Equation of State from Lattice QCD

Prasad Hegde

BNL-Bielefeld-CCNU and HotQCD collaborations

Central China Normal University,
Wuhan, China.

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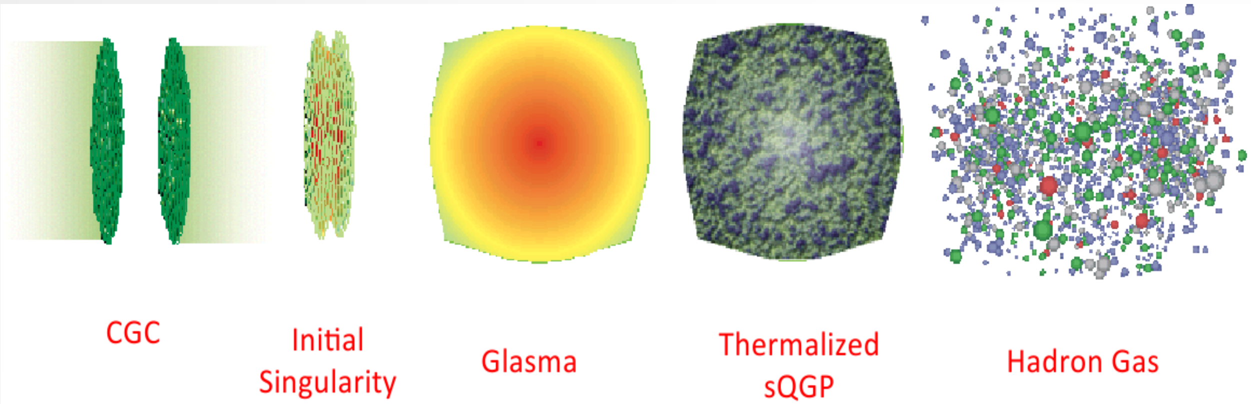
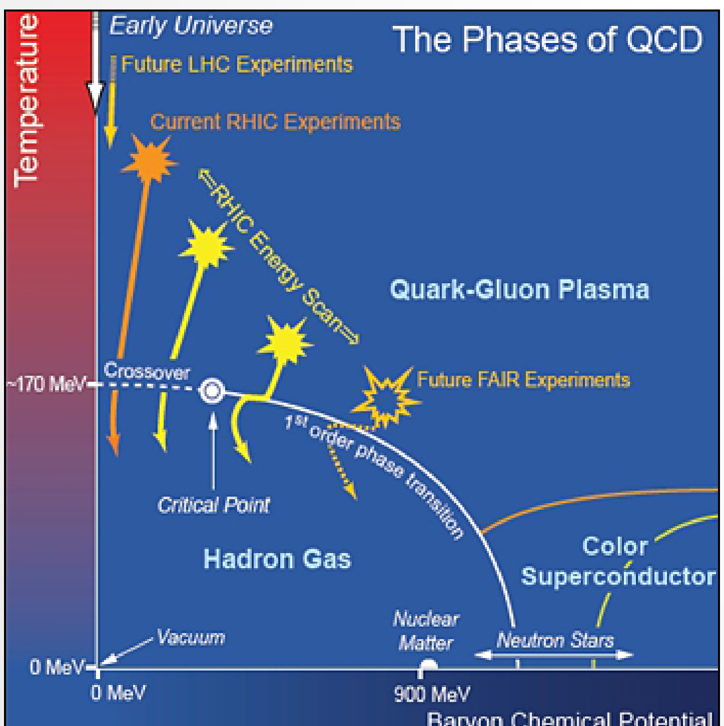
Equation of state



Sole QCD input in the hydrodynamic evolution of the QGP created in heavy ion collisions [see talk by Huichao Song].

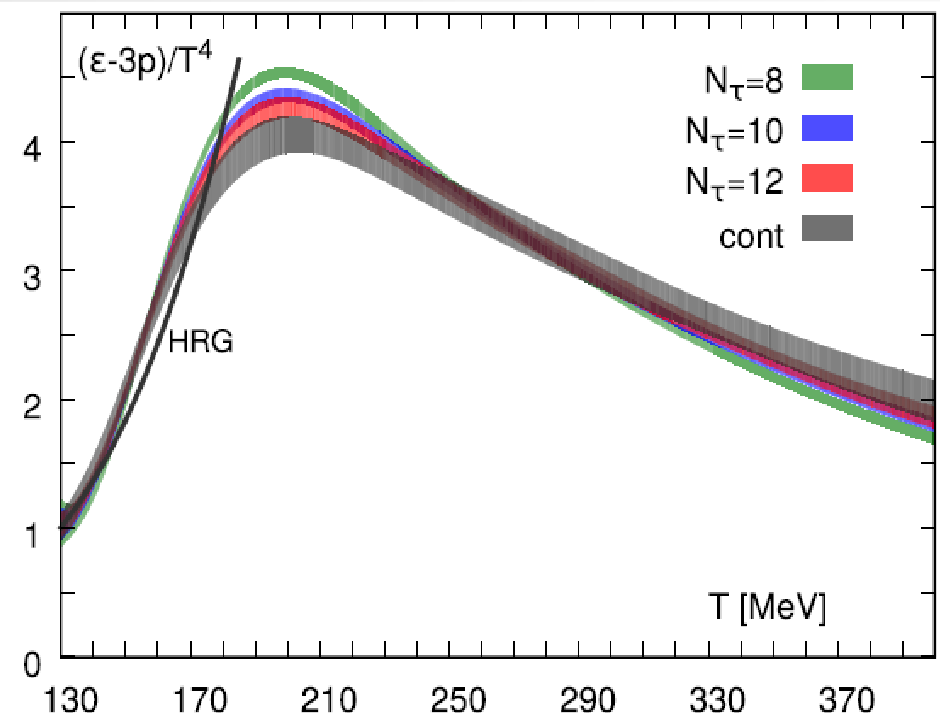


Baryochemical potential μ_B negligible (compared to the temperature) at LHC and top RHIC energies.

