Collective aspects of microscopic mean-field evolution along the fission path

Yusuke Tanimura\textsuperscript{1}, Denis Lacroix\textsuperscript{1} and Guillaume Scamps\textsuperscript{2}

\textsuperscript{1}IPN Orsay, \textsuperscript{2}Tohoku University

Nuclear fission

• Importance
  – Energy production
  – Synthesis of super heavy elements
  – Astrophysical process
  – Production of radioactive isotopes
Nuclear fission

• Importance
  – Energy production
  – Synthesis of super heavy elements
  – Astrophysical process
  – Production of radioactive isotopes

• Theoretical challenges
  – Phenomenological models in terms of a few macroscopic degrees of freedom (elongation, mass asymmetry,...) have been developed
  – Successful microscopic models are still under development
  – Complicated dynamical process of quantum many-body system
    • Quantal treatments for both single-particle and collective DOFs
    • Dynamical and non-adiabatic effects
    • Different time scales
Microscopic models for fission

Static approach

- With energy density functional (EDF) theories (Skyrme, Gogny, RMF)
- Fission paths on the potential energy surface
  - Adiabatic (no excitation)
  - Dynamics is poorly treated
Our motivation:

• Dynamical approach to fission based on **time-dependent energy density functional** (TD-EDF) theory

• Mapping TD-EDF trajectory onto a selected set of collective variables

**TD-EDF ↔ collective space**

✓ Momenta

✓ Masses

for a given set of collective variables
To Bridge TD-EDF and collective motion

Collective variable(s): $\hat{Q}_\alpha$

ex: $\hat{Q}_2 = 2z^2 - x^2 - y^2$

Mass and conjugate momentum associated with $Q_\alpha$ ?
To Bridge TD-EDF and collective motion

Collective variable(s): $\hat{Q}_\alpha$

ex: $\hat{Q}_2 = 2z^2 - x^2 - y^2$

Mass and conjugate momentum associated with $Q_\alpha$?

\[ \frac{d\langle \hat{Q}_\alpha \rangle}{dt} = \frac{\langle \hat{P}_\alpha \rangle}{M_\alpha} \]

\[ \text{Tr}(\rho(t)[\hat{Q}_\alpha, \hat{P}_\alpha]) = i\hbar \]

\[ \frac{m_N}{M_\alpha} = \text{Tr}[\rho(t)\nabla Q_\alpha \cdot \nabla Q_\alpha] \]

\[ P_\alpha = \frac{\hbar M_\alpha}{2i m_N} [(\nabla Q_\alpha) \cdot \nabla + \nabla \cdot (\nabla Q_\alpha)] \]

Application to \( ^{258}\text{Fm} \)

- TDHF + BCS (Sly4d + constant pairing)
- Starting from a point on PEC
- No spontaneous fission for \( Q_2(t=0) < 160 \text{ b} \)

Symmetric fission of \( ^{258}\text{Fm} \) with TDHF + BCS
Application to $^{258}$Fm

- TDHF + BCS
  (Sly4d + constant pairing)
- Starting from a point on PEC
- No spontaneous fission
  for $Q_2(t=0) < 160$ b

Symmetric fission of $^{258}$Fm
with TDHF + BCS
1. Collective mass and momentum

- Deviation from static one around scission
- Scission happens more smoothly

\[ \frac{m_N}{M_\alpha} = \text{Tr} [\rho \nabla Q_\alpha \cdot \nabla Q_\alpha] \]
1. Collective mass and momentum

$^{258}$Fm  $E_x = 0.0$ MeV, $t = 1187.18$ fm/$c$

$\leftarrow Q_2$ mass on dynamical path

- Deviation from static one around scission
- Scission happens more smoothly

$$\frac{m_N}{M_\alpha} = \text{Tr}[\rho \nabla Q_\alpha \cdot \nabla Q_\alpha]$$
1. Collective mass and momentum

\[ P_2 = M_2 \frac{dQ_2}{dt} \]

