Accurate Theoretical Prediction of Redshift Space Distortions

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CosKASI
Cosmology Group at
Korea Astronomy and Space Science Institute

Implication of cosmic acceleration

- Breaking down our knowledge of particle physics: we have limited knowledge of particle physics bounded by testable high energy, and our efforts to explain the cosmic acceleration turn out in vain:

    Alternative mechanism to generate fine tuned vacuum energy

    New unknown energy component

    Unification or coupling between dark sectors

- Breaking down our knowledge of gravitational physics: gravitational physics has been tested in solar system scales, and it is yet confirmed at horizon size:

    Presence of extra dimension

    Non-linear interaction to Einstein equation

- Failure of standard cosmology model: our understanding of the universe is still standing on assumption:

    Inhomogeneous models: LTB, back reaction

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Dynamical Dark Energy: modifying matter

  Alternative mechanism to generate fine tuned vacuum energy

  New unknown energy component

$$G_{\mu\nu} = 4\pi G_N T_{\mu\nu} + \Delta T_{\mu\nu}$$

  Unification or coupling between dark sectors

- Breaking down our knowledge of gravitational physics: gravitational physics has been tested in solar system scales, and it is yet confirmed at horizon size:

Geometrical Dark Energy: modifying gravity

  Presence of extra dimension

  Non-linear interaction to Einstein equation

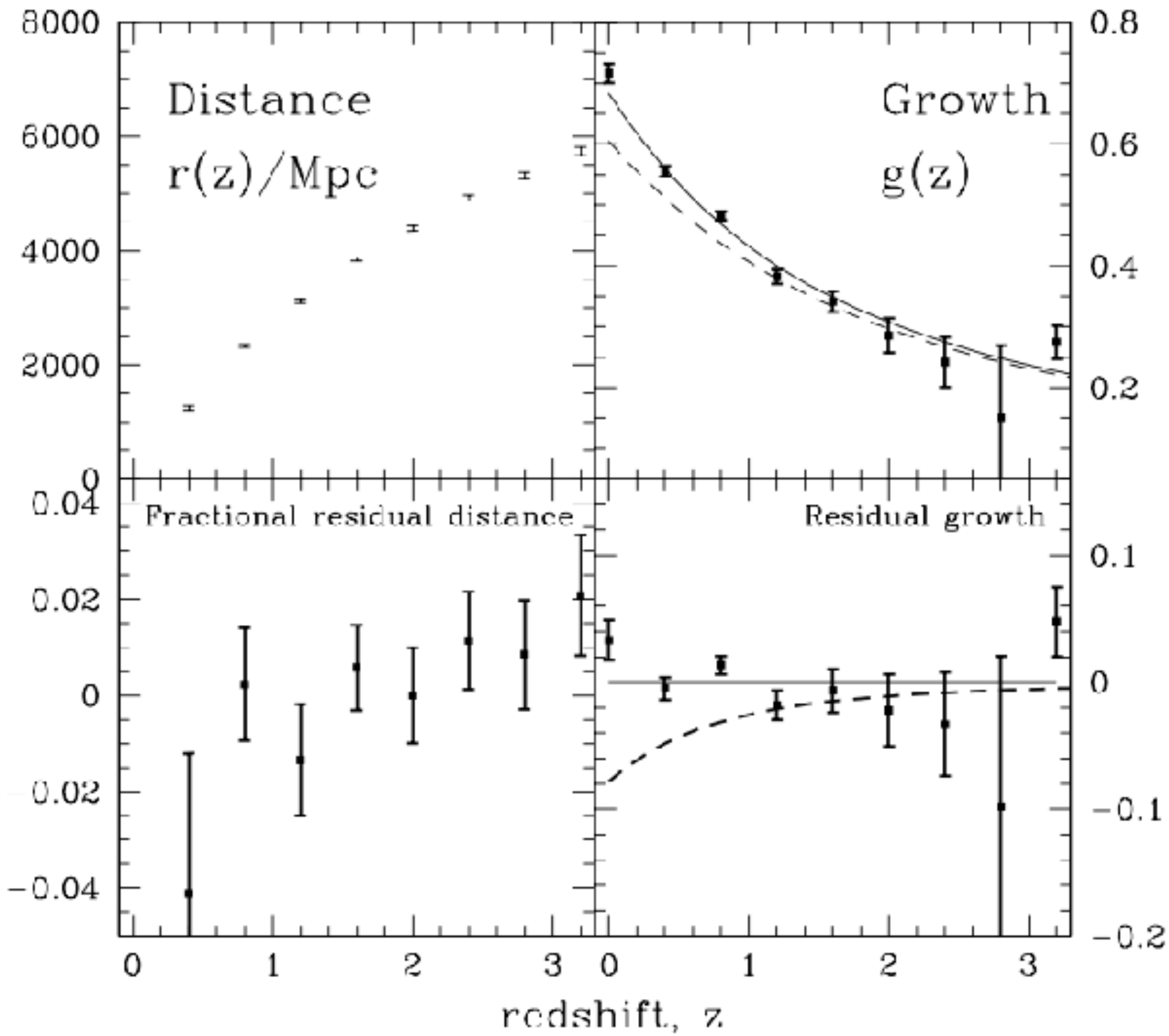
$$G_{\mu\nu} + \Delta G_{\mu\nu} = 4\pi G_N T_{\mu\nu}$$

- Failure of standard cosmology model: our understanding of the universe is still standing on assumption:

  Inhomogeneous models: LTB, back reaction

Two windows on acceleration and gravitation

Their simultaneous determination allows for a consistency test and provides sensitivity to physics beyond the standard dark energy paradigm



Galaxy clustering seen in redshift space

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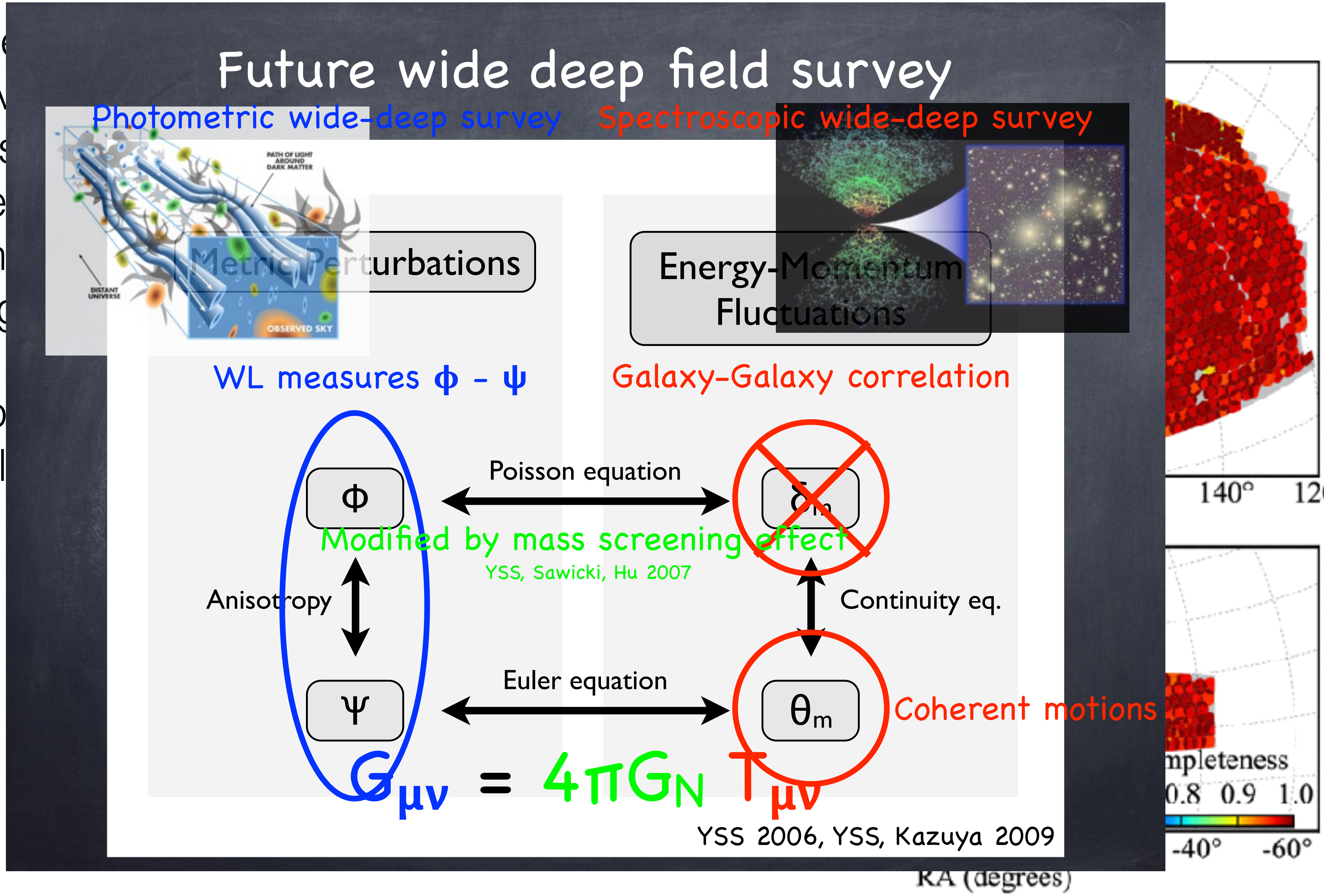

YSS, Koyama 2009

Two windows on acceleration and gravitation

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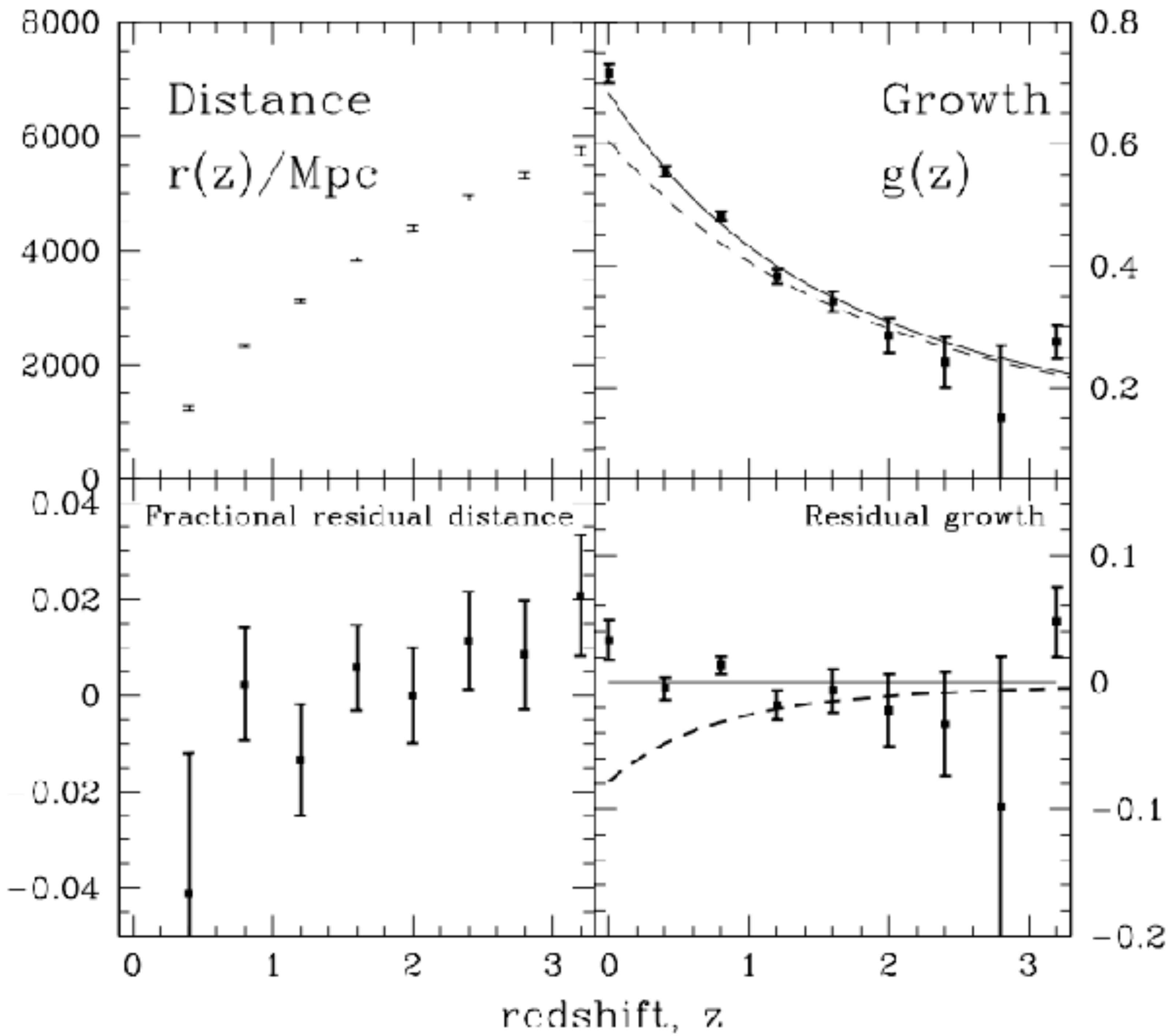
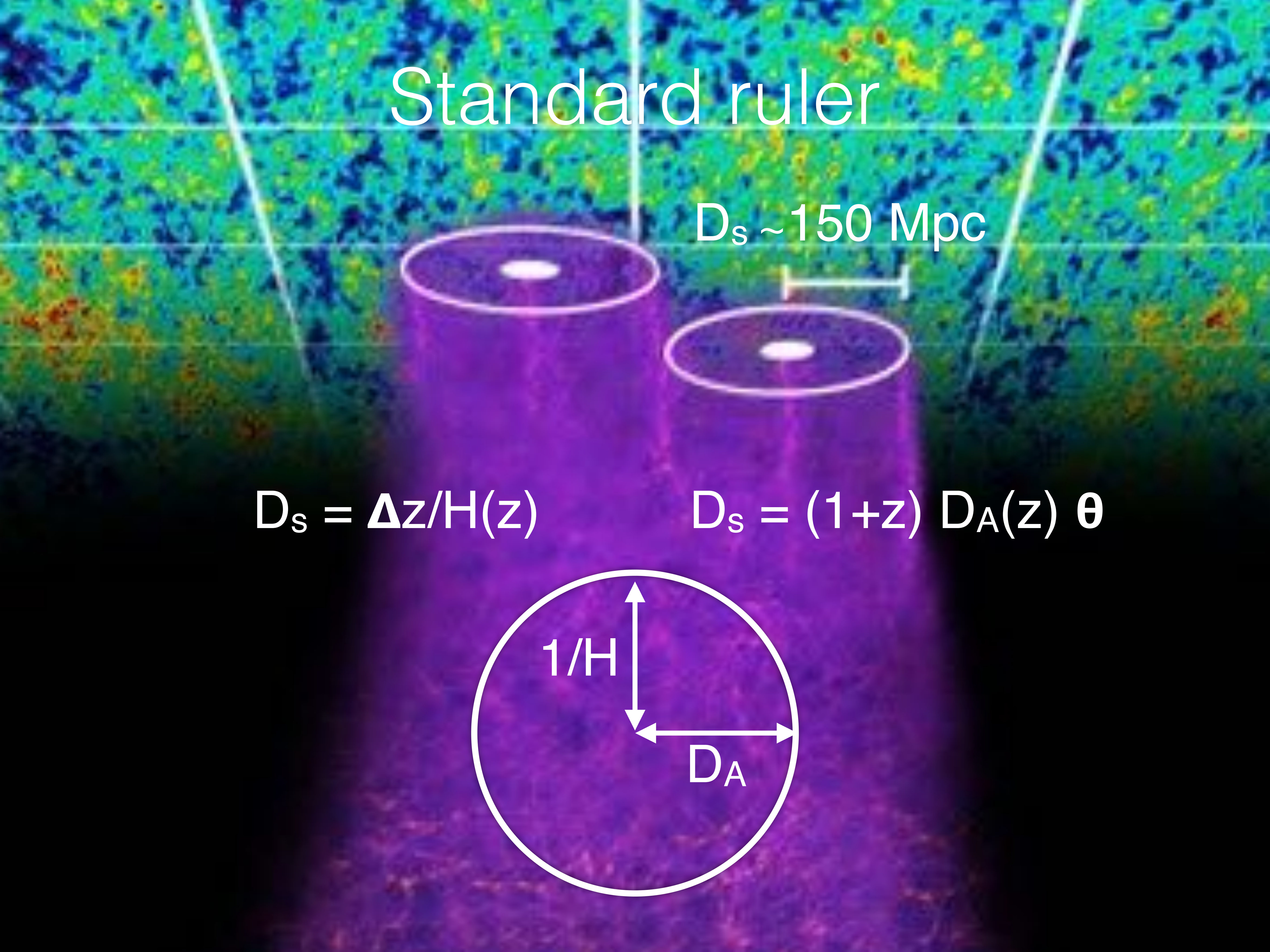
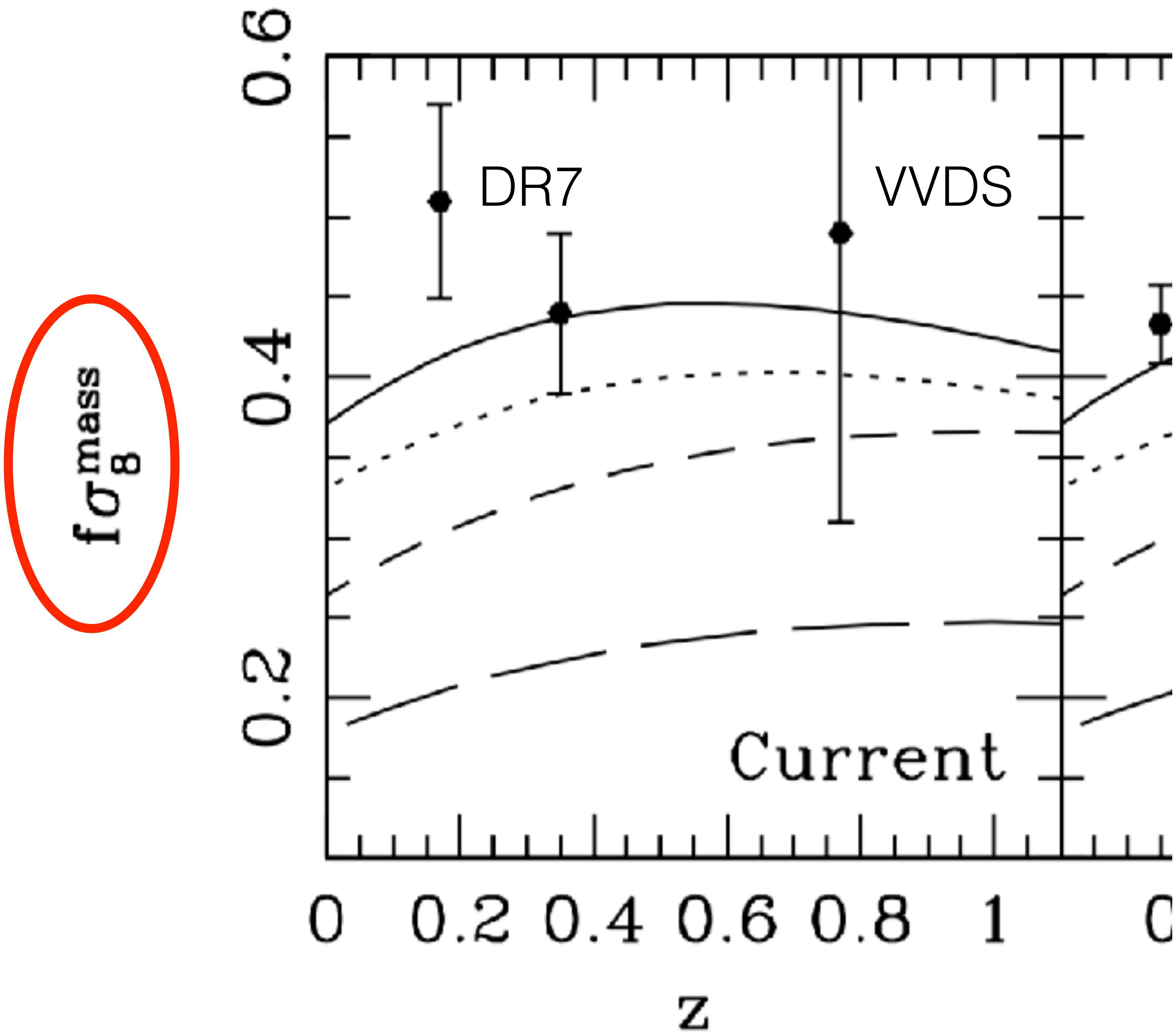




