Correlations & Fluctuations in Large & Small Systems

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Peking University

mini-symposium on "Computational Physics for High-Energy Heavy-Ion Collisions"

YITP

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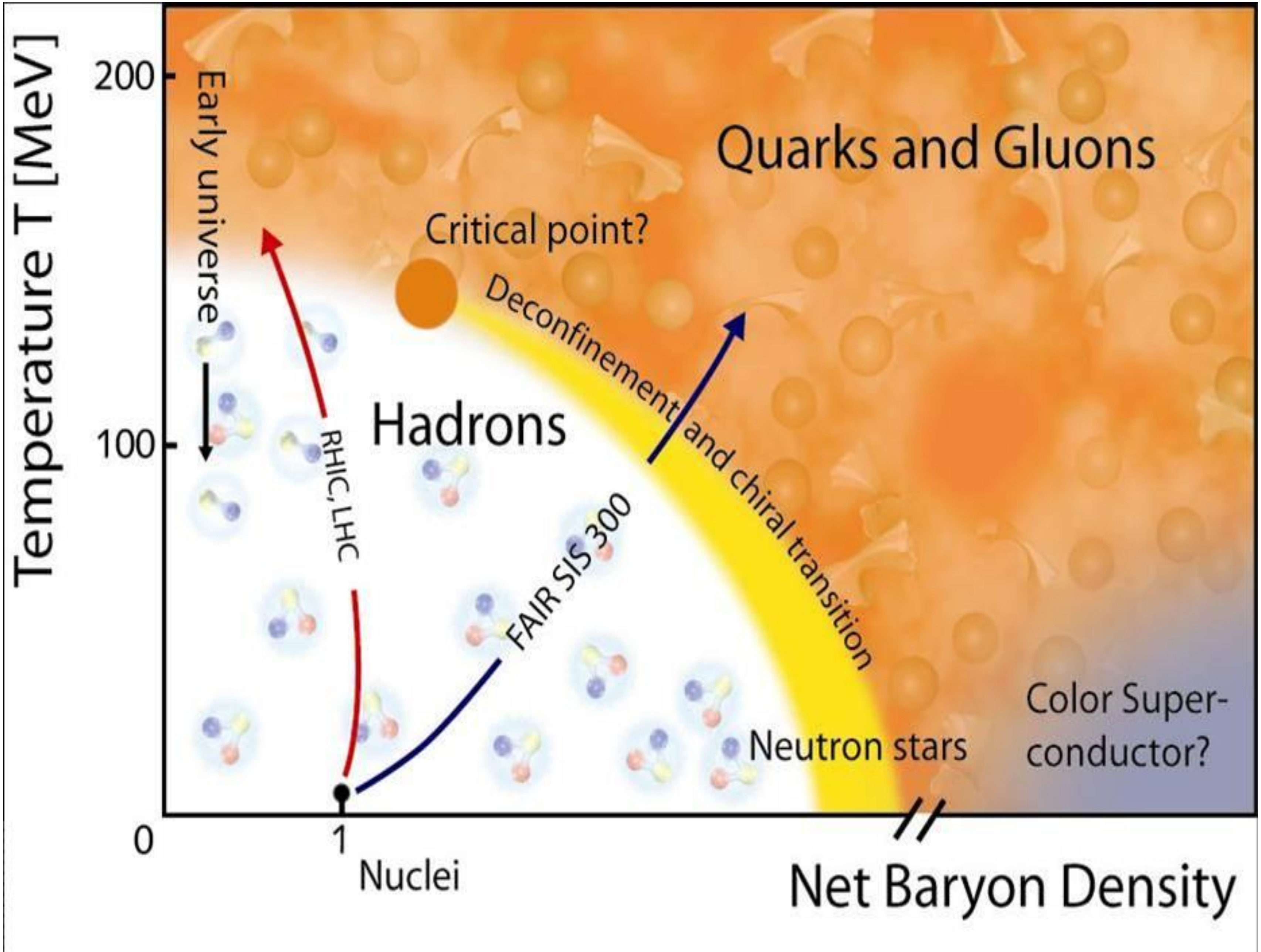
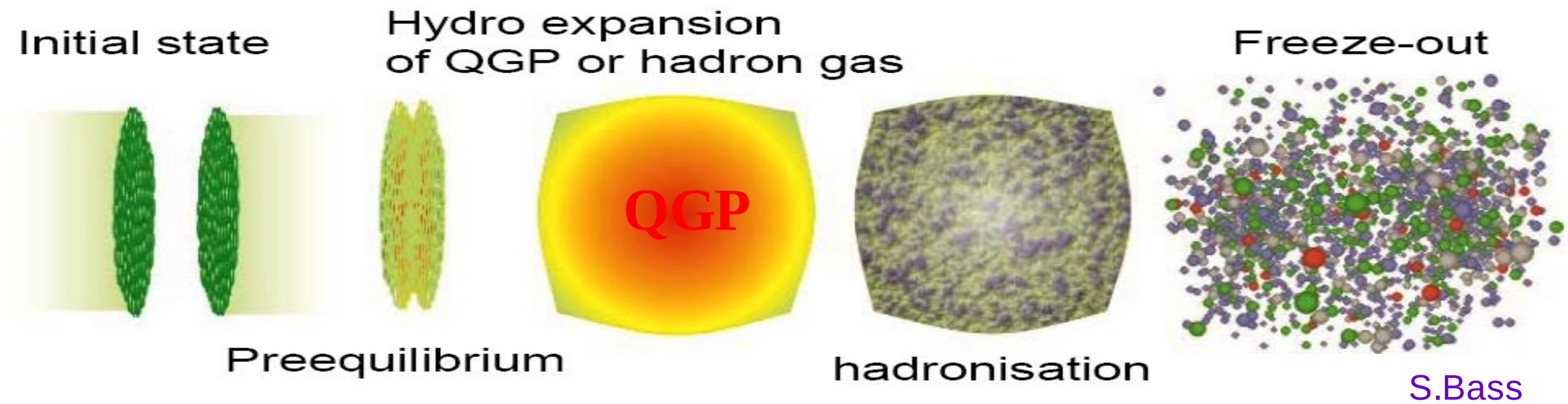
Oct. 6, 2015



Temperature T [MeV]
200
100
0
1
Nuclei
Net Baryon Density
Early universe
RHIC, LHC
FAIR SIS 300
Hadrons
Critical point?
Quarks and Gluons
Deconfinement and chiral transition
Neutron stars
Color Super-
conductor?



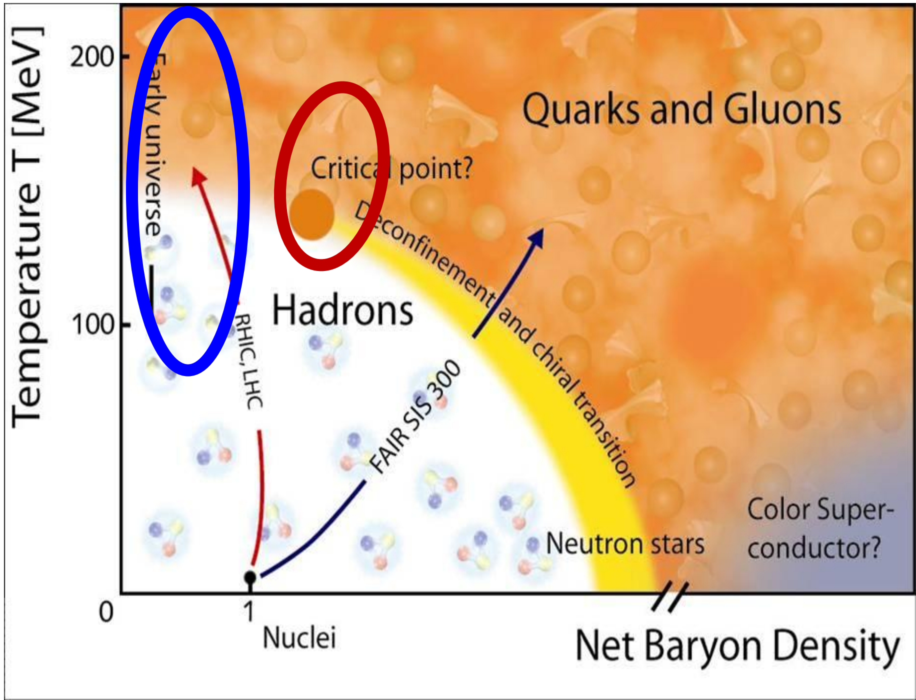
Dynamical Modeling for Relativistic Heavy Ion Collisions

Hydrodynamics & its hybrid model

- -QGP viscosity
- -initial state flucutations. & final state correlations
- -Collective flow in small systems

Dynamical modeling near the QCD critical point

- -Correlated fluctuations



Hydrodynamics & its Hybrid Model

Viscous Hydrodynamics

$$\partial_\mu T^{\mu\nu}(x) = 0$$

$$\tau_\pi \Delta^{\alpha\mu}\Delta^{\beta\nu}\dot{\pi}_{\alpha\beta} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} - \frac{1}{2}\pi^{\mu\nu}\frac{\eta T}{\tau_\pi}\partial_\lambda\left(\frac{\tau_\pi}{\eta T}u^\lambda\right)$$

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta(\partial\cdot u) - \frac{1}{2}\Pi\frac{\zeta T}{\tau_\Pi}\partial_\lambda\left(\frac{\tau_\Pi}{\zeta T}u^\lambda\right)$$

Net baryon density: Frankfurt (2014), BNL(2015)

Heat conductivity: Mcgill … … on-going work

2nd order viscous hydro (I-S)

-2+1-d: OSU, INT, Stony Brook, Pudue, Calcutta (2008), Crakow, Frankfurt(2010) …

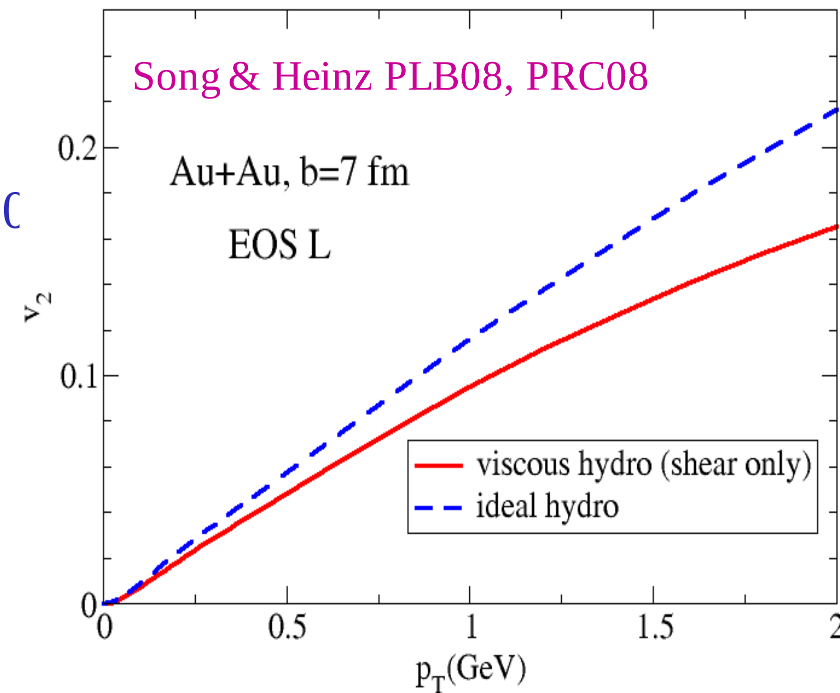
-3+1-d: Mcgill(2011), MSU(2012), Crakow(2C Nagoya(2013),Frankfurt(2013) … …

Anisotropic hydrodynamics

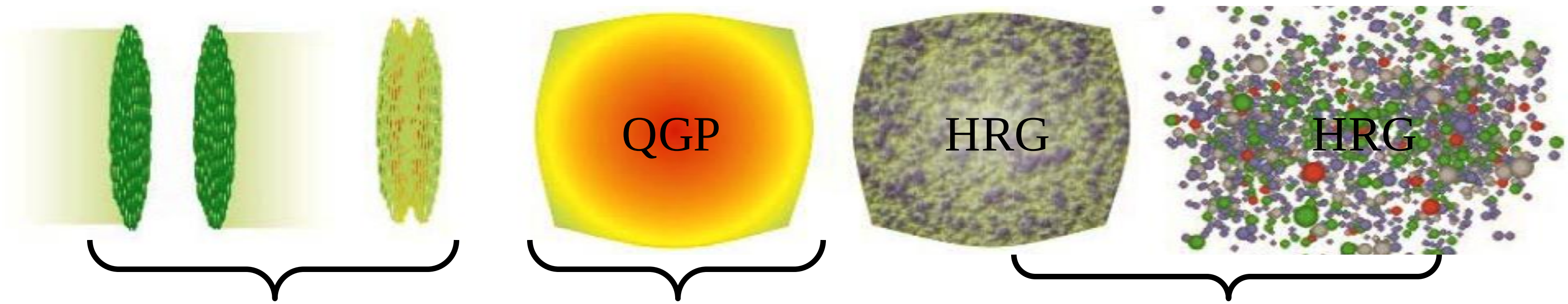
Frankfurt(2010), Cracow(2012), Kent,OSU (2013)

Hydrodynamics with thermal fluctuations

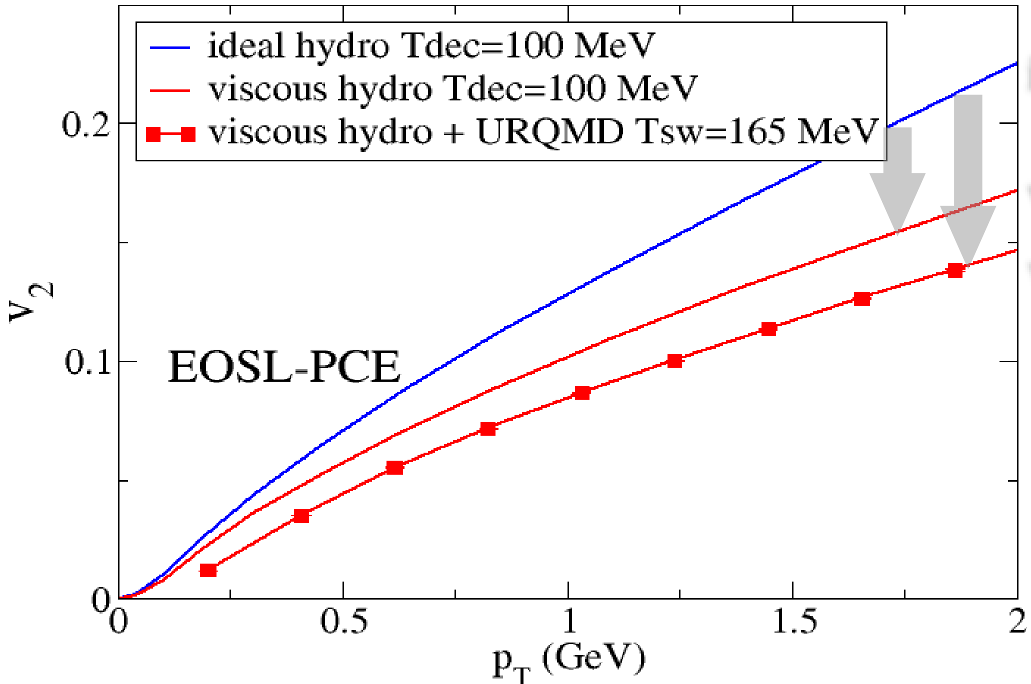
Sophia (2014)



Viscous Hydro + Hadron Cascade Hybrid Model



Initial conditions | viscous hydro | hadron cascade



ideal hydro (QGP & HRG)

viscous hydro (QGP & HRG)

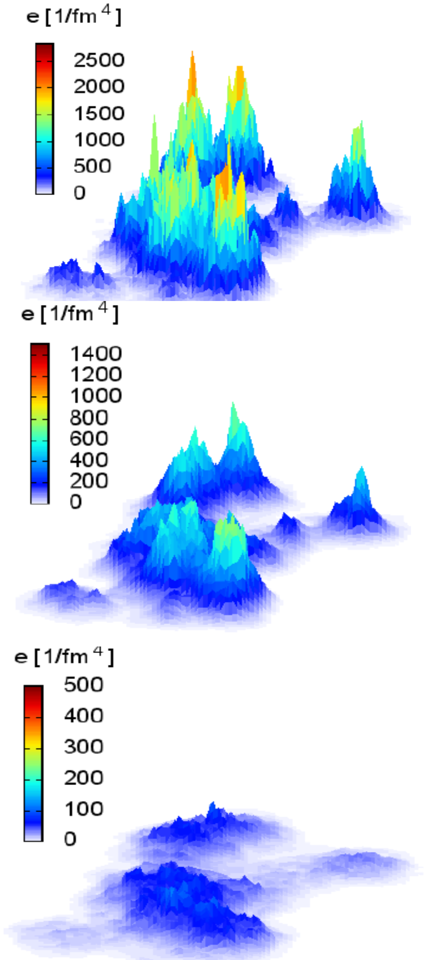
viscous hydro (QGP)
+ URQMD (HRG)

H. Song, S. Bass, U. Heinz, PRC2011

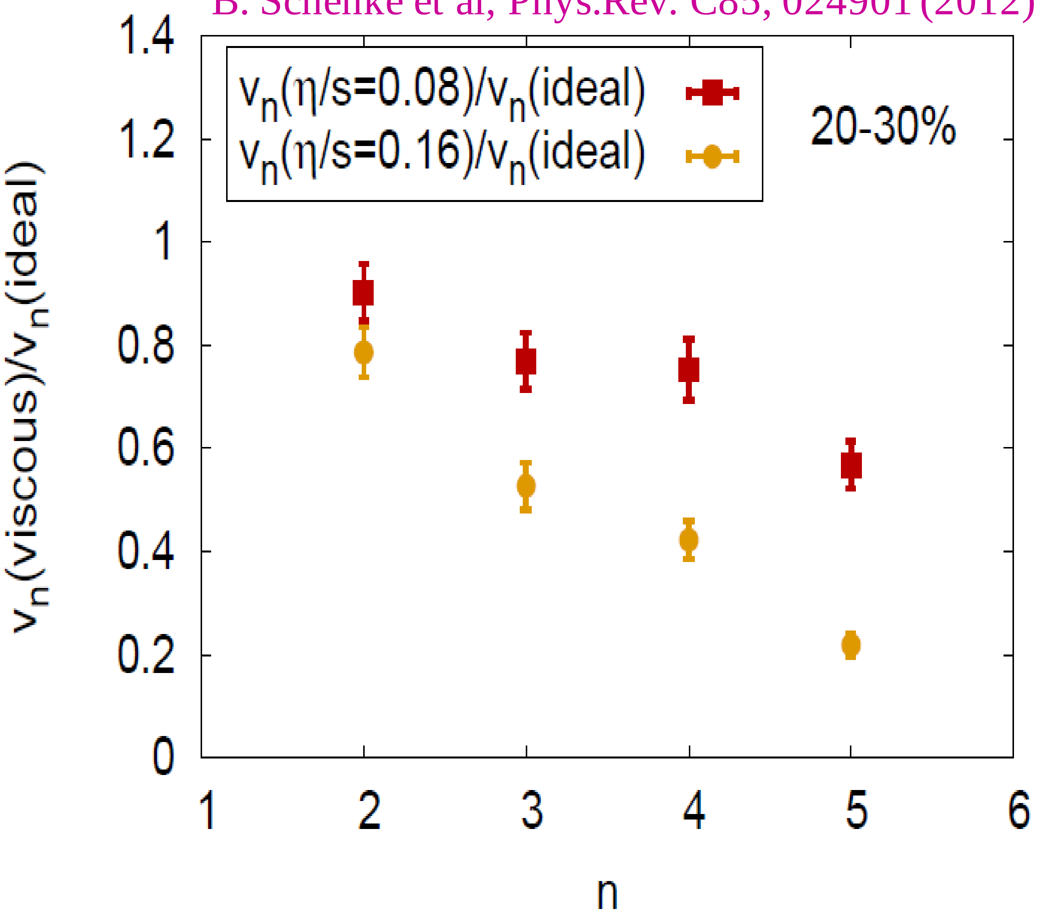
Event-by-event hydrodynamics

<u>Single shot simulations</u>: smoothed initial conditions (before 2010)

<u>E-b-E **simulations**:</u> fluctuating initial conditions (since 2010)



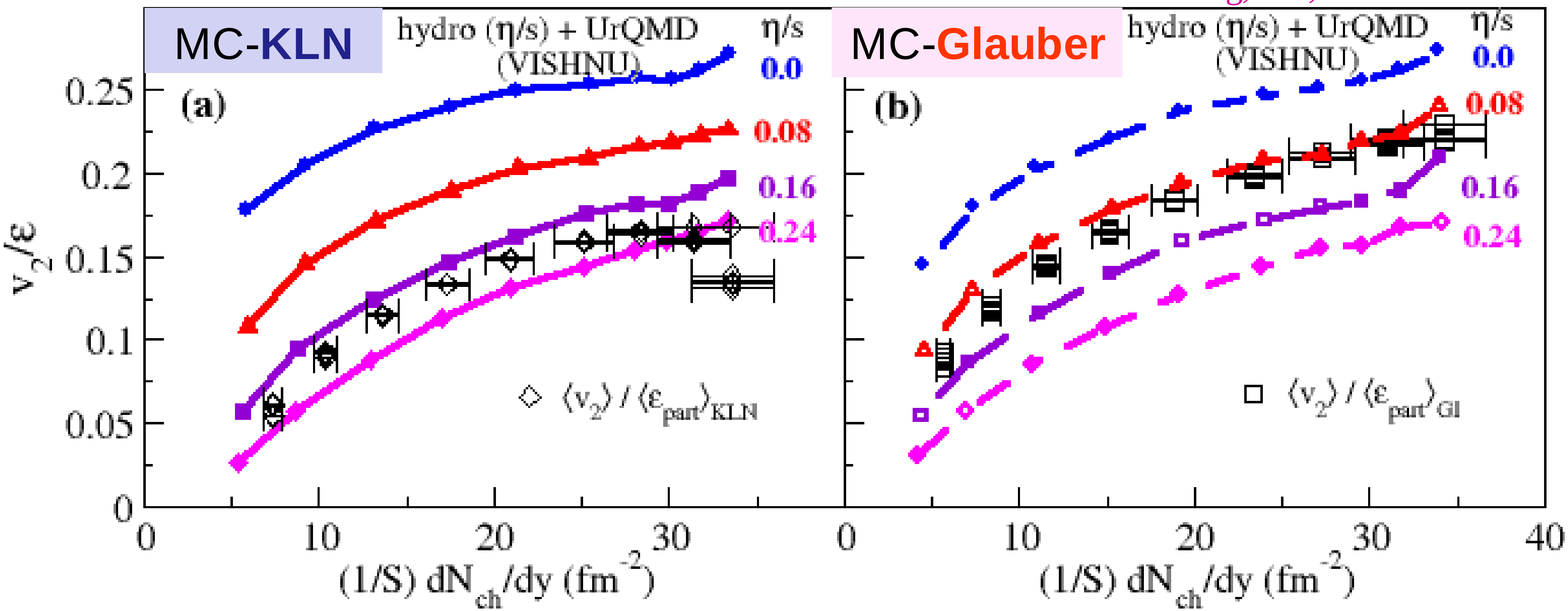
B. Schenke et al, Phys.Rev. C85, 024901 (2012)



QGP viscosity from flow data

VISHNU hybrid model & QGP viscosity

H. Song,et.al, PRL2011



$$1\times(1/4\pi)\leq(\eta/s)_{QGP}\leq 2.5\times(1/4\pi)$$

-Main uncertainties come from initial conditions

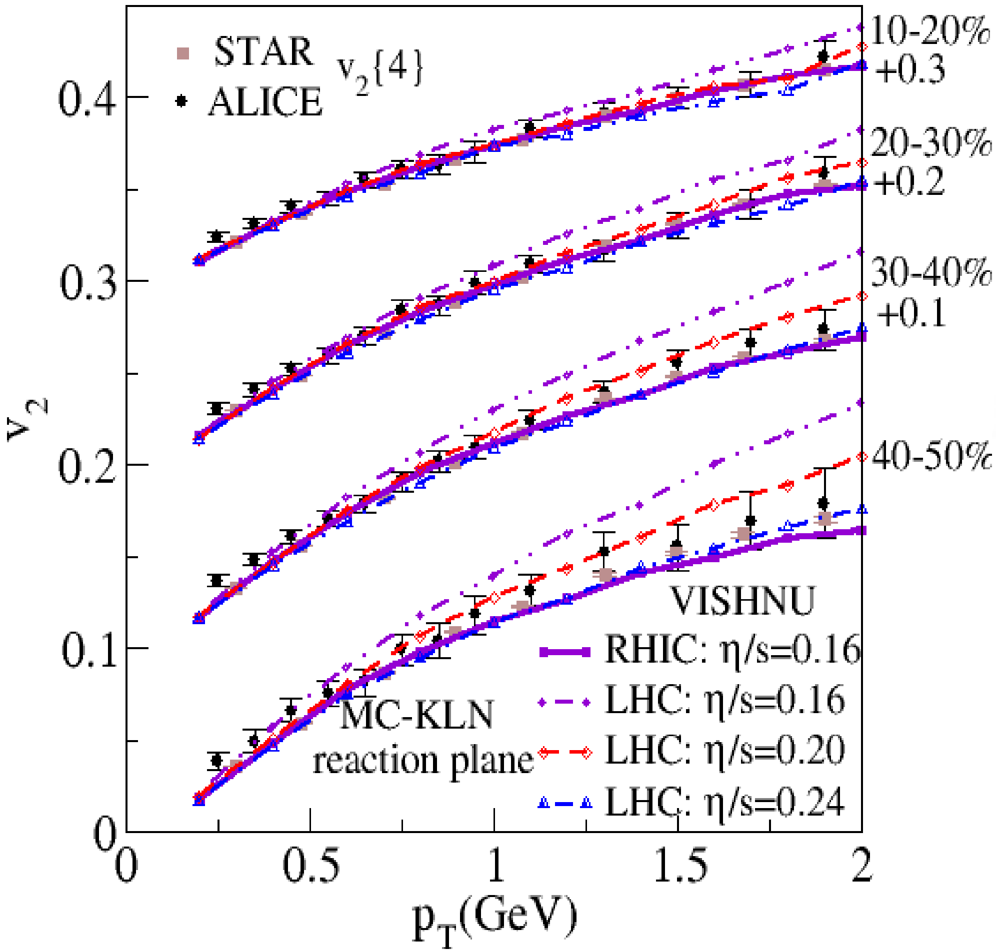
-Other uncertainties (much smaller)

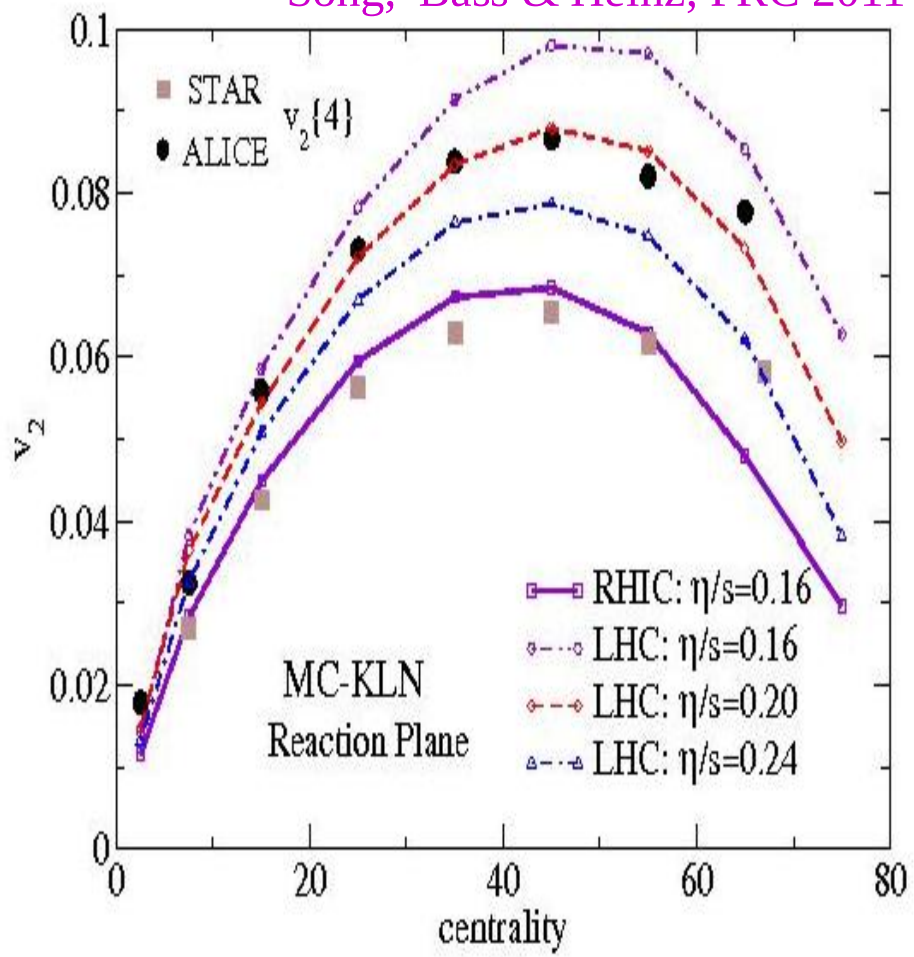
-Initial flow, bulk viscosity, single shot vs. e-b-e calculations

(each of them shift V_2 by a few percent, partial cancellation among them)

