# Hot-Dense Lattice QCD

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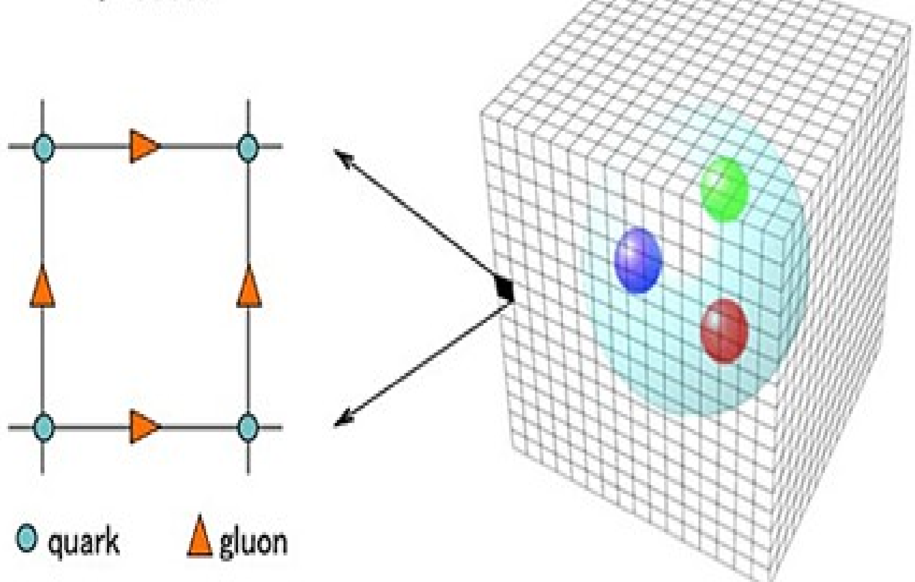
# Hot-dense Lattice QCD

QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_{q=u,d,s,c,b,t} \bar{q}\left[i\gamma^{\mu}(\partial_{\mu} - igA_{\mu}) - m_q\right]q$$




QCD on a discretized (Euclidean) space-time lattice

approximation-free, parameter-free

temperature: space =/= time
(breaking Lorentz symmetry)

density: chemical potential
coupled to conserved current in QCD Lagrangian

no free parameter
bare parameters of QCD Lagrangian fixed by reproducing physics at T=0

perform path integral numerically using Monte-Carlo technique
~ 200M dimensional integral for a modest $16\times64^3$ lattice

equilibrium & near-equilibrium properties of QCD

# Hot-dense Lattice QCD

observables: $\langle O \rangle \sim \int O \, \det(M_F[U]) \, e^{-S[U]} \, dU$

~200M dim integral for a $16 \times 64^3$ lattice

generate ensembles of U using molecular dynamics followed by Monte-Carlo, require: $M_F^{-1}$

$\mathrm{Tr}\left(M_F^{-1} \ldots M_F^{-1} \ldots M_F^{-1} \ldots\right)$

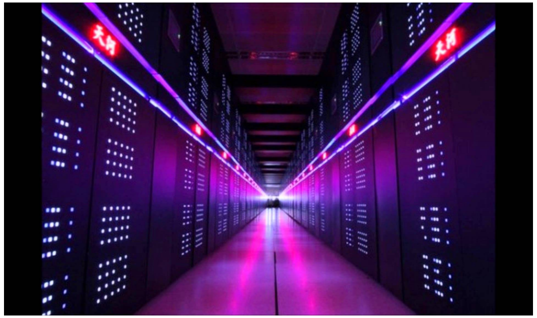
many many inversions of ~200M dim matrix $M_F$

use iterative methods, conjugate gradient etc.

# Hot-dense Lattice QCD

solve QCD exactly using supercomputers …

TOP500 rank: 1



Tianhe-2, China

TOP500 rank: 2


Titan, USA

TOP500 rank: 3

Sequoia, USA

TOP500 rank: 5


Mira, USA

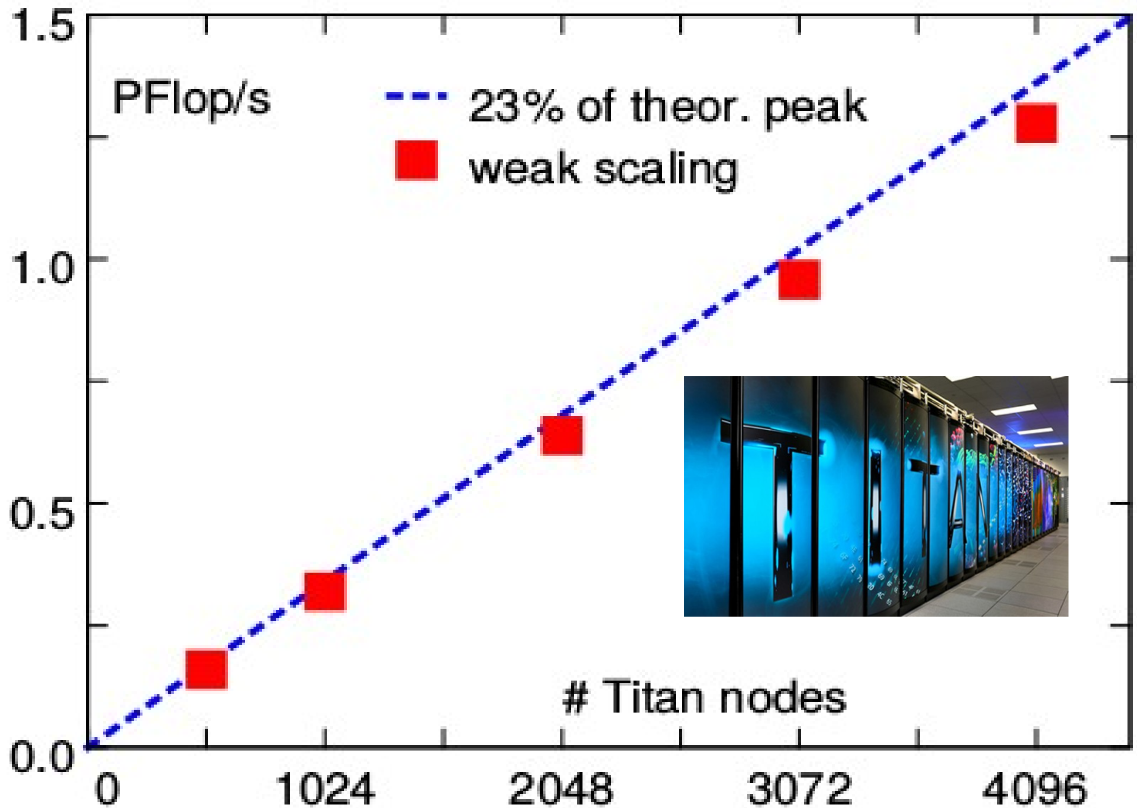
TOP500 rank: 8


Juqueen, Germany

TOP500 rank: 9

Vulcan, USA

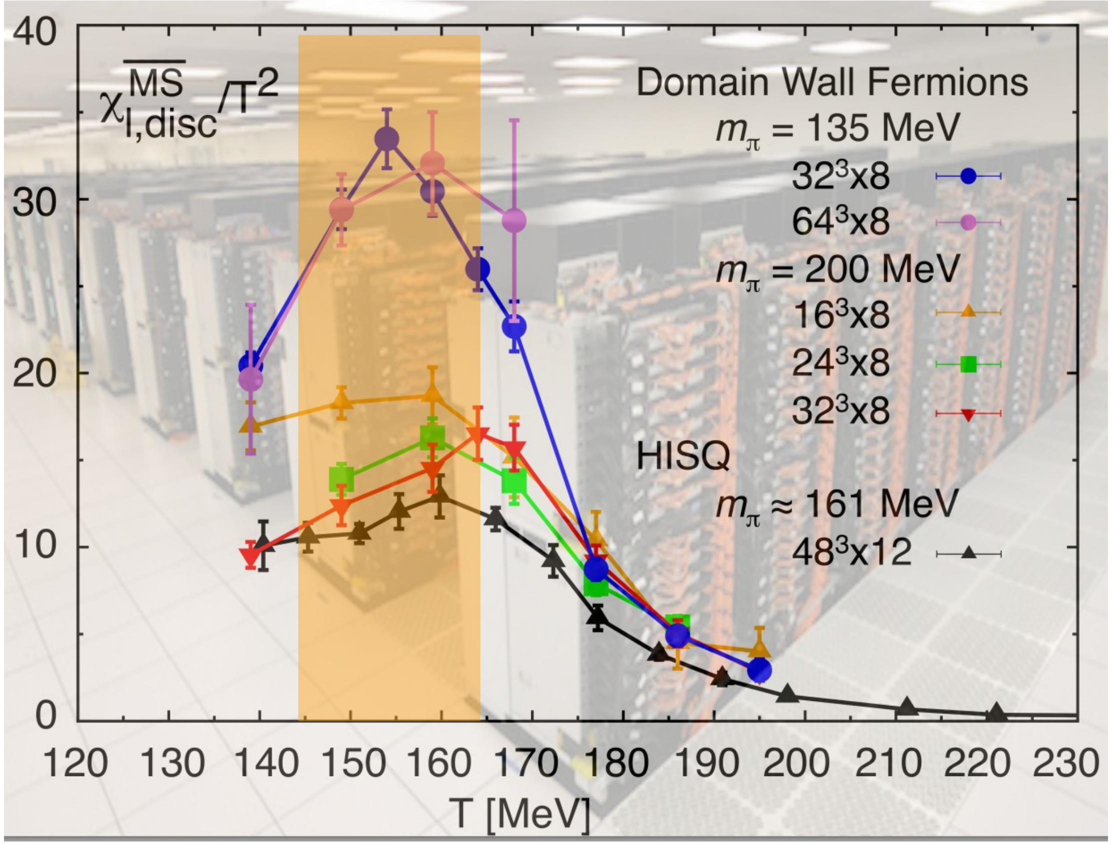
# Hot-dense Lattice QCD: code performance




