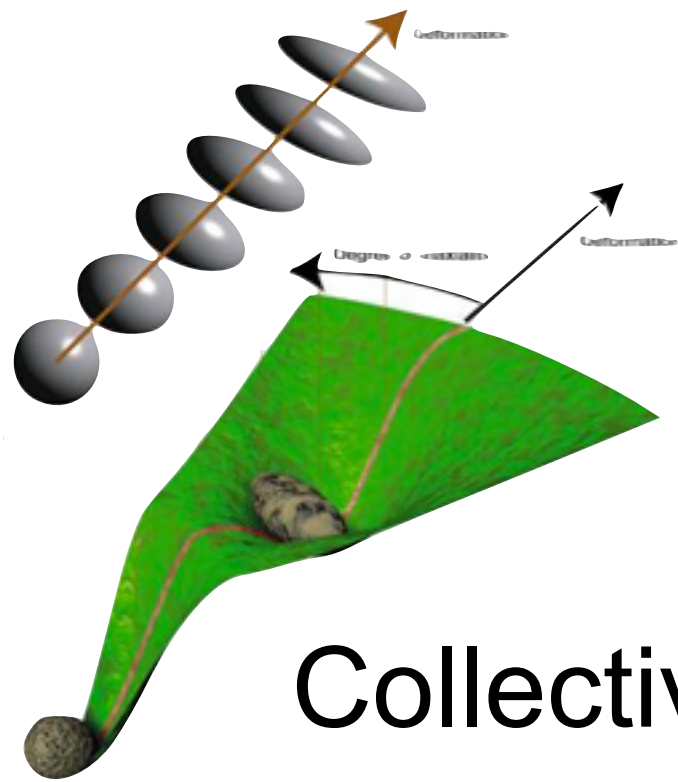


# Relativistic description for novel rotation and exotic shape in nuclei

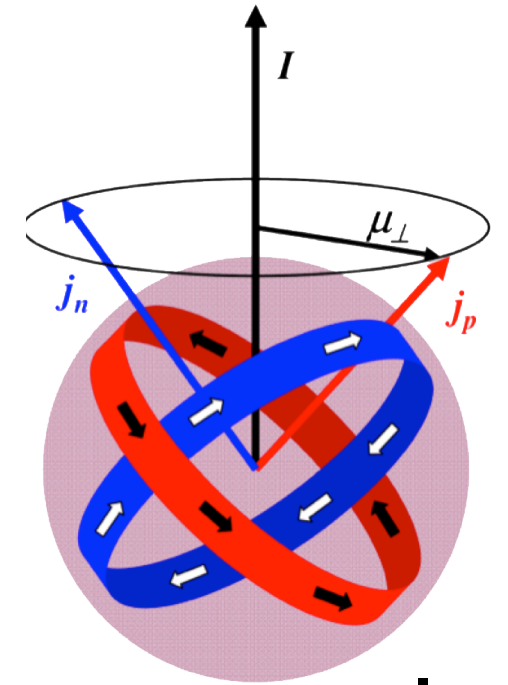
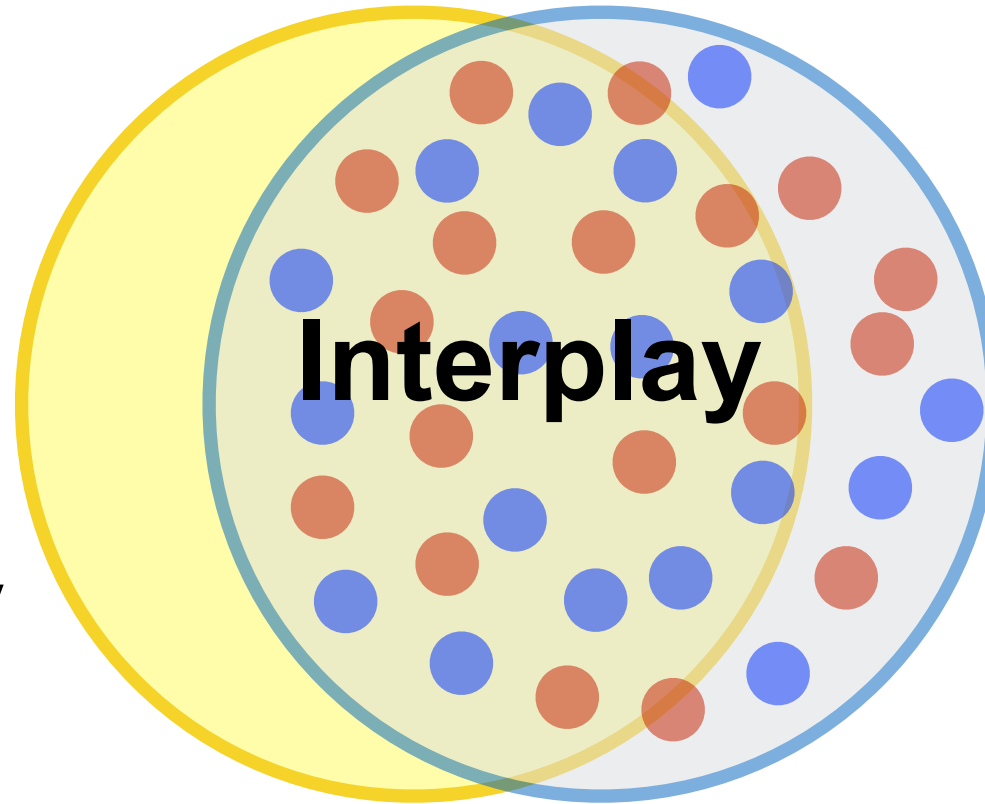
**Pengwei Zhao** (赵鹏巍)

Argonne National Laboratory

# Nuclear Rotation



Collectivity

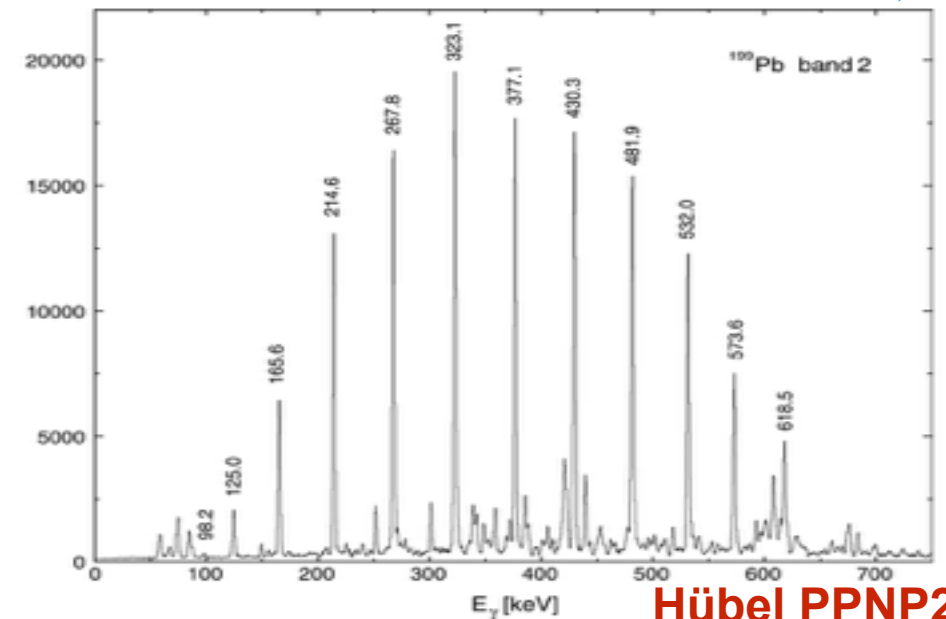
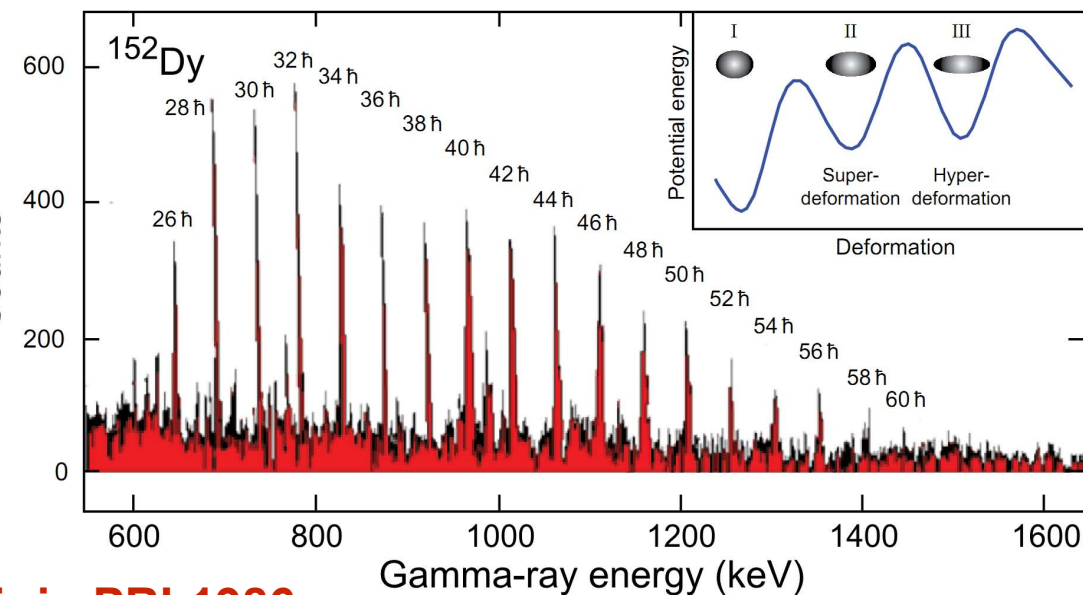


valence nucleon

exotic shape

novel rotation

nuclear structure under extreme conditions



# Outline

- ✓ Cranking covariant density functional theory
- ✓ Novel rotation:  
magnetic and antimagnetic rotation
- ✓ Exotic shape:  
stabilization of the rod shape in C isotopes
- ✓ Summary

# Tilted axis cranking DFT

- **Covariant Density Functional Theory**

Meson exchange version:

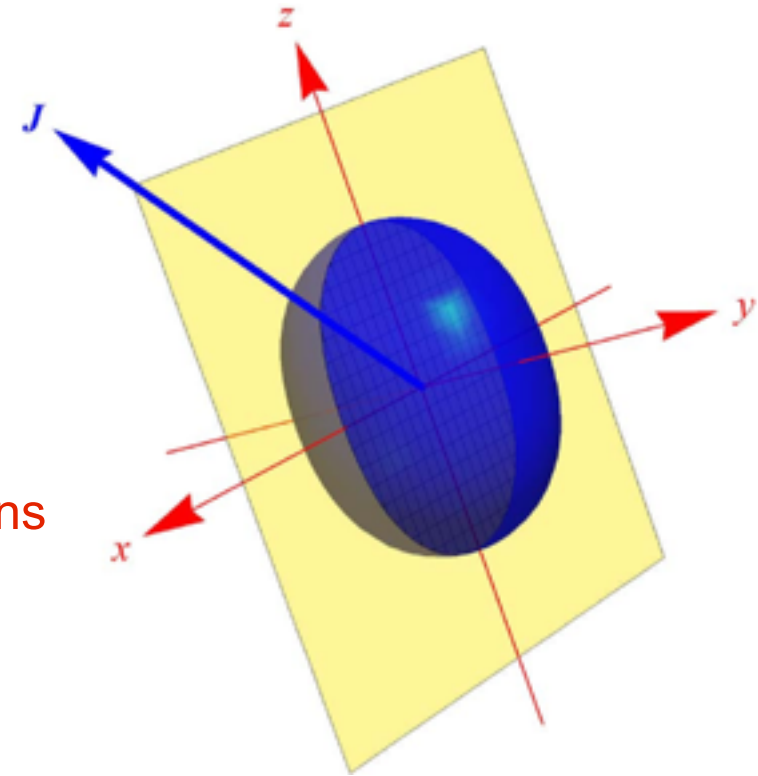
3-D Cranking: *Madokoro, Meng, Matsuzaki, Yamaji, PRC 62, 061301 (2000)*

2-D Cranking: *Peng, Meng, Ring, Zhang, PRC 78, 024313 (2008)*

Point-coupling version: Simple and more suitable for systematic investigations

2-D Cranking: *PWZ, Zhang, Peng, Liang, Ring, Meng, PLB 699, 181 (2011)*

2-D Cranking + Pairing: *PWZ, Zhang, Meng, PRC 92, 034319 (2015)*



- **Skyrme Density Functional Theory**

3-D Cranking: *Olbratowski, Dobaczewski, Dudek, Płóciennik, PRL 93, 052501(2004)*

2-D Cranking: *Olbratowski, Dobaczewski, Dudek, Rzaca-Urban, Marcinkowska, Lieder, APPB 33, 389(2002)*

## Self-consistent microscopic investigations

- fully taken into account polarization effects
- self-consistently treated the nuclear currents
- no additional parameter beyond a well-determined functional

# Covariant Density Functional Theory

## Elementary building blocks

$$(\bar{\psi} \mathcal{O}_\tau \Gamma \psi) \quad \mathcal{O}_\tau \in \{1, \tau_i\} \quad \Gamma \in \{1, \gamma_\mu, \gamma_5, \gamma_5 \gamma_\mu, \sigma_{\mu\nu}\}$$

## Densities and currents

Isoscalar-scalar  $\rho_S(\mathbf{r}) = \sum_k^{occ} \bar{\psi}_k(\mathbf{r}) \psi_k(\mathbf{r})$

Isoscalar-vector  $j_\mu(\mathbf{r}) = \sum_k^{occ} \bar{\psi}_k(\mathbf{r}) \gamma_\mu \psi_k(\mathbf{r})$

Isovector-scalar  $\vec{\rho}_S(\mathbf{r}) = \sum_k^{occ} \bar{\psi}_k(\mathbf{r}) \vec{\tau} \psi_k(\mathbf{r})$

Isovector-vector  $\vec{j}_\mu(\mathbf{r}) = \sum_k^{occ} \bar{\psi}_k(\mathbf{r}) \vec{\tau} \gamma_\mu \psi_k(\mathbf{r})$

## Energy Density Functional

$$E_{kin} = \sum_k v_k^2 \int \bar{\psi}_k (-\gamma \nabla + m) \psi_k d\mathbf{r}$$

$$E_{2nd} = \frac{1}{2} \int (\alpha_S \rho_S^2 + \alpha_V \rho_V^2 + \alpha_{tV} \rho_{tV}^2) d\mathbf{r}$$

$$E_{hot} = \frac{1}{12} \int (4\beta_S \rho_S^3 + 3\gamma_S \rho_S^4 + 3\gamma_V \rho_V^4) d\mathbf{r}$$

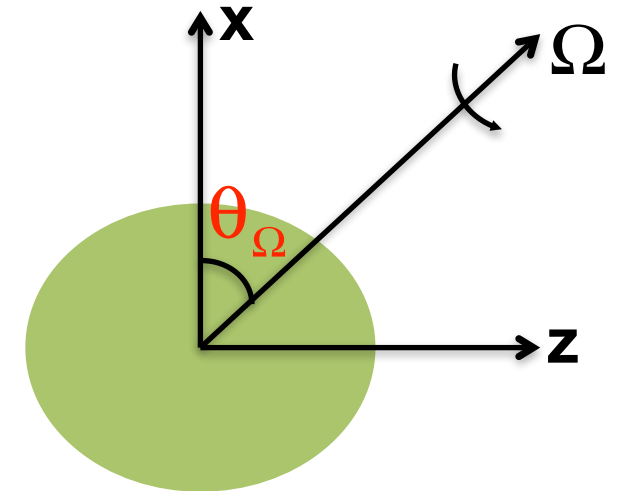
$$E_{der} = \frac{1}{2} \int (\delta_S \rho_S \Delta \rho_S + \delta_V \rho_V \Delta \rho_V + \delta_{tV} \rho_{tV} \Delta \rho_{tV}) d\mathbf{r}$$

$$E_{em} = \frac{e}{2} \int j_\mu^p A^\mu d\mathbf{r}$$

# Cranking Covariant Density Functional Theory

Transform to the frame rotating with a uniform velocity

$$x^\alpha = \begin{pmatrix} t \\ \mathbf{x} \end{pmatrix} \rightarrow \tilde{x}^\mu = \begin{pmatrix} \tilde{t} \\ \tilde{\mathbf{x}} \end{pmatrix} = \begin{pmatrix} 1 & \mathbf{0} \\ \mathbf{0} & R_x(t) \end{pmatrix} \begin{pmatrix} t \\ \mathbf{x} \end{pmatrix}$$



