



UNIVERSITÀ DEGLI STUDI DI CATANIA
INFN-LNS



**Initial state fluctuations in ultra-central collisions
in an event-by-event transport approach**

S. Plumari, G.L. Guardo, A. Puglisi,

F. Scardina, M. Ruggieri, V. Greco

Outline

- **Transport approach at fixed η/s :**
 - fix locally $\eta/s \leftrightarrow \sigma(\theta), M, T \rightarrow$ Chapman-Enskog approach.
- **Initial state fluctuations:**
 - η/s and generation of $v_n(pT)$: from RHIC to LHC
 - Correlations between ε_n and v_n
- **Conclusions**

Motivation for a kinetic approach:

$$\left\{ p^\mu \partial_\mu + \left[p_\nu F^{\mu\nu} + M \partial^\mu M \right] \partial_\mu^p \right\} f(x, p) = C_{22} + C_{23} + \dots$$

Free
streaming

Field Interaction $\rightarrow \epsilon \neq 3P$

Collisions $\rightarrow \eta \neq 0$

- Starting from 1-body distribution function and not from $T^{\mu\nu}$:

- possible to include $f(x,p)$ out of equilibrium.

M. Ruggieri et.al, PLB 727 (2013) 177

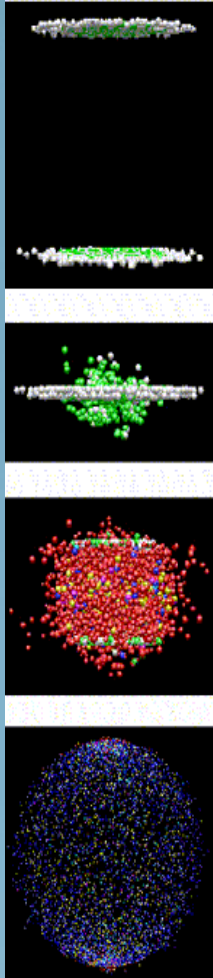
- extract information about the viscous correction δf to $f(x,p)$

S.Plumari,G.L. Guardo, V. Greco, J.Y. Ollitrault NPA 941 (2015) 87

- Valid at intermediate p_T out of equilibrium.
- Valid at high η/s (cross over region) + self consistent kinetic freeze-out

DISADVANTAGES ?!

- The relaxation times are those of kinetic theory
- Hadronization missing now but it will be possible to include:
Coalescence + Fragmentation



Parton Cascade model

For the numerical implementation of the collision integral we use the stochastic algorithm.

Z. Xu and C. Greiner, PRC 71 064901 (2005)

$$p^\mu \partial_\mu f(X, p) = C = C_{22} + C_{23} + \dots$$

$$C_{22} = \frac{1}{2E_1} \int \frac{d^3 p_2}{(2\pi)^3 2E_2} \frac{1}{v} \int \frac{d^3 p'_1}{(2\pi)^3 2E'_1} \frac{d^3 p'_2}{(2\pi)^3 2E'_2} f'_1 f'_2 |M_{1'2' \rightarrow 12}|^2 (2\pi)^4 \delta^{(4)}(p'_1 + p'_2 - p_1 - p_2)$$

Simulating a constant η/s

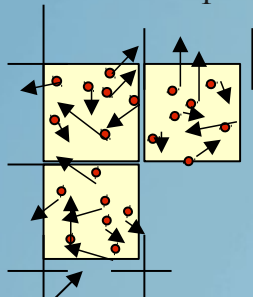
For anisotropic cross section and massless particles:

$$\eta(\vec{x}, t)/s = \frac{1}{15} \langle p \rangle \tau_\eta \longrightarrow \sigma_{tot}^{\eta/s} = \frac{1}{15} \frac{\langle p \rangle}{g(m_D/2T)n} \frac{1}{\eta/s}$$

σ is evaluated in such way to keep fixed the η/s during the dynamics according the Chapman-Enskog equation.

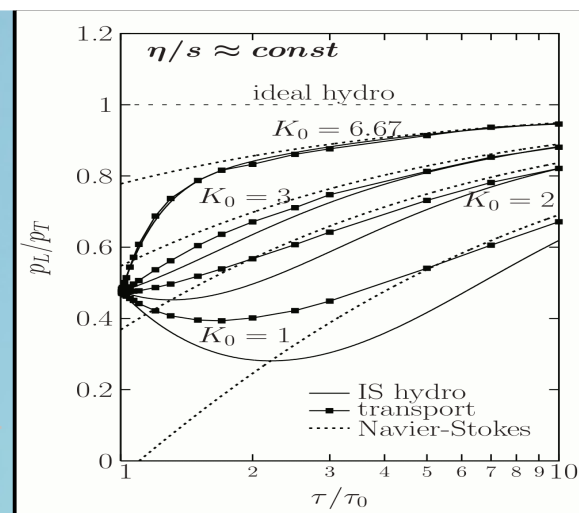
similar to D. Molnar, arXiv:0806.0026[nucl-th]

$$P_{22} = \frac{\Delta N_{coll}^{2 \rightarrow 2}}{\Delta N_1 \Delta N_2} = v_{rel} \sigma_{22} \frac{\Delta t}{\Delta^3 x}$$

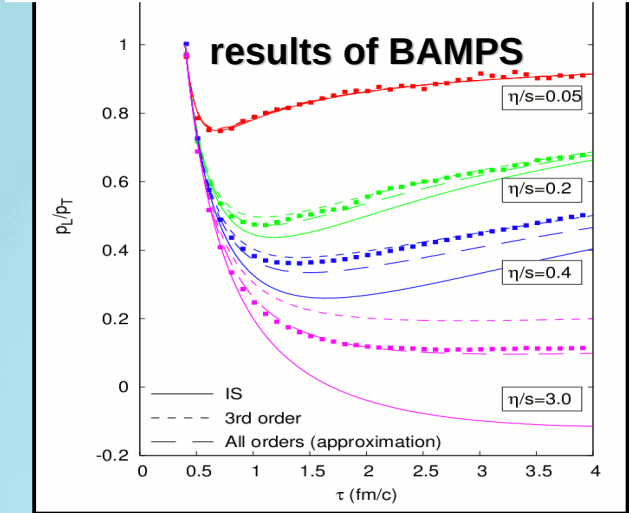


$\Delta t \rightarrow 0$
 $\Delta^3 x \rightarrow 0$ \longrightarrow right solution

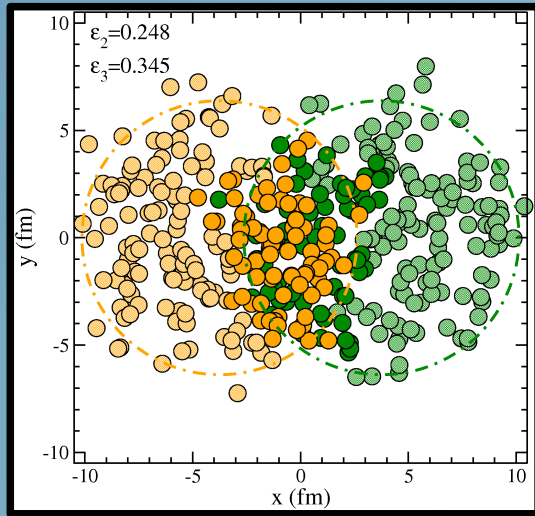
P. Huovinen and D. Molnar, PRC79(2009)



A. El, Z. Xu, C. Greiner PRC 81 (2010) 041901



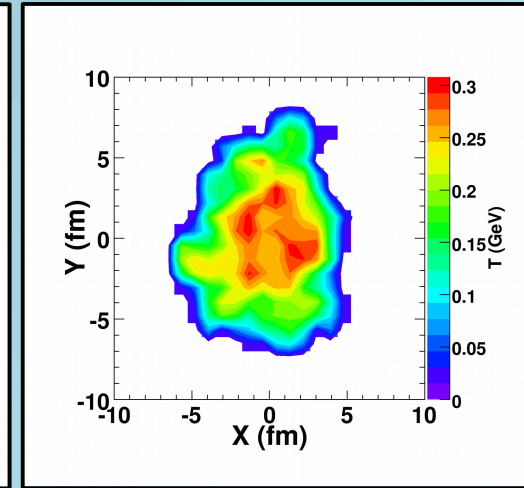
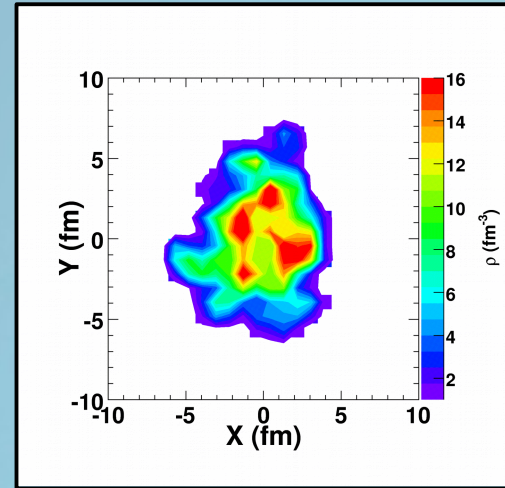
Initial State Fluctuations



smooth distribution

Monte Carlo Glauber

$$\rho_{\perp}(x, y) \propto \sum_{i=1}^{N_{part}} \exp\left\{-\left[\frac{(x-x_i)^2 + (y-y_i)^2}{2\sigma^2}\right]\right\}$$



