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Gravitational clustering of massive neutrinos around cold dark matter halos

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Introduction

Neutrinos in cosmology

Neutrinos are initially relativistic, but become non-relativistic at

$$z \simeq 200 \left(\frac{m_\nu}{0.1\ \mathrm{eV}}\right)$$

Neutrinos' free-sreaming scale :

$$k_{\mathrm{fs}} = 0.0058\Omega_{\mathrm{m}}^{1/2} \left(\frac{m_\nu}{0.1\mathrm{eV}}\right)^{1/2} h\ \mathrm{Mpc}^{-1}$$

Neutrino density fluctuations on the scale $k > k_{\mathrm{fs}}$ cannot grow.

→ modify small-scale structures.

Lesgourgues, Pastor, Rhys. Rept. 429 (2006) 307

Focus on small-scales

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Neutrinos cluster onto a gravitational potential of CDM

(their contribution to gravitational potential can be neglected)

→ Clustering neutrinos open a new window to measure their mass ?

Before that, *<u>how can we treat massive neutrinos in CDM halos</u>* ?

Vlasov v.s. Fluid

Vlasov-Poisson equations —approximation→ Fluid description

(Weak interaction can be neglected)

$$\frac{\partial f}{\partial t} + \boldsymbol{v}\cdot\nabla_{\boldsymbol{x}} f - \nabla\phi\cdot\nabla_{\boldsymbol{v}} f = 0$$

$$\left\{ \begin{aligned} \nabla^2\phi(\boldsymbol{x},t) &= 4\pi\rho(\boldsymbol{x},t) \\ \rho(\boldsymbol{x},t) &= m\int f(\boldsymbol{x},\boldsymbol{v},t)\,d^3v \end{aligned} \right.$$

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\boldsymbol{v}) = 0$$

$$\frac{\partial v_i}{\partial t} + (\boldsymbol{v}\cdot\nabla)v_i = -\nabla_i\phi - \frac{1}{\rho}\nabla_j(\rho\sigma_{ij})$$

$$\nabla^2\phi(\boldsymbol{x},t) = 4\pi\rho(\boldsymbol{x},t)$$

Goal

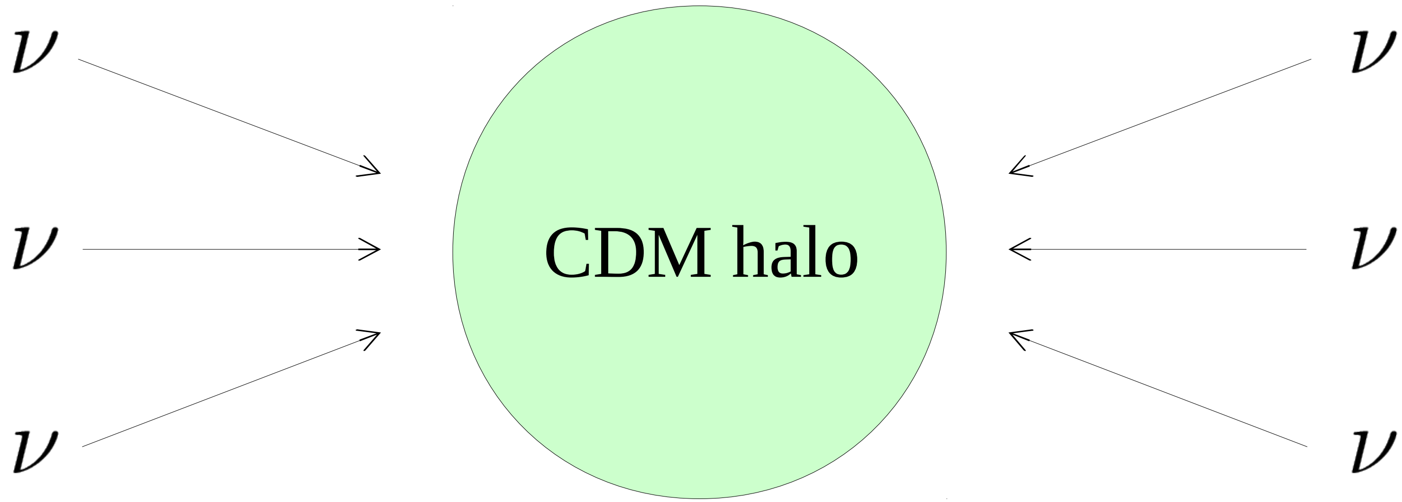
How well does the fluid approximation work for massive neutrinos clustering in a CDM halo ?

- Compute density profiles of neutrinos in fluid description and Vlasov-Poisson picture
- Show how we improve the fluid description for neutrinos

Pioneering work : Ringwald, Wong, JCAP 0412 (2004) 005

Method

Setup



- $z = 10 \rightarrow 0$
- $M_{\rm halo} = 10^{12} \sim 10^{15} M_{\odot}/h$
- Mass growth is taken into account Zhao et al., APJ 707 (2009) 354
- $m_{\nu} = 0.15, 0.3 \text{ eV}$
- Use NFW profile → gravity potential ϕ
- ν is treated as a test particle

Vlasov eq. → N-one-body approach

- Neutrinos do not contribute to the gravitaional potential.
- Gravitational potential of a CDM halo is given.

↓

No needs to solve Poisson eq.