## Rotating Density Functional

*Peng, Meng, Ring, Zhang, Phys. Rev. C 78, 024313 (2008).*

*PWZ, Zhang, Peng, Liang, Ring, Meng, Phys. Lett. B 699, 181 (2011).*

*PWZ, Peng, Liang, Ring, Meng, Phys. Rev. Lett. 107, 122501 (2011).*

*PWZ, Peng, Liang, Ring, Meng, Phys. Rev. C 85, 054310 (2012).*

*Meng, Peng, Zhang, PWZ, Front. Phys. 8, 55 (2013).*

*PWZ, Zhang, Meng, Phys. Rev. C 92, 034319 (2015).*

$$E_{kin} = \sum_k v_k^2 \int \bar{\psi}_k (-\gamma \nabla + m) \psi_k d\mathbf{r}$$

$$E_{2nd} = \frac{1}{2} \int (\alpha_S \rho_S^2 + \alpha_V \rho_V^2 + \alpha_{tV} \rho_{tV}^2) d\mathbf{r}$$

$$E_{hot} = \frac{1}{12} \int (4\beta_S \rho_S^3 + 3\gamma_S \rho_S^4 + 3\gamma_V \rho_V^4) d\mathbf{r}$$

$$E_{der} = \frac{1}{2} \int (\delta_S \rho_S \Delta \rho_S + \delta_V \rho_V \Delta \rho_V + \delta_{tV} \rho_{tV} \Delta \rho_{tV}) d\mathbf{r}$$

$$E_{em} = \frac{e}{2} \int j_\mu^p A^\mu d\mathbf{r}$$

# Kohn-Sham/Dirac Equation:

Dirac equation for single nucleon

$$\begin{pmatrix} m + S + V - \boldsymbol{\Omega} \cdot \mathbf{J} & \sigma(p - \mathbf{V}) \\ \sigma(p - \mathbf{V}) & -m - S + V - \boldsymbol{\Omega} \cdot \mathbf{J} \end{pmatrix} \begin{pmatrix} f \\ g \end{pmatrix} = \epsilon \begin{pmatrix} f \\ g \end{pmatrix}$$

$$V(r) = \alpha_V \rho_V + \gamma_V \rho_V^3 + \delta_V \Delta \rho_V + \tau_3 \alpha_{TV} \rho_{TV} + \tau_3 \delta_{TV} \Delta \rho_{TV} + e \frac{1 - \tau_3}{2} A$$

$$\mathbf{V}(r) = \alpha_V \mathbf{j}_V + \gamma_V \mathbf{j}_V^3 + \delta_V \Delta \mathbf{j}_V + \tau_3 \alpha_{TV} \mathbf{j}_{TV} + \tau_3 \delta_{TV} \Delta \mathbf{j}_{TV} + e \frac{1 - \tau_3}{2} \mathbf{A}$$

$$S(r) = \alpha_S \rho_S + \beta_S \rho_S^2 + \gamma_S \rho_S^3 + \delta_S \Delta \rho_S$$

$V(r)$  vector potential time-like  
 $\mathbf{V}(r)$  vector potential space-like

$S(r)$  scalar potential



# Kohn-Sham/RHB Equation: With Pairing

RHB equation for single quasi-nucleon

$$\begin{pmatrix} h_D - \lambda & \Delta \\ -\Delta^* & -h_D^* + \lambda \end{pmatrix} \begin{pmatrix} U_k \\ V_k \end{pmatrix} = E_k \begin{pmatrix} U_k \\ V_k \end{pmatrix}$$

$$h_D = \begin{pmatrix} m + \mathbf{S} + \mathbf{V} - \boldsymbol{\Omega} \cdot \mathbf{J} & \sigma(p - \mathbf{V}) \\ \sigma(p - \mathbf{V}) & -m - \mathbf{S} + \mathbf{V} - \boldsymbol{\Omega} \cdot \mathbf{J} \end{pmatrix}$$

$$\Delta_{ab} = \frac{1}{2} \sum_{cd} \langle ab | V^{pp} | cd \rangle_a \kappa_{cd}.$$

$\mathbf{S}(r)$  scalar potential  
 $\mathbf{V}(r)$  vector potential time-like  
 $\mathbf{V}(\mathbf{r})$  vector potential space-like



# Observables

## Total energy

$$E_{\text{tot}} = E_{\text{kin}} + E_{\text{int}} + E_{\text{cou}} + E_{\text{cm}} + E_{\text{pair}}$$

## Angular momentum

$$J = \sqrt{\langle \hat{J}_x \rangle^2 + \langle \hat{J}_z \rangle^2} \equiv \sqrt{I(I+1)}$$

## Quadrupole moments and magnetic moments

$$Q_{20} = \sqrt{\frac{5}{16\pi}} \langle 3z^2 - r^2 \rangle,$$

$$Q_{22} = \sqrt{\frac{15}{32\pi}} \langle x^2 - y^2 \rangle,$$

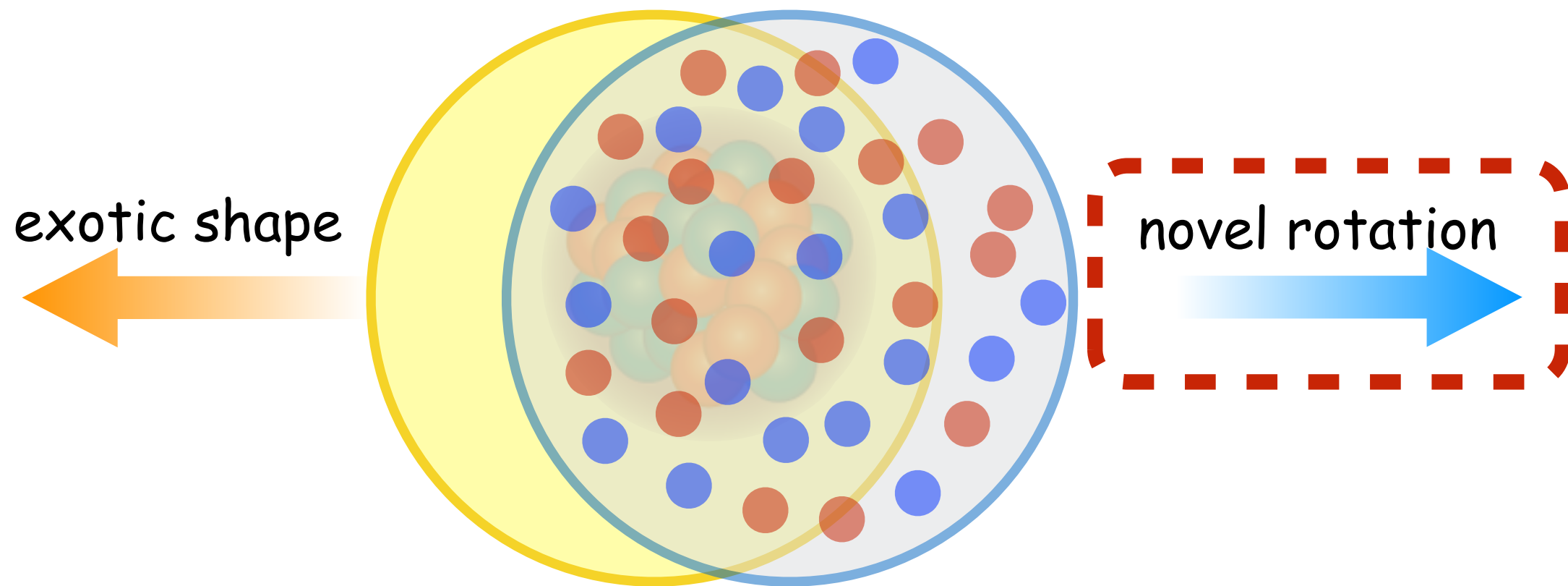
$$\mu = \sum_{i=1}^A \int d^3r \left[ \frac{mc^2}{\hbar c} q \psi_i^\dagger(\mathbf{r}) \mathbf{r} \times \boldsymbol{\alpha} \psi_i(\mathbf{r}) + \kappa \psi_i^\dagger(\mathbf{r}) \boldsymbol{\beta} \boldsymbol{\Sigma} \psi_i(\mathbf{r}) \right],$$

## B(M1) and B(E2) transition probabilities

$$B(M1) = \frac{3}{8\pi} \mu_{\perp}^2 = \frac{3}{8\pi} (\mu_x \sin \theta_J - \mu_z \cos \theta_J)^2,$$

$$B(E2) = \frac{3}{8} \left[ Q_{20}^p \cos^2 \theta_J + \sqrt{\frac{2}{3}} Q_{22}^p (1 + \sin^2 \theta_J) \right]^2,$$

# nuclear structure under extreme conditions

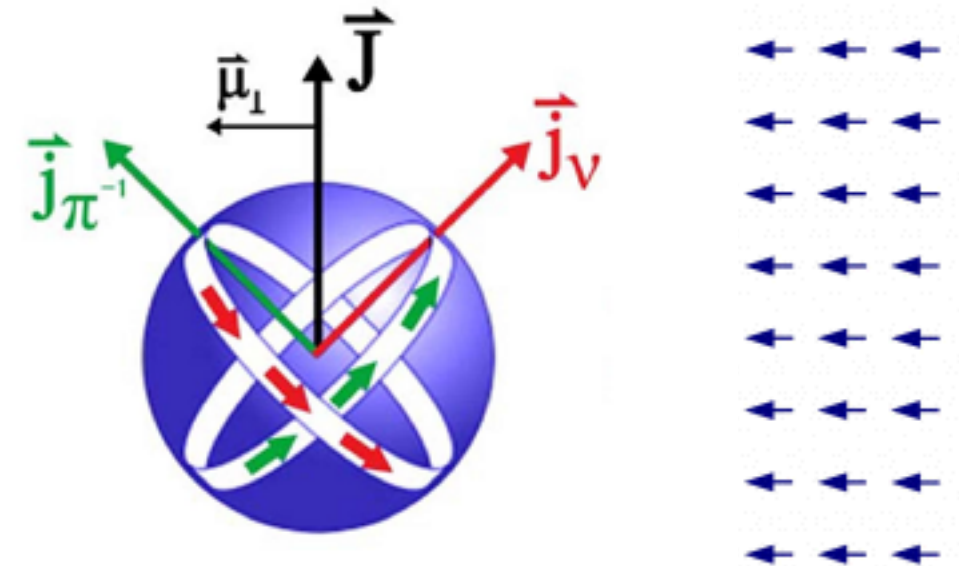


magnetic and antimagnetic rotation

# Magnetic and antimagnetic rotation

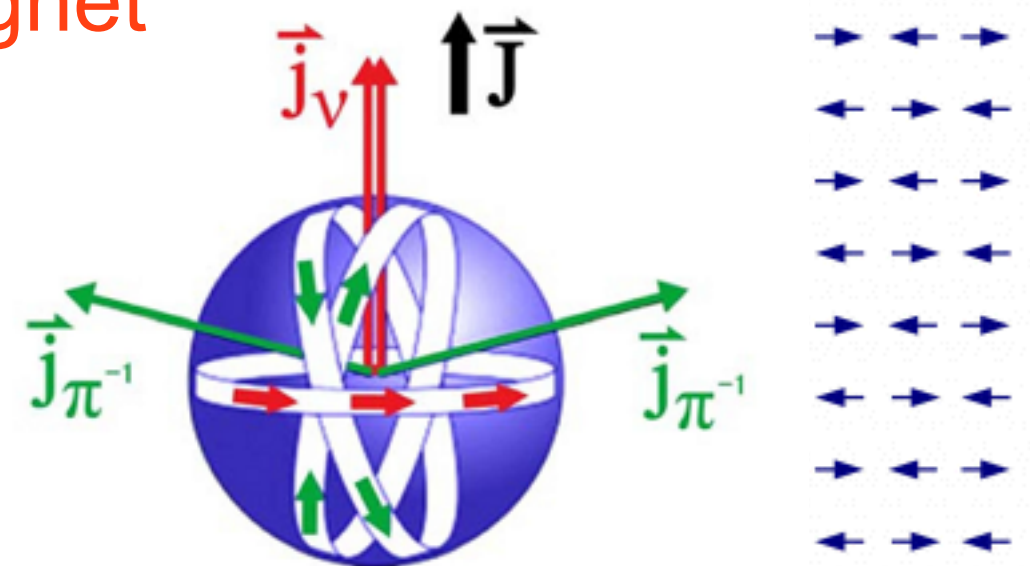
## Magnetic rotation $\longleftrightarrow$ Ferromagnet

- ✓ near spherical nuclei; weak E2 transitions
- ✓ rotational bands with  $\Delta I = 1$
- ✓ strong M1 transitions
- ✓  $B(M1)$  decrease with spin
- ✓ shears mechanism



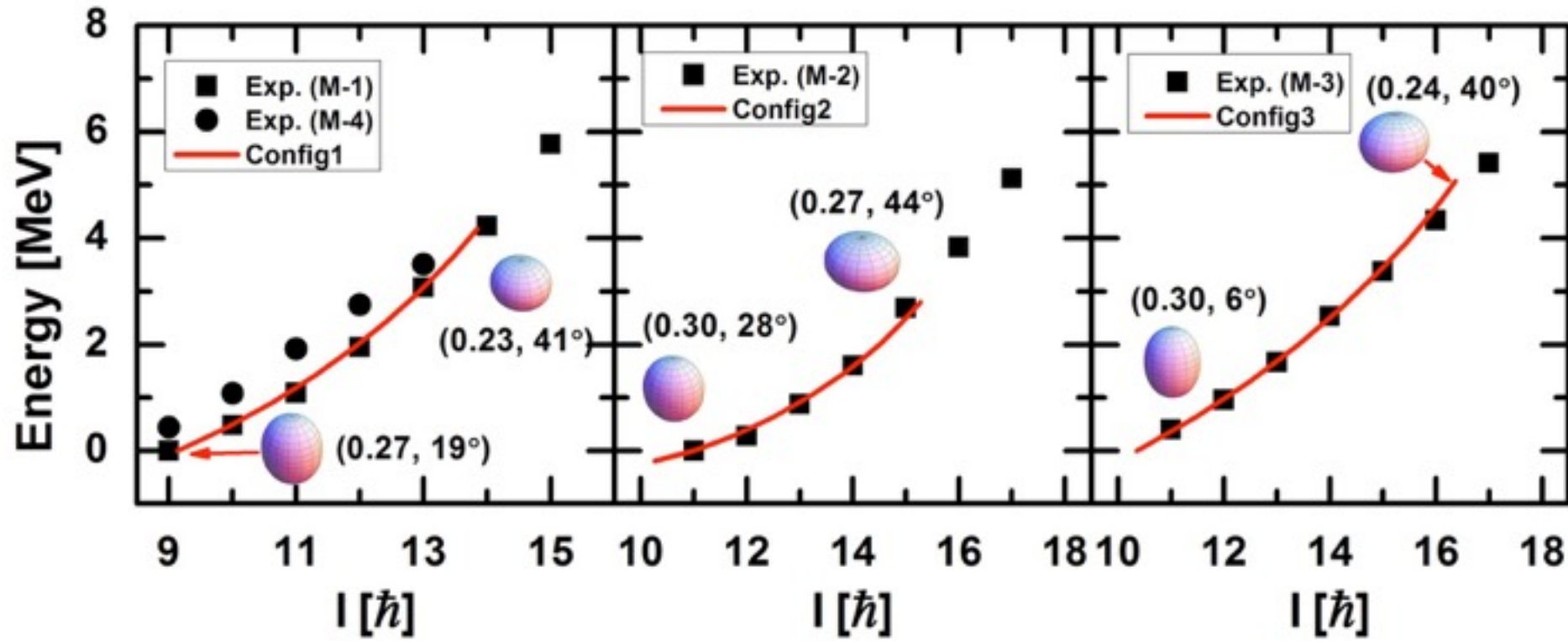
## Antimagnetic rotation $\longleftrightarrow$ Antiferromagnet

