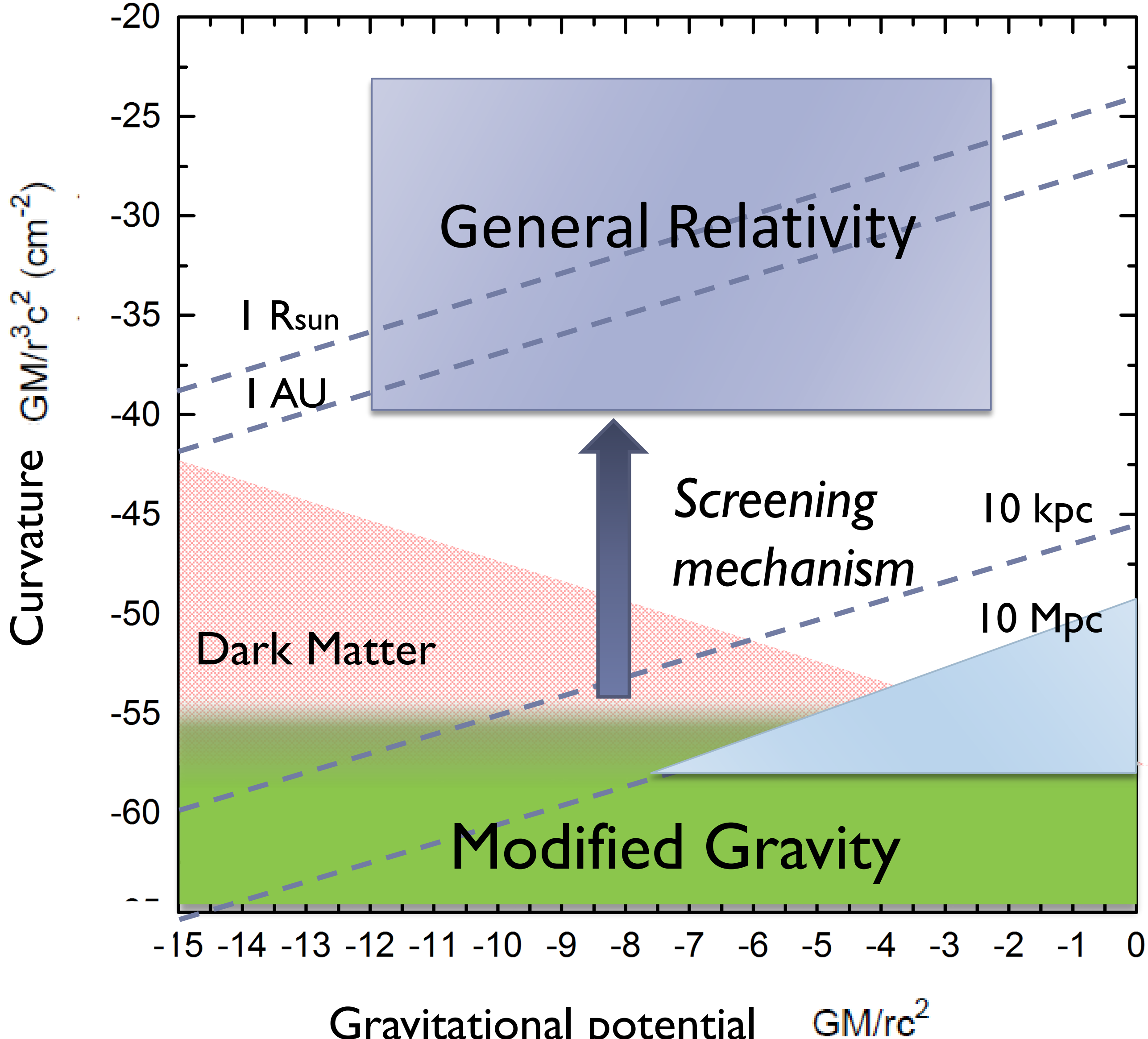
COLA with scale dependent growth: applications to modified gravity and massive neutrinos

Kazuya Koyama

Institute of Cosmology and Gravitation,
University of Portsmouth

Psaltis Living Rev. Relativity 11 (2008), 9
Baker et.al. 1412.3455



curvature

$$R = \frac{GM}{r^3c^2}$$

potential

$$\Phi = \frac{GM}{rc^2}$$

Cosmological tests of gravity

Why do we need screening mechanism?

- Brans-Dicke gravity

$$S=\int d^4x\left(\psi R-\frac{\omega_{BD}}{\psi}(\nabla\psi)^2\right)+\int d^4x\,L_{matter}$$

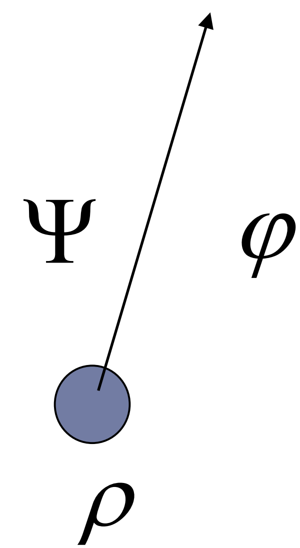
quasi-static approximations (neglecting time derivatives)

$$ds^2=-(1+2\Psi)dt^2+a(t)^2(1-2\Phi)d\vec{x}^2 \quad \psi=\psi_0+\varphi$$

$$(3+2\omega_{BD})\nabla^2\varphi=-8\pi G\rho$$

$$\nabla^2\Psi=4\pi G\rho-\frac{1}{2}\nabla^2\varphi \qquad \varphi \ \text{: fifth force}$$

$$\Phi-\Psi=-\varphi$$



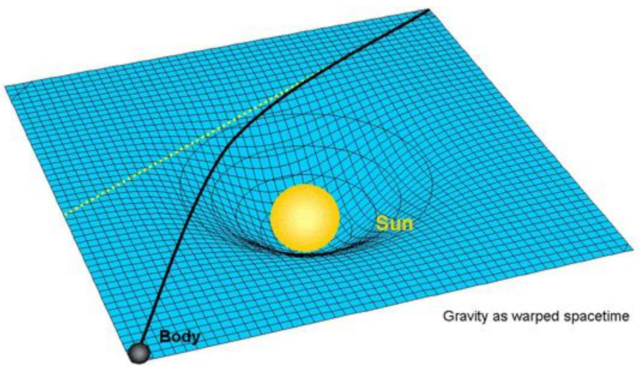
Constraints on BD parameter

- Solutions

$$(3+2\omega_{BD})\nabla^2\varphi = -8\pi G\rho$$

$$\nabla^2\Psi = -4\pi G\left(\frac{4+2\omega_{BD}}{3+2\omega_{BD}}\right)\rho, \quad G_{eff} = \left(\frac{4+2\omega_{BD}}{3+2\omega_{BD}}\right)G$$

$$\Psi = \frac{2+\omega_{BD}}{1+\omega_{BD}}\Phi \equiv \gamma^{-1}\Phi$$



- PPN parameter

$$\gamma = \frac{1+\omega_{BD}}{2+\omega_{BD}}$$

$$\gamma - 1 = (2.1 \pm 2.3)\times 10^{-5} \qquad \omega_{BD} \geq 40,000$$

This constraint excludes any detectable modifications in cosmology

Screening mechanism

- Require screening mechanism to restore GR

$$S = \int d^4x \left(\psi R - \frac{\omega_{BD}(\psi)}{\psi} (\nabla \psi)^2 + V(\psi) + N(\nabla \psi, \nabla^2 \psi) \right)$$

recovery of GR must be environmental dependent

- make the scalar short-ranged using $V(\psi)$ (*chameleon*)
- make the kinetic term large to suppress coupling to matter using $\omega_{BD}(\psi)$ (*dilaton/symmetron*) or $N(\nabla \psi)$ (*k-mouflage*) $N(\nabla^2 \psi)$ (*Vainshtein)*

Break equivalence principle

$$\int d^4x \left(B(\psi) L_{baryon} + L_{CDM} \right)$$

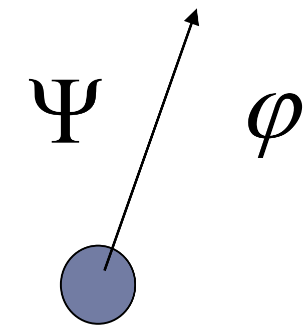
remove the fifth force from baryons (*interacting DE models* in Einstein frame)

Classification of screening mechanism

- **Classification** Joyce et.al. 1407.0059

  These screening mechanisms can be classified whether the suppression of the fifth force is determined by

