Electromagnetic structure of light nuclei

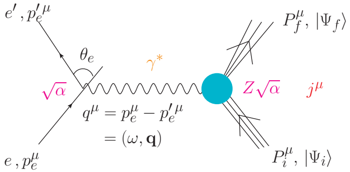
Saori Pastore @ CANHP15, Kyoto Japan - October 2015

* in collaboration with *
Bob Wiringa, Rocco Schiavilla, Steven Pieper,
Luca Girlanda, Maria Piarulli, Michele Viviani,
Laura E. Marcucci, Alejandro Kievsky

PRC78(2008)064002 - PRC80(2009)034004 - PRC84(2011)024001 - PRC87(2013)014006 - PRC87(2013)035503 - PRL111(2013)062502 - PRC90(2014)024321

Ab initio calculations of light nuclei

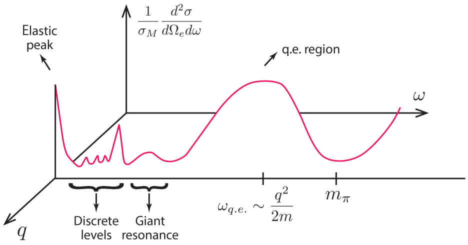
Aim: understand nuclei in terms of interactions between individual nucleons



Electromagnetic reactions are a powerful tool to test our theoretical models

- coupling constant $\alpha \sim 1/137$ allows for a perturbative treatment of the EM interaction $\rightarrow$ single photon γ exchange suffices
- calculated x-sections $\propto |\langle\Psi_f|j^{\mu}|\Psi_i\rangle|^2$ with j^{μ} nuclear EM currents $\rightarrow$ clear connection between measured x-sections and calculated properties of nuclear targets
- EXPT data (in most cases) known with great accuracy $\rightarrow$ viable EXPT constraints on theories
- For few-nucleon systems, the many-body problem can be solved exactly or within controlled approximations

Electromagnetic probes to test predictive power of nuclear theories/models



- In this talk we primarily focus on:
  EM ground state properties and transitions between low-lying states

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* Validate our theoretical understanding and control of nuclear EM structure and reactions is an essential prerequisite for studies on: *

⇒ Weak induced reactions, *e.g.*, ν-nucleus interactions
(major progress by A. Lovato, S. Gandolfi *et al.*)

⇒ Larger nuclear systems

Outline

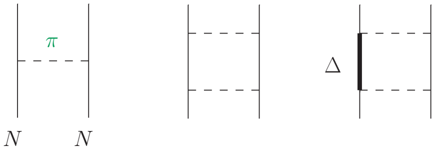
- Microscopic picture of the nucleus: the *ab initio* framework
- Many-body nuclear EM currents from χEFT
- Applications:
  - d, ^{3}H and ^{3}He EM form factors
  - Magnetic moments and EM transitions in $A \leq 10$ systems
- Summary and outlook

The Basic Model: Nuclear Potentials

- The nucleus is a system made of A non-relativistic interacting nucleons, its energy is given by

$$H = T + V = \sum_{i=1}^{A} t_i + \sum_{i<j} \upsilon_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

  where υ_{ij} and V_{ijk} are 2- and 3-nucleon interaction operators
- Realistic υ_{ij} and V_{ijk} interactions are based on EXPT data fitting and fitted parameters subsume underlying QCD
- Realistic potentials at large inter-particle distances are described in terms of one-pion-exchange, range $\sim 1/m_\pi$. Other mechanisms are, *e.g.*, two-pion exchange, range $\sim 1/2m_\pi$; Δ-excitations …



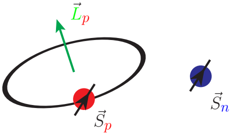
- Potentials utilized in these sets of calculations to generate nuclear wave functions $|\Psi_i\rangle$ solving $H|\Psi_i\rangle = E_i|\Psi_i\rangle$ are:
  [AV18+UIX], [AV18+IL7], [NN(N3LO)+3N(N2LO)]

The Basic Model: Nuclear Electromagnetic Currents - Impulse Approximation

- Current and charge operators describe the interaction of nuclei with external fields. They are expanded as a sum of 1−, 2−, ... nucleon operators:

$$\rho = \sum_{i=1}^{A} \rho_i + \sum_{i<j} \rho_{ij} + \dots , \qquad \mathbf{j} = \sum_{i=1}^{A} \mathbf{j}_i + \sum_{i<j} \mathbf{j}_{ij} + \dots$$

- In Impulse Approximation IA nuclear EM currents are expressed in terms of those associated with individual protons and nucleons, *i.e.*, ρ_i and $\mathbf{j}_i$



- IA picture is however incomplete; Historical evidence is the 10% underestimate of the *np* radiative capture 'fixed' by incorporating corrections from two-body meson-exchange EM currents - Riska&Brown 1972

The Basic Model: Nuclear Electromagnetic Currents

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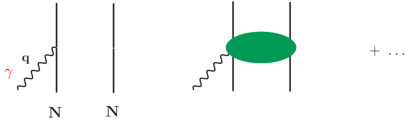
$$\rho = \sum_{i=1}^{A} \rho_i + \sum_{i<j} \rho_{ij} + \ldots\,, \qquad \mathbf{j} = \sum_{i=1}^{A} \mathbf{j}_i + \sum_{i<j} \mathbf{j}_{ij} + \ldots$$



- Longitudinal EM current operator **j** linked to the nuclear Hamiltonian via continuity eq. (**q** momentum carried by the external EM probe γ)

$$\mathbf{q}\cdot\mathbf{j} = [H, \rho] = [t_i + \upsilon_{ij} + V_{ijk}, \rho]$$

* Meson-exchange currents MEC follow once meson-exchange mechanisms are implemented to describe nuclear forces - Villars&Miyazawa 40ies

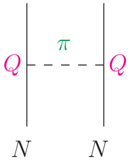
These days we have:

- Highly sophisticated MEC projected out realistic potentials
- EM currents derived from χEFTs

Nuclear χEFT Approach

S. Weinberg, Phys. Lett. **B251**, 288 (1990); Nucl. Phys. **B363**, 3 (1991); Phys. Lett. **B295**, 114 (1992)

- χEFT is a low-energy ($Q \ll \Lambda_\chi \sim 1$ GeV) approximation of QCD
- It provides effective Lagrangians describing π's, N's, Δ's, ... interactions that are expanded in powers n of a perturbative (small) parameter Q/Λ_χ



$$\mathscr{L}_{eff} = \mathscr{L}^{(0)} + \mathscr{L}^{(1)} + \mathscr{L}^{(2)} + \ldots + \mathscr{L}^{(n)} + \ldots$$

- The coefficients of the expansion, Low Energy Constants (LECs), are unknown and need to be fixed by comparison with exp data
- The systematic expansion in Q naturally has the feature

$$\langle \mathscr{O} \rangle_{1-\text{body}} > \langle \mathscr{O} \rangle_{2-\text{body}} > \langle \mathscr{O} \rangle_{3-\text{body}}$$

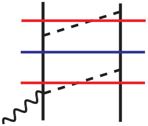
- A theoretical error due to the truncation of the expansion can be assigned

Transition amplitude in time-ordered perturbation theory

$$
\begin{aligned}
T_{\mathrm{fi}} = \langle f \mid T \mid i \rangle &= \langle f \mid H_1 \sum_{n=1}^{\infty} \left(\frac{1}{E_i - H_0 + i\eta} H_1 \right)^{n-1} \mid i \rangle \\
&= \langle f \mid H_1 \mid i \rangle + \sum_{|I\rangle} \langle f \mid H_1 \mid I \rangle \frac{1}{E_i - E_I} \langle I \mid H_1 \mid i \rangle + \ldots
\end{aligned}
$$

- A contribution with N interaction vertices and L loops scales as

$$
\underbrace{e \left(\prod_{i=1}^{N} Q^{\alpha_i - \beta_i/2} \right)}_{H_1 \,\text{scaling}} \times \underbrace{Q^{-(N-N_K-1)}\, Q^{-2N_K}}_{\text{denominators}} \times \underbrace{Q^{3L}}_{\text{loop integration}}
$$



$\alpha_i =$ number of derivatives in H_1 and $\beta_i =$ number of π's at each vertex

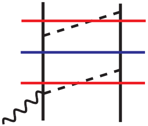
$N_K =$ number of pure nucleonic intermediate states

- Due to the chiral expansion, the transition amplitude T_{fi} can be expanded as

$$
T_{\mathrm{fi}} = T^{\mathrm{LO}} + T^{\mathrm{NLO}} + T^{\mathrm{N2LO}} + \ldots \quad \text{and} \quad T^{\mathrm{NnLO}} \sim (Q/\Lambda_\chi)^n T^{\mathrm{LO}}
$$

PRC78(2008)064002, PRC80(2009)034004, PRC84(2011)024001

Transition amplitude in time-ordered perturbation theory - bis



- N_K energy denominators scale as Q^{-2}

$$\frac{1}{E_i - E_I}|I\rangle = \frac{1}{E_i - E_N}|I\rangle \sim Q^{-2}|I\rangle$$

- $(N - N_K - 1)$ energy denominators scale Q^{-1} in the static limit; they can be further expanded in powers of $(E_i - E_N)/\omega_\pi \sim Q$

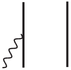
$$\frac{1}{E_i - E_I}|I\rangle = \frac{1}{E_i - E_N - \omega_\pi}|I\rangle \sim -\Big[\underbrace{\frac{1}{\omega_\pi}}_{Q^{-1}} + \underbrace{\frac{E_i - E_N}{\omega_\pi^2}}_{Q^0} + \underbrace{\frac{(E_i - E_N)^2}{\omega_\pi^3}}_{Q^1} + \dots\Big]|I\rangle$$

- Terms accounted into the Lippmann-Schwinger equation are subtracted order by order from reducible amplitudes !