- ✓ near spherical nuclei; weak E2 transitions
- ✓ rotational bands with  $\Delta I = 2$
- ✓ no M1 transitions
- ✓  $B(E2)$  decrease with spin
- ✓ two “shears-like” mechanism

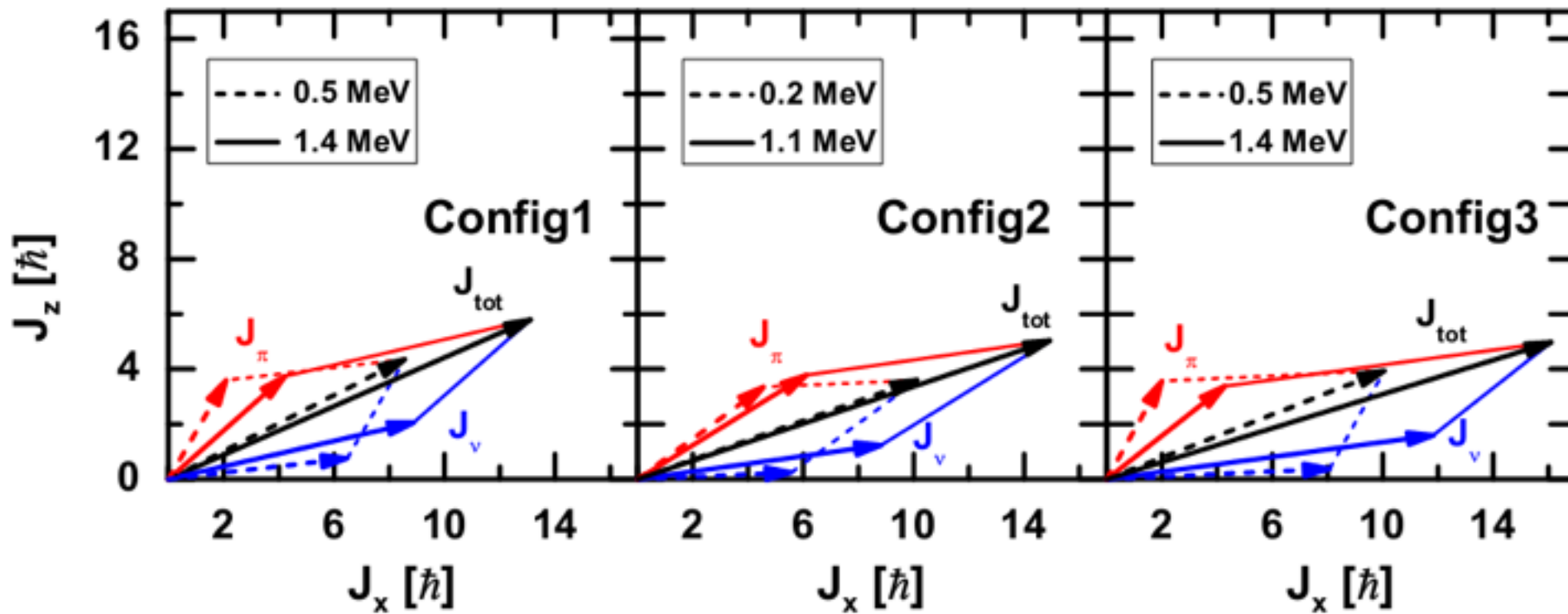




# MR in $^{60}\text{Ni}$

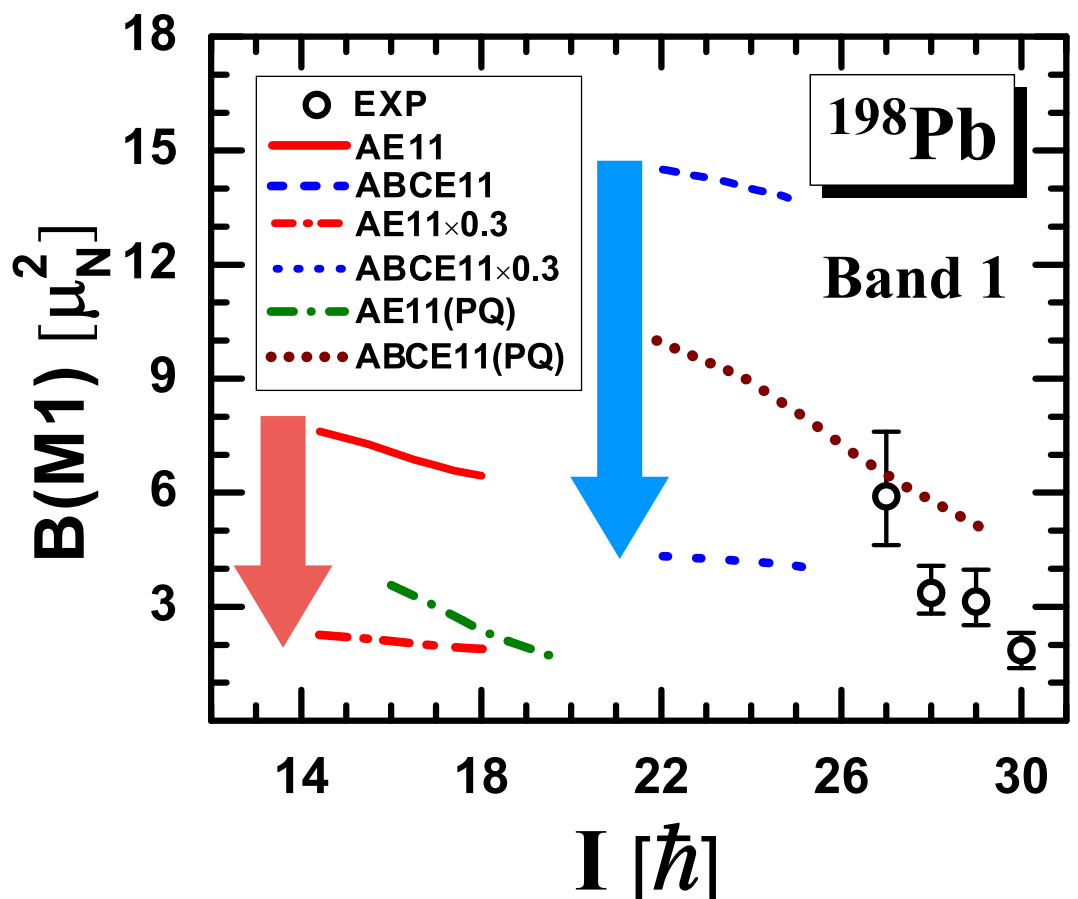
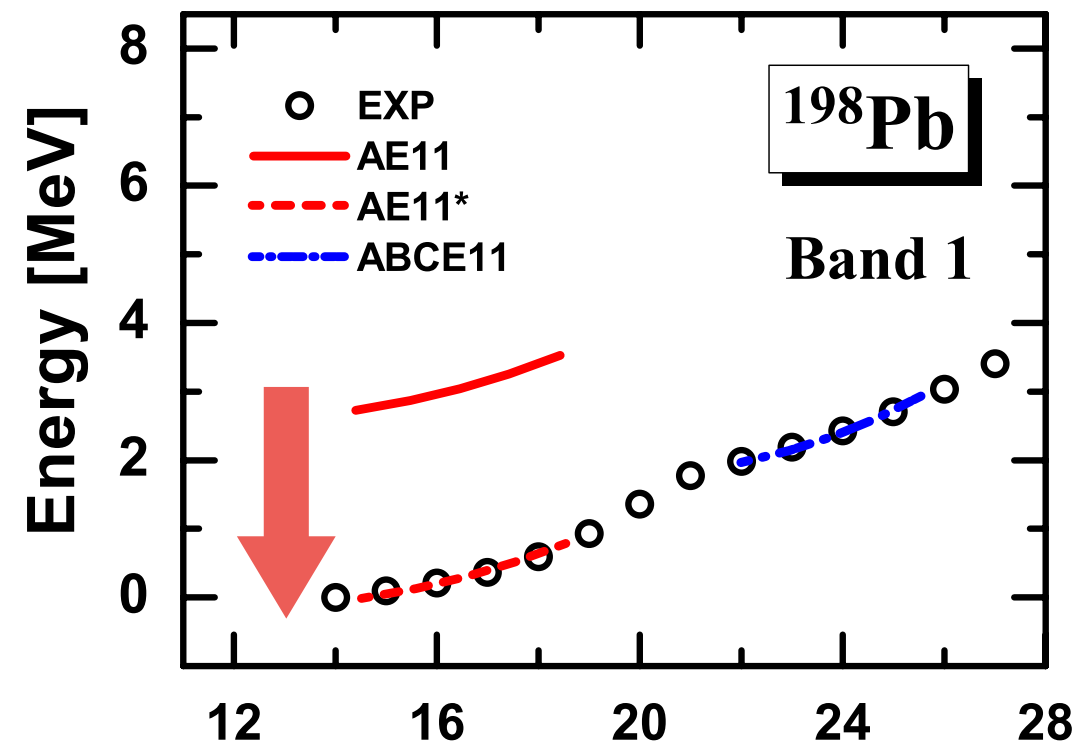


Energy & Deformation



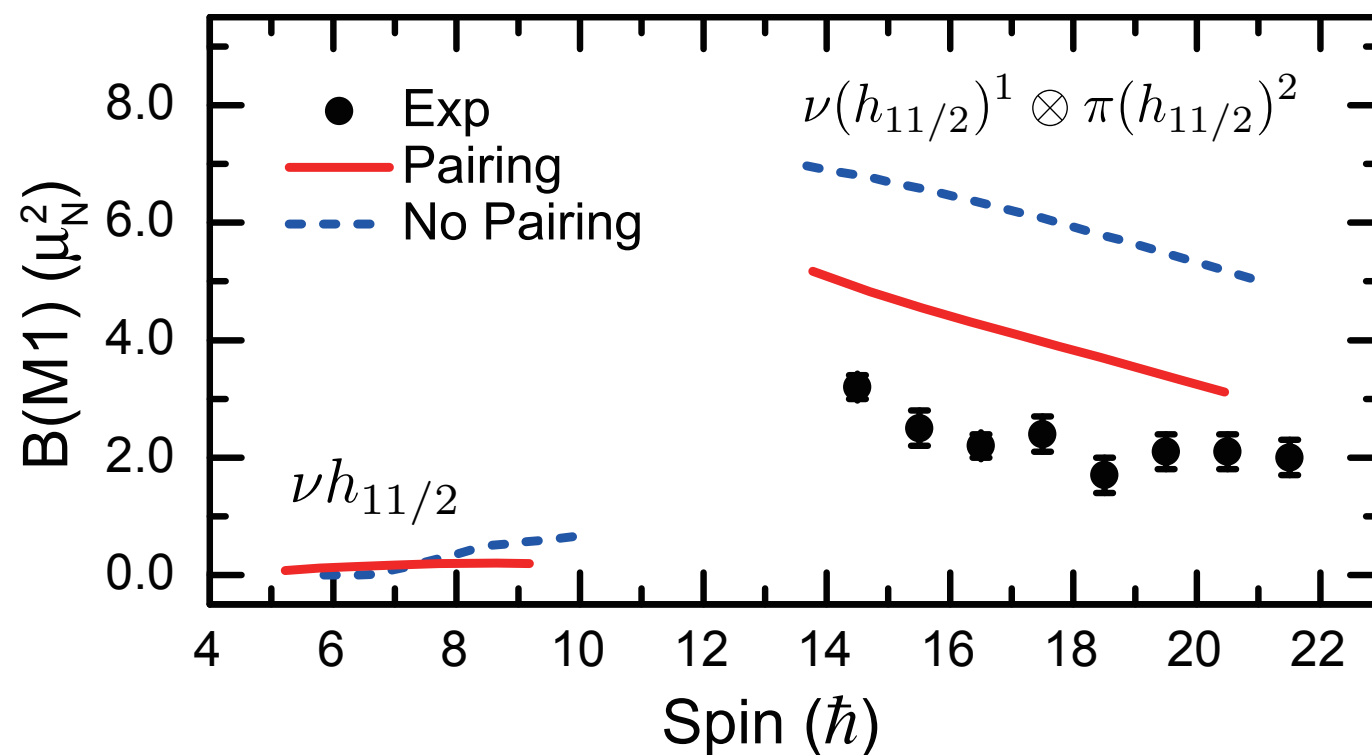
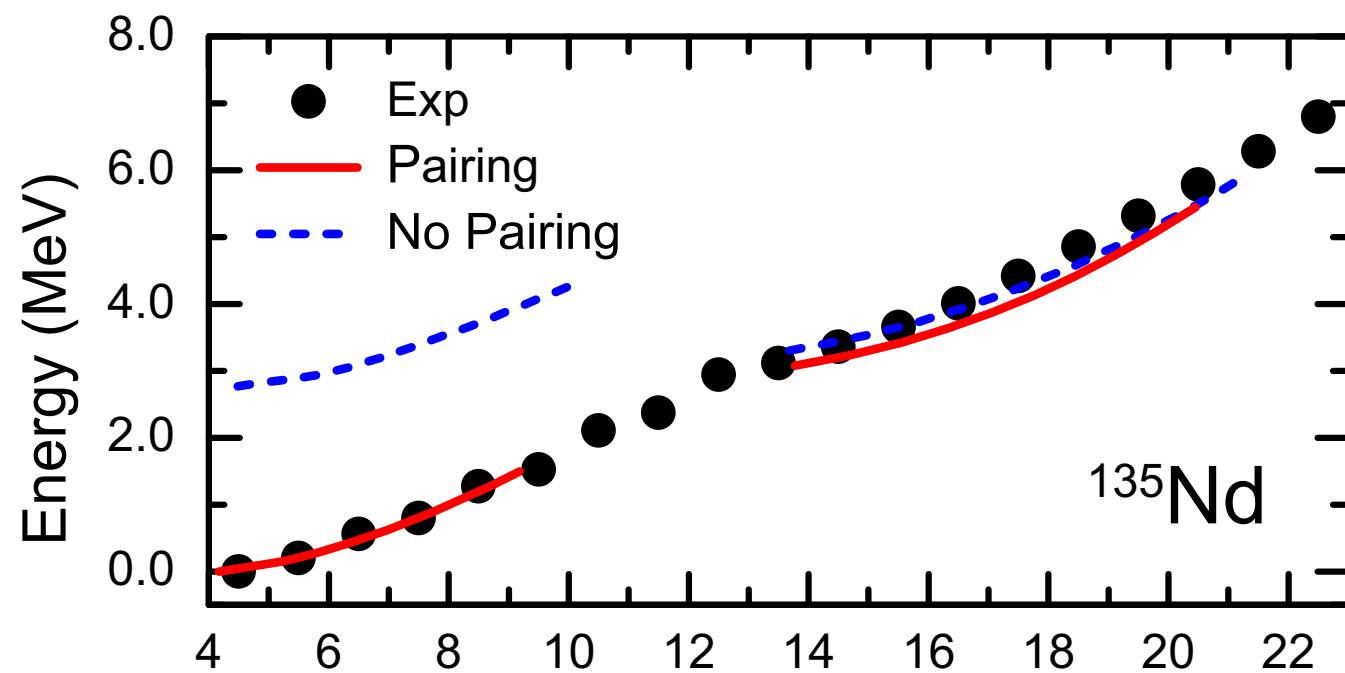
Shears mechanism

