Jet evolution in a dense QCD medium

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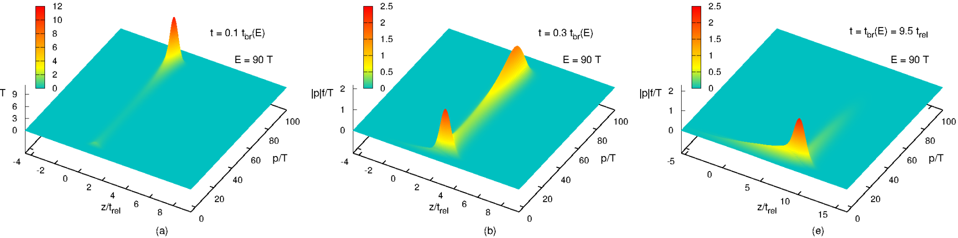
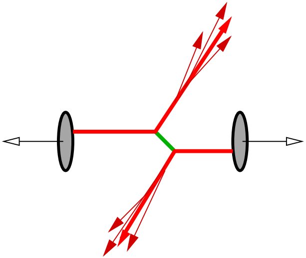
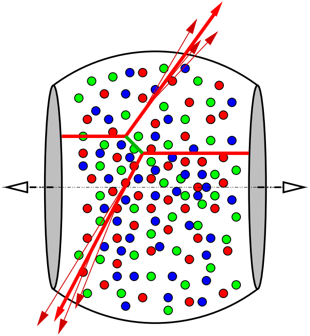
recent work with J.-P. Blaizot, F. Dominguez, Y. Mehtar-Tani, B. Wu



Jet quenching in heavy ion collisions

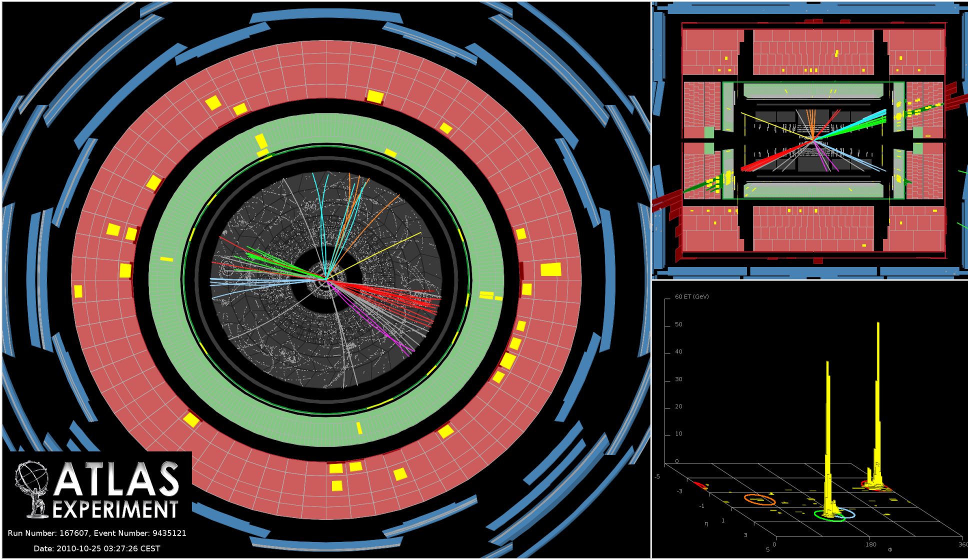
- Hard processes in QCD typically create pairs of partons which propagate back–to–back in the transverse plane
- In the vacuum (pp collision), this leads to a pair of symmetric jets
- In a dense medium (AA collision), the two jets can be differently affected by their interactions with the medium: 'di-jet asymmetry'



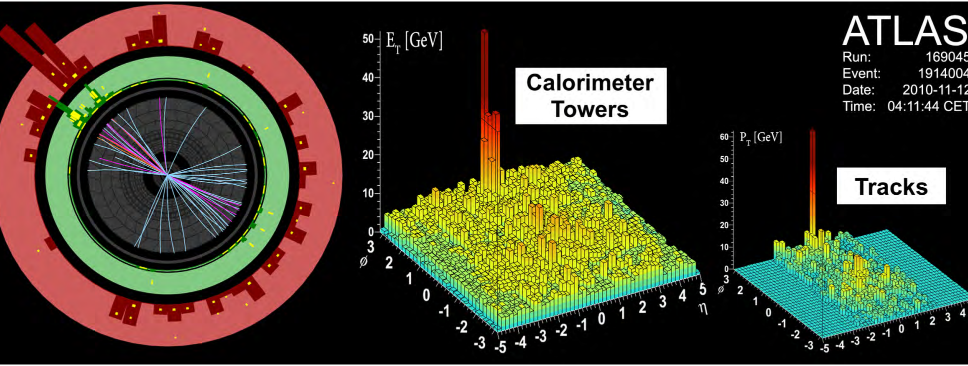


- The ensemble of medium-induced modifications: 'jet quenching'

Di–jets in p+p collisions at the LHC

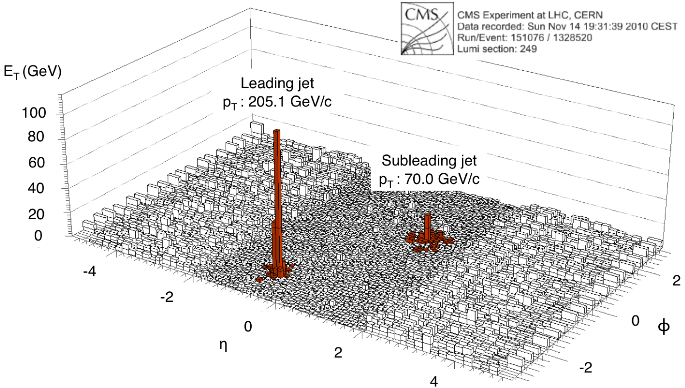


Di–jet asymmetry *(ATLAS)*



- Central Pb+Pb: 'mono–jet' events
- The secondary jet cannot be distinguished from the background: $E_{T1} \geq 100$ GeV, $E_{T2} > 25$ GeV

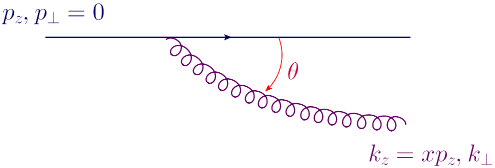
Di–jet asymmetry at the LHC *(CMS)*



- Additional energy imbalance as compared to p+p : 20 to 30 GeV
- Compare to the typical scale in the medium: $T \sim 1$ GeV (average $p_\perp$)
- A remarkable pattern for the energy loss:
  many soft ($p_\perp < 2$ GeV) hadrons propagating at large angles

A challenge for the theorists

- Can one understand these data from first principles (QCD) ?
  - how gets the energy transmitted from the leading particle to these many soft quanta ?
  - do these soft quanta thermalize ? is the medium locally heated ?
  - is all that consistent with weak coupling ?
- Very different from the branching pattern for a jet in the vacuum



