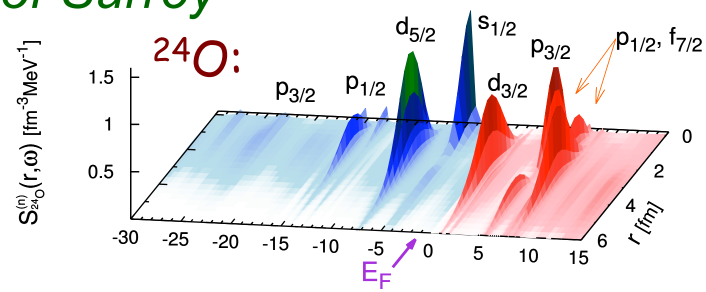
Green's function studies of medium-mass nuclei and Lattice QCD interactions

Carlo Barbieri — University of Surrey

Part I – Applications of the SCGF approach:

- *- Structure of oxygen's chain isotopes*
- *- notes on spectroscopic factors*
- *- Neutron rich Calciums and neighbors*



Phys. Rev. Lett. ***111****, 062501 (2013)*
Phys. Rev. C ***92****, 014306 (2015)*

Part II – Applications of the SCGF approach:

- *- Results for nuclear forces from LCQD*
- *- SCGF approach to handle short-range repulsion.*

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Collaborators

UNIVERSITY OF SURREY — A. Cipollone, A. Rios, F. Raimondi

cea (energie atomique • energies alternatives) — V. Somà, T. Duguet

EMMI, TECHNISCHE UNIVERSITÄT DARMSTADT — A. Carbone

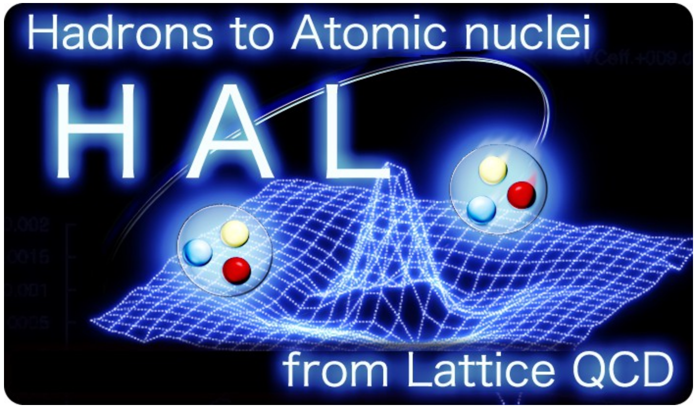
TRIUMF — P. Navratil

Universitat de Barcelona — A. Polls

Washington University in St.Louis — W.H. Dickhoff, S. Waldecker

Center for Molecular Modeling — D. Van Neck, M. Degroote

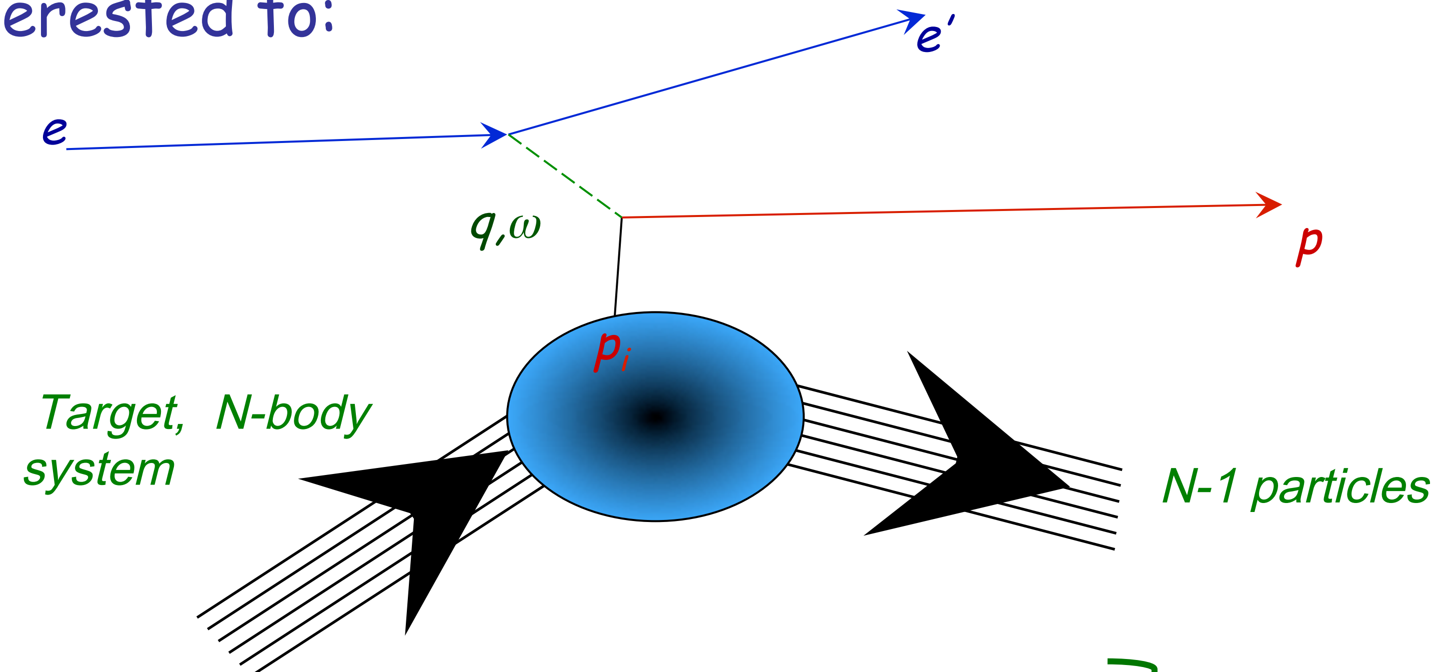
M. Hjorth-Jensen



S. Aoki,	YITP Kyoto Univ.
T. Doi, T. Hatsuda, Y. Ikeda,	RIKEN Nishina
T. Inoue,	Nihon Univ.
N. Ishii, K. Murano,	RCNP Osaka Univ
H. Nemura K. Sasaki	Univ. Tsukuba
F. Etminan	Univ. Birjand
T. Miyamoto,	Univ. Tsukuba
T. Iritani	Stony Brook Univ.
S. Gongyo	YITP Kyoto Univ.

Spectroscopy via knock out reactions-basic idea

Use a probe (ANY probe) to eject the particle we are interested to:



Basic idea:

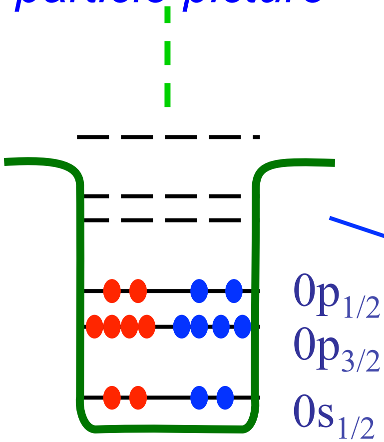
- we know, e, e' and p
- "get" *energy and momentum* of p_i: $p_i = k_e' + k_p - k_e$

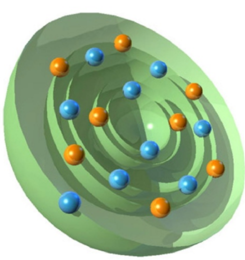
  $E_i = E_e' + E_p - E_e$

Better to choose large transferred momentum and weak probes!!!

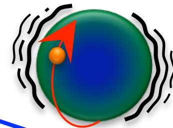
Concept of correlations

independent particle picture

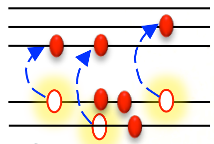




Particle-vibration coupling (PV)



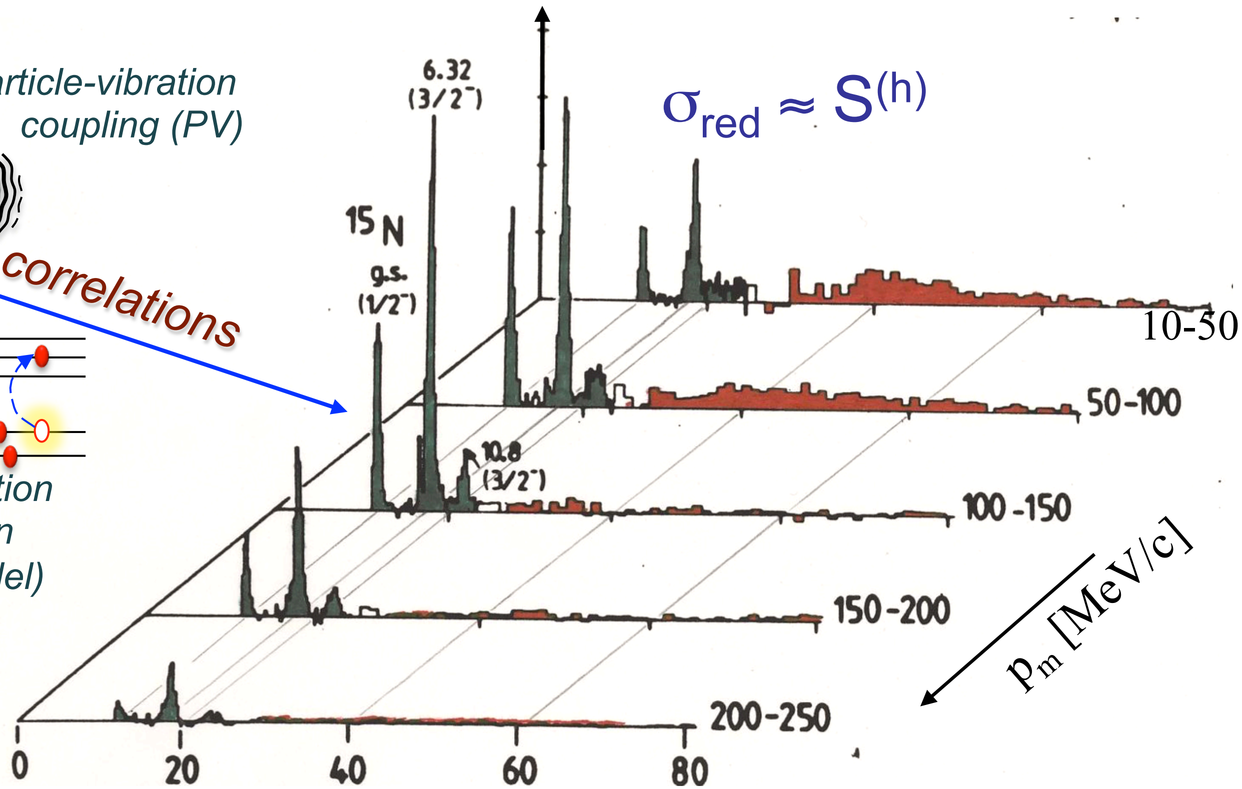
correlations



Configuration interaction (shell model)

Spectral function: distribution of momentum (p_m) and energies (E_m)

$\sigma_{red} \approx S^{(h)}$



Saclay data for $^{16}O(e,e'p)$

[Mougey et al., Nucl. Phys. A335, 35 (1980)]

Understood for a few stable closed shells:

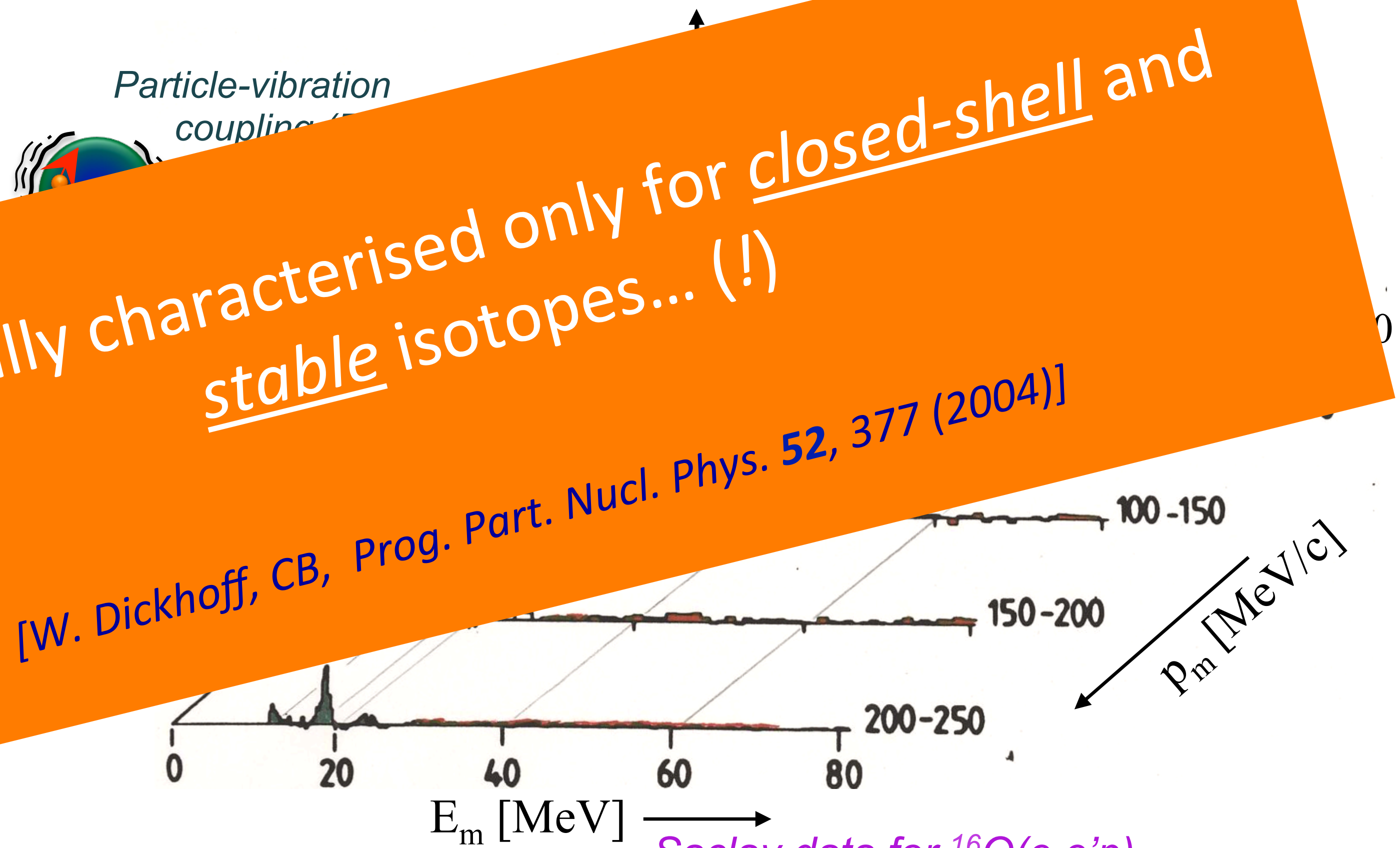
*[CB and W. H. Dickhoff, Prog. Part. Nucl. Phys **52**, 377 (2004)]*

Concept of correlations

independent particle picture

Spectral function: distribution of momentum (p_m) and

Particle-vibration



Saclay data for $^{16}O(e,e'p)$
[Mougey et al., Nucl. Phys. A335, 35 (1980)]

So far, fully characterised only for <u>closed-shell</u> and <u>stable</u> isotopes... (!)

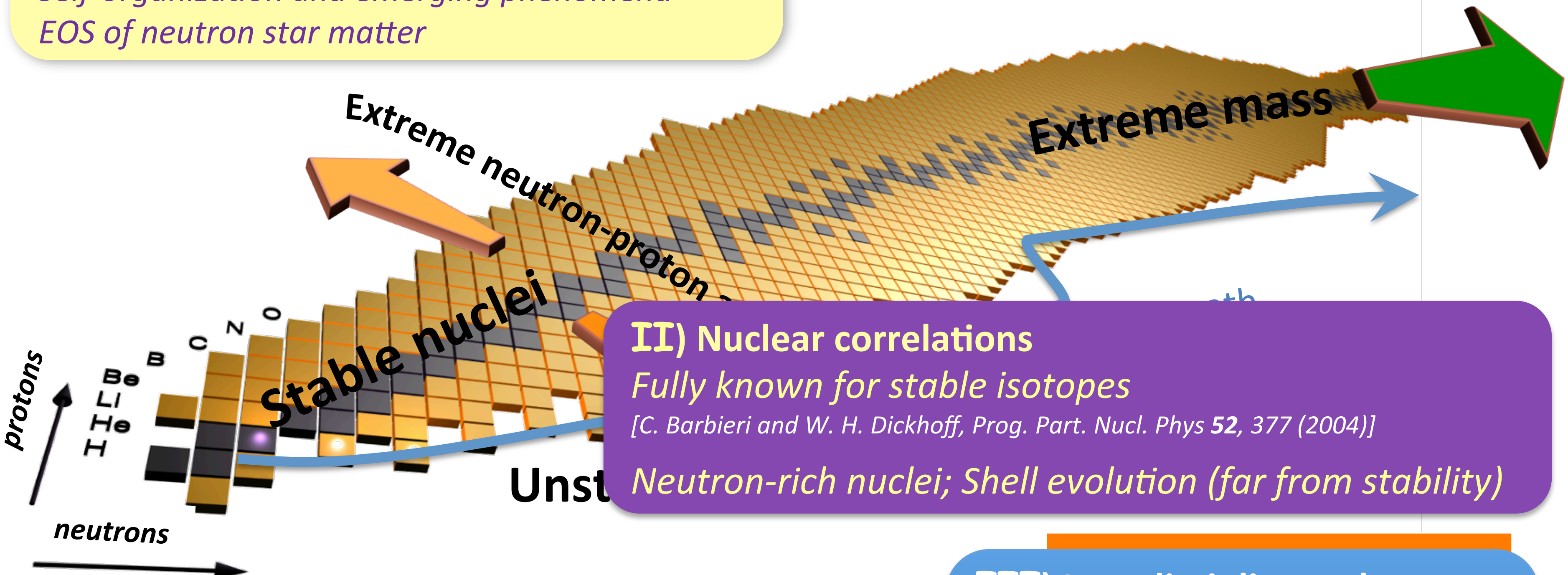
[W. Dickhoff, CB, Prog. Part. Nucl. Phys. **52**, 377 (2004)]

Understood for a few stable closed shells:
*[CB and W. H. Dickhoff, Prog. Part. Nucl. Phys **52**, 377 (2004)]*

Current Status of low-energy nuclear physics

Composite system of interacting fermions
Binding and limits of stability
Coexistence of individual and collective behaviors
Self-organization and emerging phenomena
EOS of neutron star matter

Experimental programs
RIKEN, FAIR, FRIB



II) Nuclear correlations
Fully known for stable isotopes
[C. Barbieri and W. H. Dickhoff, Prog. Part. Nucl. Phys ***52****, 377 (2004)]*
Neutron-rich nuclei; Shell evolution (far from stability)

I) Understanding the nuclear force
QCD-derived; 3-nucleon forces (3NFs)
First principle (ab-initio) predictions

III) Interdisciplinary character
Astrophysics
Tests of the standard model
Other fermionic systems:
ultracold gasses; molecules;

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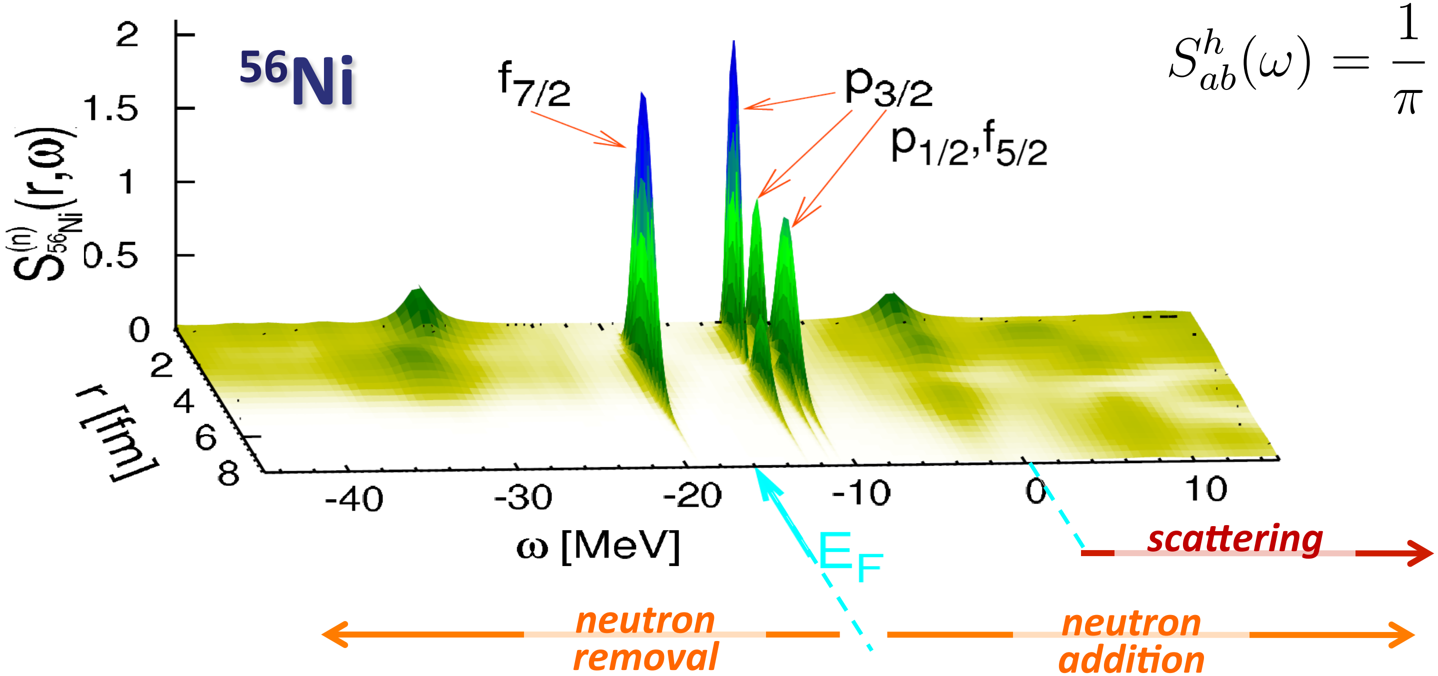
^{56}Ni neutron spectral function



$$S^h_{ab}(\omega) = \frac{1}{\pi} \, Im \, g_{ab}(\omega)$$

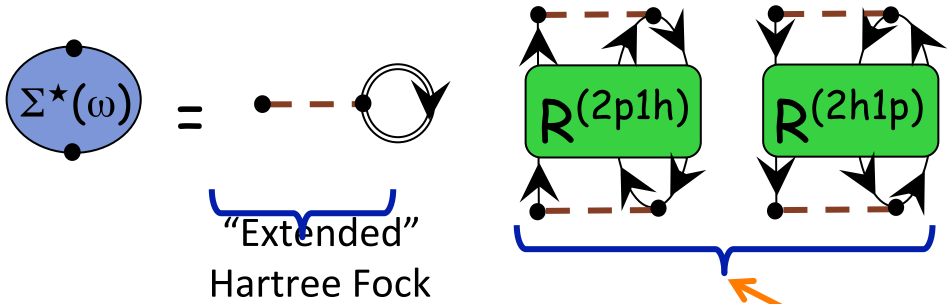
W. Dickhoff, CB, Prog. Part. Nucl. Phys. 53, 377 (2004)
CB, M.Hjorth-Jensen, Pys. Rev. C**79**, 064313 (2009)

Ab-Initio SCGF approaches

The FRPA Method in Two Words

Particle vibration coupling is the main cause driving the distribution of particle strength—on both sides of the Fermi surface…

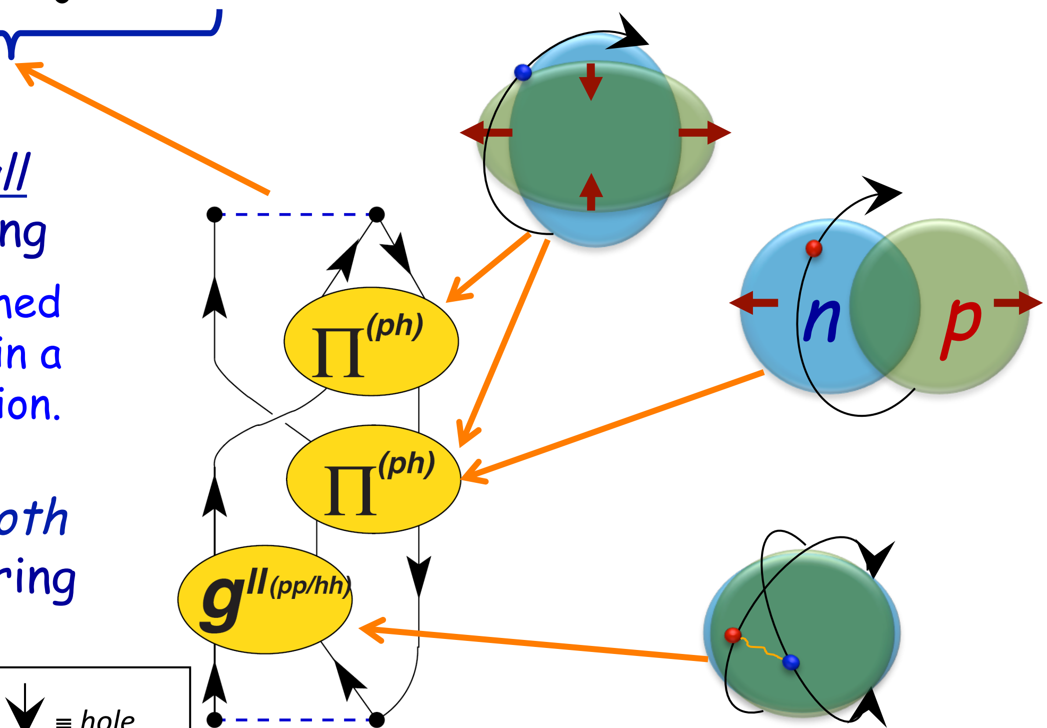
CB et al.,
Phys. Rev. C**63**, 034313 (2001)
Phys. Rev. A**76**, 052503 (2007)
Phys. Rev. C**79**, 064313 (2009)

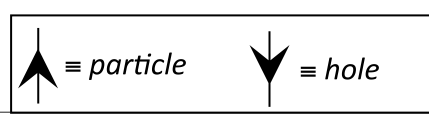


•A complete expansion requires *all* *types* of particle-vibration coupling

…these modes are all resummed exactly and to all orders in a *ab-initio* many-body expansion.

