RESPONSE FUNCTION OF THE COSMIC DENSITY POWER SPECTRUM AND RECONSTRUCTION

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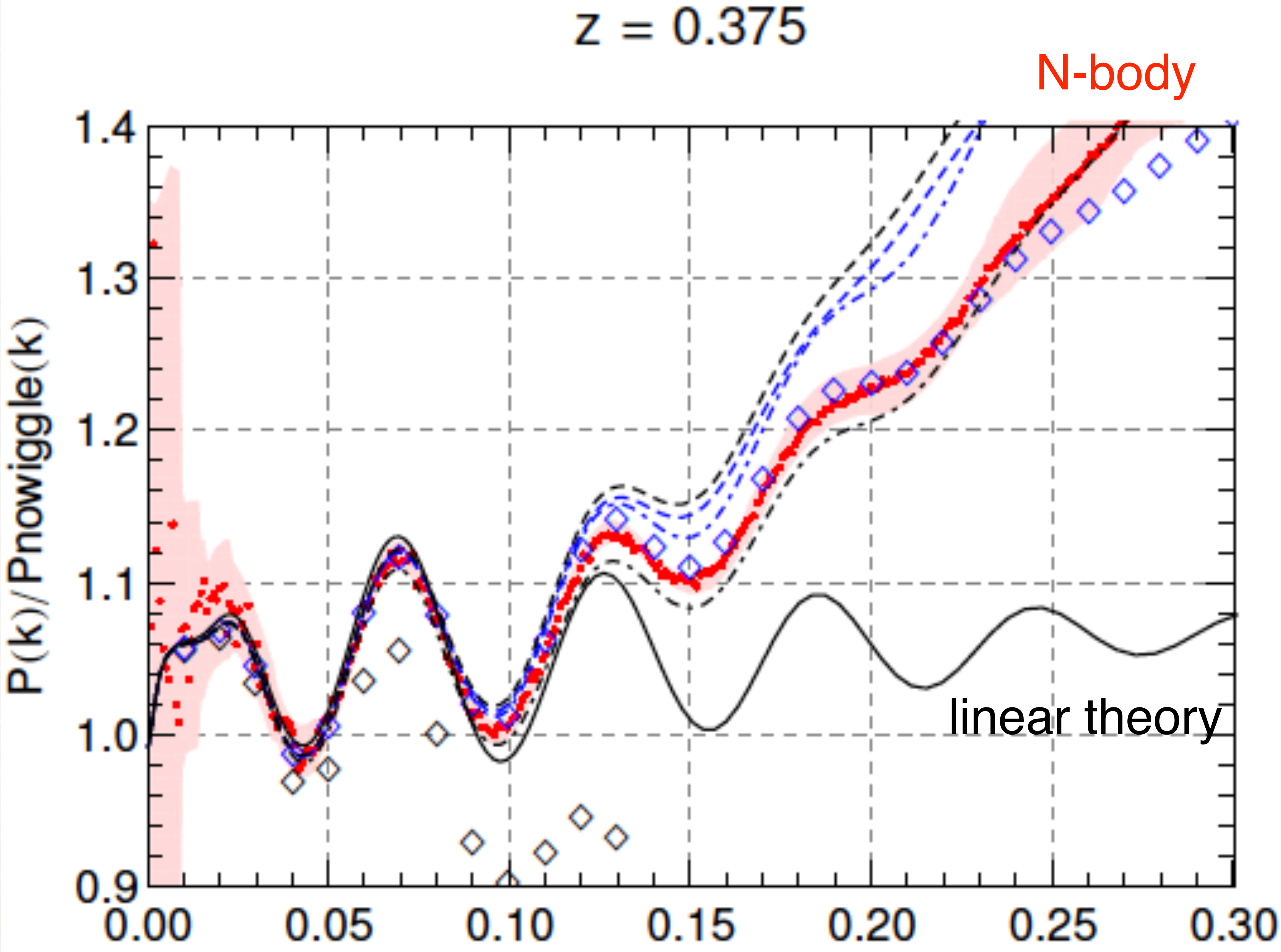
Based on PLB 762 (2016) 247 and arXiv:1708.08946
See also RESPRESSO webpage:
http://www-utap.phys.s.u-tokyo.ac.jp/~nishimichi/public_codes/respresso/

OBJECTIVES

- To extract information from large-scale clustering signals, we need
  - an accurate theory/model
    - ➡ to meet the low statistical error level of big surveys
  - quick evaluation
    - ➡ to explore multi-D cosmological parameter space
- Different approaches available
  - Perturbation theory (and its variant)
    - Accuracy: large scale good (?), small scale no
    - Speed: high loop order (2 or 3 loops) takes time
  - (N-body) simulations
    - Accuracy: good (w/ sufficient volume/realizations, only after a careful convergence study)
    - Speed: takes much more time
  - RESPRESSO !
    - A possible integration of the 2 approaches

PERTURBATION THEORY IS IN CRISIS



Blas, Garny, Konstandin ‘14

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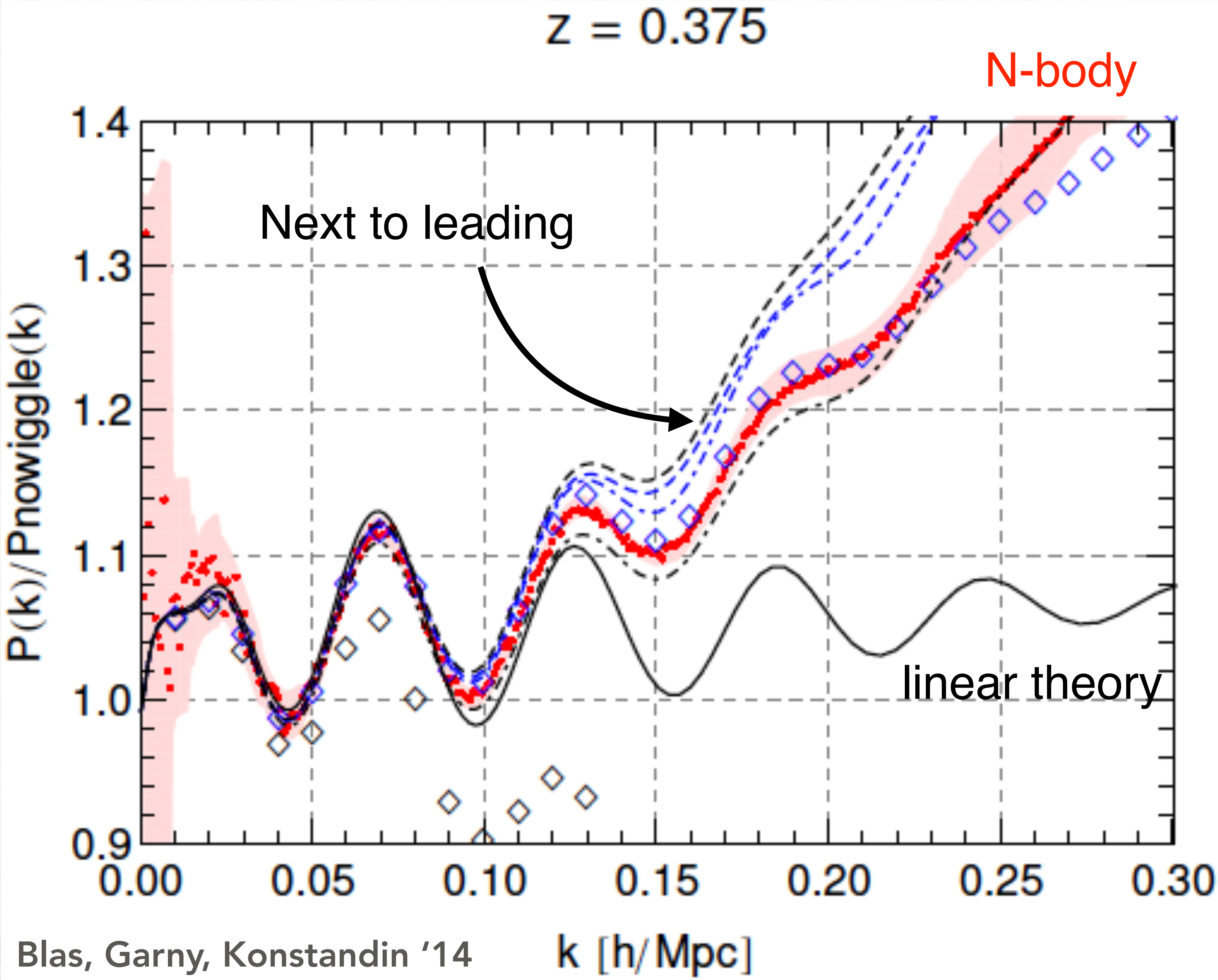

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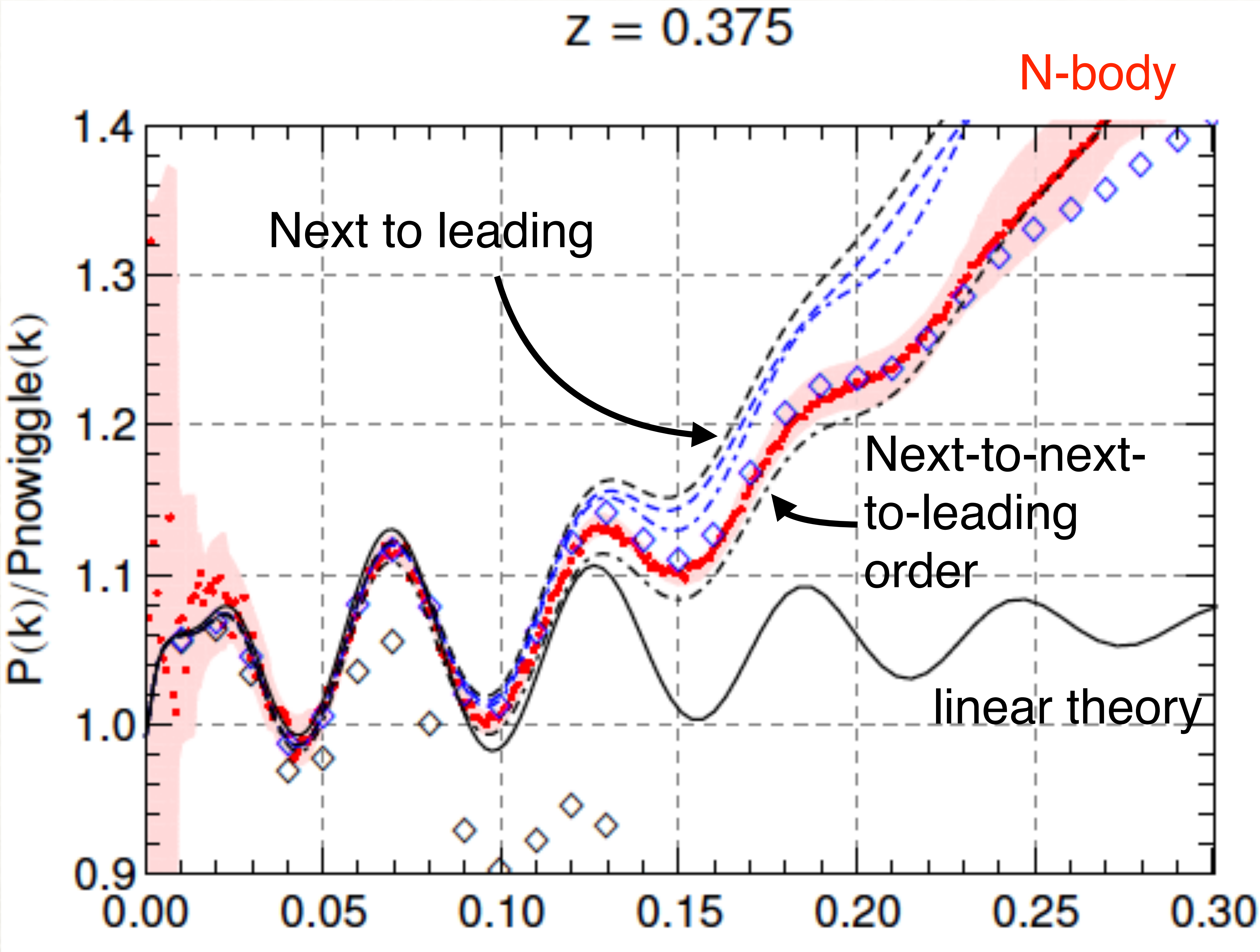