- bremsstrahlung

$$\mathrm{d}\mathcal{P} = \frac{\alpha_s C_R}{\pi} \frac{\mathrm{d}x}{x} \frac{\mathrm{d}k_\perp^2}{k_\perp^2}$$

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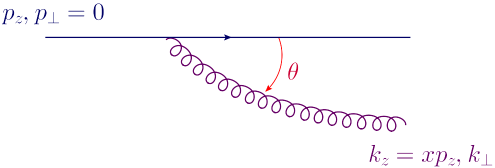

- bremsstrahlung

$$\mathrm{d}\mathcal{P} = \frac{\alpha_s C_R}{\pi}\,\frac{\mathrm{d}x}{x}\,\frac{\mathrm{d}\theta^2}{\theta^2}$$

- soft & collinear splittings

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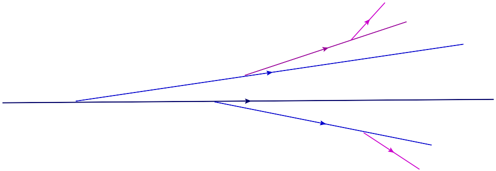
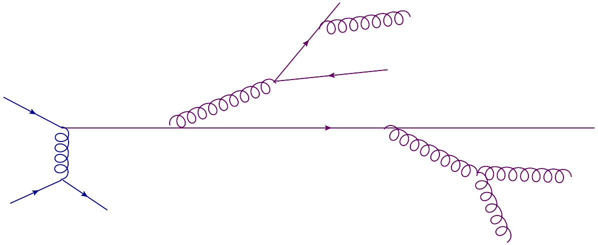


- multiple branching: DGLAP
- angular ordering (destructive interferences)
- energy remains within a narrow jet
- most energy carried by a few partons with large x

Jet evolution

- The leading particle (LP) is produced by a hard scattering
- It subsequently evolves via radiation (branchings) ...



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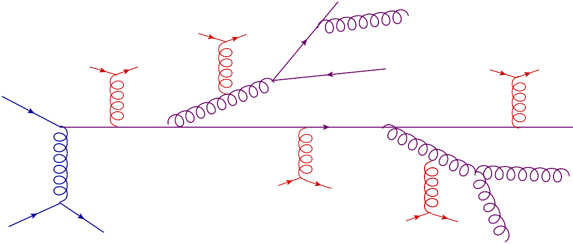

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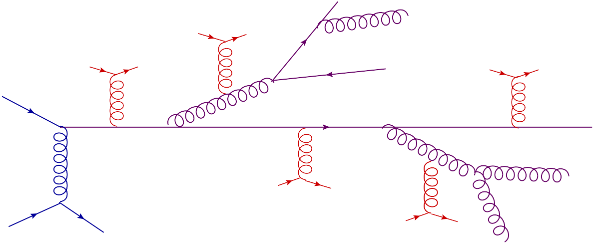
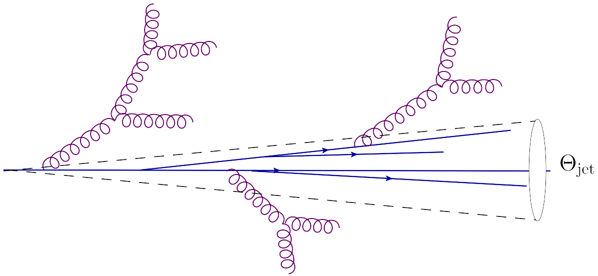


- ... and via collisions off the medium constituents
- A priori, two mechanisms for radiation:
  - triggered by the virtuality of the LP ('vacuum-like')
  - triggered by collisions ('medium-induced')
- So far, no unified description of both mechanisms (different approxs.)

Medium–induced jet evolution

- The two types of radiation are geometrically separated
  - vacuum–like: small emission angles
  - medium–induced: large angles
- The di-jet asymmetry is controlled by medium-induced radiation



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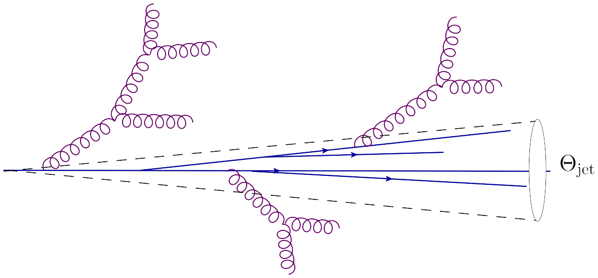


- From now on: focus on the evolution which is triggered by collisions
  - elastic collisions $2 \to 2$: energy-momentum transfer
  - inelastic collisions $2 \to 3$: medium-induced branchings

Kinetic theory for medium-induced jet evolution

- At weak coupling, this can be encoded into a kinetic equation :

  Baier, Mueller, Schiff, Son '01 ('bottom-up'); Arnold, Moore, Yaffe, '03

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\nabla_{\boldsymbol{x}}\right) f(t,\boldsymbol{x},\boldsymbol{p}) \,=\, \mathcal{C}_{\rm el}[f] + \mathcal{C}_{\rm br}[f]$$

  - $f(t,\boldsymbol{x},\boldsymbol{p})$ gluon phase–space occupation number
  - gluons dominate the cascade at high energies

- Rather natural for the elastic collisions: large mean free path

$$\lambda_{\rm m.f.p.} \sim \frac{1}{\alpha_s^2 T \ln(1/\alpha_s)} \gg t_{\rm coll} \sim \frac{1}{gT}$$

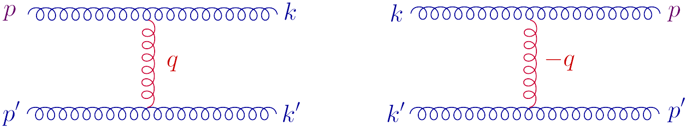
- Less obvious (but correct) for the branchings: Markovian process

  - successive emissions could interfere with each other
  - color coherence between daughter gluons is washed out by rescattering

  Blaizot, Dominguez, E.I., Mehtar-Tani (2012–13)

Elastic collisions

- The usual Boltzmann collision term adapted to QCD



- 'Gain' and 'loss' terms: the gluon p is the one which is measured

$$\mathcal{C}_{\rm el}[f] \,=\, \int_{\boldsymbol{p}',\boldsymbol{k},\boldsymbol{k}'} \frac{|\mathcal{M}|^2}{(2p)(2p')(2k)(2k')}\,\Phi[f]$$

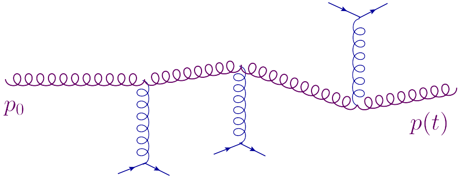
$$-\Phi[f] = f_{\boldsymbol{p}} f_{\boldsymbol{p}'}[1+f_{\boldsymbol{k}}][1+f_{\boldsymbol{k}'}] - f_{\boldsymbol{k}} f_{\boldsymbol{k}'}[1+f_{\boldsymbol{p}}][1+f_{\boldsymbol{p}'}]$$