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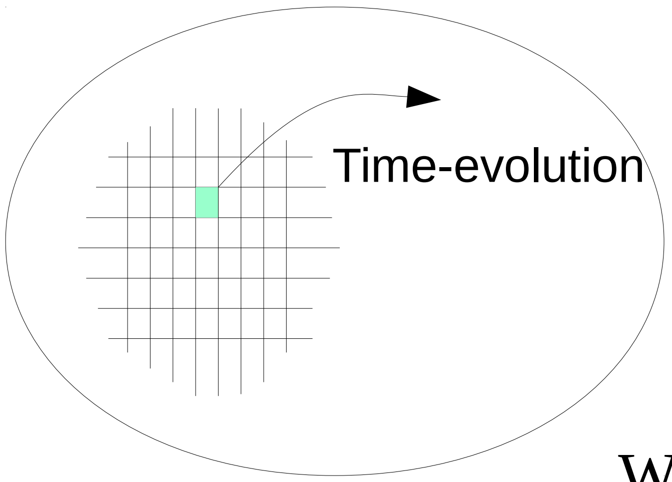
Solve the orbit of each phase-space element = N-1-body simulation

Ringwald, Wong, JCAP 0412 (2004) 005



Liouville theorem :

$$f(\boldsymbol{x},\boldsymbol{v},t)d^3\boldsymbol{x}d^3\boldsymbol{v} = f_{\rm in}(\boldsymbol{x}_{\rm in},\boldsymbol{v}_{\rm in},t_{\rm in})d^3\boldsymbol{x}_{\rm in}d^3\boldsymbol{v}_{\rm in}$$

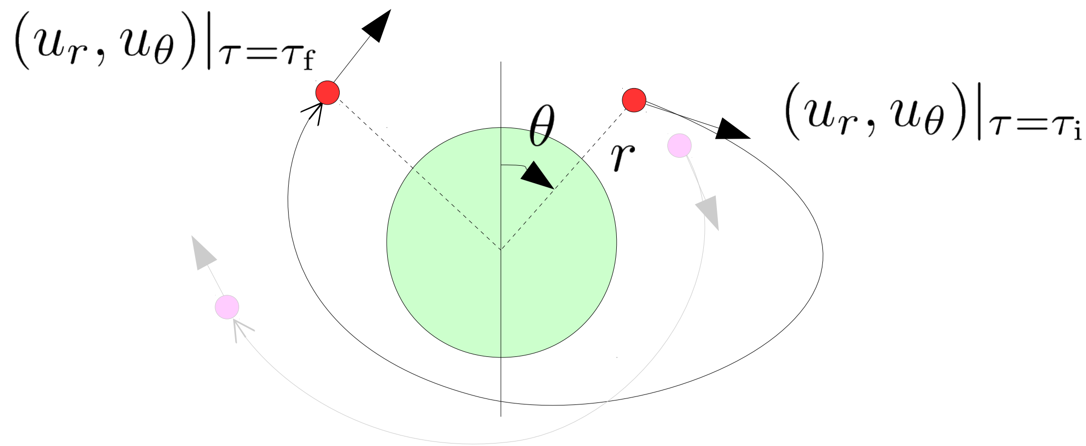
$$f(\boldsymbol{x},\boldsymbol{v},t) = f_{\rm in}(F_{\boldsymbol{x}}(\boldsymbol{x},\boldsymbol{v},t),F_{\boldsymbol{v}}(\boldsymbol{x},\boldsymbol{v},t),t_{\rm in})$$

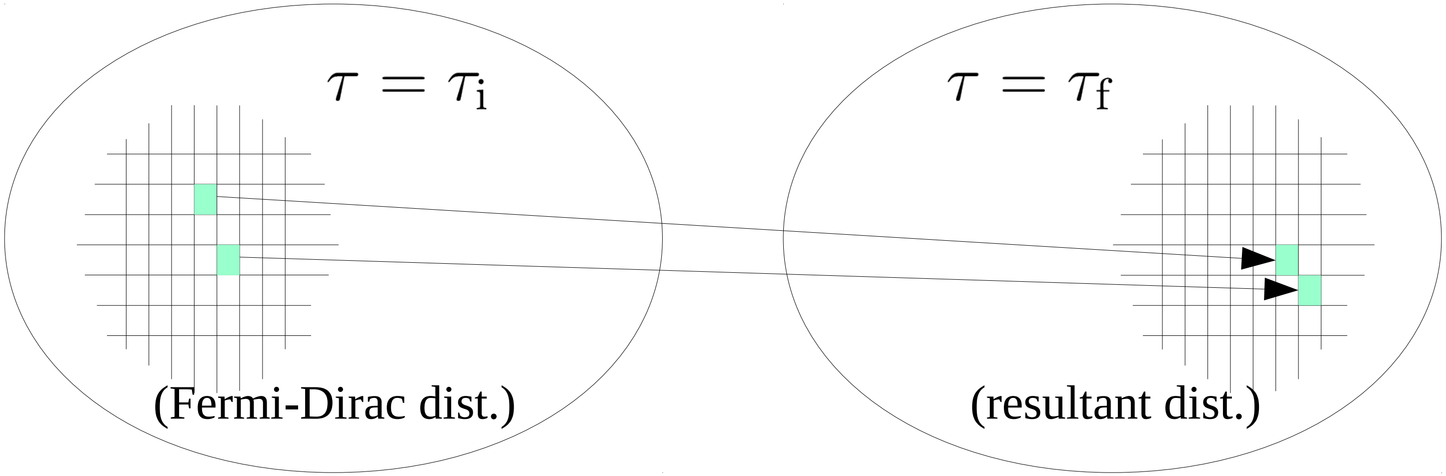
What we should to know is the mapping of phase-space positions from initial time to present time.