~25% of theoretical peak on Titan

~70K titan cores & ~6K GPU

# QCD transition at zero baryon density




$T_c = 154(9)$ MeV

physical quark masses & continuum limit

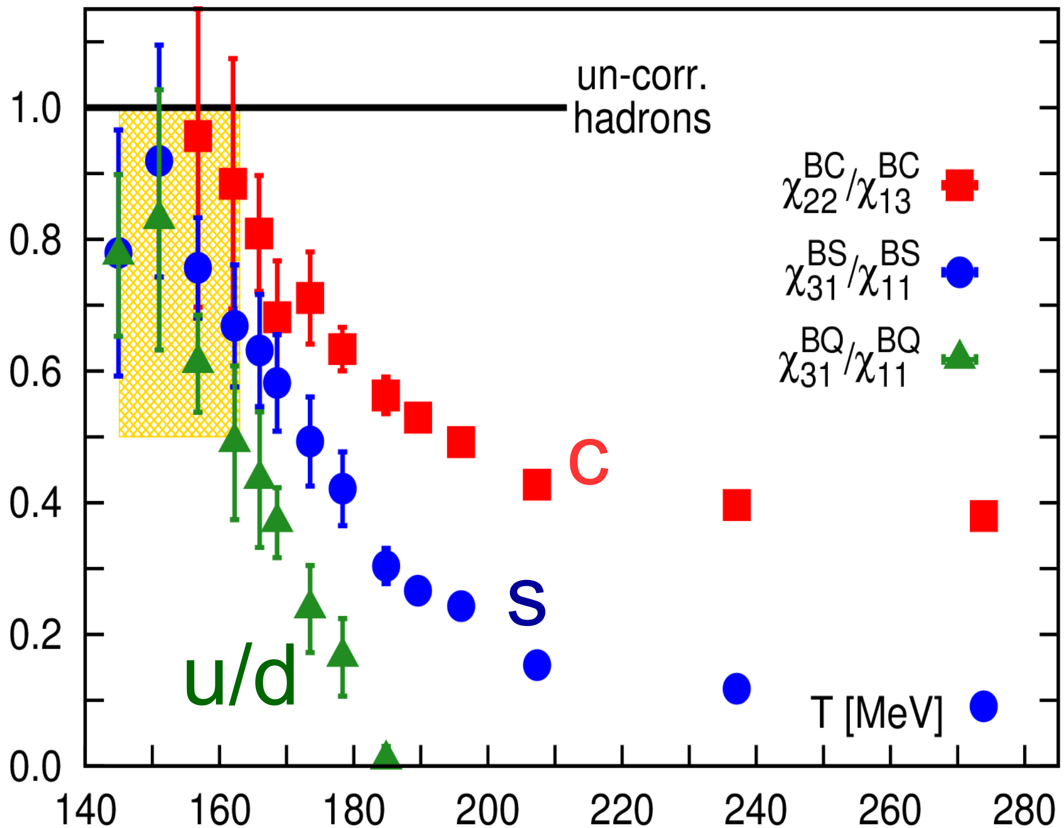
constrains 'switching' temperature of hydro calculations

chiral crossover with 3 physical pions

chiral fermion (domain wall)

HotQCD: Phys. Rev. Lett. 113 (2014) 082001

# QCD transition at zero baryon density




deconfinement & chiral crossover in same temperature range

$T_c = 154(9)$ MeV

physical quark masses & continuum limit

BNL-Bi-CCNU: Phys. Rev. Lett. 111 (2013) 082301

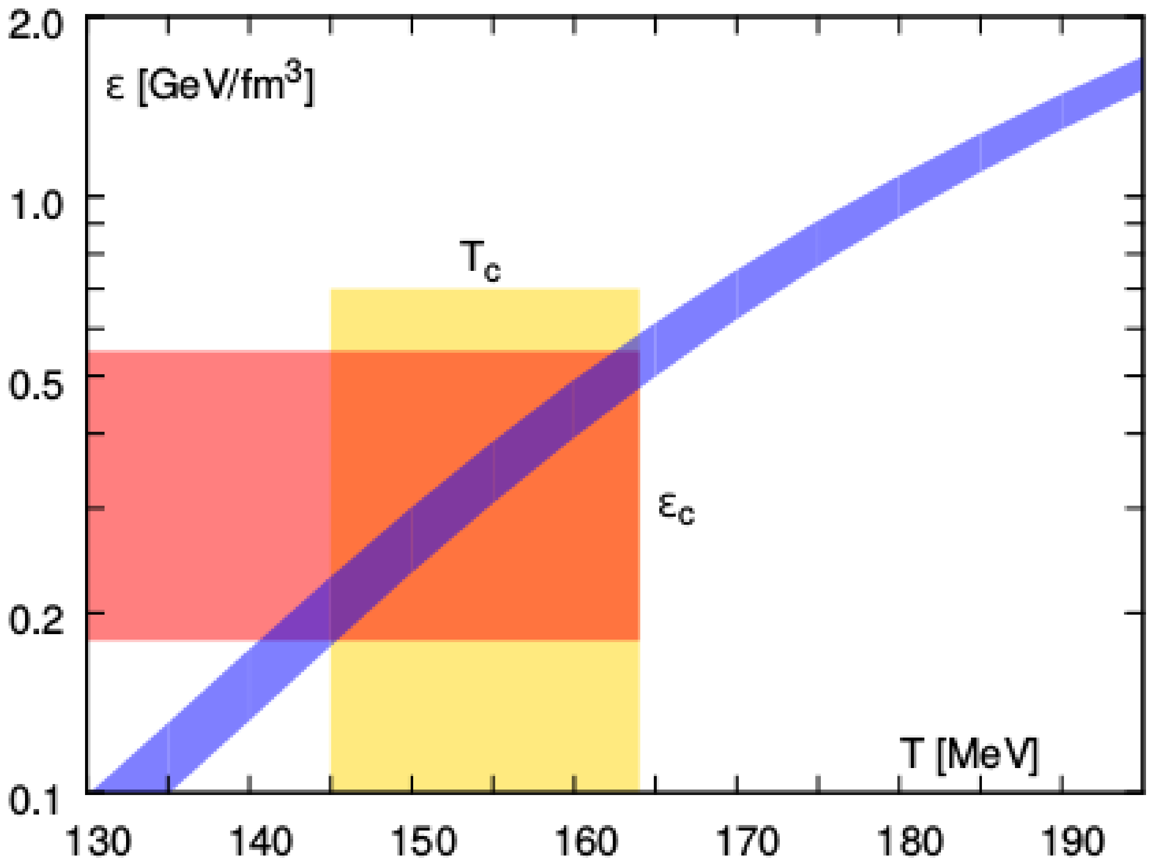
BNL-Bi-CCNU: Phys. Lett. B737 (2014) 210

appearance of fractional charges

$\chi_{BX}^{nm}/\chi_{BX}^{km} = B^{n-k}$ = 1 when B=1, DoF are hadronic

=/= 0 when B=1/3, DoF are quark like

# QCD transition at zero baryon density




HotQCD: Phys. Rev. D90 (2014) 9, 094503

critical energy density:

$$\epsilon_c = 0.18 - 0.50 \text{ GeV}/\text{fm}^3$$

$$\epsilon_c = (1.2 - 3.3)\,\rho_{nuclear}$$

physical quark masses
& continuum limit

# QCD equation of state at zero baryon density

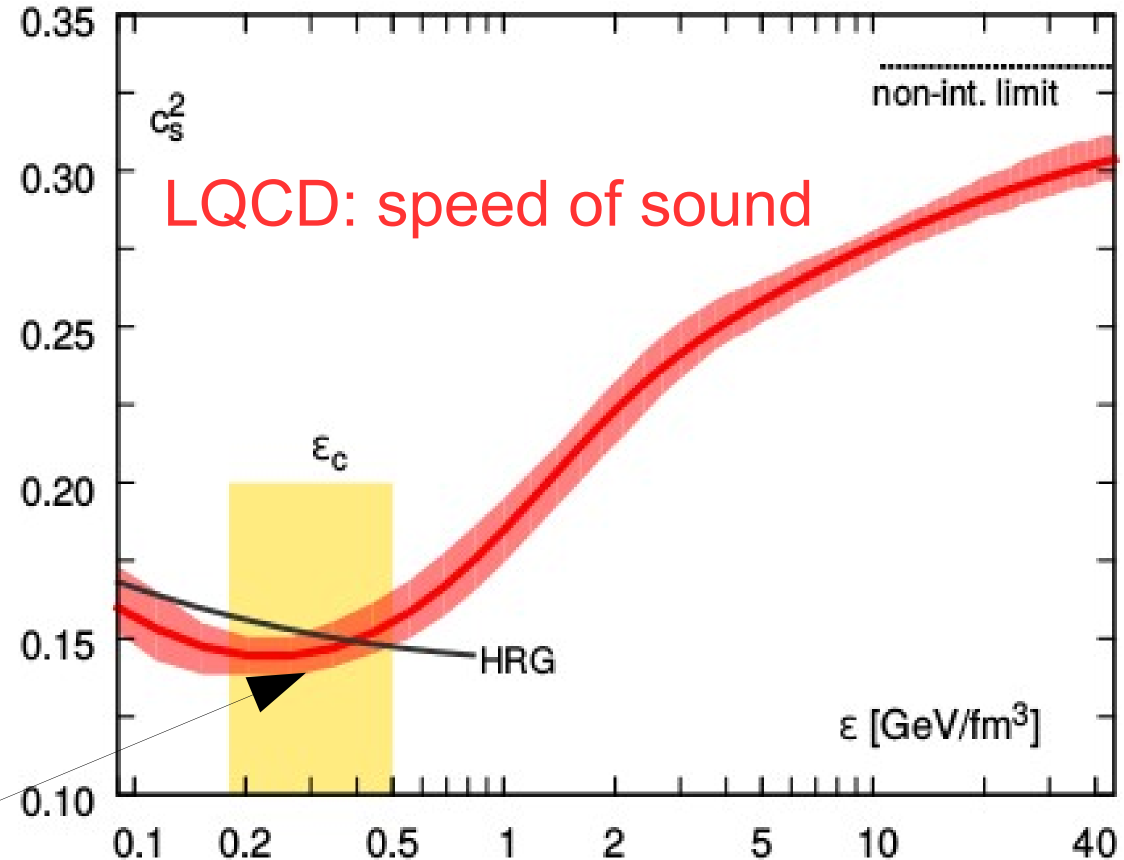
how do we know the 'nearly perfect fluid' created in HIC is QGP?