•The Self-energy $\Sigma^\star(\omega)$ yields *both* single-particle states and scattering





Gorkov and symmetry breaking approaches

V. Somà, CB, T. Duguet, , Phys. Rev. C **89**, 024323 (2014)
V. Somà, CB, T. Duguet, Phys. Rev. C **87**, 011303R (2013)
V. Somà, T. Duguet, CB, Phys. Rev. C **84**, 064317 (2011)

- Ansatz $\ldots \approx E_0^{N+2} - E_0^N \approx E_0^N - E_0^{N-2} \approx \ldots \approx 2\mu$

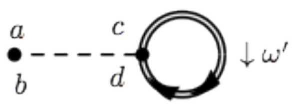
- Auxiliary many-body state $|\Psi_0\rangle \equiv \sum_N^{\text{even}} c_N \, |\psi_0^N\rangle$

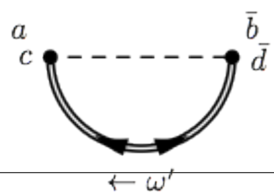
  → Mixes various particle numbers

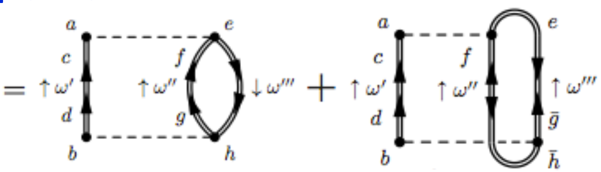
  → Introduce a "grand-canonical" potential $\Omega = H - \mu N$

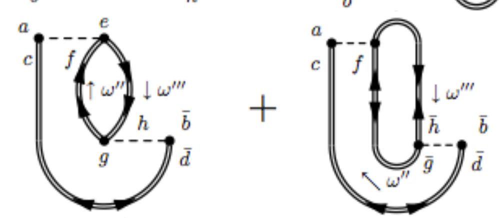
⇒ $|\Psi_0\rangle$ minimizes $\Omega_0 = \langle\Psi_0|\Omega|\Psi_0\rangle$ under the constraint $N = \langle\Psi_0|N|\Psi_0\rangle$

- This approach leads to the following Feynman diagrams:

$\Sigma_{ab}^{11\,(1)} =$ 

$\Sigma_{ab}^{12\,(1)} =$ 

$\Sigma_{ab}^{11\,(2)}(\omega) =$ 

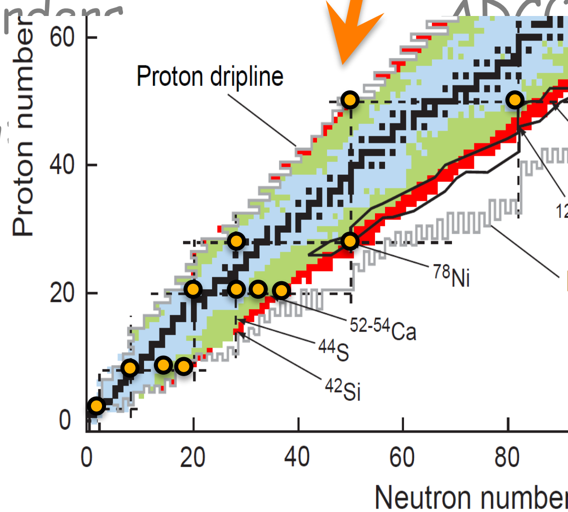
$\Sigma_{ab}^{12\,(2)}(\omega) =$ 

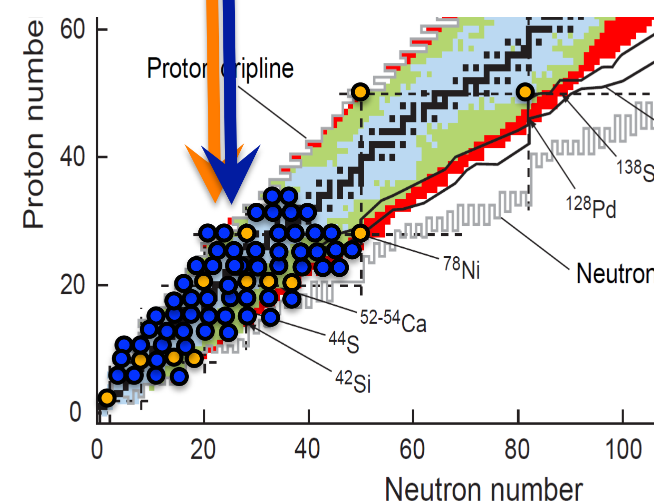
Approaches in GF theory

Truncation scheme:	Dyson formulation (closed shells)	Gorkov formulation (semi-magic)
1^{st} order:	Hartree-Fock	HF-Bogolioubov
2^{nd} order:	2^{nd} order	2^{nd} order (w/ pairing)
. . .	. . .	
3^{rd} and all-orders sums, P-V coupling:	ADC(3) FRPA etc…	G-ADC(3) …work in progress

Approaches in GF theory

Truncation scheme:	Dyson formulation (closed shells)	Gorkov formulation (semi-magic)
1st order:	Hartree-Fock	HF-Bogolioubov
2nd order:	2nd order	2nd order (w/ pairing)
. . .	. . .	
3rd and all-orders sums, P-V coupling:	ADC(3)	

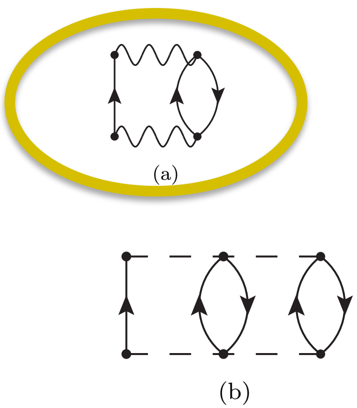




Inclusion of NNN forces

A. Carbone, CB, et al., Phys. Rev. C88, 054326 (2013)

- *Second order* PT diagrams with 3BFs:



- *Third order* PT diagrams with 3BFs:

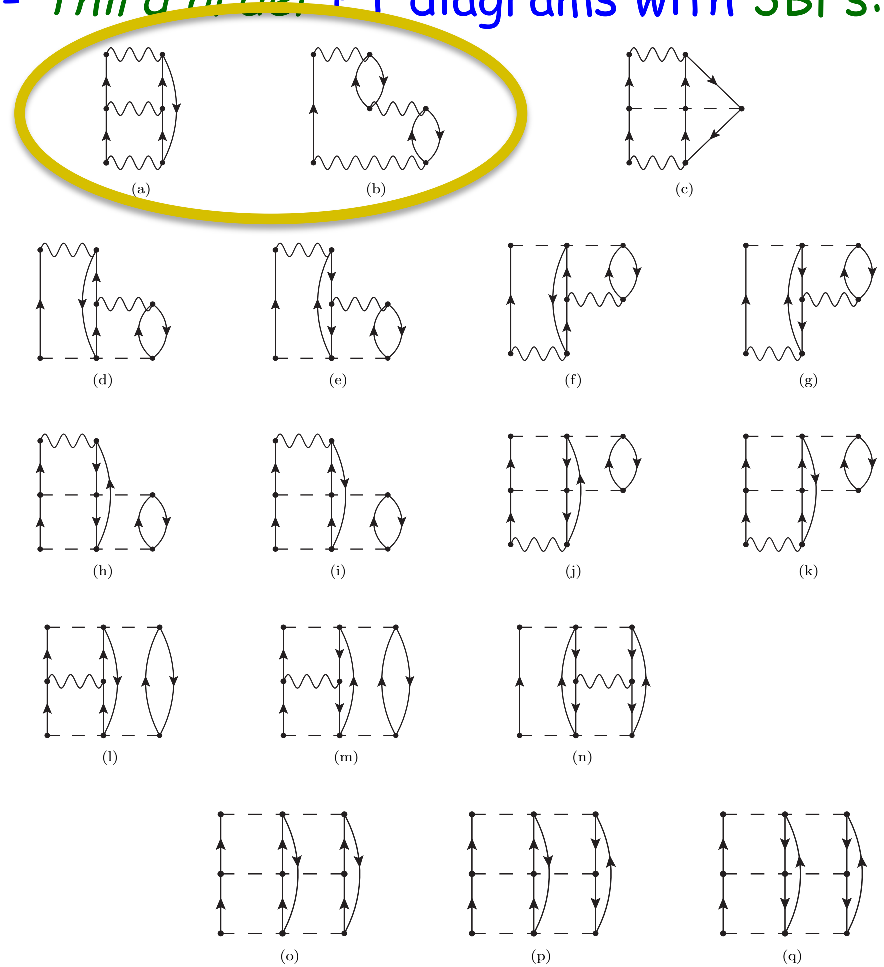


FIG. 5. 1PI, *skeleton* and *interaction irreducible* self-energy diagrams appearing at 3^{rd}-order in perturbative expansion (7), making use of the effective hamiltonian of Eq. (9).

➔ Use if effective interactions

➔ Need to correct the Koltun sum rule (for energy)

(Galitskii-Migdal-Boffi-) Koltun sumrule

- Koltun sum rule (with NNN interactions):

$$\sum_{\alpha} \frac{1}{\pi} \int_{-\infty}^{\epsilon_F^-} d\omega \, \omega \, \mathrm{Im}\, G_{\alpha\alpha}(\omega) = \langle \Psi_0^N | \hat{T} | \Psi_0^N \rangle + 2 \langle \Psi_0^N | \hat{V} | \Psi_0^N \rangle + 3 \langle \Psi_0^N | \hat{W} | \Psi_0^N \rangle$$

two-body

three-body

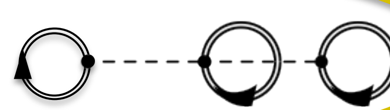
- Thus, need an extra correction:

$$E_0^N = \frac{1}{3\pi} \int_{-\infty}^{\epsilon_F^-} \mathrm{d}\omega \sum_{\alpha\beta} (2T_{\alpha\beta} + \omega \delta_{\alpha\beta}) \mathrm{Im}\, G_{\beta\alpha}(\omega) + \frac{1}{3} \langle \Psi_0^N | \widehat{V} | \Psi_0^N \rangle$$

or

$$E_0^N = \frac{1}{2\pi} \int_{-\infty}^{\epsilon_F^-} \mathrm{d}\omega \sum_{\alpha\beta} (T_{\alpha\beta} + \omega \delta_{\alpha\beta}) \mathrm{Im}\, G_{\beta\alpha}(\omega) - \frac{1}{2} \langle \Psi_0^N | \widehat{W} | \Psi_0^N \rangle$$

$$\langle \Psi_0^N | \widehat{W} | \Psi_0^N \rangle \approx \frac{1}{6}$$



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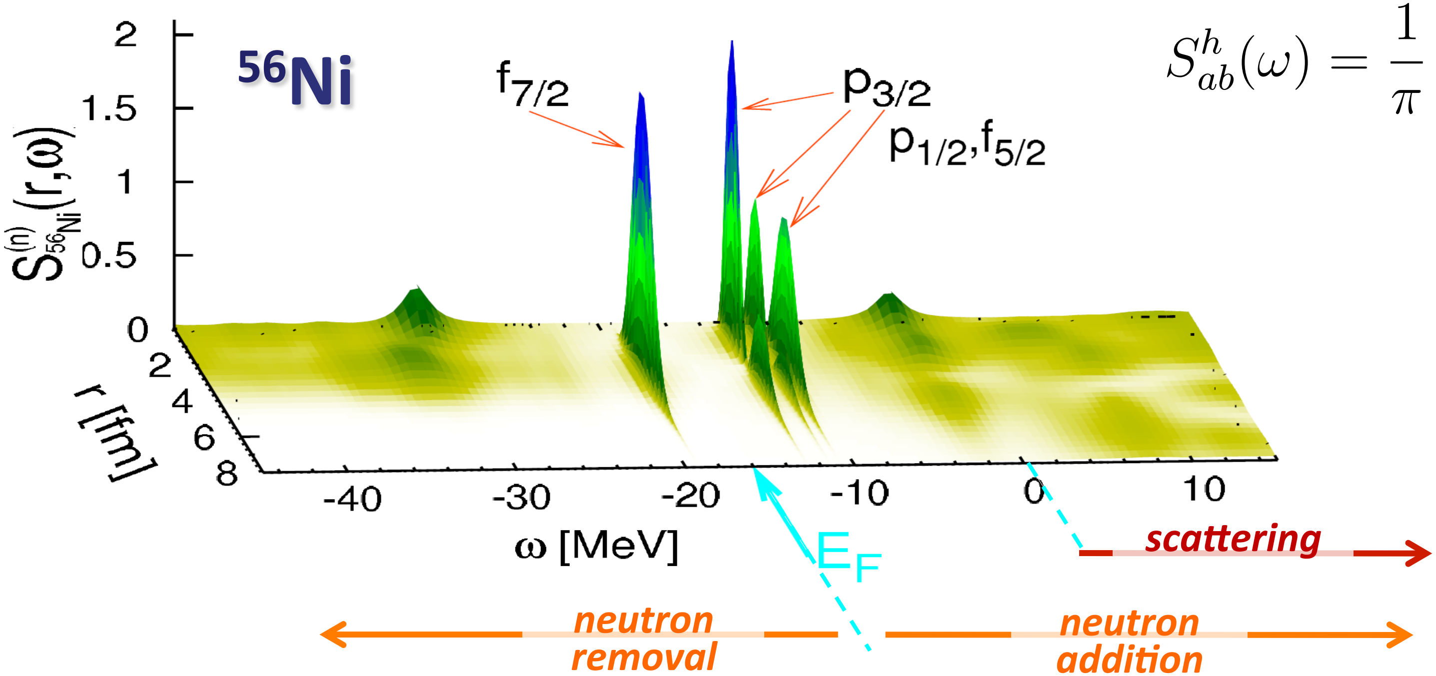
^{56}Ni neutron spectral function

$$S^h_{ab}(\omega) = \frac{1}{\pi} \, Im \, g_{ab}(\omega)$$



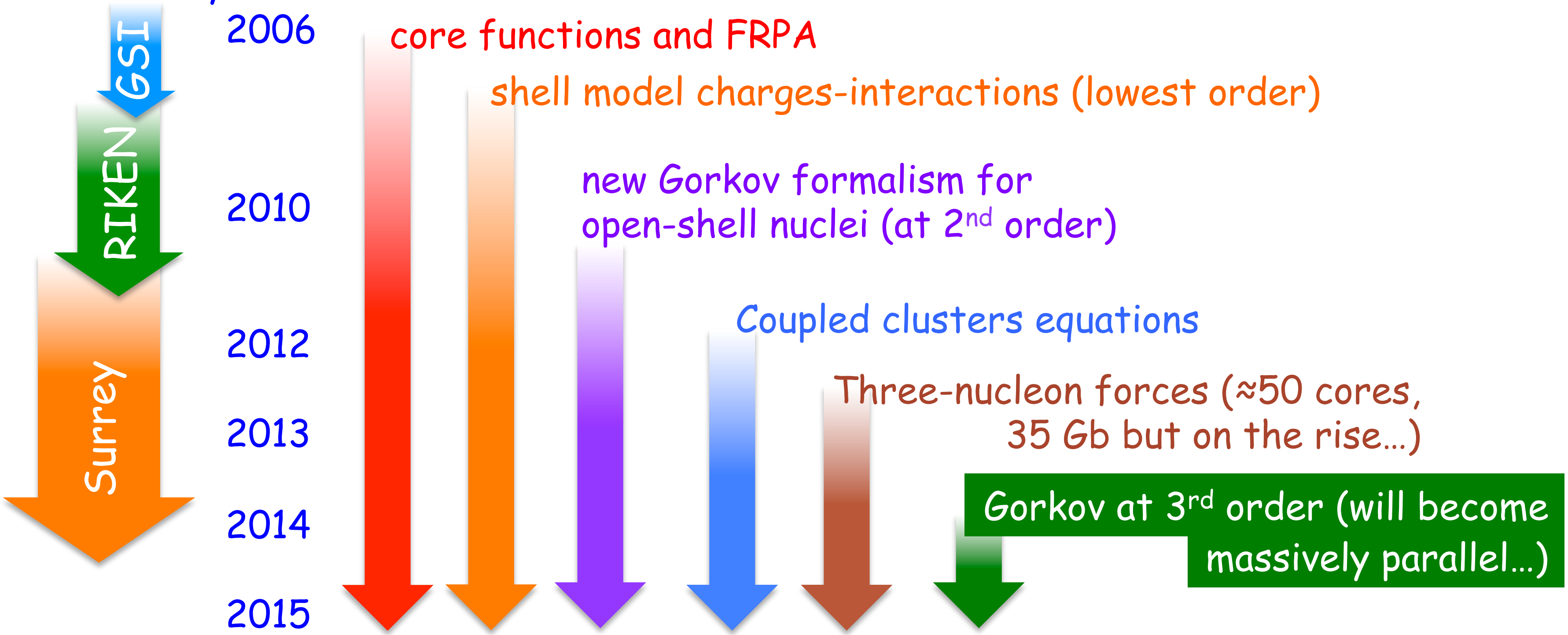
W. Dickhoff, CB, Prog. Part. Nucl. Phys. 53, 377 (2004)
CB, M.Hjorth-Jensen, Pys. Rev. C**79**, 064313 (2009)

Ab-initio Nuclear Computation & BcDor code

BoccaDorata code:
(C. Barbieri 2006-14
V. Somà 2011-14
A. Cipollone 2012-13)

- Provides a *C++ class library* for handling many-body propagators (≈40,000 lines, OpenMPI based).
- Allows to solve for nuclear spectral functions, many-body propagators, RPA responses, coupled cluster equations and effective interaction/charges for the shell model.

Code history:

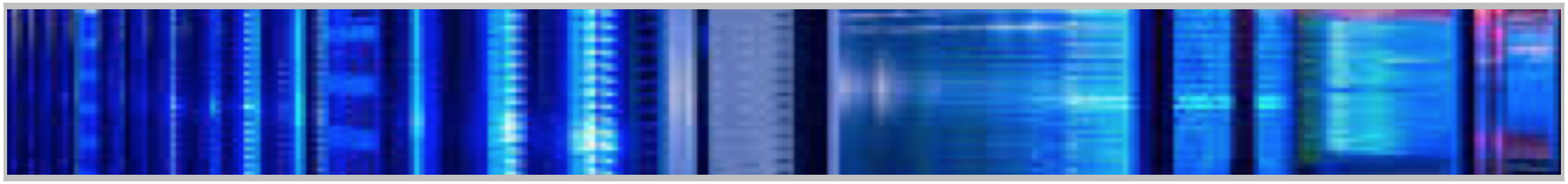


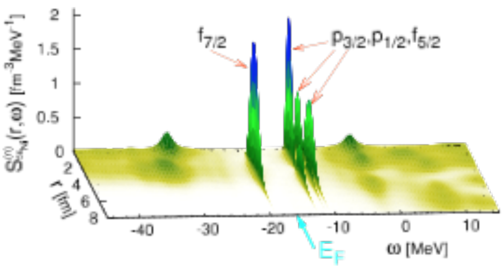
… applications …

Ab-initio Nuclear Computation & BcDor code

http://personal.ph.surrey.ac.uk/~cb0023/bcdor/

Computational Many-Body Physics





Download

Documentation

Welcome

From here you can download a public version of my self-consistent Green's function (SCGF) code for nuclear physics. This is a code in J-coupled scheme that allows the calculation of the single particle propagators (a.k.a. one-body Green's functions) and other many-body properties of spherical nuclei.

This version allows to:

- Perform Hartree-Fock calculations.
- Calculate the the correlation energy at second order in perturbation theory (MBPT2).
- Solve the Dyson equation for propagators (self consistently) up to second order in the self-energy.
- Solve coupled cluster CCD (doubles only!) equations.

When using this code you are kindly invited to follow the creative commons license agreement, as detailed at the weblinks below. In particular, we kindly ask you to refer to the publications that led the development of this software.

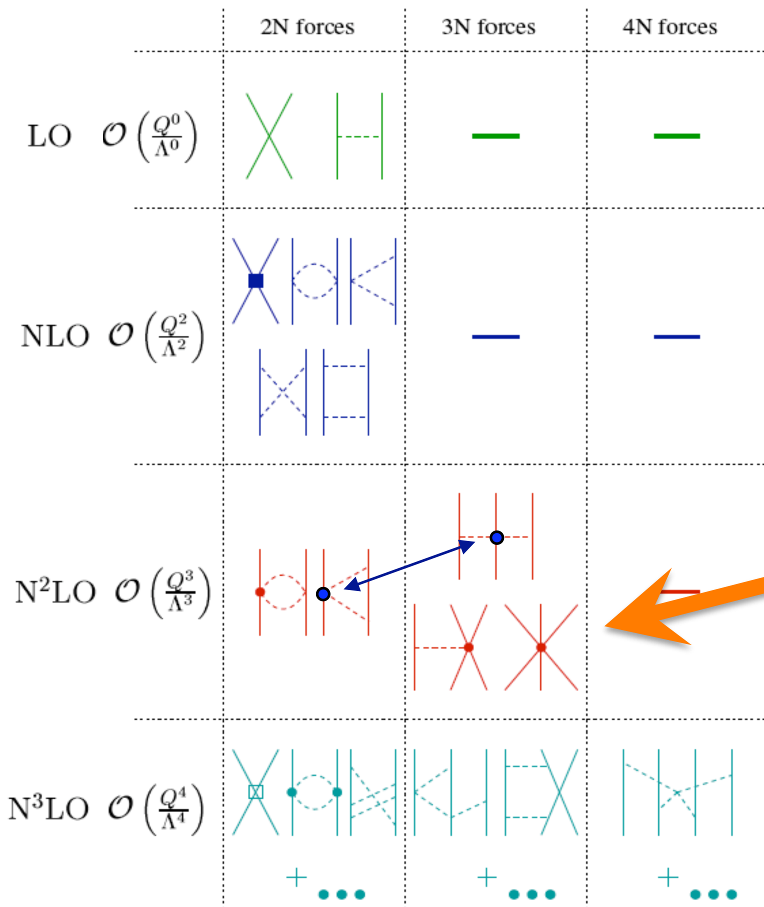
Relevant references (which can also help in using this code) are:

Prog. Part. Nucl. Phys. 52, p. 377 (2004),
Phys. Rev. A76, 052503 (2007),
Phys. Rev. C79, 064313 (2009),
Phys. Rev. C89, 024323 (2014)

Results

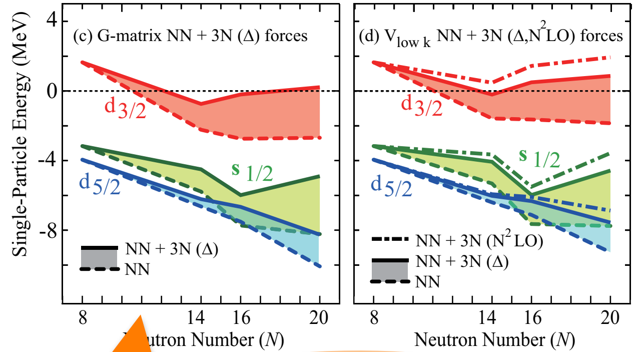
Modern realistic nuclear forces

Chiral EFT for nuclear forces:



(3NFs arise naturally at N2LO)

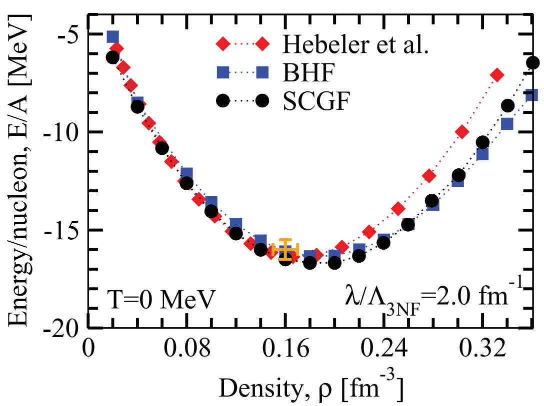
Single particle spectrum at E_{fermi}:



[T. Otsuka et al., Phys Rev. Lett **105**, 032501 (2010)]

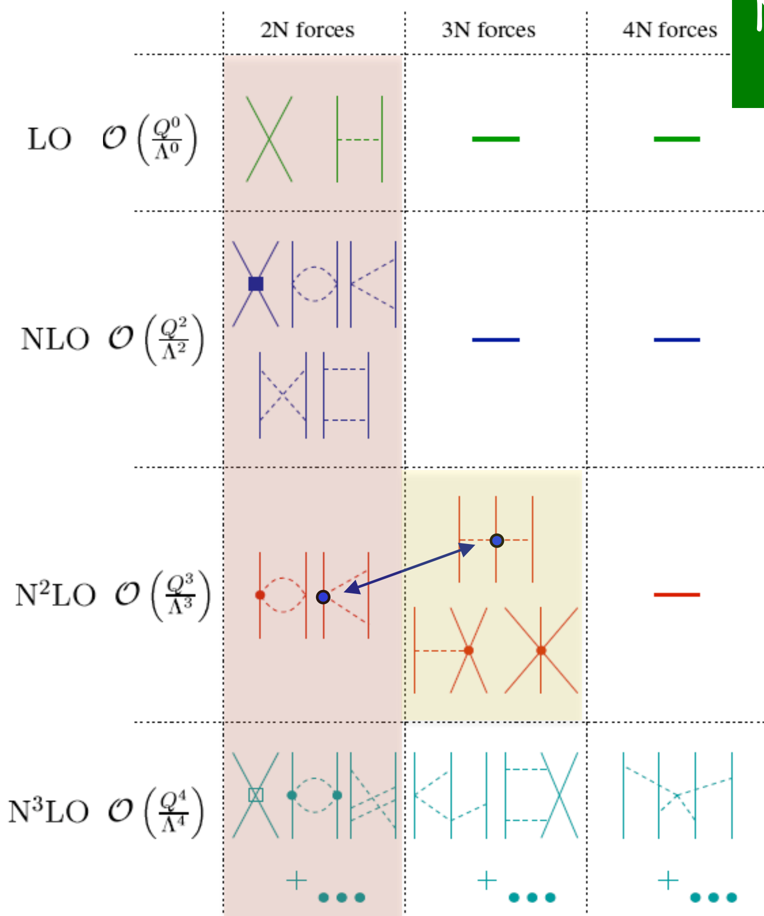
Need at LEAST 3NF!!!
("cannot" do RNB physics without…)