N-one-body simulation

$$\begin{cases} \dfrac{dr}{d\tau} = \dfrac{u_r}{a} \\ \dfrac{du_r}{d\tau} = \dfrac{u_\theta^2}{ar^3} - a\dfrac{\partial\phi}{\partial r} \\ \dfrac{d\theta}{d\tau} = \dfrac{1}{a}\dfrac{u_\theta}{r^2} \\ \dfrac{du_\theta}{d\tau} = 0 \end{cases}$$

Ringwald, Wong, JCAP 0412 (2004) 005





Time-reversal N-one-body simulation

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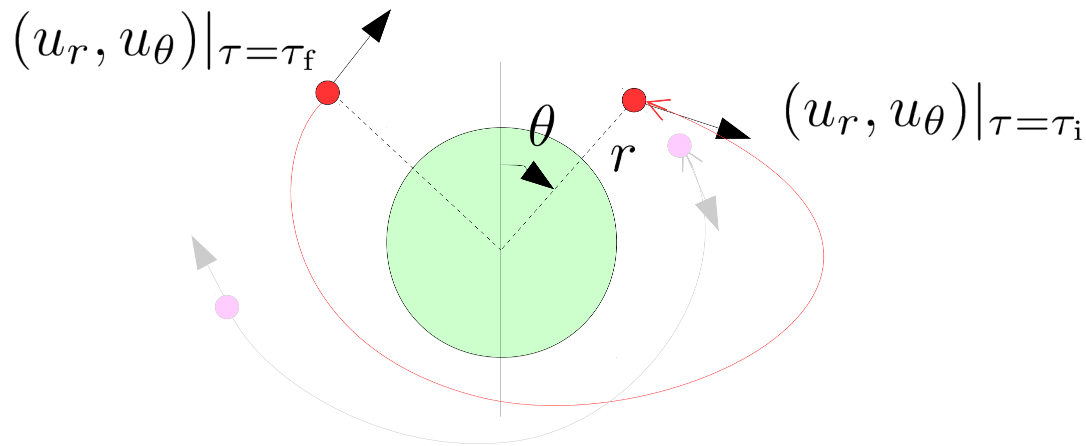
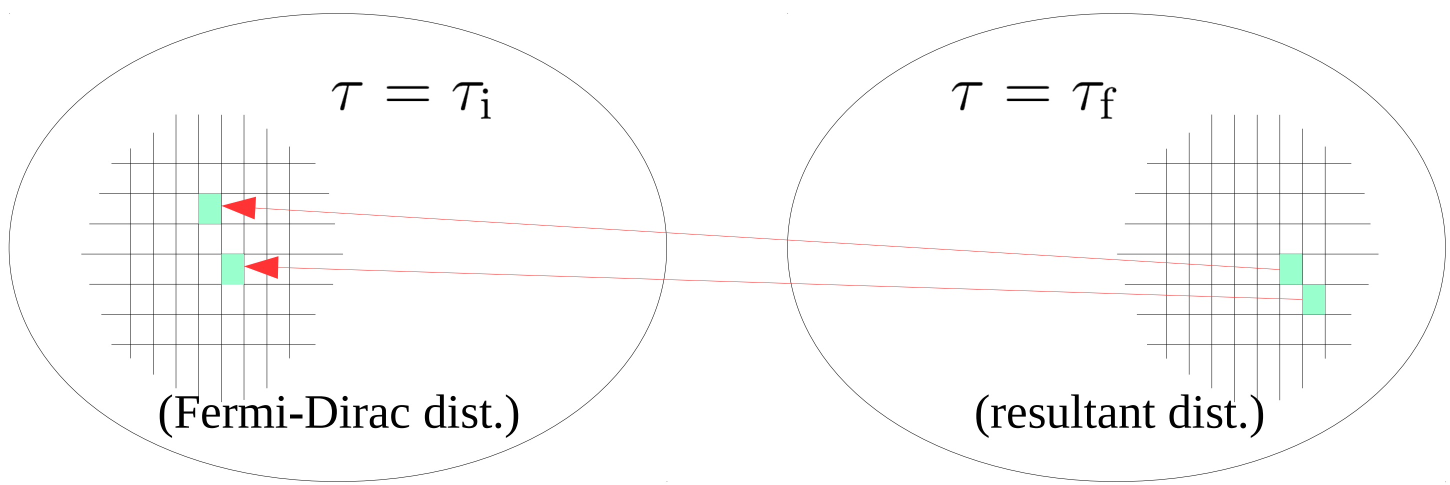
$$
\begin{cases}
\dfrac{dr}{d\tau} = \dfrac{u_r}{a} \\[2ex]
\dfrac{du_r}{d\tau} = \dfrac{u_\theta^2}{ar^3} - a\dfrac{\partial\phi}{\partial r} \\[2ex]
\dfrac{d\theta}{d\tau} = \dfrac{1}{a}\dfrac{u_\theta}{r^2} \\[2ex]
\dfrac{du_\theta}{d\tau} = 0
\end{cases}
$$



〈Time-reversal〉



We'd like to have the distribution functions in the finite range of phase space around the halo at the present time. So we first fix the range, and find their original points at $\tau = \tau_{\rm i}$ in the phase space.

