

Pairing in spin-isospin responses

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Contents / Key words

Self-consistent deformed pnQRPA for spin-isospin responses

Self-consistency:

- ✓ $T=1$ pairing and IAS
- ✓ Collectivity of GT giant resonance

Possible new type of collective mode:

- ✓ $T=0$ proton-neutron pairing vibrations

Self-consistent pnQRPA for spin-isospin responses

starting point: Skyrme EDF $\mathcal{E}[\rho(\mathbf{r}), \tilde{\rho}(\mathbf{r})]$

variation w.r.t densities

The coordinate-space Hartree-Fock-Bogoliubov eq. for ground states

J. Dobaczewski et al., NPA422(1984)103

$$\begin{pmatrix} h^q(\mathbf{r}, \sigma) - \lambda^q & \tilde{h}^q(\mathbf{r}, \sigma) \\ \tilde{h}^q(\mathbf{r}, \sigma) & -(h^q(\mathbf{r}, \sigma) - \lambda^q) \end{pmatrix} \begin{pmatrix} \varphi_{1,\alpha}^q(\mathbf{r}, \sigma) \\ \varphi_{2,\alpha}^q(\mathbf{r}, \sigma) \end{pmatrix} = E_\alpha \begin{pmatrix} \varphi_{1,\alpha}^q(\mathbf{r}, \sigma) \\ \varphi_{2,\alpha}^q(\mathbf{r}, \sigma) \end{pmatrix}$$

“s.p.” hamiltonian and pair potential: $h^q = \frac{\delta \mathcal{E}}{\delta \rho^q}, \quad \tilde{h}^q = \frac{\delta \mathcal{E}}{\delta \tilde{\rho}^q} \quad q = \nu, \pi$

 quasiparticle basis $\alpha, \beta \dots$

The proton-neutron quasiparticle RPA eq. for excited states $[\hat{H}, \hat{O}_\lambda^\dagger] |\Psi_\lambda\rangle = \omega_\lambda \hat{O}_\lambda^\dagger |\Psi_\lambda\rangle$

Collective excitation = coherent superposition of 2qp excitations:

$$\hat{O}_\lambda^\dagger = \sum_{\alpha\beta} X_{\alpha\beta}^\lambda \hat{a}_{\alpha,\nu}^\dagger \hat{a}_{\beta,\pi}^\dagger - Y_{\alpha\beta}^\lambda \hat{a}_{\beta,\pi} \hat{a}_{\alpha,\nu}$$

residual interactions derived self-consistently :

$$v_{\text{res}}(\mathbf{r}_1, \mathbf{r}_2) = \frac{\delta^2 \mathcal{E}}{\delta \rho_{1t_3}(\mathbf{r}_1) \delta \rho_{1t_3}(\mathbf{r}_2)} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + \frac{\delta^2 \mathcal{E}}{\delta \mathbf{s}_{1t_3}(\mathbf{r}_1) \delta \mathbf{s}_{1t_3}(\mathbf{r}_2)} \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$$

Recent progress

EDF-based **self-consistent** pnQRPA for **axially-deformed** nuclei

w/o any free parameters

(almost all the) arbitrary nuclei

PTEP

Prog. Theor. Exp. Phys. **2013**, 113D02 (17 pages)
DOI: 10.1093/ptep/ptt091

Spin–isospin response of deformed neutron-rich nuclei in a self-consistent Skyrme energy-density-functional approach

Kenichi Yoshida*

Matrix QRPA

PHYSICAL REVIEW C **87**, 064302 (2013)

Large-scale calculations of the double- β decay of ^{76}Ge , ^{130}Te , ^{136}Xe , and ^{150}Nd in the deformed self-consistent Skyrme quasiparticle random-phase approximation

M. T. Mustonen^{1,2,*} and J. Engel^{1,†}

Skyrme

Matrix QRPA

PHYSICAL REVIEW C **90**, 024308 (2014)

Finite-amplitude method for charge-changing transitions in axially deformed nuclei

M. T. Mustonen,^{1,*} T. Shafer,^{1,†} Z. Zenginerler,^{2,‡} and J. Engel^{1,§}

FAM-QRPA

PHYSICAL REVIEW C **89**, 044306 (2014)

Gamow-Teller strength in deformed nuclei within the self-consistent charge-exchange quasiparticle random-phase approximation with the Gogny force

M. Martini,^{1,2,3} S. Péru,³ and S. Goriely¹

Matrix QRPA

Gogny

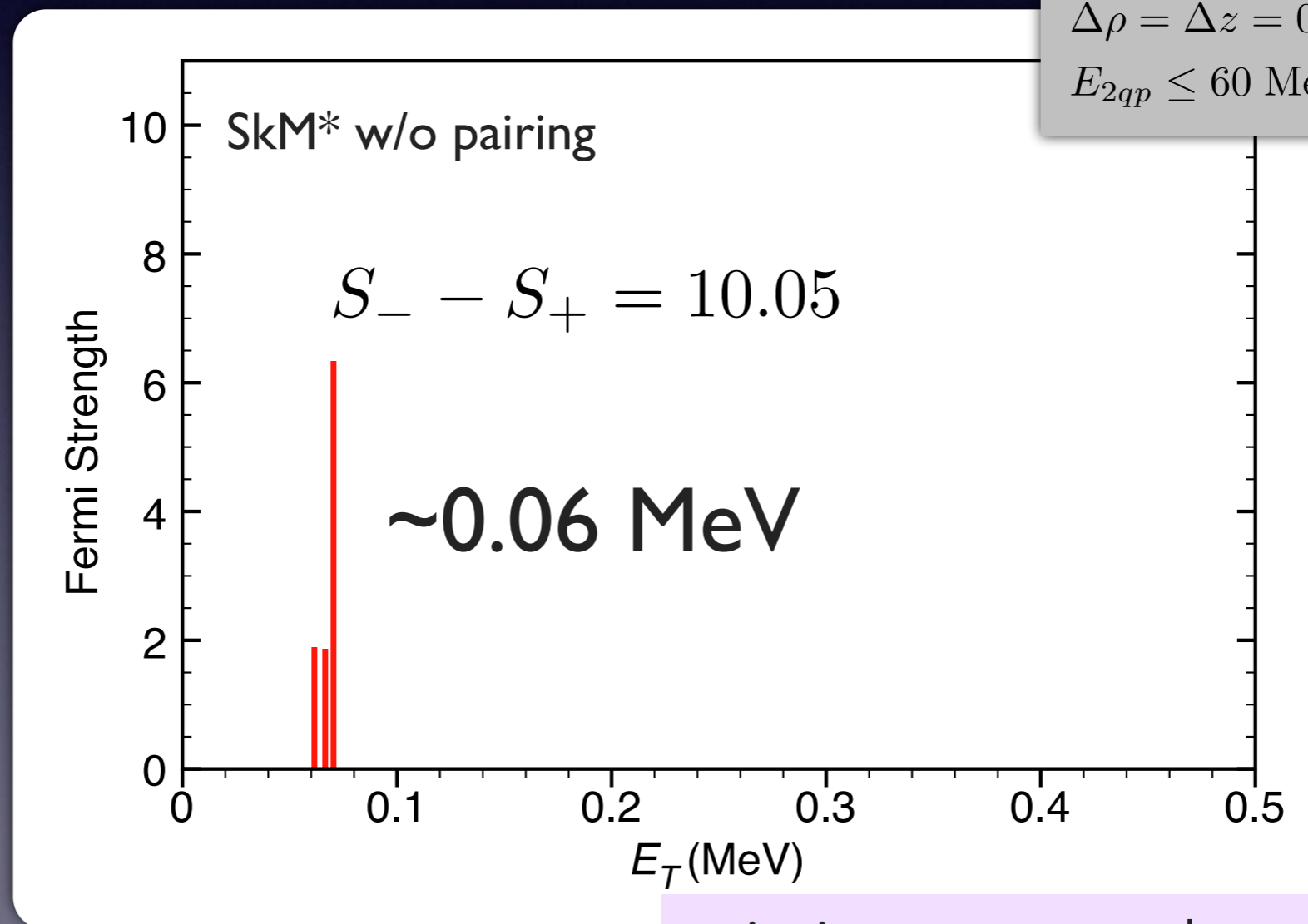
Restoration of the isospin symmetry breaking (ISB)

Even w/o the Coulomb int., the ISB occurs in $N > Z$ nuclei in a MFA

$$[H_{MF}, T_-] \neq 0 \quad \text{C. A. Engelbrecht and R. H. Lemmer, PRL24(1970)607}$$

