

# Nuclear electric dipole moment in the Gaussian expansion method

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- Introduction
- Nuclear Electric dipole moment
- Previous works ( $^2\text{H}$ ,  $^3\text{He}$ ,  $^3\text{H}$ ,  $^6\text{Li}$ ,  $^9\text{Be}$  EDM)
- $^{13}\text{C}$  EDM
- Summary

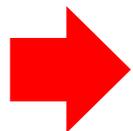
# Introduction

CP violation of Standard model is not sufficient to explain **matter/antimatter asymmetry** ...

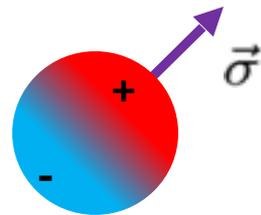
	ratio photon : matter
Prediction of Standard model:	$10^{28} : 1$
Real observed data:	$10^{10} : 1$

We need new source(s) of **large CP violation beyond the standard model !**

How to search ?



Electric dipole moment:  $\langle \vec{d} \rangle = \langle \psi | e \vec{r} | \psi \rangle$



EDM is CP-odd !

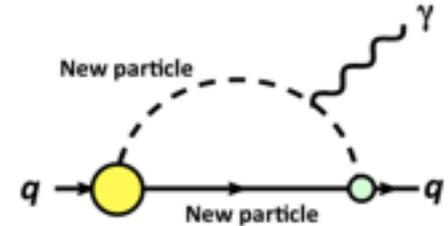
$$\left\{ \begin{array}{l} \vec{E} \quad \xrightarrow{T} \quad \vec{E} \\ \vec{\sigma} \quad \xrightarrow{T} \quad -\vec{\sigma} \end{array} \right.$$

# EDM as a sensitive probe of BSM physics

## Naïve estimation of neutron EDM:

- Coupling of new physics  $\sim O(1)$  (naturalness assumption)
- Contribute from one-loop graph
- 1 Yukawa coupling (required for chirality flip)
- $d_n/d_q \sim O(1)$  (hadron level analysis)

$$d_n = \frac{Y_q e}{4\pi^2 M_{\text{NP}}} \sim \frac{10^{-21}}{M_{\text{NP}}/\text{GeV}} e \text{ cm}$$



Exp data:  $d_n < 2.9 \times 10^{-26} e \text{ cm}$

C. A. Baker *et al.*, Phys. Rev. Lett. 97, 131801 (2006).

**$\Rightarrow$  Current exp data of Neutron EDM can probe  $M_{\text{NP}} \sim 10\text{TeV!}$**

(for  $d_n < 10^{-28} e \text{ cm}$ ,  $M_{\text{NP}} \sim 1000\text{TeV}$  can be probed: well beyond reach of LHC!)

**$\Rightarrow$  EDM is an attractive observable!**

# Nuclear EDM

# Why the nuclear EDM?

- Nuclear EDM is sensitive to **hadron level CP violation**

(hadron level CP violation is generated by CP violating operator with gluons and quarks)

- Nuclear EDM may enhance the CP violation through many-body effect

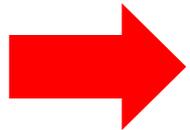
(Cluster, deformation make the parity violation easier)

V. V. Flambaum, I. B. Khriplovich and O. P. Sushkov, Phys. Lett. B162, 213 (1985);  
NY and E. Hiyama, Phys. Rev. C 91, 054005 (2015).

- Nuclear EDM does not suffer from Schiff's screening  
encountered in atomic EDM

(No electron to screen the nucleus)

- Very accurate measurement of EDM is possible using storage rings



**Nuclear EDM is a very good probe of BSM**

# EDM of charged particles using storage rings

Rotating particles in a storage ring feel very strong **central effective electric field**

The spin precession of the charged particle can be measured if magnetic moment is kept collinear to the particle momentum.  
(strong electric field normal to the precession plane)

Measurements of the EDMs of muon, **proton, deuteron,  $^3\text{He}$**  are planned.

Prospective sensitivity:

➔  $0(10^{-29})$  e cm!!

