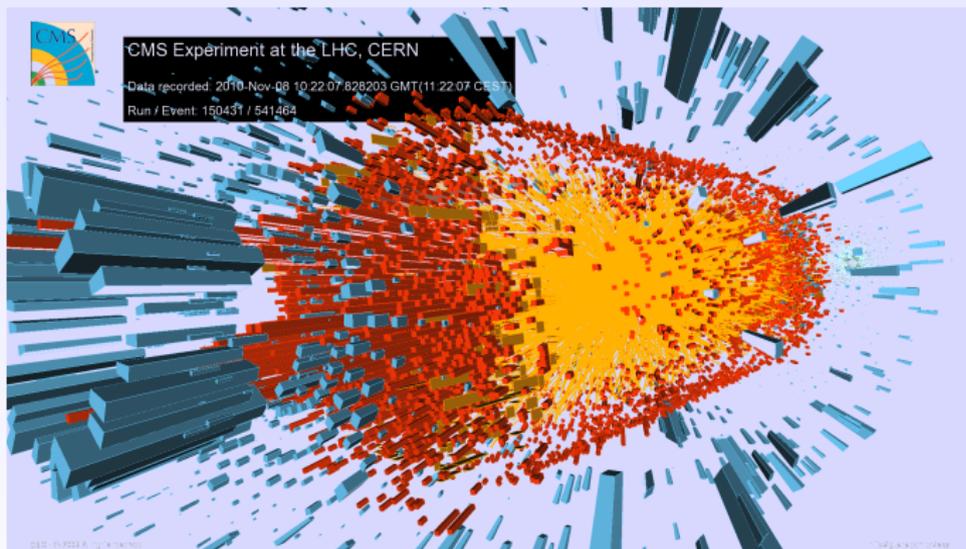


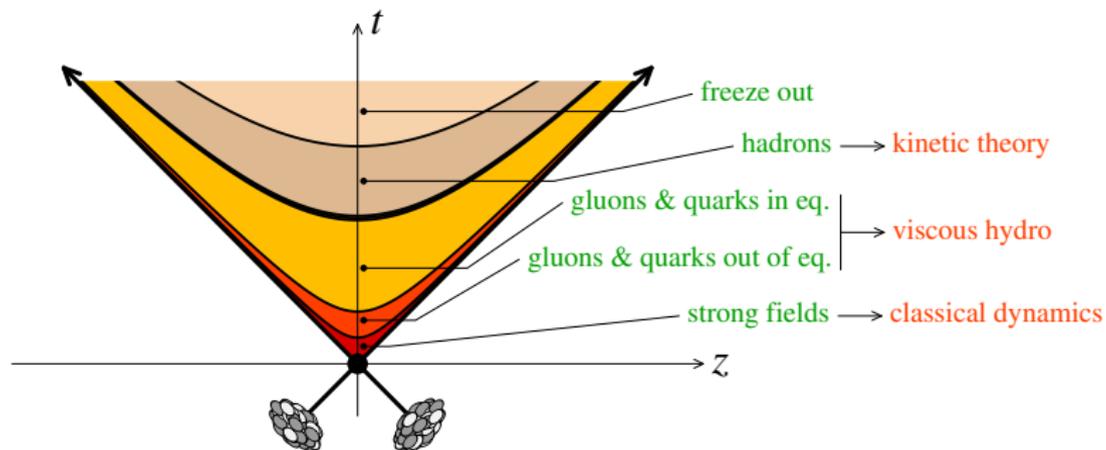
# The Early Stages of Heavy Ion Collisions

YITP, October 2015

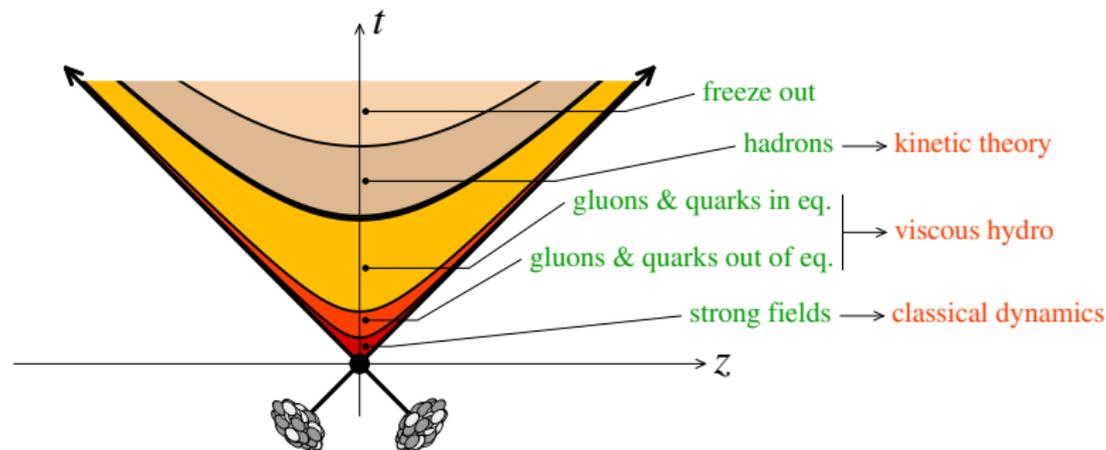
François Gelis  
IPhT, Saclay



# Stages of a nucleus-nucleus collision



## Stages of a nucleus-nucleus collision



- Well described as a nearly ideal fluid expanding into vacuum according to relativistic hydrodynamics

- Hydrodynamics is a macroscopic description based on **energy-momentum conservation** :

$$\partial_{\mu} T^{\mu\nu} = 0$$

True in any quantum field theory

Not closed : 4 equations, 10 independent components in  $T^{\mu\nu}$

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- Additional assumption : at macroscopic scales,  $T^{\mu\nu}$  is expressible in terms of  $\epsilon$  (energy density),  $P$  (pressure) and  $u^{\mu}$  (fluid velocity field)
- For a frictionless fluid :  $T_{\text{ideal}}^{\mu\nu} = (\epsilon + P) u^{\mu} u^{\nu} - P g^{\mu\nu}$

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- In general :  $T^{\mu\nu} = T_{\text{ideal}}^{\mu\nu} \oplus \underbrace{\eta \nabla^{\mu} u^{\nu} \oplus \zeta (\nabla_{\rho} u^{\rho}) \oplus \dots}_{\Pi^{\mu\nu} \equiv \text{deviation from ideal fluid}}$

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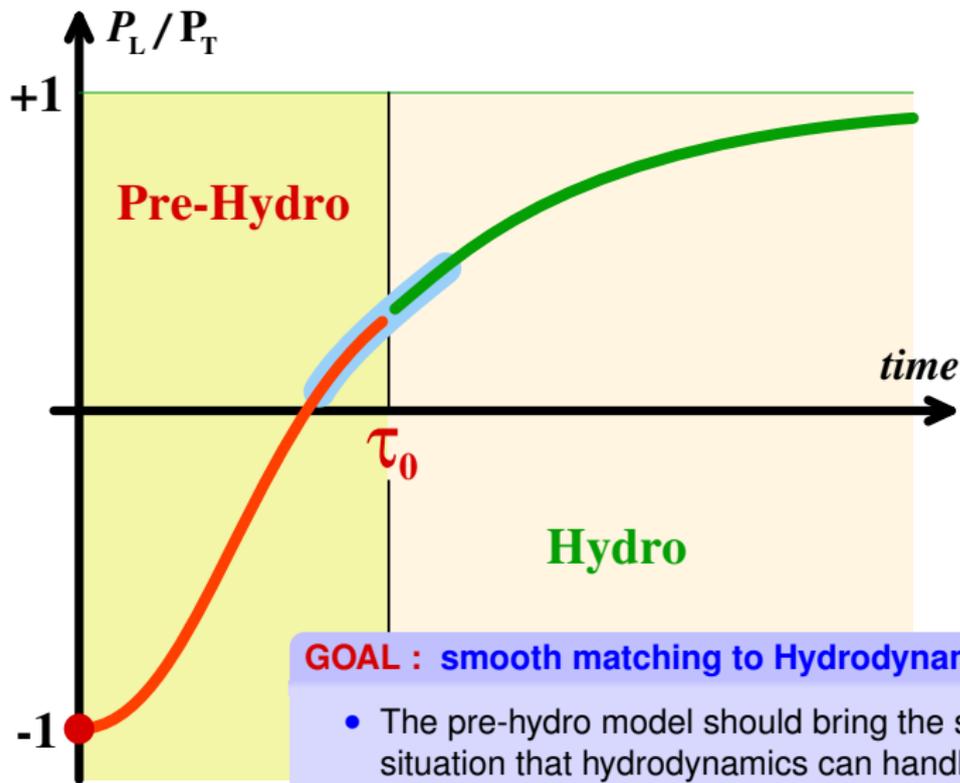
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- Microscopic inputs :  $\epsilon = f(P)$  (EoS),  $\eta, \zeta, \dots$  (transport coeff.)



