RESPONSE FUNCTION OF THE COSMIC DENSITY POWER SPECTRUM AND RECONSTRUCTION

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Based on PLB 762 (2016) 247 and arXiv:1708.08946
See also RESPRESSO webpage:
http://www-utap.phys.s.u-tokyo.ac.jp/~nishimichi/public_codes/respresso/
OBJECTIVES

• To extract information from large-scale clustering signals, we need
  • an **accurate** theory/model
    ➡ to meet the low statistical error level of big surveys
  • **quick** evaluation
    ➡ to explore multi-D cosmological parameter space

• Different approaches available
  • Perturbation theory (and its variant)
    • Accuracy: large scale good (?), small scale no
    • Speed: high loop order (2 or 3 loops) takes time
  • (N-body) simulations
    • Accuracy: good (w/ sufficient volume/realizations, only after a careful convergence study)
    • Speed: takes much more time

• **RESPRESSO**!
  • A possible integration of the 2 approaches
PERTURBATION THEORY IS IN CRISIS

Blas, Garny, Konstandin ‘14
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Blas, Garny, Konstandin '14
The success of the next-to-next-to-leading (2 loop) order calculation just an illusion?
WHY?

- Perturbation theory (PT) is fine only when the quantity of interest is small and the series expansion is convergent
  - Overdensity can reach $\gg 1$ at present
  - The fluid is assumed to follow an irrotational single-streaming flow
  - Enters to multi-streaming phase after shell crossing (even if the initial condition is cold)
- These 2 things happen together (small scale, late time)
  - The breakdown can propagate to large scales due to mode coupling
  - The way how PT breaks down is totally non-trivial —> simulations!
WHAT TO MEASURE?

- Basic things that determine the nonlinear mode-coupling structure:

\[
\{ F_n(q_1, \ldots, q_n), G_n(q_1, \ldots, q_n) \}
\]

\[
\{ \Gamma^{(n)}_\delta(q_1, \ldots, q_n), \Gamma^{(n)}_\theta(q_1, \ldots, q_n) \}
\]

\[
K(k, q) = q \frac{\delta P_{nl}(k)}{\delta P_{lin}(q)}
\]
STANDARD PT KERNEL

- Very, very basic things (not observable from sims, though…)

\[ \{ F_n(q_1, \ldots, q_n), G_n(q_1, \ldots, q_n) \} \]

continuity + Euler + Poisson eqs.

\[ \frac{\partial \delta}{\partial t} + \frac{1}{a} \nabla \cdot [(1 + \delta)v] = 0, \]
\[ \frac{\partial v}{\partial t} + Hv + \frac{1}{a} (v \cdot \nabla)v = -\frac{1}{a} \nabla \phi, \]
\[ \nabla^2 \phi = 4\pi G \bar{\rho} a^2 \delta. \]

\[ \delta(x) = \rho(x)/\bar{\rho} - 1 \]
\[ \theta(x) = \nabla \cdot v(x) \]

\[ \tilde{\delta}(k, \tau) = \sum_{n=1}^{\infty} a^n(\tau) \delta_n(k), \quad \tilde{\theta}(k, \tau) = -H(\tau) \sum_{n=1}^{\infty} a^n(\tau) \theta_n(k) \]

\[ \delta_n(k) = \int d^3q_1 \ldots \int d^3q_n \delta_D(k - q_1 \ldots q_n) F_n(q_1, \ldots, q_n) \delta_1(q_1) \ldots \delta_1(q_n), \]
\[ \theta_n(k) = \int d^3q_1 \ldots \int d^3q_n \delta_D(k - q_1 \ldots q_n) G_n(q_1, \ldots, q_n) \delta_1(q_1) \ldots \delta_1(q_n) \]
GAMMA EXPANSION (eg., RegPT)  
Bernardeau + ’09

\[
\left\{ \Gamma^{(n)}_\delta (q_1, \ldots, q_n), \Gamma^{(n)}_\theta (q_1, \ldots, q_n) \right\}
\]

• Can calibrate against sims
• Good ansatz known
• Simpler expression for the spectra
• Clear physical interpretation (Crocce & Scoccimarro ’06 for the 2pt propagator)

Bernardeau, Taruya & TN ’14

• final state (density or velocity)
• multi-point propagator

\[
\frac{1}{p!} \left\langle \frac{\delta^p \Psi_a (k, \eta)}{\delta \phi_{b_1} (k_1) \ldots \delta \phi_{b_p} (k_p)} \right\rangle = \delta_D (k - k_1 - \ldots - k_p) \Gamma^{(p)}_{ab_1 \ldots b_p} (k_1, \ldots, k_p; \eta)
\]

