Equation of State from Lattice QCD

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Equation of state

Sole QCD input in the hydrodynamic evolution of the QGP created in heavy ion collisions [see talk by Huichao Song].

Baryochemical potential $\mu_B$ negligible (compared to the temperature) at LHC and top RHIC energies.

However, with the advent of the Beam Energy Scan program at RHIC, need an EoS valid up to $\mu_B \sim 500$ MeV.

Resummed perturbation theory [N.Haque et al. JHEP 1405, 027 (2014)] does not work for temperatures below $T\sim 200$ MeV; non-perturbative methods necessary.
Pressure, energy and entropy

Recently, we presented continuum-extrapolated results for the equation of state at $\mu_B = 0$ [HotQCD, Phys.Rev.D90, 054503 (2014)].

Behavior as expected from the crossover nature of the transition i.e. smooth transition from a gaseous hadronic phase to deconfined quarks and gluons.

Around 10% difference from the non-interacting ideal gas limit even at $T \approx 400$ MeV $\approx 2.7 \, T_c$.

$p$, $s$ and $\epsilon$ well-approximated by a gas of non-interacting resonances (Hadron Resonance Gas) at temperatures below $T_c$. 
Parametrization and $\varepsilon(T_c)$

Analytic parametrization of the pressure over the entire temperature range [HotQCD, Phys.Rev.D90, 054503 (2014)].

$$\frac{p}{T^4} = \frac{1 + \tanh \left( c_t(t - t_0) \right)}{2} \times \frac{p_{id} + a_n/\bar{t} + b_n/\bar{t}^2 + c_n/\bar{t}^3 + d_n/\bar{t}^4}{1 + a_d/\bar{t} + b_d/\bar{t}^2 + c_d/\bar{t}^3 + d_d/\bar{t}^4}$$

The energy density at the phase transition is not too different from normal nuclear energy density ($\sim 0.16$ GeV/fm$^3$).
Specific heat and speed of sound

Specific heat not expected to diverge in the chiral limit since the critical exponent $\alpha = -0.21$ for the 3d-O(4) universality class.

For the same reason, the speed of sound $c_s^2 = s/C_v$ will not go to zero but only attain a minimum at the phase transition.

At finite quark mass, significant contribution from regular part of the free energy as well.
Taylor-expanding the pressure

\[ \frac{P}{T^4} = \frac{P_0}{T^4} + \frac{1}{2} \left( \frac{\mu_B}{T} \right)^2 + \frac{1}{24} \left( \frac{\mu_B}{T} \right)^4 + O \left( \frac{\mu_B}{T} \right)^6 \]

Sixth-order corrections not more than 1-5% of the second-order corrections
Validity of the expansion

\[ \frac{\mu_B}{T} = 2^{nd}, 4^{th} \text{- and } 6^{th}-\text{order contributions are } \sim 20, 23 \text{ and } 25\% \text{ resp. (expansion under control).} \]

\[ \frac{\mu_B}{T} = 3^{rd}, 4^{th} \text{- and } 6^{th}-\text{order contributions are } \sim 45, 63 \text{ and } 77\% \text{ resp. (higher orders important)!} \]

At \( T = T_c \approx 154 \text{ MeV}, c_0 \approx 1, c_2 \approx 0.05, c_4/c_2 \approx 1/24, c_6/c_4 \approx 0.1 \). Therefore:
Validity of the expansion

Thus, a 4\textsuperscript{th}-order equation of state may be safely used up to $\mu_B/T = 2$.

At $T \approx T_c = 154$ MeV, $c_0 \approx 1$, $c_2 \approx 0.05$, $c_4/c_2 \approx 1/24$, $c_6/c_4 \approx 0.1$. Therefore:
We found that an exponential+polynomial fit to the susceptibilities worked very well. From the derivatives of the ansatz, we could calculate the corrections energy and entropy densities as well.
The freeze-out curve is a phenomenological curve obtained from fitting hadron yields at different beam energies to an HRG model.

Using $T(s_{_{NN}})$ and $\mu_B(s_{_{NN}})$ as obtained from Cleymans et al. we found that $\epsilon$ remained approximately constant along the curve.

Similarly, we may also calculate “lines of constant physics” i.e. constant $\epsilon$ or $p$, as a function of $T$ and $\mu_B$. J. Cleymans et al. Phys. Rev. C73, 034905 (2006)
The equation of state is an important input in modelling heavy-ion collisions at RHIC/LHC through hydrodynamics.

The $\mu_B=0$ equation of state is useful at the LHC and at RHIC top energies, while the finite-$\mu_B$ equation of state is necessary for the Beam Energy Scan program at RHIC.

Recently, we presented continuum-extrapolated results for the $\mu_B=0$ equation of state. We have also begun working towards a finite-density equation of state that will be valid for the range of energies covered by the Beam Energy Scan program at RHIC.

Towards this end, we presented preliminary results for a fourth-order equation of state that would be valid for $\mu_B/T \leq 2$ i.e. beam energies down to $\sim 20$ GeV.