Fluid approximation

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Moments

$$\rho(\boldsymbol{x},t) = \int d^3\boldsymbol{v}\, f(\boldsymbol{x},\boldsymbol{v},t) \longrightarrow \rho(\boldsymbol{x},t) = \overline{\rho}(t)\left[1+\delta(\boldsymbol{x},t)\right]$$

$$\boldsymbol{v}(\boldsymbol{x},t) = \frac{1}{\rho(\boldsymbol{x},t)}\int d^3\boldsymbol{v}\,\frac{\boldsymbol{v}}{a} f(\boldsymbol{x},\boldsymbol{v},t)$$

$$\sigma_{ij}(\boldsymbol{x},t) + v_i(\boldsymbol{x},t)v_j(\boldsymbol{x},t) = \frac{1}{\rho(\boldsymbol{x},t)}\int d^3\boldsymbol{v}\,\frac{v_i v_j}{a^2} f(\boldsymbol{x},\boldsymbol{v},t)$$

Fluid equations in spherical coordinates

$$\begin{cases} \dfrac{\partial\delta}{\partial t} + \dfrac{1}{ar^2}\dfrac{\partial}{\partial r}\left[r^2(1+\delta)v_r\right] = 0 \\ \dfrac{\partial v_r}{\partial t} + Hv_r + \dfrac{1}{a}v_r\dfrac{\partial v_r}{\partial r} \\ \qquad = -\dfrac{1}{a}\dfrac{\partial\phi}{\partial r} - \dfrac{1}{a}\dfrac{1}{1+\delta}\dfrac{\partial}{\partial r}\left[(1+\delta)\sigma_r^2\right] - 2\dfrac{\beta\sigma_r^2}{r} \end{cases}$$

$$\sigma_i^2 = \sigma_{ii}$$

How to treat velocity dispersion

$$\frac{\partial v_r}{\partial t} + H v_r + \frac{1}{a} v_r \frac{\partial v_r}{\partial r} = -\frac{1}{a}\frac{\partial \phi}{\partial r} - \frac{1}{a}\frac{1}{1+\delta}\frac{\partial}{\partial r}\left[(1+\delta)\sigma_r^2\right] - 2\frac{\beta \sigma_r^2}{r}$$

$$\beta = 1 - \frac{\sigma_t^2}{2\sigma_r^2}$$

anisotropy

1) determined from FD dist.

$$\sigma_r^2 = \frac{\sigma_\nu^2}{3} \qquad \sigma_\nu^2 = \frac{15\zeta(5)}{\zeta(3)}\left(\frac{k_B T_{\nu,0}}{m_\nu a}\right)^2$$

2) given by N-one-body simulations

$$\sigma_r^2(r,t) = \frac{1}{\rho(r,t)}\int d^3\boldsymbol{v}\ \{\boldsymbol{v}\cdot\hat{\boldsymbol{r}} - v_r\}^2 f(\boldsymbol{r},\boldsymbol{v},t)$$

$$\sigma_t^2(r,t) = \frac{1}{\rho(r,t)}\int d^3\boldsymbol{v}\ \{\boldsymbol{v} - (\boldsymbol{v}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}}\}^2 f(\boldsymbol{r},\boldsymbol{v},t)$$

$$v_r(r,t) = \frac{1}{\rho(r,t)}\int d^3\boldsymbol{v}\,(\boldsymbol{v}\cdot\hat{\boldsymbol{r}}) f(\boldsymbol{r},\boldsymbol{v},t)$$

Results

Result : phase space distribution

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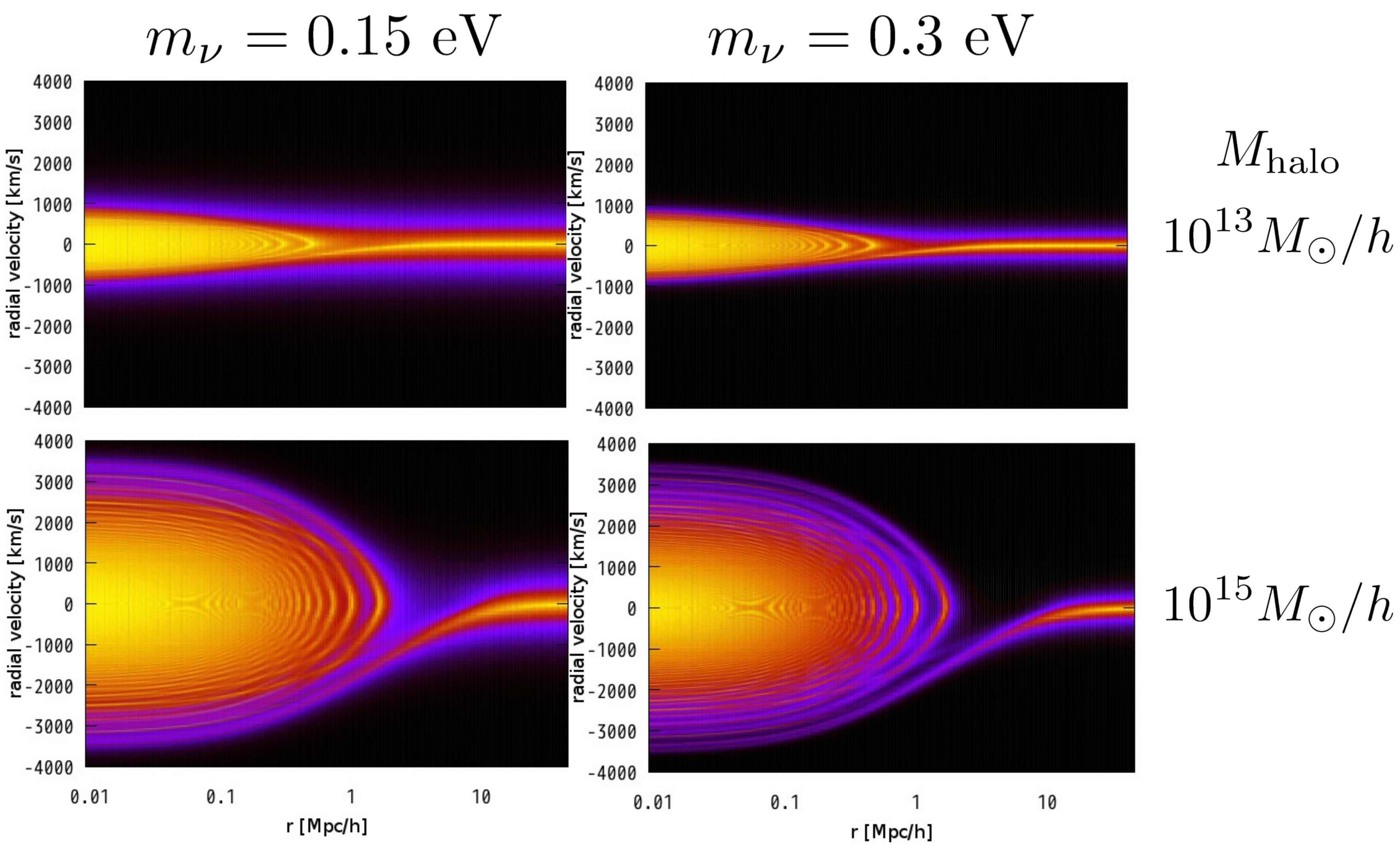