Bernardeau, Taruya & TN ’14

standard PT

Exp. asymptote
VIRTUE OF RESUMMATION

- Better convergence: everything is positive definite
- BAO almost done at the lowest order

![Diagram](image-url)
MORE INTUITIVE QUANTITY? RESPONSE!

large scale structure gravitational evolution

Input

\((\Omega_m, h, \ldots; z)\)

Output

\(P_{nl}(k)\)
MORE INTUITIVE QUANTITY? RESPONSE!

large scale structure gravitational evolution

Input

$\left(\Omega_m, h, \ldots; z\right)$

$P_{\text{lin}}(k)$

To a very good approximation

Output

$P_{\text{nl}}(k)$
large scale structure gravitational evolution

Input

\[ P_{\text{lin}}(k) \]

Output

\[ P_{\text{nl}}(k) \]

\[ K(k, q) = q \frac{\delta P_{\text{nl}}(k)}{\delta P_{\text{lin}}(q)} \]
large scale structure gravitational evolution

\[ K(k, q) = q \frac{\delta P_{nl}(k)}{\delta P_{\text{lin}}(q)} \]
MORE INTUITIVE QUANTITY? RESPONSE!

large scale structure gravitational evolution

Input

\[ P_{\text{lin}}(k) \]

\[ + \delta P_{\text{lin}}(k) \]

Output

\[ P_{\text{nl}}(k) \]

\[ K(k, q) = q \frac{\delta P_{\text{nl}}(k)}{\delta P_{\text{lin}}(q)} + \delta P_{\text{nl}}(k) \]
I want to study this mode at some late time $t$.

The impact of wave mode $q$ at the initial time $t_0$ is given by the response function:

$$K(k, q) = q \frac{\delta P_{nl}(k)}{\delta P_{lin}(q)}$$
RESPONSE FUNCTION: THE FIRST TRIAL

- From order-by-order to the full order discussion possible
- Can estimate the derivative from a simulation ensemble

\[
\hat{K}_{i,j} P_{j}^{\text{lin}} \equiv \frac{P_{i}^{\text{nl}}[P_{+,j}^{\text{lin}}] - P_{i}^{\text{nl}}[P_{-,j}^{\text{lin}}]}{\Delta \ln P^{\text{lin}} \Delta \ln q}
\]

![Graph showing the initial and final states of wave modes](image)

<table>
<thead>
<tr>
<th>L9-N8</th>
<th>L9-N9</th>
<th>L10-N9</th>
</tr>
</thead>
<tbody>
<tr>
<td>512 256</td>
<td>512 512</td>
<td>31 15 1 30</td>
</tr>
</tbody>
</table>

TABLE I: Initial wave mode q [h/Mpc] and volume of the simulations. We hereafter discuss the comparison with PT combination results. The vertical error bars of filled triangles depict the statistical scatter, while those of open triangles or solid lines (lower triangles or dashed lines). Positive (negative) values of \( \Delta K_{i,j} P_{j}^{\text{lin}} \) are shown by upper triangle or solid line, while lower triangle or dashed line. We consider 15 or 30 runs used in this analysis. They are in different bins are created with exactly the same random numbers. Initial conditions are created using a parallel code developed in [11]. We follow the time evolution of the matter distribution using the smoothing e-function of the Cloud-in-Cell (CIC) method. Initial conditions and the error in the tree-force calculation, the sum of the transient shifts shown in the table are determined to minimize the relative constant in units of \( 100 \cdot \Omega_{m}^{0.1} \). It is straightforward to show that the estimator for the linear power spectrum and we set a log-equal bin.

FIG. 1: Results of the comparison of the kernel function measured from simulations. We plot \( \Delta K_{i,j} P_{j}^{\text{lin}} \) as a function of \( k \) at fixed final state and fixed value of fixed final wavenumber \( q_{f} \). We show by vertical arrows the position of the kernel presented in each panel. We show the three simulation results by filled symbols and lines (lower triangles or dashed lines). Positive (negative) values of \( \Delta K_{i,j} P_{j}^{\text{lin}} \) are shown by upper triangle or solid line, while lower triangle or dashed line. We show by vertical arrows the position of the kernel presented in each panel. We show the three simulation results by filled symbols and lines (lower triangles or dashed lines). Positive (negative) values of \( \Delta K_{i,j} P_{j}^{\text{lin}} \) are shown by upper triangle or solid line, while lower triangle or dashed line.
RESPONSE FUNCTION: SIM VS PT

- SPT (2-loop) $\gg$ N-body @ high $q$
- This is exactly where PT breaks down
- What N-body tells us is:
  “Physics at strongly nonlinear regime does not propagate to large scales”