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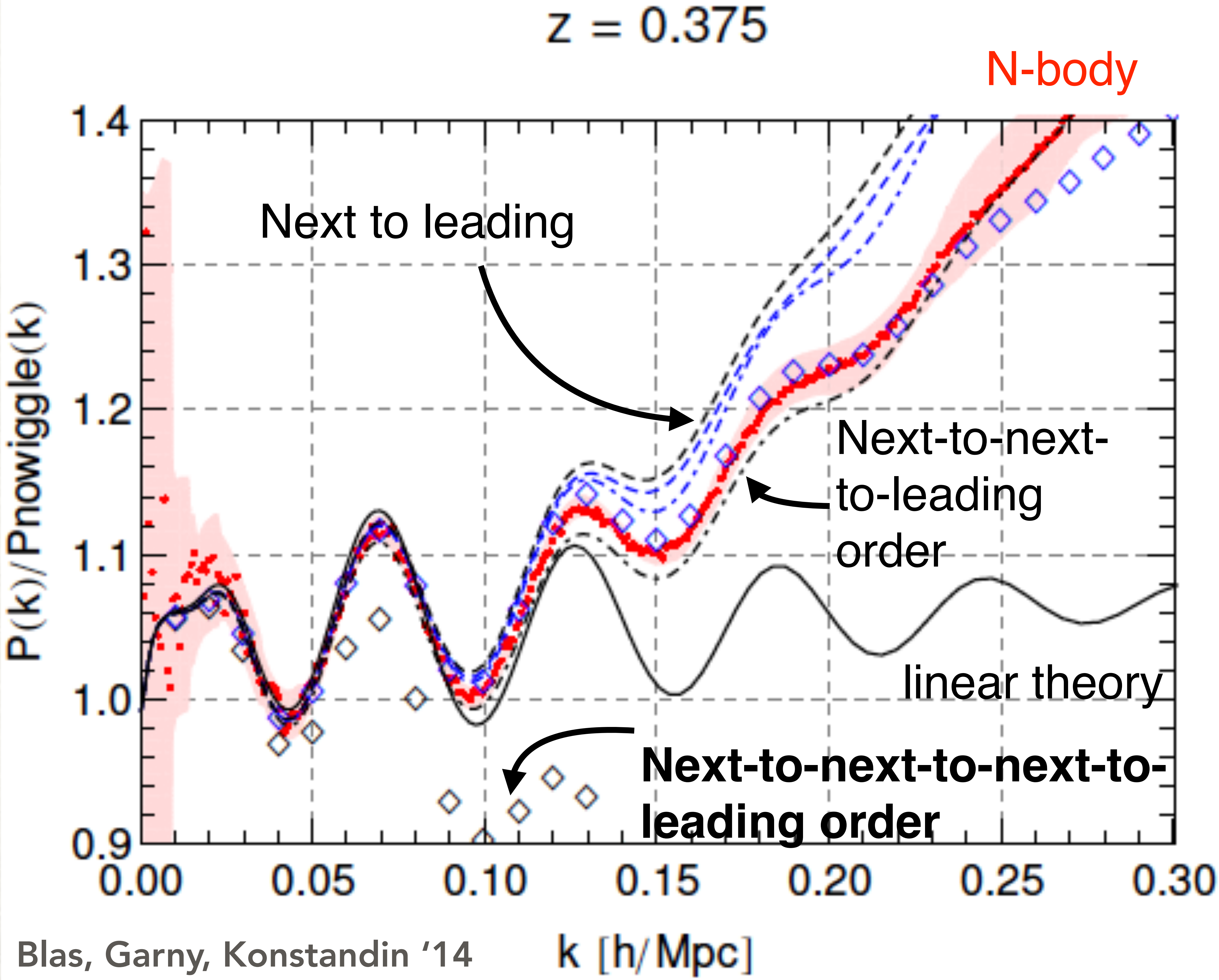

Blas, Garny, Konstandin '14

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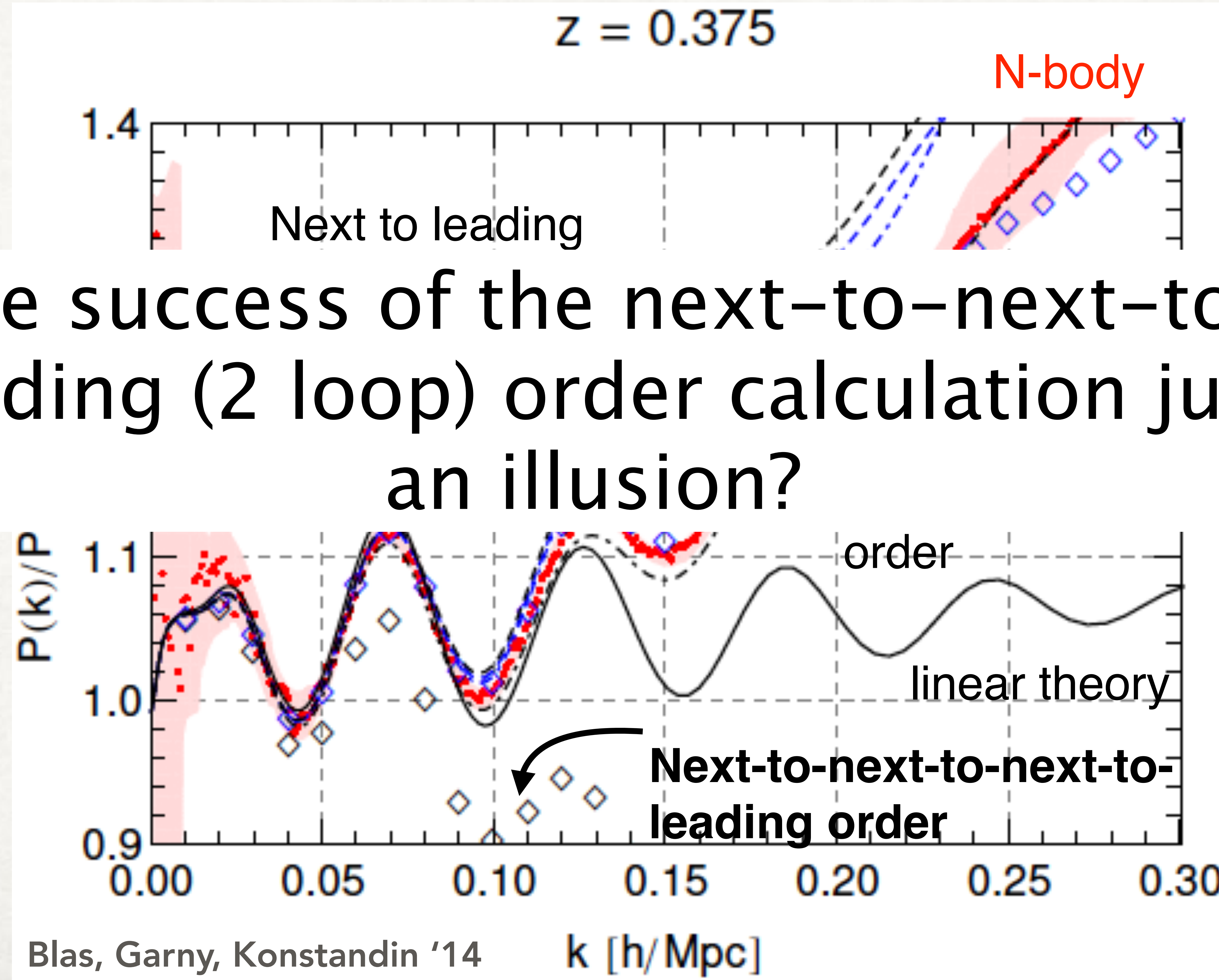


The success of the next–to–next–to–leading (2 loop) order calculation just an illusion?

Blas, Garny, Konstandin '14

WHY?

- Perturbation theory (PT) is fine only when the quantity of interest is small and the series expansion is convergent
    - Overdensity can reach >> 1 at present
- The fluid is assumed to follow an irrotational single-streaming flow
    - Enters to multi-streaming phase after shell crossing (even if the initial condition is cold)
- These 2 things happen together (small scale, late time)
    - The breakdown can propagate to large scales due to mode coupling
    - The way how PT breaks down is totally non-trivial —> simulations!

WHAT TO MEASURE?

- Basic things that determine the nonlinear mode-coupling structure:

$$\boxed{\{F_n(\mathbf{q}_1, \ldots, \mathbf{q}_n), G_n(\mathbf{q}_1, \ldots, \mathbf{q}_n)\}}$$

$$\boxed{\left\{\Gamma_\delta^{(n)}(\mathbf{q}_1, \ldots, \mathbf{q}_n), \Gamma_\theta^{(n)}(\mathbf{q}_1, \ldots, \mathbf{q}_n)\right\}}$$

$$\boxed{K(k,q) = q\,\frac{\delta P_{nl}(k)}{\delta P_{lin}(q)}}$$

STANDARD PT KERNEL

- Very, very basic things (not observable from sims, though...)

$$\boxed{\{F_n(\mathbf{q}_1,\ldots,\mathbf{q}_n),G_n(\mathbf{q}_1,\ldots,\mathbf{q}_n)\}}$$

continuity + Euler + Poisson eqs.

$$\delta(\mathbf{x})=\rho(\mathbf{x})/\bar{\rho}-1$$

$$\theta(\mathbf{x})=\nabla\cdot\mathbf{v}(\mathbf{x})$$

✓ $$\frac{\partial\delta}{\partial t}+\frac{1}{a}\nabla\cdot[(1+\delta)\boldsymbol{v}]=0,$$

✓ $$\frac{\partial\boldsymbol{v}}{\partial t}+H\boldsymbol{v}+\frac{1}{a}(\boldsymbol{v}\cdot\nabla)\boldsymbol{v}=-\frac{1}{a}\nabla\phi,$$

✓ $$\nabla^2\phi=4\pi\mathcal{G}\bar{\rho}a^2\delta.$$

$$\tilde{\delta}(\mathbf{k},\tau)=\sum_{n=1}^{\infty}a^n(\tau)\delta_n(\mathbf{k}),\qquad\tilde{\theta}(\mathbf{k},\tau)=-\mathcal{H}(\tau)\sum_{n=1}^{\infty}a^n(\tau)\theta_n(\mathbf{k})$$

$$\delta_n(\mathbf{k})=\int\mathrm{d}^3\mathbf{q}_1\ldots\int\mathrm{d}^3\mathbf{q}_n\,\delta_D(\mathbf{k}-\mathbf{q}_{1\ldots n})F_n(\mathbf{q}_1,\ldots,\mathbf{q}_n)\delta_1(\mathbf{q}_1)\ldots\delta_1(\mathbf{q}_n),$$

$$\theta_n(\mathbf{k})=\int\mathrm{d}^3\mathbf{q}_1\ldots\int\mathrm{d}^3\mathbf{q}_n\,\delta_D(\mathbf{k}-\mathbf{q}_{1\ldots n})G_n(\mathbf{q}_1,\ldots,\mathbf{q}_n)\delta_1(\mathbf{q}_1)\ldots\delta_1(\mathbf{q}_n)$$

GAMMA EXPANSION (eg., RegPT)

Bernardeau + '09

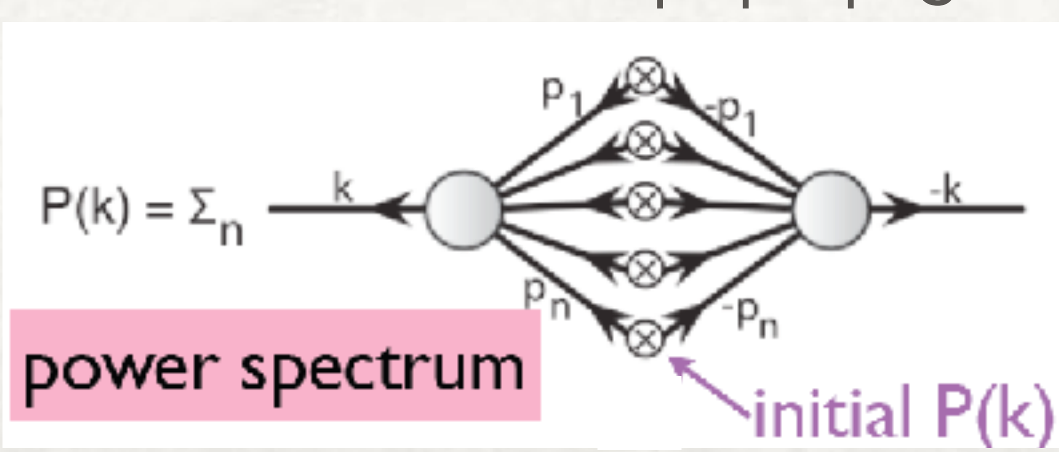
$$\left\{ \Gamma_\delta^{(n)}(\mathbf{q}_1,\dots,\mathbf{q}_n), \Gamma_\theta^{(n)}(\mathbf{q}_1,\dots,\mathbf{q}_n) \right\}$$

final state (density or velocity) multi-point propagator

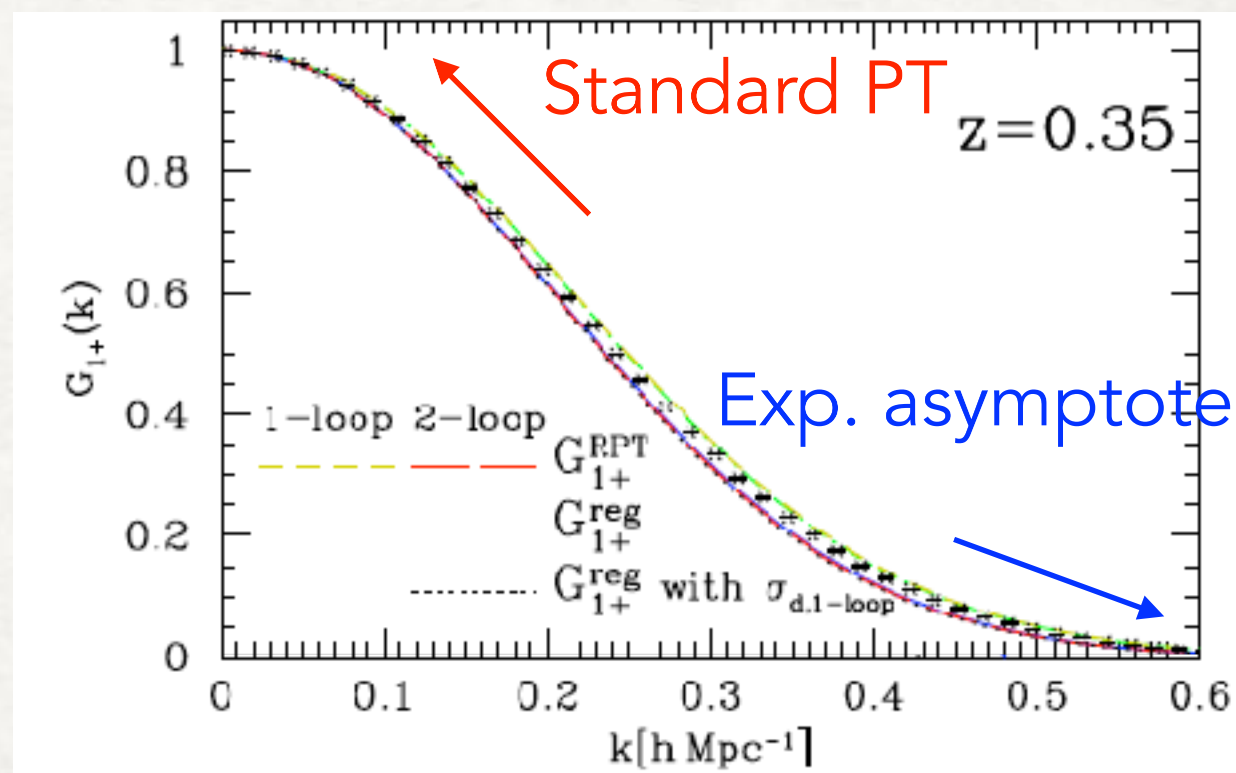
$$\frac{1}{p!}\left\langle \frac{\delta^p \Psi_a(\mathbf{k},\eta)}{\delta\phi_{b_1}(\mathbf{k}_1)\dots\delta\phi_{b_p}(\mathbf{k}_p)}\right\rangle = \delta_{\rm D}(\mathbf{k}-\mathbf{k}_1-\dots-\mathbf{k}_p)\,\Gamma^{(p)}_{ab_1\dots b_p}(\mathbf{k}_1,\dots,\mathbf{k}_p;\eta)$$

initial state

- Can calibrate against sims
- Good ansatz known
- Simpler expression for the spectra
- Clear physical interpretation (Crocce & Scoccimarro '06 for the 2pt propagator)