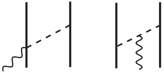
PRC78(2008)064002, PRC80(2009)034004, PRC84(2011)024001

χEFT EM current up to $n = 1$ (or up to N3LO)

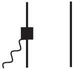
LO : $\mathbf{j}^{(-2)} \sim e\mathbf{Q}^{-2}$



NLO : $\mathbf{j}^{(-1)} \sim e\mathbf{Q}^{-1}$



N^{2}LO : $\mathbf{j}^{(-0)} \sim e\mathbf{Q}^{0}$



* Two-body charge operators enter at N3LO and do not depend on LECs *

- LO = IA
  N2LO = IA(relativistic-correction)
- Strong contact LECs at N3LO fixed from fits to *np* phases shifts
  PRC**68**, 041001 (2003)
- Unknown EM LECs enter the N3LO contact and tree-level currents

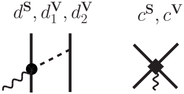
- No three-body EM currents at this order !!!
- NLO and N3LO loop-contributions lead to purely isovector operators

N^{3}LO: $\mathbf{j}^{(1)} \sim e\mathbf{Q}$

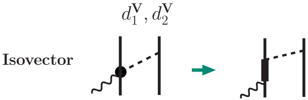
unknown LEC's →

PRC78(2008)064002, PRC80(2009)034004, PRC84(2011)024001

χEFT EM currents at N3LO: fixing the EM LECs



Five LECs: d^S, d_1^V, and d_2^V could be determined by pion photo-production data on the nucleon



d_2^V and d_1^V are known assuming Δ-resonance saturation

Left with 3 LECs: Fixed in the $A = 2 - 3$ nucleons' sector

- Isoscalar sector:
  - d^S and c^S from EXPT μ_d and $\mu_S(^3\mathrm{H}/^3\mathrm{He})$
- Isovector sector:
  - model I = c^V from EXPT $npd\gamma$ xsec.

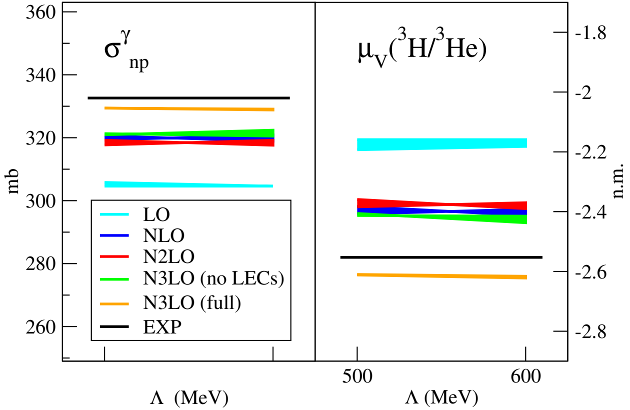
    or
  - model II = c^V from EXPT $\mu_V(^3\mathrm{H}/^3\mathrm{He})$ m.m. $\leftarrow$ our choice

Note that:
χEFT operators have a power law behavior $\rightarrow$ introduce a regulator to kill divergencies at large Q, *e.g.*, $C_\Lambda = e^{-(Q/\Lambda)^n}$, ...and also, pick n large enough so as to not generate spurious contributions

$$C_\Lambda \sim 1 - \left(\frac{Q}{\Lambda}\right)^n + \ldots$$

Predictions with χEFT EM currents for $A = 2$–3 systems

np capture xsec. (using model II) / μ_V of $A = 3$ nuclei (using model I)
bands represent nuclear model dependence (N3LO/N2LO – AV18/UIX)

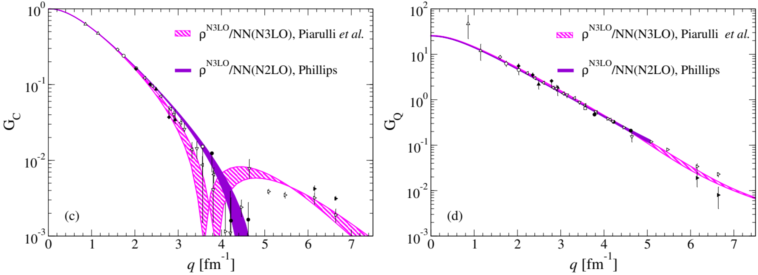


- *npd*γ xsec. and $\mu_V(^3\text{H}/^3\text{He})$ m.m. are within 1% and 3% of EXPT
- Two-body currents important to reach agreement with exp data
- Negligible dependence on the cutoff entering the regulator $exp(-(k/\Lambda)^4)$

PRC87(2013)014006

Applications:
EM form factors of nuclei with $A = 2$ and 3

Predictions with χEFT EM Currents for the Deuteron Charge and Quadrupole f.f.'s

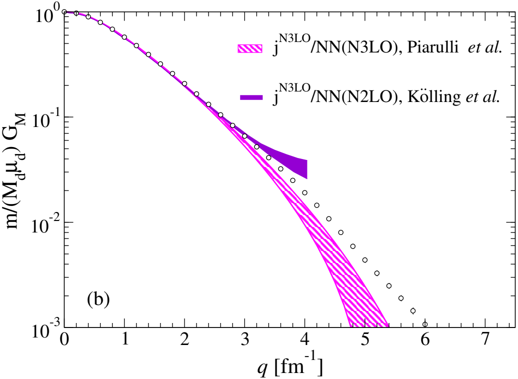


Λ MeV	$< r_d >$ (fm)	$< r_d >$ EXP	Q_d (fm^2)	Q_d (fm^2) EXP
500	1.976	1.9734(44)	0.285	0.2859(3)
600	1.968		0.282	

- Calculations include nucleonic f.f.'s taken from EXPT data
- Sensitivity to the cutoff used to regularize divergencies in the matrix elements is given by the bands' thickness

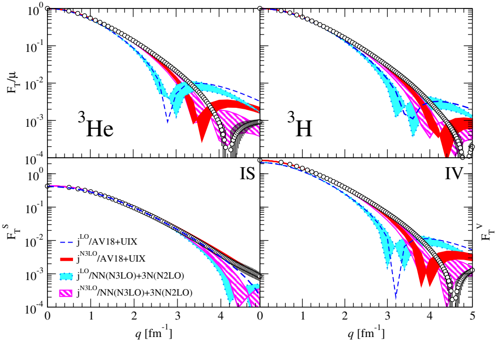
J.Phys.G34(2007)365 & PRC87(2013)014006

Predictions with χEFT EM Currents for the Deuteron Magnetic f.f.



PRC86(2012)047001 & PRC87(2013)014006

Predictions with χEFT EM Currents for ^{3}He and ^{3}H Magnetic f.f.'s



LO/N3LO with AV18+UIX – LO/N3LO with χ-potentials NN(N3LO)+3N(N2LO)

- ^{3}He/^{3}H m.m.'s used to fix EM LECs; $\sim$ 15% correction from two-body currents
- Two-body corrections crucial to improve agreement with EXPT data

	^{3}He $< r >_{EXP} = 1.976 \pm 0.047$ fm		^{3}H $< r >_{EXP} = 1.840 \pm 0.181$ fm	
Λ	500	600	500	600
LO	2.098 (2.092)	2.090 (2.092)	1.924 (1.918)	1.914 (1.918)
N3LO	1.927 (1.915)	1.913 (1.924)	1.808 (1.792)	1.794 (1.797)

PRC87(2013)014006

Calculations with EM Currents from χEFT with π's and N's

- Park, Min, and Rho *et al.* (1996)

  applications to:
  magnetic moments and M1 properties of A=2–3 systems, and
  radiative captures in A=2–4 systems by Song, Lazauskas, Park *at al.*
  (2009-2011) within the hybrid approach

  ......
  * Based on EM χEFT currents from NPA596(1996)515

- Meissner and Walzl (2001);
  Kölling, Epelbaum, Krebs, and Meissner (2009–2011)

  applications to:
  d and ^{3}He photodisintegration by Rozpedzik *et al.* (2011); e-scattering (2014);
  d magnetic f.f. by Kölling, Epelbaum, Phillips (2012);
  radiative $N-d$ capture by Skibinski *et al.* (2014)

  ......
  * Based on EM χEFT currents from PRC80(2009)045502 &
  PRC84(2011)054008 and consistent χEFT potentials from UT method

- Phillips (2003-2007)

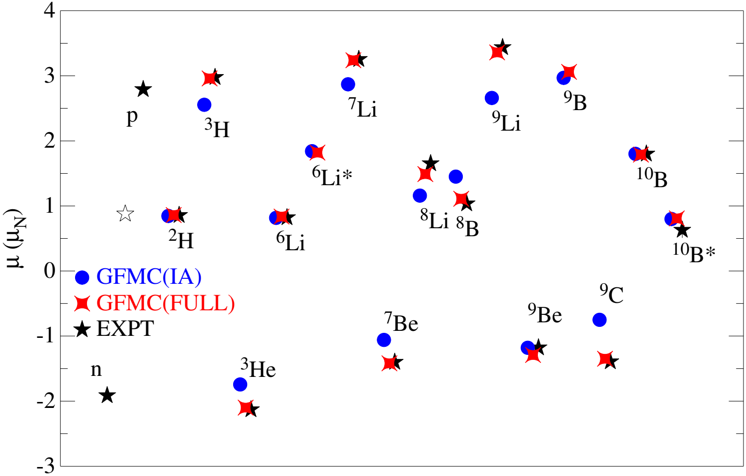
  applications to deuteron static properties and f.f.'s

  ......