## Conditions for hydrodynamics

- The difference between  $P_L$  and  $P_T$  should not be too large (for the expansion to make sense)
- The ratio  $\eta/s$  should be very small (fits require  $\eta/s \sim 0.1$ ) (for an efficient transfer from spatial to momentum anisotropy)

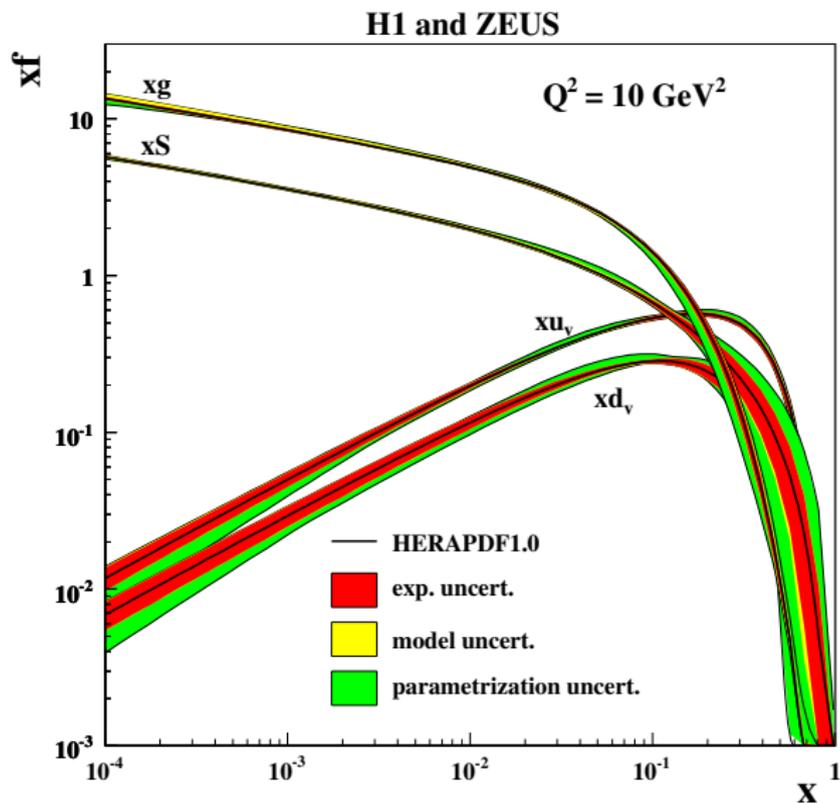


**GOAL : smooth matching to Hydrodynamics**

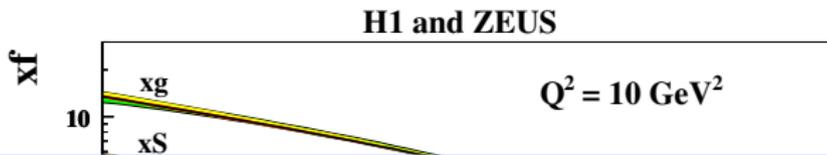
- The pre-hydro model should bring the system to a situation that hydrodynamics can handle
- Pre-hydro and hydro should agree over some range of time  $\Rightarrow$  no  $\tau_0$  dependence

# **Color Glass Condensate in Heavy Ion Collisions**

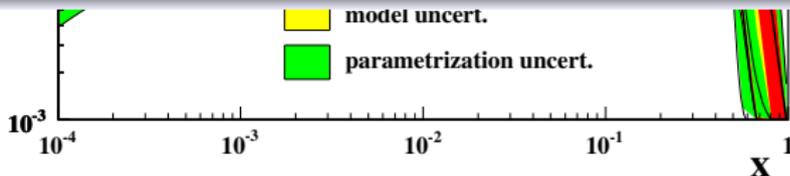
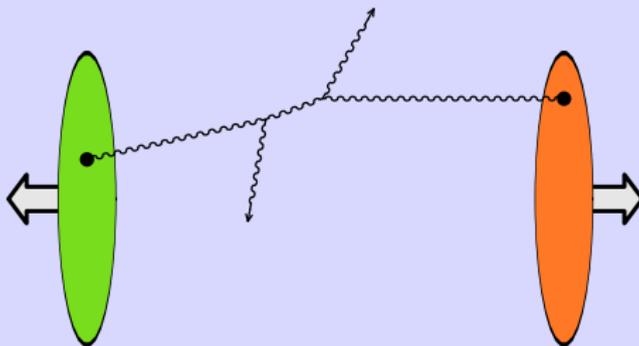
## Parton distributions in a nucleon



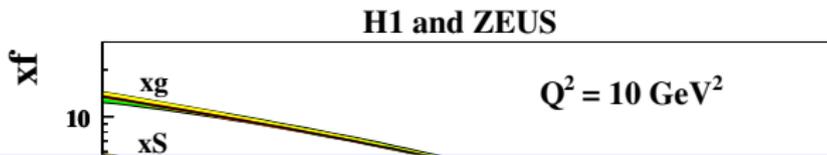
# Parton distributions in a nucleon



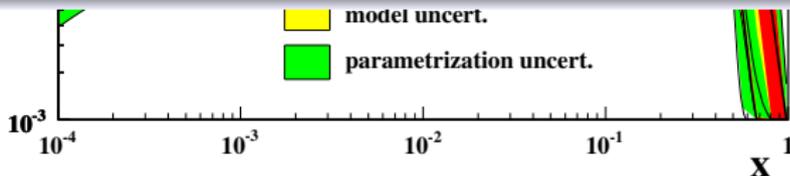
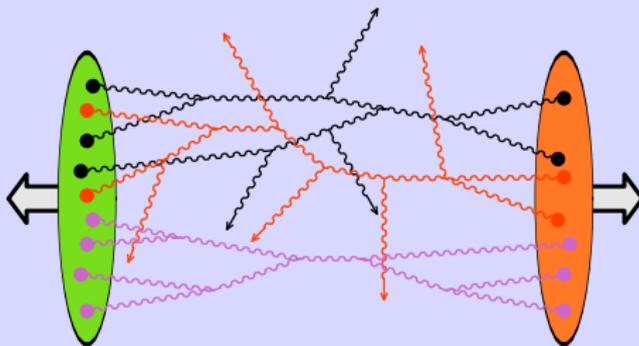
Large  $x$  : dilute, dominated by single parton scattering

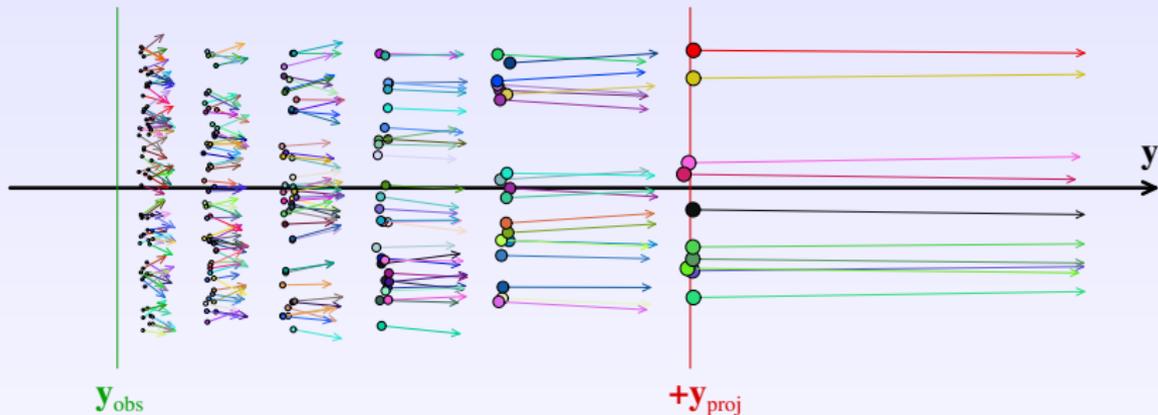


# Parton distributions in a nucleon

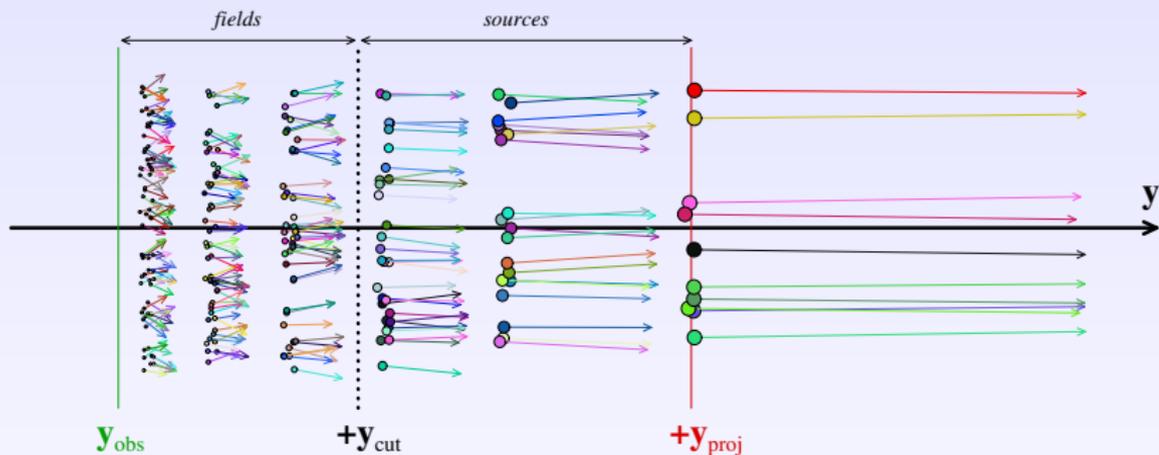


Small  $x$  : dense, multi-parton interactions become likely



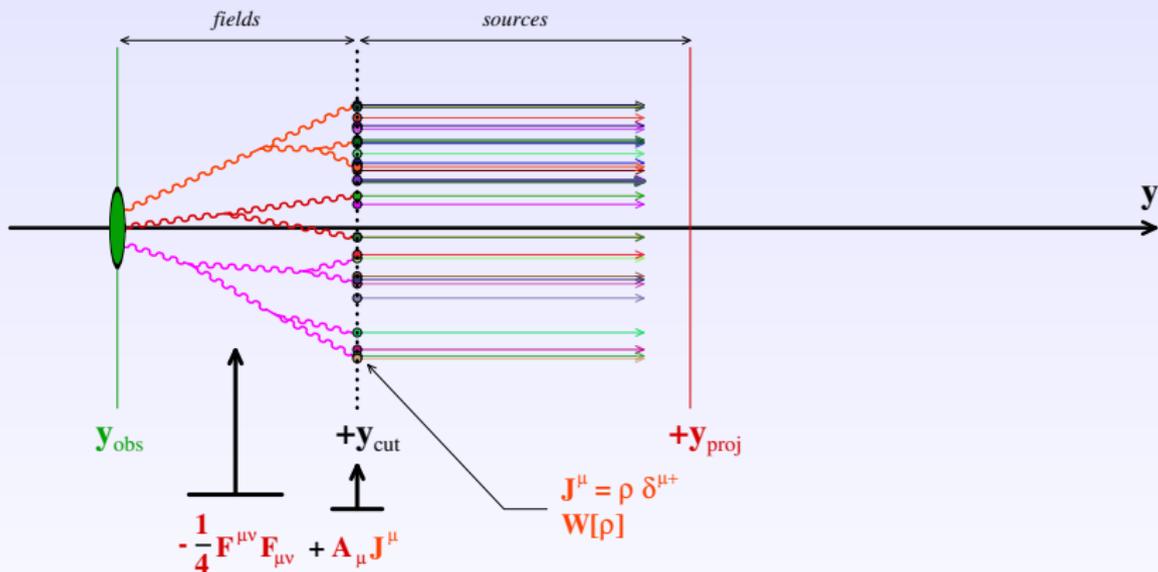


- $p_{\perp}^2 \sim Q_s^2 \sim \Lambda_{\text{QCD}}^2 e^{\lambda(y_{\text{proj}} - y)}$  ,  $p_z \sim Q_s e^{y - y_{\text{obs}}}$
- Fast partons : frozen dynamics, negligible  $p_{\perp} \Rightarrow$  **classical sources**
- Slow partons : evolve with time  $\Rightarrow$  **gauge fields**