  - Potential (*chameleon, dilaton/symmetron*) Ψ
  - Gradient of potential (*k-mouflage*) $\nabla_i \Psi$
  - Curvature (*Vainshtein*) $\nabla_i \nabla_j \Psi$



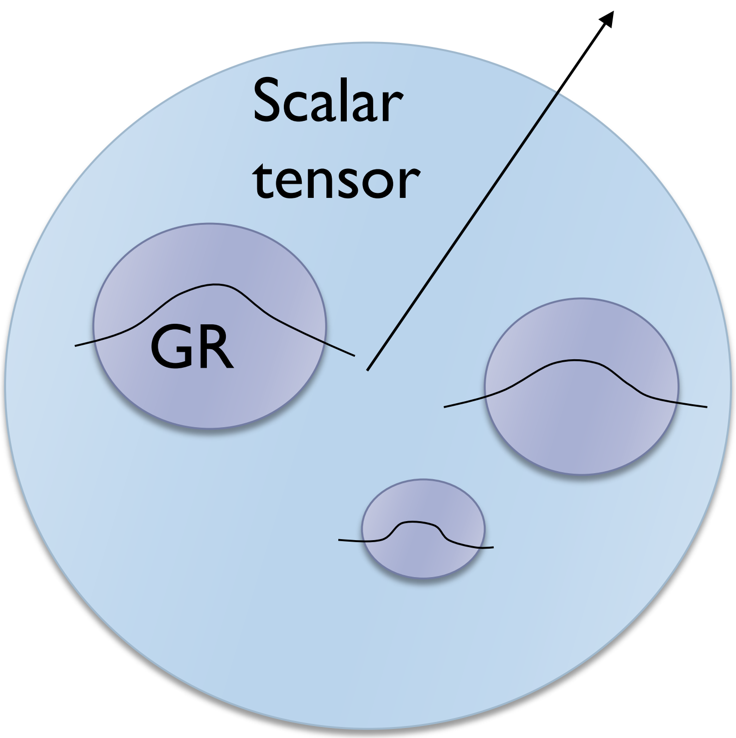
$$\begin{cases} \Psi = \dfrac{GM}{r} \\ \varphi = 2\beta^2 \, \varepsilon \dfrac{GM}{r} \end{cases}$$

$$F_\varphi = 2\,\beta^2\, F_{Newton}\, \varepsilon(\Psi, \nabla\Psi, \nabla^2\Psi)$$

$$2\beta^2 = \frac{1}{3 + 2\omega_{BD}}$$

Behaviour of gravity

There regimes of gravity



In most models, the scalar mode obeys non-linear equations describing the transition from the scalar tensor theory on large scales to GR on small scales

$$\rho_{crit} \approx 10^{-29} g / cm^3,$$
$$\rho_{galaxy} \approx 10^{-24} g / cm^3,$$
$$\rho_{solar} \approx 10 g / cm^3$$

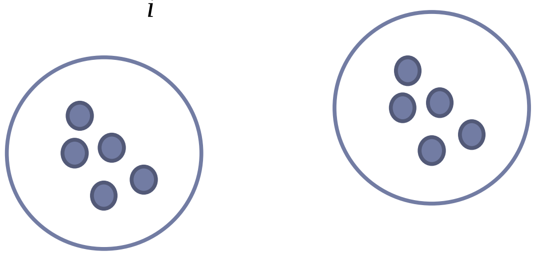
Understandings of non-linear clustering require N-body simulations where the non-linear scalar equation needs to be solved

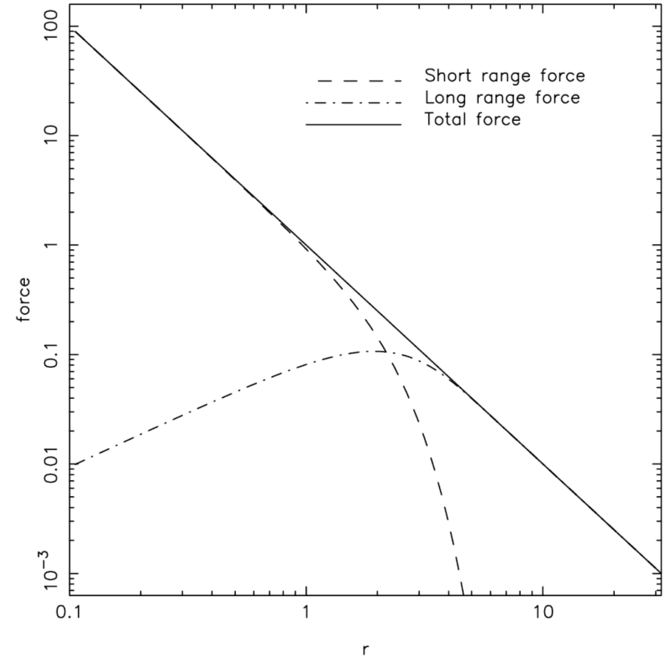
N-body simulations

- GR

  superposition of forces

$$\vec{F}(r) = \sum_i m_i \; \vec{f}_i(r - r_i)$$





- Modified gravity models

  the non-linear nature of the scalar field equation implies that the superposition rule does not hold

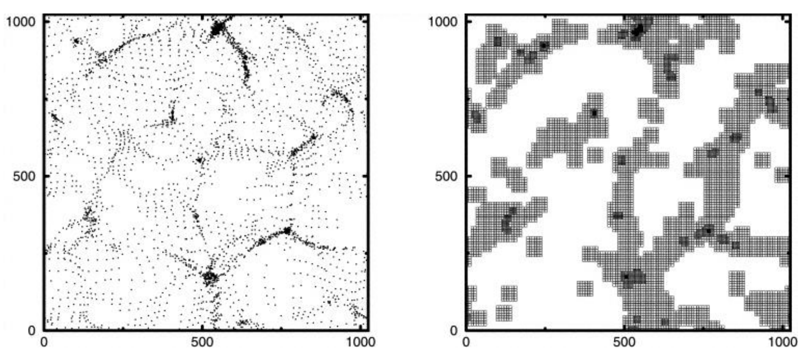
  - It is required to solve the non-linear scalar equation directly on a mesh a computational challenge!
  - The breakdown of the superposition rule has interesting consequences

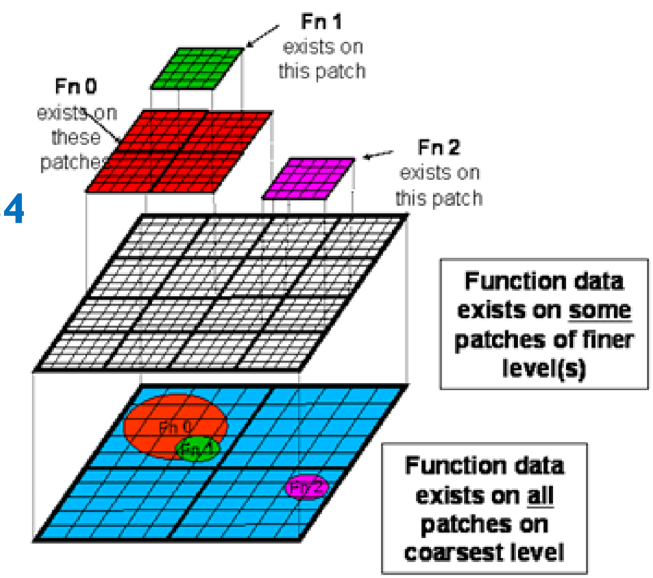
N-body Simulations for MG

- Multi-level adaptive mesh refinement
- a scalar field solver using a non-linear Gauss Seidel relaxation

ECOSMOG Li, Zhao, Teyssier, KK JCAP1201 (2012) 051
MG-GADGET Puchwein, Baldi, Springel MNRAS (2013) 436 348
ISIS Llinares, Mota, Winther A&A (2014) 562 A78
DGPM, Schmidt PRD80, 043001