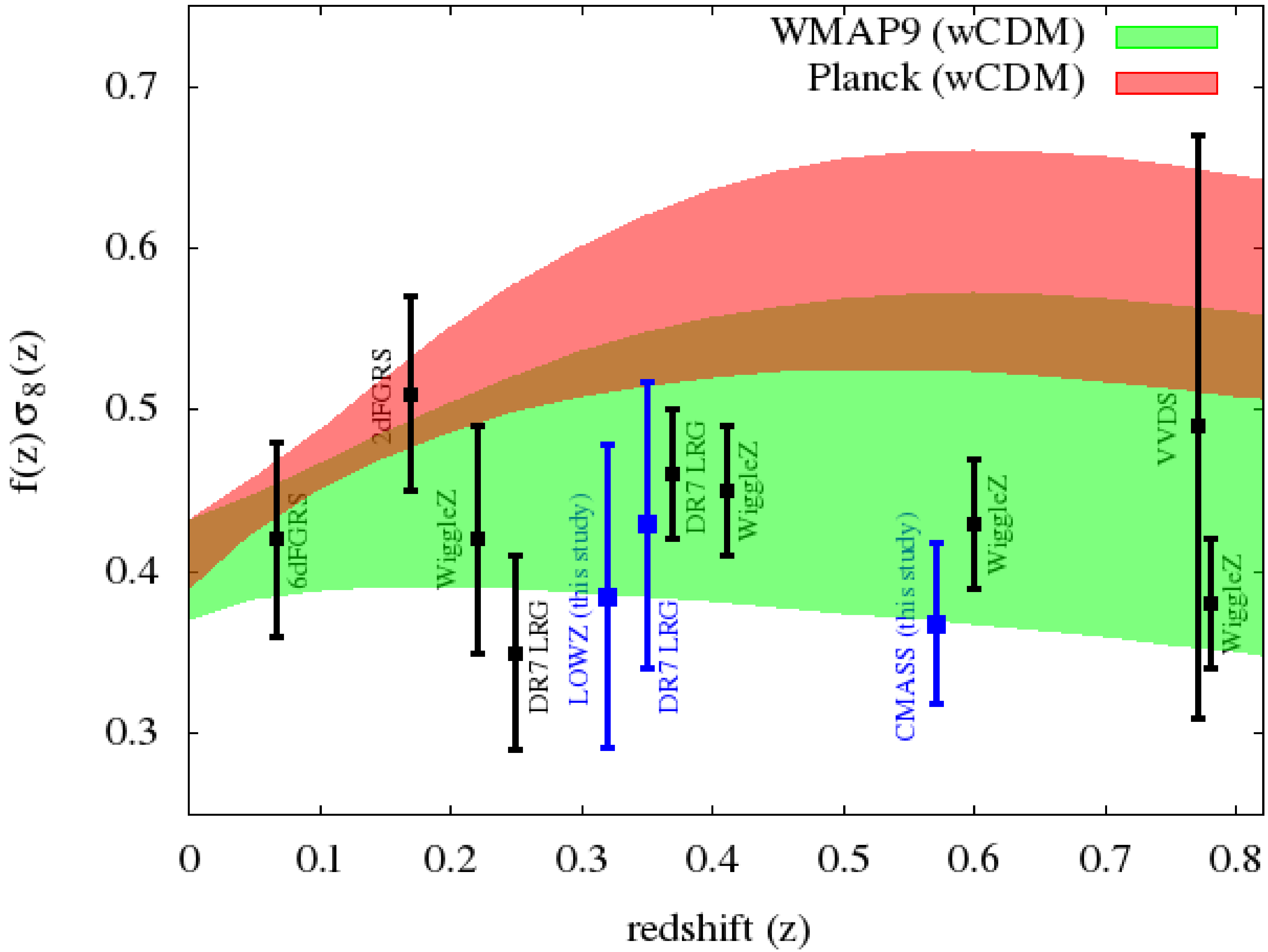
Standard ruler
$D_s \sim 150$ Mpc
$D_s = \Delta z / H(z)$
$D_s = (1+z)\, D_A(z)\, \theta$
1/H
D_A

Cosmological probe of coherent motion

The first measured $f\sigma_8$

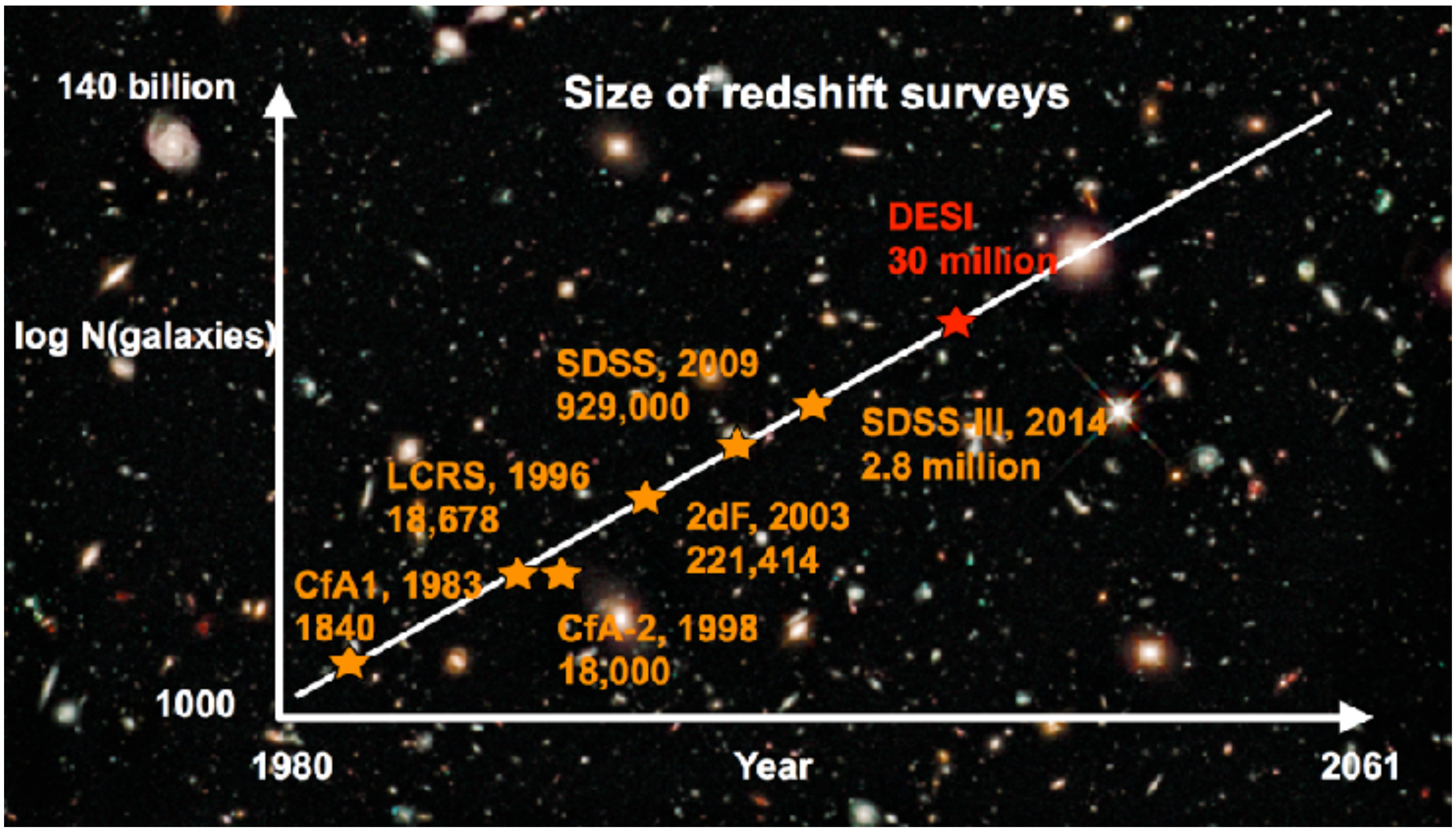


Cosmological probe of coherent motion



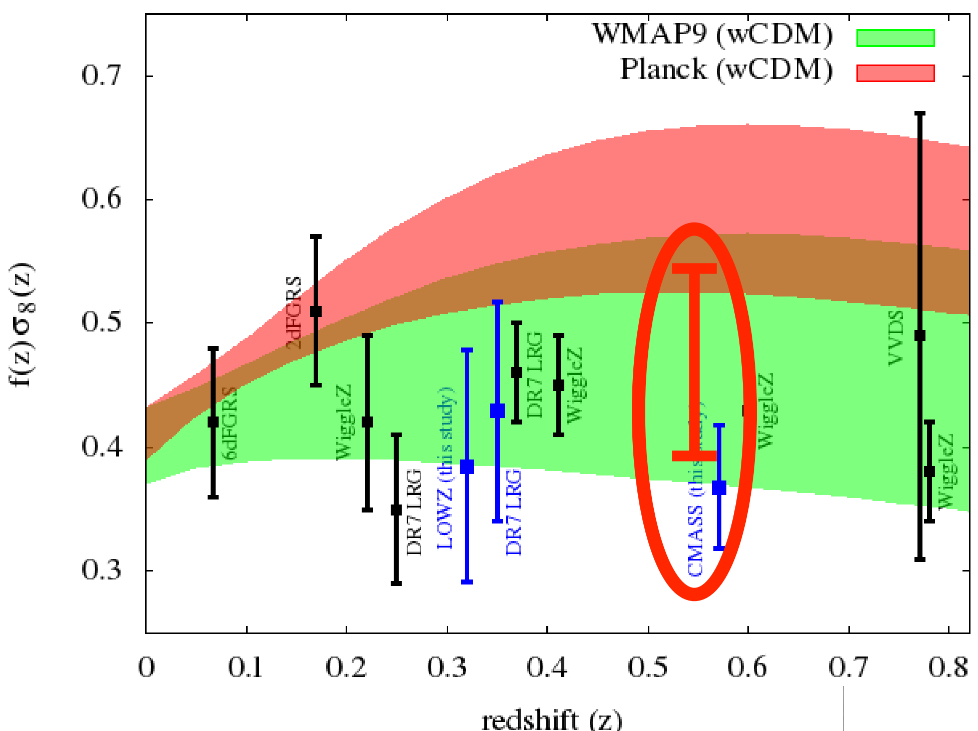
Motivation

High precision experiments planned to be launched



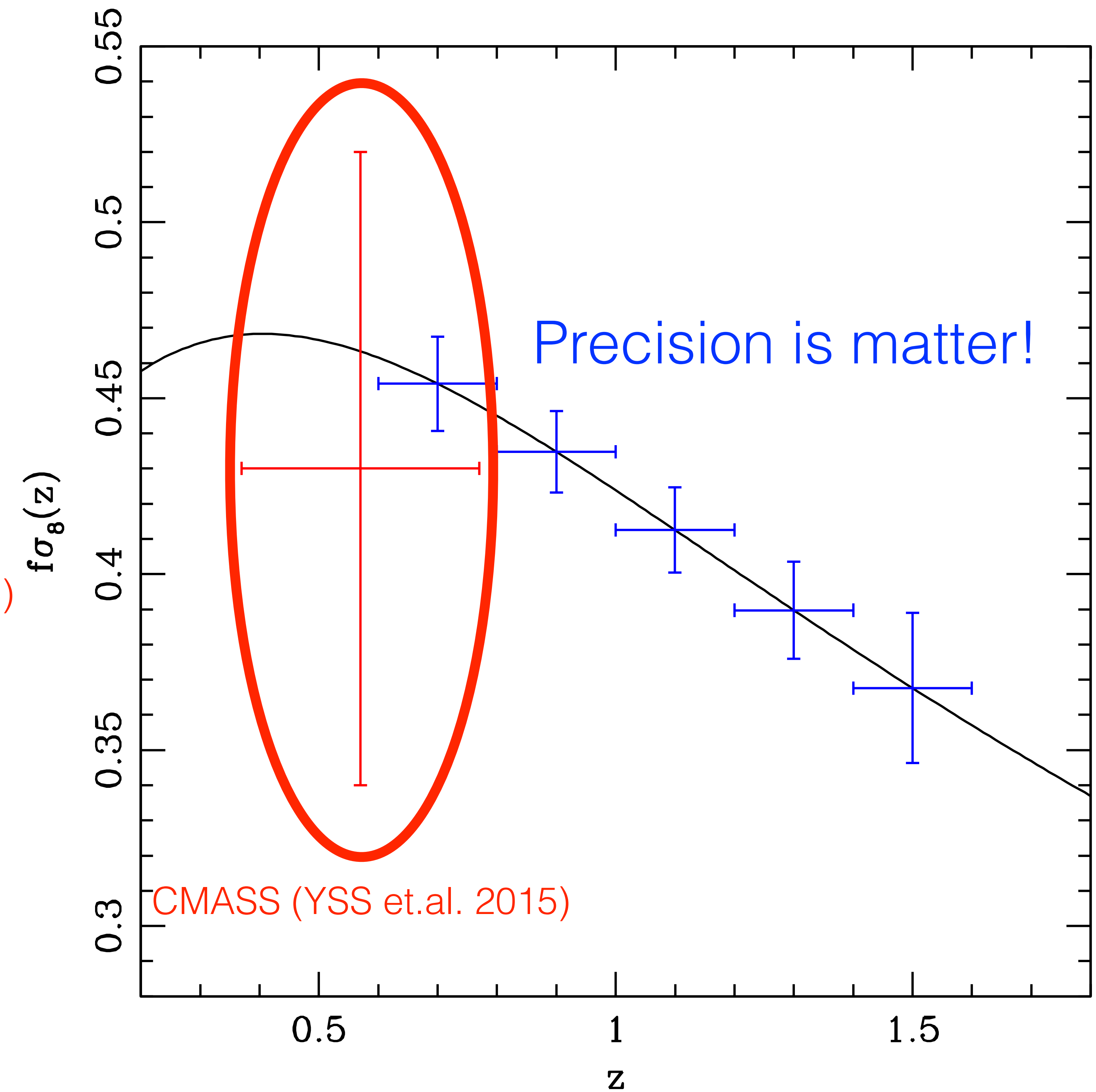
We answer to it by providing theoretical prediction within 1% accuracy

Cosmological probe of coherent motion



CMASS (Chuan et.al. 2015)

CMASS (YSS et.al. 2015)

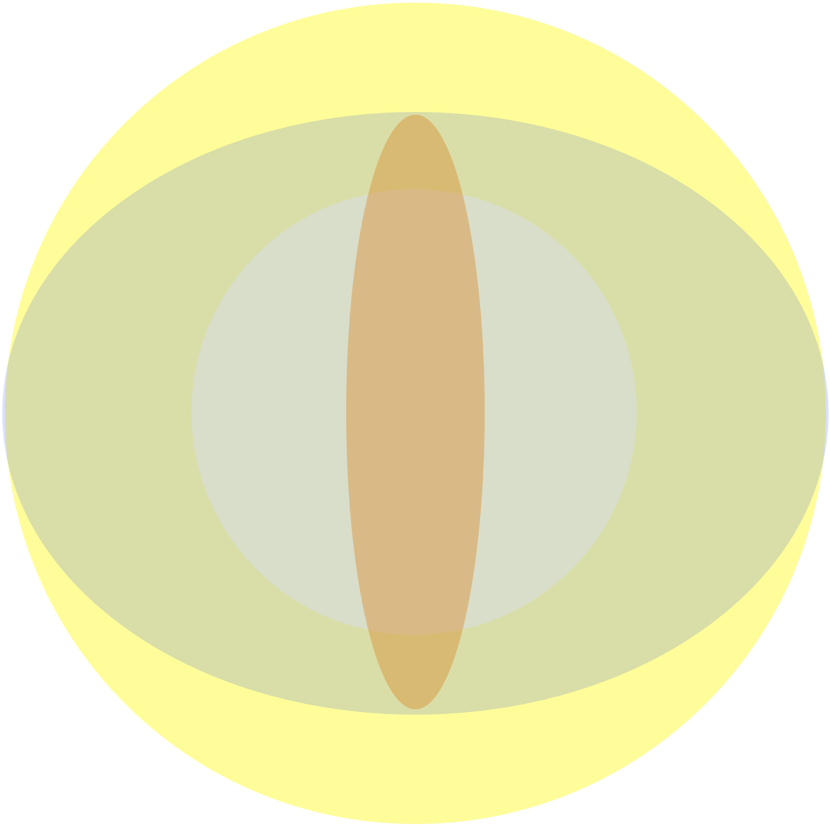


YSS, Taruya, Akira 2015

Power spectrum in redshift space

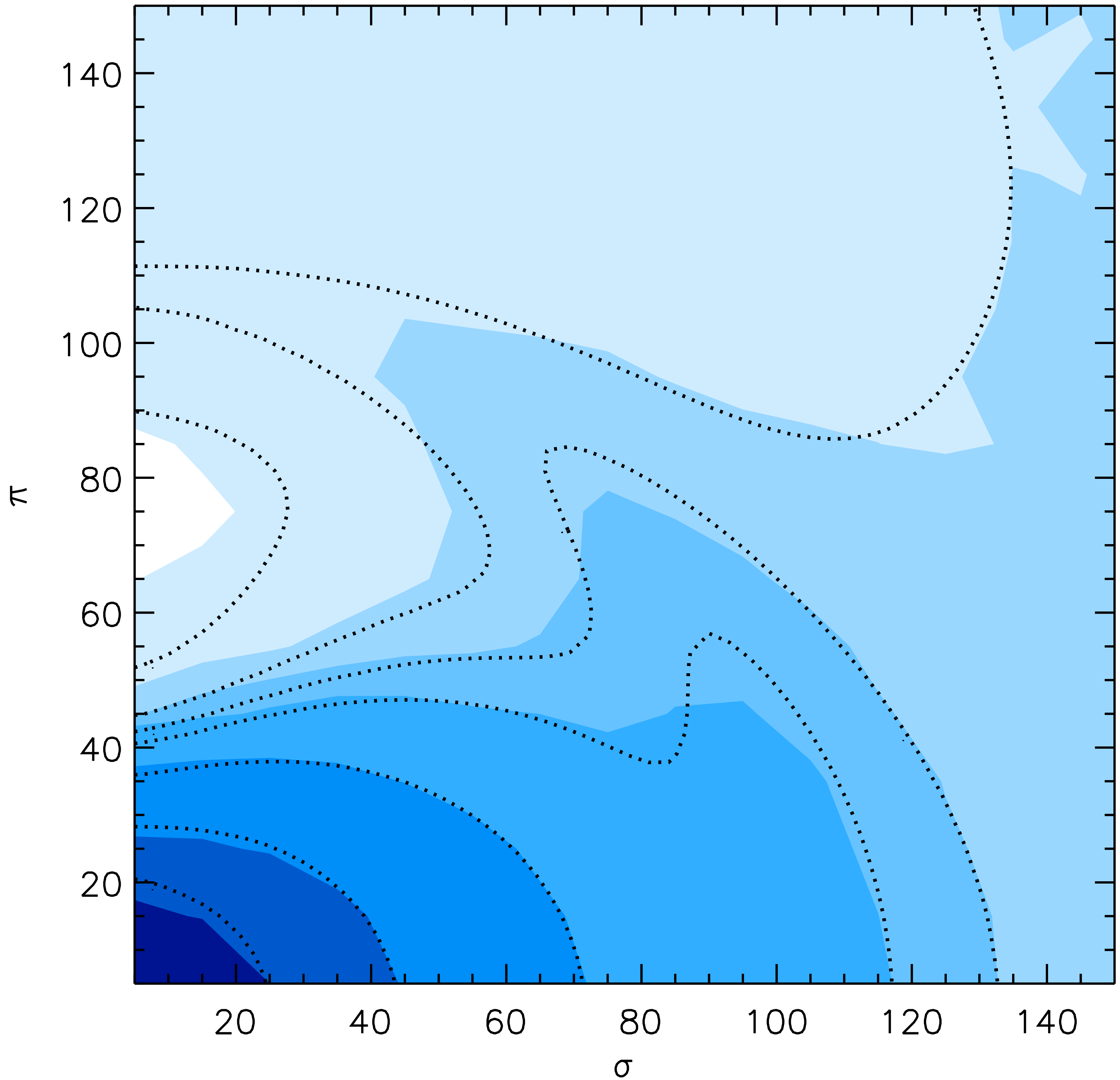
Squeezing effect at large scales

(Kaiser 1987)



$$P_s(k,\boldsymbol{\mu}) = P_{\delta\delta}(k) + 2\boldsymbol{\mu}^2 P_{\delta\Theta}(k) + \boldsymbol{\mu}^4 P_{\Theta\Theta}(k)$$

Anisotropy correlation without corrections



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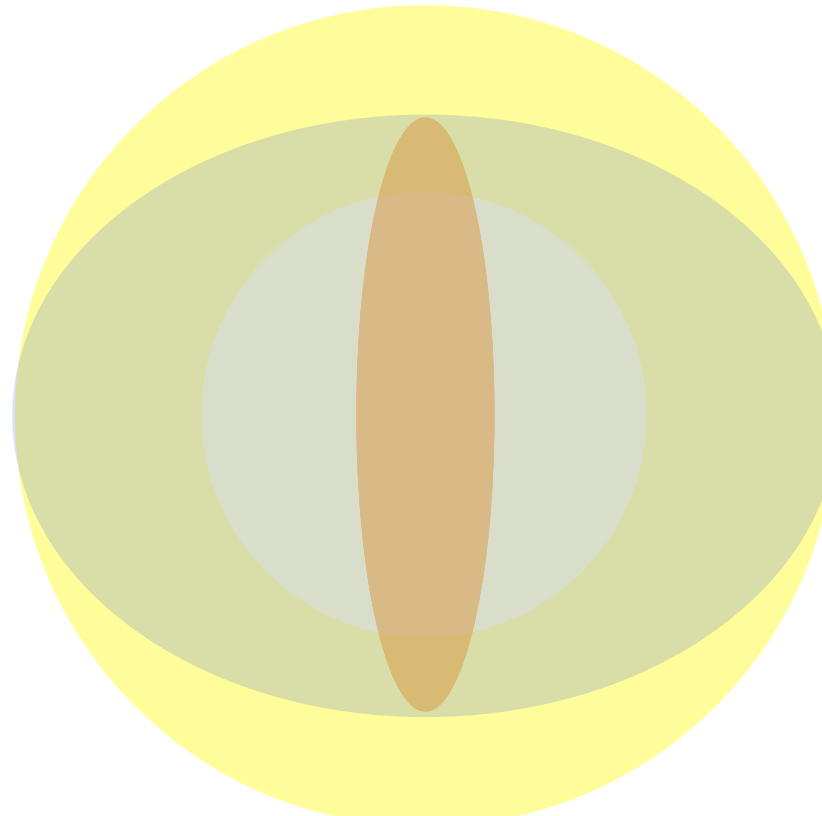