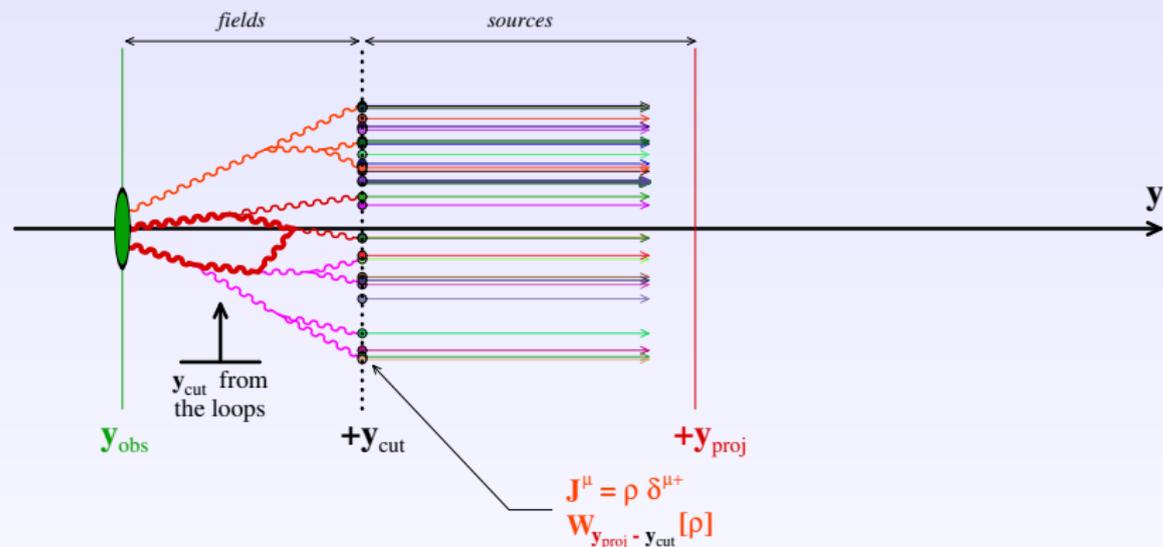
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- Fast partons : frozen dynamics, negligible  $p_\perp \Rightarrow$  **classical sources**
- Slow partons : evolve with time  $\Rightarrow$  **gauge fields**

# Cancellation of the cutoff dependence



- The cutoff  $y_{\text{cut}}$  is arbitrary and should not affect the result
- The probability density  $W[\rho]$  changes with the cutoff
- Loop corrections cancel the cutoff dependence from  $W[\rho]$

[Jalilian-Marian, Kovner, Leonidov, Weigert (1998)]  
[Balitsky (1996)] [Iancu, Leonidov, McLerran (2001)]

## B-JIMWLK equation at Leading Log

$$\frac{\partial W_Y[\rho]}{\partial Y} = \frac{1}{2} \int_{\vec{x}_\perp, \vec{y}_\perp} \underbrace{\frac{\delta}{\delta \rho_a(\vec{x}_\perp)} \chi_{ab}(\vec{x}_\perp, \vec{y}_\perp) \frac{\delta}{\delta \rho_b(\vec{y}_\perp)}}_{\mathcal{H} \text{ (JIMWLK Hamiltonian)}} W_Y[\rho]$$

- Mean field approx. (BK equation) : [Kovchegov (1999)]
- Langevin form of B-JIMWLK : [Blaizot, Iancu, Weigert (2003)]
- First numerical solution : [Rummukainen, Weigert (2004)]

[Jalilian-Marian, Kovner, Leonidov, Weigert (1998)]  
[Balitsky (1996)] [Iancu, Leonidov, McLerran (2001)]

B-JIMWLK

Recent developments :

Running coupling correction

[Lappi, Mäntysaari (2012)]

B-JIMWLK equation at Next to Leading Log

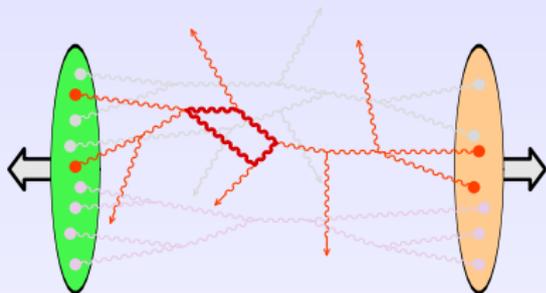
[Kovner, Lublinsky, Mulian (2013)]

• Me [Caron-Huot (2013)][Balitsky, Chirilli (2013)]

• Langevin form of B-JIMWLK : [Blaizot, Iancu, Weigert (2003)]

• First numerical solution : [Rummukainen, Weigert (2004)]

$$\mathcal{S} = \underbrace{-\frac{1}{4} \int F_{\mu\nu} F^{\mu\nu}}_{\text{slow gluons}} + \underbrace{\int (J_1^\mu + J_2^\mu) A_\mu}_{\text{fast partons}}$$



**In the saturated regime:**  $J^\mu \sim g^{-1}$

$$g^{-2} g^{\# \text{ of external gluons}} g^{2 \times (\# \text{ of loops})}$$

- No dependence on the number of sources  $J^\mu$ 
  - ▷ infinite number of graphs at each order in  $g^2$

**Example : expansion of  $T^{\mu\nu}$  in powers of  $g^2$**

$$T^{\mu\nu} \sim \frac{1}{g^2} \left[ c_0 + c_1 g^2 + c_2 g^4 + \dots \right]$$

### [FG, Venugopalan (2006)]

- The Leading Order is the sum of all the tree diagrams  
Expressible in terms of **classical solutions of Yang-Mills equations** :

$$\mathcal{D}_\mu \mathcal{F}^{\mu\nu} = J_1^\nu + J_2^\nu$$

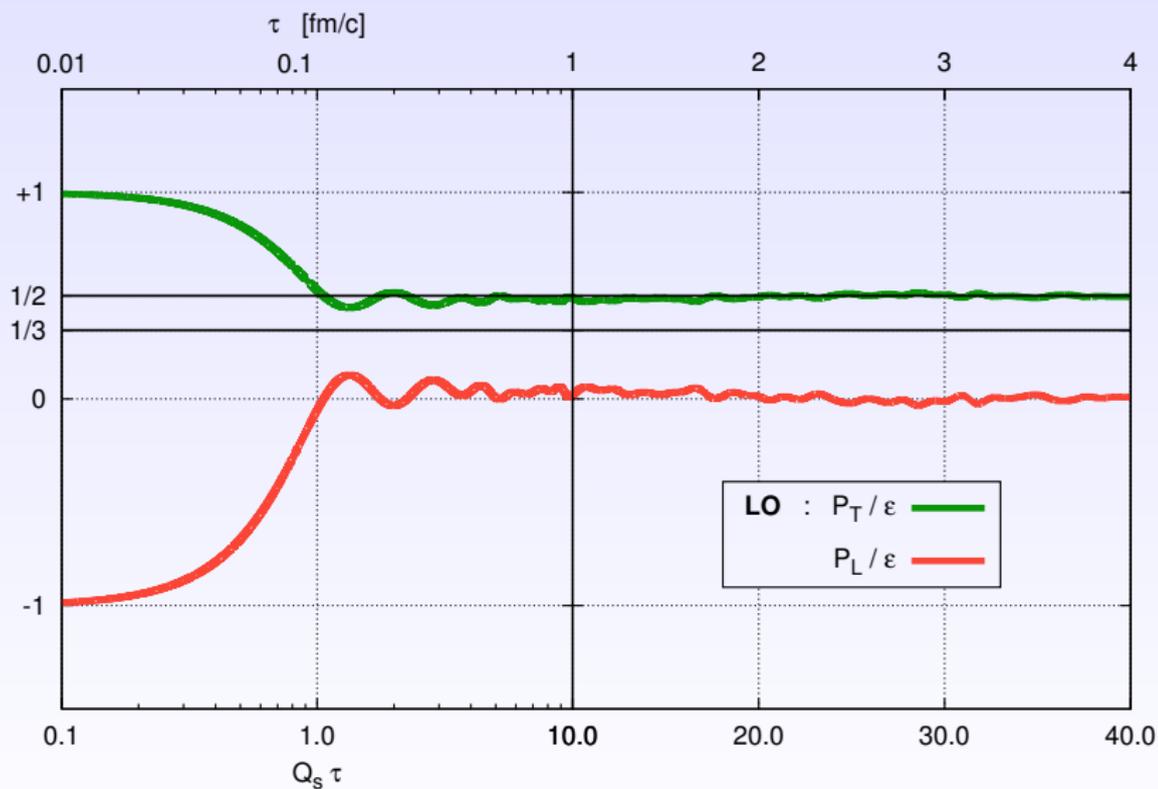
- Boundary conditions :  $\lim_{x^0 \rightarrow -\infty} \mathcal{A}^\mu(x) = 0$

(WARNING : this is not true for exclusive observables!)

