

# Numerical simulations in high energy nuclear collisions

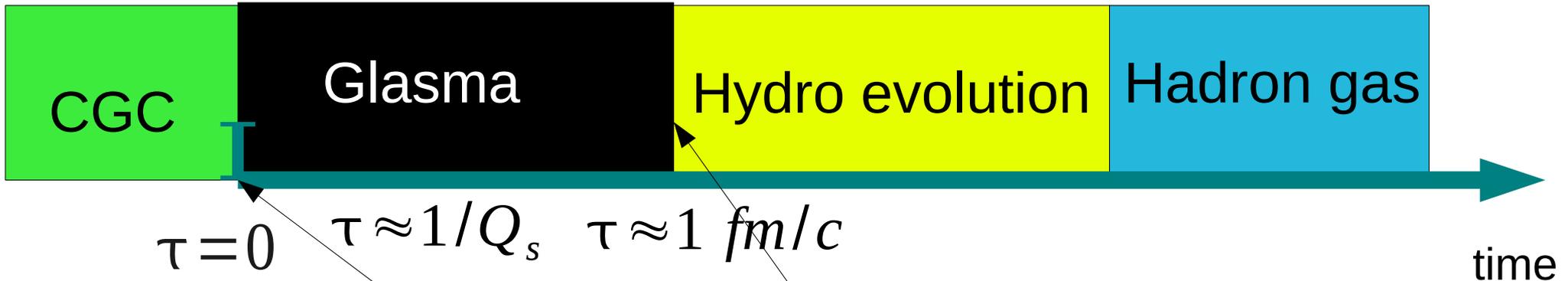
Yasushi Nara (Akita International Univ.)

- Event generators for high energy nuclear collisions
- Recent development for hadronic transport model for phase transition region (with Akira Ohnishi (YITP, Kyoto U.)
- Results for directed flow.

# High energy heavy ion collisions

Initial state interactions

Final state interactions



DGLAP, BFKL evolution  
JIMWLK, BK evolution

Initial condition for hydro

Initial condition for non-equilibrium evolution

KB equation  
Classical YM  
Boltzmann equation

Hydro simulations

Hadronic transport models

Numerical approaches are essential tools for nuclear collisions

# Gluon production based on CGC

- **x-evolution + Solving classical Yang-Mills equation**

CYM + IP-sat model, Schenke, Tribedy, Venugopalan

CYM+JIMWLK evolution, Lappi, Phys.Lett.B703(2011)325

- **rcBK evolution + Based on kt-factorization formula**

Albacete, Armesto, Mihano, Salgado 2009 for HERA fit

$$\frac{dN_g}{d^2 x_t dy} = \frac{4 \pi N_c}{N_c^2 - 1} \int \frac{d^2 p_t}{p_t^2} \int d^2 k_t \alpha_s \varphi(x_1, k_t^2) \varphi(x_2, (p_t - k_t)^2)$$

Forward particle production: Dumitru Hayashigaki Jalilian-Marian (DHJ)

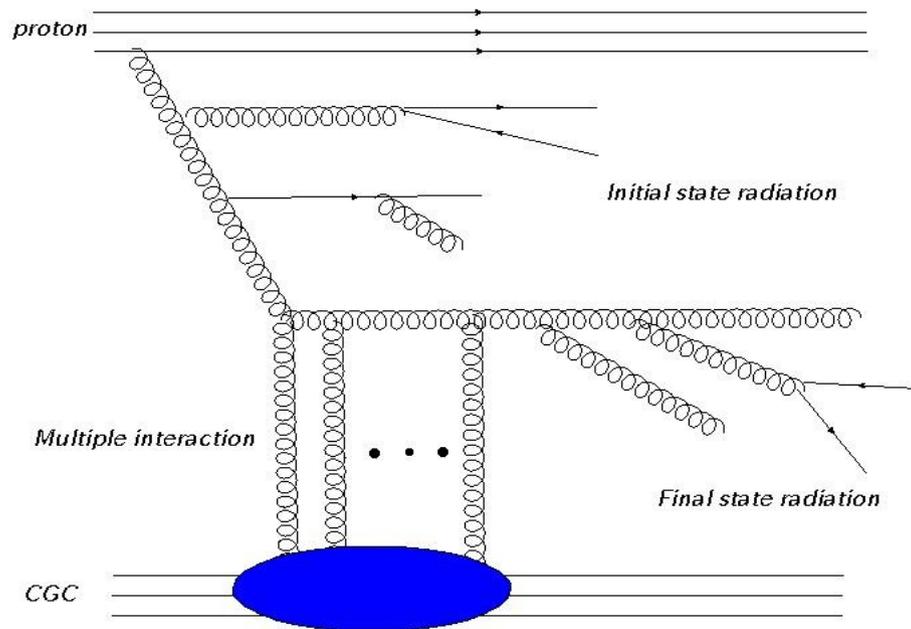
$$\frac{dN}{dy_h d^2 p_\perp} = \frac{K}{(2\pi)^2} \sum_{i=q,g} \int_{x_F}^1 \frac{dz}{z^2} x_1 f_{i/p}(x_1, p_\perp^2) N_i(p_\perp/z, x_2) D_{h/i}(z, p_\perp^2)$$

**RcBK integro- differential equation: 50-170 times faster on GPU**

# Monte-Carlo Event Generator for DHJ approach

Phys.Rev.D91,014006(2015)

$gg \rightarrow g, gq \rightarrow q$  with initial and final state radiations

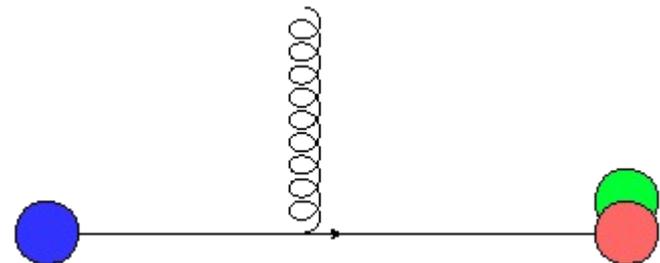


Gluons and quarks are generated according to the DHJ formula.

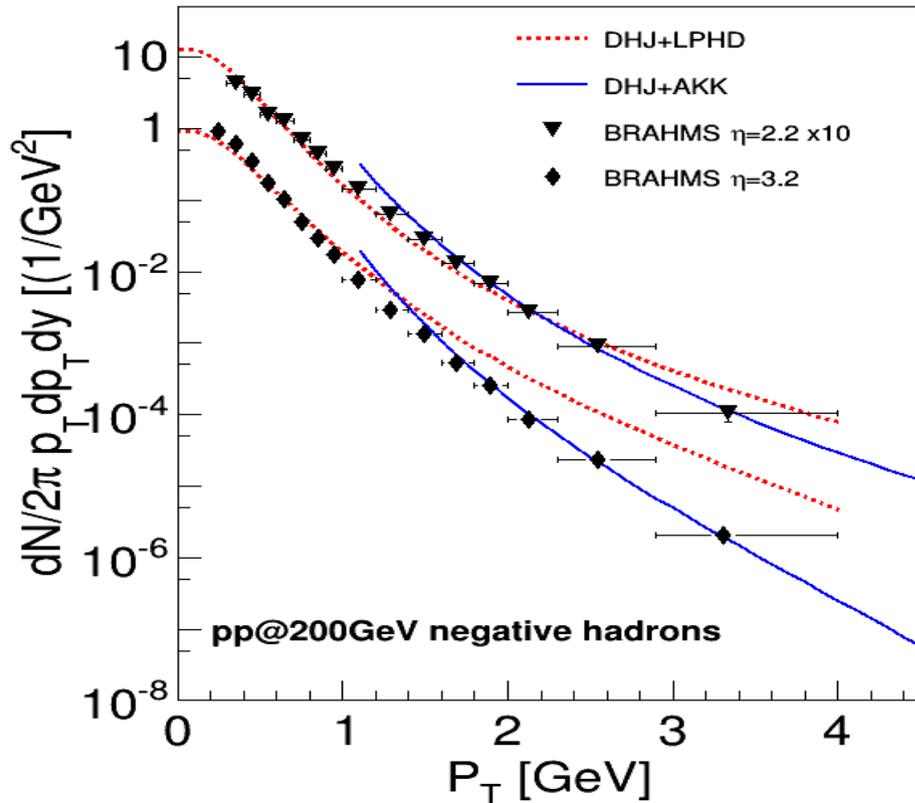
$$\frac{dN}{dyd^2p_{\perp}} = \frac{K}{(2\pi)^2} f_{i/p}(x_1, p_{\perp}^2) N_i(p_{\perp}, x_2)$$

Hadrons are produced by the Lund string fragmentation model

How do you simulate x-evolution in MC?  
SMALLX (CCFM), RAPGAP  
CASCADE (CCFM evolution)  
LDC(Linked Dipole Chain), DIPSY

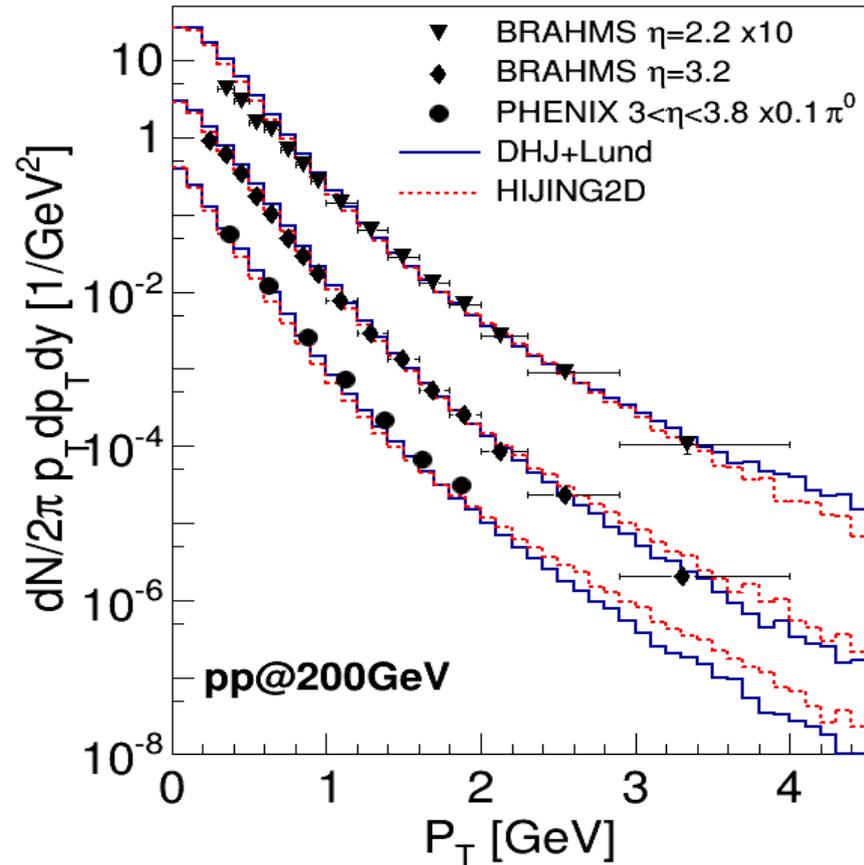


# DHJ+LPHD or FF v.s. MC-DHJ



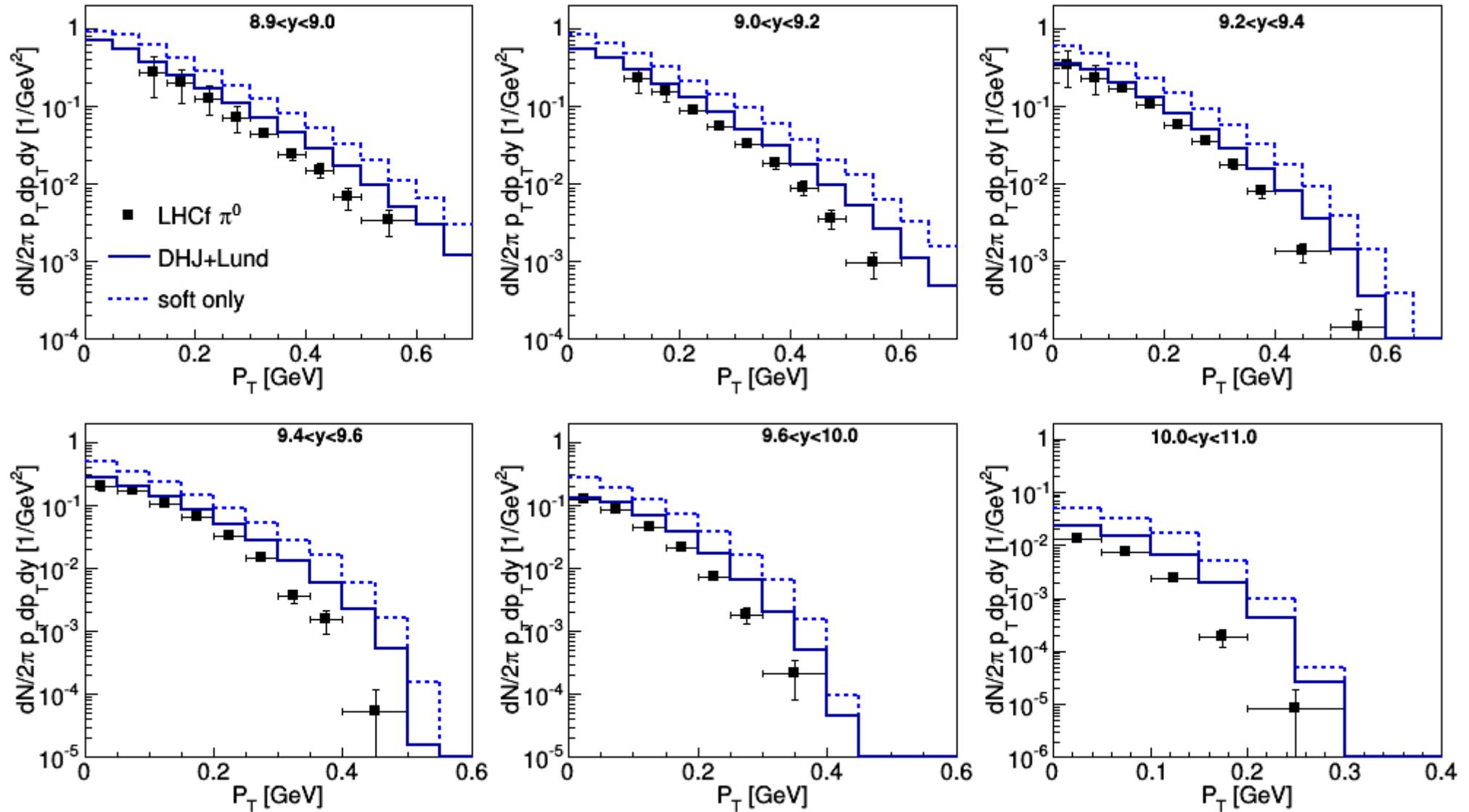
Within kt-factorization approach,  
High pt hadrons are well  
described by  
the [Fragmentation function](#),

Low pt hadrons including  
multiplicity are well described  
by the [parton-hadron duality](#).