→ IAS appears as a NG mode in the pnRPA

Ex. ^{90}Zr ($N-Z=10$) w/o Coulomb

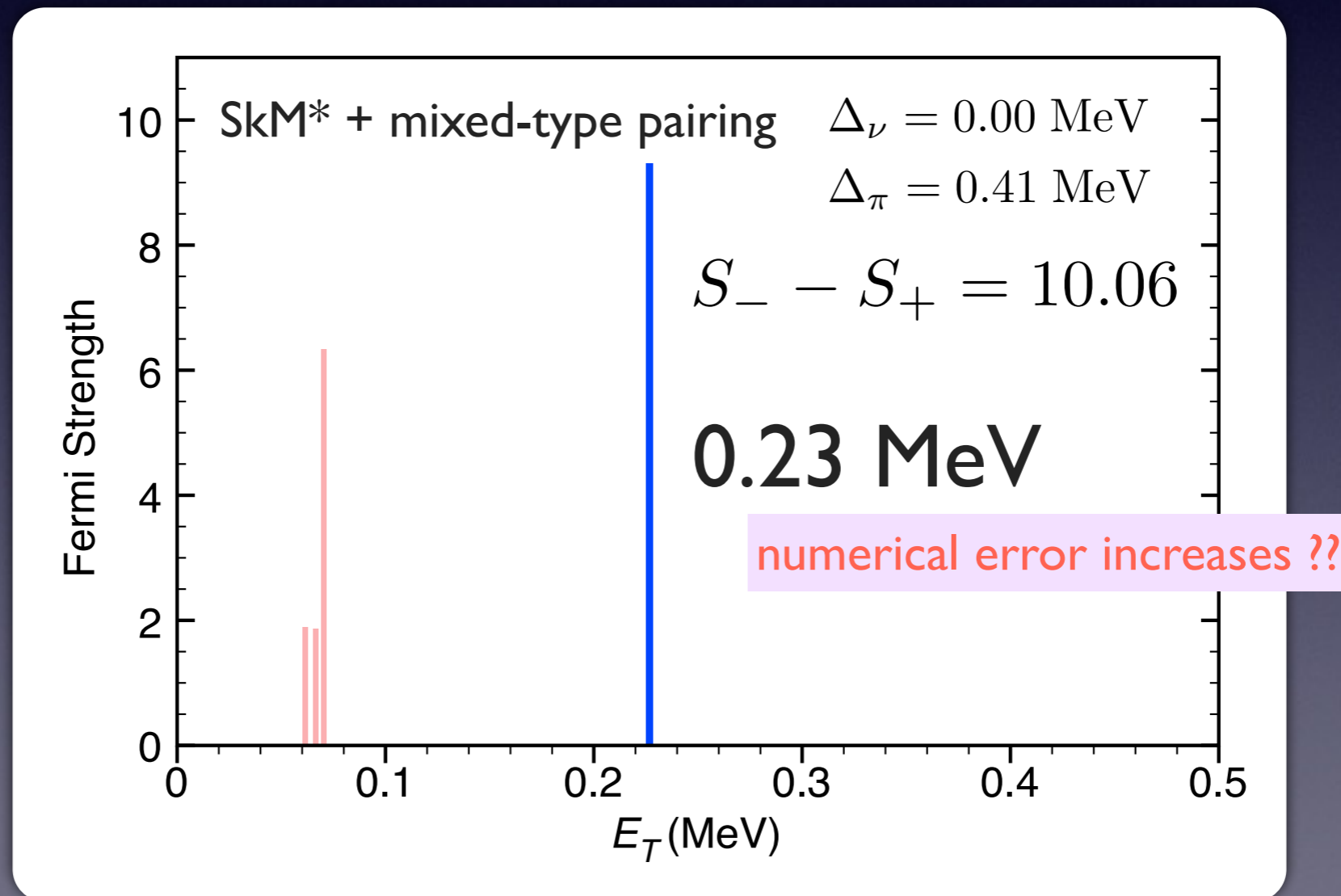


$$\rho_{\max} \times z_{\max} = 14.7 \text{ fm} \times 14.4 \text{ fm}$$
$$\Delta\rho = \Delta z = 0.6 \text{ fm}$$
$$E_{2qp} \leq 60 \text{ MeV}$$

excitation energy w.r.t the gs of ^{90}Zr

Restoration of the isospin symmetry breaking (ISB)

Ex. ^{90}Zr (N-Z=10) w/o Coulomb, w/ pairing

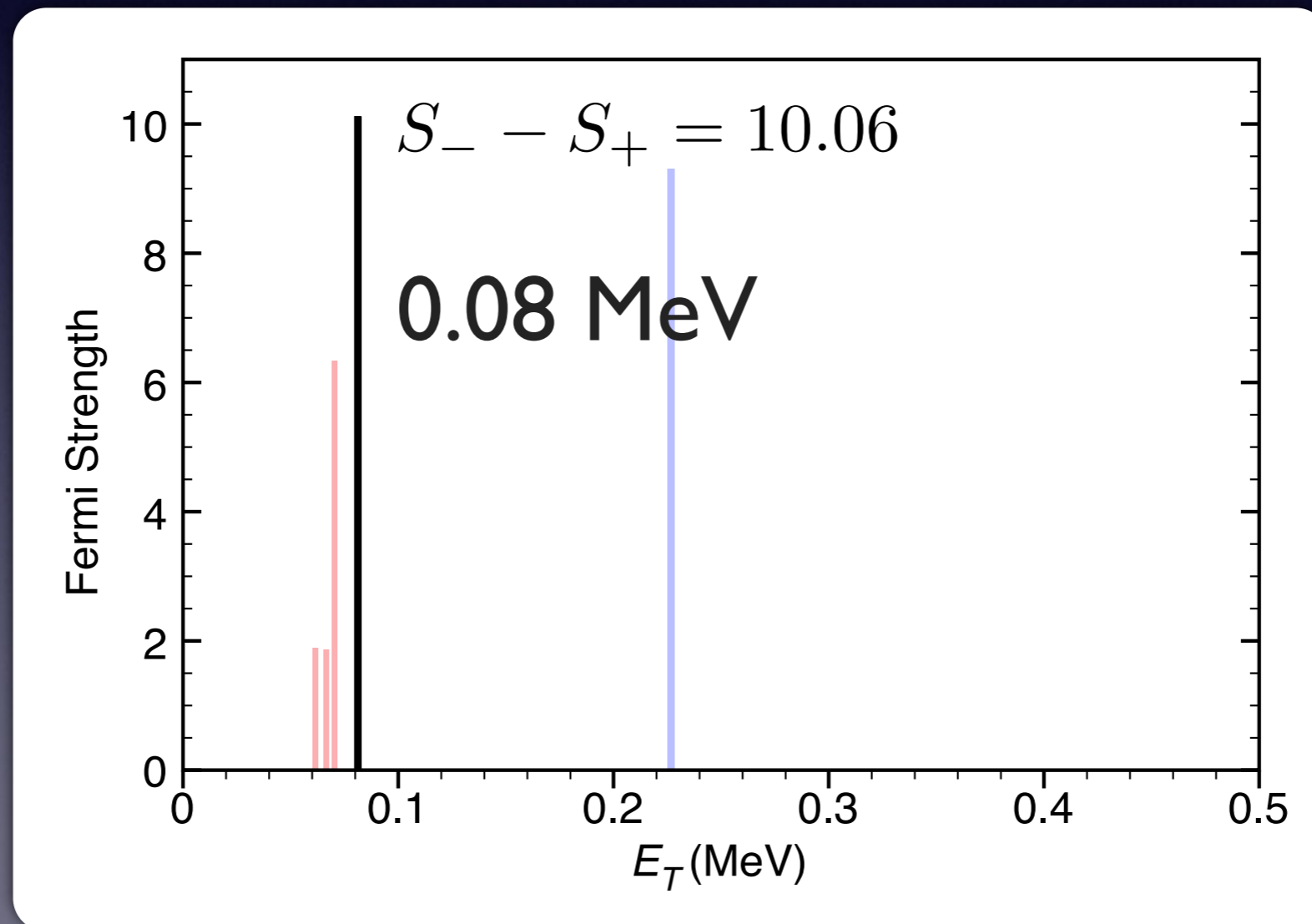


Restoration of the isospin symmetry breaking (ISB)

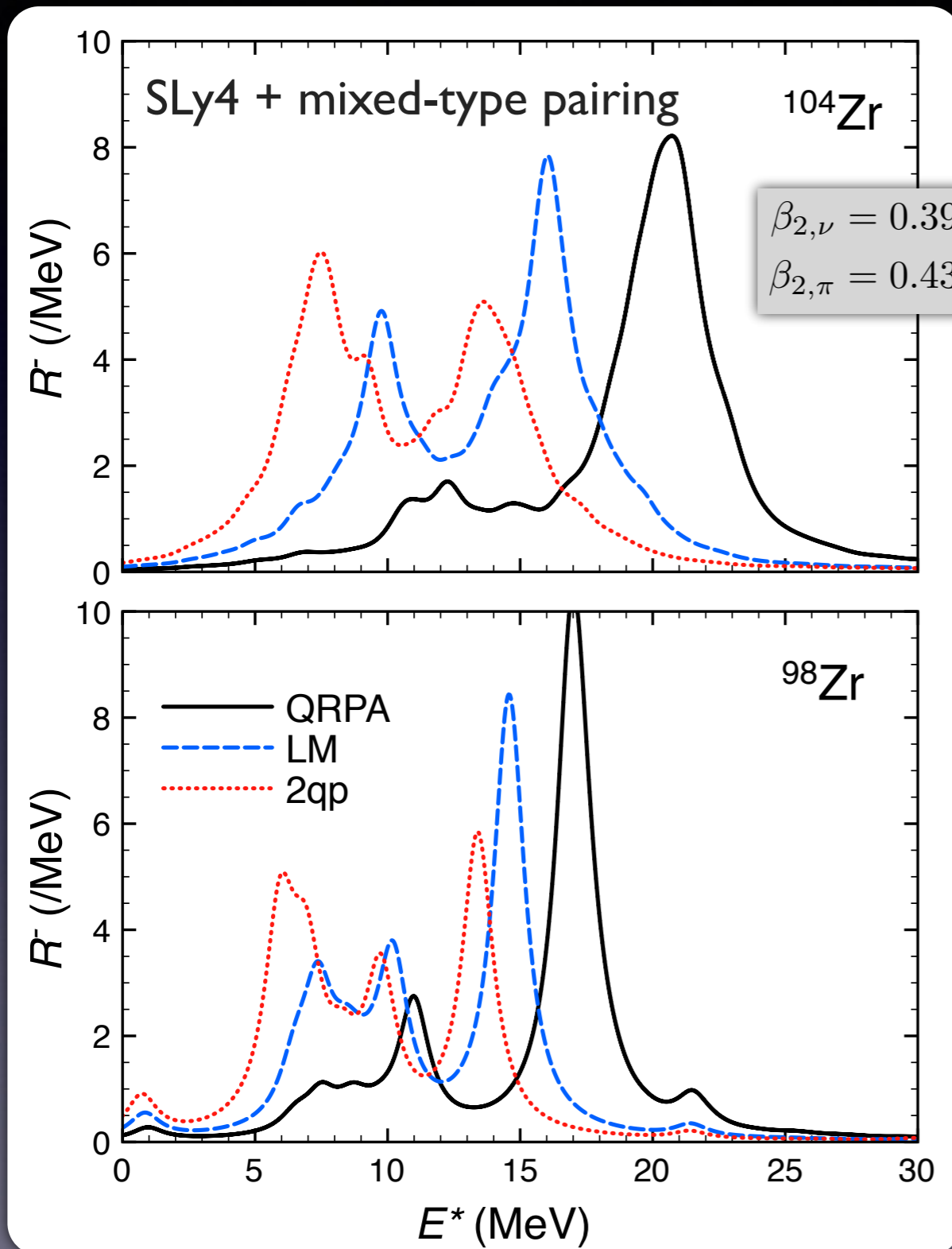
Ex. ^{90}Zr ($N-Z=10$) w/o Coulomb

inclusion of the $S=0$ pairing interaction in the pnQRPA

$$v_{pp}(\mathbf{r}, \mathbf{r}') = V_0 \left[1 - \frac{1}{2} \frac{\rho(\mathbf{r})}{\rho_0} \right] \delta(\mathbf{r} - \mathbf{r}')$$



GTGR: the need of self-consistency



✓ the collectivity generated by the Landau-Migdal approximation is weak

$$v_{\text{ph}}(\mathbf{r}_1 \mathbf{r}_2) = N_0^{-1} [f'_0 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + g'_0 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2] \delta(\mathbf{r}_1 - \mathbf{r}_2)$$

LM parameter:

M. Bender et al., PRC65(2002)054322



the self-consistent treatment of the static and dynamic calculations is needed for a quantitative description of the GTGR

proton-neutron pairing vibrations

✓ collectivity of $T=0$ and $T=1$ types

Ref: PRC90(2014)031303R

Pairing vibration and condensation (of neutrons)

cf. Bès and Broglia

neutron-pair operator; a probe to see the collectivity

$$\hat{P}_{T=1, T_z=1, S=0} \equiv \frac{1}{2} \sum_{\sigma, \sigma'} \sum_{\tau, \tau'} \int d\mathbf{r} \hat{\psi}(\mathbf{r} \sigma \tau) \delta_{\sigma, \sigma'} \langle \tau | \tau_+ | \tau' \rangle \hat{\psi}(\mathbf{r} \bar{\sigma}' \bar{\tau}') = \sqrt{2} \int d\mathbf{r} \hat{\psi}_\nu(\mathbf{r} \downarrow) \hat{\psi}_\nu(\mathbf{r} \uparrow)$$

$$\hat{\psi}(\mathbf{r} \bar{\sigma} \bar{\tau}) = (-2\sigma)(-2\tau) \hat{\psi}(\mathbf{r} - \sigma - \tau)$$

pairing condensation: order parameter

$$q \equiv \langle \hat{P}_{T=1, T_z=1, S=0} \rangle = \sqrt{2} \int d\mathbf{r} \tilde{\rho}_\nu(\mathbf{r})$$

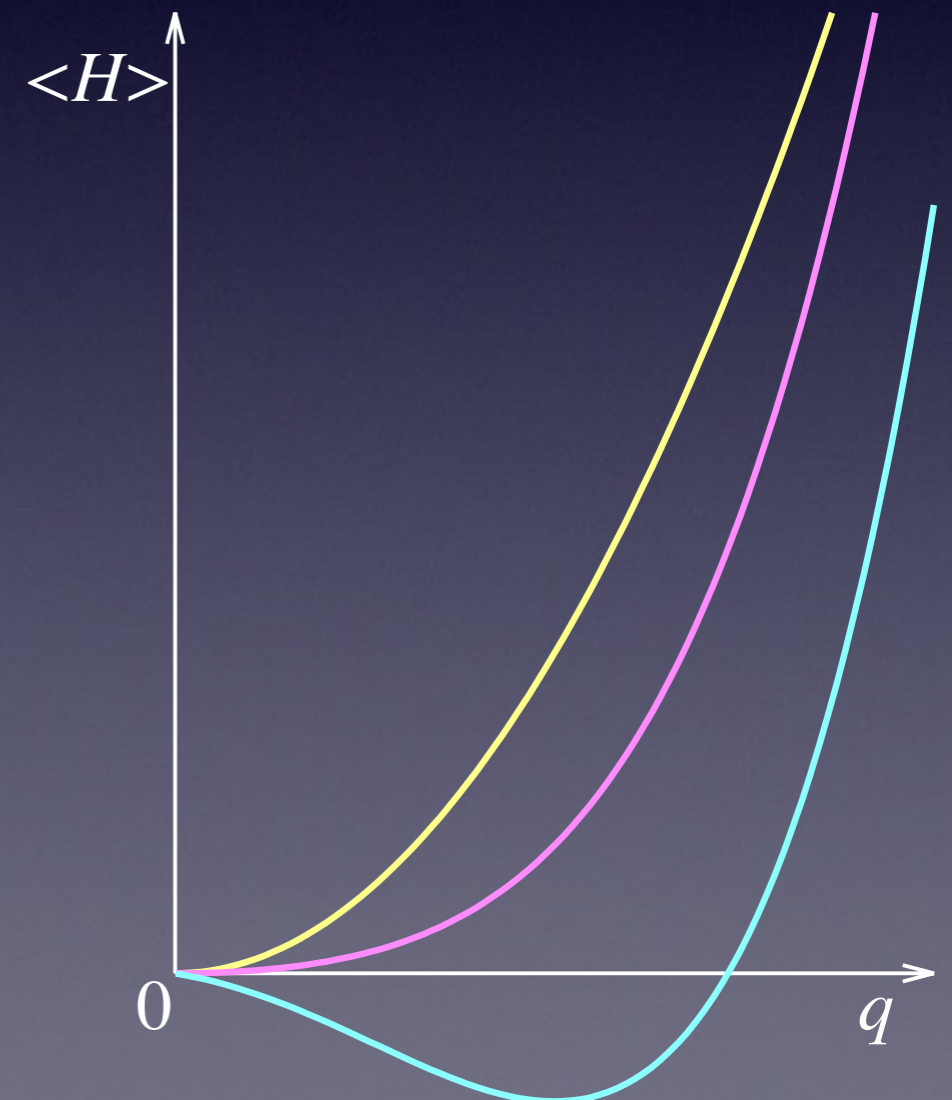
pairing gap: $\Delta \sim \int d\mathbf{r} \tilde{h}(\mathbf{r}) \tilde{\rho}(\mathbf{r})$

pairing vibration;
precursory soft mode: $|\lambda\rangle$

w/ an enhanced transition strength

$$|\langle \lambda | \hat{P}_{T=1, T_z=1, S=0} | \rangle|^2$$

is seen in normal nuclei ($q=0$)



Proton-neutron pairing collectivity

$T=1$ ($T_z=0$), $S=0$ pair

$$\hat{P}_{T=1, T_z=0, S=0} \equiv \frac{1}{2} \sum_{\sigma, \sigma'} \sum_{\tau, \tau'} \int d\mathbf{r} \hat{\psi}(\mathbf{r} \sigma \tau) \delta_{\sigma, \sigma'} \langle \tau | \tau_0 | \tau' \rangle \hat{\psi}(\mathbf{r} \bar{\sigma}' \bar{\tau}')$$

strong collectivity is expected as in nn and pp pairings

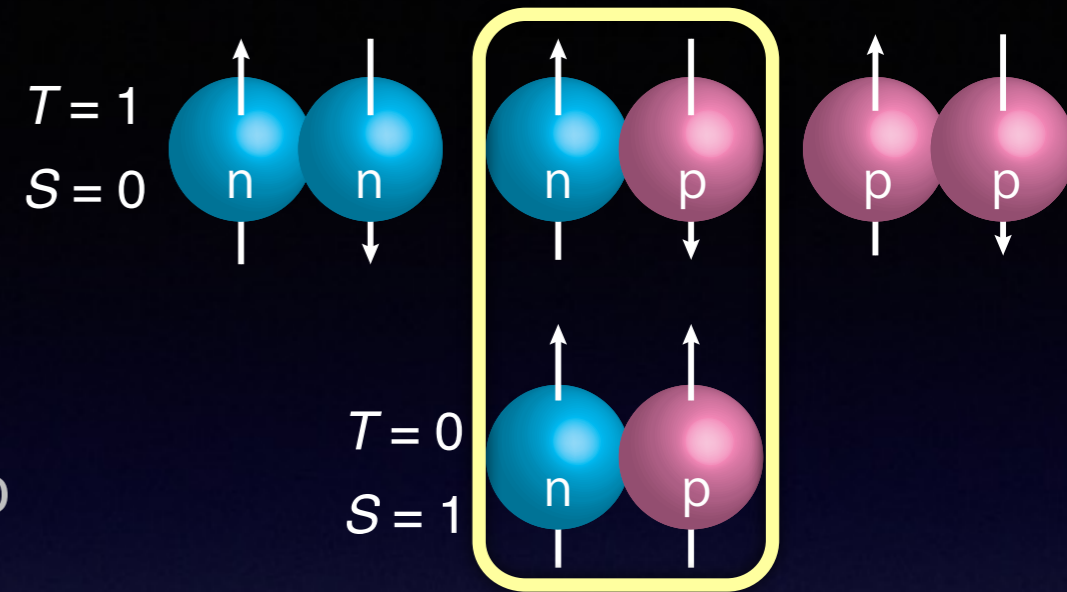
$T=0$, $S=1$ ($S_z=0, \pm 1$) pair

$$\hat{P}_{T=0, S=1} \equiv \frac{1}{2} \sum_{\sigma, \sigma'} \sum_{\tau, \tau'} \int d\mathbf{r} \hat{\psi}(\mathbf{r} \sigma \tau) \delta_{\tau, \tau'} \langle \sigma | \sigma | \sigma' \rangle \hat{\psi}(\mathbf{r} \bar{\sigma}' \bar{\tau}')$$

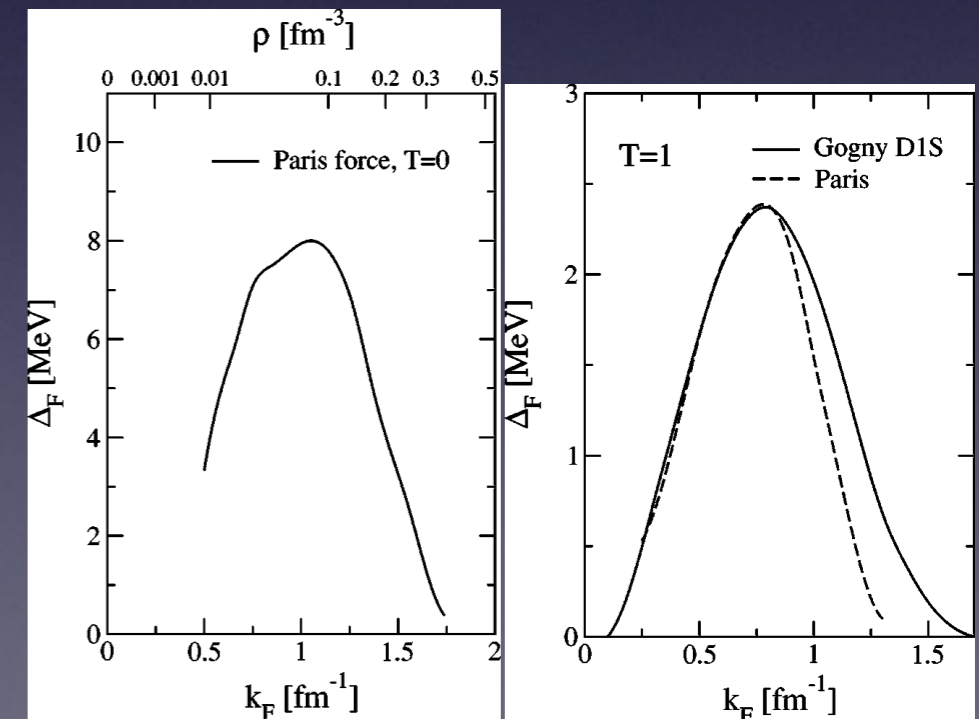
many works on the possible occurrence of the condensation, but largely unknown

“no experimental evidence so far”

S. Frauendorf and A. O. Macchiavelli,
Prog. Part. Nucl. Phys. 78 (2014) 24



$$\Delta_{01} > \Delta_{10}$$



Pairing phase diagram: Pairing vibration and rotation

G.G.Dussel et al., NPA450(1986)164

two-level solvable model:

$$H = 2N_2 - X_{10} \sum_{\mu} \sum_{l,l'=1,2} D_{\mu l}^+ D_{\mu l'} - X_{01} \sum_{\mu} \sum_{l,l'=1,2} P_{\mu l}^+ P_{\mu l'}$$