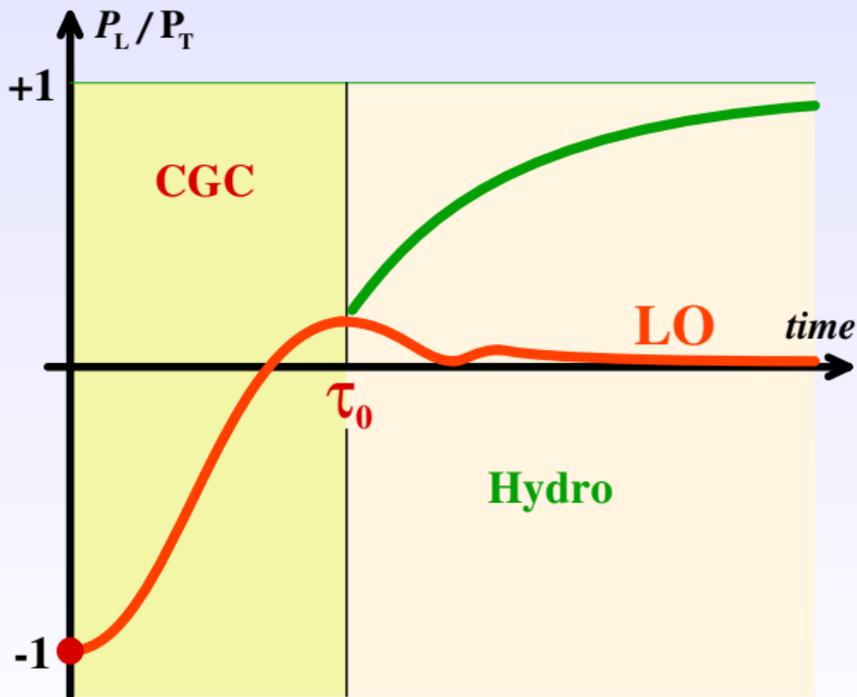
### Components of the energy-momentum tensor at **LO** :

$$T_{LO}^{00} = \frac{1}{2} \underbrace{[\mathbf{E}^2 + \mathbf{B}^2]}_{\text{class. fields}} \quad T_{LO}^{0i} = [\mathbf{E} \times \mathbf{B}]^i$$

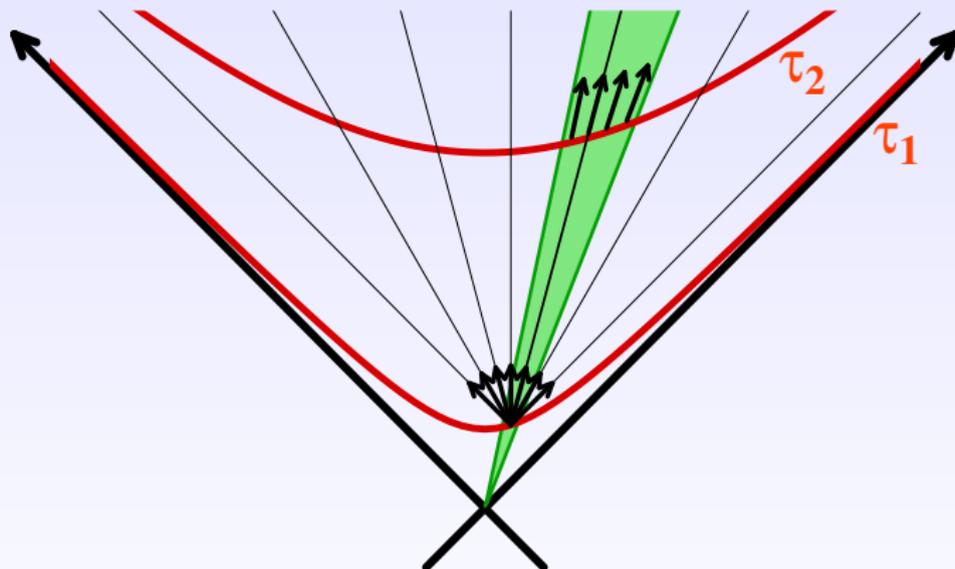
$$T_{LO}^{ij} = \frac{\delta^{ij}}{2} [\mathbf{E}^2 + \mathbf{B}^2] - [\mathbf{E}^i \mathbf{E}^j + \mathbf{B}^i \mathbf{B}^j]$$



# CGC at LO : unsatisfactory matching to hydrodynamics





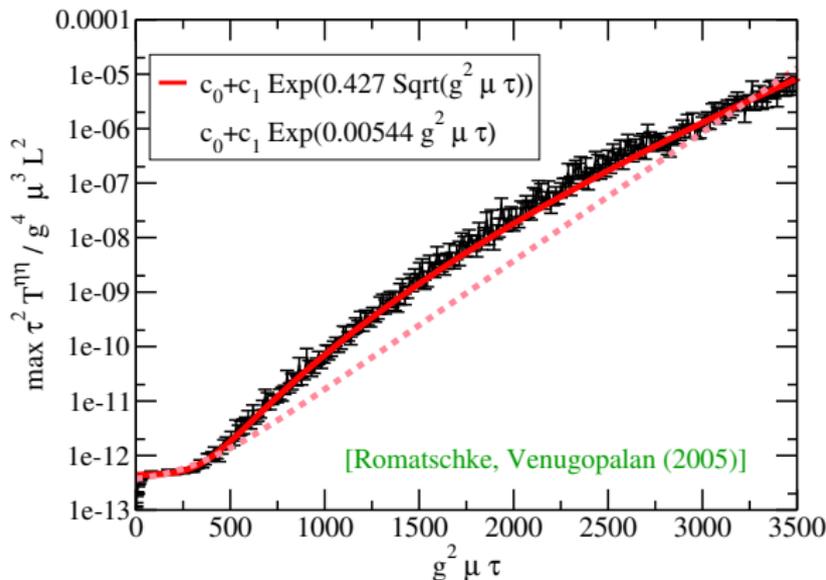


- CGC at LO is very close to free streaming

**Does it get better at  
Next-to-Leading Order?**

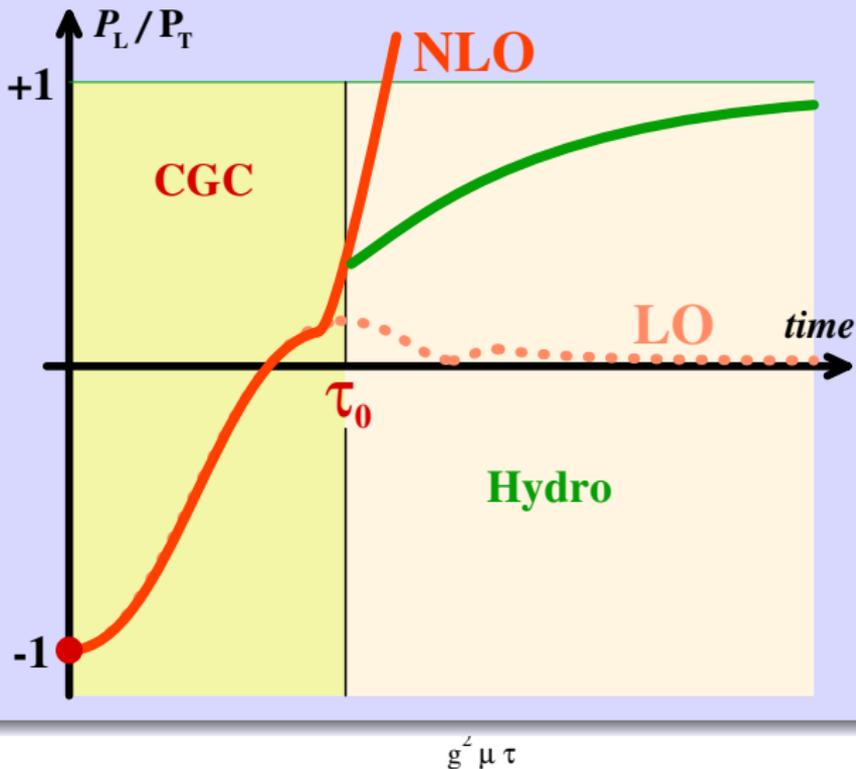
## CGC at NLO : instabilities

[Mrowczynski (1988), Romatschke, Strickland (2003), Arnold, Lenaghan, Moore (2003), Rebhan, Romatschke, Strickland (2005), Arnold, Lenaghan, Moore, Yaffe (2005), Romatschke, Rebhan (2006), Bodeker, Rummukainen (2007), Fujii, Itakura (2008),..., **Attems, Rebhan, Strickland (2012)**, **Fukushima (2013)**]



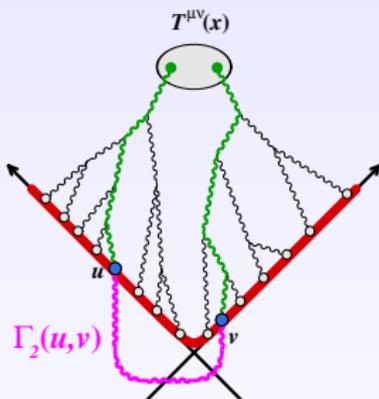
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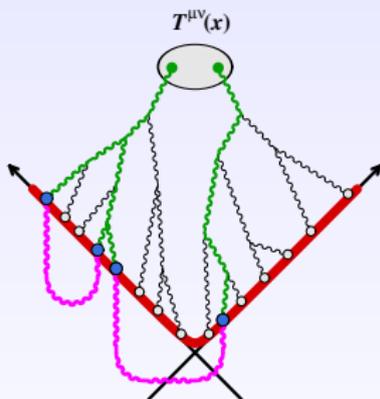
**Beyond NLO :**  
**Classical Statistical  
Approximation**

Loop  $\sim g^2$  , Small perturbation  $\sim e^{\sqrt{\mu\tau}}$



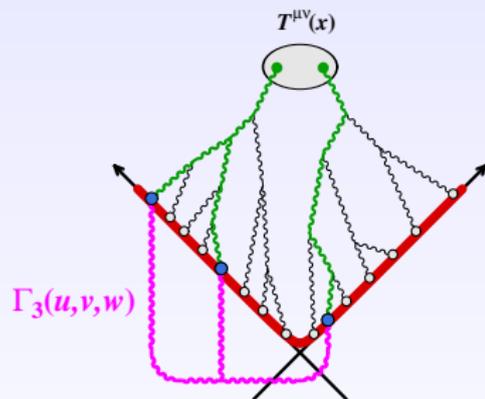
- 1 loop :  $(ge^{\sqrt{\mu\tau}})^2$

Loop  $\sim g^2$  , Small perturbation  $\sim e^{\sqrt{\mu\tau}}$



- 1 loop :  $(ge^{\sqrt{\mu\tau}})^2$
- 2 disconnected loops :  $(ge^{\sqrt{\mu\tau}})^4$