More realistic model:  
event generator version of DHJ  
describes the data in a unified way.

# Comparison of LHCf data



# Event generators in high energy physics

HepForge: development environment for high energy physics software  
<https://www.hepforge.org/>

PYTHIA, HERWIG++, SHERPA and so on

OSCAR: Open Standard Codes and Routines  
[https://karman.physics.purdue.edu/OSCAR/index.php/Main\\_Page](https://karman.physics.purdue.edu/OSCAR/index.php/Main_Page)

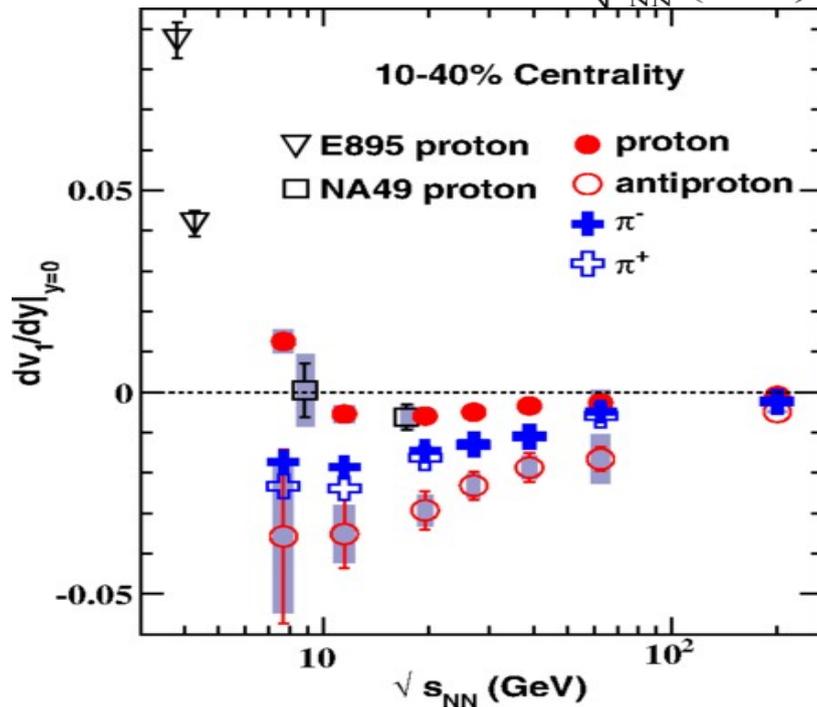
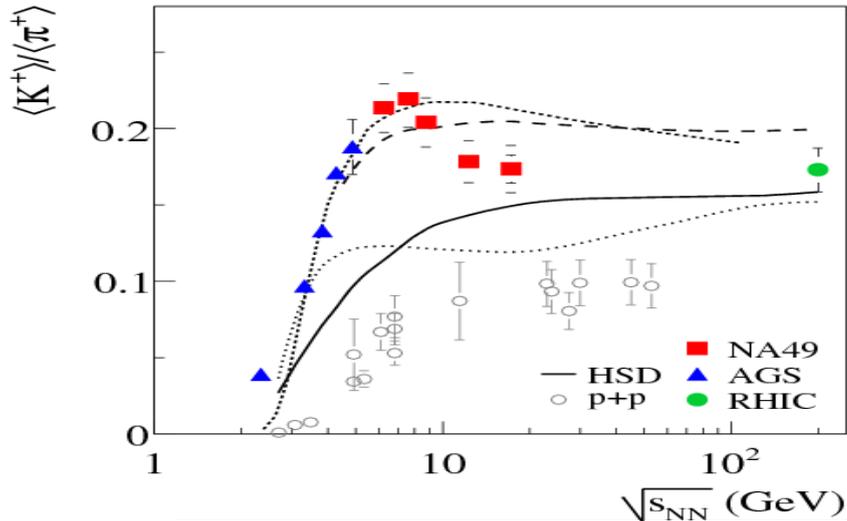
Event generators mainly for cosmic ray physics

DPMJET 3  
QGSJET II  
SIBYLL 2.1  
EPOS3

# Microscopic transport models (event generator for nuclear collisions)

- **UrQMD 3.4** Frankfurt **public**  
resonance model N\*,D\*, string pQCD, PYTHIA6.4
- **PHSD** Giessen (Cassing) **upon request**  
D(1232),N(1440),N(1530), string, pQCD, FRITIOF7.02
- **GiBUU 1.6** Giessen (Mosel) **public**  
resonance model N\*,D\*, string, pQCD,PYTHIA6.4
- **AMPT** **public**  
HIJING+ZPC+ART
- **JAM** **public**  
resonance model N\*,D\*, string, pQCD, PYTHIA6.1

# Search for phase transition

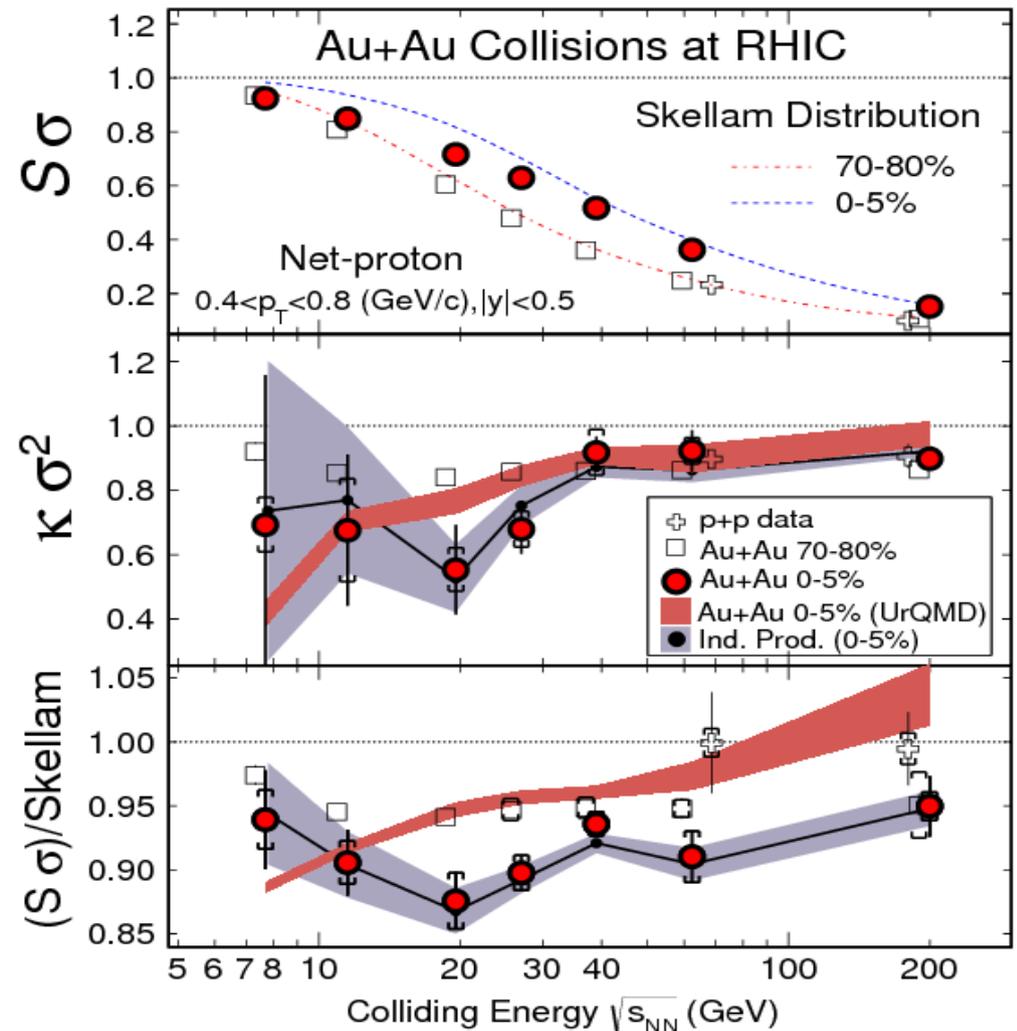


RHIC beam scan, FAIR, NICA, J-PARC

STAR, PRL 112, 032302 (2014)

$$\sigma = \langle \delta N^2 \rangle, S\sigma = \langle \delta N^3 \rangle / \sigma^2, K\sigma^2 = \langle \delta N^4 \rangle / \sigma^2 - 3\sigma^2$$

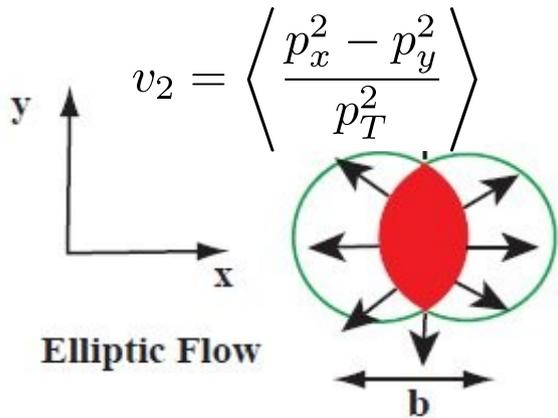
Event-by-event fluctuations



# Determination of EOS at high density from an anisotropic flow in heavy ion collisions

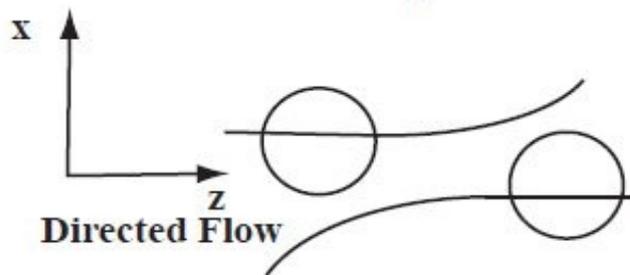
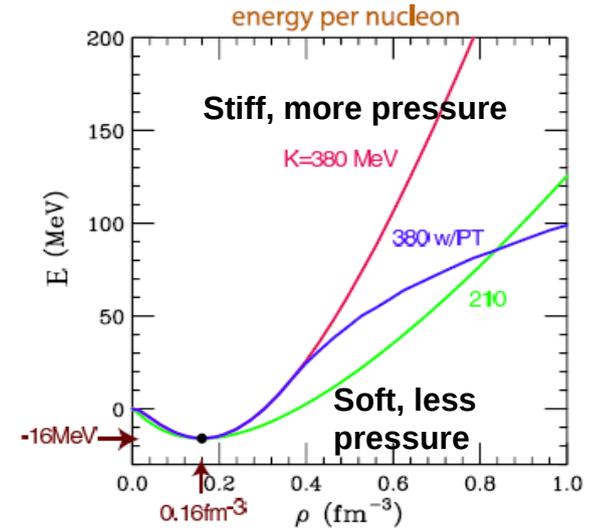
Fourier decomposition of single particle inclusive spectra:

$$\frac{dN}{d^2p_T} = \frac{d^2N}{2\pi dp_T dy} (1 + 2v_1 \cos(\phi) + 2v_2 \cos(2\phi) + \dots)$$



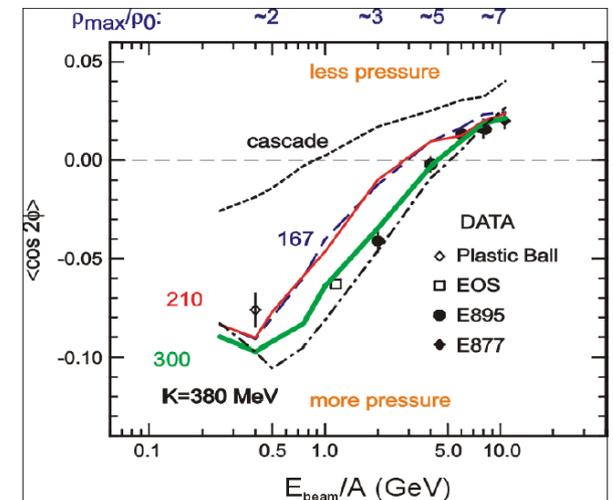
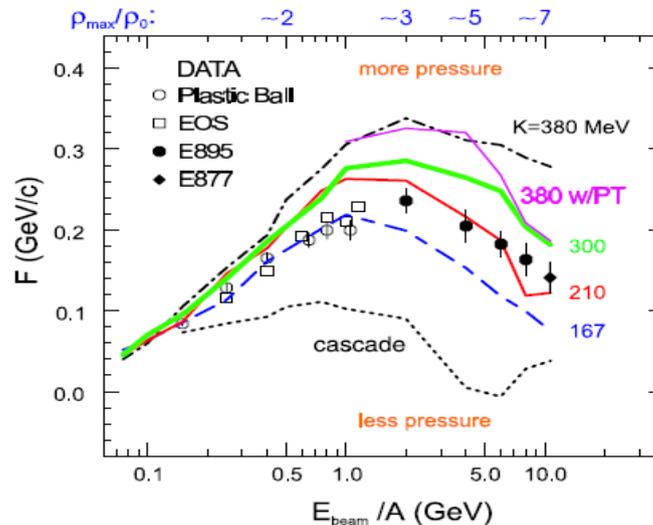
$$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_T^2} \right\rangle \quad v_1 = \left\langle \frac{p_x}{p_T} \right\rangle \quad F = \left. \frac{dv_1}{dy} \right|_{y=0}$$