# MR in $^{198}\text{Pb}$



# Pairing Effects

# $^{135}\text{Nd}$



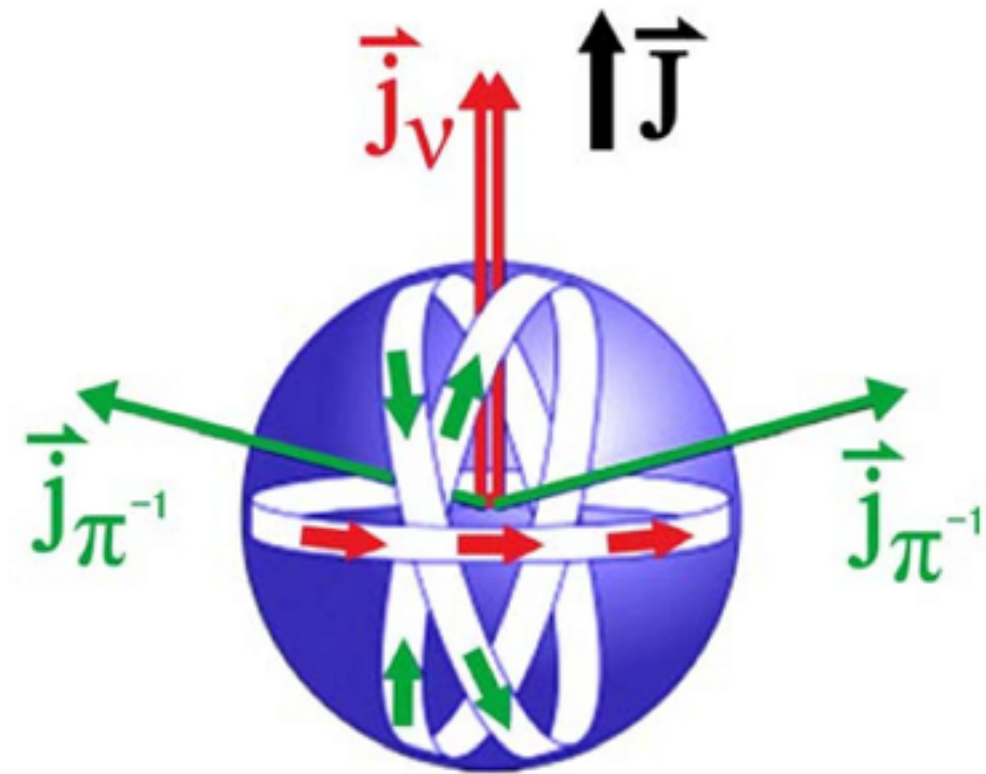
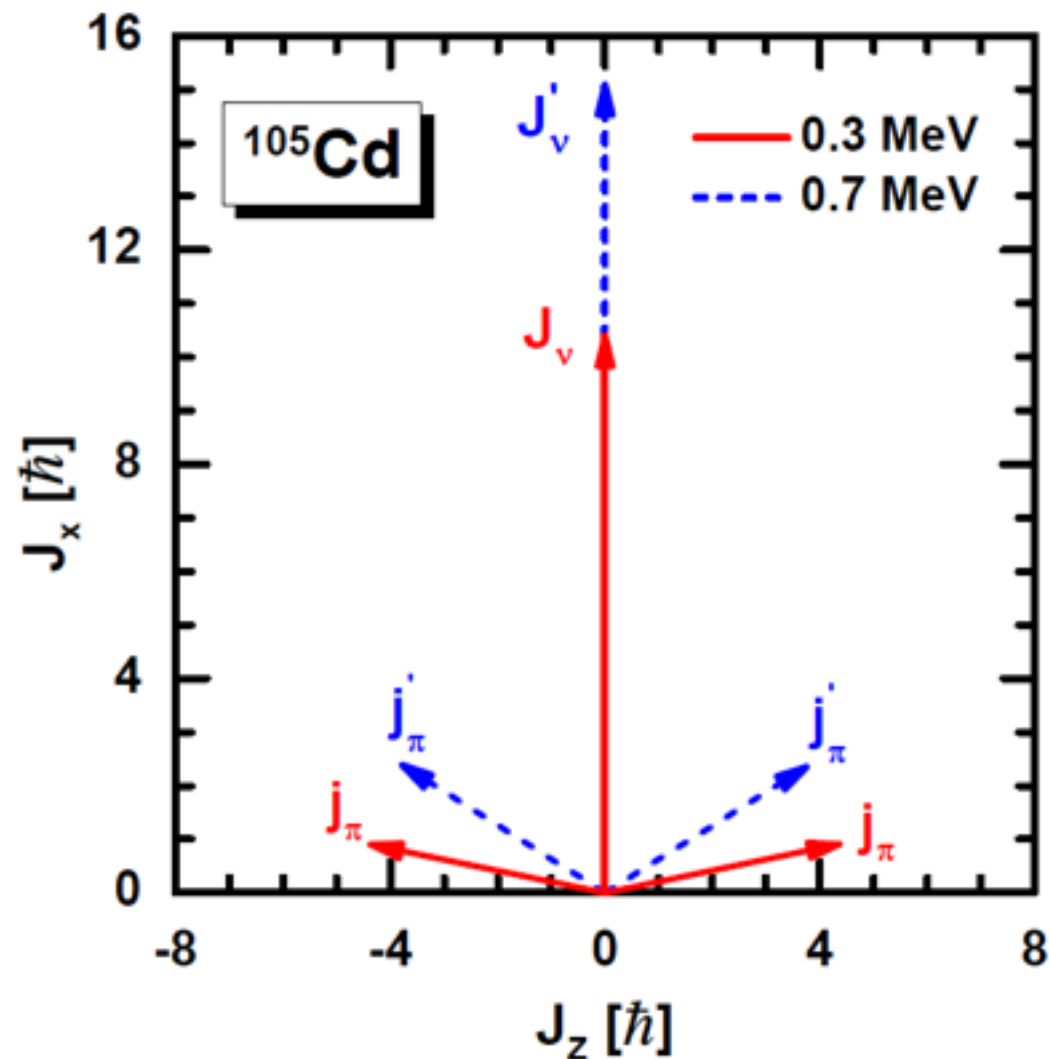
PWZ, Zhang, Meng, Phys. Rev. C 92, 034319 (2015)

More calculations in the future...



# AMR in $^{105}\text{Cd}$

## Two shears-like mechanism



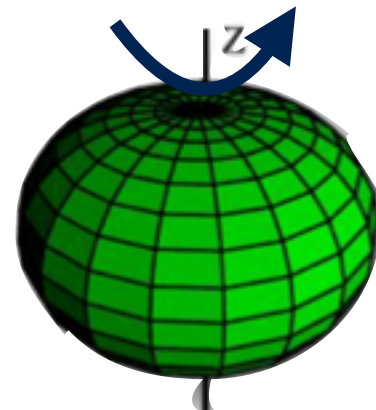
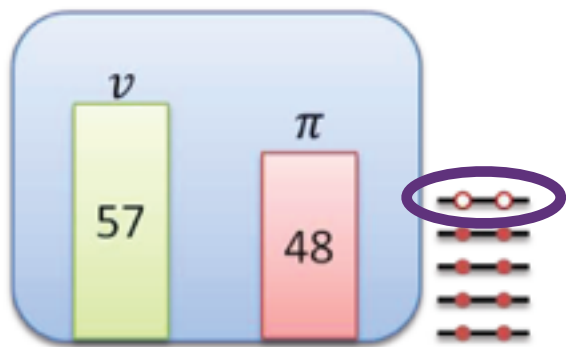
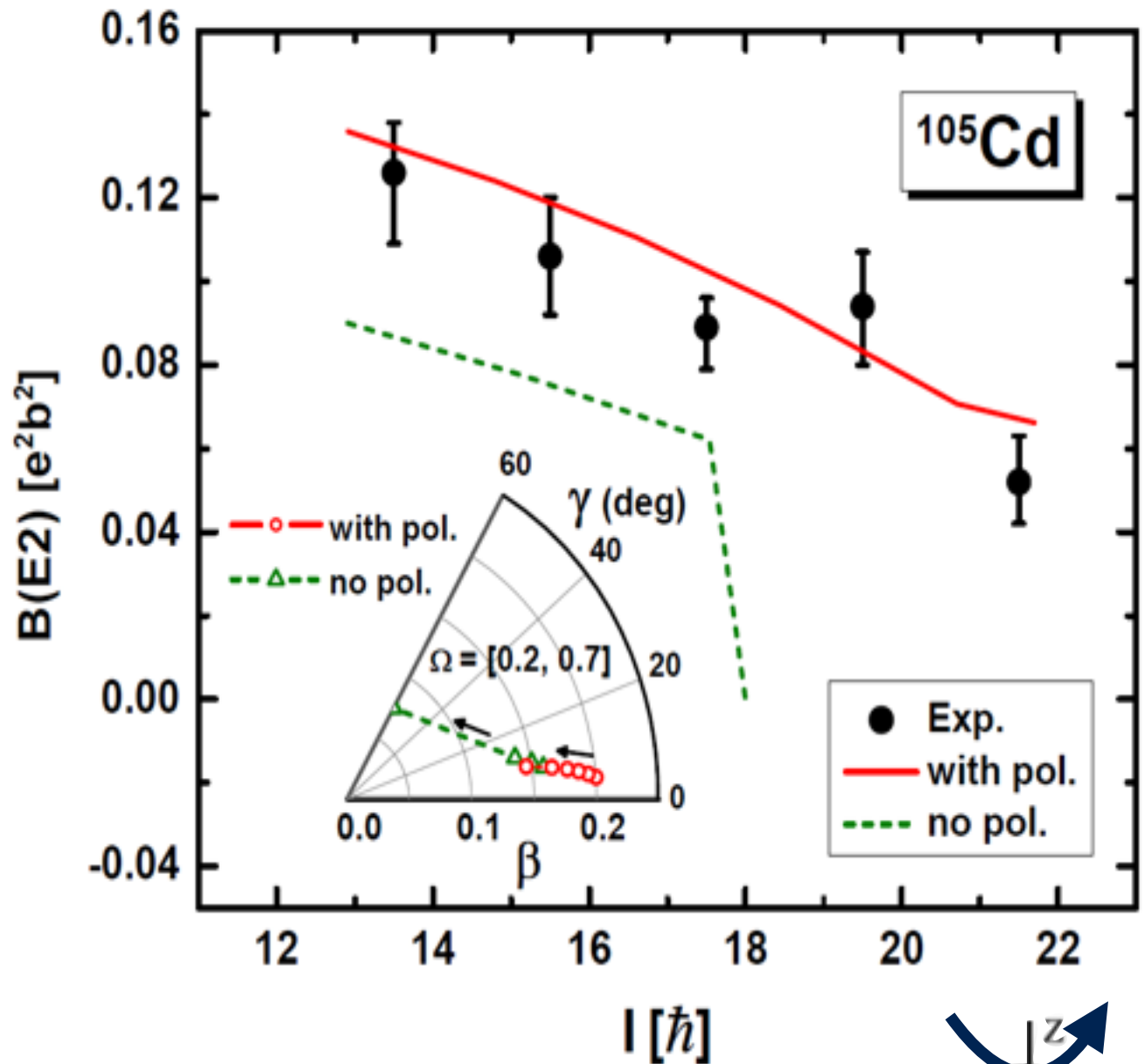
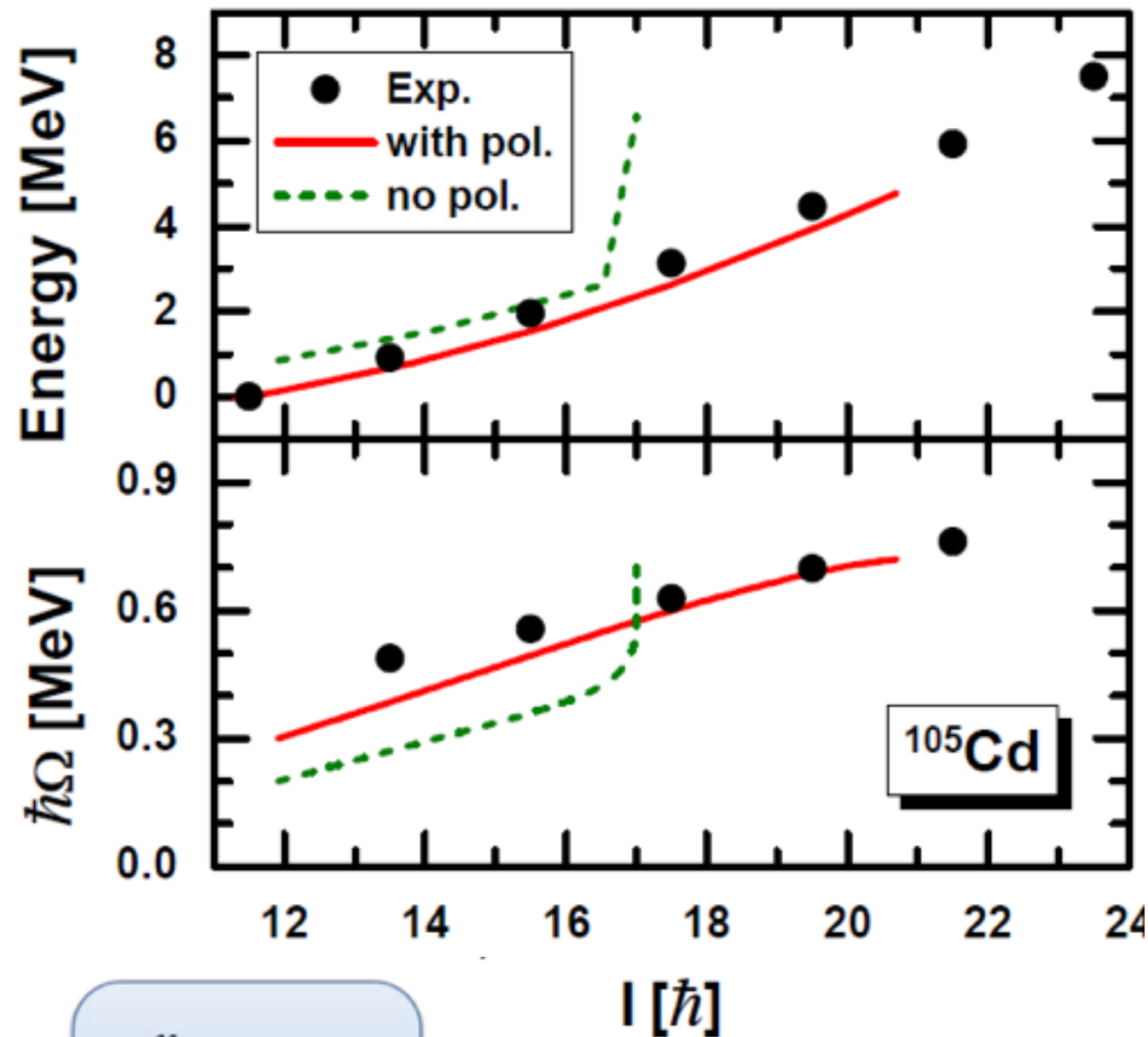
PWZ, Peng, Liang, Ring, Meng PRL 107, 122501(2011)

- ✓ The two proton angular momentum are pointing opposite to each other and are nearly perpendicular to the neutron angular momentum. They form the blades of the two shears.
- ✓ Increasing  $\Omega$ , the two proton blades towards to each other and generates the total angular momentum.

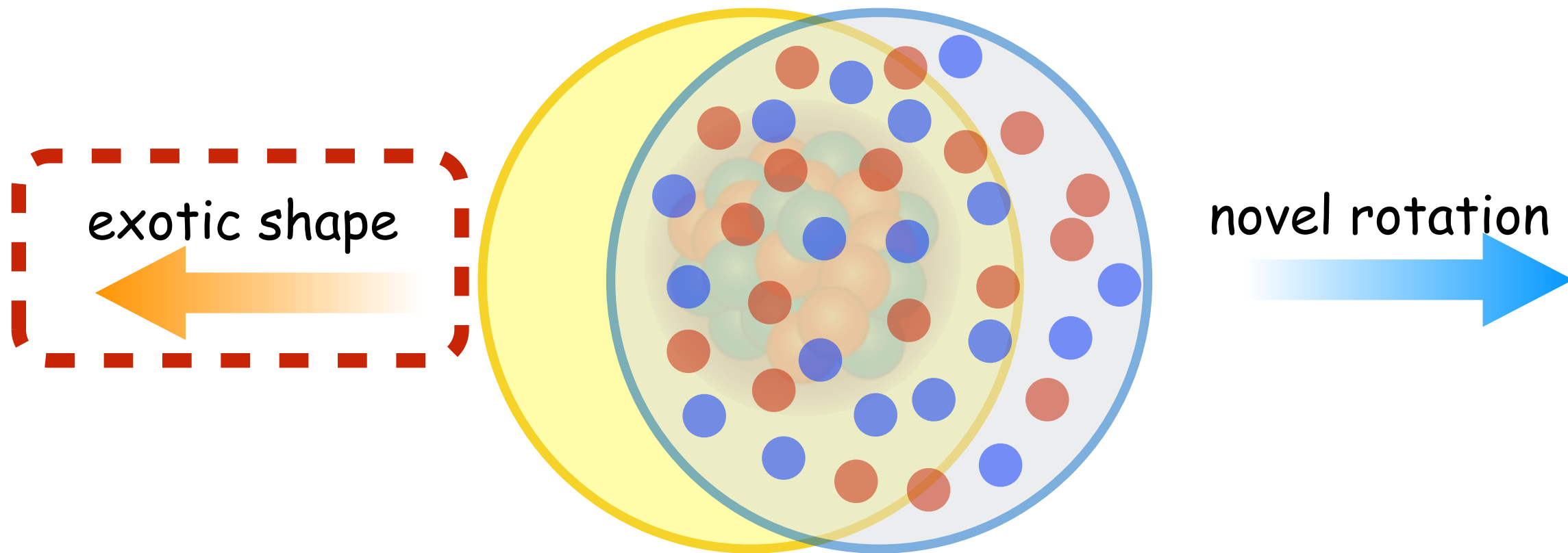


# AMR in $^{105}\text{Cd}$

## Energy and $B(E2)$



# nuclear structure under extreme conditions



Rod shape in C isotopes

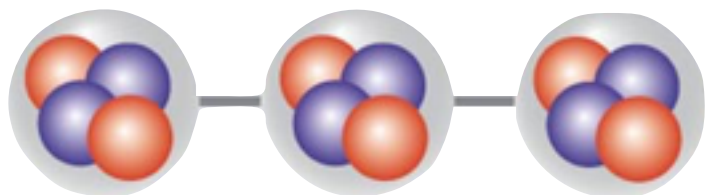
# Exotic deformation

Strongly deformed states [towards a hyper-deformation](#) might exist in light  $N = Z$  nuclei due to a cluster structure.