- 5 conserved quantities: particle number and the 4–momentum
- Detailed balance $\Longrightarrow$ local thermal equilibrium: $\mathcal{C}_{\rm el}[f_{\rm loc}] = 0$

$$f_{\rm eq}(p) = \frac{1}{{\rm e}^{\beta(p-\mu)} - 1} \,\rightarrow\, f_{\rm loc}(x,\boldsymbol{p}) = \frac{1}{{\rm e}^{\beta(x)[p-\boldsymbol{p}\cdot\boldsymbol{u}(x)-\mu(x)]} - 1}$$

Test particles (elastic collisions only)

- The jet particles are relatively hard to start with: $p_z \gg T$
- Small angle scattering $\Longrightarrow$ Langevin (or Fokker-Planck) dynamics
- Energy loss ('drag') & momentum broadening ('diffusion')



$$\mathcal{C}_{\rm el}[f] \simeq \nabla_{\boldsymbol{p}} \cdot \left[\frac{\hat{q}}{4} \left(\nabla_{\boldsymbol{p}} + \eta \boldsymbol{v} \right) f \right]$$

- Jet quenching parameter: squared momentum transfer per unit time

$$\hat{q} \simeq \frac{m_D^2}{\lambda} \sim \alpha_s^2 T^3 \ln \frac{1}{\alpha_s}$$

- The medium is in thermal equilibrium $\Rightarrow$ Einstein relation: $\hat{q} = 4T\eta$
- Eventually, a test particle thermalizes: $f_p \propto {\rm e}^{-p/T}$

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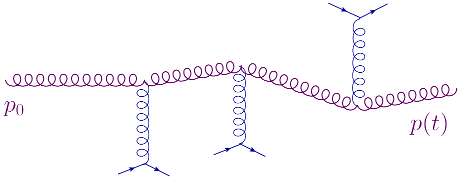

$$\langle p_z(t) \rangle \simeq p_0 - \eta t$$

$$\langle \Delta p^2(t) \rangle \simeq \frac{3}{2}\,\hat{q} t$$

- first, it loses most of its energy via drag, over a time $t_{\rm drag}(p_0)$

$$t_{\rm drag}(p_0) \simeq \frac{p_0}{\eta} = \frac{4 p_0 T}{\hat{q}}$$

- then, it approaches a thermal distribution, over an additional time $t_{\rm rel}$

$$t_{\rm rel} \equiv \frac{4T^2}{\hat{q}} \sim \frac{1}{\alpha_s^2 T \ln(1/\alpha_s)}$$

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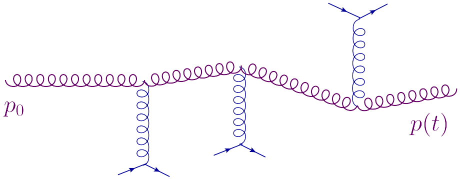

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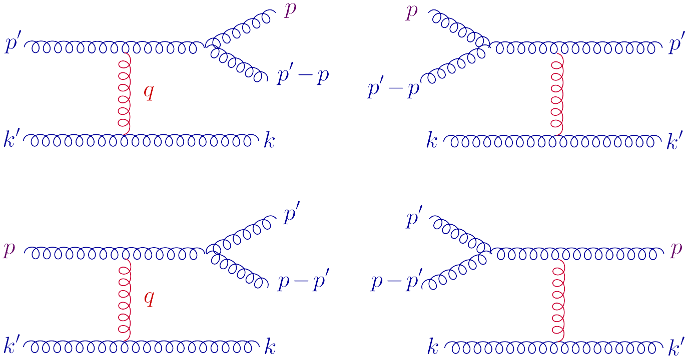
- then, it approaches a thermal distribution, over an additional time $t_{\rm rel}$

- Overall duration controlled by the 1st stage: collisional energy loss

Inelastic collisions: Medium-induced branchings

- The prototype: $2 \to 3$ (single scattering)



- 'Gain' - recombination; 'loss' - recombination
- Particle number is obviously not conserved
- Fixed point: zero chemical potential: $f_{\mathrm{loc}}(x, \boldsymbol{p}) = \frac{1}{\mathrm{e}^{\beta(x)[p - \boldsymbol{p}\cdot\boldsymbol{u}(x)]} - 1}$

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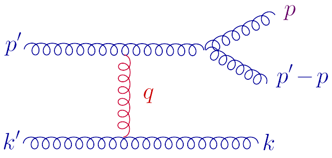


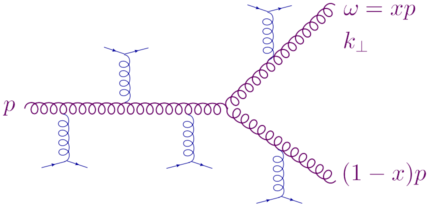
- Formally, suppressed by α_s compared to elastic collisions
- Actually, enhanced by the would-be mass-shell singularity of the intermediate gluon (in the limit of a soft momentum transfer $q \to 0$)
- Cured by resummation of multiple soft scattering (coherent emission)
  - Landau-Pomeranchuk-Migdal effect in QCD
  - dominated by small $q \Longrightarrow$ quasi-collinear splitting

'BDMPSZ' : Baier, Dokshitzer, Mueller, Peigné, Schiff; Zakharov (96-97)

Aurenche, Gelis, Moore, and Zaraket '02; Arnold, Moore, Yaffe, '03

Multiple scattering

- Important when formation time $t_{\rm f}$ is larger than mean free path λ
- $t_{\rm f}$: quantum mechanical duration of a gluon emission



$$t_{\rm f} \simeq \frac{1}{\Delta E} \simeq \frac{x(1-x)p}{k_\perp^2}$$

- large when $k_\perp \to 0$

- $k_\perp$ cannot be arbitrarily small: it accumulates via collisions

$$k_\perp^2 \simeq \hat{q} t_{\rm f} \Longrightarrow t_{\rm f}(x,p) \simeq \sqrt{\frac{x(1-x)p}{\hat{q}}}$$

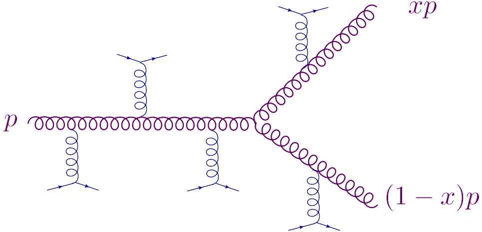
- The BDMPSZ branching rate:

$$\frac{{\rm d}\mathcal{P}}{{\rm d}x{\rm d}t} = \frac{\alpha_s}{\pi}\frac{P_{g\to g}(x)}{t_{\rm f}(x,p)} \simeq \frac{\alpha_s N_c}{\pi}\sqrt{\frac{\hat{q}}{p}}\frac{1}{[x(1-x)]^{3/2}}$$