Dissipation before the scission (static)
Assume a classical motion with dissipative force

\[ F(Q_2) \sim \text{force coming from dynamical potential and friction} \]

\[ \begin{align*}
\dot{Q}_2 &= \frac{P_2}{M_2} \\
\dot{P}_2 &= -\frac{\partial V_{\text{coll}}}{\partial Q_2} + \frac{1}{2} \frac{\partial M_2}{\partial Q_2} \dot{Q}_2^2 + \gamma(Q_2) \dot{Q}_2 \\
\dot{P}_2 - \frac{1}{2} \frac{\partial M_2}{\partial Q_2} \dot{Q}_2^2 &= -\frac{\partial V_{\text{coll}}}{\partial Q_2} + \gamma(Q_2) \dot{Q}_2
\end{align*} \]

cf. K. Washiyama and D. Lacroix, PRC78, 024610 ('08)
2. More analysis of motion in $Q_2$ space

\[ \dot{P}_2 - \frac{1}{2} \frac{\partial M_2}{\partial Q_2} \dot{Q}_2^2 = - \frac{\partial V_{\text{coll}}}{\partial Q_2} + \gamma(Q_2) \dot{Q}_2 \]

$F(Q_2)$ ~ force coming from dynamical potential and friction

- Reduction of outward force compared to the static path
  - fragments stick together more and/or
  - friction against separation (dissipation)

- Dissipation occurs before scission
2. More analysis of motion in $Q_2$ space

Define "dynamical potential" (work done by $F$ on $Q_2$)

$$V^{\text{dyn}}(Q_2) \equiv V_C(Q_2^{\text{max}}) + \int_{Q_2}^{Q_2^{\text{max}}} dQ_2' F(Q_2')$$
2. More analysis of motion in $Q_2$ space

Define “dynamical potential” (work done by $F$ on $Q_2$)

$$V^{\text{dyn}}(Q_2) \equiv V_C(Q_2^{\max}) + \int_{Q_2}^{Q_2^{\max}} dQ_2' F(Q_2')$$

Potential **along adiabatic path**

Work by $F$ **along dynamic path**

Difference = Dissipated Energy
3. Generalization to several DOFs

Diagonalize the inertia tensor:

\[
\frac{m_N}{M_{\alpha\beta}} = \text{Tr}[\rho \nabla Q_\alpha \nabla Q_\beta]
\]

\[
\frac{1}{M} \rightarrow \frac{1}{M'} = W \frac{1}{M} W^T
\]

\[
Q' \rightarrow W Q
\]

\[
(P/M)' \rightarrow W(P/M)
\]

\[
\Rightarrow \langle [Q'_\alpha, P'_\beta] \rangle = \delta_{\alpha\beta} i\hbar
\]
3. Generalization to several DOFs

Diagonalize the inertia tensor:

$$\frac{m_N}{M_{\alpha\beta}} = \text{Tr}[\rho \nabla Q_\alpha \nabla Q_\beta]$$

$$\frac{1}{M} \rightarrow \frac{1}{M'} = W \frac{1}{M} W^T$$

$$Q' \rightarrow WQ$$

$$(P/M)' \rightarrow W(P/M)$$

Collective kinetic energy

$$E_{\text{kin}}^{\{\alpha\}} = \sum_\alpha \frac{P^{i2}_\alpha}{2M'_{\alpha}}$$  \hspace{1cm} \text{selected set}$$

$$E_{\text{kin}}^{\text{tot}} = \int d^3r \frac{\dot{\mathbf{j}}^2}{\rho}$$  \hspace{1cm} \text{total}
Summary and perspectives

- We have developed a method to extract information in collective space from TD-EDF theory
- Our goal: **unified microscopic approach for fission**
- Next steps
  - beyond-mean-field effects with configuration mixing
    - mapping onto $\{Q_\alpha, P_\alpha\}$ space
    - quantum/thermal fluctuation of collective DOFs
  - obtain physical observables
    - fragment mass/charge distribution
    - kinetic energy of fragments
    - ...
  - compare them with data
    and other theoretical approaches