✓ [Linear-chain structure](#) of three- $\alpha$  clusters was suggested about 60 years ago to explain the [Hoyle state](#). **Morinaga PR 1956**

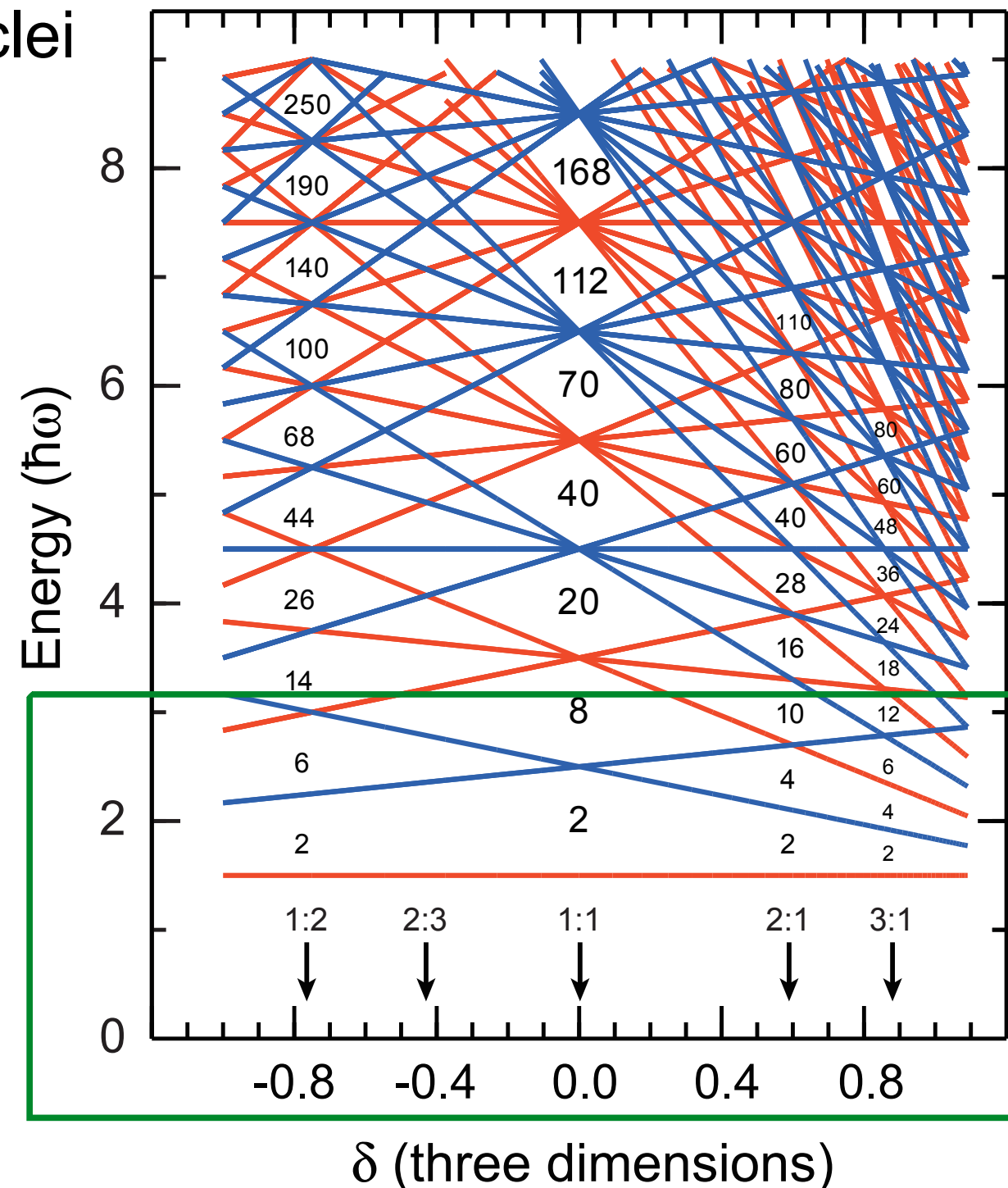
✓ However, Hoyle state was later found to be a [mixing](#) of the linear-chain configuration and other configurations, and recently reinterpreted as an  [\$\alpha\$ -condensate-like](#) state.

**Fujiwara PTP1980; Tohsaki PRL 2001; Suhara PRL 2014**



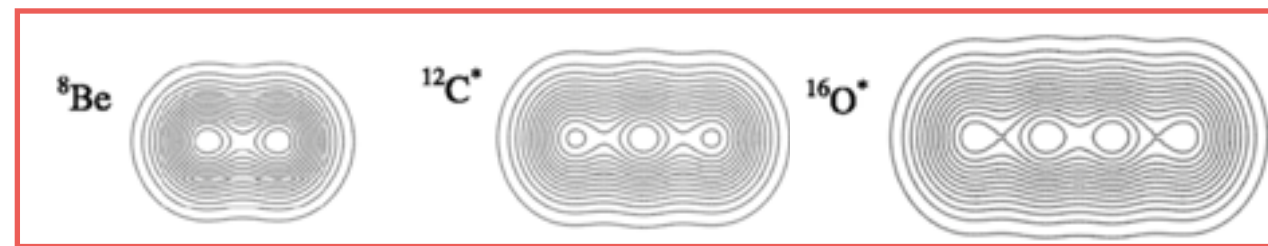
Cluster structure  
in light nuclei

## Harmonic oscillator



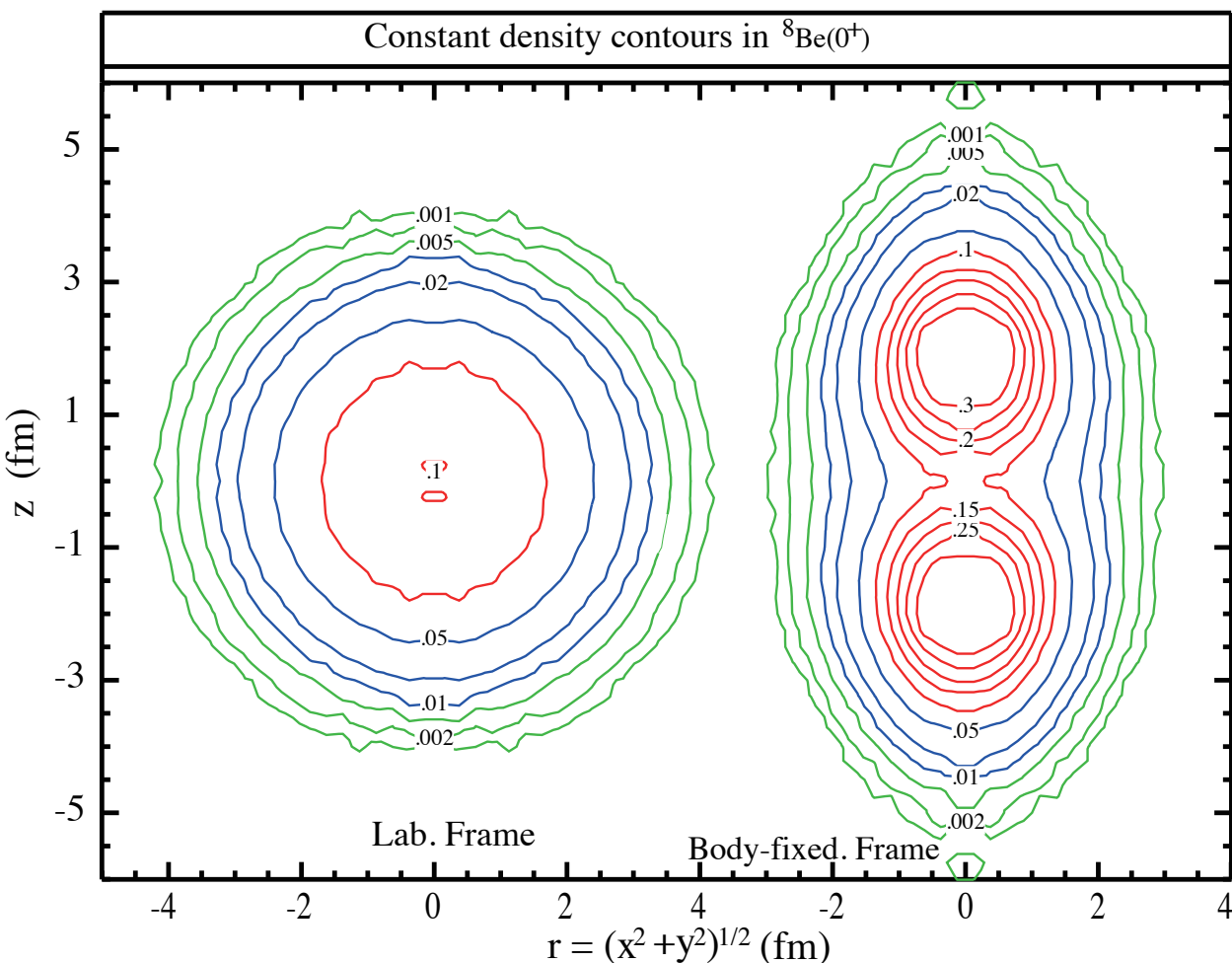
# Alpha cluster chain and rod shape

Harmonic oscillator density



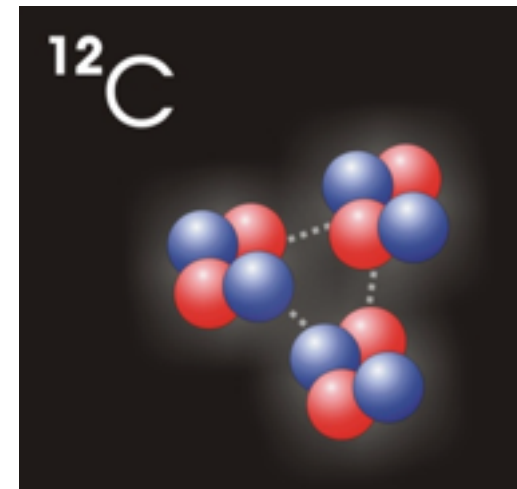
Freer RPP 2007

Be-8 Ground state

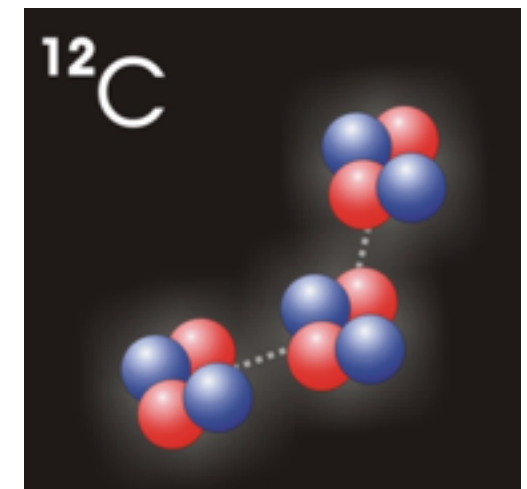


Green Function Monte Carlo Wiringa PRC 2000

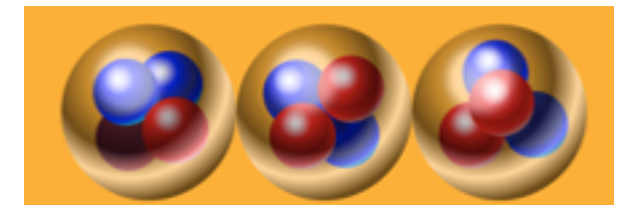
Ground



Hoyle



No firm evidence



Because of

- ✓ antisymmetrization effects
- ✓ weak-coupling nature

It is difficult to stabilize the rod-shaped configuration in nuclear systems.

