Why there is no Newtonian backreaction

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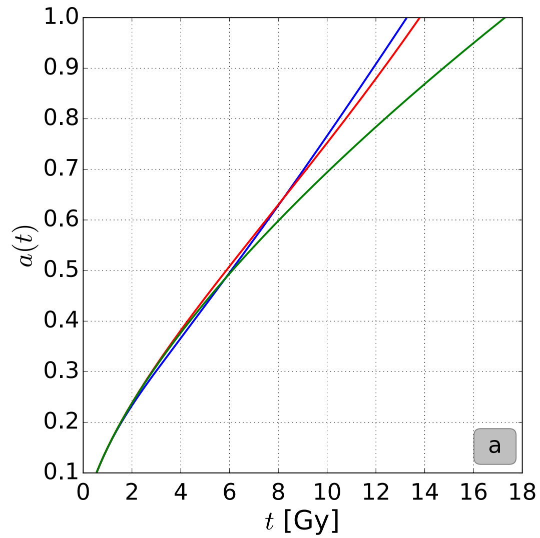
Conventional Framework for Cosmological Dynamics

- Homogeneous background with scale factor a(t)
  - $a'' = -(4\pi/3)\, G\, \rho_b\, a$ (' = d/dt) Friedmann eq
- Structure (in e.g. N-body calc.) obeys
  - $\mathbf{x}'' + 2\,(a'/a)\,\mathbf{x}' + \boldsymbol{\nabla}\phi\,/\,a^2 = 0$ where
    - $\mathbf{x} = \mathbf{r}\,/\,a(t)$ are "conformal" coords, and
    - $\boldsymbol{\nabla}^2\phi = 4\pi\, G\, (\rho - \rho_b)\, a^2$
- No feedback (or "backreaction") of $\delta\rho$ on evolution of a(t)
- G.F.R. Ellis (1984...): is this legitimate?
  - explored by Buchert & Ehlers '97 plus many others

Racz et al 2017: Modified N-body calculations

- They assume the conventional <u>structure</u> equations:
  - $\mathbf{x}'' + 2\,(a'/a)\,\mathbf{x}' + \boldsymbol{\nabla}\phi\,/\,a^2 = 0$
  - $\boldsymbol{\nabla}^2\phi = 4\pi\, G\,(\rho - \rho_b)\, a^2$



- but evolve a(t) according to $a \rightarrow a + a'\delta t$
  - with a' obtained by averaging <u>local expansion</u>:<a'/a> invoking "separate universe" approximation
  - "Strong backreaction" based on Newtonian physics
- Big effect: a(t) very similar to ΛCDM concordance model
  - "<u>concordance cosmology without dark energy</u>"

Racz et al. world view

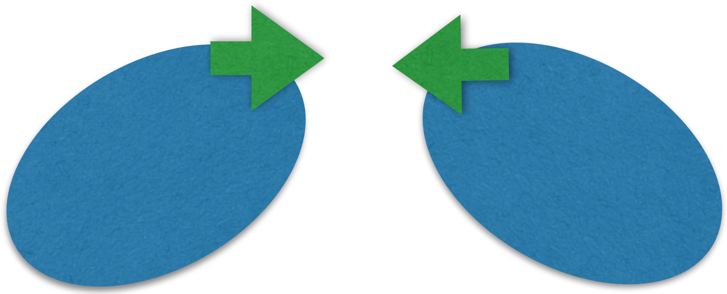
- *"N-body simulations integrate Newtonian dynamics with a changing GR metric that is calculated from averaged quantities"*
- *"changing GR metric"*: FRW metric: expansion factor a(t)
  - a(t) comes from strong-field GR physics
    - so we don't really understand it except in highly idealised (e.g. homogeneous) situations
    - hence legitimate to propose alternative ansatz?
  - a(t) - the "expansion of space" - affects the small-scale dynamics of structure

Is it legitimate to modify the Friedmann equation?

- Does emergence of structure really "backreact" on a(t)?
- Can address this in <u>Newtonian</u> gravity. Relevant as:
  - Accurate description of the local universe (v << c)
    - aside from effects from BHs
  - this is where we observe e.g. $H_0 = 70$ km/s/Mpc!
    - not $H_0 \sim 35$ km/s/Mpc expected w/o dark energy, Ω_k
  - At z = 0.1 relativistic corrections ~ 0.01
- If backreaction is important at > 1% level Newtonian analysis should show it

Why we might expect backreaction - tidal torques

- Neighbouring structures exert torques on each other
  - happens as structures reach δ ~ 1
  - a non-linear (2nd order) effect
  - purely Newtonian
  - explains spin of galaxies
- can this affect expansion?
  - it does in the local group
  - do internal degrees of freedom couple to (i.e. exchange energy with) universal expansion



