Study of low-frequency octupole vibrations of rotating superdeformed nuclei by means of cranked RPA calculation with Skyrme energy density functional

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in collaboration with

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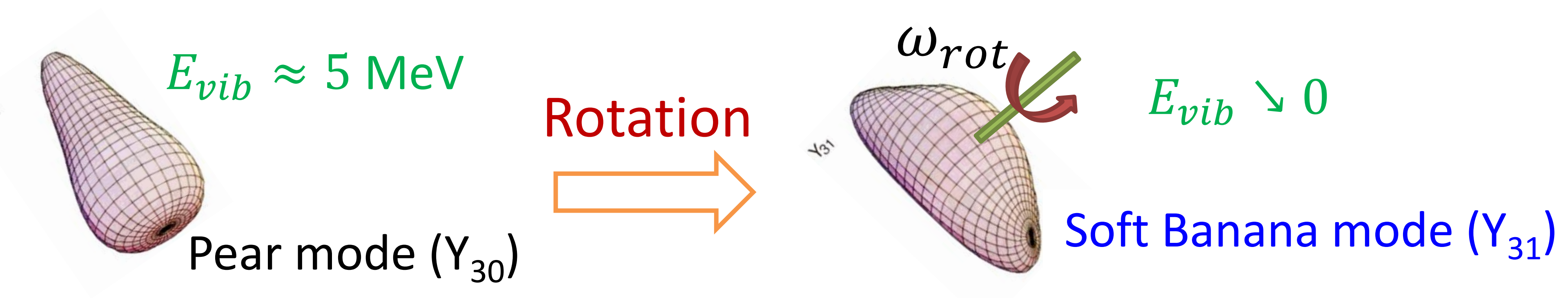


Outline

First part (method):

- Fourier-series expansion method ($\vec{k}$-space rep.)

  (*Fast/small-size DFT calc. for deformed unstable nuclei*)

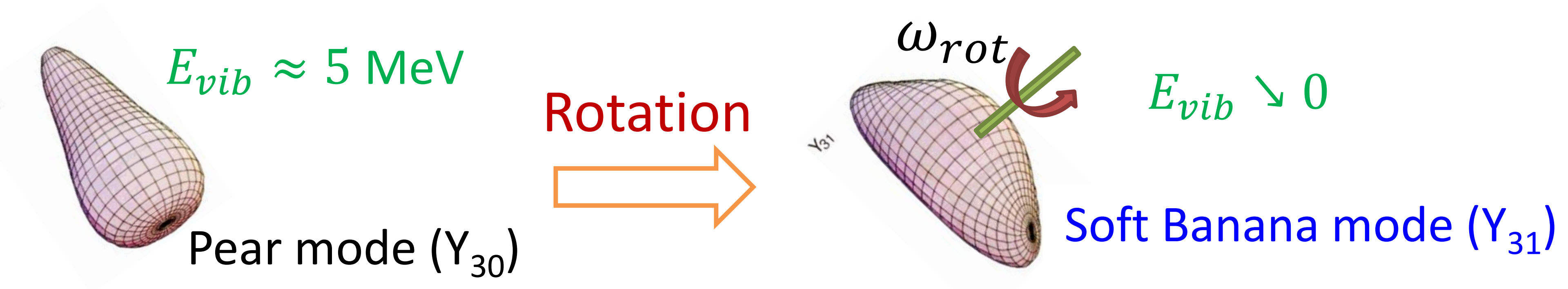
  ➢ Convergence, Comparison to $\vec{r}$-space representation

- Application to cranked RPA with Skyrme-EDF

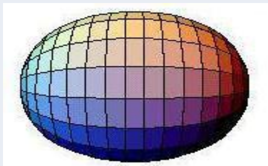
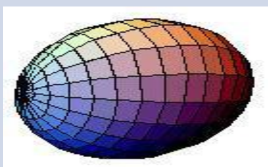
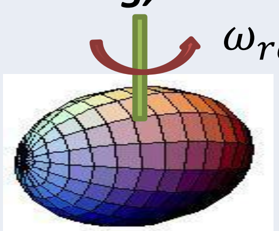
Second part (application):

- Octupole vibrations of *Super*- and *Hyper*deformed states around ^{40}Ca

  ➢ Rotational effect, Soft banana mode (Y_{31} on SD, HD)



Deformed RPA with Skyrme-EDF (extract)

Nuclear shape	*Normal-fluid* nuclei	*Superfluid* nuclei
Axially symmetric 	**$\vec{r}$-space rep. $\varphi(\vec{r})$** **Yoshida *et al.*, PRC78 (2008)** **Terasaki *et al.*, PRC82 (2010)**	
Triaxial 	**$\vec{r}$-space rep. $\varphi(\vec{r})$** **Imagawa *et al.*, PRC67 (2003)** **Inakura *et al.*, NPA768 (2006)**	***None***
Rotating, Triaxial ω_{rot} 	**$\vec{r}$-space rep. $\varphi(\vec{r})$** ***None**** **$\vec{k}$-space rep. $\hat{\varphi}(\vec{k})$** ***Present work***	***None***

Next challenge

Future

Extension of RPA to Local-RPA with Skyrme-EDF

LARGE amplitude dynamics:

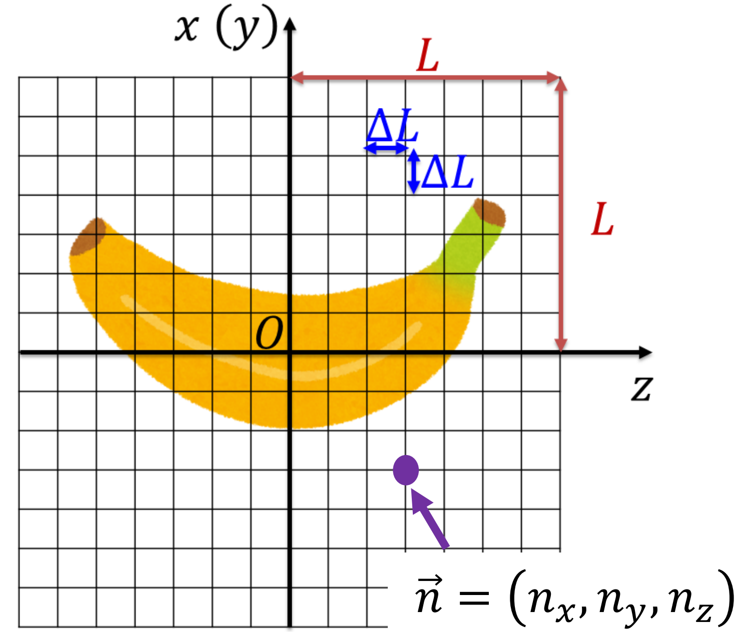
Shape transition, shape coexistence, fission dynamics, …

* RPA with *Woods-Saxon pot.* : H. Ogasawara*, et al.*, Prog. Theor. Phys. 121 (2009)

** Time-dependent approach (including FAM) : USA, France, Germany, Japan, …

3D coordinate-space rep. ($\vec{r}$-space rep.)

$$\varphi_{\vec{n}} \equiv \varphi(\vec{r} = \Delta L \vec{n})$$
$$\vec{n} = (n_x, n_y, n_z)$$
$$n_x, n_y, n_z = 0, \pm 1, , \cdots, \pm N_{max}$$



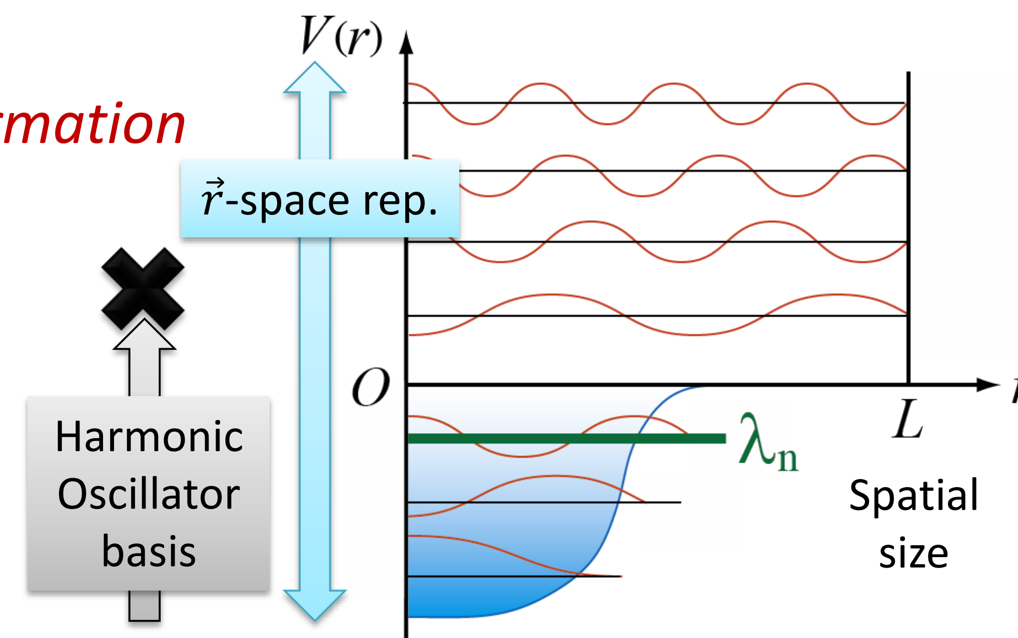
Advantage

- Unstable nuclei
  Weakly, unbound single-particle states
- Exotic shapes
  e.g., non-axial octupole deformation
- Simple coding

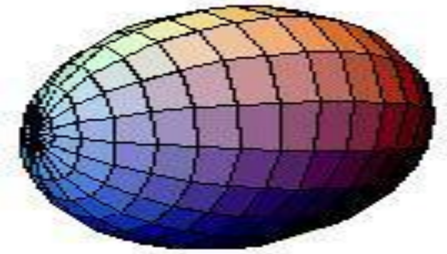
Disadvantage

- Large computational effort (Time, memory)

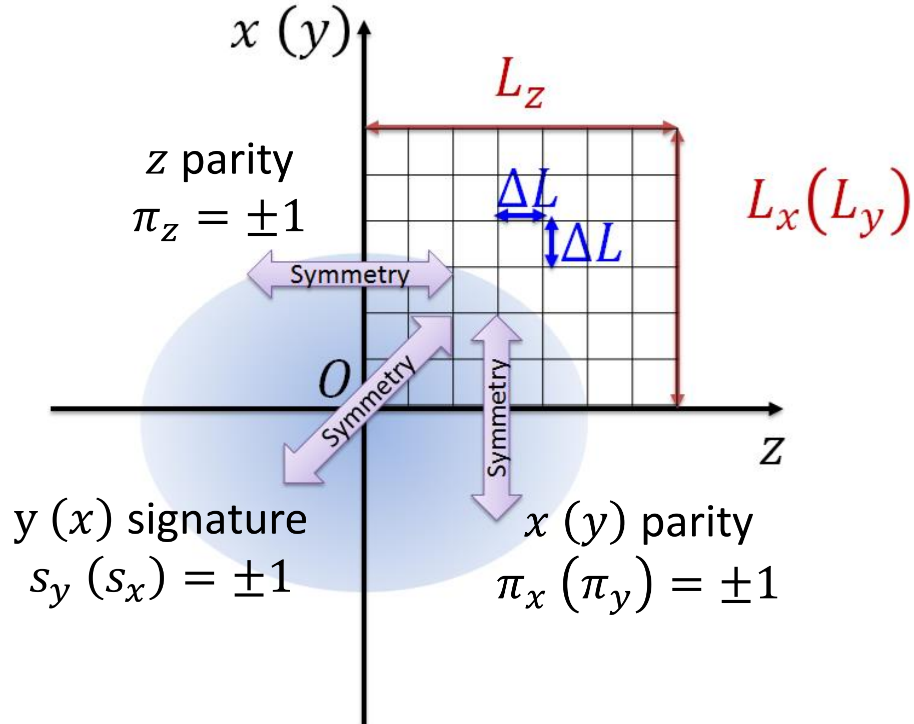
Cf. Harmonic Oscillator basis



Reduction of the computational effort



Triaxial shape



1/8 coodinate space

$n_x, n_y, n_z \geq 0$

$\varphi_{\vec{n}} \equiv \varphi(\vec{r} = \Delta L \vec{n})$

$\vec{n} = (n_x, n_y, n_z)$

P.Bonche, H.Flocard, P.H.Heenen, S.J.Krieger, M.S.Weiss
Nucl.Phys. A443, 39 (1985)

