac{9}{2}$$

A. Puglisi et al. arxiv:1407.2559



- ★ Independent of k-factor and $\alpha_s(T)$
- ★ Sensitive only on C^q
- ★ Increases near T_c
- ★ Constant value for $T \gg T_c$
- ★ Conformal Theory prediction: flat behavior
- ★ σ_{el}/T extra T-dependence
- ★ Understanding the relative role of quarks and gluons in the QGP
- ★ Lattice results interpretation

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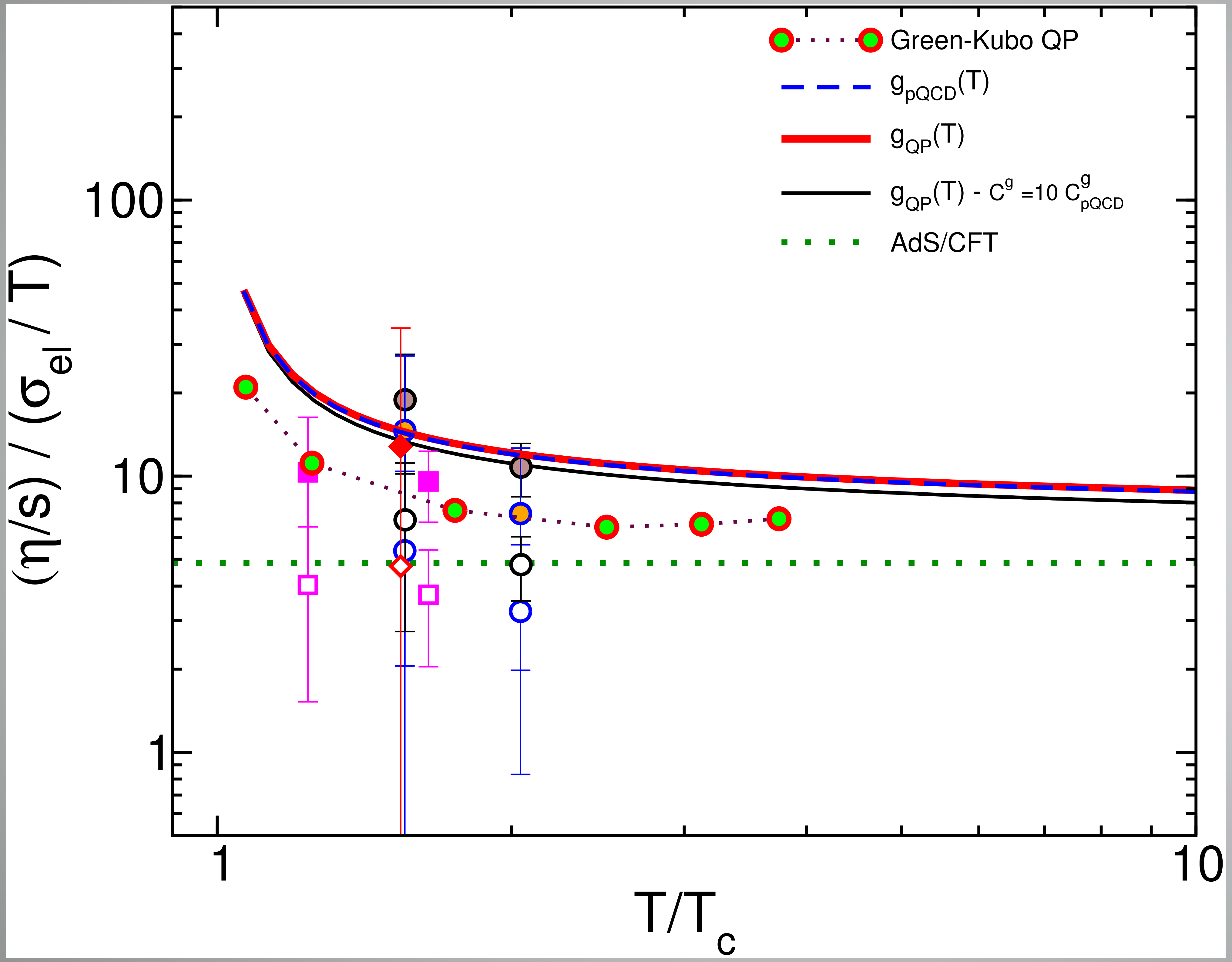