Better Experiment possible:  $d\mu < 10^{-24}$  ecm

$$\vec{\omega} = a_\mu \vec{B} + \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) \vec{\beta} \times \vec{E} + \frac{\eta}{2} (\vec{\beta} \times \vec{B} + \vec{E})$$

Essence: Cancel counteracting effects of g-2 precession!  
Can work also for any charged particle

(prepared at BNL)

➔ **EDM of light nuclei is accurately measurable!**

# Nuclear EDM from nucleon level CP violation

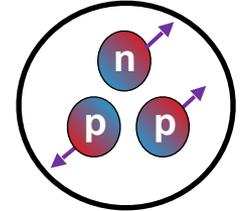
Two leading contributions to be evaluated:

## 1) Nucleon's intrinsic EDM:

Contribution from the **nucleon EDM**

$$D^{(1)} = \langle \psi | \sum_{i=1}^A \frac{1}{2} [(d_p + d_n) + (d_p - d_n) \tau_i^z] \sigma_i^z | \psi \rangle$$

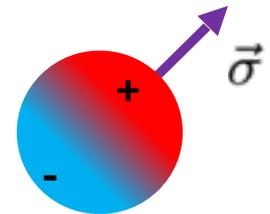
⇒ Spin expectation value (CP-even)



## 2) Polarization of the nucleus:

Contribution from the **P, CP-odd nuclear force**

$$D^{(\text{pol})} = \langle 0 | \hat{D}_z | \tilde{0} \rangle + \text{c.c.}, \quad \hat{D}_z = \frac{e}{2} \sum_{i=1}^A (1 + \tau_i^z) z_i$$



⇒ EDM generated by the CP-even ⇌ CP-odd mixing

# Nuclear EDM from nucleon level CP violation

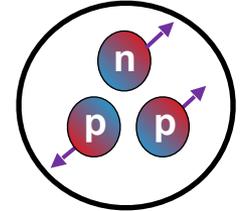
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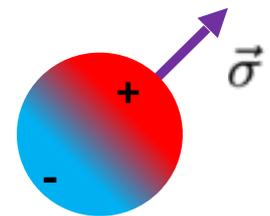


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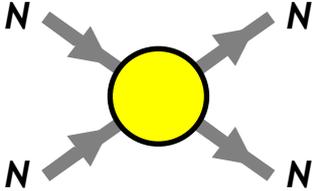
**May be enhanced by many-body effect!**

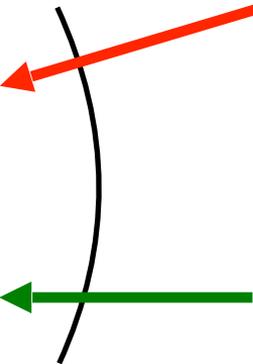
# Nuclear EDM (polarization) from CP-odd nuclear force

Electric dipole operator requires CP mixing to have finite expectation value

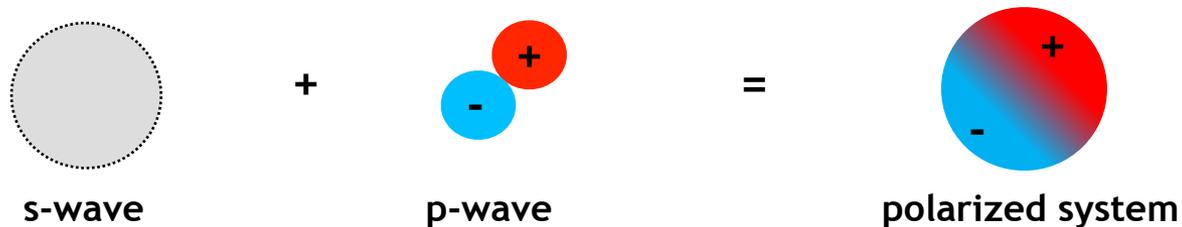
Total hamiltonian:

$$H = \begin{pmatrix} H_{\text{realistic}} & H_{\mathcal{P}\mathcal{T}} \\ H_{\mathcal{P}\mathcal{T}} & H_{\text{realistic}} \end{pmatrix}$$

  
P, CP-odd nuclear force

  
P, CP-even realistic nuclear force (e.g. Av18,xEFT,...)