Saturation of nuclear matter:

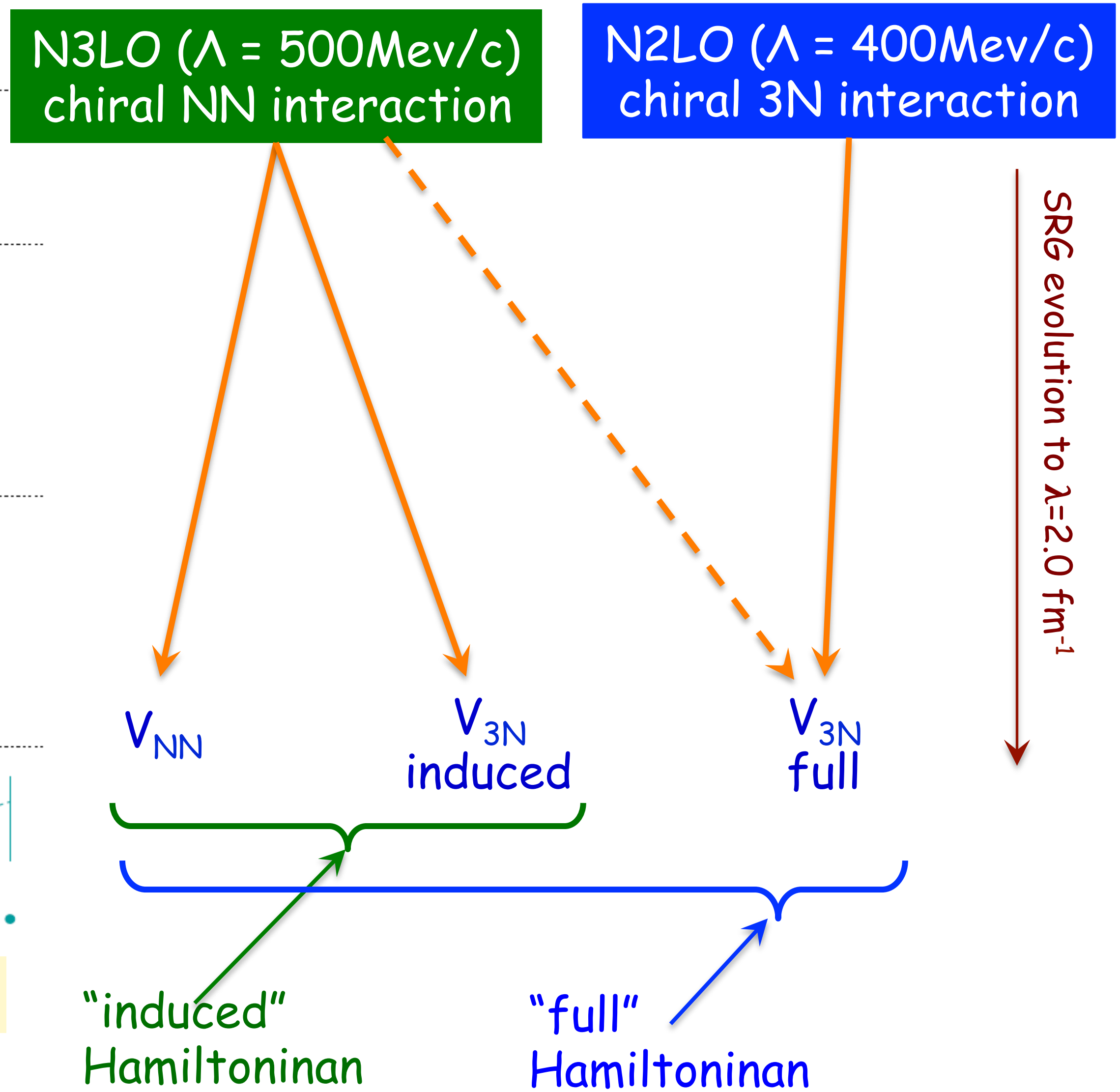


[A. Carbone et al., Phy.s Rev. C **88**, 044302 (2013)]

Chiral Nuclear forces - SRG evolved

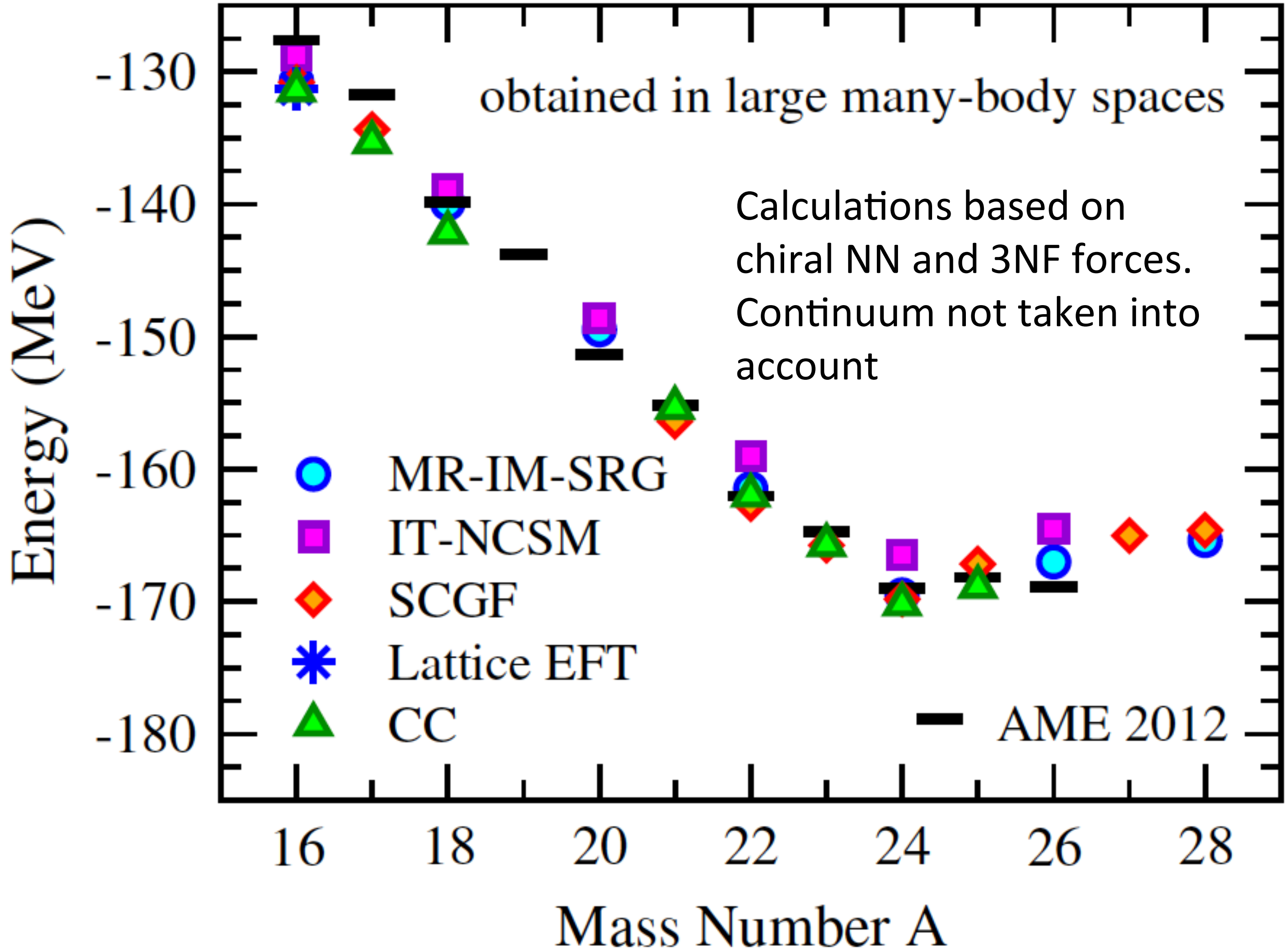


(3NFs arise naturally at N2LO)



[Jurgenson, Navrátil, Furnstahl, Phys. Rev. Lett. **103**, 082501 (2009); Hebeler, Phys. Rev. C 85, 021002 (2012)]

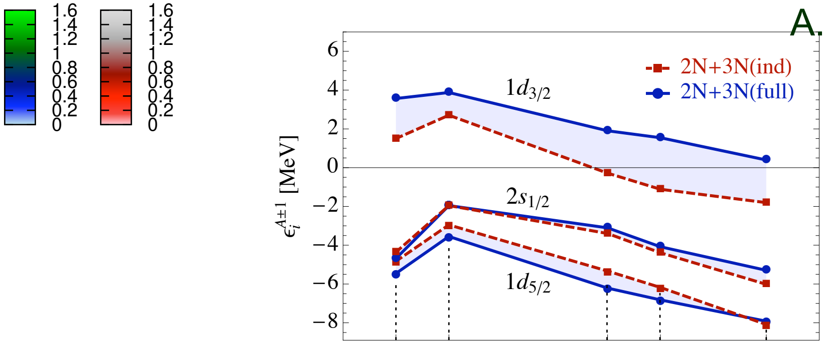
Benchmark of ab-initio methods in the oxygen isotopic chain

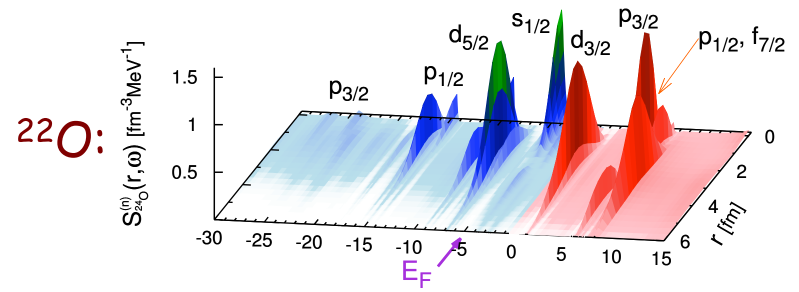


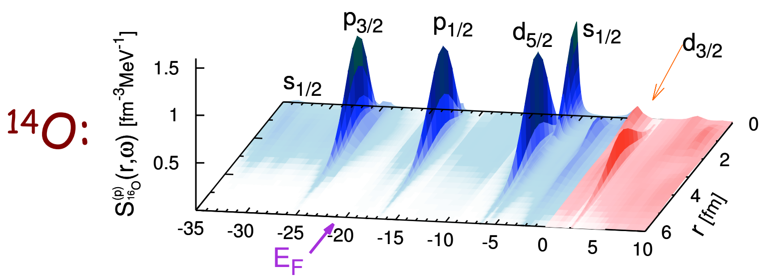
Hebeler, Holt, Menendez, Schwenk, Ann. Rev. Nucl. Part. Sci. in press (2015)

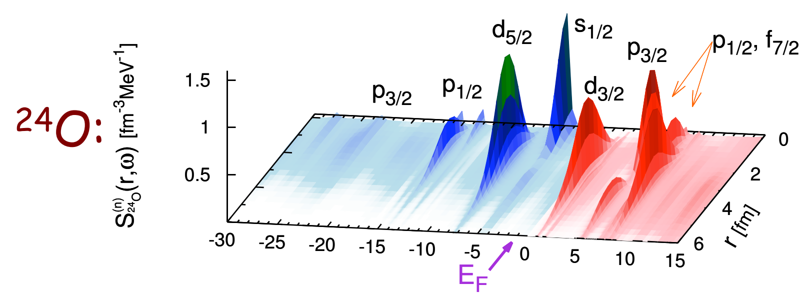
Neutron spectral function of Oxygens

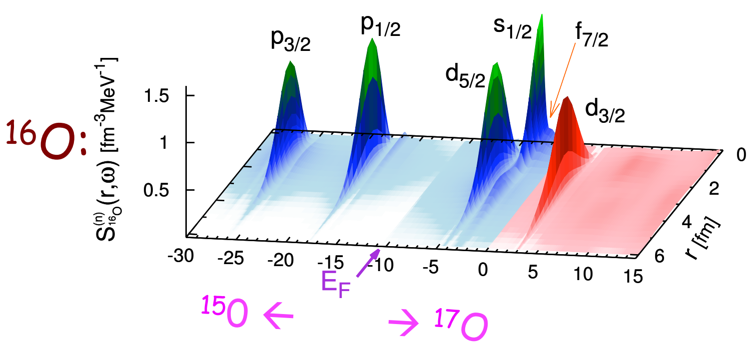
A. Cipollone, CB, P. Navrátil, *Phys. Rev. C* ***92****, 014306 (2015)*

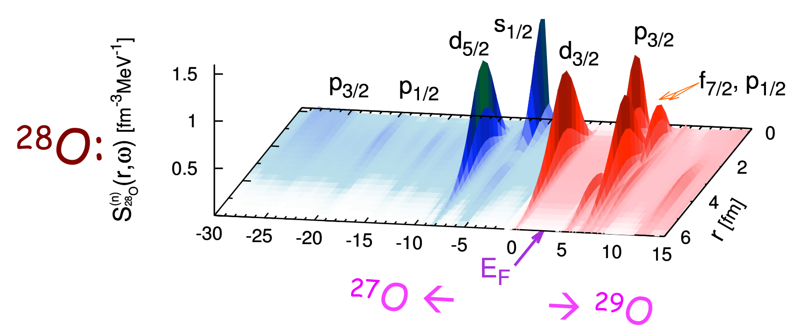












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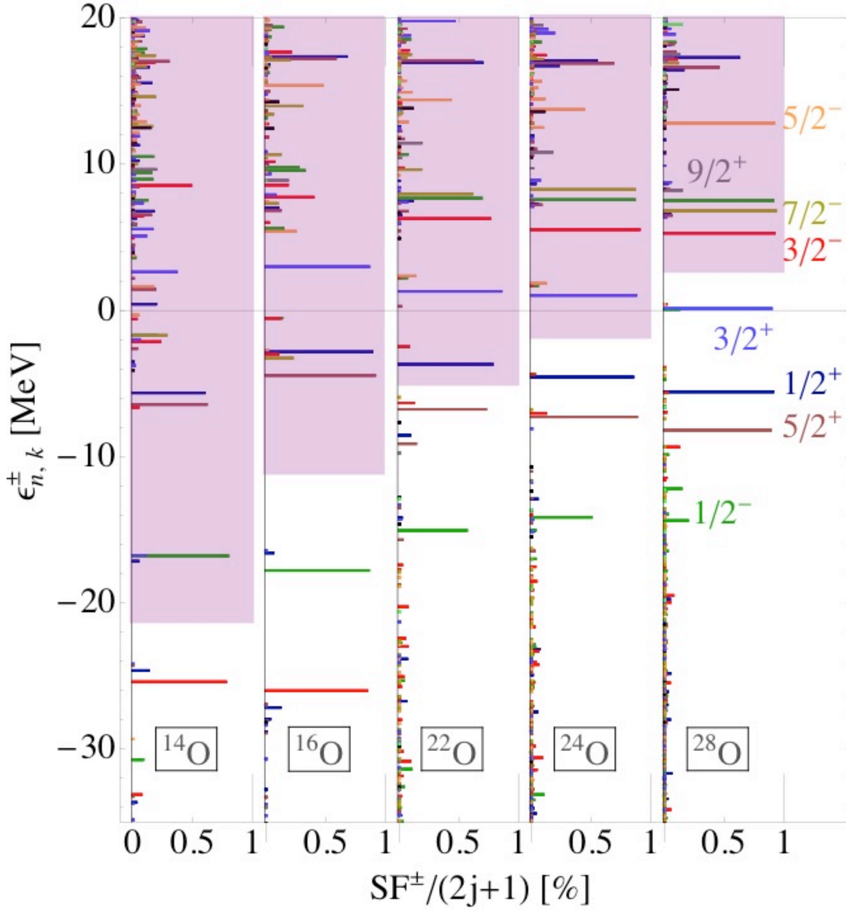
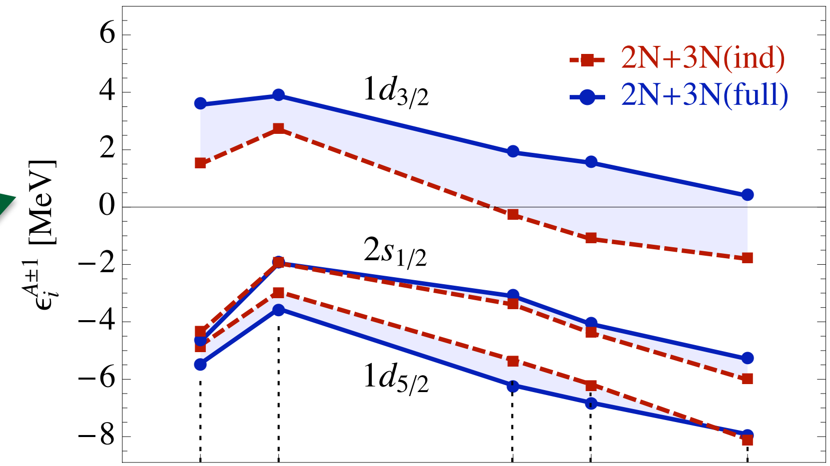
Results for the N-O-F chains

A. Cipollone, CB, P. Navrátil, Phys. Rev. Lett. **111**, 062501 (2013)
and arXiv:1412.3002 [nucl-th] (2014)



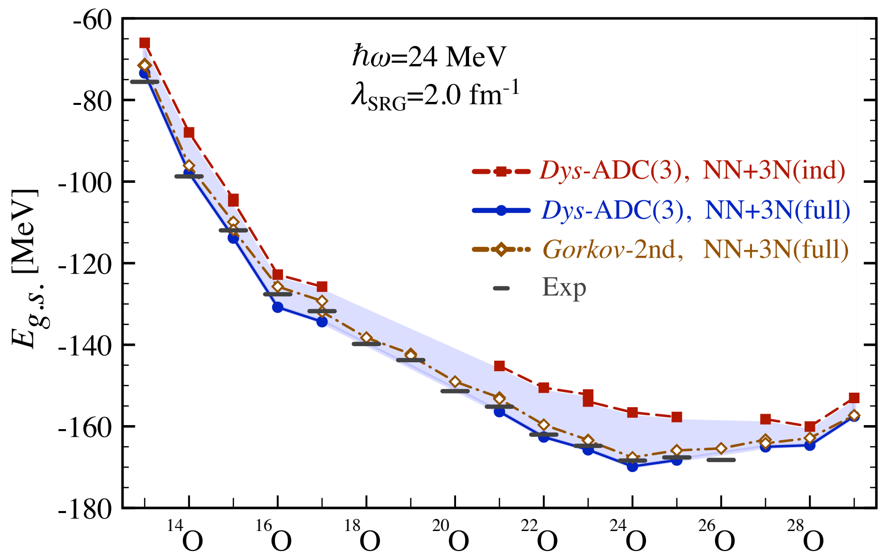


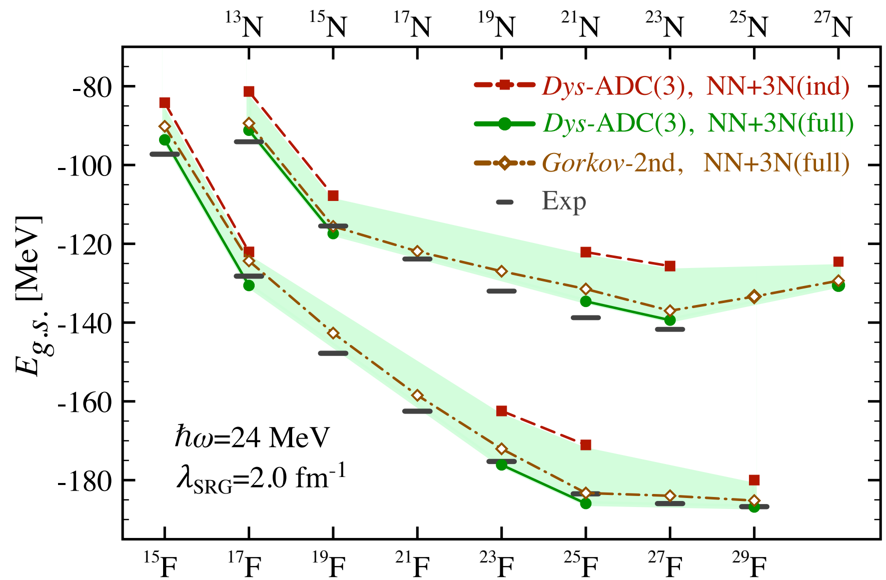
→ *$d_{3/2}$ raised* by genuine 3NF

→ cf. microscopic shell model [Otsuka et al, PRL**105**, 032501 (2010).]

Results for the N-O-F chains

A. Cipollone, CB, P. Navrátil, Phys. Rev. Lett. **111**, 062501 (2013)
and Phys. Rev. C **92**, 014306 (2015)





→ 3NF crucial for reproducing binding energies and driplines around oxygen

→ cf. microscopic shell model [Otsuka et al, PRL**105**, 032501 (2010).]

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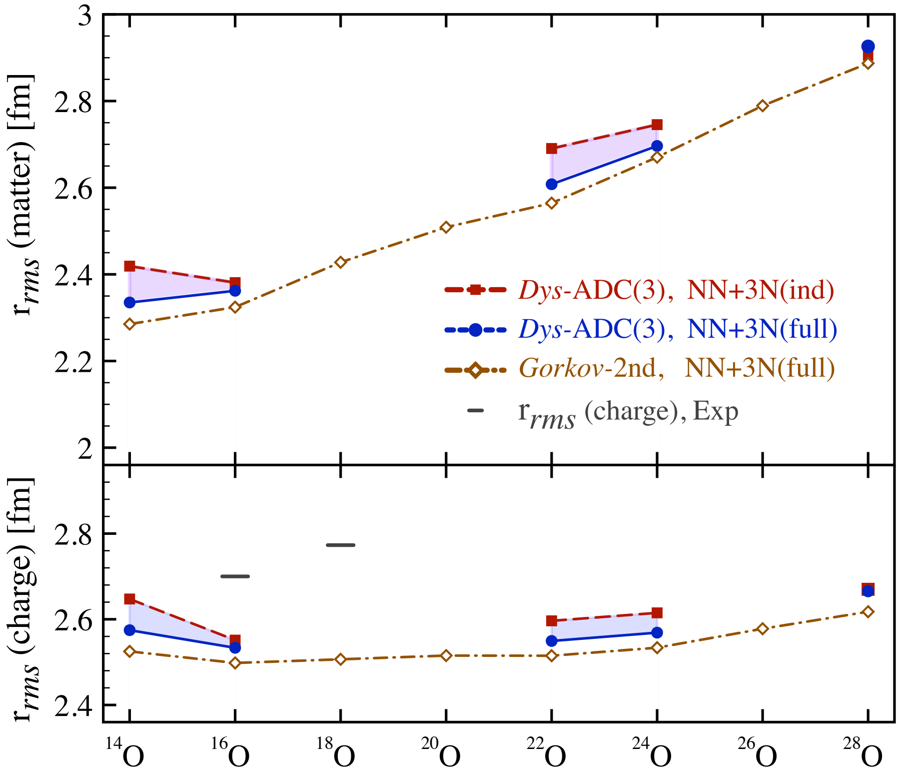
N3LO (Λ = 500Mev/c) chiral NN interaction evolved to 2N + 3N forces ($2.0 fm^{-1}$)
N2LO (Λ = 400Mev/c) chiral 3N interaction evolved ($2.0 fm^{-1}$)

Results for the oxygen chain

A. Cipollone, CB, P. Navrátil, Phys. Rev. C **92**, 014306 (2015)



→ Single particle spectra slightly to spread and

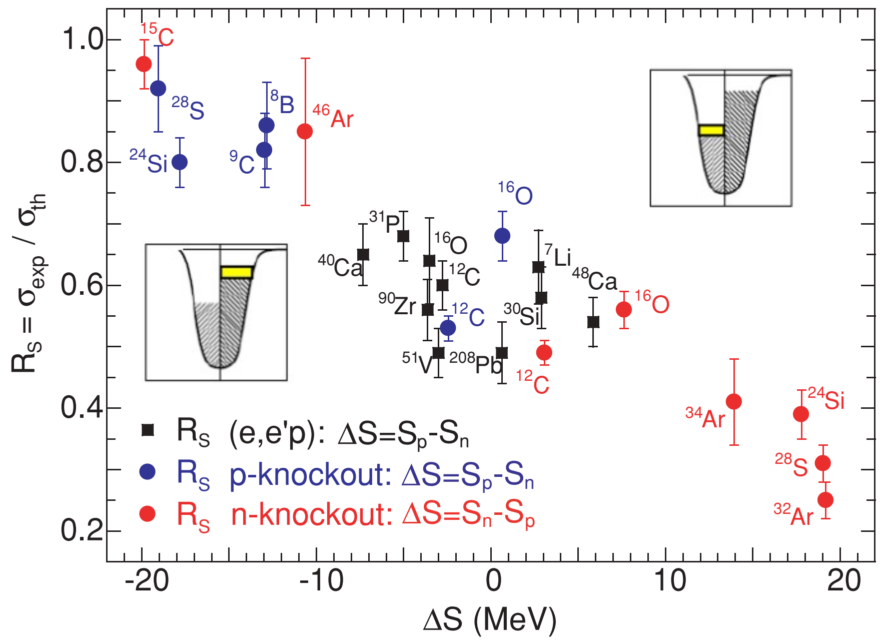
→ systematic underestimation of radii

Spectroscopic Factors

Spectroscopic factors @ limits of stability

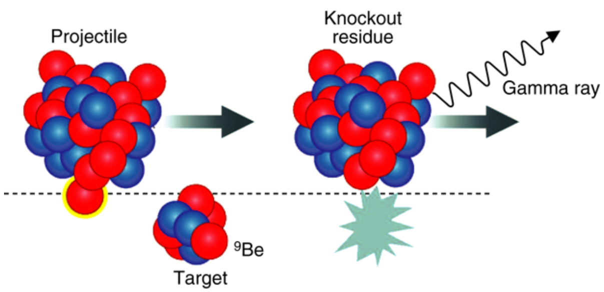
Weakling bound nucleons

Strongly bound nucleons



[Phys. Rev. C77, 044306 (2008)]

High energy knock-out in inverse kinematics



ORIGIN UNCLEAR ? ? ?