Democratic branchings

$$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}x\mathrm{d}t} = \frac{1}{t_{\rm br}(p)} \frac{1}{[x(1-x)]^{3/2}}\,, \qquad t_{\rm br}(p) \equiv \frac{1}{\bar{\alpha}}\sqrt{\frac{p}{\hat{q}}}$$

- Probability $\Delta\mathcal{P}$ for a branching with $x \geq x_0$ to occur during Δt



$$\Delta\mathcal{P} \sim \frac{\Delta t}{\sqrt{x_0}\, t_{\rm br}(p)}$$

$\Delta\mathcal{P} \sim 1$ when :

$$\Delta t \simeq \sqrt{x_0}\, t_{\rm br}(p)$$

- Δt can be arbitrarily small if $x_0 \ll 1$: prompt soft emissions
- $\Delta t \sim t_{\rm br}(p)$ if $x_0 \sim \mathcal{O}(1)$: democratic branching

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$$\frac{d\mathcal{P}}{dx\,dt} = \frac{1}{t_{\rm br}(p)} \frac{1}{[x(1-x)]^{3/2}}\,, \qquad t_{\rm br}(p) \equiv \frac{1}{\bar{\alpha}}\sqrt{\frac{p}{\hat{q}}}$$

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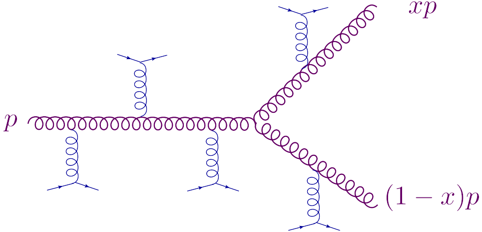

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- At early times $\Delta t \ll t_{\rm br}(p)$: asymmetric splittings $(x \ll 1)$
- When $\Delta t \sim t_{\rm br}(p)$, the particle disappears via a democratic branching

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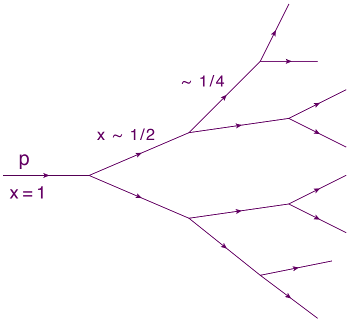

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$\Delta\mathcal{P} \sim 1$ when :

$$\Delta t \simeq \sqrt{x_0}\, t_{\rm br}(p)$$

- The daughter gluons are softer, so they disappear even faster
- $t_{\rm br}(p) \simeq$ the lifetime of the democratic cascade initiated by p

Hard gluons: only branchings

- Recall: a gluon would lose its energy p via drag after a time $t_{\rm drag}(p)$

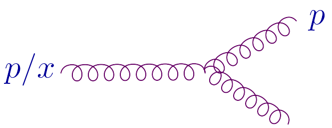
$$t_{\rm drag}(p) \simeq \frac{4pT}{\hat{q}} \sim \sqrt{\frac{p}{T}}\, t_{\rm br}(p) \gg t_{\rm br}(p) \quad \text{when } p \gg T$$

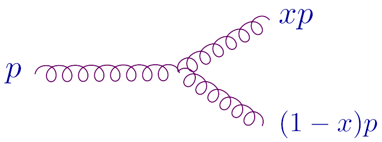
  - thermalization effects are irrelevant for the hard gluons

- Kinetic equation with branchings alone
  - energy flow within the democratic cascade

- Time evolution of the gluon spectrum: $D(p,t) \equiv p\frac{{\rm d}N}{{\rm d}p}$

$$\frac{\partial}{\partial t} D(p,t) = \frac{1}{t_{\rm br}(E)} \int \frac{{\rm d}x}{[x(1-x)]^{\frac{3}{2}}} \left[\sqrt{x}\, D\left(\frac{p}{x}, t\right) - \frac{1}{2} D(p,t)\right]$$





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- Linear equation: 'gain' - 'loss', but no recombinations
  - gluon occupation numbers are small when $p \gg T$
- Initial condition: $D(p, t=0) = E\delta(p-E)$

Wave turbulence

J.-P. Blaizot, E. I., Y. Mehtar-Tani, PRL 111, 052001 (2013)

- Exact analytic solution:

$$D(x,\tau) = \frac{\tau}{\sqrt{x}(1-x)^{3/2}}\, \mathrm{e}^{-\pi\frac{\tau^2}{1-x}}\,, \qquad x \equiv \frac{p}{E}\,, \quad \tau \equiv \frac{t}{t_{\rm br}(E)}$$

- Scaling spectrum at small x (Kolmogorov-Zakharov spectrum)

$$D(x,\tau) \simeq \frac{\tau}{\sqrt{x}}\, \mathrm{e}^{-\pi\tau^2} \qquad \text{for } x \ll 1$$

- Turbulent fixed point for the branching collision term
  - energy flux is independent of x (that is, of the parton generation)
  - the very definition of weak (linear) turbulence

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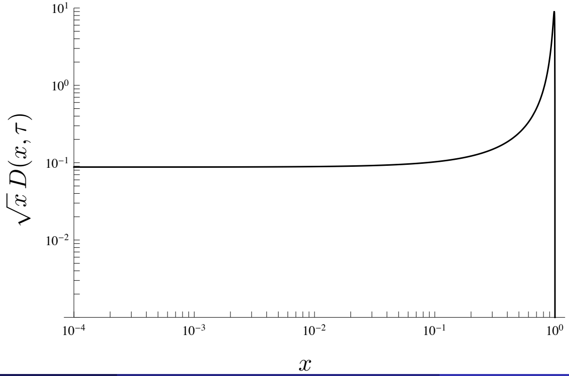
- Early times $t \ll t_{\rm br}(E)$: the broadening of the leading particle



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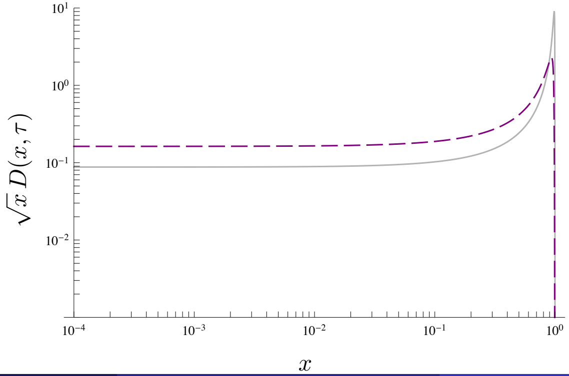

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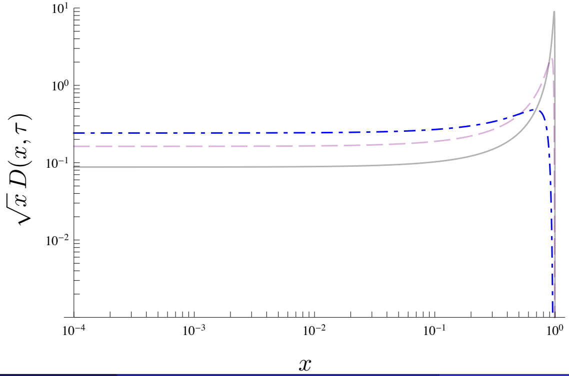