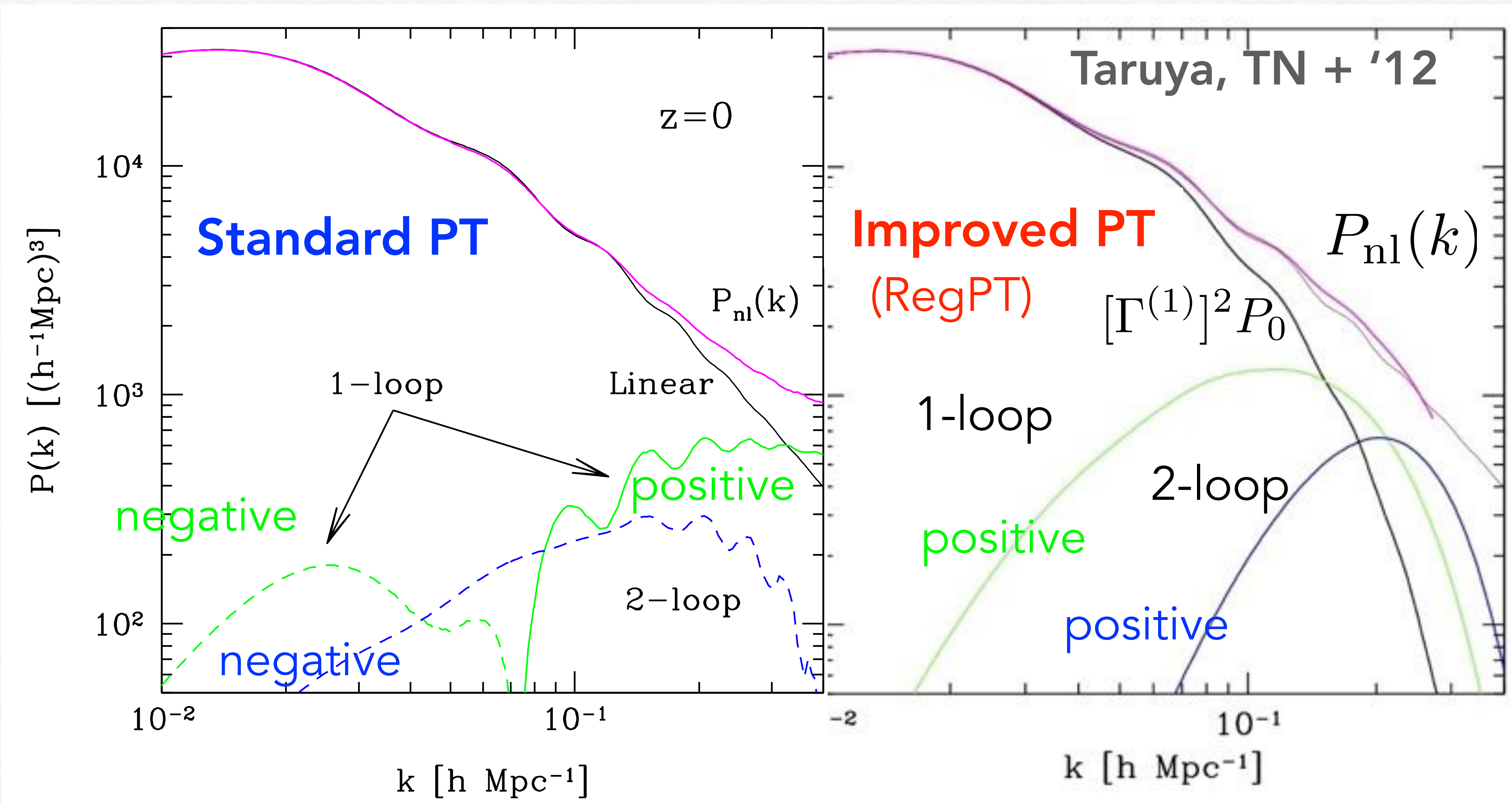


Bernardeau, Taruya & TN '14



VIRTUE OF RESUMMATION

- Better convergence: everything is positive definite
- BAO almost done at the lowest order



MORE INTUITIVE QUANTITY? RESPONSE!

large scale structure gravitational evolution

Input

$$(\Omega_m, h, ...; z)$$

$$P_{nl}(k)$$

Output

MORE INTUITIVE QUANTITY? RESPONSE!

large scale structure gravitational evolution

Input

$$(\Omega_m, h, ...; z)$$

$$P_{lin}(k)$$

$$P_{nl}(k)$$

To a very good approximation

Output

MORE INTUITIVE QUANTITY? RESPONSE!

large scale structure gravitational evolution

Input

$$P_{lin}(k)$$

$$P_{nl}(k)$$

Output

$$\boxed{K(k,q) = q\,\frac{\delta P_{nl}(k)}{\delta P_{lin}(q)}}$$

MORE INTUITIVE QUANTITY? RESPONSE!

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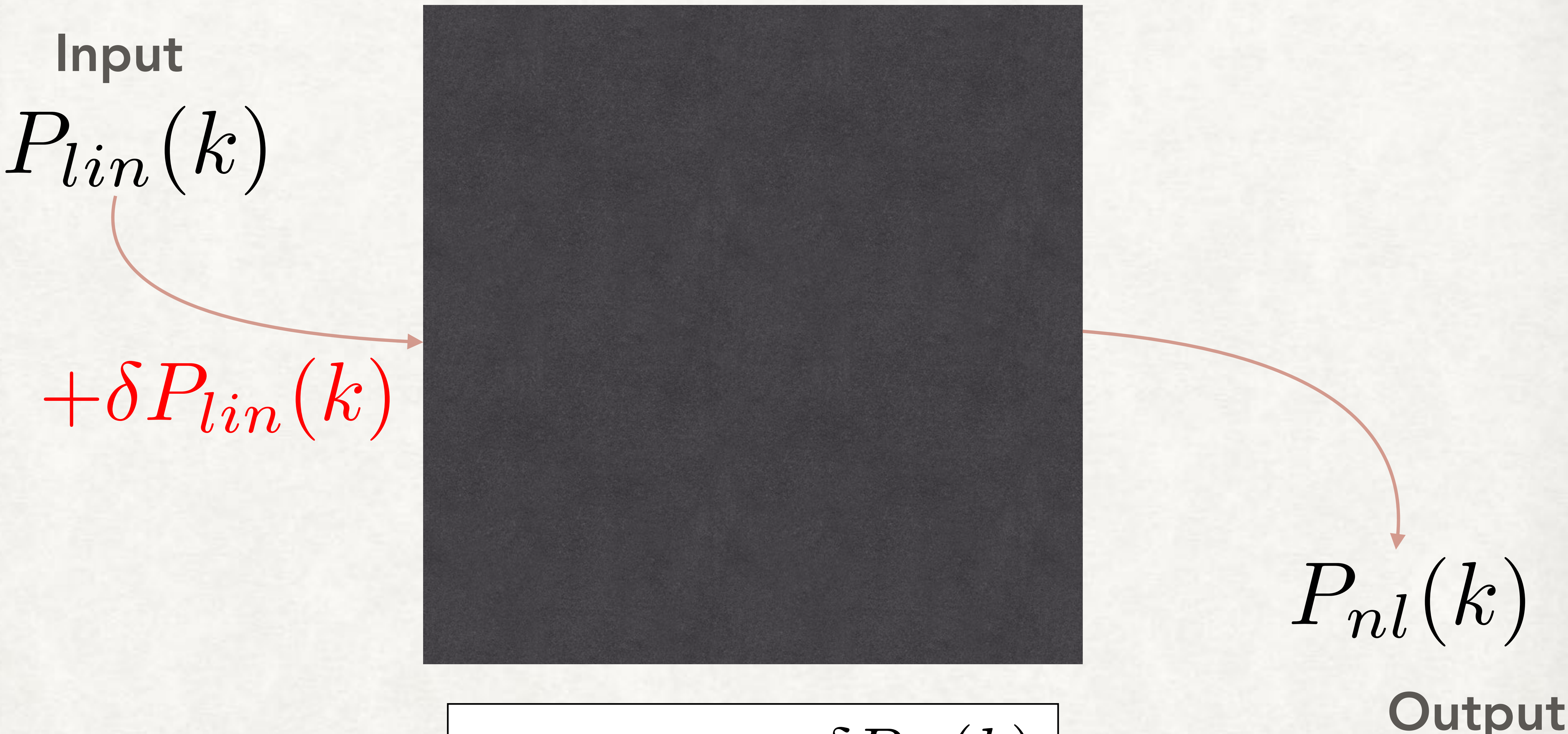


$$\boxed{K(k,q) = q\,\frac{\delta P_{nl}(k)}{\delta P_{lin}(q)}}$$

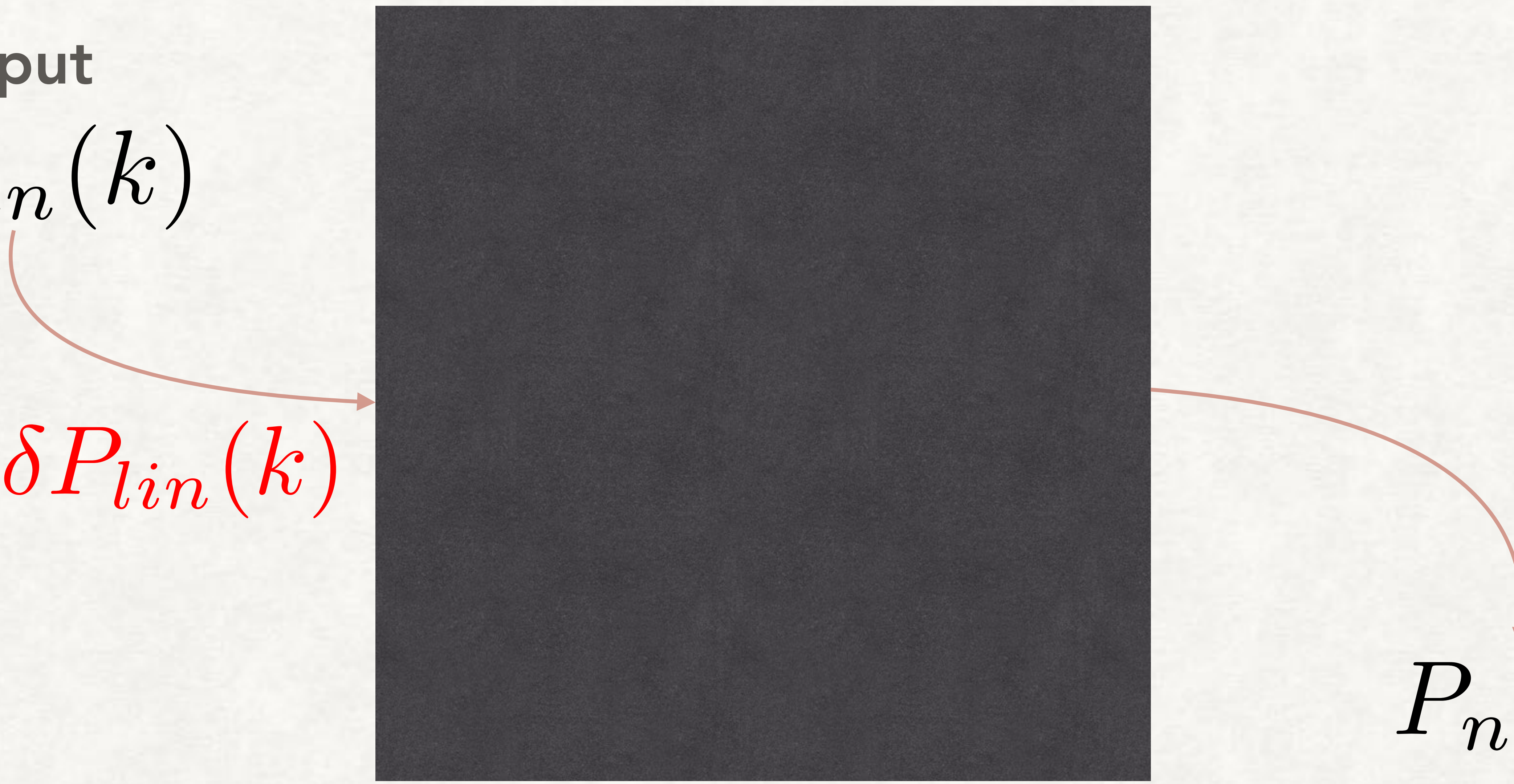
MORE INTUITIVE QUANTITY? RESPONSE!