# How can we stabilize linear chain configurations?

## Two important mechanisms

- ✓ Adding valence neutrons

Itagaki, PRC2001; Maruhn, NPA2010

- ✓ Rotating the system

Ichikawa, PRL2011





# How can we stabilize linear chain configurations?

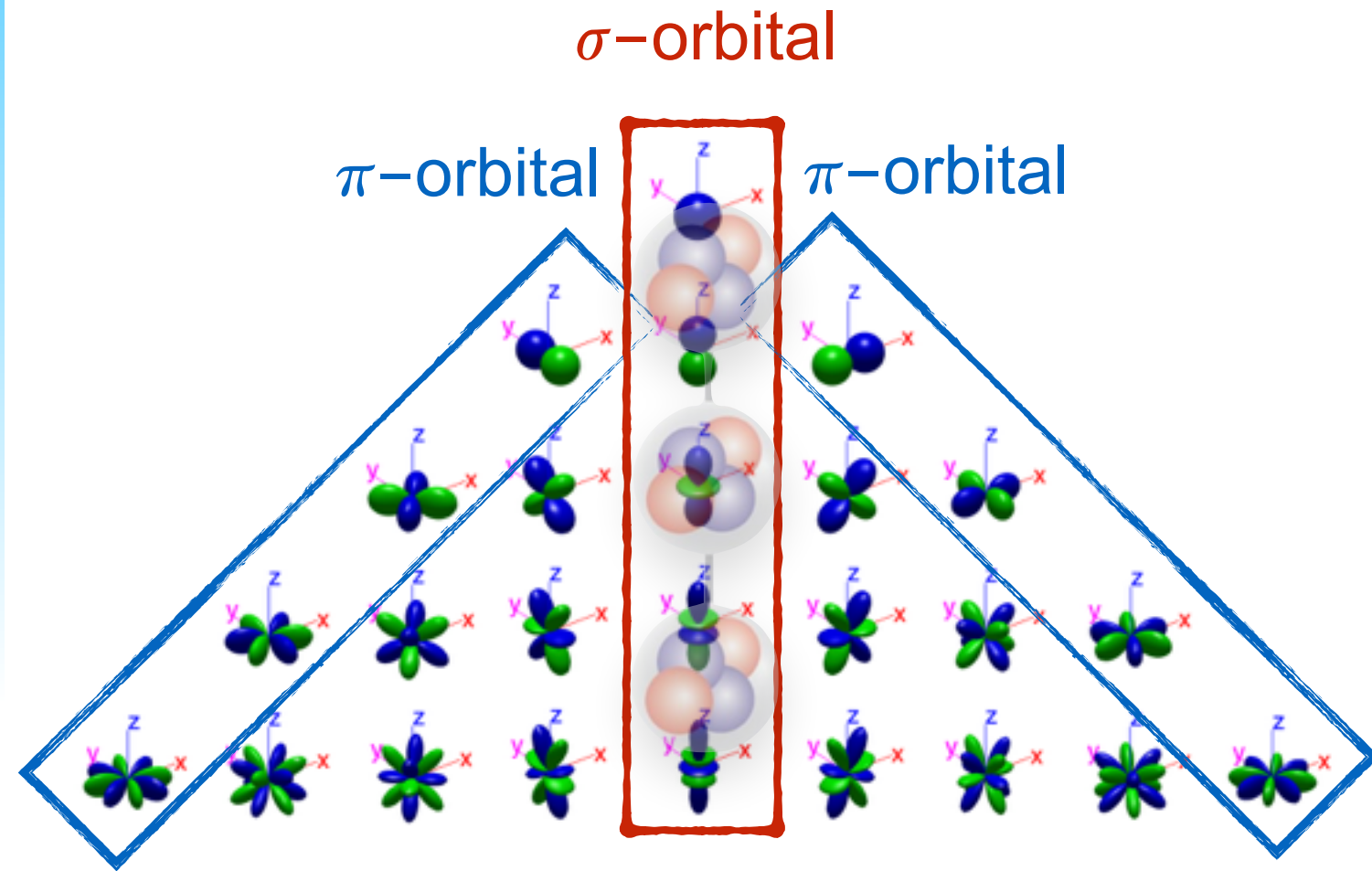
## Two important mechanisms

- ✓ Adding valence neutrons

Itagaki, PRC2001; Maruhn, NPA2010

- ✓ Rotating the system

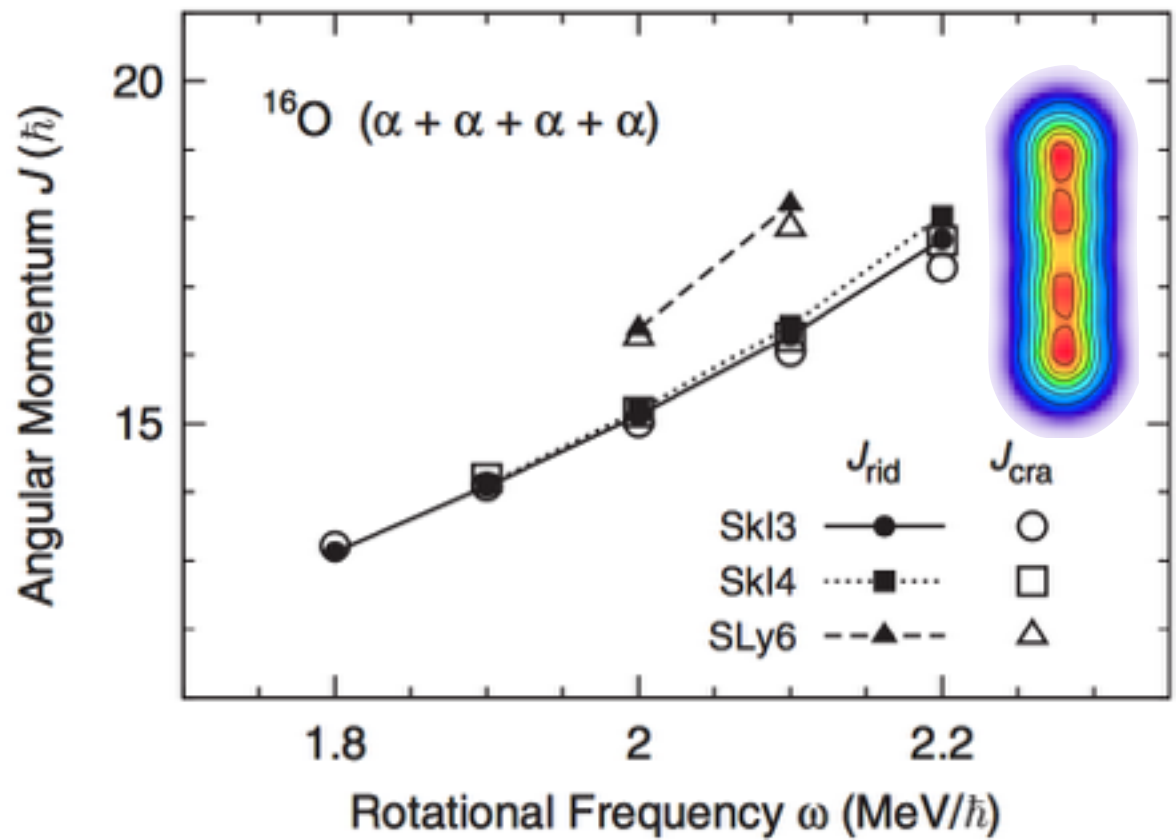
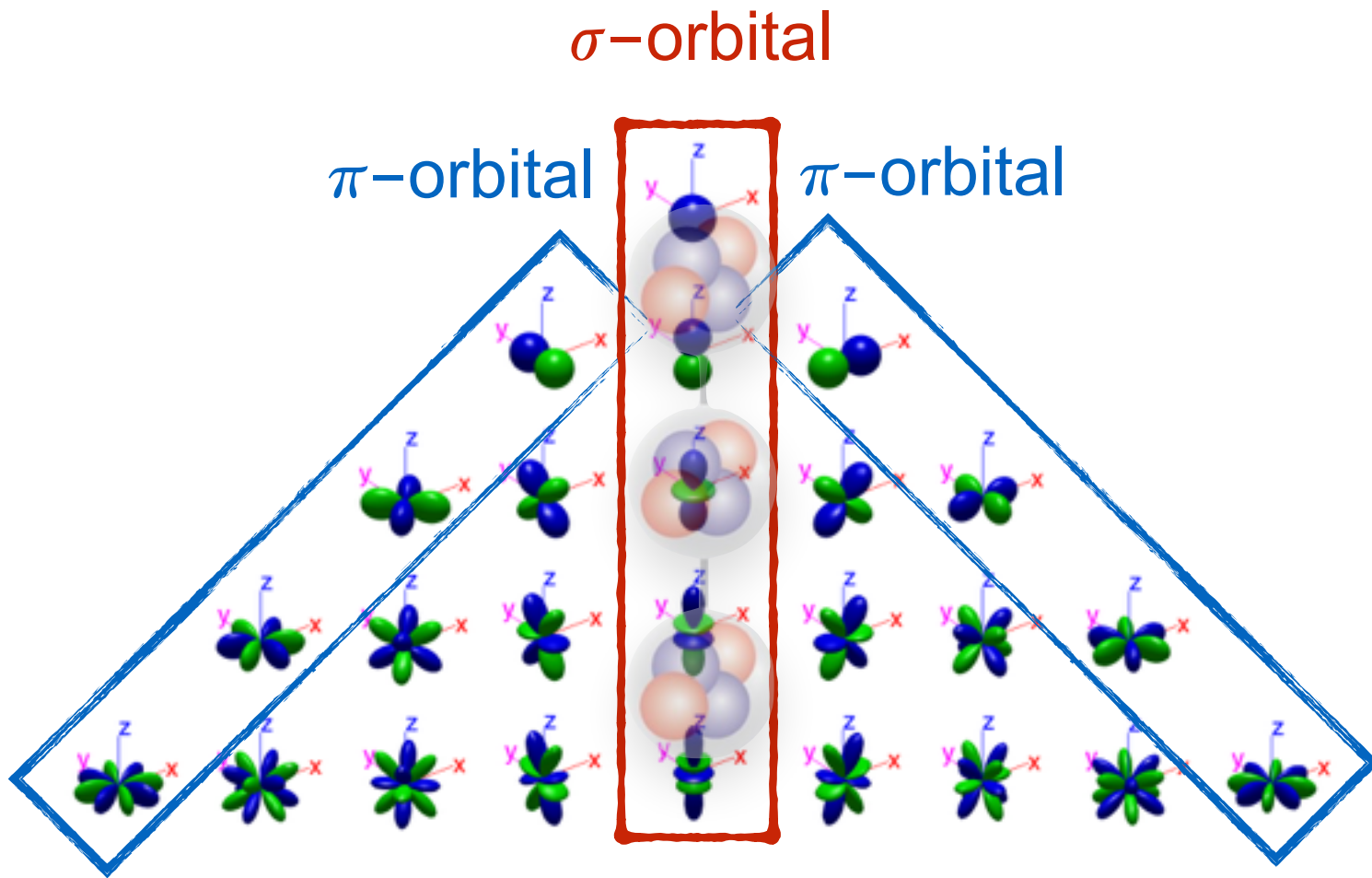
Ichikawa, PRL2011



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Ichikawa, PRL2011

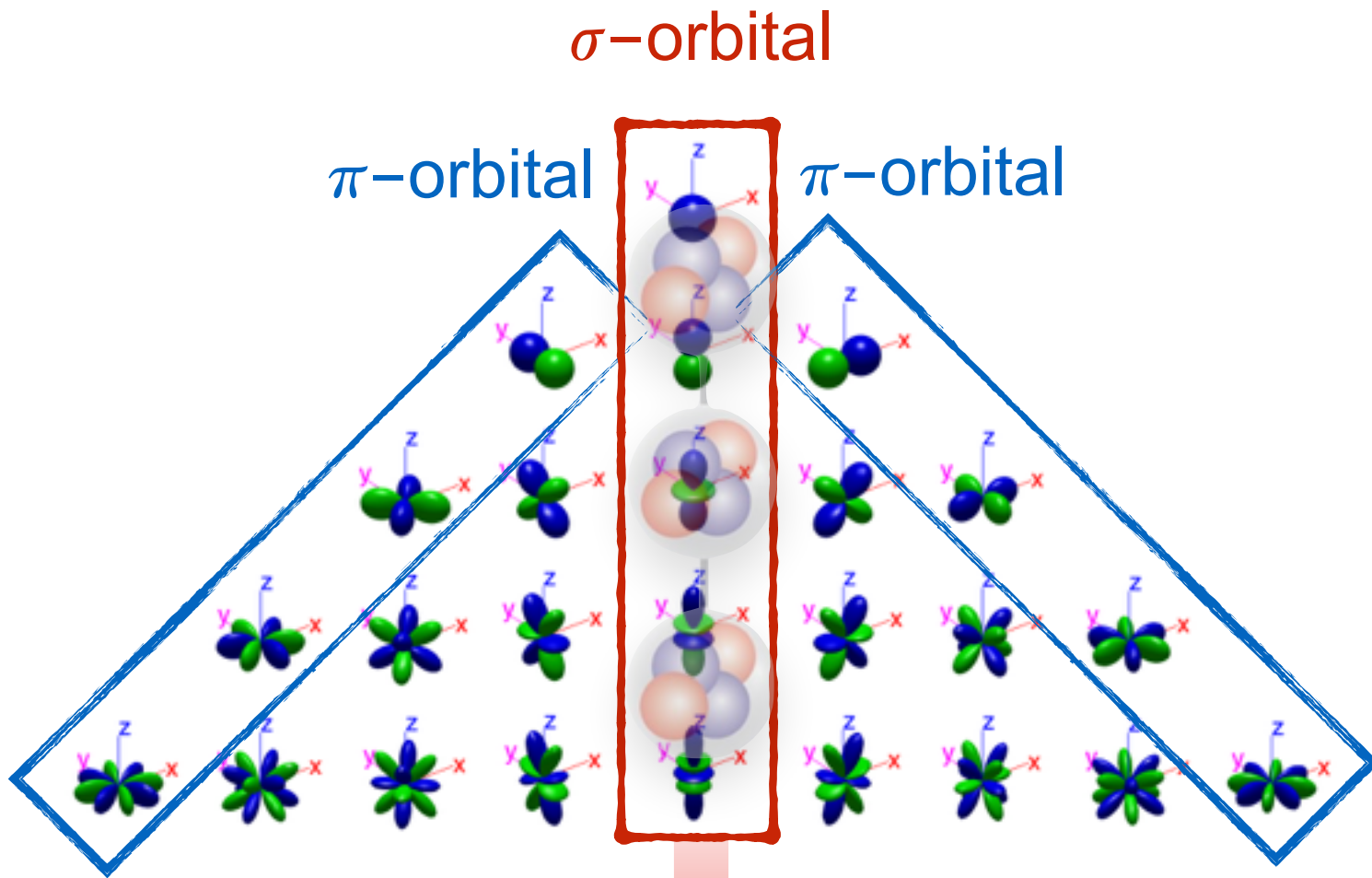
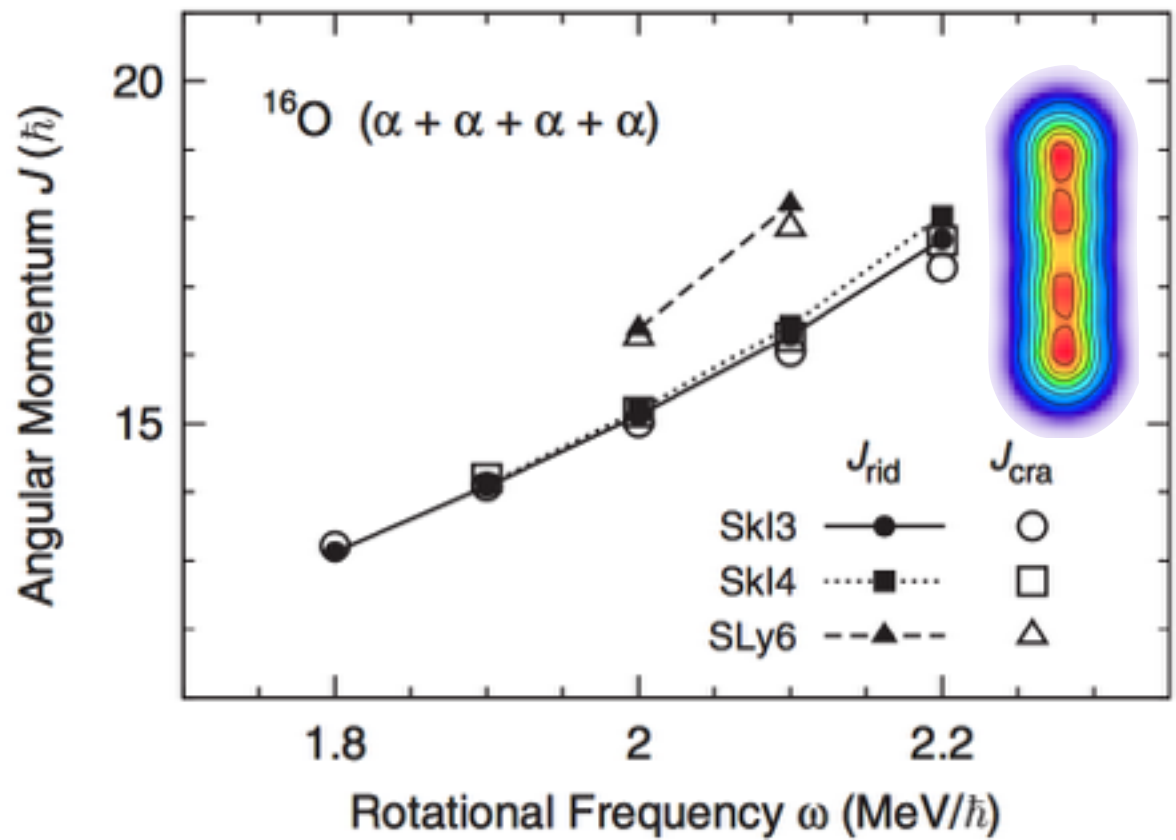