- *Challenged* by recent experiments

- May be correlations or scattering analysis

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Quenching of absolute spectroscopic factors

[CB, *Phys. Rev. Lett.* **103**, 202520 (2009)]

Overall quenching of *spectroscopic factors* is driven by:

SRC → *~10%*

part-vibr. coupling → *dominant*

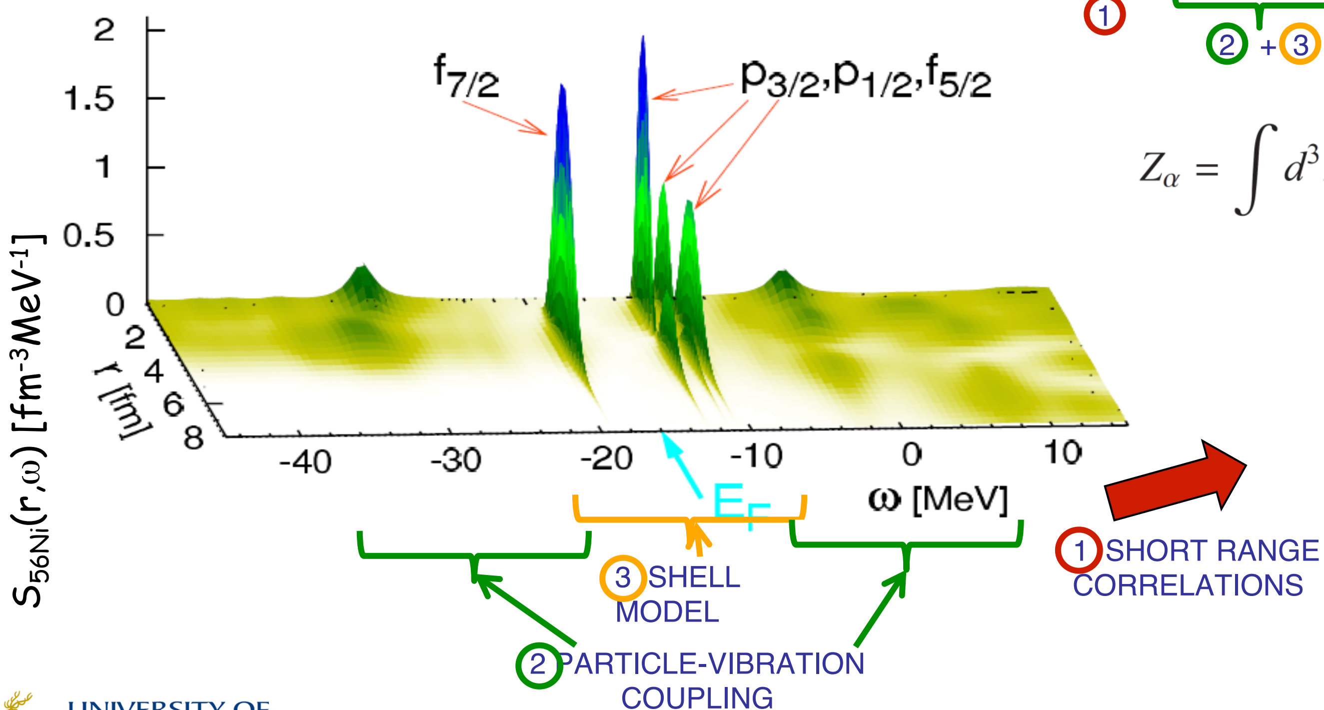
"shell-model" → *in open shell*

...with analogous conclusions for ^{48}Ca

	10 osc. shells			Exp. [30]	$1p0f$ space		
	FRPA (SRC)	full FRPA	FRPA $+\Delta Z_\alpha$		FRPA	SM	ΔZ_α
^{57}Ni:							
$\nu 1p_{1/2}$	0.96	0.63	0.61		0.79	0.77	-0.02
$\nu 0f_{5/2}$	0.95	0.59	0.55		0.79	0.75	-0.04
$\nu 1p_{3/2}$	0.95	0.65	0.62	0.58(11)	0.82	0.79	-0.03
^{55}Ni:							
$\nu 0f_{7/2}$	0.95	0.72	0.69		0.89	0.86	-0.03
	(1)	(2) + (3)			(3)		

^{57}Ni, ^{55}Ni

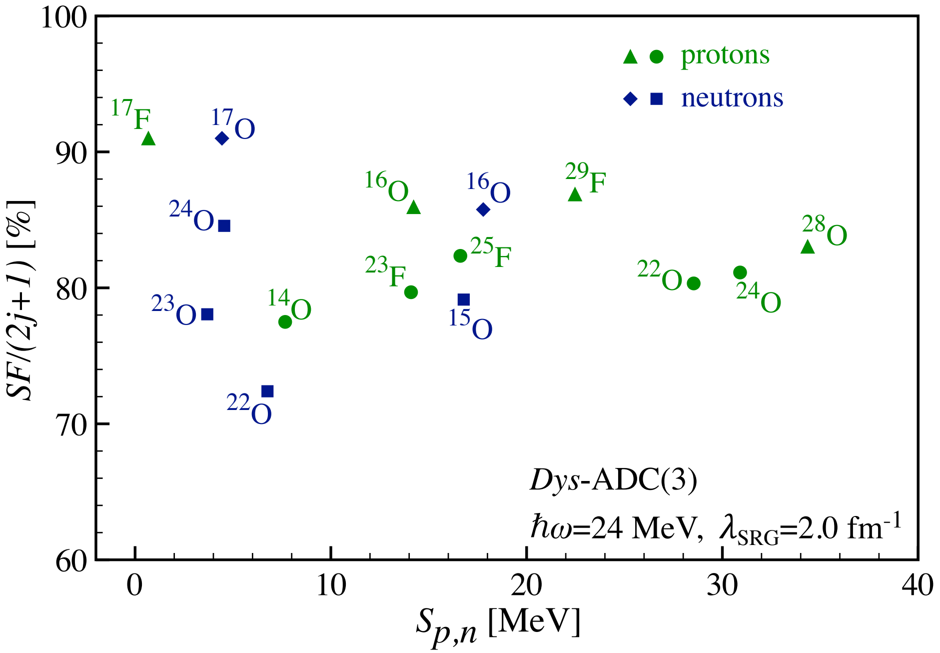
$$Z_\alpha = \int d^3r \, |\psi_\alpha^{overlap}(\mathbf{r})|^2 = \left. \frac{1}{1 - \frac{\partial \Sigma_{\hat\alpha\hat\alpha}(\omega)}{\partial \omega}} \right|_{\omega=\varepsilon_\alpha}$$



Z/N asymmetry dependence of SFs - Theory

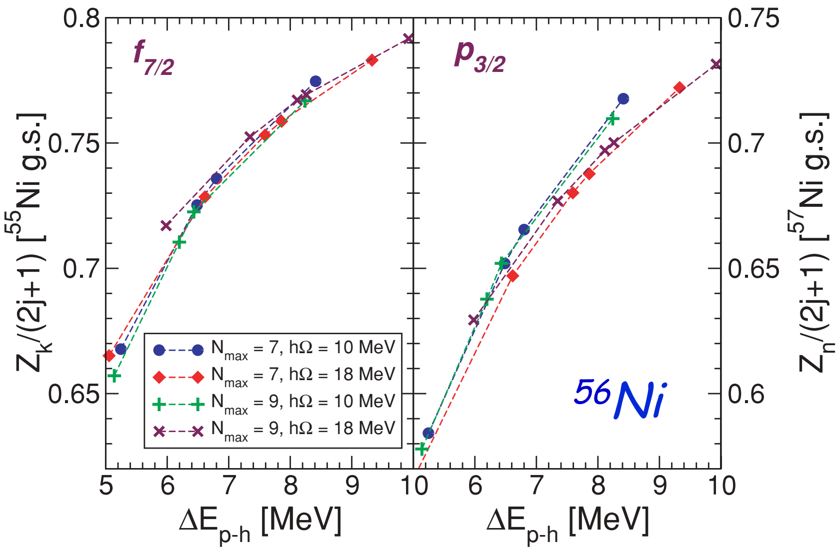
Ab-initio calculations explain the Z/N dependence but the effect is much lower than suggested by direct knockout

Effects of continuum become important at the driplines



arXiv:1412.3002 [nucl-th] (2014)

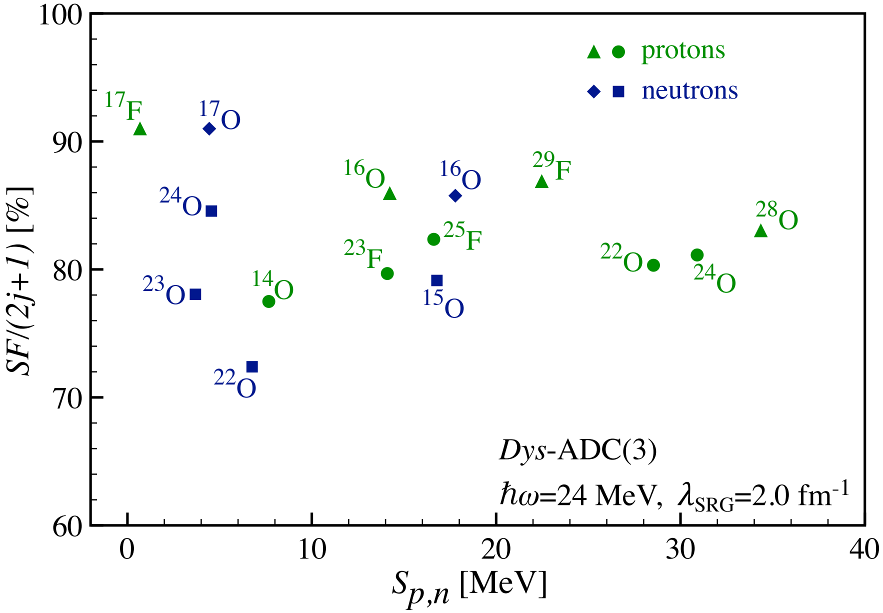
Spectroscopic factor are strongly correlated to p-h gaps:



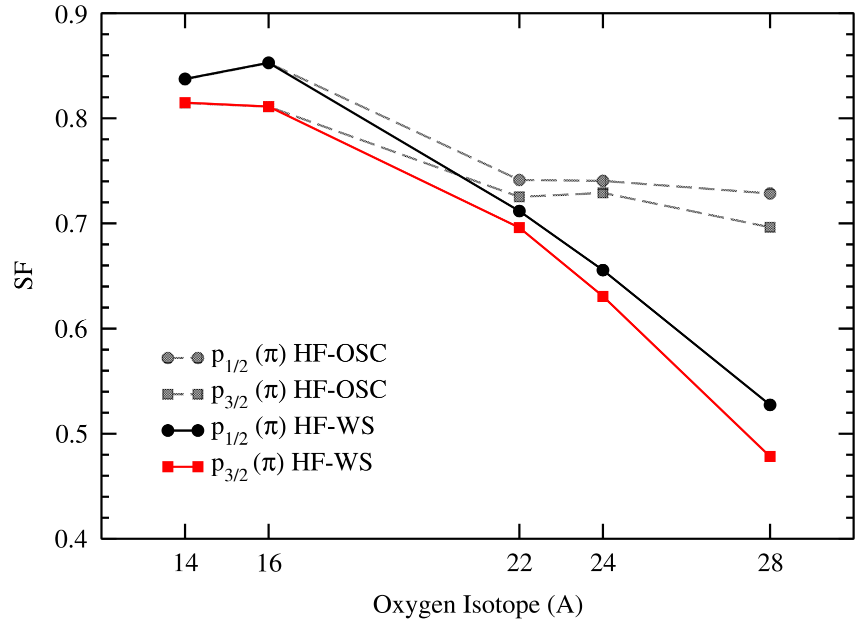
Z/N asymmetry dependence of SFs - Theory

Ab-initio calculations explain the Z/N dependence but the effect is much lower than observed

Effects of continuum become important at the driplines



arXiv:1412.3002 [nucl-th] (2014)



[Hagen et al.
Phys. Rev. Lett. 107, 032501 (2011)]

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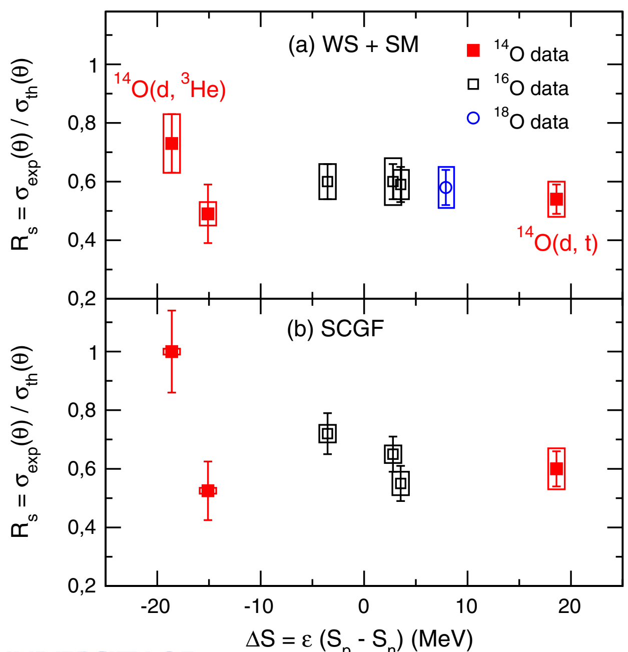
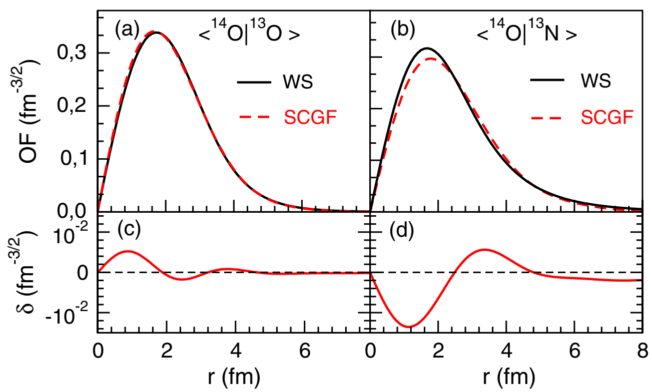
Single nucleon transfer in the oxygen chain

[F. Flavigny et al, PRL**110**, 122503 (2013)]

→ Analysis of ^{14}O(d,t)^{13}O and ^{14}O(d,^{3}He)^{13}N transfer reactions @ SPIRAL

Reaction	E^* (MeV)	J^π	$R_{\rm rms}^{\rm HFB}$ (fm)	r_0 (fm)	$C^2S_{\rm exp}$ (WS)	$C^2S_{\rm th}$ $0p+2\hbar\omega$	R_s (WS)	$C^2S_{\rm exp}$ (SCGF)	$C^2S_{\rm th}$ (SCGF)	R_s (SCGF)
^{14}O (d, t) ^{13}O	0.00	$3/2^-$	2.69	1.40	1.69 (17)(20)	3.15	0.54(5)(6)	1.89(19)(22)	3.17	0.60(6)(7)
^{14}O (d, ^{3}He) ^{13}N	0.00	$1/2^-$	3.03	1.23	1.14(16)(15)	1.55	0.73(10)(10)	1.58(22)(2)	1.58	1.00(14)(1)
	3.50	$3/2^-$	2.77	1.12	0.94(19)(7)	1.90	0.49(10)(4)	1.00(20)(1)	1.90	0.53(10)(1)
^{16}O (d, t) ^{15}O	0.00	$1/2^-$	2.91	1.46	0.91(9)(8)	1.54	0.59(6)(5)	0.96(10)(7)	1.73	0.55(6)(4)
^{16}O (d, ^{3}He) ^{15}N [19,20]	0.00	$1/2^-$	2.95	1.46	0.93(9)(9)	1.54	0.60(6)(6)	1.25(12)(5)	1.74	0.72(7)(3)
	6.32	$3/2^-$	2.80	1.31	1.83(18)(24)	3.07	0.60(6)(8)	2.24(22)(10)	3.45	0.65(6)(3)
^{18}O (d, ^{3}He) ^{17}N [21]	0.00	$1/2^-$	2.91	1.46	0.92(9)(12)	1.58	0.58(6)(10)			





- Overlap functions and strengths from GF
- Rs independent of asymmetry

Knockout & transfer experiments

- Neutron removal from proton- and neutron- Ar isotopes @ NSCL:

Isotopes	lj^{π}	Sn(MeV)	ΔS (MeV)	(theo.) SF(LB-SM)	(expt.) SF(JLM + HF)	Rs(JLM + HF)	(expt.) SF(CH89)	Rs(CH89)
^{34}Ar	$s1/2^+$	17.07	12.41	1.31	0.85 ± 0.09	0.65 ± 0.07	1.10 ± 0.11	0.84 ± 0.08
^{36}Ar	$d3/2^+$	15.25	6.75	2.10	1.60 ± 0.16	0.76 ± 0.08	2.29 ± 0.23	1.09 ± 0.11
^{46}Ar	$f7/2^-$	8.07	−10.03	5.16	3.93 ± 0.39	0.76 ± 0.08	5.29 ± 0.53	1.02 ± 0.10

[Lee *et al.* 2010]

$\Delta S = Sn - Sp$

Gorkov GF NN

	Sn (MeV)	ΔS (MeV)	SF
^{34}Ar	33.0	18.6	1.46
^{36}Ar	27.7	7.5	1.46
^{46}Ar	16.0	-22.3	5.88

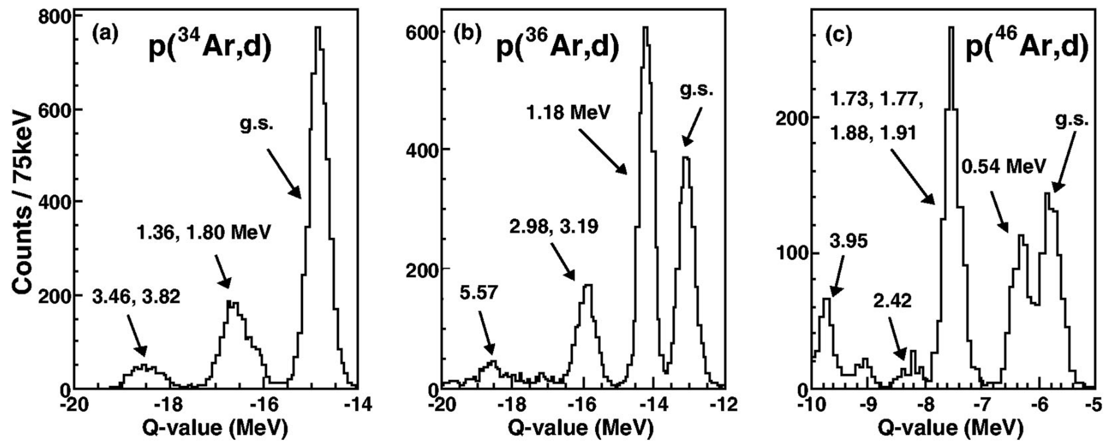
Gorkov GF NN + 3N

	Sn (MeV)	ΔS (MeV)	SF
^{34}Ar	22.4	15.5	1.56
^{36}Ar	15.3	7.2	1.54
^{46}Ar	6.5	-15.7	6.64

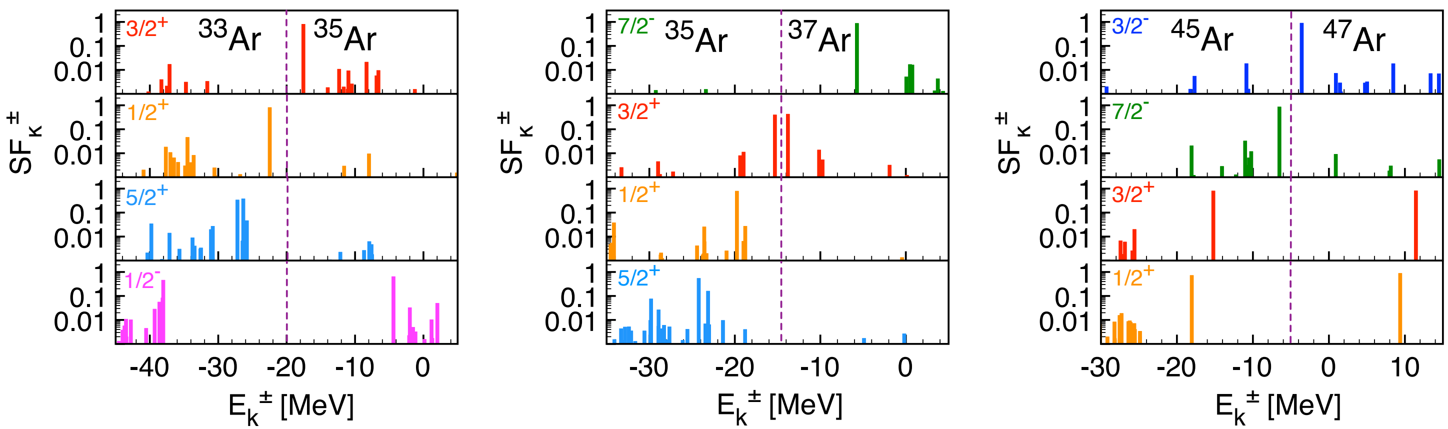
V.Somà, CB, et al, Eur. Phys. Jour.: Web. of Conf. **66**, 02005 (2014).

Knockout & transfer experiments

- Neutron removal from proton- and neutron- Ar isotopes @ NSCL:



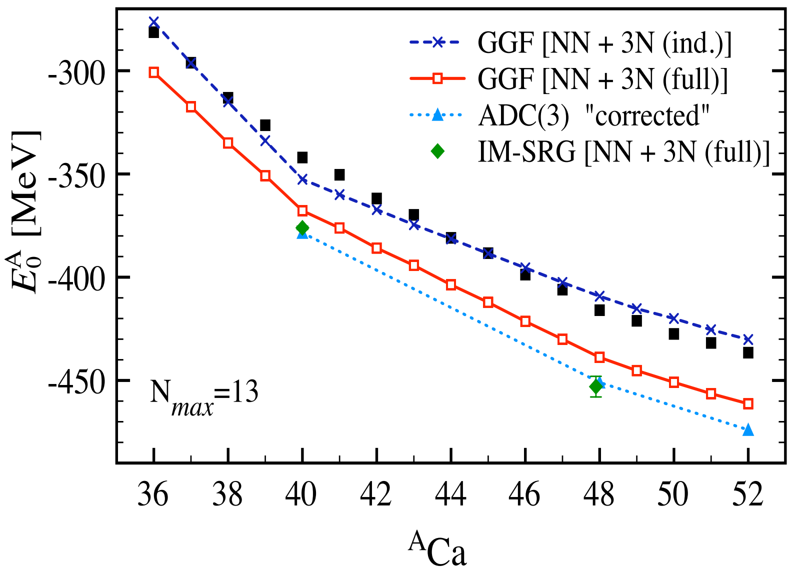
[Lee *et al.* 2010]

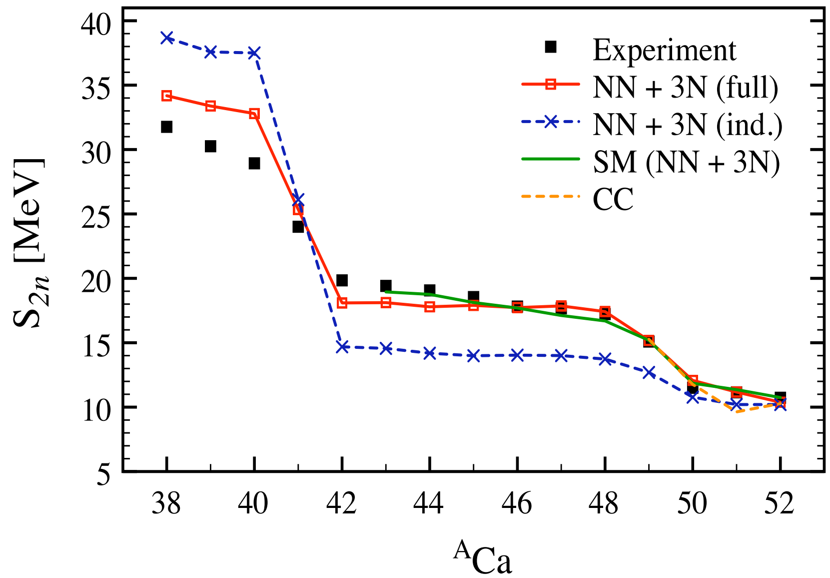


V.Somà, CB, et al, Eur. Phys. Jour.: Web. of Conf. **66**, 02005 (2014).

Calcium isotopic chain

Ab-initio calculation of the whole Ca: *induced* and full 3NF investigated





→ *induced* and full 3NF investigated

→ genuine (N2LO) 3NF needed to reproduce the energy curvature and S_{2n}

→ N=20 and Z=20 gaps overestimated!

→ Full 3NF give a correct trend but over bind!