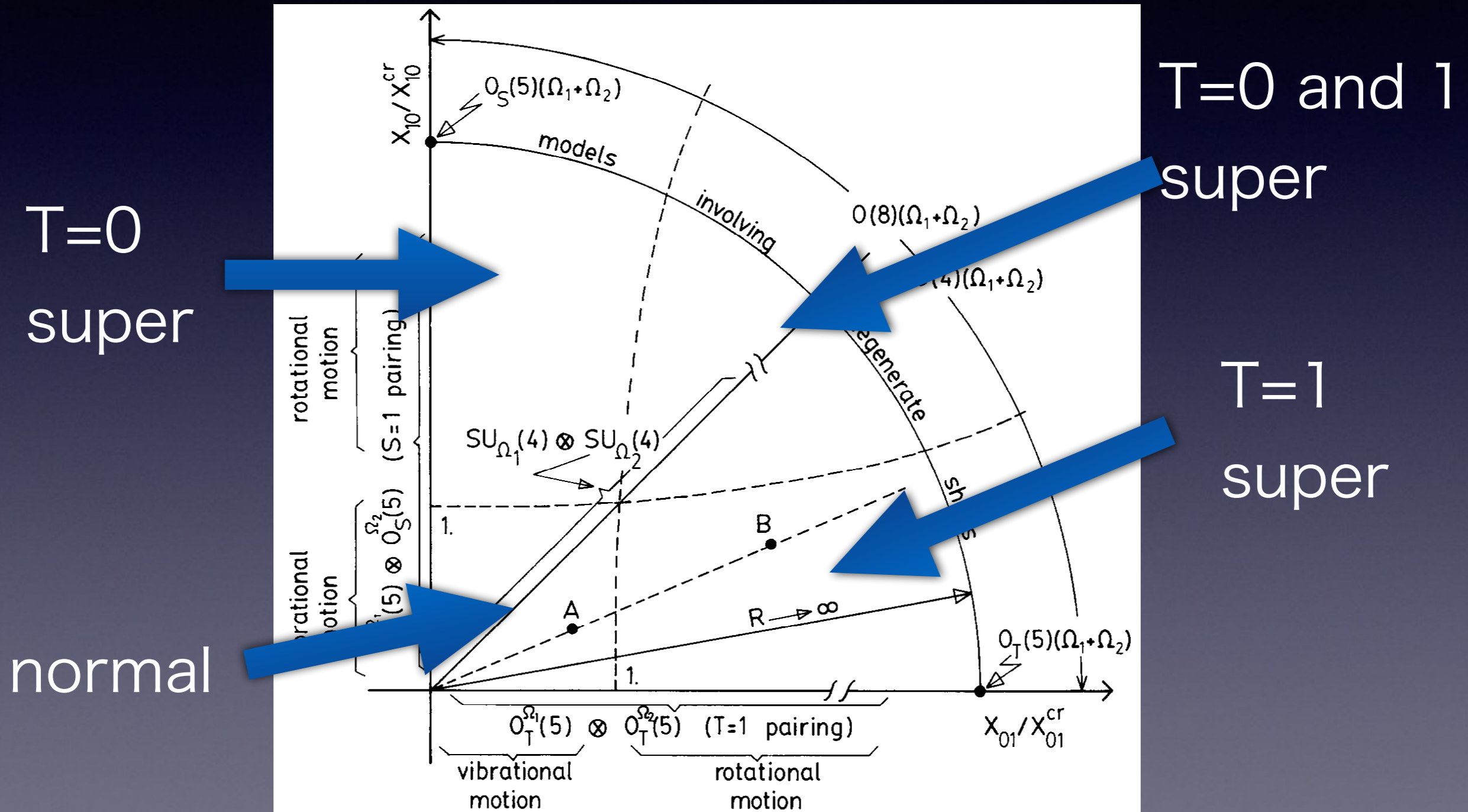


Fig. 1. The two-dimensional space of phases of the model. Various limiting schemes are indicated.

Interactions employed for pn-pairing vibrations in fp-shell nuclei

KSB(HFB) eq:

SGII + surface pairing

$$V_0 = -520 \text{ MeV fm}^3$$

^{44}Ti

$$\Delta_n = 1.82 \text{ MeV}$$

$$\Delta_p = 1.87 \text{ MeV}$$

pnQRPA eq:

p-h channel: SGII

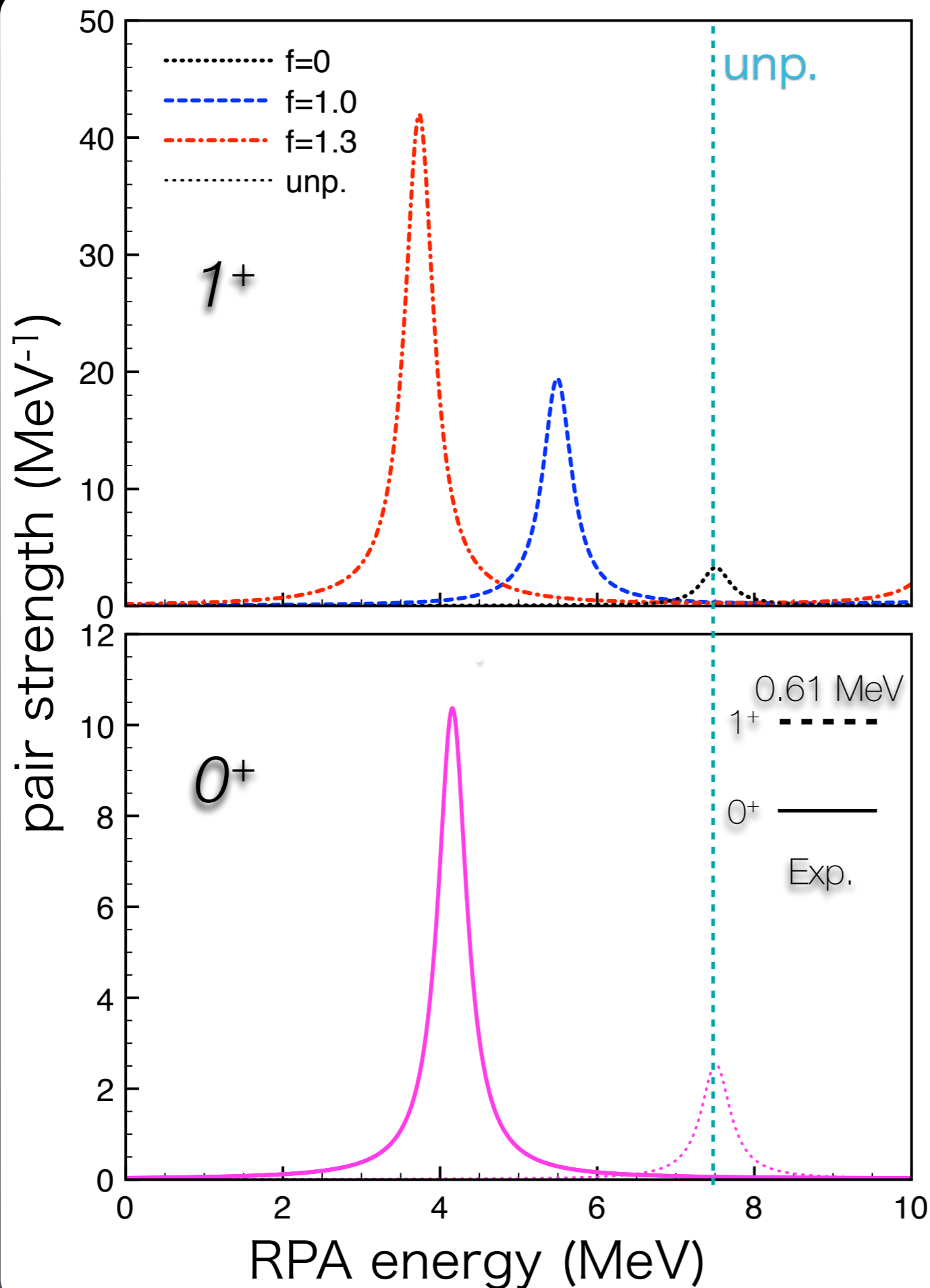
p-p channel:

$$S=0: \quad v_{pp}(\mathbf{r}, \mathbf{r}') = V_0 \left[1 - \frac{\rho(\mathbf{r})}{\rho_0} \right] \delta(\mathbf{r} - \mathbf{r}') \quad \text{self-consistent}$$

$$S=1: \quad v_{pp}(\mathbf{r}, \mathbf{r}') = fV_0 \left[1 - \frac{\rho(\mathbf{r})}{\rho_0} \right] \delta(\mathbf{r} - \mathbf{r}') \quad \text{“arbitrary”}$$