Non-linear corrections

Higher order polynomials

Finger of God effect

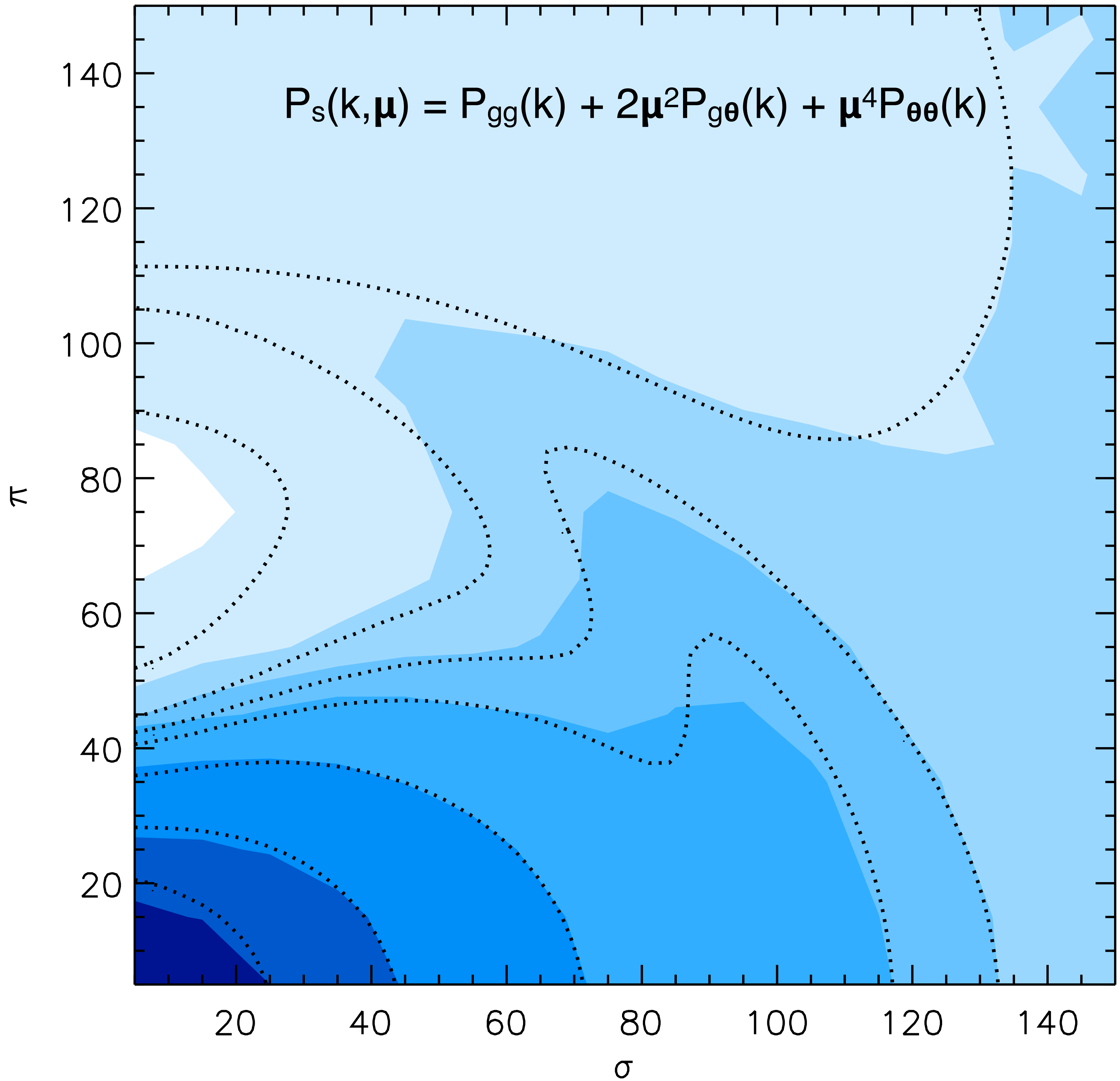
$$P_s(k,\boldsymbol{\mu}) = P_{\delta\delta}(k) + 2\boldsymbol{\mu}^2 P_{\delta\Theta}(k) + \boldsymbol{\mu}^4 P_{\Theta\Theta}(k)$$

$$\downarrow$$

$$P_s(k,\mu) = [P_{\delta\delta}(k) + 2\boldsymbol{\mu}^2 P_{\delta\Theta}(k) + \boldsymbol{\mu}^4 P_{\Theta\Theta}(k) + \text{Corrections} \ldots]$$

Fisher 1995; Scoccimarro 2004; Reid, White 2009; Taruya, Nishimichi, Saito 2010; Okumura, Seljak et.al 2010, 2011; Zhang et.al. 2011; Zheng, Song 2016

Anisotropy correlation without corrections



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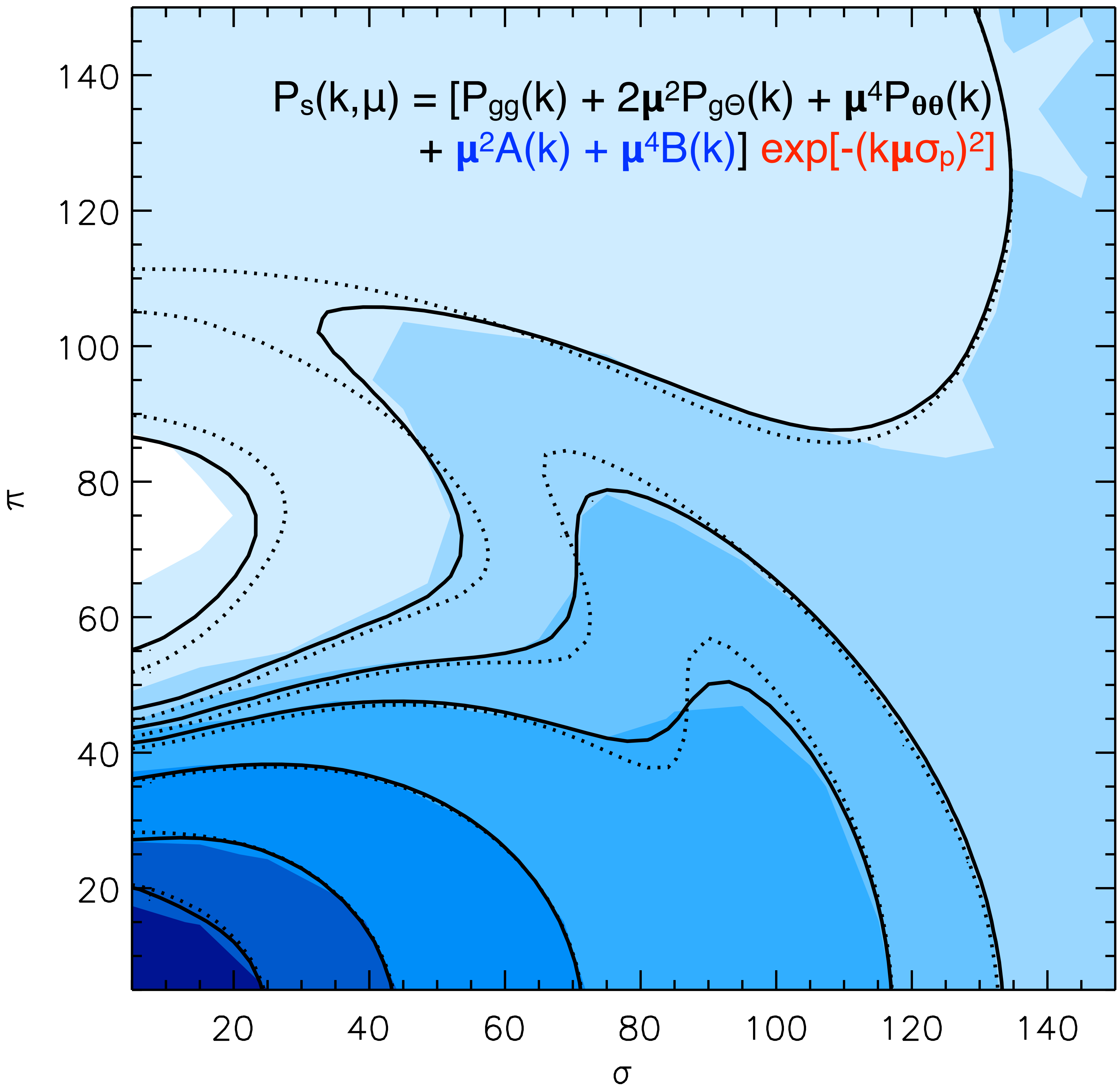

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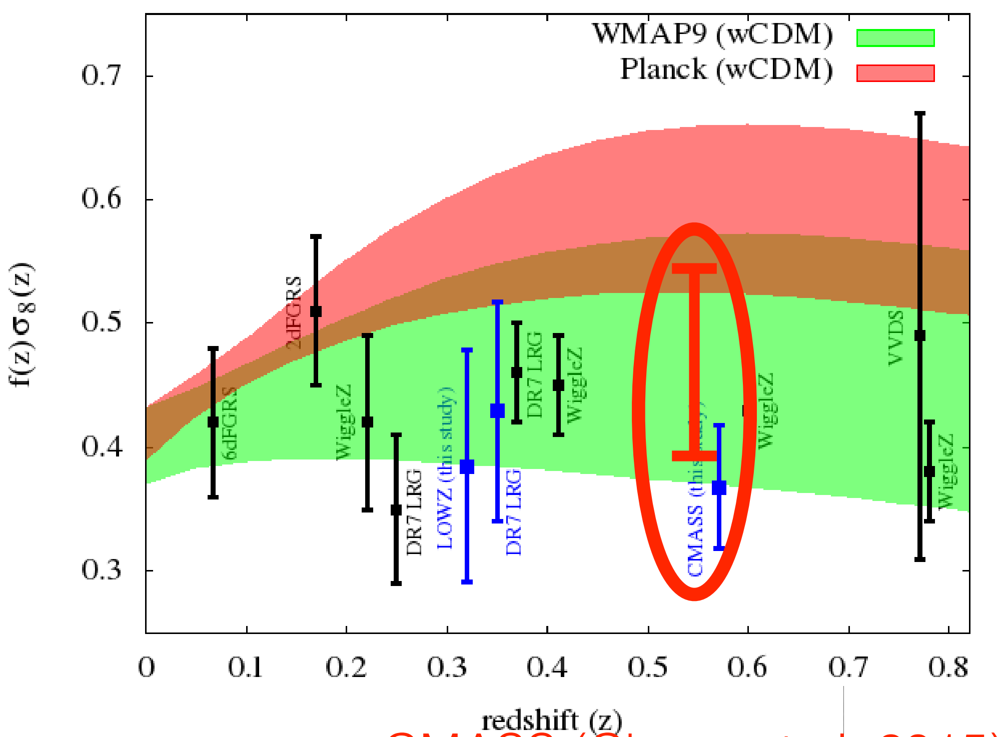

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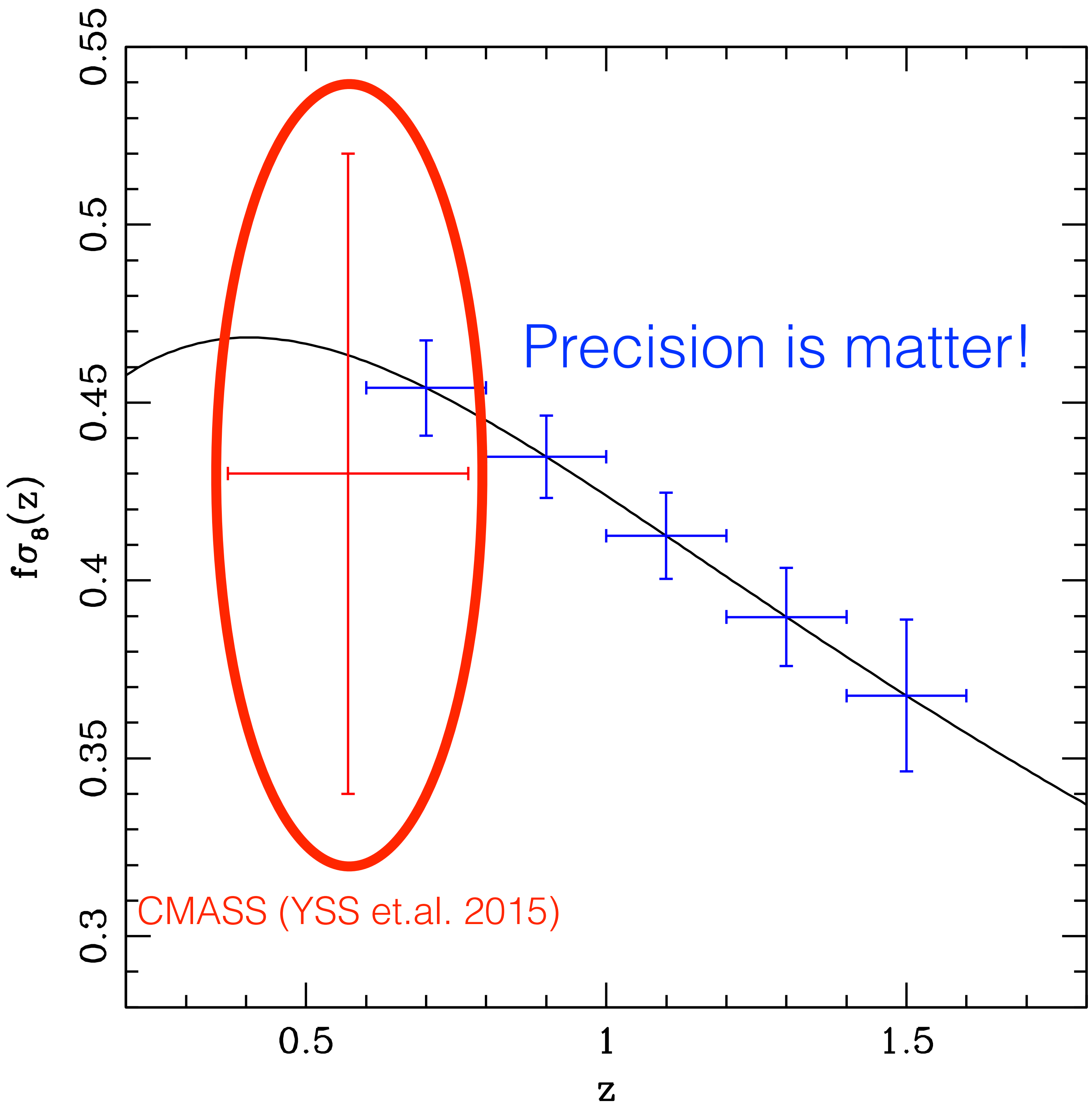


YSS, Taruya, Akira 2015

Mapping of clustering from real to redshift spaces

$$P_s(k,\boldsymbol{\mu}) = \int d^3x\ e^{ikx} \langle \boldsymbol{\delta\delta} \rangle$$

$$\Downarrow$$

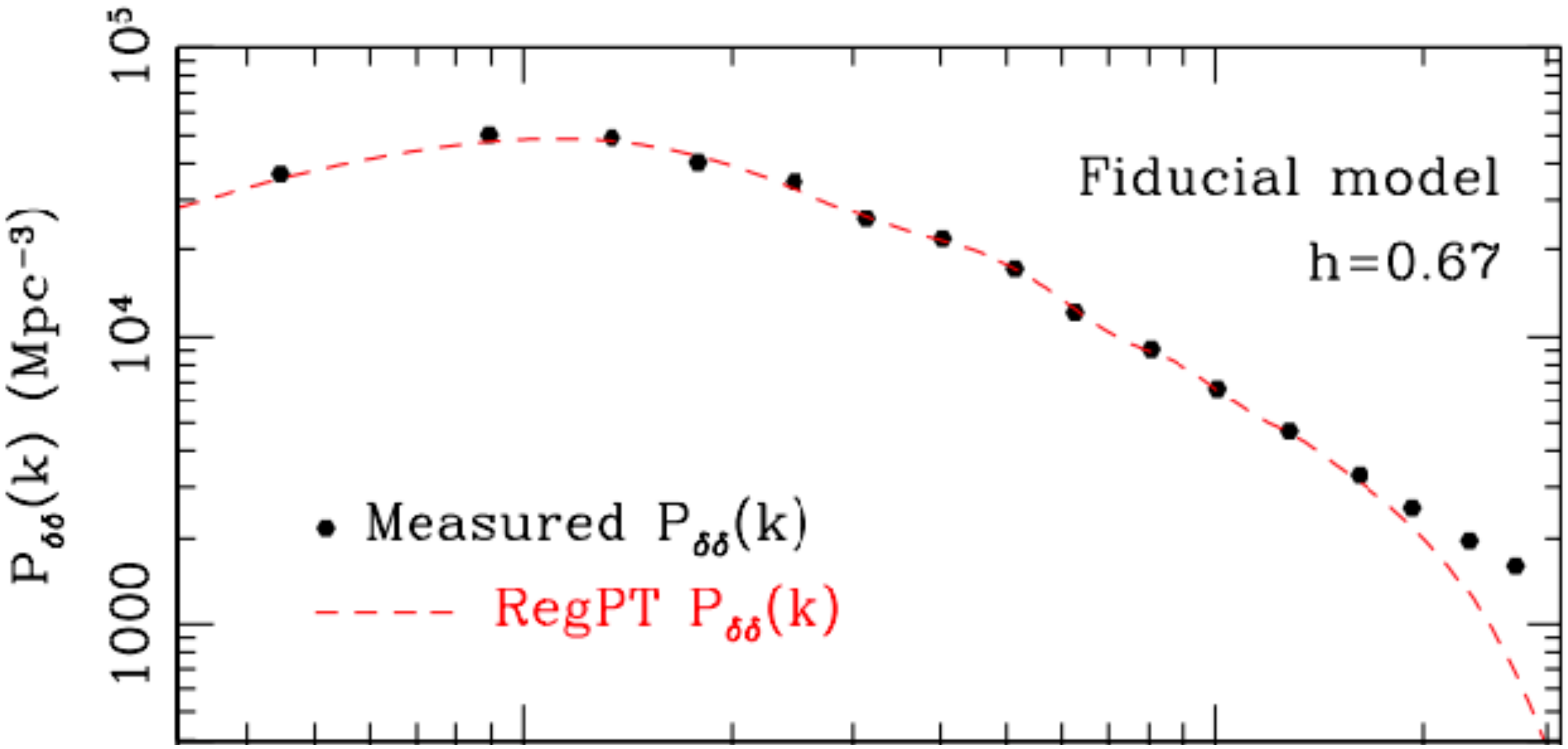
$$P_s(k,\mu) = \int d^3x\ e^{ikx} \langle e^{jv} (\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta) \rangle$$

$$= \int d^3x\ e^{ikx} \exp\{\langle e^{jv} \rangle_c\} [\langle e^{jv}(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)\rangle_c + \langle e^{jv}(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)\rangle_c \langle e^{jv}(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)\rangle_c]$$

- We understand RSD as a mapping from real to redshift space including stochastic quantity of peculiar velocity
- The mapping contains the contribution from two point correlation functions depending on separation distance, such as the cross correlation of density and velocity and the velocity auto correlation.
- The mapping also contains the contribution from one point correlation function of peculiar velocity which can be given by a functional form in terms of velocity dispersion σ_p.

Taruya, Nishimichi, Saito 2010

Non linear corrections



- We compare the theoretical predictions from RegPT and the measured spectrum of density fluctuations. Both are consistent up to quasi linear scale.
- As this correction is not relevant to RSD mapping, we will discuss it at later part of this talk when we need to explain the growth function projection.