# How can we stabilize linear chain configurations?

## Two important mechanisms

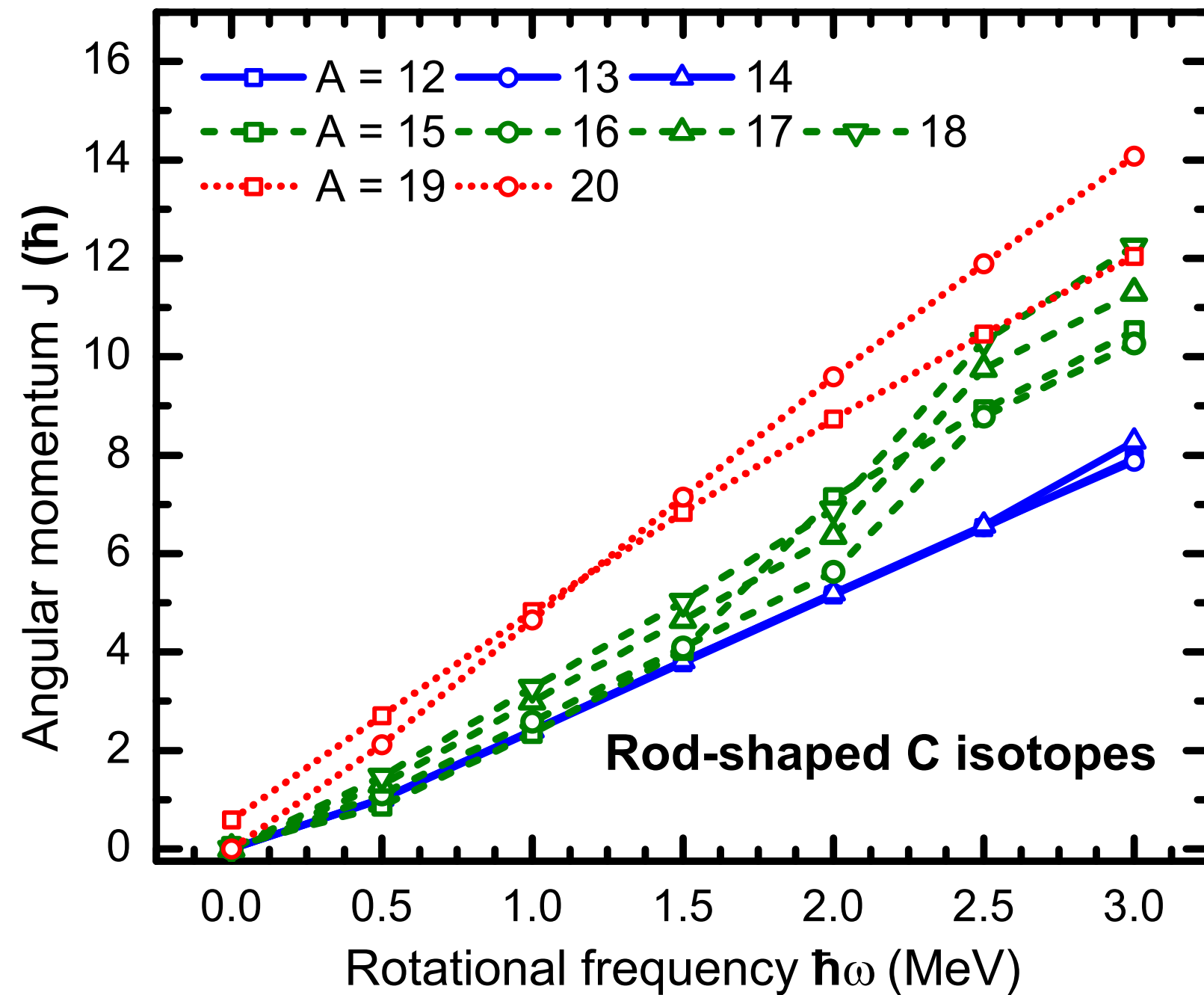
- ✓ Adding valence neutrons  
Itagaki, PRC2001; Maruhn, NPA2010
- ✓ Rotating the system  
Ichikawa, PRL2011



Coherent effects exist?  
They facilitate the stabilization?

# Angular momentum

DD-ME2, 3D HO basis with  $N = 12$  major shells



➤ C-12, C-13, C-14

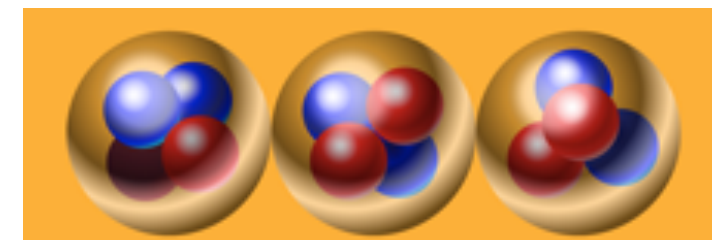
constant moments of inertia (MOI); like a rotor

➤ C-15, C-16, C-17, C-18

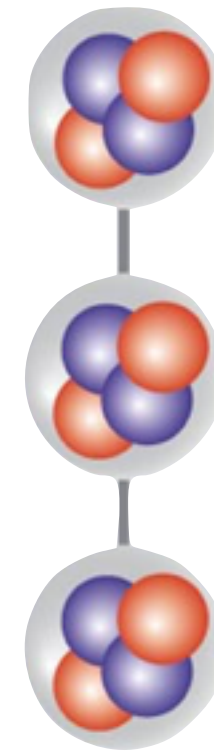
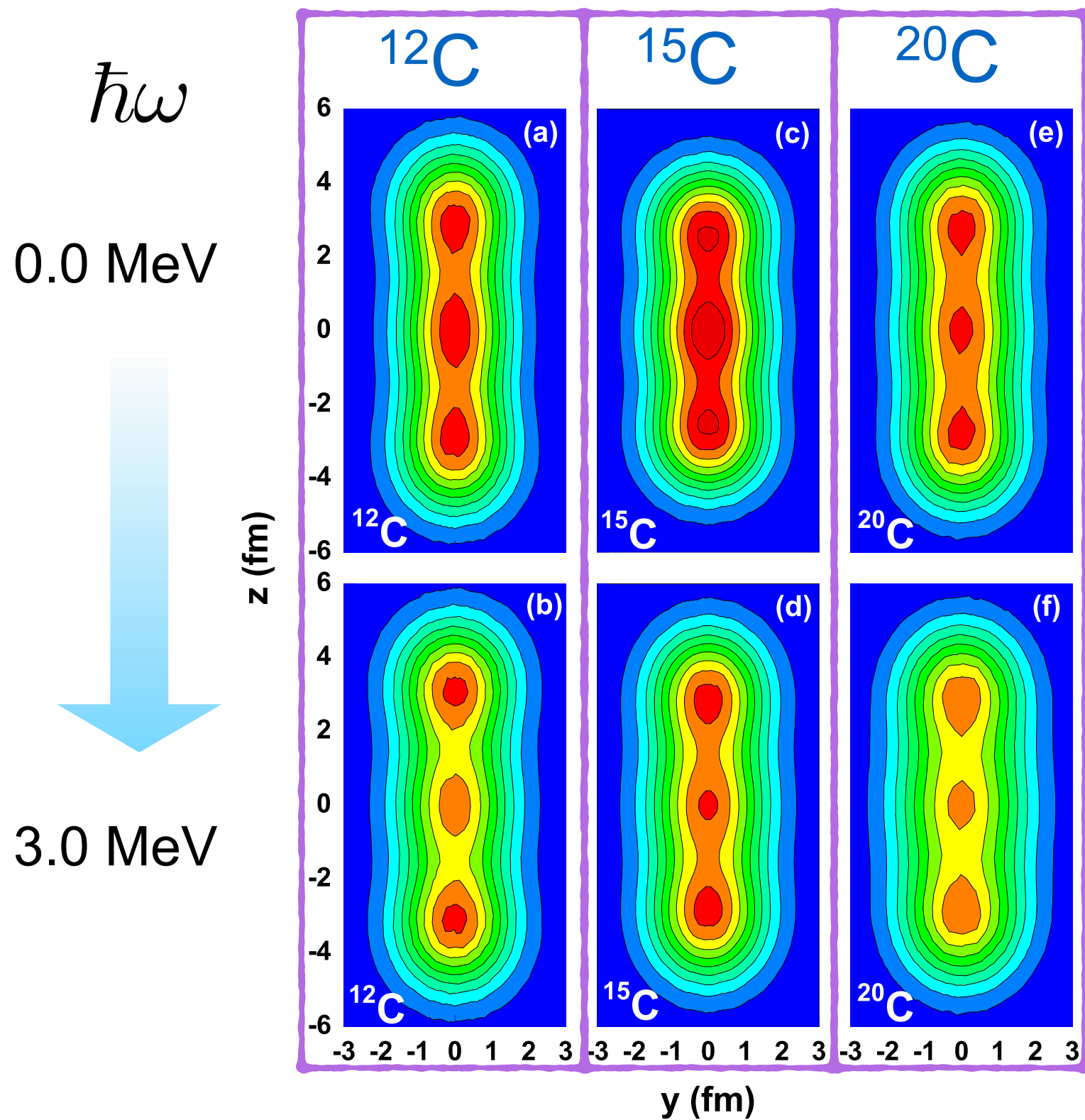
abrupt increase of MOI;  
some changes in structure

➤ C-19; C-20

constant moments of inertia;  
much larger



# Proton density distribution

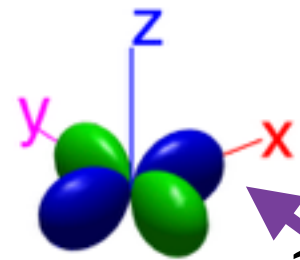


Very large deformation  
Very clear clustering

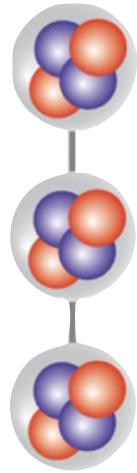
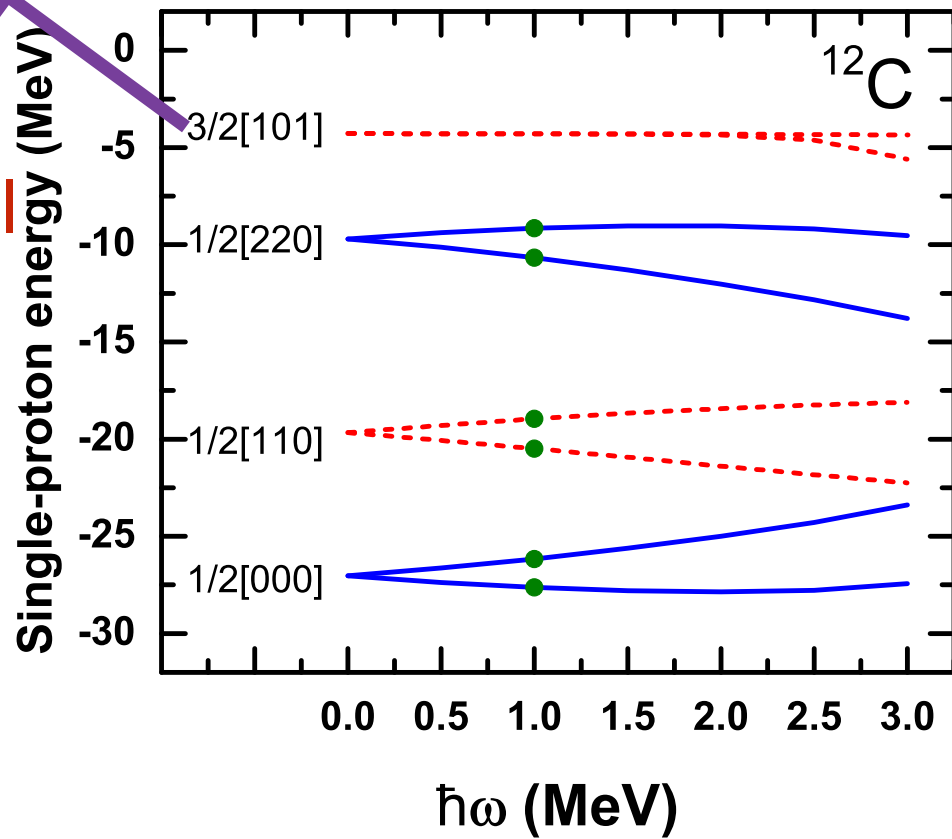
Rod shape are obtained in all isotopes by tracing the corresponding rod-shaped configuration.

# Single-proton energy

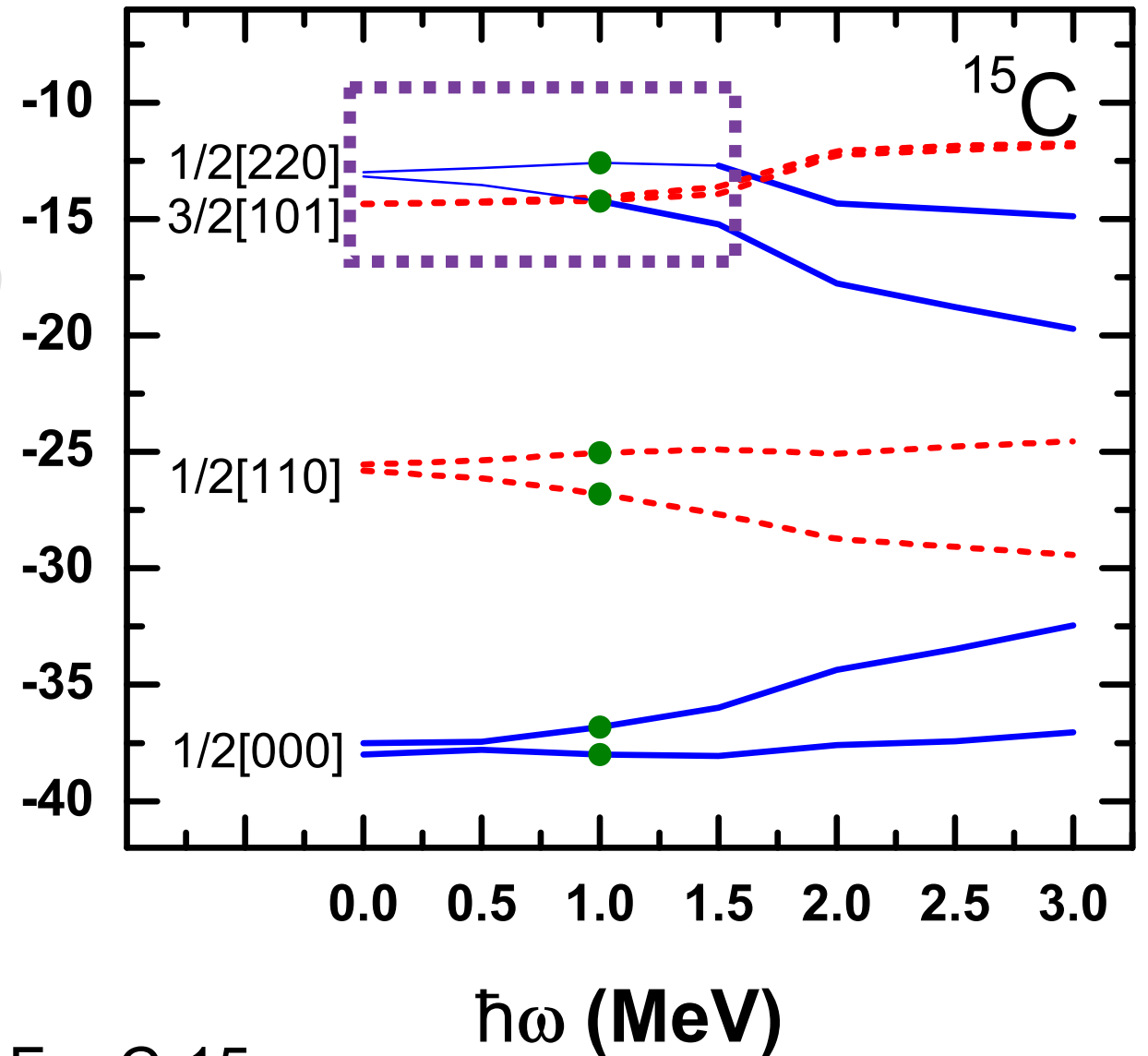
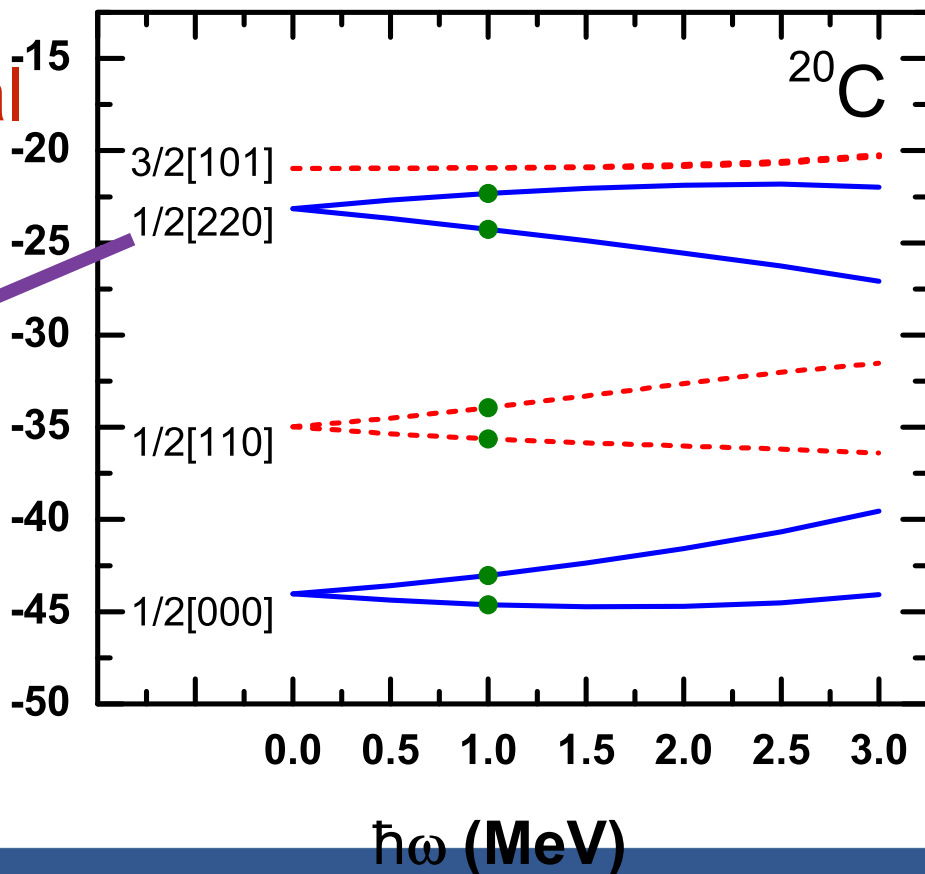
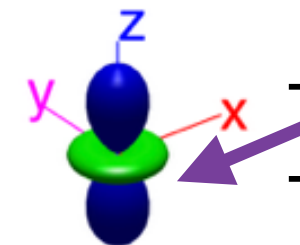
Rotating effects