large scale structure gravitational evolution

Input

$P_{lin}(k)$

$+\delta P_{lin}(k)$



Output

$P_{nl}(k)$

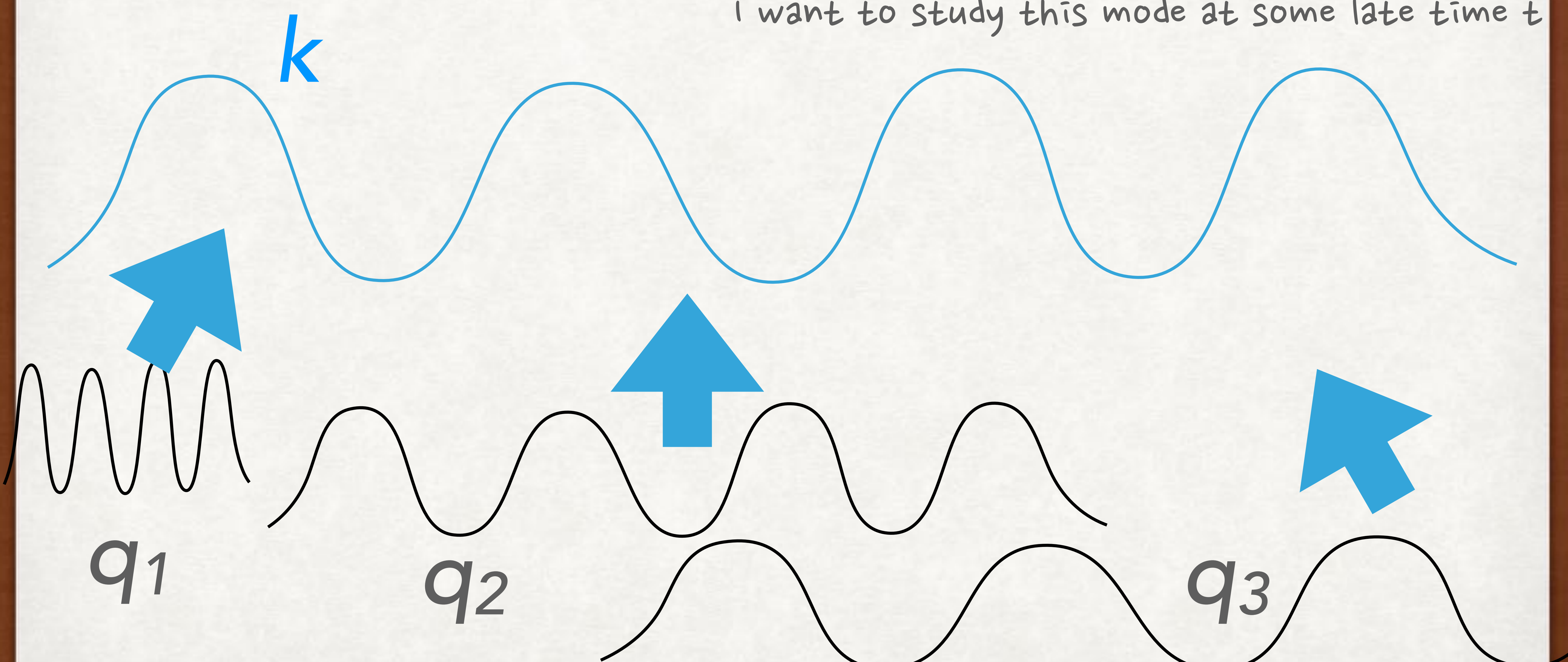
$+\delta P_{nl}(k)$

$$\boxed{K(k,q) = q\,\frac{\delta P_{nl}(k)}{\delta P_{lin}(q)}}$$

RESPONSE FUNCTION

$$K(k,q) = q\,\frac{\delta P_{nl}(k)}{\delta P_{lin}(q)}$$

I want to study this mode at some late time t



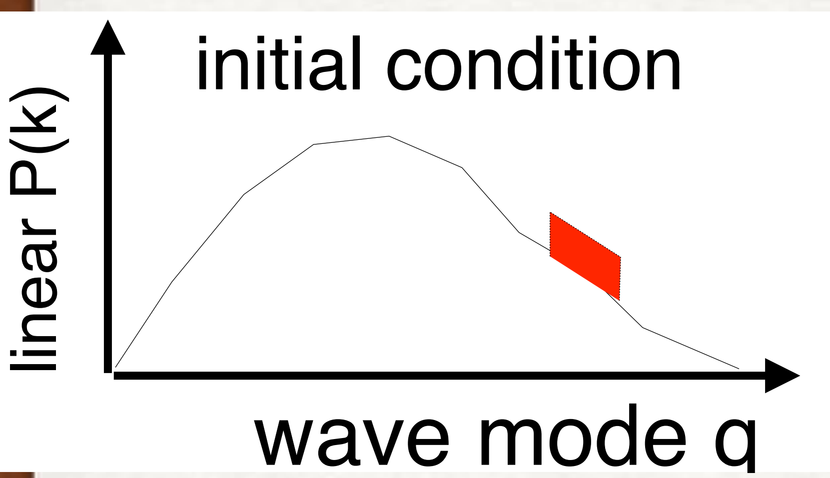
what is the impact from wave mode q at the initial time t_0?

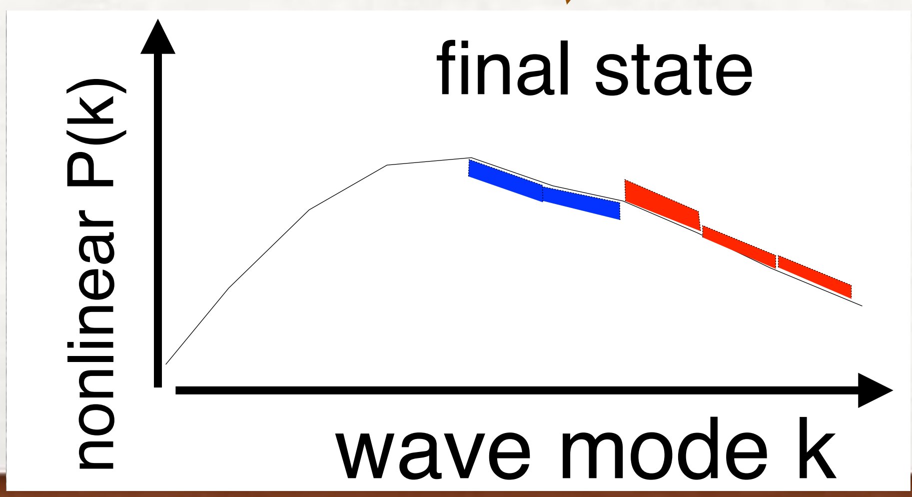
RESPONSE FUNCTION: THE FIRST TRIAL

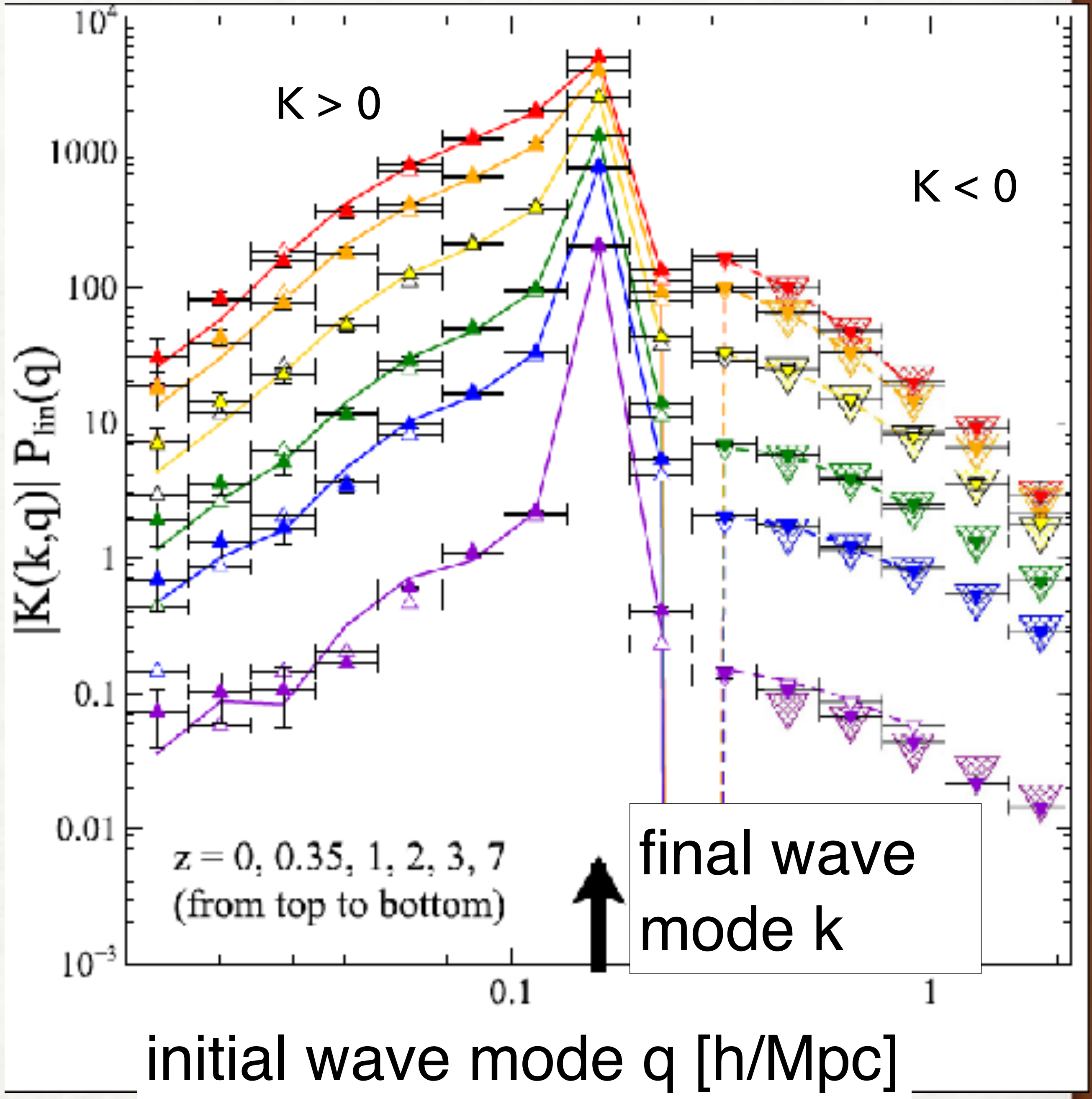
TN, Bernardeau, Taruya '16 PLB

- From order-by-order to the full order discussion possible
- Can estimate the derivative from a simulation ensemble

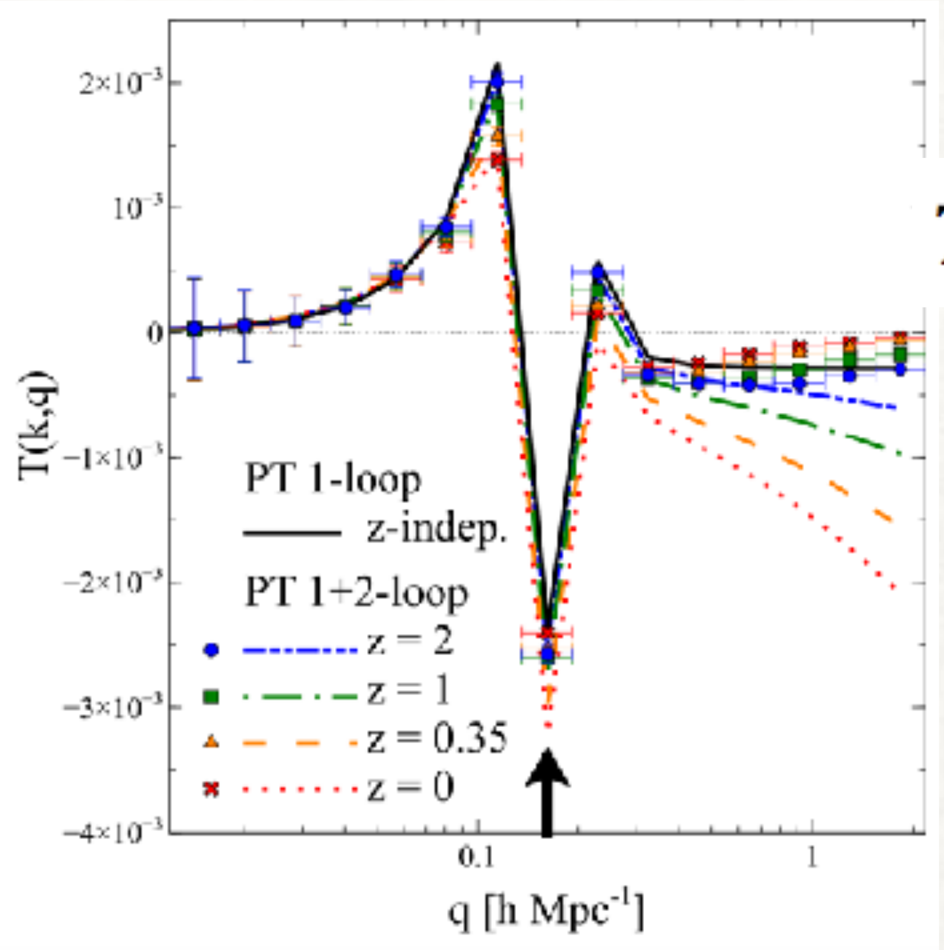
$$\hat{K}_{i,j}P_j^{\rm lin} \equiv \frac{P_i^{\rm nl}[P_{+,j}^{\rm lin}] - P_i^{\rm nl}[P_{-,j}^{\rm lin}]}{\Delta \ln P^{\rm lin} \Delta \ln q}$$