V_2 and QGP viscosity at the LHC

Song, Bass & Heinz, PRC 2011



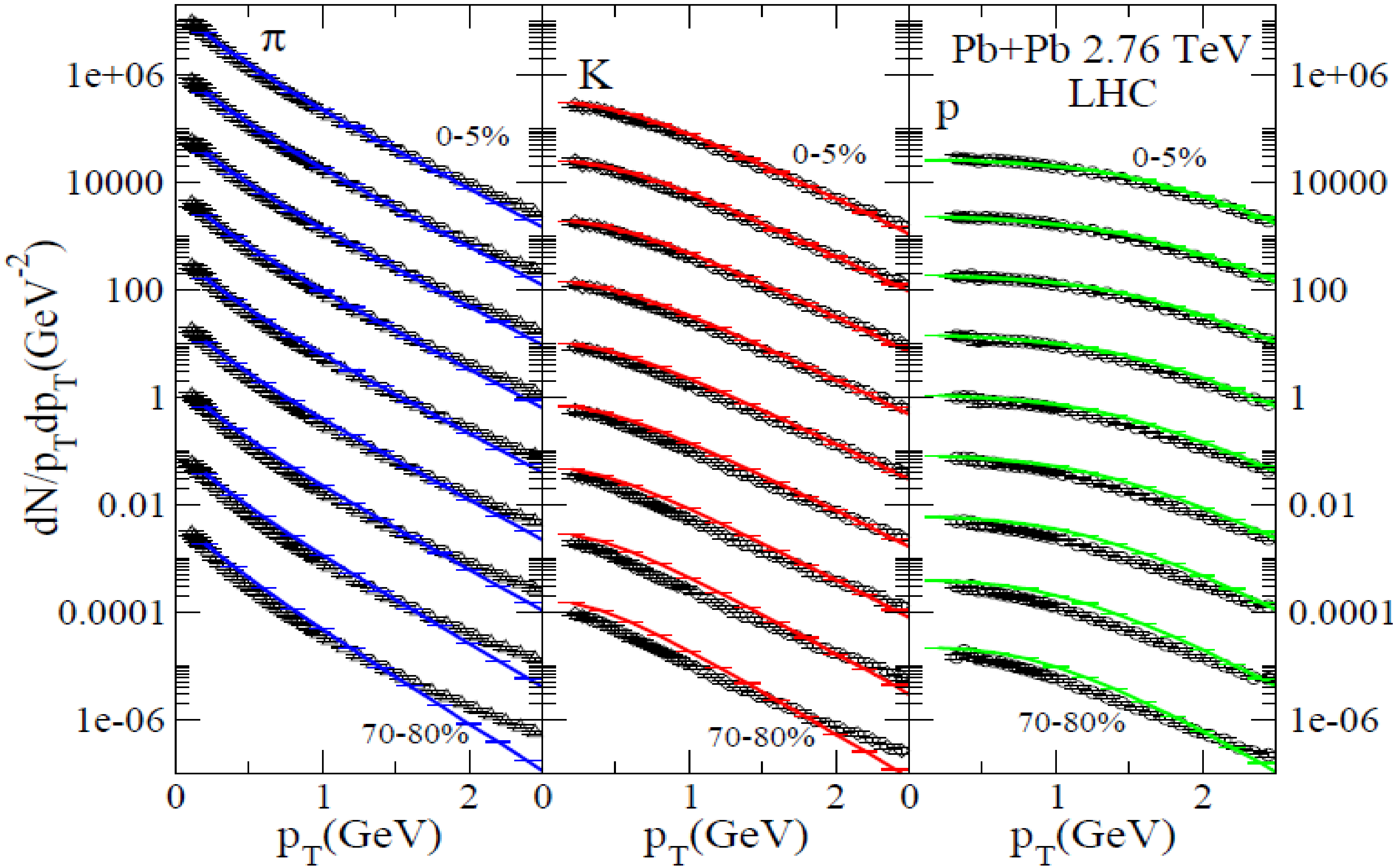


The average QGP viscosity is roughly the same at RHIC and LHC

Please also refer to C. Gale, et al., ArXiv: 1209.5330 [nucl-th]

LHC: spectra for identified hadrons

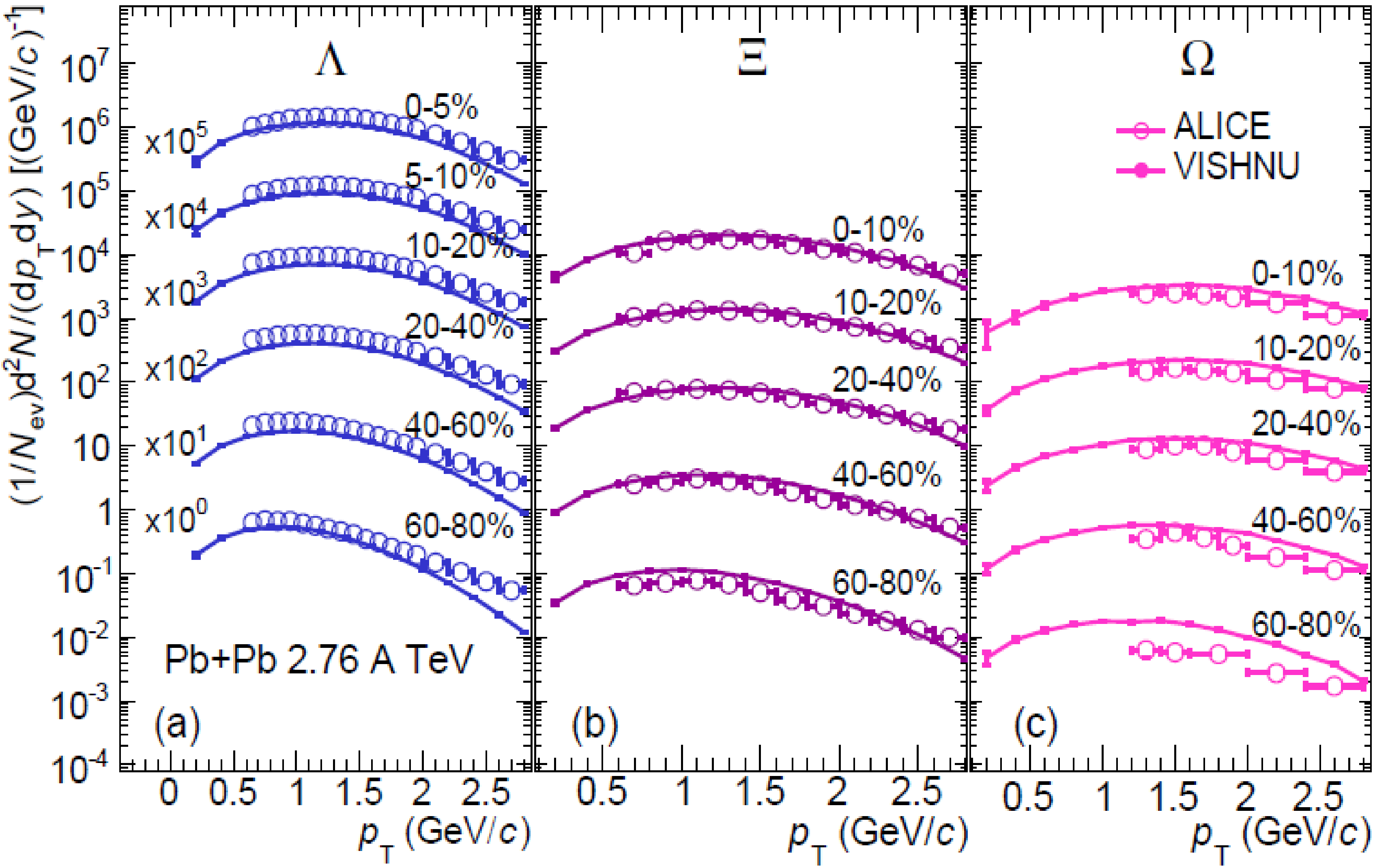
Song, Bass & Heinz,PRC 2014



-a nice fit of spectra for pions, kaons and proton

Spectra of Strange & Multi-strange Hadrons

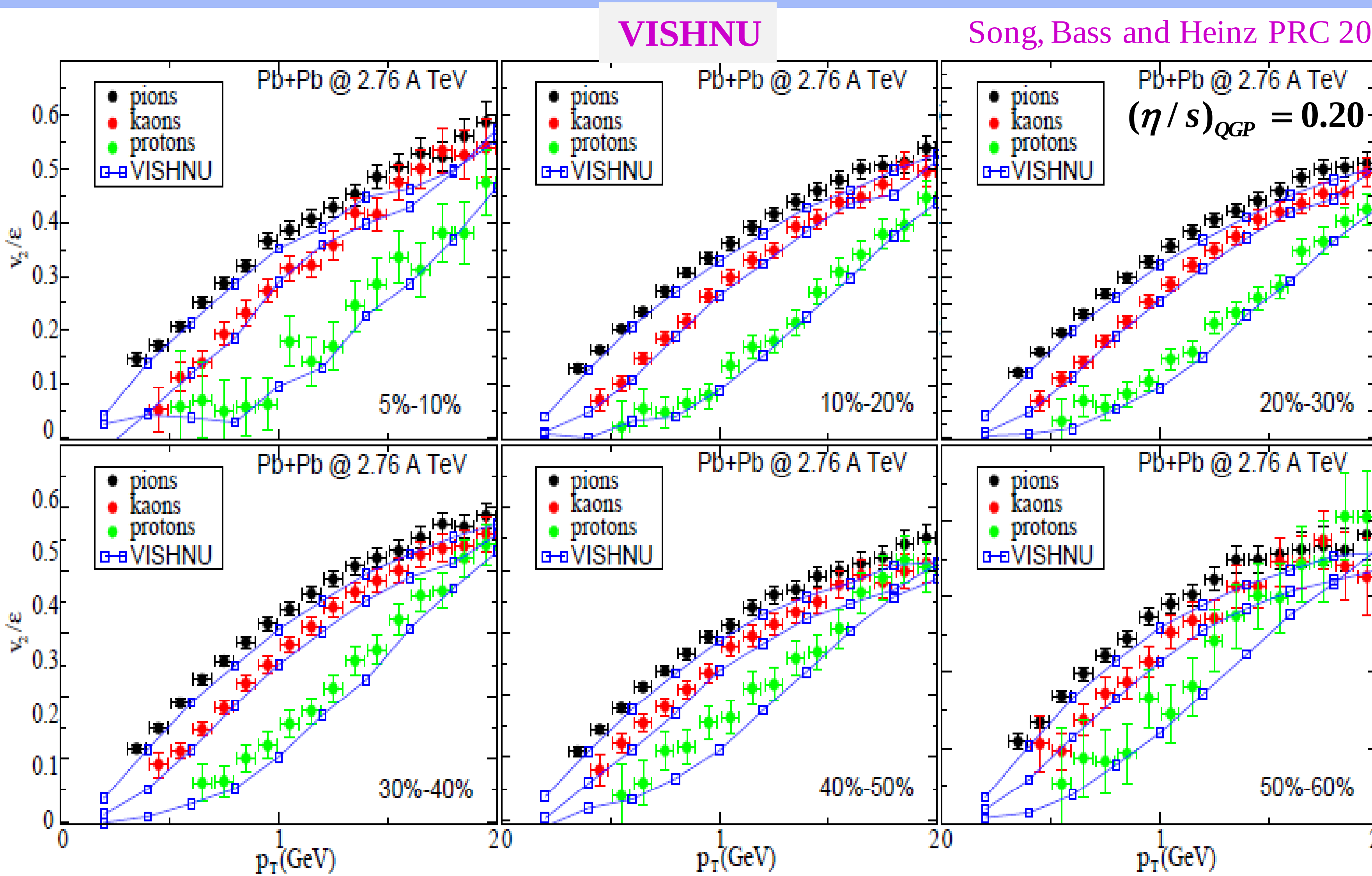
Zhu, Meng , Song, Liu PRC 2015



-a nice fit of spectra for Lambda, Xi, and Omega

$V_2(p_T)$ for pions, kaons & protons at LHC

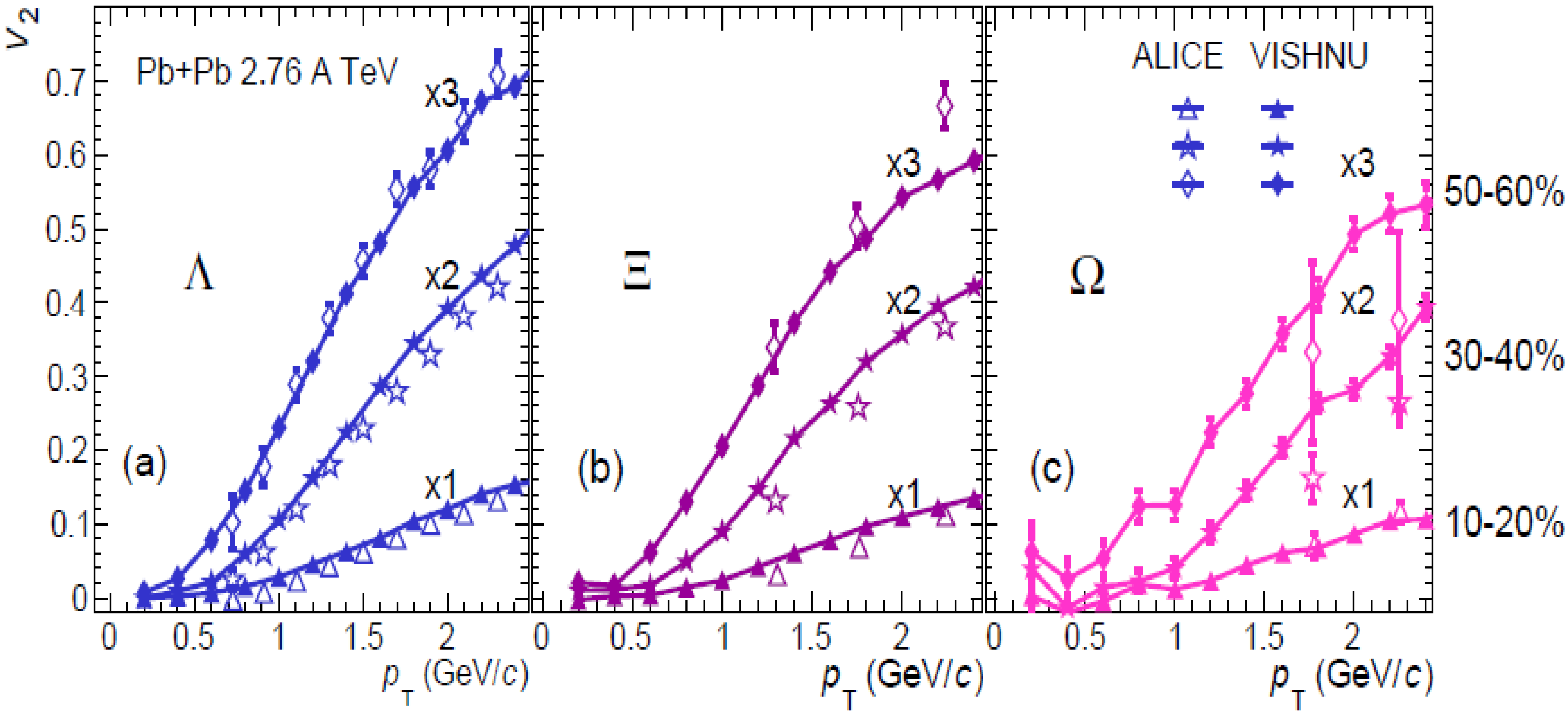
Song, Bass and Heinz PRC 2014



A very nice fit of $V_2(P_T)$ for all centrality bins at LHC from VISHNU hybrid model

v_2 of Strange & Multi-strange Hadrons

Zhu, Meng , Song, Liu PRC 2015

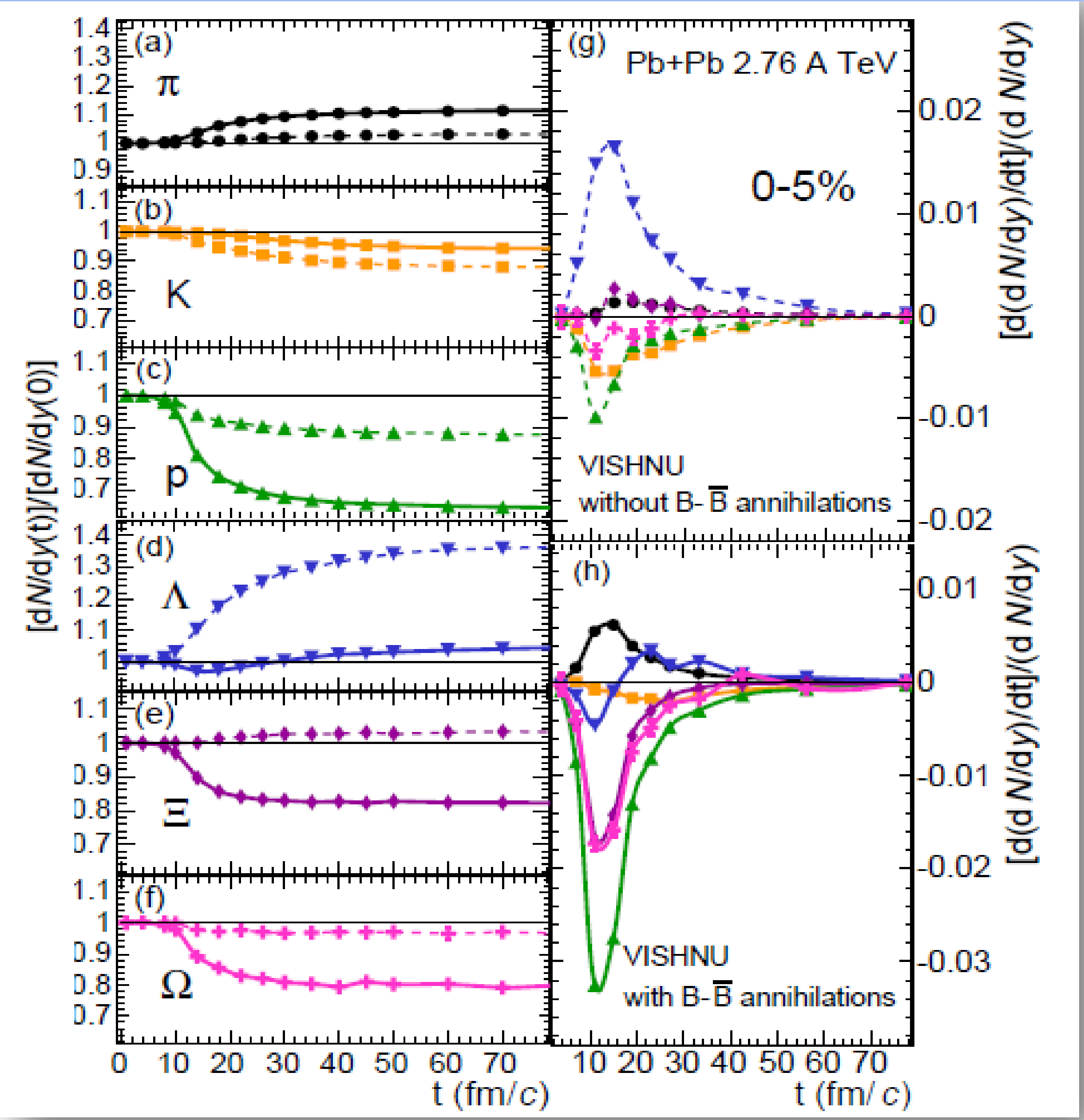


-Nice descriptions of Lambda, Xi and Omega v_2 at various centralities

Chemical & Thermal freeze-out

at the LHC

Chemical freeze-out for various hadrons

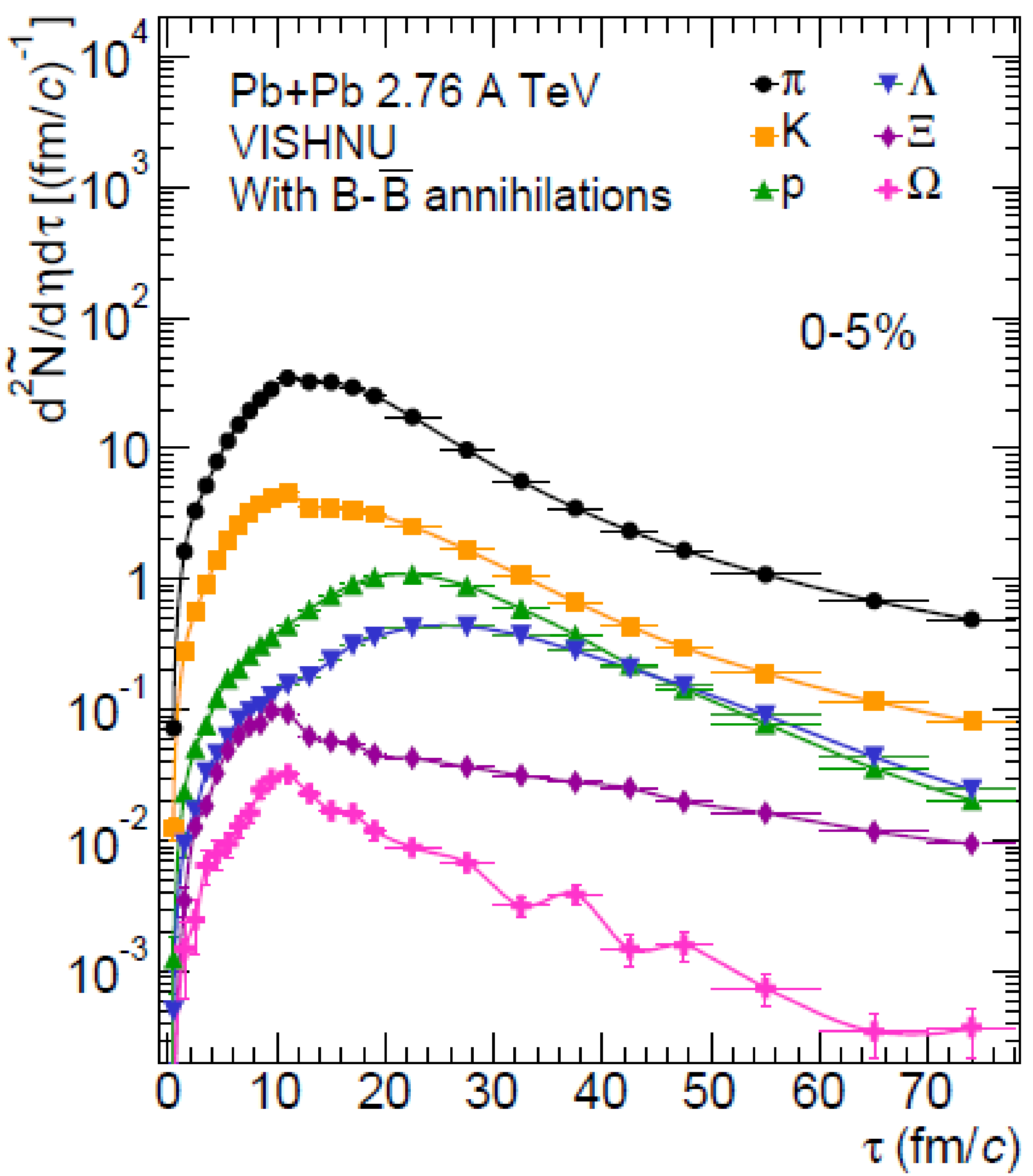


Zhu, Meng , Song, Liu PRC 2015

-Earlier Chemical freeze-out of Xi and Omega!

-Different hadrons may have different effective chemical freeze-out temperature

Thermal freeze-out of various hadrons



Zhu, Meng , Song, Liu PRC 2015

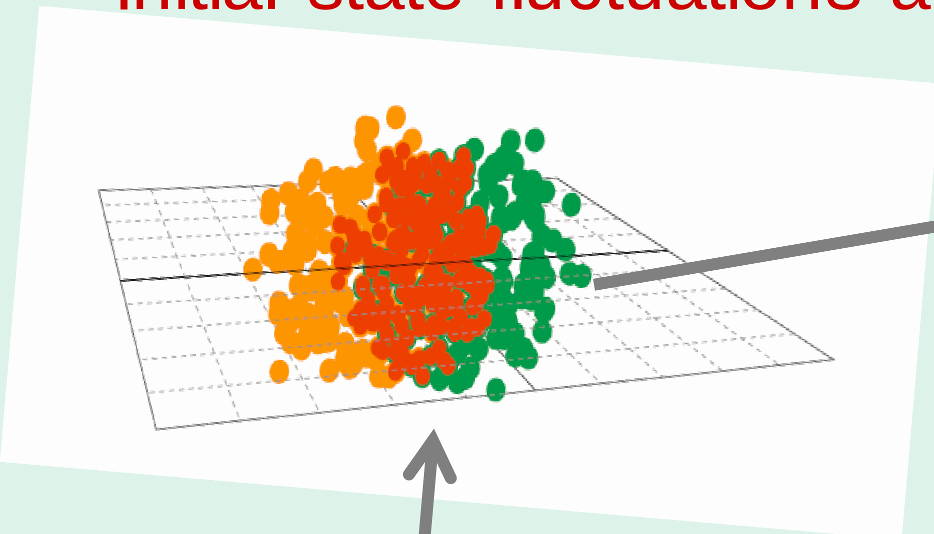
-thermal freeze-out time distributions widely spread for various hadrons

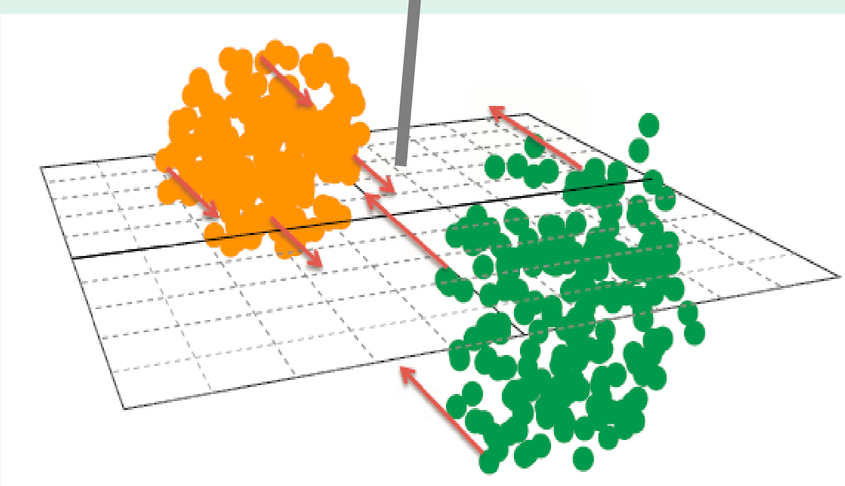
-Earlier thermal freeze-out of Xi and Omega!

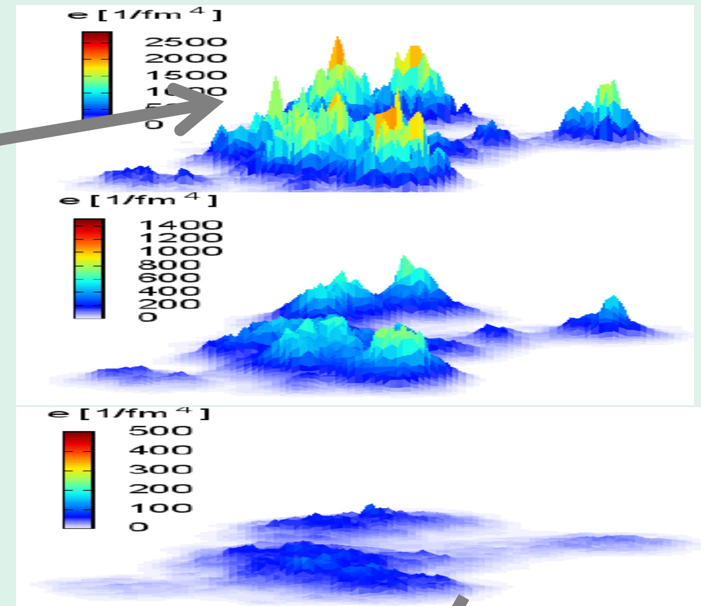
Please also refer to S. Takeuchi, et.al, arXiv: 1505.05961 [nucl-th]

EbE-Simulations

-Initial state fluctuations and final state correlations







$$E\frac{dN}{d^3p}=\frac{dN}{dyp_T dp_T d\varphi}$$

$$=\frac{1}{2\pi}\frac{dN}{dyp_T dp_T}[1+2v_1(p_T,b)\cos(\varphi)+2v_2(p_T,b)\cos(2\varphi)$$

$$+2v_3(p_T,b)\cos(3\varphi)......]$$

Initialization & Pre-equilibrium

-fluctuations of nucleon positions: **MC-Glauber, MC-KLN**

-fluctuations of color charges (in the framework of CGC):

IP-Glasma: B. Schenke et al., Phys.Rev. C85, 024901 (2012).

Correlated Fluctuation: B. Muller & A. Schafer, Phys.Rev. D85,114030 (2012).

-fluctuations of local gluon numbers (in the famework of MC-KLN):

Multiplicity fluctuations: A. Dumitru and Y. Nara, Phys. Rev. C 85, 034907 (2012).

-Pre-equilibriums:

URQMD initialization: H.Petersen & M. Bleicher, Phys. Rev. C81, 044906, (2010).

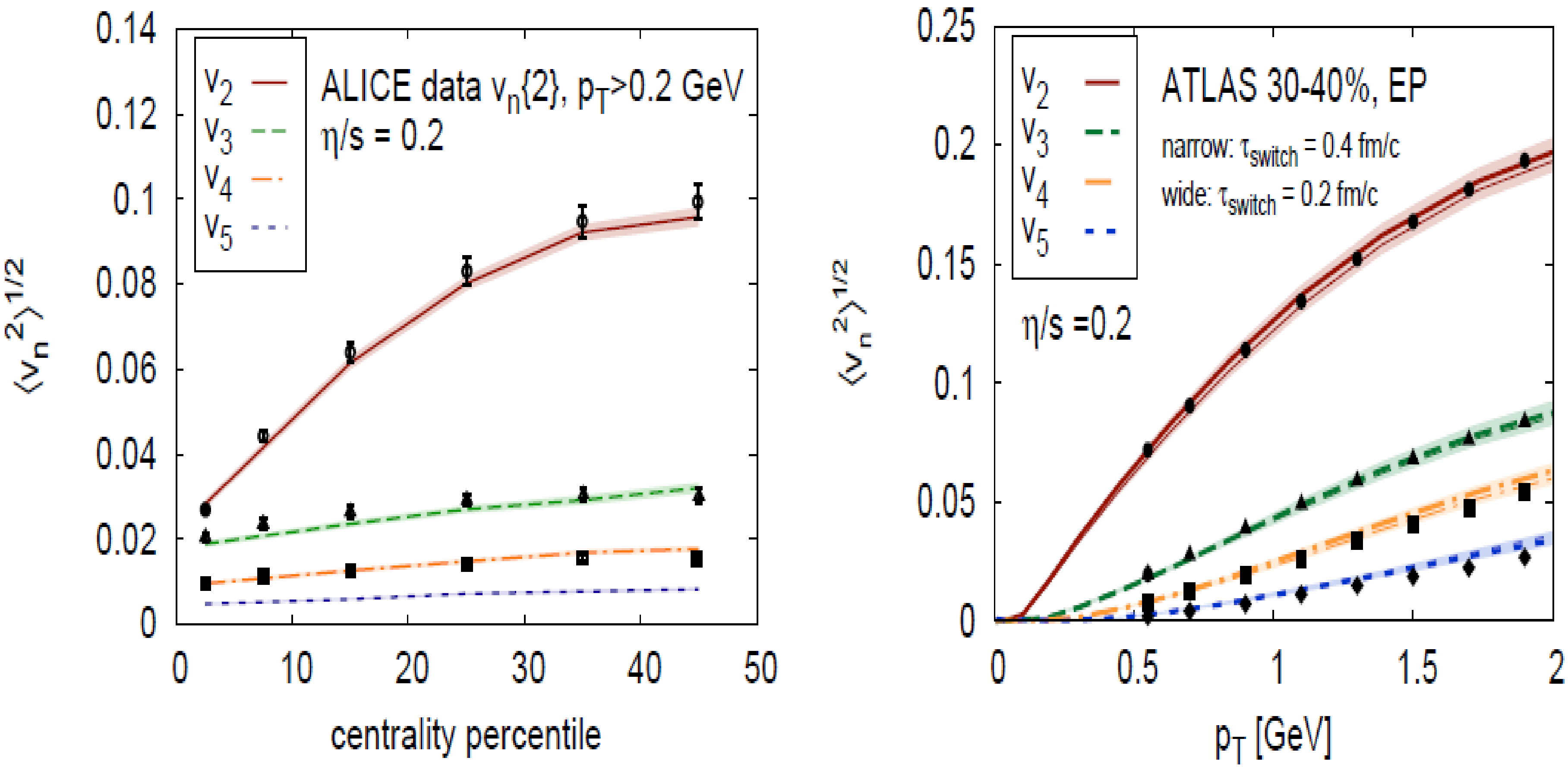
AMPT initialization: L. Pang, Q.Wang & X.Wang, Phys.Rev. C86, 024911(2012).