Inhomogeneous Newtonian cosmology

- Lay down particles on a uniform grid in a big uniformly expanding sphere ($\mathbf{v} = H\mathbf{r}$)
- Perturb the particles off the grid $\mathbf{r} \rightarrow \mathbf{r} + \delta\mathbf{r}$
  - plus related velocity perturbations to generate "growing mode" of structure
- $\mathbf{g}(\mathbf{r})$ can be decomposed into:
  - homogenous field sourced by mean density ρ
  - inhomogeneous field sourced by $\delta\rho$ (little dipoles)
- equations of motions $\mathbf{r}'' = \mathbf{g}$ can be re-scaled
  - gives the equations that are solved in N-body codes

Newtonian gravity in re-scaled coordinates

N-particles of mass m: $$\ddot{\mathbf{r}}_i = Gm \sum_{j \neq i} \frac{\mathbf{r}_j - \mathbf{r}_i}{|\mathbf{r}_j - \mathbf{r}_i|^3}.$$

With **r** = a(t) **x** for arbitrary a(t)

$$\ddot{\mathbf{x}}_i + 2\frac{\dot{a}}{a}\dot{\mathbf{x}}_i = \frac{Gm}{a^3} \sum_{j \neq i} \frac{\mathbf{x}_j - \mathbf{x}_i}{|\mathbf{x}_j - \mathbf{x}_i|^3} - \frac{\ddot{a}}{a}\mathbf{x}_i.$$

initial conditions: $\mathbf{x} = \mathbf{r}/a$ and $\dot{\mathbf{x}} = ((H - \dot{a}/a)\mathbf{r} + \delta\dot{\mathbf{r}})/a$

Defining $n(\mathbf{x}) \equiv \sum_i \delta(\mathbf{x} - \mathbf{x}_i)$ and $\delta n \equiv n - \overline{n}$

$$\ddot{\mathbf{x}}_i + 2\frac{\dot{a}}{a}\dot{\mathbf{x}}_i - \frac{Gm}{a^3} \int d^3x \; \delta n(\mathbf{x}) \frac{\mathbf{x} - \mathbf{x}_i}{|\mathbf{x} - \mathbf{x}_i|^3}$$
$$= -\left(\frac{\ddot{a}}{a} + \frac{4\pi G m \overline{n}}{3a^3}\right)\mathbf{x}_i.$$

Exactly equivalent to the usual equations in **r**-coords

Newtonian cosmology:

$$\ddot{\mathbf{x}}_i + 2\frac{\dot{a}}{a}\dot{\mathbf{x}}_i - \frac{Gm}{a^3}\int d^3x\ \delta n(\mathbf{x})\frac{\mathbf{x}-\mathbf{x}_i}{|\mathbf{x}-\mathbf{x}_i|^3}$$
$$= -\left(\frac{\ddot{a}}{a} + \frac{4\pi Gm\overline{n}}{3a^3}\right)\mathbf{x}_i.$$

with ICs

$\mathbf{x} = \mathbf{r}/a \quad \text{and} \quad \dot{\mathbf{x}} = ((H - \dot{a}/a)\mathbf{r} + \delta\dot{\mathbf{r}})/a$

- 3N equations for N particles
  - there is no extra equation of motion for a(t)
- But we may choose a(t) to obey Friedmann equation
  - an "auxiliary relation"
- Gives conventional expansion + structure equations
  - a(t) suffers no backreaction from structure emergence
  - a(t) is just a "book-keeping" factor - no physical effect

Part 1: summary/conclusions

- A different perspective on the conventional equations for structure growth (Dmietriev & Zel'dovich '63)
    - fully non-linear & exact (but Newtonian) description
- a(t) is arbitrary, but extra terms appear in equations of motion if a(t) does not obey Friedmann's equation
    - physical quantities invariant under choice of a(t)
- No coupling of expansion to internal structure via tidal torques
    - can also be understood from scaling with radius/mass

Relation to Buchert & Ehlers '97 "kinematic BR"

- Matter modelled as <u>pressure-free Newtonian fluid</u>
  - unrealistic, but maybe a useful "toy model"
- Consider a specific volume $V = a^3$ containing mass M
- Raychaudhuri equation (expansion θ, vorticity ω, shear σ)
  - $a''/a + (4\pi/3)\, GM/a^3 = Q$
    - with $Q = 2(\langle\theta^2\rangle - \langle\theta\rangle^2)/3 + 2\langle\omega^2-\sigma^2\rangle$
    - 2nd order - no linear effect!
- Naively a big effect (individual terms in $Q \sim G\rho$)
  - but...