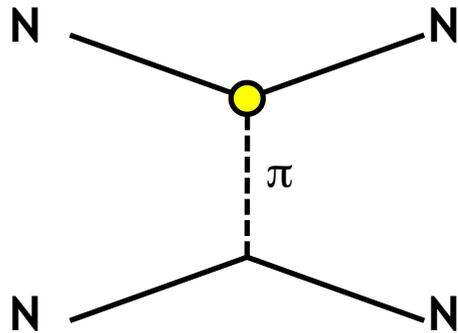
CP-odd N-N interactions mixes opposite parity states



Parity mixing  $\Rightarrow$  Polarized ground state!

# P, CP-odd nuclear force from one pion exchange

P, CP-odd nuclear force : we assume one-pion exchange process



$$\sim \frac{1}{q^2 - m_\pi^2} \bar{N} N \bar{N} i \gamma_5 N$$

## ● P, CP-odd Hamiltonian (3-types):

$$H_{\not{P}\not{C}} = -\frac{g_{\pi NN}}{8\pi m_p} \left[ \underbrace{\bar{g}_{\pi NN}^{(0)}}_{\text{Isoscalar}} \tau_a \cdot \tau_b + \underbrace{\bar{g}_{\pi NN}^{(2)}}_{\text{Isotensor}} (\tau_a \cdot \tau_b - 3\tau_a^z \tau_b^z) \right] (\vec{\sigma}_a - \vec{\sigma}_b) + \underbrace{\bar{g}_{\pi NN}^{(1)}}_{\text{Isovector}} (\tau_a^z \vec{\sigma}_a - \tau_b^z \vec{\sigma}_b) \cdot \vec{\nabla}_a \frac{e^{-m_\pi r_{ab}}}{r_{ab}},$$

## ● 4 important properties:

- Coherence in nuclear scalar density : enhanced in nucleon number
- One-pion exchange : suppress long distance contribution
- Spin dependent interaction : closed shell has no EDM
- Derivative : contribution from the surface

## ● What is expected:

- Polarization effect grows in A for small nuclei
- May have additional enhancements with **cluster**, deformation, ...

# What we want to do

⇒ Nucleon level CPV is unknown and small : linear dependence

⇒ Linear coefficients depends on the nuclear structure

⇒ We want to find nuclei with large enhancement factors

⇒ We must calculate the nuclear structure with nucleon level CPV

Dependence of nuclear EDM on nucleon level CP violation must be written as:

$$d_A = \left( c_0 \bar{g}_{\pi NN}^{(0)} + c_1 \bar{g}_{\pi NN}^{(1)} + c_2 \bar{g}_{\pi NN}^{(2)} \right) \times 10^{-13} e \text{ cm}$$

Unknown parameter of CP violation beyond standard model

Depends on the nuclear structure!

⇒ We want to evaluate **red factors** and find interesting nuclei!

**Previous works**  
**(<sup>2</sup>H, <sup>3</sup>He, <sup>3</sup>H, <sup>6</sup>Li, <sup>9</sup>Be EDM)**

## Our previous work

We have previously evaluated the EDM of light nuclei

⇒ EDM of  $^2\text{H}$ ,  $^3\text{He}$ ,  $^3\text{H}$ ,  $^6\text{Li}$ , and  $^9\text{Be}$  nuclei

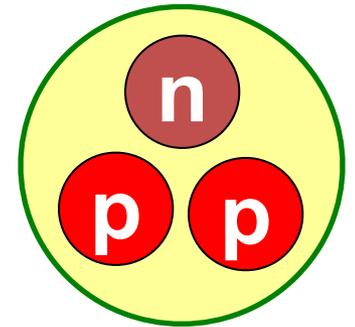
NY and E. Hiyama, Phys. Rev. C 91, 054005 (2015).