n=2

n=3

n=4

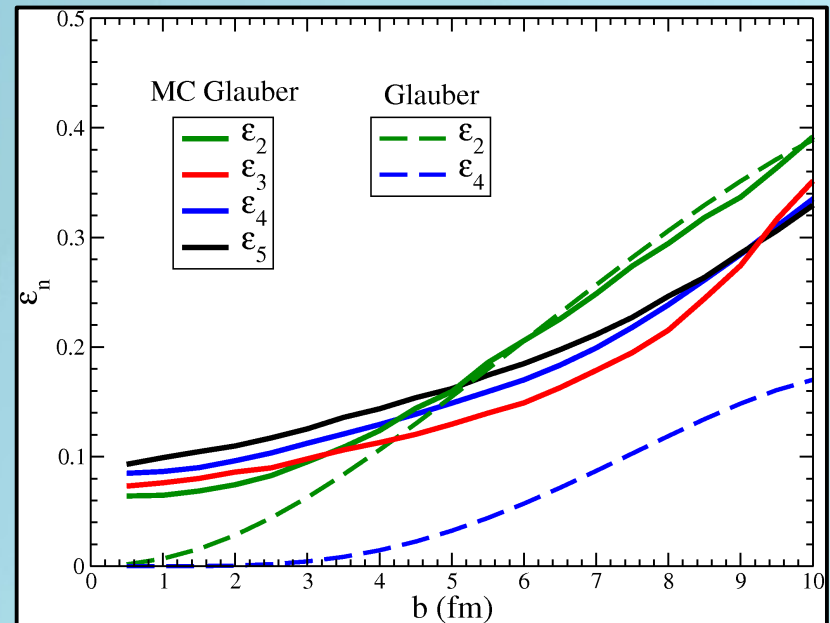
n=5

n=6

Characterization of the initial profile in terms of Fourier coefficients

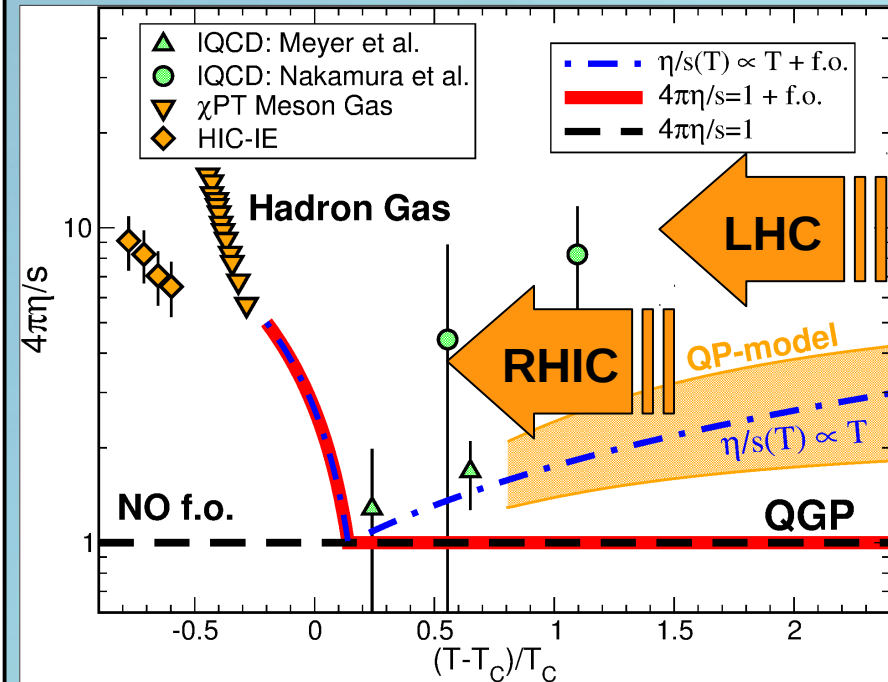
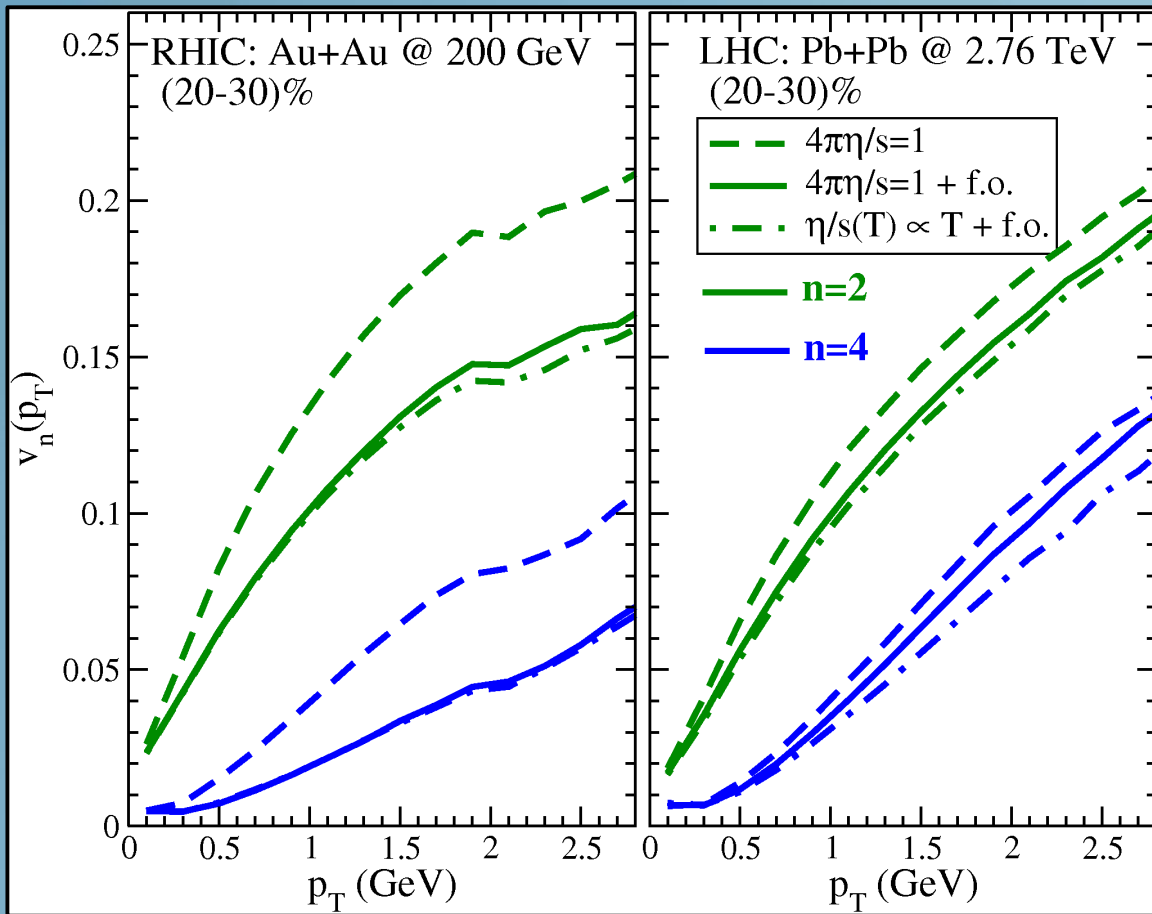
$$\epsilon_n = \frac{\langle r_{\perp}^n \cos[n(\varphi - \Phi_n)] \rangle}{\langle r_{\perp}^n \rangle} \quad \Phi_n = \frac{1}{n} \arctan \frac{\langle r_{\perp}^n \sin(n\varphi) \rangle}{\langle r_{\perp}^n \cos(n\varphi) \rangle}$$

$$r_{\perp} = \sqrt{x^2 + y^2}, \quad \varphi = \arctan(y/x)$$



G-Y. Qin, H. Petersen, S.A. Bass and B. Muller, PRC82,064903 (2010).
H.Holopainen, H. Niemi and K.J. Eskola, PRC83, 034901 (2011).

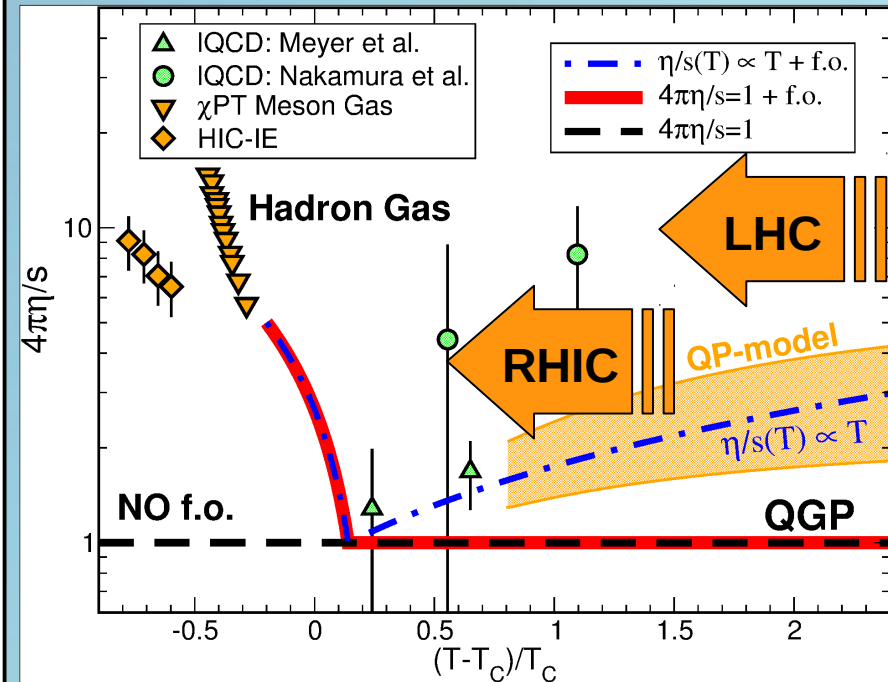
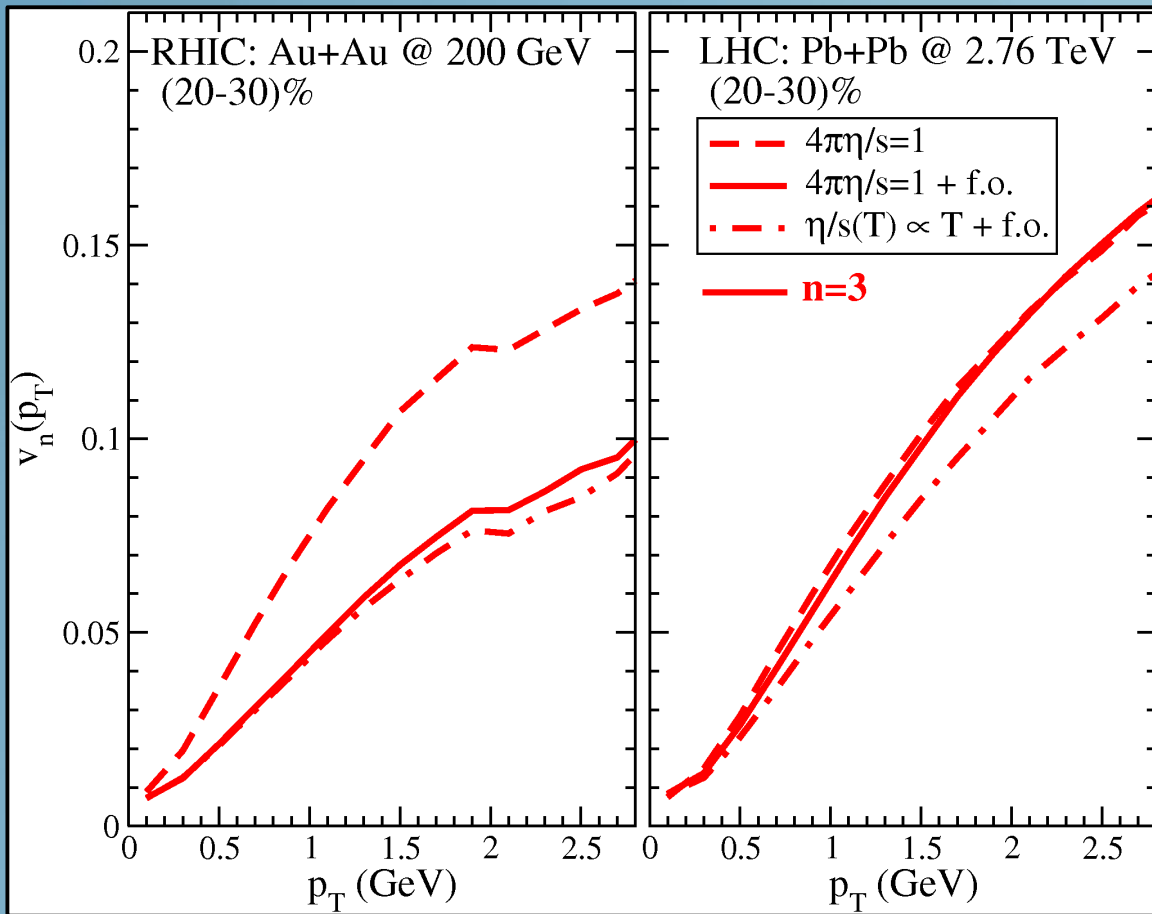
Initial State Fluctuations: $v_n(p_T)$ and η/s