However, with the advent of the Beam Energy Scan program at RHIC, need an EoS valid up to μ_B ~ 500 MeV.

Resummed perturbation theory [N.Haque *et al*. JHEP 1405, 027 (2014)] does not work for temperatures below T~200 MeV; non-perturbative methods necessary.

Pressure, energy and entropy

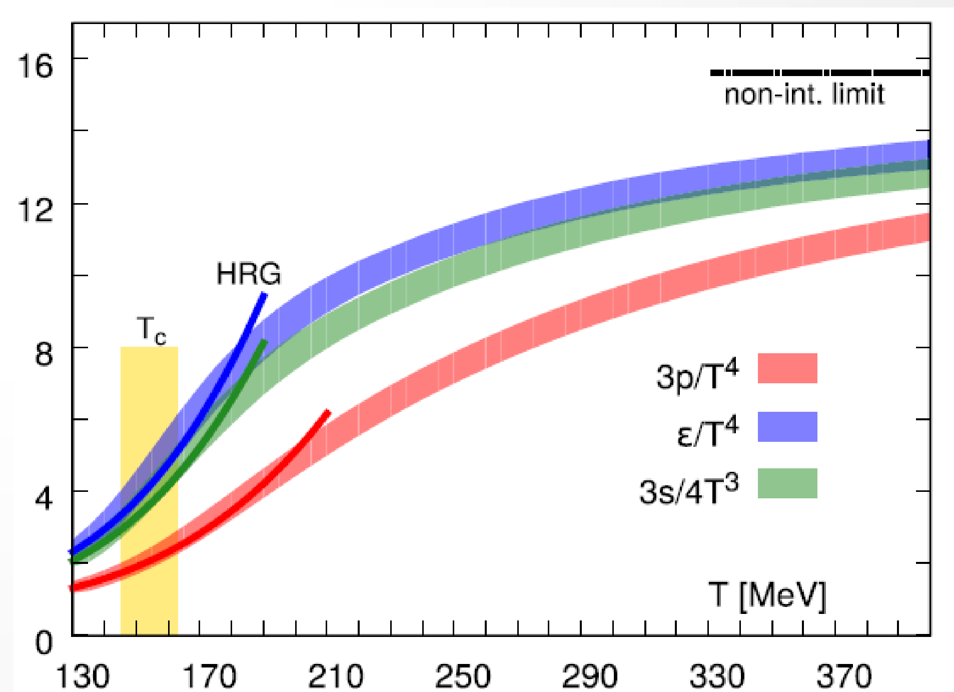


Recently, we presented continuum-extrapolated results for the equation of state at μ_B = 0 [HotQCD, Phys.Rev.D90, 054503 (2014)].

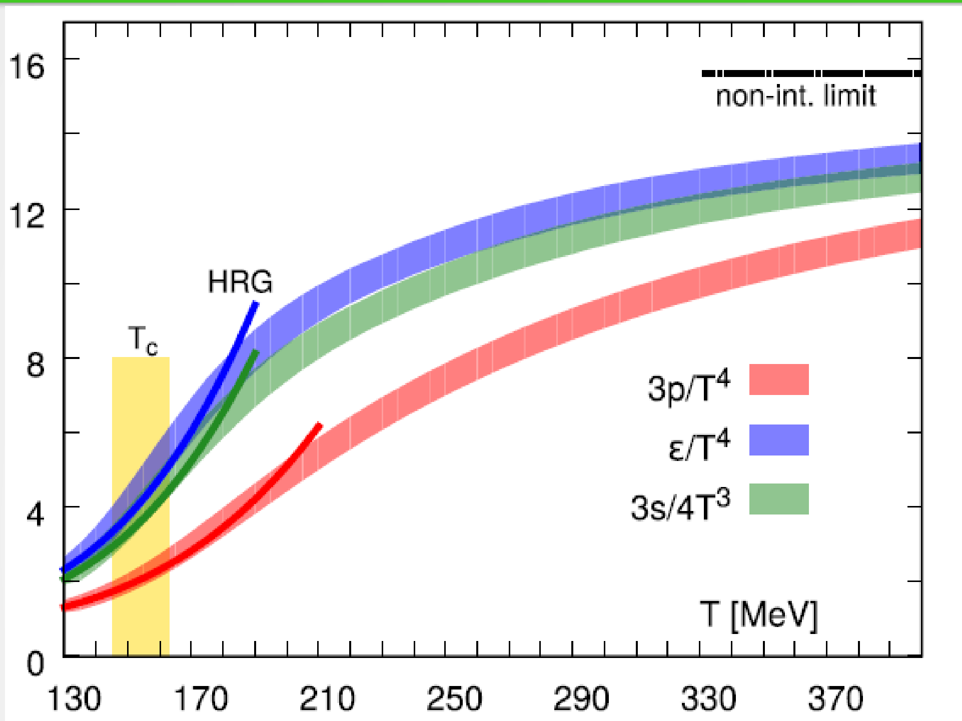
Behavior as expected from the crossover nature of the transition *i.e.* smooth transition from a gaseous hadronic phase to deconfined quarks and gluons.