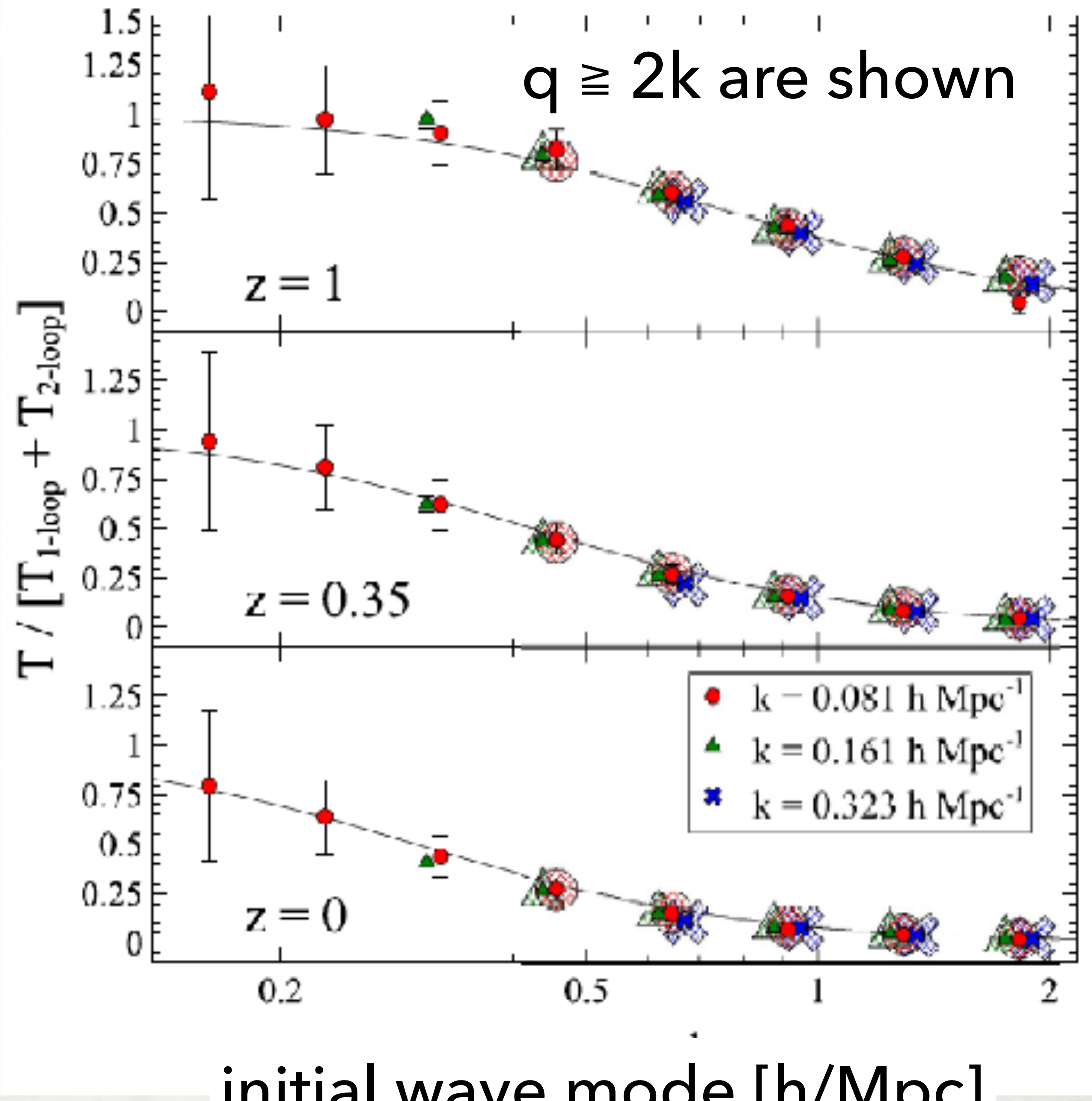


RESPONSE FUNCTION: SIM VS PT



Rescaled quantity:

$$T(k,q) \equiv [K(k,q) - K^{\rm lin}(k,q)]/[q P^{\rm lin}(k)]$$



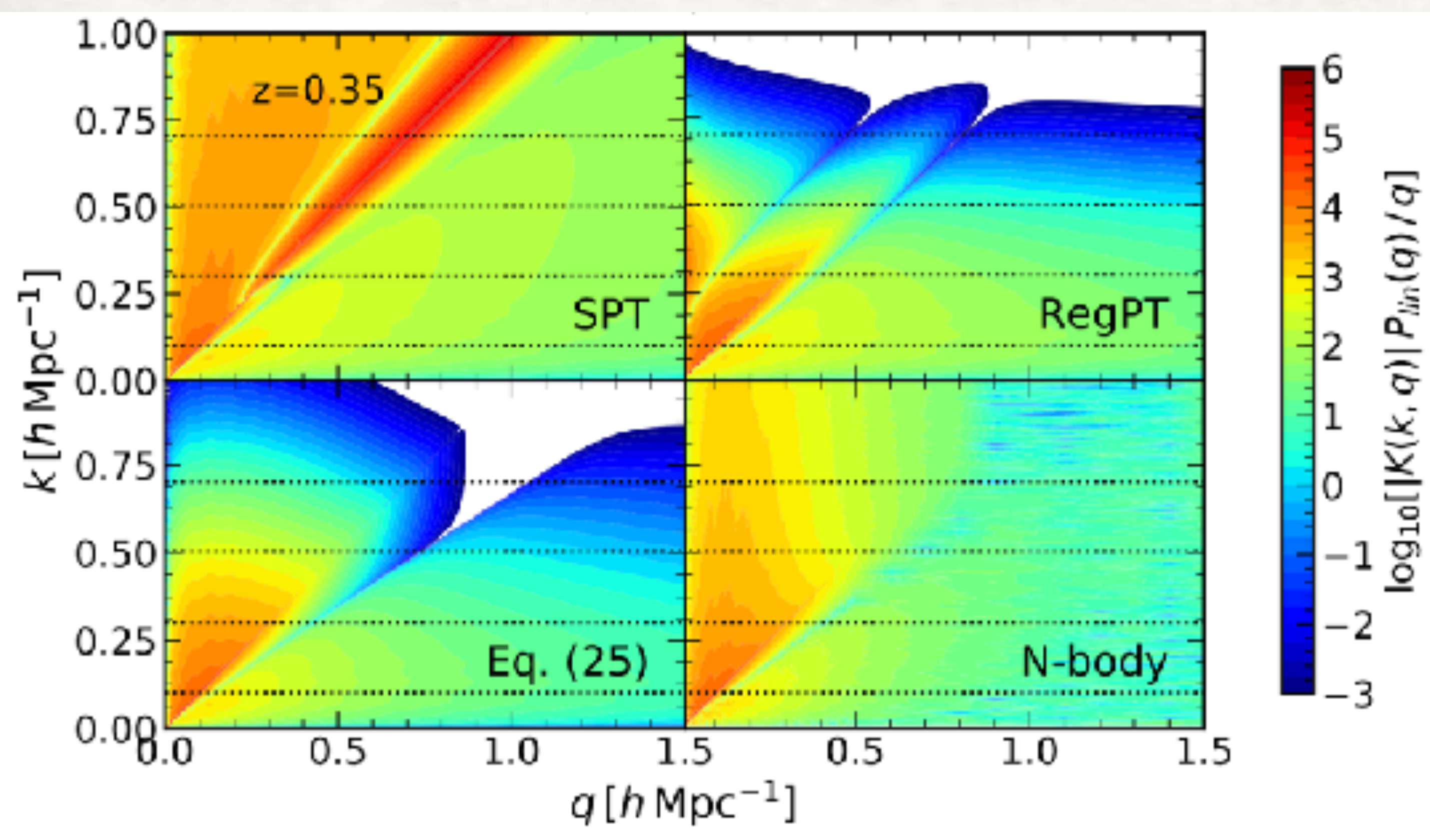
- SPT (2-loop) >> N-body @ high q
- This is exactly where PT breaks down
- What N-body tells us is:

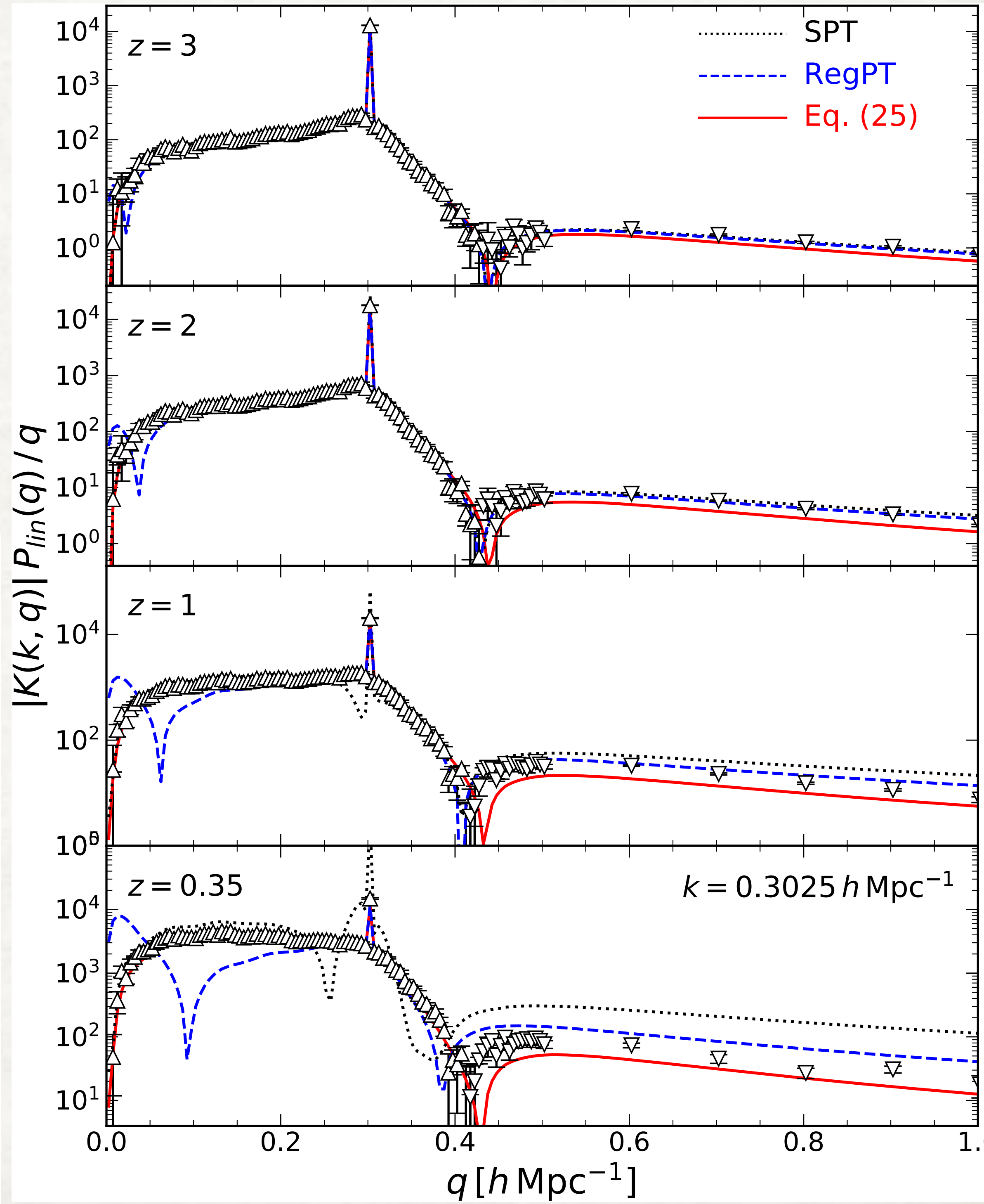
"Physics at strongly nonlinear regime does not propagate to large scales"

HIGH RES RESPONSE FUNCTION

- 1400 N=512^3 simulations to study fine structures of the response function
- Vs 2-loop calculation based on different schemes (SPT and RegPT)
- New phenomenological model introduced