S. Plumari, G. L. Guardo, F. Scardina and V. Greco, arXiv:1507.05540

- $v_n(p_T)$ at RHIC is more sensitive to the value of the η/s at low temperature. $v_4(p_T)$ and $v_3(p_T)$ are more sensitive to the value of η/s than the $v_2(p_T)$.
- At LHC energies $v_n(p_T)$ is more sensitive to the value of η/s in the QGP phase (compare solid and dot-dashed lines).

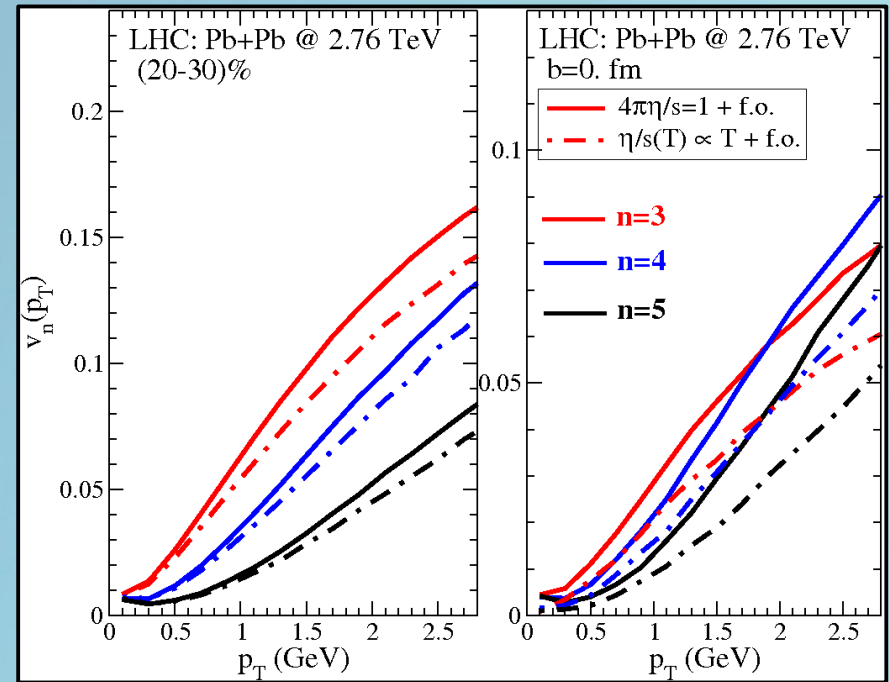
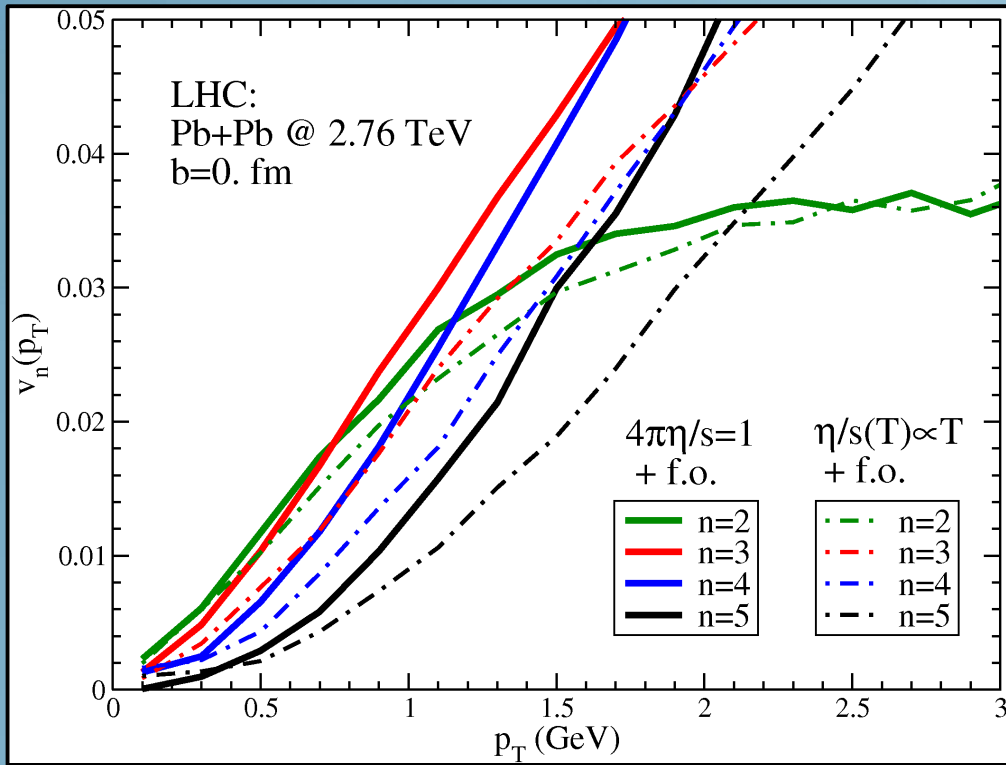
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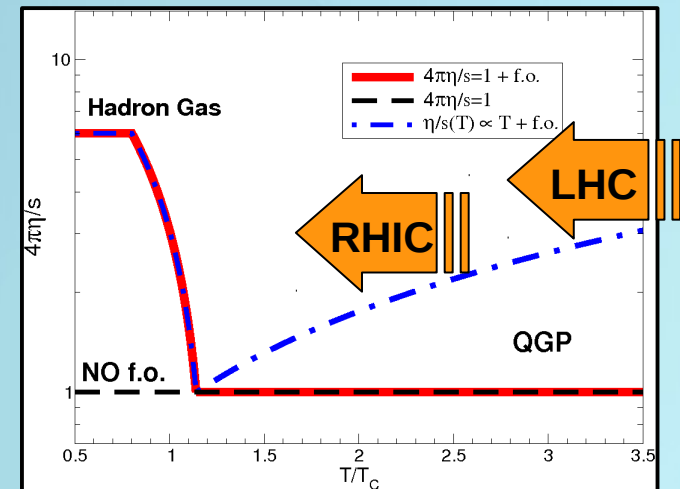
S. Plumari, G. L. Guardo, F. Scardina and V. Greco, arXiv:1507.05540