Buchert & Ehlers '97

- "<u>Generalised Friedmann equation</u>": $a''/a + GM/a^3 = Q$
  - $Q = 2(\langle\theta^2\rangle - \langle\theta\rangle^2)/3 + 2\langle\omega^2 - \sigma^2\rangle$
  - Q=0 is *"conspiracy assumption"*
- But *"Q is a divergence"*: $Q = V^{-1}\int \mathbf{dA}.(\mathbf{u}(\nabla.\mathbf{u})-(\mathbf{u}.\nabla)\mathbf{u})$
  - so no global effect for periodic BCs - *"by construction"*
- No surprise that $a''/a \neq -GM/a^3$ for an individual region
  - fluctuations affect acceleration a'' and M
  - but <u>local</u>, not "backreaction of $\delta\rho$ on global expansion"
- If $\langle Q\rangle_{V\to\infty}$!= 0 would imply a conflict - this is <u>not</u> the case

Do B&E claim Newtonian backreaction?

- Q = 0 requires *"conspiracy"* - but *"the average motion may be approximately given by the Friedmann equation on a scale which is larger than the largest existing inhomogeneities"*

- Later works: E.g. Buchert & Rasanen 2011 review

  - *"..linear theory ... effect vanishes by construction ... in Newtonian ... true also in non-perturbative regime"*

  - *"When we impose periodic BCs .... Q is strictly zero"*

  - but *"If backreaction is substantial then current Newtonian simulations (and analytic studies) are inapplicable".*

- *"To get Newtonian cosmology unique there is no other way, relevant for cosmology, than introducing a background and packing the deviations on a torus, that's our result."*

How large is $Q = (3 a''/a + 4\pi GM/a^3)$?

- $Q = Q_1 + Q_2 = V^{-1}\int \mathbf{dA}.(\mathbf{u}(\nabla.\mathbf{u})-(\mathbf{u}.\nabla)\mathbf{u}) - (3/2V^2)(\int \mathbf{dA}.\mathbf{u})^2$
  - **u** is peculiar velocity wrt <u>global</u> H
- If structure is a stat. homog. and isotropic random process
  - $\langle \mathbf{u}(\nabla.\mathbf{u})-(\mathbf{u}.\nabla)\mathbf{u}\rangle_{ensemble} = 0$ (Monin and Iaglom, 1975)
  - so Q_1 is pure fluctuation
  - $|Q_1| \sim \langle u^2\rangle/r^2$ independent of coherence length λ
- Second term is systematic: $Q_2 \sim \langle u^2\rangle \lambda^2 / r^4$
- Both are <u>very small</u> ($\ll H^2$) for large V

Is there relativistic backreaction?

- Buchert etc: *"GR backreaction"* is non-zero - and large
- But local universe should be accurately Newtonian
  - errors ~ $v^2/c^2 \rightarrow$ ~1% accuracy within $z = 0.1$
  - and that's where we measure H_0
  - so very hard to believe there are >> 1% effects
- Q: Are there even very small effects on expansion history coming from non-relativistic effects?

Is there *relativistic* backreaction?