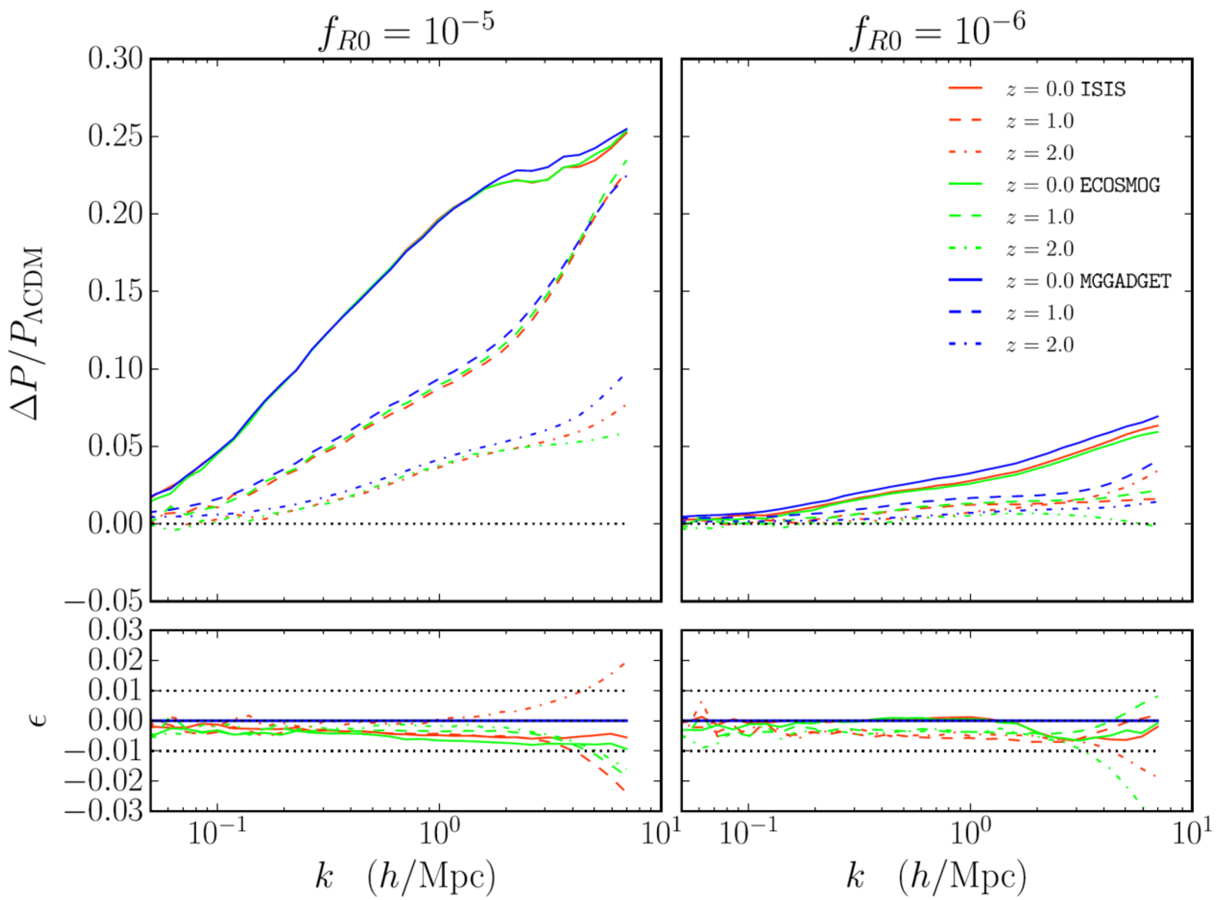
Modified Gravity Simulations comparison project
Winther, Shcmidt, Barreira et.al. arXiv: 1506.06384





Example f(R) gravity

$$f(R) \simeq -2\kappa^2 \rho_\Lambda - f_{R0} \frac{R_0^2}{R}$$



Three independent codes agree within 1% up to $k = 3h/\mathrm{Mpc}$

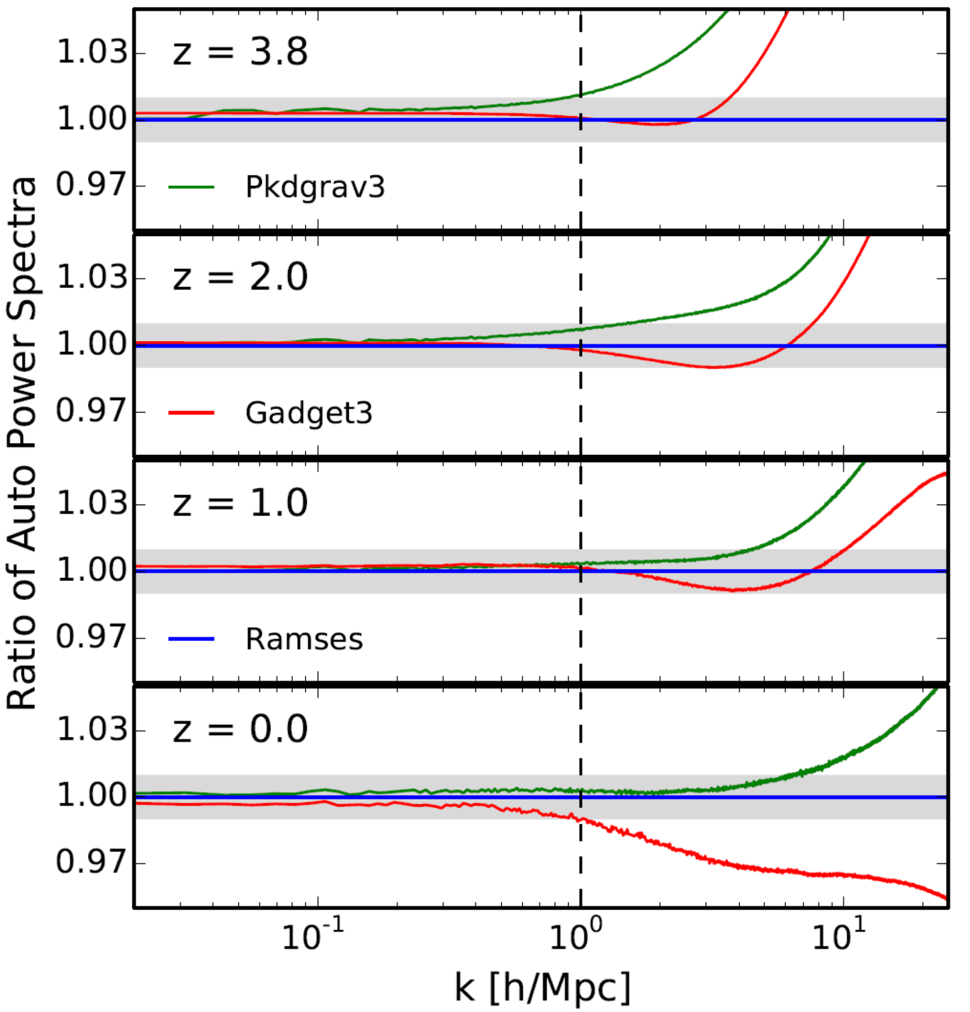
Winther et.al. 1506. 06384

Comparisons in LCDM

- Comparisons between three codes

  RAMSES (AMR)
  Gadget3 (tree-PM)
  Pkgrav3 (tree)

  Schneider et.al. 1503.05920



Problems of MG simulations

- Computing time

  Solving non-linear scalar filed equation can be very slow
  (it can take 5-20 times more time than LCDM simulations)

- Various ideas to speed-up simulations
  - f(R) (chameleon) Bose et.al. 1611.09375

    In some models, the discretised equation can be solved analytically removing the necessity to solve non-linear equation
  - nDGP (Vainshtein) Barreira et.al. 1511.08200

    For the computation of dark matter statistics, there is no need to use refined meshes for the scalar as the scalar force is suppressed in high density regions

COmoving Lagrangian Acceleration method

- **COLA** Tassev, Zaldarriaga, Eisenstein 1301.0322

  The idea is to augment simulations with perturbation theory

  geodesic equation

  $$\partial_t^2 \boldsymbol{x} = -\nabla\Phi$$

  Compute the large scale displacement using 2nd order Lagrangian perturbations (2LPT) and solve the residual using normal N-body methods such as Particle-Mesh (PM)

  $$\partial_t^2 \boldsymbol{x}_{\rm res} = -\nabla\Phi - \partial_t^2 \boldsymbol{x}_{\rm LPT} \ , \quad \text{with } \boldsymbol{x}_{\rm res} \equiv \boldsymbol{x} - \boldsymbol{x}_{\rm LPT}$$

  It is possible to use large time steps

2LPT

- Poisson equation

$$\mathcal{F}_{\mathbf{x}}[\nabla^2_{\mathbf{x}}\Phi](\vec{k},a) = \kappa\,\mu(k,a)\delta^{\mathrm{E}}(\vec{k},a) \; + a^4H^2\int\frac{\mathrm{d}^3k_1\mathrm{d}^3k_2}{(2\pi)^3}\delta^{(1)}(\vec{k_1},a)\delta^{(1)}(\vec{k_2},a)\gamma_2^{\mathrm{E}}(\vec{k},\vec{k_1},\vec{k_2},a)$$