Multi-stream develops in the CDM halo's potential.

Result : velocity dispersion

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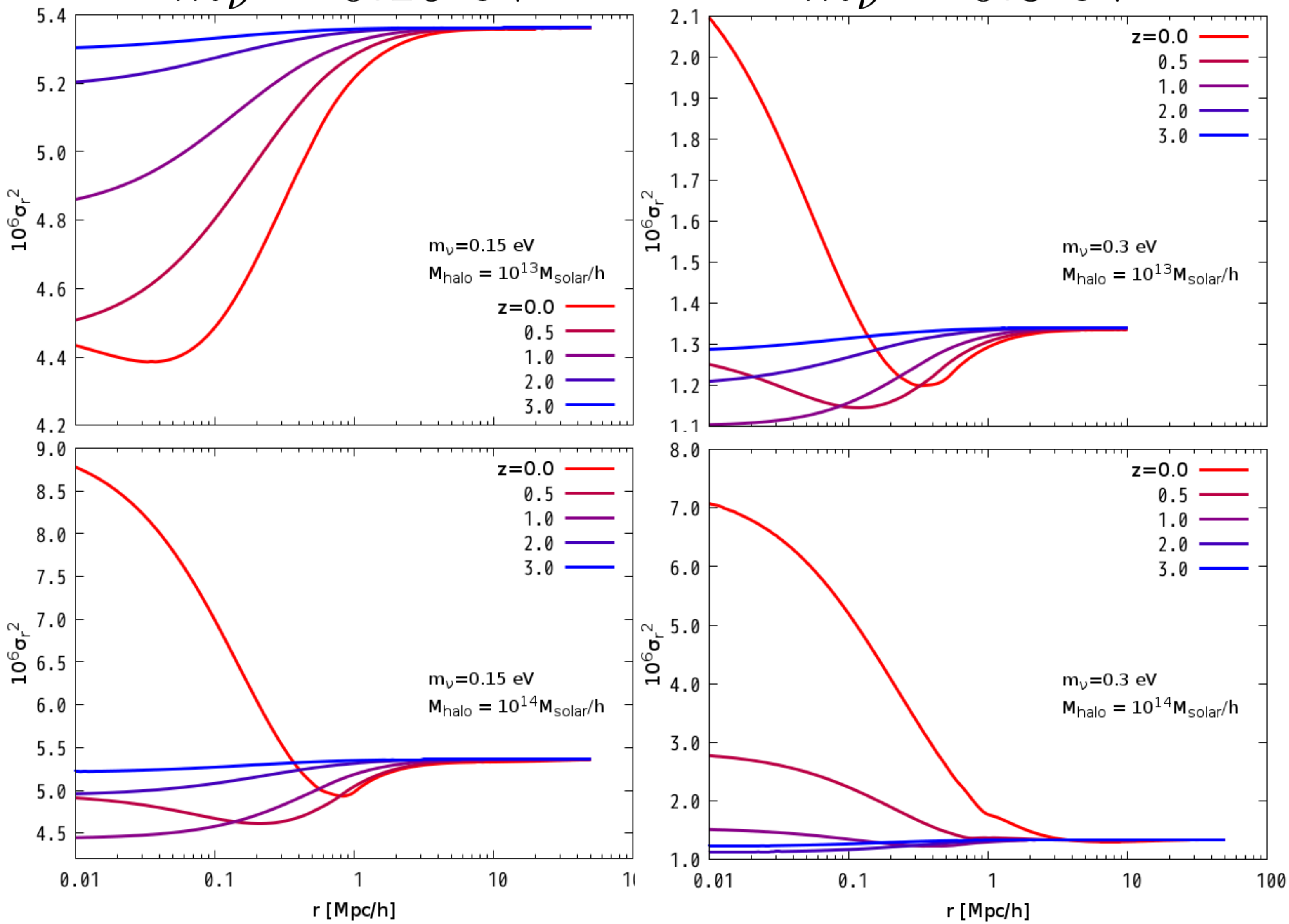
$m_\nu = 0.15$ eV $\quad m_\nu = 0.3$ eV



$M_{\rm halo}$

10^{13} $M_\odot/h$

10^{14} $M_\odot/h$

Velocity dispersion σ_r^2 depends on time as well as spatial coordinate.