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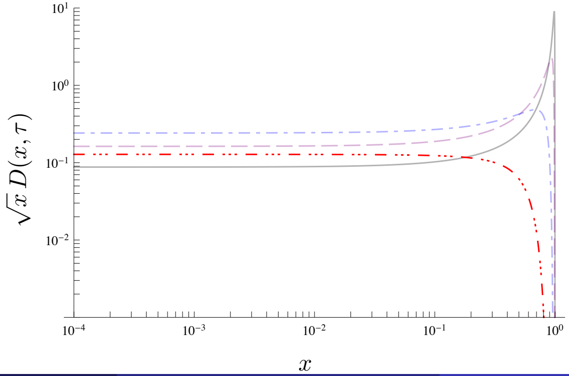
- Later times $t \gtrsim t_{\rm br}(E)$: the spectrum is suppressed at any x



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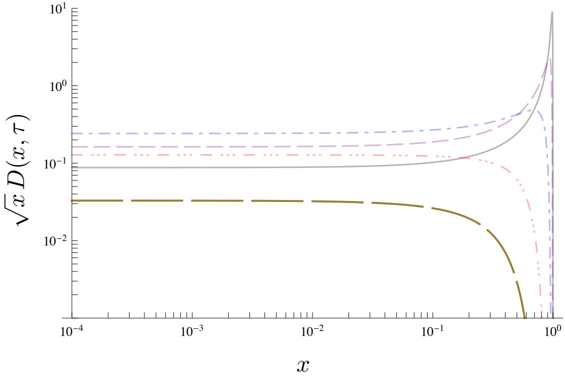
$$D(x,\tau) = \frac{\tau}{\sqrt{x}(1-x)^{3/2}}\, e^{-\pi\frac{\tau^2}{1-x}}, \qquad x \equiv \frac{p}{E}, \quad \tau \equiv \frac{t}{t_{\rm br}(E)}$$

- The energy accumulates at $x = 0$: 'condensate'



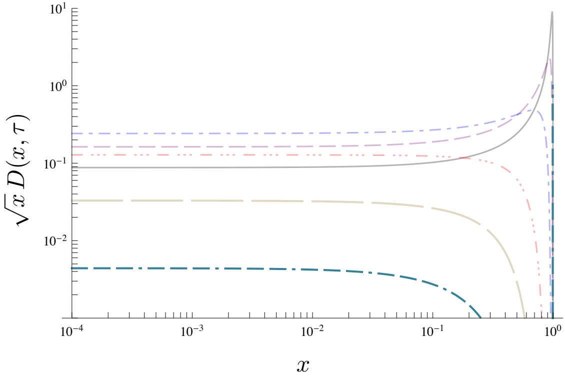
Wave turbulence

J.-P. Blaizot, E. I., Y. Mehtar-Tani, PRL 111, 052001 (2013)

- Exact analytic solution:

$$D(x,\tau) \;=\; \frac{\tau}{\sqrt{x}(1-x)^{3/2}}\; e^{-\pi\frac{\tau^2}{1-x}}\,, \qquad x \equiv \frac{p}{E}\,, \quad \tau \equiv \frac{t}{t_{\rm br}(E)}$$

- Medium treated as a 'perfect sink' : instantaneous thermalization



Elastic collisions + branchings

- For the soft gluons with $p \lesssim T$, elastic collisions become essential
- The dynamics is correctly encoded in the complete kinetic equation

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v} \cdot \nabla_{\boldsymbol{x}}\right) f(t, \boldsymbol{x}, \boldsymbol{p}) \;=\; \mathcal{C}_{\rm el}[f] + \mathcal{C}_{\rm br}[f]$$

- But this seems too complicated, even for numerical studies (?)
- So far, numerical solutions only for homogeneous situations
  - QGP thermalization starting with CGC initial conditions
  - formation of a Bose-Einstein condensate

Huang and Liao (2013); Kurkela and Lu (2014); Kurkela and Zhu (2015)

- The jet problem is further complicated by its strong inhomogeneity

$$f(t=0, \boldsymbol{x}, \boldsymbol{p}) \;=\; \frac{(2\pi)^3}{2(N_c^2-1)}\, \delta^{(3)}(\boldsymbol{x})\, \delta(p_z - E)\, \delta^{(2)}(\boldsymbol{p}_\perp)$$

A longitudinal kinetic equation

(E.I. and Bin Wu, arXiv:1506.07871, to appear in JHEP)

- A tractable equation can be obtained for the longitudinal dynamics

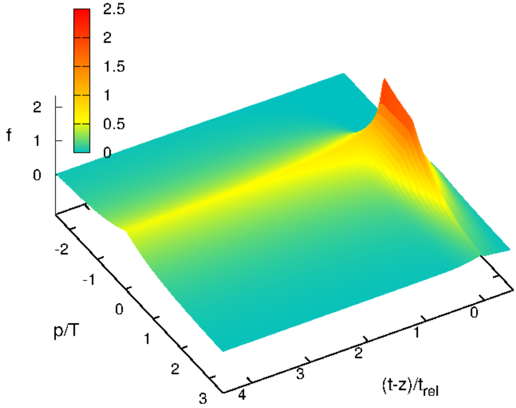
$$\left(\frac{\partial}{\partial t} + v_z \frac{\partial}{\partial z}\right) f(t,z,p_z) = \frac{\hat{q}}{4}\frac{\partial}{\partial p_z}\left[\left(\frac{\partial}{\partial p_z} + \frac{v_z}{T}\right) f\right]$$

$$+\frac{1}{t_{\rm br}(p_z)}\int_{p_*}\frac{{\rm d}x}{[x(1-x)]^{\frac{3}{2}}}\left[\frac{1}{\sqrt{x}} f\left(\frac{p_z}{x}\right) - \frac{1}{2} f(p_z)\right]$$

- Strictly correct when $p \gg T$
- Recombinations modeled by an IR cutoff on branchings: $p_* \sim T$
- This captures the right time scales to parametric accuracy
- Physical picture:
  - hard gluons ($p_0 \gg T$) first degrade their energy via democratic branchings, down to $p \sim T$, then thermalize via elastic collisions

The steady source approximation

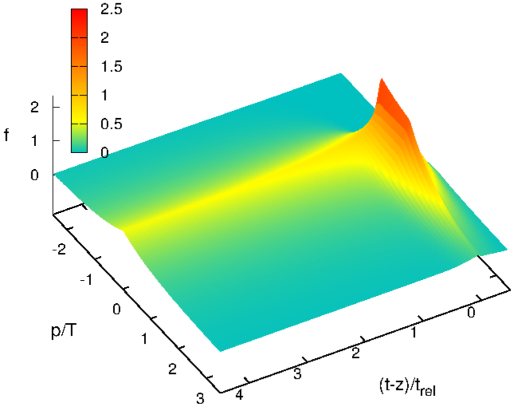
- The branching process $\approx$ a source of gluons with $p = p_*$
- The Fokker-Planck equation in $D = 1 + 1$ can be solved exactly