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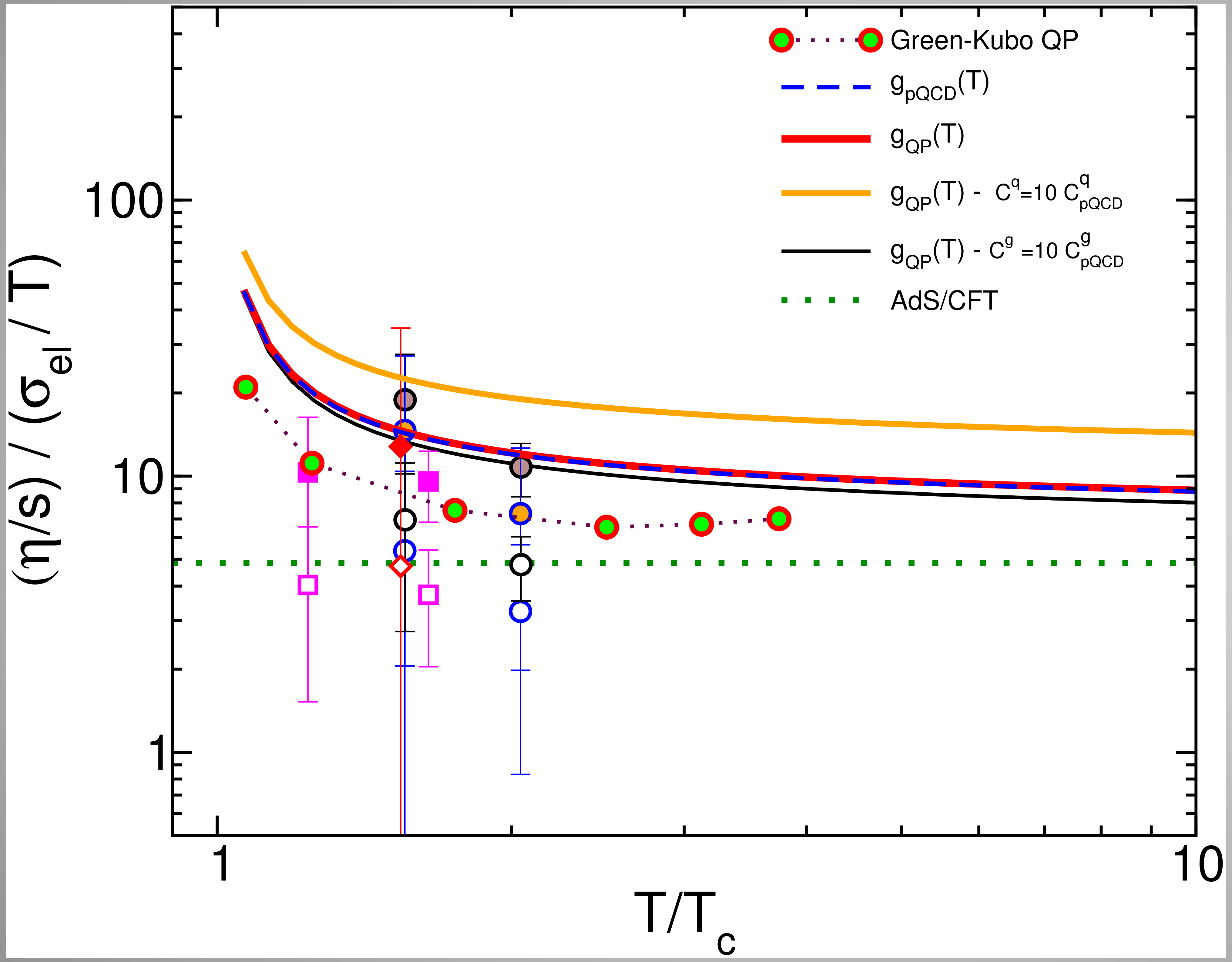