Moving on to larger nuclear systems:
magnetic moments and transitions in $A \leq 10$ nuclei

Magnetic Moments in $A \leq 10$ Nuclei

Predictions for $A > 3$ nuclei

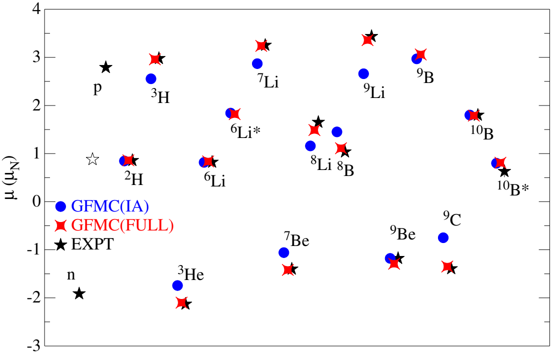


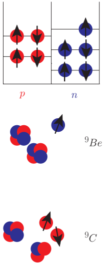
- $\mu(\text{IA}) = \mu_N \sum_i [(L_i + g_p S_i)(1+\tau_{i,z})/2 + g_n S_i (1-\tau_{i,z})/2]$
- GFMC calculations based on H = T + AV18 + IL7 $\rightarrow$ hybrid framework

PRC87(2013)035503

Magnetic Moments in $A \leq 10$ Nuclei - bis

Predictions for $A > 3$ nuclei





- $\mu_N(\text{IA}) = \sum_i [(L_i + g_p S_i)(1+\tau_{i,z})/2 + g_n S_i (1-\tau_{i,z})/2]$
- ^{9}C (^{9}Li) dominant spatial symmetry [s.s.] = [432] = [α,^{3}He(^{3}H),$pp(nn)$] $\rightarrow$ Large MEC
- ^{9}Be (^{9}B) dominant spatial symmetry [s.s.] = [441] = [$\alpha, \alpha, n(p)$]

PRC87(2013)035503

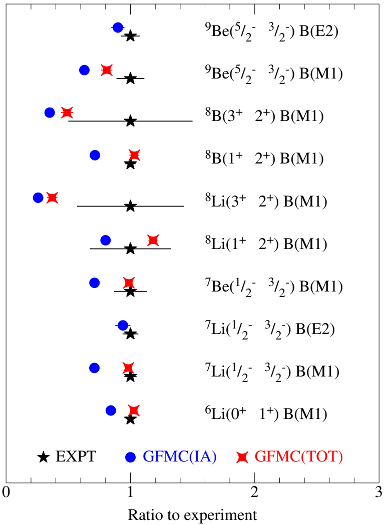
EM Transitions in $A \leq 9$ Nuclei

- Two-body EM currents bring the theory in a better agreement with the EXP
- Significant correction in $A = 9$, $T = 3/2$ systems. Up to $\sim 40\%$ correction found in ^{9}C m.m.
- Major correction ($\sim 60 - 70\%$ of total MEC) is due to the one-pion-exchange currents at NLO – purely isovector

One M1 prediction: $^9\text{Li}(1/2 \to 3/2)$*

$\Gamma(\text{IA}) = 0.59(2)$ eV
$\Gamma(\text{TOT}) = 0.79(3)$ eV

+ a number of B(E2)s in IA



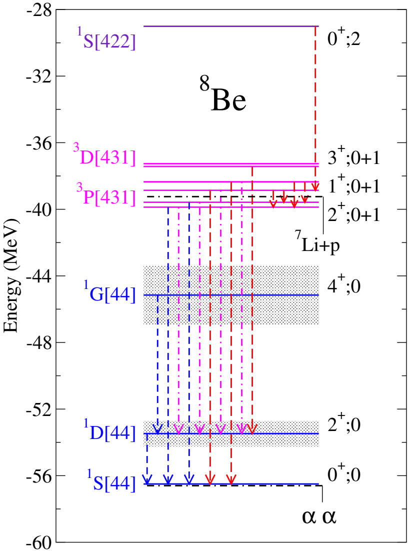
*Ricard-McCutchan *et al.* TRIUMF proposal 2014 - ongoing data analysis

PRC87(2013)035503

^{8}Be Energy Spectrum

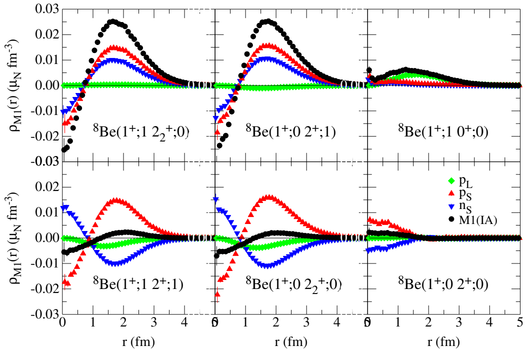
- 2^+ and 4^+ broad states at ~ 3 MeV and ~ 11 MeV
- isospin-mixed states at ~ 16 MeV, ~ 17 MeV, ~ 19 MeV
- M1 transitions
- E2 transitions
- E2 + M1 transitions

$J^\pi;T$	GFMC	Iso-mixed	Experiment
0^+	-56.3(1)		-56.50
2^+	+ 3.2(2)		+ 3.03(1)
4^+	+11.2(3)		+11.35(15)
$2^+;0$	+16.8(2)	+16.746(3)	+16.626(3)
$2^+;1$	+16.8(2)	+16.802(3)	+16.922(3)
$1^+;1$	+17.5(2)	+17.67	+17.640(1)
$1^+;0$	+18.0(2)	+18.12	+18.150(4)
$3^+;1$	+19.4(2)	+19.10	+19.07(3)
$3^+;0$	+19.9(2)	+19.21	+19.235(10)



PRL111(2013)062502 & PRC90(2014)024321

One-body M1 transitions densities



- [s.s.]-conserving transitions are enhanced due to overlap between large components of the initial and final w.f.'s
- Isospin-conserving transitions are suppressed w.r.t. isospin-changing transitions due to a cancellation between proton and neutron spin magnetization terms

$$\mathrm{M1(IA)} = \mu_N \sum_i [(L_i + g_p S_i)(1+\tau_{i,z})/2 + g_n S_i (1-\tau_{i,z})/2]$$

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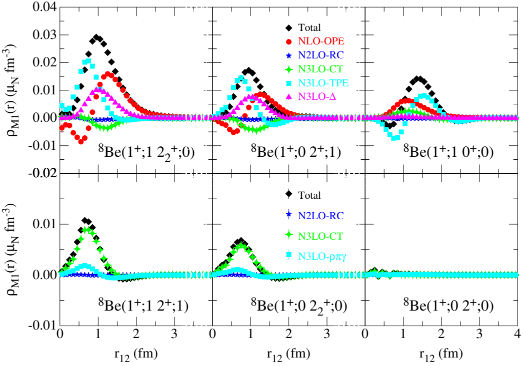
Two-body M1 transitions densities



$(J_i,T_i)\rightarrow(J_f,T_f)$	IA	NLO-OPE	N2LO-RC	N3LO-TPE	N3LO-CT	N3LO-Δ	MEC
$(1^+;1)\rightarrow(2_2^+;0)$	2.461 (13)	0.457 (3)	-0.058 (1)	0.095 (2)	-0.035 (3)	0.161 (21)	0.620 (5)

PRC90(2014)024321

M1 transitions in ^{8}Be isospin-mixed states

- 2^+, 1^+, and 3^+ states are isospin mixed, with mixing coefficients $\alpha_J^2 + \beta_J^2 = 1$

$$\begin{aligned} \psi^a &= \alpha_J\, \psi_{\mathrm{T}=0} + \beta_J\, \psi_{\mathrm{T}=1} \\ \psi^b &= \beta_J\, \psi_{\mathrm{T}=0} - \alpha_J\, \psi_{\mathrm{T}=1} \end{aligned}$$

- Mixing angles α, β are from experimental decay widths as $\Gamma^a/\Gamma^b = \alpha_J^2/\beta_J^2$
  - ($\alpha_2 \sim 0.77$, β_2) well known through EXP α-decay widths, which is the only channel energetically allowed and available via $T = 0$
  - ($\alpha_1 \sim 0.21$, β_1) and ($\alpha_3 \sim 0.41$, β_3) involve multiple decay channels $\rightarrow$ hard to extract them with great accuracy (Barker NUCL. PHYS. **83**, 418 (1966))