EPOS/NEXUS initialization: K. Werner et al., Phys. Rev. C83:044915, (2011).

… … … …

The Success of IP-Glasma + MUSIC

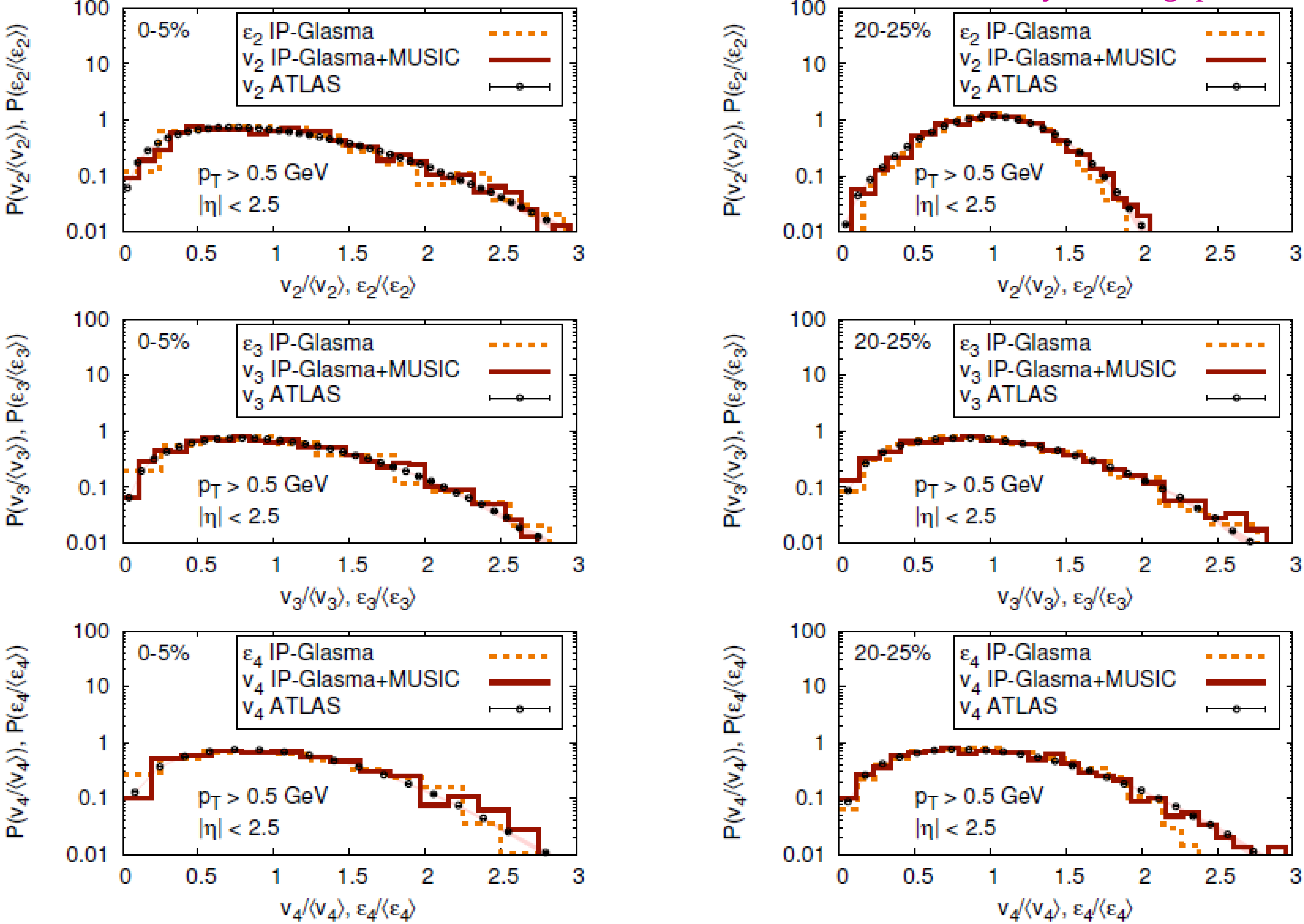
Gale, Jeon ,Schenke, Tribedy &Venugopalan PRL2013



-IP–Glasma: sub-nucleonic color fluctuations + CYM dynamics

-nice descriptions of integrated and differential V_n

Gale, Jeon ,Schenke, Tribedy &Venugopalan PRL2013

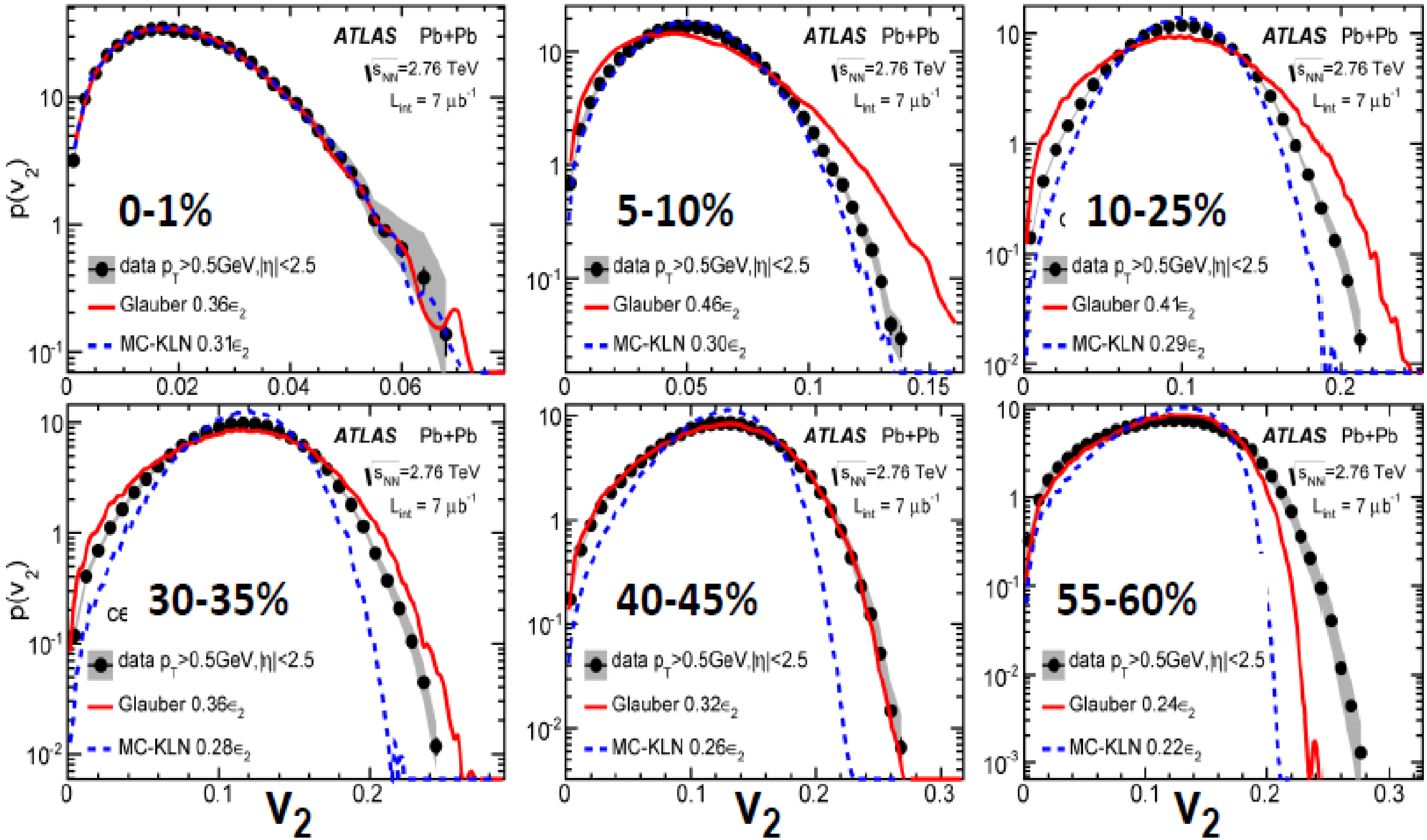


-In hydrodynamics, P(v_n) follows P(ε_n)

-A nice description of P(v_n) with IP-Glasma initialization

ε_n & V_n distributions (MC-Glauber vs. MC-KLN)

G. Aad et al. [ATLAS Collaboration], JHEP 1311, 183 (2013)

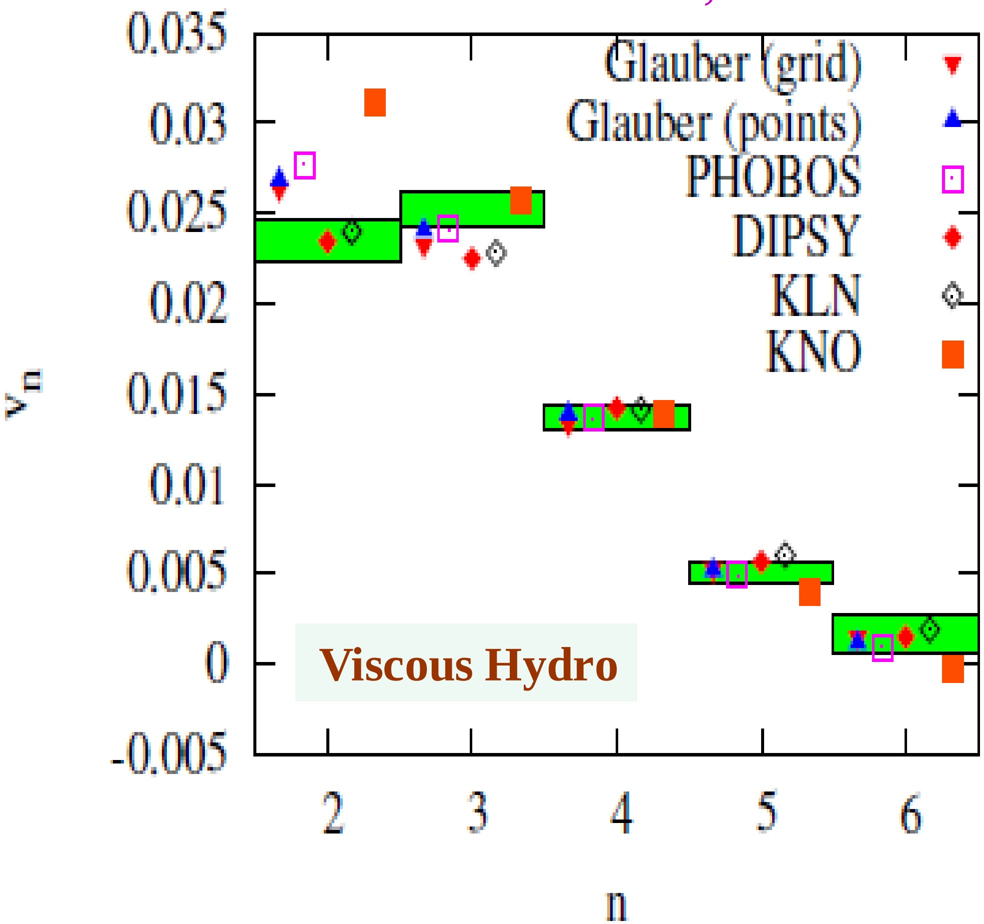


-Neither MC-Glauber nor MC-KLN works for all centralities

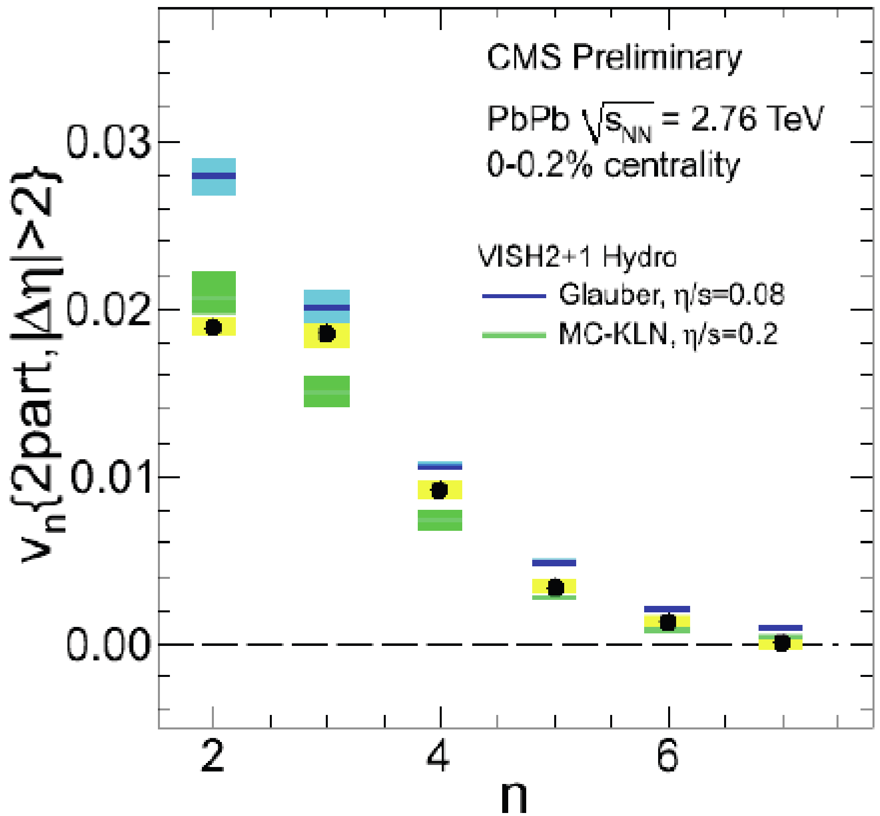
More flow observables

Extracting η/s from V_n in ultra-central collisions

Luzum & Ollitrault, NPA 2013



Shen, Qiu and Heinz 2013

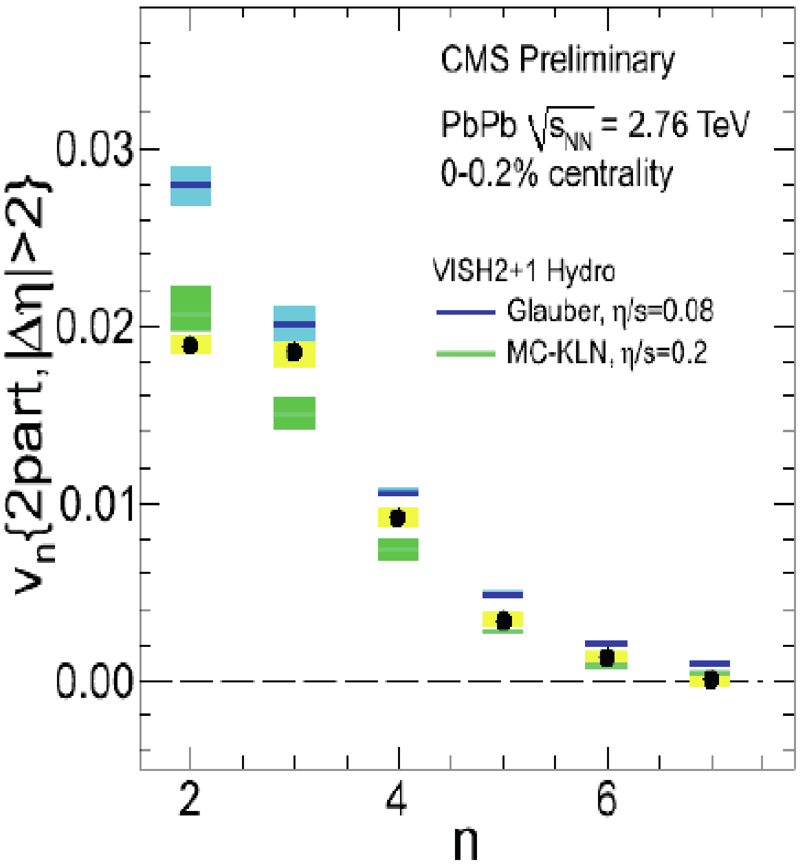


-In most central collisions, fluctuation effects are dominant (Geometry effects are suppressed)

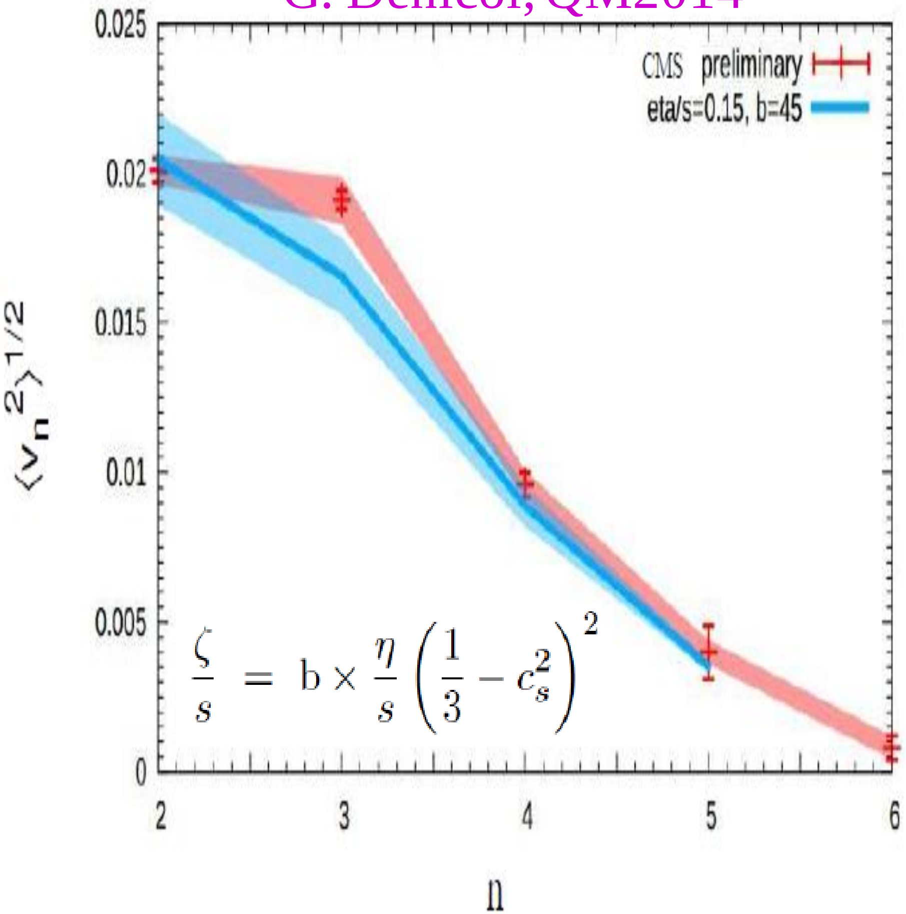
-can not simultaneously fit V_2 and V_3 with single η/s (MC-Glauber & MC-KLN)

Ultracentral Collisions: bulk visc. & NN correlations

Shen, Qiu and Heinz 2013



G. Denicol, QM2014

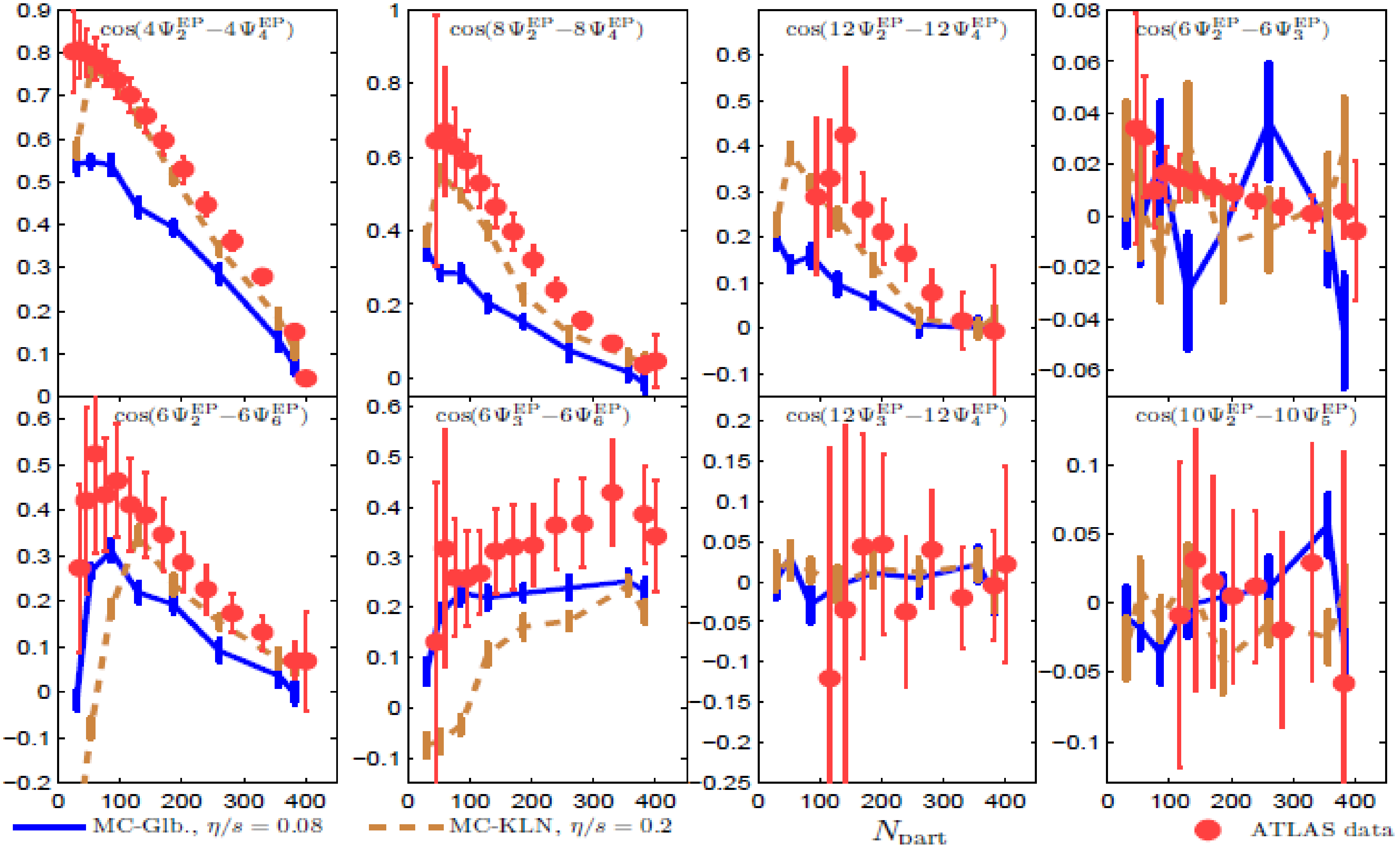


-MC-Glauber & MC-KLN: can not simultaneously fit V_2 and V_3

-IP+Glasma + NN correlations + bulk viscosity nicely reproduces V_n in ultra-central collisions

Higher Order Event Plane Correlations

Qiu & Heinz, PLB(2012)



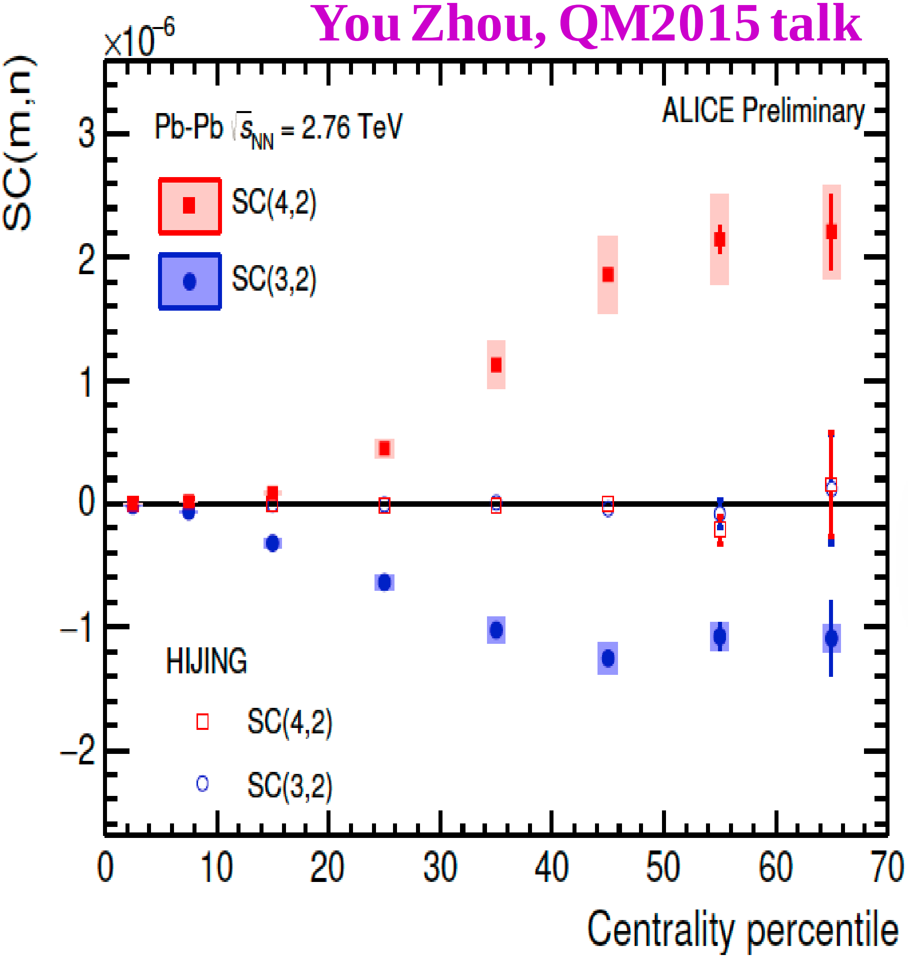
EXP. data:[ATLAS Collaboration],
CERN preprint ATLAS-CONF-2012-049

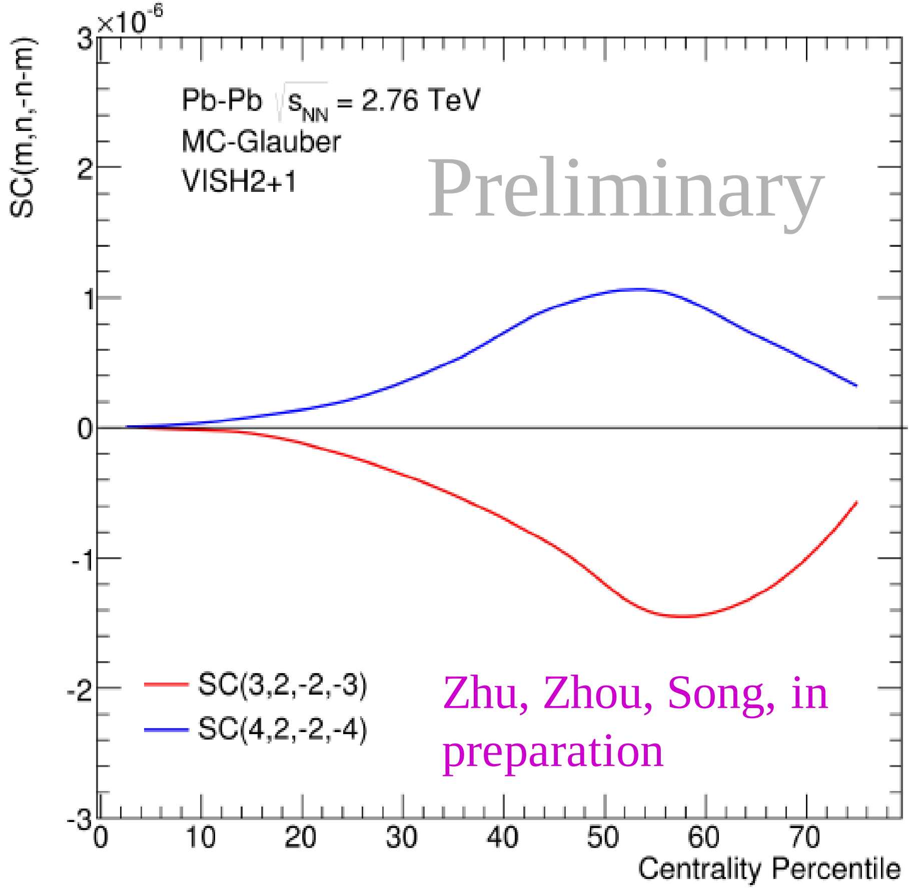
Pure e-b-e viscous hydro simulations :