Rescaled quantity:

$$T(k, q) = \left[ K(k, q) - K^{\text{lin}}(k, q) \right] / \left[ qP^{\text{lin}}(k) \right]$$

$q \geq 2k$ are shown
HIGH RES RESPONSE FUNCTION

- 1400 N=512^3 simulations to study fine structures of the response function
- Vs 2-loop calculation based on different schemes (SPT and RegPT)
- New phenomenological model introduced

III. RESPONSE FUNCTION FROM PERTURBATION THEORY

In this section, we present analytical calculations of the response function based on perturbation theory (PT). The results are confronted with the response function measured from N-body simulations. As we will see below, the predictions made with the standard and reg-summed PT treatments do not perfectly match the simulation results, but in several different regimes, they quantitatively explain the measured results of response function. We discuss the reasons for these, and then propose a simple PT model that incorporates all the necessary ingredients.

TN, Bernardeau, Taruya ’17 (arXiv:1708.08946)
RESPONSE FUNCTION AT $q \ll k$

- Response function goes to zero from simulations
  - Extended galilean invariance
- This is nicely explained by SPT
- Not the case for RegPT
- Peaky structure decay as time goes by
- SPT behaves weirdly at late time
- RegPT has its strength in this regime
- Efficiently captures mode transfer between nearby modes
We need phenomenology here anyway!
**OUR MODEL**

\[
K_{\text{model}}(k, q) = \left[ (1 + \beta_{k,q} + \frac{1}{2} \beta_{k,q}^2) K_{\text{tree}}(k, q) + (1 + \beta_{k,q}) K_{1\text{-loop}}(k, q) + K_{2\text{-loop}}(k, q) \right] D(\beta_{k,q}),
\]

\[
D(x) = \begin{cases} 
\exp(-x), & \text{if } K_{\text{model}}(k, q) > 0, \\
\frac{1}{1 + x}, & \text{if } K_{\text{model}}(k, q) < 0.
\end{cases}
\]

\[
\beta_{k,q} = \alpha_k + \alpha_q
\]

Regularize both in \( k \) and \( q \)

\[
\alpha_k = \frac{1}{2} k^2 \sigma_d^2; \quad \sigma_d^2 = \int \frac{dq}{6\pi^2} P_{\text{lin}}(q),
\]

- Well-behaved over all \( q \)
- Eventually fails at high \( k \)
From the definition of a functional derivative

\[ P_{nl}(k; p_1) \approx P_{nl}(k; p_0) + \int d\ln q \ K(k, q) \]
\[ \times [P_{\text{lin}}(q; p_1) - P_{\text{lin}}(q; p_0)], \]

Use this to predict \( P_{nl} \) for cosmological model \( p_1 \) given \( P_{nl} \) for another model \( p_0 \)
STARTING POINT: SIMULATION DATABASE

- $P_{nl}$ database for the fiducial Planck 2015 cosmology from $10 \times 2048^3$ sims
- Cosmic variance suppressed with Angulo-Pontzen technique
- Fractional error < 0.1%
- Can smoothly interpolate over $k$ and time
A SIMPLE IMPLEMENTATION

- Double the reliable k range from the pure RegPT prediction
MORE EXTREME MODELS

- Employ multi-steps

![Diagram](image-url)
MORE EXTREME MODELS

- Employ multi-steps
RESPRESSO PYTHON PACKAGE AVAILABLE!

(Rapid and Efficient SPectrum calculation based on RESponSe functiOn)

```
In [1]: %pylab inline
   import respresso

In [3]: respresso_obj = respresso.respresso_core()
   Hello. This is RESPRESSO.
   Load precomputed data files...
   RESPRESSO ready.

In [6]: respresso_obj.set_target(plin_targetSpl)

In [7]: respresso_obj.find_path()

In [9]: kwave = respresso_obj.get_kinternal()
   pnl_rec = respresso_obj.reconstruct()
```

http://www-utap.phys.s.u-tokyo.ac.jp/~nishimichi/public_codes/respresso/
SUMMARY

• P(k) to a 2D quantity K(k,q): more physical insight
• Difficulty in perturbative approaches
  • Suppress small to large scale mode transfer!
  • SPT and RegPT have good and bad behavior in different regimes
• RESPRESSO package available
  • Response function is a natural interpolator over the cosmological parameter space
  • Can go to k ≈ 0.44 (0.35) h/Mpc at z=1 (0.5) within 1%
  • You can put your own simulation data if you do not like mine ;)

http://www-utap.phys.s.u-tokyo.ac.jp/~nishimichi/public_codes/respresso/