hydrodynamics

conservation laws

QGP enters only through equation of state

physical quark masses
& continuum limit

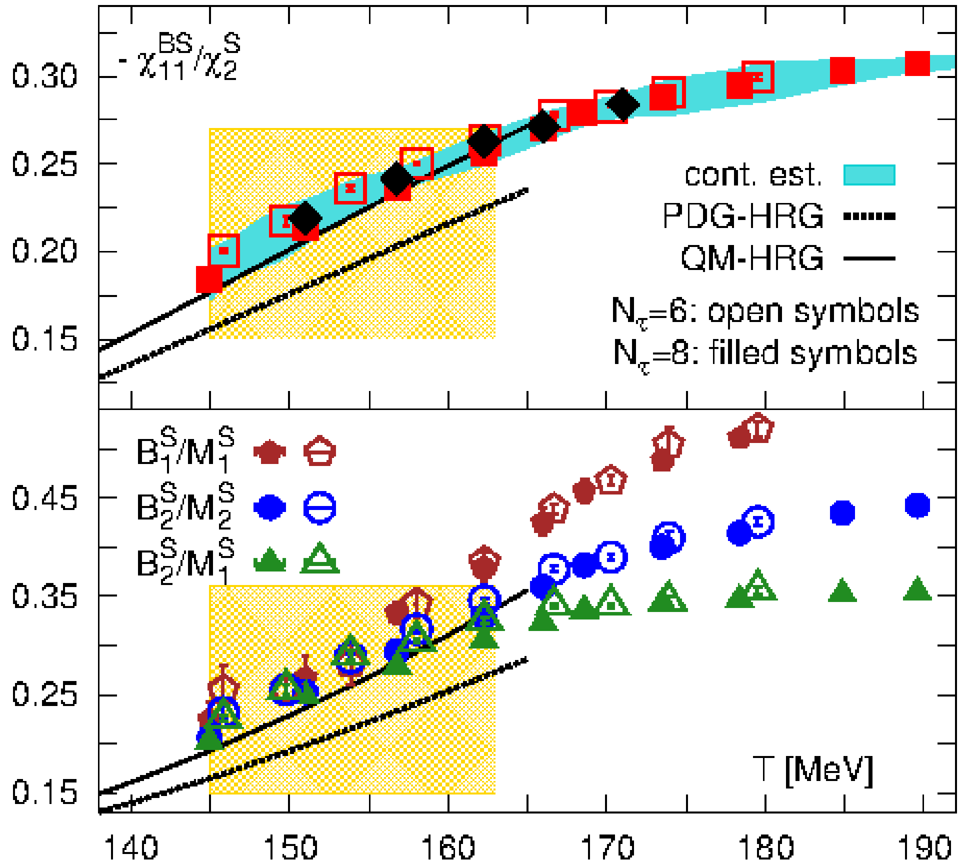



softest point
of EoS

HotQCD: Phys. Rev. D90 (2014) 9, 094503

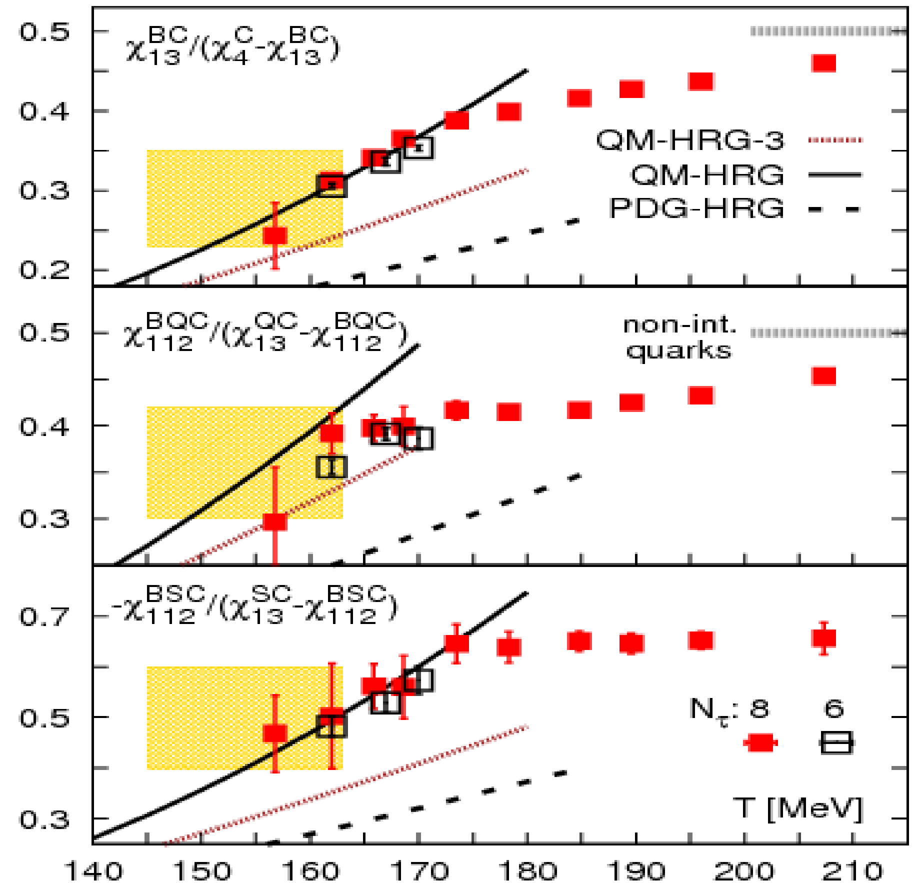
# Matter at the edge of transition

strange sector




BNL-Bi-CCNU: Phys. Rev. Lett. 113 (2014) 7, 072001

charm sector

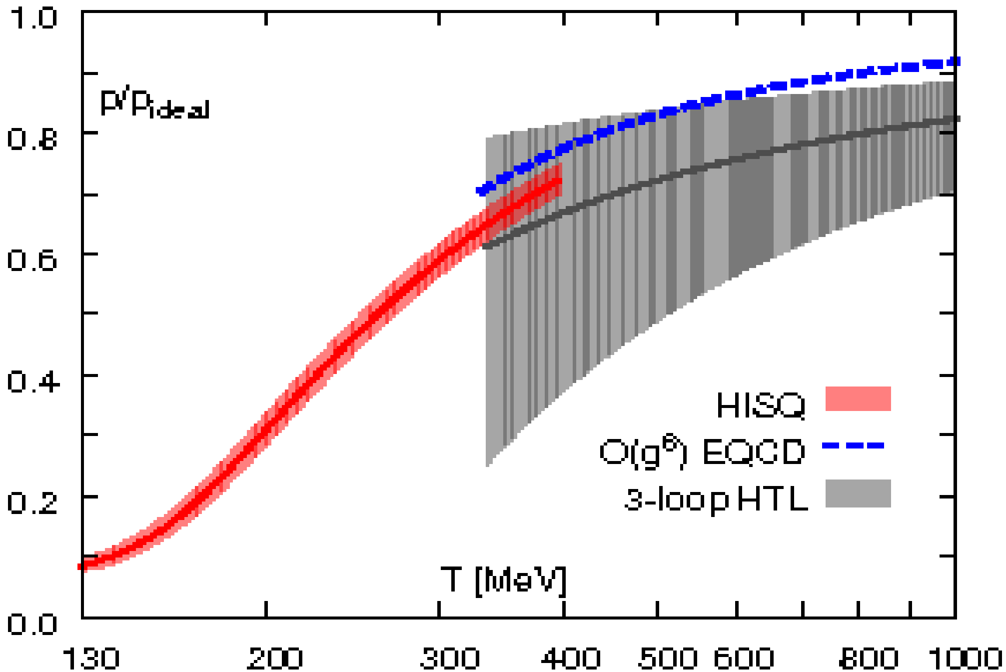



BNL-Bi-CCNU: Phys. Lett. B737 (2014) 210-215

hints of additional, yet unobserved hadrons?