- A front $\propto \delta(t - z)$: gluons with $T \lesssim p < p_*$
  - gluons recently injected that had no time to thermalize

The steady source approximation

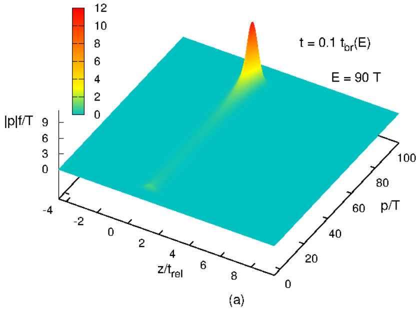
- The branching process $\approx$ a source of gluons with $p = p_*$
- The Fokker-Planck equation in $D = 1 + 1$ can be solved exactly



- A tail at $z \lesssim t - t_{\rm rel}$: $f_p \propto {\rm e}^{-|p|/T}$
  - gluons in thermal equilibrium with the medium

Numerical studies of the full dynamics (1)

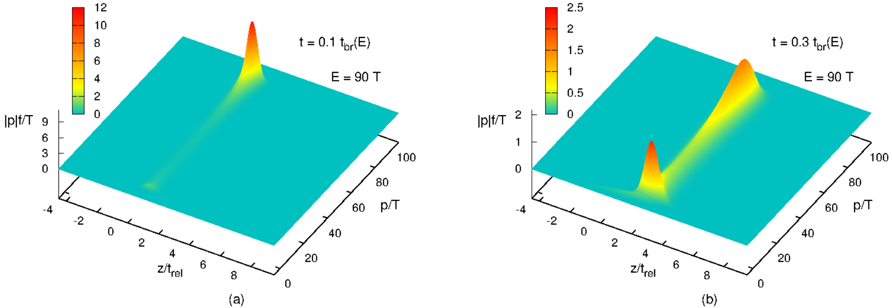
- Numerical solutions to the longitudinal kinetic equation
- Parameters: $T = 0.5\,\text{GeV}$, $t_{\rm rel} = 1\,\text{fm}$, $E = 90\,T$, $t_{\rm br}(E) \simeq 9.5\,\text{fm}$



- $t = 0.1\,t_{\rm br}(E) \simeq 1$ fm is representative for the 'leading jet' at the LHC
- $t = 0.3\,t_{\rm br}(E) \simeq 3$ fm : the 'subleading jet' (partially quenched)
- With increasing time, the jet substructure is softening and broadening

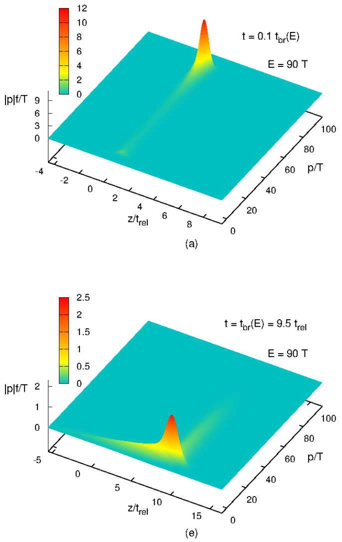
Numerical studies of the full dynamics (1)

- Numerical solutions to the longitudinal kinetic equation
- Parameters: $T = 0.5\,\text{GeV}$, $t_{\rm rel} = 1\,\text{fm}$, $E = 90\,T$, $t_{\rm br}(E) \simeq 9.5\,\text{fm}$



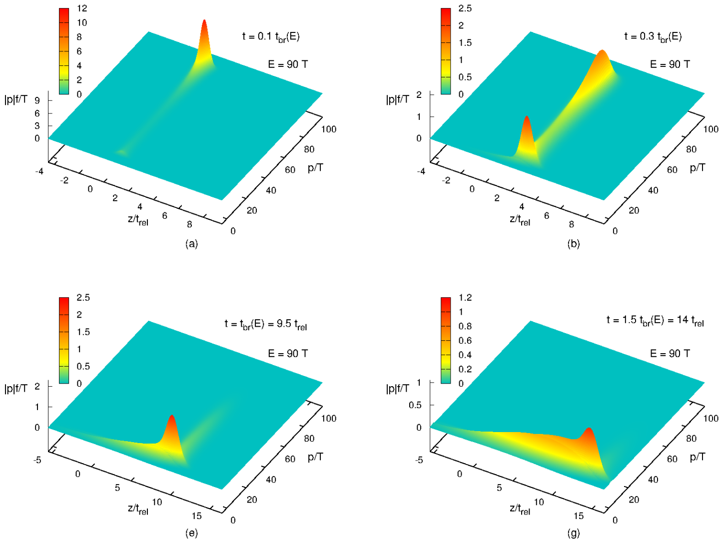
- $t = 0.3\,t_{\rm br}(E) \simeq 3\,\text{fm}$
    - the LP peak still visible around $p = E$
    - a second peak emerges near $p = T$ (branchings)
    - this second peak develops a thermalized tail at $z < t$ (collisions)

Numerical studies of the full dynamics (2)



- $t = t_{\rm br}(E)$: the leading particle disappears (democratic branching)

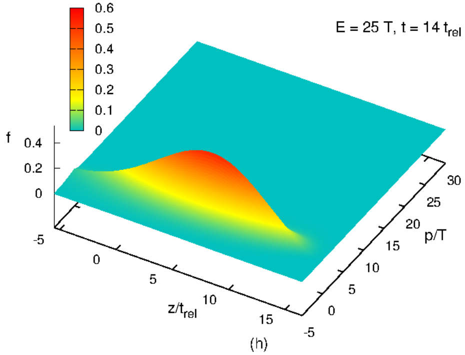
Numerical studies of the full dynamics (2)

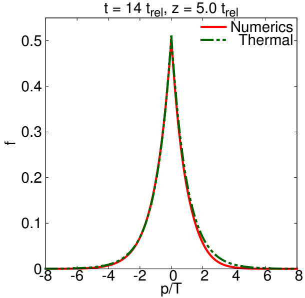


- $t = 1.5\, t_{\rm br}(E) \simeq 15$ fm : very large medium (almost fully quenched)

The late stages: thermalizing a mini-jet

- At late times $t \gg t_{\rm br}(E)$, the jet is fully quenched
  - no trace of the leading particle, just a thermalized tail
  - the emergence of hydrodynamics : spatial diffusion