changing “f” to see an effect of the residual interaction

cf. C. Bai et al., PLB719(2013)116



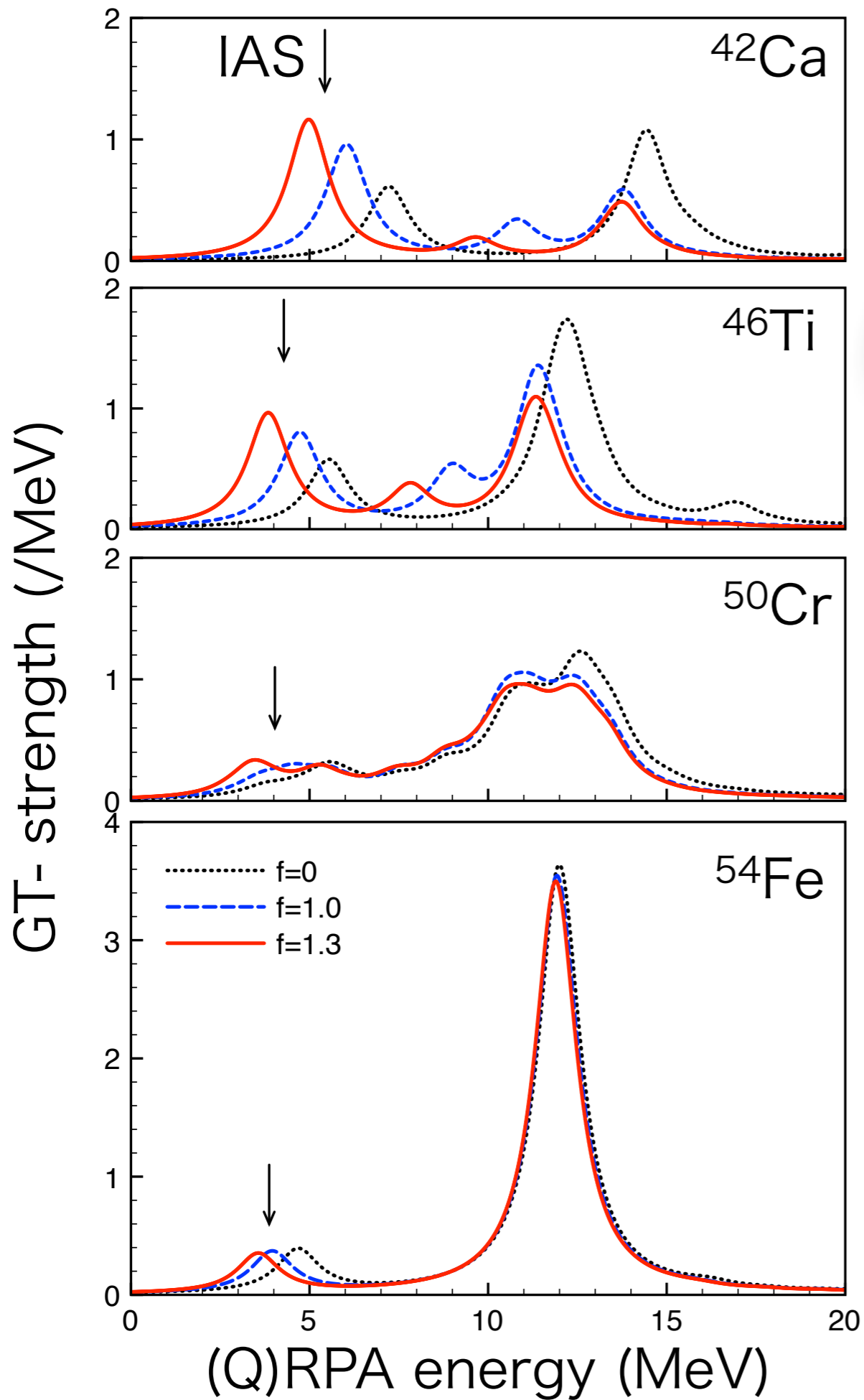
f=1.3

^{42}Sc	$J^\pi = 1^+$	$J^\pi = 0^+$	
configuration	$E_\alpha + E_\beta$	$M_{\alpha\beta}^{S=1, S_z=0}$	$M_{\alpha\beta}^{S=0}$
$\pi 1f_{7/2} \otimes \nu 1f_{7/2}$	7.5	1.70	2.85
$\pi 1f_{7/2} \otimes \nu 1f_{5/2}$	15.2	0.62	
$\pi 1f_{5/2} \otimes \nu 1f_{7/2}$	14.7	0.51	
$\pi 2p_{3/2} \otimes \nu 2p_{3/2}$	16.1	0.17	0.22
$\pi 1d_{3/2} \otimes \nu 1d_{3/2}$	4.2	0.25	0.48
$\pi 2s_{1/2} \otimes \nu 2s_{1/2}$	6.6	0.25	
$\pi 1d_{3/2} \otimes \nu 1d_{5/2}$	10.1	0.32	
$\pi 1d_{5/2} \otimes \nu 1d_{3/2}$	10.2	0.32	
$\pi 1d_{5/2} \otimes \nu 1d_{5/2}$	16.1	0.16	0.31

Transition matrix element

$$\langle \lambda | \hat{P}_{T,S}^\dagger | 0 \rangle = \sum_{\alpha\beta} M_{\alpha\beta}^{T,S}$$

- ✓ coherent superposition of $(f)^2$ excitation
- ✓ sizable hole-hole excitations

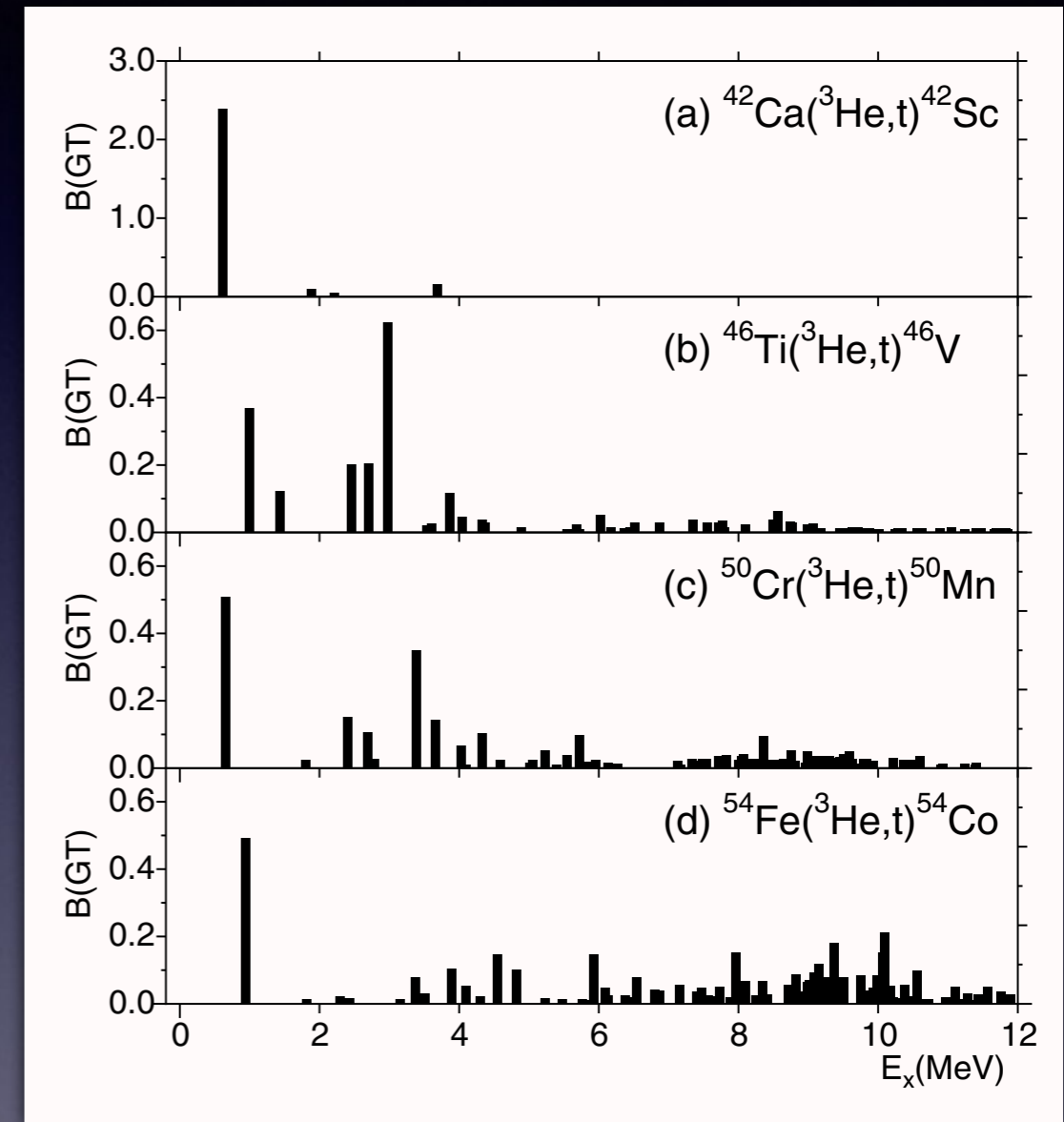


Y. Fujita et al, PRL112(2014)112502

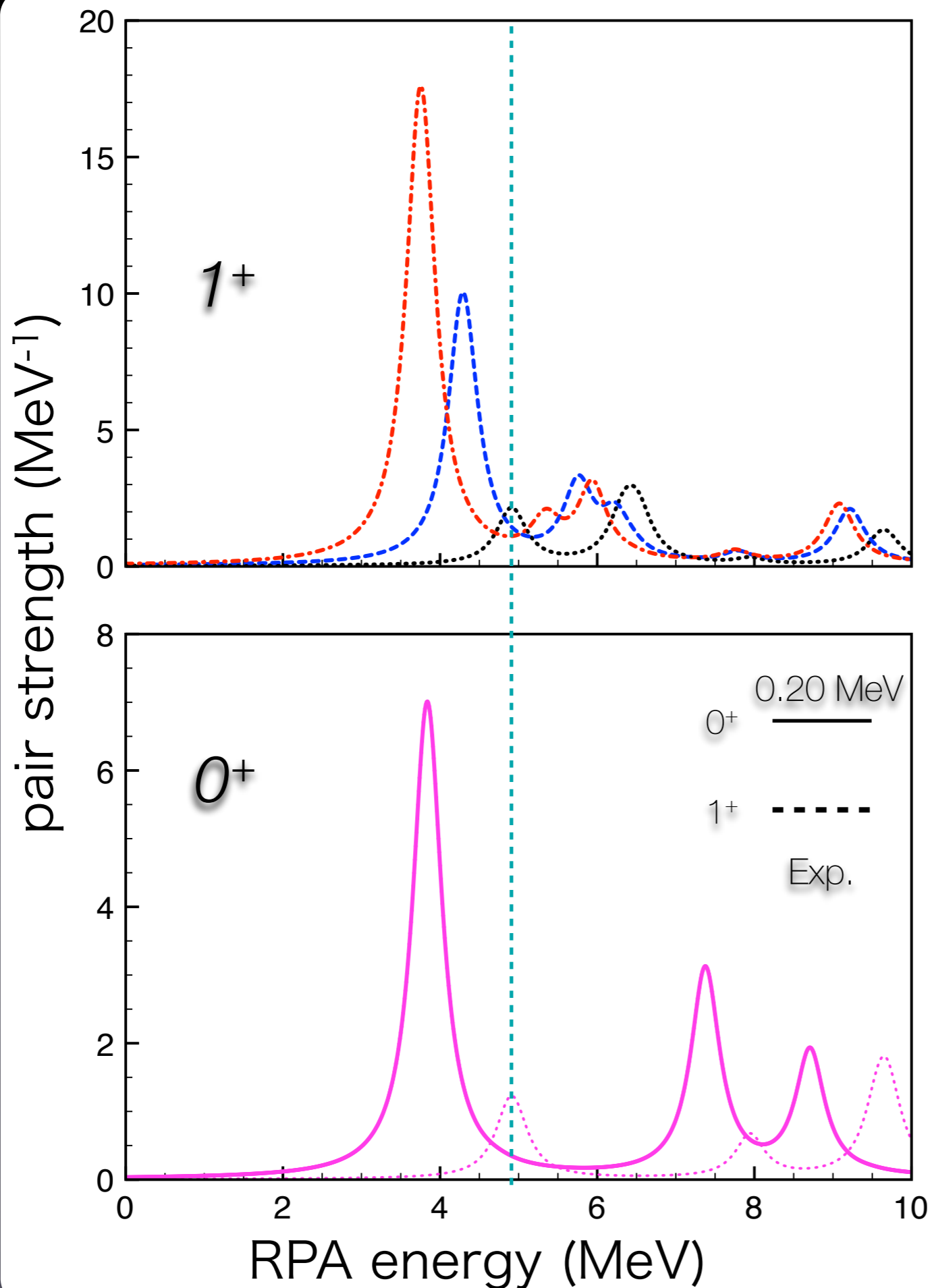
C. L. Bai et al, PRC90(2014)054335

“Low-energy super GT state” in ^{42}Sc

pn-pairing more effective



T. Adachi, Y. Fujita et al.,
NPA788 (2007) 70c



$f=1.3$

^{58}Cu	$J^\pi = 1^+$	$J^\pi = 0^+$	
configuration	$E_\alpha + E_\beta$	$M_{\alpha\beta}^{S=1, S_z=0}$	$M_{\alpha\beta}^{S=0}$
$\pi 2p_{3/2} \otimes \nu 2p_{3/2}$	4.5	1.28	1.90
$\pi 2p_{1/2} \otimes \nu 2p_{3/2}$	6.4	0.39	
$\pi 2p_{3/2} \otimes \nu 2p_{1/2}$	6.5	0.37	
$\pi 2p_{1/2} \otimes \nu 2p_{1/2}$	7.9		0.26
$\pi 1f_{5/2} \otimes \nu 1f_{5/2}$	9.7	0.15	0.55
$\pi 1f_{7/2} \otimes \nu 1f_{7/2}$	5.1	0.17	0.50