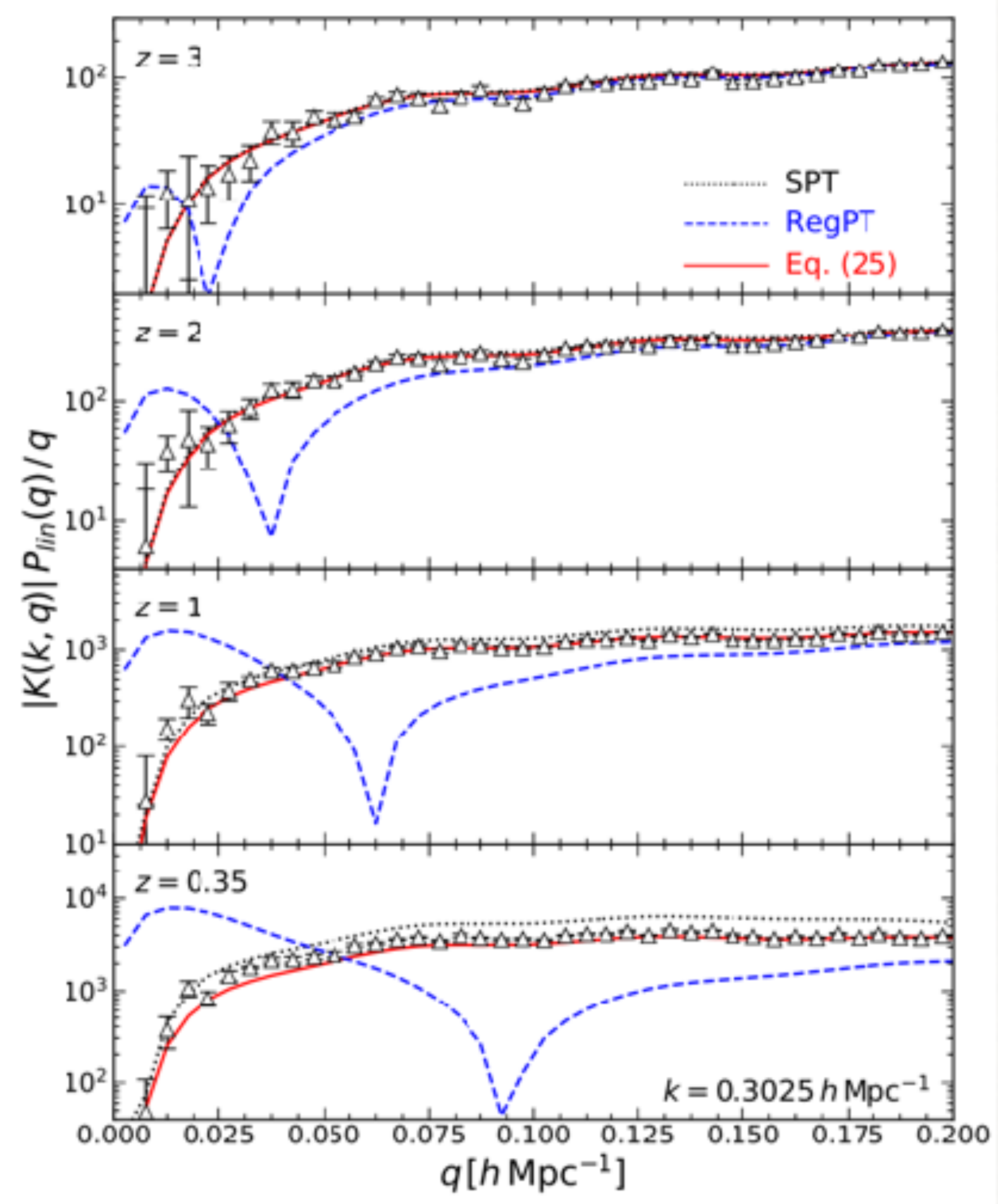
TN, Bernardeau, Taruya '17 (arXiv:1708.08946)





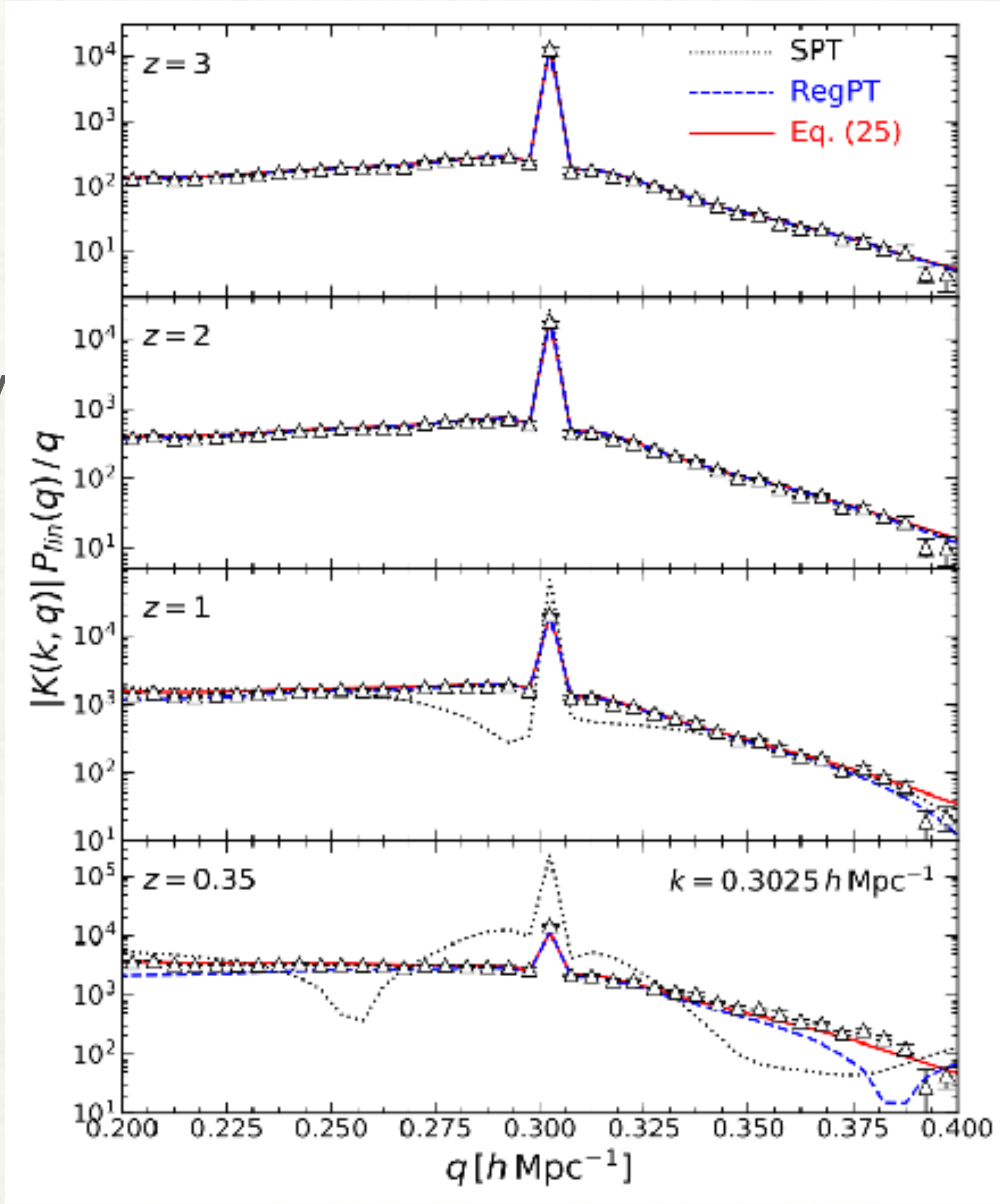
RESPONSE FUNCTION AT q << k

- Response function goes to zero from simulations
  - **Extended galilean invariance**
- This is nicely explained by SPT
- Not the case for RegPT



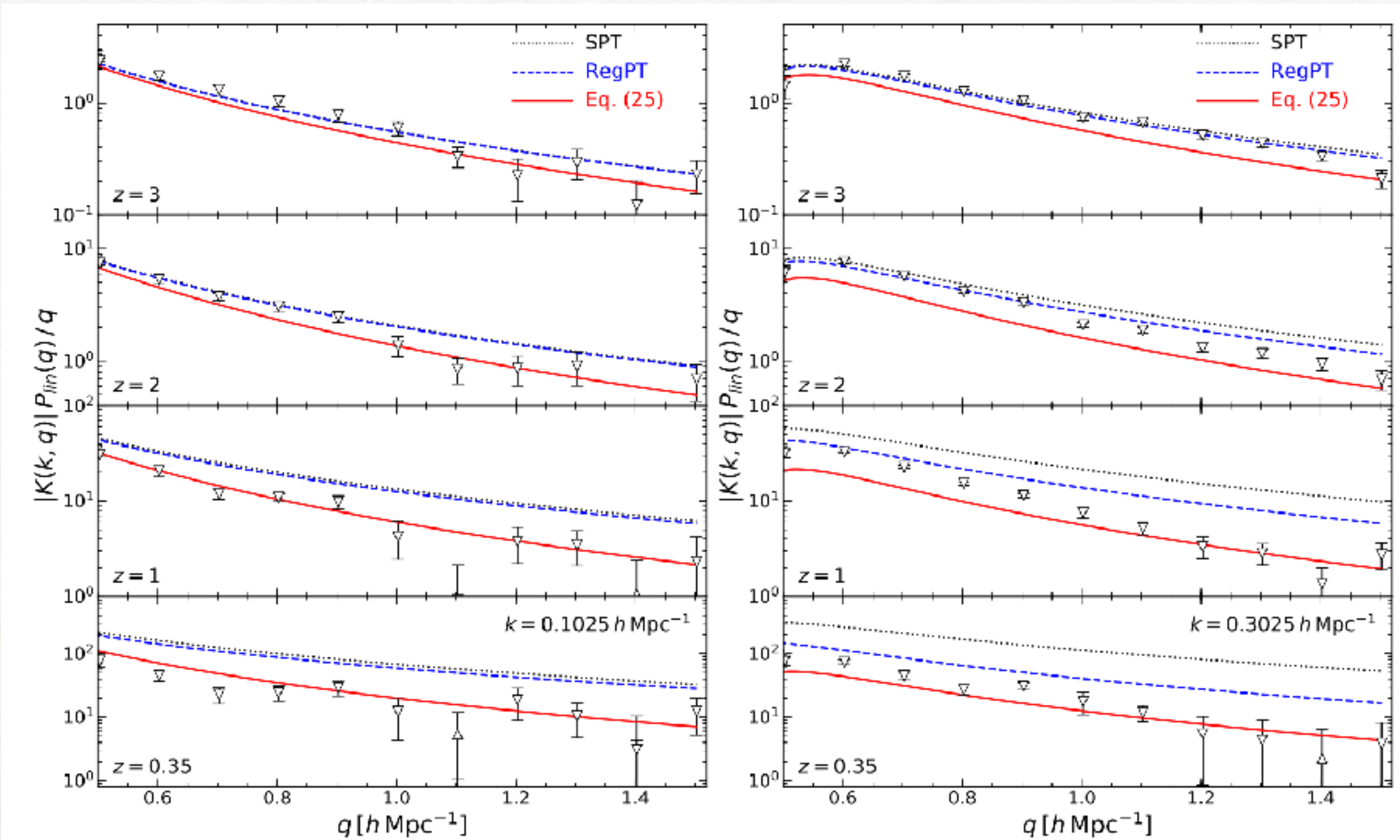
RESPONSE FUNCTION AT q~k

- Peaky structure decay as time goes by
- SPT behaves weirdly at late time
- RegPT has its strength in this regime
  - Efficiently captures mode transfer between nearby modes



RESPONSE FUNCTION AT k << q

- We need phenomenology here anyway!



OUR MODEL

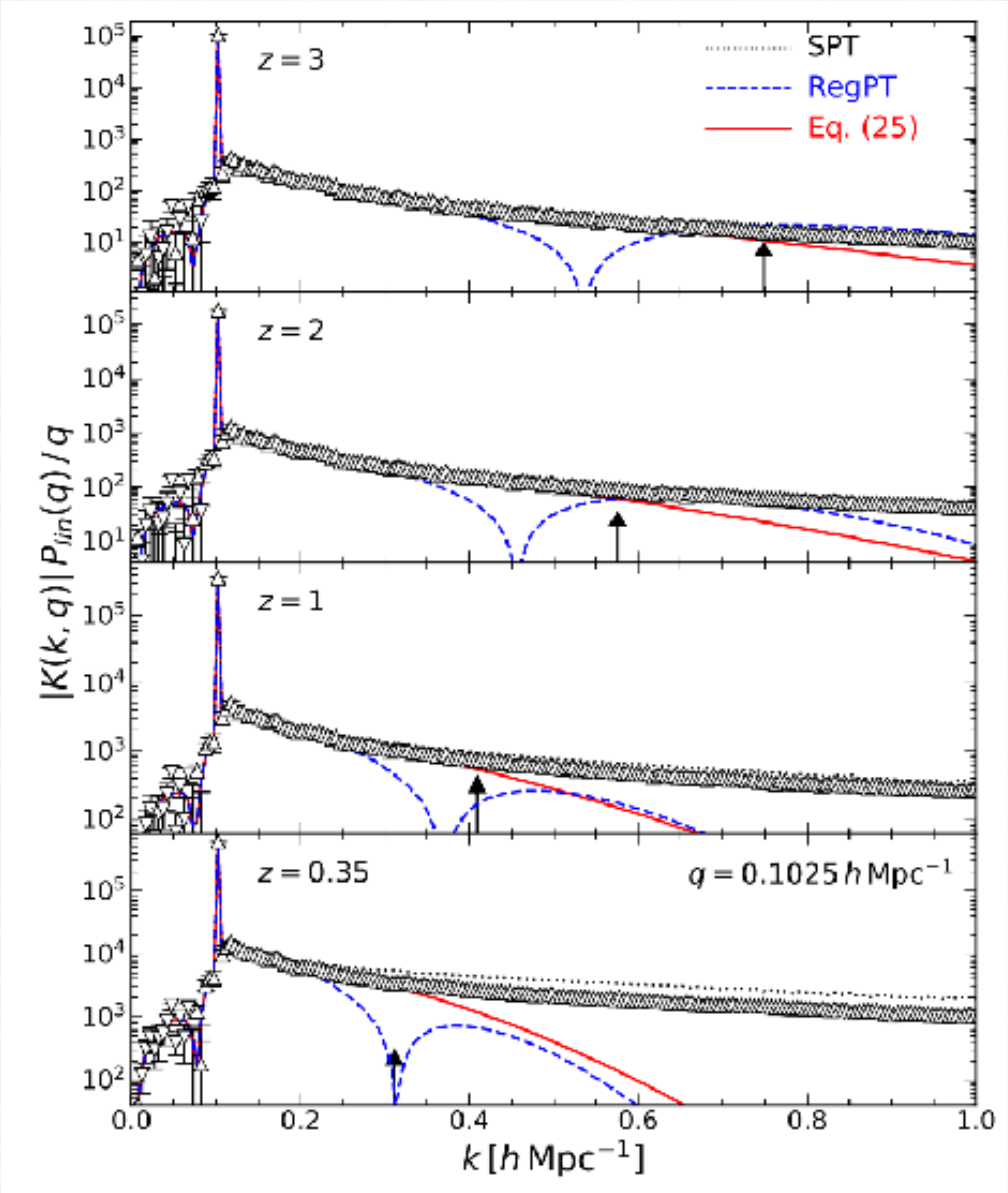
$$K_{\rm model}(k,q) = \Big[\big(1+\beta_{k,q}+\frac{1}{2}\beta_{k,q}^2\big)K^{\rm SPT}_{\rm tree}(k,q) + (1+\beta_{k,q})\,K^{\rm SPT}_{\text{1-loop}}(k,q) + K^{\rm SPT}_{\text{2-loop}}(k,q)\Big]D(\beta_{k,q}),$$

$$D(x) = \begin{cases} \exp(-x)\,, & \text{if } K_{\rm model}(k,q) > 0, \\ \dfrac{1}{1+x}, & \text{if } K_{\rm model}(k,q) < 0. \end{cases}$$