Around 10% difference from the non-interacting ideal gas limit even at T ≈ 400 MeV ≈ 2.7 T_c.

p, s and ε well-approximated by a gas of non-interacting resonances (Hadron Resonance Gas) at temperatures below T_c.



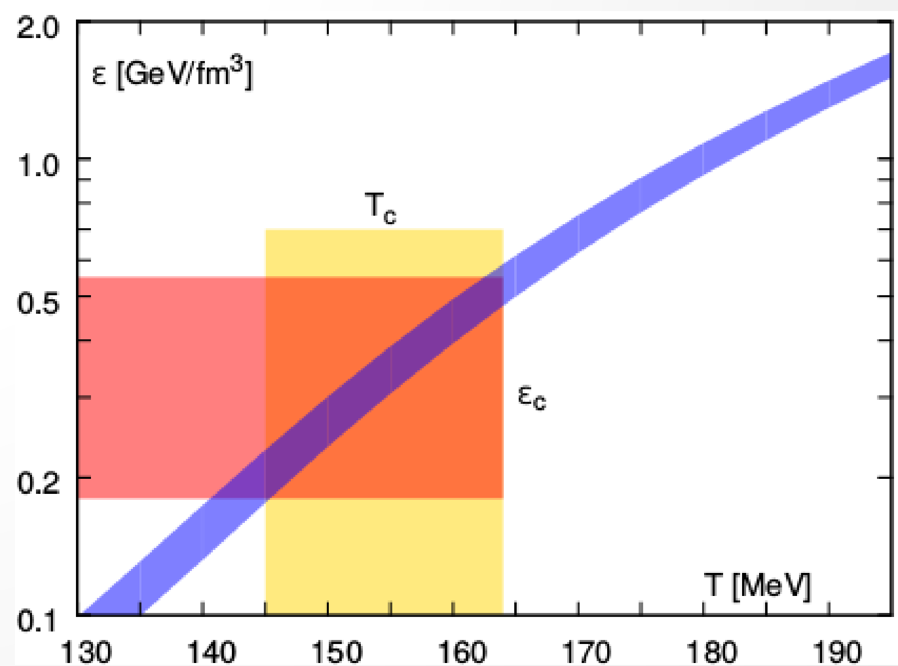
Parametrization and ε(T_c)



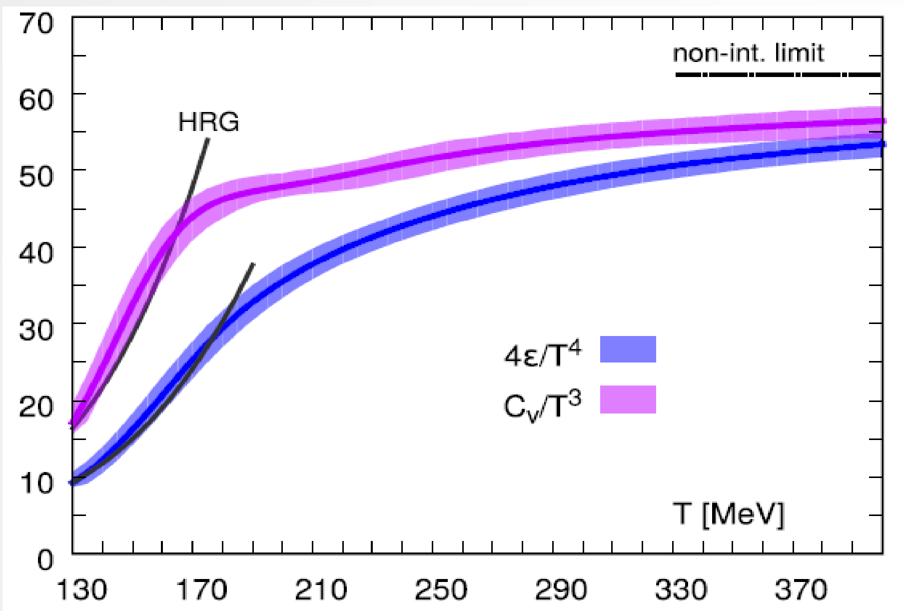
Analytic parametrization of the pressure over the entire temperature range [HotQCD, Phys.Rev.D90, 054503 (2014)].

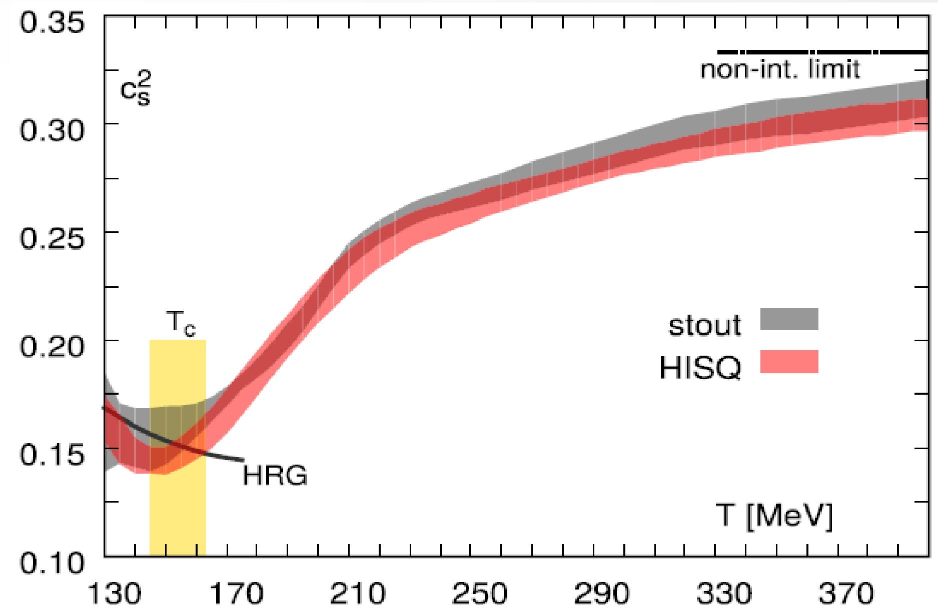
$$\frac{p}{T^4} = \frac{1+\tanh\left(c_t(t-t_0)\right)}{2} \times \frac{p_{\mathrm{id}} + a_n/\bar{t} + b_n/\bar{t}^2 + c_n/\bar{t}^3 + d_n/\bar{t}^4}{1 + a_d/\bar{t} + b_d/\bar{t}^2 + c_d/\bar{t}^3 + d_d/\bar{t}^4}$$