Lagrangian space

$$\mathcal{F}_{\mathbf{q}}[\nabla^2_{\mathbf{x}}\Phi](\vec{k},a) = \kappa\,\mu(k,a)\delta(\vec{k},a) + a^4H^2\int\frac{\mathrm{d}^3k_1\mathrm{d}^3k_2}{(2\pi)^3}\delta^{(1)}(\vec{k_1},a)\delta^{(1)}(\vec{k_2},a)\gamma_2(\vec{k},\vec{k_1},\vec{k_2},a)$$

Displacement $\vec{\Psi}^{(i)} = \vec{\nabla}_{\mathbf{q}}\phi^{(i)}$

$$\gamma_2 = \gamma_2^{\mathrm{E}} + \tfrac{3}{2}\Omega_m(a)\left[\mu(k,a)-\mu(k_1,a)\right]\frac{\vec{k_1}\cdot\vec{k_2}}{k_2^2}$$

$$\phi^{(1)}(\vec{k},\tau) = D_1(k,\tau)\phi^{(1)}(\vec{k},\tau_{\mathrm{ini}}) \qquad \frac{d^2D_1}{d\tau^2} - \kappa\boxed{\mu(k,a)}D_1 = 0$$

$$\phi^{(2)}(\vec{k},\tau) = -\frac{1}{2k^2}\int\frac{\mathrm{d}^3k_1\mathrm{d}^3k_2}{(2\pi)^3}\delta_D(\vec{k}-\vec{k}_{12})\;\delta^{(1)}(\vec{k_1},\tau_{\mathrm{ini}})\delta^{(1)}(\vec{k_2},\tau_{\mathrm{ini}})D_2(\vec{k},\vec{k_1},\vec{k_2},\tau)$$

$$\frac{d^2D_2}{d\tau^2} - \kappa\,\mu(k,a)D_2 = -\kappa\,\mu(k,a)D_1(k_1,\tau)D_1(k_2,\tau)$$
$$\times\left(1-\left(\frac{2\mu(k_1,a)-\mu(k,a)}{\mu(k,a)}\right)\frac{(\vec{k_1}\cdot\vec{k_2})^2}{k_1^2k_2^2}+\frac{2a^4H^2}{\kappa\,\mu(k,a)}\gamma_2(\vec{k},\vec{k_1},\vec{k_2},a)\right)$$

2LPT and PM

- LCDM approximation

$$\phi^{(2)}(\vec{k},\tau) = \hat{D}_2(k,\tau)\phi^{(2)}(\vec{k},\tau_{\rm ini})$$

$$\phi^{(2)}(\vec{k},\tau_{\rm ini}) = -\frac{1}{2k^2}\int \frac{{\rm d}^3k_1{\rm d}^3k_2}{(2\pi)^3}\delta_D(\vec{k}-\vec{k}_{12})\ \delta^{(1)}(\vec{k_1},\tau_{\rm ini})\delta^{(1)}(\vec{k_2},\tau_{\rm ini})\left(1-\frac{(\vec{k_1}\cdot\vec{k_2})^2}{k_1^2k_2^2}\right)$$

$$\frac{d^2\hat{D}_2}{d\tau^2} - \kappa\,\mu(k,a)\hat{D}_2 = -\,\kappa\,\mu(k,a)D_1^2(k,a)\left(1+\frac{2a^4H^2}{\kappa\,\mu}\gamma_2(k,k/\sqrt{2},k/\sqrt{2},a)\right)$$

- Particle mesh part

We include screening effects by the following approximation

$$F_\varphi = 2\,\beta^2\,F_{Newton}\,\varepsilon(\Psi,\nabla\Psi,\nabla^2\,\Psi)$$

Examples

- f(R)

$$\mu(k,a) = 1 + \frac{1}{3}\frac{k^2}{k^2 + a^2 m^2(a)} \qquad m^2(a) = \frac{1}{3 f_{RR}(a)} = \frac{H_0^2(\Omega_m + 4\Omega_\Lambda)}{2|f_{R0}|}\left(\frac{\Omega_m a^{-3} + 4\Omega_\Lambda}{\Omega_m + 4\Omega_\Lambda}\right)^3$$

$$\gamma_2^{\mathrm{E}} = -\frac{9\Omega_m^2}{48 a^6 |f_{R0}|^2}\left(\frac{k}{aH}\right)^2 \times \frac{(\Omega_m a^{-3} + 4\Omega_\Lambda)^5}{(\Omega_m + 4\Omega_\Lambda)^4}\frac{1}{\Pi(k,a)\Pi(k_1,a)\Pi(k_2,a)}$$

$$\Pi(k,a) = \left(\frac{k}{aH_0}\right)^2 + \frac{(\Omega_m a^{-3} + 4\Omega_\Lambda)^3}{2|f_{R0}|(\Omega_m + 4\Omega_\Lambda)^2}$$

$$\vec{F}_\phi = \frac{1}{3}\cdot \vec{F}_{\mathrm{Newton}} \cdot \epsilon_{\mathrm{screen}}(\Phi_N) \qquad \epsilon_{\mathrm{screen}}(\Phi_N) = \mathrm{Min}\left[1, \left|\frac{3 f_R(a)}{2\Phi_N}\right|\right] \qquad f_R(a) = f_{R0}\left(\frac{\Omega_m + 4\Omega_\Lambda}{\Omega_m a^{-3} + 4\Omega_\Lambda}\right)^2$$

- nDGP

$$\mu(k,a) = 1 + \frac{1}{3\beta_{\mathrm{DGP}}(a)} \qquad \beta_{\mathrm{DGP}}(a) = 1 + 2 r_c H(a)\left(1 + \frac{\dot{H}}{3H^2}\right)$$

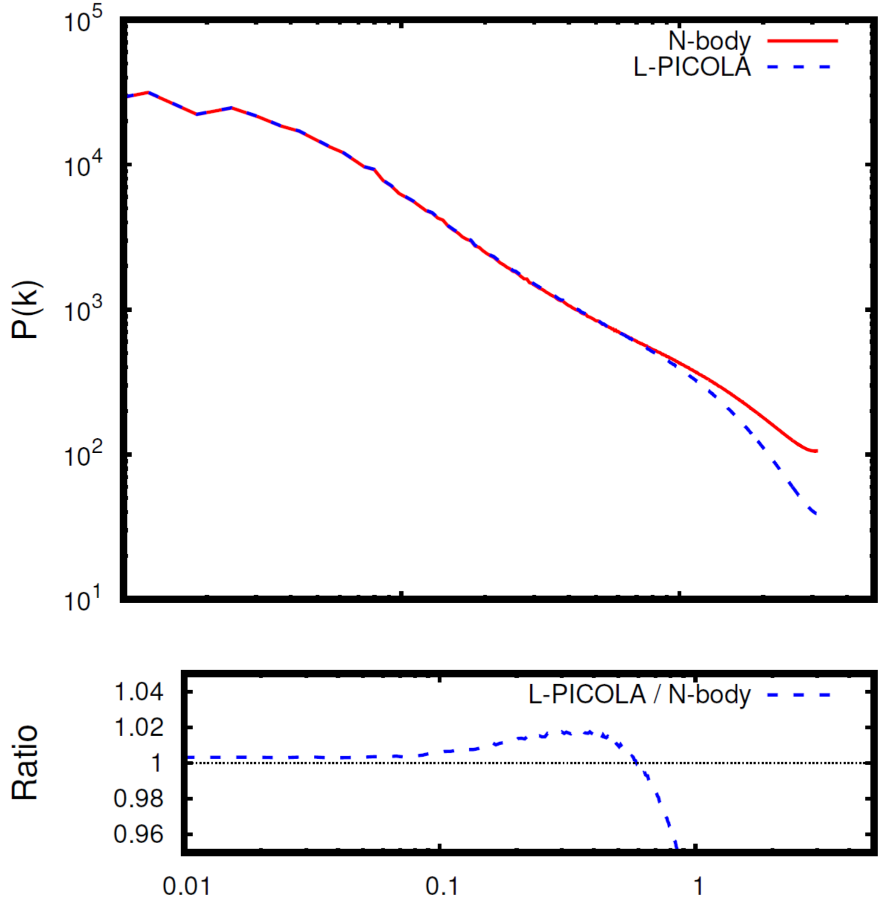
$$\gamma_2^{\mathrm{E}} = -\left(\frac{H_0}{H}\right)^2 \frac{(r_c H_0)^2 \Omega_m^2}{6\beta_{\mathrm{DGP}}^3(a) a^6}\left(1 - \frac{(\vec{k}_1\cdot\vec{k}_2)^2}{k_1^2 k_2^2}\right)$$

$$\vec{F}_\phi = \frac{1}{3\beta_{\mathrm{DGP}}(a)}\cdot \vec{F}_{\mathrm{Newton}} \cdot \epsilon_{\mathrm{screen}}(\rho) \qquad \epsilon_{\mathrm{screen}}(\rho) = \frac{2\sqrt{1+x}}{x} \qquad x = \frac{8(r_c H_0)^2 \Omega_m}{9\beta_{\mathrm{DGP}}^2(a)}\frac{\rho}{\bar{\rho}}$$