$\beta_{k,q} = \alpha_k + \alpha_q$ Regularize both in k and q

$$\alpha_k = \frac{1}{2}k^2\sigma_{\rm d}^2\,; \quad \sigma_{\rm d}^2 = \int \frac{dq}{6\pi^2}\,P_{\rm lin}(q),$$

- Well-behaved over all q
- Eventually fails at high k



PRACTICAL USAGE? RECONSTRUCTION

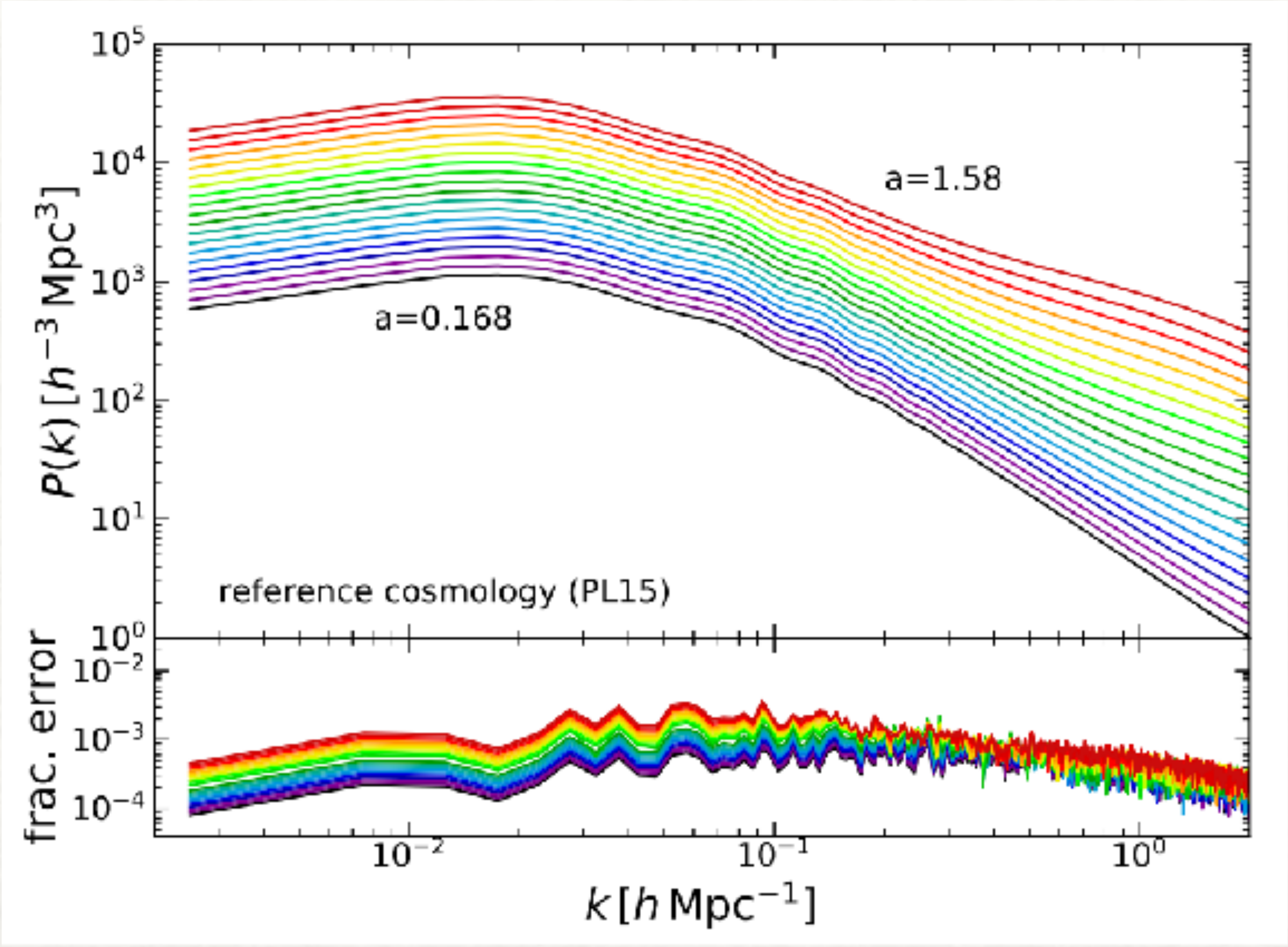
- From the definition of a functional derivative

$$P_{\rm nl}(k;\boldsymbol{p}_1) \approx P_{\rm nl}(k;\boldsymbol{p}_0) + \int {\rm d}\ln q\, K(k,q) \times \left[P_{\rm lin}(q;\boldsymbol{p}_1) - P_{\rm lin}(q;\boldsymbol{p}_0)\right],$$

- Use this to predict P_{nl} for cosmological model $\mathbf{p_1}$ given P_{nl} for another model $\mathbf{p_0}$

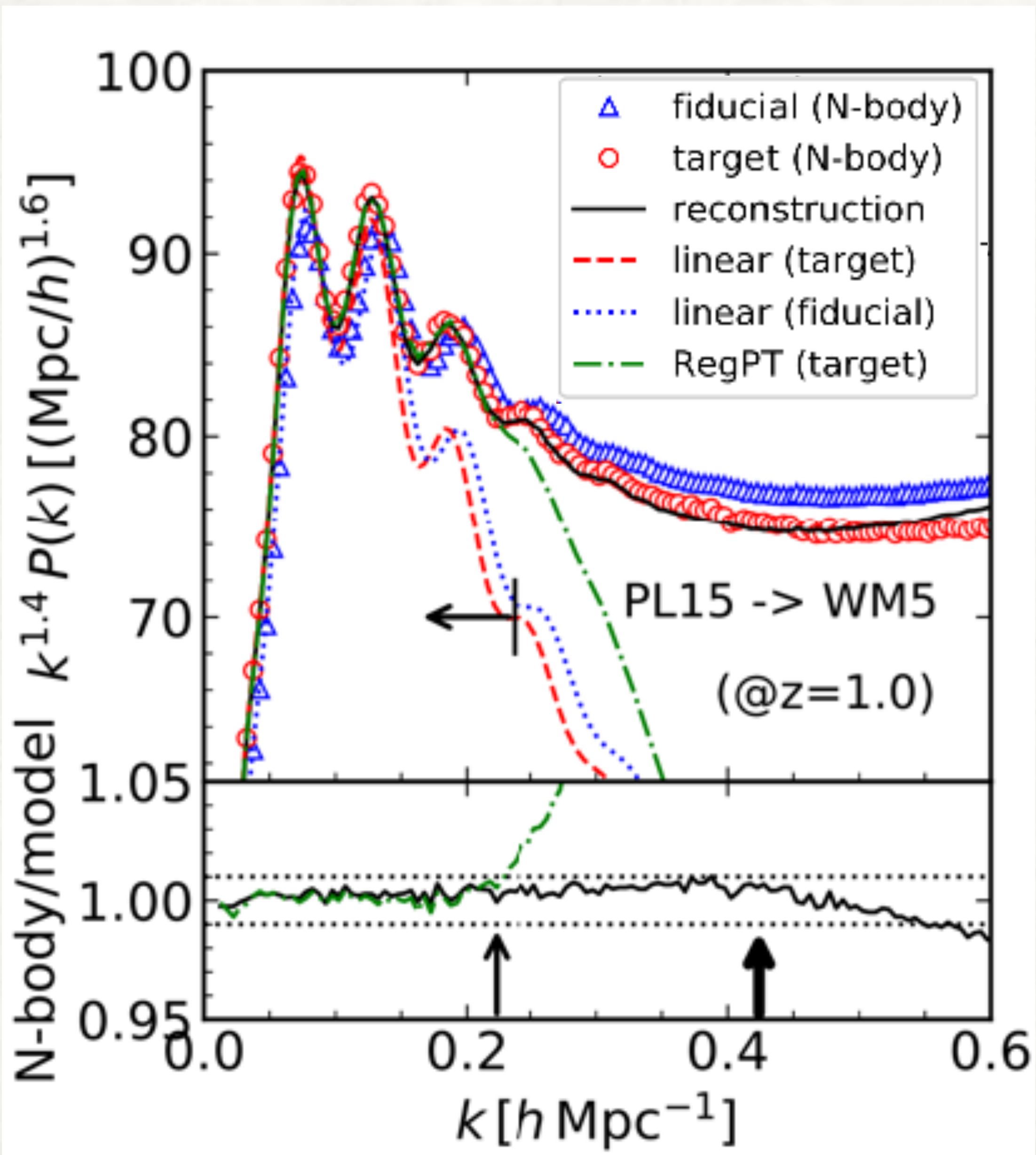
STARTING POINT: SIMULATION DATABASE

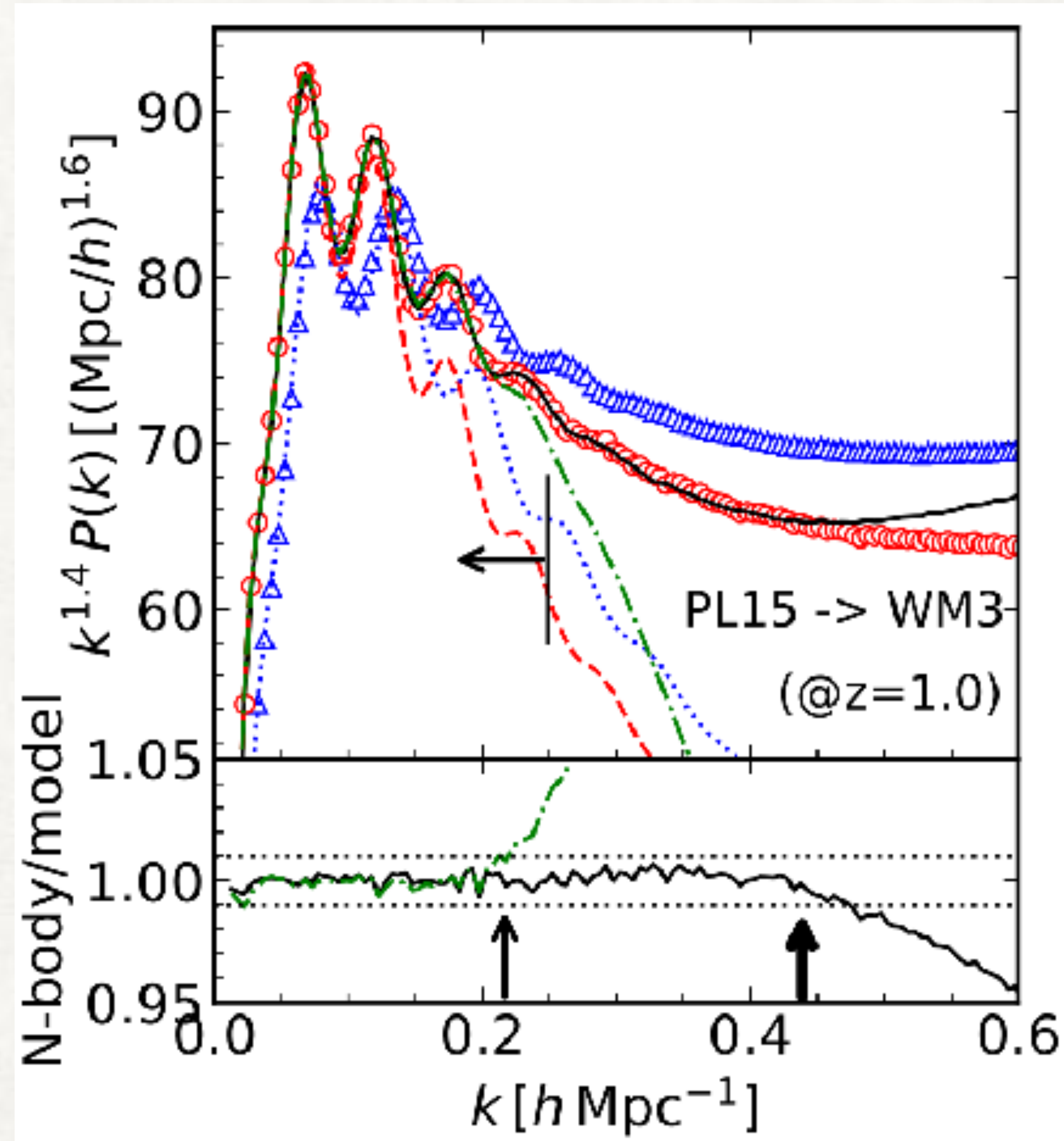
- P_{nl} database for the fiducial Planck 2015 cosmology from 10 x 2048^3 sims
  - Cosmic variance suppressed with Angulo-Pontzen technique
  - Fractional error < 0.1%
- Can smoothly interpolate over k and time