Result : neutrino density profile $(m_\nu = 0.15 \text{ eV})$

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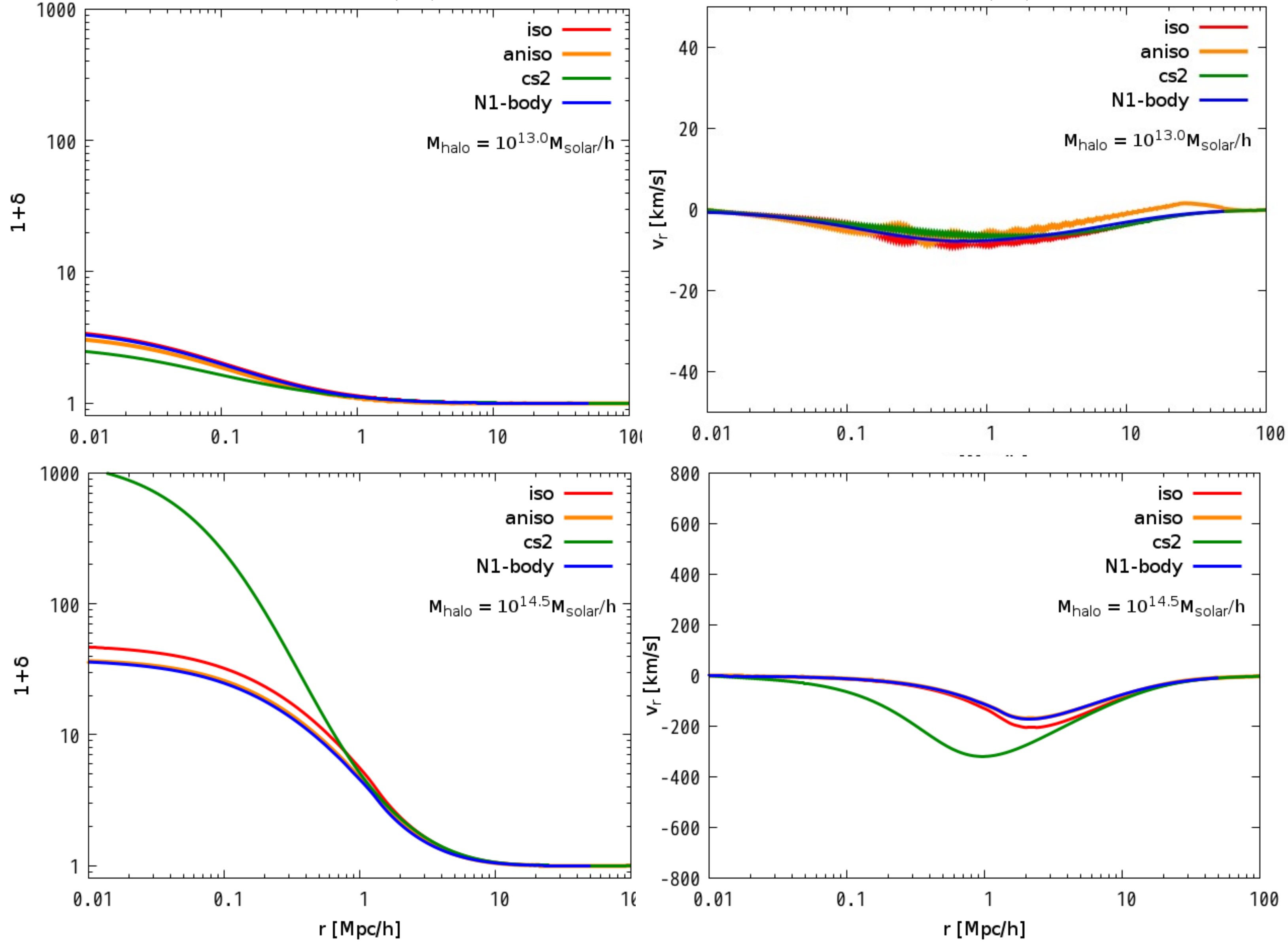
$1+\delta(r)$ $v_r(r)$