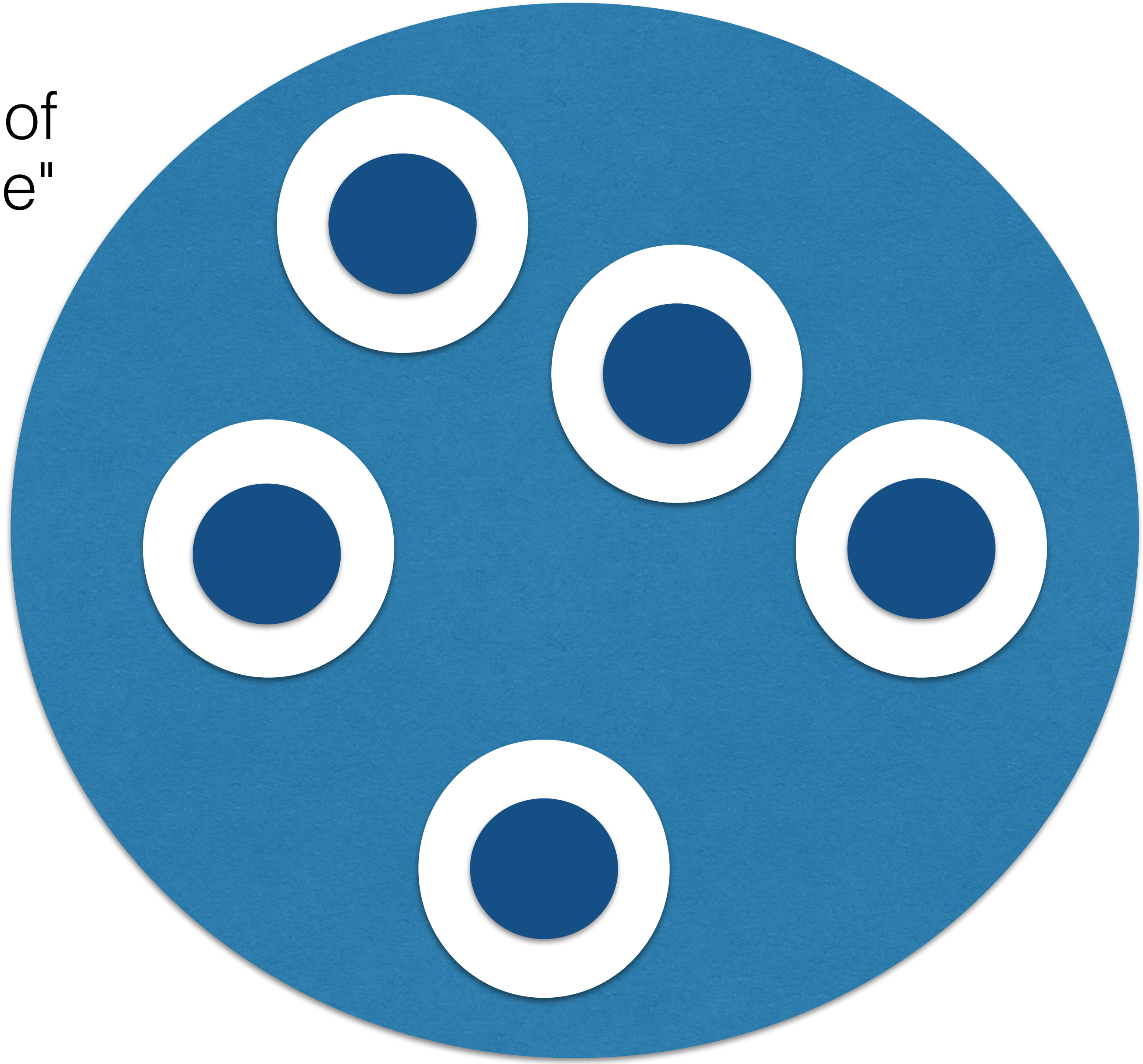
- Averaging of Einstein equations: **G** = **T**
- FRW: metric **g** -> **G** and **T** = diag(ρ, P, P, P) are diagonal
  - **G** = **T** and **∇.T** = 0 -> Friedmann equations
- with inhomogeneity < **G** > = < **T** >?
  - *"averaging problem"* widely discussed in BR literature
- what about internal pressure P of clusters?
  - or internal pressure in stars, other compact objects
- Do those give Friedmann equations with non-zero P?
  - and hence deviation from Newtonian expansion law?

Averaging of Einstein equations: < **G** > = < **T** >?

- Consider e.g. stars with internal pressure P
  - does that give Friedmann equations with non-zero P?
- No. Stars have Schwarzschild exterior with mass m
    - space integral of the stress pseudo-tensor
      - includes rest mass, motions, P, binding energy
      - but is independent of time
- Conservation of stars implies $\rho \sim a^{-3}$
  - which demands P = 0 in the Friedmann equations

Relativistic BR from large-scale structure?

- Einstein-Straus '45
  - "What is the effect of expansion of space"
- -> Swiss-cheese
- Fully non-linear
- Interesting pert^n to e.g. proper mass
- but background expansion is exactly unperturbed
- small effects on D(z)



Backreaction from inter-galactic pressure

- Stars & DM ejected from galaxies by merging SMBHs
  - intergalactic pressure $P = n\ m\ \sigma_v^2$
  - and P in the background of GWs emitted
- Homogeneous (in conformal coords) pressure is a <u>flux of energy</u> with non-zero divergence in real space
  - 1st law ... PdV work .... : $\rho' = -(\rho + P/c^2)\ V' / V$
    - but a <u>very small effect</u>
- relies on pressure being extended throughout space
  - no effect from internal pressure in bound systems that are surrounded by empty space

Summary

- A different perspective on the DZ equations. There is no dynamical equation for a(t). a(t) is arbitrary. But there is no freedom to modify F-equation w/o changing structure eqs. Conventional system of equations is exact.

- Clarification of "generalised Friedmann equation". Periodic BCs is not the issue. $|Q_1| \sim \langle v^2 \rangle / r^2$ and $\langle Q_1 \rangle = 0$ (Monin and Iaglom). $\langle Q_2 \rangle \sim \langle v^2 \rangle \lambda^2 / r^4$. Both are v. small and tend to zero for large r.

- Discussion of relativistic backreaction. Averaging of stress-energy for systems with internal pressure does not introduce non-zero P in Freidmann equations. Exact non-linear solutions show no backreaction. Intergalactic P does backreact, but P is weak and positive.