A SIMPLE IMPLEMENTATION

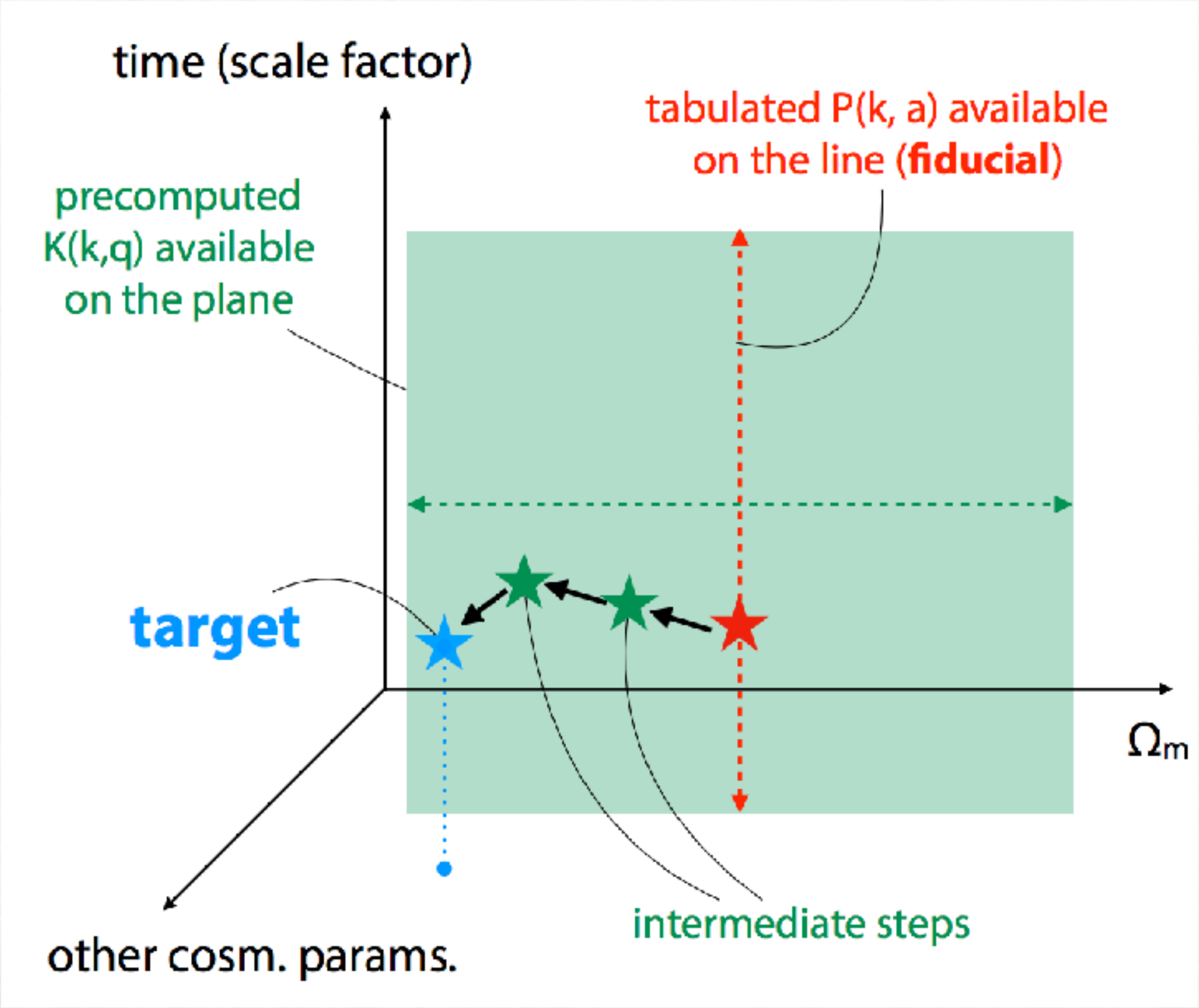
- Double the reliable k range from the pure RegPT prediction





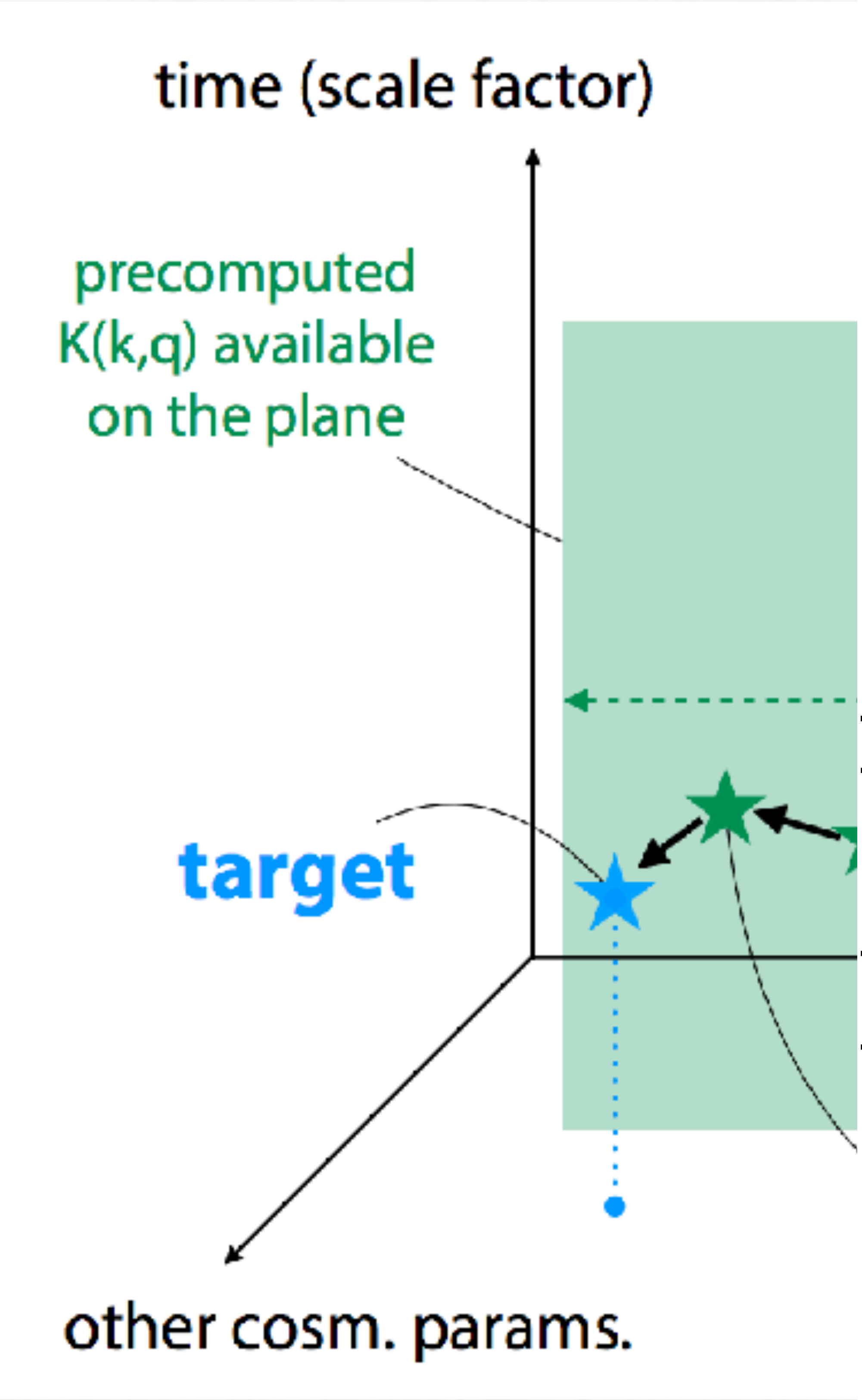
MORE EXTREME MODELS

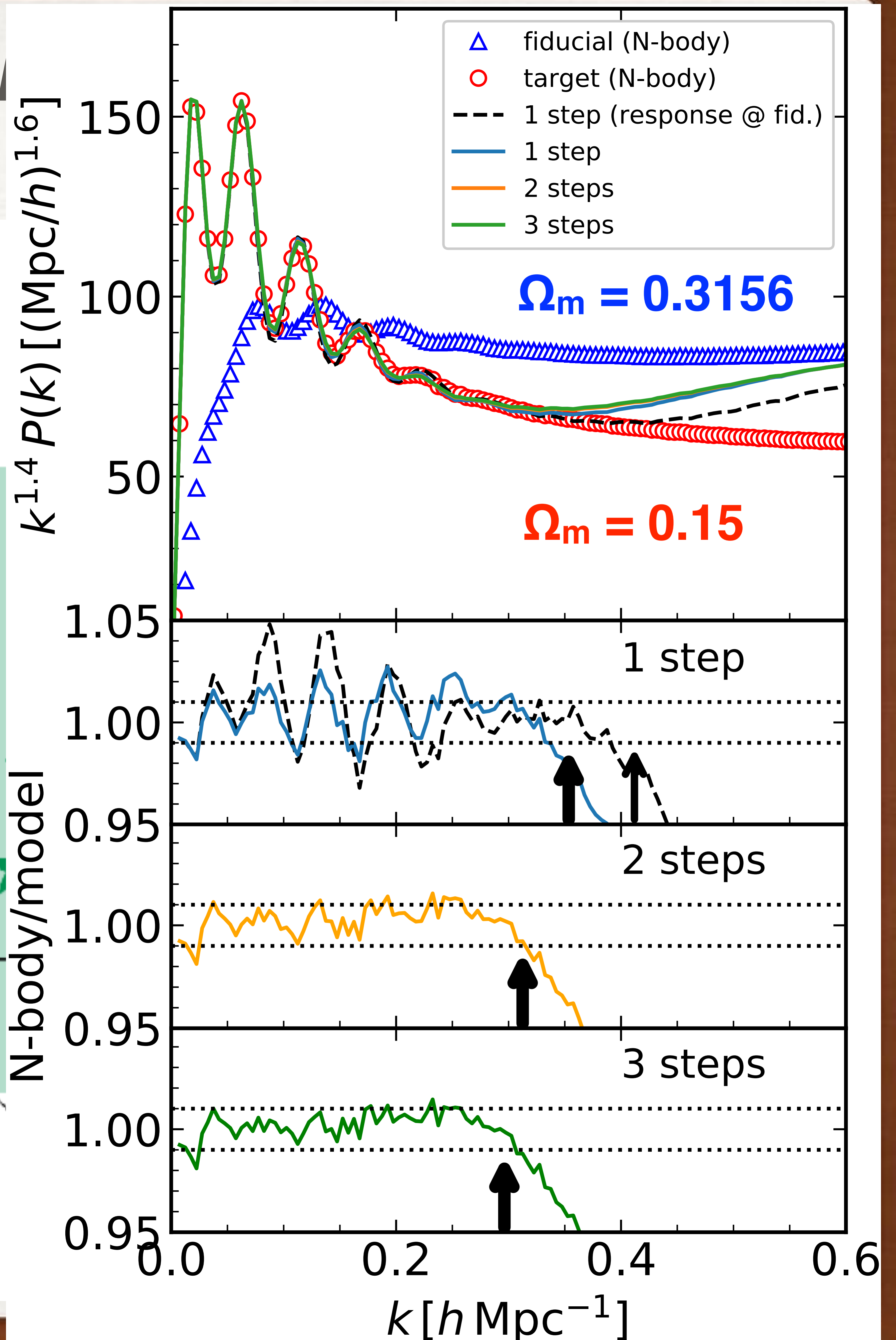
- Employ multi-steps



MORE EXTRE

- Employ multi-steps





RESPRESSO PYTHON PACKAGE AVAILABLE!

(Rapid and Efficient SPectrum calculation based on RESponSe functiOn)

```
In [1]: %pylab inline
        import respresso
```

```
In [3]: respresso_obj = respresso.respresso_core()

        Hello. This is RESPRESSO.
        Load precomputed data files...
        RESPRESSO ready.
```

```
In [6]: respresso_obj.set_target(plin_target_spl)
```

```
In [7]: respresso_obj.find_path()
```

```
In [9]: kwave = respresso_obj.get_kinternal()
        pnl_rec = respresso_obj.reconstruct()
```

http://www-utap.phys.s.u-tokyo.ac.jp/~nishimichi/public_codes/respresso/

SUMMARY

- P(k) to a 2D quantity K(k,q): more physical insight
- Difficulty in perturbative approaches
    - Suppress small to large scale mode transfer!
    - SPT and RegPT have good and bad behavior in different regimes
- RESPRESSO package available
    - Response function is a natural interpolator over the cosmological parameter space
    - Can go to k ~ 0.44 (0.35) h/Mpc at z=1 (0.5) within 1%
    - You can put your own simulation data if you do not like mine ;)

http://www-utap.phys.s.u-tokyo.ac.jp/~nishimichi/public_codes/respresso/