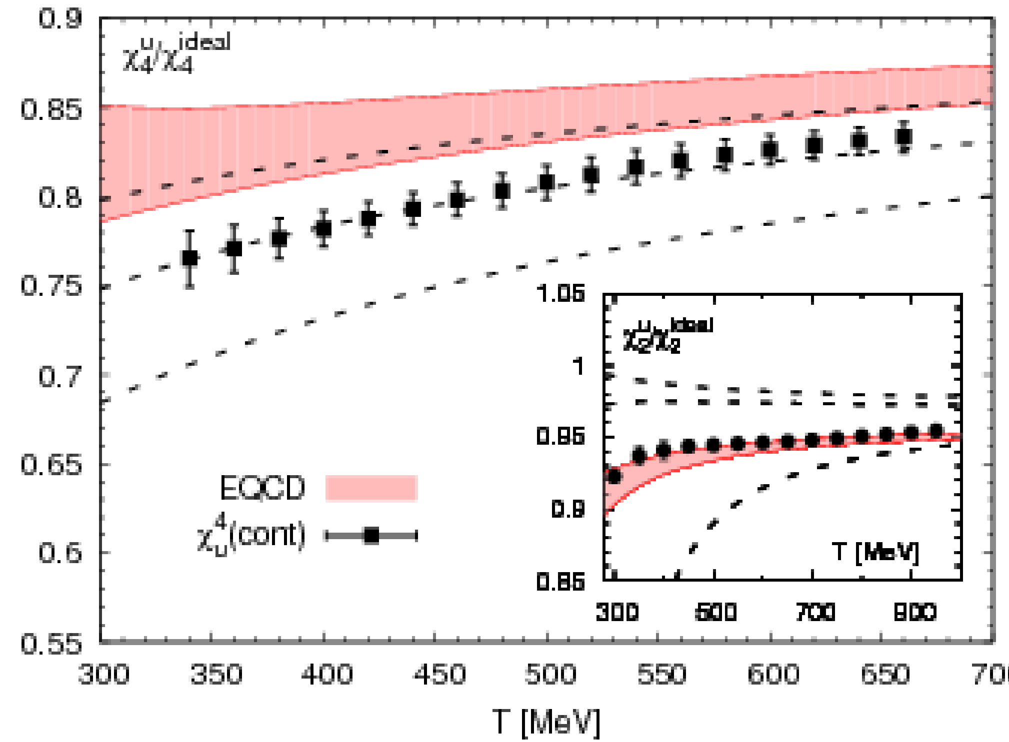
# Weakly interacting regime of QGP

pressure




HotQCD: Phys. Rev. D90 (2014) 9, 094503

quark number susceptibilities

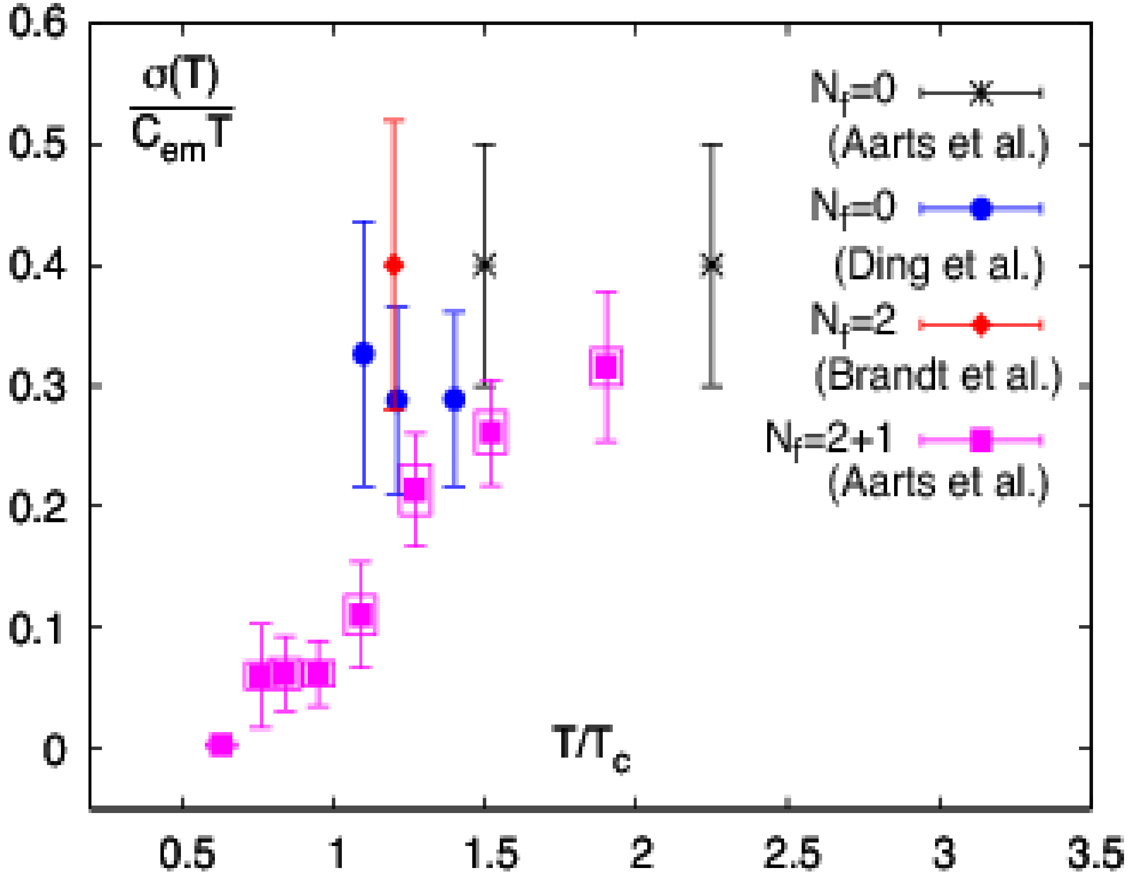



BNL-Bi-CCNU: arXiv:1507.06637

agreements with weak coupling calculations: $T \gtrsim 2T_c$

# Transport properties of QCD

electrical conductivity




H.-T. Ding, F. Karsch, SM: arXiv:1504.05274

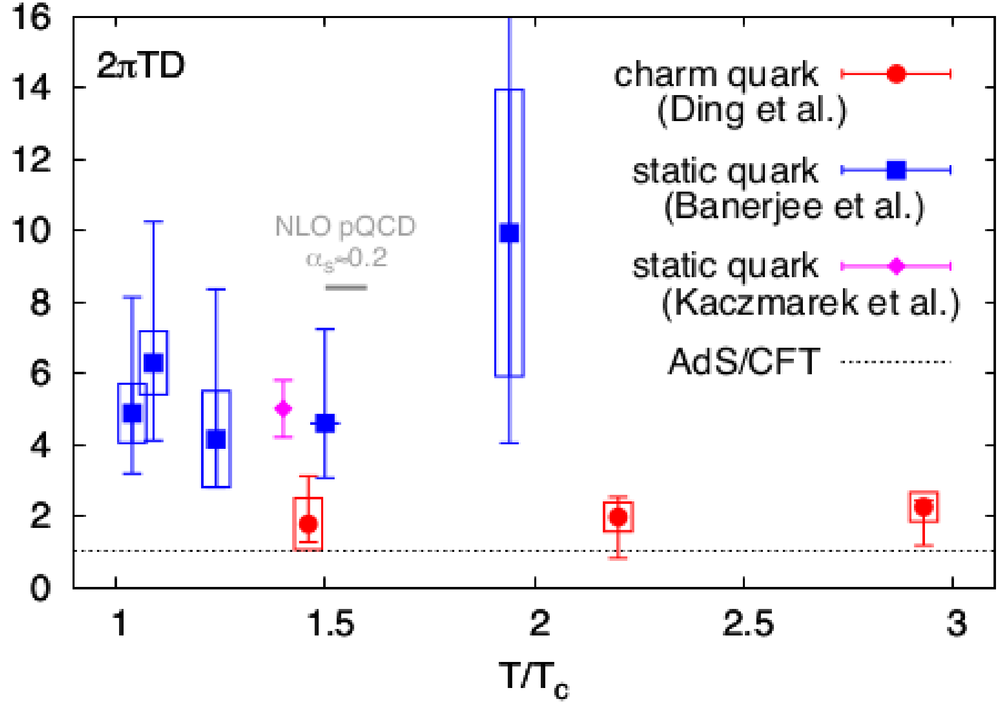
ultra-soft photon emission rate is proportional to electrical conductivity