Typical number of grid points

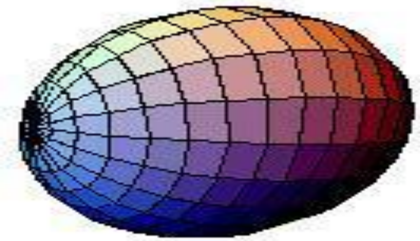
RPA for SD in ^{40}Ca: T.Inakura *et al.*, Nucl.Phys. A768 (2006)

$N_{grid}^{(\vec{r})}$=5625 points

$\Delta L = 0.6$ fm, $L_x = L_y = 8.7$ fm, $L_z = 14.7$ fm

Fourier-series expansion method ($\vec{k}$-space rep.)

$$\varphi(\vec{r}) = \sum_{\vec{n}} \hat{\varphi}_{\vec{n}} f_{n_x}^{(\pi_x)}(x) f_{n_y}^{(\pi_y)}(y) f_{n_z}^{(\pi_z)}(z)$$

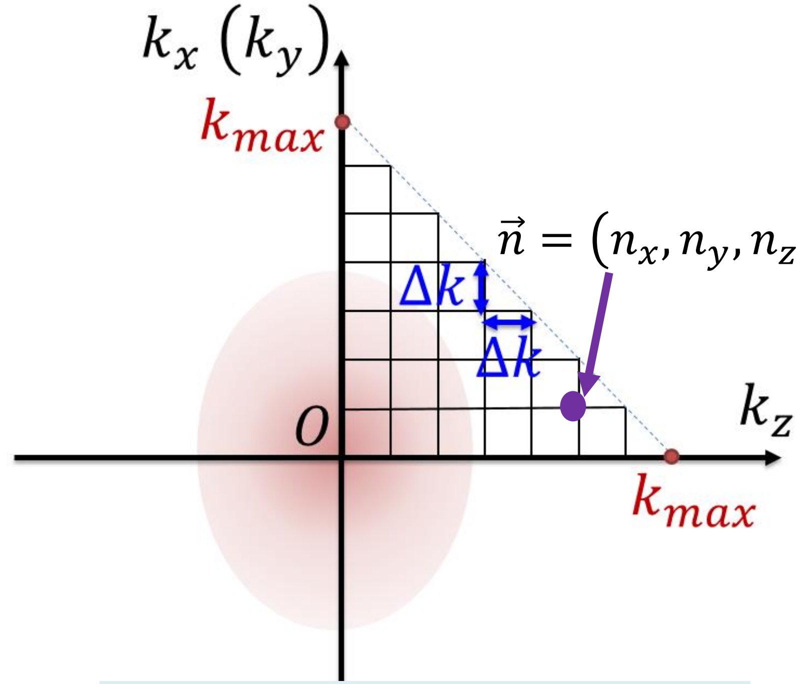


Triaxial shape

$$\vec{k} = \Delta k \vec{n} = \Delta k (n_x, n_y, n_z)$$

$$f_n^{(+)}(x) = \frac{1}{\sqrt{(1+\delta_{n,0})L}} \cos k_n x$$

$$f_n^{(-)}(x) = \frac{1}{\sqrt{L}} \sin k_n x$$



$$0 \le k_x, k_y, k_z \le k_{max}$$
$$k_x + k_y + k_z \le k_{max}$$

Essential to calculate within $\vec{k}$-space:

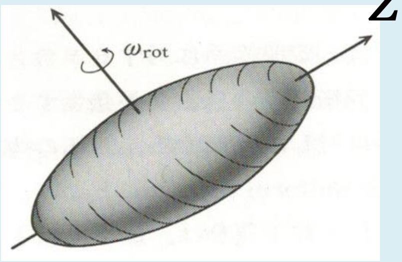
ρ^{α}, $V_{Coulomb}$, …

Skyrme-RPA calculation *for rotation nuclei*

Microscopic description of coupling between *rotation* and *vibration*

Cranked mean-field calculation

$$h' = h_{Skyrme} - \omega_{rot} \hat{j}_x$$



- Skyrme-EDF: SLy4
- Triaxial shape & NO time-reversal symmetry
- Fourier-series expansion method $\left(\vec{k}\text{–space}\right)$

RPA matrix equation

$$\sum_{p'h'} \begin{pmatrix} A_{php'h'} & B_{php'h'} \\ -B^*_{php'h'} & -A^*_{php'h'} \end{pmatrix} \begin{pmatrix} f^{(\lambda)}_{p'h'} \\ g^{(\lambda)}_{p'h'} \end{pmatrix} = E_\lambda \begin{pmatrix} f^{(\lambda)}_{ph} \\ g^{(\lambda)}_{ph} \end{pmatrix}$$

- Residual interaction

  Landu-Migdal approximation of Skyrme interaction
- Energy cutoff: $\varepsilon_p - \varepsilon_h < 40$ MeV

Computer (SR16000 in YITP Kyoto)

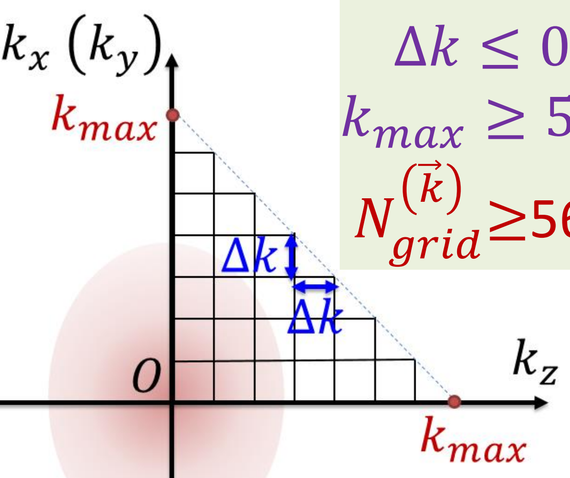
- Non-pallarel use, Memory < 1.5 GB (small size computing)

Convergence: mean-field solutions

Ground state of ^{40}Ca

Skyrme	E_{tot} [MeV]	$E_{tot}^{(Ref)}$ [MeV]	R_{ch} [fm]	$R_{ch}^{(Ref)}$ [fm]
SLy4	-344.30	-344.23	3.485	3.493
SkM*	-340.90	-341.05	3.489	3.499

- $\Delta k = 0.32\ fm^{-1}$, $k_{max} = 5.76\ fm^{-1}$
- Reference values (spherical code)
  E. Chabanat et al., NPA635 (1998) 231



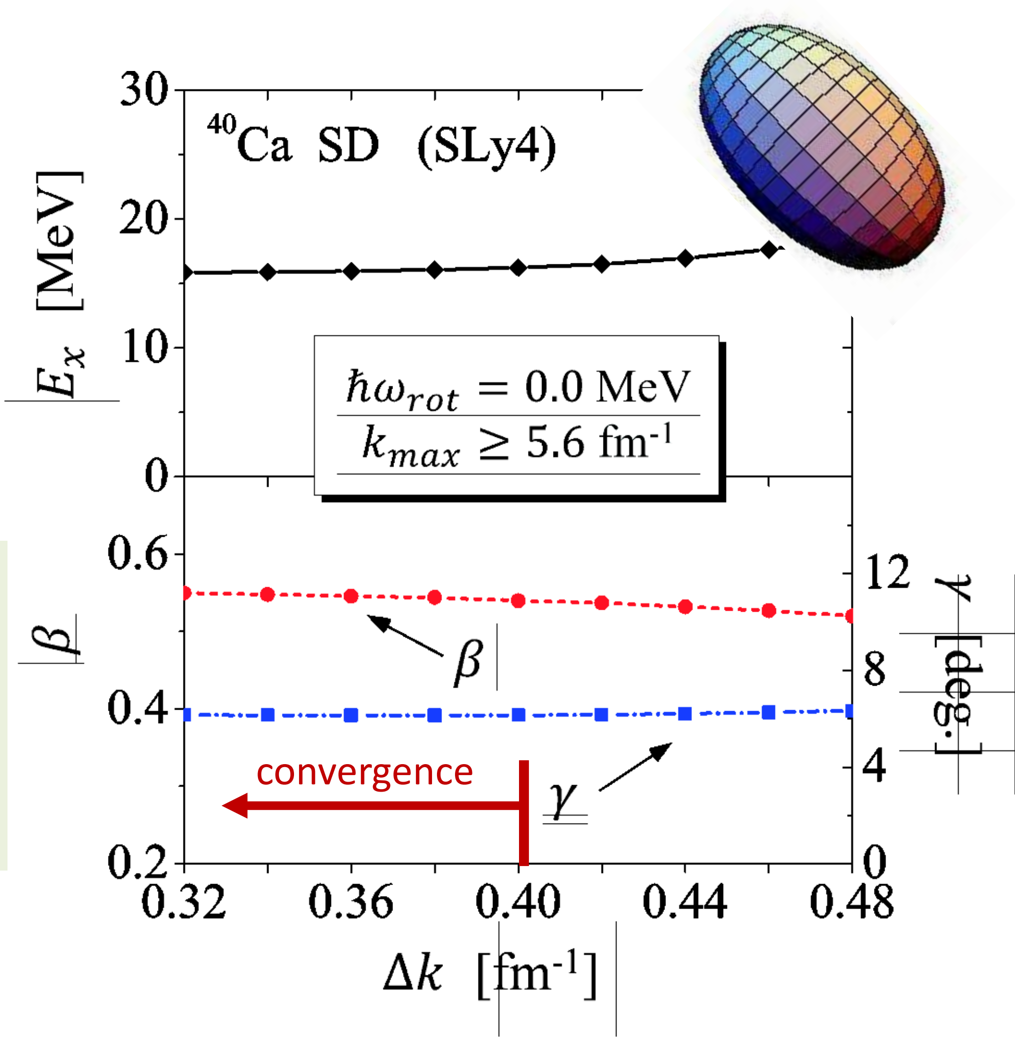
For convergence

$\Delta k \leq 0.40$ fm^{-1}

$k_{max} \geq 5.6$ fm^{-1}

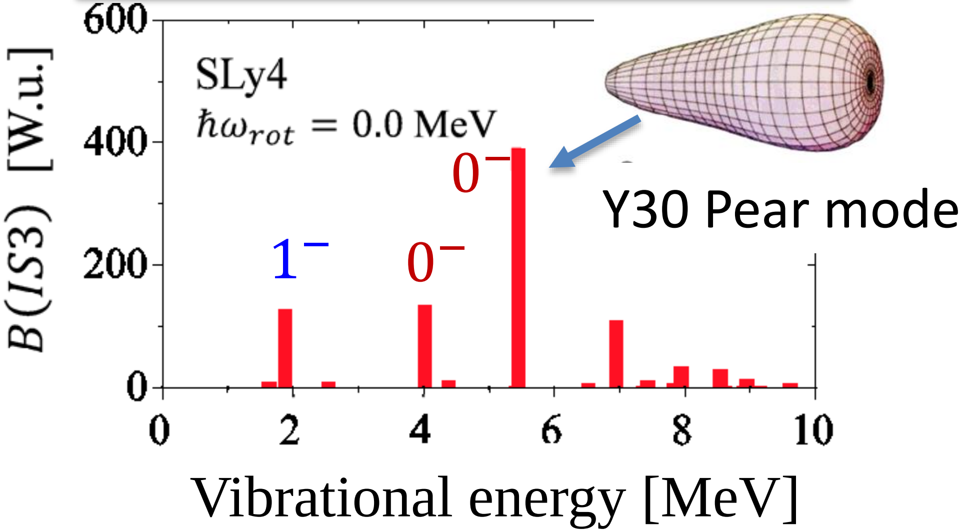
$N_{grid}^{(\vec{k})} \geq 560$ grid points