$\pi$ -orbital



$\sigma$ -orbital



For C-15:

Low spin: deexcitations easily happen

High spin: More stable against deexcitations

# Valence neutron density distribution

$A_C - {}^{12}C$

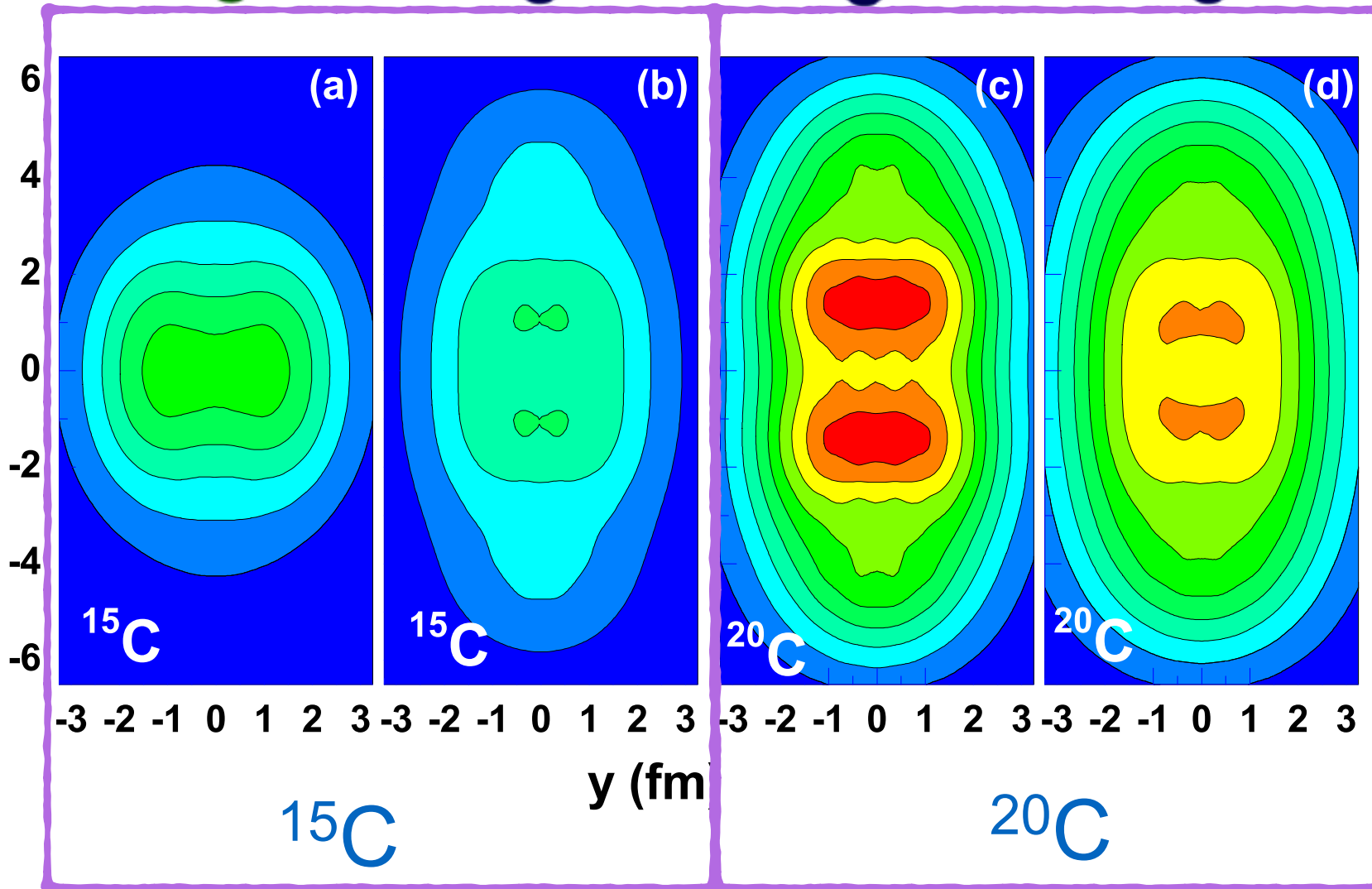
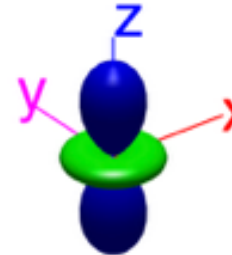
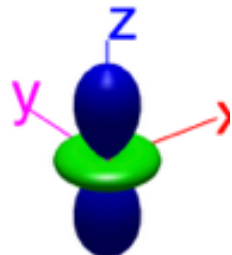
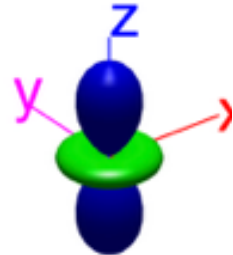
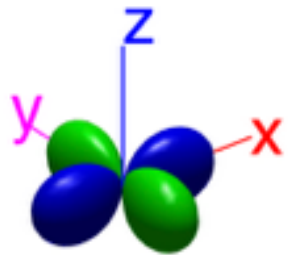
Isospin effects

$\pi$ -orbital

$\sigma$ -orbital

$\sigma$ -orbital

$\sigma$ -orbital



**C-15:** valence neutrons

Low spin:  $\pi$ -orbital; proton unstable

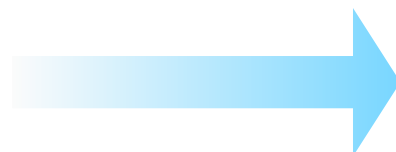
High spin:  $\sigma$ -orbital; proton stable

**C-20:** valence neutrons

Low spin:  $\sigma$ -orbital; proton stable

High spin:  $\sigma$ -orbital; proton stable

$\hbar\omega$  0.0 MeV

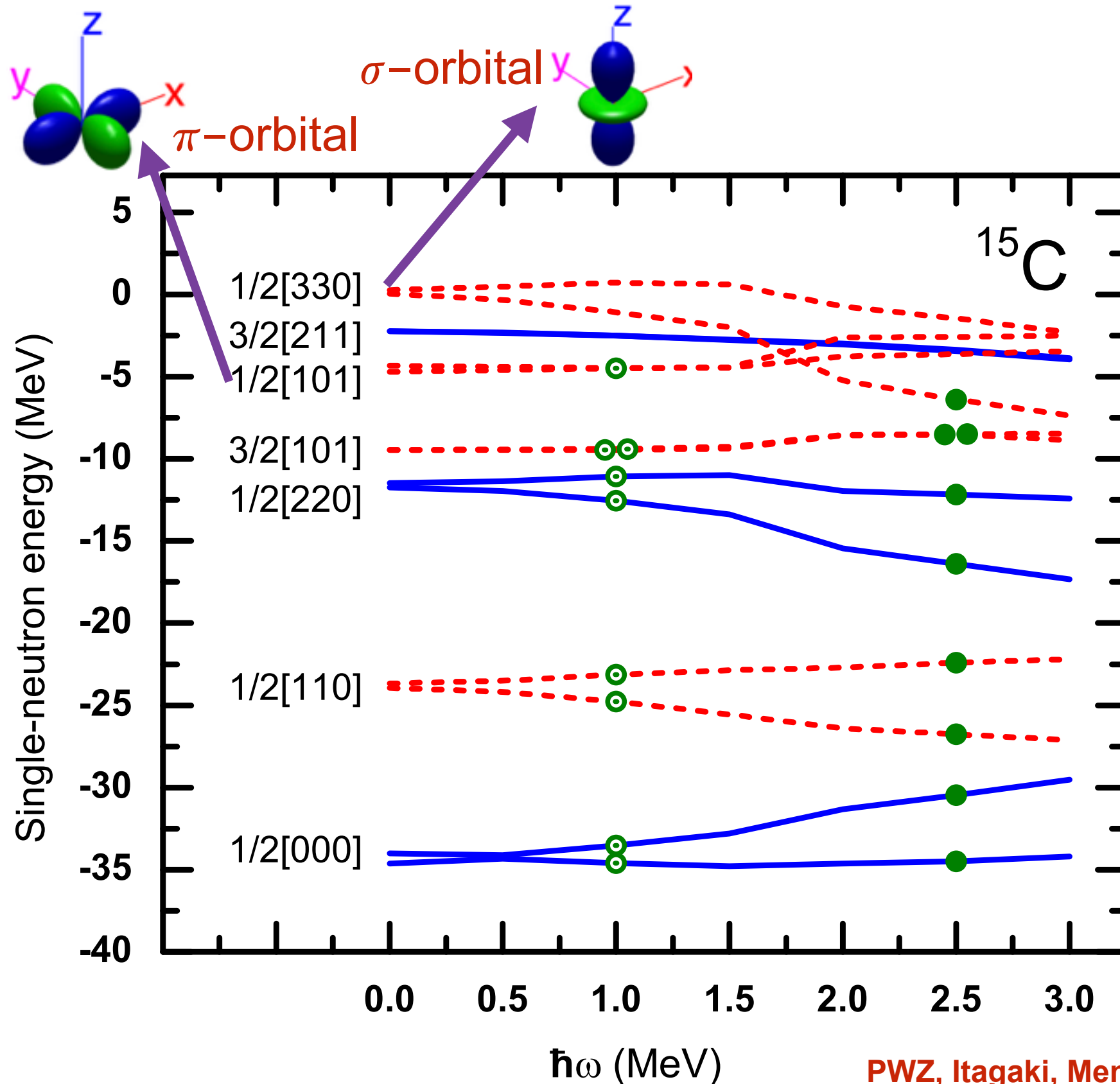


3.0 MeV

PWZ, Itagaki, Meng, Phys. Rev. Lett. 115, 022501 (2015)



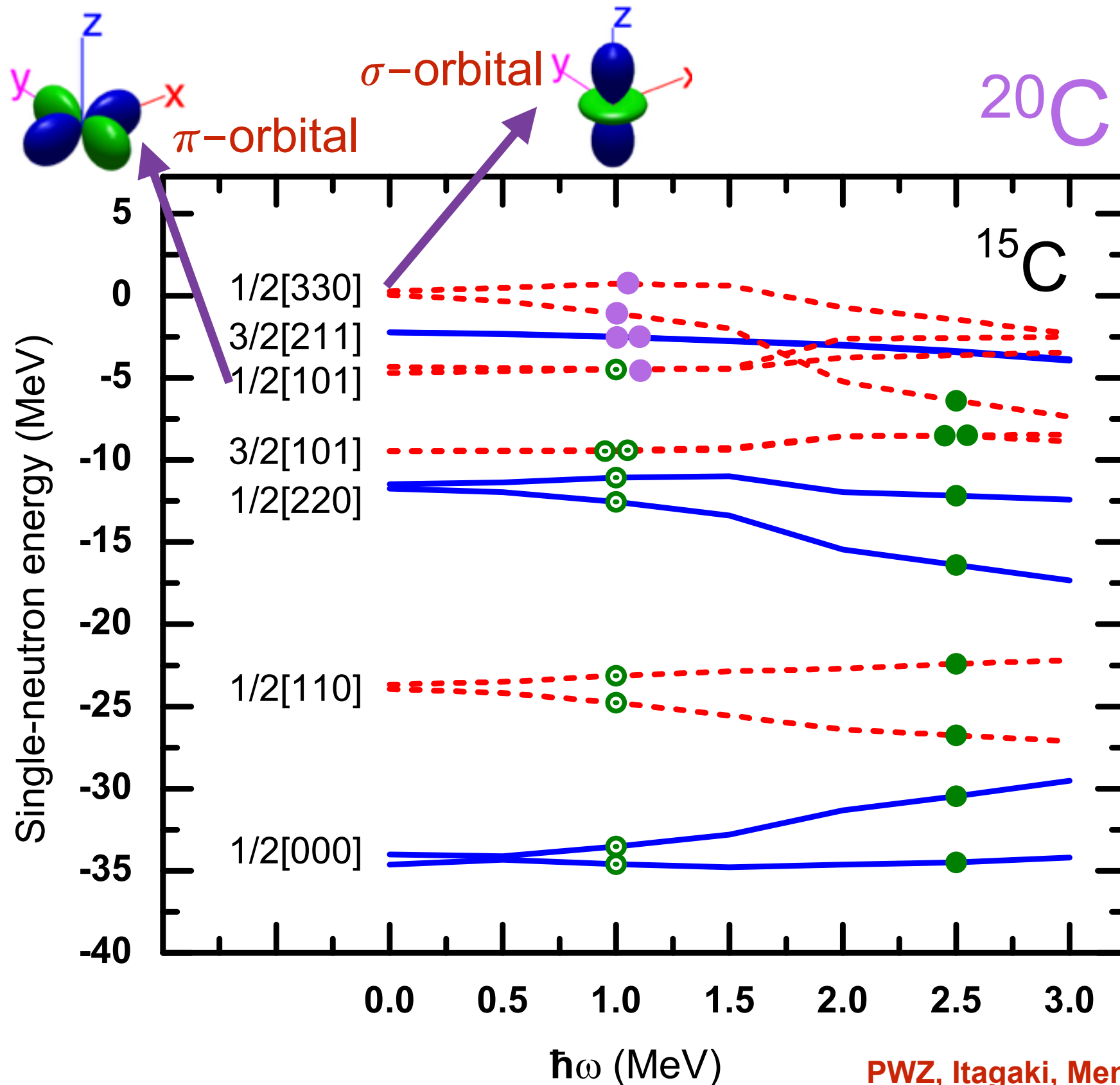
# Single-neutron energy



Spin and Isospin  
Coherent Effects

Rotation makes the  $\sigma$  valence neutron orbital lower and easier to be occupied, and thus pull down the  $\sigma$  proton orbitals.

# Single-neutron energy



Spin and Isospin Coherent Effects

Rotation makes the sigma valence neutron orbital lower and easier to be occupied, and thus pull down the sigma proton orbitals.



# Summary

Covariant density functional theory has been extended to describe rotational excitations.

- Both **MR and AMR** and their mechanism could be described well.
- Pairing correlation could **improve** descriptions (**more examples**)
- Two mechanisms to stabilize the rod shape, **rotation** (high spin) and **adding neutrons** (high Isospin), **coherently** work in C isotopes
- **Coherent Effects:**
  - Rotation** makes the **sigma valence neutron orbital** lower, and thus
    1. pull down the sigma proton orbitals
    2. enhances the prolate deformation of protons

## In collaboration with

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# Thank you for your attention!