MG-PICOLA

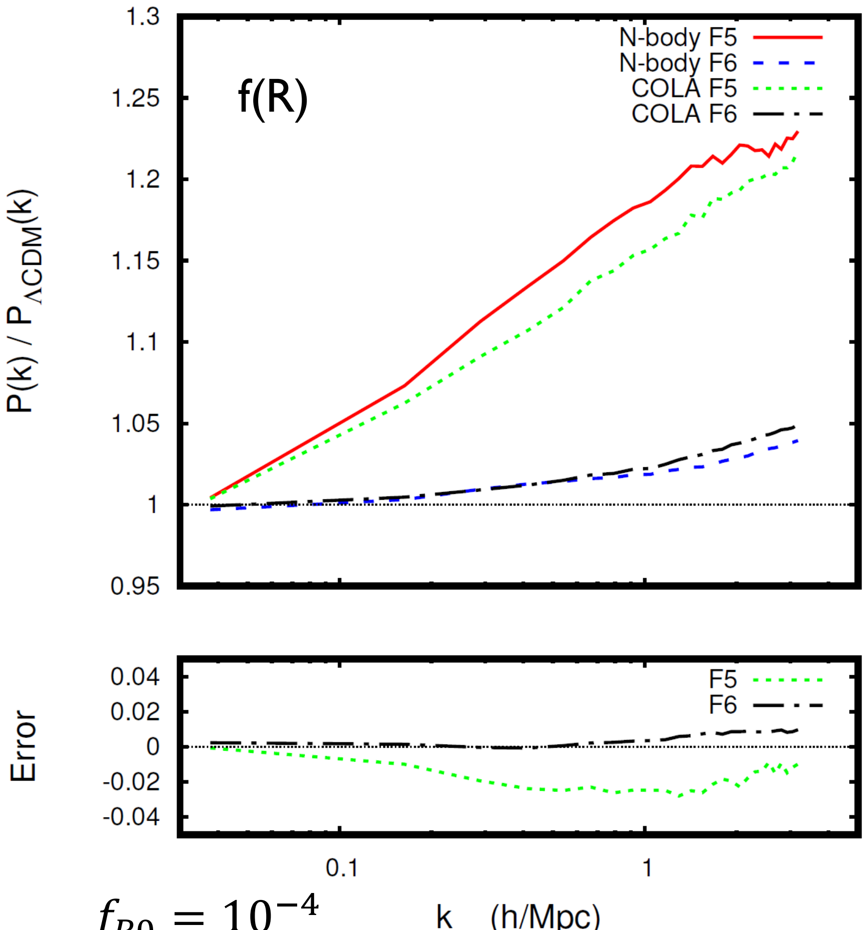
Winther, Koyama, Manera, Wright, Zhao 1703.00879
based on L-PICOLA Howlett et.al. 1506.03737

- Comparison with an AMR code (RAMSES)