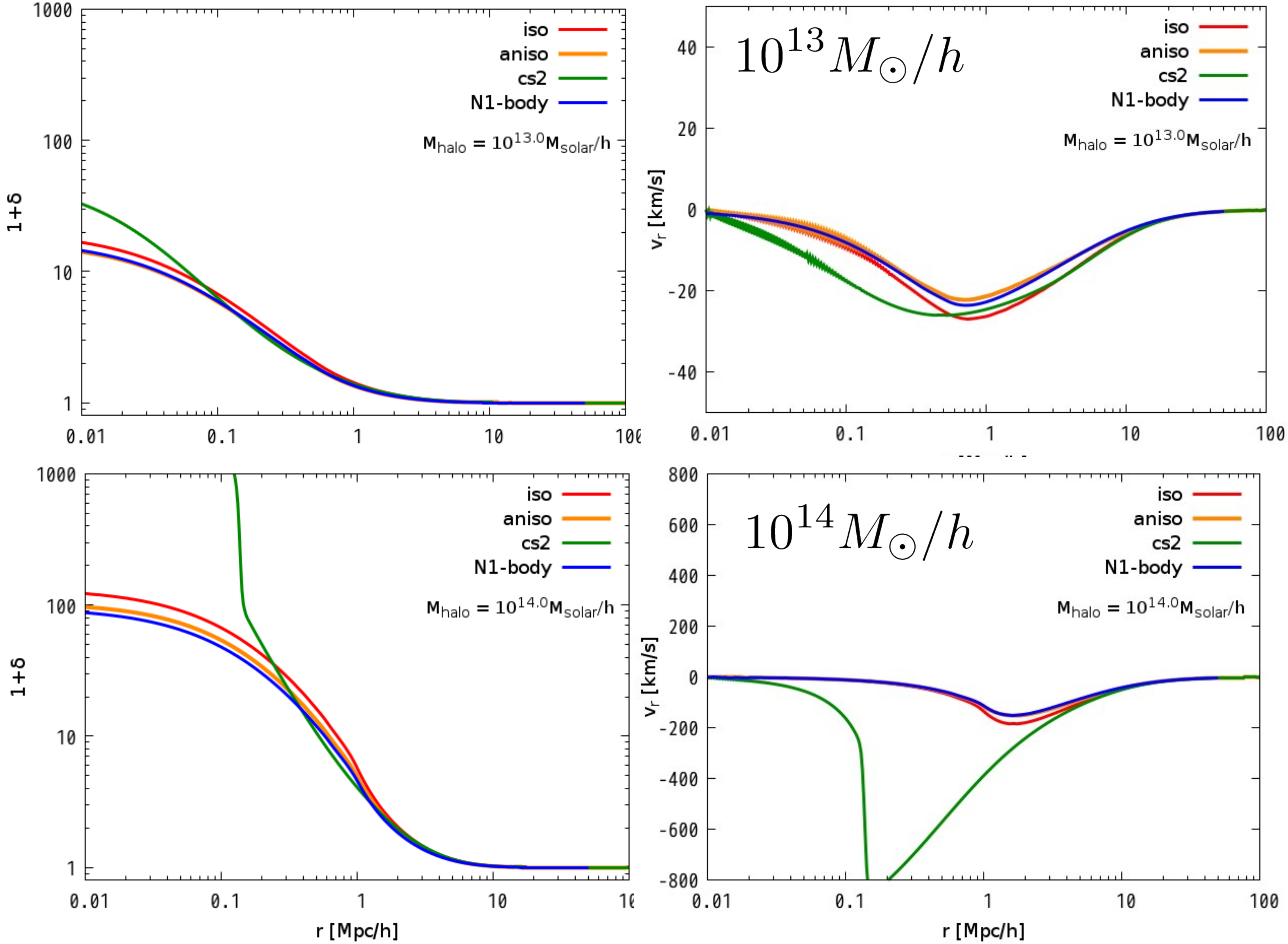
$10^{13} M_\odot/h$

$10^{15} M_\odot/h$

Fluid description over-estimates the density at centre of halo

Result : neutrino density profile $(m_\nu = 0.3 \text{ eV})$

$1+\delta(r)$ $\quad$ $v_r(r)$



10^{13} $M_\odot/h$

10^{14} $M_\odot/h$

Infall velocity becomes supersonic, and then fluid description breaks down.

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$$M_{\rm halo} = 10^{14.5} M_\odot/h$$