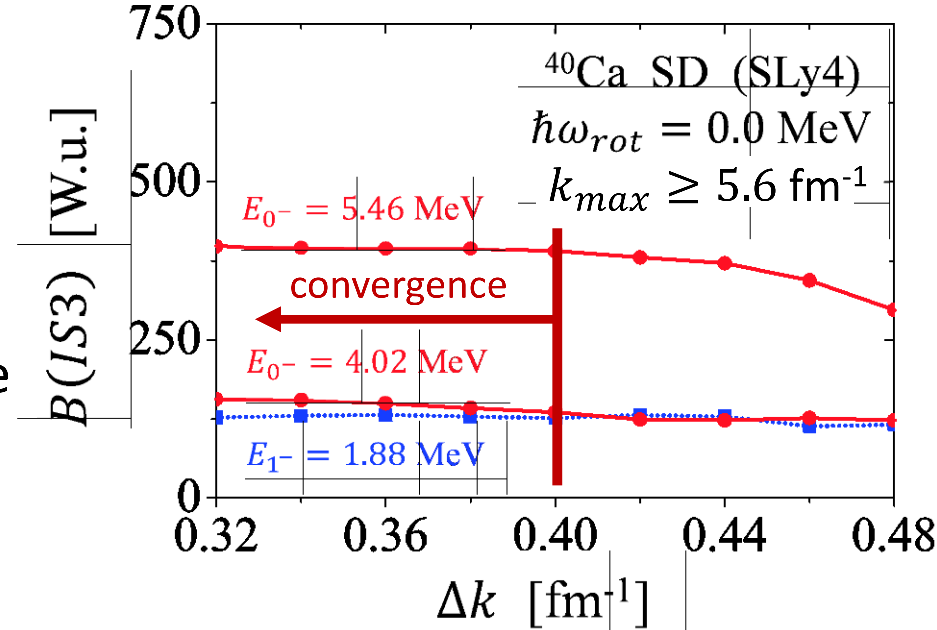
Band head of SD in ^{40}Ca

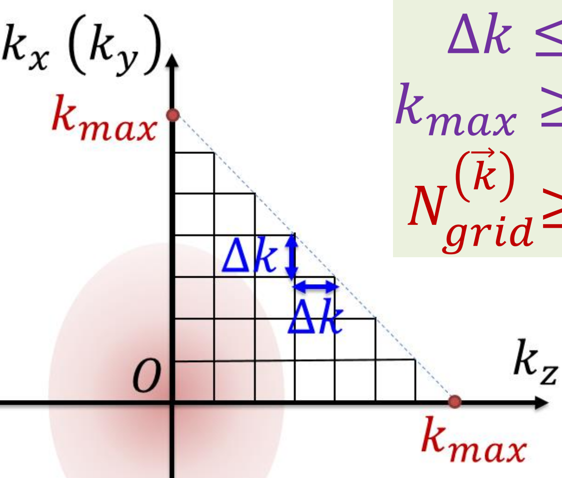


Convergence: RPA calculation

Band head of SD in ^{40}Ca





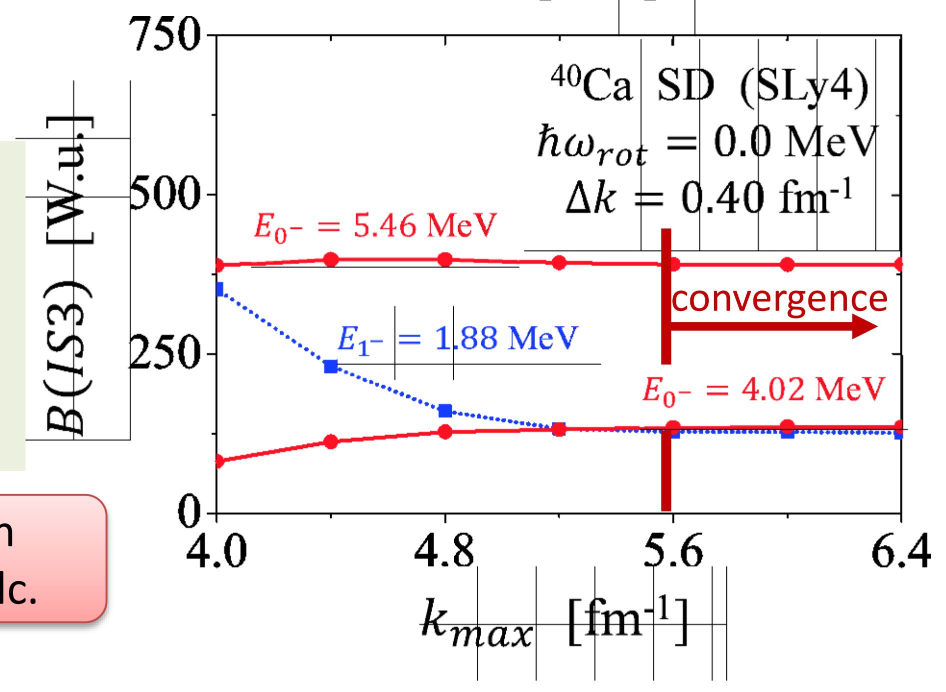


For reasonable results,

$\Delta k \leq 0.40$ fm^{-1}

$k_{max} \geq 5.6$ fm^{-1}

$N_{grid}^{(\vec{k})} \geq 560$ grid points



The same condition with mean-field calc.

Numerical complexity: Advantage / Disadvantage

$$F(\vec{r}) = \sum_{\vec{n}} \hat{F}_{\vec{n}} f_{n_x}^{(\pi_x)}(x) f_{n_y}^{(\pi_y)}(y) f_{n_z}^{(\pi_z)}(z), \qquad G(\vec{r}) = \cdots$$

Operations	$\vec{r}$-space ($N_{grid}^{(\vec{r})} \approx 5600$)		$\vec{k}$-space ($N_{grid}^{(\vec{k})} \approx 560$)
$\frac{\partial^m F(\vec{r})}{\partial x^m}$	Numerical: $\mathrm{O}\left(N_{grid}^{(\vec{r})}\right)$	$\gg$	Analytic: $\mathrm{O}(1)$
$I^{(m)} = \int x^m F(\vec{r}) d\vec{r}$ and similar integrals	Numerical: $\mathrm{O}\left(N_{grid}^{(\vec{r})}\right)$	$\gg$	Analytic / numerical: $\mathrm{O}(1)$ $I^{(0)} \propto F_{\vec{k}=\vec{0}}$, $I^{(m)} \propto \left.\frac{\partial^m F_{\vec{n}}}{\partial (k_x)^m}\right\vert_{\vec{k}=\vec{0}}$ and similar expressions
$\int F(\vec{r}) G(\vec{r}) d\vec{r}$	Numerical: $\mathrm{O}\left(N_{grid}^{(\vec{r})}\right)$	$>$	Analytic (Orth-normality of $f_n^{(+)}(x)$): $\mathrm{O}\left(N_{grid}^{(\vec{k})}\right)$
$F(\vec{r}) G(\vec{r})$	$\mathrm{O}\left(N_{grid}^{(\vec{r})}\right)$	$<$	$\mathrm{O}\left(\left[N_{grid}^{(\vec{k})}\right]^2\right) \approx \mathrm{O}\left(50 N_{rid}^{(\vec{r})}\right)$

Note: $\delta(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{ikx} dx$, $\frac{d^m \delta(k)}{dk^m} = \frac{i^m}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x^m e^{ikx} dx$

Comment: Number of *ph*-configurations

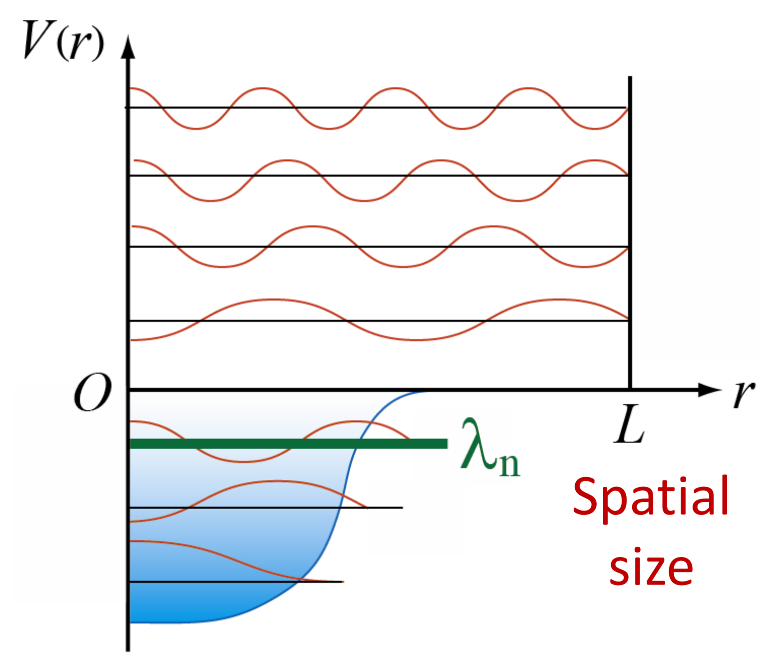
$\Delta k \approx 0.40$ fm^{-1}
Reasonable results
for $\vec{k}$-space calculation

$L \approx 7.8$ fm
Too small
for $\vec{r}$-space calculation

Inakura *et al.*, Nucl.Phys. A768 (2006)
$L_x = L_y = 8.7$ fm, $L_z = 14.7$ fm

LOW level density of
discretized-continuum states

SMALL number of
ph-configurations
(~1000(n), ~1000(p),
$\varepsilon_p - \varepsilon_h < 40$ MeV)



Spatial
size

Second part (application):

- Octupole vibrations of *Super*- and *Hyper*deformed states around ^{40}Ca
  - Rotational effect, Soft banana mode (Y_{31} on SD, HD)