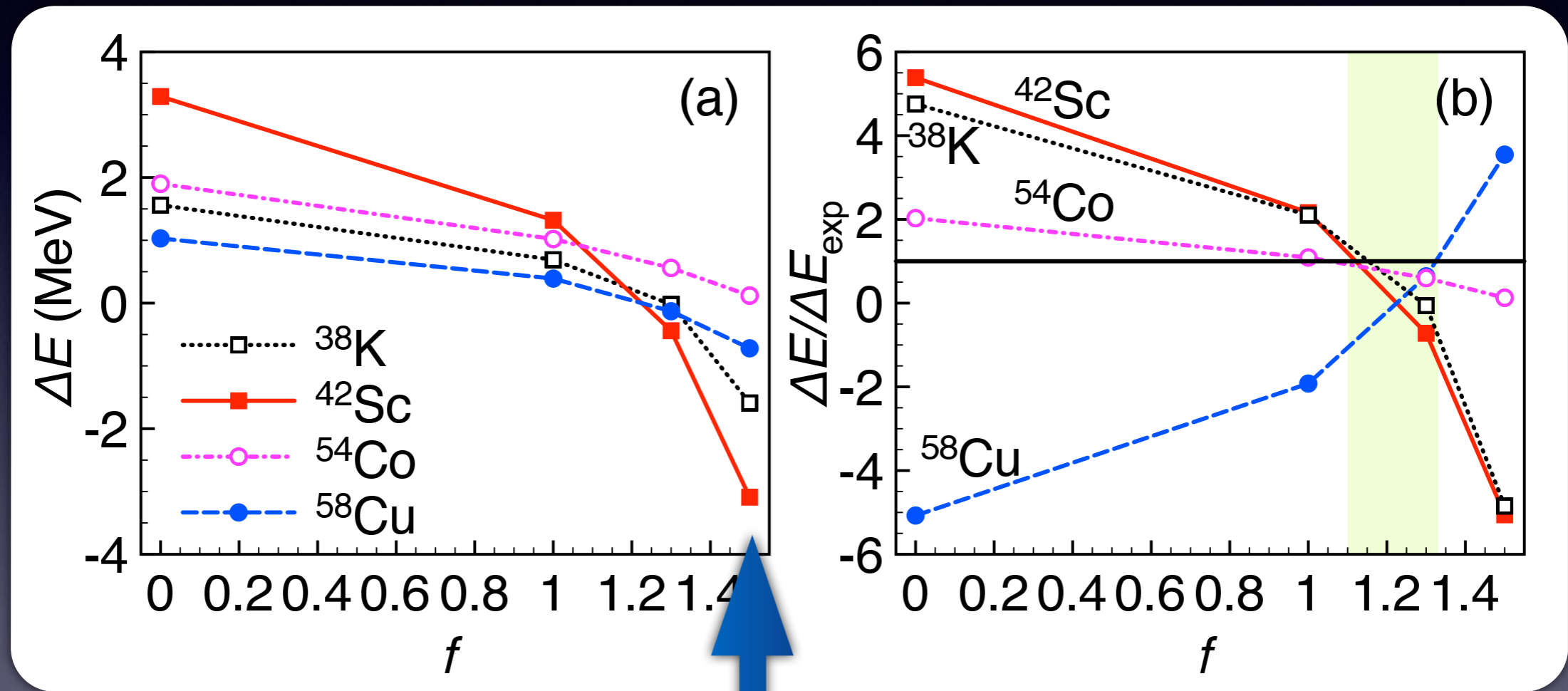
- ✓ coherent superposition of $(p)^2$ and $(f_{5/2})^2$ excitations
- ✓ $(f_{7/2})^2$ excitation as a ground-state correlation



weaker collectivity than
in ^{40}Ca

Collective pn-pairing vibration mode precursory to the T=0 pairing condensation

$$\Delta E = \omega_{1+} - \omega_{0+}$$

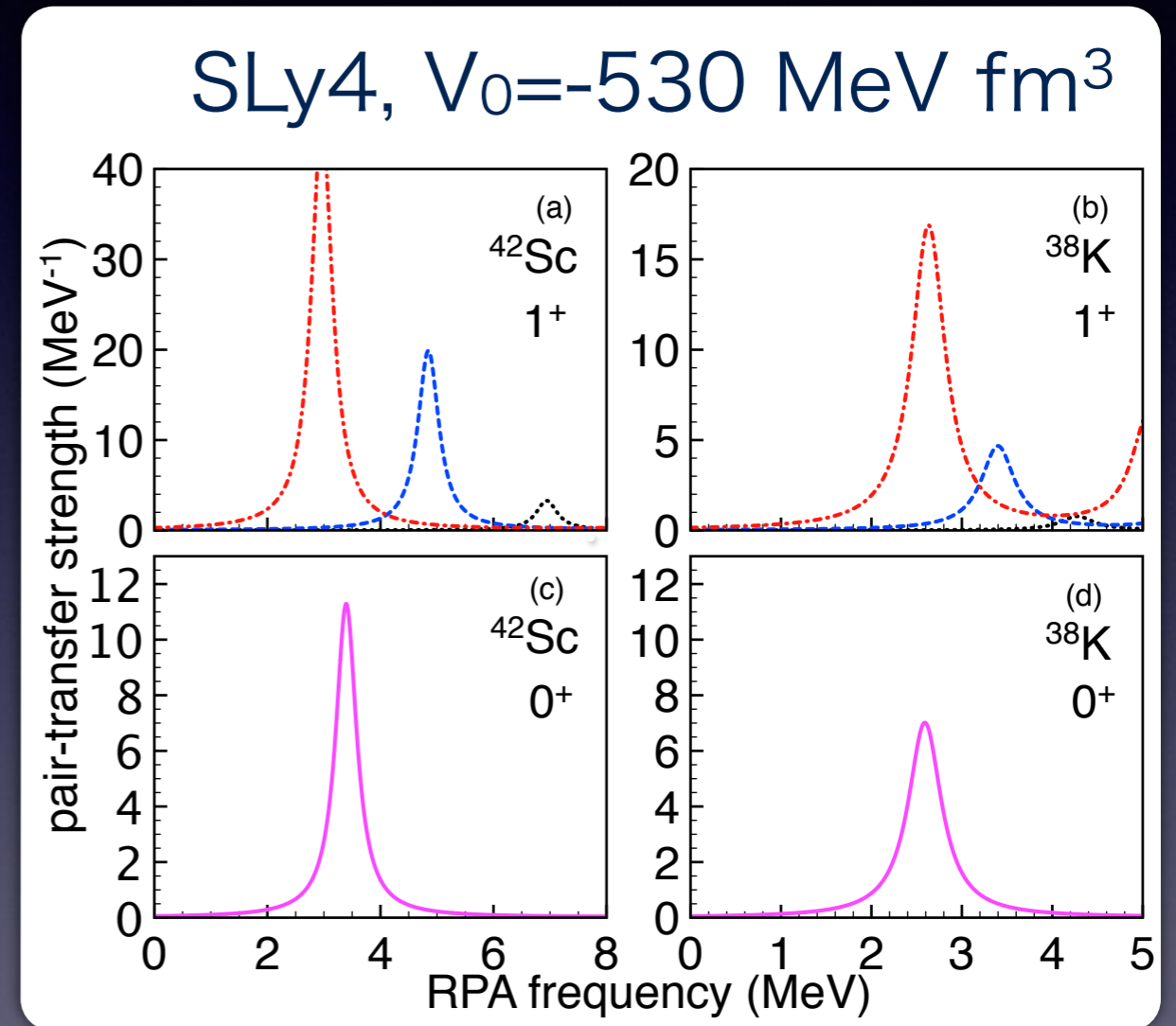
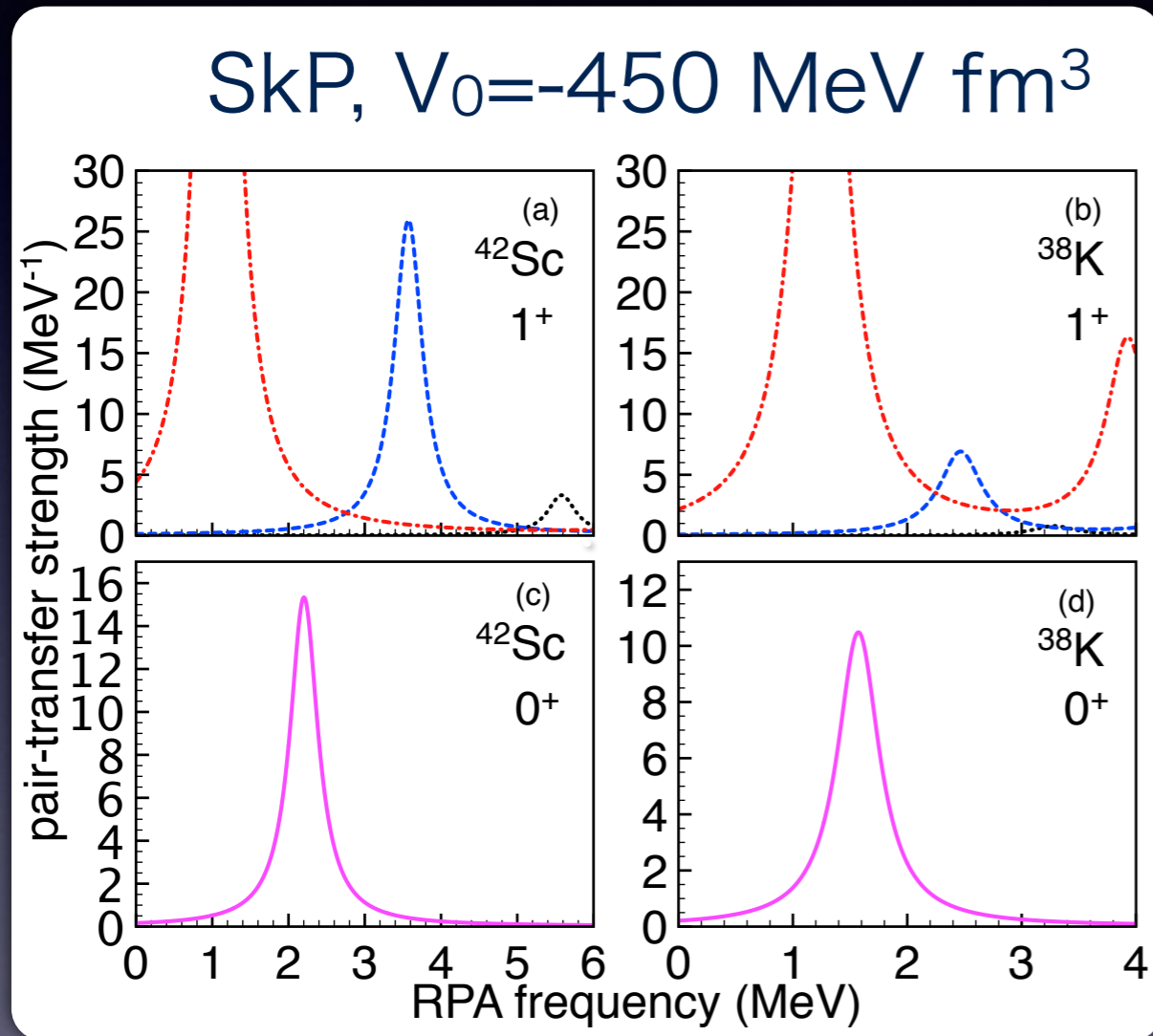