Mapping of clustering from real to redshift spaces

$$P_s(k,\boldsymbol{\mu}) = \int d^3x\, e^{ikx} \langle \boldsymbol{\delta\delta} \rangle$$

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The contribution from two point correlations

$$P_s = \int d^3x\, e^{ikx} \exp\{\langle e^{jv}\rangle_c\} \left[\langle e^{jv}(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)\rangle_c + \langle e^{jv}(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)\rangle_c \langle e^{jv}(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)\rangle_c\right]$$

- The contribution from the cross correlation between density and velocity fields

$$\langle e^{jv}(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)\rangle_c + \langle e^{jv}(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)\rangle_c \langle e^{jv}(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)\rangle_c$$

$$= j^0 \langle(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)\rangle_c$$

$$+ j^1\langle v(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)\rangle_c$$

$$+ j^2\langle v(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)\rangle_c\langle v(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)\rangle_c$$

$$+ j^2\langle vv(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)\rangle_c$$

$$+ j^2\langle vv\rangle_c\langle(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)\rangle_c$$

$$+ O(> j^3)$$

The contribution from two point correlations

$$P_s = \int d^3x\, e^{ikx} \exp\{\langle e^{jv}\rangle_c\} [\langle e^{jv}(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)\rangle_c + \langle e^{jv}(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)\rangle_c \langle e^{jv}(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)\rangle_c]$$

- We truncate the infinite polynomials above j^2 order, then the following terms are defined as;

$$A(k,\boldsymbol{\mu}) = j^1 \int d^3x\, e^{ikx} \langle v(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)\rangle_c$$

$$B(k,\boldsymbol{\mu}) = j^2 \int d^3x\, e^{ikx} \langle v(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)\rangle_c \langle v(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)\rangle_c$$

$$T(k,\boldsymbol{\mu}) = j^2 \int d^3x\, e^{ikx} \langle vv(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)\rangle_c$$

The contribution from two point correlations

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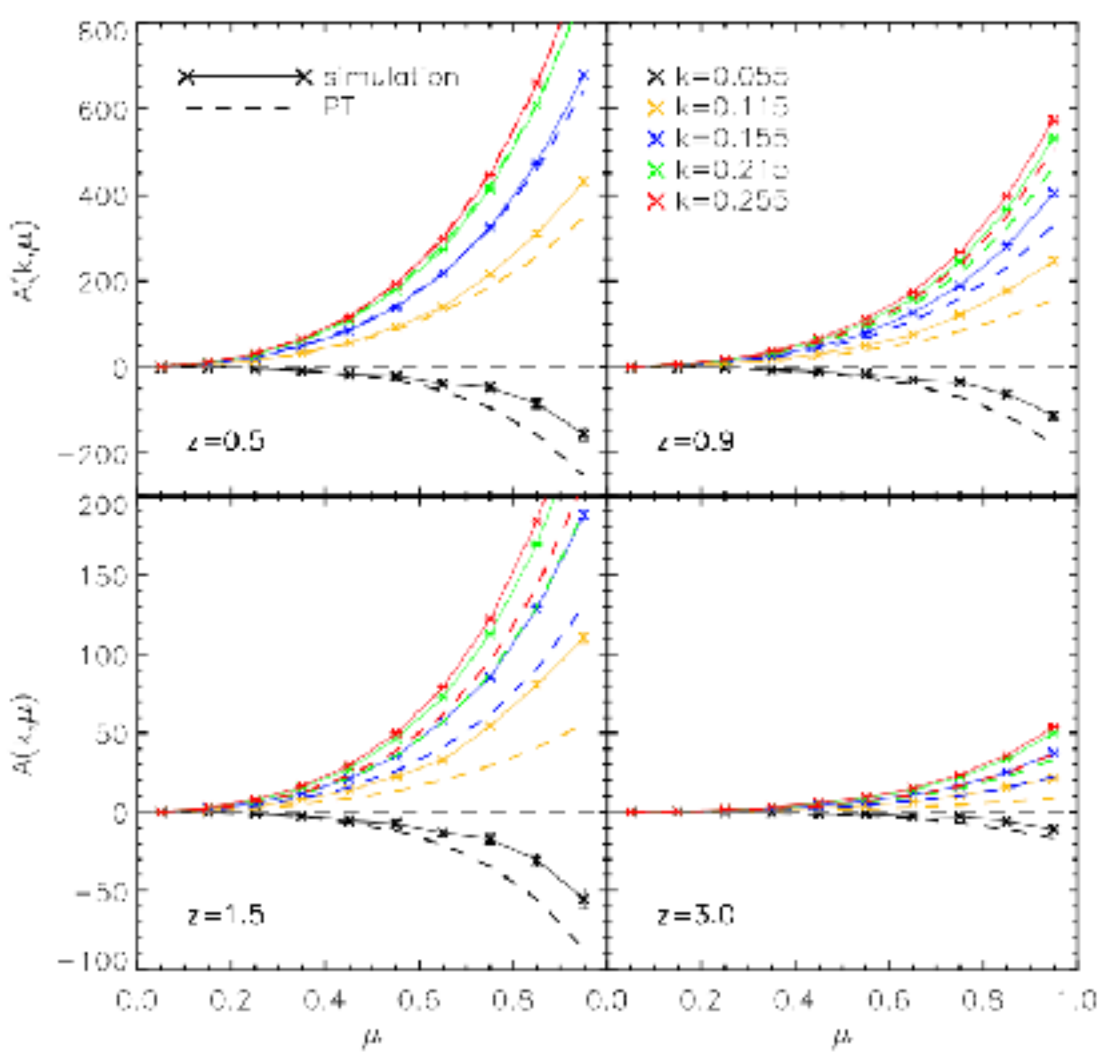
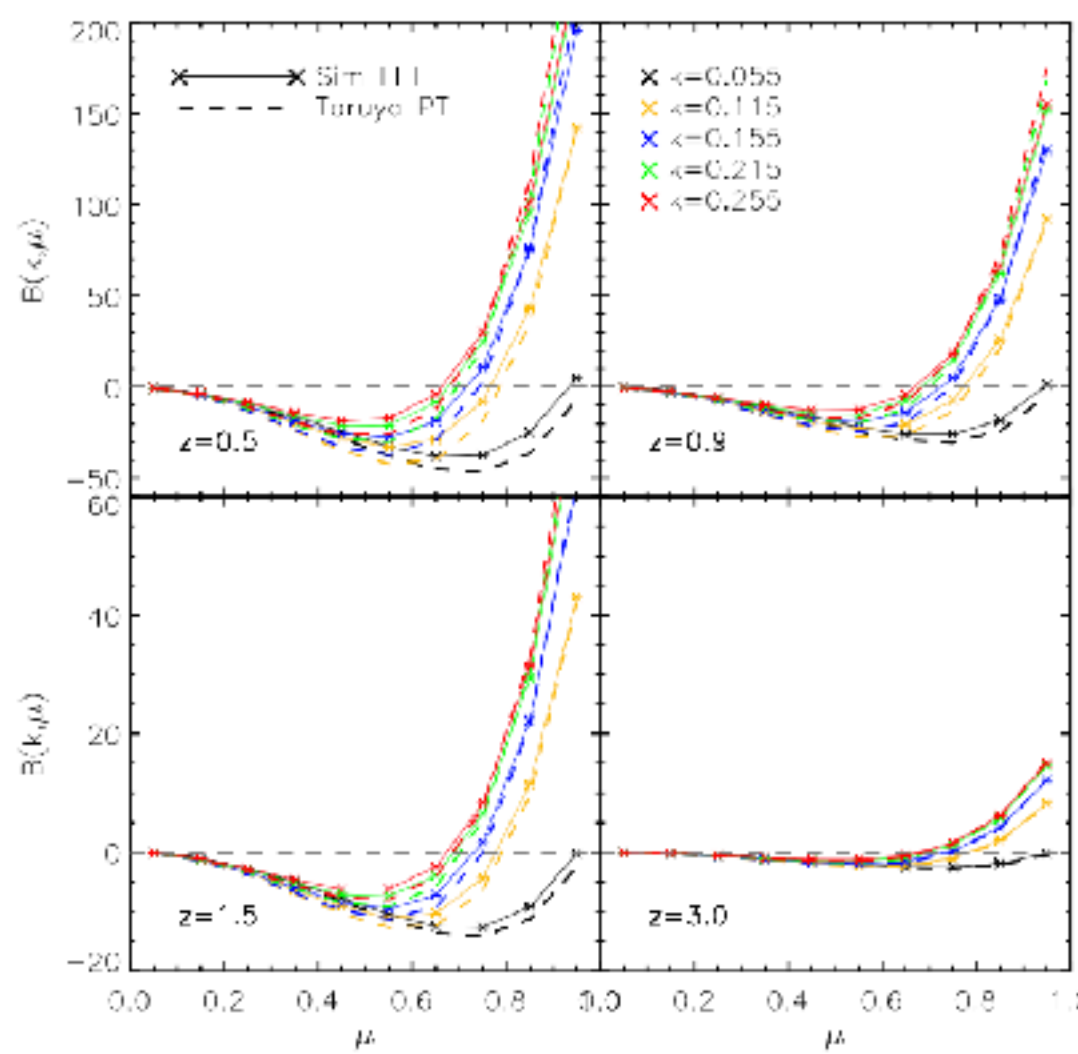
- The theoretical predictions of A and B are acceptable, while the measured A and B are better to be exploited;





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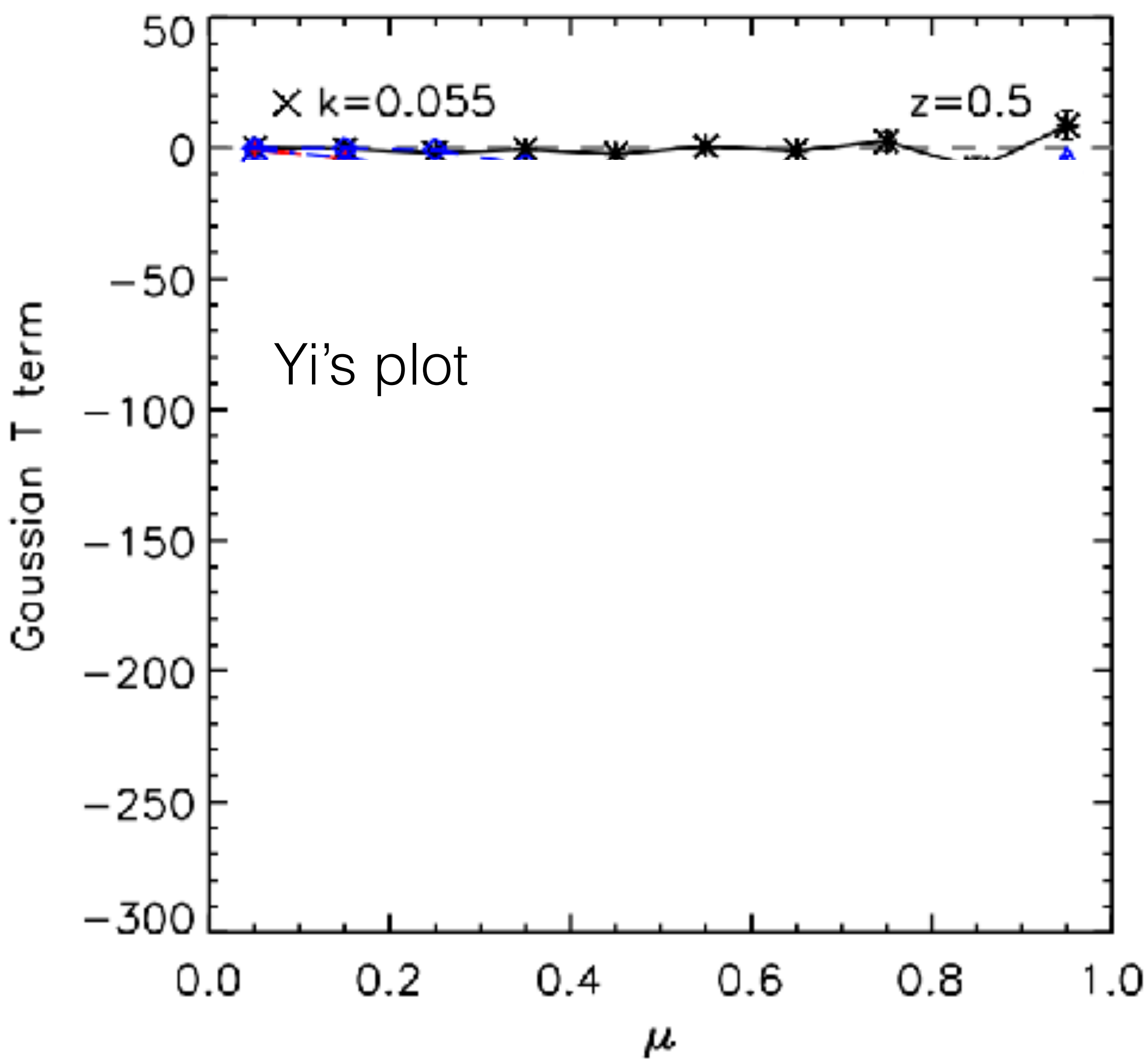
- We are not able to predict the full theoretical T expression at this moment



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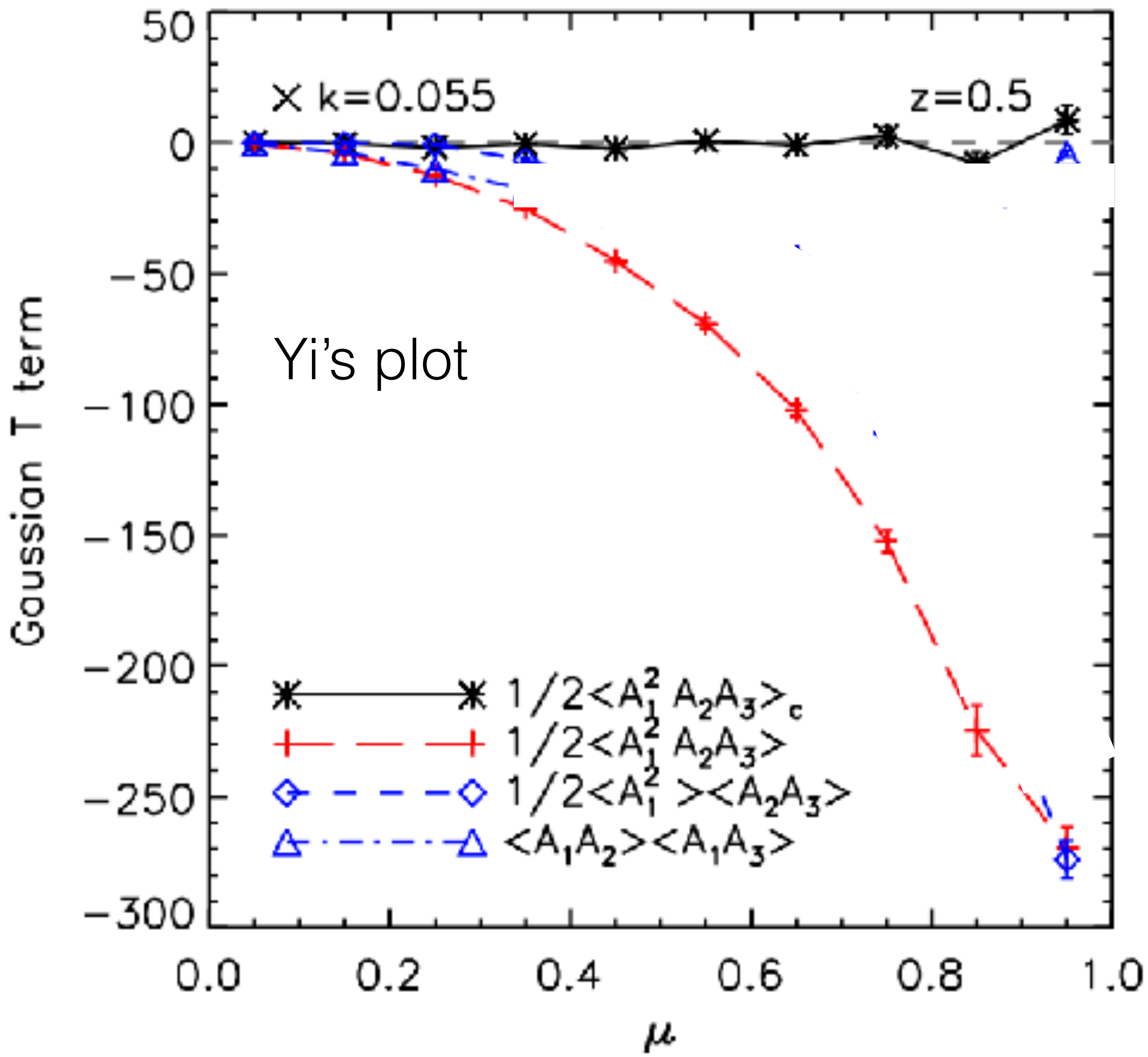

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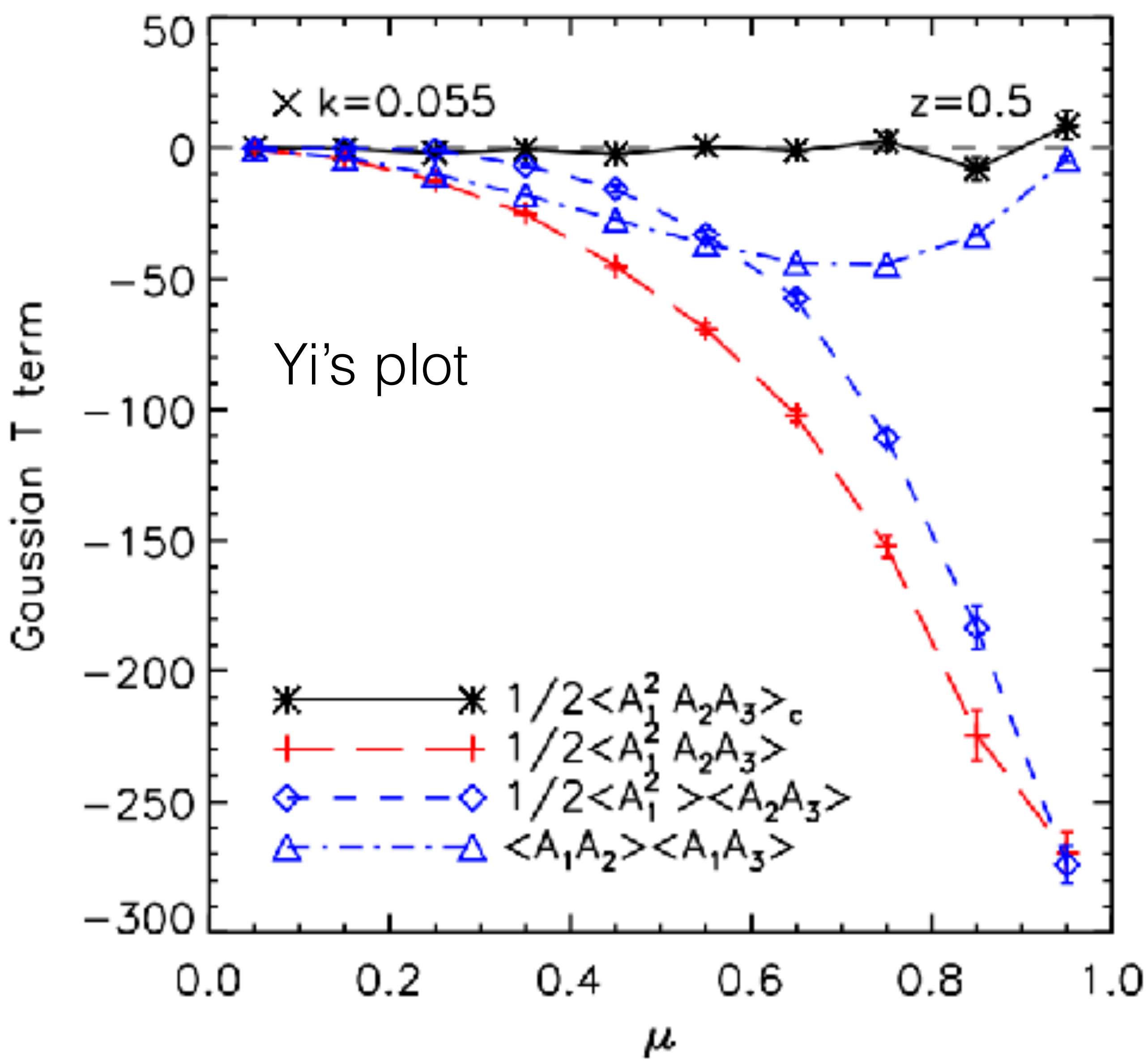

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- The term contains both one and two point correlation contributions, and we are going to separate those

$$
\begin{aligned}
\exp\left\{\langle e^{j_1 A_1}\rangle_c\right\} &= \exp\left\{\sum_{n=1}^{\infty} j_1^n \frac{\langle A_1^n\rangle_c}{n!}\right\} = \exp\left\{\sum_{n=1}^{\infty} j_1^{2n} \frac{\langle (u_z(\boldsymbol{r}) - u_z(\boldsymbol{r}'))^{2n}\rangle_c}{(2n)!}\right\} \\
&= \exp\left\{\sum_{n=1}^{\infty} j_1^{2n} \frac{2\langle u_z(\boldsymbol{r})^{2n}\rangle_c}{(2n)!}\right\} \times \exp\left\{\sum_{n=1}^{\infty} j_1^{2n} \frac{\langle (u_z(\boldsymbol{r}) - u_z(\boldsymbol{r}'))^{2n}\rangle_c - \langle u_z(\boldsymbol{r})^{2n}\rangle_c - \langle u_z(\boldsymbol{r}')^{2n}\rangle_c}{(2n)!}\right\} \\
&= D_{\rm 1pt}^{\rm FoG}(k\mu) \times D_{\rm corr}^{\rm FoG}(k\mu, \boldsymbol{x}).
\end{aligned}
$$

$$
\begin{aligned}
D_{\rm corr}^{\rm FoG}(k\mu, \boldsymbol{x}) &\equiv \exp\left\{\sum_{n=1}^{\infty} j_1^{2n} \frac{\langle (u_z(\boldsymbol{r}) - u_z(\boldsymbol{r}'))^{2n}\rangle_c - \langle u_z(\boldsymbol{r})^{2n}\rangle_c - \langle u_z(\boldsymbol{r}')^{2n}\rangle_c}{(2n)!}\right\} \\
&= \exp\left\{-j_1^2 \langle u_z(\boldsymbol{r}) u_z(\boldsymbol{r}')\rangle_c + \sum_{n=2}^{\infty} j_1^{2n} \frac{\langle (u_z(\boldsymbol{r}) - u_z(\boldsymbol{r}'))^{2n}\rangle_c - \langle u_z(\boldsymbol{r})^{2n}\rangle_c - \langle u_z(\boldsymbol{r}')^{2n}\rangle_c}{(2n)!}\right\}.
\end{aligned}
$$