determines how fast initially produced magnetic field decays inside QGP

calculations needed for physical quark masses using large lattice sizes

# Transport properties of QCD

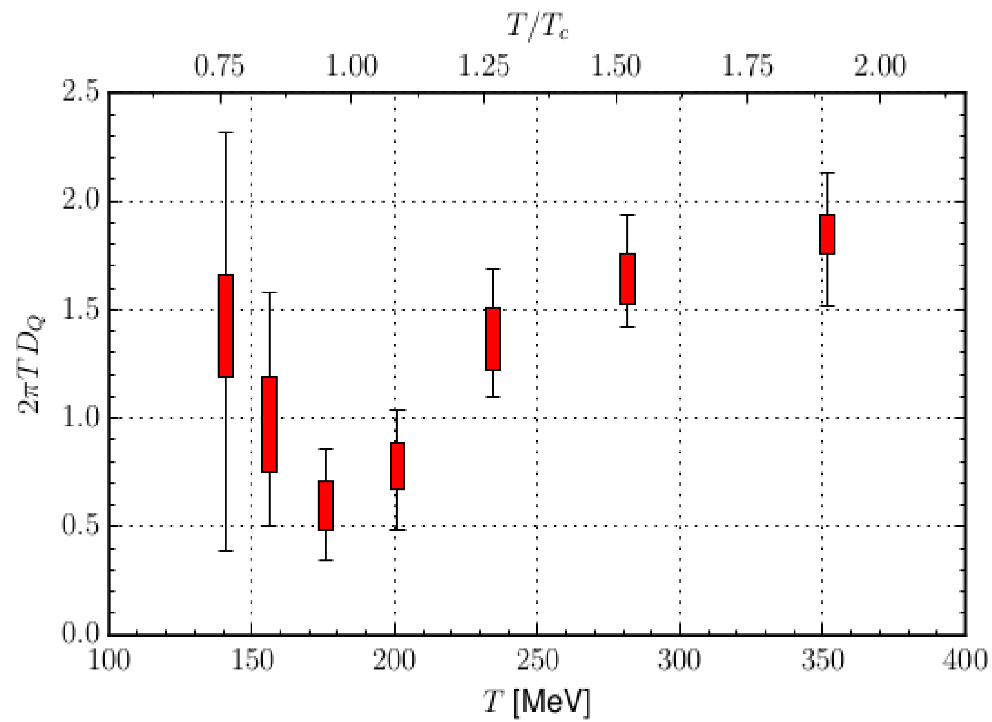
## heavy quark diffusion constant




H.-T. Ding, F. Karsch, SM: arXiv:1504.05274

QCD input for understanding thermalization and flow of heavy quarks

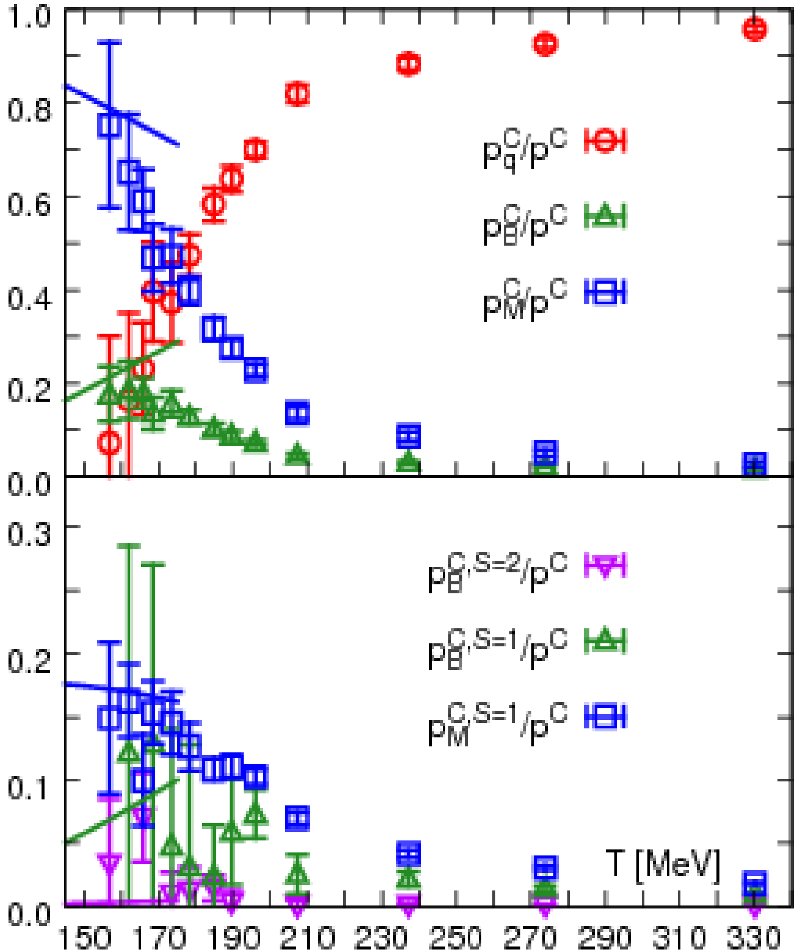
## charge diffusion constant




Aarts et. al.: JHEP 1502 (2015) 186

need to include light dynamical fermions

# Heavy quark bound states in QGP




charm fluctuations and correlations are consistent with a non-interacting gas of charm quasi quark, meson & baryon-like excitations in QGP

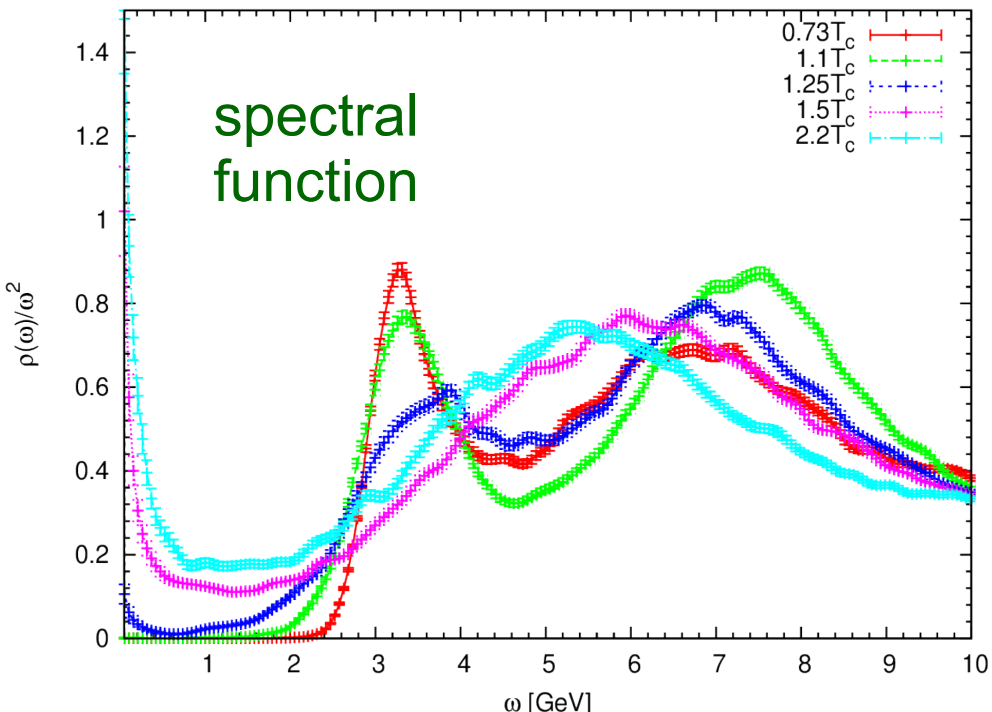
$T_c < T \lesssim 1.3\, T_c$ presence of charm meson & baryon-like excitations in QGP ?

important for simultaneous description of D meson nuclear modification factor & elliptic flow