Using **Infinitesimally shifted  
Gaussian expansion method**

E. Hiyama *et al.*, Prog. Part. Nucl. Phys. 51, 223 (2003).

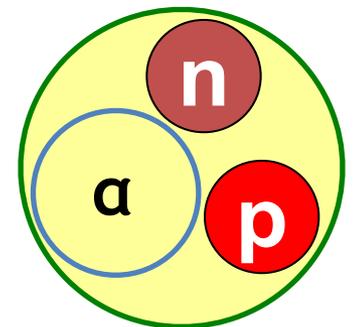
●  $^2\text{H}$ ,  $^3\text{He}$ ,  $^3\text{H}$  in ab initio evaluation (Av18)

⇒ **Good agreement with other works!**



●  $^6\text{Li}$ ,  $^9\text{Be}$  in cluster approximation  
+Orthogonality Condition Model (OCM)

⇒ **Cluster structure may enhance EDM!**



## A sophisticated method to calculate few-body system

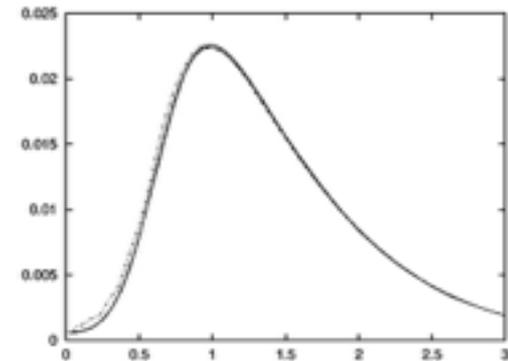
E. Hiyama *et al.*, Prog. Part. Nucl. Phys. **51**, 223 (2003).

● **Basis function:** 
$$\phi_{lm}(\mathbf{r}) = \sum_n N_{nl} \sum_k C_{lm,k} e^{-\nu_n(\mathbf{r}-\mathbf{D}_{lm,k})^2}$$

● **Variational method**

● **Successful in the benchmark calculation of  $^4\text{He}$  binding energy**

H. Kamada *et al.*, Phys. Rev. C **64**, 044001 (2001).



$^4\text{He}$  Density distribution

● **It is applied in many subjects:**  
Nuclei, Hypernuclei, atoms, hadrons, astrophysics, ...

**We expect accurate calculation of nuclear EDM!**

# Ab initio evaluations ( $^2\text{H}$ , $^3\text{He}$ )

## Deuteron EDM:

Group	Nuclear force	$C_0$	$C_1$	$C_2$
<b>Liu &amp; Timmermans</b> <small>Liu et al., PRC 70, 055501 (2004)</small>	Av18	0	$1.43 \times 10^{-2} e \text{ fm}$	0
<b>GEM (our work)</b>	Av18	0	$1.45 \times 10^{-2} e \text{ fm}$	0

## $^3\text{He}$ EDM:

Group	Nuclear force	isoscalar ( $c_0$ )	isovector ( $c_1$ )	isotensor ( $c_2$ )
<b>Faddeev</b> <small>Bsaisou et al., JHEP 1503 (2015) 104</small>	$\text{N}^2\text{LO}$ chiral EFT	$0.0079 e \text{ fm}$	$0.0101 e \text{ fm}$	$0.0169 e \text{ fm}$
<b>GEM (our work)</b> <small>NY, E. Hiyama, PRC 91, 054005 (2015)</small>	Av18	$0.0060 e \text{ fm}$	$0.0108 e \text{ fm}$	$0.0168 e \text{ fm}$

**$\Rightarrow$  Consistent with previous works!**

# Cluster approximation : ${}^6\text{Li}$ and ${}^9\text{Be}$ EDM

## ● ${}^6\text{Li}$ EDM:

Nuclear force	$\langle\sigma\rangle$	$\langle\sigma\tau\rangle$	isoscalar ( $c_0$ )	isovector ( $c_1$ )	isotensor ( $c_2$ )
Av8'+cluster model	0.88	—	—	0.028 e fm	—

${}^6\text{Li}$  EDM is made of 2 comparable components:

- Deuteron cluster polarization : slightly smaller than deuteron EDM
- CP-odd  $\alpha$ -N interaction effect

Compare with deuteron EDM (  $c_1 = 0.0145$  e fm ) :

**$\Rightarrow {}^6\text{Li}$  enhances the CP-odd effect ! (twice deuteron EDM)**

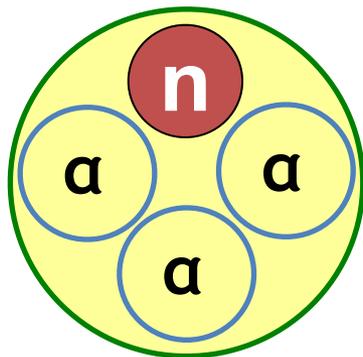
## ● ${}^9\text{Be}$ EDM:

Nuclear force	$\langle\sigma\rangle$	$\langle\sigma\tau\rangle$	isoscalar ( $c_0$ )	isovector ( $c_1$ )	isotensor ( $c_2$ )
Cluster model	0.38	-0.38	—	0.014 e fm	—

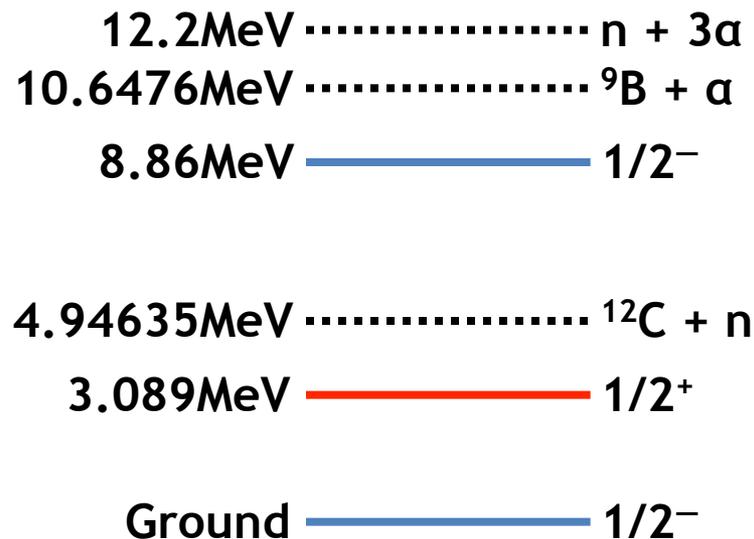
Similar contribution as  ${}^6\text{Li}$  for the polarization due to CP-odd  $\alpha$ -N interaction

**$^{13}\text{C}$  EDM**

# Structure of $^{13}\text{C}$



$^{13}\text{C}$  : 4-body in cluster approximation



Energy levels of  $^{13}\text{C}$

Study of  $^{13}\text{C}$  EDM has many motivations:

- $1/2^+$  state is close to ground state (3.1 MeV, bound)
- Ground state has shell structure, but  $1/2^+$  state is neutron halo
- Ground state is dominated by  $l=1$  state (core+valence picture)

➡ Object of study : Let us study the EDM of  $^{13}\text{C}$ !

# Setup of the cluster model

We treat the  $^{13}\text{C}$  nucleus as a 4-body system of nucleons and  $\alpha$  clusters ( $^4\text{He}$  nucleus).

T. Yamada, Y. Funaki, to appear in Phys. Rev. C (2015) [arXiv:1503.04261].

## ● $\alpha$ - $\alpha$ interaction:

Folding of Schmid-Wildermuth  $NN$  force (include Coulomb)

Pauli exclusion taken into account via OCM

E. W. Schmid and K. Wildermuth, Nucl. Phys. 26, 462 (1961).

## ● $N$ - $\alpha$ interaction:

Fitted to reproduce the  $\alpha$ - $N$  scattering phase shift at low energy

Pauli exclusion taken into account via OCM (0s excluded)

H. Kanada *et al.*, Prog. Theor. Phys. 61, 1327 (1979).

## ● 3-, 4-body interactions:

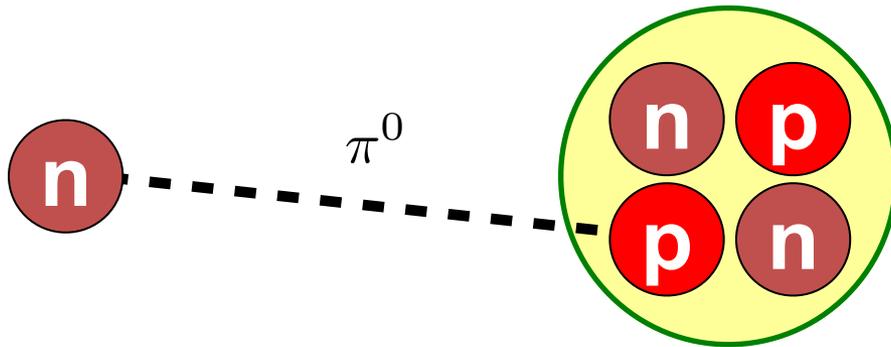
Phenomenological 3- $\alpha$  force with angular momentum dependence