$F(k,\boldsymbol{\mu}) = j^2 \int d^3x\, e^{ikx}\, \langle vv\rangle_c \langle(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)\rangle_c$

The contribution from two point correlations

$$P_s = \int d^3x\, e^{ikx} \exp\{\langle e^{jv}\rangle_c\} [\langle e^{jv}(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)\rangle_c + \langle e^{jv}(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)\rangle_c \langle e^{jv}(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)\rangle_c]$$

$$\Downarrow$$

$$P_s = D_{1pt}(k\boldsymbol{\mu}\sigma_p) \int d^3x\, e^{ikx}[P_{\delta\delta}(k) + 2\boldsymbol{\mu}^2 P_{\delta\Theta}(k) + \boldsymbol{\mu}^4 P_{\Theta\Theta}(k) + A(k,\boldsymbol{\mu}) + B(k,\boldsymbol{\mu}) + T(k,\boldsymbol{\mu}) + F(k,\boldsymbol{\mu})]$$

- We would like to test whether higher order contributions of j^n ($n>2$) is no longer contaminating mapping above threshold scale or not, by using the following residual test;

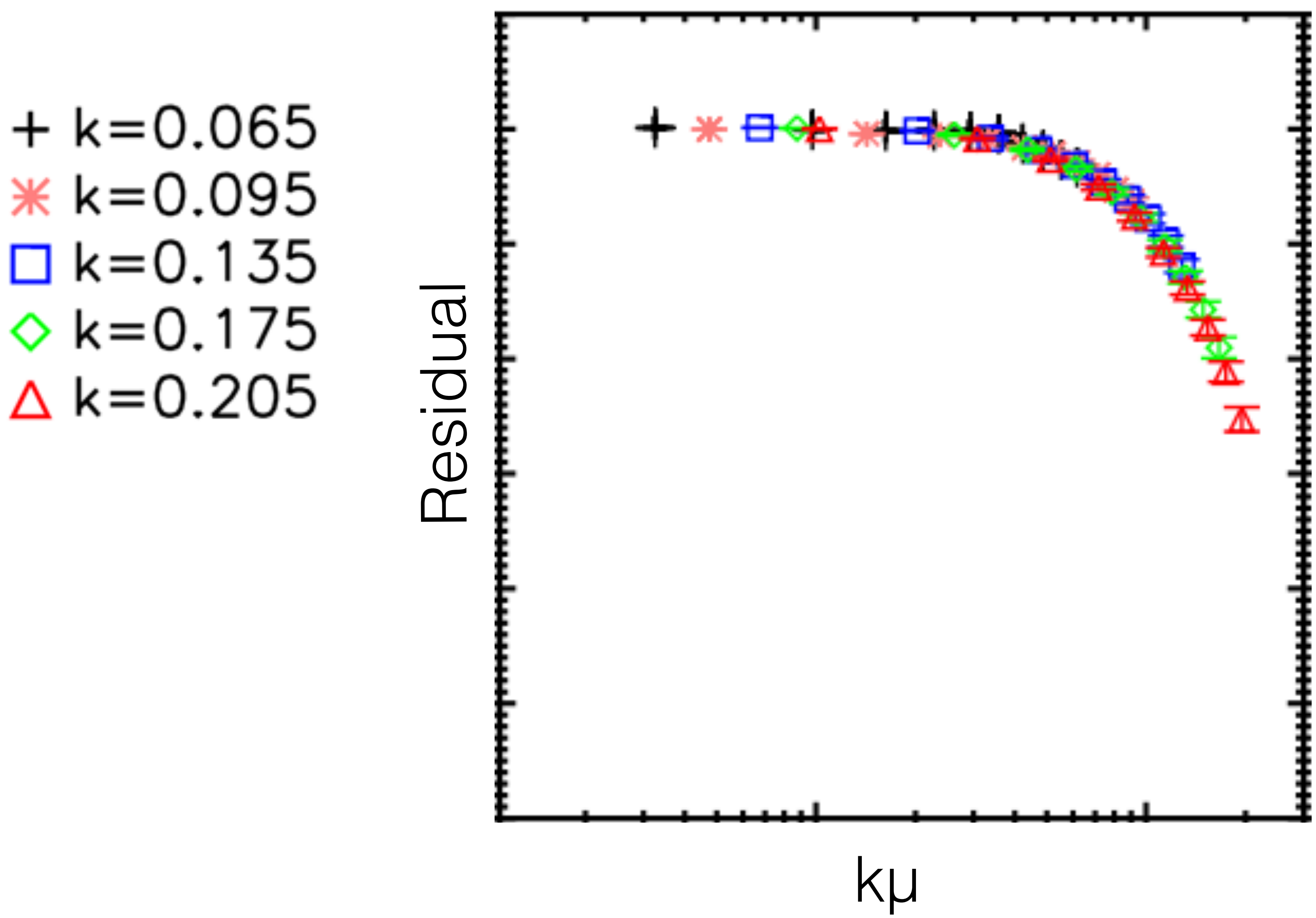
$$D_{1pt}(k\boldsymbol{\mu}\sigma_p) = P_s / \int d^3x\, e^{ikx}[P_{\delta\delta}(k) + 2\boldsymbol{\mu}^2 P_{\delta\Theta}(k) + \boldsymbol{\mu}^4 P_{\Theta\Theta}(k) + A(k,\boldsymbol{\mu}) + B(k,\boldsymbol{\mu}) + T(k,\boldsymbol{\mu}) + F(k,\boldsymbol{\mu})]$$

- If the truncation of correlated parts of perturbations is complete, then the measured residual would not show the explicit k dependence, but it will depend on kμ

The contribution from two point correlations

$D_{1pt} = P_s(k,\mu)/[P_{\delta\delta}(k) + 2\mu^2 P_{\delta\Theta}(k) + \mu^4 P_{\Theta\Theta}(k) + A(k,\mu) + B(k,\mu) + T(k,\mu) + F(k,\mu)]$

- The residual term which is the subtraction of the measured spectrum by the perturbed terms including halo density fluctuations is well fitting to Gaussian FoG function as well



Yi Zheng, YSS 2016

The contribution from two point correlations

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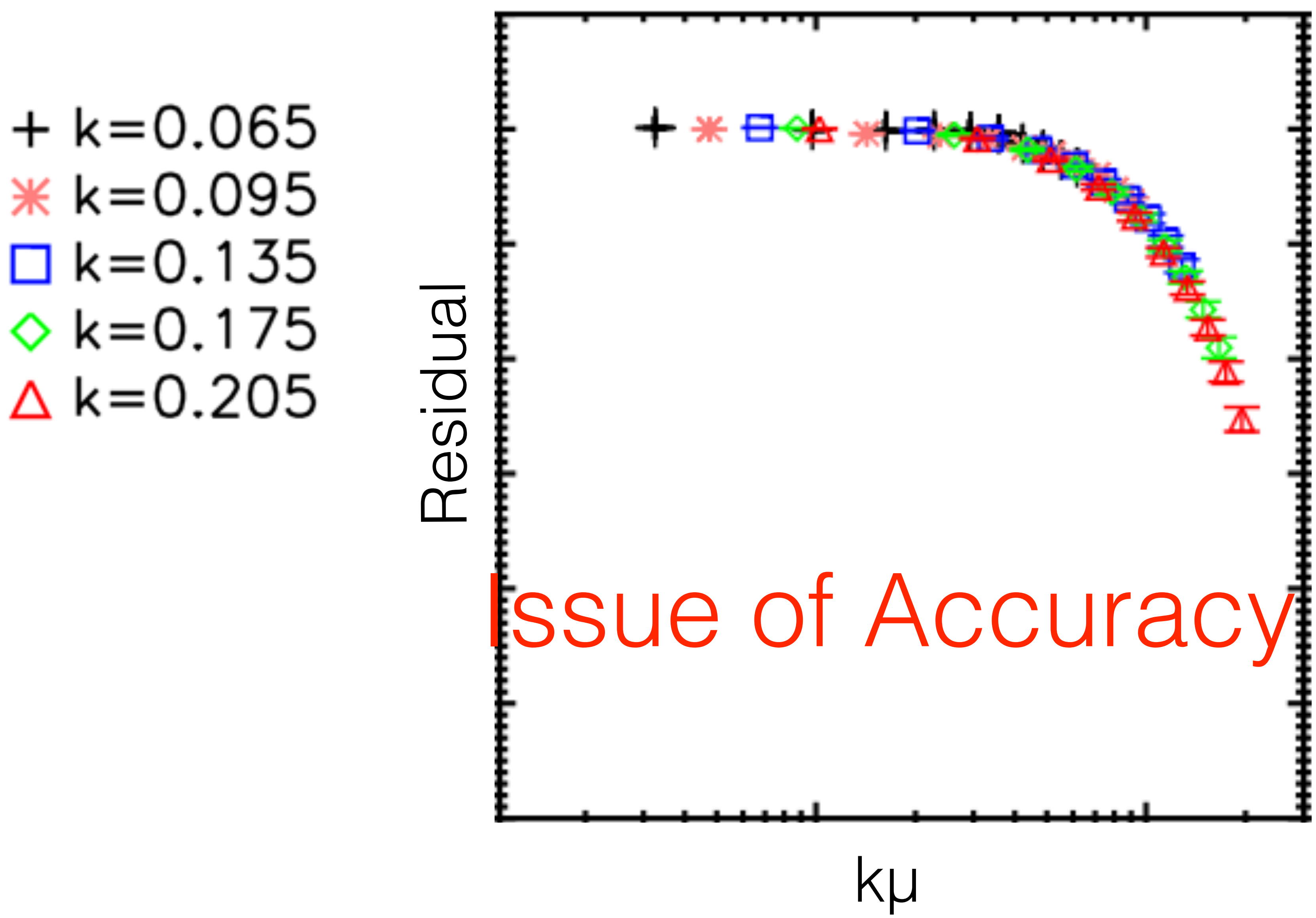

Yi Zheng, YSS 2016

Mapping of clustering from real to redshift spaces

$$P_s(k,\boldsymbol{\mu}) = \int d^3x\, e^{ikx} \langle \boldsymbol{\delta\delta} \rangle$$

↓

$$P_s(k,\mu) = \int d^3x\, e^{ikx} \langle e^{jv} (\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta) \rangle$$

$$= \int d^3x\, e^{ikx} \exp\{\langle e^{jv} \rangle_c\} \left[\langle e^{jv}(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta)(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta) \rangle_c + \langle e^{jv}(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta) \rangle_c \langle e^{jv}(\boldsymbol{\delta}+\boldsymbol{\mu}^2\Theta) \rangle_c \right]$$

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The contribution from one point correlations

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⬇

$$P_s = D_{1pt}(k\boldsymbol{\mu}\sigma_p) \int d^3x\, e^{ikx}[P_{\delta\delta}(k) + 2\boldsymbol{\mu}^2 P_{\delta\Theta}(k) + \boldsymbol{\mu}^4 P_{\Theta\Theta}(k) + A(k,\boldsymbol{\mu}) + B(k,\boldsymbol{\mu}) + T(k,\boldsymbol{\mu}) + F(k,\boldsymbol{\mu})]$$

- The residual one point correlation function contribution can be identified as FoG effect, and it is also expanded into the infinite loop in terms of σ_p

$$D_{1\mathrm{pt}}^{\mathrm{FoG}}(k\mu) = \exp\left\{ j_1^2 \sigma_z^2 + 2 \sum_{n=2}^{\infty} j_1^{2n} \sigma_z^{2n} \frac{K_{2n}}{(2n)!} \right\}$$

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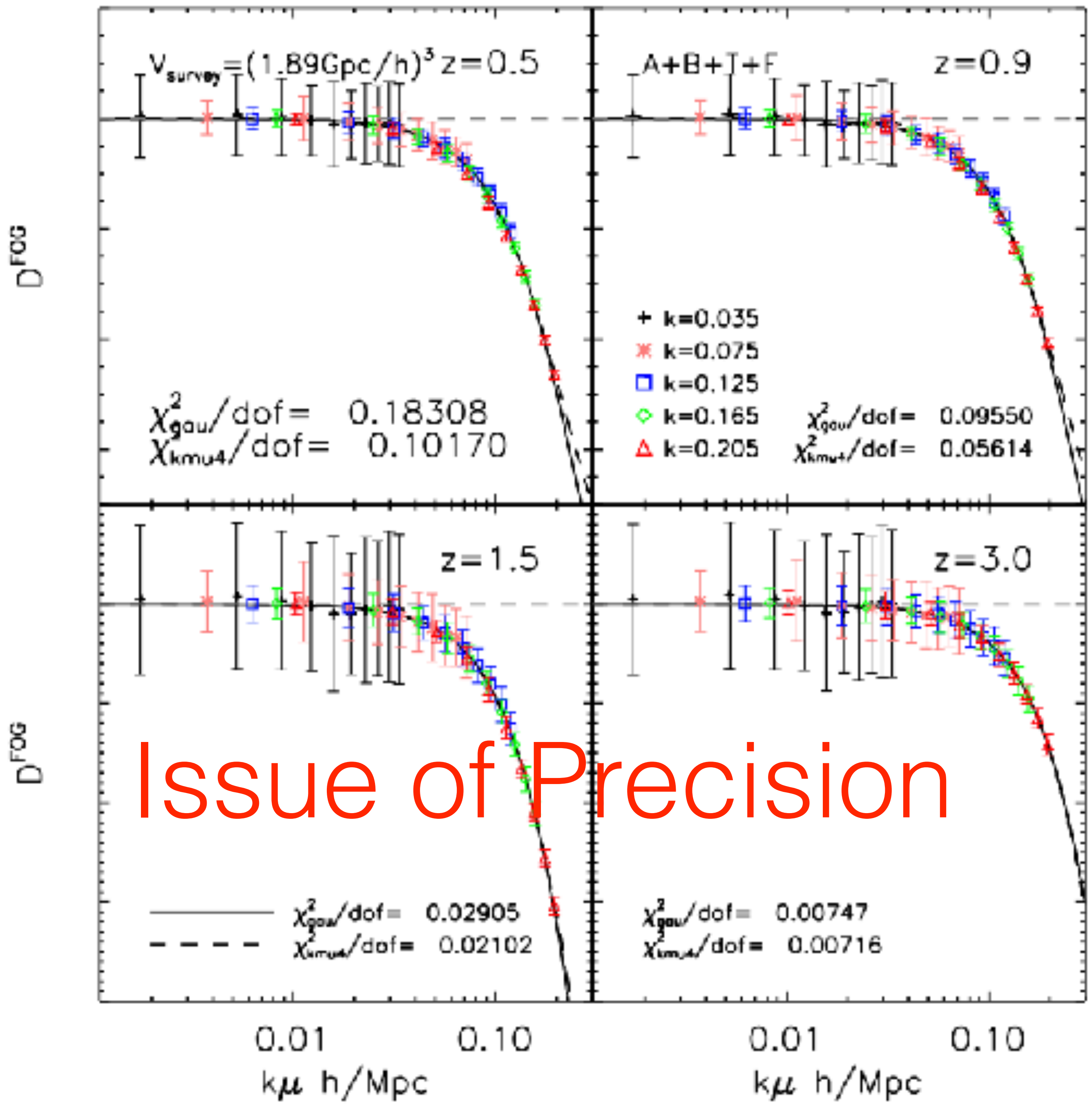
Well approximated to Gaussian

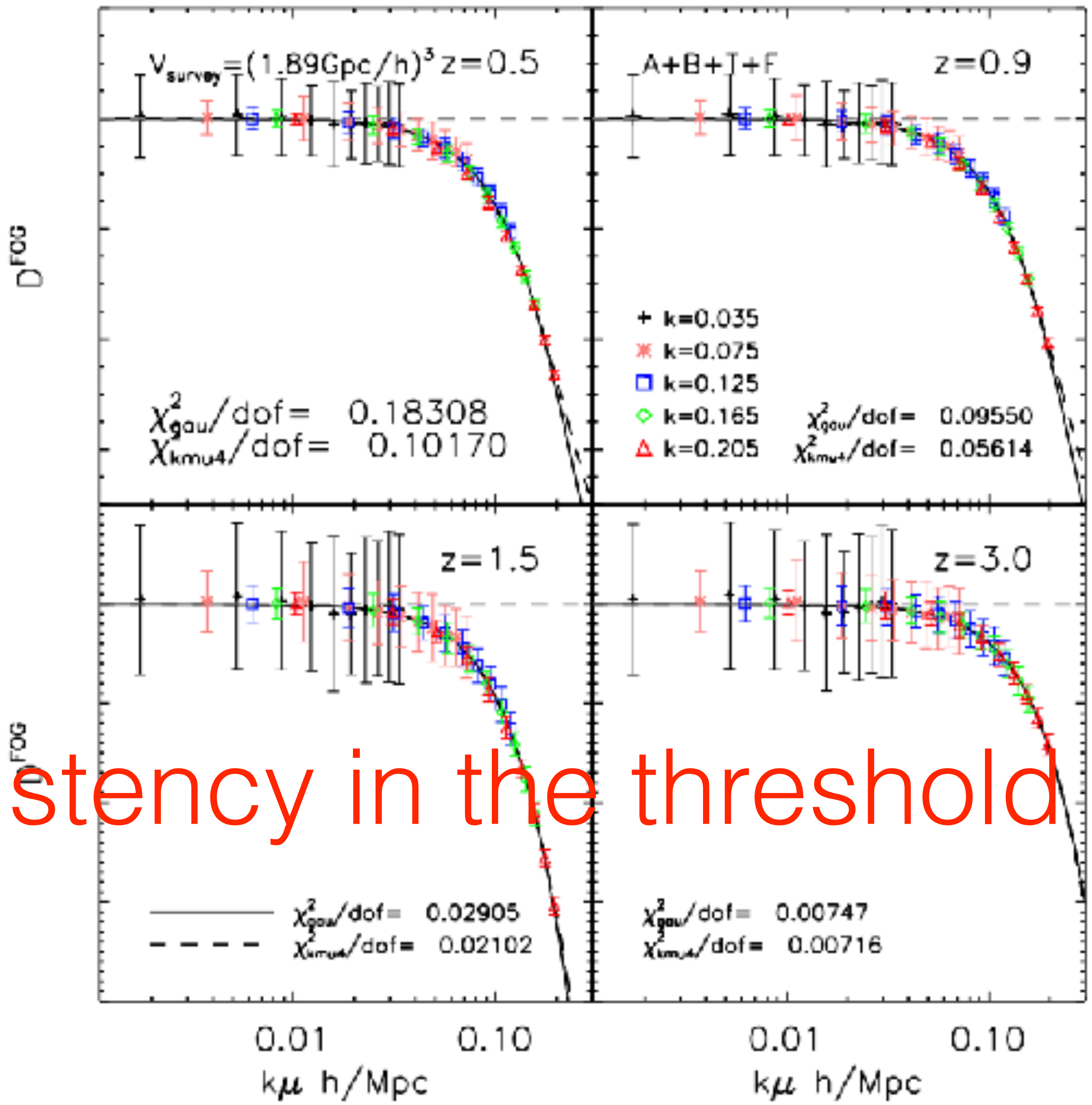


Issue of Precision

The contribution from one point correlations

$$D^{\rm FoG}_{\rm 1pt}(k\mu) = \exp\left\{ j_1^2\sigma_z^2 + 2\sum_{n=2}^{\infty} j_1^{2n}\sigma_z^{2n}\frac{K_{2n}}{(2n)!}\right\}$$