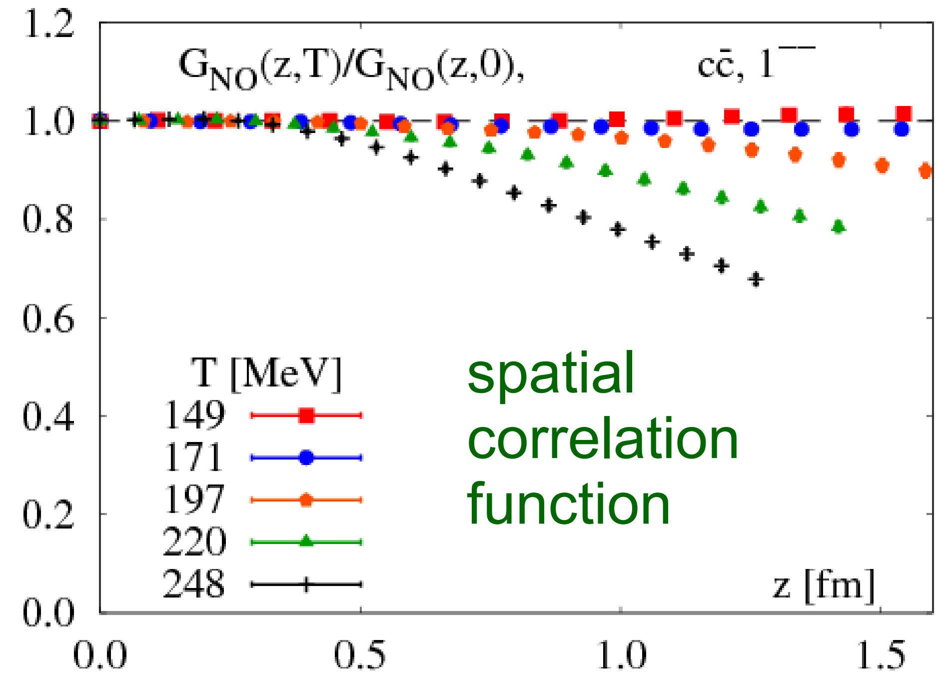
SM, Petreczky, Sharma: arXiv:1509.08887

# Heavy quark bound states in QGP

## $J/\Psi$




H. Ohno et. al.: QM-15



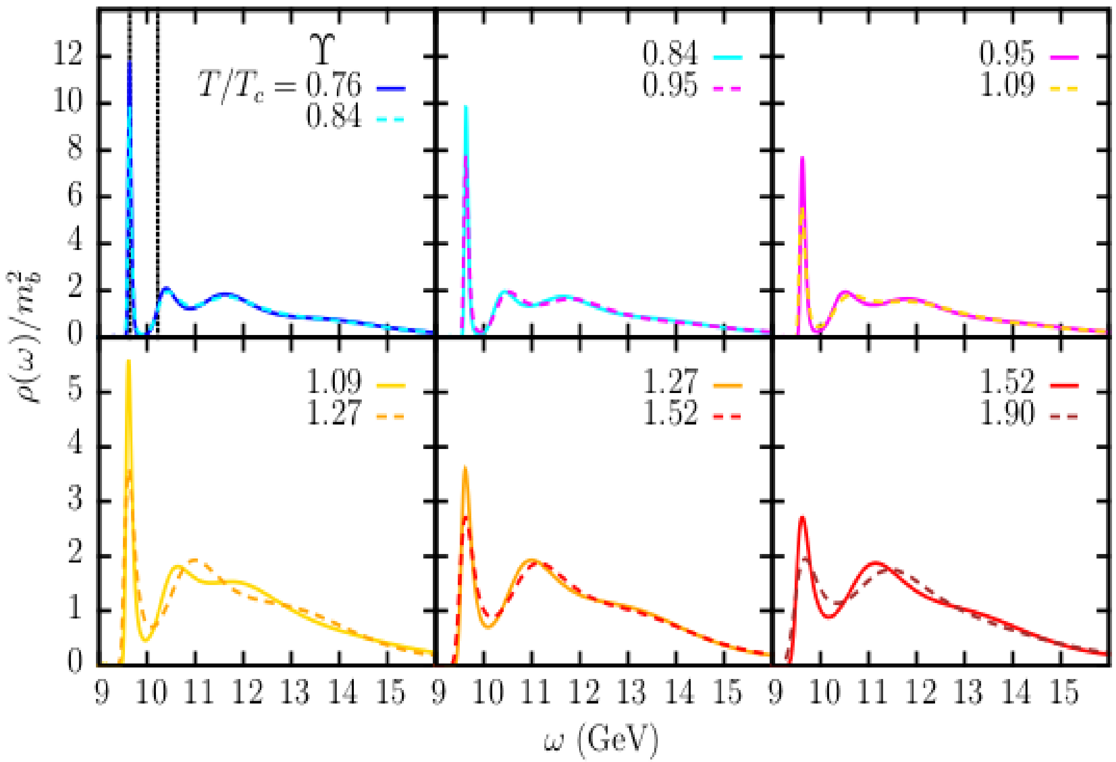

Y. Maezawa et. al.: Phys. Rev. D91 (2015) 5, 054503

nearly unmodified till $T \sim 1.1\, T_c$

melts for $T \gtrsim 1.25\, T_c$

# Heavy quark bound states in QGP

$\Upsilon(1S)$




NRQCD spectral function

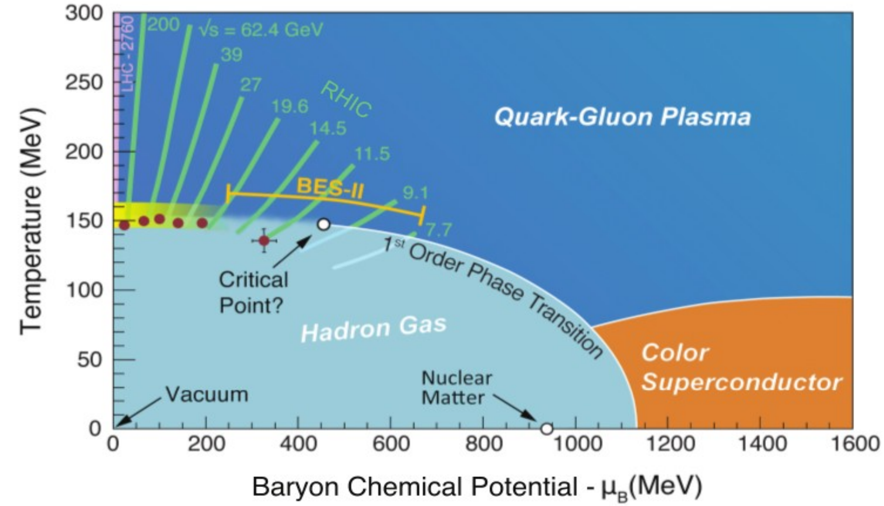
survive at least till

$T = 2T_c$

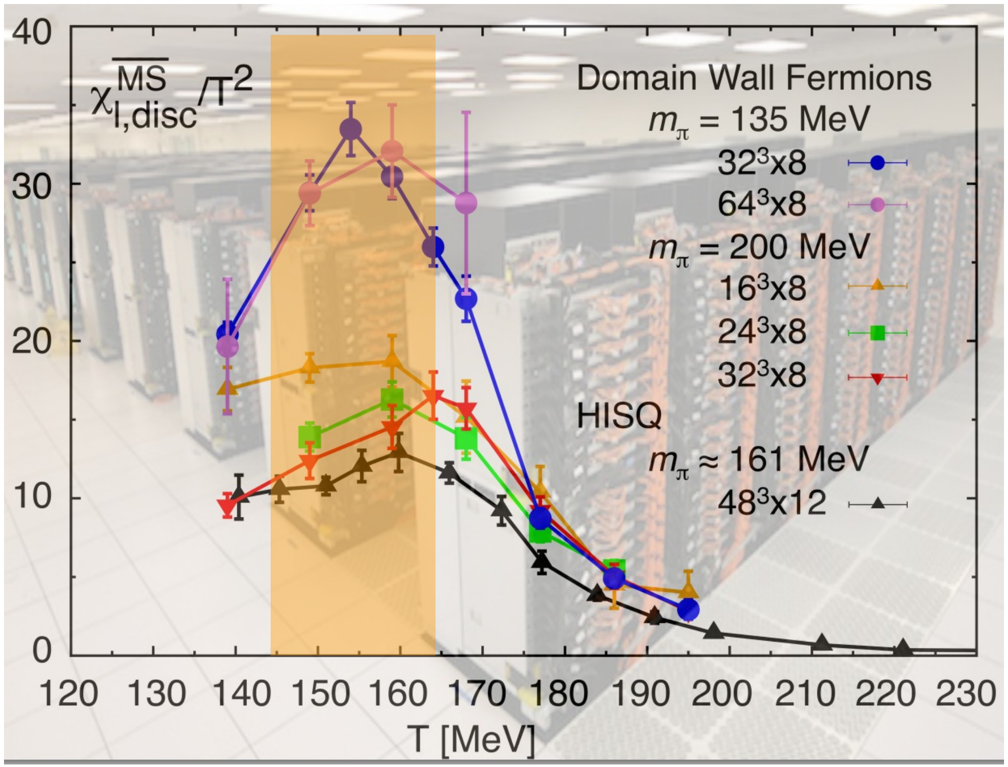
Aarts et. al.: JHEP 1407 (2014) 097

# QCD phase diagram at non-zero baryon density

necessary condition for existence of QCD critical point: QCD transition is a crossover for $\mu_B \geqslant 0$




crossover at $\mu_B = 0$




with exact chiral symmetry & chiral anomaly on the lattice

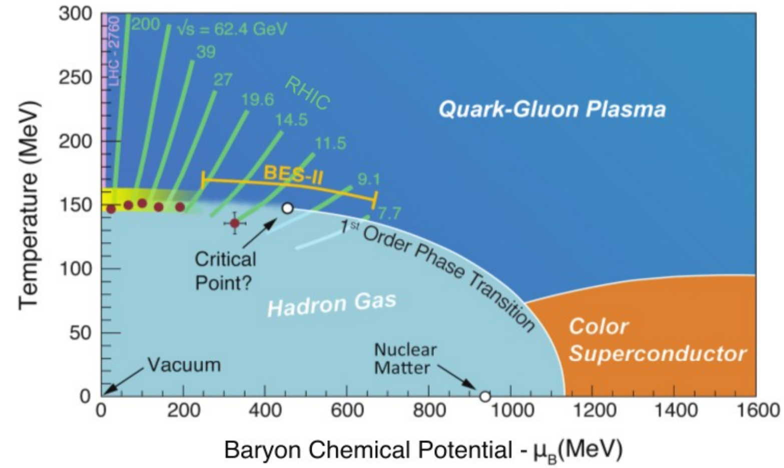
chiral fermion (domain wall)

practically no volume dependence of chiral susceptibility even with 8 times increased volume

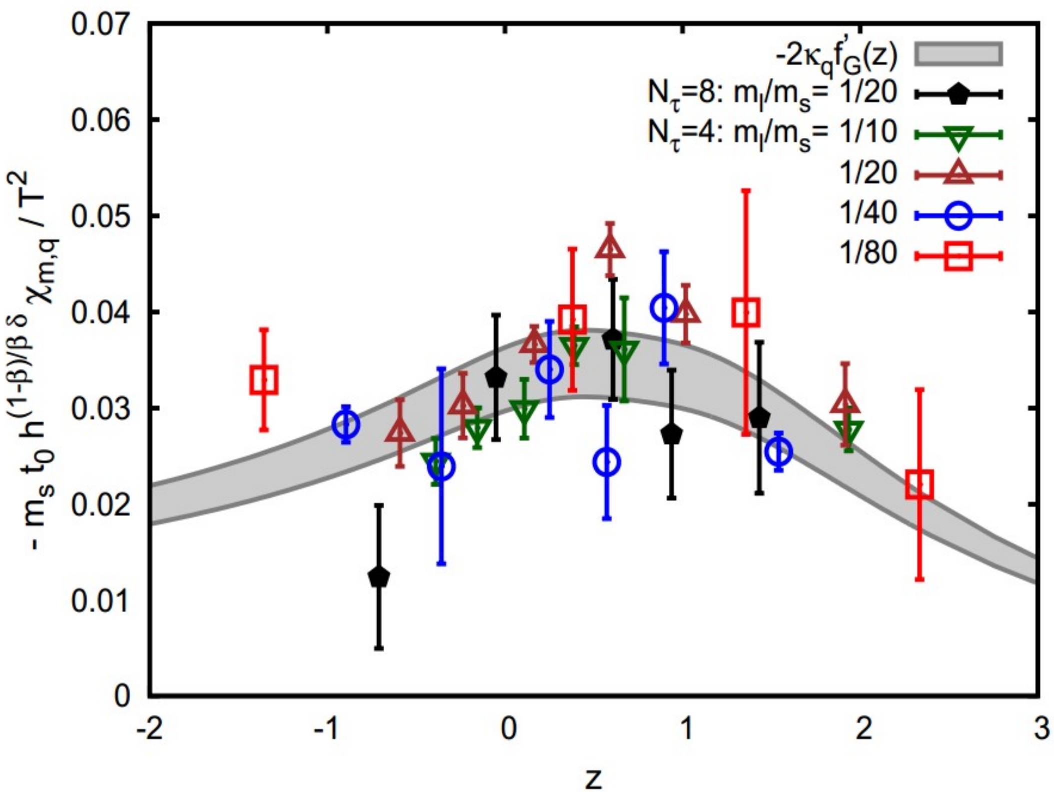
HotQCD: Phys. Rev. Lett. 113 (2014) 082001