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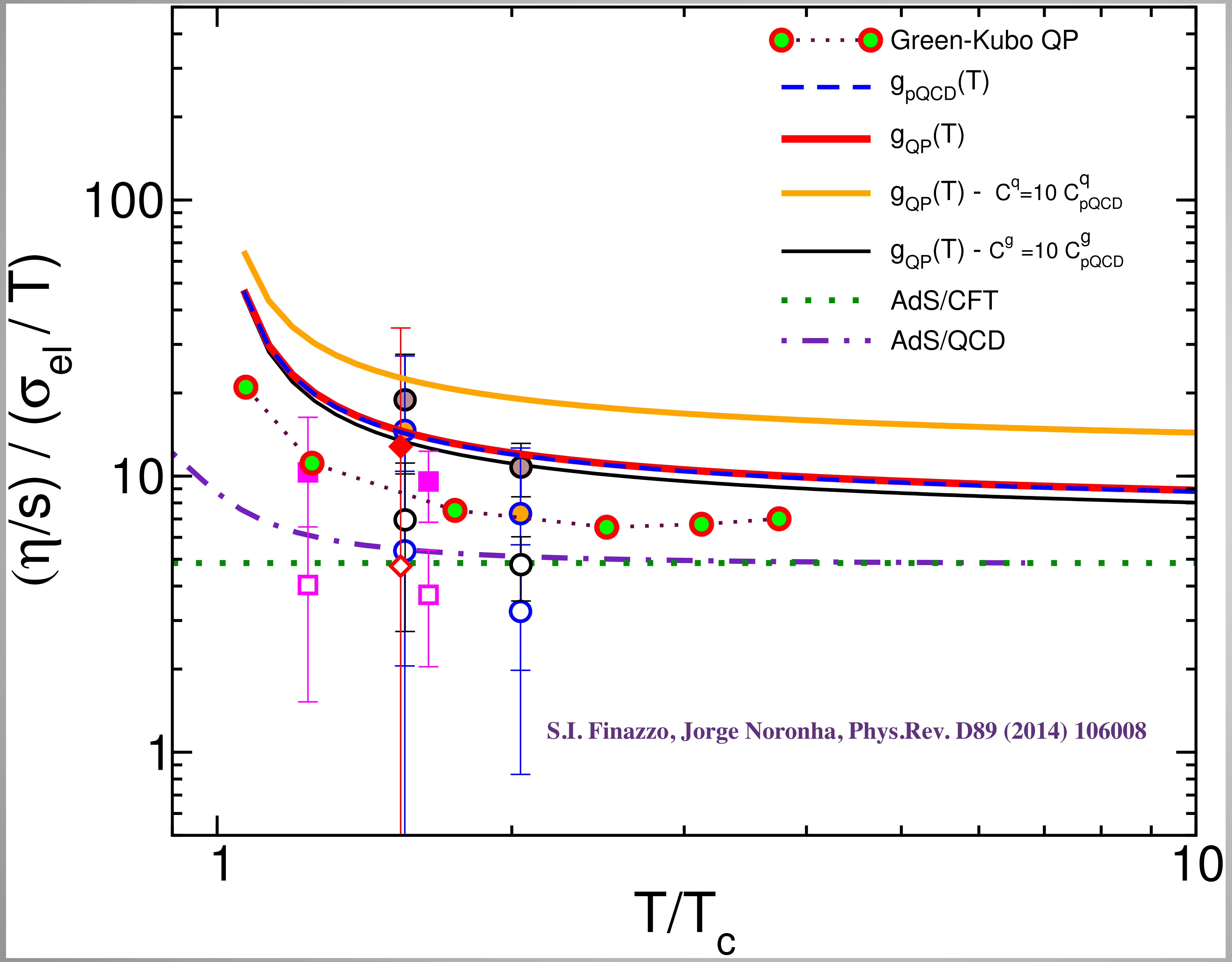

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Conclusions

Shear Viscosity

- η/s predicted $\simeq 5/4\pi$
- upscaling the coupling by k-factor in order to reproduce $\eta/s \simeq 1/4\pi$
- What does happen to σ_{el} using the same relaxation times?

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- σ_{el}/T consistent with the minimum $\eta/s = 1/4\pi$
- extra T-dependence
- $\varepsilon - 3P > 0$ as the origin of extra T-dependence
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