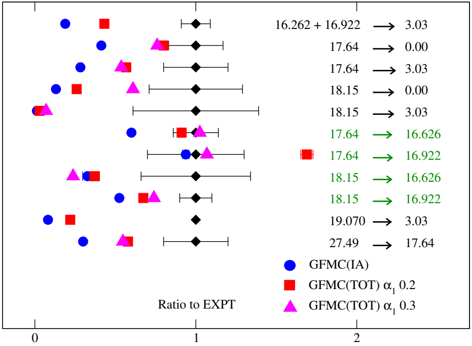
Example: $1^+; T = 0+1 \rightarrow 2^+; T = 0+1$, [431] $\rightarrow$ [431]

$< JT\|M1\|JT >$	IA	TOT	$E_i \rightarrow E_f$ [MeV]	B(M1)$_{\mathrm{IA}}$	B(M1)$_{\mathrm{TOT}}$	EXP [μ_N^2]
$< 10\|M1\|20 >$	0.17(0)	0.19(0)	18.15→16.626	0.56(1)	0.62(1)	1.88(46)
$< 10\|M1\|21 >$	2.60(1)	2.89(1)	18.15→16.922	1.56(2)	2.01(2)	2.89(33)
$< 11\|M1\|20 >$	2.29(1)	2.91(1)	17.64→16.626	1.65(2)	2.54(3)	2.65(25)
$< 11\|M1\|21 >$	0.14(0)	0.18(1)	17.64→16.922	0.25(1)	0.46(1)	0.30(7)

MEC contribute $\sim$ 20–30% of the total m.e.'s

PRC90(2014)024321 & PRC90(2014)024321

M1 Transition Widths / EXPT



- Predictions for [s.s.]-conserving transitions are in fair agreement with EXPT
- The theoretical description for this system is unsatisfactory, however, MEC provide a $\sim 20-30\%$ correction to the calculated matrix elements improving the agreement with EXPT data

PRC90(2014)024321

Benchmark calculations of ^{3}He Zemach Moments*

Quote: Precise moments are useful observables for the comparison with theoretical calculations, … in particular for light nuclei where very accurate *ab initio* calculations can be performed. I. Sick - PRC90(2014)064002

$$\langle r\rangle_{(2)} \propto -\int_0^\infty \frac{dq}{q^2}\left[G_E G_M - 1\right], \qquad \langle r^3\rangle_{(2)} \propto \int_0^\infty \frac{dq}{q^4}\left[G_E^2 - 1 + q^2R^2/3\right]$$

	VMC(IA)	VMC(TOT)	GFMC(IA)	GFMC(TOT)	EXPT
$\langle r\rangle_{(2)}$	2.522	2.477	2.504	2.454	2.528 ± 0.016 fm
$\langle r^3\rangle_{(2)}$	27.40	n.a.	29.30	n.a.	28.15 ± 0.70 fm^3
$\langle r_{ch}^2\rangle^{1/2}$	1.967	n.a.	1.970	n.a.	1.973 ± 0.014 fm
$\langle r_m^2\rangle^{1/2}$	2.000	1.962	2.019	1.942	1.976 ± 0.047 fm
$\langle r_{ch}^4\rangle$	19.8	n.a.	30.0	n.a.	32.9 ± 1.60 fm^4
$\langle\mu\rangle$	-1.775	-2.134	-1.767	-2.129	-2.127 μ_N

* in collaboration with S. Bacca, C. Ji *et al.*

Preliminary!!!

Summary

The microscopic description of nuclei successfully reproduces EXPT data provided that many-body effects in nuclear interactions and EM currents are accounted for.
J.Phys.G41(2014)123002 - S.Bacca&S.P.

- Two-body EM currents from χEFT tested in $A \leq 10$ nuclei
- Two-body corrections can be sizable and improve on theory/EXPT agreement
- EM structure of $A = 2$–3 nuclei well reproduced with chiral charge and current operators for $q \lesssim 3m_\pi$
- $\sim 40\%$ two-body correction found in ^{9}C's m.m.
- $\sim$ 20-30% corrections found in M1 transitions in low-lying states of ^{8}Be

Outlook

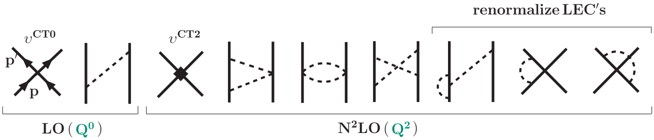
The microscopic description of nuclei successfully reproduces EXPT data provided that many-body effects in nuclear interactions and EM currents are accounted for.
J.Phys.G41(2014)123002 - S.Bacca&S.P.

* EM structure and dynamics of light nuclei
    - Charge and magnetic form factors of $A \leq 10$ systems
    - M1/E2 transitions in light nuclei
    - Radiative captures, photonuclear reactions ...
    - Fully consistent χEFT calculations now attainable within QMC computational schemes
    - Role of Δ-resonances in 'MEC' (EM current consistent with the chiral 'Δ-full' NN potential developed by M. Piarulli et al. PRC91(2015)024003)

* Electroweak structure and dynamics of light nuclei
    - Test axial currents (chiral and conventional) in light nuclei (χEFT axial currents derived by A. Baroni et al. arXiv:1509.07039)
    - Many-body effects in ν-d pion-production at threshold (in preparation)
    - Study ν-scattering off $A > 12$ nuclei

EXTRA SLIDES

NN Potential at NLO (or $Q^{n=2}$)

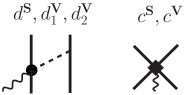


- Contact potential at LO (or $Q^{n=0}$) depends on 2 LECs
- Contact potential at NLO (or $Q^{n=2}$) depends on 7 additional LECs

NN potentials with π's and *N*'s

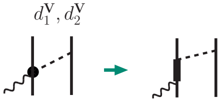
- van Kolck *et al.* (1994–96)
- Kaiser, Weise *et al.* (1997–98)
- Epelbaum, Glöckle, Meissner (1998–2015)
- Entem and Machleidt (2002–2015) $\leftarrow$
- …

χEFT EM currents at N3LO: fixing the EM LECs



Five LECs: d^S, d_1^V, and d_2^V could be determined by pion photo-production data on the nucleon

Isovector



d_2^V and d_1^V are known assuming Δ-resonance saturation

Left with 3 LECs: Fixed in the $A = 2 - 3$ nucleons' sector

- Isoscalar sector:
  - d^S and c^S from EXPT μ_d and $\mu_S(^3\mathrm{H}/^3\mathrm{He})$
- Isovector sector:
  - model I = c^V from EXPT $npd\gamma$ xsec.

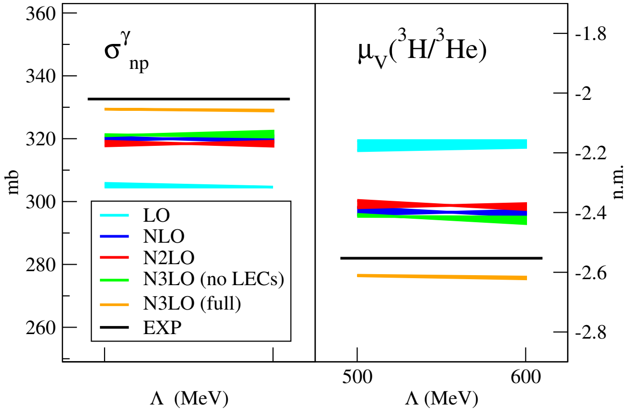
    or
  - model II = c^V from EXPT $\mu_V(^3\mathrm{H}/^3\mathrm{He})$ m.m. $\leftarrow$ our choice

Note that:
χEFT operators have a power law behavior $\rightarrow$ introduce a regulator to kill divergencies at large Q, *e.g.*, $C_\Lambda = e^{-(Q/\Lambda)^n}$, ...and also, pick n large enough so as to not generate spurious contributions

$$C_\Lambda \sim 1 - \left(\frac{Q}{\Lambda}\right)^n + \ldots$$

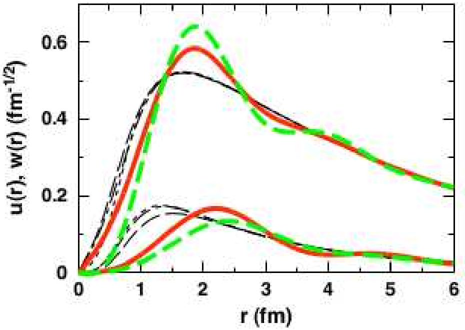
Predictions with χEFT EM currents for $A = 2$–3 systems

np capture xsec. (using model II) / μ_V of $A = 3$ nuclei (using model I)
bands represent nuclear model dependence (N3LO/N2LO – AV18/UIX)



- $npd\gamma$ xsec. and $\mu_V(^3\text{H}/^3\text{He})$ m.m. are within 1% and 3% of EXPT
- Two-body currents important to reach agreement with exp data
- Negligible dependence on the cutoff entering the regulator $exp(-(k/\Lambda)^4)$