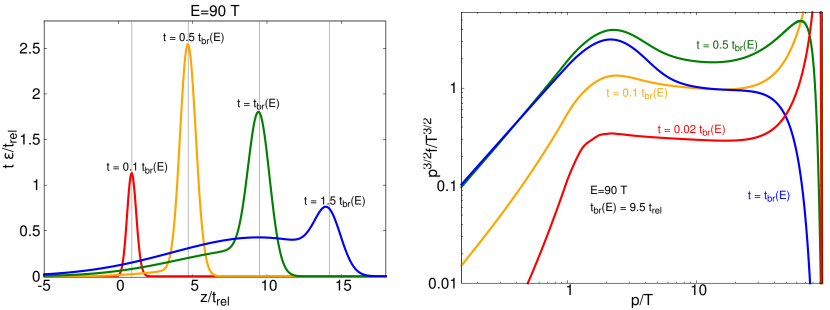


$$f(t,z,p) \simeq \mathrm{e}^{-\frac{|p|}{T}}\, \mathrm{e}^{-\frac{z^2}{4tt_{\rm rel}}}$$

Conclusions

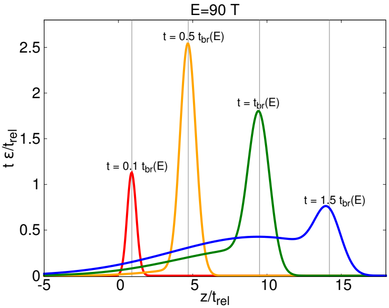
- A simple, yet detailed, 2-step, picture for jet quenching from pQCD
  - copious production of 'mini-jets' via democratic branchings
  - thermalization of the soft branching products with $p \sim T$
- Characteristic, 'front + tail', structure of the (partially) quenched jet
  - 'front' : the leading particle, but also soft gluons radiated at late times
  - 'tail' : soft partons in local thermal equilibrium with the medium
- Energy loss by the jet towards the medium is identified as the energy carried by the thermalized tail.
- Many approximations, need for more detailed/precise studies: coupled evolution of jet + medium in D= 1 + 3 + 3
- Well defined problem, but one definitely needs better calculations

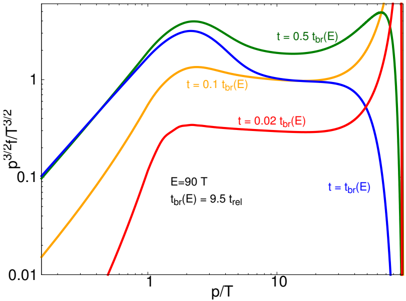
Energy distribution



- Left: z-distribution of the energy density $\varepsilon(t,z) = \int \mathrm{d}p\, |p|\, f(t,z,p)$
  - even for $t = t_{\rm br}(E)$, most of the energy is still carried by the front ...
  - but the respective 'front' is mostly made with soft gluons $(p \sim T)$
  - branching products which did not yet have the time to thermalize
  - a thermalized tail is clearly visible at $z < t$

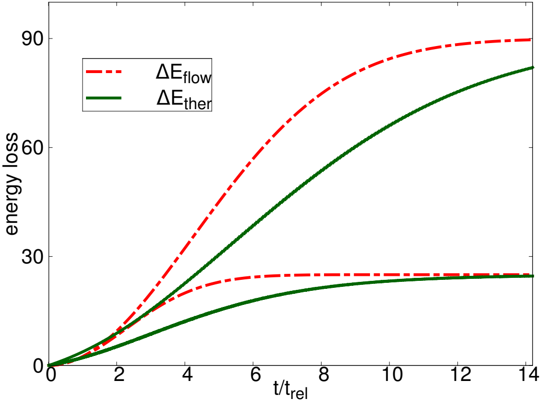
Energy distribution





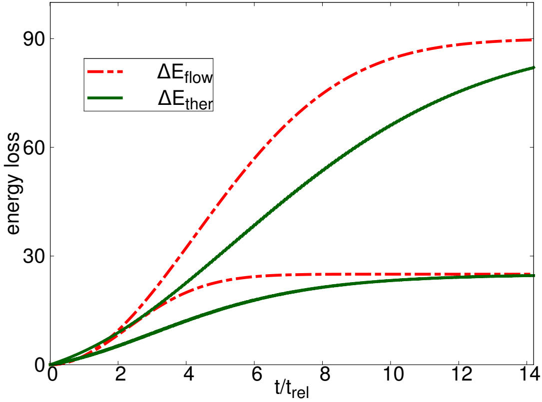
- Left: z-distribution of the energy density $\varepsilon(t, z) = \int \mathrm{d}p\, |p|\, f(t, z, p)$
- Right: p-distribution near the front: $f(t, z, p)$ for $z = t$
  - scaling window at $t \ll t_{\rm br}(E)$: $p^{3/2} f \approx$ const. $\Longrightarrow$ wave turbulence
  - for $t \gtrsim 0.5\, t_{\rm br}(E)$, some pile-up is visible around $p = T$
  - thermalization is efficient but certainly not instantaneous

Energy loss towards the medium



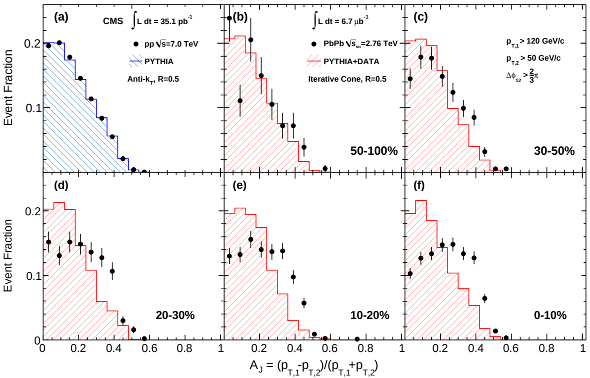
- Upper curves: $E = 90\,T$; lower curves: $E = 25\,T$
- $\Delta E_{\rm ther}$: the energy carried by the thermalized tail ($t - z \geq t_{\rm rel}$)
- $\Delta E_{\rm flow} = \nu\omega_{\rm br}(t) \propto t^2$ with $\nu \simeq 4.96$: the energy carried away by the turbulent flow (ideal branching process)

Energy loss towards the medium



- Upper curves: $E = 90\,T$; lower curves: $E = 25\,T$
- The energy carried by the thermalized tail is ...
  - somewhat smaller than the 'flow' energy: medium is not a 'perfect sink'
  - ... but still substantial: $6 \div 20$ GeV for $L = 3 \div 6$ fm

Di–jet asymmetry : A_J (CMS)

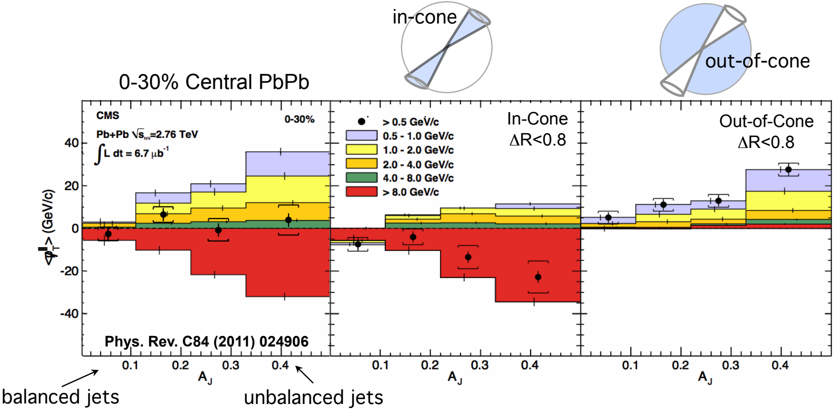


- Event fraction as a function of the di–jet energy imbalance in p+p (a) and Pb+Pb (b–f) collisions for different bins of centrality

$$A_J = \frac{E_1 - E_2}{E_1 + E_2} \qquad (E_i \equiv p_{T,i} = \text{ jet energies})$$

- N.B. A pronounced asymmetry already in the p+p collisions !