The energy density at the phase transition is not too different from normal nuclear energy density (~0.16 GeV/fm³).



Specific heat and speed of sound



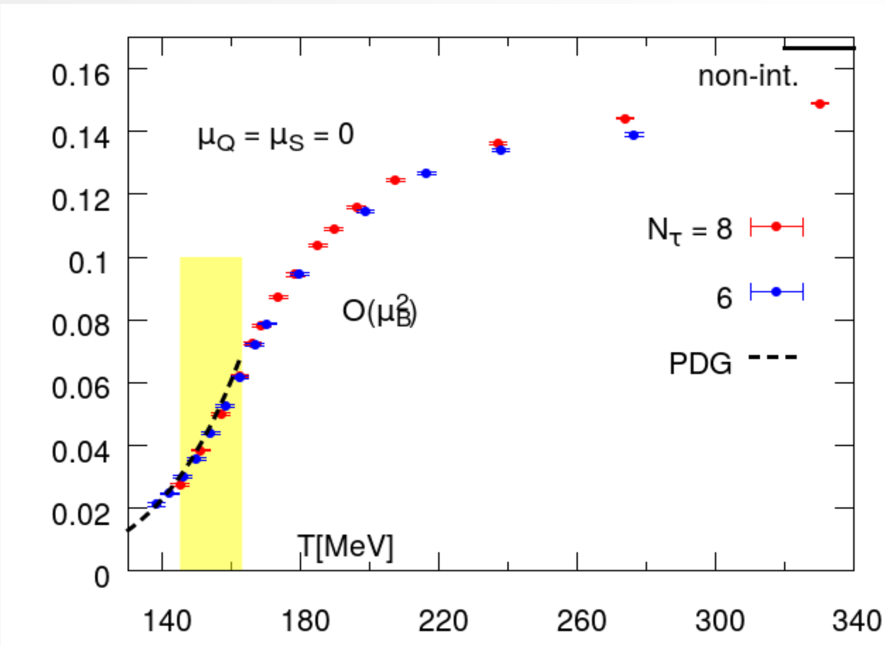


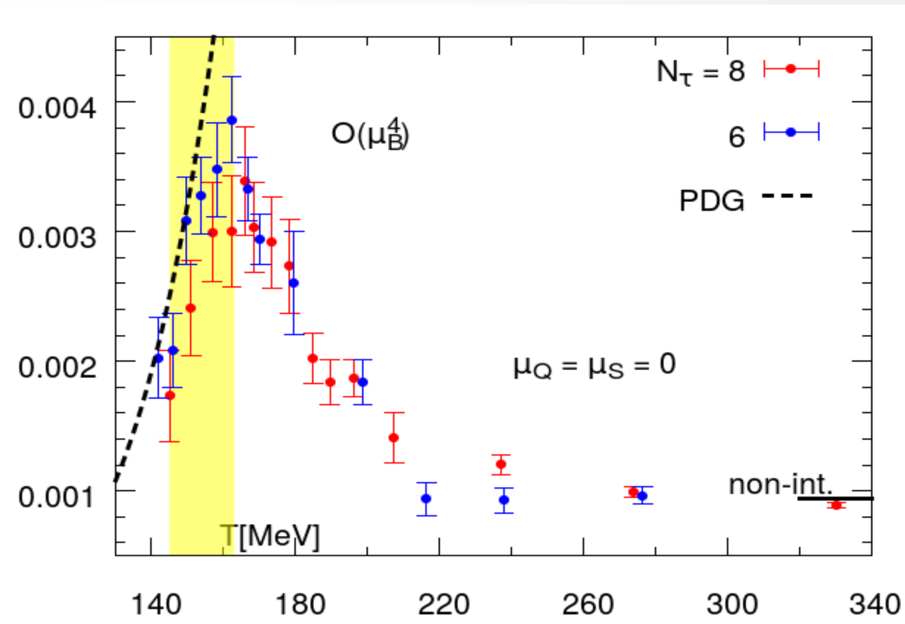
Specific heat not expected to diverge in the chiral limit since the critical exponent α = -0.21 for the 3d-O(4) universality class.

For the same reason, the speed of sound $c_s^2 = s/C_v$ will not go to zero but only attain a minimum at the phase transition.