Soft banana mode in SD, HD

$$E_{vib} \searrow 0$$

$$\left(\omega_{rot} \nearrow \omega_{rot}^{(critical)}\right)$$

Precursor of *Static* banana shape

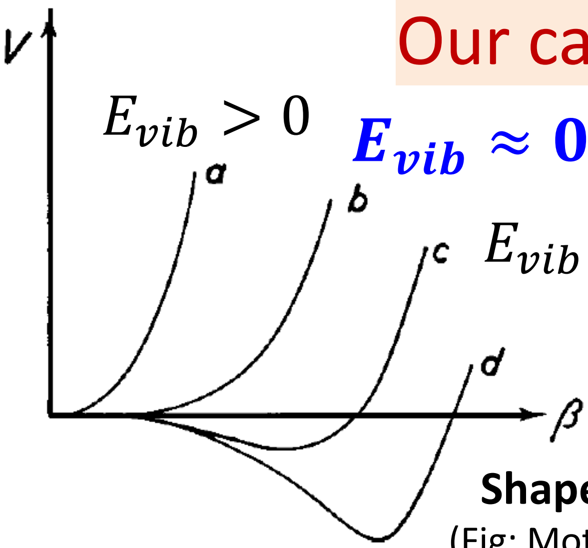
Usually N (Z) $\nearrow$

Our case $\omega_{rot} \nearrow$

$E_{vib} > 0$

$E_{vib} \approx 0$ **(Soft mode)**

$E_{vib} = imag.$



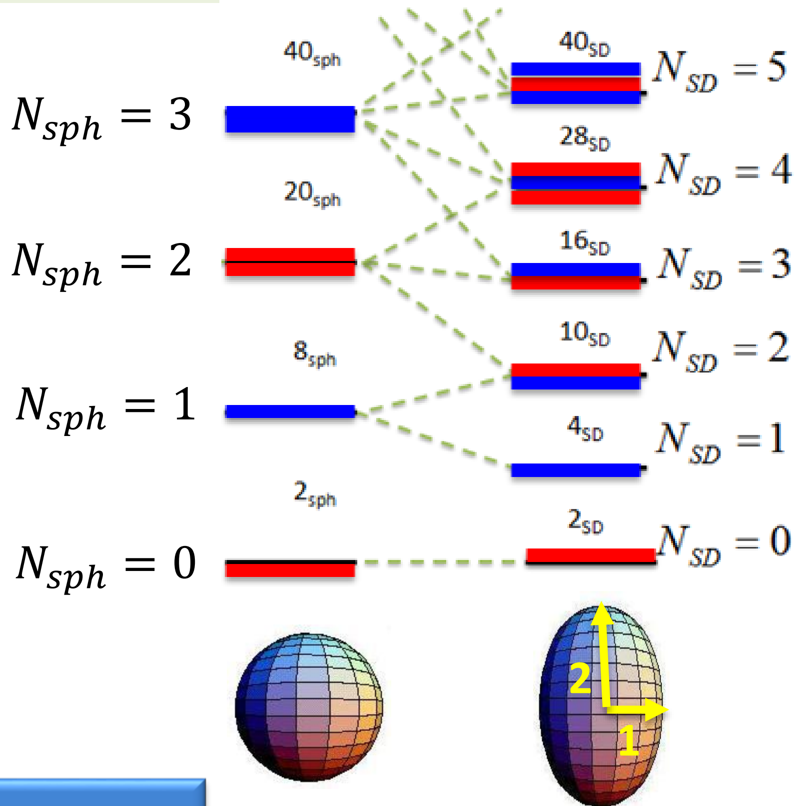
Shape transition

(Fig: Mottelson's Novel lecture)

Why we study *octupole vibration* in SD?

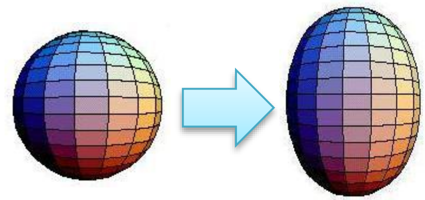
Harmonic oscillator potential

$$V(\vec{r}) = \frac{1}{2} m\omega_{\perp}^2 (x^2 + y^2) + \frac{1}{2} m\omega_z^2 z^2$$



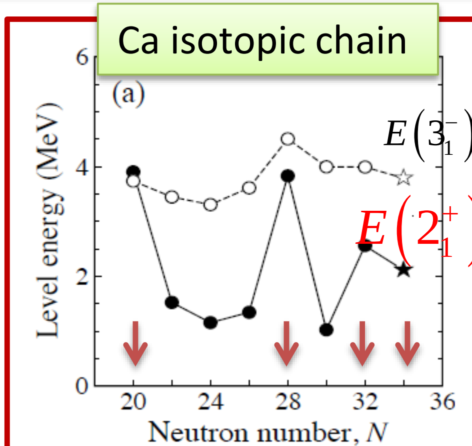
Degeneracy of levels	Same parity	+ & – parity
Low-lying modes	*Quadrupole*	*Octupole*

Quadrupole modes in spherical state



$E(2^+), B(E2)$

Shell structure



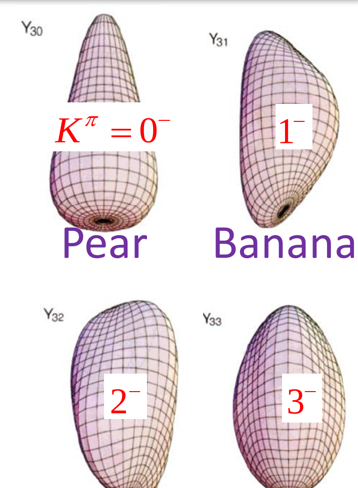
D. Steppenbeck et al,
J. Phys. Conf. Ser. 445 012012 (2013)

Octupole modes in SD state

$E(K^-), B(E1), B(E3)$

Shell structure

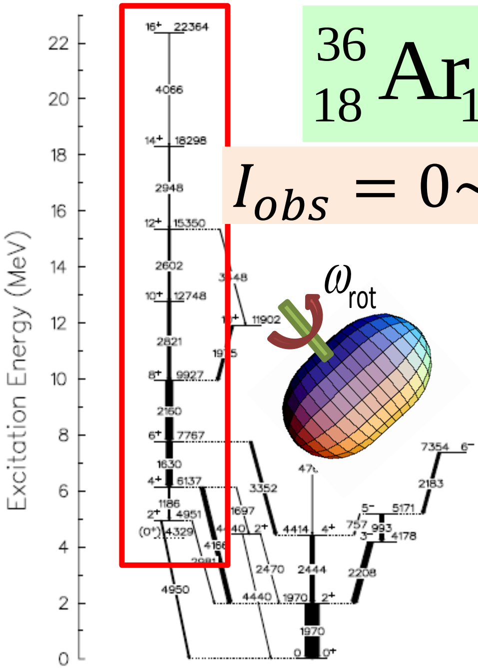
Especially $\epsilon_i(\omega_{rot})$



Superdeformation in ^{40}Ca region

$^{36}_{18}\mathrm{Ar}_{18}$

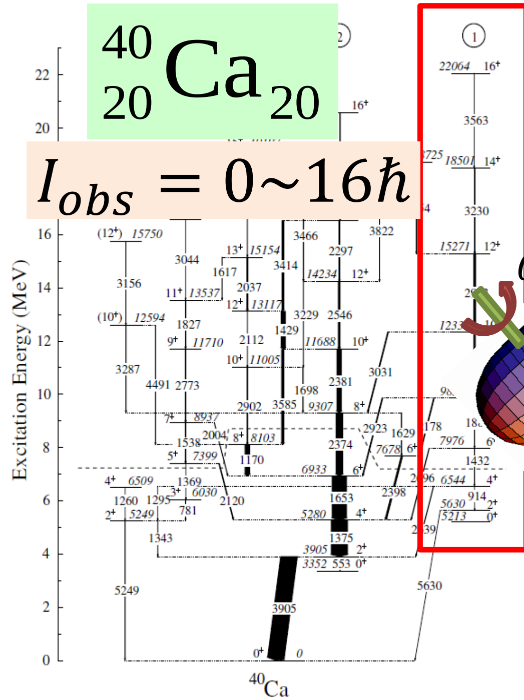
$I_{obs} = 0 \sim 16\hbar$



C.E. Svensson *et al.*,
Phys. Rev. Lett. 85, 2693 (2000)

$^{40}_{20}\mathrm{Ca}_{20}$

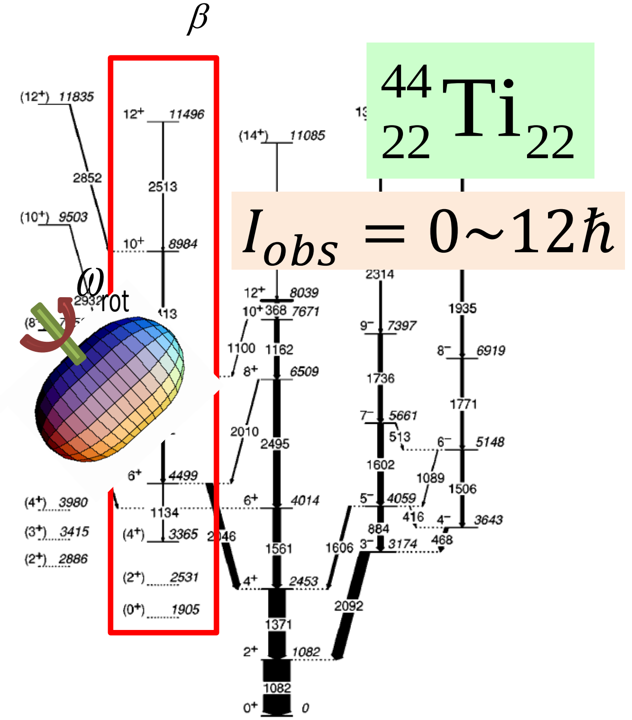
$I_{obs} = 0 \sim 16\hbar$



E. Ideguchi *et al.*,
Phys. Rev. Lett. 87, 222501 (2001)

$^{44}_{22}\mathrm{Ti}_{22}$

$I_{obs} = 0 \sim 12\hbar$

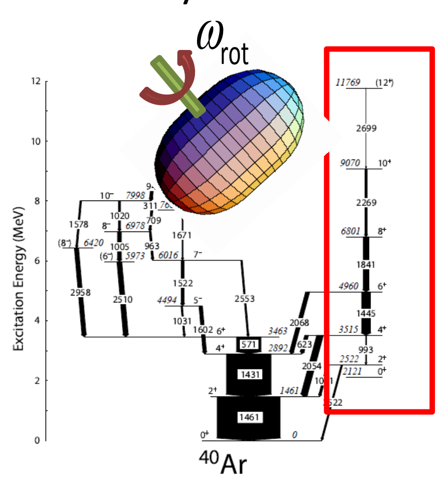


C.D.O'Leary et al.,
Phys.Rev. C61, 064314 (2000)

$^{40}_{18}\mathrm{Ar}_{22}$ $(N > Z)$

$I_{obs} = 0 \sim 12\hbar$

E.Ideguchi *et al.*,
Phys.Lett. B 686, 18 (2010)