# QCD phase diagram at non-zero baryon density

necessary condition for existence of QCD critical point: QCD transition is a crossover for $\mu_B \gtrsim 0$



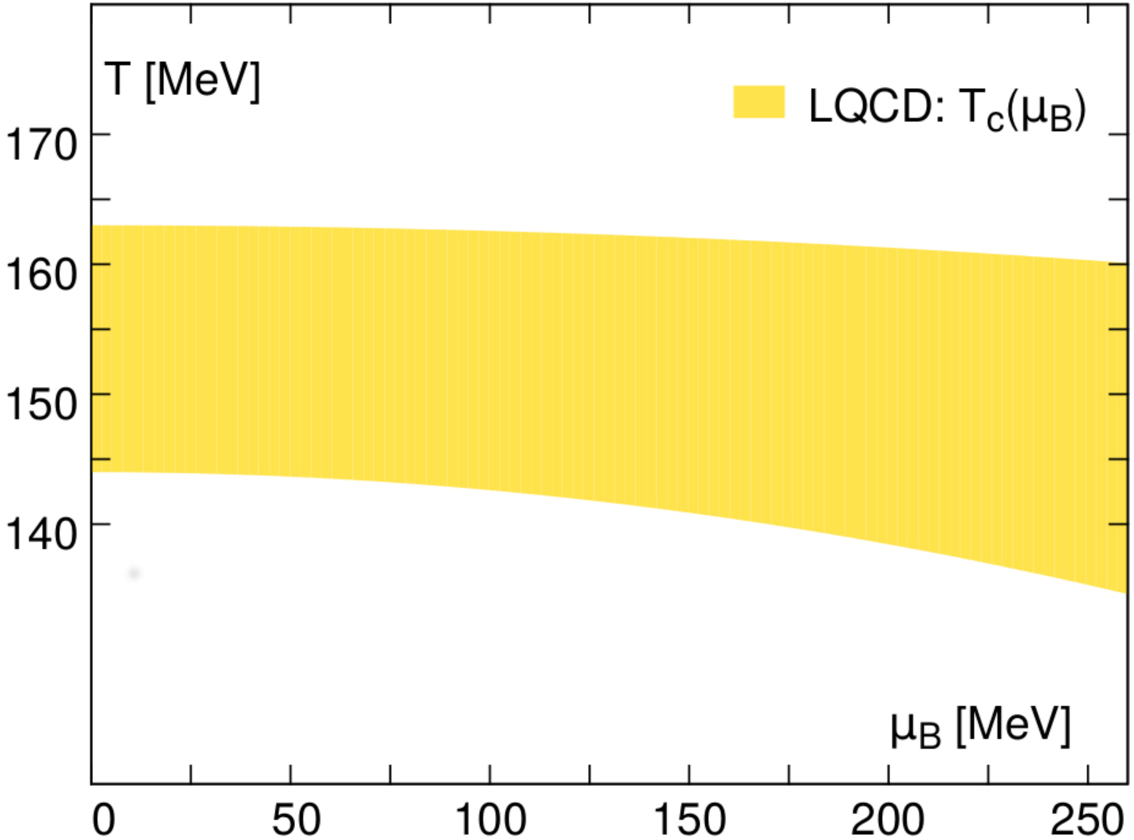

crossover for small $\mu_B \gtrsim 0$




2nd order O(N) chiral scaling behavior of the order parameter for $\mu_B > 0$

BNL-Bi: Phys.Rev. D83 (2011) 014504

# QCD phase diagram at non-zero baryon density



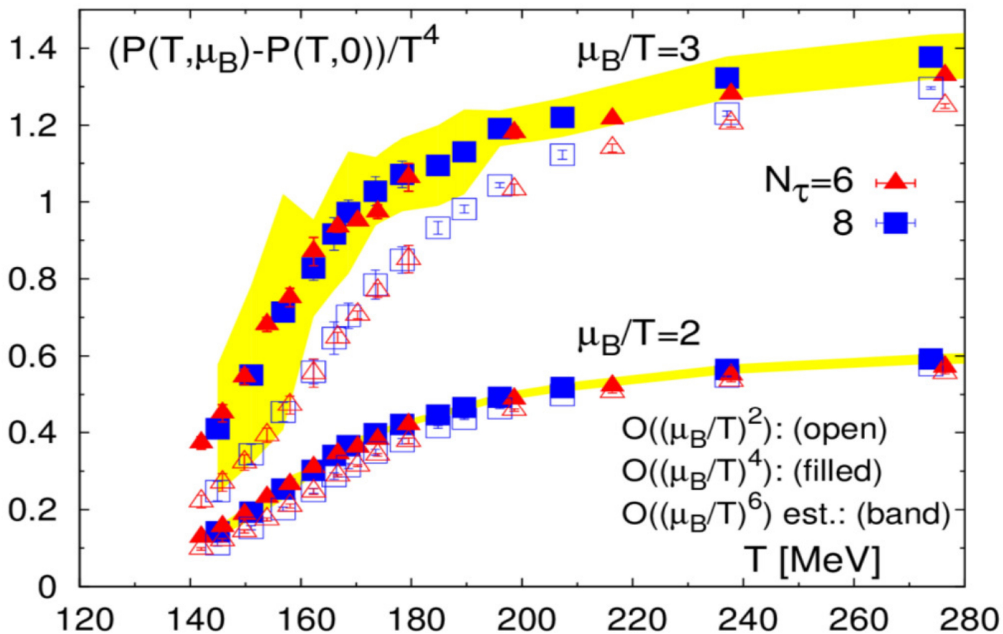

crossover temperature:

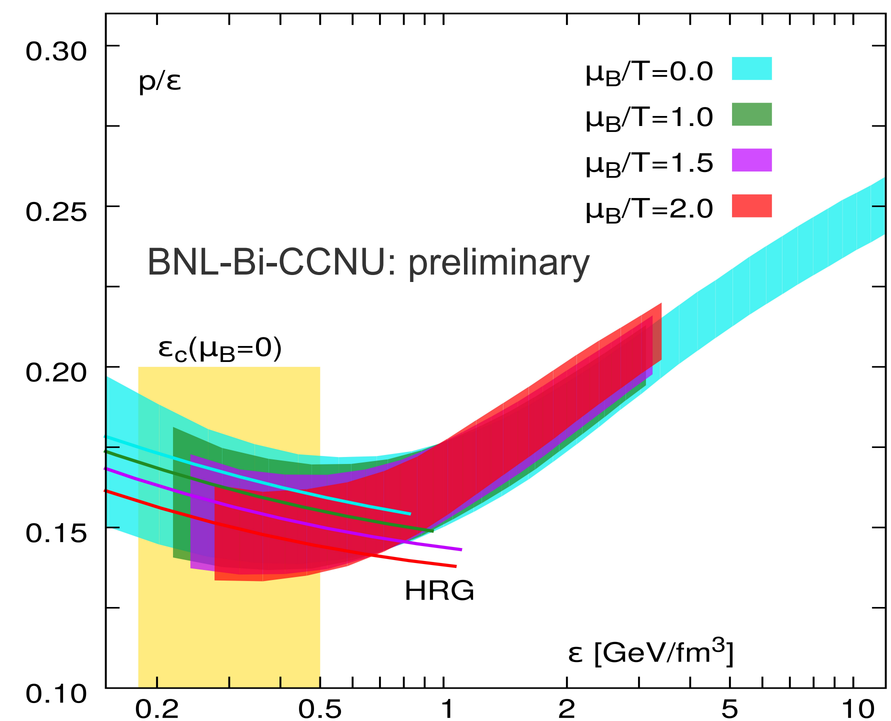
$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa_B \left( \frac{\mu_B}{T_c(0)} \right)^2$$

$$\kappa_B = 0.007 - 0.02$$

BNL-Bi: Phys.Rev. D83 (2011) 014504
W-B: JHEP 1104 (2011) 001
Bonati et. al.: arXiv:1507.03571
W-B: arXiv:1507.07510
Cea et. al.: arXiv:1508.07599

# Equation of state at non-zero baryon density




Taylor expansion method:

$$\frac{p(\mu_B,T)}{T^4} = \sum_n \frac{1}{n!}\chi_n^B(T)\left(\frac{\mu_B}{T}\right)^n$$

$6^{th}$ order expansion is controlled for $\mu_B/T \lesssim 2$

breakdown of the $6^{th}$ order expansion for T<Tc, T>Tc OK

# Cumulants of conserved charge fluctuations

LQCD: conserved charge susceptibilities

$$\chi_n^X(T,\mu_X) = \frac{\partial^n\left(p(T,\mu_X)/T^4\right)}{\partial(\mu_X/T)^n}$$

$$\chi_n^X(T,\mu_X) = \sum_n \frac{1}{k!}\chi_{k+n}^X(T)\left(\frac{\mu_X}{T}\right)^n$$

can be compared directly with experimentally measured cumulants of charge fluctuations

$$\frac{M_Q(\sqrt{s})}{\sigma_Q^2(\sqrt{s})} = \frac{\chi_1^Q(T,\mu_B)}{\chi_2^Q(T,\mu_B)}$$

$$\frac{S_Q(\sqrt{s})\,\sigma_Q^3(\sqrt{s})}{M_Q(\sqrt{s})} = \frac{\chi_3^Q(T,\mu_B)}{\chi_1^Q(T,\mu_B)}$$