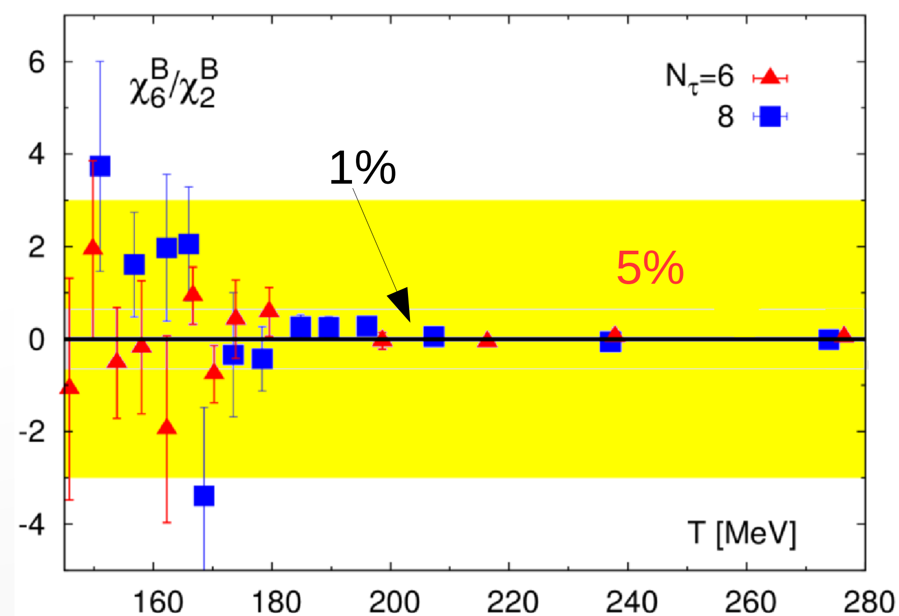
At finite quark mass, significant contribution from regular part of the free energy as well.

Taylor-expanding the pressure



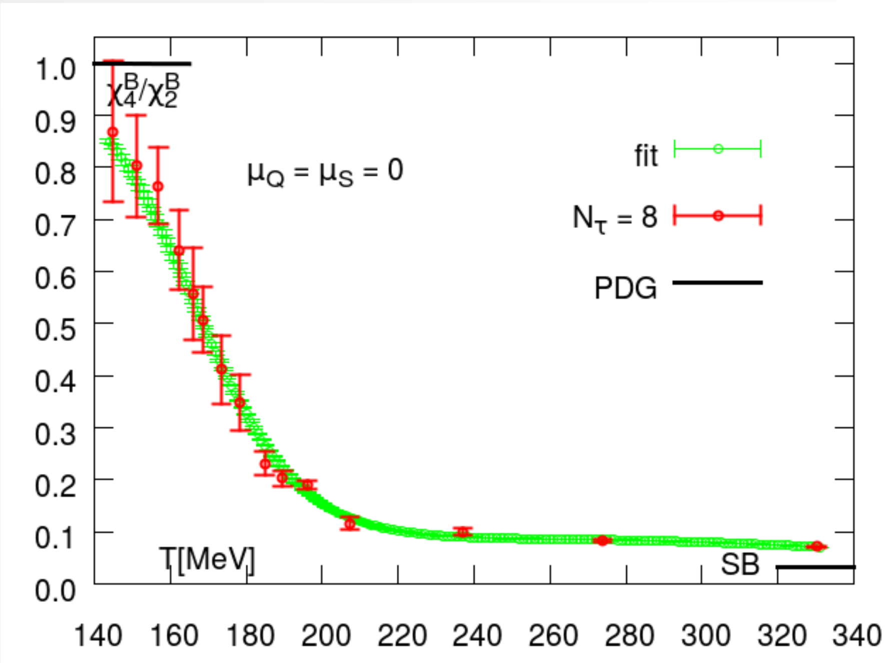


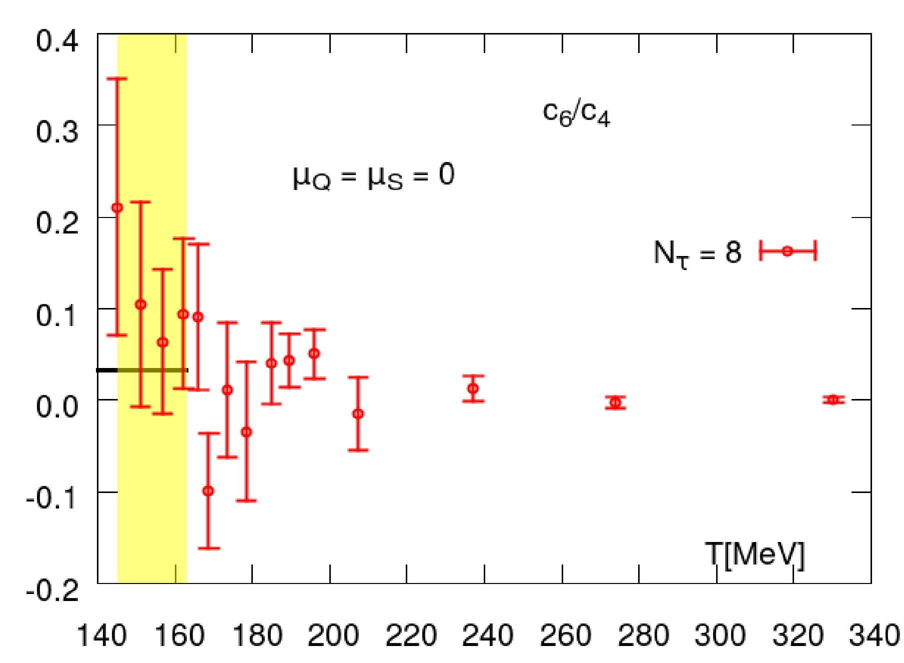
$$\frac{P}{T^4} = \frac{P_0}{T^4} + \frac{1}{2}\left(\frac{\mu_B}{T}\right)^2 + \frac{1}{24}\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}\left(\frac{\mu_B}{T}\right)^6$$