Energy imbalance @ large angles: $R = 0.8$



- No missing energy : $E_{\rm Lead}^{\rm in+out} = E_{\rm SubLead}^{\rm in+out}$
- In-Cone : $E_{\rm Lead}^{\rm in} > E_{\rm SubLead}^{\rm in}$: di-jet asymmetry, hard particles
- Out-of-Cone : $E_{\rm Lead}^{\rm out} < E_{\rm SubLead}^{\rm out}$: soft hadrons @ large angles

An exact solution to the 1-D FP equation

- The Green's function for ultrarelativistic FP in D=1+1+1

$$\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial z}\right) f = \frac{\hat{q}}{4}\frac{\partial}{\partial p}\left(\frac{\partial}{\partial p} + \frac{v}{T}\right) f\,, \qquad f(t=0,z,p) = \delta(z)\delta(p-p_0)$$

$$f(t,z,p>0) = \frac{\mathrm{e}^{-\frac{p_0-p}{2}-\frac{t}{4}}}{2\sqrt{\pi t}}\left[\mathrm{e}^{-\frac{(p-p_0)^2}{4t}} - \mathrm{e}^{-\frac{(p+p_0)^2}{4t}}\right]\delta(t-z)$$

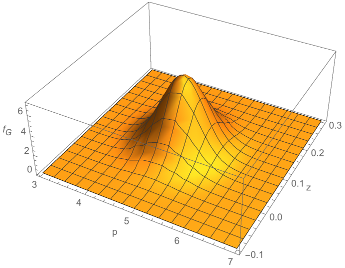
$$+\frac{\mathrm{e}^{-\frac{(p+p_0-z)^2}{4t}-p}}{8\sqrt{\pi}t^{5/2}}\left[t(t+2)-(p+p_0-z)^2\right]\times$$

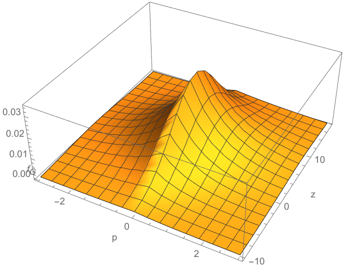
$$\times\ \mathrm{erfc}\left(\frac{1}{2}\sqrt{t-\frac{z^2}{t}}\left(\frac{p+p_0}{t+z}-1\right)\right)$$

$$+\frac{(t+z)(p+p_0+t-z)}{4\pi t^2\sqrt{t^2-z^2}}\,\mathrm{e}^{-\frac{(p+p_0)^2}{2(t+z)}+\frac{p_0-p}{2}-\frac{t}{4}}$$

Limiting behaviors

- Physics transparent at both small and large times: $t_{\rm drag}(p_0) = \frac{p_0}{T}\, t_{\rm rel}$





- small times: $t_{\rm rel} \lesssim t \ll t_{\rm drag}(p_0)$

$$f(t,z,p) \propto {\rm e}^{-\frac{(p-\langle p(t)\rangle)^2}{\hat{q}t}}\, \delta(t-z)$$

$$\langle p(t)\rangle = p_0 - \frac{t}{t_{\rm rel}}\, T$$

- energy loss & diffusion

- large times: $t > t_{\rm drag}(p_0) \gg t_{\rm rel}$

$$f \simeq {\rm e}^{-\frac{|p|}{T}}\, \exp\left\{-\frac{\big(z-(p_0/T)t_{\rm rel}\big)^2}{4tt_{\rm rel}}\right\}$$

- equilibrium & spatial diffusion

- plots: $p_0 = 5T$, $t_1 = t_{\rm rel}$, $t_2 = 20t_{\rm rel}$

Formation time & emission angle

$$t_{\mathrm{f}}(\omega) \simeq \sqrt{\frac{\omega}{\hat{q}}} \qquad \& \qquad \theta_{\mathrm{f}}(\omega) \simeq \frac{\sqrt{\hat{q} t_{\mathrm{f}}}}{\omega} \sim \left(\frac{\hat{q}}{\omega^3}\right)^{1/4}$$

- This mechanism applies so long as

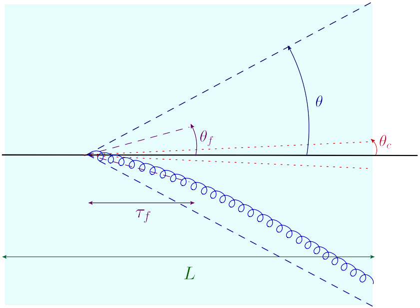
$$\lambda \ll t_{\mathrm{f}}(\omega) \le L \implies T \ll \omega \le \omega_c \equiv \hat{q} L^2$$

- Soft gluons : short formation times & large emission angles

$$\omega \ll \omega_c \implies t_{\mathrm{f}}(\omega) \ll L$$

$$\theta_{\mathrm{f}}(\omega) \gg \theta_c$$

$$\theta(\omega) \simeq \frac{\sqrt{\hat{q} L}}{\omega} \gg \theta_{\mathrm{f}}(\omega)$$



- The emission angle keeps increasing with time, via rescattering

Formation time & emission angle

$$t_{\rm f}(\omega) \simeq \sqrt{\frac{\omega}{\hat q}} \qquad \& \qquad \theta_{\rm f}(\omega) \simeq \frac{\sqrt{\hat q t_{\rm f}}}{\omega} \sim \left(\frac{\hat q}{\omega^3}\right)^{1/4}$$

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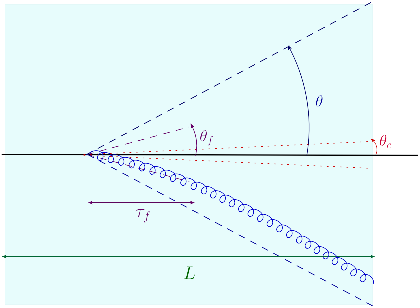
$$\omega \ll \omega_c \implies t_{\rm f}(\omega) \ll L$$

$$\theta_{\rm f}(\omega) \gg \theta_c$$

$$\theta(\omega) \simeq \frac{\sqrt{\hat q L}}{\omega} \gg \theta_{\rm f}(\omega)$$



- Emissions can effectively be treated as collinear