Expt.: mean : $M_Q$

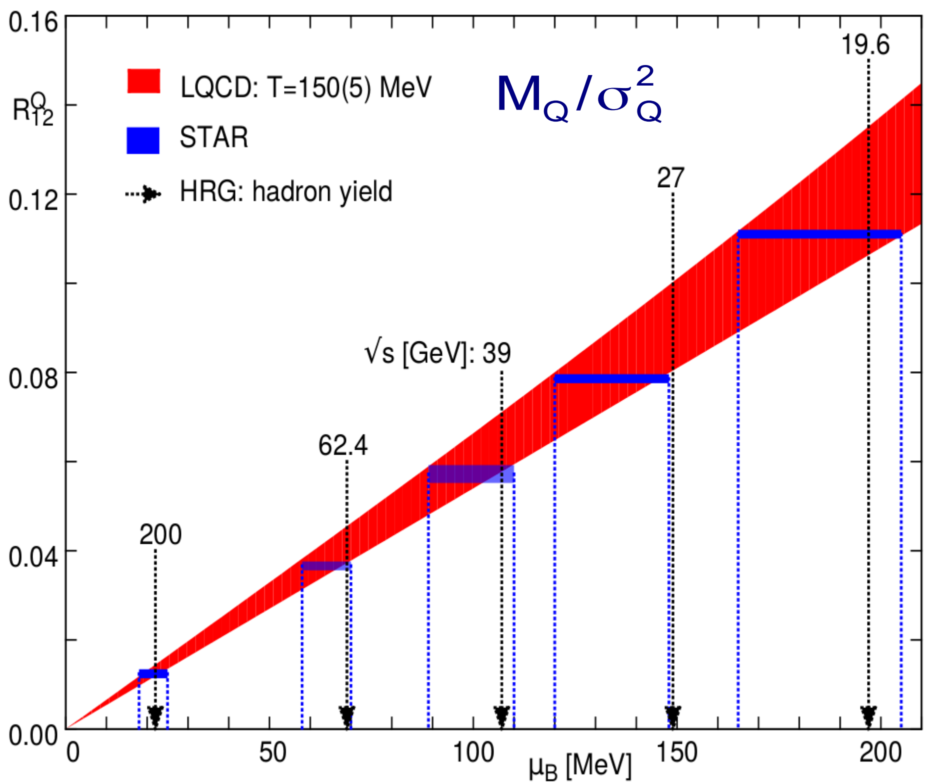
variance : $\sigma_Q^2$

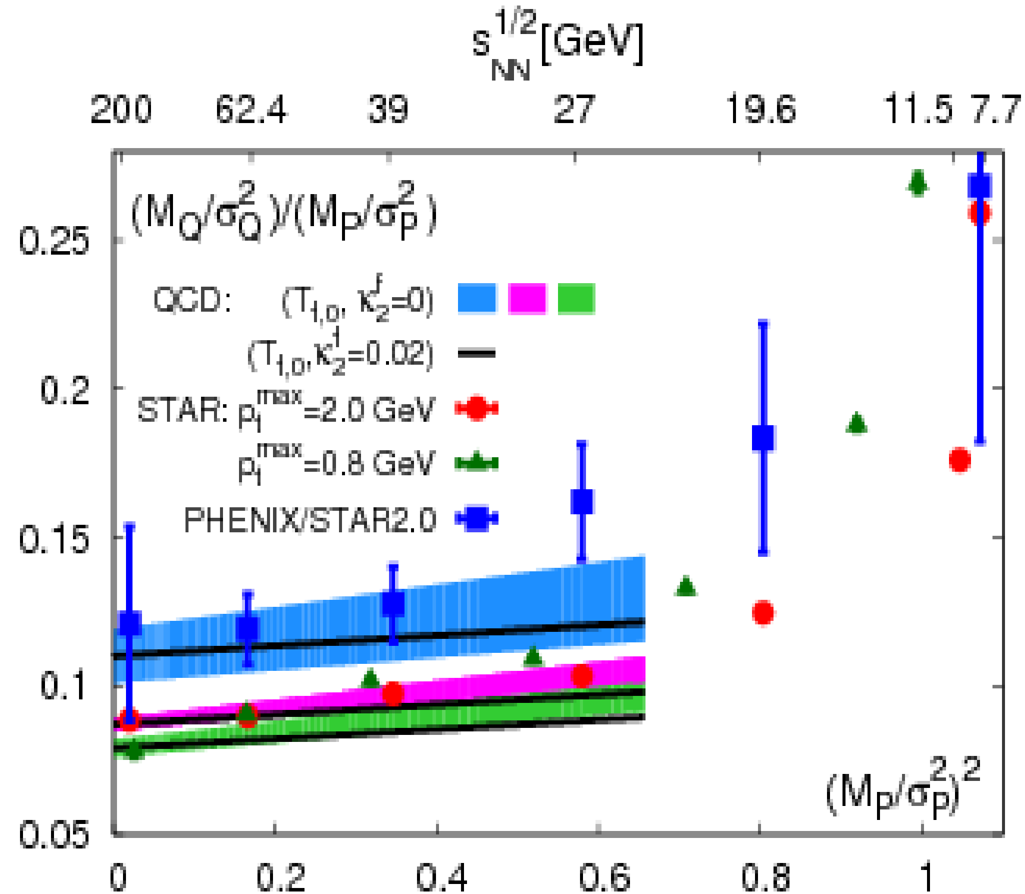
skewness : $S_Q$

can be used to extract freeze-out parameters

BNL-Bi: Phys. Rev. Lett. 109, 192302 (2012)

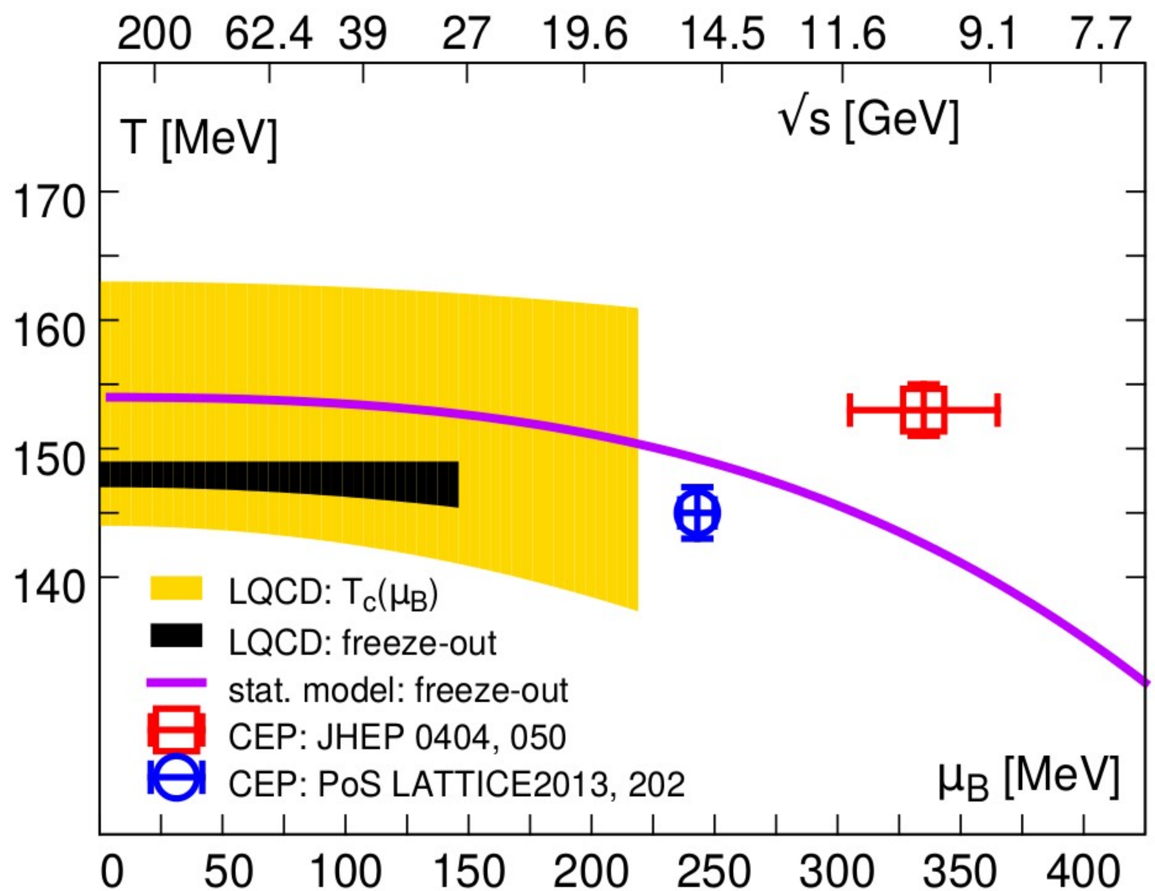
# Charge fluctuations, LQCD and freeze-out in HIC




BNL-Bi: Phys. Rev. Lett. 109, 192302 (2012)
SM: PoS CPOD2013, 039 (2013)
BNL-Bi-CCNU: arXiv:1509.05786

# Dense LQCD: as we stand ...



# Summary

QCD transition & EoS at zero baryon density:

- ✔ QCD calculations: physical quark masses, continuum limit

LQCD at non-zero baryon density:

- ✔ EoS controlled for $\mu_B/T \lesssim 2$
- ✔ direct comparison between (L)QCD calculations and HIC expt.
  freeze-out parameters & more
- ✔ indirect evidence for unobserved strange baryons
- ✔ location of the QCD ciritical point remains a challenge
- ✔ need calculations of higher order cumulants: feasible in coming years

Transport, heavy quarks & other observables:

- ✔ observables calculated from fermionic correlation functions
  have demonstrated to be feasible
- ✔ need inclusion of light dynamical fermions & very large lattices:
  feasible in coming years
- ✔ observables involving gluonic correlation functions still challenging:
  viscosities & jet-quenching parameter