Loop  $\sim g^2$  , Small perturbation  $\sim e^{\sqrt{\mu\tau}$



- 1 loop :  $(ge^{\sqrt{\mu\tau}})^2$
- 2 disconnected loops :  $(ge^{\sqrt{\mu\tau}})^4$
- 2 entangled loops :  $g(ge^{\sqrt{\mu\tau}})^3 \triangleright$  subleading

## Leading terms

- All disconnected loops to all orders  
 $\triangleright$  exponentiation of the 1-loop result



$$\begin{aligned}
 T_{\text{resummed}}^{\mu\nu} &= \underbrace{T_{\text{LO}}^{\mu\nu} + T_{\text{NLO}}^{\mu\nu}}_{\text{in full}} + \underbrace{T_{\text{NNLO}}^{\mu\nu} + \dots}_{\text{partially}} \\
 &= \int [D\mathbf{a}] \exp \left[ -\frac{1}{2} \int_{\mathbf{u}, \mathbf{v}} \mathbf{a}(\mathbf{u}) \Gamma_2^{-1}(\mathbf{u}, \mathbf{v}) \mathbf{a}(\mathbf{v}) \right] T_{\text{LO}}^{\mu\nu} [\mathcal{A}_{\text{init}} + \mathbf{a}]
 \end{aligned}$$

- The exponentiation of the 1-loop result collects all the terms with the worst time behavior
- Equivalent to Gaussian fluctuations of the initial field + classical time evolution
- Note: This is the Wigner distribution of a coherent state

- Von Neumann equation for the density operator :

$$\frac{\partial \hat{\rho}_\tau}{\partial \tau} = i\hbar [\hat{H}, \hat{\rho}_\tau]$$

- Wigner transform :

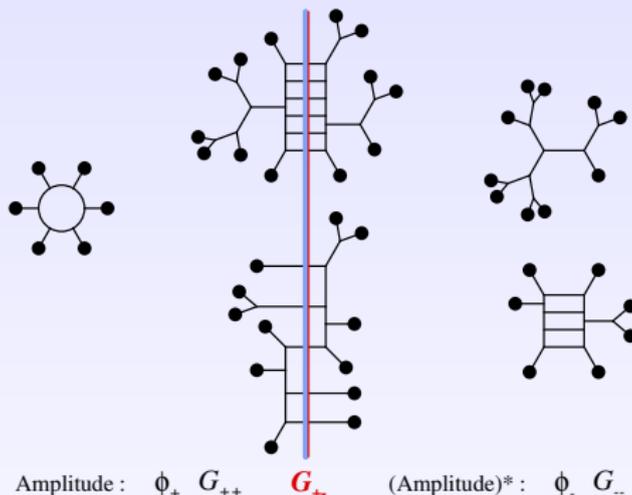
$$W_\tau(\mathbf{x}, \mathbf{p}) \equiv \int d\mathbf{s} e^{i\mathbf{p}\cdot\mathbf{s}} \langle \mathbf{x} + \frac{\mathbf{s}}{2} | \hat{\rho}_\tau | \mathbf{x} - \frac{\mathbf{s}}{2} \rangle$$

$$\mathcal{H}(\mathbf{x}, \mathbf{p}) \equiv \int d\mathbf{s} e^{i\mathbf{p}\cdot\mathbf{s}} \langle \mathbf{x} + \frac{\mathbf{s}}{2} | \hat{H} | \mathbf{x} - \frac{\mathbf{s}}{2} \rangle \quad (\text{classical Hamiltonian})$$

- Moyal equation (equivalent to Von Neumann) :

$$\begin{aligned} \frac{\partial W_\tau}{\partial \tau} &= \mathcal{H}(\mathbf{x}, \mathbf{p}) \frac{2}{i\hbar} \sin \left( \frac{i\hbar}{2} \left( \overleftarrow{\partial}_\mathbf{p} \overrightarrow{\partial}_\mathbf{x} - \overleftarrow{\partial}_\mathbf{x} \overrightarrow{\partial}_\mathbf{p} \right) \right) W_\tau(\mathbf{x}, \mathbf{p}) \\ &= \underbrace{\{\mathcal{H}, W_\tau\}}_{\text{Poisson bracket}} + \mathcal{O}(\hbar^2) \end{aligned}$$

- Classical time evolution  $\iff \mathcal{O}(\hbar^2)$  error
- $\mathcal{O}(\hbar^1)$  corrections come from the initial condition



- From the Schwinger-Keldysh formalism, define new fields as:

$$\phi_1 \equiv \frac{1}{2}(\phi_+ + \phi_-) \quad \phi_2 \equiv \phi_+ - \phi_-$$

- Vertices :  $\phi_1^3 \phi_2$   $\phi_1 \phi_2^3$  (odd terms in  $\phi_2$  only)
- $\phi_2$  encodes quantum interferences

### CSA : drop the vertex $\phi_1\phi_2^3$

- No  $\phi_1\phi_2^3$  vertex  $\implies$  classical time evolution
- Differences with the original QFT start appearing at 2 loops
- Equivalent to classical runs averaged over the initial conditions

### Ensemble of initial classical fields

- This approximation does not specify the initial fields: controlled by the observable under consideration (e.g.  $\langle 0_{\text{in}} | T^{\mu\nu} | 0_{\text{in}} \rangle$ )
- Initial 2-point correlations encoded in  $G_{11}$ . Generically:

$$G_{11}(\mathbf{p}) \sim \left( f_0(\mathbf{p}) + \frac{1}{2} \right) \delta(p^2)$$

quasiparticles  $\leftrightarrow$   $\quad$   $\leftrightarrow$  vacuum zero point fluctuations

- The  $1/2$  generates the 1-loop quantum corrections
- Dropping the  $1/2$  : nothing quantum left

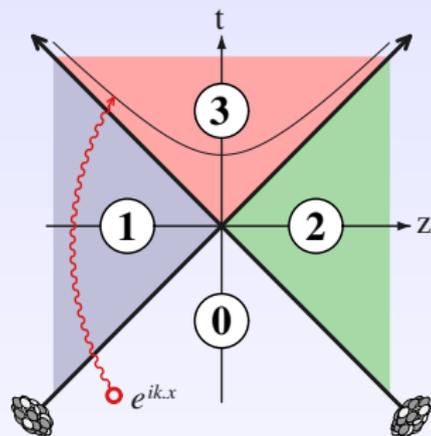
[Epelbaum, FG (2013)]

CGC at  $\tau \ll Q_s^{-1}$  (1-loop accurate)

$$\langle \mathcal{A}^\mu \rangle = \mathcal{A}_{LO}^\mu \quad \text{Var.} = \frac{1}{2} \int_{\text{modes } \mathbf{k}} \mathbf{a}_{\mathbf{k}}(\mathbf{u}) \mathbf{a}_{\mathbf{k}}^*(\mathbf{v})$$

$$\left[ \mathcal{D}_\rho \mathcal{D}^\rho \delta_\mu^\nu - \mathcal{D}_\mu \mathcal{D}^\nu + ig \mathcal{F}_\mu{}^\nu \right] \mathbf{a}_{\mathbf{k}}^\mu = 0$$

$$\lim_{x^0 \rightarrow -\infty} \mathbf{a}_{\mathbf{k}}(x) = e^{ik \cdot x}$$



[Epelbaum, FG (2013)]

CGC at  $\tau \ll Q_s^{-1}$  (1-loop accurate)

$$\langle \mathcal{A}^\mu \rangle = \mathcal{A}_{LO}^\mu \quad \text{Var} = \frac{1}{L} \int \alpha_k(\mathbf{u}) \alpha_k^*(\mathbf{v})$$

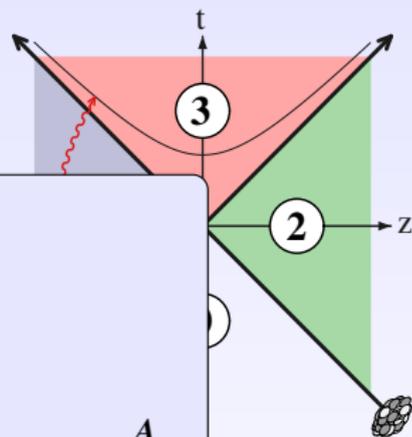
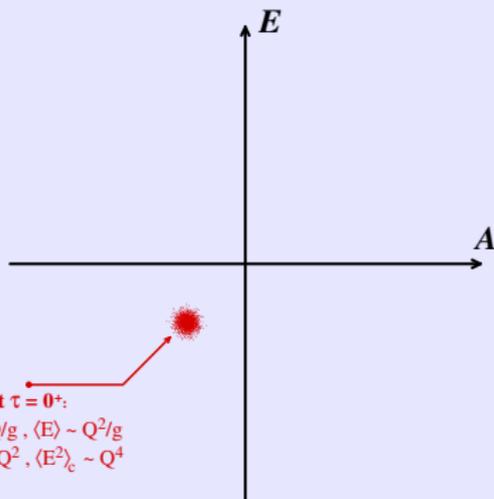
$$\left[ \mathcal{D}_\rho \mathcal{D}^\rho \delta_{\mu\nu}^y \right]$$

$$\lim_{x^0 \rightarrow -\infty} \alpha_k(x)$$

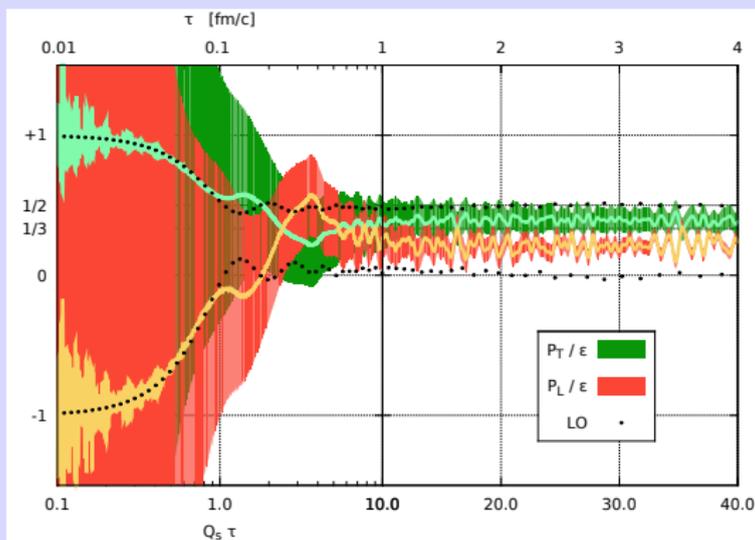
CGC at  $\tau = 0^+$ :

$$\langle A \rangle \sim Q/g, \quad \langle E \rangle \sim Q^2/g$$

$$\langle A^2 \rangle_c \sim Q^2, \quad \langle E^2 \rangle_c \sim Q^4$$



$g = 0.5$



- Nearly constant  $P_L/P_T$  for  $Q_s \tau \gtrsim 2$

[Berges, Boguslavski, Schlichting, Venugopalan (2013)]