-qualitatively reproduce the measured event plane correlations

V_m Correlations via cumulants

You Zhou, QM2015 talk



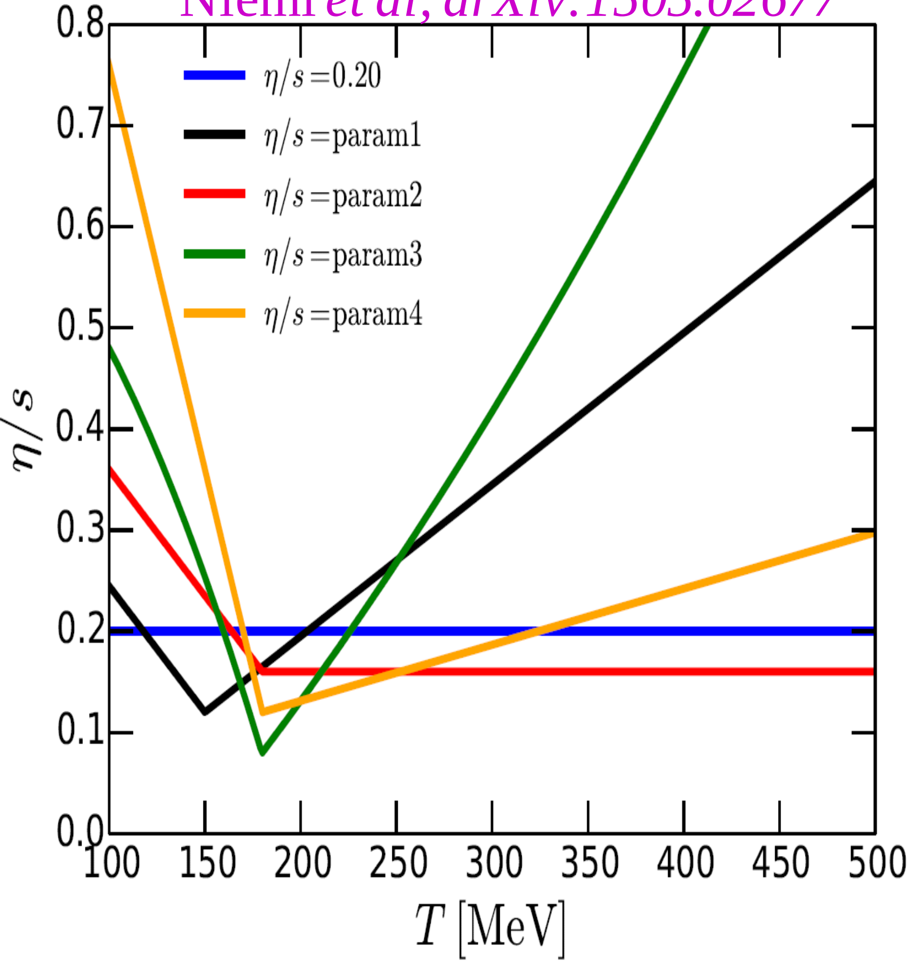


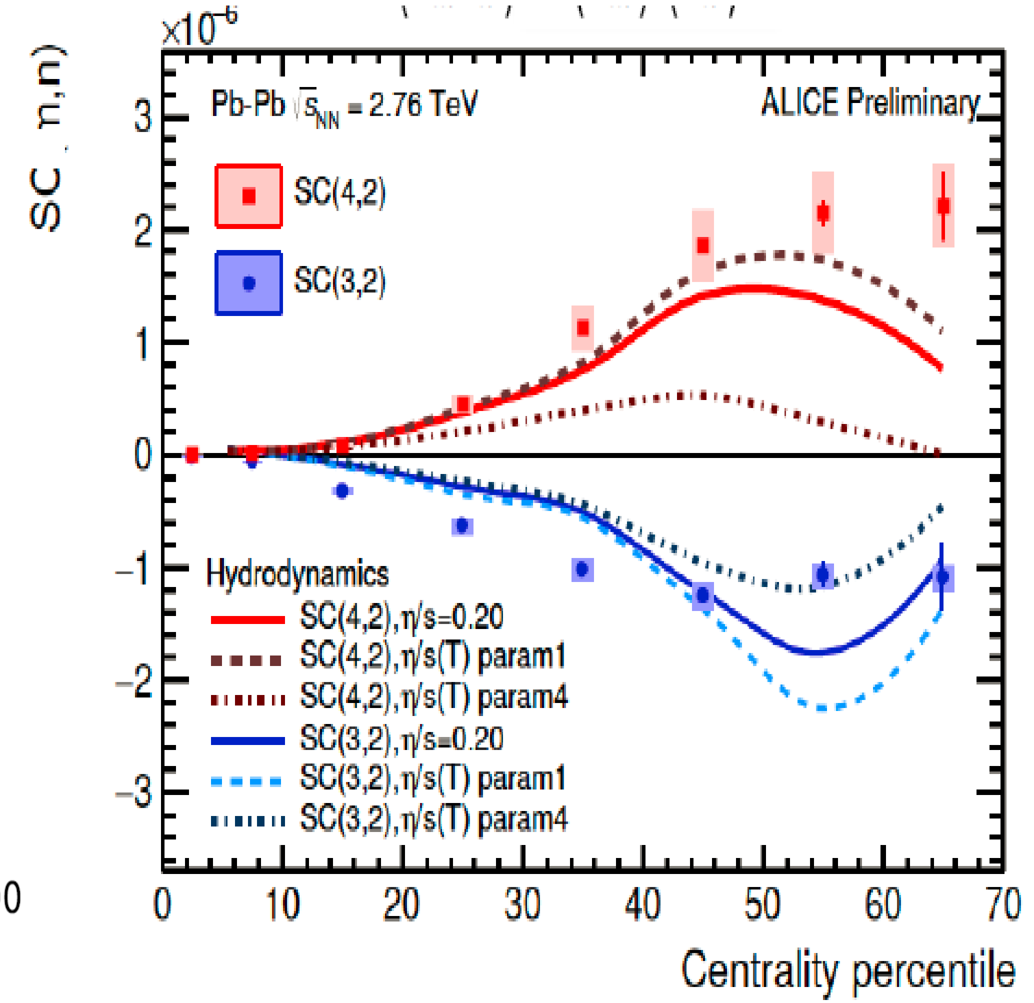
Symmetric 2-harmonic 4-particle Cumulants, SC(m,n)

$$SC(m,n) = \left\langle v_m^2 v_n^2 \right\rangle - \left\langle v_m^2 \right\rangle \left\langle v_n^2 \right\rangle$$

V_m correlations via cumulants

Niemi et al, arXiv:1505.02677





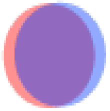
$$SC(m,n) = \langle v_m^2 v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle$$

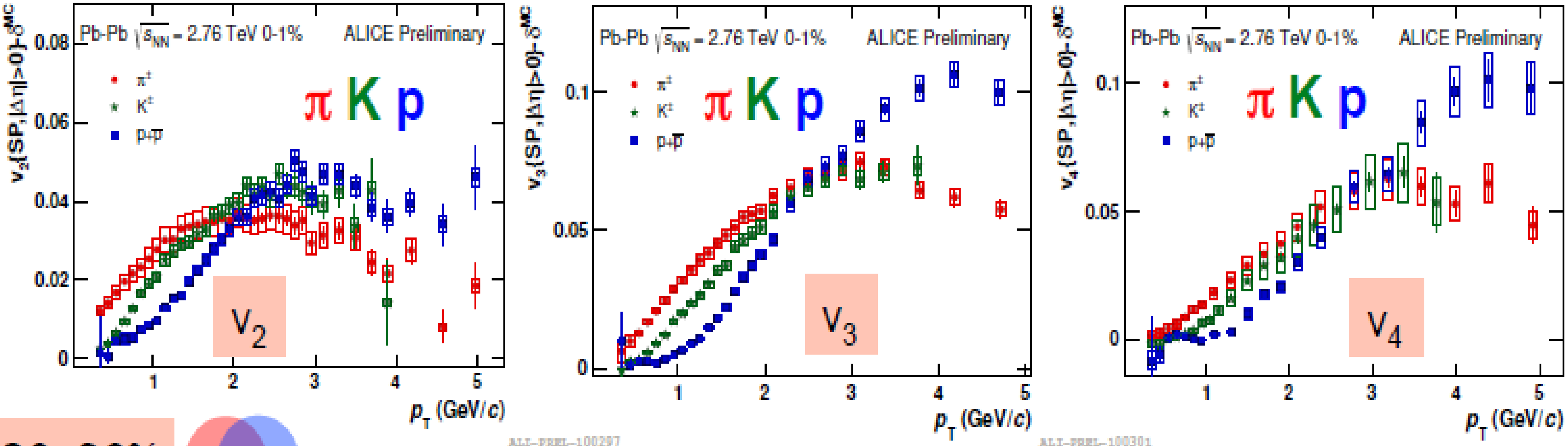
-provide strong constraint on $\eta / s(T)$

V2, V3, V4 for identified hadrons

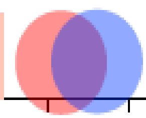
N. Mohammadi, QM2015 talk

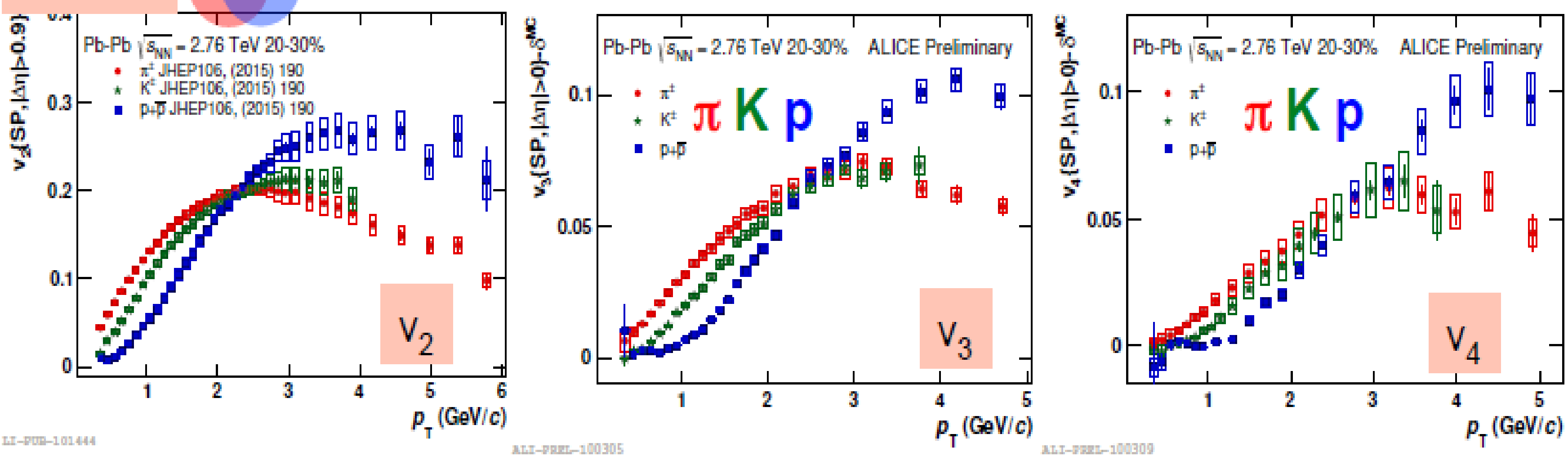
0-1%





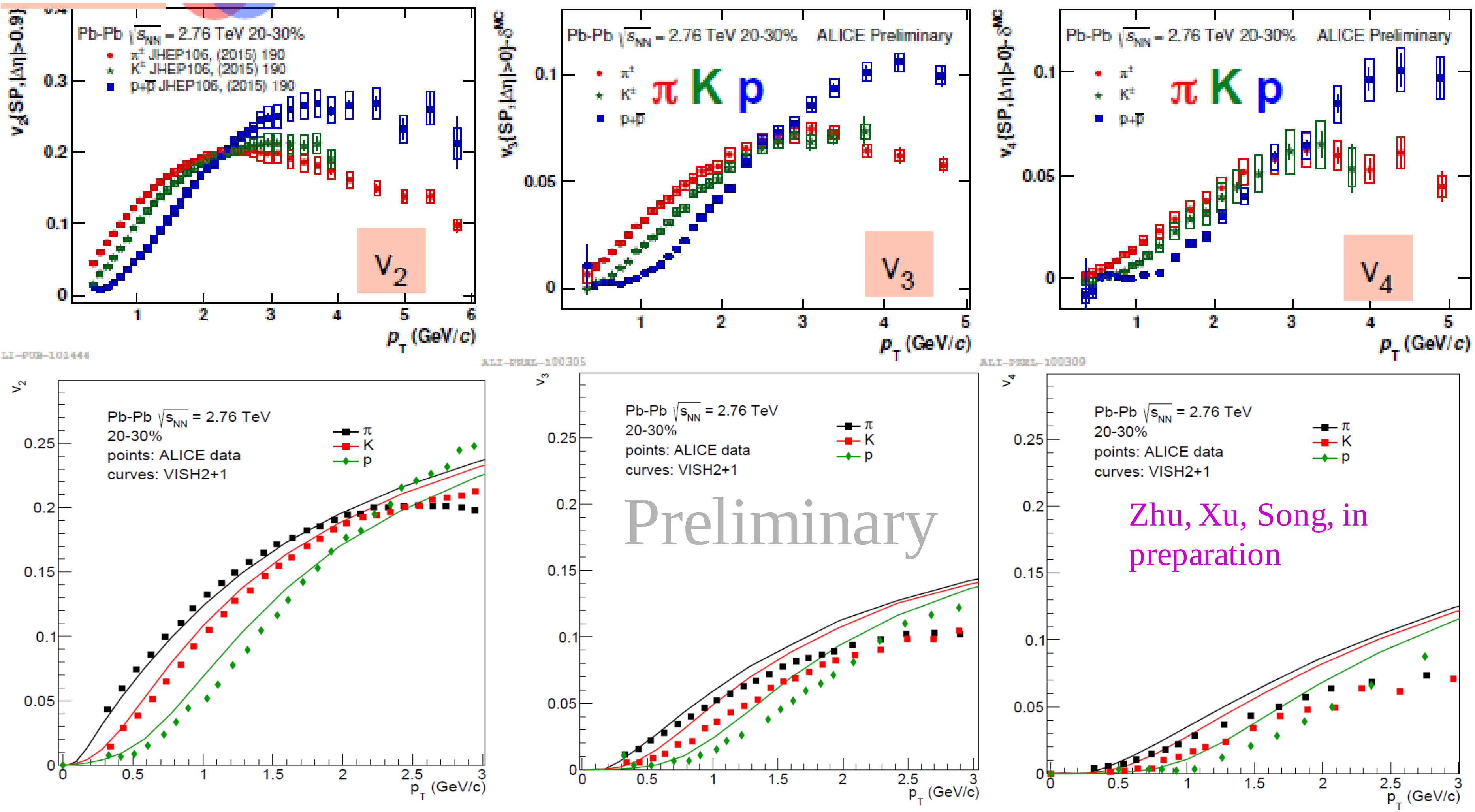
20-30%





V2, V3, V4 for identified hadrons

N. Mohammadi, QM2015 talk



- Mass ordering of higher flow harmonics: interaction between radial & anisotropic flow

Massive Data evaluation

Exp Observables

- particle yields
- spectra
- elliptic flow
- triangular flow & higher order flow harmonics
- event by event v_n distributions
- higher-order event plane correlations

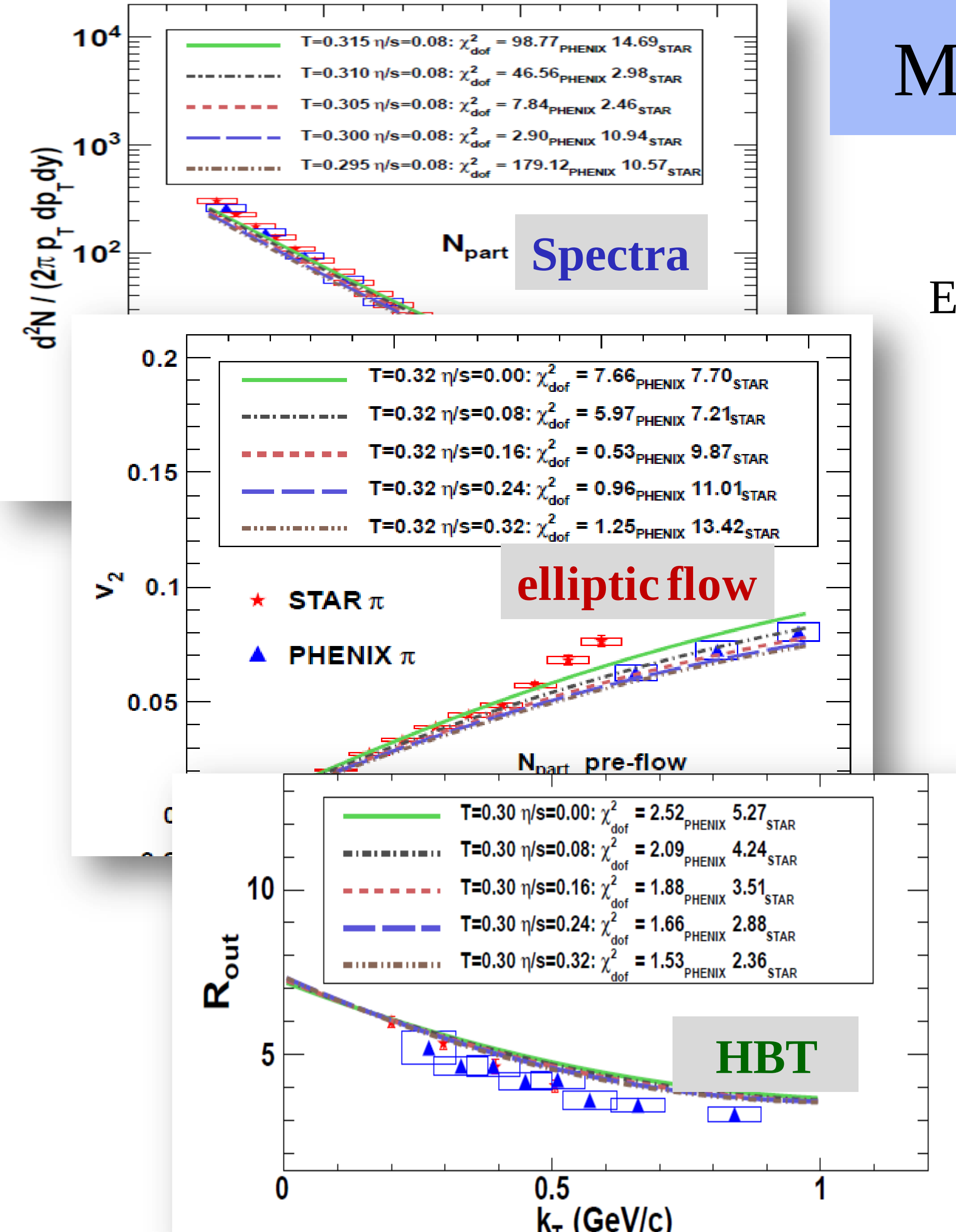
… ... … …

Theoretical Inputs:

- type of initial conditions
- initial flow
- starting time
- EoS
- shear viscosity
- bulk viscosity
- relaxation times
- freeze-out/switching cond.

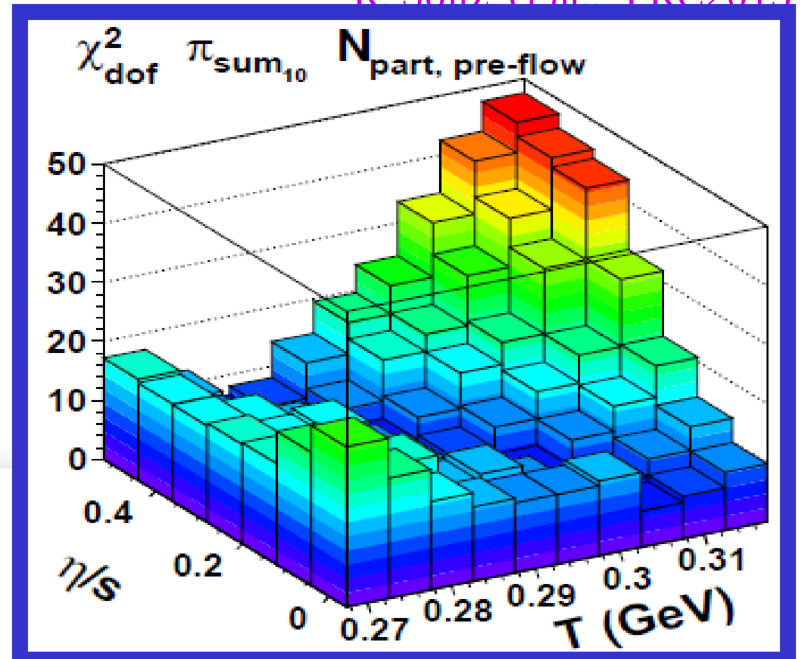
… … … …

Massive data evaluation



Early CHIMERA Results
(Comprehensive Heavy Ion Model
Evaluation and Reporting Algorithm)

R. Soltz, et al., PRC2013

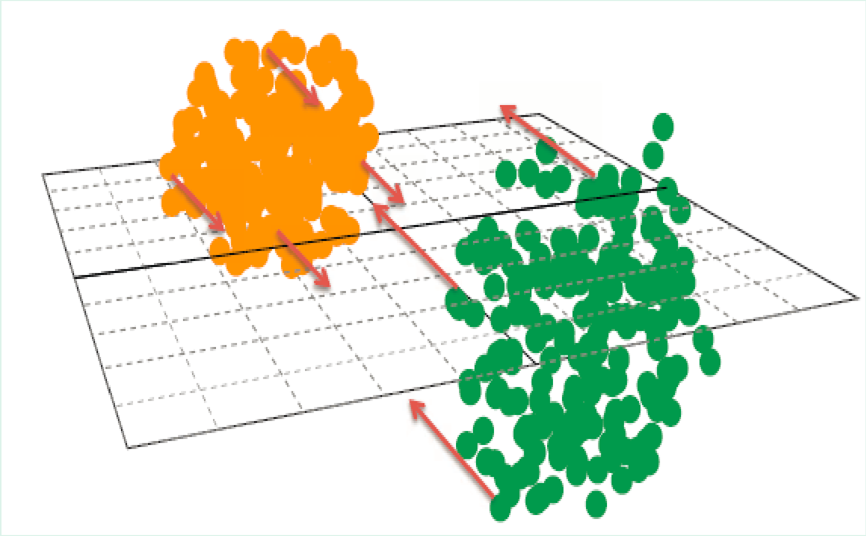


An extension to 6-dimentional parameter space, please refer to MSU-DUKE collaboration, arXiv: 1303.5769, Bernhard, QM2015 Poster

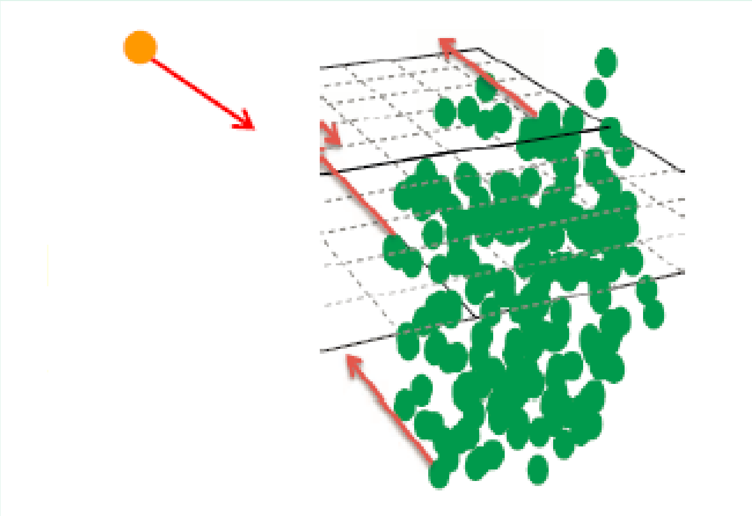
Fluctuations and Correlations in smaller systems

-p+Pb collisions at 5 TeV

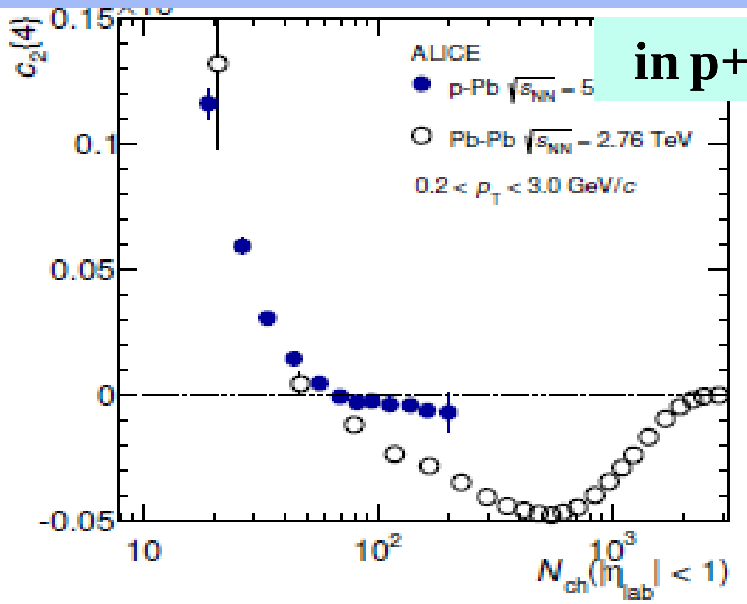
Pb+Pb



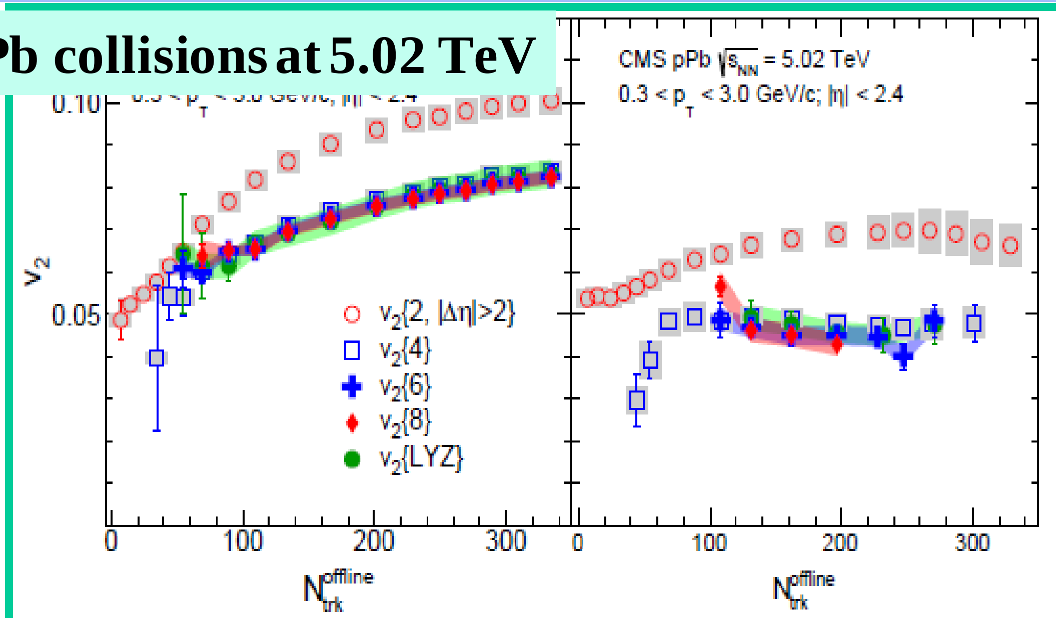
p+Pb

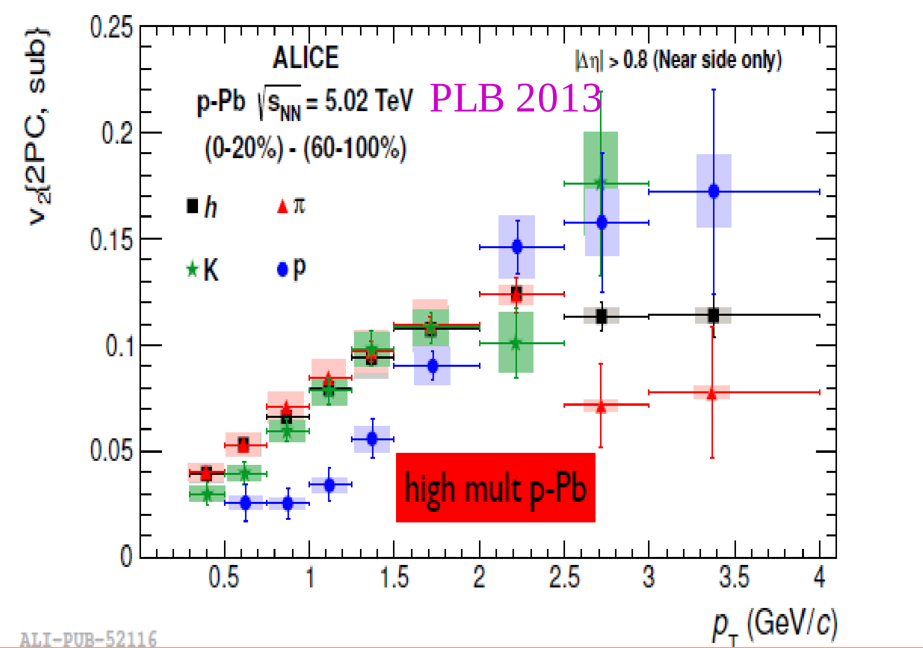


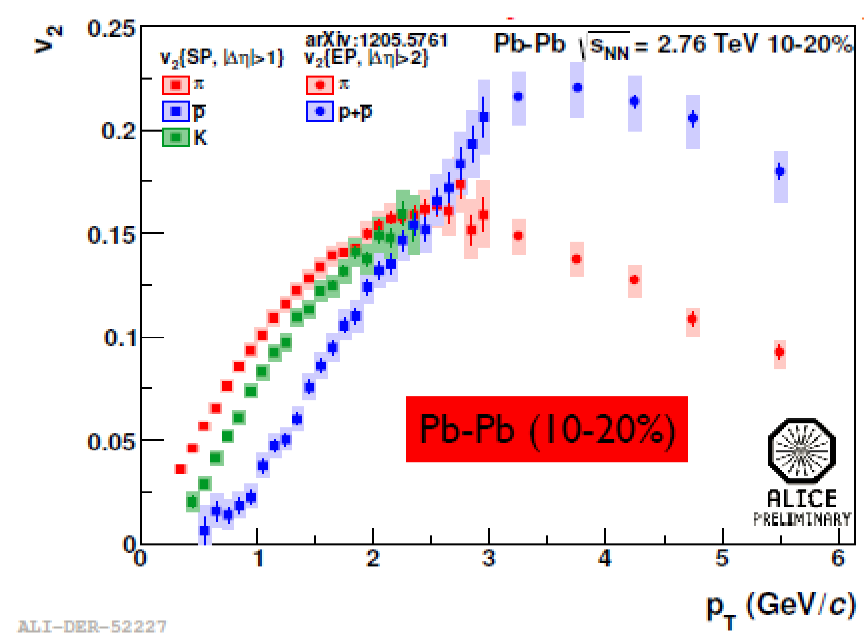
Collective flow -- Experimental Observations