$$m_\nu = 0.3\ {\rm eV}$$

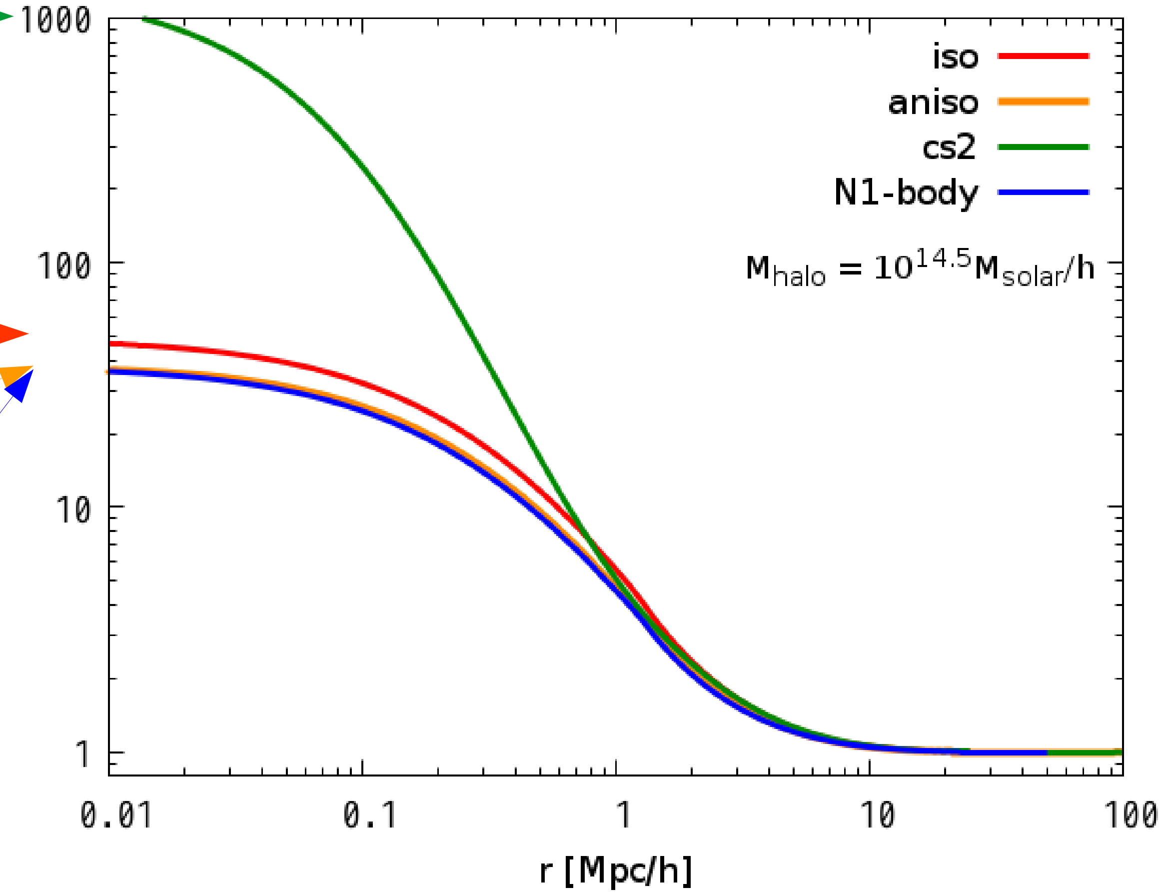
Fluid

Fluid + σ_r^2

Fluid + $\sigma_r^2 + \beta$

N-one-body

Good coincidense!



Fluid desctription is highly improved if we take into account σ_r^2 and $\beta = 1 - \sigma_t^2/(2\sigma_r^2)$ computed from N-one-body simulations.

Result : total mass of clustering neutrinos

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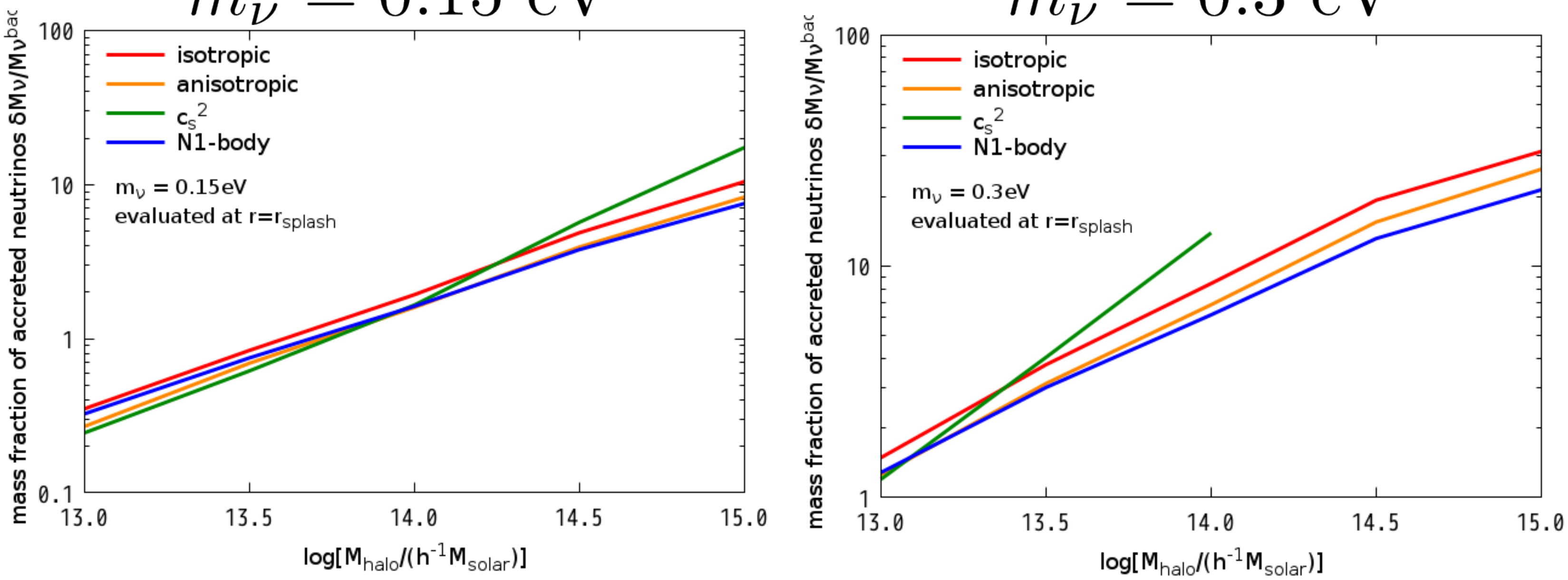
$$M_\nu(r) = 4\pi\overline{\rho}\int_0^r [1+\delta_\nu(r)]\, r^2\, dr \longrightarrow \frac{M_\nu - \overline{M}_\nu}{\overline{M}_\nu}$$

$m_\nu = 0.15$ eV $\qquad$ $m_\nu = 0.3$ eV



Breakdown of fluid description leads to quite large deviation from N-one-body results. But it can be improved.

How well does the fluid approximation work for massive neutrinos clustered in a CDM halo ?

N-one-body vs. Fluid

- Density profile of clustering neutrinos highly deviates from that obtained by N-one-body approach.

- Naive fluid description leads to the supersonic flow for large $M_{\rm halo}$ or m_ν , and thus breaks down.

- Fluid description can be improved by calibrating σ_r^2 and β from N-one-body simulations.