Dense gas of free gluons

$$\langle \mathcal{A}^\mu \rangle = 0 \quad \text{Var.} = \int_{\text{modes } \mathbf{k}} f_0(\mathbf{k}) a_{\mathbf{k}}(\mathbf{u}) a_{\mathbf{k}}^*(\mathbf{v}) \quad a_{\mathbf{k}}(\mathbf{x}) \equiv e^{i\mathbf{k} \cdot \mathbf{x}}$$

$$f_0(\mathbf{k}) \sim (n_0/g^2) \times \theta(Q - \sqrt{k_\perp^2 + \xi_0 k_z^2})$$

BBSV :

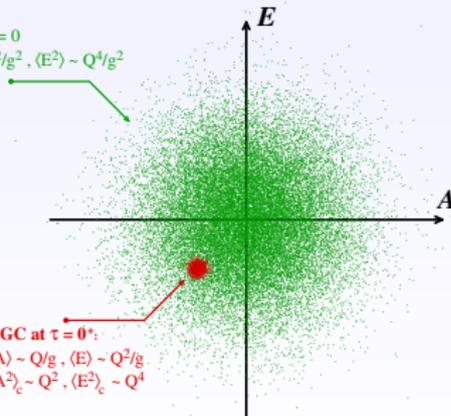
$$\langle A \rangle, \langle E \rangle = 0$$

$$\langle A^2 \rangle \sim Q^2/g^2, \langle E^2 \rangle \sim Q^4/g^2$$

CGC at  $\tau = 0^+$ :

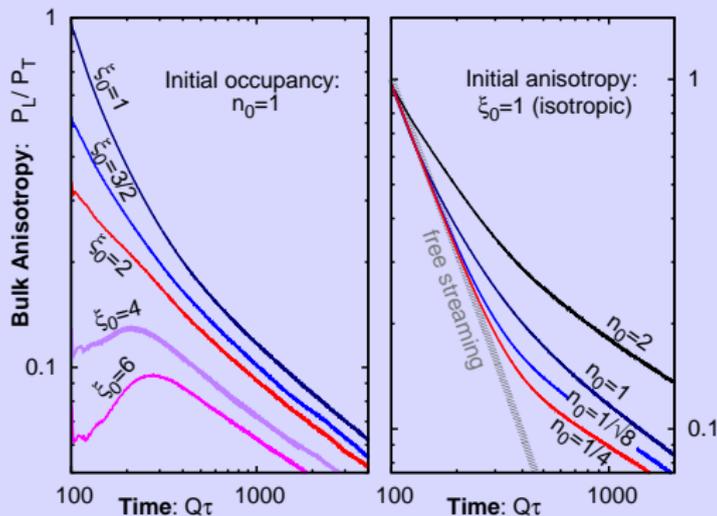
$$\langle A \rangle \sim Q/g, \langle E \rangle \sim Q^2/g$$

$$\langle A^2 \rangle_c \sim Q^2, \langle E^2 \rangle_c \sim Q^4$$





Same result for all  $g$ 's



- Self-similar evolution with scaling laws consistent with the **small angle scattering** analysis of **Baier, Mueller, Schiff, Son (2002)** :

$$f(t, p_{\perp}, p_z) \sim \tau^{-2/3} f_s(\tau^0 p_{\perp}, \tau^{1/3} p_z) \quad \frac{P_L}{P_T} \sim \tau^{-2/3}$$

# **Lessons from Kinetic Theory**

- Boltzmann equation for the elastic process  $12 \rightarrow 34$  :

$$\partial_t f_1 = \frac{1}{\omega_1} \int_{2,3,4} \delta^{(4)}(P_1+P_2-P_3-P_4) |\mathcal{M}_{12,34}|^2 \left[ f_3 f_4 (1+f_1)(1+f_2) - f_1 f_2 (1+f_3)(1+f_4) \right]$$

- Kinetic analogue of **CSA<sub>part</sub>** : keep only the cubic terms in  $f$

$$f_3 f_4 (1+f_1)(1+f_2) - f_1 f_2 (1+f_3)(1+f_4) \rightarrow f_3 f_4 (f_1 + f_2) - f_1 f_2 (f_3 + f_4)$$

- Kinetic analogue of **CSA<sub>vac</sub>** : from the cubic terms, do  $f \rightarrow f + \frac{1}{2}$

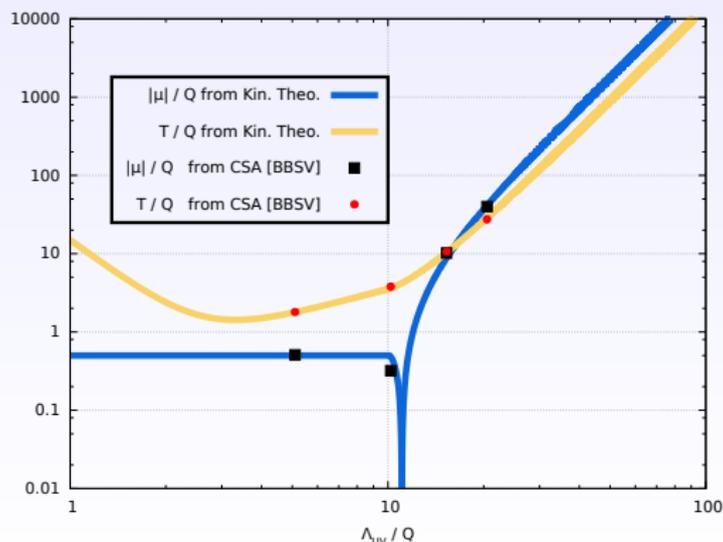
$$f_3 f_4 (1+f_1)(1+f_2) - f_1 f_2 (1+f_3)(1+f_4) \rightarrow \left(\frac{1}{2} + f_3\right)\left(\frac{1}{2} + f_4\right)(1+f_1+f_2) - \left(\frac{1}{2} + f_1\right)\left(\frac{1}{2} + f_2\right)(1+f_3+f_4)$$

- The Boltzmann equation can be used to assess the effect of these two approximations by comparing their solutions with that of the unapproximated equation

[Berges, Boguslavski, Schlichting, Venugopalan (2013)]

[Epelbaum, FG, Tanji, Wu (2014)]

- $CSA_{vac}$  is a nonrenormalizable approximation of the original QFT
- Dependence on the UV cutoff. Can also be seen in kinetic theory
- At late times,  $f(p) = \frac{T}{\omega_p - \mu} - \frac{1}{2}$ , but  $T$  and  $\mu$  depend on  $\Lambda_{UV}$

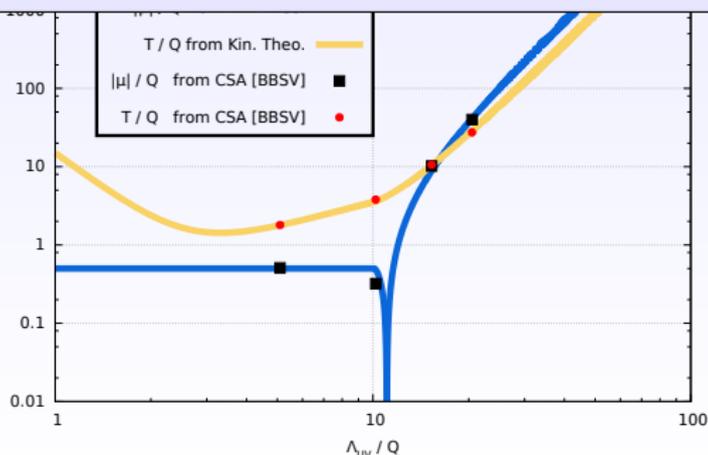


[Berges, Boguslavski, Schlichting, Venugopalan (2013)]

[Epelbaum, FG, Tanji, Wu (2014)]

- $CSA_{vac}$  is a nonrenormalizable approximation of the original QFT
- Dependence on the UV cutoff. Can also be seen in kinetic theory

- Strong cutoff dependence if  $\Lambda_{UV} \gg$  physical scales
- Mild sensitivity if  $\Lambda_{UV} \sim [3 - 6] \times$  (physical scales)

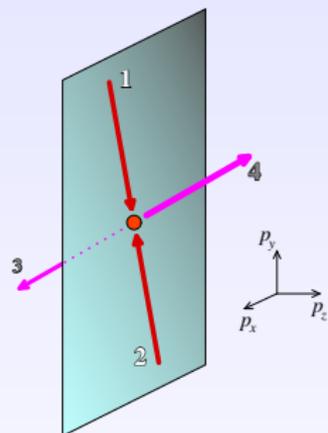


[Epelbaum, FG, Jeon, Moore, Wu (2015)]

- Kinetic version of CSA<sub>part</sub> :

$$\partial_t f_4 \sim g^4 \int_{123} \cdots [f_1 f_2 (f_3 + f_4) - f_3 f_4 (f_1 + f_2)] \\ + \cdots [f_1 f_2 - f_3 f_4]$$

[Epelbaum, FG, Jeon, Moore, Wu (2015)]