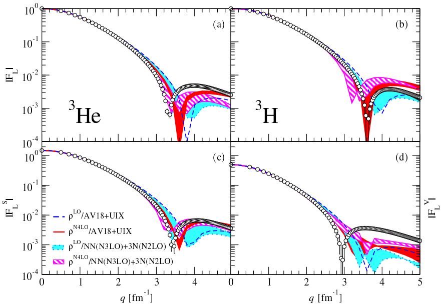
Deuteron wave functions



from Entem&Machleidt 2011 Review

- Entem&Machleidt N3LO
- Epelbaum *et al.* 2005
- black lines = conventional potentials, *i.e.* AV18, CD-Bonn, Nijm-I

^{3}He and ^{3}H charge f.f.'s



- Excellent agreement up to $q \simeq 2$ fm^{-1}
- N3LO and N4LO comparable

	^{3}He $< r >_{EXP} = 1.959 \pm 0.030$ fm		^{3}H $< r >_{EXP} = 1.755 \pm 0.086$	
Λ	500	600	500	600
LO	1.966 (1.950)	1.958 (1.950)	1.762 (1.743)	1.750 (1.743)
N4LO	1.966 (1.950)	1.958 (1.950)	1.762 (1.743)	1.750 (1.743)

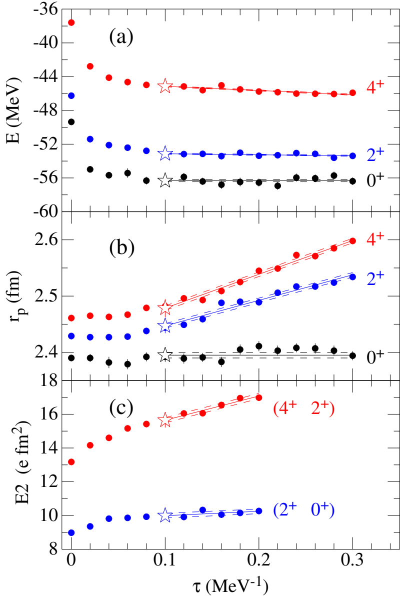
E2 transitions in ^{8}Be

- 2^+ and 4^+ broad rotational states at ~ 3 MeV and ~ 11 MeV
- $4^+ \rightarrow 2^+$ transition recently measured at BARC*, Mumbai
- Calculational challenge: 2^+ and 4^+ states tend to break up into two α as τ increases
- Results obtained by linear fitting the GFMC points and extrapolating at $\tau = 0.1$ MeV where stability is observed in the g.s. energy propagation

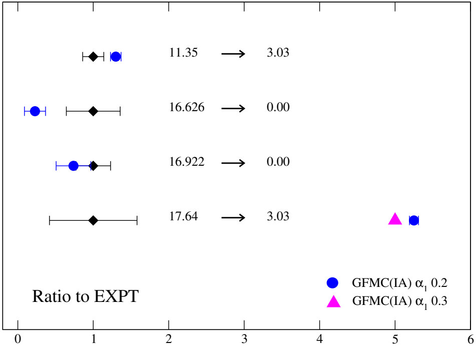
$J^\pi;T$	E [MeV]	B(E2) [e^2 fm^4]
0^+	-56.3(1)	
2^+	+ 3.2(2)	20.0 (8)– [$2^+ \rightarrow 0^+$]
4^+	+11.2(3)	27.2(15)– [$4^+ \rightarrow 2^+$]*

*Bhabha Atomic Research Centre

*EXPT B(E2) = 21 $\pm$ 2.3 e^2 fm^4



- We attempt to evaluate a number of E2 transitions (predictions not shown in the figure)
- Complications are due to large cancellations among large m.e.'s $\rightarrow$ E2s very sensitive to small components
- One more complication: make sure that the first and second $(J^\pi, T) = (2^+, 0)$ states are orthogonal



  * We orthogonalize the second $(J^\pi, T) = (2^+, 0)$ via

$$|\Psi^{2_2^+}(\text{ortho})\rangle_G = |\Psi^{2_2^+}\rangle_G - {}_G\langle\Psi^{2_2^+}|\Psi^{2^+}\rangle_V |\Psi^{2^+}\rangle_G$$

Anomalous magnetic moment of ^{9}C

Mirror nuclei spin expectation value

- Charge Symmetry Conserving (CSC) picture ($p \longleftrightarrow n$) $^\diamond$

$$< \sigma_z >= \frac{\mu(T_z=+T)+\mu(T_z=-T)-J}{(g_s^p+g_s^n-1)/2} = \frac{2\mu(\mathrm{IS})-J}{0.3796}$$

- For $A=9$, $T=3/2$ mirror nuclei: ^{9}C and ^{9}Li
  EXP $<\sigma_z>= 1.44$ while THEORY $<\sigma_z>\sim 1$ (assuming CSC)
  possible cause: Charge Symmetry Breaking (CSB)
- Three different predictions for $<\sigma_z>$ with CSC w.f.'s (*) and CSB w.f.'s

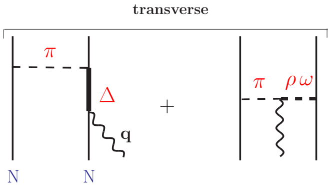
$<\sigma_z>$	Symmetry	IA	TOT	EXP
CSB	$^9\mathrm{Li}(\frac{3}{2}^-;\frac{3}{2}),{}^9\mathrm{C}(\frac{3}{2}^-;\frac{3}{2})$	1.05(1)	1.31(11)	1.44
CSC	$^9\mathrm{Li}(\frac{3}{2}^-;\frac{3}{2}),{}^9\mathrm{C}(\frac{3}{2}^-;\frac{3}{2})*$	0.95 (11)	1.00 (11)	
CSC	$^9\mathrm{Li}(\frac{3}{2}^-;\frac{3}{2})*,{}^9\mathrm{C}(\frac{3}{2}^-;\frac{3}{2})$	1.00 (1)	1.05 (1)	

- Need both CSB in the w.f.'s and MEC!

$^\diamond$ Utsuno – PRC**70**, 011303(R) (2004)

- Current operator **j** constructed so as to satisfy the continuity equation with a realistic Hamiltonian
- Short- and intermediate-behavior of the EM operators inferred from the nuclear two- and three-body potentials

$$\mathbf{j} = \mathbf{j}^{(1)} + \mathbf{j}^{(2)}(v) +$$



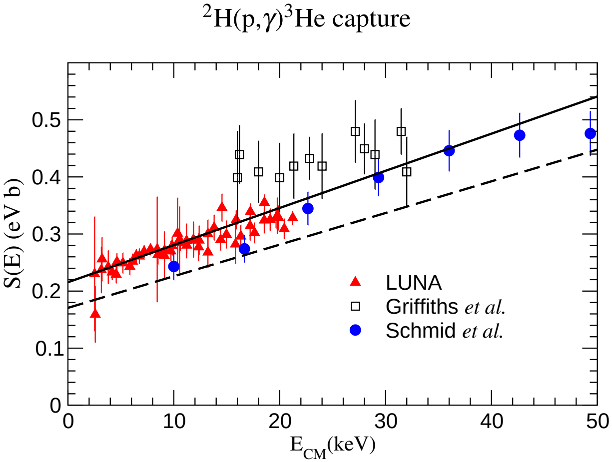
$$+ \ \mathbf{j}^{(3)}(V)$$

$*$ also referred to as Standard Nuclear Physics Approach (SNPA) currents

- Long range part of $\mathbf{j}(v)$ corresponds to OPE seagull and pion-in-flight EM currents

Currents from nuclear interactions - Marcucci *et al.* PRC**72**, 014001 (2005)

Satisfactory description of a variety of nuclear EM properties [see Marcucci *et al.* (2005) and (2008)]



- Isoscalar magnetic moments are a few % off (10% in A=7 nuclei)

Magnetic moments in $A \leq 9$ nuclei: SNPA vs χEFT

	A	s.s.	IA	TOT SNPA	TOT χEFT*	EXP
IS	7	[43]	0.902 (3)	0.833 (12)	0.906 (7)	0.929
IV		[43]	– 3.944 (5)	– 4.587 (18)	– 4.670 (9)	– 4.654
IS	8	[431]	1.289 (8)	1.160 (15)	1.299 (9)	1.344
IV		[431]	0.182 (8)	– 0.129 (15)	– 0.139 (9)	– 0.310
IS	9	[432]	0.994 (15)	0.922 (32)	1.038 (21)	1.024
IV		[432]	– 1.095 (10)	– 1.371 (21)	– 1.532 (15)	– 1.610

Preliminary results

Overall improvement of isoscalar (IS) component of the magnetic moment

$$\mu = \mu_S + \tau_z \mu_V$$

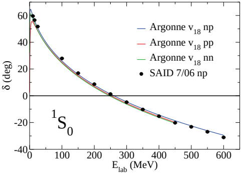
NUCLEAR HAMILTONIAN

$$H = \sum_i K_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

K_i: Non-relativistic kinetic energy, m_n-m_p effects included

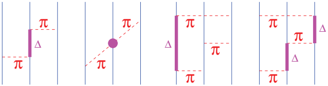
Argonne v$_{18}$: $v_{ij} = v_{ij}^{\gamma} + v_{ij}^{\pi} + v_{ij}^{I} + v_{ij}^{S} = \sum v_p(r_{ij}) O_{ij}^p$