P. Danielewicz, R. Lacey, W.G. Lynch,  
Science 298 (2002) 1592



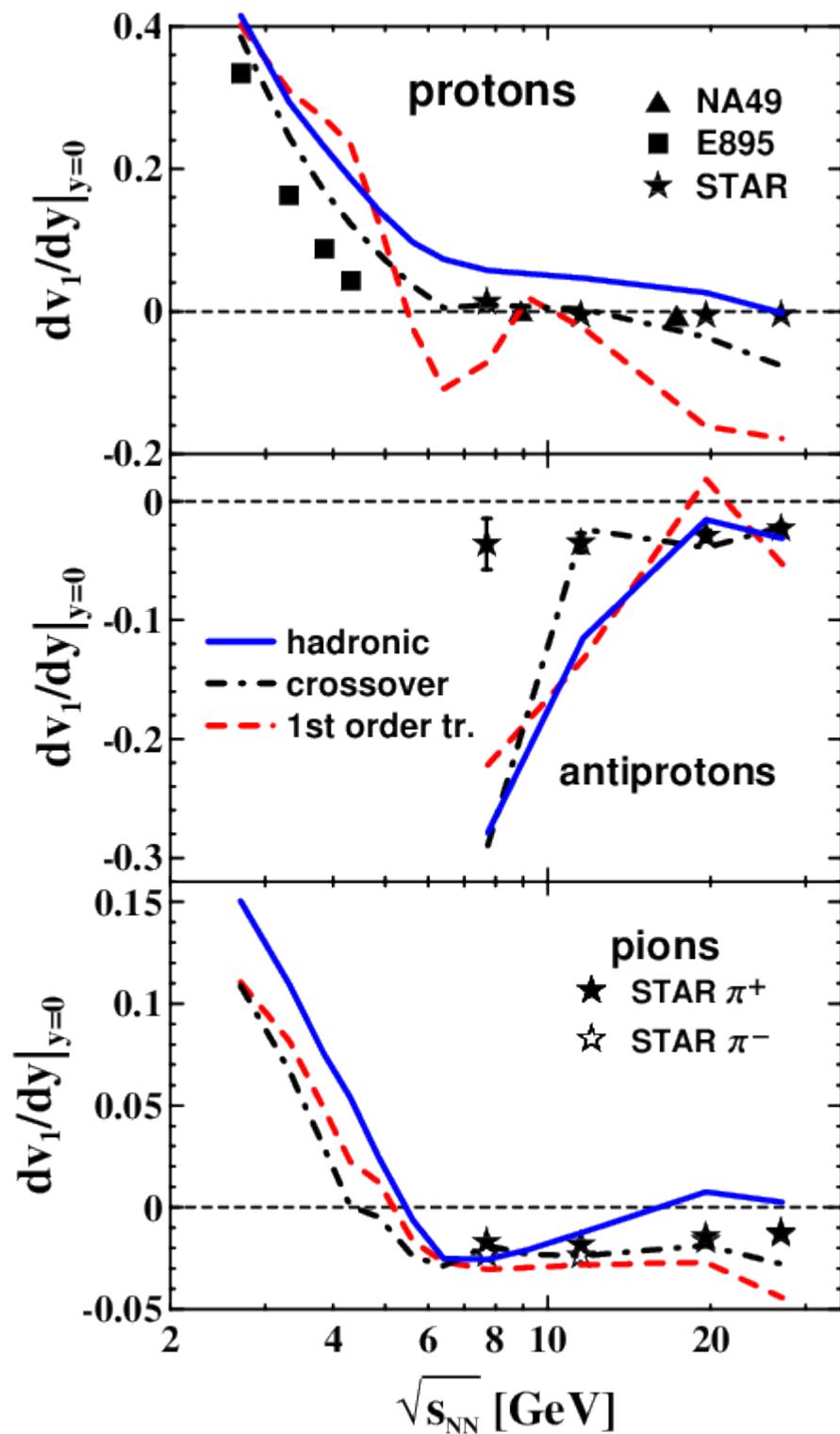
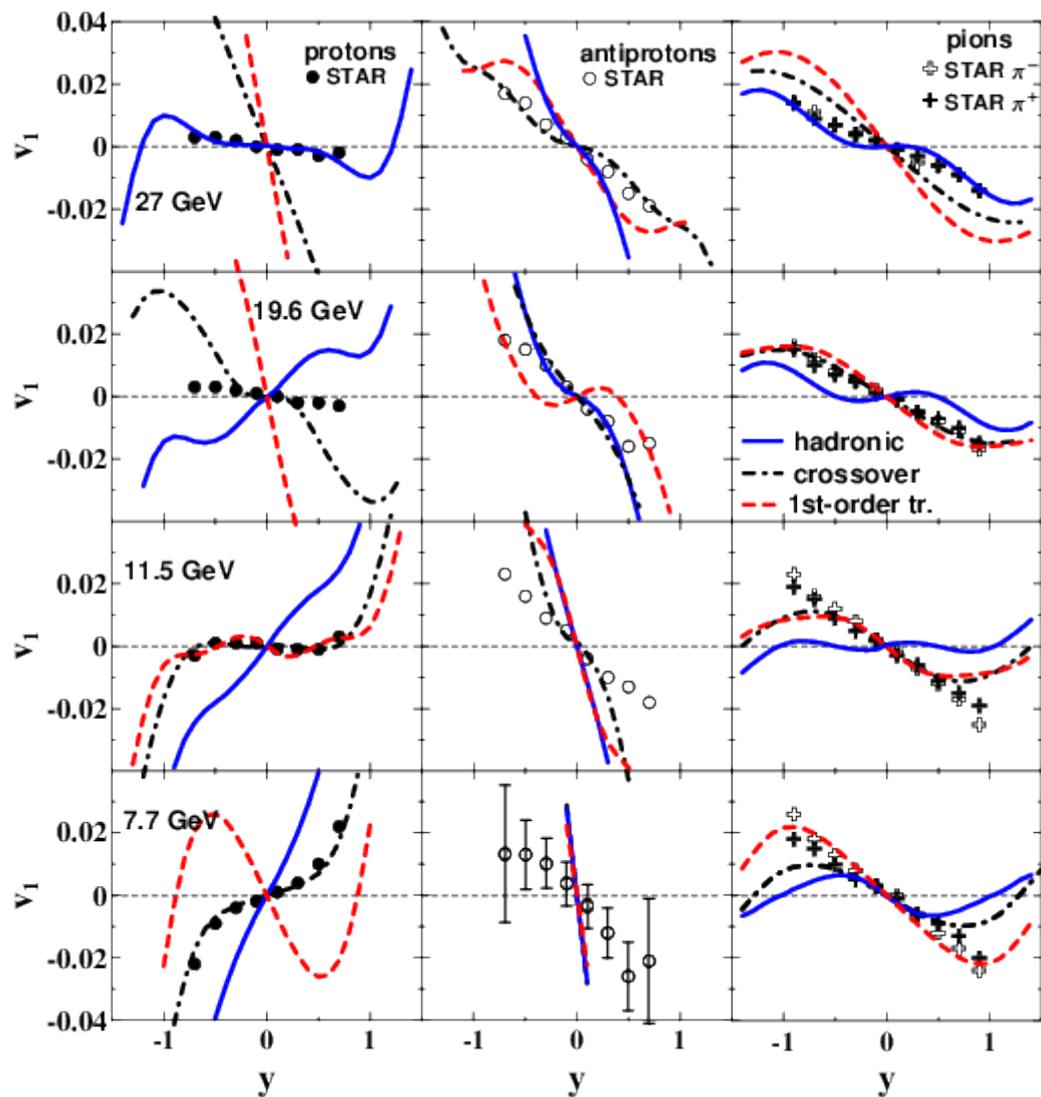
$$F = \left. \frac{\langle p_x/A \rangle}{d(y/y_{cm})} \right|_{y/y_{cm}=1}$$

In-plane flow,  $v_1$



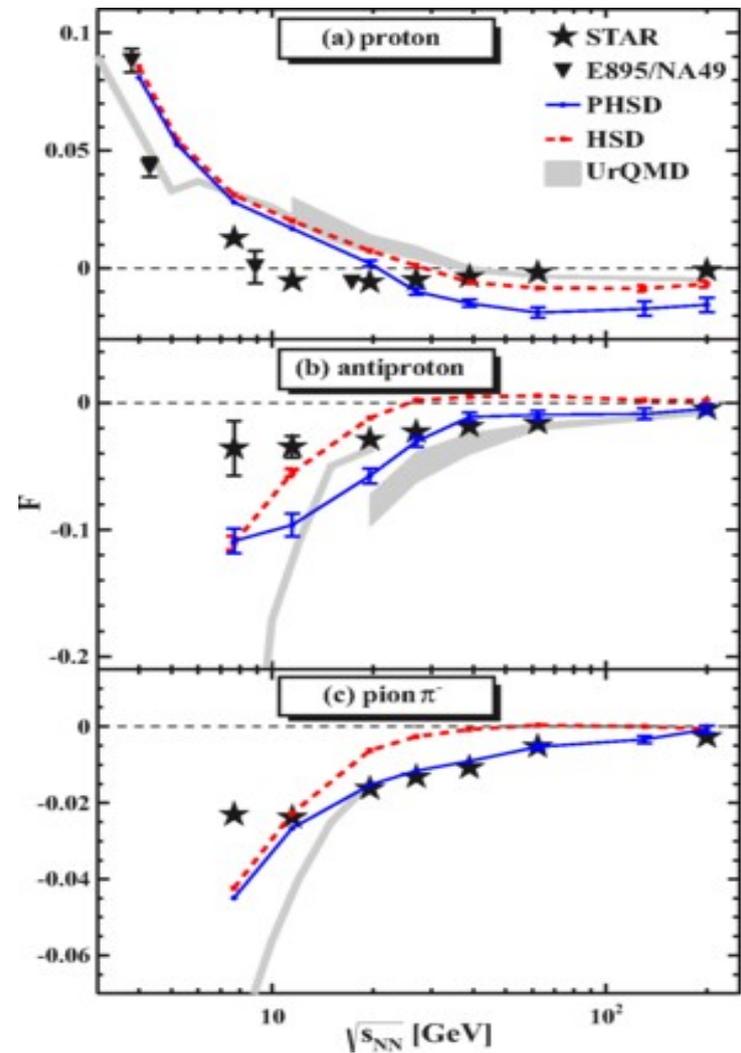
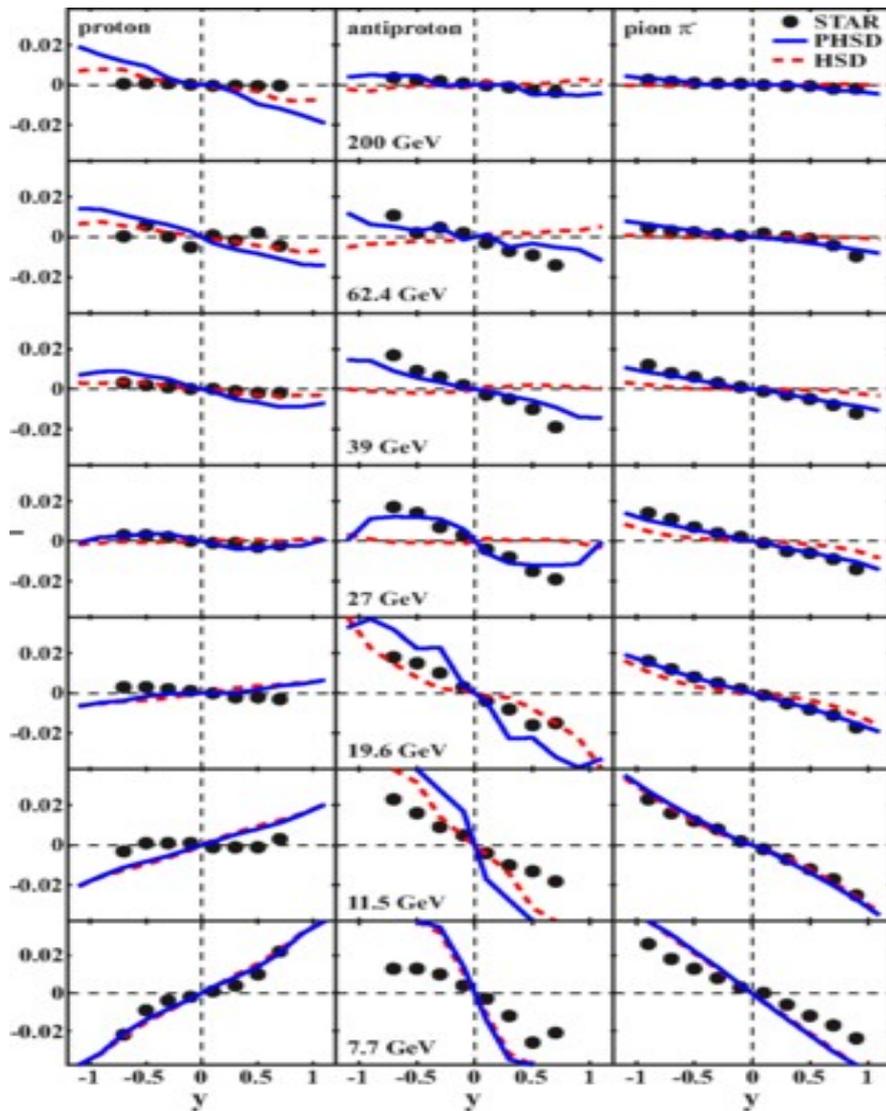
# V1 from hydrodynamics

Y. B. Ivanov and A. A. Soldatov, Phys. Rev. C91, no. 2, 024915 (2015)

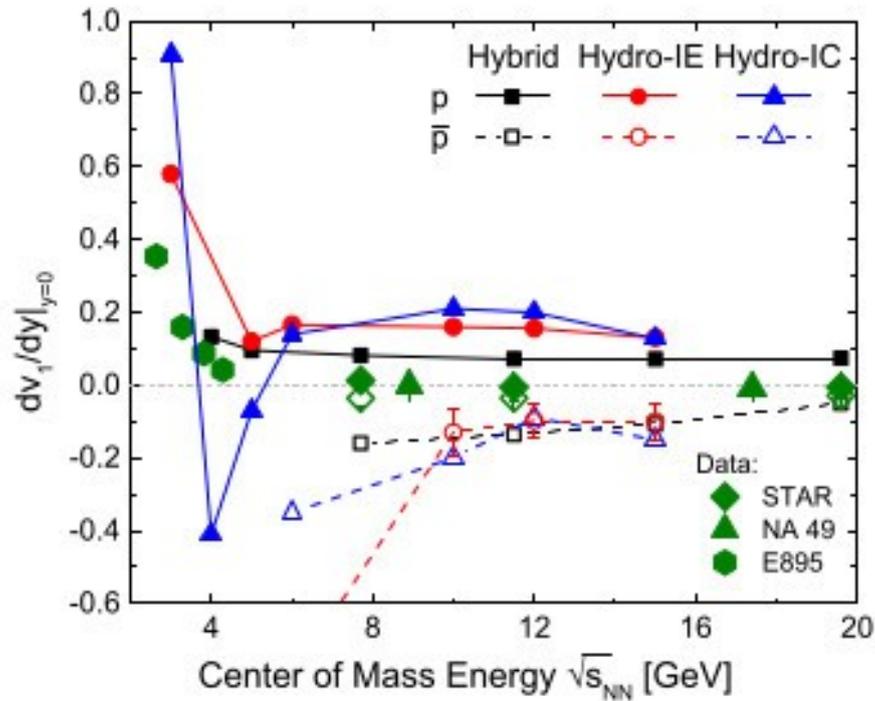


# PHSD/HSD predictions

V. P. Konchakovski, W. Cassing, Y. B. Ivanov and V. D. Toneev,  
*Phys. Rev. C*90, no. 1, 014903 (2014)



# UrQMD+hydro+UrQMD results



The values of the slopes are always positive.