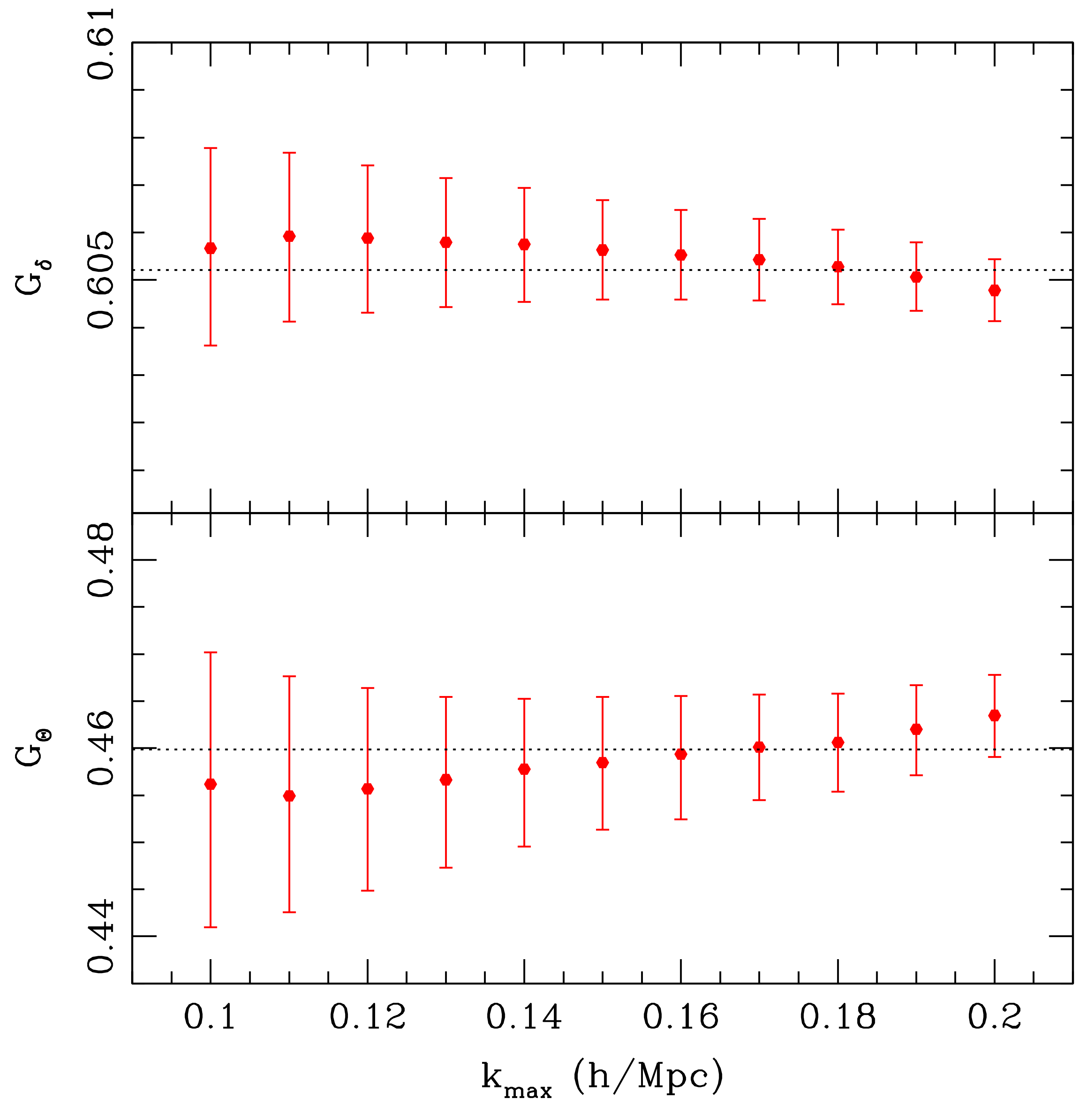
Well approximated to Gaussian



Consistency in the threshold scale

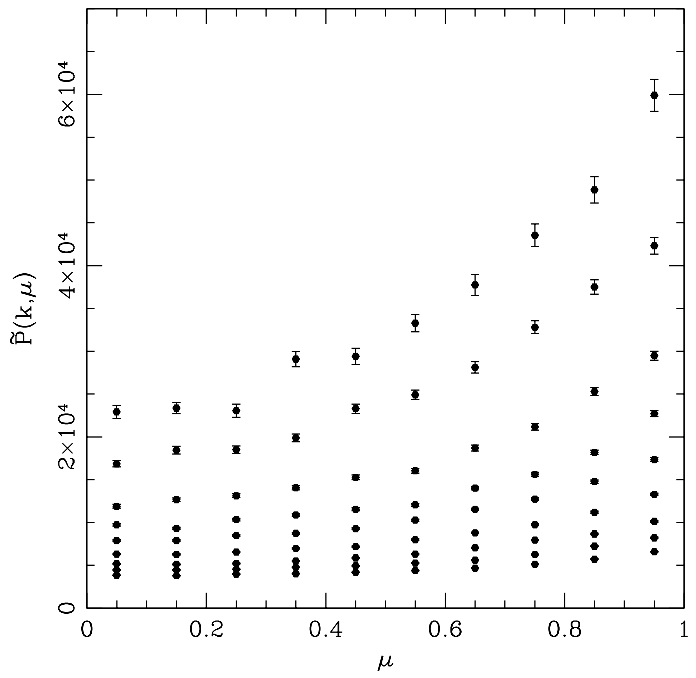
The threshold scale

The measured growth functions are consistent with the residual test



The contribution from all higher order polynomials

The differences between the best fit and observed spectra are presented



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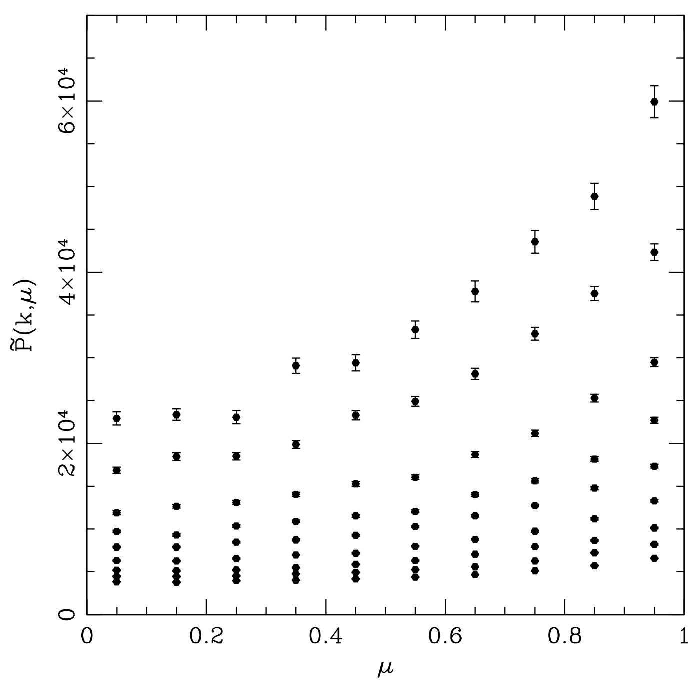

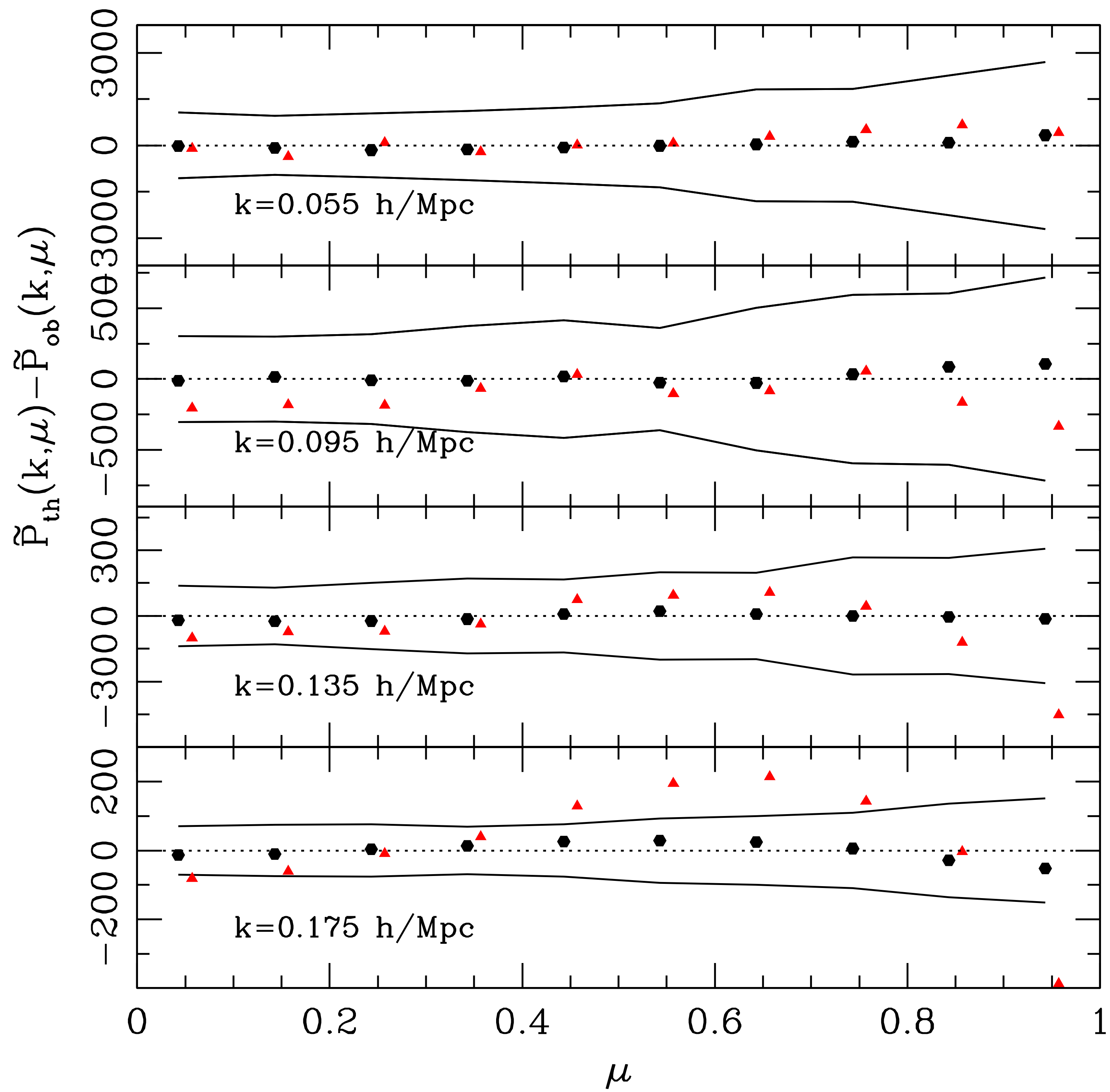

A+B

A+B+T

Open new window to test cosmological models

(D_A, H^{-1}, G_δ, G_Θ, FoG)

Standard model

Cold dark matter

Massless neutrino

New physics

Quintessence dark energy

Phantom dark energy

Λ

Open new window to test cosmological models

(D_A, H^{-1}, G_δ, G_Θ, FoG, New, New, …)

Standard model

Cold dark matter

Massless neutrino

Hot or warm dark matter

Massive neutrino

Interacting dark matter

Unified dark matter

Λ

New physics

Quintessence dark energy

Phantom dark energy

Decaying vacuum

Chameleon type gravity

Dilaton or Symmetron

Vainstein type gravity

Inhomogeneity of universe

non-Friedman universe

Precise determination on Ω_Λ

(D_A, H^{-1}, G_δ, G_Θ, FoG)

Standard model

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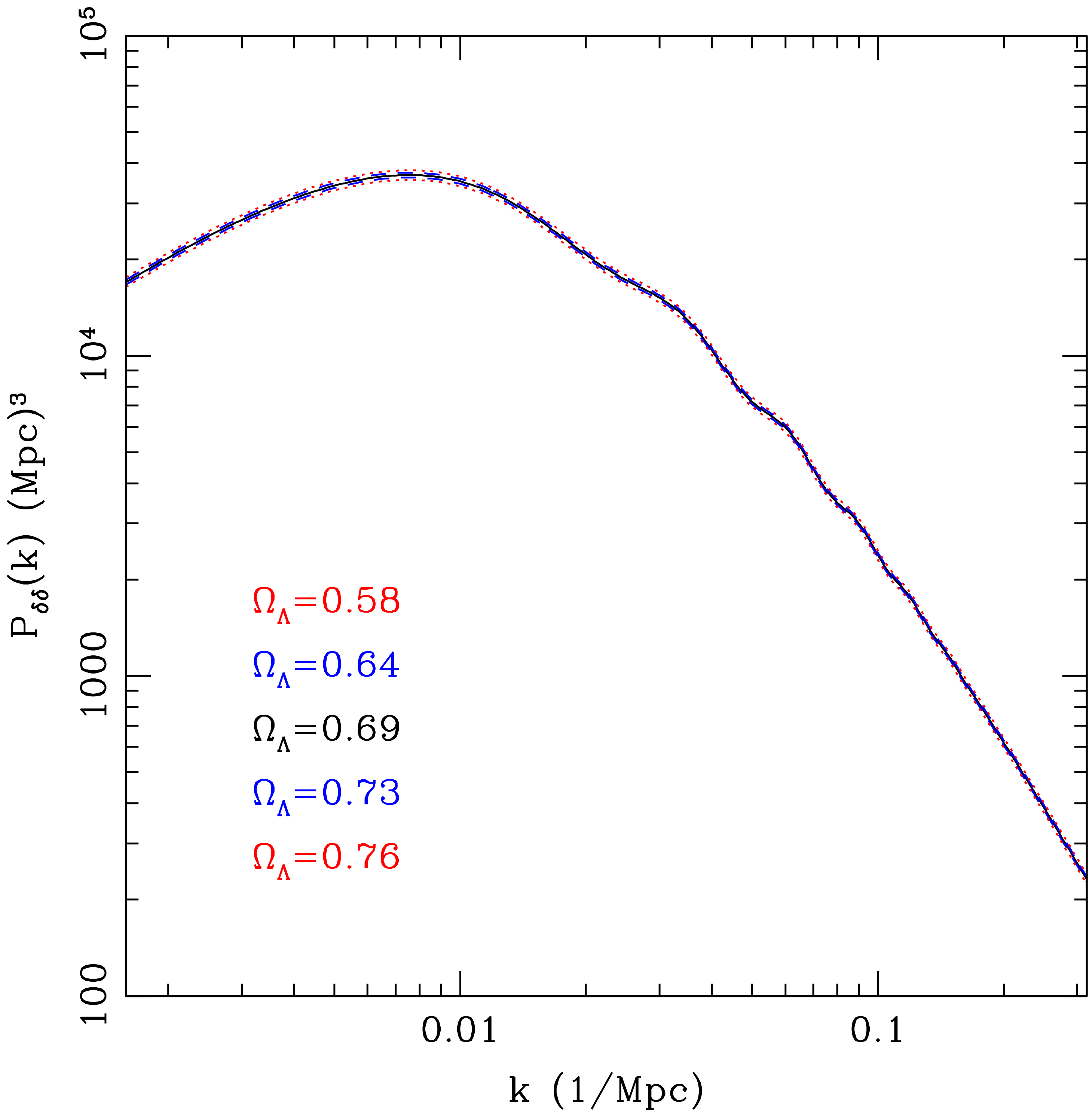
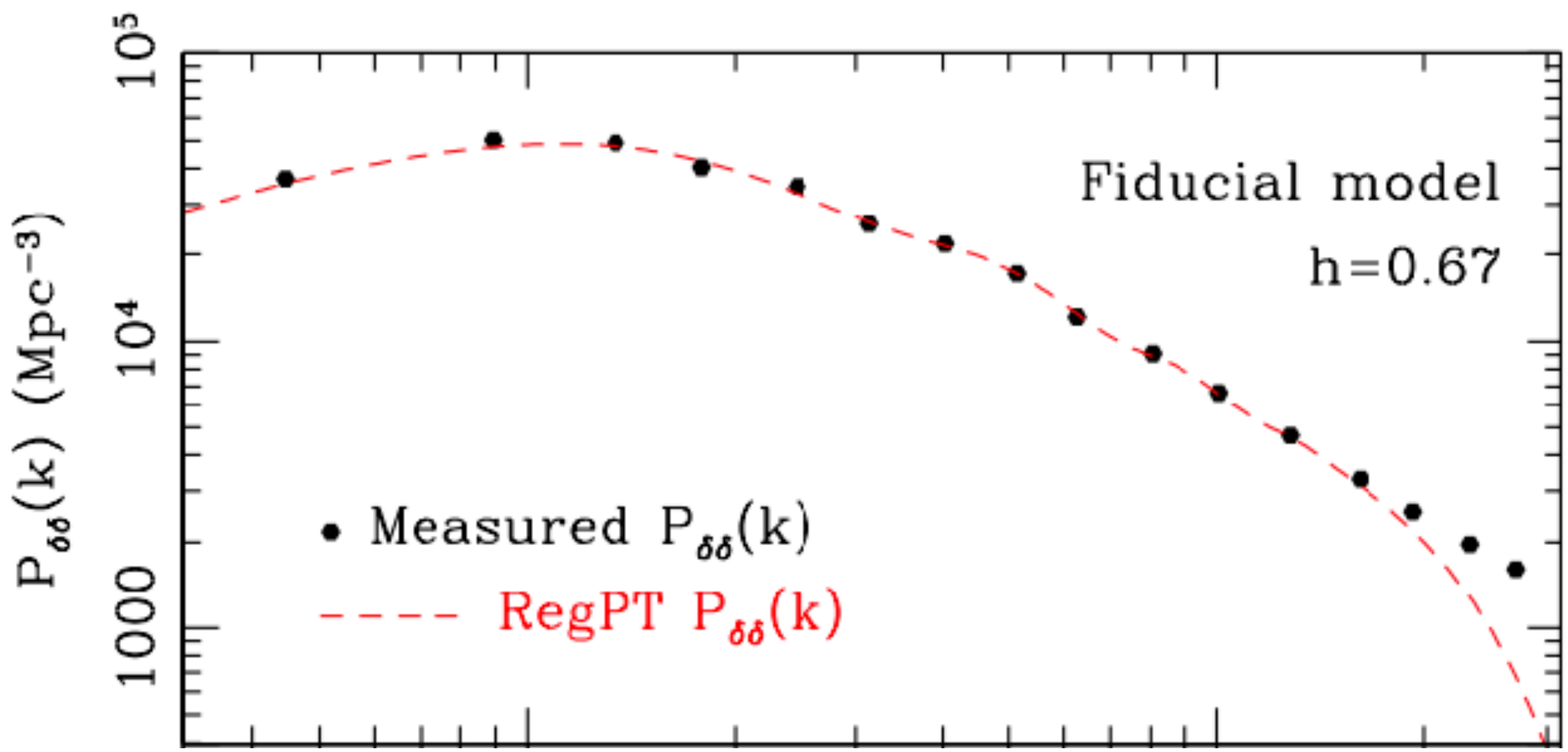
The measured spectra with different Ω_Λ

We vary Ω_Λ coherently with BAO statistics, i.e. the observed sound horizon is fixed



Non linear corrections



- We compare the theoretical predictions from RegPT and the measured spectrum of density fluctuations. Both are consistent up to quasi linear scale.
- As this correction is not relevant to RSD mapping, we will discuss it at later part of this talk when we need to explain the growth function projection.