approaching the critical point to the T=0 pairing condensation

$$f_c = 1.53 \text{ } (^{40}\text{Ca})$$

Interaction dependence:

no qualitative difference



SkP: lower energy due to high effective mass

^{42}Sc $B(M1)_{\text{exp}}=6.3\pm 0.3 \mu_N^2$: PRC75(2007)064321

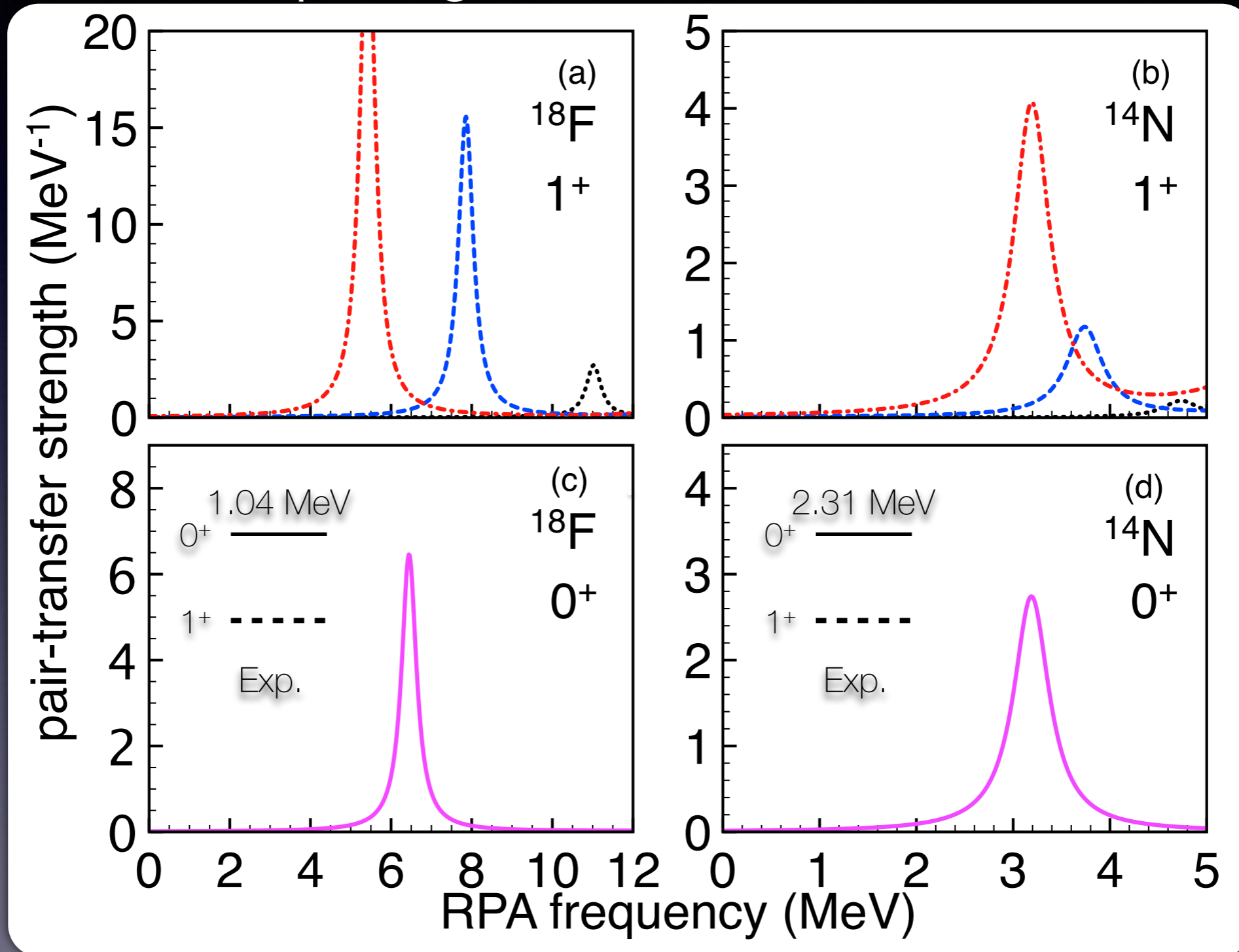
	core+2N in single-j(*)	3-body model (pnTDA)**	pnRPA (f=1.3)	pnRPA (f=1.0)
$\sum_i g_l(i) \vec{l}(i)$	3.40	2.91	2.54	2.71
$g_s^{\text{IV}} \sum_i \tau_z(i) \vec{s}(i)$	5.34	6.34	6.77	6.35
$g_s^{\text{IS}} \sum_i \vec{s}(i)$	0	2×10^{-3}	-4×10^{-2}	-2×10^{-2}
$B(M1 \downarrow) (\mu_N^2)$	6.08	6.81	6.90	6.53

(*)Lisetskiy et al.,PRC60(1999)064310

(**)Tanimura et al., PTEP(2014)053D02

pn-pairing vibrations in ^{16}O : another example of LS-closed system

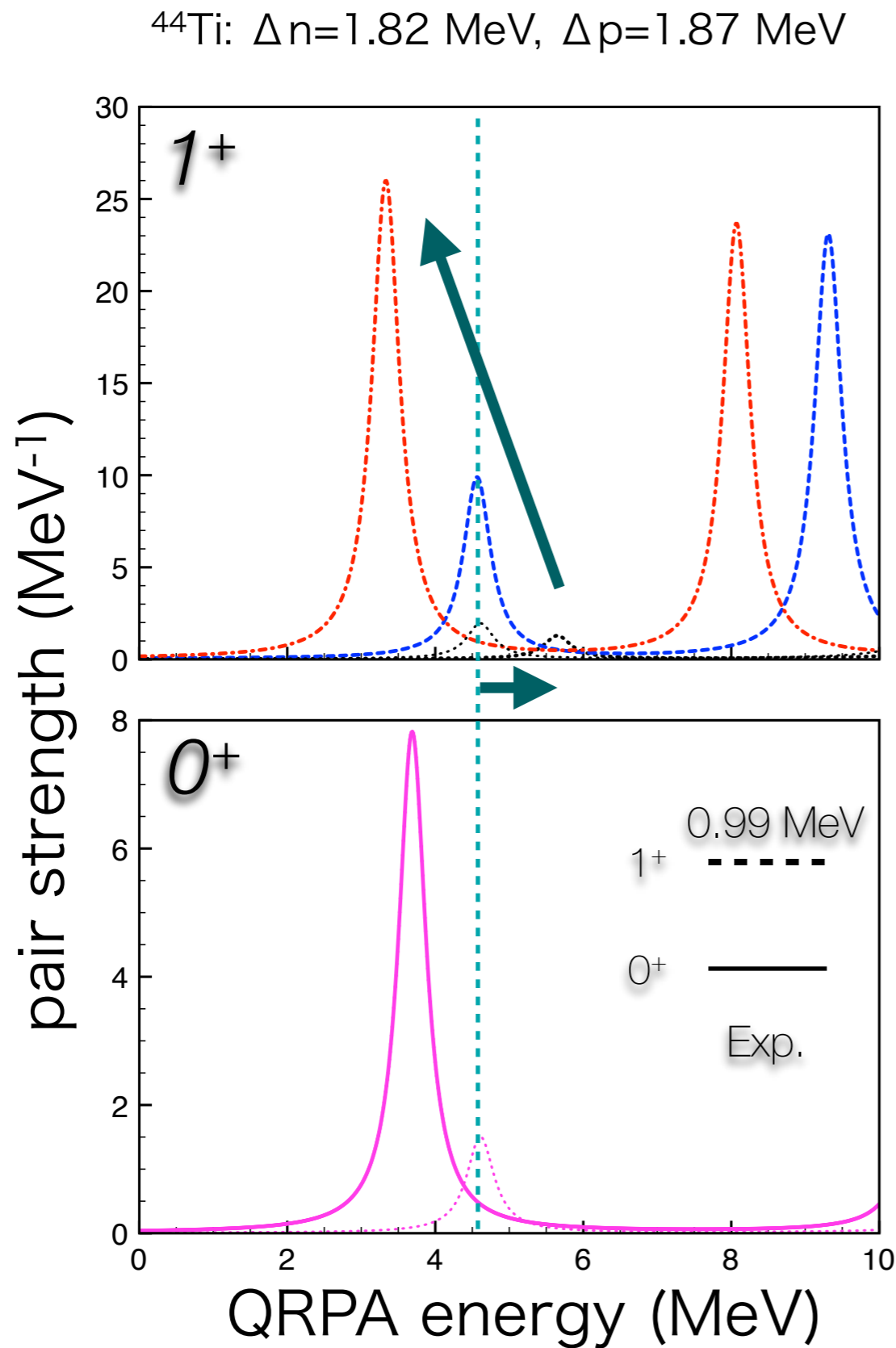
SGLI + surface pairing: $V_0 = -490 \text{ MeV fm}^3$