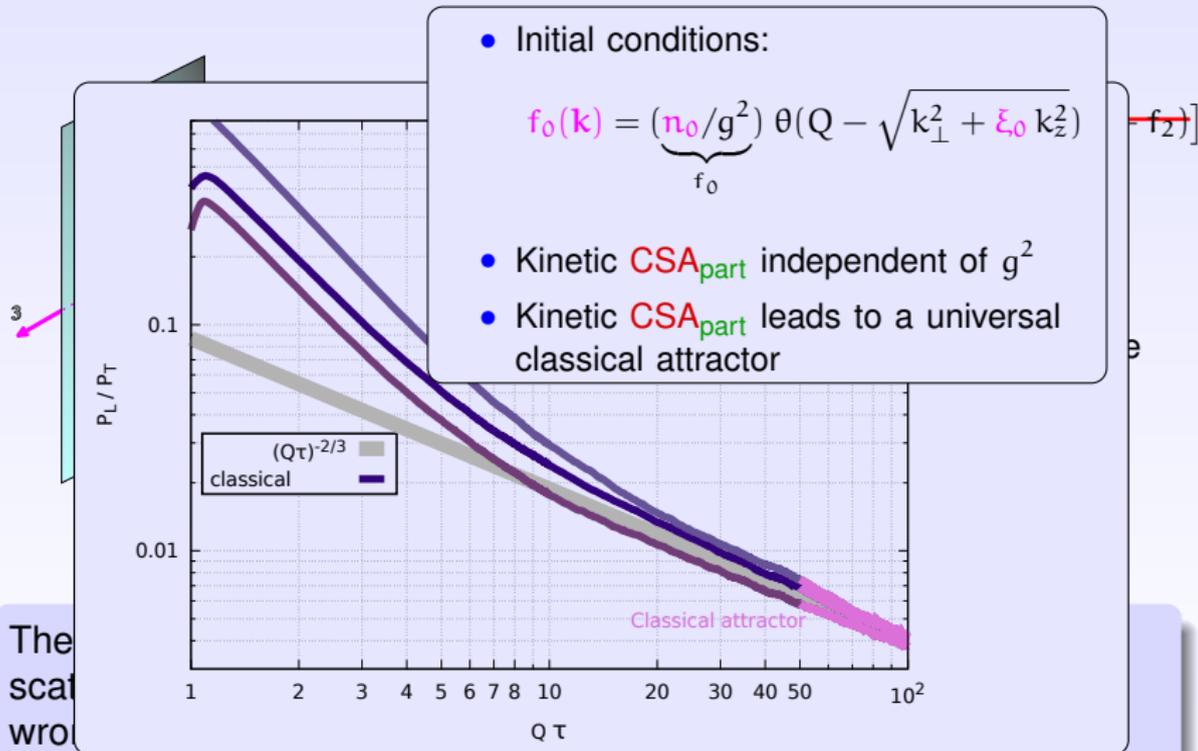
- Kinetic version of CSA<sub>part</sub> :

$$\partial_t f_4 \sim g^4 \int_{123} \dots [f_1 f_2 (f_3 + f_4) - f_3 f_4 (f_1 + f_2)] + \dots [f_1 f_2 - f_3 f_4]$$

- If the distribution becomes very anisotropic, trying to produce the particle 4 at large angle results in  $f_3 \approx f_4 \approx 0 \Rightarrow$  nothing left
- The same argument applies also to any inelastic  $n \rightarrow n'$  scattering

The CSA without vacuum fluctuations underestimates large angle scatterings when the distribution is anisotropic, and may lead to wrong conclusions regarding isotropization

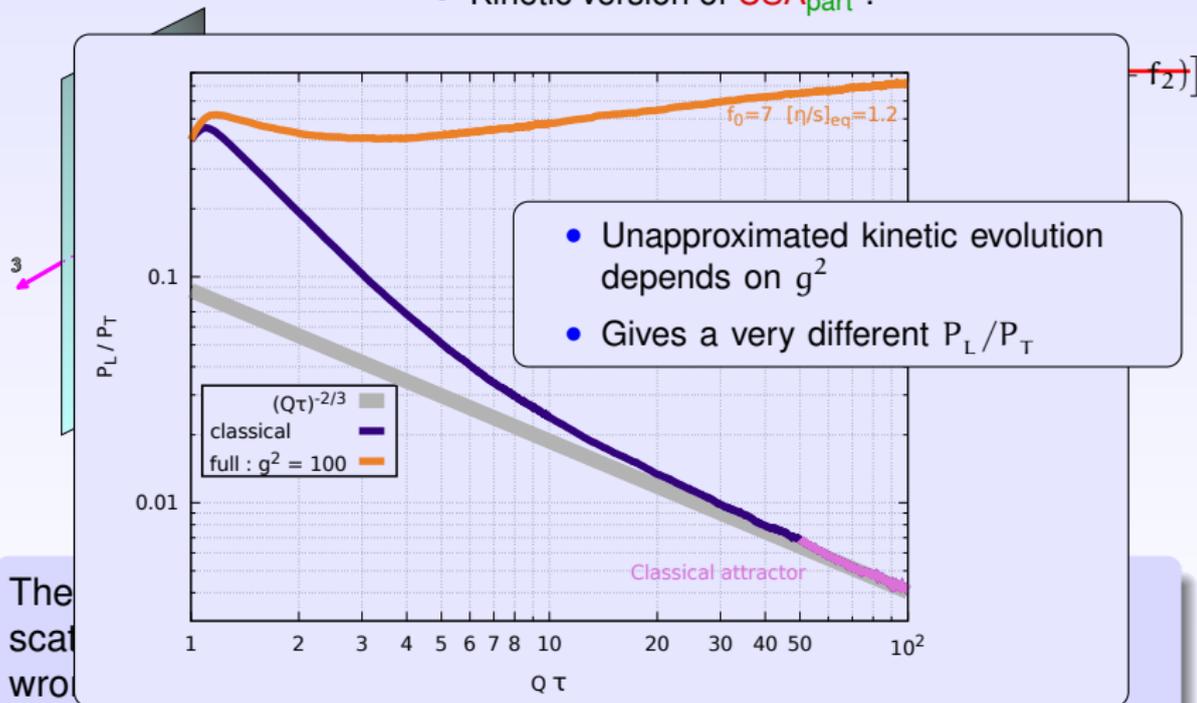
[Epelbaum, FG, Jeon, Moore, Wu (2015)]





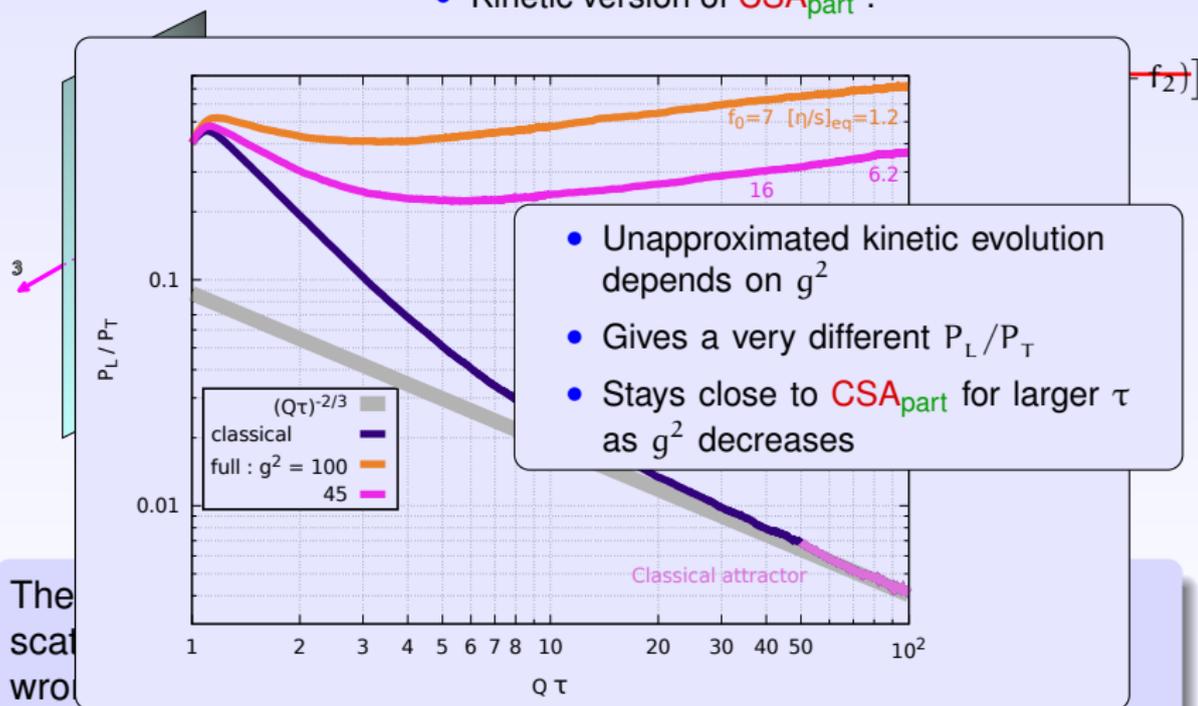
[Epelbaum, FG, Jeon, Moore, Wu (2015)]

- Kinetic version of CSA<sub>part</sub> :