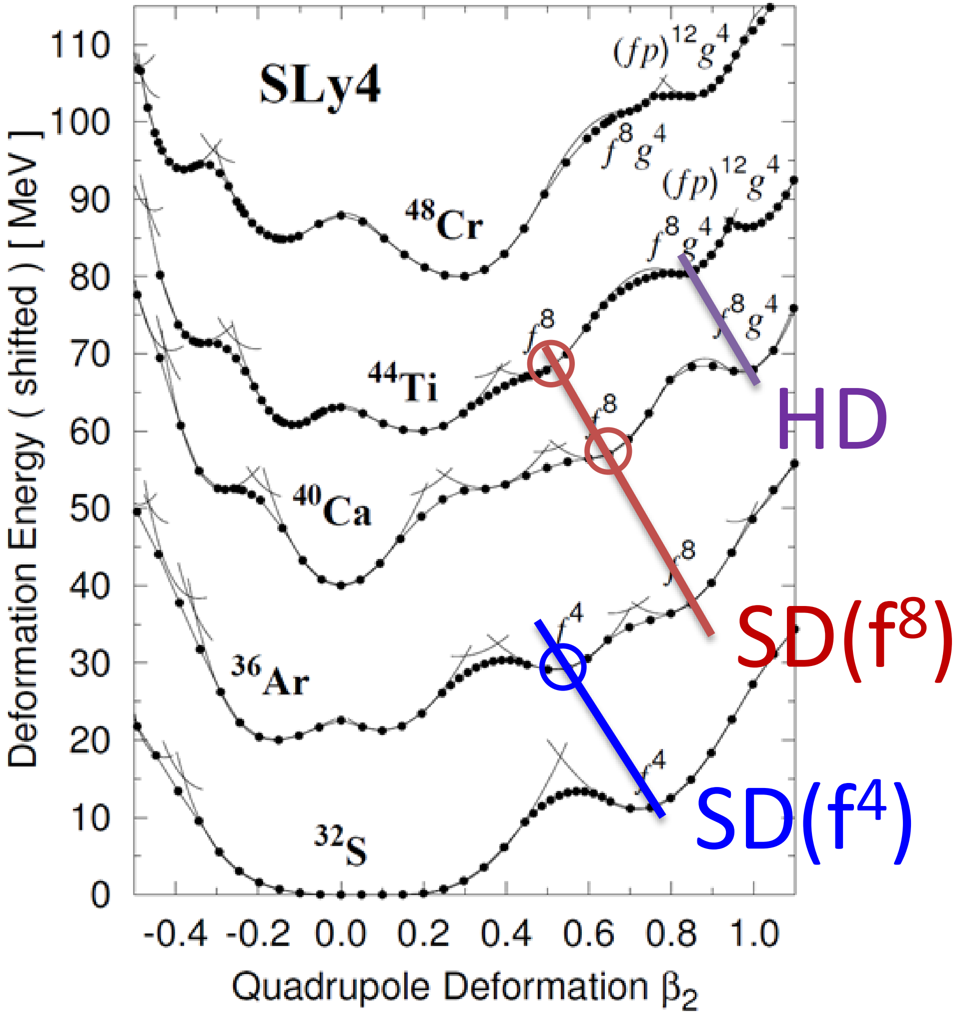
NEW !

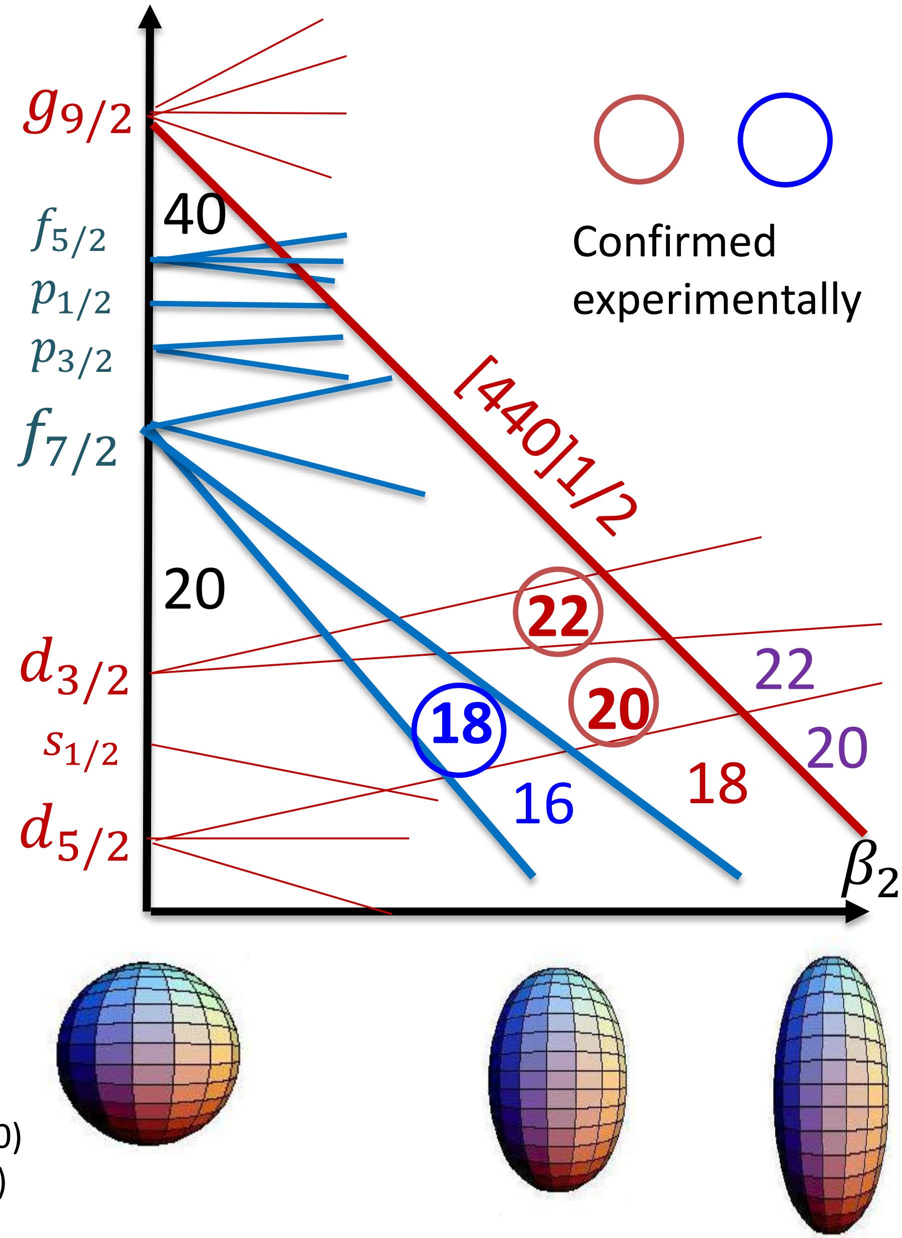
Ideguchi *et al.* (CAGRA campaign)

SD in ^{44}Ti, ^{41}Ca are being analyzed

$K^{\pi} = 2^{+}$ band, Octupole correlation,…

Super- and *Hyper*deformtion in ^{40}Ca region





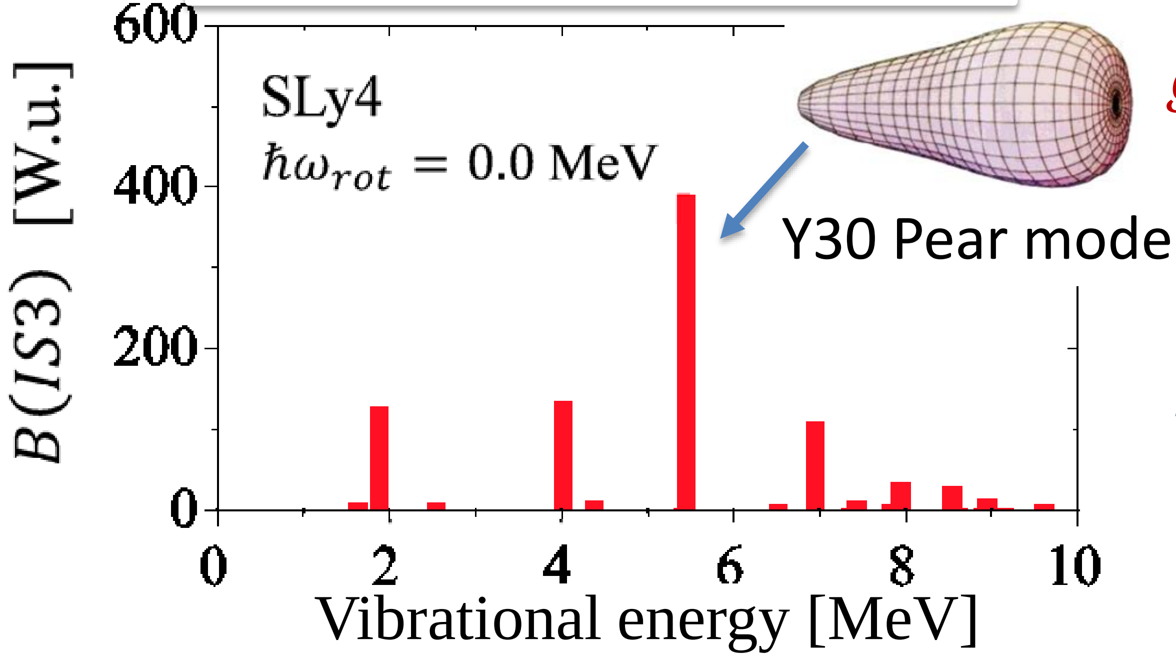
Theory T. Inakura *et al*. (2002) [Skyrme-HF]

Experiment

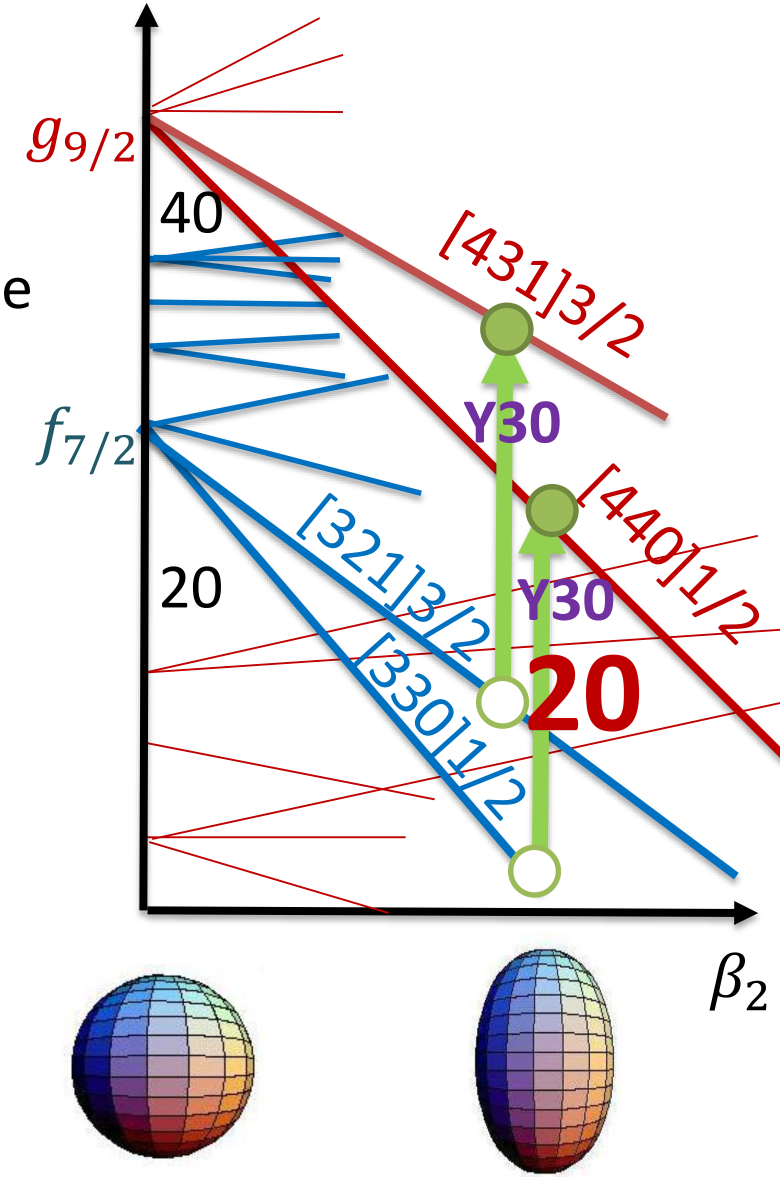
^{36}Ar SD(f^4): C.E. Svensson *et al*., Phys. Rev. Lett. 85, 2693 (2000)
^{40}Ca SD(f^8): E. Ideguchi *et al*., Phys. Rev. Lett. 87, 222501 (2001)
^{44}Ti SD(f^8): C.D.O'Leary *et al*., Phys.Rev. C61, 064314 (2000)