pn-pairing vibrations in the open-shell nuclei

w/ T=I pairing condensation

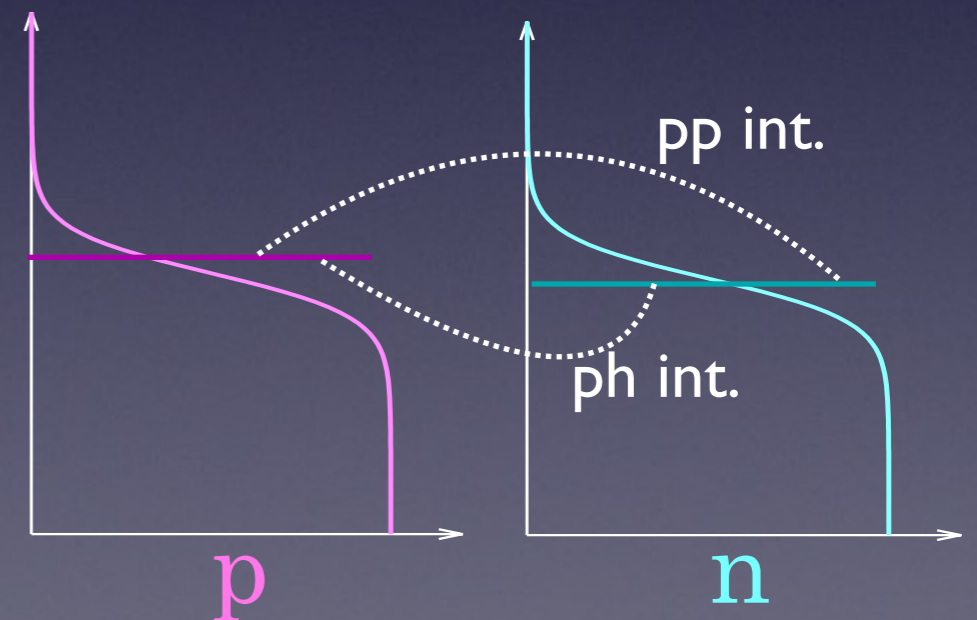
^{44}Ti
↓
 ^{46}V



repulsive ph interaction
(GT-type)



attractive pp interaction



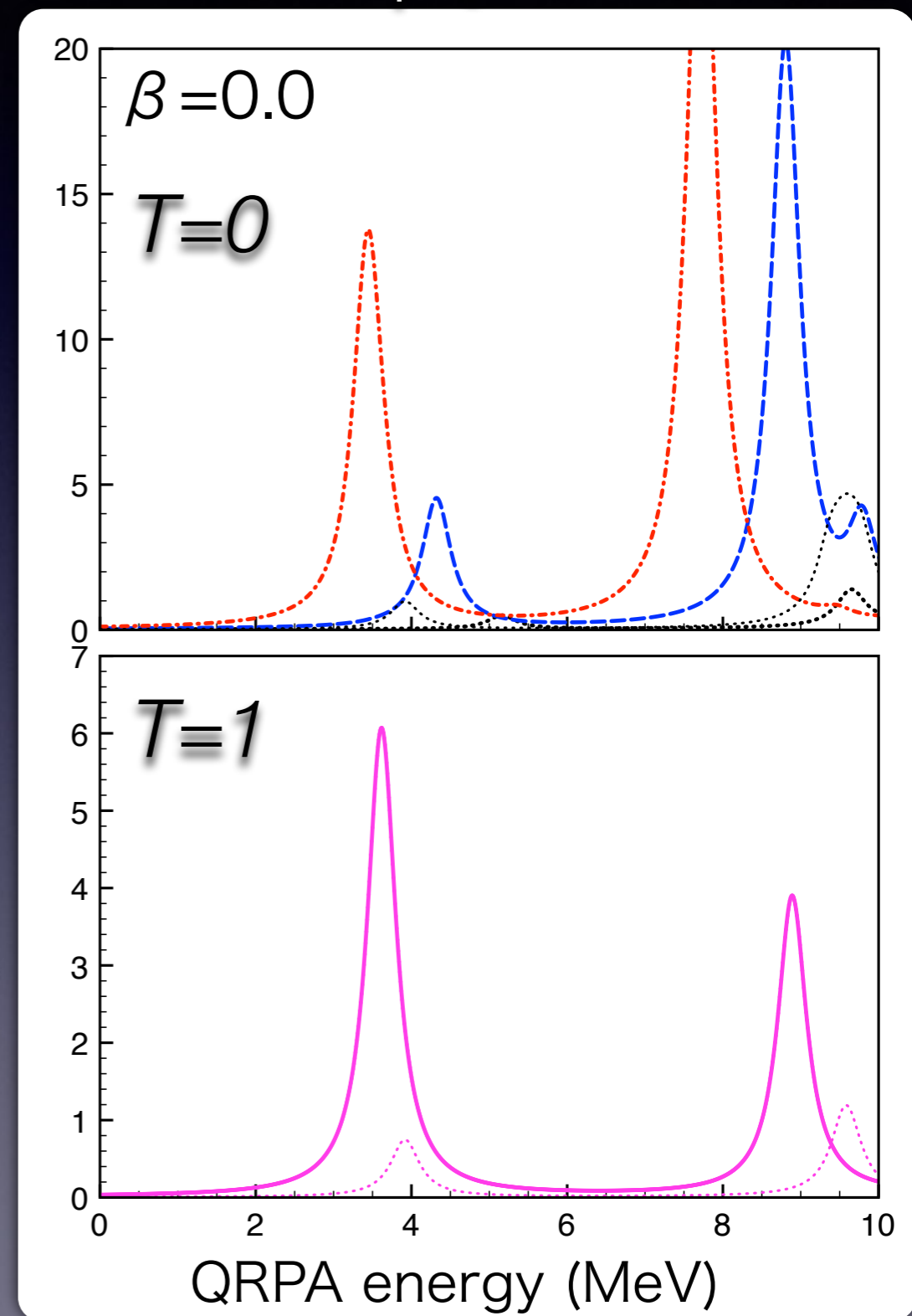
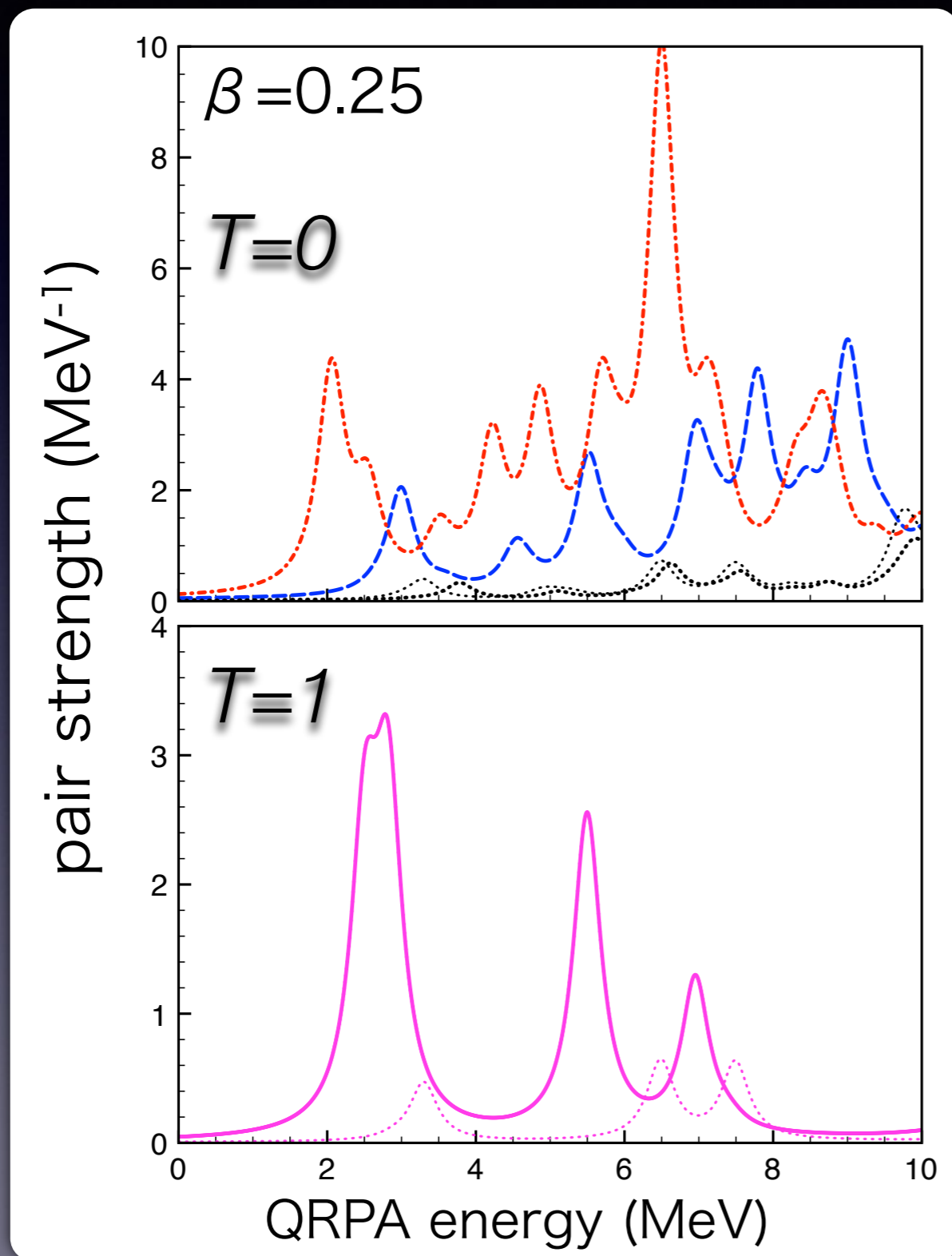
“ $^{44}\text{Ti} + 2\text{qp}$ excitation”

pn-pairing vibrations in the mid-shell nuclei

w/ $T=1$ pairing condensation and quadrupole def.

^{48}Cr
↓
 ^{50}Mn

constrained HFB+pnQRPA



Summary

- ✓ self-consistent deformed pnQRPA
 - investigation of a rich variety of excitation modes
 - quest for new types of collective mode

Possible occurrence of a new kind of collective mode associated with the spin-triplet pairing condensation

In LS-closed nuclei, the spin-orbit partners have a coherent contribution to the collective mode.

We can study the nature of $T=0$ pairing in nuclei even if they are in the “normal” phase.

We don't need to stick about an emergence of the spin-triplet pairing condensation.