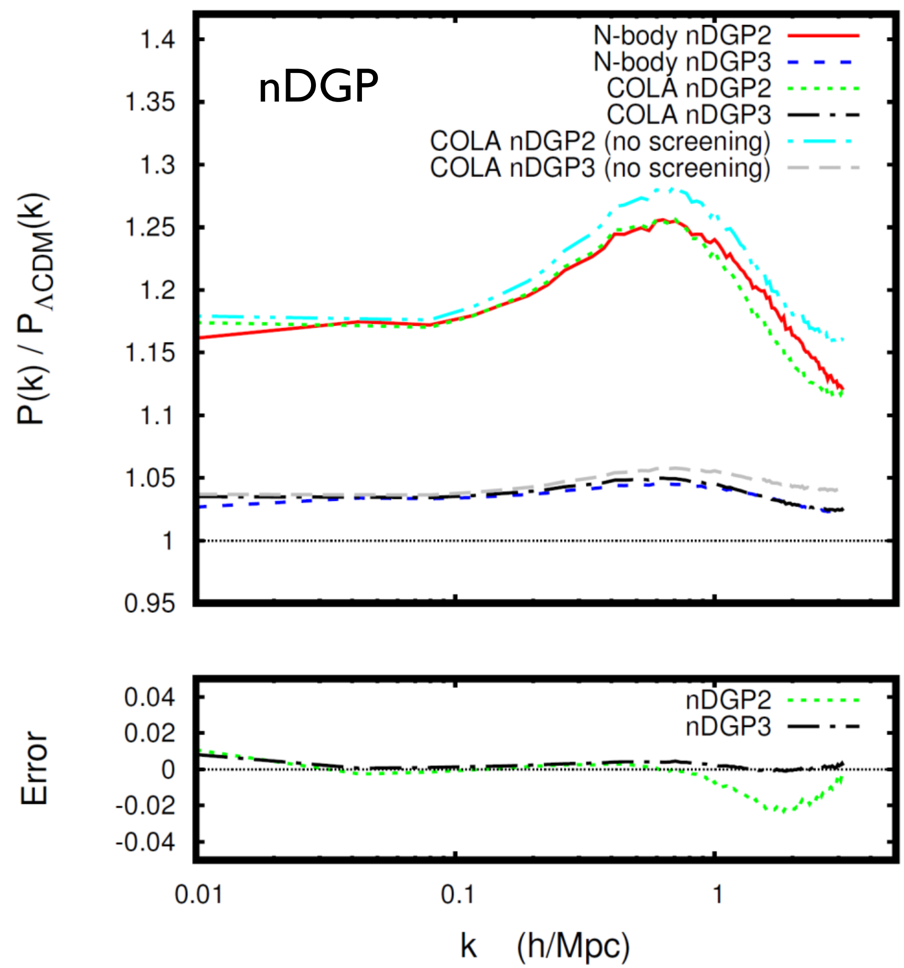


Modified gravity examples

- Power spectrum

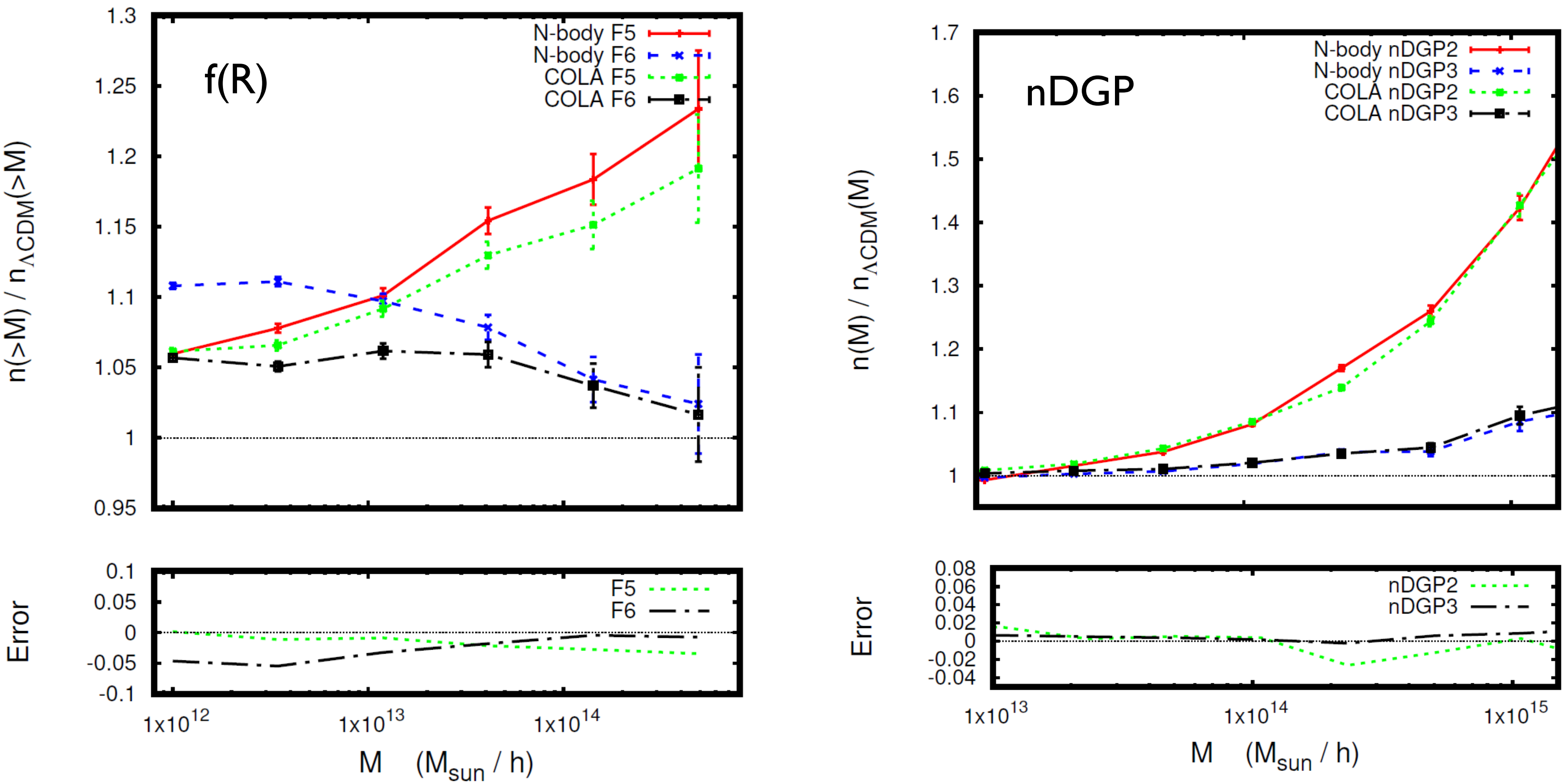


$f_{R0} = 10^{-4}$



Halo mass function

- Halo mass function



Massive neutrinos

Wright, Winther, Koyama 1705.08165

- First order

$$\bar{\rho}_{\rm cb}\delta^{(1)}_{\rm cb} + \bar{\rho}_\nu\delta^{(1)}_\nu = \left(\frac{\bar{\rho}_{\rm cb}}{\bar{\rho}_{\rm m}} + \frac{\bar{\rho}_\nu}{\bar{\rho}_{\rm m}}\frac{\delta^{(1)}_\nu}{\delta^{(1)}_{\rm cb}}\right)\bar{\rho}_{\rm m}\delta^{(1)}_{\rm cb} = \left(f_{\rm cb} + f_\nu\frac{D_{1,\nu}(k,\tau)}{D_{1,{\rm cb}}(k,\tau)}\right)\bar{\rho}_{\rm m}\delta^{(1)}_{\rm cb}$$

$$= \bar{\rho}_{\rm m}\,\mu_{m_\nu}(k,\tau)\delta^{(1)}_{\rm cb}$$

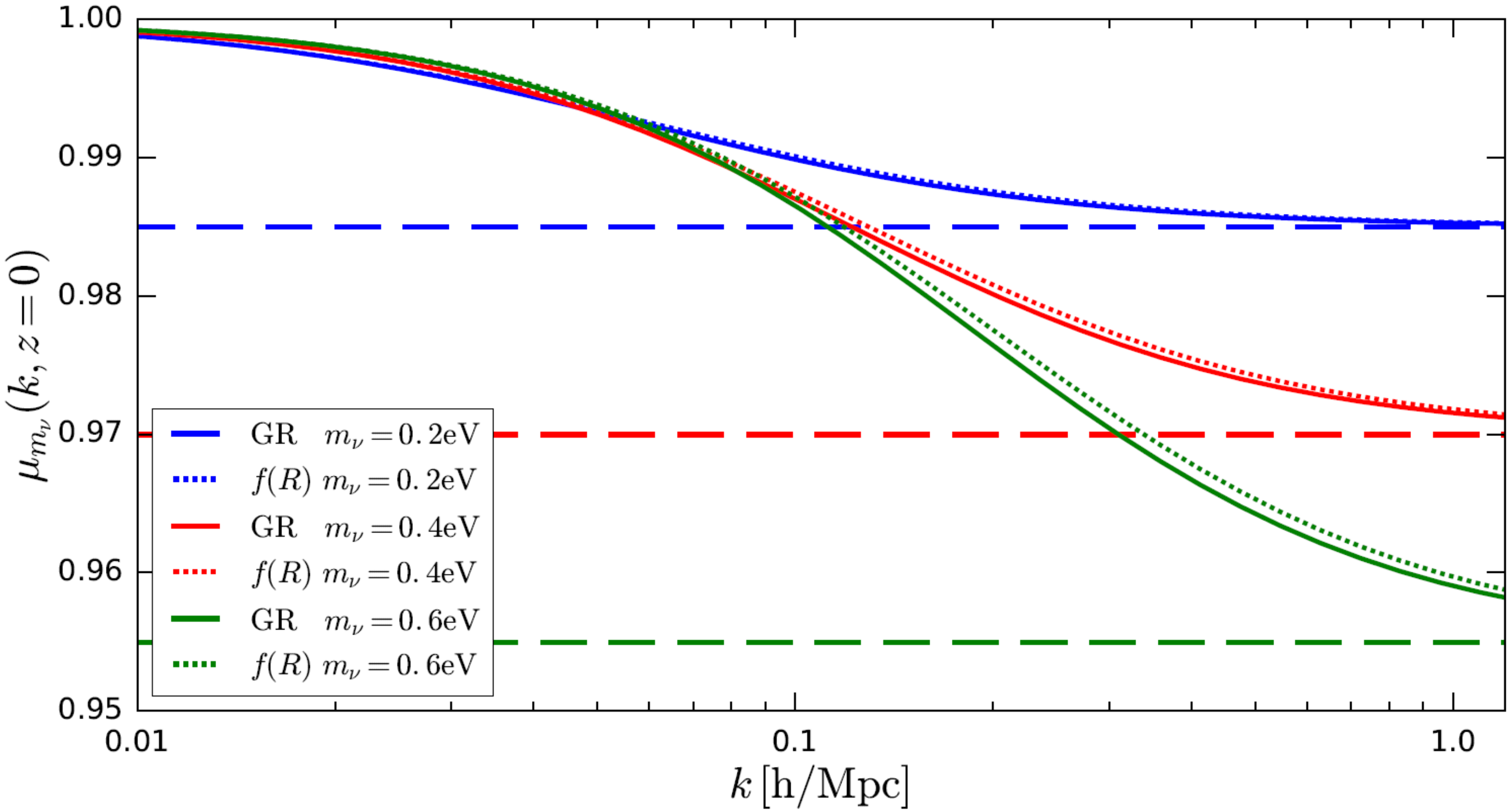
$$\left(\frac{d^2}{d\tau^2} - \kappa\,\mu_{m_\nu}(k,\tau)\right)\phi^{(1)}_{\rm cb}(k,\tau) = 0$$

Massive neutrinos act like modified gravity to CDM/baryons

$\mu_{m_\nu}(k,\tau)$ can be computed using the linear Boltzmann code or the Eiseinstein-Hu fitting formula

- Non-linear order

We assume massive neutrinos remain linear while CDM/baryons become fully non-linear



1.00
0.99
0.98
0.97
0.96
0.95
0.01
0.1
1.0
$k\,[\mathrm{h/Mpc}]$
$\mu_{m_\nu}(k, z=0)$
GR $m_\nu = 0.2\mathrm{eV}$
$f(R)$ $m_\nu = 0.2\mathrm{eV}$
GR $m_\nu = 0.4\mathrm{eV}$
$f(R)$ $m_\nu = 0.4\mathrm{eV}$
GR $m_\nu = 0.6\mathrm{eV}$
$f(R)$ $m_\nu = 0.6\mathrm{eV}$

PM part

- Initial conditions
  - Generate massive neutrinos initial conditions using the same phases as CDM

$$\delta_\nu(\vec{k},\tau_{\rm ini}) = \delta_{\rm cb}(\vec{k},\tau_{\rm ini})\frac{T_\nu(k,\tau_{\rm ini})}{T_{\rm cb}(k,\tau_{\rm ini})} = \delta_{\rm cb}(\vec{k},\tau_{\rm ini})\frac{D_{1,\nu}(k,\tau_{\rm ini})}{D_{1,{\rm cb}}(k,\tau_{\rm ini})}$$