- 18 spin, tensor, spin-orbit, isospin, etc., operators
- full EM and strong CD and CSB terms included
- predominantly local operator structure
- fits Nijmegen PWA93 data with χ^2/d.o.f.=1.1



Wiringa, Stoks, & Schiavilla, PRC **51**, (1995)

Urbana & Illinois: $V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^{3\pi} + V_{ijk}^{R}$



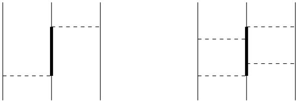
- Urbana has standard 2π P-wave + short-range repulsion for matter saturation
- Illinois adds 2π S-wave + 3π rings to provide extra T=3/2 interaction
- Illinois-7 has four parameters fit to 23 levels in $A \leq 10$ nuclei

Pieper, Pandharipande, Wiringa, & Carlson, PRC **64**, 014001 (2001)
Pieper, AIP CP **1011**, 143 (2008)

courtesy of R.B.Wiringa

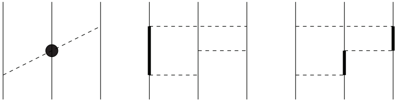
THREE-NUCLEON POTENTIALS

Urbana $V_{ijk} = V_{ijk}^{2\pi P} + V_{ijk}^{R}$



Carlson, Pandharipande, & Wiringa, NP **A401**, 59 (1983)

Illinois $V_{ijk} = V_{ijk}^{2\pi P} + V_{ijk}^{2\pi S} + V_{ijk}^{3\pi \Delta R} + V_{ijk}^{R}$



Pieper, Pandharipande, Wiringa, & Carlson, PRC **64**, 014001 (2001)

Illinois-7 has 4 strength parameters fit to 23 energy levels in $A \leq 10$ nuclei.
In light nuclei we find (thanks to large cancellation between $\langle K \rangle$ & $\langle v_{ij} \rangle$):

$$\langle V_{ijk} \rangle \sim (0.02 \text{ to } 0.07)\ \langle v_{ij} \rangle \sim (0.15 \text{ to } 0.5)\ \langle H \rangle$$

We expect $\langle \mathcal{V}_{ijkl} \rangle \sim 0.05\ \langle V_{ijk} \rangle \sim (0.01 \text{ to } 0.03)\ \langle H \rangle \sim 1$ MeV in ^{12}C .

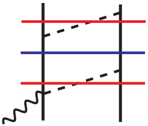
Transition amplitude in time-ordered perturbation theory

$$T_{\mathrm{fi}} = \langle f \mid T \mid i \rangle = \langle f \mid H_1 \sum_{n=1}^{\infty} \left(\frac{1}{E_i - H_0 + i\eta} H_1 \right)^{n-1} \mid i \rangle$$

$$= \langle f \mid H_1 \mid i \rangle + \sum_{|I\rangle} \langle f \mid H_1 \mid I \rangle \frac{1}{E_i - E_I} \langle I \mid H_1 \mid i \rangle + \ldots$$

- A contribution with N interaction vertices and L loops scales as

$$\underbrace{e \left(\prod_{i=1}^{N} Q^{\alpha_i - \beta_i/2} \right)}_{H_1\,\text{scaling}} \times \underbrace{Q^{-(N-N_K-1)}\, Q^{-2N_K}}_{\text{denominators}} \times \underbrace{Q^{3L}}_{\text{loop integration}}$$



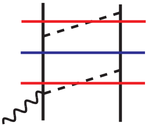
α_i = number of derivatives in H_1 and β_i = number of π's at each vertex

N_K = number of pure nucleonic intermediate states

- Due to the chiral expansion, the transition amplitude T_{fi} can be expanded as

$$T_{\mathrm{fi}} = T^{\mathrm{LO}} + T^{\mathrm{NLO}} + T^{\mathrm{N2LO}} + \ldots \quad \text{and} \quad T^{\mathrm{NnLO}} \sim (Q/\Lambda_\chi)^n T^{\mathrm{LO}}$$

Power counting



- N_K energy denominators scale as Q^{-2}

$$\frac{1}{E_i - E_I}|I\rangle = \frac{1}{E_i - E_N}|I\rangle \sim Q^{-2}|I\rangle$$

- $(N - N_K - 1)$ energy denominators scale Q^{-1} in the static limit; they can be further expanded in powers of $(E_i - E_N)/\omega_\pi \sim Q$

$$\frac{1}{E_i - E_I}|I\rangle = \frac{1}{E_i - E_N - \omega_\pi}|I\rangle \sim -\Big[\underbrace{\frac{1}{\omega_\pi}}_{Q^{-1}} + \underbrace{\frac{E_i - E_N}{\omega_\pi^2}}_{Q^0} + \underbrace{\frac{(E_i - E_N)^2}{\omega_\pi^3}}_{Q^1} + \dots\Big]|I\rangle$$

- Terms accounted into the Lippmann-Schwinger Eq. are subtracted from the reducible amplitude
- EM operators depend on the off-the-energy shell prescription adopted for the non-static OPE and TPE potentials
- Ultimately, the EM operators are unitarily equivalent: Description of physical systems is not affected by this ambiguity

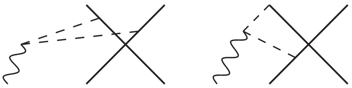
Magnetic moment at N^3LO

- Magnetic moment operator due to two-body current density $\mathbf{J}(\mathbf{x})$

$$\boldsymbol{\mu}(\mathbf{R},\mathbf{r}) = \frac{1}{2}\left[\mathbf{R}\times\int \mathrm{d}\mathbf{x}\,\mathbf{J}(\mathbf{x}) + \int \mathrm{d}\mathbf{x}\,(\mathbf{x}-\mathbf{R})\times\mathbf{J}(\mathbf{x})\right]$$

- Sachs' and translationally invariant magnetic moments

$$\begin{aligned}
\boldsymbol{\mu}_{\mathrm{Sachs}}(\mathbf{R},\mathbf{r}) &= -i\,\frac{\mathbf{R}}{2}\times\int \mathrm{d}\mathbf{x}\,\mathbf{x}\,[\rho(\mathbf{x})\,,\,\upsilon_{12}] \\
\boldsymbol{\mu}_{\mathrm{T}}(\mathbf{r}) &= -\frac{i}{2}\int_{\mathbf{k}} \mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}}\,\nabla_q\times\mathbf{j}(\mathbf{q},\mathbf{k})\Big|_{\mathbf{q}=0}
\end{aligned}$$



g) i)

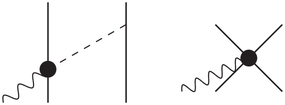
- Non-static corrections entering single-nucleon operators accounted into the derivation of current i)

$$
\begin{aligned}
i)\,\mathrm{OLD} &= i\frac{e\,g_A^2}{F_\pi^2}\,\tau_{1,z}\int \frac{\mathbf{q}_1-\mathbf{q}_2}{\omega_1^3\,\omega_2^3}\,\frac{\omega_1^2+\omega_1\,\omega_2+\omega_2^2}{\omega_1+\omega_2}\Big[C_S\,\boldsymbol{\sigma}_1\cdot(\mathbf{q}_1\times\mathbf{q}_2)\\
&- C_T\,\boldsymbol{\sigma}_2\cdot(\mathbf{q}_1\times\mathbf{q}_2)\Big]+1\rightleftharpoons 2\\
i)\,\mathrm{NEW} &= 2\,i\,\frac{e\,g_A^2\,C_T}{F_\pi^2}\,\tau_{1z}\int_{\mathbf{q}_1,\mathbf{q}_2}\frac{\omega_1^2+\omega_1\,\omega_2+\omega_2^2}{\omega_1^3\,\omega_2^3\,(\omega_1+\omega_2)}\,(\mathbf{q}_1-\mathbf{q}_2)\,\boldsymbol{\sigma}_2\cdot\mathbf{q}_2\times\mathbf{q}_1+1\rightleftharpoons 2
\end{aligned}
$$

- i) NEW in agreement with Kölling 2009/2011* but for a factor of 2, which has no impact because (i + g) = 0

* PRC**80**, 045502 (2009)/ PRC **84**, 054008 (2011)

2009 EM current vs 2011 EM currents p. 2/2



- A different derivation in Kölling 2009/2011* leads to an additional term $\sim (\boldsymbol{\sigma}_i \times \mathbf{q}) \times \mathbf{q}$ in the N3LO current at tree level, which however does not contribute to the magnetic moment
- The N3LO contact current of Pastore 2009 is in agreement with that of Kölling 2011 after Fierz-reordering, apart from differences in the term $\propto C_5$:

$$\mathbf{j}_{\mathrm{ct}}^{\mathrm{N3LO}} = -\frac{i\,C_5}{4}\,(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \times (e_1\,\mathbf{k}_1 + e_2\,\mathbf{k}_2)$$

* PRC**80**, 045502 (2009)/ PRC **84**, 054008 (2011)