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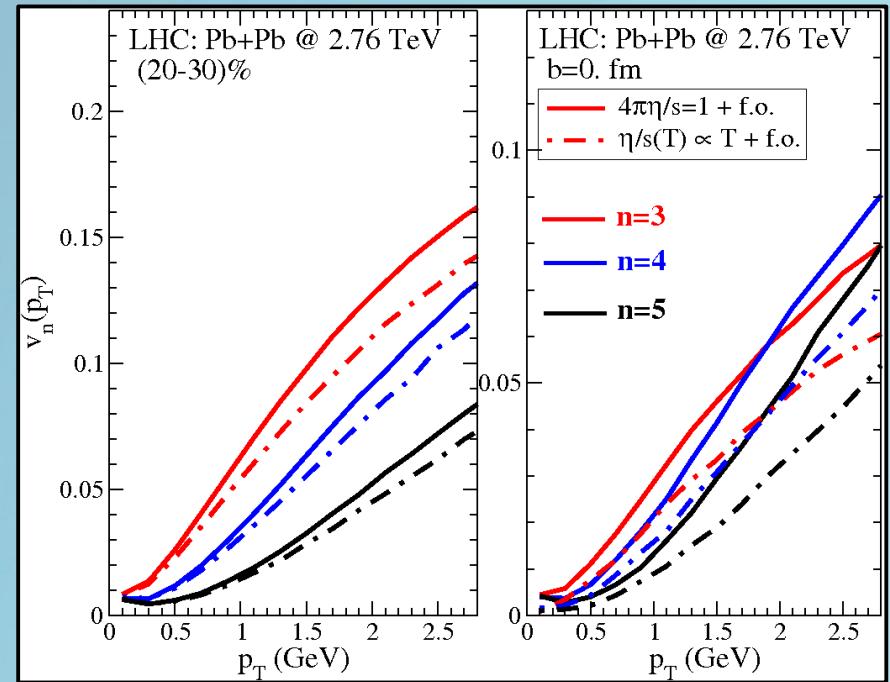
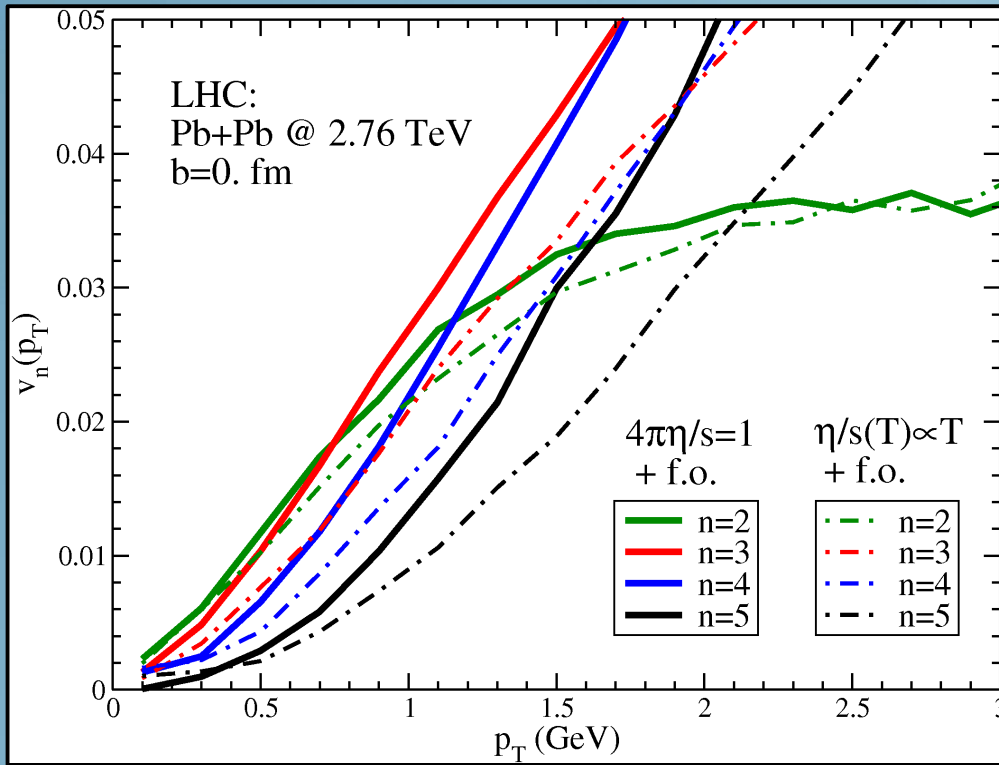
Initial State Fluctuations: $v_n(p_T)$ for central collisions



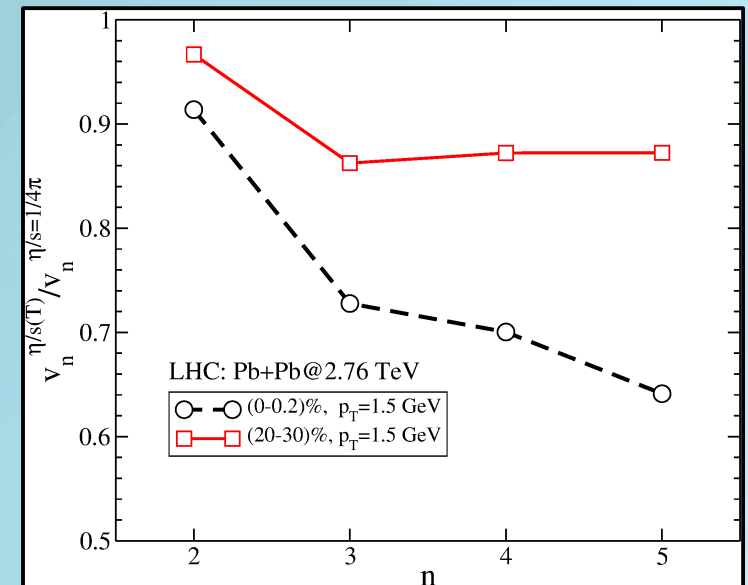
- At low p_T $v_n(p_T) \propto p_T^n$. v_2 for higher p_T saturates while v_n for $n > 3$ increase linearly with p_T .
- For central collisions viscous effect are more relevant. For $n > 2$ the $v_n(p_T)$ are more sensitive to the η/s ratio in the QGP phase.



Initial State Fluctuations: $v_n(p_T)$ for central collisions



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RHIC:
Au+Au @ 200 GeV

Initial State Fluctuations: v_n vs ϵ_n

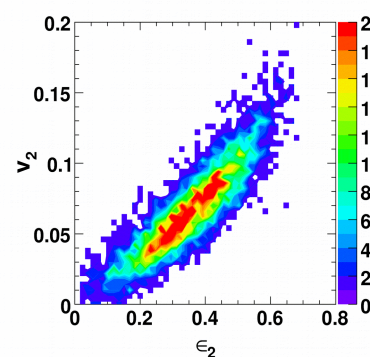
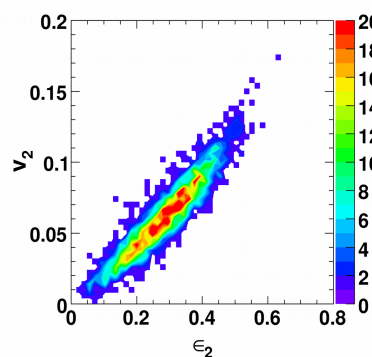
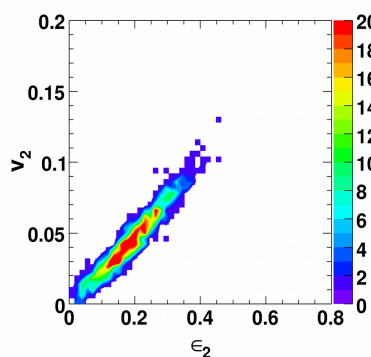
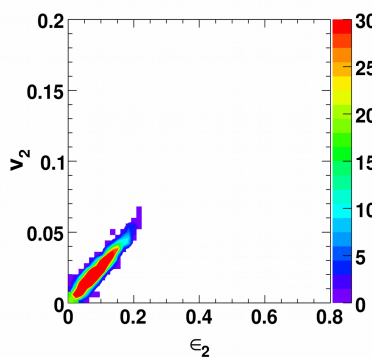
(0-10)%

(10-20)%

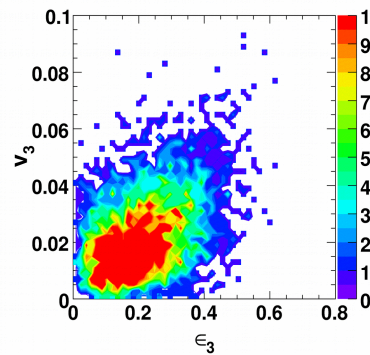
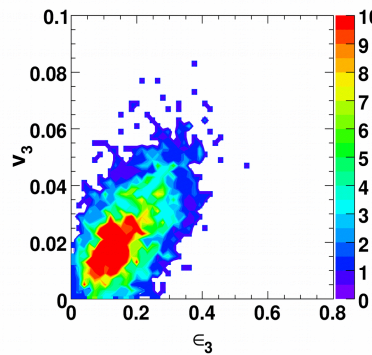
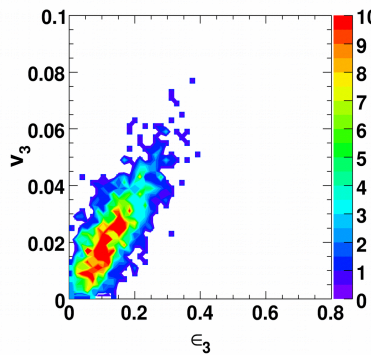
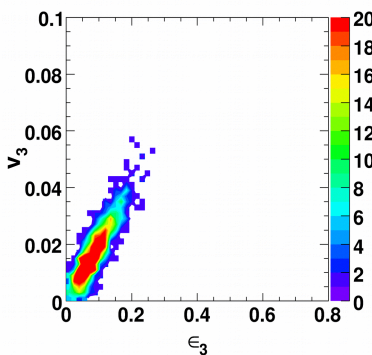
(20-30)%

(30-40)%

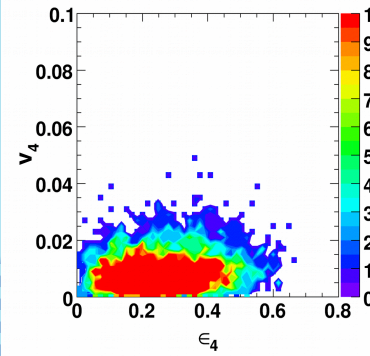
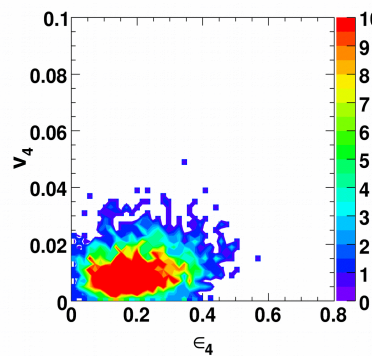
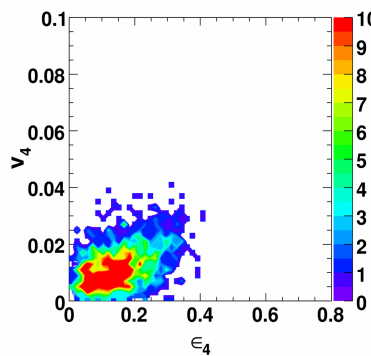
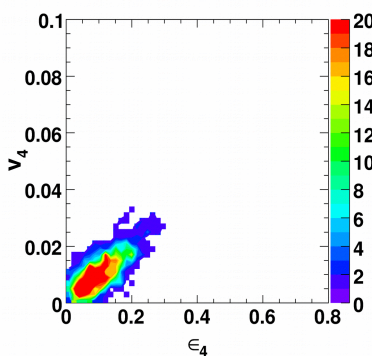
$n=2$