J. Steinheimer et al. PRC89, 054913(2014)

# Hadronic transport Approach

Purpose: Effects of hadron mean field potential on the directed flow  $v_1$

JAM hadronic cascade model : resonance and string excitation

Mean field by the framework of the Relativistic Quantum Molecular Dynamics

Nuclear cluster formation by phase space coalescence.

Statistical decay of nuclear fragment

# Relativistic QMD/Simplified (RQMD/S)

RQMD based on Constraint Hamiltonian Dynamics

Sorge, Stoecker, Greiner, Ann. Phys. 192 (1989), 266.

RQMD/S: Tomoyuki Maruyama, et al. Prog. Theor. Phys. 96(1996),263.

Single particle energy:  $p_i^0 = \sqrt{\mathbf{p}_i^2 + m_i^2 + 2m_i V_i}$

$$\dot{\mathbf{r}}_i = \frac{\mathbf{p}_i}{p_i^0} + \sum_j \frac{m_j}{p_j^0} \frac{\partial V_j}{\partial \mathbf{p}_i} \quad \dot{\mathbf{p}}_i = - \sum_j \frac{m_j}{p_j^0} \frac{\partial V_j}{\partial \mathbf{r}_i}$$

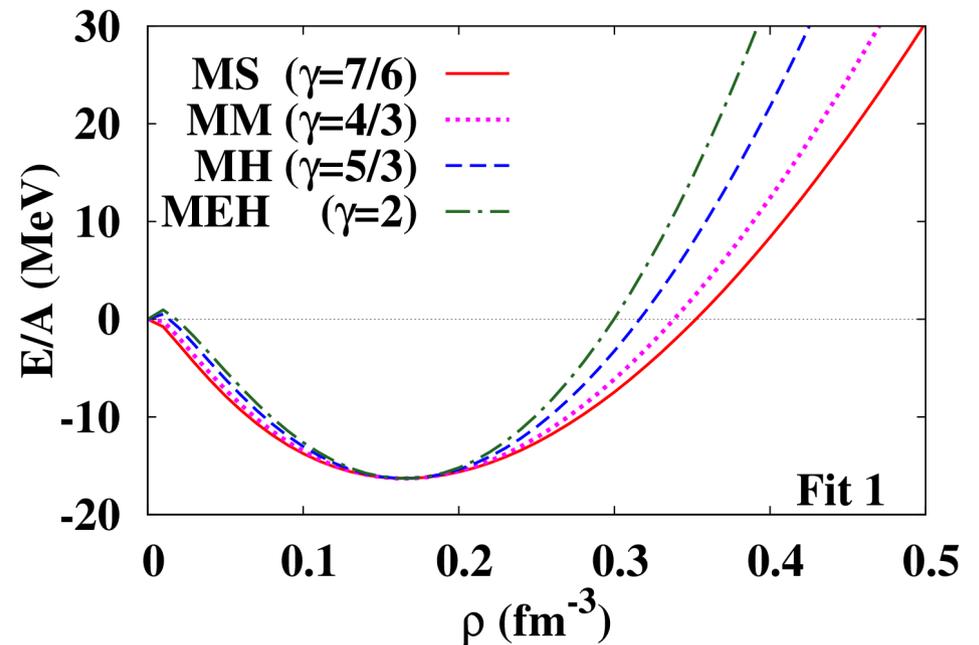
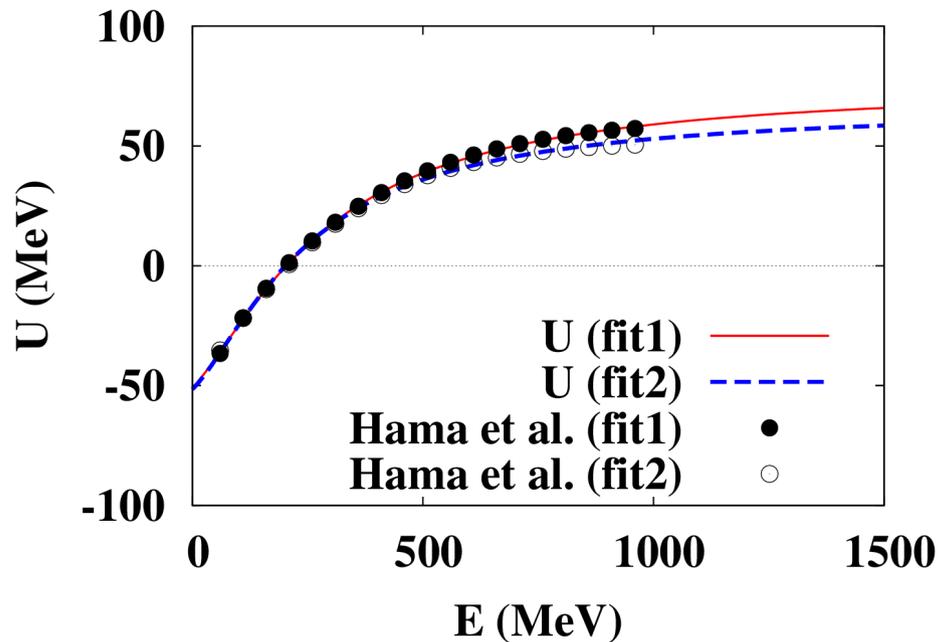
Arguments of potential  $\mathbf{r}_i - \mathbf{r}_j$  and  $\mathbf{p}_i - \mathbf{p}_j$  are replaced by the distances in the two-body c.m.

# Mean field potential

Skyrme type density dependent + Lorentzian momentum dependent potential

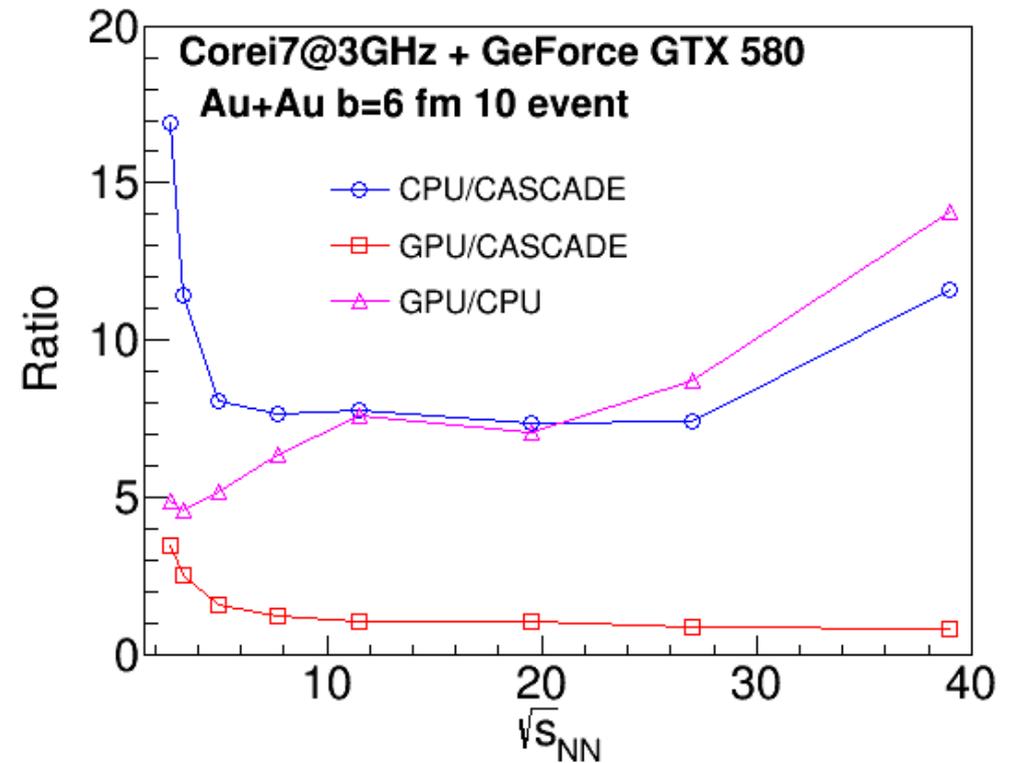
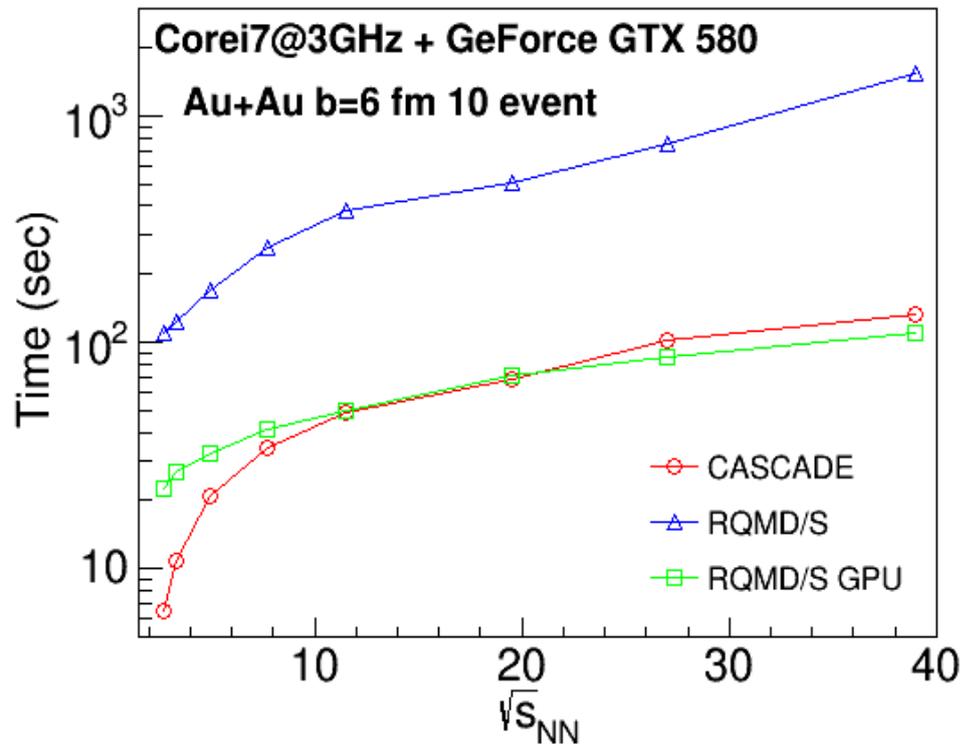
$$V = \sum_i V_i = \int d^3r \left[ \frac{\alpha}{2} \left( \frac{\rho}{\rho_0} \right)^2 + \frac{\beta}{\gamma+1} \left( \frac{\rho}{\rho_0} \right)^{\gamma+1} \right] + \sum_k \int d^3r d^3p d^3p' \frac{C_{ex}^{(k)}}{2\rho_0} \frac{f(\mathbf{r}, \mathbf{p}) f(\mathbf{r}, \mathbf{p}')}{1 + (\mathbf{p} - \mathbf{p}')^2 / \mu_k^2}$$

| Type | $\alpha$<br>(MeV) | $\beta$<br>(MeV) | $\gamma$ | $C_{ex}^{(1)}$<br>(MeV) | $C_{ex}^{(2)}$<br>(MeV) | $\mu_1$<br>(fm <sup>-1</sup> ) | $\mu_2$<br>(fm <sup>-1</sup> ) | $K$<br>(MeV) |
|------|-------------------|------------------|----------|-------------------------|-------------------------|--------------------------------|--------------------------------|--------------|
| MH1  | -12.25            | 87.40            | 5/3      | -383.14                 | 337.41                  | 2.02                           | 1.0                            | 371.92       |
| MS1  | -208.89           | 284.04           | 7/6      | -383.14                 | 337.41                  | 2.02                           | 1.0                            | 272.6        |