Phenomenological  $\alpha\alpha N$  force to fit  $^9\text{Be}$  ground-state energy

Phenomenological 3 $\alpha$ - $N$  force to fit  $^{13}\text{C}$  ground-state energy

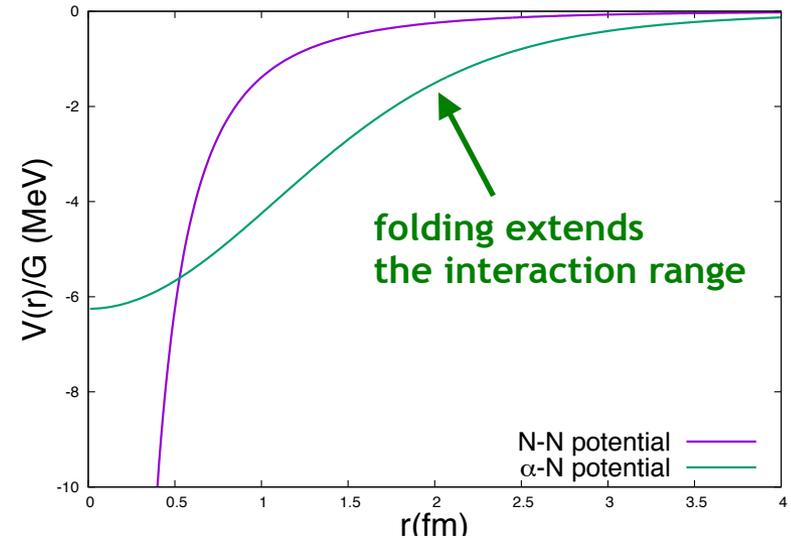
# CP-odd nuclear force (CP-odd $\alpha$ -N interaction)

Integrate the CP-odd N-N interaction with the  $^4\text{He}$  nucleon density  
( $\alpha$  cluster is indestructible)



Gaussian approximation of density:

$$\rho_\alpha(r) = A e^{-\frac{r^2}{b}} \quad \text{Spread : } b = (1.358 \text{ fm})^2$$



Only isovector CP-odd nuclear force is relevant in N- $\alpha$  interaction  
(Isoscalar and isotensor CP-odd nuclear forces cancel by folding)

# Calculation of $^{13}\text{C}$ EDM

- We have used 120 channels, with 343 bases for each.

Total number of bases : 41160

- Reproduce the binding energy of  $^{13}\text{C}$ :

1/2- (Ground) : 12.4 MeV

1/2+ : 9.4 MeV

Energy difference:  $E(\text{GS}) - E(1/2+) = 3 \text{ MeV}$

- Reproduce the root mean square radius of  $^{13}\text{C}$ :

1/2- (Ground) :  $\langle \sqrt{r^2} \rangle = 2.4 \text{ fm}$

- Root mean square radius of core-valence distance:

1/2- (Ground) :  $\langle \sqrt{r_1^2} \rangle = 2.81 \text{ fm} \Rightarrow \text{Compact}$

1/2+ :  $\langle \sqrt{r_1^2} \rangle = 3.95 \text{ fm} \Rightarrow \text{Neutron-halo}$