  - We evolve this using the linear Boltzmann code

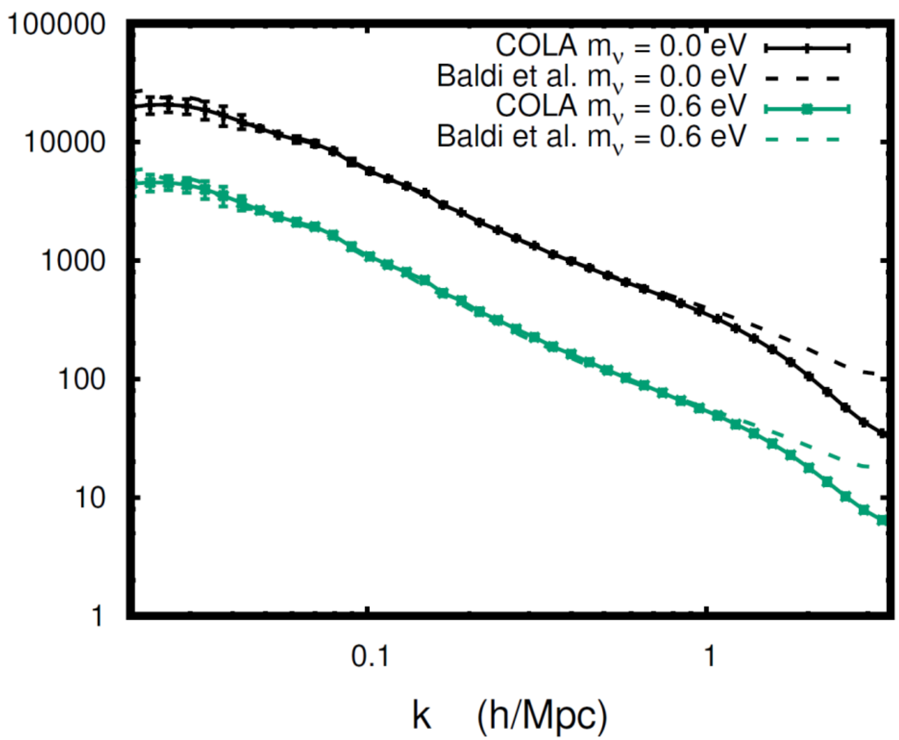
$$\delta_\nu(\vec{k},\tau) = \delta_\nu(\vec{k},\tau_{\rm ini})\frac{T_\nu(k,\tau)}{T_\nu(k,\tau_{\rm ini})} = \delta_\nu(\vec{k},\tau_{\rm ini})\frac{D_{1,\nu}(k,\tau)}{D_{1,\nu}(k,\tau_{\rm ini})}$$

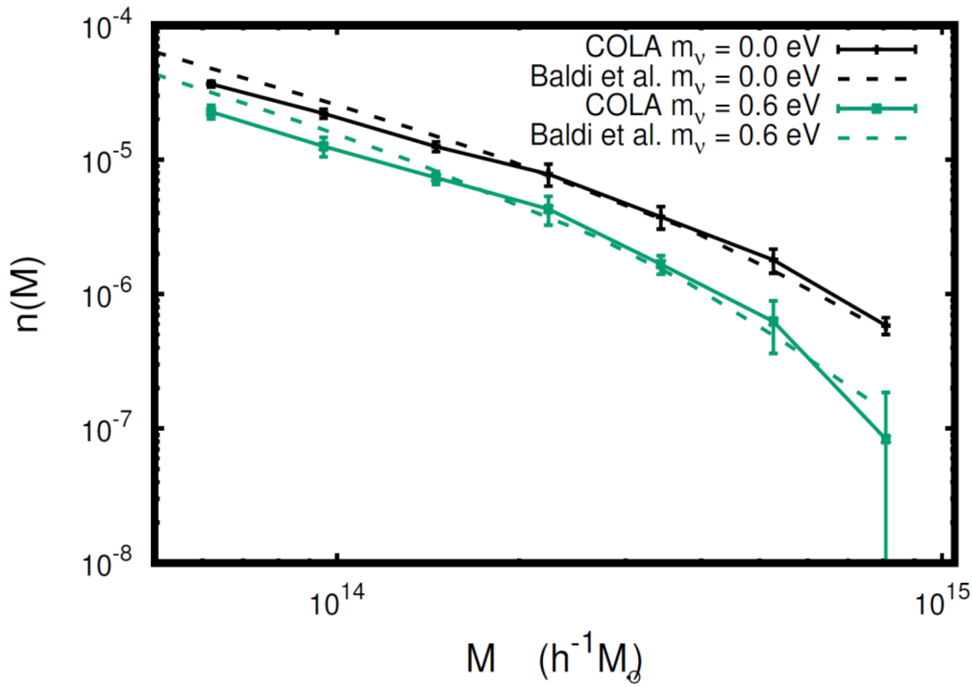
  - The Newtonian potential

$$-k^2\Phi(\vec{k},\tau) = \frac{3}{2}\Omega_{\rm m}a\left[f_{cb}\delta_{\rm cb}(\vec{k},\tau) + f_\nu\delta_\nu(\vec{k},\tau)\right]$$

Comparisons with N-body

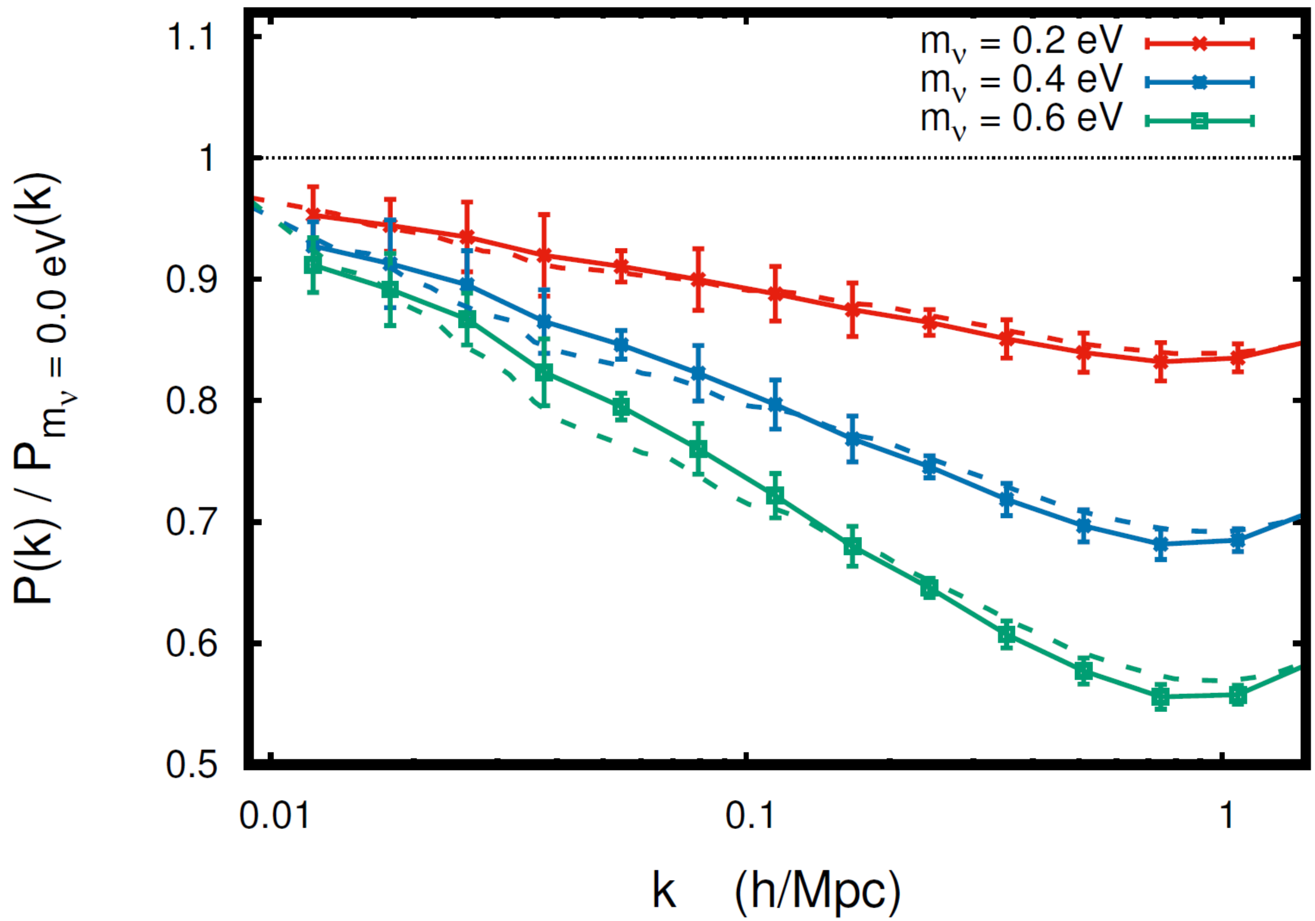
- Comparison with particle based N-body simulations (Gadget) Baldi et.al. 1311.2588