# CUDA implementation

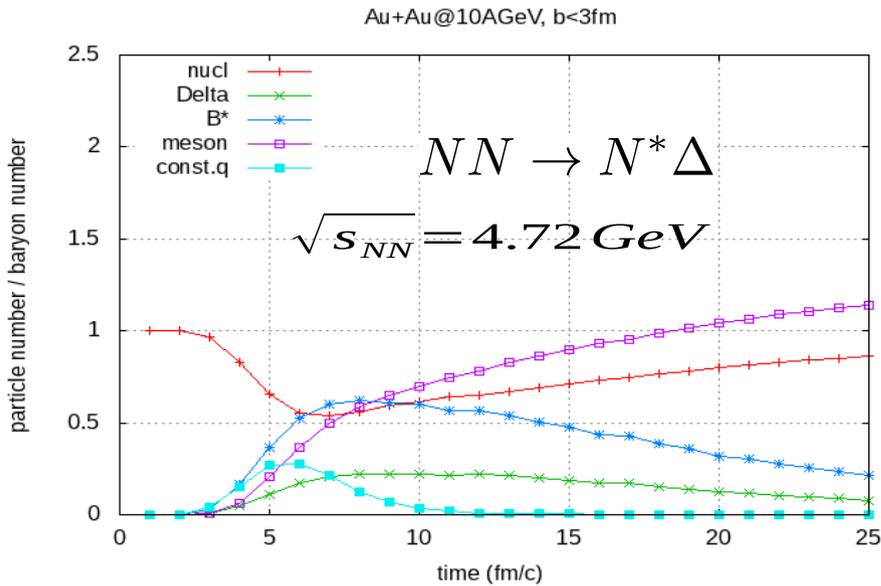
Execution time: RQMD/S = CASCADE on GPU



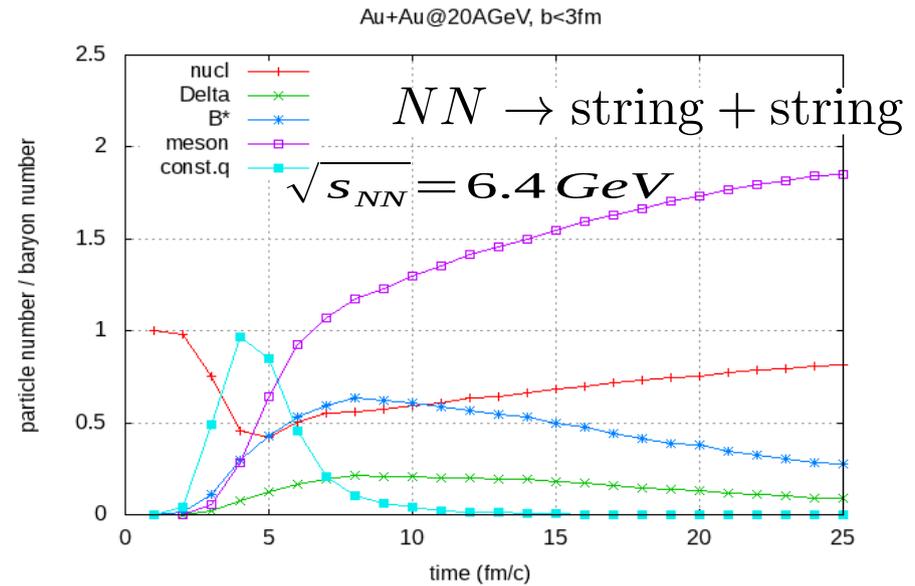
5-14 times faster with GPU

# How to treat mean-field for excited matter?

Hadronic resonance dominant



constituent quark dominant due to string

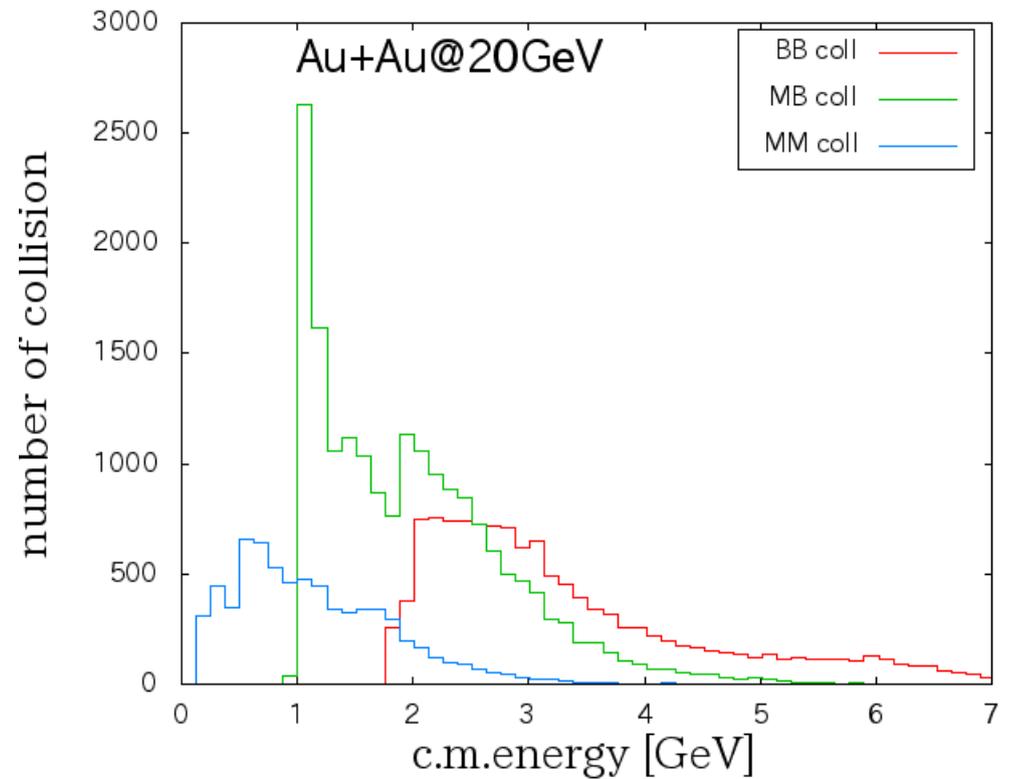
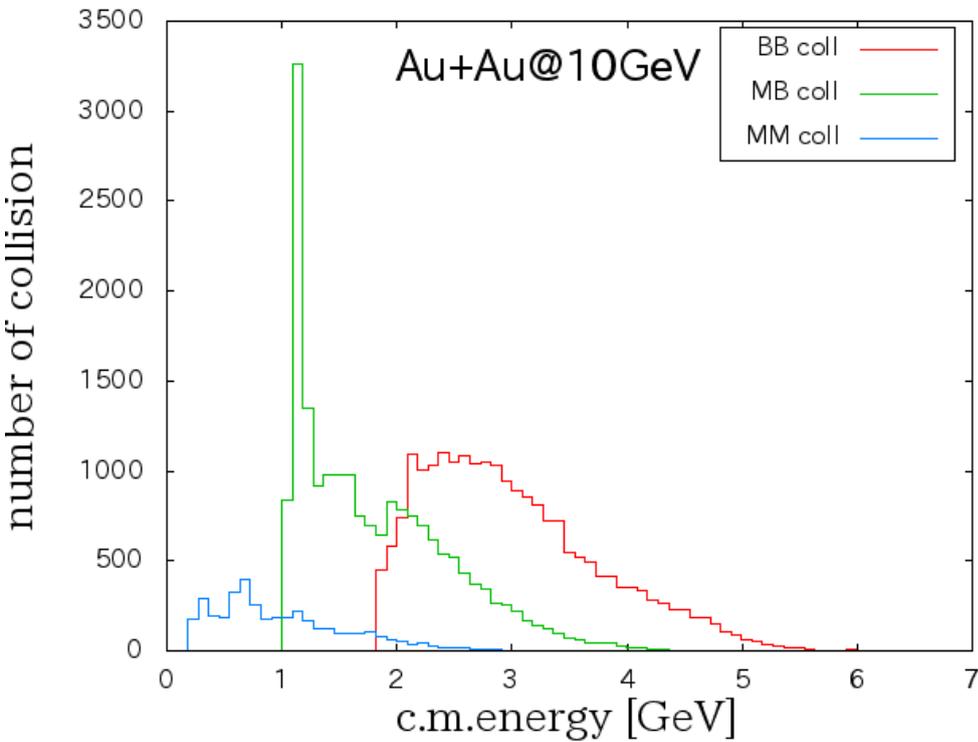


Model 1 JAM/M: potential for all formed baryon

Model 2 JAM/Mq: potentials for quarks inside the pre-formed baryon

Model 3: JAM/Mf: both formed and pre-formed baryons

# Collision spectrum

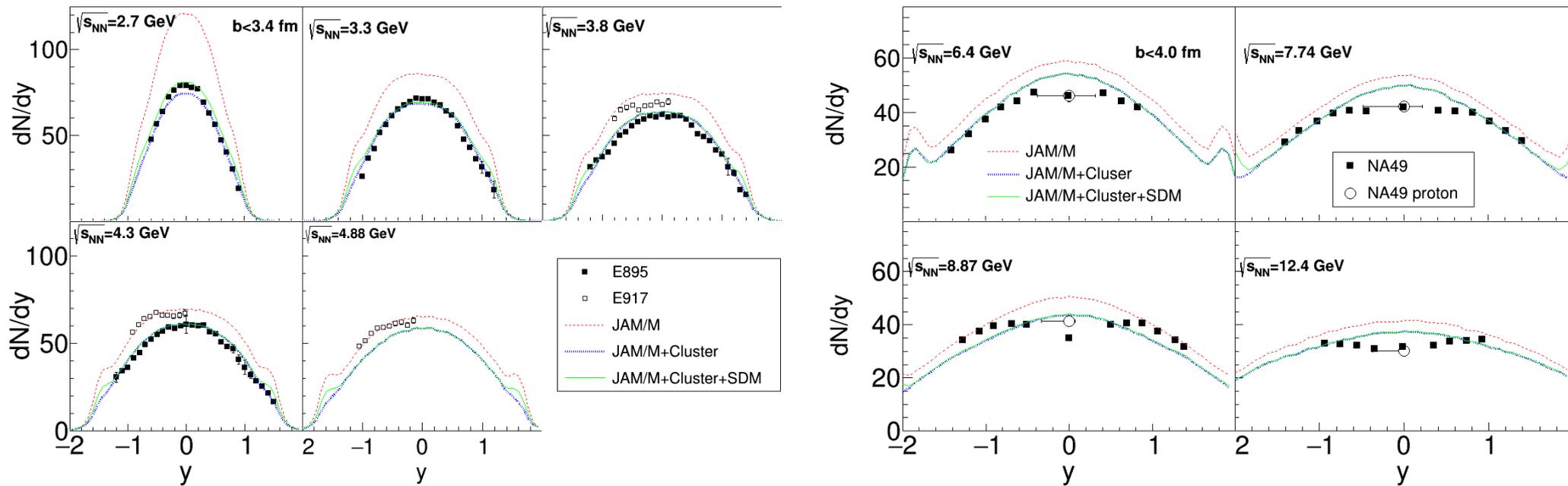


Re-scattering among produced particle is very important

# Proton rapidity distributions

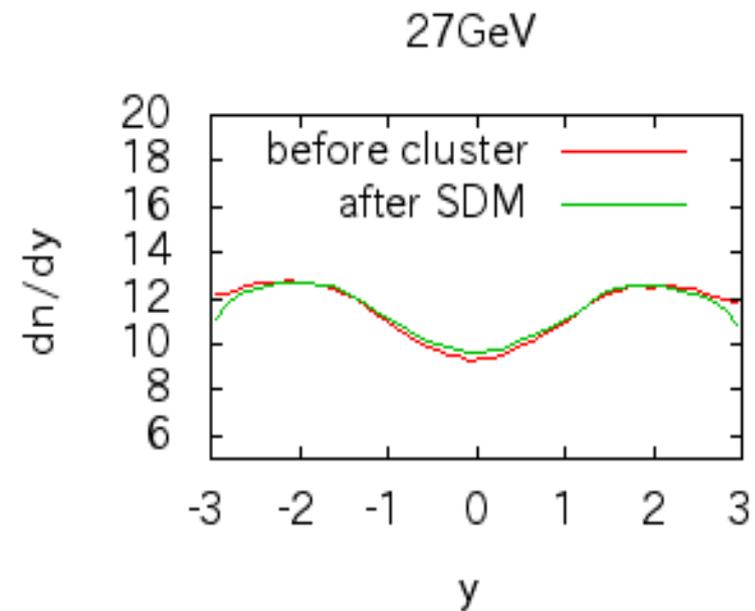
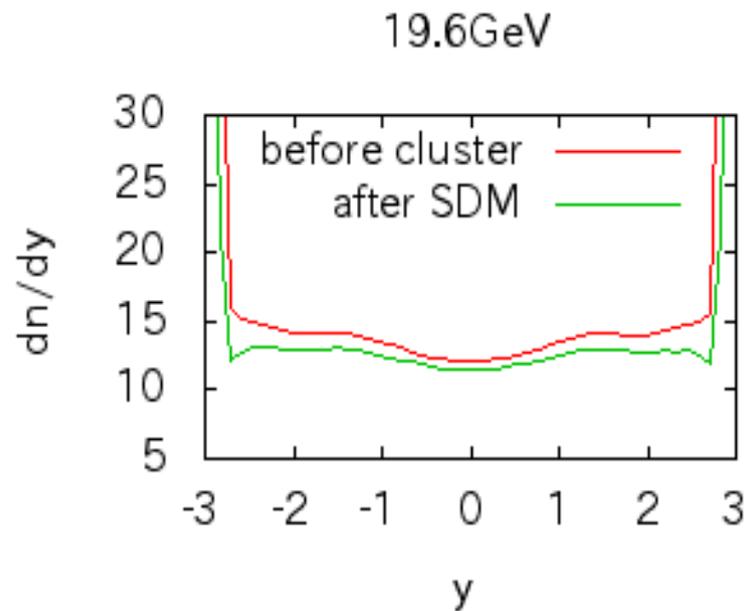
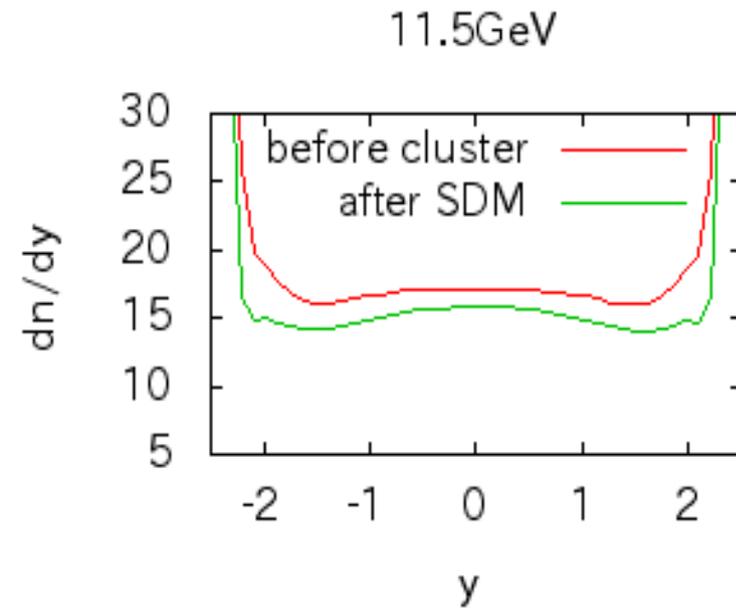
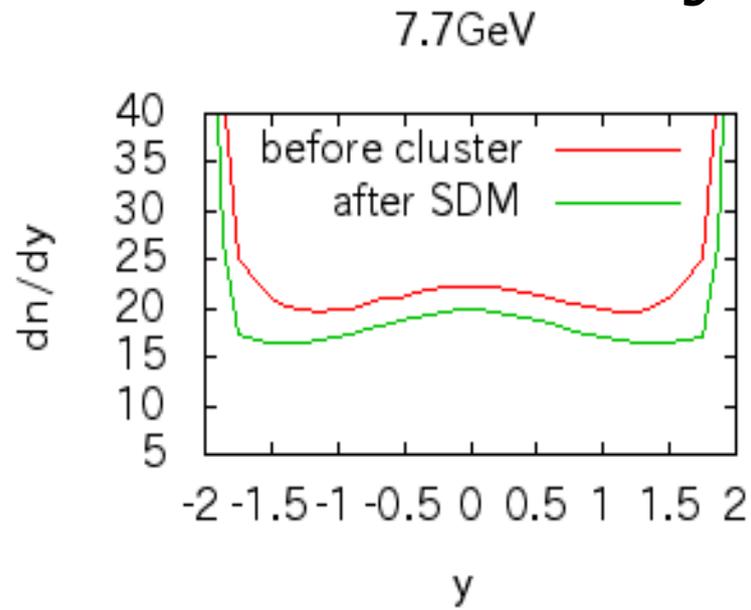
Effect of nuclear clustering on the proton distribution was first pointed out by Q. Li, Y. Wang, X. Wnag, C Shen and M. Bleicher, hep-ph 1507.06033. within the UrQMD model.