Octupole vibration of SD at $\omega_{rot} = 0$ in ^{40}Ca

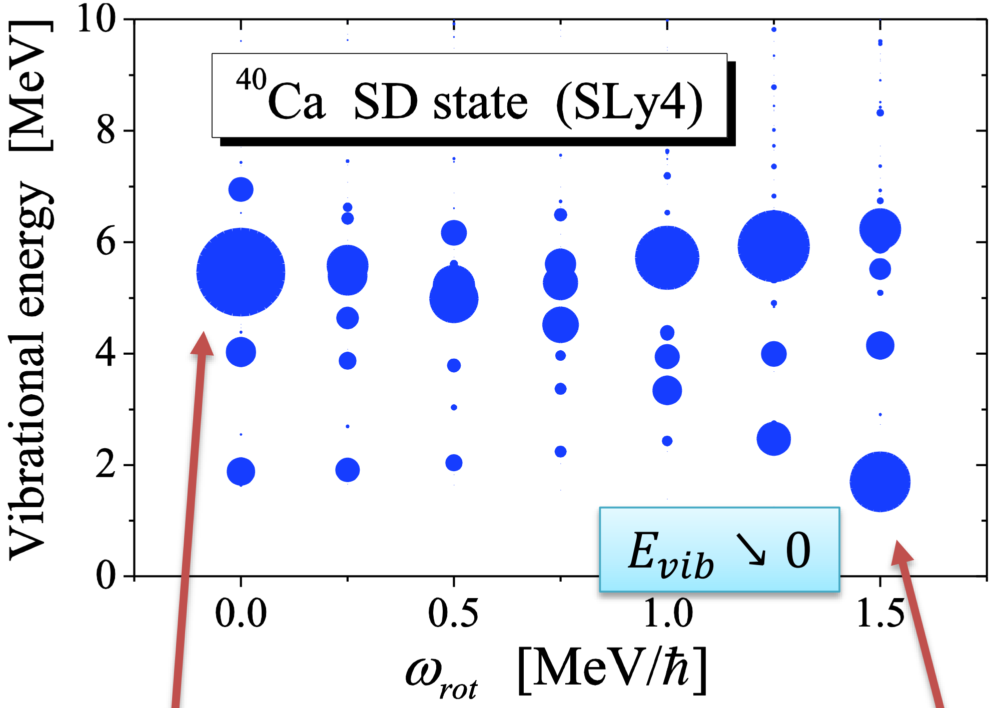
Band head of SD in ^{40}Ca

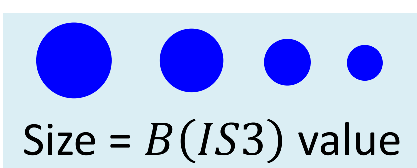


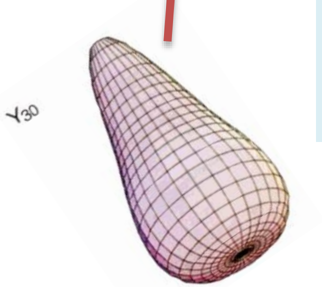
E_{vib} [MeV]	1.9	4.0	**5.4**
$B(Q^{IS}3)$ [W.u.]	127	134	**390**
Dominant component	$K = 1$ 99.6%	$K = 0$ 99.5%	$\boldsymbol{K = 0}$ **98.2 %**



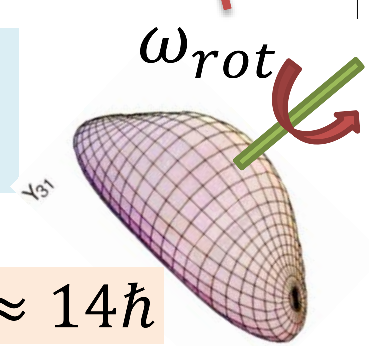
Soft banana mode at $\hbar\omega_{rot} \approx 1.5$ MeV





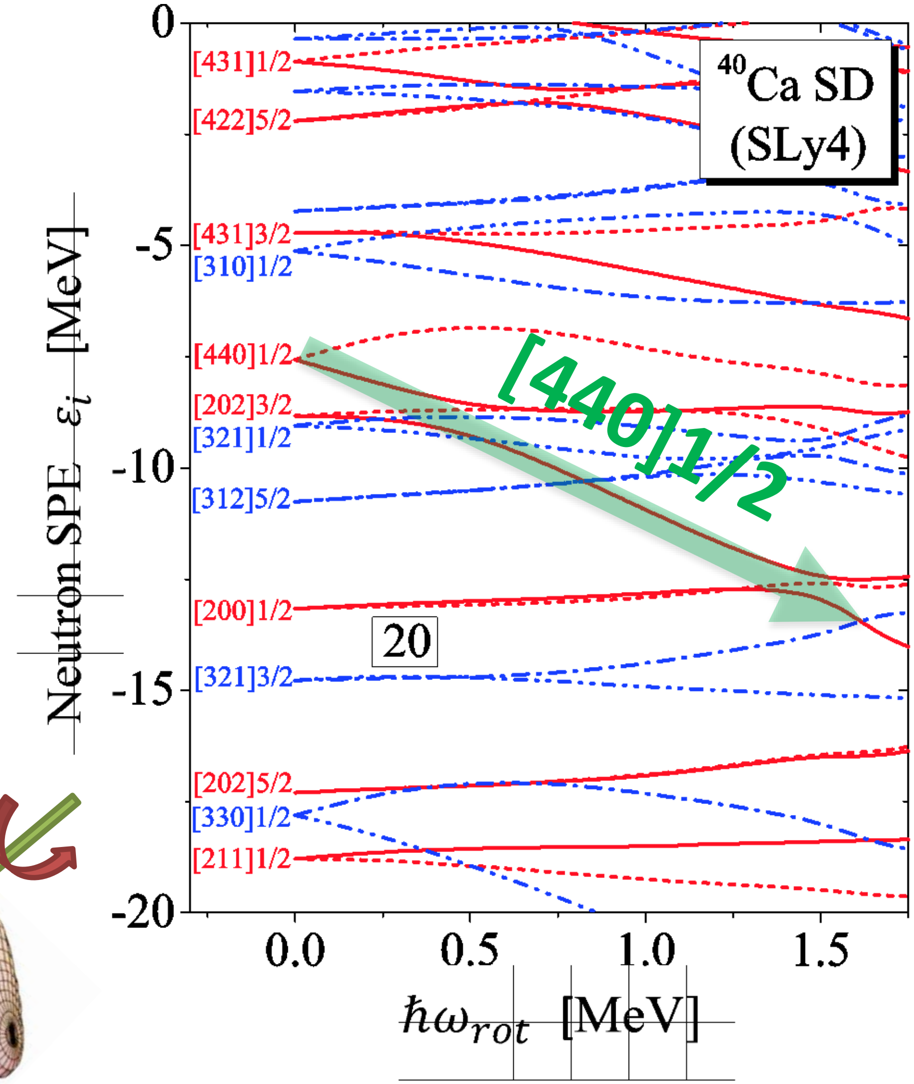


Pear mode
(Y_{30}: 98%)

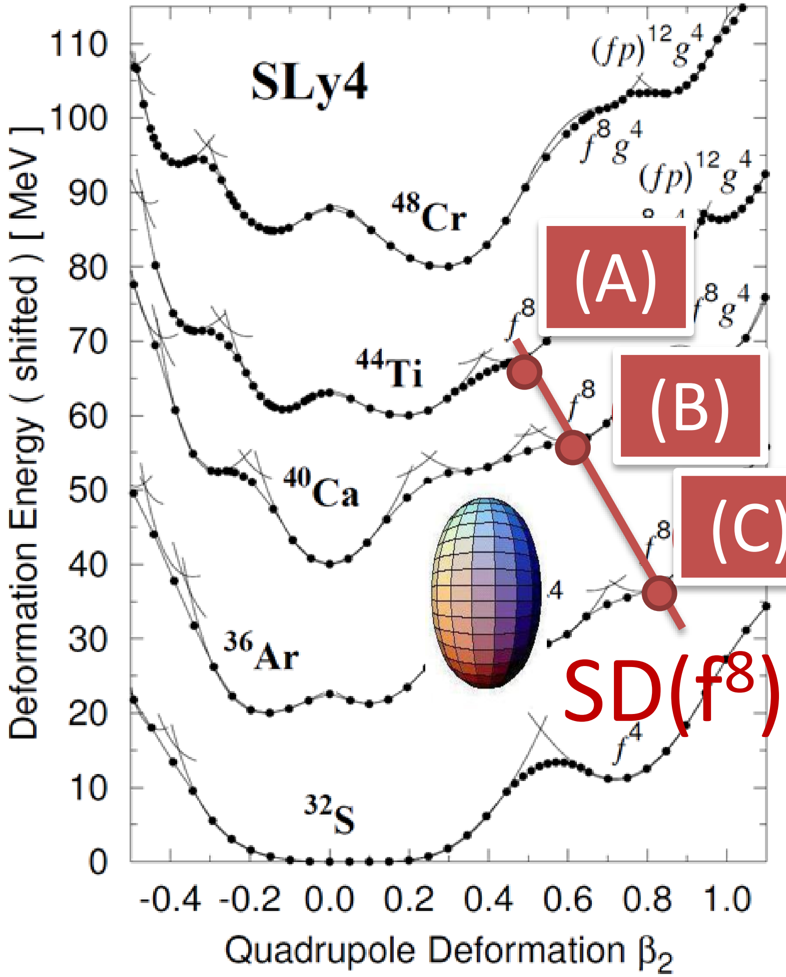


$J_x \approx 14\hbar$

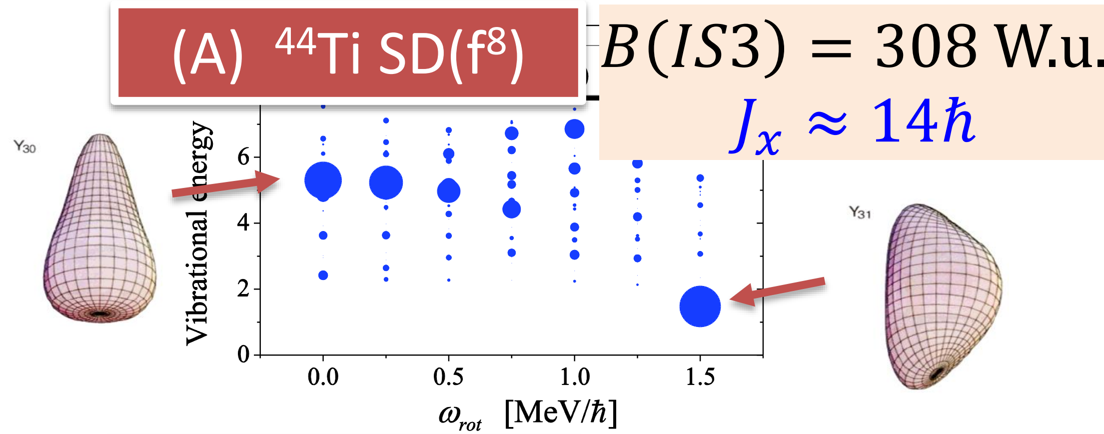
Soft Banana mode
(Y_{31}: 78%)