- comparison with Kölling *et al.*:

  i) LO, NLO, N2LO, N3LO TPE, N3LO CT, N3LO TREE m.m.'s agree, but for the N3LO CT term $\propto C_5$
  ii) currents associated with one loop corrections to the OPE are missing in these calculations of m.m.'s; renormalization of OPE currents has been carried out in Kölling 2011*

- comparison with Park *et al.***:

  i) Sachs' m.m. is missing (no problem in two-body systems),
  ii) TPE box contribution at N3LO generates an extra term $\propto (\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)_z$

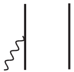
* Ultimately, in actual calculations these differences are presumably mitigated by fitting LECs to experimental data

* loop corrections to OPE: terms $\propto L(k)$ in Eq. (4.28) of Kölling 2011;

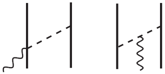
** NP **A596**, 515, (1996)

EM current up to $n = 1$ (or up to N3LO) - bis

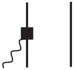
LO : $\mathbf{j}^{(-2)} \sim \mathbf{e}\mathbf{Q}^{-2}$



NLO : $\mathbf{j}^{(-1)} \sim \mathbf{e}\mathbf{Q}^{-1}$



$\mathrm{N}^2\mathrm{LO}$: $\mathbf{j}^{(-0)} \sim \mathbf{e}\mathbf{Q}^{0}$



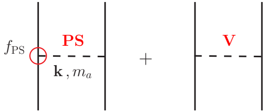
- $n = -2, -1, 0,$ and 1-(loops only): depend on known LECs namely g_A, F_π, and proton and neutron μ
- $n = 0$: $(Q/m_N)^2$ relativistic correction to $\mathbf{j}^{(-2)}$
- unknown LECs enter the $n = 1$ contact and tree-level currents (the latter originates from a $\gamma\pi N$ vertex of order eQ^2)

- divergencies associated with loop integrals are reabsorbed by renormalization of contact terms
- loops contributions lead to purely isovector operators
- $\mathbf{j}^{(n\leq 1)}$ satisfies the CCR with χEFT two-nucleon potential $\upsilon^{(n\leq 2)}$

$\mathrm{N}^3\mathrm{LO}$: $\mathbf{j}^{(1)} \sim \mathbf{e}\mathbf{Q}$

unknown LEC′s →

$v^{\mathrm{ME}} =$

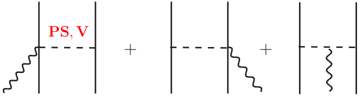


- Exploiting the meson exchange (ME) mechanism, one assumes that the static part υ_0 of υ is due to pseudoscalar (PS) and vector (V) exchanges
- υ^{ME} is expressed in terms of 'effective propagators' υ_{PS}, υ_V, υ_{VS}, fixed such to reproduce υ_0, for example

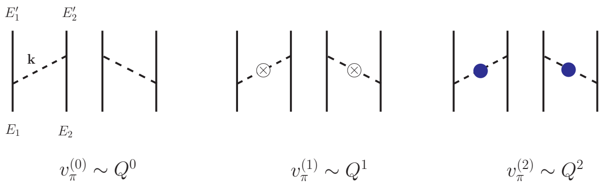
$$\upsilon_{PS} = [\upsilon^{\sigma\tau}(k) - 2\upsilon^{t\tau}(k)]/3$$

  with $\upsilon^{\sigma\tau}$ and $\upsilon^{t\tau}$ components of υ_0
- The current operator is obtained by taking the non relativistic reduction of the ME Feynman amplitudes and replacing the bare propagators with the 'effective' ones

$\mathbf{j}^{(2)}(v_0) =$



OPEP beyond the static limit



On-the-energy-shell, non-static OPEP at N2LO (Q^2) can be equivalently written as

$$
\begin{aligned}
\upsilon_\pi^{(2)}(\nu = 0) &= \upsilon_\pi^{(0)}(\mathbf{k})\,\frac{(E_1' - E_1)^2 + (E_2' - E_2)^2}{2\,\omega_k^2} \\
\upsilon_\pi^{(2)}(\nu = 1) &= -\upsilon_\pi^{(0)}(\mathbf{k})\,\frac{(E_1' - E_1)\,(E_2' - E_2)}{\omega_k^2} \\
\upsilon_\pi^{(0)}(\mathbf{k}) &= -\frac{g_A^2}{F_\pi^2}\,\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2\,\frac{\boldsymbol{\sigma}_1 \cdot \mathbf{k}\,\boldsymbol{\sigma}_2 \cdot \mathbf{k}}{\omega_k^2}
\end{aligned}
$$

$\upsilon_\pi^{(2)}(\nu)$ corrections are different off-the-energy-shell $(E_1 + E_2 \neq E_1' + E_2')$

- TPE contributions are affected by the choice made for the parameter ν

From amplitudes to potentials

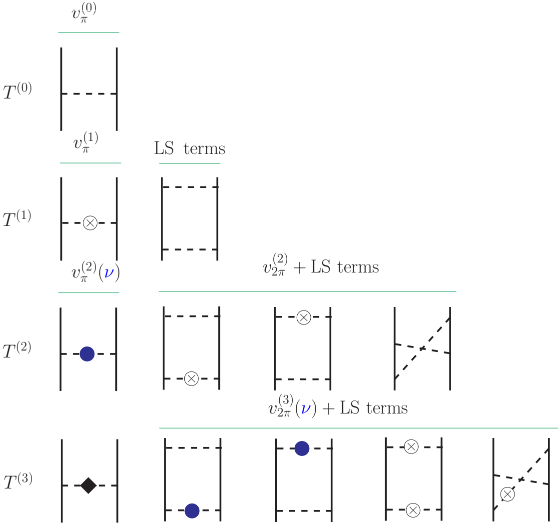
The two-nucleon potential $\upsilon = \upsilon^{(0)} + \upsilon^{(1)} + \upsilon^{(2)} + \dots$ (with $\upsilon^{(n)} \sim Q^n$) is iterated into the Lippmann-Schwinger (LS) equation *i.e.*

$$\upsilon + \upsilon\, G_0\, \upsilon + \upsilon\, G_0\, \upsilon\, G_0\, \upsilon + \dots\ , \qquad\qquad G_0 = 1/(E_i - E_I + i\,\eta)$$

$\upsilon^{(n)}$ is obtained subtracting from the transition amplitude $T_{\rm fi}^{(n)}$ terms already accounted for into the LS equation

$$\begin{aligned}
\upsilon^{(0)} &= T^{(0)}\ , \\
\upsilon^{(1)} &= T^{(1)} - \left[\upsilon^{(0)}\, G_0\, \upsilon^{(0)}\right]\ , \\
\upsilon^{(2)} &= T^{(2)} - \left[\upsilon^{(0)}\, G_0\, \upsilon^{(0)}\, G_0\, \upsilon^{(0)}\right] - \left[\upsilon^{(1)}\, G_0\, \upsilon^{(0)} + \upsilon^{(0)}\, G_0\, \upsilon^{(1)}\right]\ , \\
\upsilon^{(3)}(\nu) &= T^{(3)} - \left[\upsilon^{(0)}\, G_0\, \upsilon^{(0)}\, G_0\, \upsilon^{(0)}\, G_0\, \upsilon^{(0)}\right] - \left[\upsilon^{(1)}\, G_0\, \upsilon^{(0)}\, G_0\, \upsilon^{(0)} + \text{permutations}\right] \\
&\quad \underbrace{- \left[\upsilon^{(1)}\, G_0\, \upsilon^{(1)}\right] - \left[\upsilon^{(2)}(\nu)\, G_0\, \upsilon^{(0)} + \upsilon^{(0)}\, G_0\, \upsilon^{(2)}(\nu)\right]}_{\text{LS terms}}
\end{aligned}$$

From amplitudes to potentials: an example with OPE and TPE only



- To each $v_\pi^{(2)}(\nu)$ corresponds a $v_{2\pi}^{(3)}(\nu)$

Unitary equivalence of $\upsilon_\pi^{(2)}(\nu)$ and $\upsilon_{2\pi}^{(3)}(\nu)$

- Different off-the-energy-shell parameterizations lead to unitarily equivalent two-nucleon Hamiltonians

$$H(\nu) = t^{(-1)} + \upsilon_\pi^{(0)} + \upsilon_{2\pi}^{(2)} + \upsilon_\pi^{(2)}(\nu) + \upsilon_{2\pi}^{(3)}(\nu)$$

  $t^{(-1)}$ is the kinetic energy, $\upsilon_\pi^{(0)}$ and $\upsilon_{2\pi}^{(2)}$ are the static OPEP and TPEP

- The Hamiltonians are related to each other via

$$H(\nu) = \mathrm{e}^{-iU(\nu)} H(\nu=0)\, \mathrm{e}^{+iU(\nu)} , \qquad i\,U(\nu) \simeq i\,U^{(0)}(\nu) + i\,U^{(1)}(\nu)$$

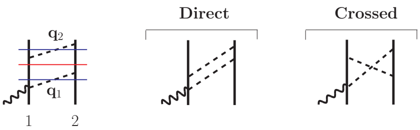
  from which it follows

$$H(\nu) = H(\nu=0) + \left[t^{(-1)} + \upsilon_\pi^{(0)}\, , i\,U^{(0)}(\nu)\right] + \left[t^{(-1)}, i\,U^{(1)}(\nu)\right]$$