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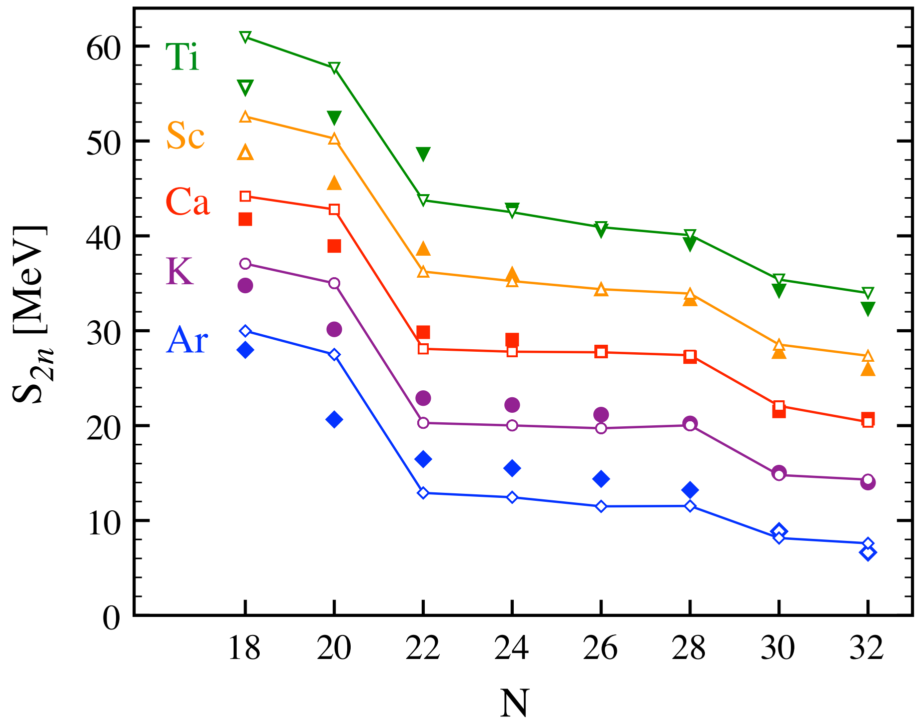
V. Somà, CB *et al.* Phys. Rev. C**89**, 061301R (2014)

Neighbouring Ar, K, Ca, Sc, and Ti chains

V. Somà, CB *et al.* Phys. Rev. C**89**, 061301R (2014)

Two-neutron separation energies predicted by chiral NN+3NF forces:



→ First *ab-initio* calculation over a contiguous portion of the nuclear chart—open shells are now possible through the Gorkov-GF formalism

Neighbouring Ar, K, Ca, Sc, and Ti chains

V. Somà, CB *et al.* Phys. Rev. C**89**, 061301R (2014)

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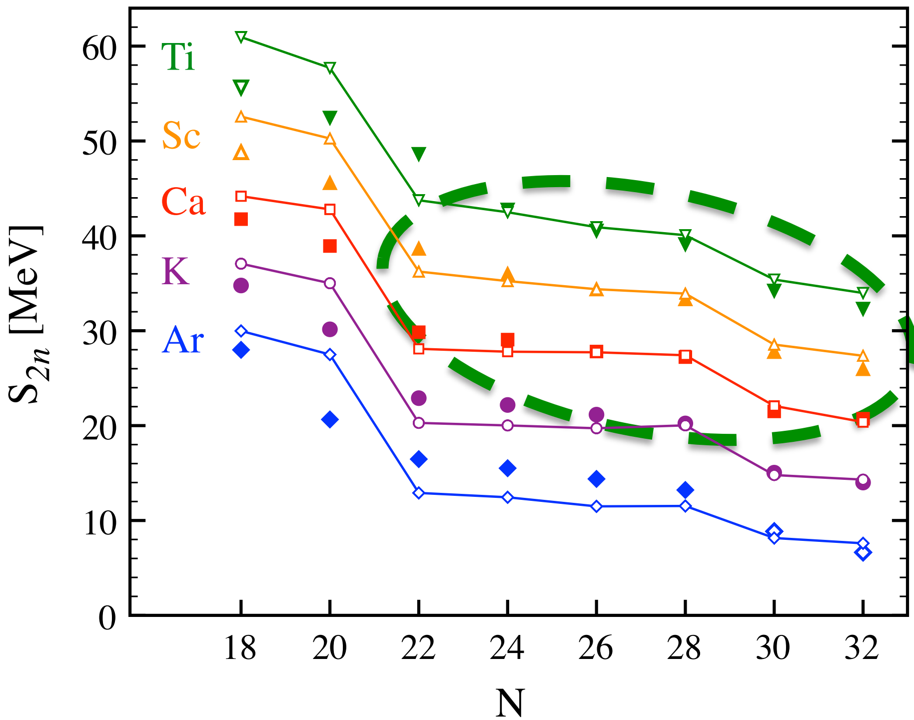

Works well in the *pf* shell

→ First *ab-initio* calculation over a contiguous portion of the nuclear chart—open shells are now possible through the Gorkov-GF formalism

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Neighbouring Ar, K, Ca, Sc, and Ti chains

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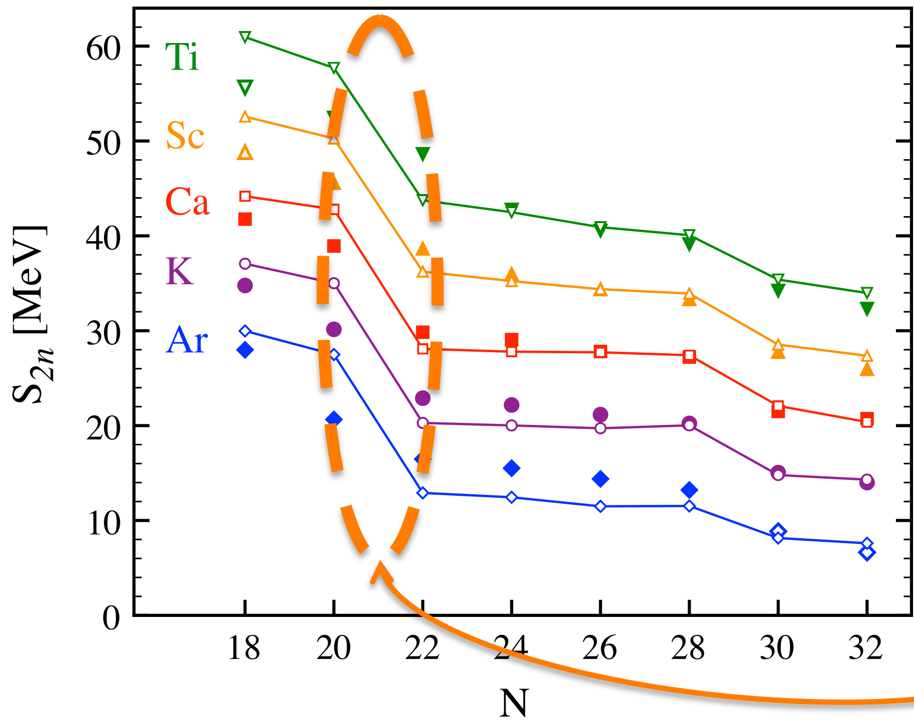

Over estimated
N=20 and Z=20 gaps

→ First *ab-initio* calculation over a contiguous portion of the nuclear chart—open shells are now possible through the Gorkov-GF formalism

Neighbouring Ar, K, Ca, Sc, and Ti chains

V. Somà, CB *et al.* Phys. Rev. C**89**, 061301R (2014)

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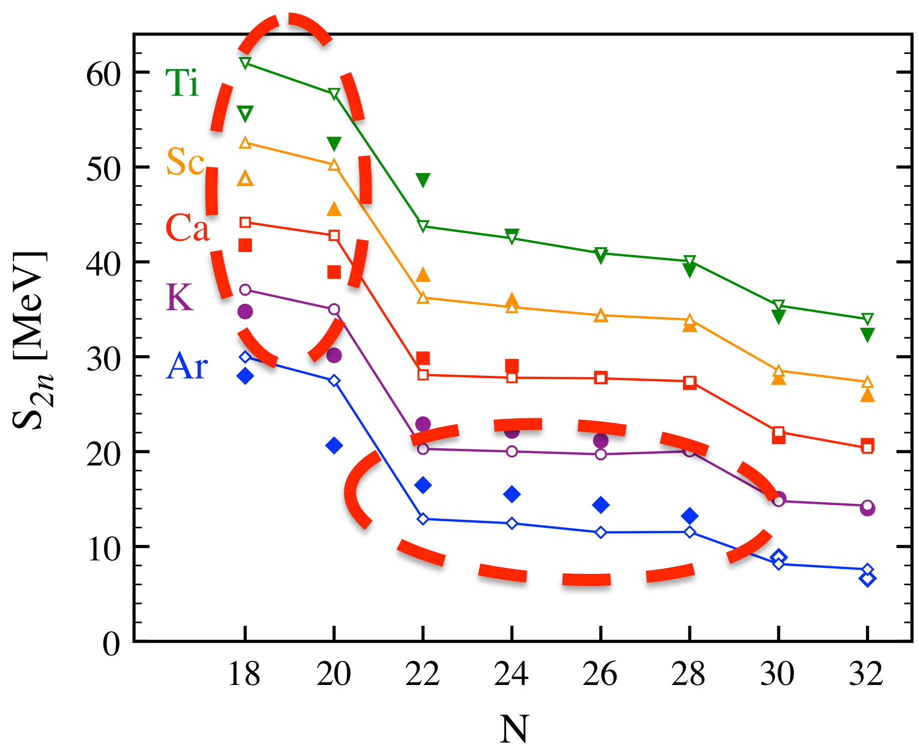
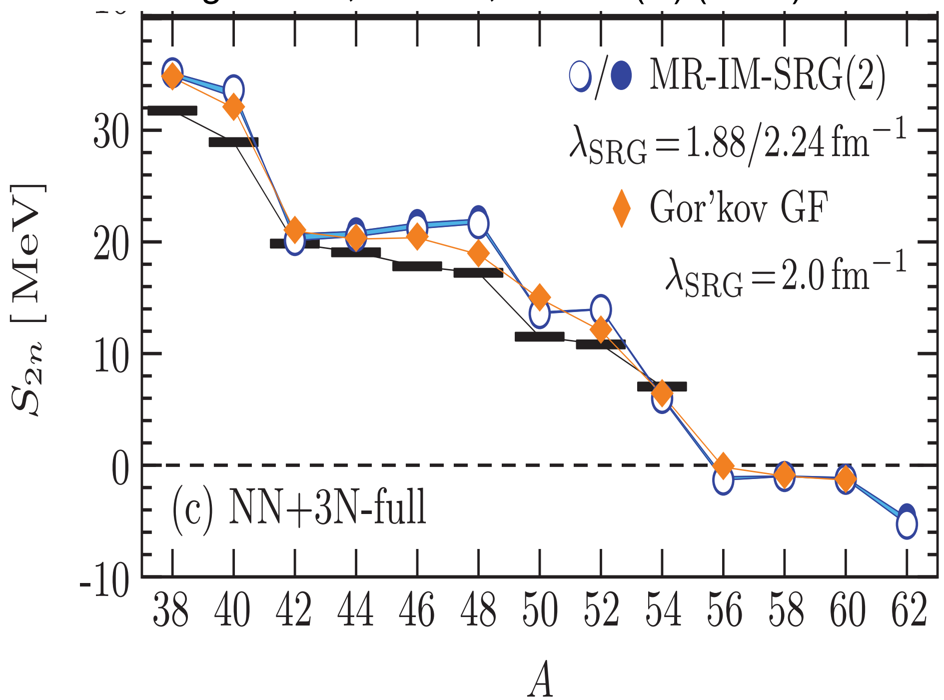


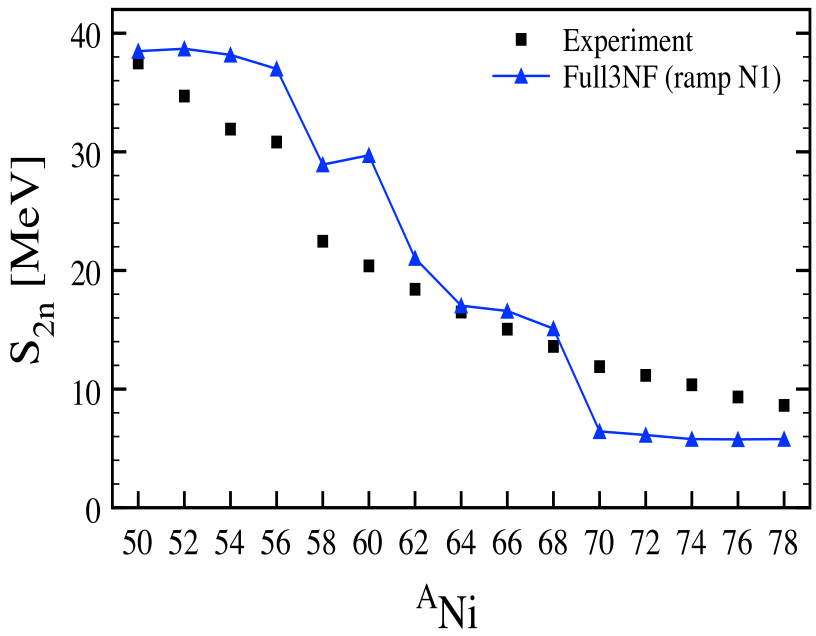
Lack of deformation due to quenched cross-shell quadrupole excitations

→ First *ab-initio* calculation over a contiguous portion of the nuclear chart—open shells are now possible through the Gorkov-GF formalism

Ca and Ni isotopic chains

Hergert et al, PRC**90**, 041302(R) (2014)



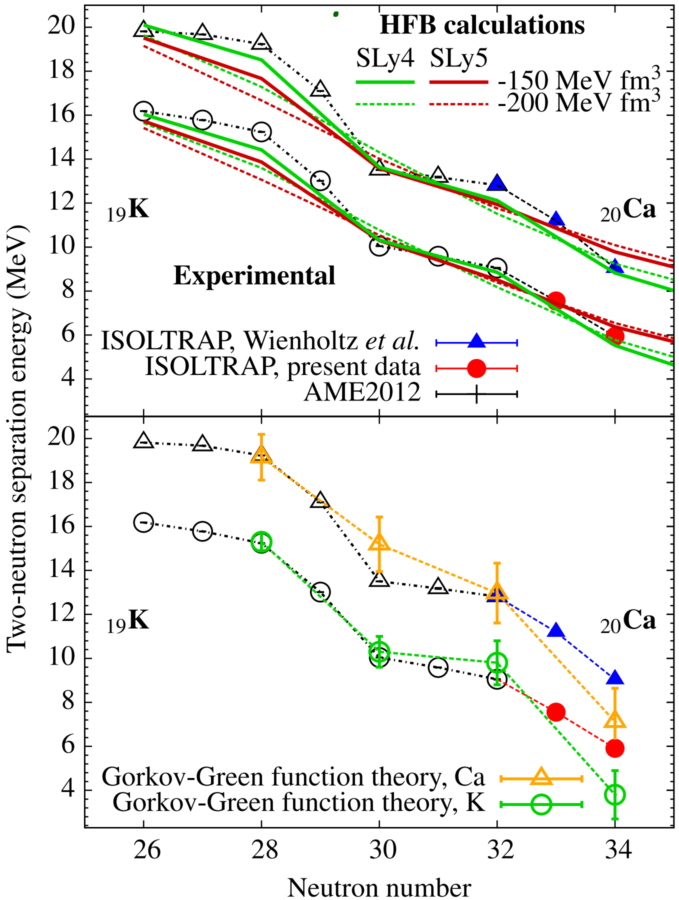


→ Large J in free space SRG matter (must pay attention to its convergence)

→ Overall conclusions regarding over binding and S_{2n} remain but details change

Two-neutron separation energies for neutron rich K isotopes

*M. Rosenbusch, et al., PRL**114**, 202501 (2015)*



Measurements
@ ISOLTRAP

Theory tend to overestimate the gap at N=34, but overall good

→ Error bar in predictions are from extrapolating the many-body expansion to convergence of the model space.

Inversion of $d_{3/2}$—$s_{1/2}$ at N=28

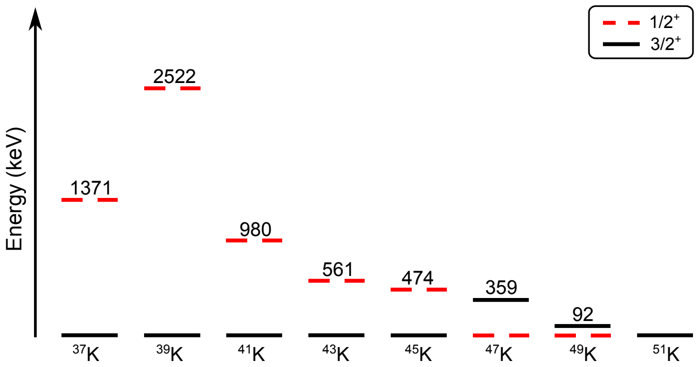


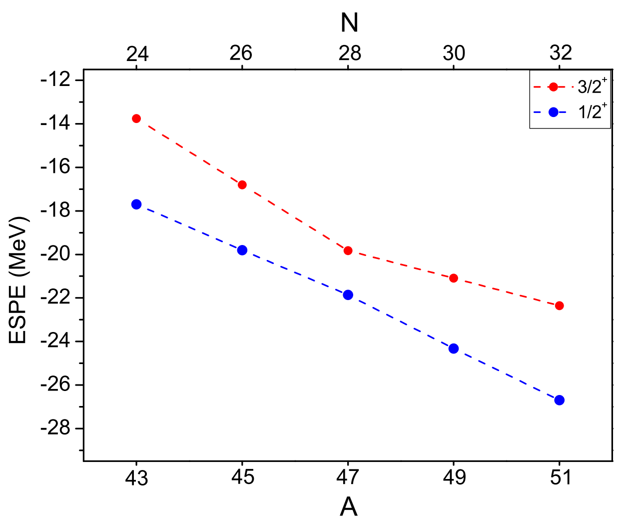
FIG. 1. (color online) Experimental energies for $1/2^+$ and $3/2^+$ states in odd-A K isotopes. Inversion of the nuclear spin is obtained in ^{47,49}K and reinversion back in ^{51}K. Results are

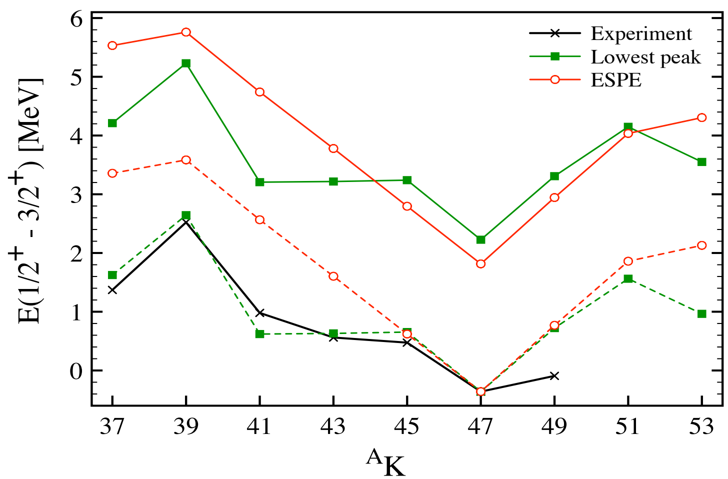
J. Papuga, et al., Phys. Rev. Lett. **110**, 172503 (2013);
Phys. Rev. C **90**, 034321 (2014)

^{A}K isotopes

Laser spectroscopy @ ISOLDE

Change in separation described by chiral NN+3NF:





ESPE: "centroid" energies

(Gorkov calculations at 2nd order)

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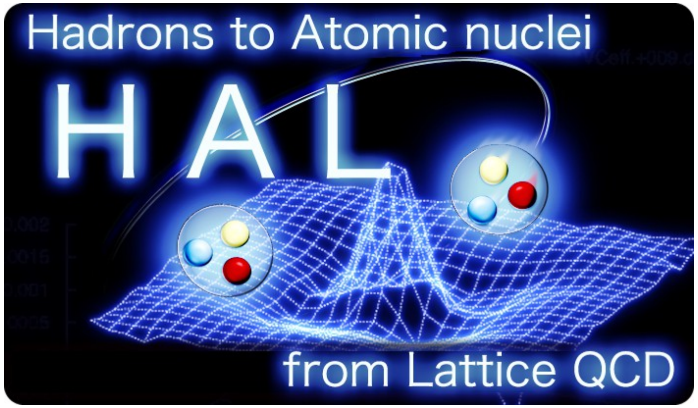
Study of nuclear interactions from Lattice QCD

In collaboration with:



Why should we investigate LQCD interactions?

- *It gives complimentary insight to the EFT approach:*
    - *Allows to approach physical interaction from heavy quark masses (opposite direction than the chiral limit).*
    - *Can study implications of SU(3) limit.*
- *No need to fit to experiment. No LEC constants.*
- *Provides consistent interactions in the Hyperon sector.*

- *It is very fundamental approach (QCD), and an alternative to Chiral-EFT.*

Challenges and limitations:

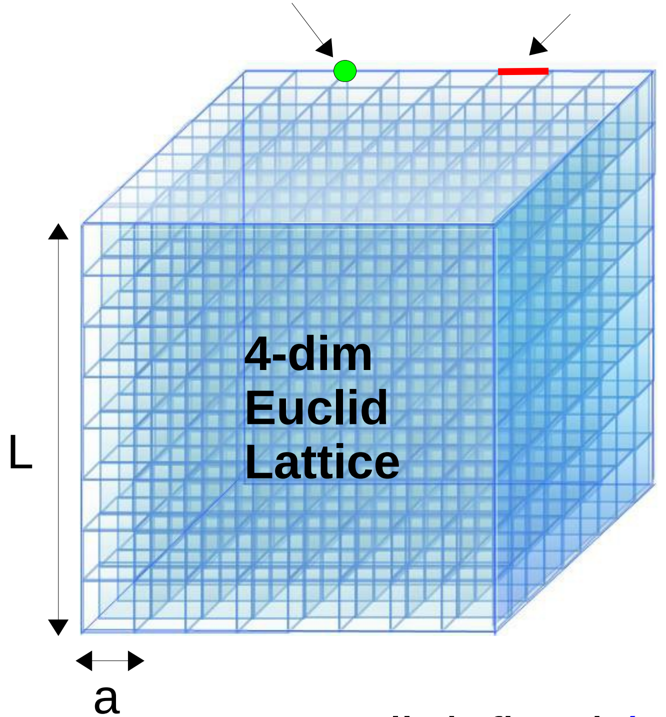
- *Mostly LO terms of the NN force exploited so far (but being improved).*
- *Physical pion mass limit requires efforts (but underway).*
- *NNN only barely addressed.*
- *Strong short-range repulsion is a challenge to ab-initio approaches.*

Lattice QCD

$$L = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \bar{q}\gamma^\mu \left(i\partial_\mu - g t^a A^a_\mu \right) q - m\bar{q}q$$

quarks q
on the sites

gluons $U = e^{iaA_\mu}$
on the links



Vacuum expectation value

$$\langle O(\bar{q}, q, U) \rangle$$

path integral

$$= \int dU\, d\bar{q}\, dq\, e^{-S(\bar{q},q,U)}\, O(\bar{q}, q, U)$$

$$= \int dU \det D(U) e^{-S_U(U)}\, O(D^{-1}(U))$$

quark propagator

$$= \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} O(D^{-1}(U_i))$$

$\{ U_i \}$: ensemble of gauge conf. U
generated w/ probability $\det D(U)\, e^{-S_U(U)}$

- Well defined (regularized)
- Manifest gauge invariance
- Fully non-perturvative
- Highly predictive

HAL Method

S. Aoki, T. Hatsuda, N. Ishii, Prog. Theo. Phys. 123 89 (2010)
N. Ishii etal. [HAL QCD coll.] Phys. Lett. B712 , 437 (2012)

NBS wave function $\varphi_{\vec{k}}(\vec{r}) = \sum_{\vec{x}} \langle 0 | B_i(\vec{x}+\vec{r},t) B_j(\vec{x},t) | B=2, \vec{k} \rangle$

Define a common potential U for all E eigenstates by a "Schrödinger" eq.

$$\left[-\frac{\nabla^2}{2\mu} \right] \varphi_{\vec{k}}(\vec{r}) + \int d^3\vec{r}\,' U(\vec{r},\vec{r}\,') \varphi_{\vec{k}}(\vec{r}\,') = E_{\vec{k}} \varphi_{\vec{k}}(\vec{r})$$

Non-local but
energy independent
below inelastic threshold

Measure 4-point function in LQCD

$$\psi(\vec{r},t) = \sum_{\vec{x}} \langle 0 | B_i(\vec{x}+\vec{r},t) B_j(\vec{x},t)\, J(t_0) | 0 \rangle = \sum_{\vec{k}} A_{\vec{k}} \varphi_{\vec{k}}(\vec{r}) e^{-W_{\vec{k}}(t-t_0)} + \cdots$$

$$\left[2M_B - \frac{\nabla^2}{2\mu} \right] \psi(\vec{r},t) + \int d^3\vec{r}\,' U(\vec{r},\vec{r}\,') \psi(\vec{r}\,',t) = -\frac{\partial}{\partial t} \psi(\vec{r},t)$$

∇ expansion
& truncation

$$U(\vec{r},\vec{r}\,') = \delta(\vec{r}-\vec{r}\,') V(\vec{r},\nabla) = \delta(\vec{r}-\vec{r}\,')[V(\vec{r}) + \cancel{\nabla + \nabla^2} \ldots]$$