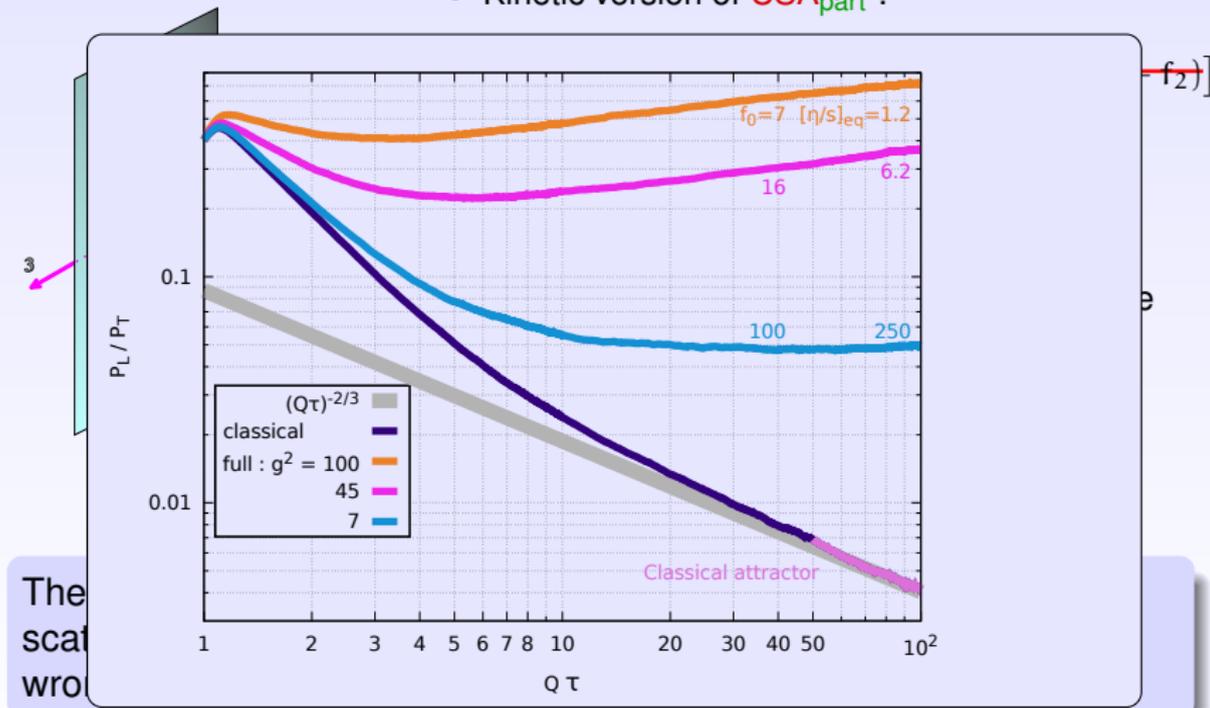
[Epelbaum, FG, Jeon, Moore, Wu (2015)]

- Kinetic version of CSA<sub>part</sub> :



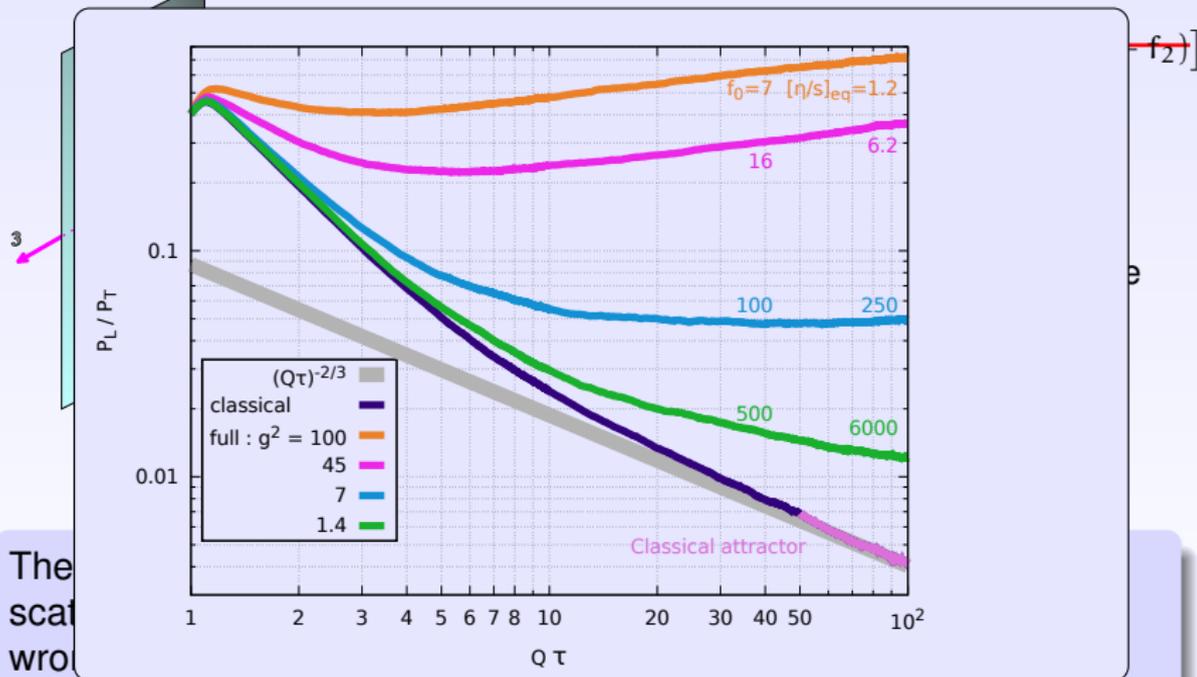
[Epelbaum, FG, Jeon, Moore, Wu (2015)]

- Kinetic version of CSA<sub>part</sub> :



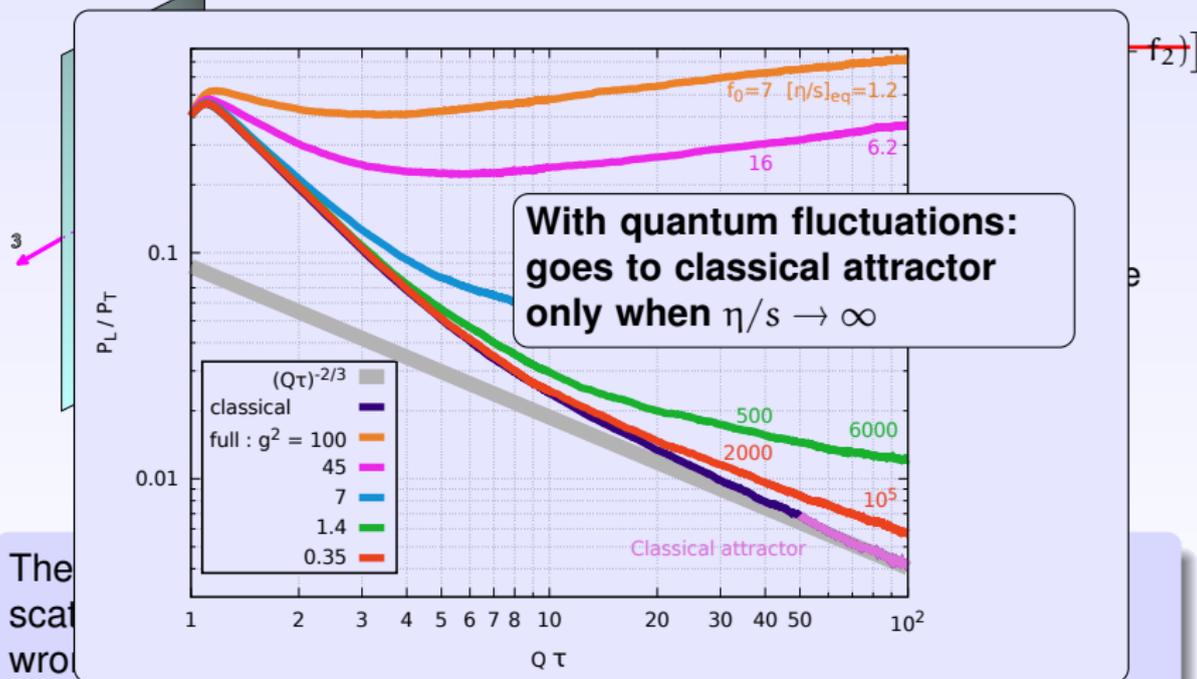
[Epelbaum, FG, Jeon, Moore, Wu (2015)]

- Kinetic version of CSA<sub>part</sub> :

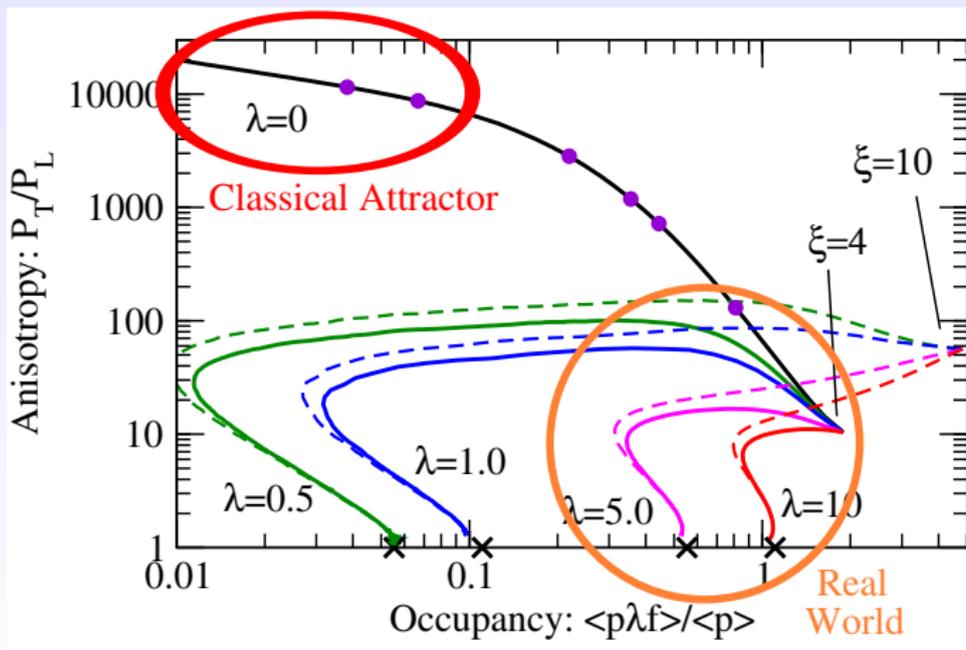


[Epelbaum, FG, Jeon, Moore, Wu (2015)]

- Kinetic version of CSA<sub>part</sub> :



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# Summary

## Summary

- LO : no pressure isotropization, NLO : instabilities
- Resummation beyond NLO : Classical statistical approximation
- Two implementations... and two different results :
  - $CSA_{vac}$  : vacuum-like CGC initial conditions  
roughly constant  $P_L/P_T$ ,  
non-renormalizable approximation, very sensitive to UV cutoff
  - $CSA_{part}$  : particle-like initial conditions  
universal classical attractor,  $P_L/P_T$  decreases forever,  
underestimates large angle scatterings,  
breaks long before reaching the attractor even for quite large  $\eta/s$
- In the present situation, classical field simulations need to be corroborated and validated by other approaches
- Highly needed : ways to overcome the problems of the classical statistical approximation (Kinetic theory, 2-PI,...)