Effect of massive neutrinos

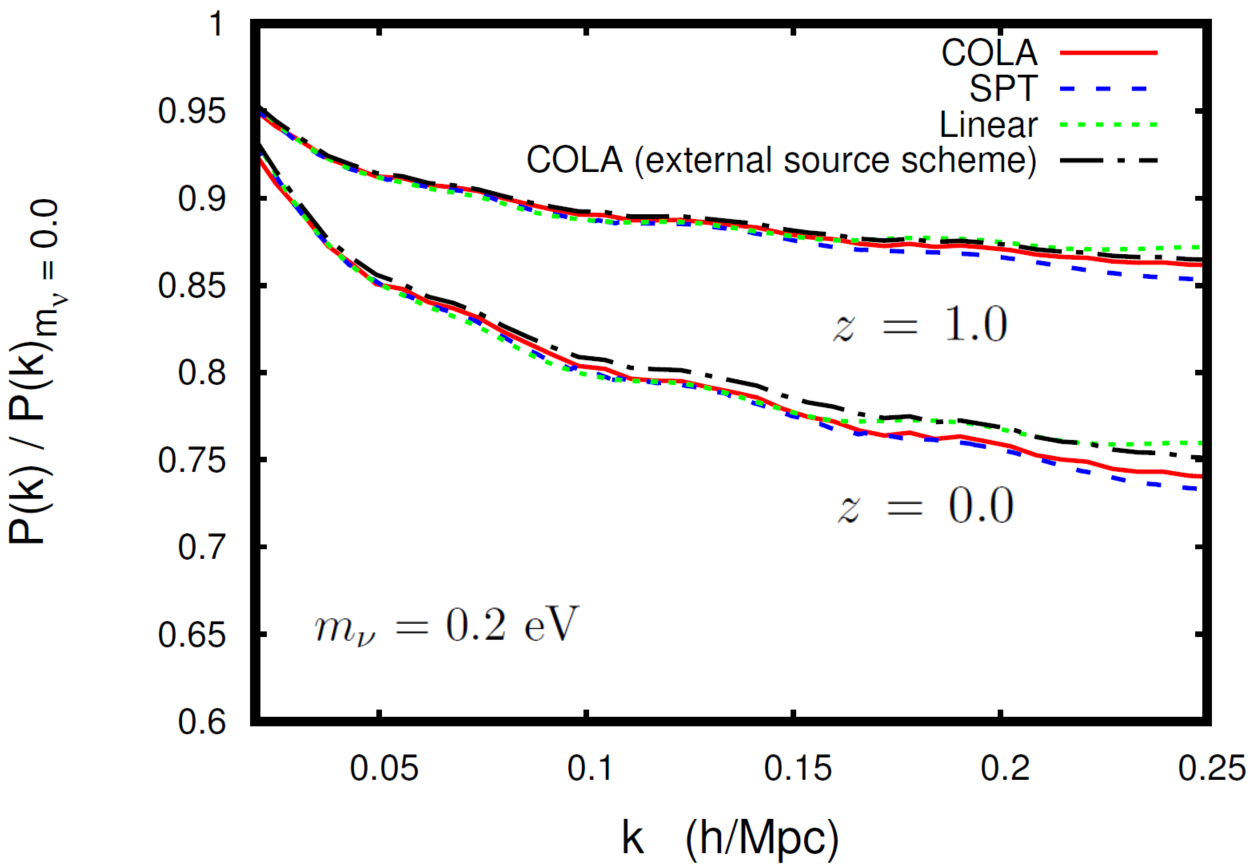
- LCDM



Effect of massive neutrinos

- Comparison with SPT

Saito, Takada & Taruya 0801.0607, Blas et.al. 1408.2995
Levi and Vlah 1605.0941



External source scheme

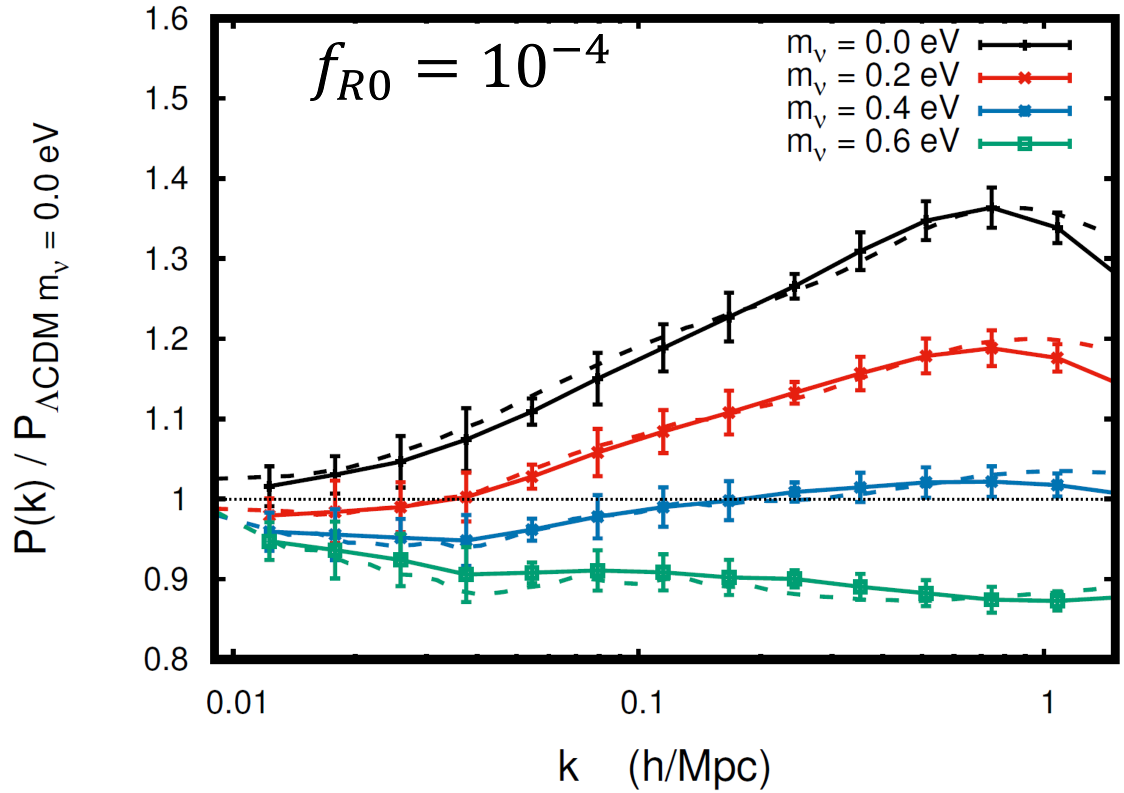
$$\delta_\nu = \delta_{cb} \frac{\delta_\nu^{\rm lin}}{\delta_{cb}^{\rm lin}}$$

Massive neutrinos in modified gravity

- Modified gravity

  Inclusion of modified gravity is computationally cheap

  (comparison with MG-Gadget Baldi et.al. 1311.2588)



degeneracy

Motohashi, Staronbinsky & Yokoyama 1203.6828

Summary

- **Screening mechanism**

  GR is recovered on small scales due to the non-linearity of the scalar mode

- **N-body simulations**

  N-body simulations have been developed to understand non-linear clustering of dark matter in the presence of screening

- **Fast methods**

  Fast approximated methods are required to create many galaxy mocks

  We provided such a method using COLA by augmenting simulations by perturbation theory

  Inclusion of massive neutrinos using the linear approximation for the neutrino density is computationally cheap

- **Other applications**

  Inclusion of general relativistic effects in Newtonian simulations?