in p+Pb collisions at 5.02 TeV

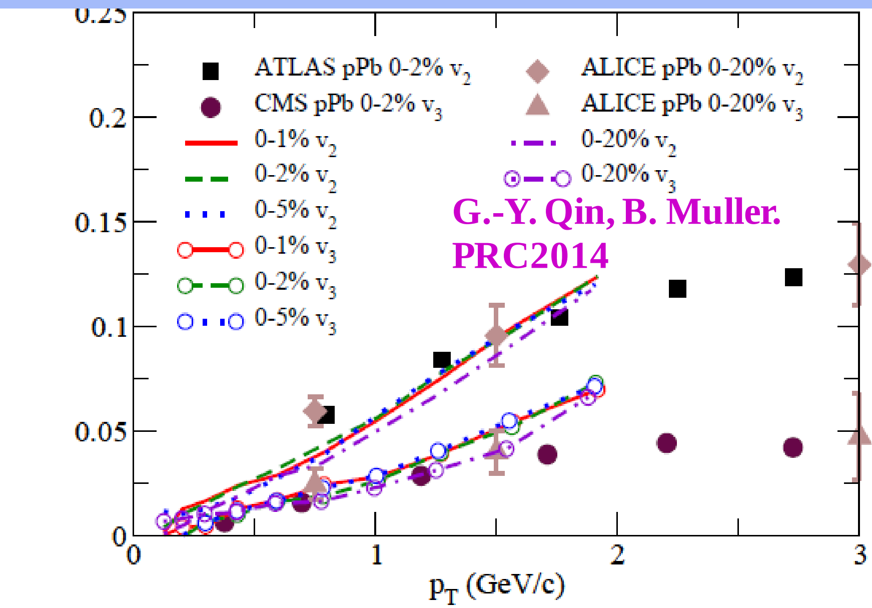


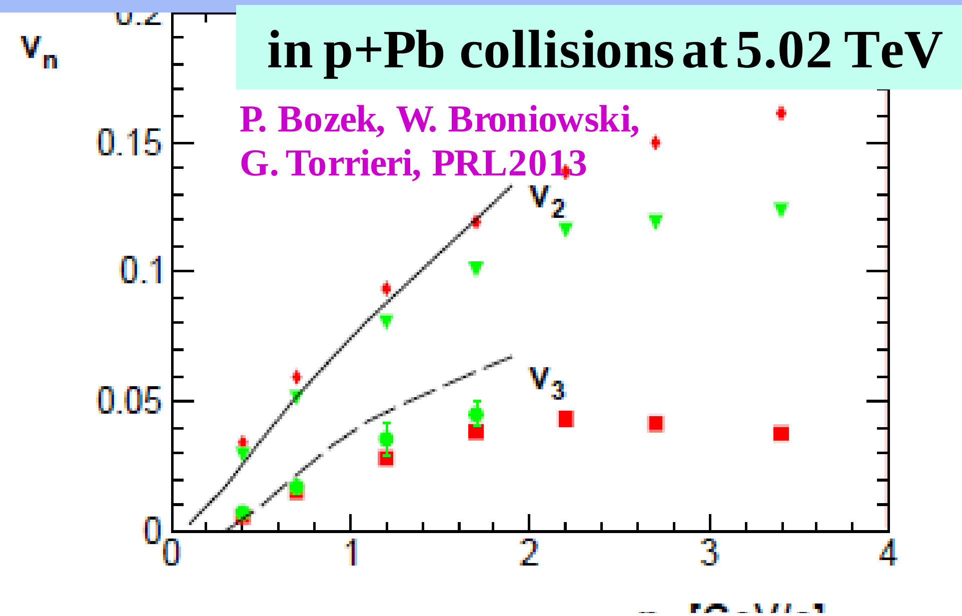


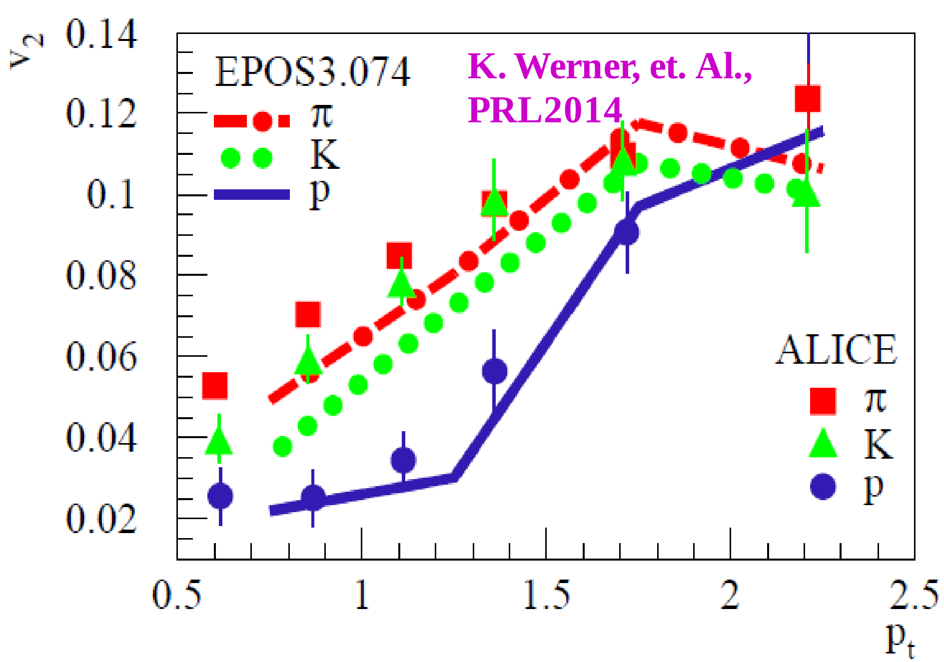


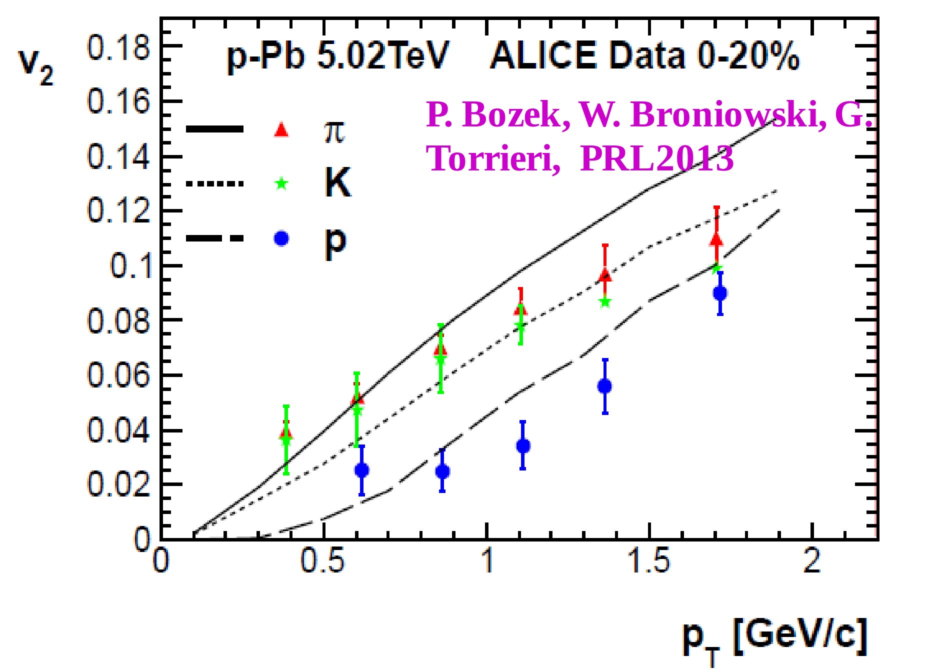
Collective flow? -- Hydrodynamics Simulations

in p+Pb collisions at 5.02 TeV





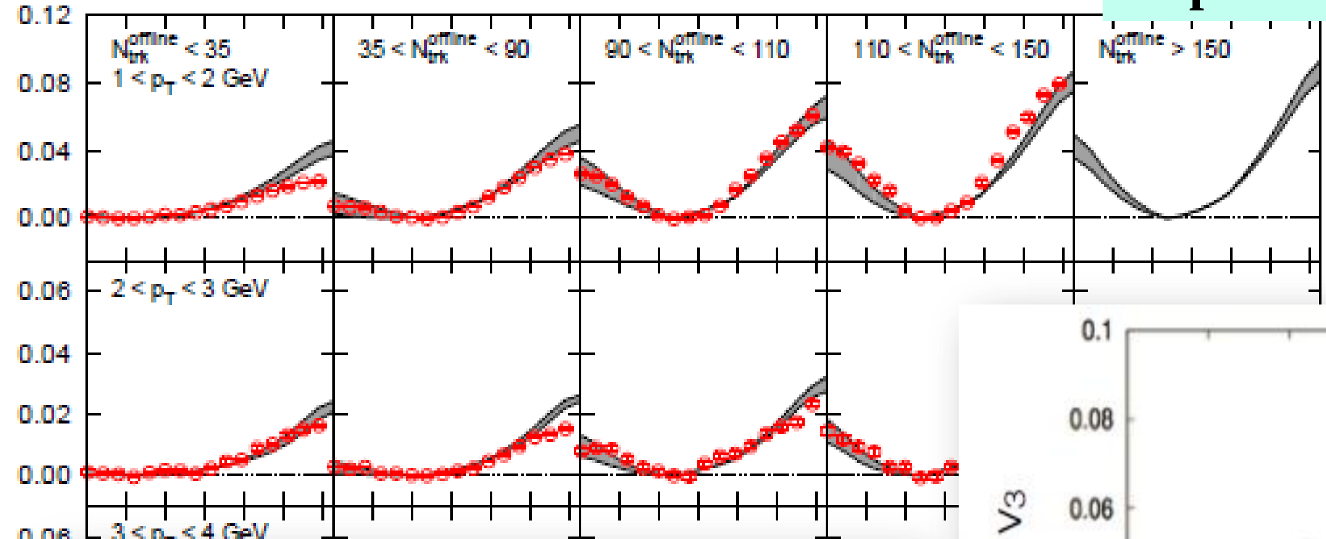




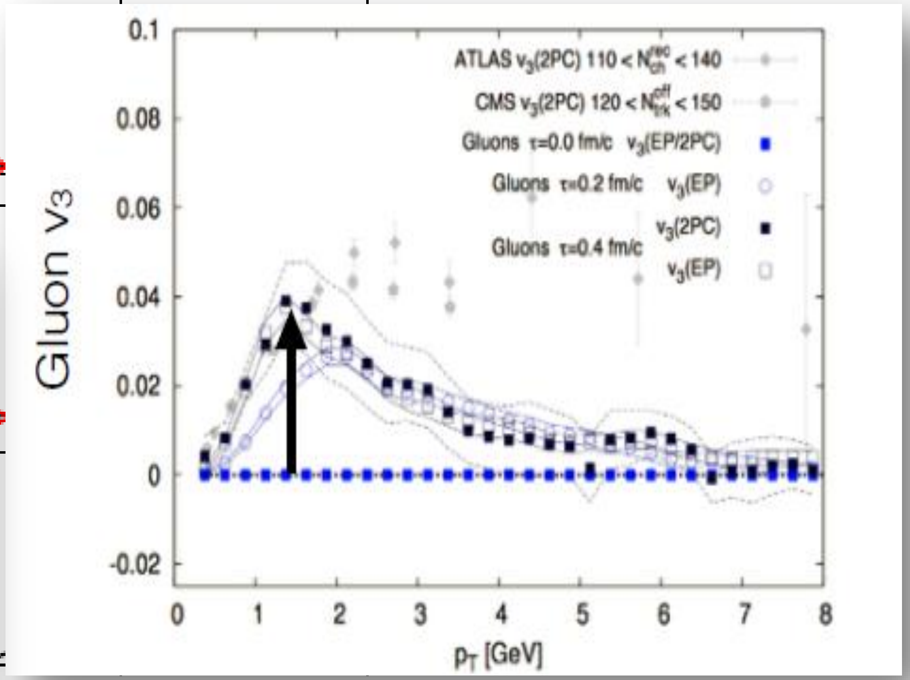
Correlations from initial state

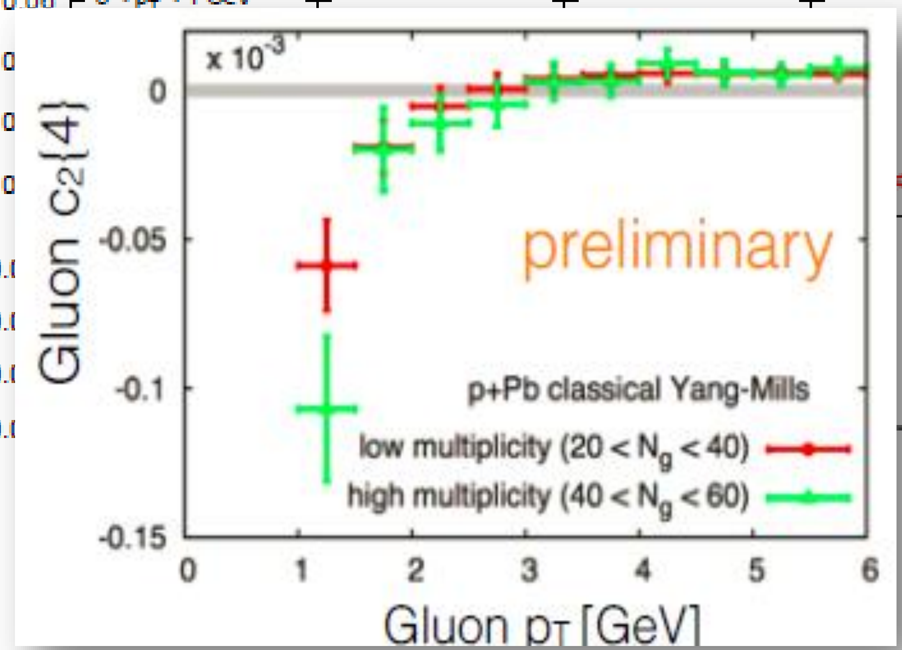
Dusling & Venugopalan PRD 2013

in p+Pb collisions at 5.02 TeV



Schenke, Schlichting, Venugopalan, PLB2015





Including final state re-scattering via CYM evolution generates a positive v3 on the time scale of a single scattering

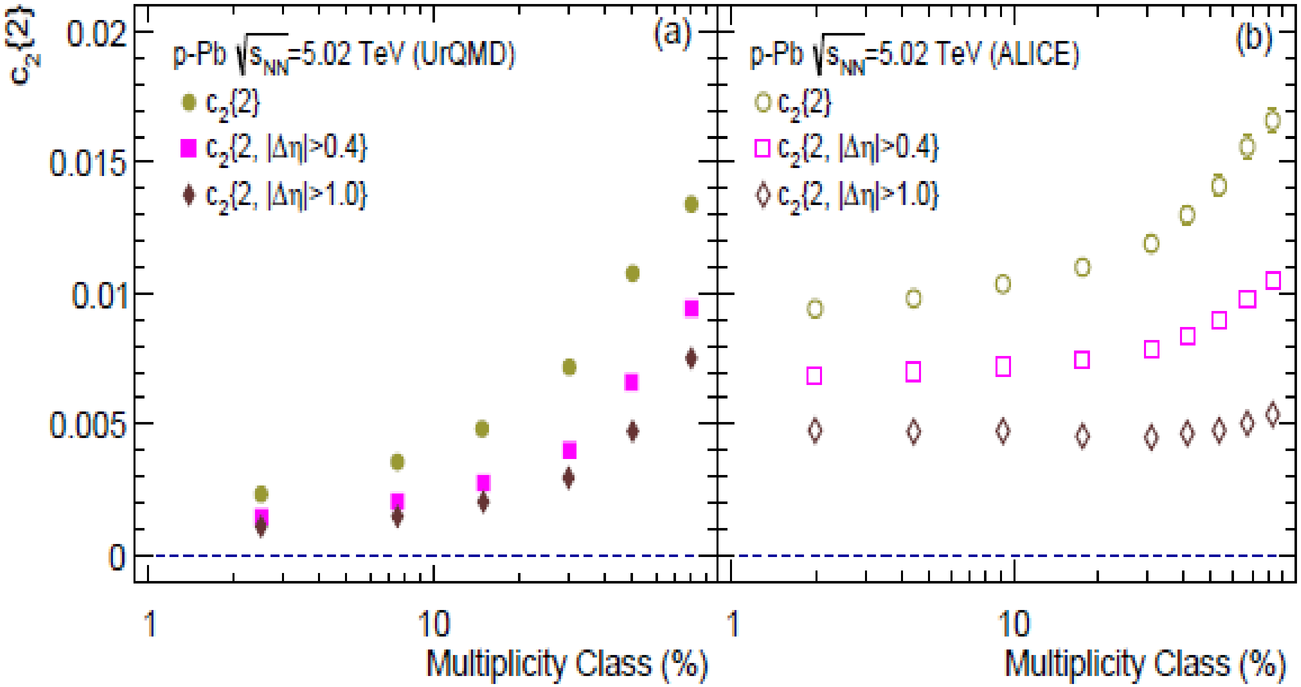
Where dose the correlations (collective flow) in 5.02 TeV p-Pb collisions come from?

- -Initial State?
- -QGP ?
- -Hadronic matter ?

UrQMD Baseline Calculations

Zhou, Zhu, Li, Song, PRC 2015

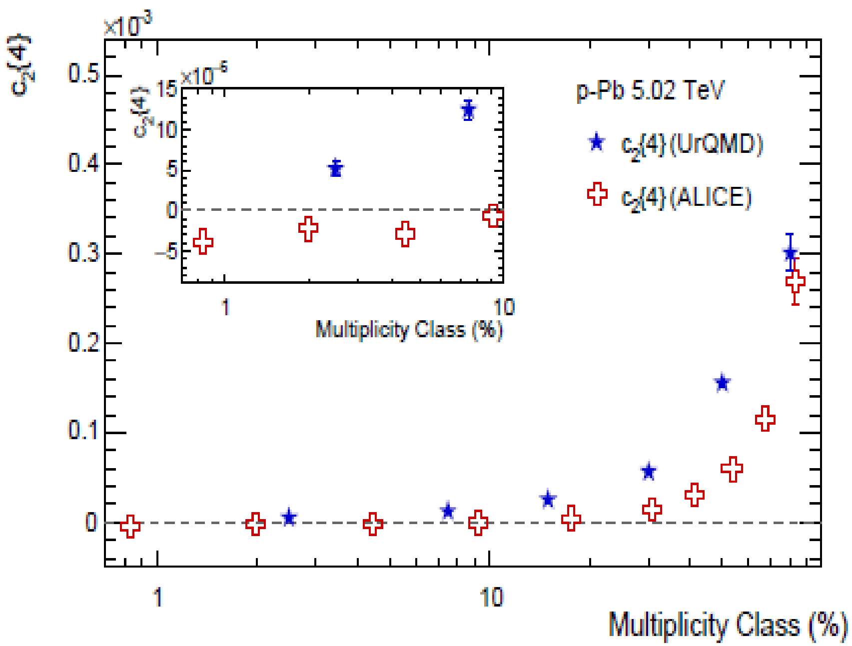
Assumption: p-Pb collisions only produce hadronic systems without reach the thresh hold of the QGP formation

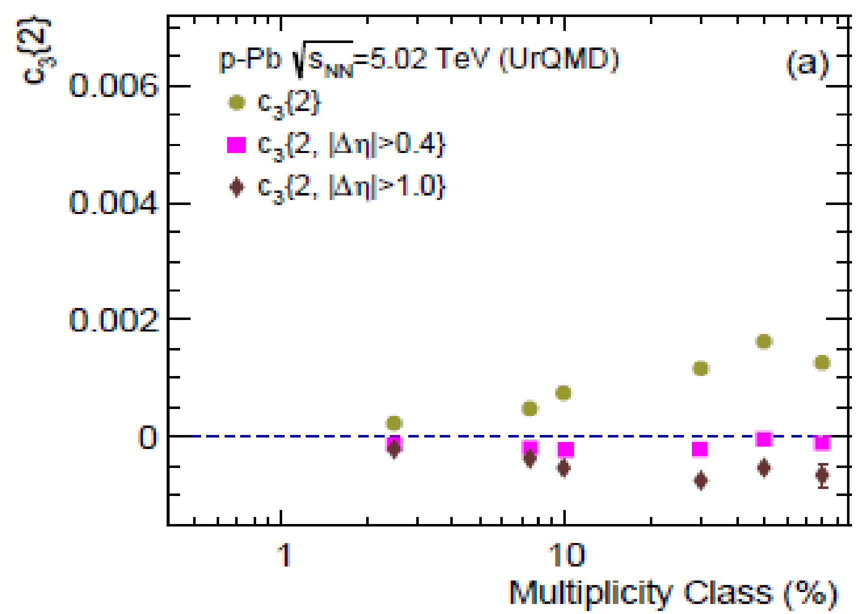


Zhou, Zhu, Li, Song, PRC2015

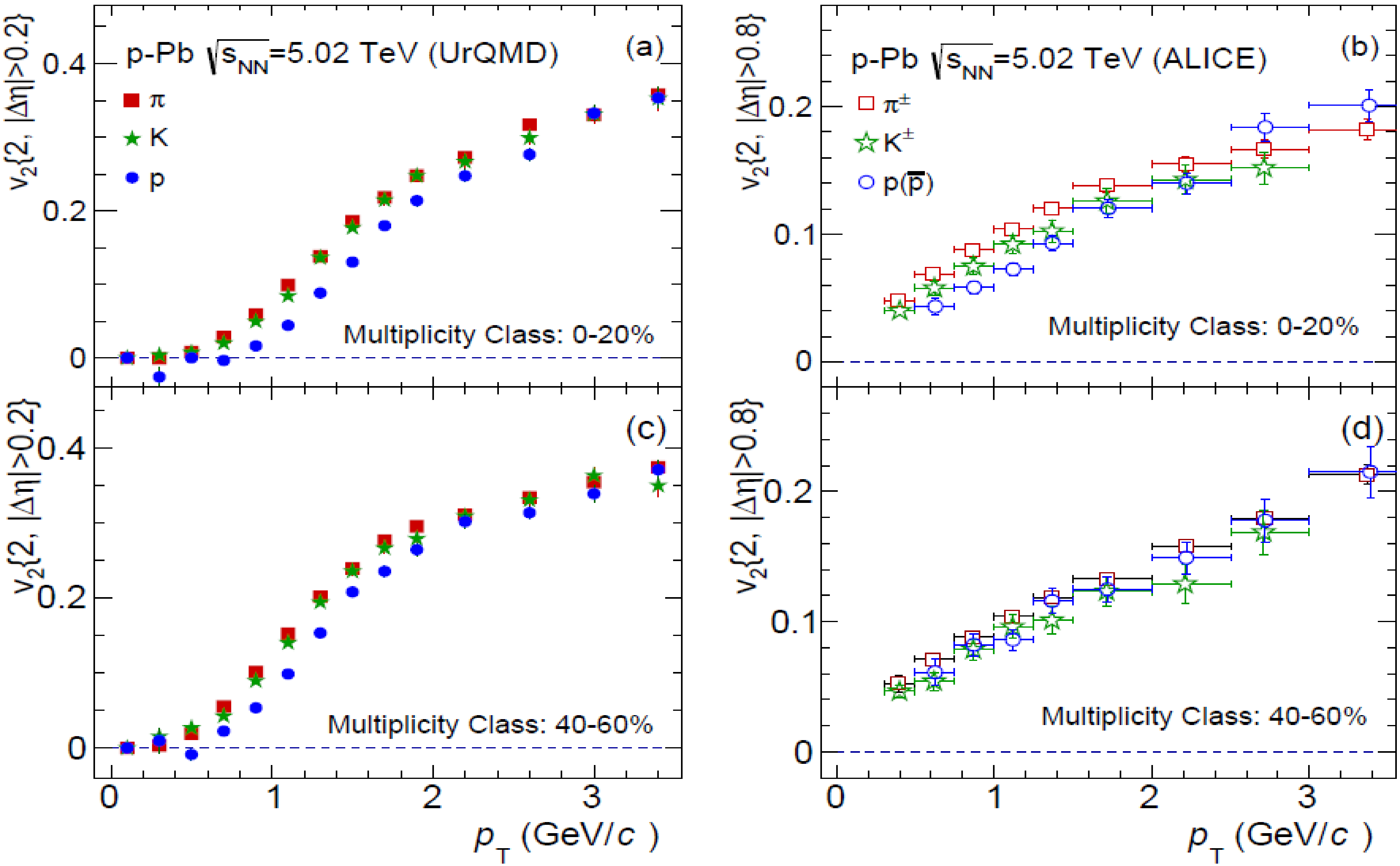
-The UrQMD systems are largely influenced by non-flow effects

-To reproduce the flow data, effects from initial state and/or QGP are needed





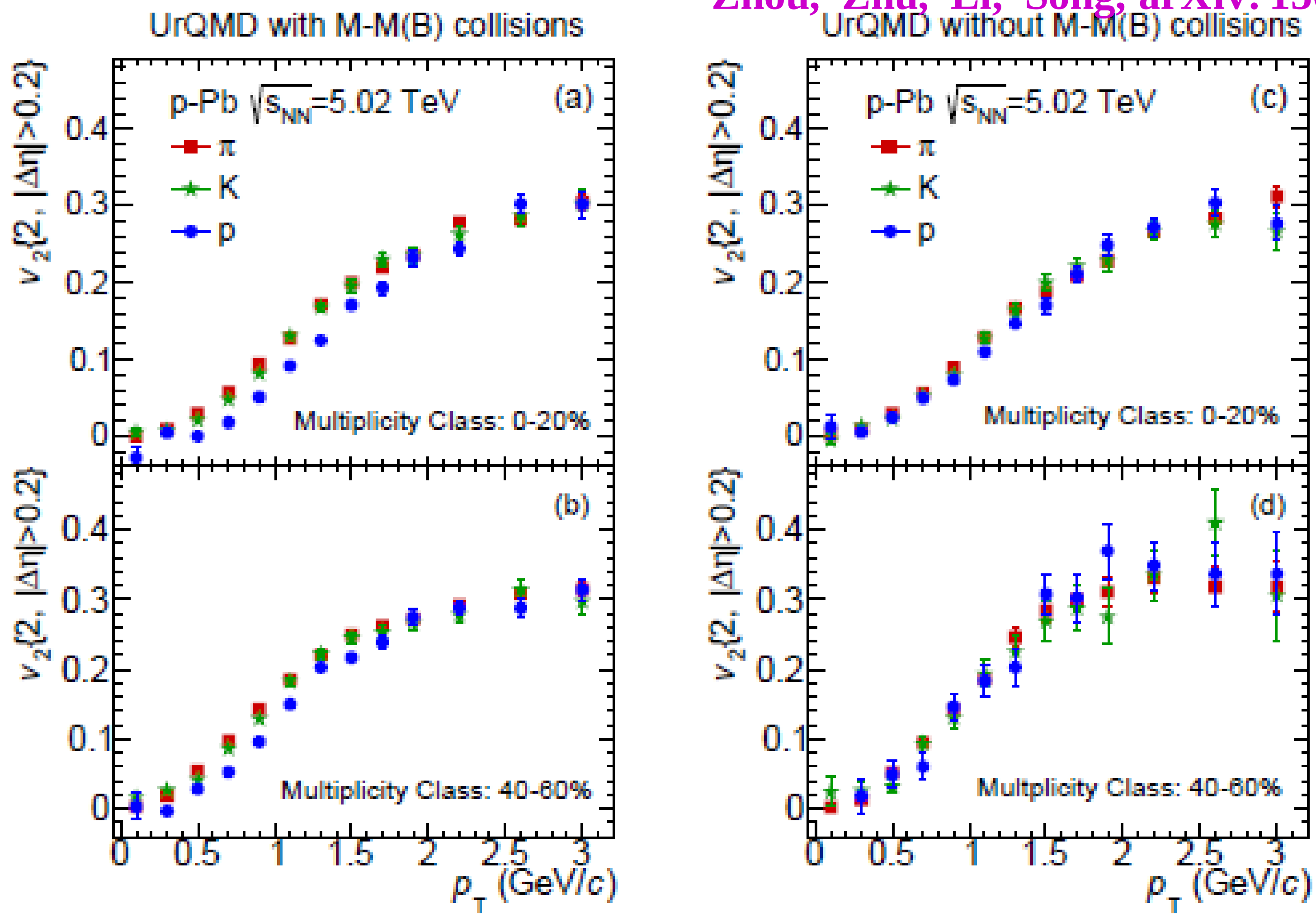
V_2 mass ordering in p+Pb collisions at 5.02 TeV



v_2 mass ordering is produced by UrQMD, similar to the ALICE data

Hadronic interactions & v2 mass ordering

Zhou, Zhu, Li, Song, arXiv: 1503.06986

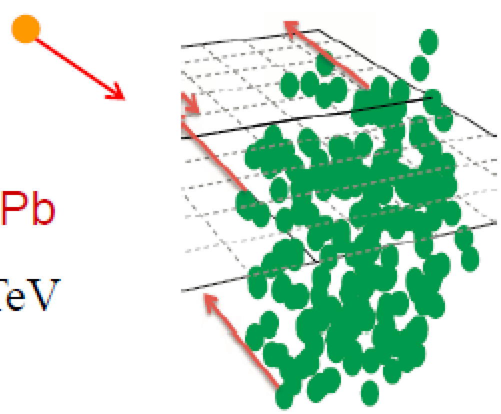


-Hadronic interaction can generate a mass ordering for 2- particle correlations

-Additive quark model: different M-M M-B cross-sections

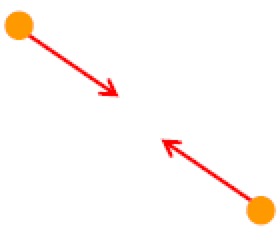
Fluctuations and Correlations in even smaller systems