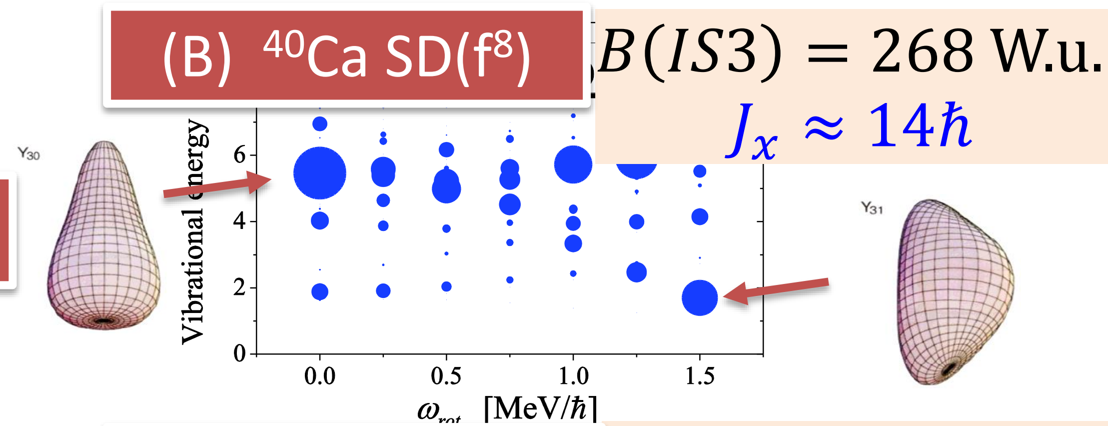


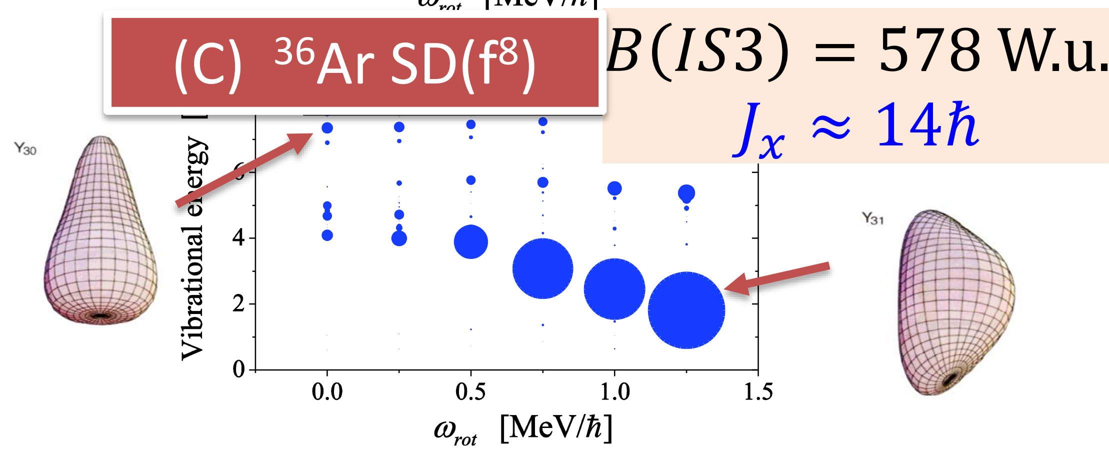
Octupole vibrations in SD(f⁸) band (x-sig. α=-1)



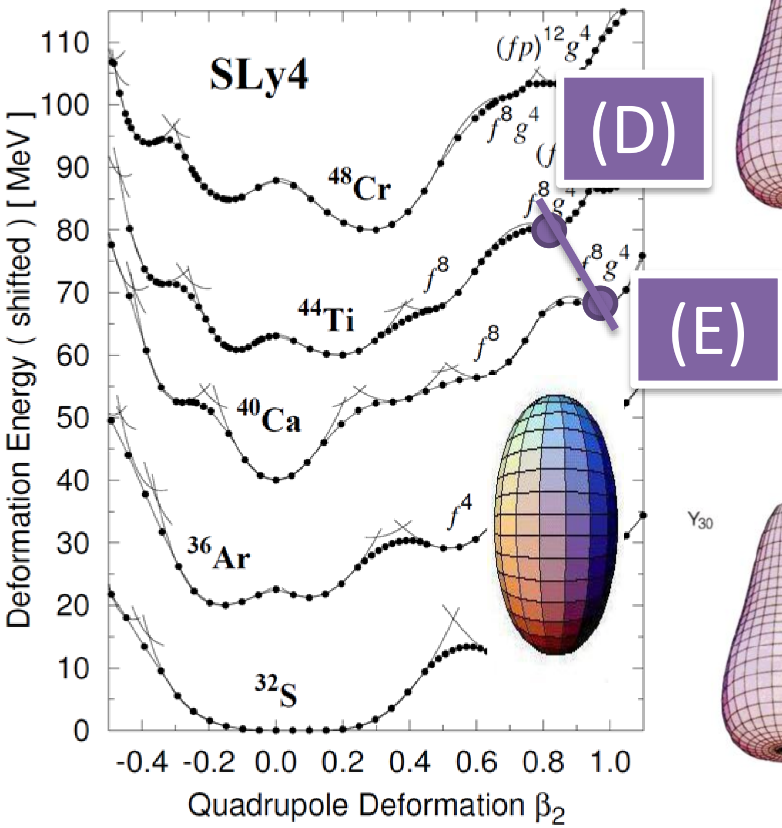
[Skyrme-HF] T. Inakura *et al*. (2002)



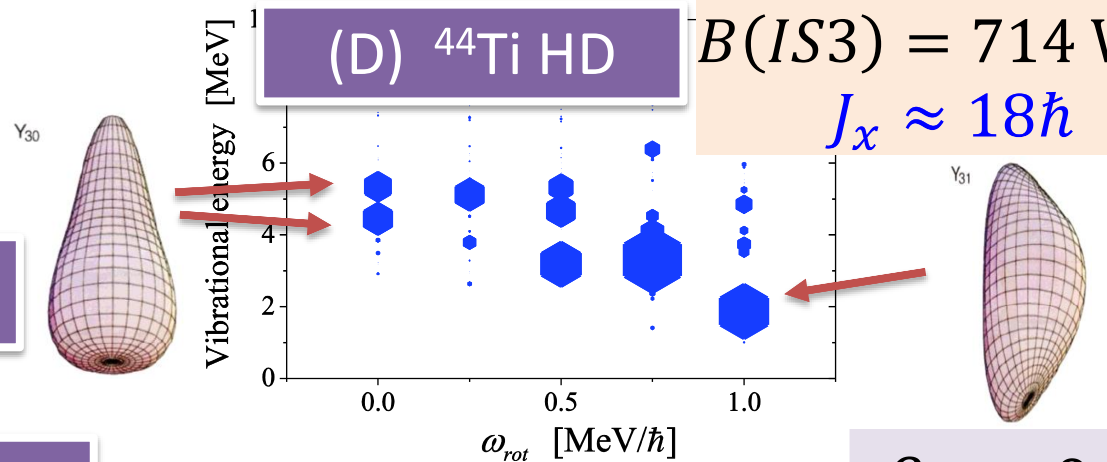




Octupole vibrations of Hyperdeformation (x-sig. α=-1)



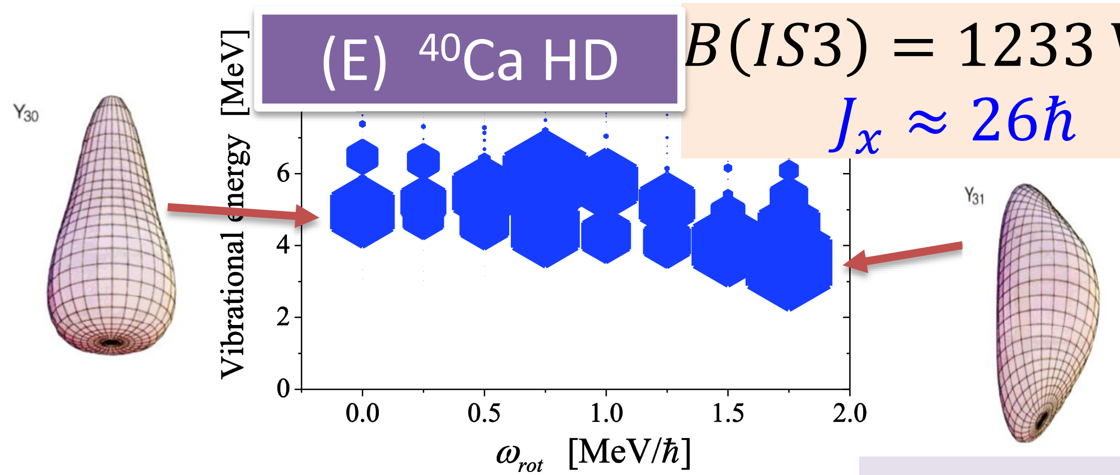
[Skyrme-HF] T. Inakura *et al*. (2002)



$B(IS3) = 714$ W.u.

$J_x \approx 18\hbar$

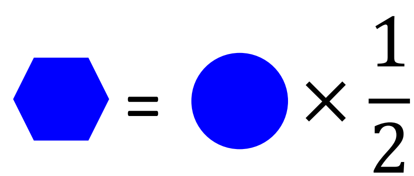
$\beta_2 \approx 0.85$



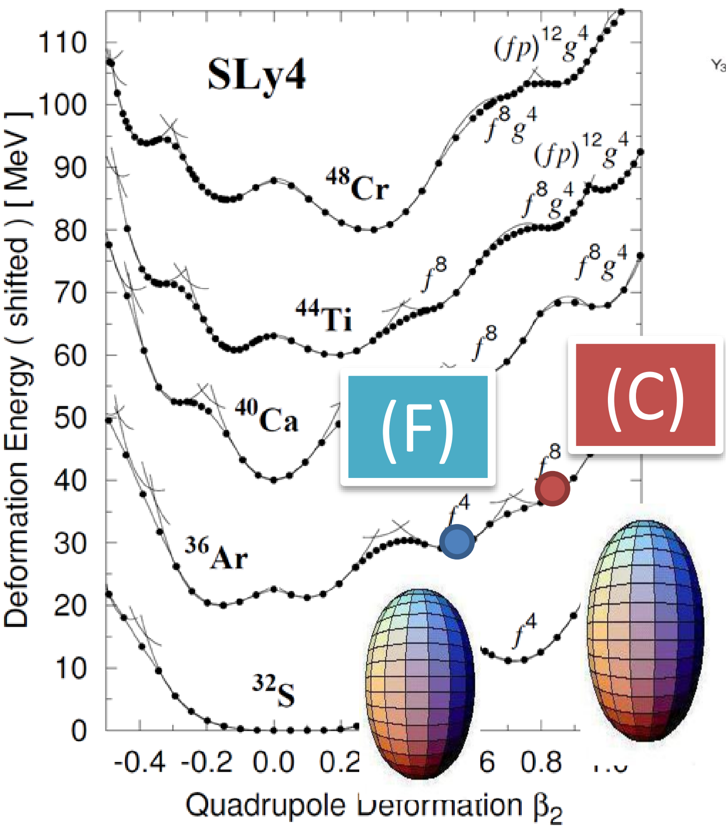
$B(IS3) = 1233$ W.u.