Sixth-order corrections not more than 1-5% of the second-order corrections

Validity of the expansion





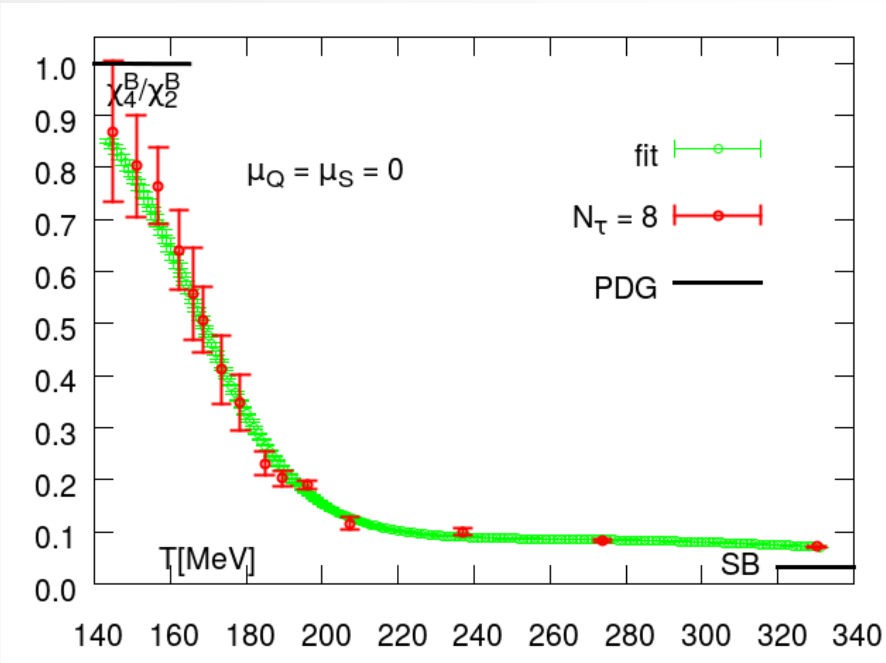
$$\frac{p}{T^4} = c_0 + c_2\hat{\mu}^2\left(1 + \frac{c_4}{c_2}\hat{\mu}^2\left(1 + \frac{c_6}{c_4}\hat{\mu}^2\right)\right)$$

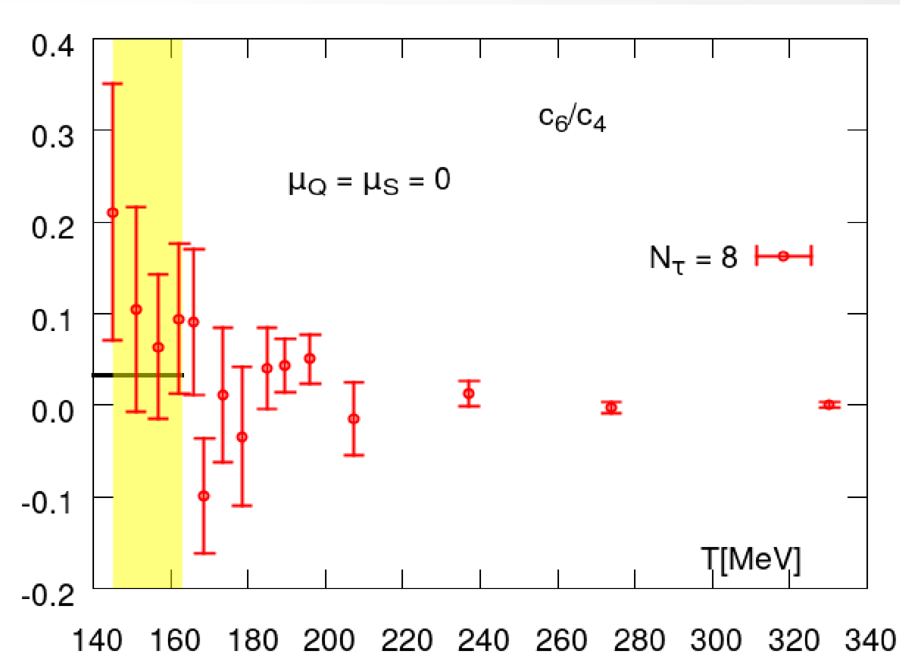
At $T = T_c \approx 154$ MeV, $c_0 \approx 1$, $c_2 \approx 0.05$, $c_4/c_2 \approx 1/24$, $c_6/c_4 \approx 0.1$. Therefore:

$\mu_B/T = 2$: 2^{nd}-, 4^{th}- and 6^{th}-order contributions are ~20, 23 and 25% resp. (expansion under control).

$\mu_B/T = 3$: 2^{nd}-, 4^{th}- and 6^{th}-order contributions are ~45, 63 and 77% resp. **(higher orders important)!**