$n=3$

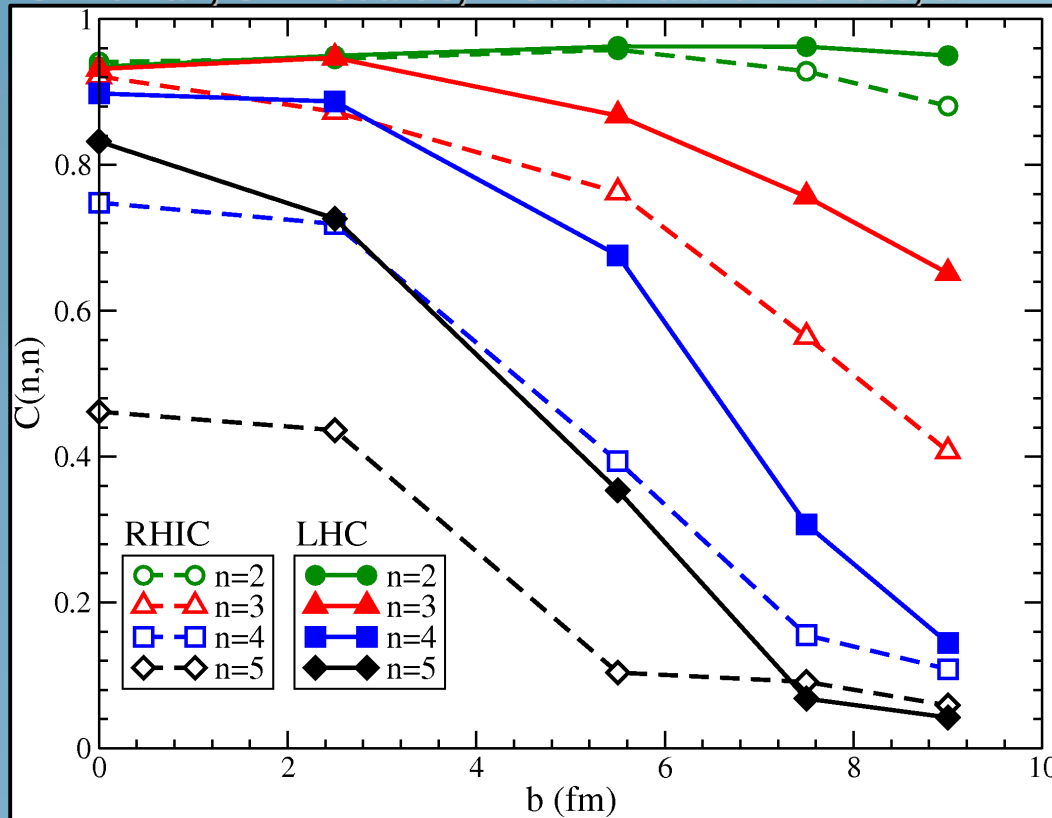


$n=4$



Initial State Fluctuations: v_n vs ϵ_n

S. Plumari, G. L. Guardo, F. Scardina and V. Greco, arXiv:1507.05540



$$C(n, m) = \left\langle \frac{(v_n - \langle v_n \rangle)(\epsilon_m - \langle \epsilon_m \rangle)}{\sigma_{v_n} \sigma_{\epsilon_m}} \right\rangle$$

B.H. Alver, C. Gombeaud, M. Luzum and J.-Y. Ollitrault, Phys.Rev. C82 (2010) 034913.

H. Petersen, G.-Y. Qin, S.A. Bass and B. Muller, Phys.Rev. C82 (2010) 041901.

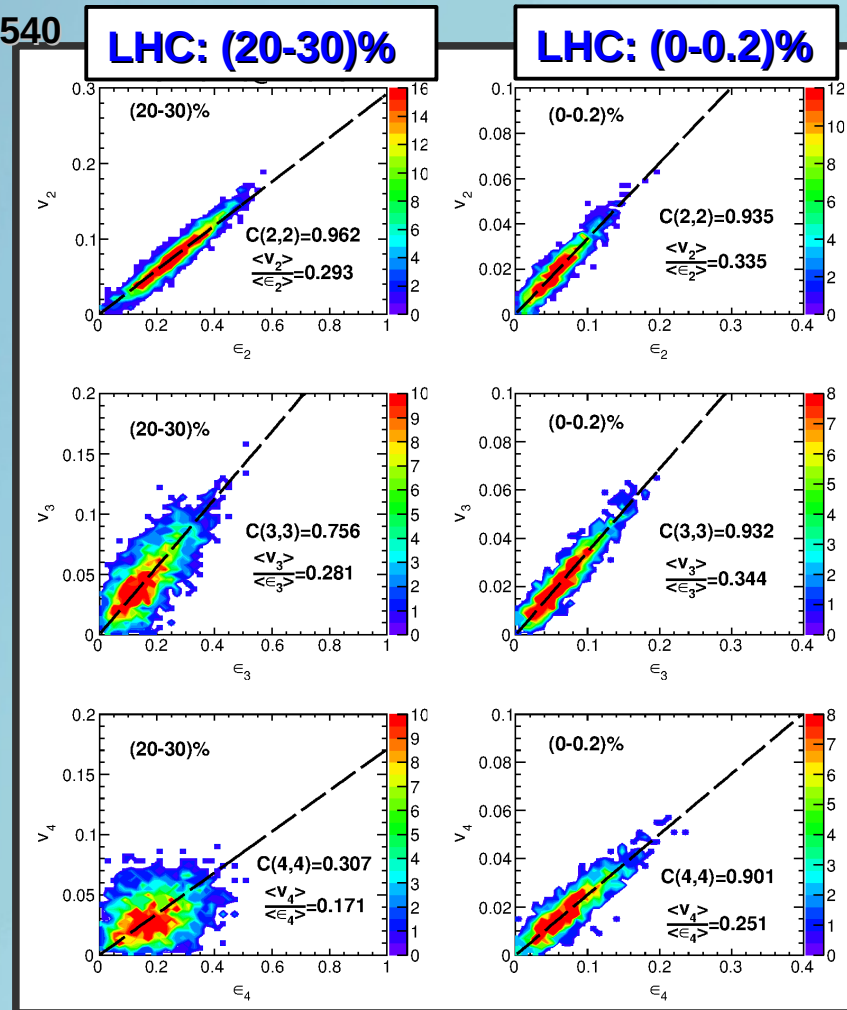
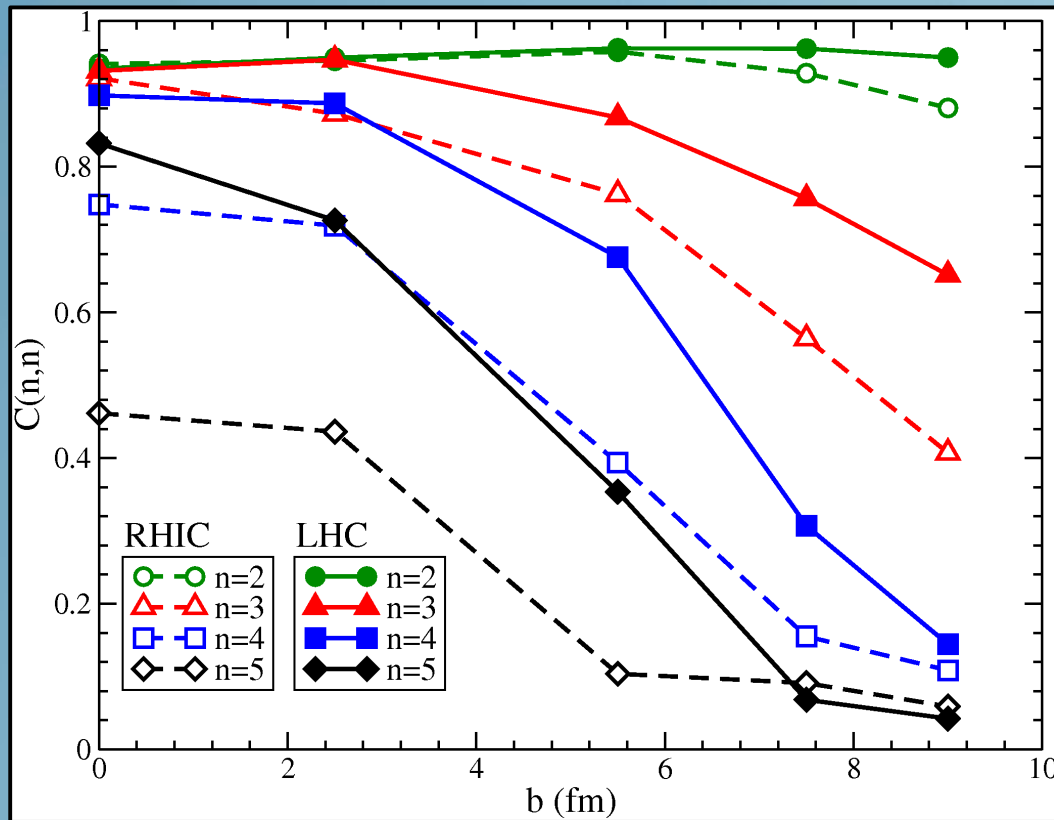
Z. Qiu and U. W. Heinz, Phys.Rev. C84 (2011) 024911.

H. Niemi, G.S. Denicol, H. Holopainen and P. Huovinen, Phys.Rev. C87 (2013) 5, 054901.

- At LHC v_n are more correlated to ϵ_n than at RHIC.
- v_2 and v_3 linearly correlated to the corresponding eccentricities ϵ_2 and ϵ_3 respectively.
- $C(4,4) < C(2,2)$ for all centralities. v_4 and ϵ_4 weak correlated similar to hydro calculations:
 F.G. Gardim, F. Grassi, M. Luzum and J.Y. Ollitrault NPA904 (2013) 503.
 H. Niemi, G.S. Denicol, H. Holopainen and P. Huovinen PRC87(2013) 054901.
- For central collisions v_n are strongly correlated to ϵ_n : $v_n \propto \epsilon_n$ for $n=2,3,4$.

Initial State Fluctuations: v_n vs ϵ_n

S. Plumari, G. L. Guardo, F. Scardina and V. Greco, arXiv:1507.05540



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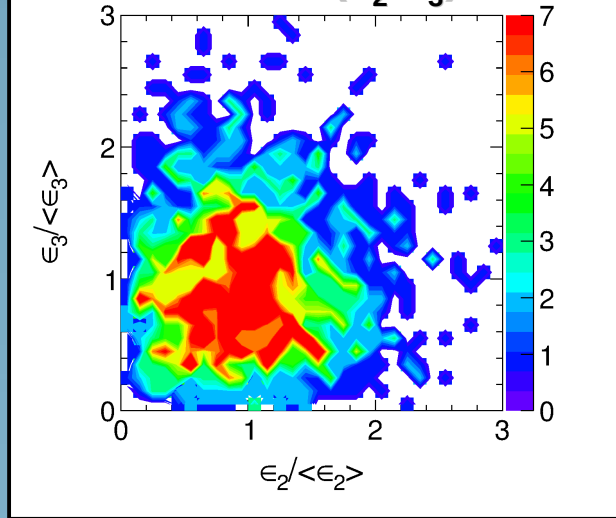
H. Niemi, G.S. Denicol, H. Holopainen and P. Huovinen PRC87(2013) 054901.