-p+p collisions at 13 TeV



p+Pb

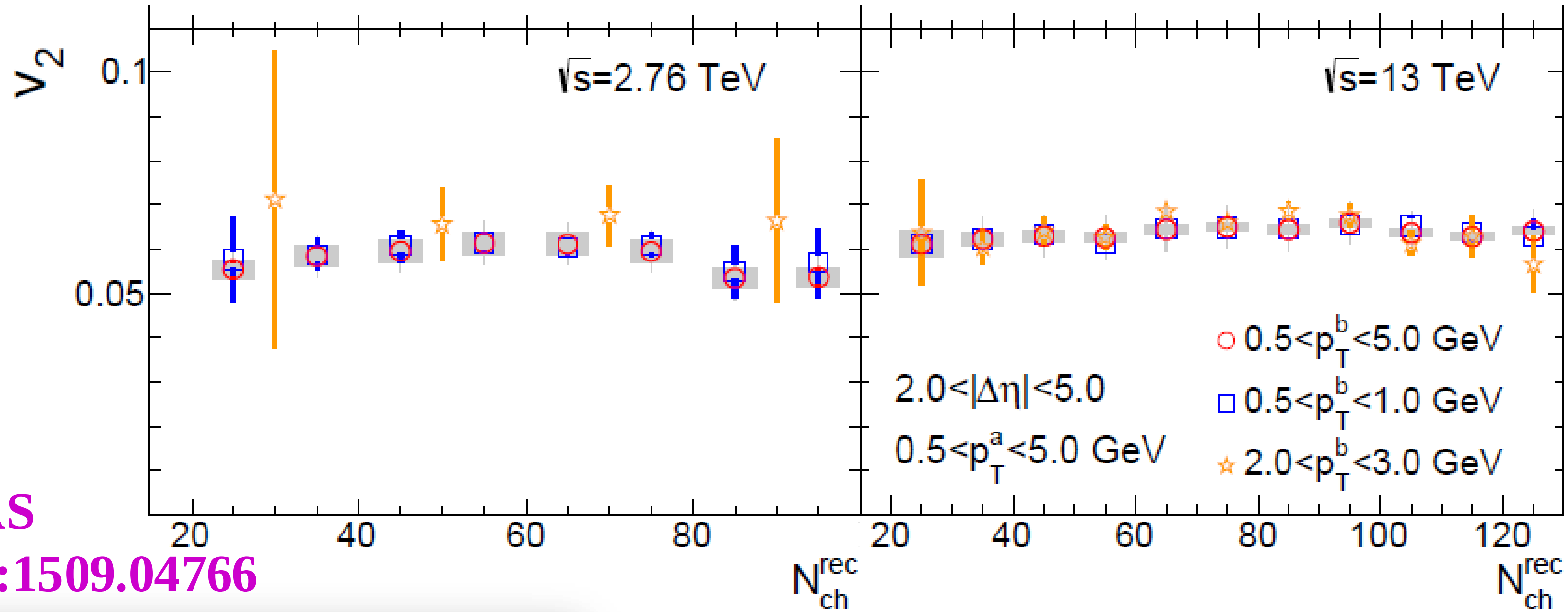
5 TeV



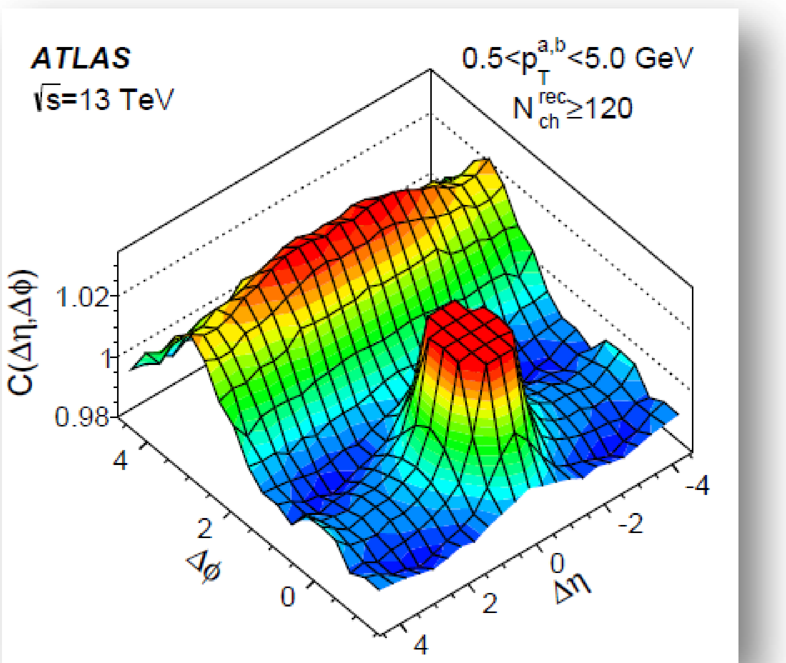
p+p

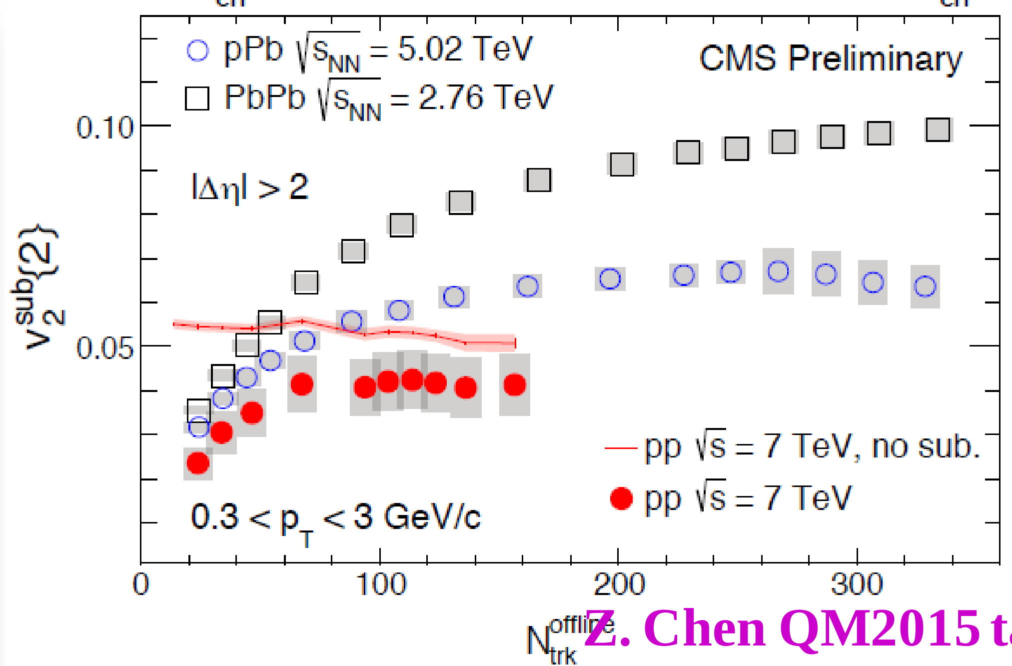
13 TeV

Correlations and flow in 13 TeV p+p



ATLAS
arXiv:1509.04766

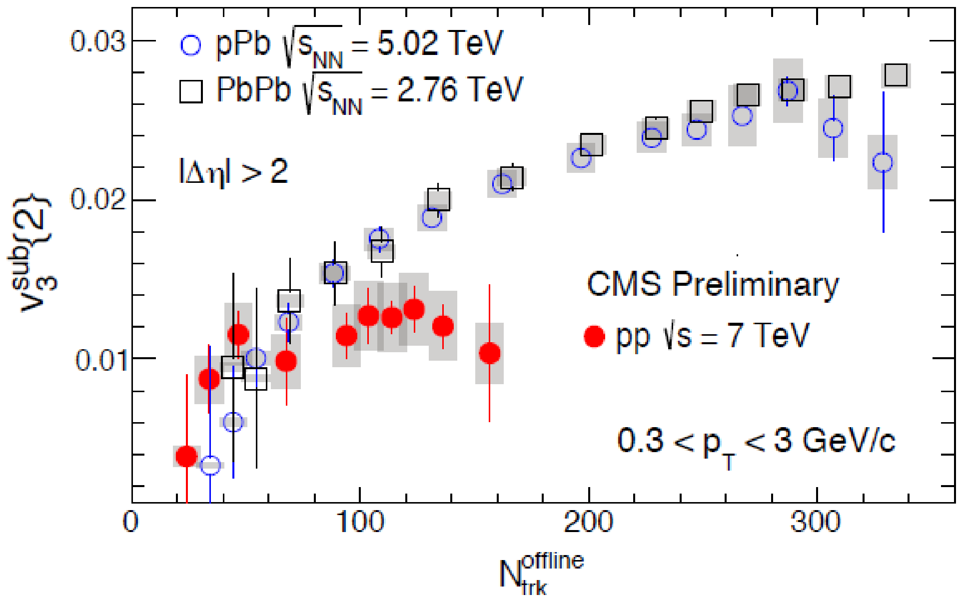


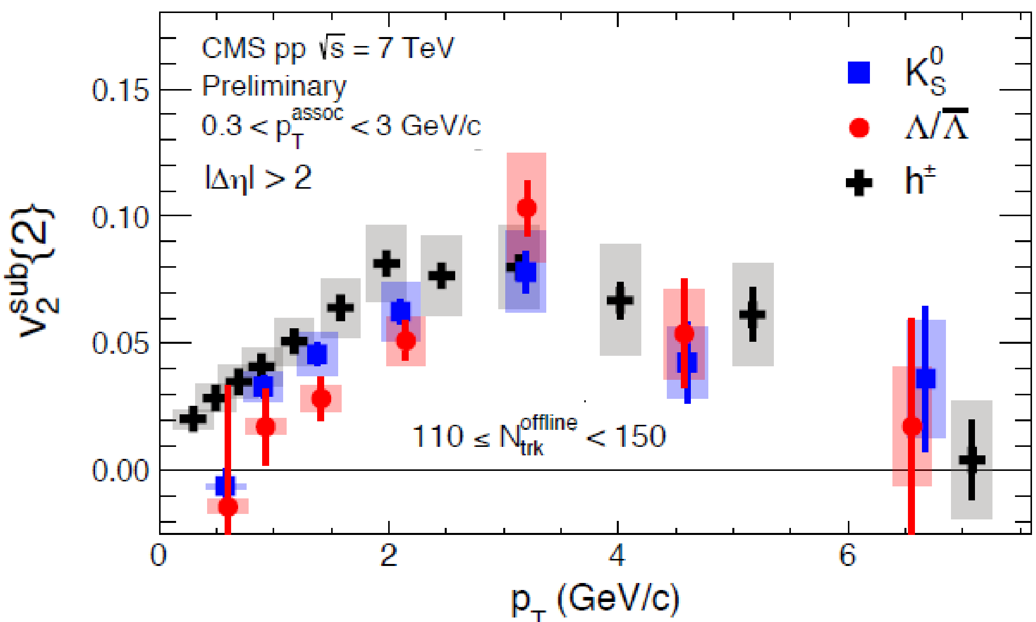


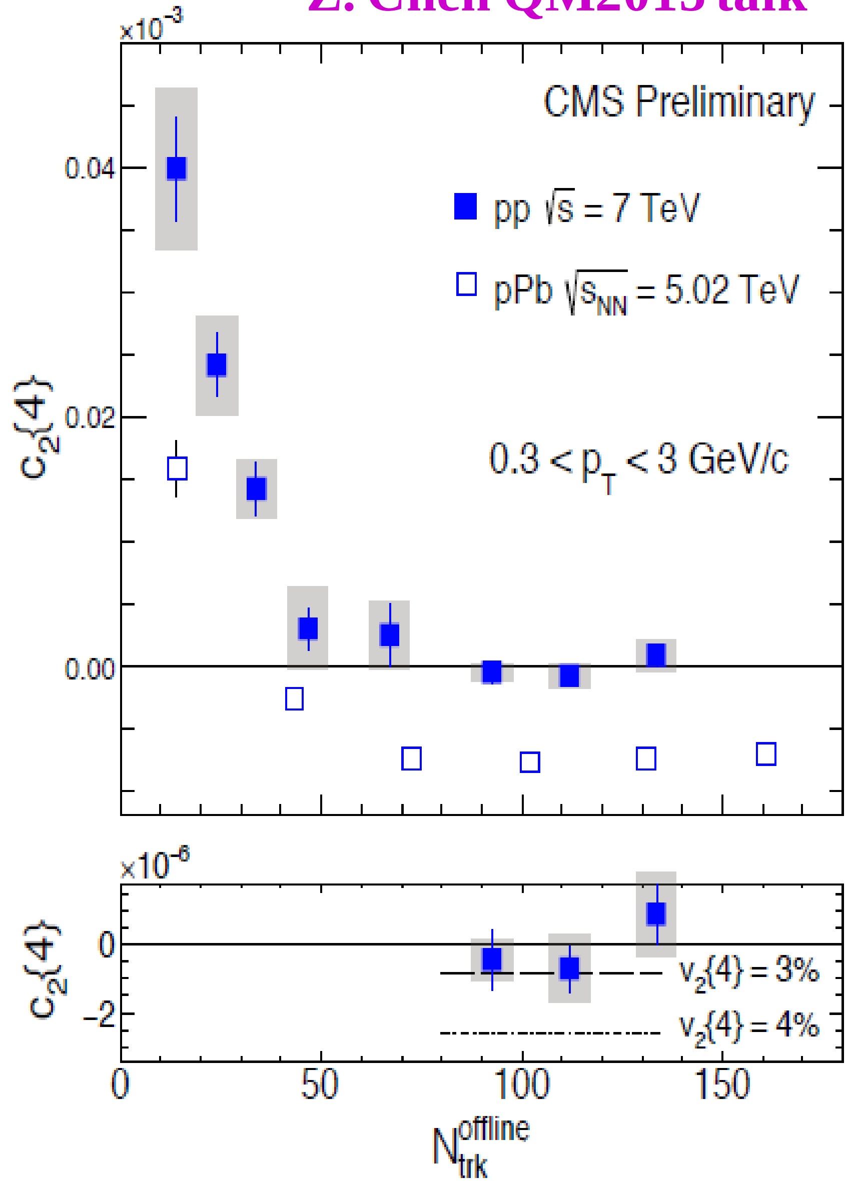
Z. Chen QM2015 talk

Correlations and flow in 13 TeV p+p (from CMS)

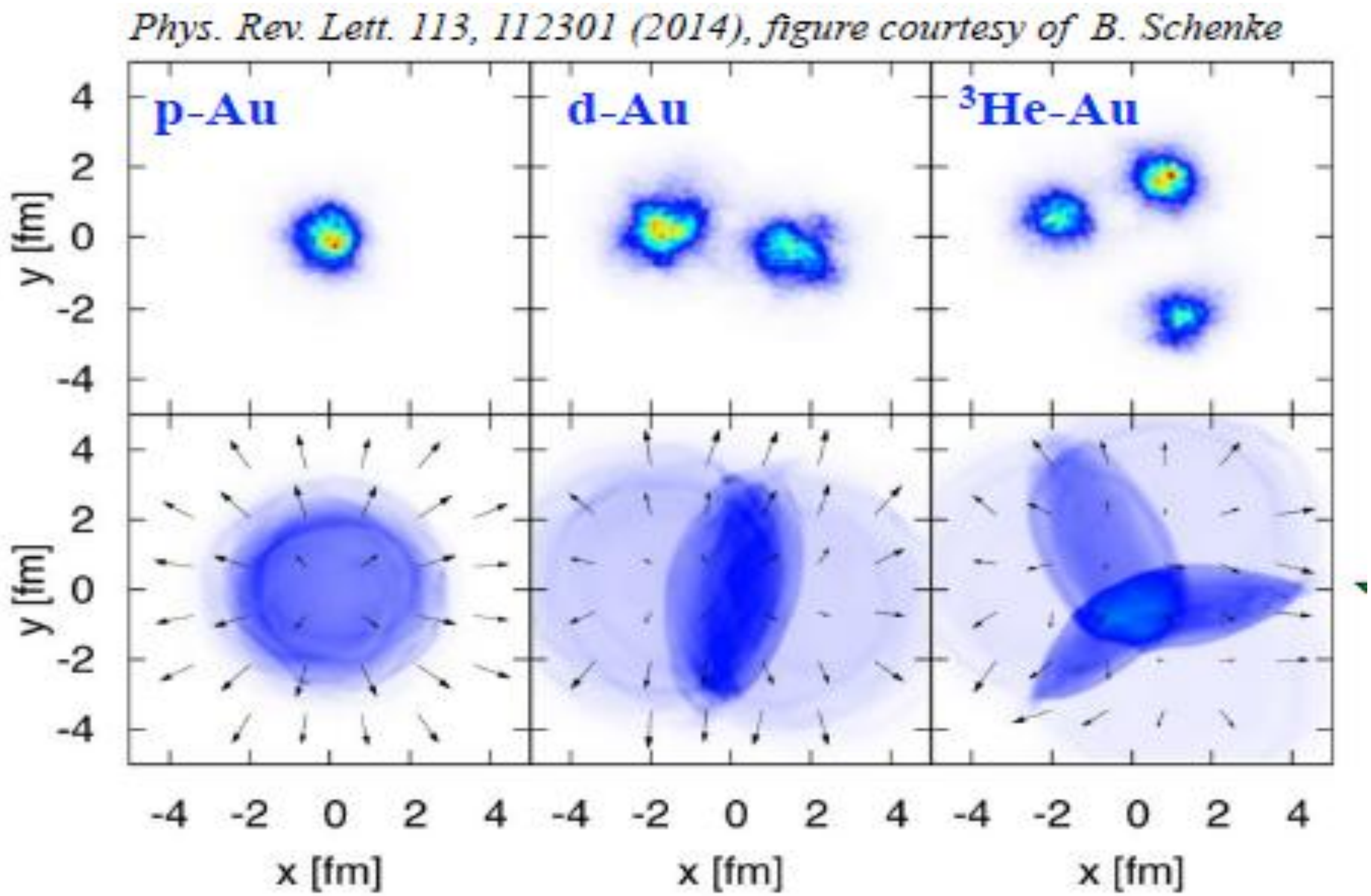
Z. Chen QM2015 talk

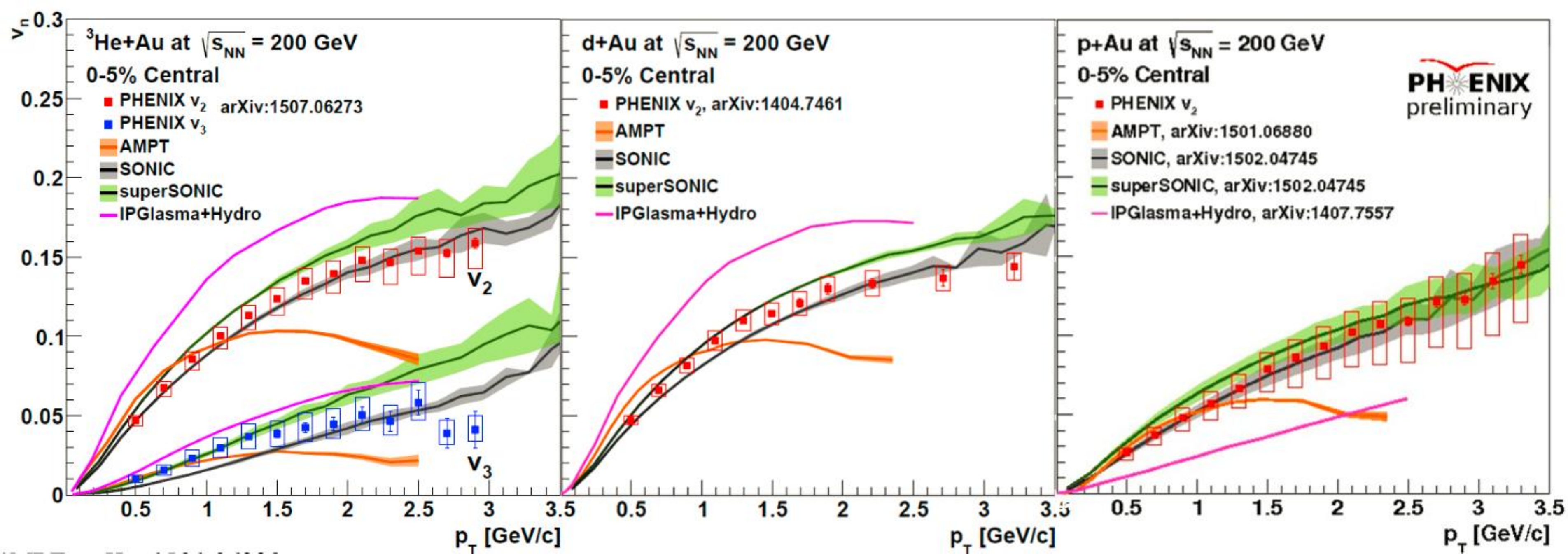




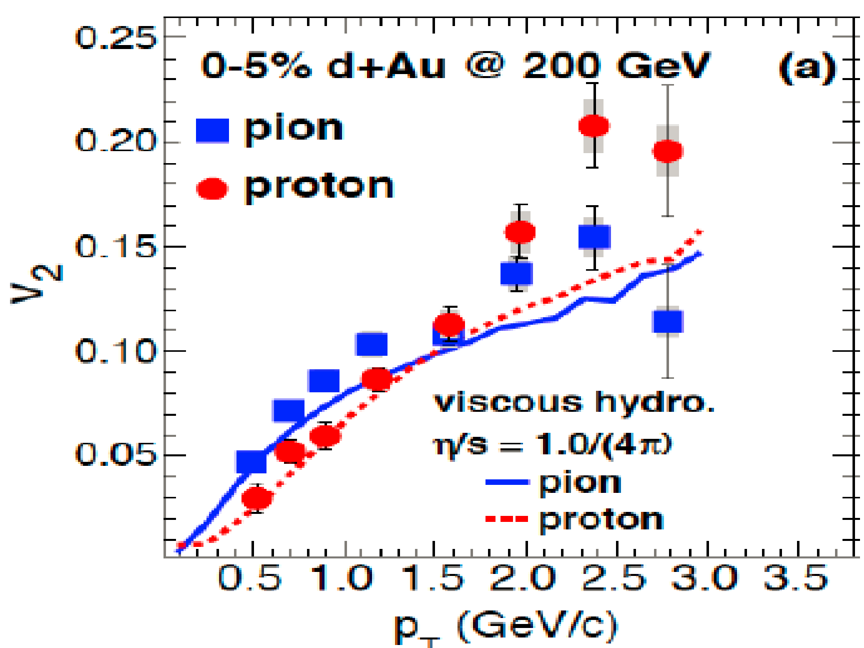


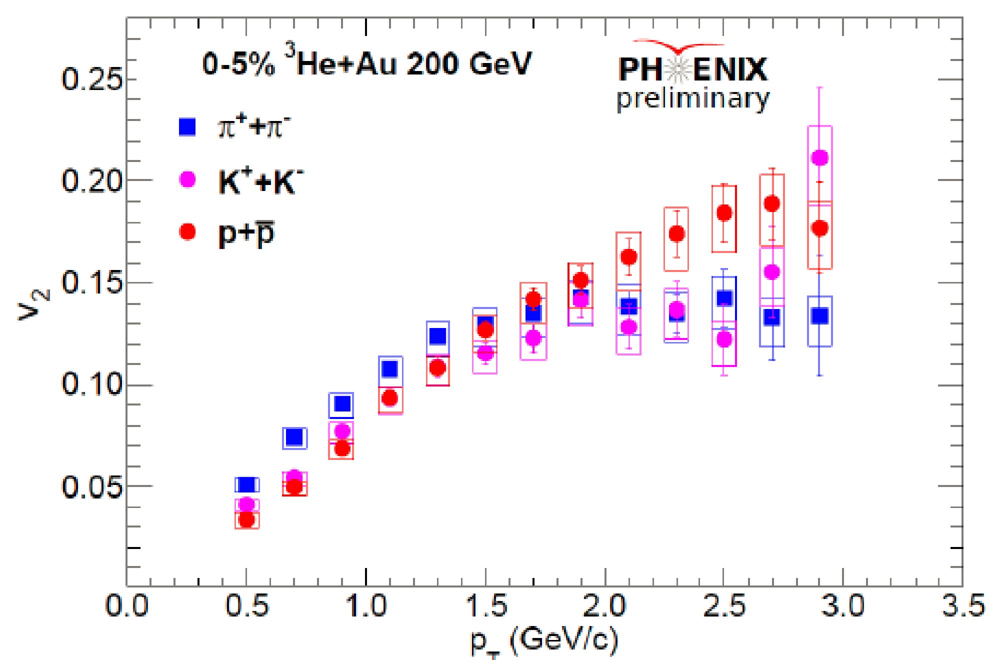
Fluctuations and correlations --more small systems at RHIC



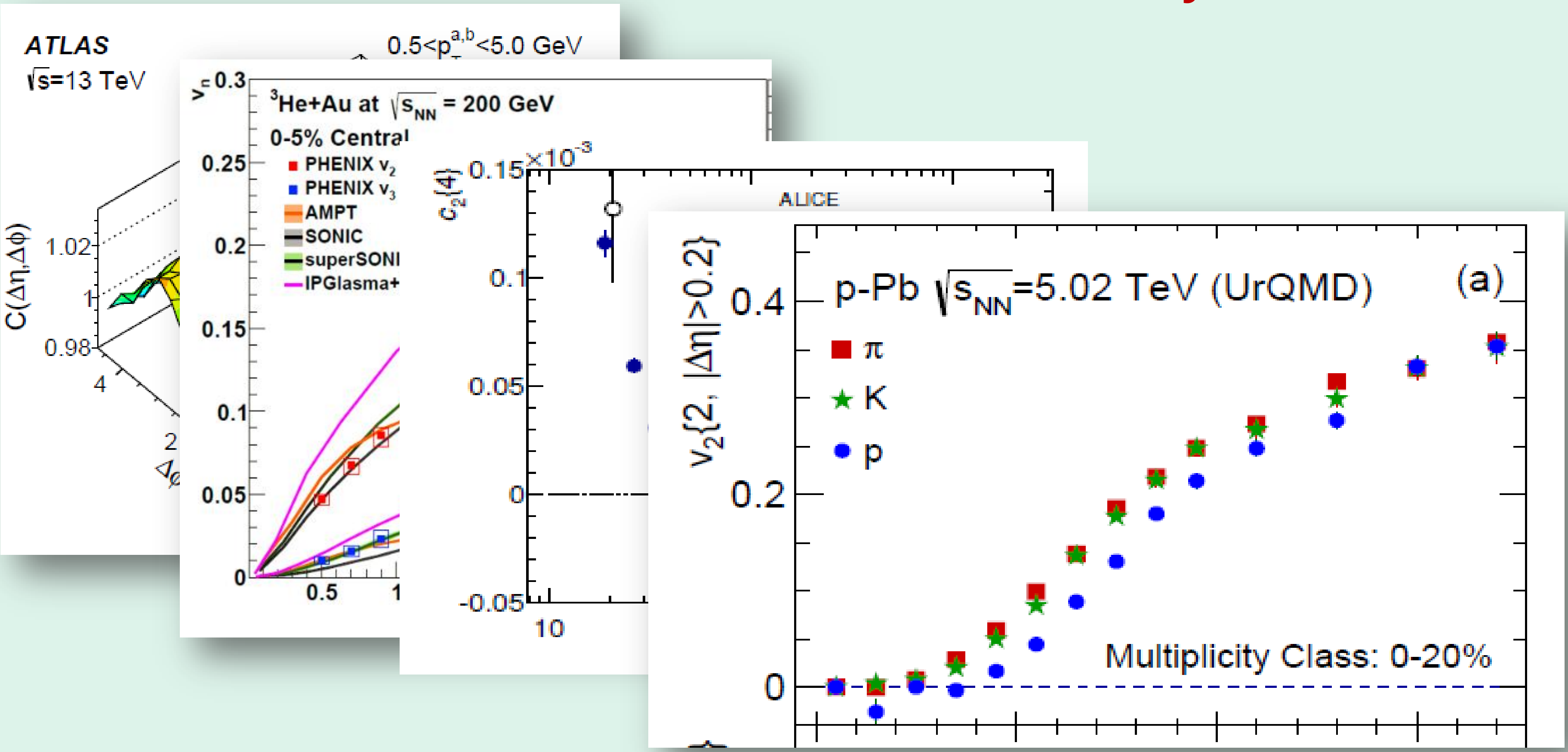


AMPT: arXiv:1501.06880 SONIC: arXiv:1502.04745 IP+Hydro:arXiv:1407:7557





Fluctuations and Correlations in small systems

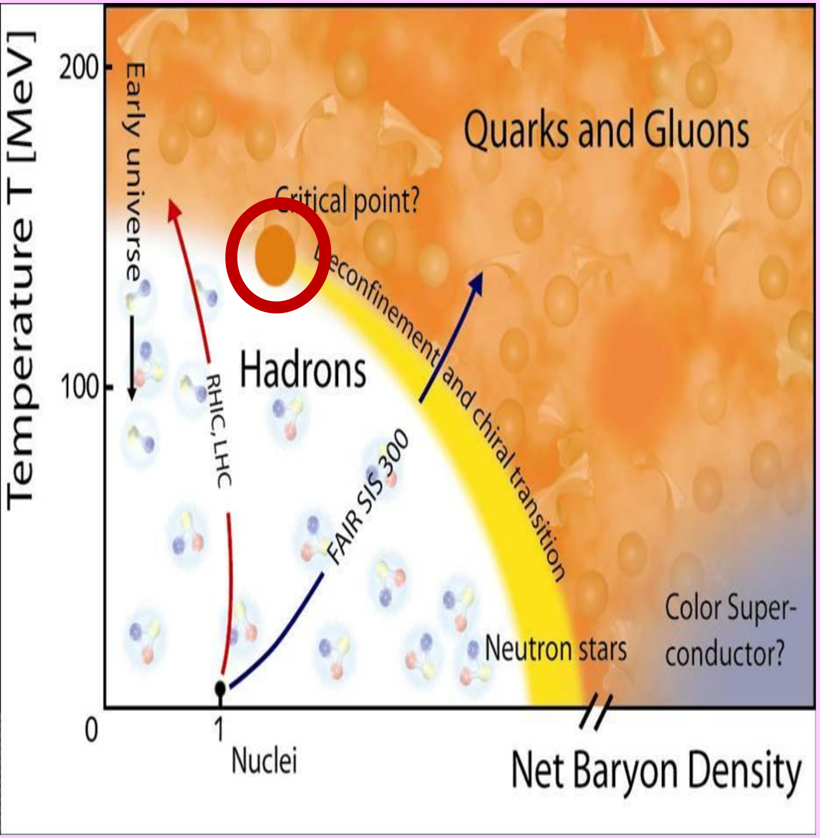


Many many related flow measurements for different small colliding system,

-What is the solid flow signal?