The growth function dependence of non-linearity

- Non-linear spectrum: we use the perturbative theory and the simulation measurement, in order to classify the different growth function dependences

$$\bar{P}_{XY}(k,z) = \bar{P}^{\rm th}_{XY}(k,z) + \bar{P}^{\rm res}_{XY}(k,z),$$

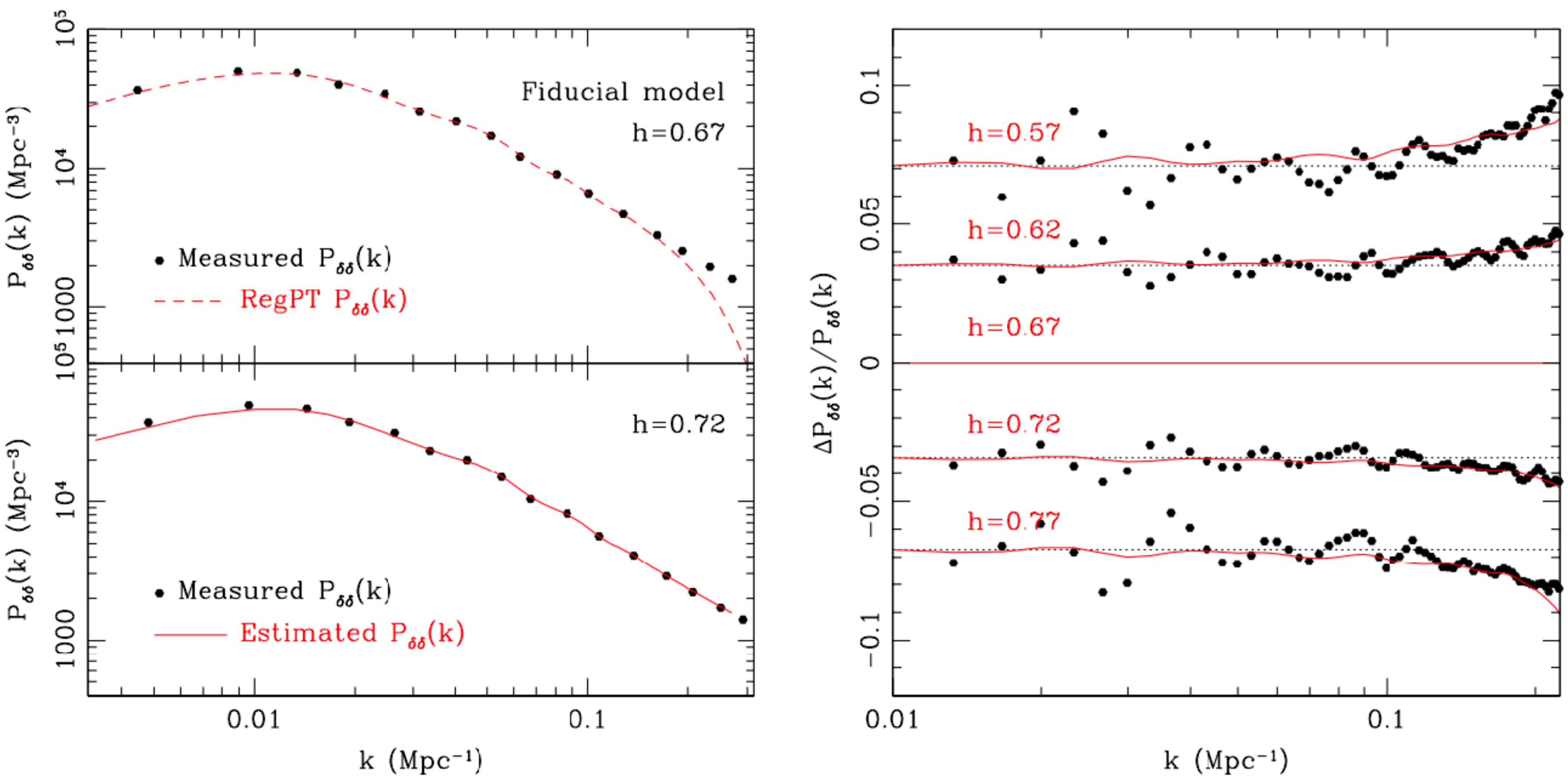
$$\begin{aligned}\bar{P}_{XY}(k,z) &= \bar{\Gamma}^{(1)}_X(k,z)\bar{\Gamma}^{(1)}_Y(k,z)\bar{P}^i(k)\\ &+2\int \frac{d^3\vec{q}}{(2\pi)^3}\bar{\Gamma}^{(2)}_X(\vec{q},\vec{k}-\vec{q},z)\bar{\Gamma}^{(2)}_Y(\vec{q},\vec{k}-\vec{q},z)\bar{P}^i(q)\bar{P}^i(|\vec{k}-\vec{q}|)\\ &+6\int \frac{d^3\vec{p}d^3\vec{q}}{(2\pi)^6}\bar{\Gamma}^{(3)}_X(\vec{p},\vec{q},\vec{k}-\vec{p}-\vec{q},z)\bar{\Gamma}^{(3)}_Y(\vec{p},\vec{q},\vec{k}-\vec{p}-\vec{q},z)\bar{P}^i(p)\bar{P}^i(q)\bar{P}^i(|\vec{k}-\vec{p}-\vec{q}|),\end{aligned}$$

$$\begin{aligned}\bar{P}^{\rm res}_{XY} = \bar{G}_X\bar{G}_Y\bar{G}^4_\delta \Big\{ &\left[\bar{\mathcal{O}}^{(1)}_{Y,5} + {\rm higher}\right]\bar{P}^i + \left[\bar{\mathcal{O}}^{(1)}_{X,5} + {\rm higher}\right]\bar{P}^i,\\ &+\int\left[\bar{\mathcal{O}}^{(2)}_{Y,4}\bar{F}^{(2)}_Y + {\rm higher}\right]\bar{P}^i\bar{P}^i + \int\left[\bar{\mathcal{O}}^{(2)}_{X,4}\bar{F}^{(2)}_X + {\rm higher}\right]\bar{P}^i\bar{P}^i,\\ &+\int\int\left[\bar{\mathcal{O}}^{(3)}_{Y,3}\bar{F}^{(3)}_Y + {\rm higher}\right]\bar{P}^i\bar{P}^i\bar{P}^i + \int\int\left[\bar{\mathcal{O}}^{(3)}_{X,3}\bar{F}^{(3)}_X + {\rm higher}\right]\bar{P}^i\bar{P}^i\bar{P}^i\Big\}.\end{aligned}$$

YSS, Yi, Oh, Taruya 2017 prepared

The growth function dependence of non-linearity

- Non-linear spectrum: we use the perturbative theory and the simulation measurement, in order to classify the different growth function dependences



YSS, Yi, Oh, Taruya 2017 prepared

The growth function dependence of higher order polynomial

- Higher order polynomials: we split different growth function dependent terms in pieces, and apply zeroth order growth function multiplication

$$\bar{A}(k,\mu) = j_1 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \; \langle A_1 A_2 A_3 \rangle_c = \sum_{n=1}^{6} \bar{A}_n$$

$$\bar{A}_1 = 2j_1 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \; \langle \bar{u}_z(\boldsymbol{r})\bar{\delta}(\boldsymbol{r})\bar{\delta}(\boldsymbol{r}')\rangle_c$$

$$\bar{A}_2 = j_1 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \; \langle u_z(\boldsymbol{r})\delta(\boldsymbol{r})\,\nabla_z u_z(\boldsymbol{r}')\rangle_c$$

$$\bar{A}_3 = j_1 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \; \langle u_z(\boldsymbol{r})\,\nabla_z u_z(\boldsymbol{r})\delta(\boldsymbol{r}')\rangle_c$$

$$\bar{A}_4 = 2j_1 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \; \langle u_z(\boldsymbol{r})\,\nabla_z u_z(\boldsymbol{r})\,\nabla_z u_z(\boldsymbol{r}')\rangle_c$$

$$\bar{A}_5 = j_1 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \; \langle -\delta(\boldsymbol{r})u_z(\boldsymbol{r}')\,\nabla_z u_z(\boldsymbol{r}')\rangle_c$$

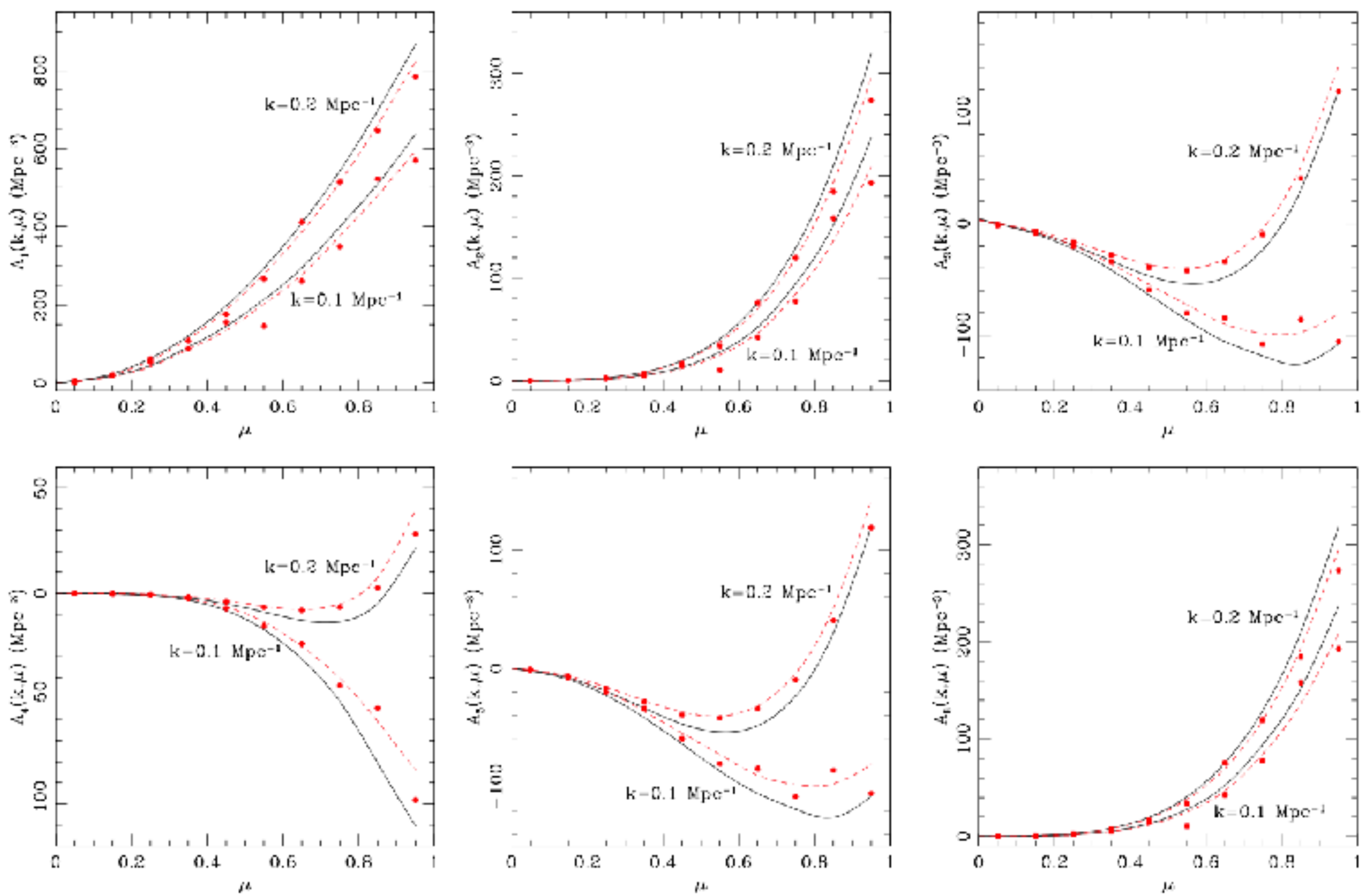
$$\bar{A}_6 = j_1 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \; \langle -\nabla_z u_z(\boldsymbol{r})u_z(\boldsymbol{r}')\delta(\boldsymbol{r}')\rangle_c$$

$$\begin{aligned}\bar{A}(k,\mu) &= \sum_{n=1}^{6} A_n \\ &= \left(G_\delta/\bar{G}_\delta\right)^2\left(G_\Theta/\bar{G}_\Theta\right)\bar{A}_1 + \left(G_\delta/\bar{G}_\delta\right)\left(G_\Theta/\bar{G}_\Theta\right)^2\bar{A}_2 \\ &+ \left(G_\delta/\bar{G}_\delta\right)\left(G_\Theta/\bar{G}_\Theta\right)^2\bar{A}_3 + \left(G_\Theta/\bar{G}_\Theta\right)^3\bar{A}_4 \\ &+ \left(G_\delta/\bar{G}_\delta\right)\left(G_\Theta/\bar{G}_\Theta\right)^2\bar{A}_5 + \left(G_\delta/\bar{G}_\delta\right)\left(G_\Theta/\bar{G}_\Theta\right)^2\bar{A}_6\end{aligned}$$

YSS, Yi, Oh, Taruya 2017 prepared

The growth function dependence of higher order polynomial

- Higher order polynomials: we split different growth function dependent terms in pieces, and apply zeroth order growth function multiplication



YSS, Yi, Oh, Taruya 2017 prepared

The growth function dependence of higher order polynomial

- Higher order polynomials: we split different growth function dependent terms in pieces, and apply zeroth order growth function multiplication

$$\bar{B}(k,\mu) = j_1^2 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle A_1 A_2\rangle_c \langle A_1 A_3\rangle_c = \sum_{n=1}^{4} \bar{\mathcal{B}}_n$$

$$\bar{\mathcal{B}}_1 = j_1^2 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle -u_z(\boldsymbol{r}')\delta(\boldsymbol{r})\rangle_c \langle u_z(\boldsymbol{r})\delta(\boldsymbol{r}')\rangle_c$$

$$\bar{\mathcal{B}}_2 = j_1^2 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle -u_z(\boldsymbol{r}')\delta(\boldsymbol{r})\rangle_c \langle u_z(\boldsymbol{r})\,\nabla_z u_z(\boldsymbol{r}')\rangle_c$$

$$\bar{\mathcal{B}}_3 = j_1^2 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle -u_z(\boldsymbol{r}')\,\nabla_z u_z(\boldsymbol{r})\rangle_c \langle u_z(\boldsymbol{r})\delta(\boldsymbol{r}')\rangle_c$$

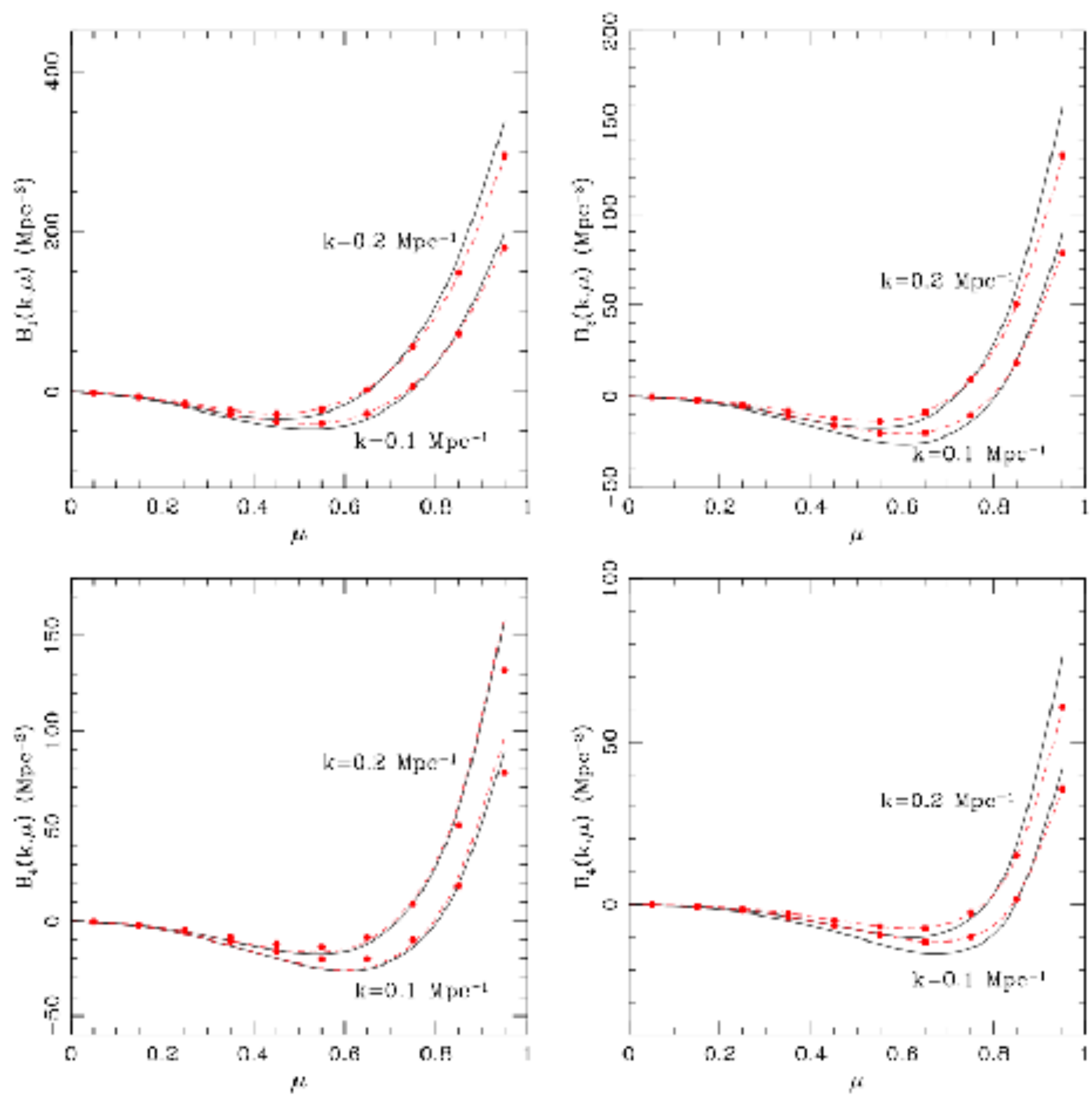
$$\bar{\mathcal{B}}_4 = j_1^2 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle -u_z(\boldsymbol{r}')\,\nabla_z u_z(\boldsymbol{r})\rangle_c \langle u_z(\boldsymbol{r})\,\nabla_z u_z(\boldsymbol{r}')\rangle_c$$

$$\begin{aligned}\bar{B}(k,\mu) &= \sum_{n=1}^{4} \mathcal{B}_n \\ &= \left(G_\delta/\bar{G}_\delta\right)^2 \left(G_\Theta/\bar{G}_\Theta\right)^2 \bar{\mathcal{B}}_1 + \left(G_\delta/\bar{G}_\delta\right) \left(G_\Theta/\bar{G}_\Theta\right)^3 \bar{\mathcal{B}}_2 \\ &+ \left(G_\delta/\bar{G}_\delta\right) \left(G_\Theta/\bar{G}_\Theta\right)^3 \bar{\mathcal{B}}_3 + \left(G_\Theta/\bar{G}_\Theta\right)^4 \bar{\mathcal{B}}_4\end{aligned}$$

YSS, Yi, Oh, Taruya 2017 prepared

The growth function dependence of higher order polynomial

- Higher order polynomials: we split different growth function dependent terms in pieces, and apply zeroth order growth function multiplication



YSS, Yi, Oh, Taruya 2017 prepared

The growth function dependence of higher order polynomial

- Higher order polynomials: we split different growth function dependent terms in pieces, and apply zeroth order growth function multiplication

$$\bar{T}(k,\mu) = \frac{1}{2} j_1^2 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle A_1^2 A_2 A_3 \rangle_c, = \sum_{n=1}^{7} \bar{\mathcal{T}}_n$$

$$\bar{\mathcal{T}}_1 = j_1^2 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle u_z(\boldsymbol{r}) u_z(\boldsymbol{r}) \delta(\boldsymbol{r}) \delta(\boldsymbol{r}') \rangle_c$$

$$\bar{\mathcal{T}}_2 = j_1^2 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle u_z(\boldsymbol{r}) u_z(\boldsymbol{r}) \delta(\boldsymbol{r})\, \nabla_z u_z(\boldsymbol{r}') \rangle_c$$

$$\bar{\mathcal{T}}_3 = j_1^2 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle u_z(\boldsymbol{r}) u_z(\boldsymbol{r})\, \nabla_z u_z(\boldsymbol{r}) \delta(\boldsymbol{r}') \rangle_c$$

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$$\bar{\mathcal{T}}_5 = \frac{1}{2} j_1^2 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle -2 u_z(\boldsymbol{r}') u_z(\boldsymbol{r}) \delta(\boldsymbol{r}) \delta(\boldsymbol{r}') \rangle_c$$

$$\bar{\mathcal{T}}_6 = j_1^2 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle -2 u_z(\boldsymbol{r}') u_z(\boldsymbol{r}) \delta(\boldsymbol{r})\, \nabla_z u_z(\boldsymbol{r}') \rangle_c$$

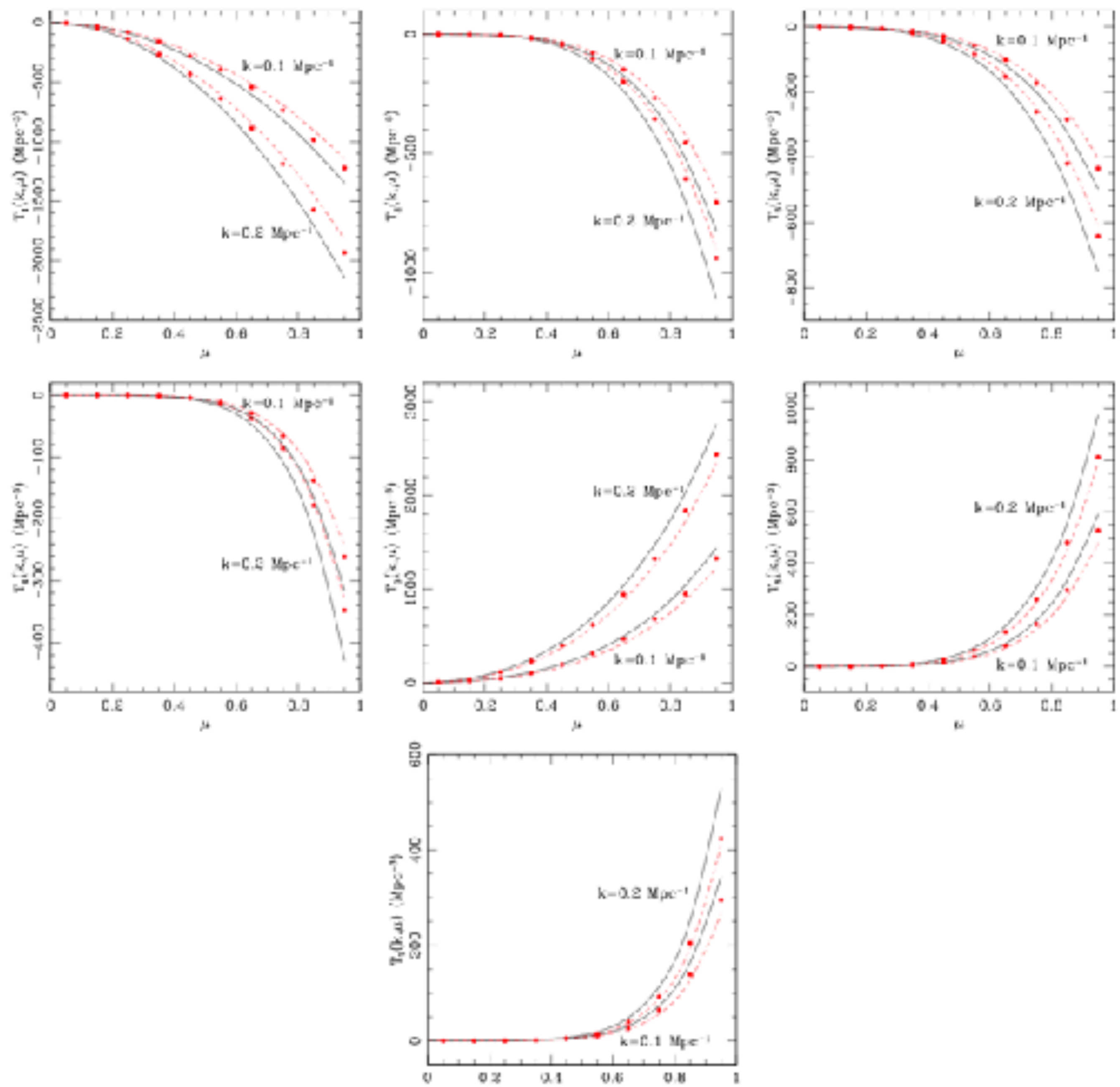
$$\bar{\mathcal{T}}_7 = \frac{1}{2} j_1^2 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle -2 u_z(\boldsymbol{r}') u_z(\boldsymbol{r})\, \nabla_z u_z(\boldsymbol{r})\, \nabla_z u_z(\boldsymbol{r}') \rangle_c$$

$$\begin{aligned} \bar{T}(k,\mu) &= \sum_{n=1}^{3} \mathcal{T}_n \\ &= \left(G_\delta/\bar{G}_\delta\right)^2 \left(G_\Theta/\bar{G}_\Theta\right)^2 \bar{\mathcal{T}}_1 + \left(G_\delta/\bar{G}_\delta\right) \left(G_\Theta/\bar{G}_\Theta\right)^3 \bar{\mathcal{T}}_2 + \left(G_\delta/\bar{G}_\delta\right) \left(G_\Theta/\bar{G}_\Theta\right)^3 \bar{\mathcal{T}}_3 \\ &+ \left(G_\Theta/\bar{G}_\Theta\right)^4 \bar{\mathcal{T}}_4 + \left(G_\delta/\bar{G}_\delta\right)^2 \left(G_\Theta/\bar{G}_\Theta\right)^2 \bar{\mathcal{T}}_5 + \left(G_\delta/\bar{G}_\delta\right) \left(G_\Theta/\bar{G}_\Theta\right)^3 \bar{\mathcal{T}}_6 + \left(G_\Theta/\bar{G}_\Theta\right)^4 \bar{\mathcal{T}}_7 \end{aligned}$$