Validity of the expansion



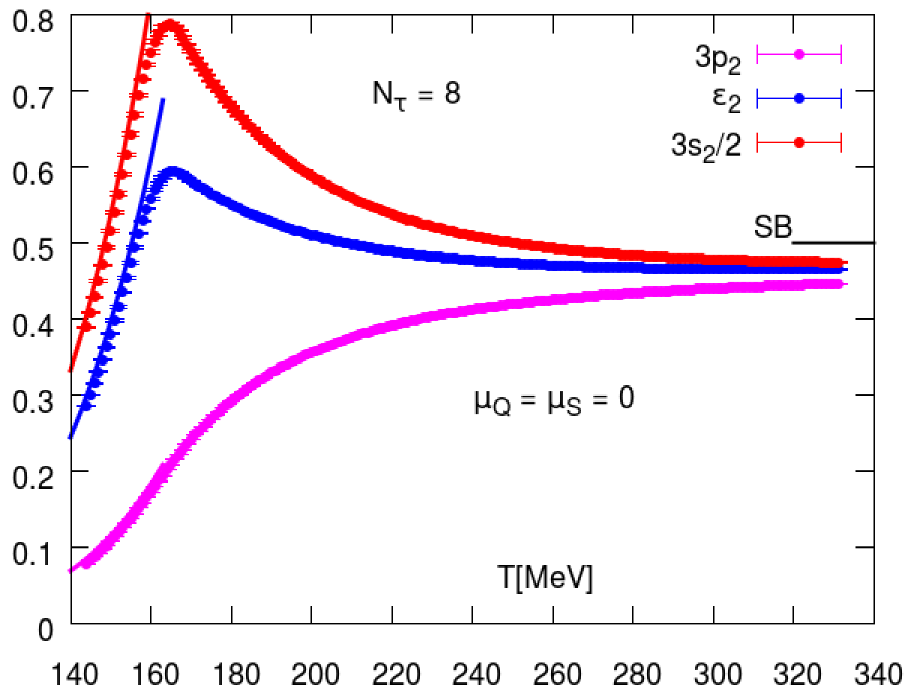


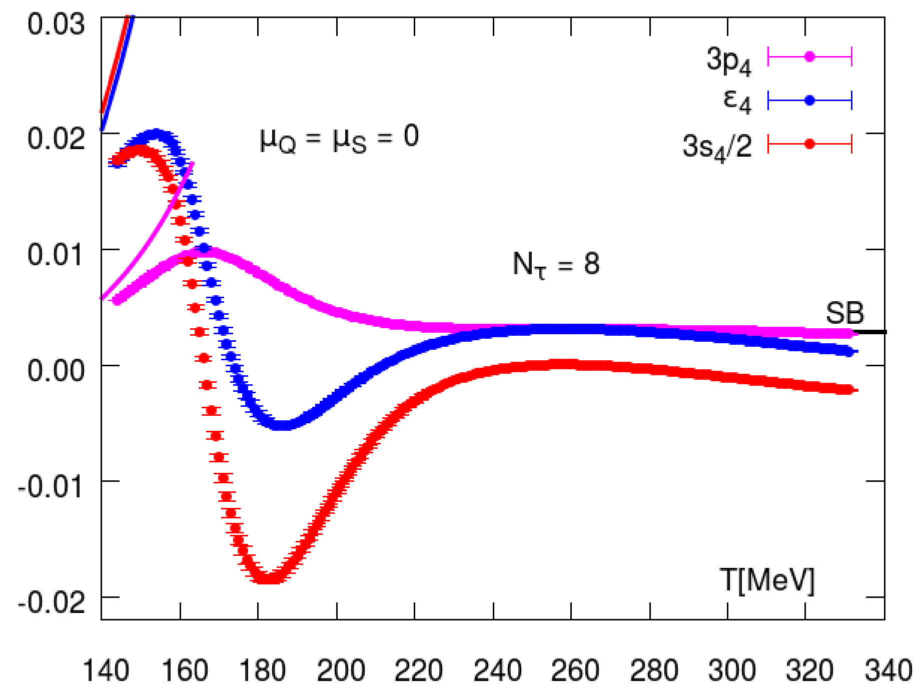
$$\frac{p}{T^4} = c_0 + c_2\hat{\mu}^2\left(1 + \frac{c_4}{c_2}\hat{\mu}^2\left(1 + \frac{c_6}{c_4}\hat{\mu}^2\right)\right)$$

Thus, a 4[th]-order equation of state may be safely used up to $\mu_B/T = 2$.

At $T \approx T_c = 154$ MeV, $c_0 \approx 1$, $c_2 \approx 0.05$, $c_4/c_2 \approx 1/24$, $c_6/c_4 \approx 0.1$. Therefore:

Energy and entropy densities



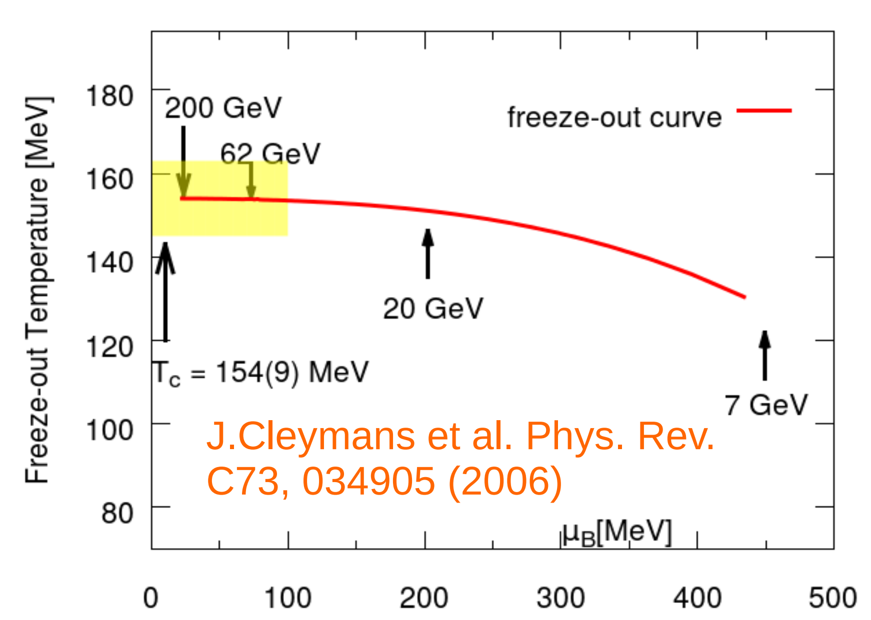


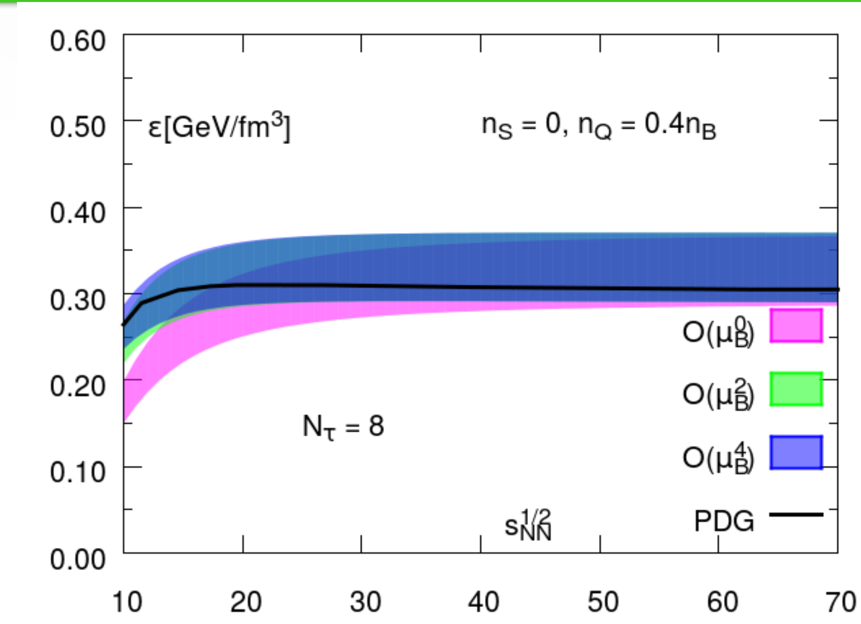
$$\frac{\varepsilon}{T^4} = \sum_{n=0}^{\infty} \left(\frac{\mu_B}{T}\right)^n \left\{T\frac{dc_n}{dT} + 3c_n\right\}$$

$$\frac{s}{T^3} = \sum_{n=0}^{\infty} \left(\frac{\mu_B}{T}\right)^n \left\{T\frac{dc_n}{dT} + (4-n)c_n\right\}$$

We found that an exponential+polynomial fit to the susceptibilities worked very well. From the derivatives of the ansatz, we could calculate the corrections energy and entropy densities as well.

The freeze-out curve and lines of constant physics

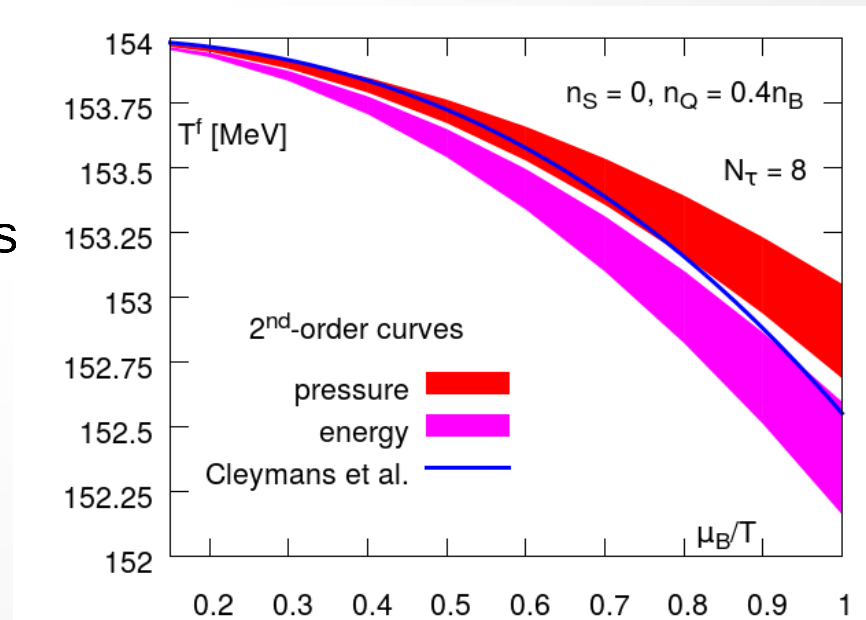




The freeze-out curve is a phenomenological curve obtained from fitting hadron yields at different beam energies to an HRG model.

Using $T(s_{NN})$ and $\mu_B(s_{NN})$ as obtained from Cleymans *et al.* we found that ε remained approximately constant along the curve.

Similarly, we may also calculate “lines of constant physics” *i.e.* constant ε or p, as a function of T and μ_B.



Conclusions

The equation of state an important input in modelling heavy-ion collisions at RHIC/LHC through hydrodynamics.

The μ_B=0 equation of state is useful at the LHC and at RHIC top energies, while the finite-μ_B equation of state is necessary for the Beam Energy Scan program at RHIC.

Recently, we presented continuum-extrapolated results for the μ_B=0 equation of state. We have also begun working towards a finite-density equation of state that will be valid for the range of energies covered by the Beam Energy Scan program at RHIC.

Towards this end, we presented preliminary results for a fourth-order equation of state that would be valid for $\mu_B/T \leq 2$ i.e. beam energies down to ~20 GeV.