Therefor, in the leading

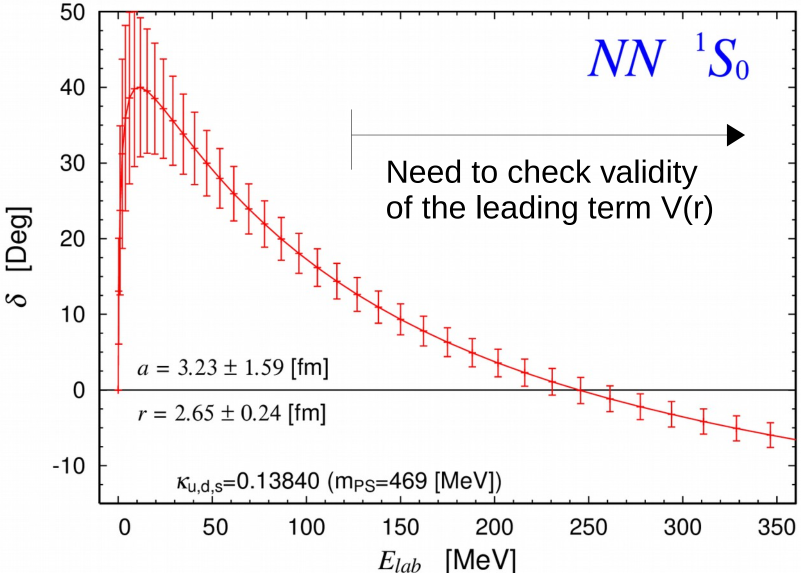
$$V(\vec{r}) = \frac{1}{2\mu} \frac{\nabla^2 \psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$$

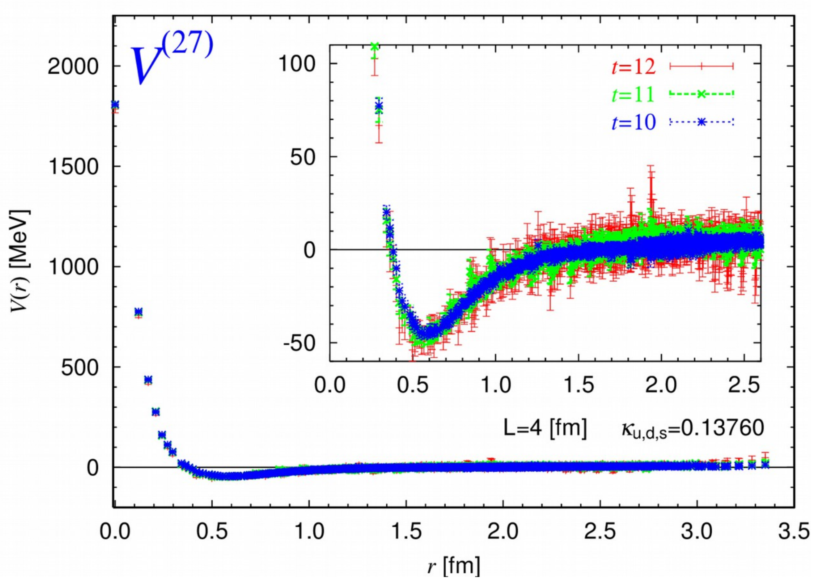
Slide, courtesy of T. Inoue (see Oct. 8th talk)

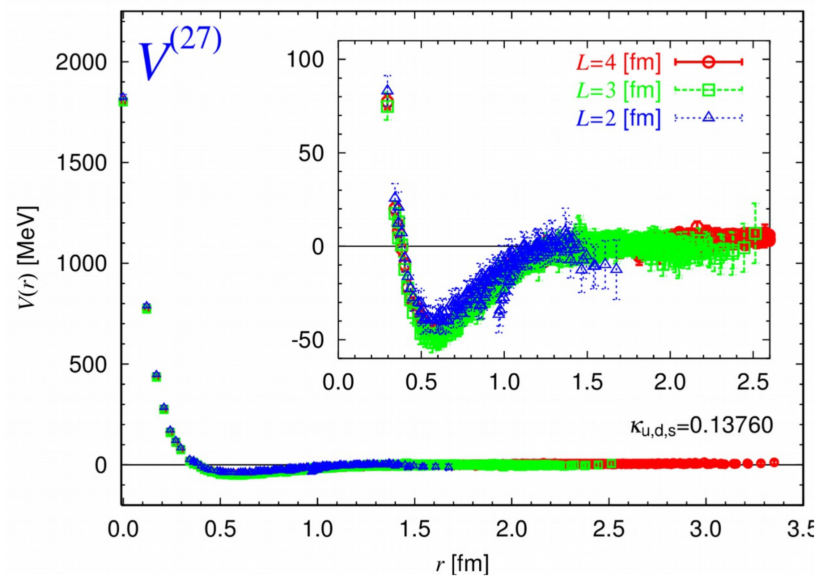
HAL Method

Advantages:

- ✓ No need to separate E eigenstate. Just need to measure $\psi(\vec{r}, t)$
- ✓ Then, potential can be extracted.
- ✓ Demand a minimal lattice volume. No need to extrapolate to V=∞.
- ✓ Can output more observables.
- ✓ One can address *large nuclei* too!!

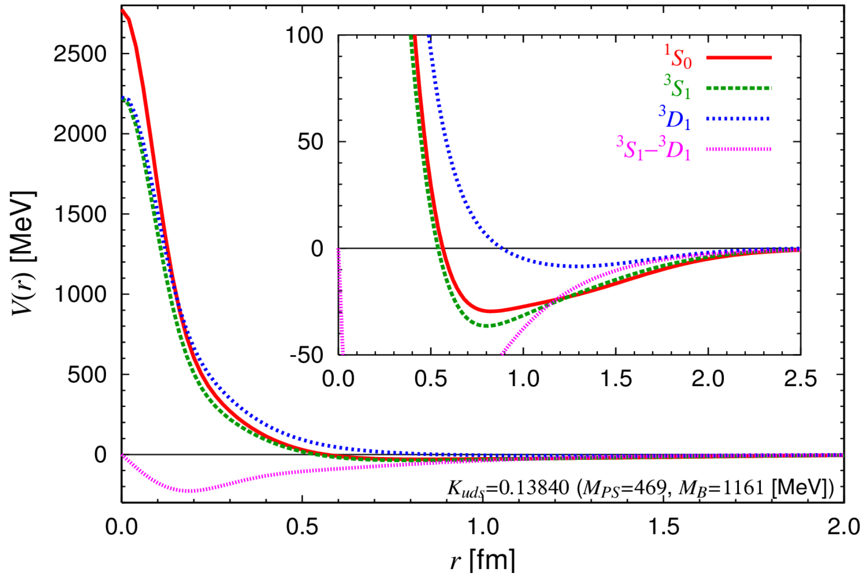


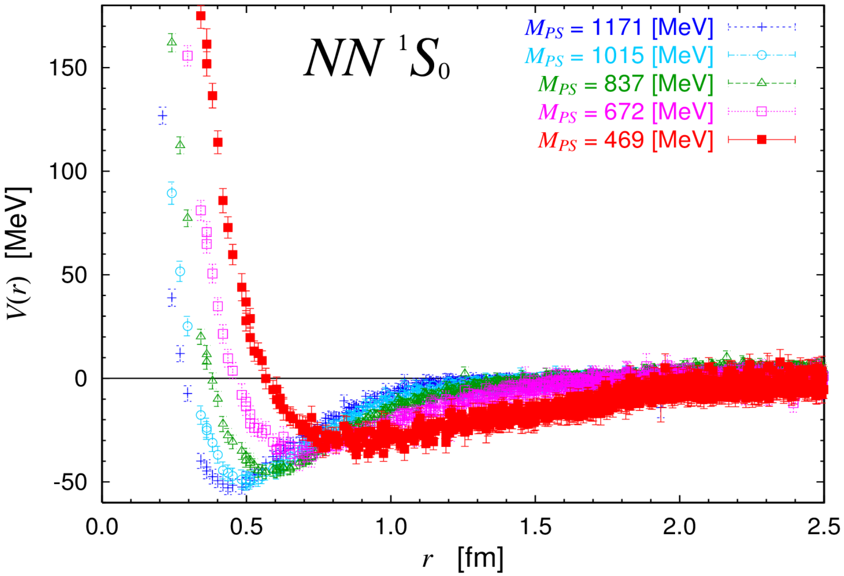




Pictures, courtesy of T. Inoue (see Oct. 8th talk)

Two-Nucleon HAL potentials





- Left: NN potentials in partial waves at the lightest m_q.
  - Repulsive core & attractive pocket & strong tensor force.
  - Similar to phenomenological potentials qualitatively. e.g. AV18
  - Least χ^2 fit of data which give central value of observable.
  - Higher orders in velocity expansions are not available yet. We restrict us to these leading order potentials.
- Right: Quark mass dependence of $V(r)$ of NN $^1S_0$.
  - Potentials become stronger as m_q decrease.

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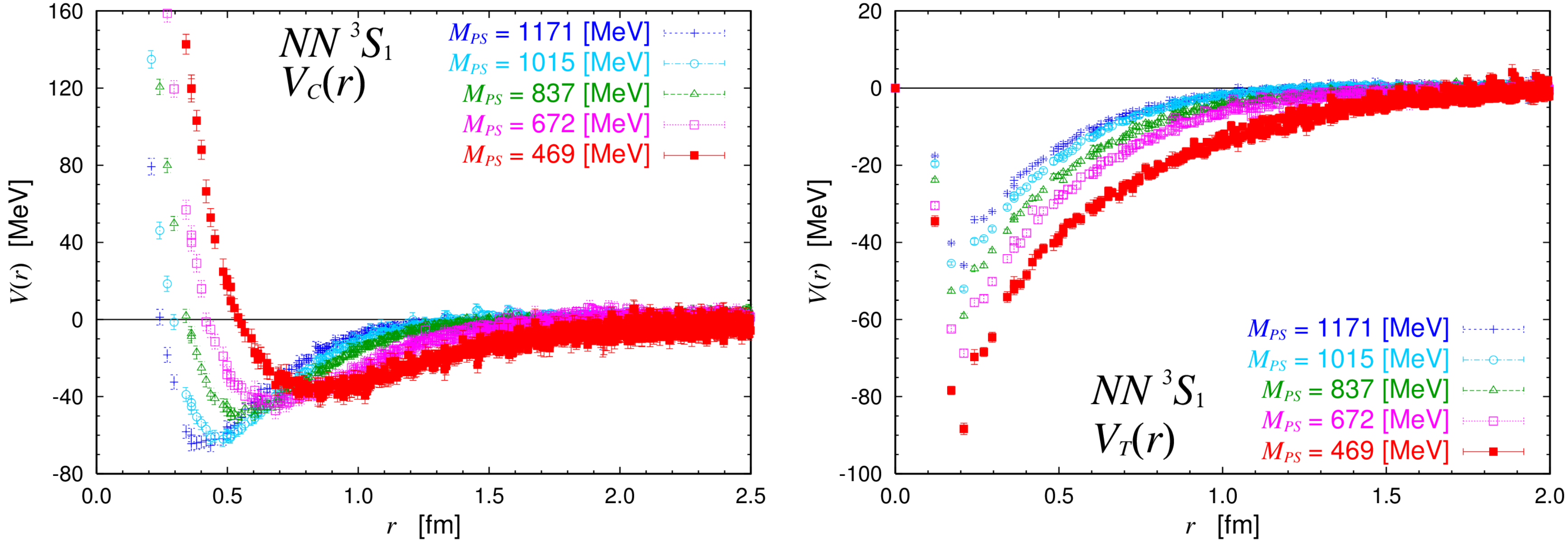
Pictures, courtesy of T. Inoue (see Oct. 8th talk)

Two-Nucleon HAL potentials



- Quark mass dependence of potentials in NN $^3S_1$
- All components get bigger as quark mass decrease.

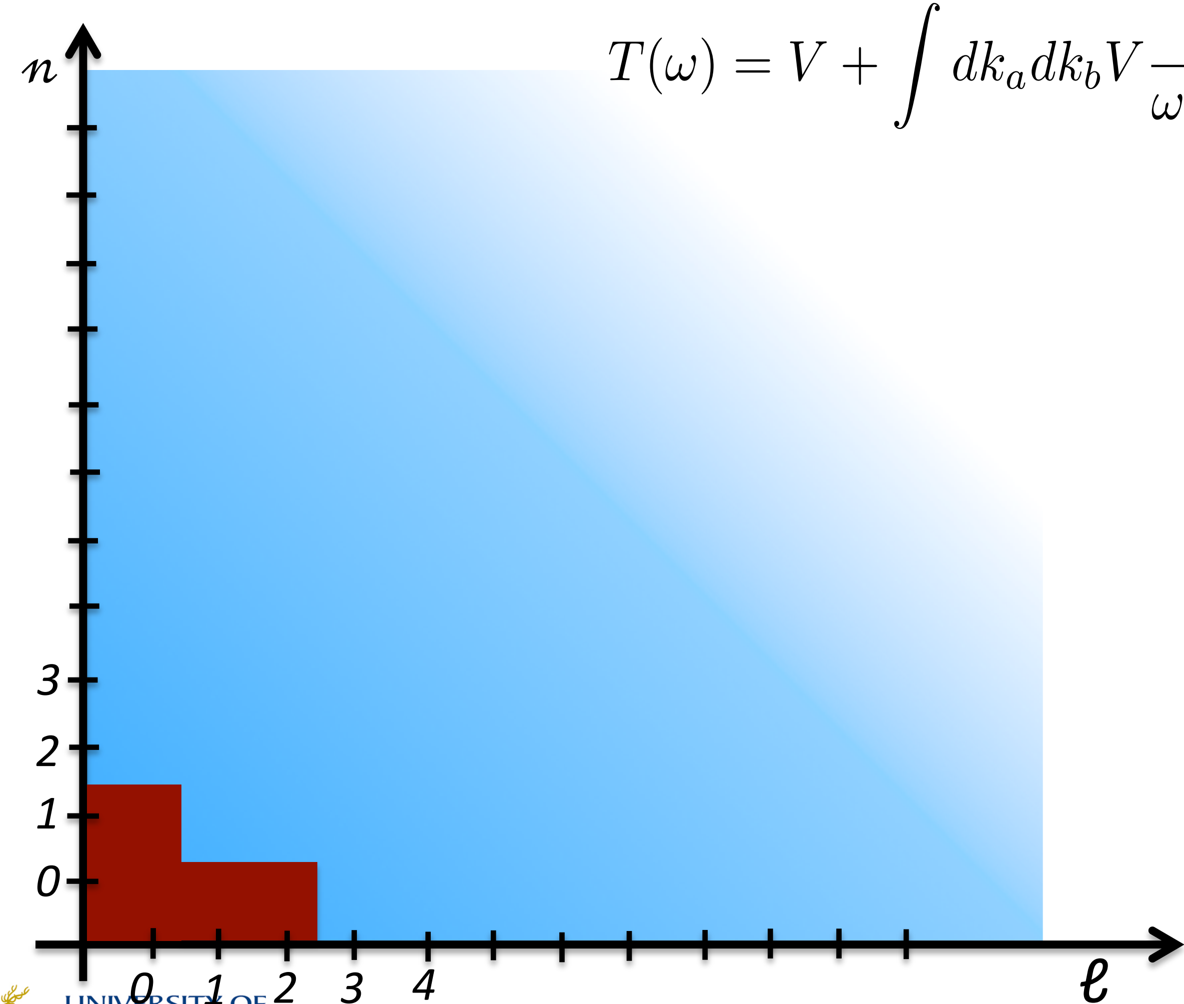
Application of microscopic (Ab-Initio) SCGF to potentials with hard cores.

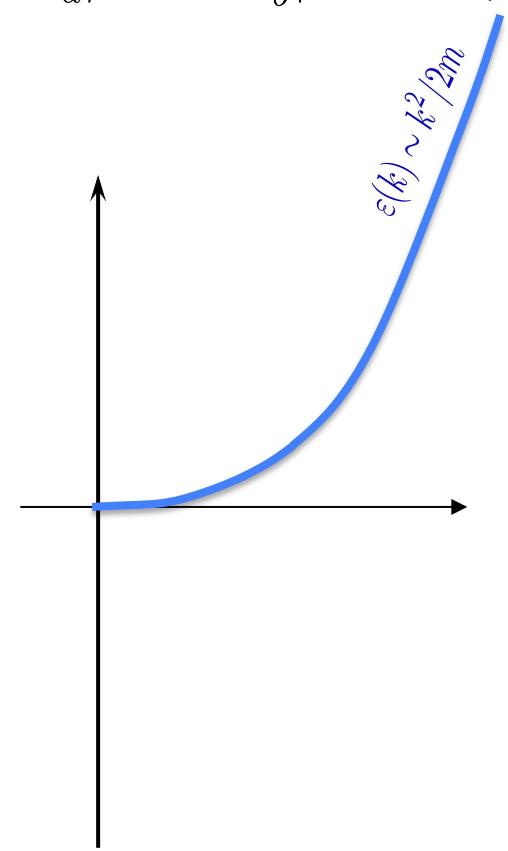
How do we do it?? → *With a G-matrix!*

Analysis of Brueckner HF

Scattering of two nucleon in free space:

$$T(\omega) = V + \int dk_a dk_b V \frac{1}{\omega - k_a^2/2m - k_b^2/2m + i\eta} T(\omega)$$

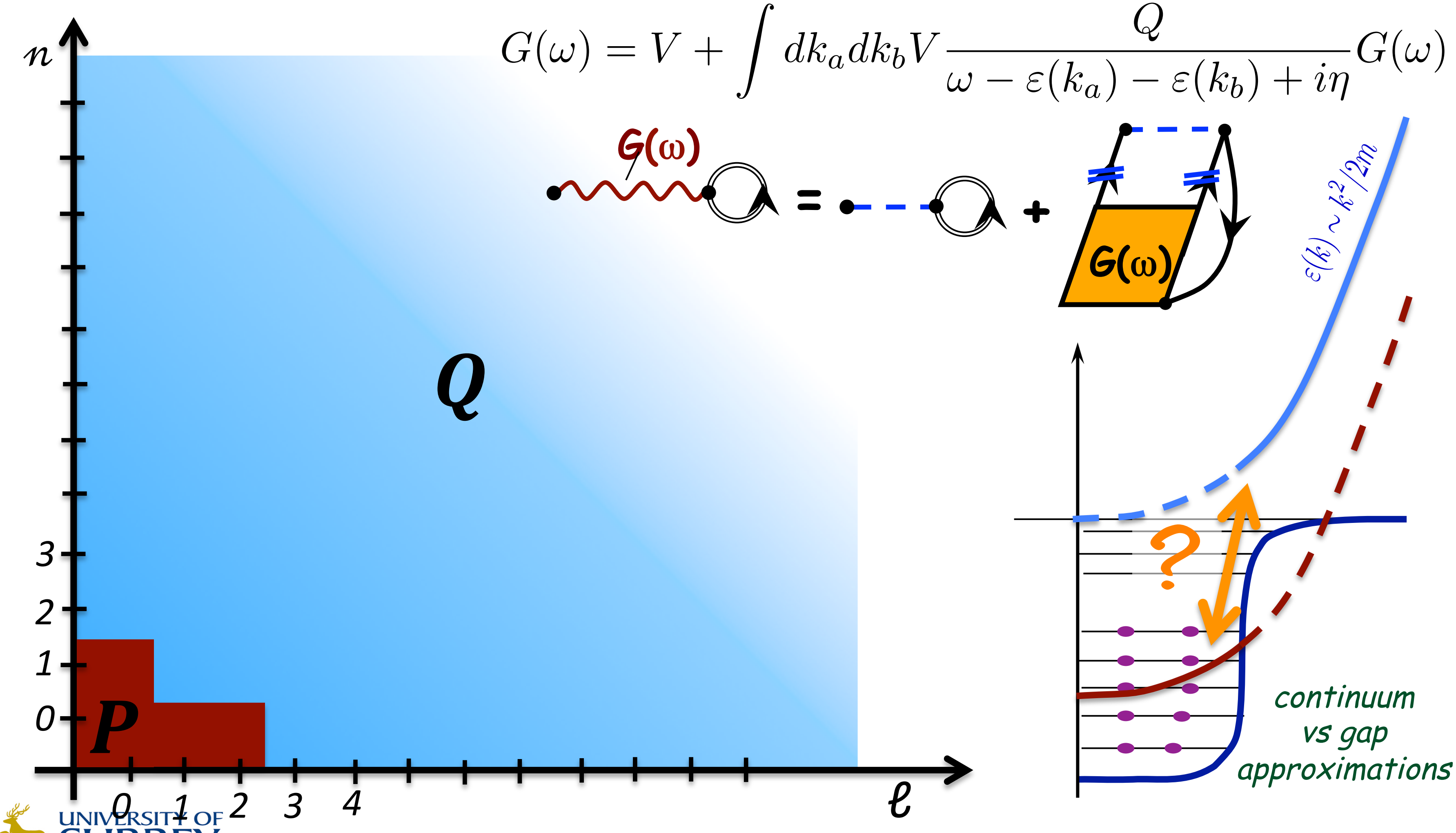




Analysis of Brueckner HF

Scattering of two nucleons outside the Fermi sea (➔BHF):

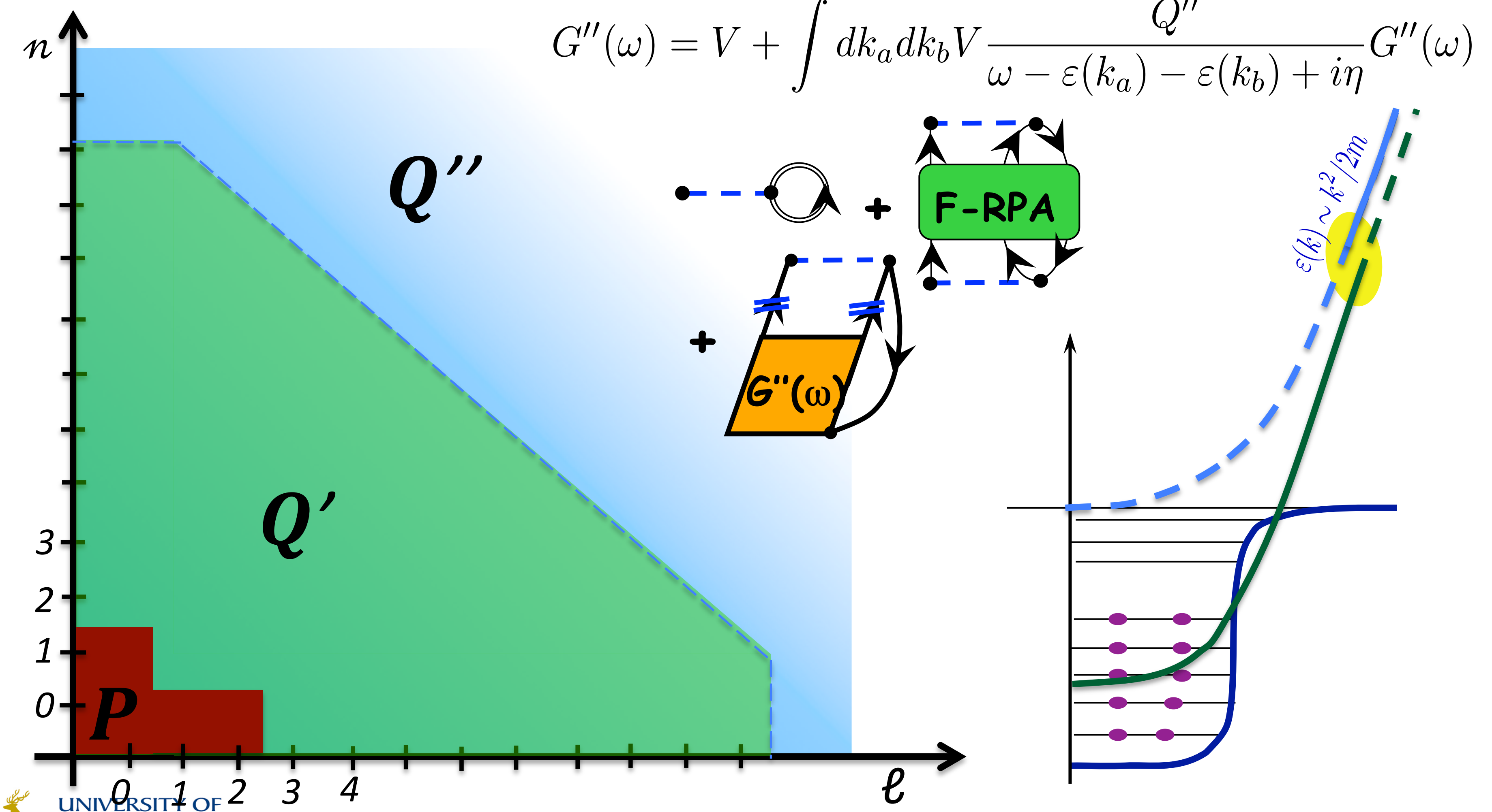
$$G(\omega) = V + \int dk_a dk_b V \frac{\hat{Q}}{\omega - \varepsilon(k_a) - \varepsilon(k_b) + i\eta} G(\omega)$$



Mixed SCGF-Brueckner approach

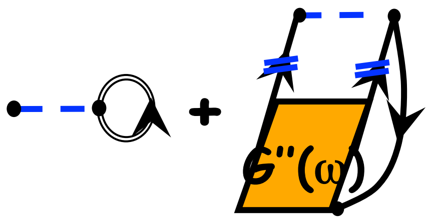
Solve full many-body dynamics in model space (P+Q') and the Goldstone's ladders outside it (i.e. in Q'' only):

$$G''(\omega) = V + \int dk_a dk_b V \frac{\hat{Q}''}{\omega - \varepsilon(k_a) - \varepsilon(k_b) + i\eta} G''(\omega)$$

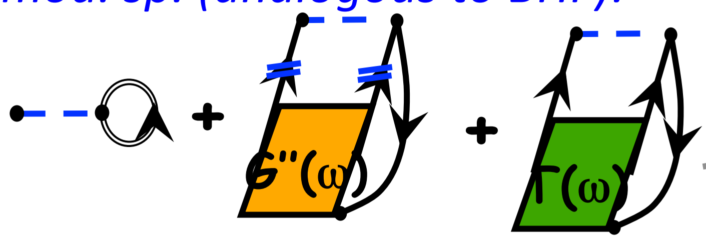


Different levels of approximation:

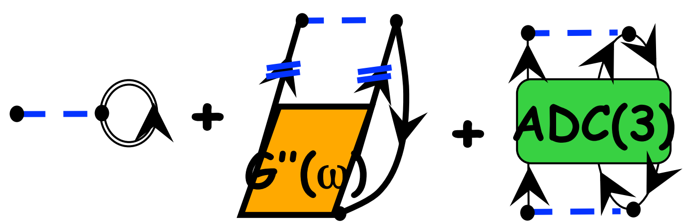
Goldstone's ladders outside the model space (i.e. in Q'' only):

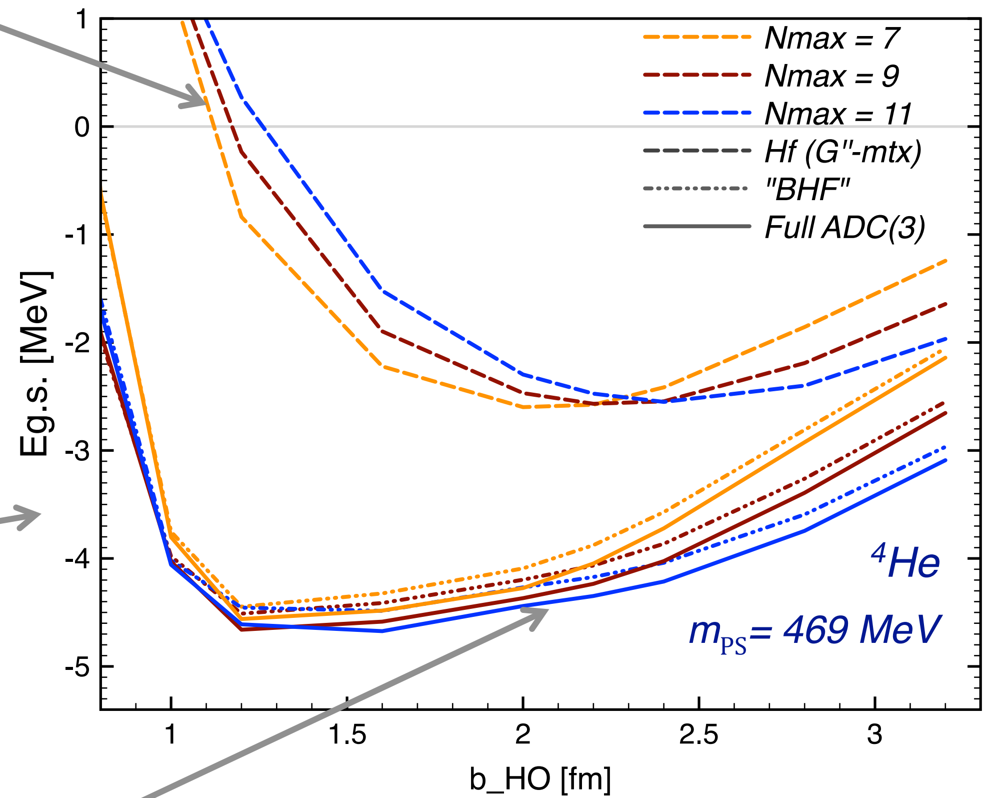


All ladders inside and outside the mod. sp. (analogous to BHF):



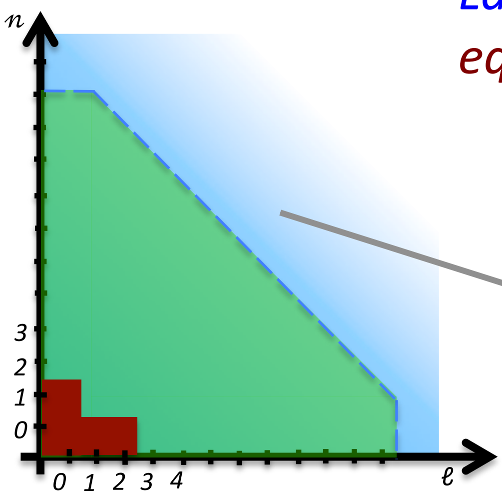
Full many-body dynamics [at ADC(3)]

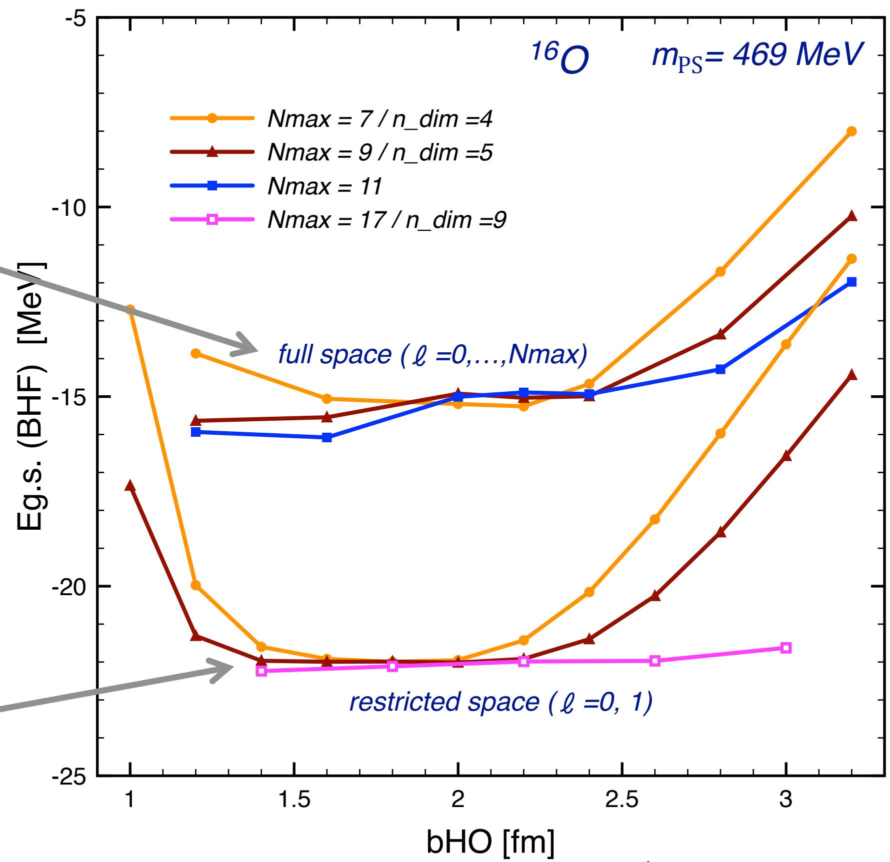


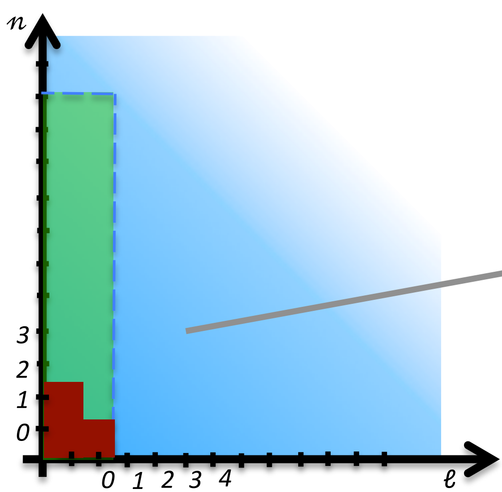


Sensitivity of BHF of the ε(k) spectrum

Ladders calculated inside and outside the model are NOT equivalent because of the different ε(k) spectrum:





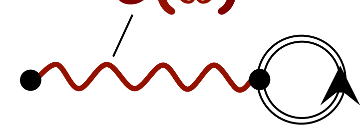


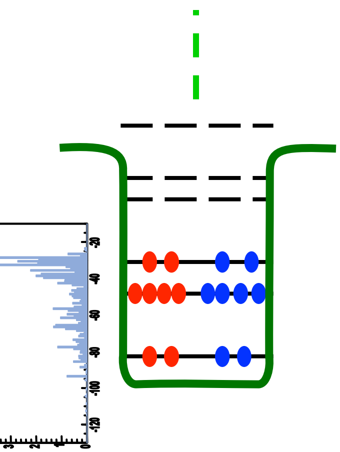
$$G''(\omega) = V + \int dk_a dk_b V \frac{\hat{Q}''}{\omega - \varepsilon(k_a) - \varepsilon(k_b) + i\eta} G''(\omega)$$

Treating short-range corr. with a G-matrix

- The short-range core can be treated by summing ladders outside the model space:

$$\Sigma^{\mathrm{MF}}_{\alpha\beta}(\omega) = i \sum_{\gamma\delta} \int \frac{d\omega'}{2\pi} G_{\alpha\gamma,\delta\beta}(\omega+\omega') g_{\delta\gamma}(\omega') \quad =$$





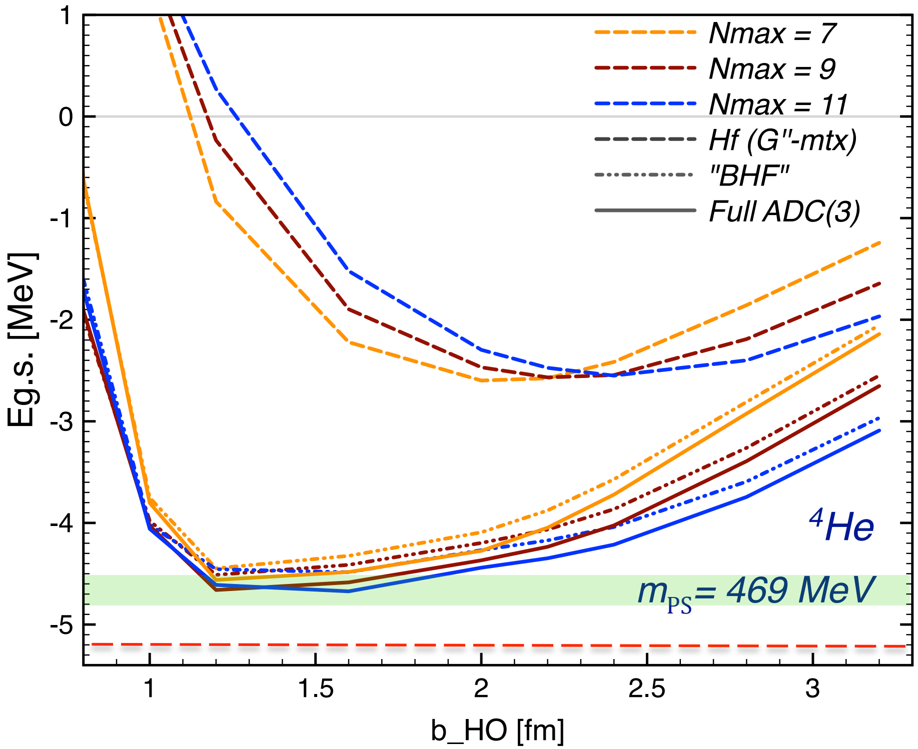
$$\Sigma^{\star}(\mathbf{r},\mathbf{r}';\omega) = \Sigma^{MF}(\mathbf{r},\mathbf{r}';\omega) + \tilde{\Sigma}(\mathbf{r},\mathbf{r}';\omega) \;.$$

$$Z_\alpha = \int d\mathbf{r}\, |\psi^{A\pm1}_\alpha(\mathbf{r})|^2 = \left. \frac{1}{1 - \frac{\partial \Sigma^{\star}_{\hat{\alpha}\hat{\alpha}}(\omega)}{\partial\omega}} \right|_{\omega=\pm(E^{A\pm1}_\alpha - E^A_0)}$$

Two contributions to the derivative:

- $\Sigma^{\mathrm{MF}}_{\alpha\beta}(\omega)$ is due to scattering to (high-k) states in the Q space
- $\tilde{\Sigma}(\mathbf{r},\mathbf{r}';\omega)$ accounts for low-energy (long range) correlations

Benchmark on ⁴He



Can benchmark the Gmtx+ADC(3) method on light 4He, where exact solutions are possible:

	G(ω) + ADC(3)	Exact
HALQCD @ m_π=469MeV	4.7(2) MeV	5.09 MeV[1]
Argonne v8'	25(1) MeV	25.91 MeV[2]

➔ *Can expect accuracy on binding energies at about 10%*

[1]H. Nemura *et al.*, Int. J. Mod. Phys. E **23**, 1461006 (2014)
[2]H. Kamada *et al.*, Phys. Rev. C**64** 044001 (2001).

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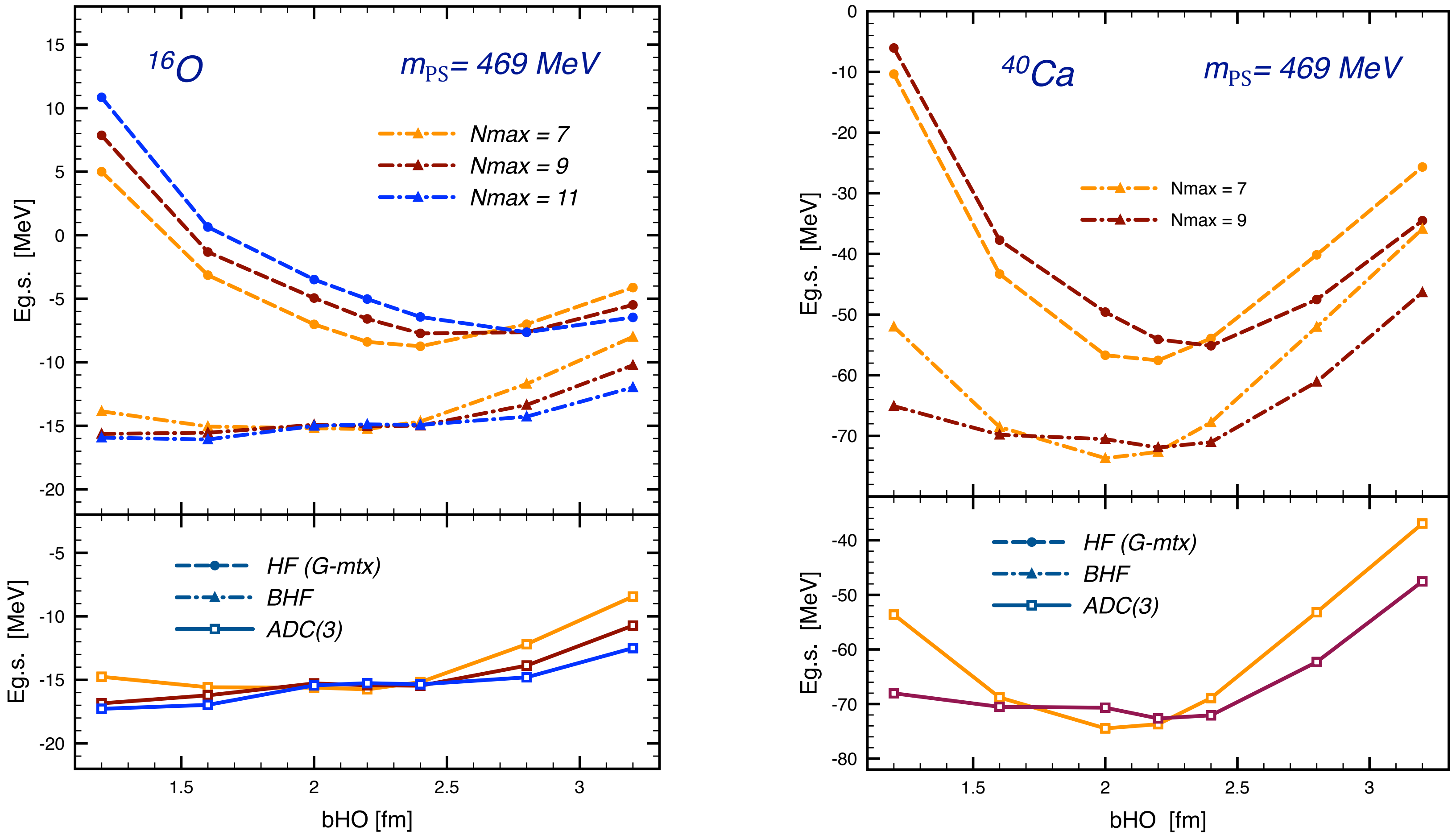
Binding of ^{16}O and ^{40}Ca:



Binding energies are ~15 MeV ^{16}O and 70-75MeV for ^{40}Ca. Possibly being underestimated by 10%

➔ 16O at m_π= 469 MeV is unstable toward 4-α breakup*!*

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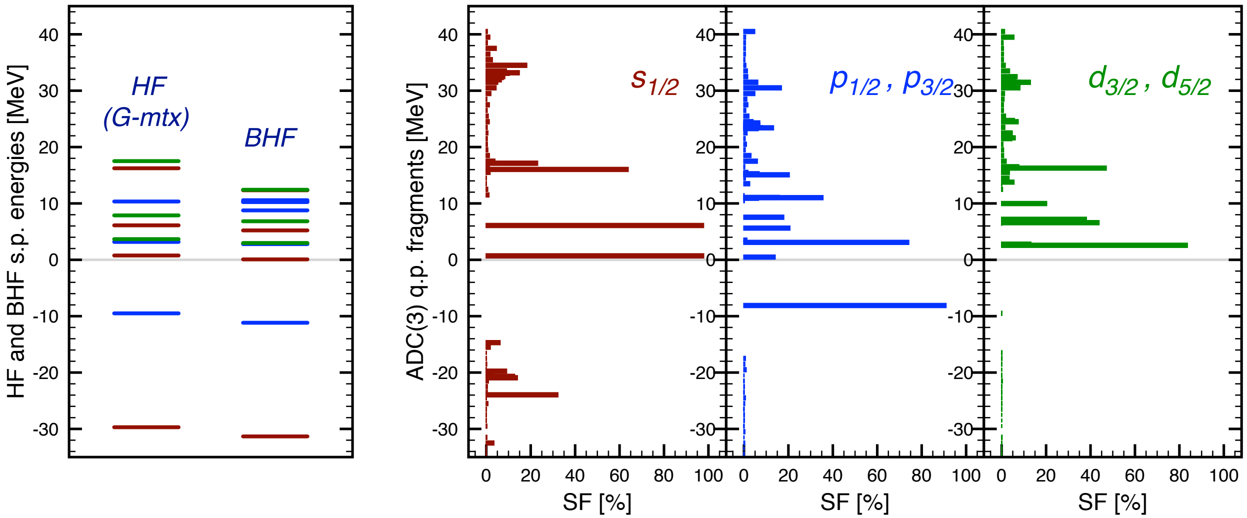
Spectral strength in ^{16}O and ^{40}Ca:



Particle-hole gaps

^{16}O

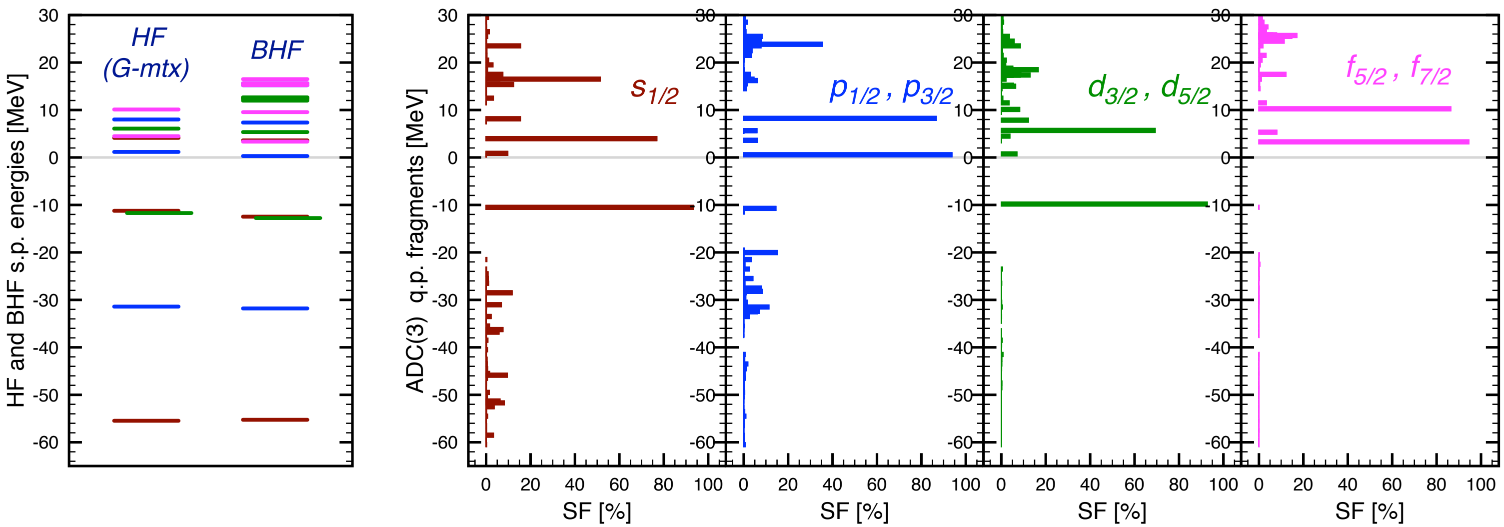
m_π= 469 MeV: ~8 MeV

Expt (phys m_π): 11.5 MeV

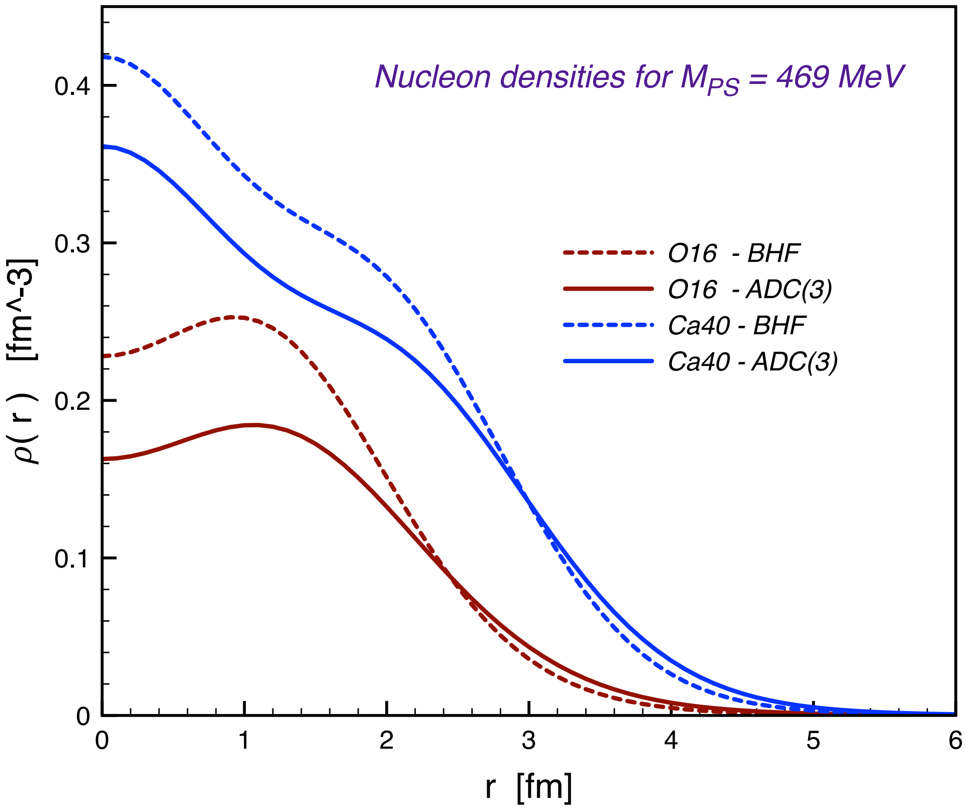
^{40}Ca

m_π= 469 MeV: ~10 MeV

Expt (phys m_π): 7.5 MeV



Matter distribution of ^{16}O and ^{40}Ca:



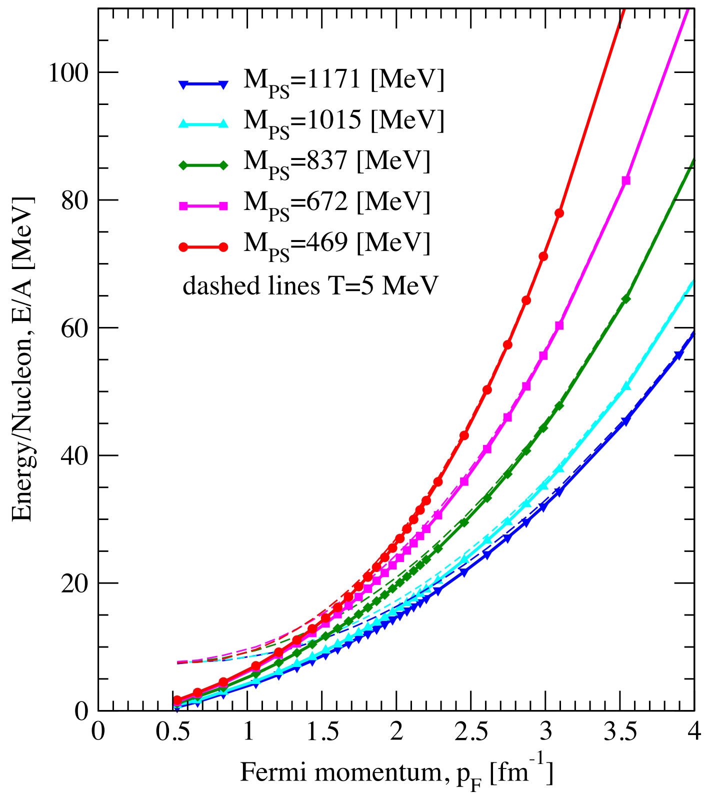
Calculated matter radii at m_{π}= 469 MeV are:

	^{16}O	^{40}Ca
"BHF"	2.33 fm	2.78 fm
ADC(3)	2.60 fm	2.97 fm
r_{charge} (expt.)	2.73 fm	3.48 fm

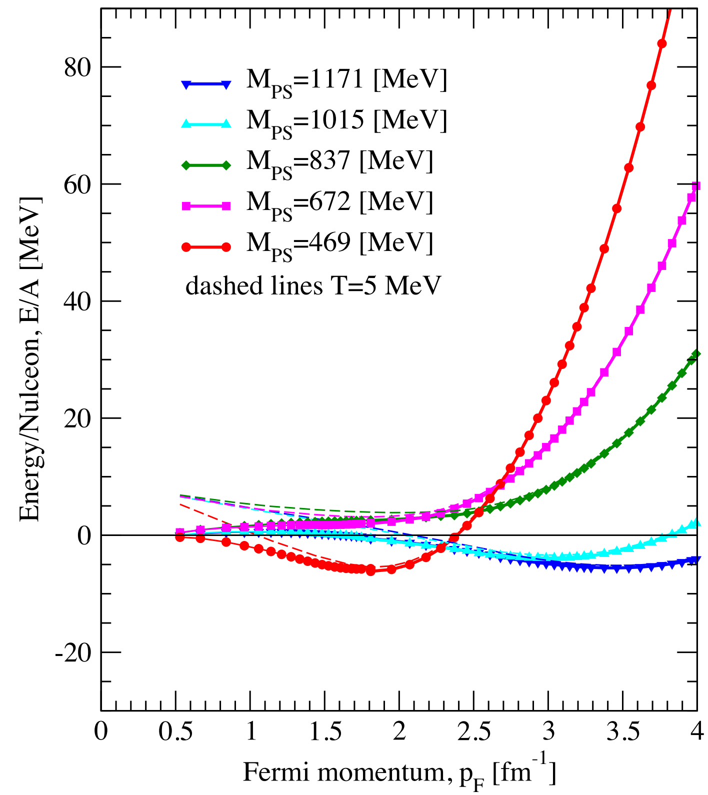
➔ Radii discrepancy worsens with increasing A

Infinite matter

Pure Neutron Matter



Symmetric Nuclear Matter



PNM unbound as usual, but less stiff

SNM saturates at 469 MeV but under bound and at higher densities that physical.