- Predictions for physical observables are unaffected by off-the-energy-shell effects

Technical issue II - Recoil corrections at N^{3}LO

$$\mathbf{j}^{\mathrm{N^3LO}} \quad =$$



- Reducible contributions

$$\begin{aligned}
\mathbf{j}_{\mathrm{red}} \quad &\sim \quad \int \upsilon^{\pi}(\mathbf{q}_2)\,\frac{1}{E_i - E_I}\,\mathbf{j}^{\mathrm{NLO}}(\mathbf{q}_1) \\
&- \quad \int 2\,\frac{\omega_1+\omega_2}{\omega_1\,\omega_2}\,V_{\pi NN}(2,\mathbf{q}_2)\,V_{\pi NN}(2,\mathbf{q}_1)\,V_{\pi NN}(1,\mathbf{q}_2)\,V_{\gamma\pi NN}(1,\mathbf{q}_1)
\end{aligned}$$

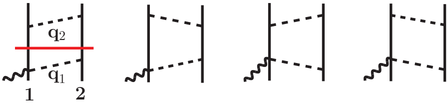
- Irreducible contributions

$$\begin{aligned}
\mathbf{j}_{\mathrm{irr}} \quad &= \quad \int 2\,\frac{\omega_1+\omega_2}{\omega_1\,\omega_2}\,V_{\pi NN}(2,\mathbf{q}_2)\,V_{\pi NN}(2,\mathbf{q}_1)\,V_{\pi NN}(1,\mathbf{q}_2)\,V_{\gamma\pi NN}(1,\mathbf{q}_1) \\
&- \quad \int 2\,\frac{\omega_1^2+\omega_2^2+\omega_1\,\omega_2}{\omega_1\,\omega_2(\omega_1+\omega_2)}\left[V_{\pi NN}(2,\mathbf{q}_2),V_{\pi NN}(2,\mathbf{q}_1)\right]_{-}V_{\pi NN}(1,\mathbf{q}_2)V_{\gamma\pi NN}(1,\mathbf{q}_1)
\end{aligned}$$

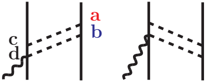
- Observed partial cancellations at N^{3}LO between recoil corrections to reducible diagrams and irreducible contributions

The box diagram: an example at N^{3}LO

Reducible



Irreducible direct



Irreducible crossed

$$\text{direct} = f_d(\omega_1,\omega_2)\, V_a\, V_b\, V_c\, V_d$$

$$\text{crossed} = f_c(\omega_1,\omega_2)\, V_b\, V_a\, V_c\, V_d \qquad V_b V_a = V_a V_b - [V_a, V_b]_-$$

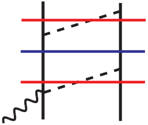
$$\begin{aligned} \text{irreducible} &= [f_d(\omega_1,\omega_2) + f_c(\omega_1,\omega_2)]\, V_a\, V_b\, V_c\, V_d \\ &- f_c(\omega_1,\omega_2)\, [V_a, V_b]_-\, V_c\, V_d \end{aligned}$$

Transition amplitude in time-ordered perturbation theory

$$
\begin{aligned}
T_{\rm fi} = \langle f \mid T \mid i \rangle \quad &= \quad \langle f \mid H_1 \sum_{n=1}^{\infty} \left(\frac{1}{E_i - H_0 + i\eta} H_1 \right)^{n-1} \mid i \rangle \\
&= \quad \langle f \mid H_1 \mid i \rangle + \sum_{|I\rangle} \langle f \mid H_1 \mid I \rangle \frac{1}{E_i - E_I} \langle I \mid H_1 \mid i \rangle + \ldots
\end{aligned}
$$

- A contribution with N interaction vertices and L loops scales as

$$
\underbrace{e \left(\prod_{i=1}^{N} Q^{\alpha_i - \beta_i/2} \right)}_{H_1 \, \text{scaling}} \times \underbrace{Q^{-(N-N_K-1)} \, Q^{-2N_K}}_{\text{denominators}} \times \underbrace{Q^{3L}}_{\text{loop integration}}
$$



α_i = number of derivatives in H_1 and β_i = number of π's at each vertex

N_K = number of pure nucleonic intermediate states

- $(N - N_K - 1)$ energy denominators expanded in powers of $(E_i - E_N)/\omega_\pi \sim Q$

$$
\frac{1}{E_i - E_I} |I\rangle = \frac{1}{E_i - E_N - \omega_\pi} |I\rangle \sim - \left[\underbrace{\frac{1}{\omega_\pi}}_{Q^{-1}} + \underbrace{\frac{E_i - E_N}{\omega_\pi^2}}_{Q^0} + \underbrace{\frac{(E_i - E_N)^2}{\omega_\pi^3}}_{Q^1} + \ldots \right] |I\rangle
$$

- Due to the chiral expansion, the transition amplitude $T_{\rm fi}$ can be expanded as

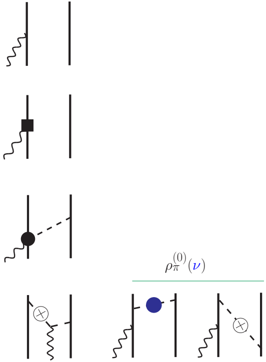
$$
T_{\rm fi} = T^{\rm LO} + T^{\rm NLO} + T^{\rm N2LO} + \ldots \quad \text{and} \quad T^{\rm NnLO} \sim (Q/\Lambda_\chi)^n T^{\rm LO}
$$

EM charge up to $n = 0$ (or up to N3LO)

LO : $\rho^{(-3)}$

N^{2}LO : $\rho^{(-1)}$

N^{3}LO : $\rho^{(0)}$



- $n = -3$

  $\rho^{(-3)}(\mathbf{q}) = e\,(2\pi)^3\delta(\mathbf{p}_1 + \mathbf{q} - \mathbf{p}_1')\,(1+\tau_{1,z})/2 + 1 \rightleftharpoons 2$

- $n = -1$:

  $(Q/m_N)^2$ relativistic correction to $\rho^{(-3)}$

- $n = 0$:

  i) 'static' tree-level current (originates from a $\gamma\pi N$ vertex of order $e\,Q$)

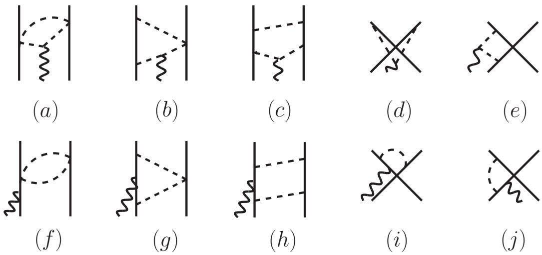
  ii) 'non-static' OPE charge operators, $\rho_\pi^{(0)}(\nu)$ depends on $\upsilon_\pi^{(2)}(\nu)$

- $\rho_\pi^{(0)}(\nu)$'s are unitarily equivalent

$$\rho_\pi^{(0)}(\nu) \quad = \quad \rho_\pi^{(0)}(\nu = 0) + \left[\rho^{(-3)}\,,\, i\,U^{(0)}(\nu)\right]$$

- No unknown LECs up to this order (g_A, F_π)

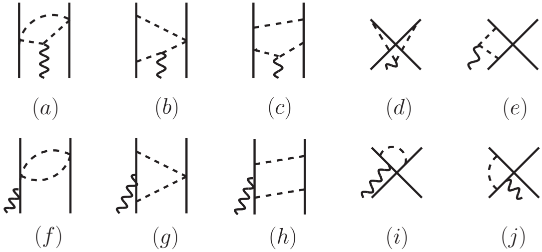
N^{4}LO : $\rho^{(1)}$



- (a), (f), (d), and (i) vanish
- Divergencies associated with (b) + (g), (c) + (h), and (e) + (j) cancel out—as they must since there are no counter-terms at N4LO
- $\rho_{\rm h}^{(1)}(\nu)$ depends on the parametrization adopted for $\upsilon_{\pi}^{(2)}(\nu)$ and $\upsilon_{2\pi}^{(3)}(\nu)$
- $\rho_{\rm h}^{(1)}(\nu)$'s are unitarily equivalent

$$\rho_{\rm h}^{(1)}(\nu) \quad = \quad \rho_{\rm h}^{(1)}(\nu=0) + \left[\rho^{(-3)}\, , i\, U^{(1)}(\nu)\right]$$

N^{4}LO : $\rho^{(1)}$



- Charge operators (ν-dependent included) up to $n=1$ satisfy the condition

$$\rho^{(n>-3)}(\mathbf{q}=0)=0$$

which follows from charge conservation

$$\rho(\mathbf{q}=0)=\int \mathrm{d}\mathbf{x}\,\rho(\mathbf{x})=e\,\frac{(1+\tau_{1,z})}{2}+1\rightleftharpoons 2=\rho^{(-3)}(\mathbf{q}=0)$$

- $\rho^{(1)}$ does not depend on unknown LECs and it is purely isovector