- For central collisions v_n are strongly correlated to ϵ_n : $v_n \propto \epsilon_n$ for $n=2,3,4$.

Correlations: (ϵ_n, ϵ_m) vs (v_n, v_m) in (0-0.2)%

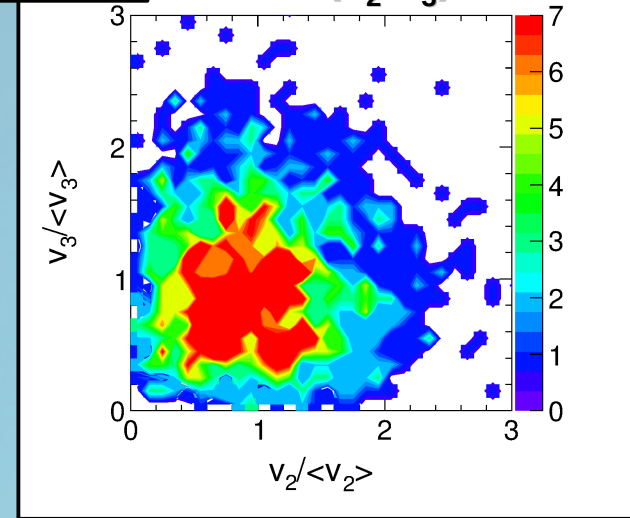
(0-0.2)%

$C(\epsilon_2, \epsilon_3) = 0.009$



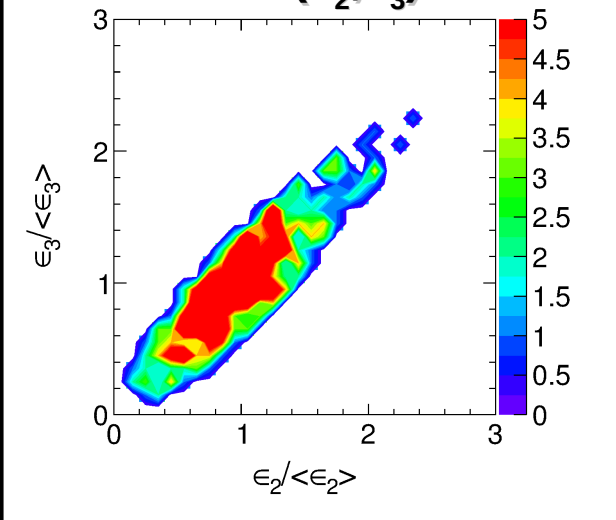
(0-0.2)%

$C(v_2, v_3) = 0.015$



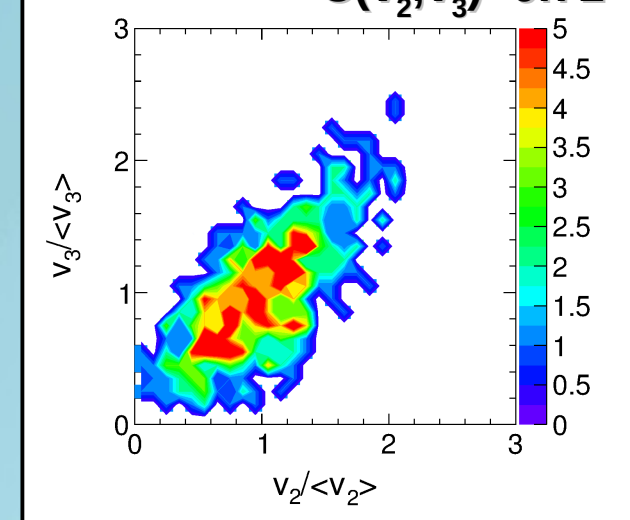
(0-0.2)%

$C(\epsilon_2, \epsilon_3) = 0.87$



(0-0.2)%

$C(v_2, v_3) = 0.71$



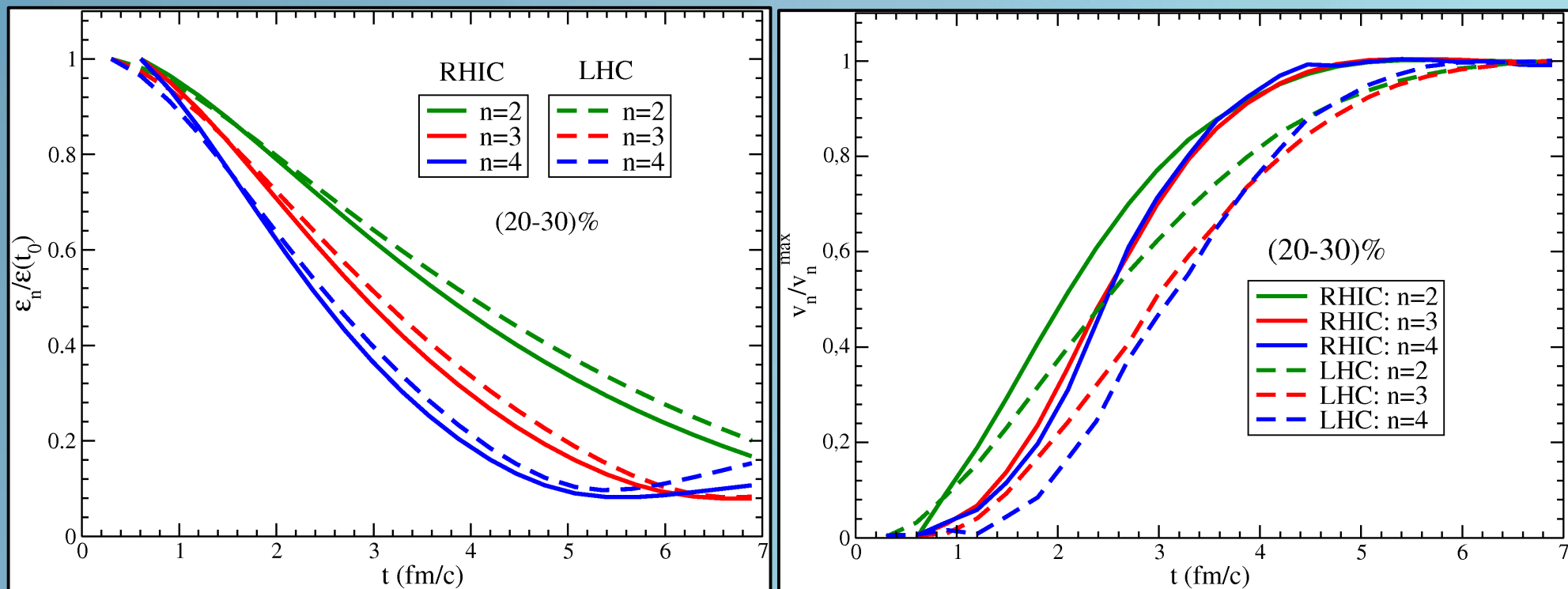
- Final correlations in (v_n, v_m) reflect initial correlations in (ϵ_n, ϵ_m)
- For (20-30)%: $C(v_n, v_m) = 0.38$ and $C(\epsilon_n, \epsilon_m) = 0.78$ differ a factor 2

Conclusions

Transport at fixed η/s :

- Enhancement of $\eta/s(T)$ in the cross-over region affect differently the expanding QGP from RHIC to LHC. LHC nearly all the v_n from the QGP phase.
- At LHC there is a stronger correlation between v_n and ε_n than at RHIC for all n .
- Ultra central collisions:
 - $v_n \propto \varepsilon_n$ for $n=2,3,4$ strong correlation $C(n,n) \approx 1$
 - $v_n(p_T)$ much more sensitive to $\eta/s(T)$
 - degree of correlation increase with the collision energy
 - correlations in (v_n, v_m) reflect the initial correlations in $(\varepsilon_n, \varepsilon_m)$

Initial State Fluctuations: time evolution of $\langle v_n \rangle$ and ε_n



- The time evolution for ε_n is faster for large n . At very early times $\varepsilon_n(t) = \varepsilon_n(t_0) - \alpha_n t^{n-2}$.
- $\langle v_n \rangle$ shows an opposite behaviour: $\langle v_n \rangle$ develops later for large n . At very early times $\langle v_n \rangle \propto t^{n+1}$.
- Different v_n can probe different values of $\eta/s(T)$ during the expansion of the fireball.

$\eta/s(T)$ around to a phase transition

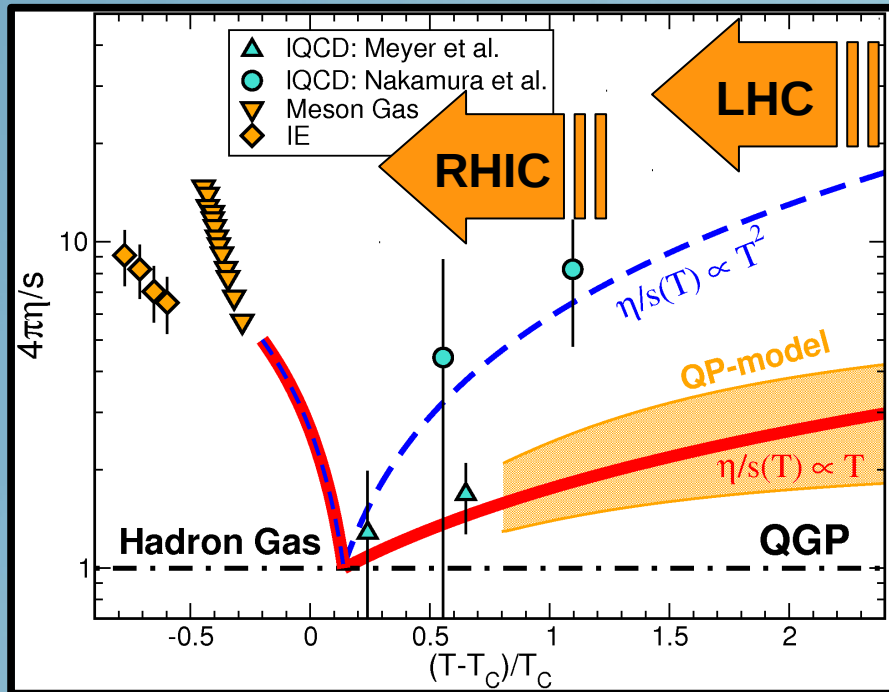
- Quantum mechanism

$$\Delta E \cdot \Delta t \geq 1 \rightarrow \eta/s \approx \frac{1}{15} \langle p \rangle \cdot \tau > \frac{1}{15}$$