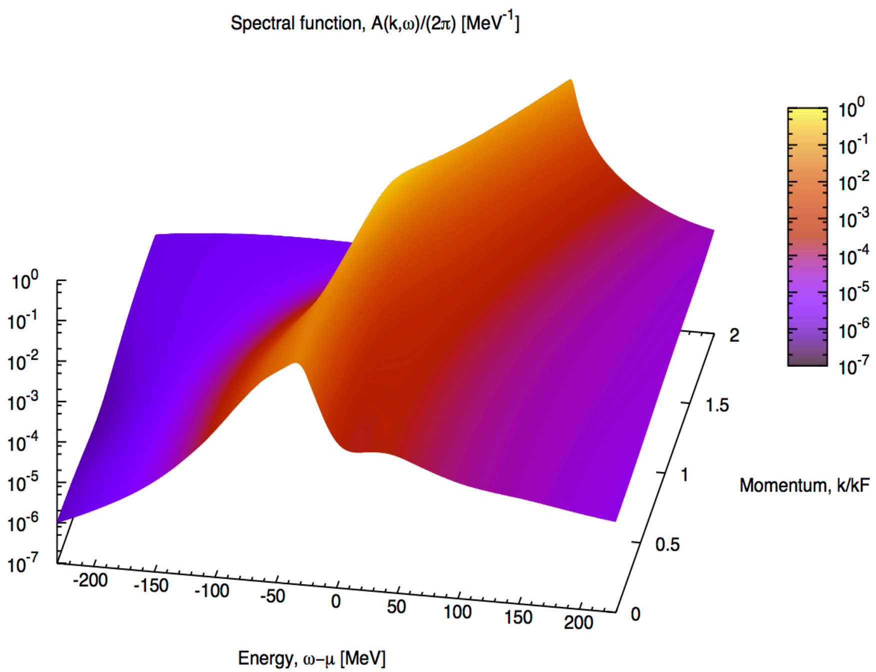
T. Inoue *et al.*, Phys. Rev. Lett. **111** 112503 (2013).

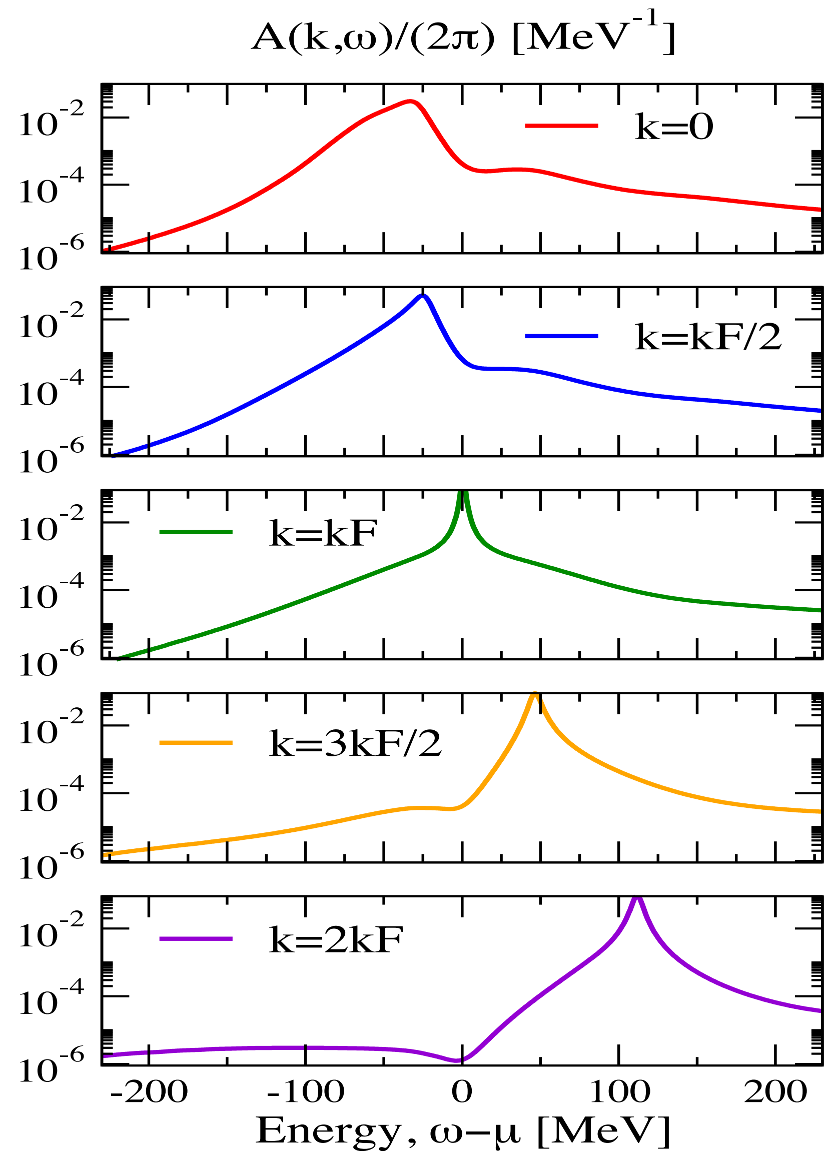
Finite-T results by A. Carbone, priv. comm.

SCGF in infinite SNM @ m_π=469MeV

Single particle spectral distribution behaves as usual.

BHF results and binding remain confirmed in SCGF calculations.





Results by A. Carbone, priv. comm.

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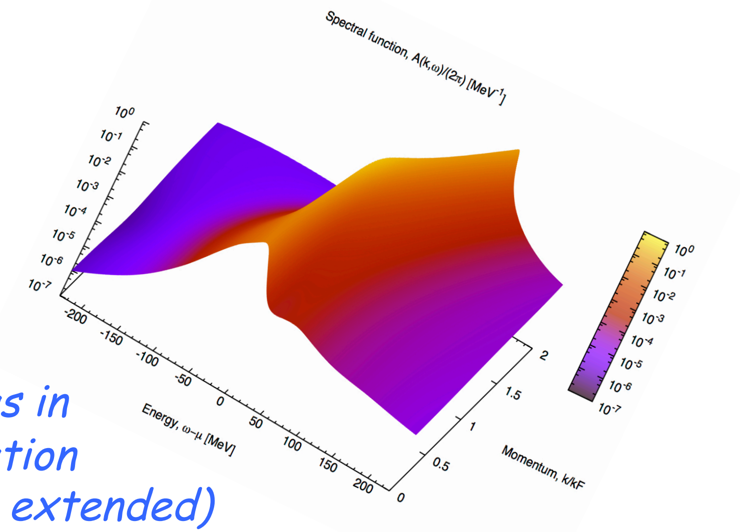
Conclusions

Mid-masses and chiral interactions:

→ *Leading order 3NF are crucial to predict many important features that are observed experimentally (drip lines, saturation, orbit evolution, etc…)*

→ *Experimental binding is predicted accurately up to the lower sd shell (A≈30) but deteriorates for medium mass isotopes (Ca and above) with roughly 1 MeV/A over binding.*

Thank you for your attention!!!



HALQCD Nuclear forces:

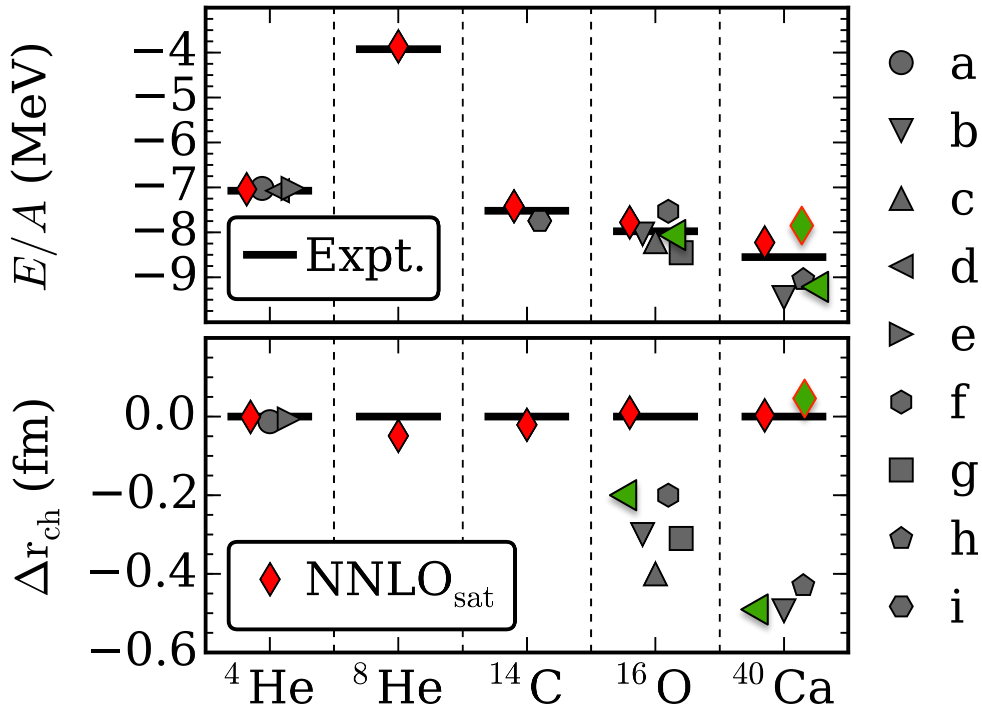
→ *Approaching the physical pion mass quickly*

→ *Strong short range repulsion requires new ideas in ab-initio many-body methods. Diagram resummation through G-matrix is a workable approach (to be extended)*

→ *At m_π=469MeV, closed shell 4He, 16O and 40Ca are bound. But oxygen is unstable toward 4-α break up, calcium stays bound. Underestimation of radii increases with A do to large saturation density (as for EM(500)+NLO3NF).*

NNLO-sat : a global fit up to A≈24

A. Ekström *et al.* Phys. Rev. C**91**, 051301(R) (2015)



- Constrain NN phase shifts

- Constrain radii and energies up to A≲24

➔ Provides saturation up to large masses!

NNLOsat (V2 + W3) -- *Grkv 2nd ord.*

From **SCGF:**

V2-N3LO(500) + W3-NNLO(400MeV/c) w/ SRG at 2.0 fm^{-1}

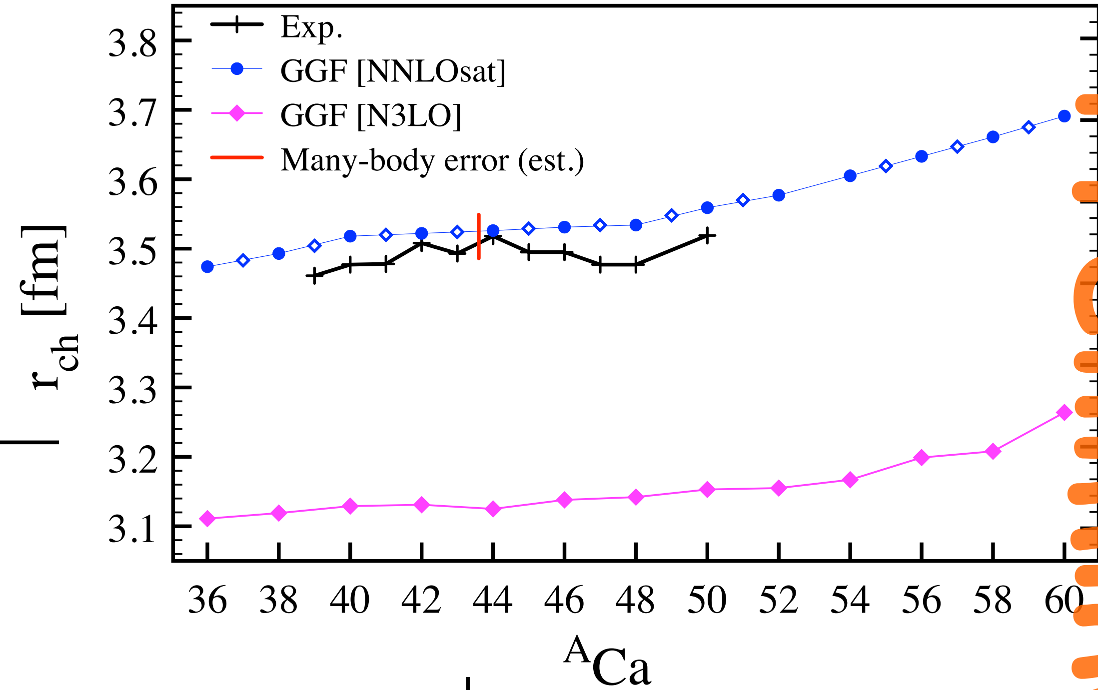
A. Cipollone, CB, P. Navrátil, Phys. Rev. Lett. **111**, 062501 (2013)

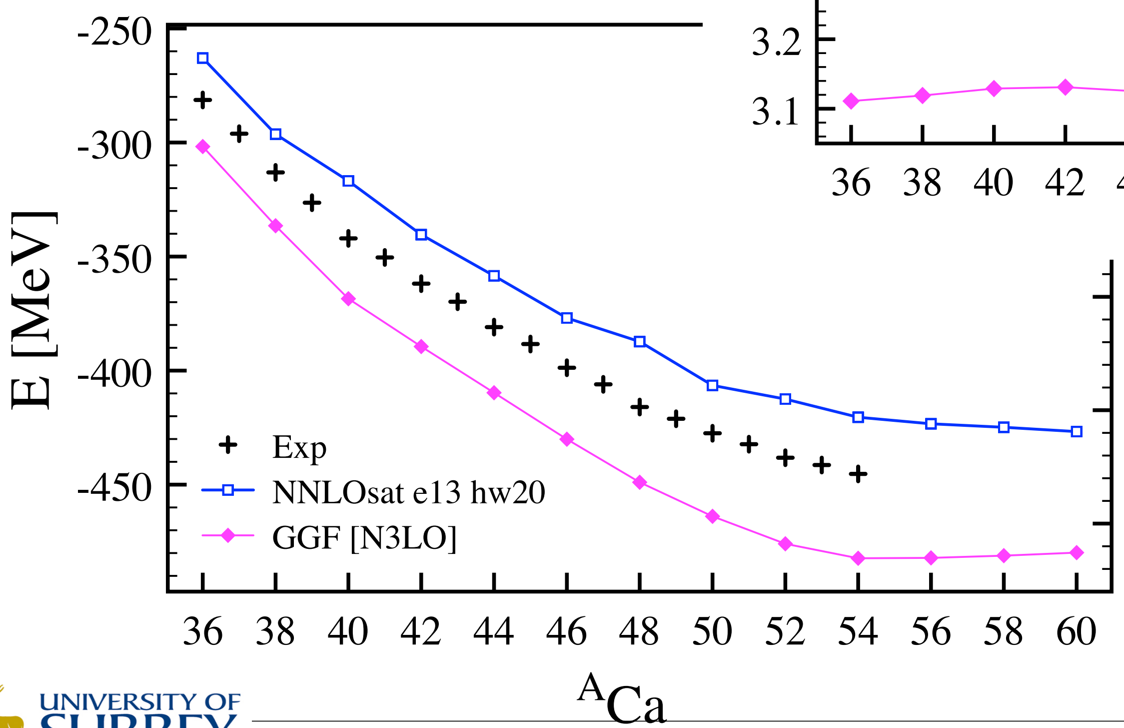
V. Somà, CB *et al.* Phys. Rev. C**89**, 061301R (2014)

BE and charge radii in ^{A}Ca

2^{nd} order GGF 'correct' to give a slight under binding and larger radii

Radii of even-odd are possible





Preliminary

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BE and charge radii in ^{A}Ca

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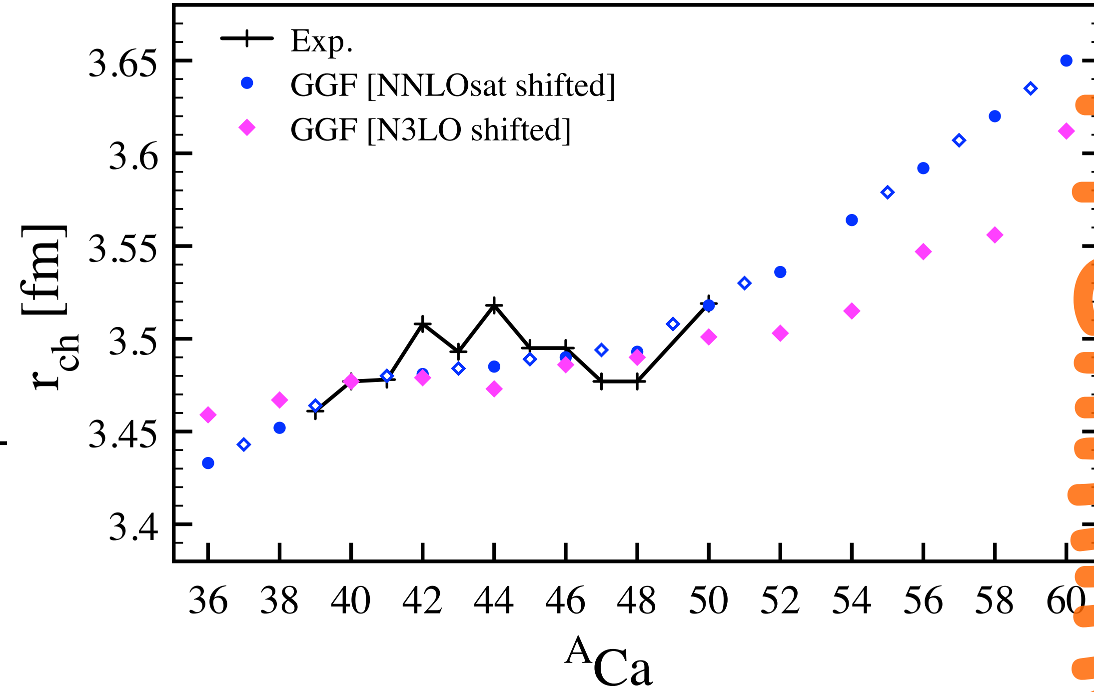
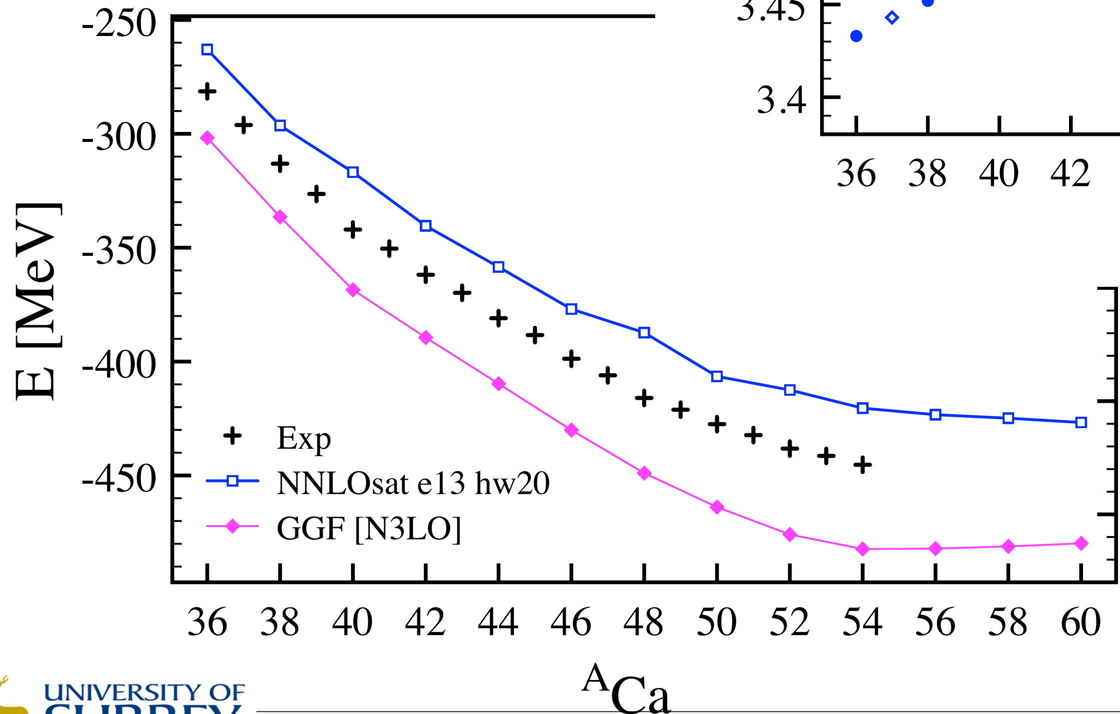




NNLO sat improves trend of radii

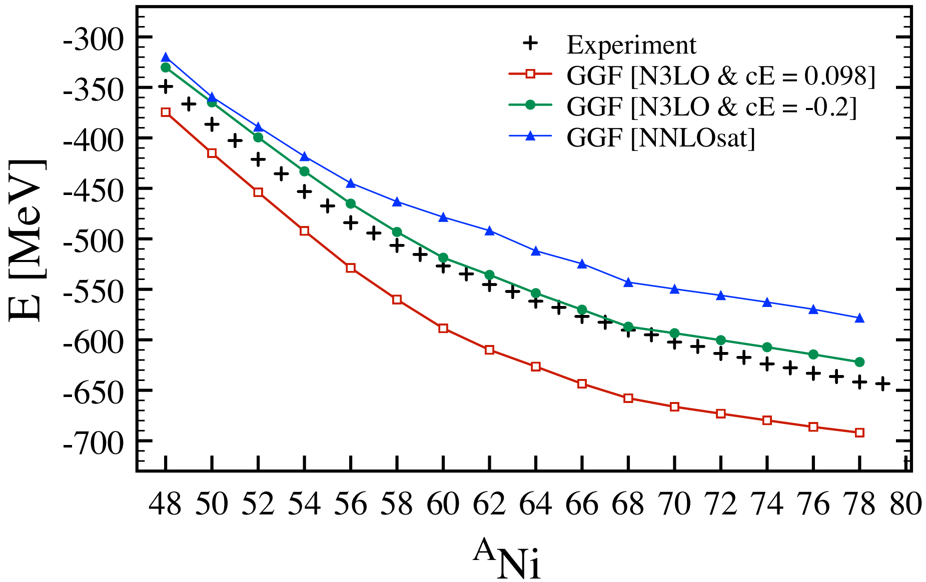
radii od $^{42-46}Ca$ require shell model…

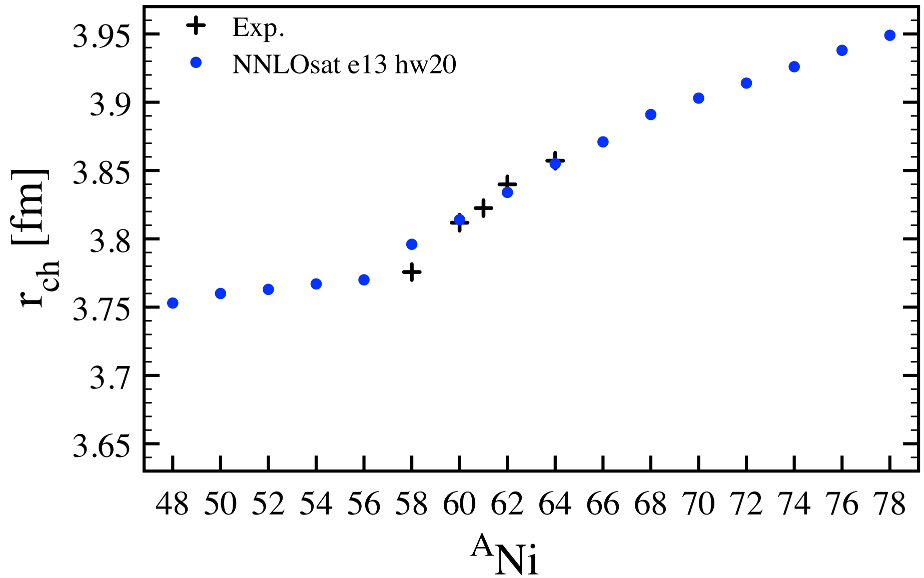
Preliminary

BE and charge radii in ^{A}Ni

Similar quality of Ni isotopes

Up to A=78





Preliminary