-Why hydrodynamics work well for such small system

Correlated fluctuations near the QCD critical point



Initial State Fluctuations

-QGP fireball evolutions smear-out the initial fluctuations

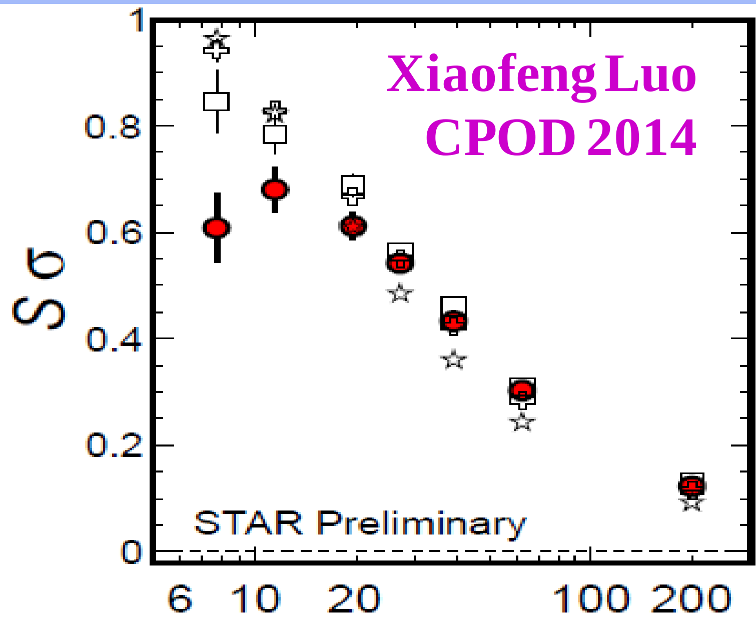
-uncorrelated (in general)

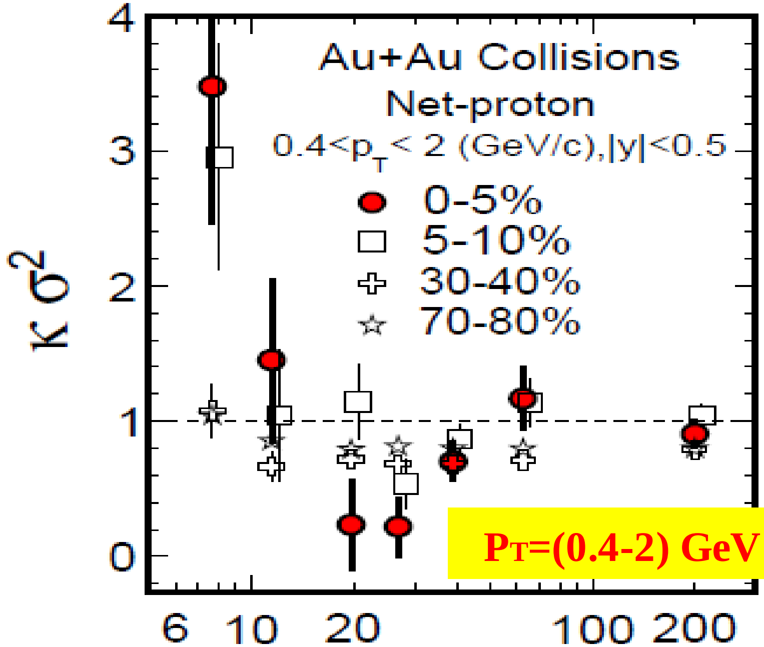
Fluctuations near the critical point

-dramatically increase near Tc

-Strongly correlated

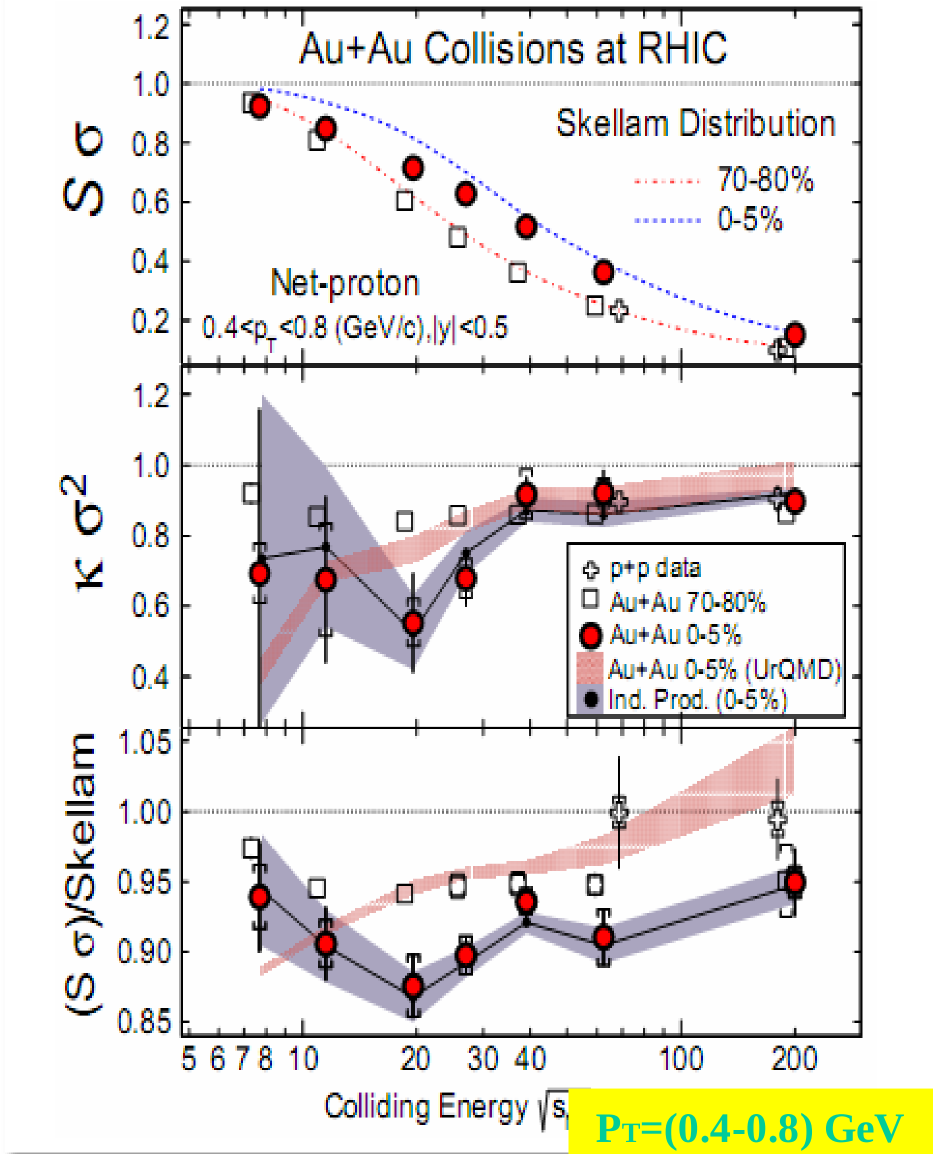
STAR BES: Cumulant ratios





PT=(0.4-2) GeV

STAR PRL 2014



PT=(0.4-0.8) GeV

Theoretical predictions on critical fluctuations

Stephanov PRL 2009

$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\}, \qquad \Omega = \int d^3x \left[\frac{1}{2}(\nabla\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \cdots\right]$$

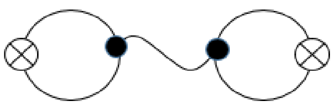
$$\langle\sigma_0^2\rangle = \frac{T}{V}\xi^2 \qquad \langle\sigma_0^3\rangle = \frac{2\lambda_3 T}{V}\xi^6; \qquad \langle\sigma_0^4\rangle_c = \frac{6T}{V}[2(\lambda_3\xi)^2 - \lambda_4]\xi^8.$$

Critical Fluctuations of particles :

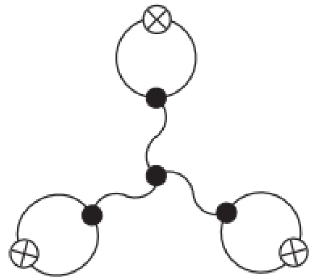
$$\langle(\delta N)^2\rangle \sim \xi^2$$

$$\langle(\delta N)^3\rangle \sim \xi^{4.5}$$

$$\langle(\delta N)^4\rangle \sim \xi^7$$

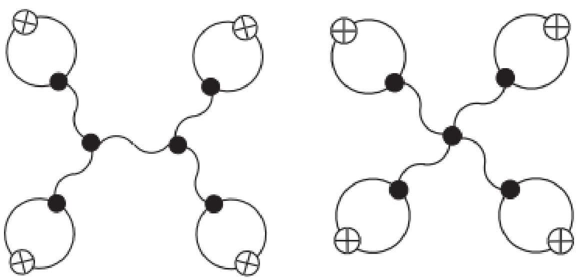


$\sim \xi^2$



$\sim \xi^{4.5}$

$3\times$



$\sim \xi^7$

At critical point : $\xi \sim \infty$ **(infinite medium)**

Finite size & finite evolution time: $\xi < \mathrm{O}(2\text{-}3\mathrm{fm})$

It is important to address the effects from dynamical evolutions

Dynamical Modeling near the QCD critical point

Chiral Hydrodynamics (I)

K. Paech, H. Stocker and A. Dumitru, PRC2003

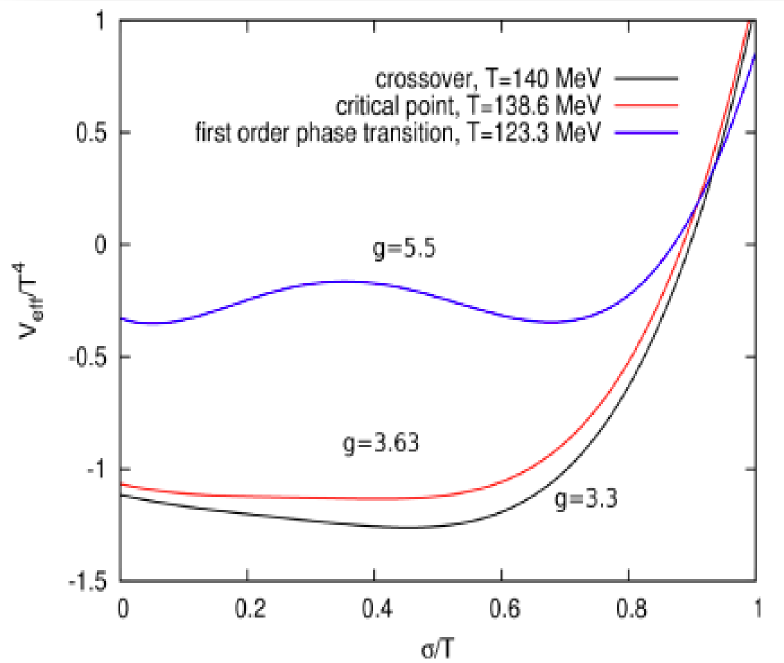
$$L = \bar{q}[i\gamma - g(\sigma + i\gamma_5 \tau\pi)]q + \frac{1}{2}[\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \pi \partial^\mu \pi] - U(\sigma, \pi)$$

$$\begin{cases} \partial_\mu \partial^\mu \sigma + \dfrac{\partial U_{eff}}{\partial \sigma} + g < \bar{q}q >= 0 \\ \partial_\mu T^{\mu\nu}_{fluid} = S^\nu \qquad S^\nu = -(\partial^2 u + \dfrac{\partial U_{eff}}{\partial u})\partial^\nu \sigma \end{cases}$$

the order of the phase transition is in charged by the coupling g.

σ order parameter

quark & anti-quark is treated as the heat bath (fluid), which interact with the chiral field via effective mass $g\sigma$



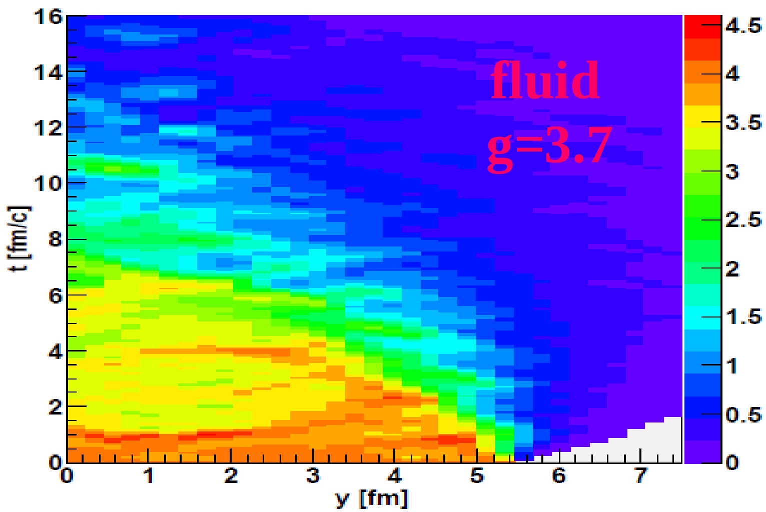
Chiral Hydrodynamics (II)

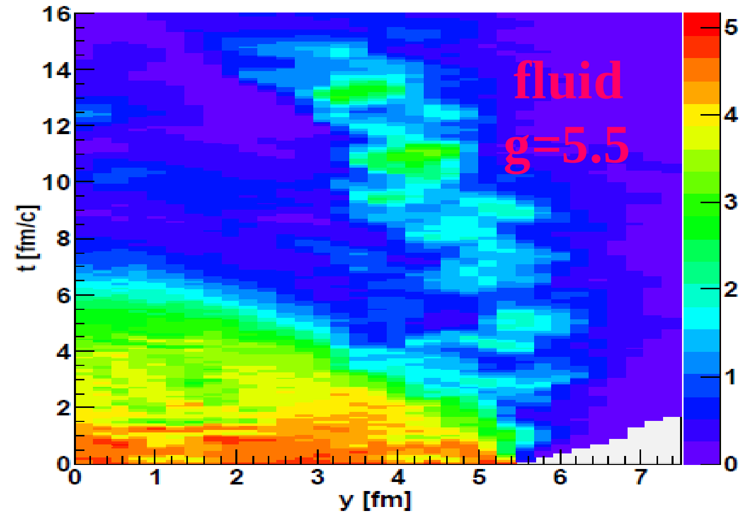
K. Paech, H. Stocker and A. Dumitru, PRC2003

$$L = \bar{q}[i\gamma - g(\sigma + i\gamma_5 \tau\pi)]q + \frac{1}{2}[\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \pi \partial^\mu \pi] - U(\sigma, \pi)$$

$$\begin{cases} \partial_\mu \partial^\mu \sigma + \dfrac{\partial U_{eff}}{\partial \sigma} + g < \bar{q} q >= 0 \\ \partial_\mu T^{\mu\nu}_{fluid} = S^\nu \end{cases} \qquad S^\nu = -(\partial^2 u + \frac{\partial U_{eff}}{\partial u}) \partial^\nu \sigma$$

(fluctuation is introduced by initial state)

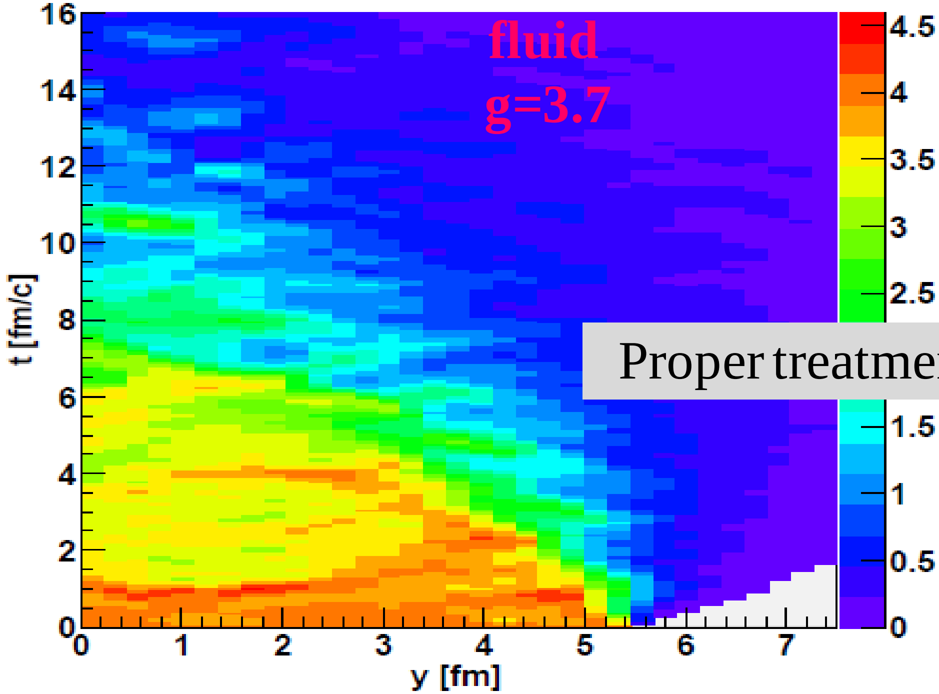




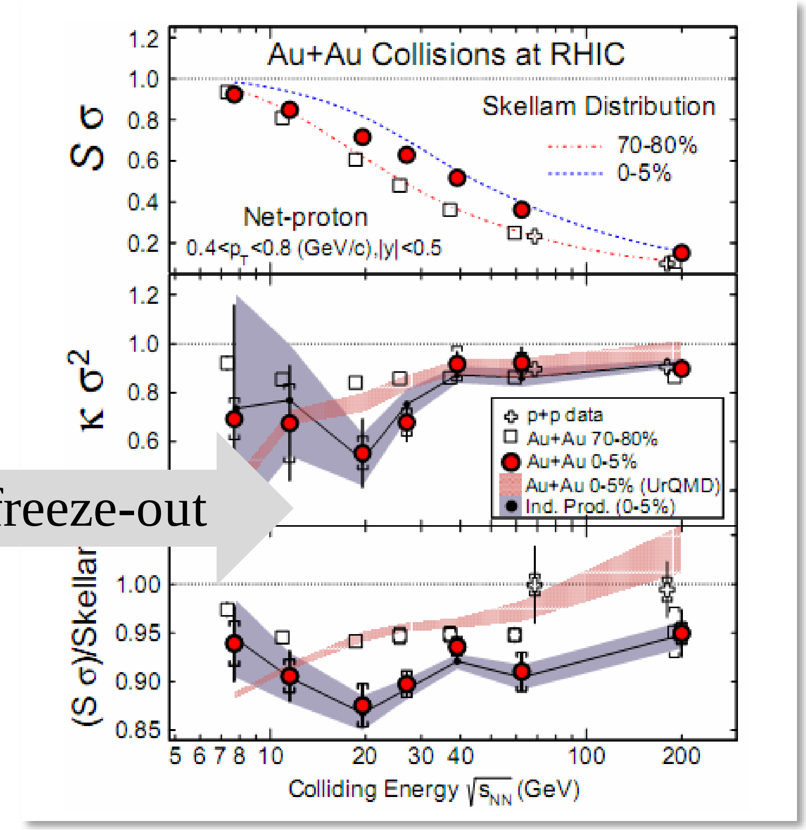
-Chiral fluid dynamics with dissipation & noise Nahrgang, et al., PRC 2011

-Chiral fluid dynamics with a Polyakov loop (PNJL)

Herold, et al., PRC 2013



Proper treatment of freeze-out



From dynamical evolution to experimental observables, it is important to properly treat the freeze-out procedure with external field

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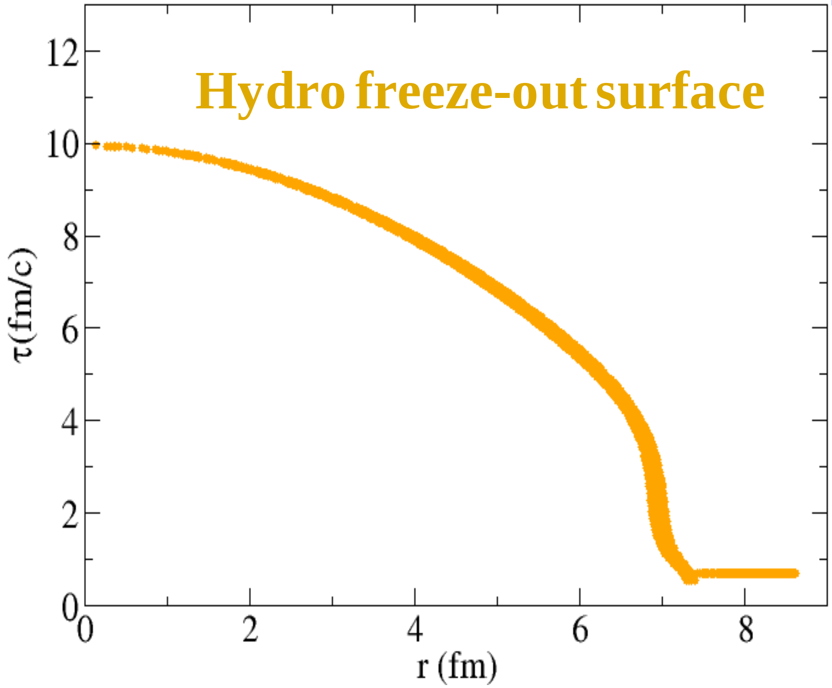
Correlated fluctuations along the freeze-out surface near T_c

-theoretical models

Particle emissions near Tc with external field



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Particle emissions in traditional hydro

$$E\frac{dN}{d^3p} = \int_\Sigma \frac{p_\mu d\sigma^\mu}{2\pi^3} f(x,p)$$

Particle emissions with external field

$$M \longrightarrow g\sigma(x)$$

$$f(x,p) = f_0(x,p)[1 - g\sigma(x)/(\gamma T)]$$
$$= f_0 + \delta f$$

$$\langle \delta f_1 \delta f_2 \rangle_\sigma = f_{01} f_{02} f_{03} \left(\frac{g^2}{\gamma_1 \gamma_2} \frac{1}{T^3} \right) \langle \sigma_1 \sigma_2 \rangle_c ,$$

$$\langle \delta f_1 \delta f_2 \delta f_3 \rangle_\sigma = f_{01} f_{02} f_{03} \left(- \frac{g^3}{\gamma_1 \gamma_2 \gamma_3} \frac{1}{T^3} \right) \langle \sigma_1 \sigma_2 \sigma_3 \rangle_c ,$$

$$\langle \delta f_1 \delta f_2 \delta f_3 \delta f_4 \rangle_\sigma = f_{01} f_{02} f_{03} f_{04} \left(\frac{g^4}{\gamma_1 \gamma_2 \gamma_3 \gamma_4} \frac{1}{T^4} \right) \langle \sigma_1 \sigma_2 \sigma_3 \sigma_4 \rangle_c .$$

$$\begin{aligned}
\langle \delta f_1 \delta f_2 \rangle_\sigma &= f_{01} f_{02} f_{03} \left(\frac{g^2}{\gamma_1 \gamma_2} \frac{1}{T^3} \right) \langle \sigma_1 \sigma_2 \rangle_c \,, \\
\langle \delta f_1 \delta f_2 \delta f_3 \rangle_\sigma &= f_{01} f_{02} f_{03} \left(-\frac{g^3}{\gamma_1 \gamma_2 \gamma_3} \frac{1}{T^3} \right) \langle \sigma_1 \sigma_2 \sigma_3 \rangle_c \,, \\
\langle \delta f_1 \delta f_2 \delta f_3 \delta f_4 \rangle_\sigma &= f_{01} f_{02} f_{03} f_{04} \left(\frac{g^4}{\gamma_1 \gamma_2 \gamma_3 \gamma_4} \frac{1}{T^4} \right) \langle \sigma_1 \sigma_2 \sigma_3 \sigma_4 \rangle_c \,.
\end{aligned}$$

For stationary & infinite medium:

$$\begin{aligned}
\left\langle (\delta N)^2 \right\rangle_c &= \left(\frac{g_i}{(2\pi)^3} \right)^2 \int d^3p_1 d^3x_1 \int d^3p_2 d^3x_2 \frac{f_{01} f_{02}}{\gamma_1 \gamma_2} \frac{g^2}{T^2} \langle \sigma_1 \sigma_2 \rangle_c \,, \\
\left\langle (\delta N)^3 \right\rangle_c &= \left(\frac{g_i}{(2\pi)^3} \right)^3 \int d^3p_1 d^3x_1 \int d^3p_2 d^3x_2 \int d^3p_3 d^3x_3 \frac{f_{01} f_{02} f_{03}}{\gamma_1 \gamma_2 \gamma_3} \left(-\frac{g^3}{T^3} \langle \sigma_1 \sigma_2 \sigma_3 \rangle_c \right) , \\
\left\langle (\delta N)^4 \right\rangle_c &= \left(\frac{g_i}{(2\pi)^3} \right)^4 \int d^3p_1 d^3x_1 \int d^3p_2 d^3x_2 \int d^3p_3 d^3x_3 \int d^3p_4 d^3x_4 \frac{f_{01} f_{02} f_{03} f_{04}}{\gamma_1 \gamma_2 \gamma_3 \gamma_4} \frac{g^4}{T^4} \langle \sigma_1 \sigma_2 \sigma_3 \sigma_4 \rangle_c \,.
\end{aligned}$$

--the results in Stephanov PRL09 are reproduced

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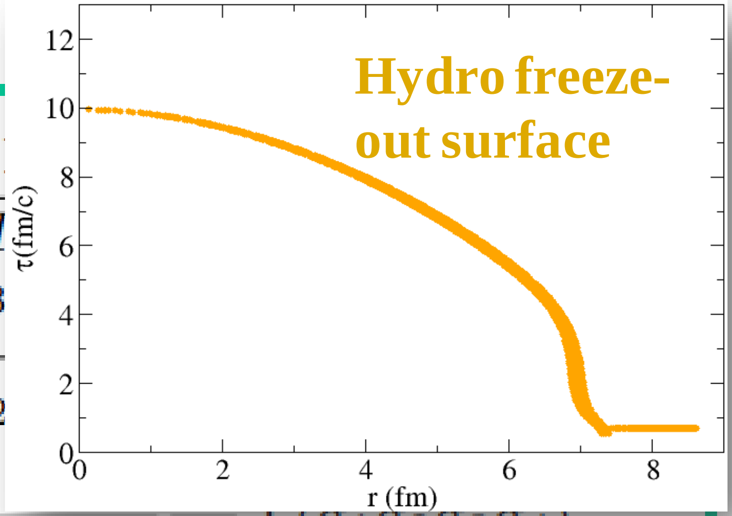
$$\langle \delta f_1 \delta f_2 \rangle_\sigma = f_{01} f_{02} f_{03} \left(\frac{g^2}{\gamma_1 \gamma_2} \frac{\cdots}{T \cdots} \right.$$

$$\langle \delta f_1 \delta f_2 \delta f_3 \rangle_\sigma = f_{01} f_{02} f_{03} \left(- \frac{g^3}{\gamma_1 \gamma_2 \cdots} \right.$$

$$\langle \delta f_1 \delta f_2 \delta f_3 \delta f_4 \rangle_\sigma = f_{01} f_{02} f_{03} f_{04} \left(\frac{\cdots}{\gamma_1 \gamma_2 \gamma_3 \gamma_4} \frac{\cdots}{T^4} \right) \langle \sigma_1 \sigma_2 \sigma_3 \sigma_4 \rangle_c .$$



CORRELATED particle emissions along the freeze-out surface

$$\left\langle (\delta N)^2 \right\rangle_c = \left(\frac{g_i}{(2\pi)^3} \right)^2 \left(\prod_{i=1,2} \left(\frac{1}{E_i} \int d^3 p_i \int_{\Sigma_i} p_{i\mu} d\sigma_i^\mu d\eta_i \right) \right) \frac{f_{01} f_{02}}{\gamma_1 \gamma_2} \frac{g^2}{T^2} \langle \sigma_1 \sigma_2 \rangle_c ,$$

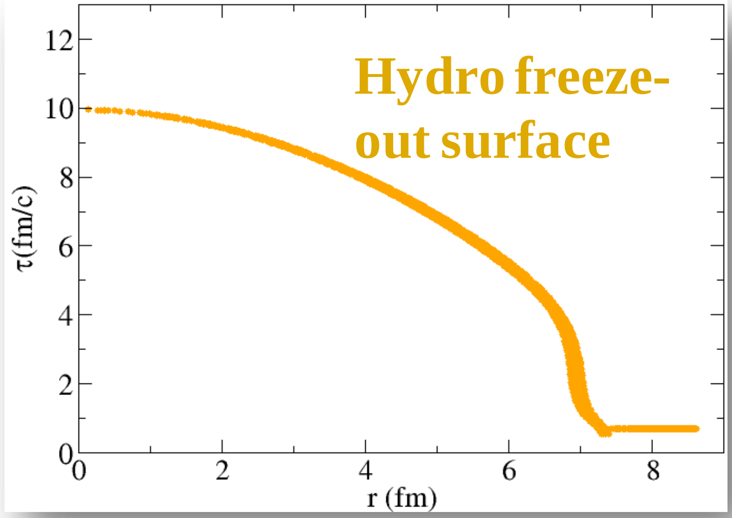
$$\left\langle (\delta N)^3 \right\rangle_c = \left(\frac{g_i}{(2\pi)^3} \right)^3 \left(\prod_{i=1,2,3} \left(\frac{1}{E_i} \int d^3 p_i \int_{\Sigma_i} p_{i\mu} d\sigma_i^\mu d\eta_i \right) \right) \frac{f_{01} f_{02} f_{03}}{\gamma_1 \gamma_2 \gamma_3} \left(- \frac{g^3}{T^3} \langle \sigma_1 \sigma_2 \sigma_3 \rangle_c \right) ,$$

$$\left\langle (\delta N)^4 \right\rangle_c = \left(\frac{g_i}{(2\pi)^3} \right)^4 \left(\prod_{i=1,2,3,4} \left(\frac{1}{E_i} \int d^3 p_i \int_{\Sigma_i} p_{i\mu} d\sigma_i^\mu d\eta_i \right) \right) \frac{f_{01} f_{02} f_{03} f_{04}}{\gamma_1 \gamma_2 \gamma_3 \gamma_4} \frac{g^4}{T^4} \langle \sigma_1 \sigma_2 \sigma_3 \sigma_4 \rangle_c$$

--partially include the evolution effects and volume effects

For simplicity: We assume that the correlated sigma field only influence the particle emissions near Tc, which does not influence the evolution of the bulk matter

Input: hydro freeze-out surface; hydro has been tuned to fit dN/dy, spectra and V2



CORRELATED particle emissions along the freeze-out surface

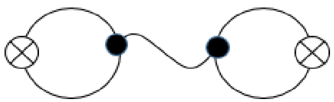
$$\left\langle (\delta N)^2 \right\rangle_c = \left(\frac{g_i}{(2\pi)^3}\right)^2 \left(\prod_{i=1,2} \left(\frac{1}{E_i}\int d^3p_i \int_{\Sigma_i} p_{i\mu} d\sigma_i^{\mu} d\eta_i\right)\right) \frac{f_{01} f_{02}}{\gamma_1\gamma_2} \frac{g^2}{T^2} \langle \sigma_1\sigma_2\rangle_c \,,$$

$$\left\langle (\delta N)^3 \right\rangle_c = \left(\frac{g_i}{(2\pi)^3}\right)^3 \left(\prod_{i=1,2,3} \left(\frac{1}{E_i}\int d^3p_i \int_{\Sigma_i} p_{i\mu} d\sigma_i^{\mu} d\eta_i\right)\right) \frac{f_{01} f_{02} f_{03}}{\gamma_1\gamma_2\gamma_3} \left(-\frac{g^3}{T^3} \langle \sigma_1\sigma_2\sigma_3\rangle_c\right) ,$$

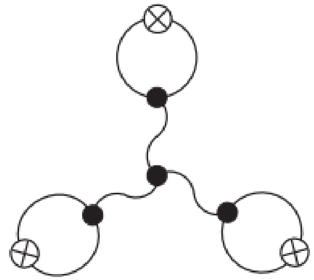
$$\left\langle (\delta N)^4 \right\rangle_c = \left(\frac{g_i}{(2\pi)^3}\right)^4 \left(\prod_{i=1,2,3,4} \left(\frac{1}{E_i}\int d^3p_i \int_{\Sigma_i} p_{i\mu} d\sigma_i^{\mu} d\eta_i\right)\right) \frac{f_{01} f_{02} f_{03} f_{04}}{\gamma_1\gamma_2\gamma_3\gamma_4} \frac{g^4}{T^4} \langle \sigma_1\sigma_2\sigma_3\sigma_4\rangle_c$$

--partially include the evolution effects and volume effects

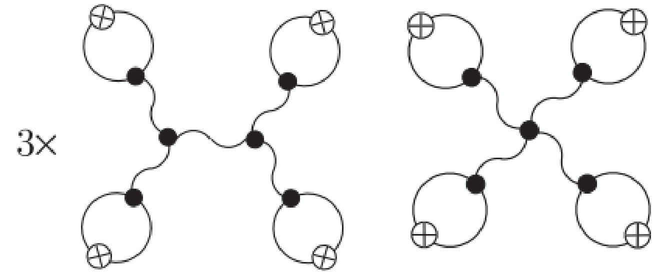
The choice of input parameters



$g_{\sigma pp}$ ξ



$g_{\sigma pp}$ ξ λ_3



$g_{\sigma pp}$ ξ λ_3 λ_4

- $g_{\sigma pp} \sim (0, 10)$

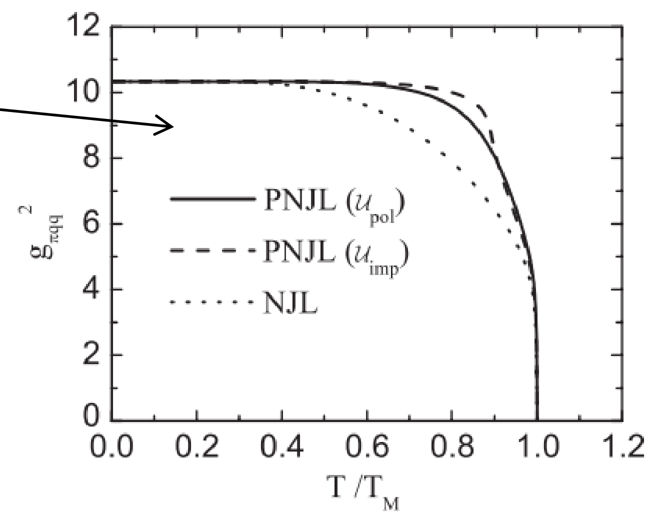
  phenomenological model

- $\xi \sim$ 3fm

  near the critical point, critical slowing down

- $\lambda_3 \sim (0, 8)$, $\lambda_4 \sim (4, 20)$

  lattice simulation of the effective potential around critical point.