# How nuclear EDM is sensitive to CP violation??

EDM	isoscalar ( $c_0$ )	isovector ( $c_1$ )	isotensor ( $c_2$ )
<b><math>^{199}\text{Hg}</math> atom</b> Ban et al., PRC 82, , 015501 (2010) Dzuba et al., PRA 80, 032120 (2009)	<b><math>4.7 \times 10^{-6}</math> e fm</b>	<b><math>-1.8 \times 10^{-6}</math> e fm</b>	<b><math>7.5 \times 10^{-6}</math> e fm</b>
<b><math>^{225}\text{Ra}</math> atom</b> Dobaczewski et al., PRL 94, 232502 (2005) Dzuba et al., PRA 80, 032120 (2009)	<b>0.00088 e fm</b>	<b>-0.0052 e fm</b>	<b>0.0035 e fm</b>
<b>Neutron</b> (Chiral analysis)	<b>0.01 e fm</b>	—	<b>- 0.01 e fm</b>
<b>Deuteron</b> Liu et al., PRC 70, 055501 (2004)	—	<b>0.0145 e fm</b>	—
<b><math>^3\text{He}</math> nucleus</b> Bsaisou et al., JHEP 1503 (2015) 104 NY and EH, PRC 91, 054005 (2015)	<b>0.0060 e fm</b>	<b>0.0108 e fm</b>	<b>0.0168 e fm</b>
<b><math>^6\text{Li}</math> nucleus</b> NY and EH, PRC 91, 054005 (2015)	—	<b>0.028 e fm</b>	—
<b><math>^9\text{Be}</math> nucleus</b> NY and EH, PRC 91, 054005 (2015)	—	<b>0.014 e fm</b>	—
<b><math>^{13}\text{C}</math> nucleus</b>	—	<b>-0.00085 e fm</b>	—

Preliminary

Our result

## Why small $^{13}\text{C}$ EDM?

- Small overlap between shell-like  $1/2^-$  state and neutron halo like  $1/2^+$  state  
(EDM is larger for large overlap between opposite parity states)
  - Orbital angular momentum and spin are antiparallel:  
⇒ Suppresses the EDM
- ➡ This is a very good guide for searching nuclei with large EDM**

### A sensitive nucleus must have:

- Ground and opposite parity excited states with large overlap
- Same sign orbital angular momentum and spin in ground state
- Cluster structure : low opposite parity excitation energy

## Summary:

- We have studied the EDM of the  $^{13}\text{C}$  nucleus in the cluster approximation, using the Gaussian Expansion Method.
- Result :  $d_{13\text{C}} = -8.5 \times 10^{-4} G_{\pi}^{(1)} \text{ e fm}$   
Relatively small EDM: may be understood by the small overlap between shell-like  $1/2^-$  state and neutron halo like  $1/2^+$  state.

## Future subjects:

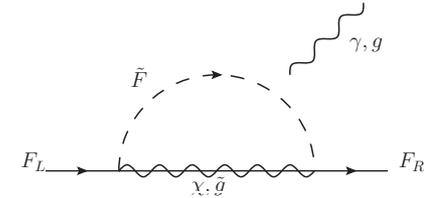
- Study of  $^{13}\text{N}$  EDM.
- Further study of EDM of light nuclei: find sensitive nuclei respecting the analysis of this work.
- We are waiting for experiments!

# Sensitivity to new physics beyond standard model

If the EDM of light nuclei can be measured at  $O(10^{-29})$  e cm:

## ● Supersymmetric model:

⇒ Can probe 10 TeV scale SUSY breaking



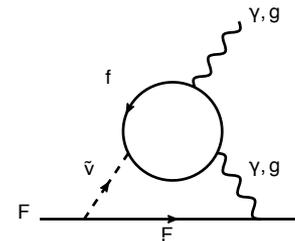
## ● Models with 4-quark interactions:

⇒ Can probe PeV scale physics  
(Left-right symmetric model, ...)



## ● Models with Barr-Zee type diagrams:

⇒ Can probe PeV scale physics  
(Higgs doublet models, RPV SUSY, ...)



**➔ EDM is an attractive observable  
in the search for BSM physics!**

**End**

# Orthogonality condition model (OCM)

Simple way to include the effect of antisymmetrization (Pauli exclusion) in cluster model

## ● N- $\alpha$ interaction:

Repulsion of the 0s state:

$$V_{\text{Pauli}} = \lim_{\lambda \rightarrow \infty} \sum_{f=0s} |\phi_f(\mathbf{r}_{\alpha\alpha})\rangle \langle \phi_f(\mathbf{r}'_{\alpha\alpha})| \lambda$$

## ● $\alpha$ - $\alpha$ interaction:

Repulsion of the 0s, 1s, 0d states.

$$V_{\text{Pauli}} = \lim_{\lambda \rightarrow \infty} \sum_{f=0s,1s,0d} \lambda |\phi_f(\mathbf{r}_{\alpha\alpha})\rangle \langle \phi_f(\mathbf{r}'_{\alpha\alpha})|$$

In our calculation, we have taken  $\lambda \sim 10^4$  MeV