Coalescence parameter  $R_0=4\text{fm}$ ,  $P_0=0.3\text{ GeV}/c$

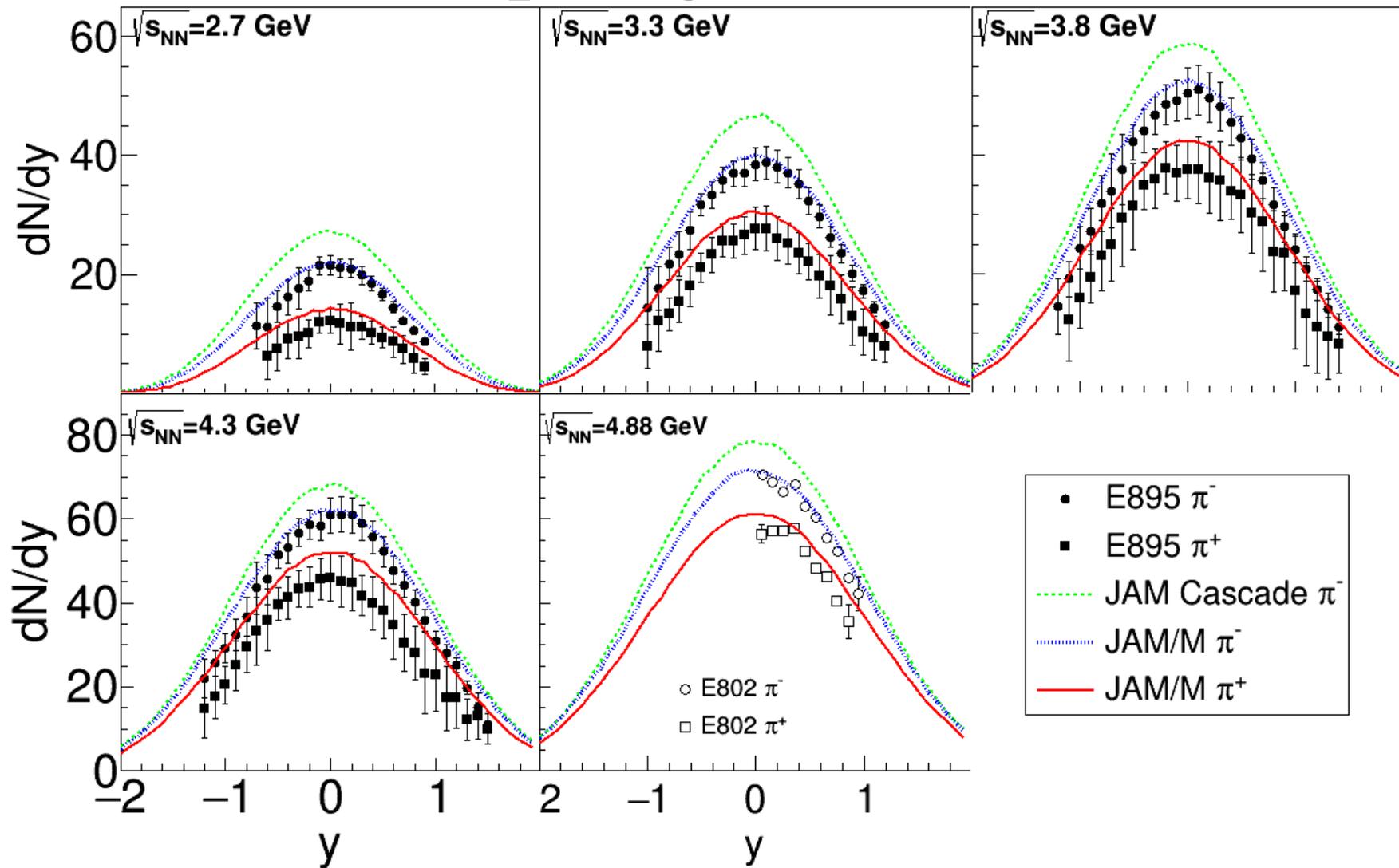


Cluster formation reduces proton  $dN/dy$  by around 20%.  
Statistical decay of nuclear cluster is important only at  $E_{\text{lab}} = 2\text{ AGeV}$  for the proton rapidity distribution.

# Proton $dN/dy$ for semi-central

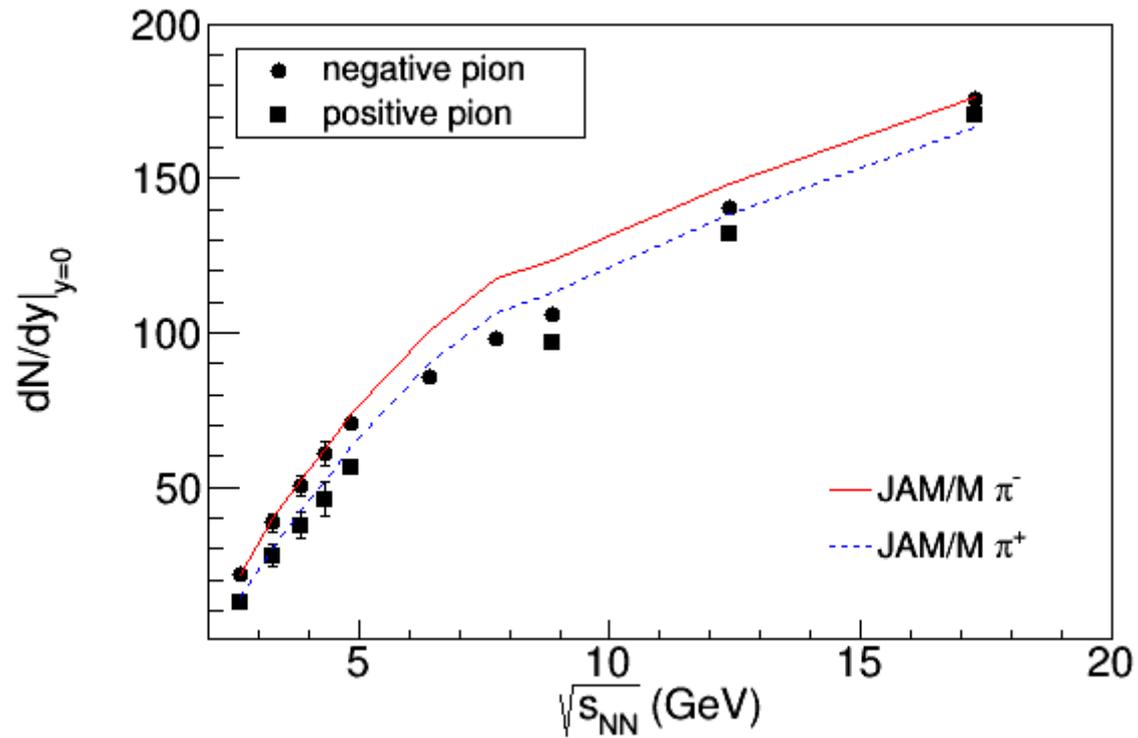


# Pion rapidity distributions

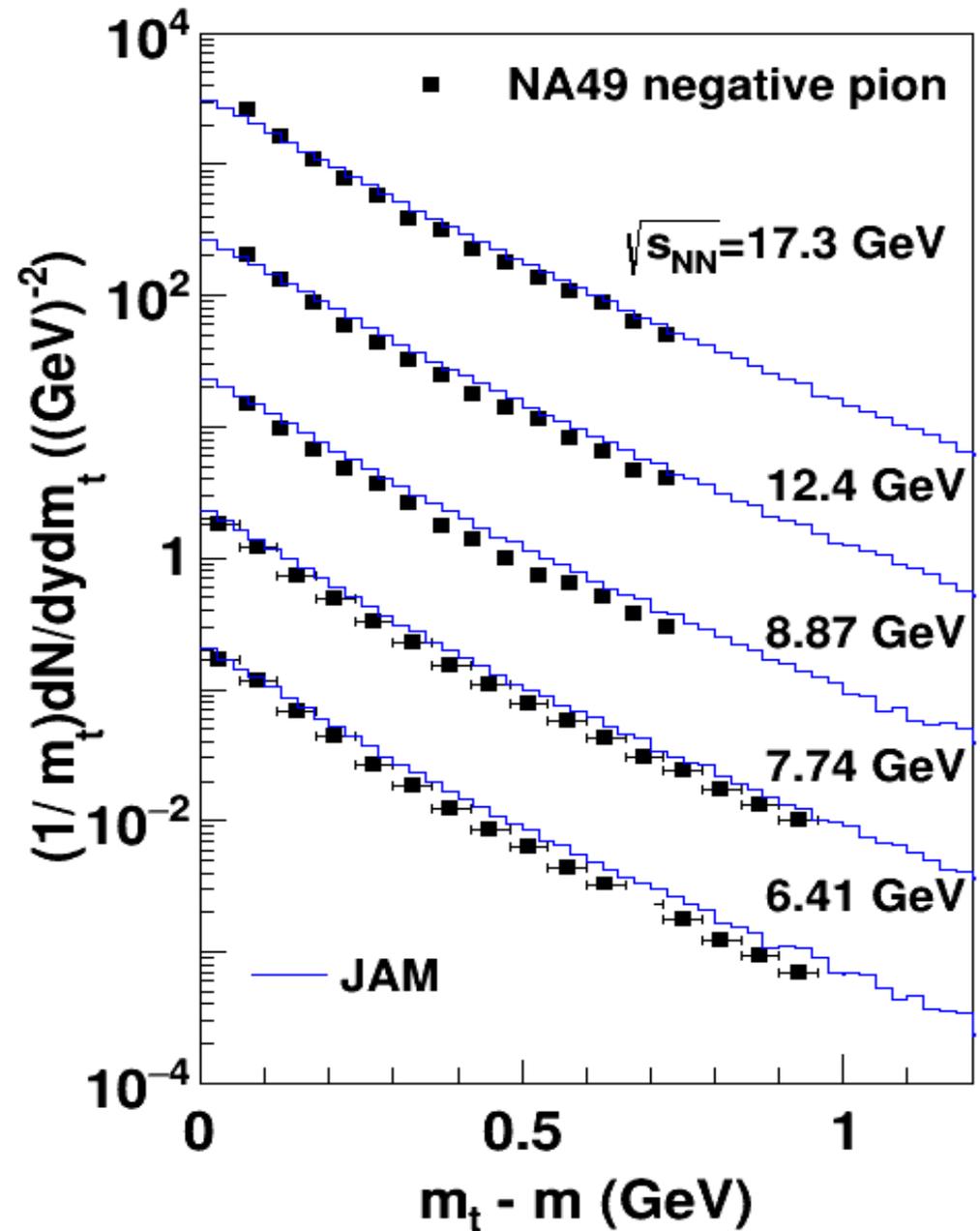
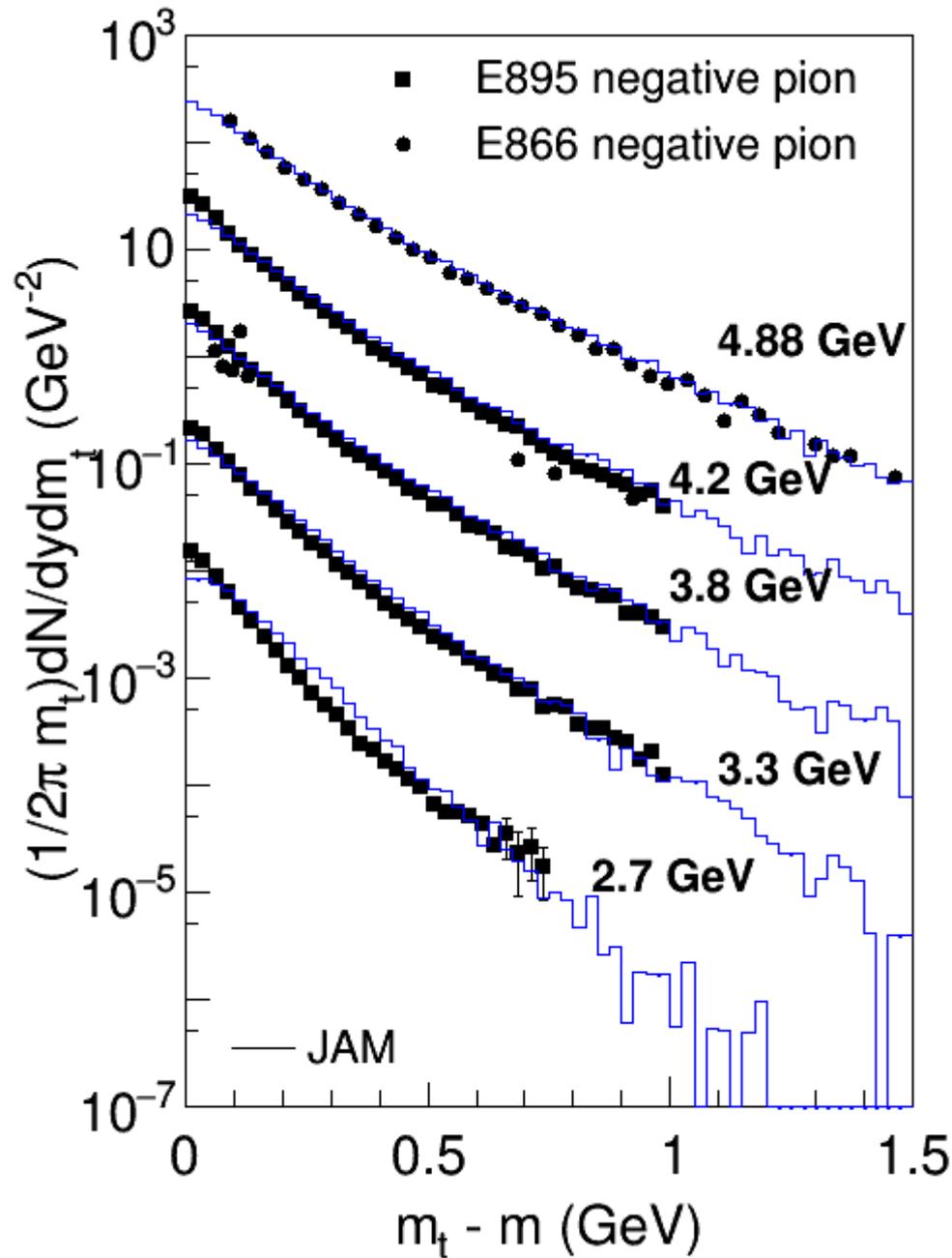


Nuclear mean field reduces the pion yield.