- AdS/CFT suggest a lower bound $\eta/s = 1/(4\pi) \sim 0.08$

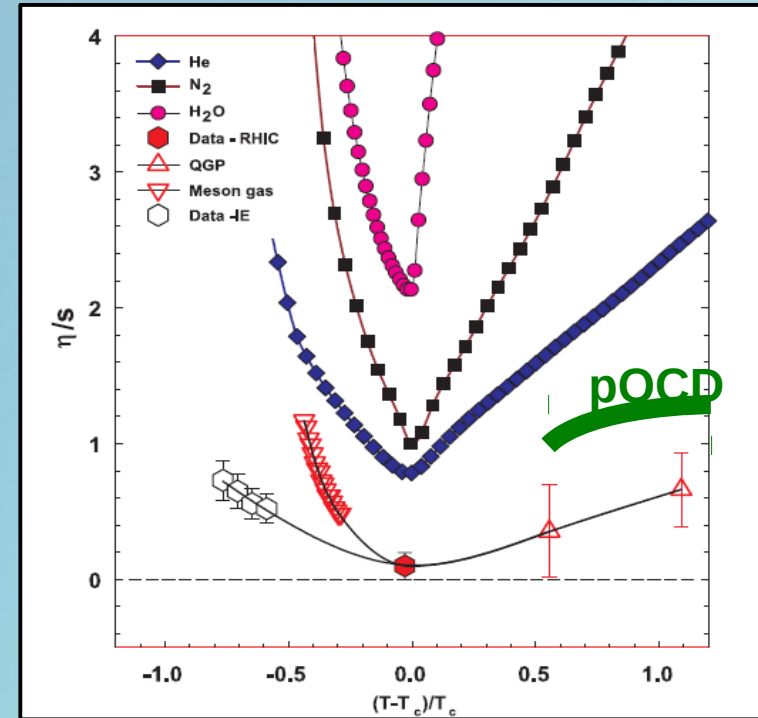
- From pQCD: $\eta/s \sim \frac{1}{g^4 \ln(1/g)} \Rightarrow \eta/s \sim 1$

P. Arnold et al., JHEP 0305 (2003) 051.



S. Plumari et al., J. Phys.: Conf. Ser. 420 012029 (2013).
arXiv:1209.0601.

P. Kovtun et al., Phys.Rev.Lett. 94 (2005) 111601.
L. P. Csernai et al., Phys.Rev.Lett. 97 (2006) 152303.
R. A. Lacey et al., Phys.Rev.Lett. 98 (2007) 092301.



- LQCD some results for quenched approx. large error bars
- Quasi-Particle models seem to suggest a $\eta/s \sim T^\alpha$ $\alpha \sim 1 - 1.5$.
- Chiral perturbation theory \rightarrow Meson Gas
- Intermediate Energies - IE ($\mu_B > T$)

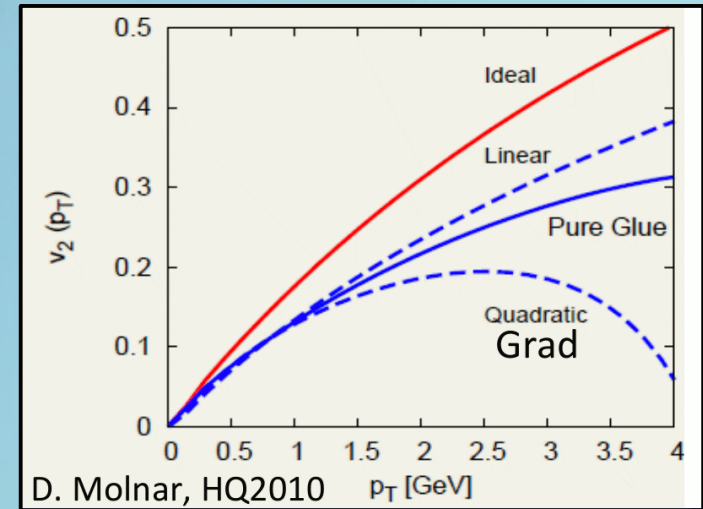
From Transport to Hydro: extraction of viscous corrections to $f(x,p)$ and $v_n(p_T)$. (work in collaboration with J.Y. Ollitrault)

$$f(x,p) = f^{(0)}(x,p) + \delta f(x,p)$$

$$T^{\mu\nu} = T^{(0)\mu\nu} + \delta T^{\mu\nu} \leftarrow f^{(0)} + \delta f$$

A common choice for δf – the Grad ansatz

$$\delta f \propto \Gamma_s f^{(0)} p^\alpha p^\beta \langle \nabla_\alpha u_\beta \rangle \propto p_T^2$$



BUT it doesn't care about the microscopic dynamics

In general in the limit $\sigma \rightarrow \infty$, $f(\sigma)$ can be expanded in power of $1/\sigma$.

$$f(\sigma) \underset{\sigma \rightarrow \infty}{\approx} f^{(0)} + \frac{1}{\sigma} \delta f + O\left(\frac{1}{\sigma^2}\right) \quad \longrightarrow \quad v_n(p_T) \underset{\sigma \rightarrow \infty}{\approx} v_n^{(0)}(p_T) + \frac{1}{\sigma} \delta v_n + O\left(\frac{1}{\sigma^2}\right)$$

PURPOSE: evaluate the ideal hydrodynamics limit $f^{(0)}$, $v_n^{(0)}$ and the viscous corrections δf and δv_n solving the Relativistic Boltzmann eq for large values of the cross section σ

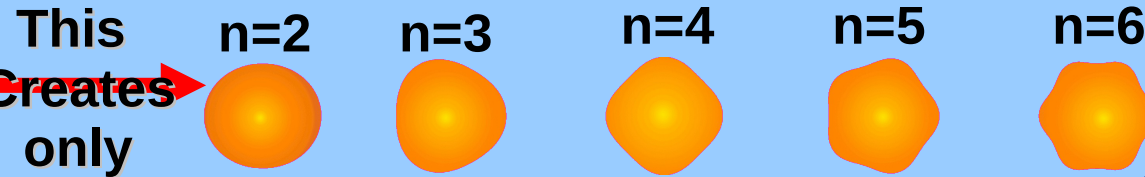
From Transport to Hydro: extraction of viscous corrections to $f(x,p)$ and $v_n(p_T)$. (work in collaboration with J.Y. Ollitrault)

Coordinate space (x,y)

We start with an initial azimuthally symmetric profile (optical Glauber model).

Then we deform the initial distribution ($\alpha \ll 1$)

$$z = x + iy \rightarrow z + \delta z \equiv z - \alpha \bar{z}^{n-1} \quad 2\pi/n \text{ symmetry}$$



Momentum space

Thermal distribution:
 $dN/d^3 p \propto \exp(-p/T)$

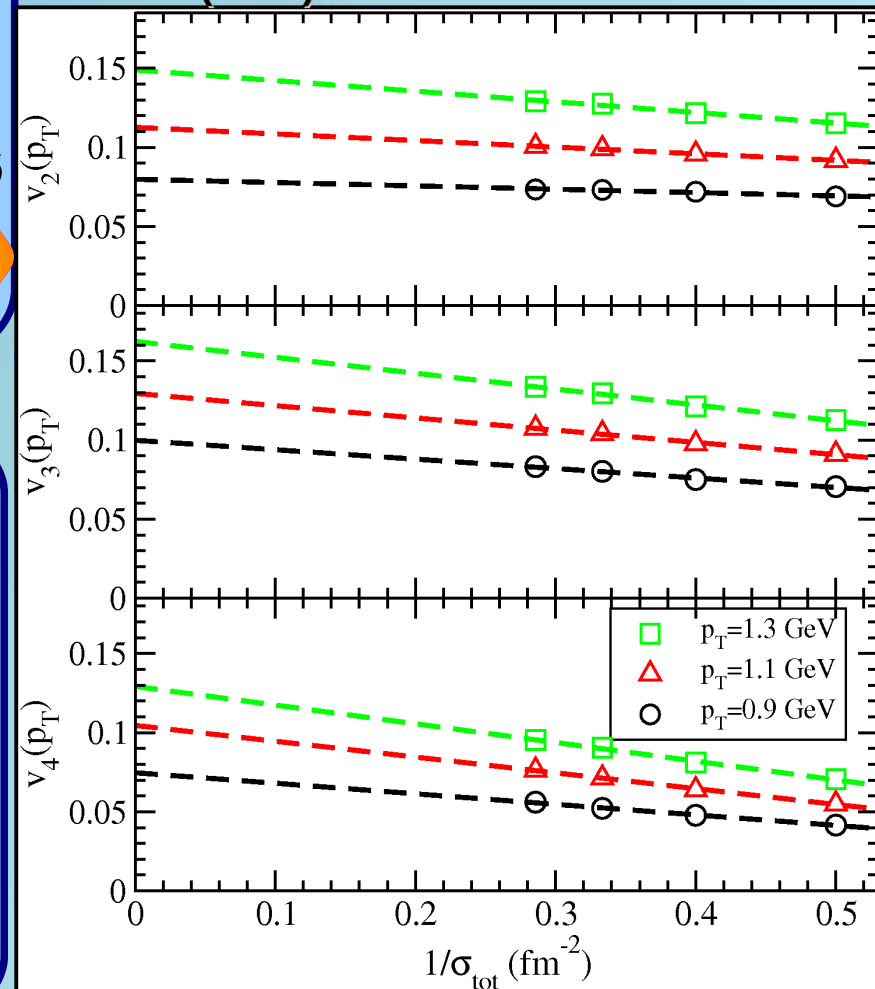
Constant distribution:
 $dN/d^3 p \propto \theta(p_0 - p)$

We assume initially the same local $T^{\mu\nu}(x)$

$$f(\sigma) \underset{\sigma \rightarrow \infty}{\approx} f^{(0)} + \frac{1}{\sigma} \delta f + O\left(\frac{1}{\sigma^2}\right)$$

$$v_n(p_T) \underset{\sigma \rightarrow \infty}{\approx} v_n^{(0)}(p_T) + \frac{1}{\sigma} \delta v_n + O\left(\frac{1}{\sigma^2}\right)$$