A. Andronic, et al. NPA (2006); M. A. Stephanov, Phys. Rev. Lett. 102, 032301 (2009); S. P. Klevansky, Rev. Mod. Phys, Vol, 64, No.3 (1992); W. Fu, Y-x, Liu, Phys. Rev. D 79, 074011(2009); M. M. Tsypin, Phys. Rev. Lett. 73, 2015 (1994); M. M. Tsypin, Phys. Rev. B 55, 8911 (1997).; B. Berdnikov and K. Rajagopal, Phys. Rev. D 61, 105017 (2000).

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Correlated fluctuations along the freeze-out surface near T_c

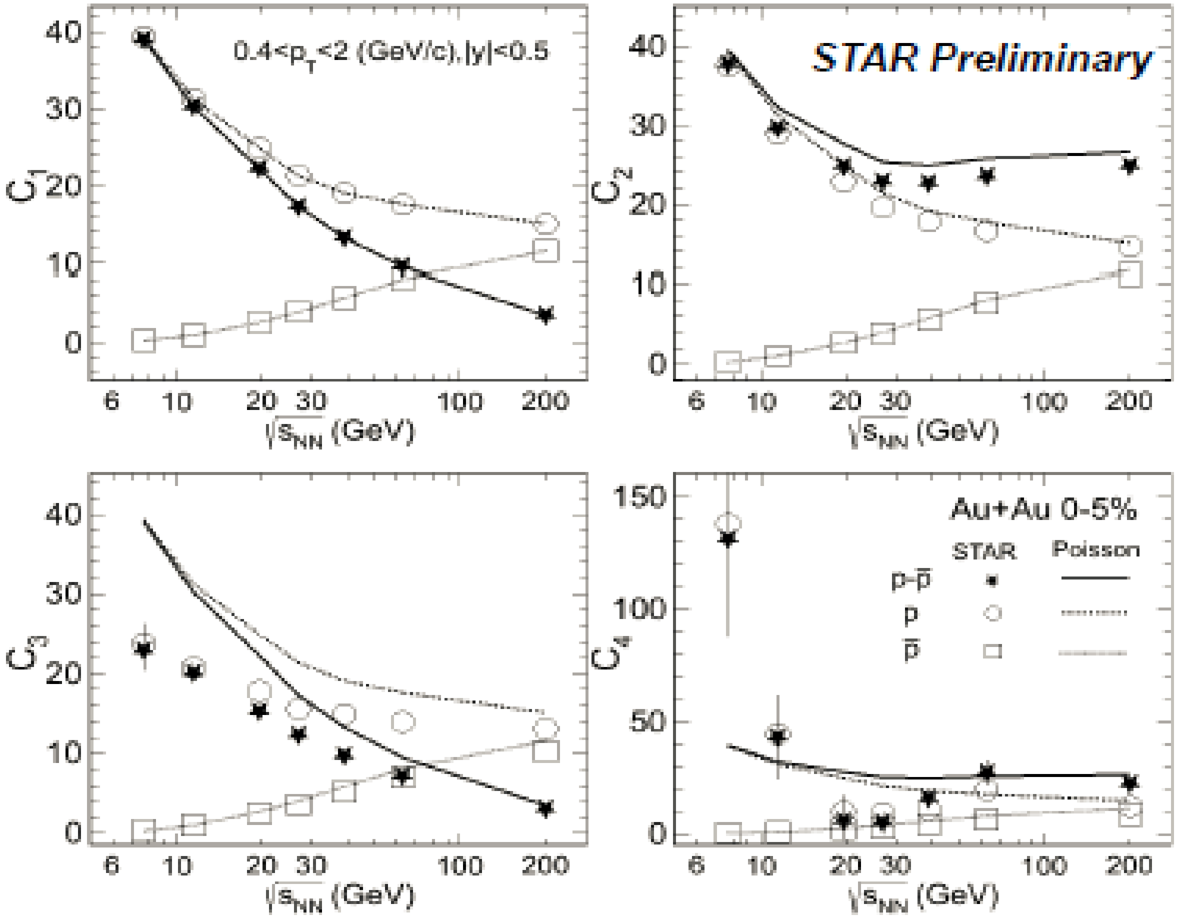
-comparison with the experimental data

STAR data vs Thermal fluctuation baselines

P_T=(0.4-2) GeV

Xiaofeng Luo CPOD 2014

Cumulants vs. Poisson



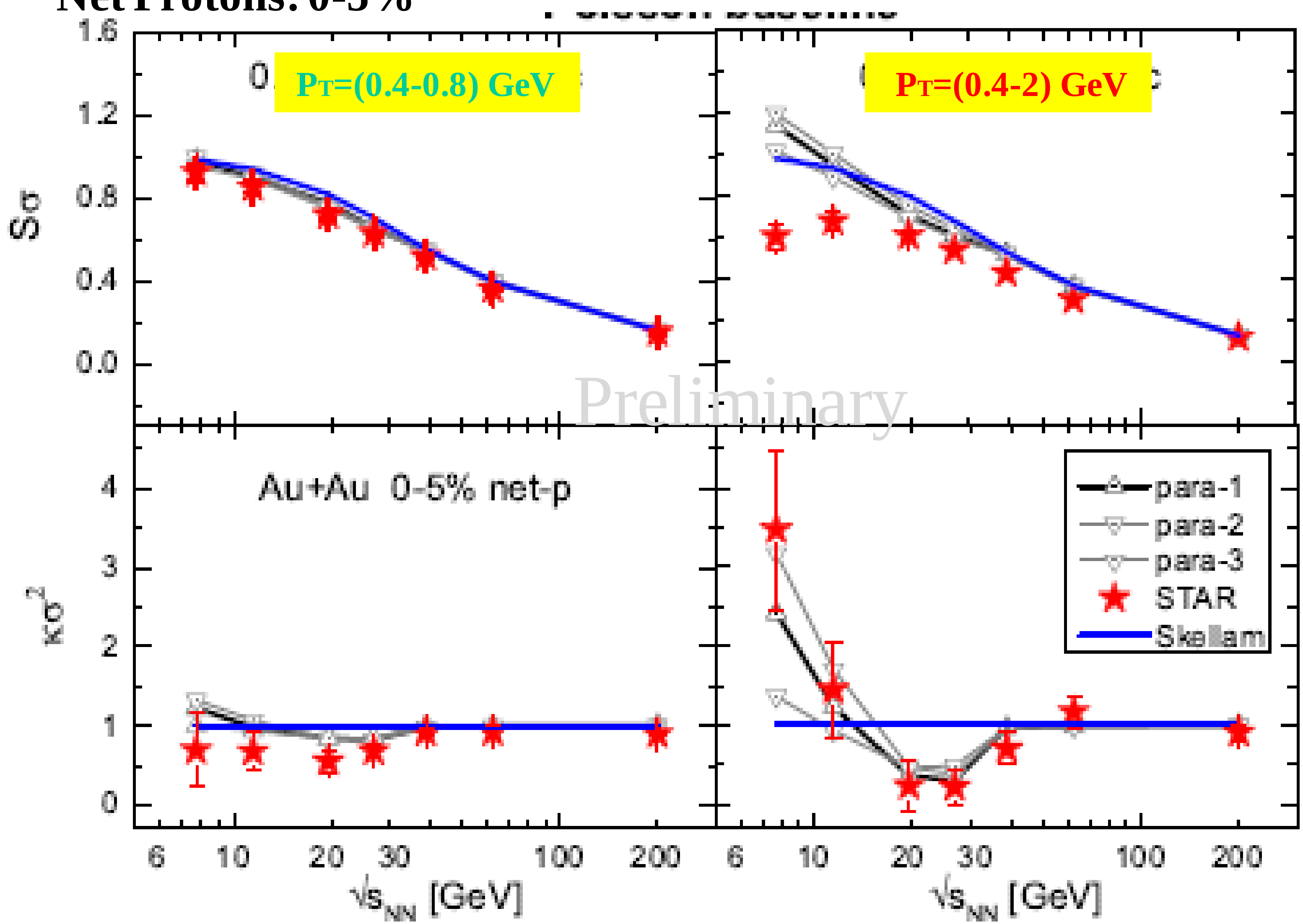
Fluctuations measured in experiment: critical fluct. + thermal fluct. + …

The higher order cumulants shows large deviations from Poisson expectations

$\kappa\sigma^2, S\sigma$: (Model + Poisson baselines)

Jiang, Li & Song in preparation

Net Protons: 0-5%

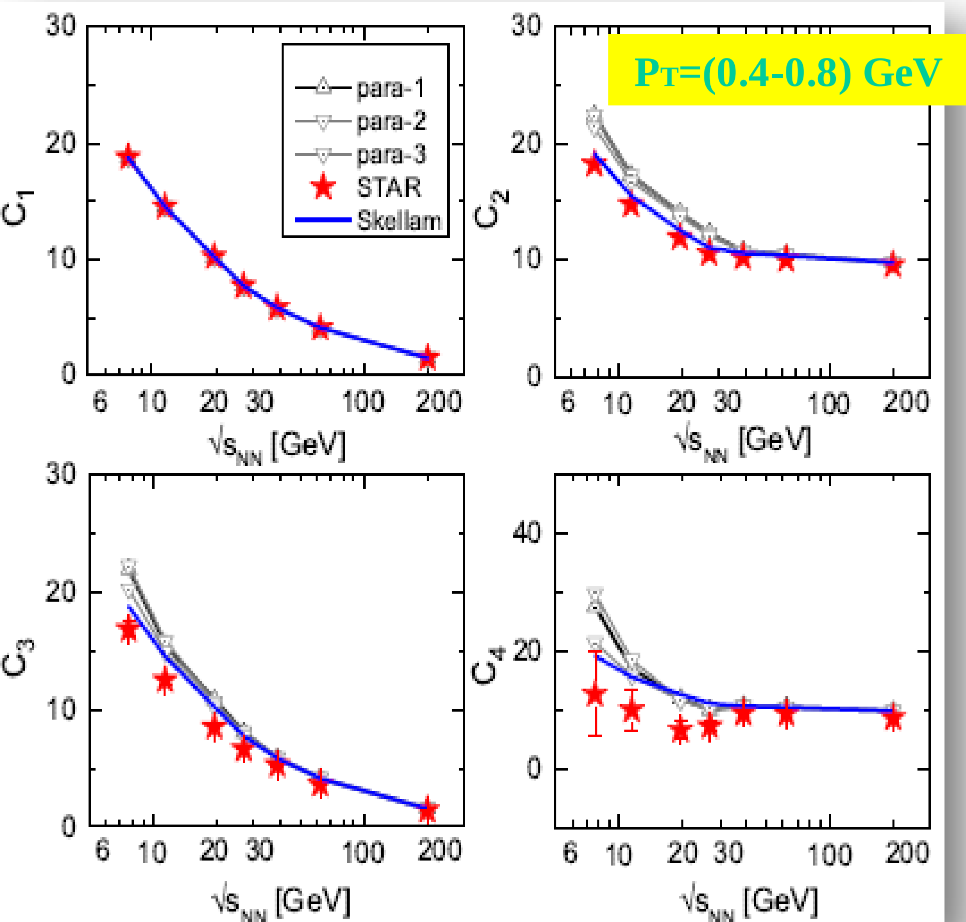


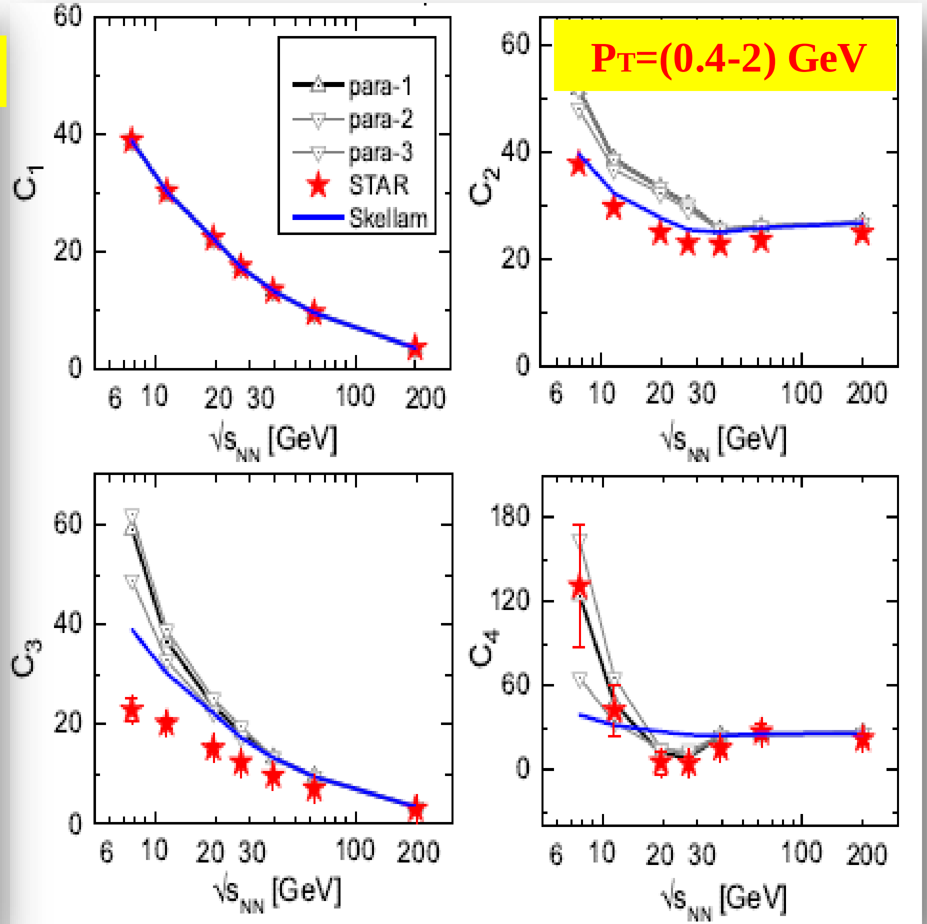
C1 C2 C3 C4: (Model + Poisson baselines)

Net Protons 0-5%

Preliminary

Jiang, Li & Song in preparation

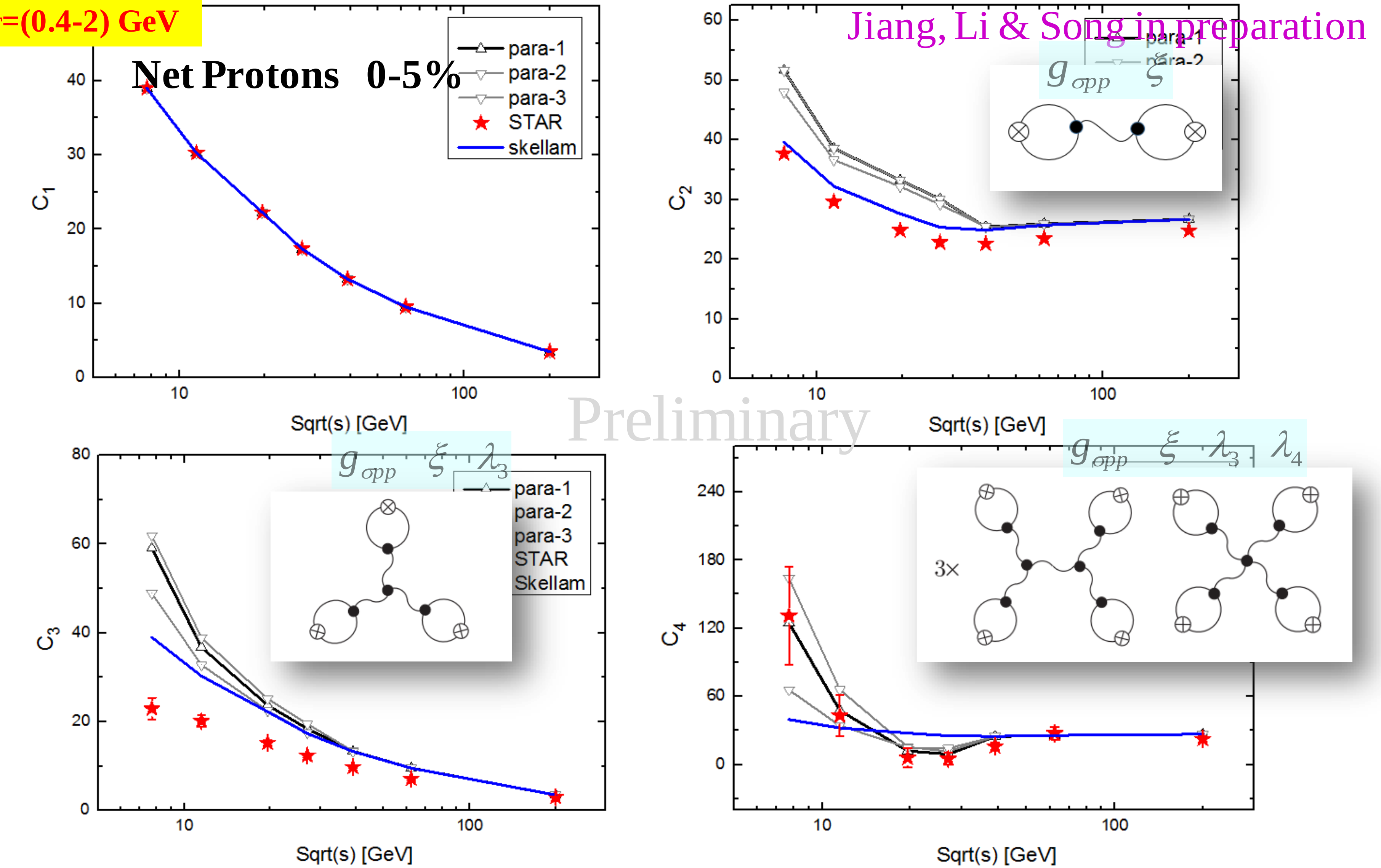




Critical fluctuations give positive contribution to C2 , C3; well above the poisson baselines, can NOT explain/describe the C2 , C3 data

C_1 C_2 C_3 C_4: Pt-(0.4-2) GeV (Model + Poisson baselines)

P_T=(0.4-2) GeV



In this model (and also Stephanov PRL09 framework) critical fluctuations's contributions to C_2 , C_3 are always positive

Effects from dynamical evolutions

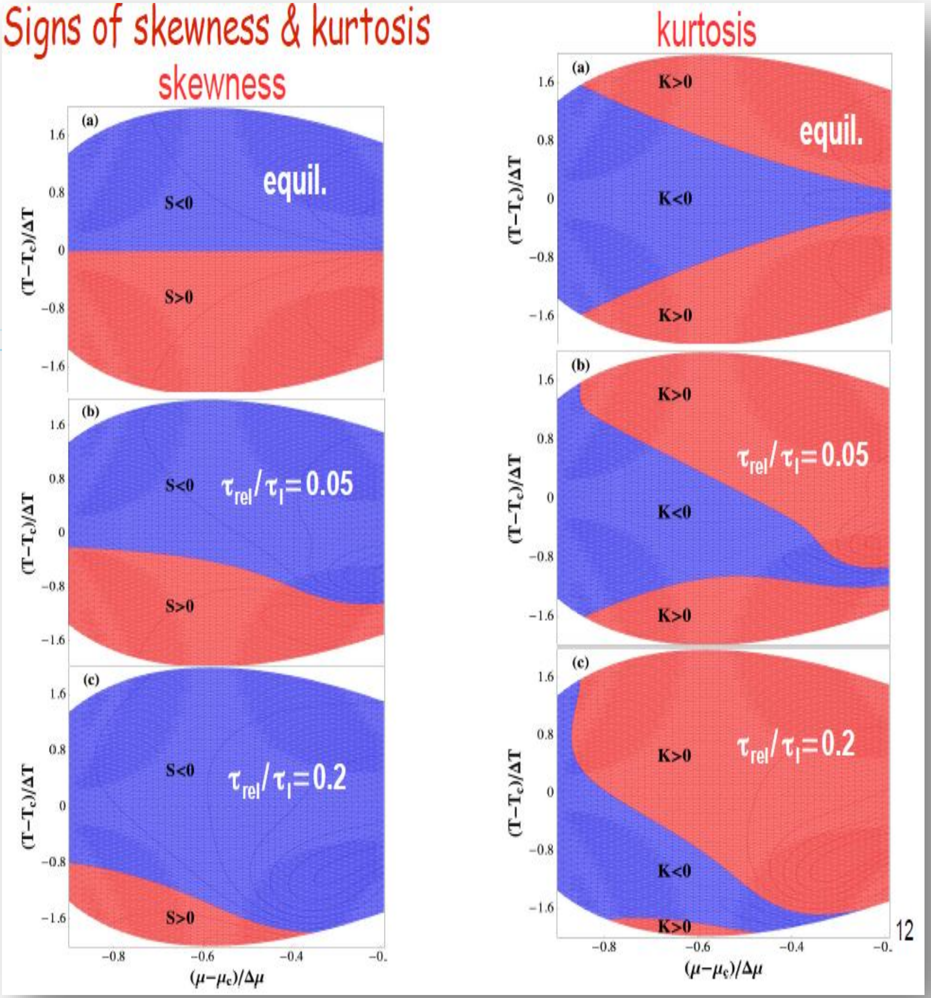
$$\partial_\tau P(\sigma;\tau) = \frac{1}{m_\sigma^2 \tau_{\mathrm{eff}}}\left[\partial_\sigma\left[\partial_\sigma \Omega_0(\sigma) + V_4^{-1}\partial_\sigma\right]P(\sigma;\tau)\right]$$

near-equilibrium limit: $\delta\kappa_n = \kappa_n - \kappa_n^{\mathrm{eq}}$

$$\partial_\tau \kappa_2 = -2\tau_{\mathrm{eff}}^{-1} a_2 \delta\kappa_2$$

$$\partial_\tau \kappa_3 = -3\tau_{\mathrm{eff}}^{-1}\left[a_2\delta\kappa_2 + a_3\delta\kappa_3\right]$$

$$\partial_\tau \kappa_4 = -4\tau_{\mathrm{eff}}^{-1}\left[a_2\delta\kappa_2 + a_3\delta\kappa_3 + a_4\delta\kappa\right.$$



S. Mukherjee, R. Venugopalan, Y. Yin, PRC92 (2015)

sign of non-Gaussian cumulants can be different from equilibrium one

Summary

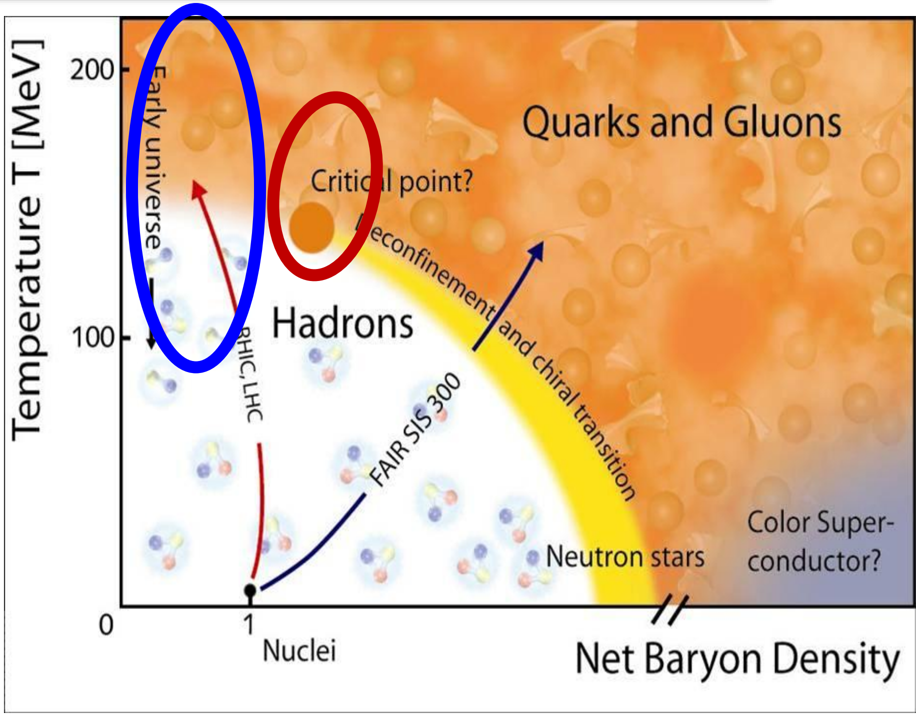
Dynamical Modeling for Relativistic Heavy Ion Collisions

Hydrodynamics & its hybrid model

- -QGP viscosity
- -initial state flucutations. & final state correlations
- -Collective flow in small systems

Dynamical modeling near the QCD critical point

- -Correlated fluctuations



-more & precise experimental data at different colliding systems provide valuable information on the properties of the QGP and the QCD phase diagram

-Sophisticated dynamical model are need to be further developed

Thank You

Boltzmann approach with external field

Stephanov PRD 2010

$$\mathcal{S} = \int d^3x \frac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma - U(\sigma)) - \int ds M(\sigma),$$

$$\begin{cases} \partial^2\sigma + dU/d\sigma + (dM/d\sigma)\int_p f/\gamma = 0. \\ \frac{p^\mu}{M}\frac{\partial f}{\partial x^\mu} + \partial^\mu M \frac{\partial f}{\partial p^\mu} + \mathcal{C}[f] = 0, \end{cases}$$

-analytical solution with perturbative expansion, please refer to Stephanov PRD 2010

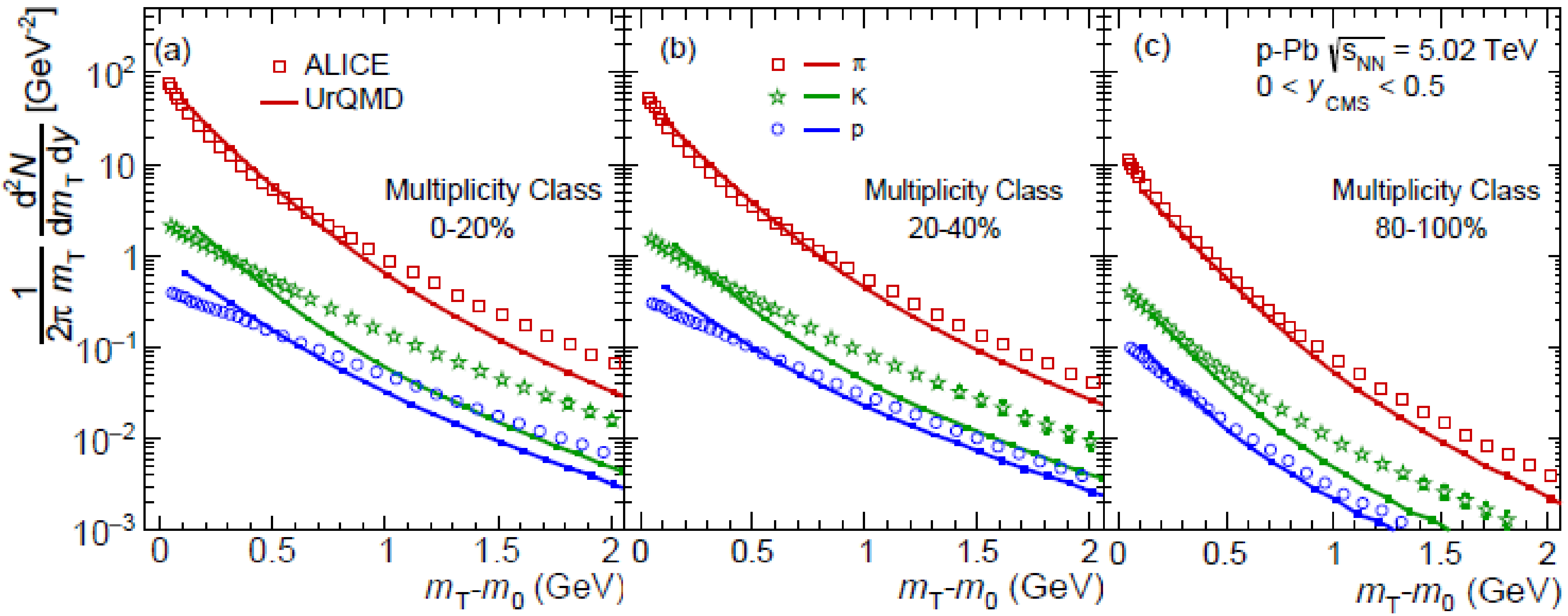
Stationary solution for the Boltamann equation with external field

$$f_\sigma(\boldsymbol{p}) = e^{\mu/T} e^{-\gamma(\boldsymbol{p})M/T}.$$

Effective particle mass: $M = M(\sigma) = g\sigma$

m_T spectra & radial flow

Zhou, Zhu, Li, S ong, PRC2015



-Broken of m_T-scaling in both ALICE and UrQMD

-The UrQMD systems can not generate the amount of radial flow as observed in experiment