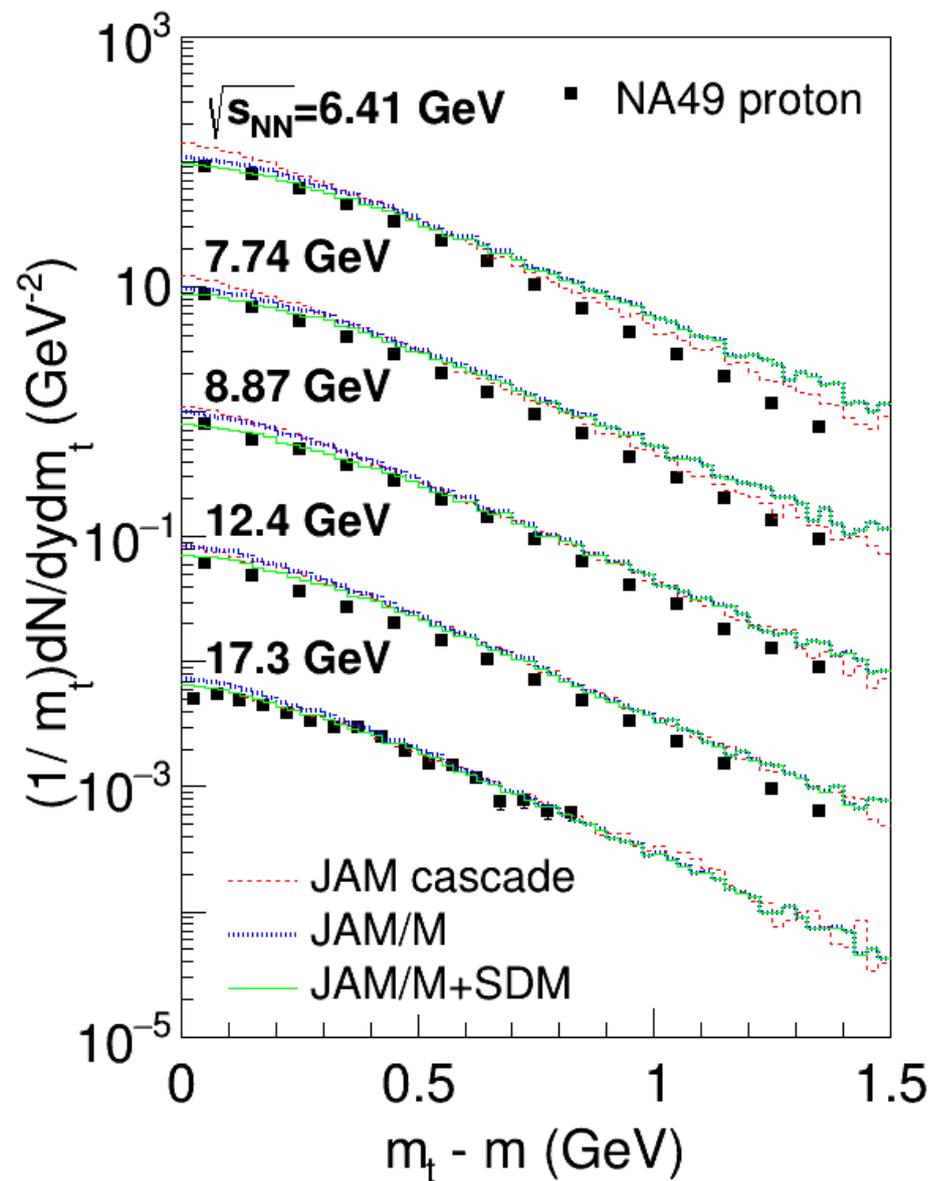
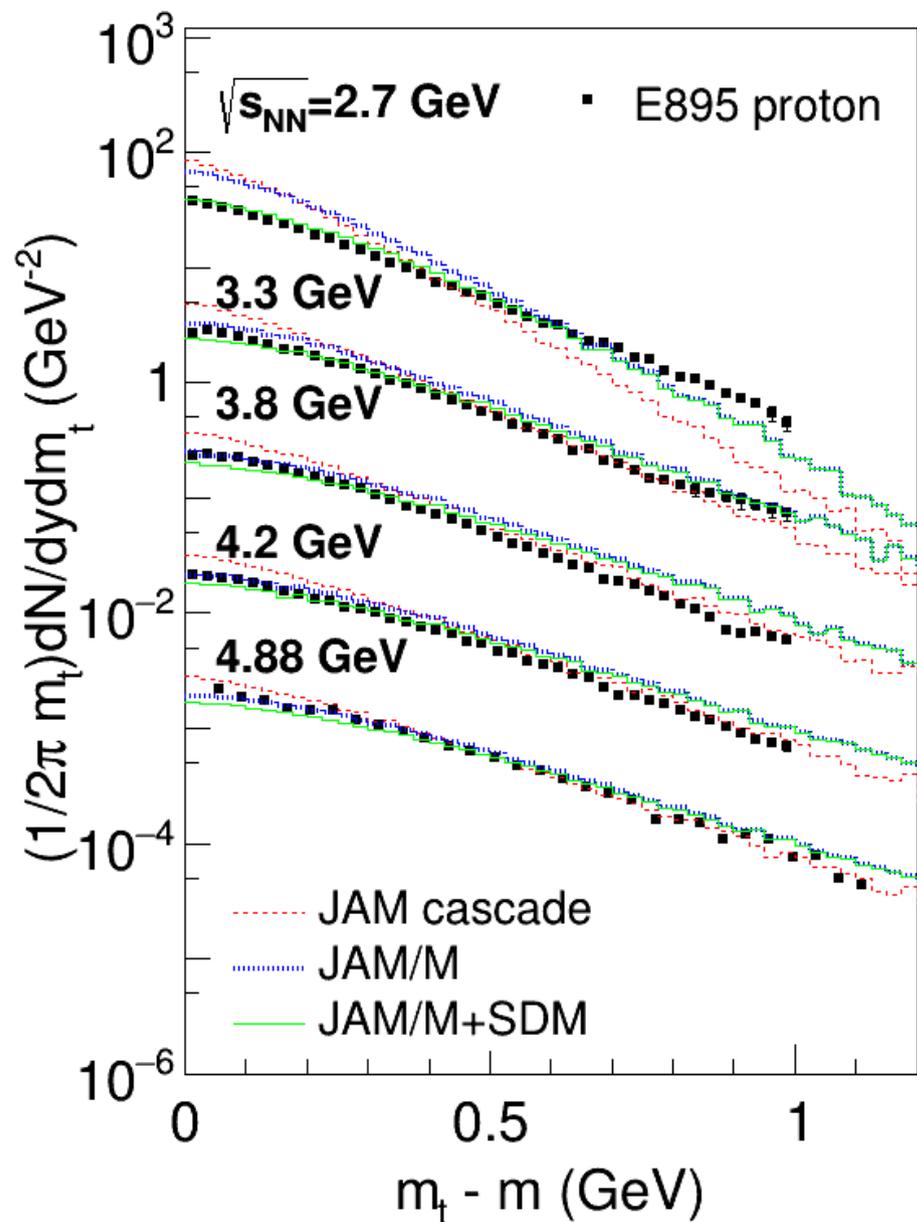
# Pion yield at mid-rapidity



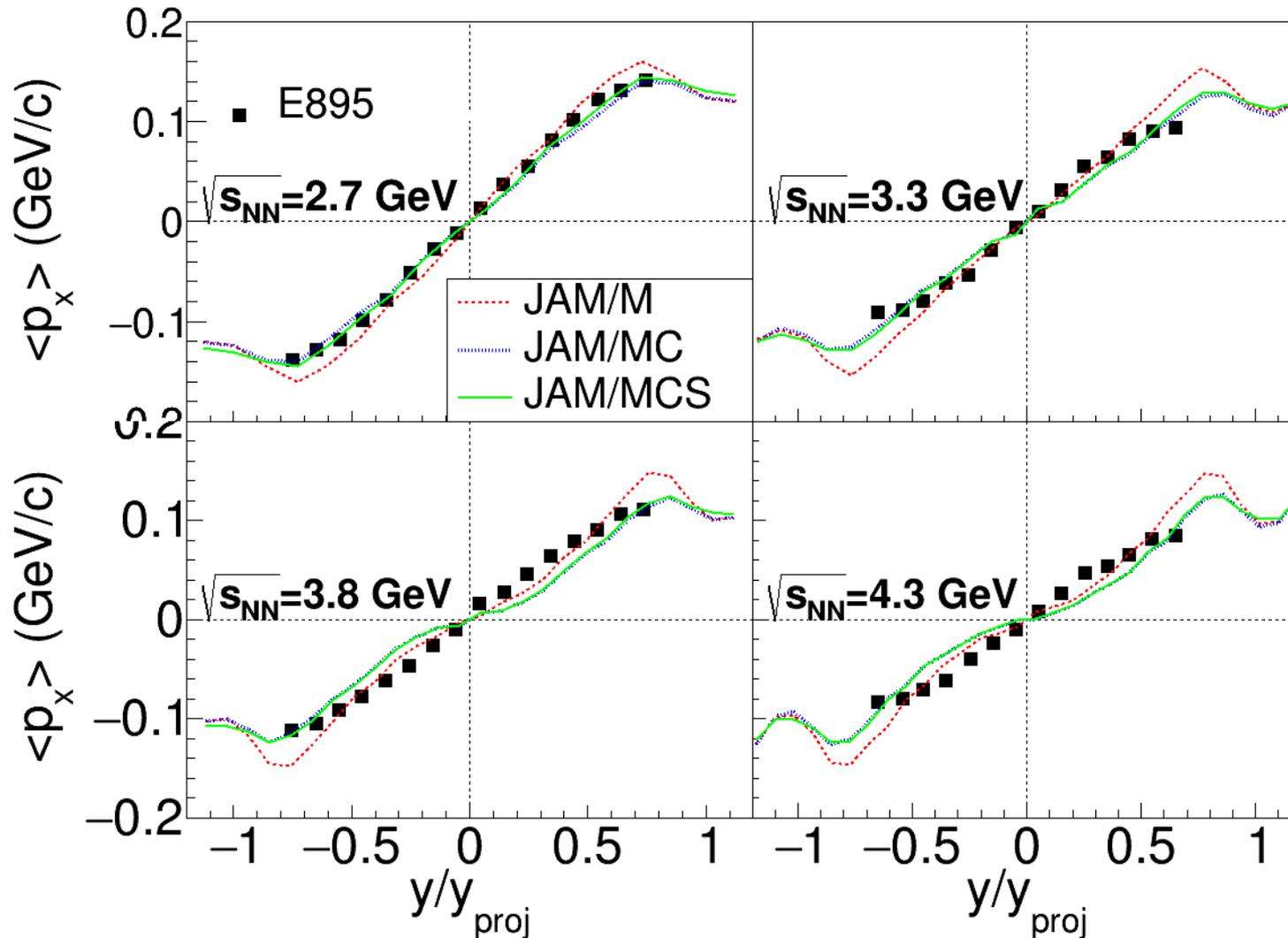
# Pion transverse mas distribution



# Proton distributions



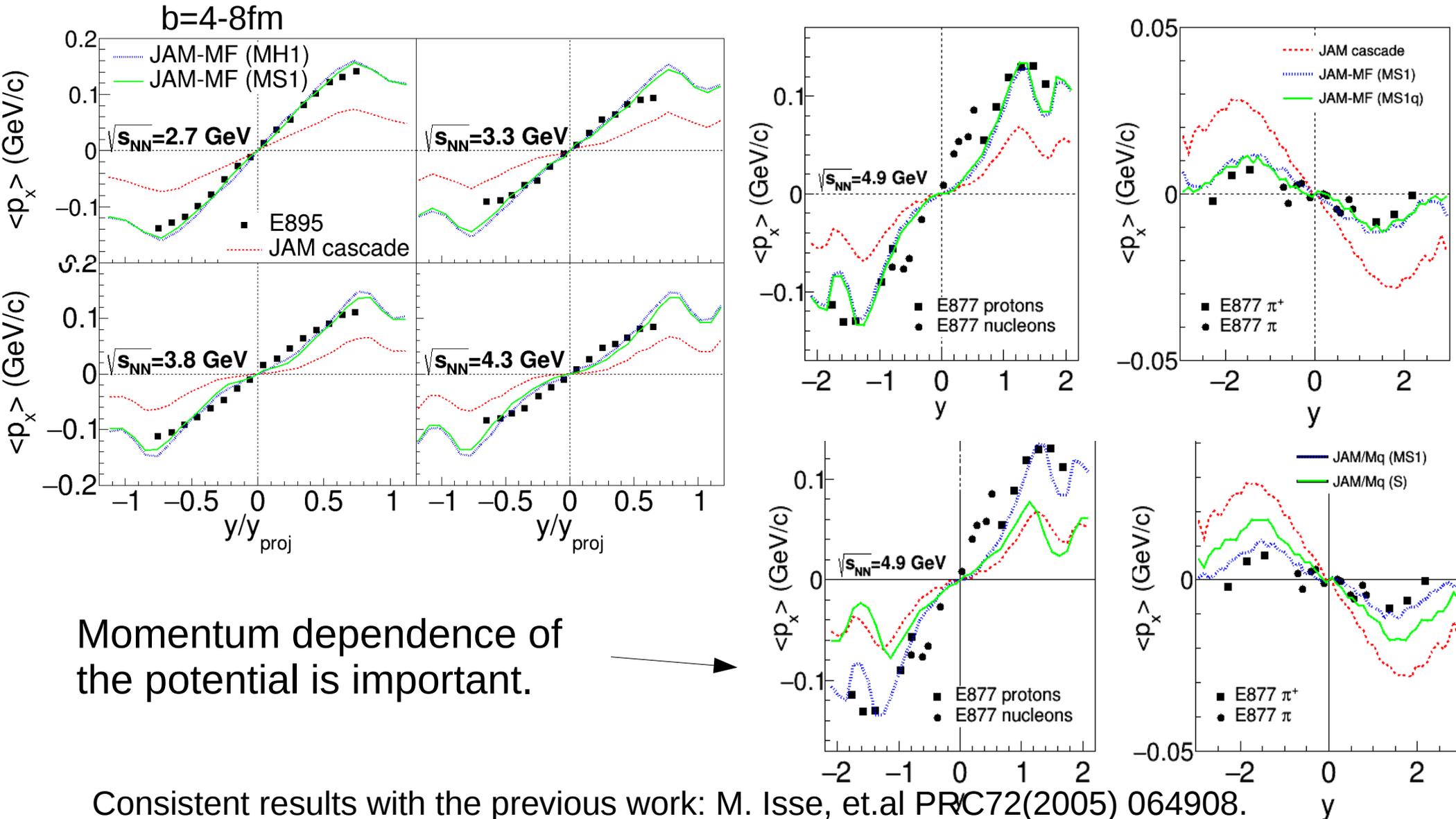
# Effect of cluster and its decay on the directed flow