$J_x \approx 26\hbar$

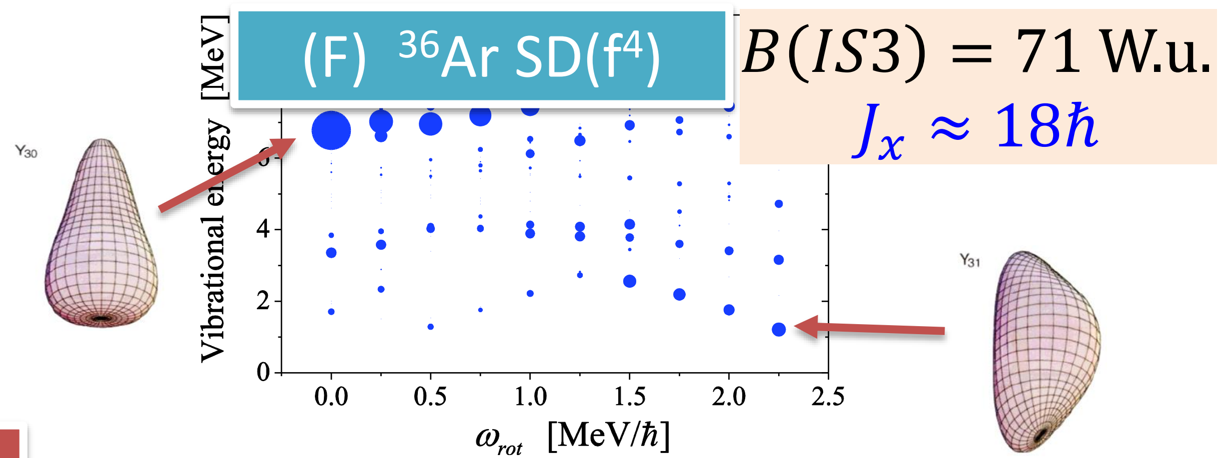
$\beta_2 \approx 0.96$

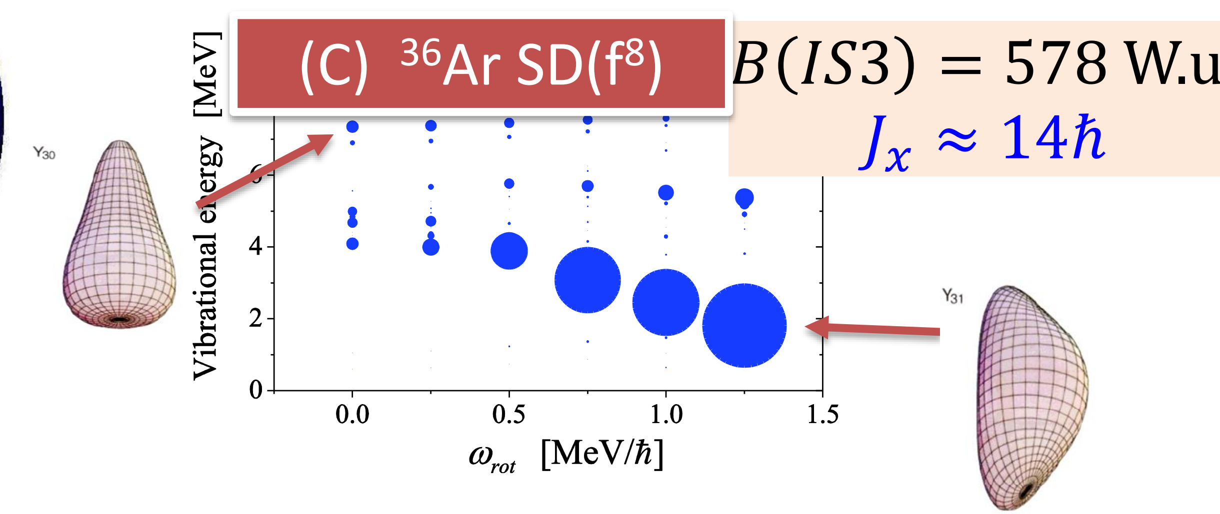


SD(f^4) and SD(f^8) band (x-sig. α=-1)

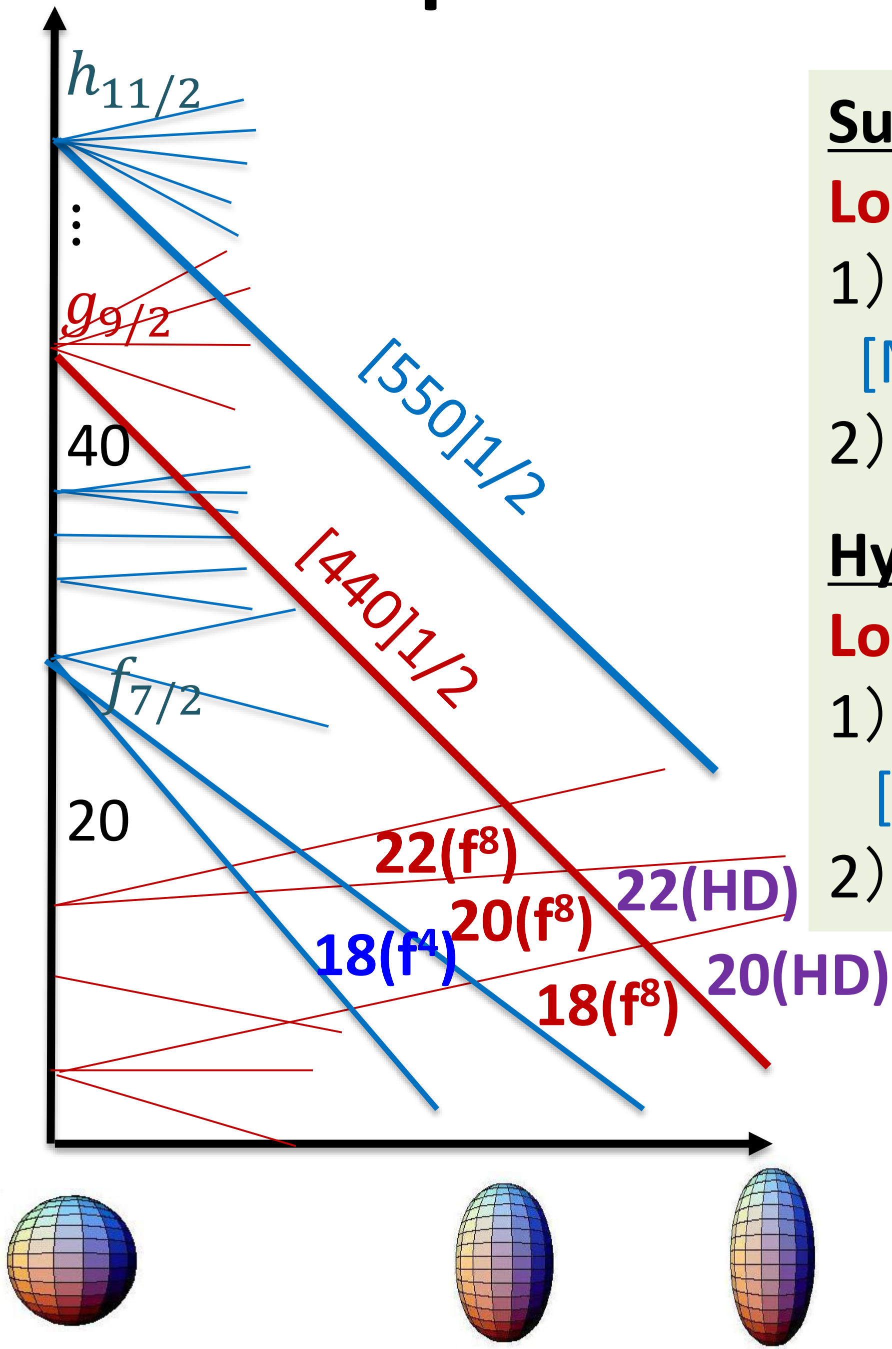


[Skyrme-HF] T. Inakura *et al.* (2002)





Microscopic mechanism of *soft banana mode*



<u>Superdeformation</u>

Lowering of [440]1/2 orbits

1）Larger deformation

[Many p-h config. in SD(f8) than SD(f4)]

2）Rotational alignment

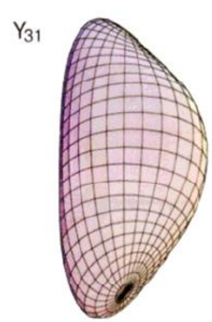
<u>Hyperdeformation</u>

Lowering of [550]1/2 orbits

1）[440]1/2 orbits are hole state

[Large p-h matrix elements]

2） Rotational alignment

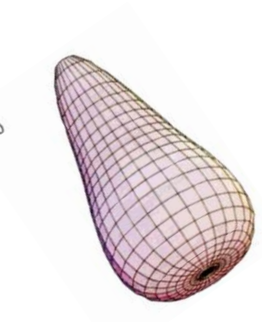


Soft banana mode

（Precursor of static shape)

Summary

1. **Fourier-series expansion method ($\vec{k}$-space rep.)** **is applied to Skyrme-RPA calculation *for rotating nuclei***
2. ***Soft banana mode*** **appears in *Super-* and *Hyper*deformed band in ^{40}Ca region → *Rotational effect is essential.***
   → *Static banana shape is expected.*

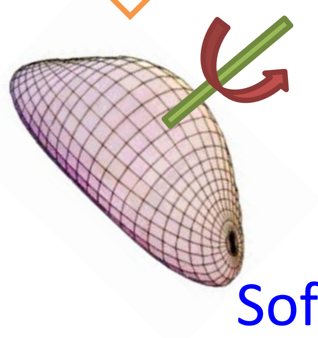


$E_{vib} \approx 5$ MeV

Pear mode (Y_{30})

Rotation

$\omega_{rot} \nearrow \omega_{rot}^{(critical)}$



$E_{vib} \searrow 0$

Soft Banana mode (Y_{31})

	Spin region for ***sofe banana mode*** **(Cf. Observed maximum spin)**
^{36}Ar SD(f^8)	$J_x \approx 14\hbar$
^{40}Ca SD(f^8)	$J_x \approx 14\hbar$ ($I_{obs} = 16\hbar$)
^{44}Ti SD(f^8)	$J_x \approx 14\hbar$ ($I_{obs} = 12\hbar$)
^{36}Ar SD(f^4)	$J_x \approx 18\hbar$ ($I_{obs} = 16\hbar$)
^{40}Ca HD	$J_x \approx 26\hbar$
^{44}Ti HD	$J_x \approx 18\hbar$