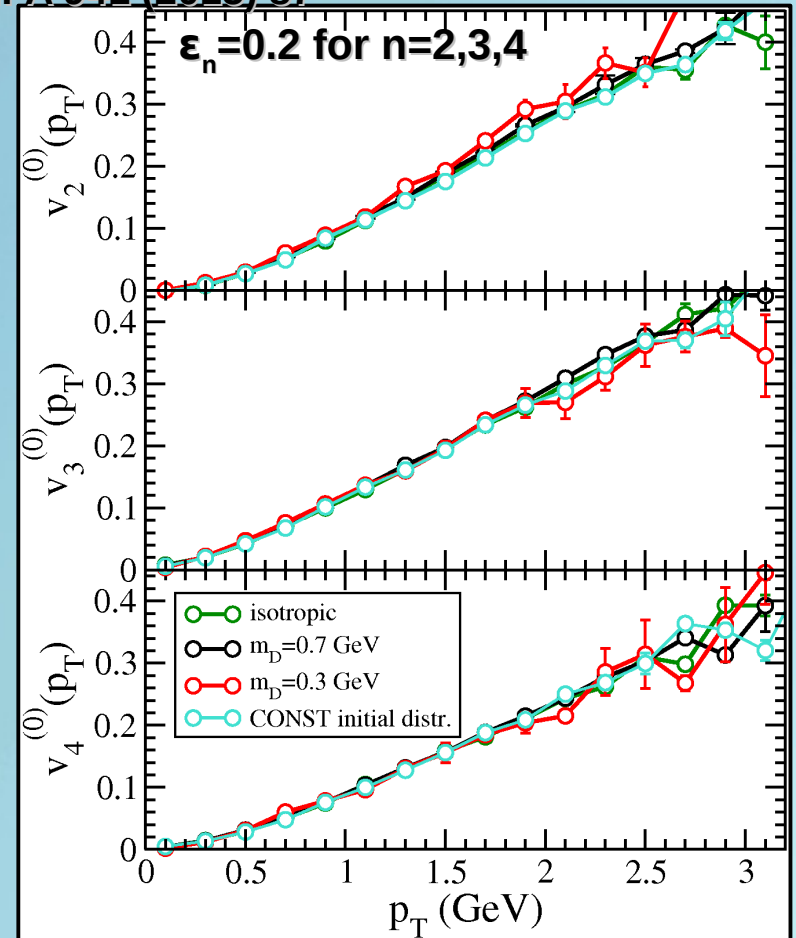
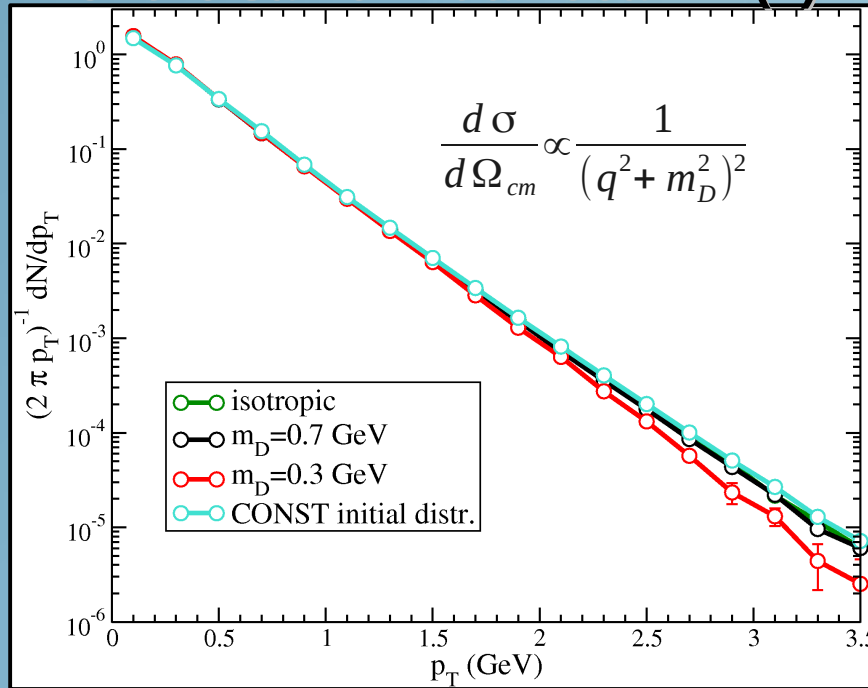
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For the same initial local $T^{\mu\nu}(x)$:

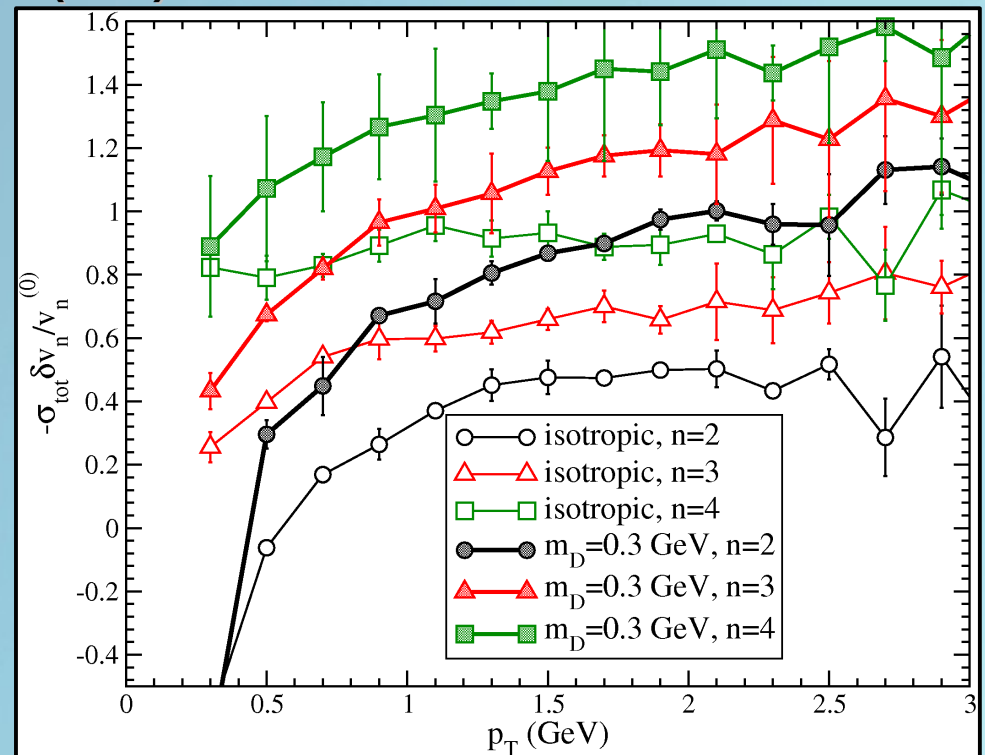
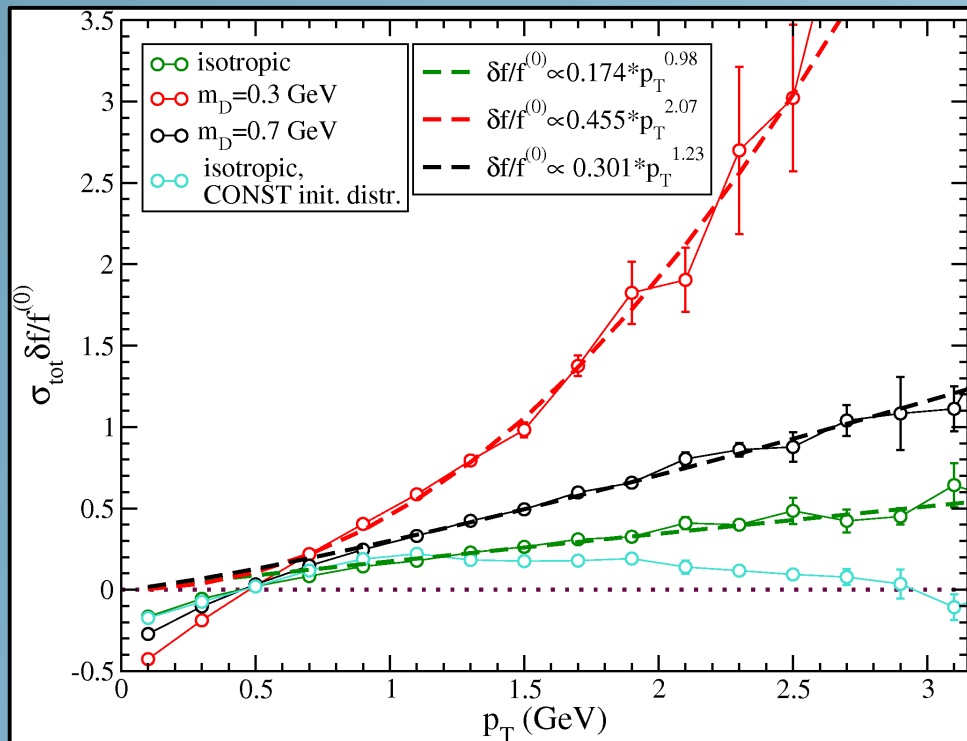


For $\sigma \rightarrow \infty$ we find the ideal Hydro limit:

- $f^{(0)}$ is an exponential decreasing function.
- $f^{(0)}$ doesn't depends on microscopical details (i.e. m_D).
- Universal behavior of $v_n^{(0)}(p_T)$
- $v_n^{(0)}(p_T)/\epsilon_n$ is approximatively the same for all n and p_T .

From Transport to Hydro: extraction of viscous corrections to $f(x,p)$ and $v_n(p_T)$. (work in collaboration with J.Y. Ollitrault)

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In δf and δv_n it is encoded the information about the microscopical details

- $\delta f(p_T)/f^{(0)} \propto p_T^\alpha$ with $\alpha = 1. - 2.$ and $\alpha(m_D)$.

For isotropic σ similar to R.S. Bhalerao et al. PRC 89, 054903 (2014)

- Larger is n larger is the viscous correction to $v_n(p_T)$
- Scaling: for $p_T > 1.5$ GeV $\rightarrow -\delta v_n(p_T)/v_n^{(0)} \propto n$