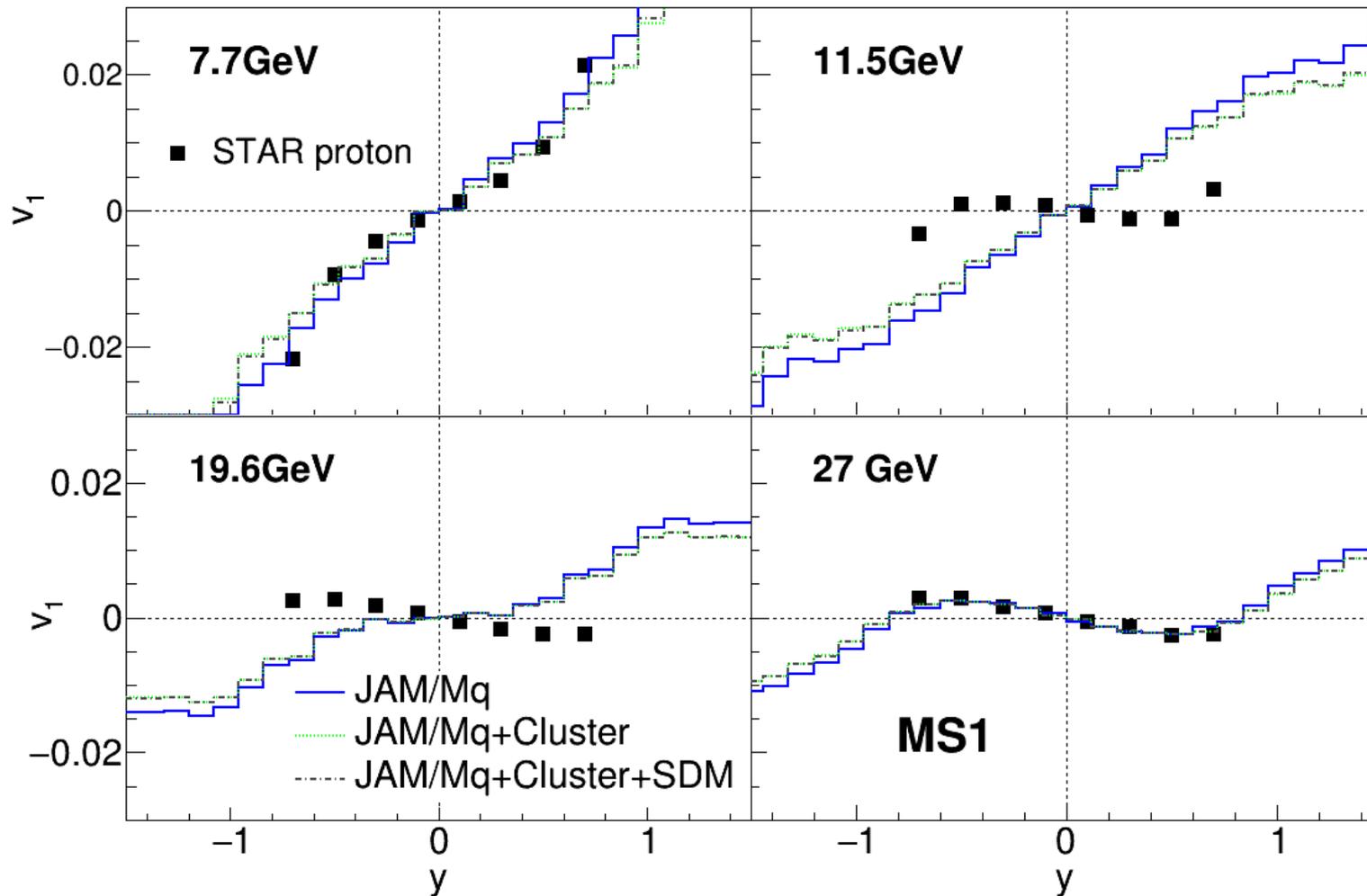
Effect of cluster is 10% on the  $v_1$

# JAM/RQMD results at AGS energies

Significant mean-field effect on the directed flow

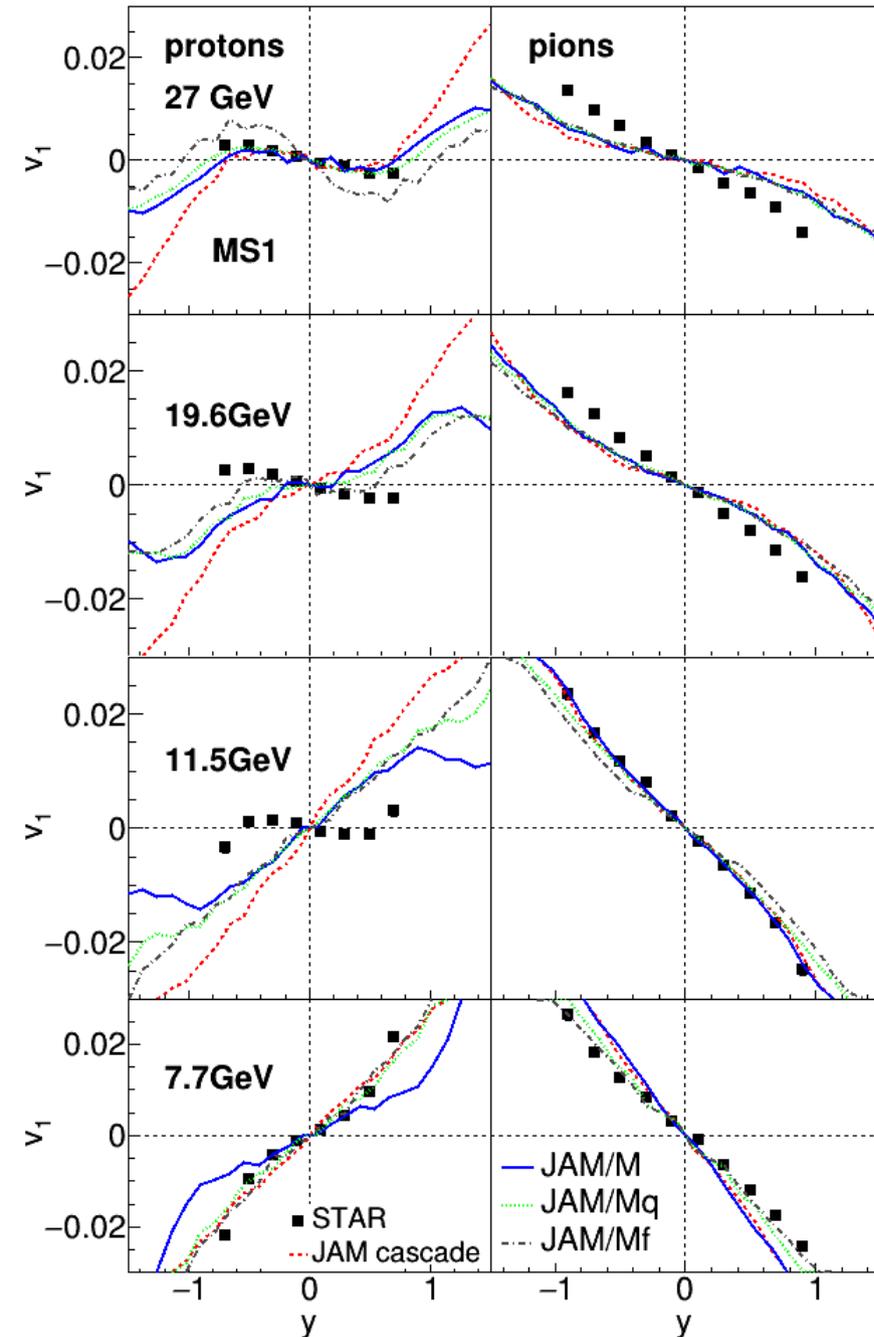


# JAM/M at STAR energies



Effect of the nuclear cluster formation is about 15%.  
No effect of statistical decay of nuclear fragment on  $v_1$

# Comparison of $v_1$



Effects of potential on the  $v_1$  is significant

Hadronic approach does not reproduce the correct beam energy dependence of the directed flow.

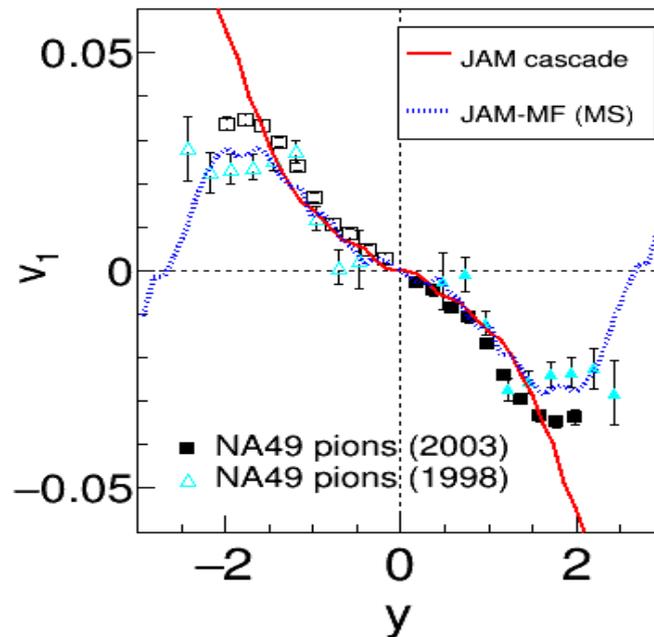
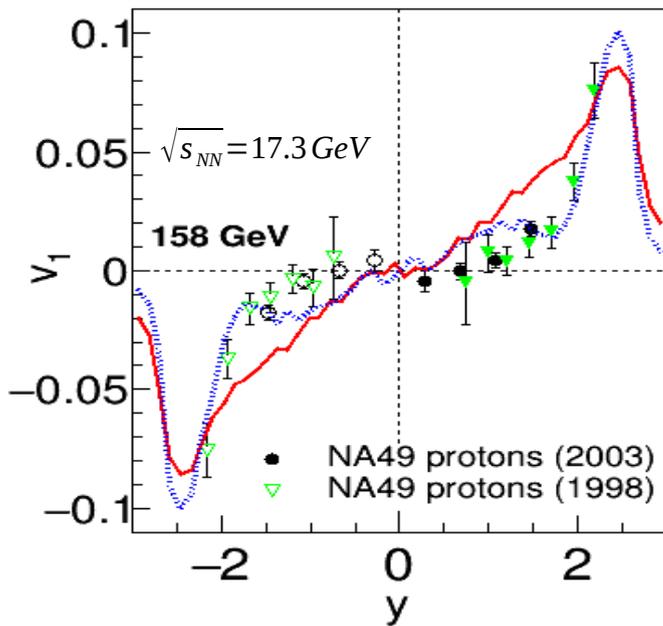
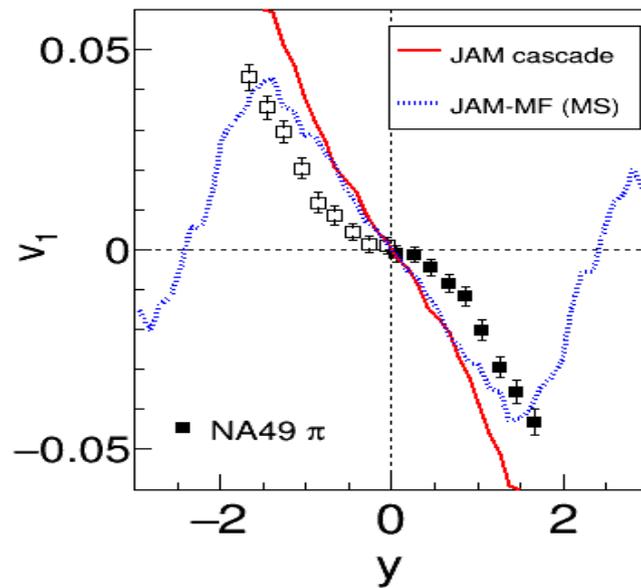
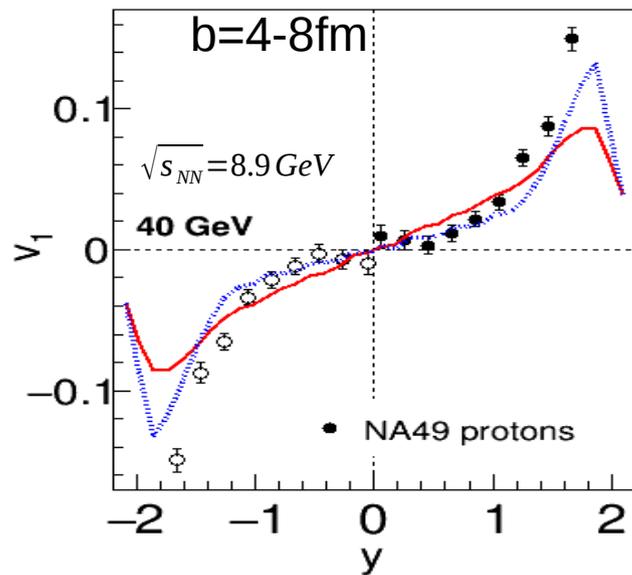
Something happens around 10-20GeV?

JAM/M: only formed baryons feel potential forces  
JAM/Mq: pre-formed hadron feel potential with factor  $2/3$  for diquark, and  $1/3$  for quark  
JAM/Mf: both formed and pre-formed hadrons feel potential forces.

# Summary

- Remarkable progress of the models for numerical simulation of high energy nuclear collisions such as CGC + hydro + Boltzmann approach.
- Reliable models for phase transition region must be developed.
- Hadronic transport model JAM with nuclear mean field followed by formation of nuclear cluster and its statistical decay.  
JAM + mean field + nuclear cluster formation + statistical decay
- JAM/M predicts the transition of proton directed flow from positive to negative. However, transition point is inconsistent with the STAR data  $F < 0$  at 11.5 GeV, but  $F > 0$  for JAM/M.
  - ★ Effects of cluster formation on the net-baryon distribution
  - ★ Hydrodynamics + Boltzmann(JAM) + mean field approach?

# JAM-MF at SPS energies



# Hadronic Cross sections in JAM

$$\begin{aligned}\sigma_{tot}(s) &= \sigma_{el}(s) + \sigma_{ch}(s) + \sigma_{ann}(s) \\ &+ \sigma_{t-R}(s) + \sigma_{s-R}(s) \quad : \text{Resonance} \\ &+ \sigma_{t-S}(s) + \sigma_{s-S}(s) \quad : \text{String}\end{aligned}$$

## Resonance production (absorption)

$$\sigma_{t-R}(s) : NN \leftrightarrow N\Delta, \quad NN \leftrightarrow N^*\Delta^*, \dots$$

$$\sigma_{s-R}(s) : \pi N \leftrightarrow \Delta, \quad \bar{K}N \leftrightarrow Y^*, \dots$$

## String formation

$$\sigma_{t-S}(s) : NN \rightarrow \text{String} + \text{String},$$

$$\sigma_{s-S}(s) : \pi N \rightarrow \text{String}$$