YSS, Yi, Oh, Taruya 2017 prepared

The growth function dependence of higher order polynomial

- Higher order polynomials: we split different growth function dependent terms in pieces, and apply zeroth order growth function multiplication



YSS, Yi, Oh, Taruya 2017 prepared

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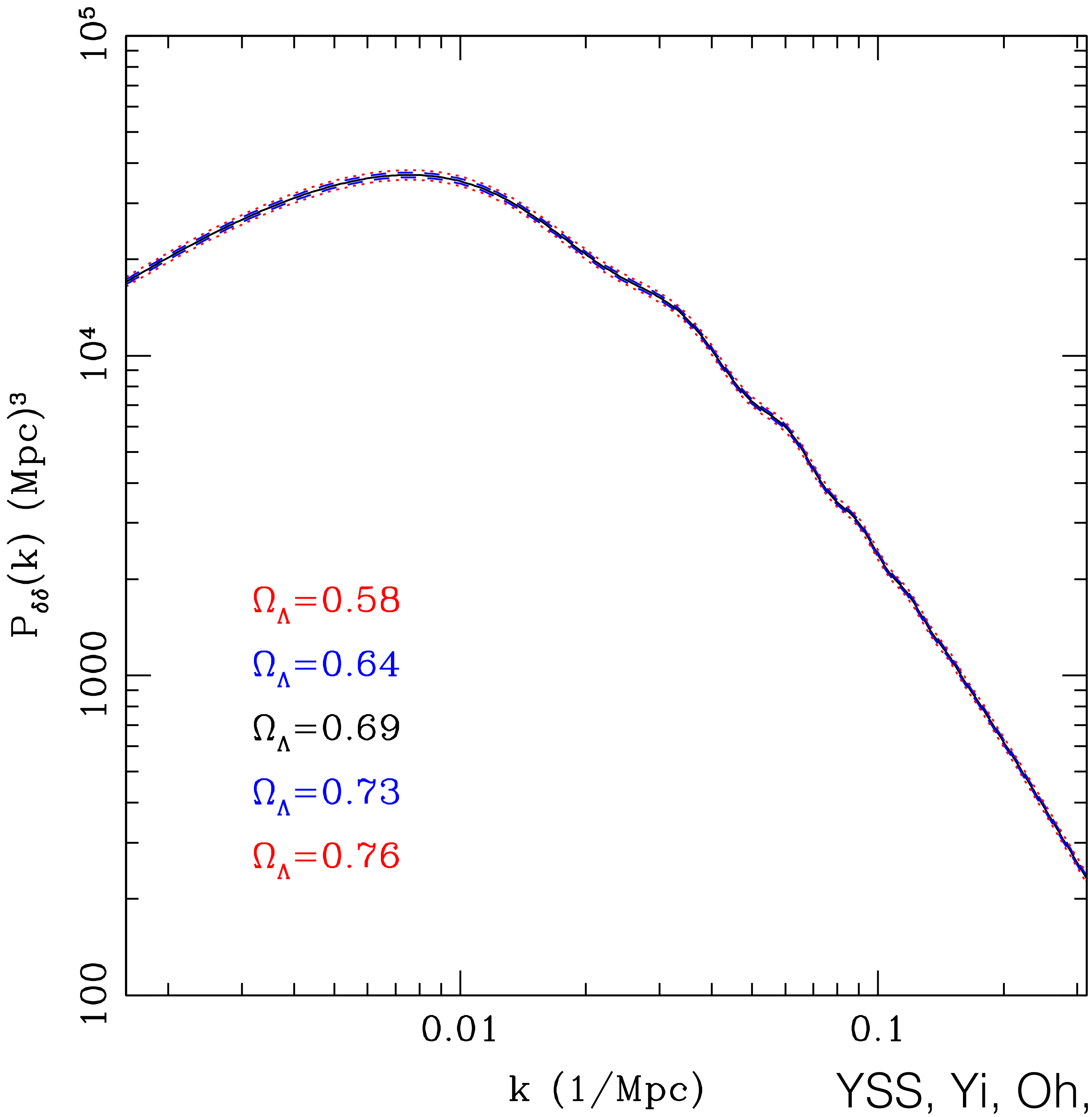
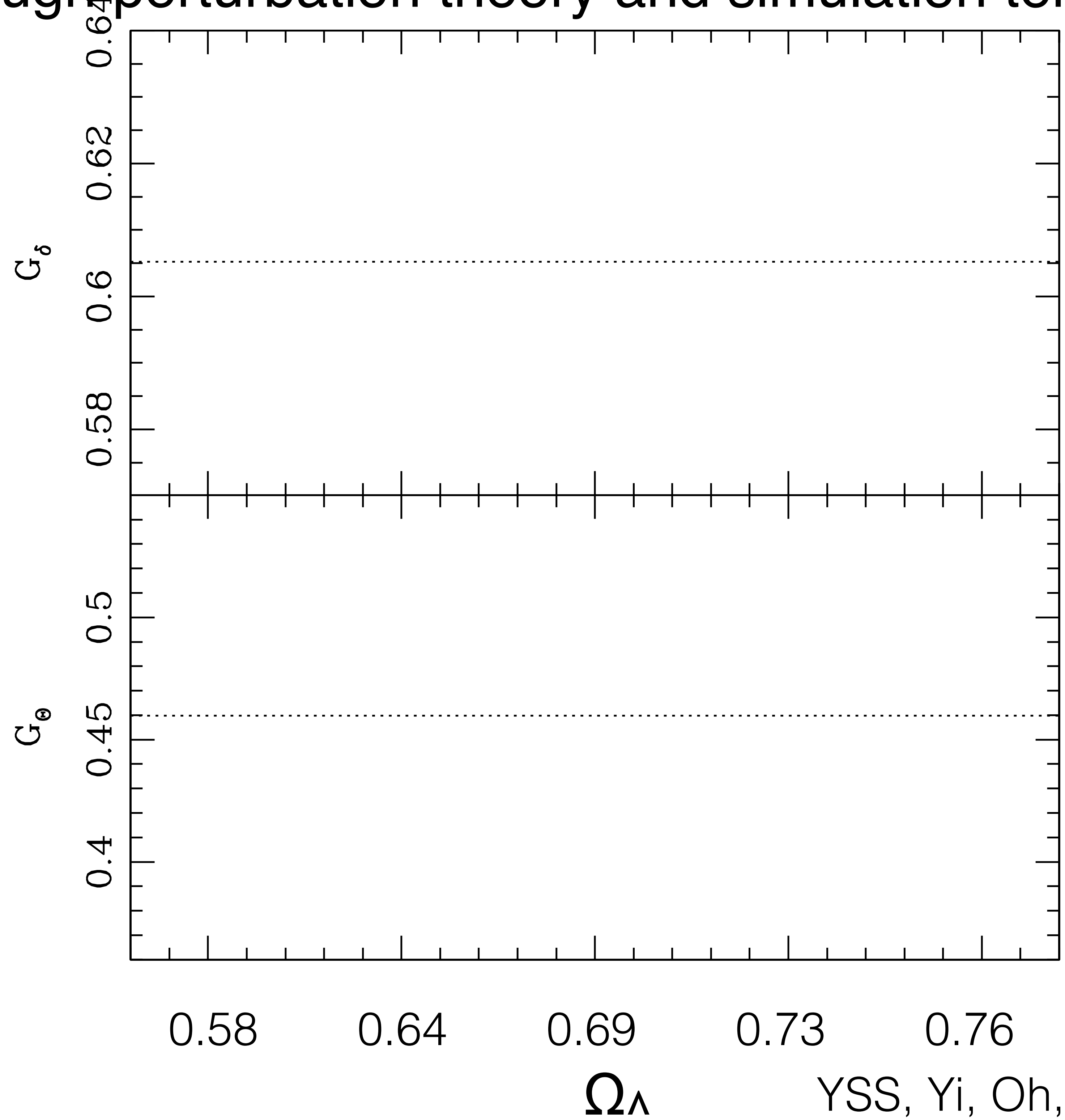
We vary Ω_Λ coherently with BAO statistics, i.e. the observed sound horizon is fixed



YSS, Yi, Oh, Taruya 2017 prepared

The accurate measurement of growth functions

We achieve the 1% accuracy measurement after a long journey through perturbation theory and simulation template



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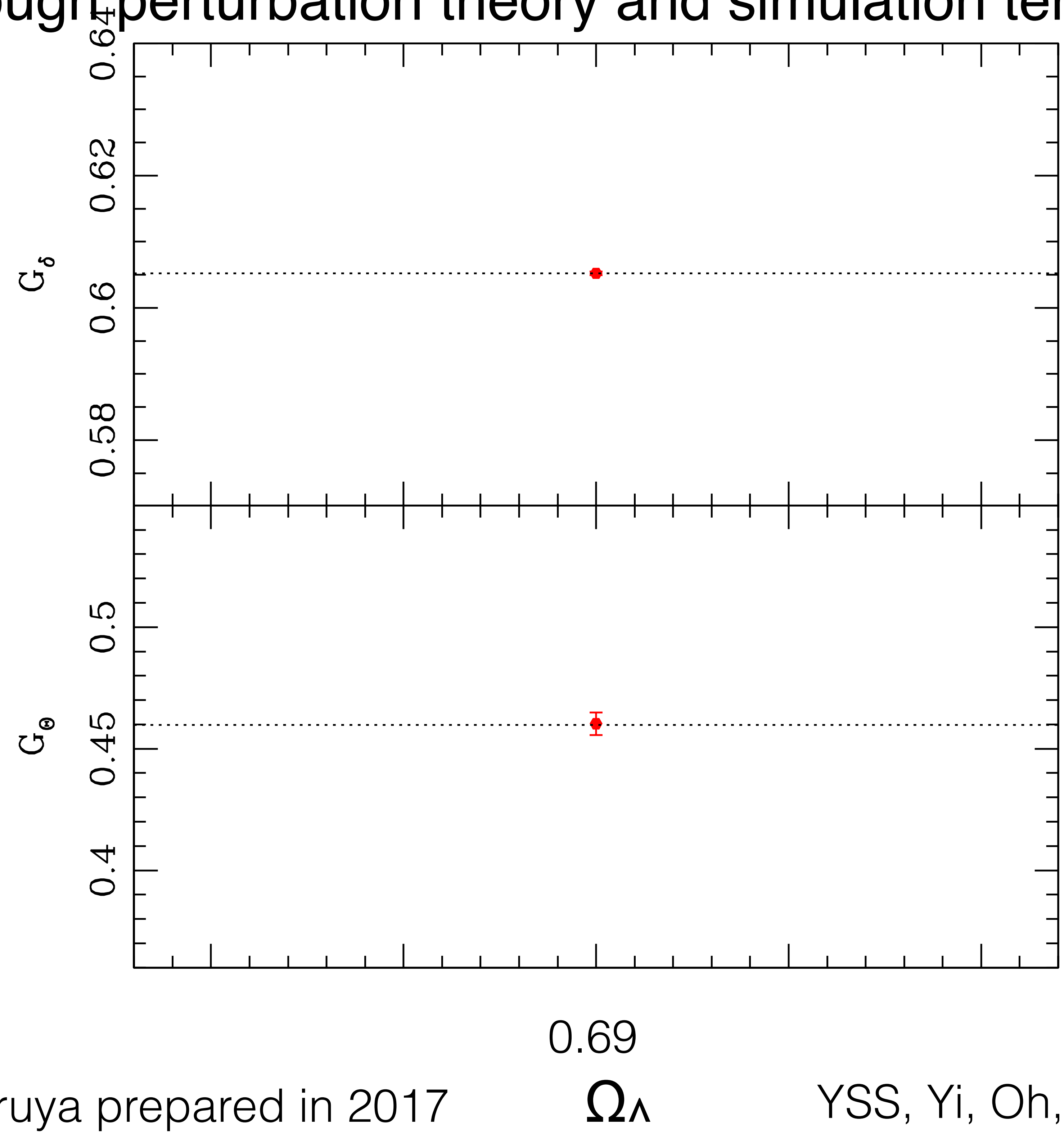

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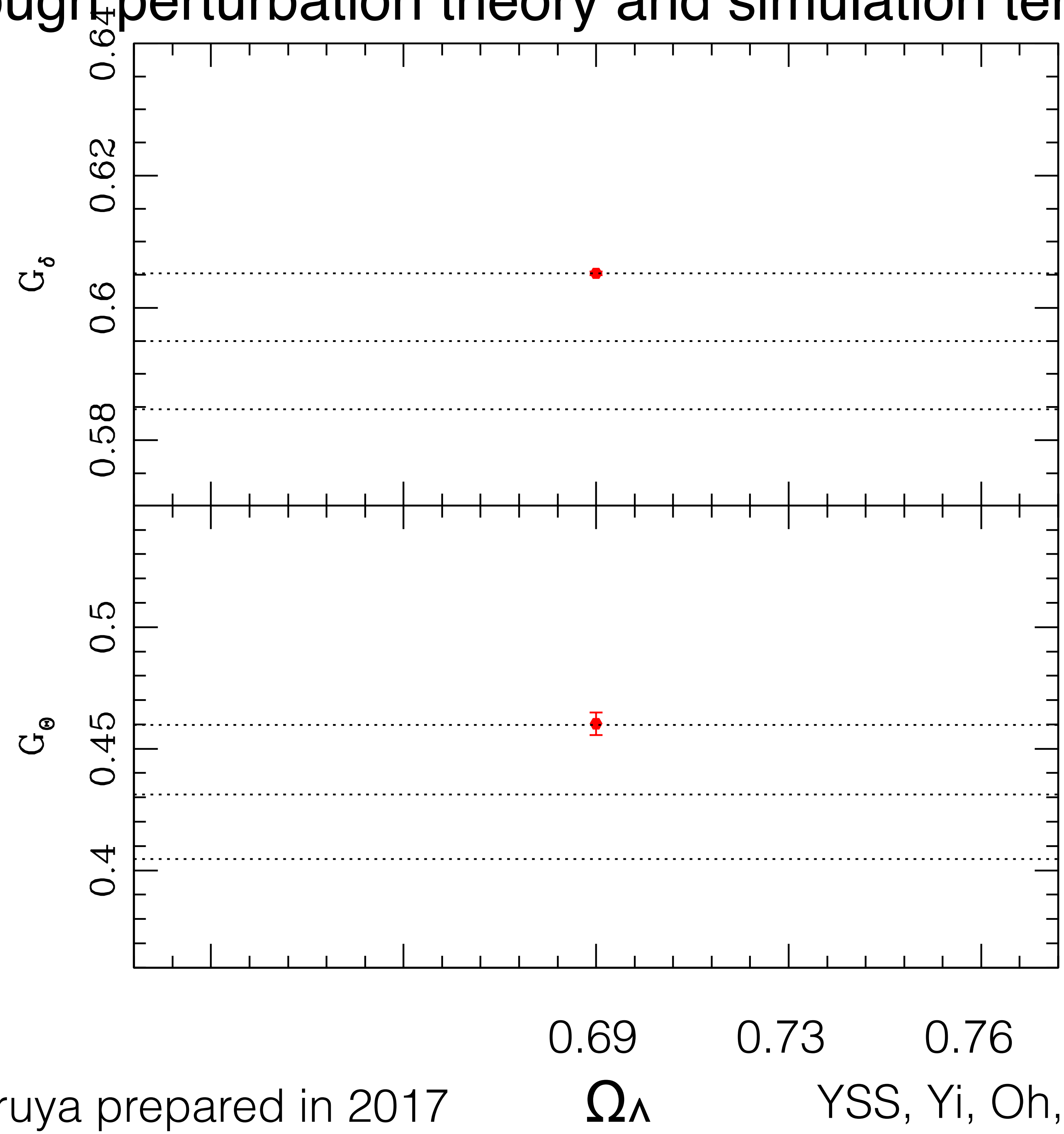

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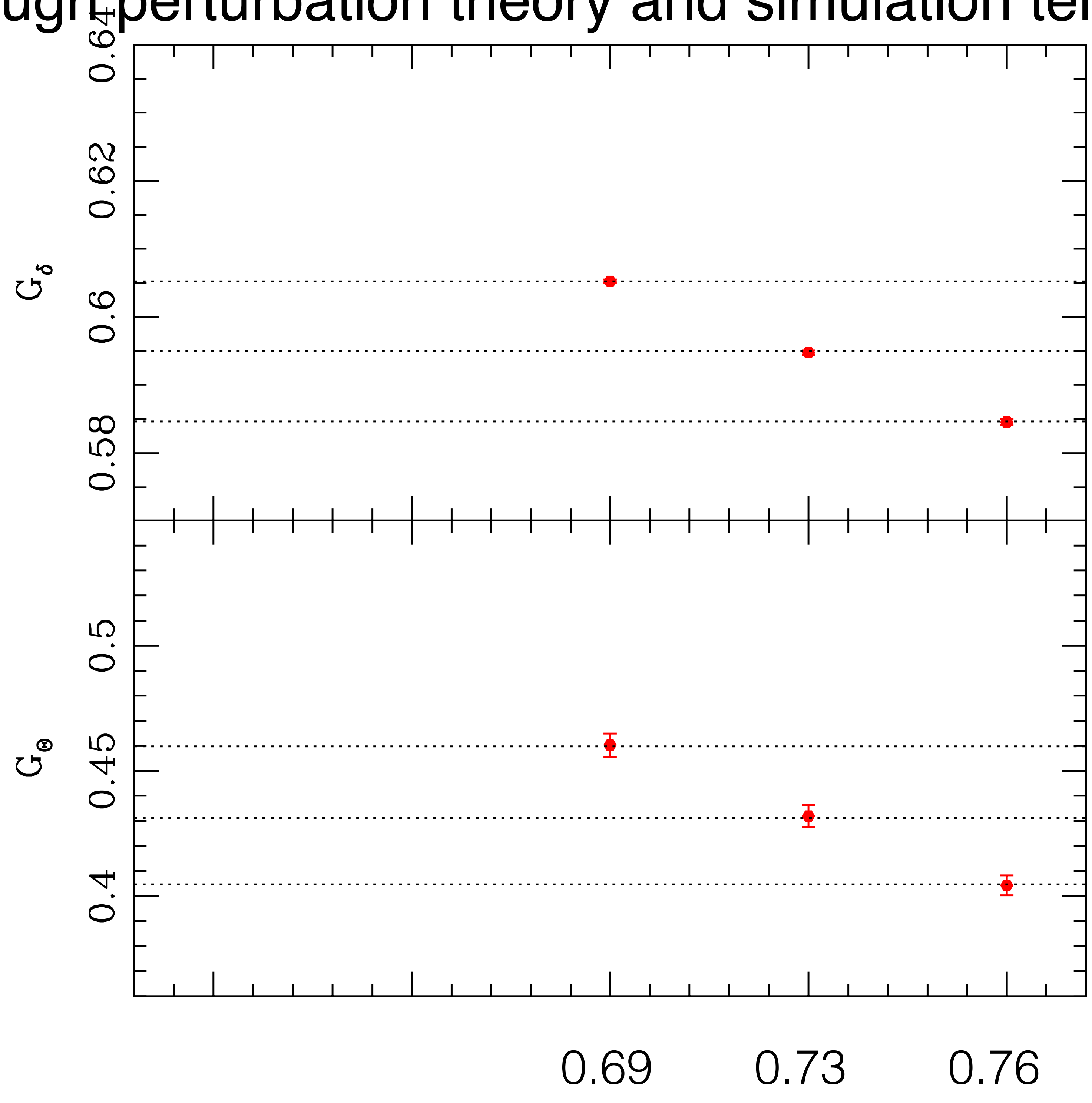

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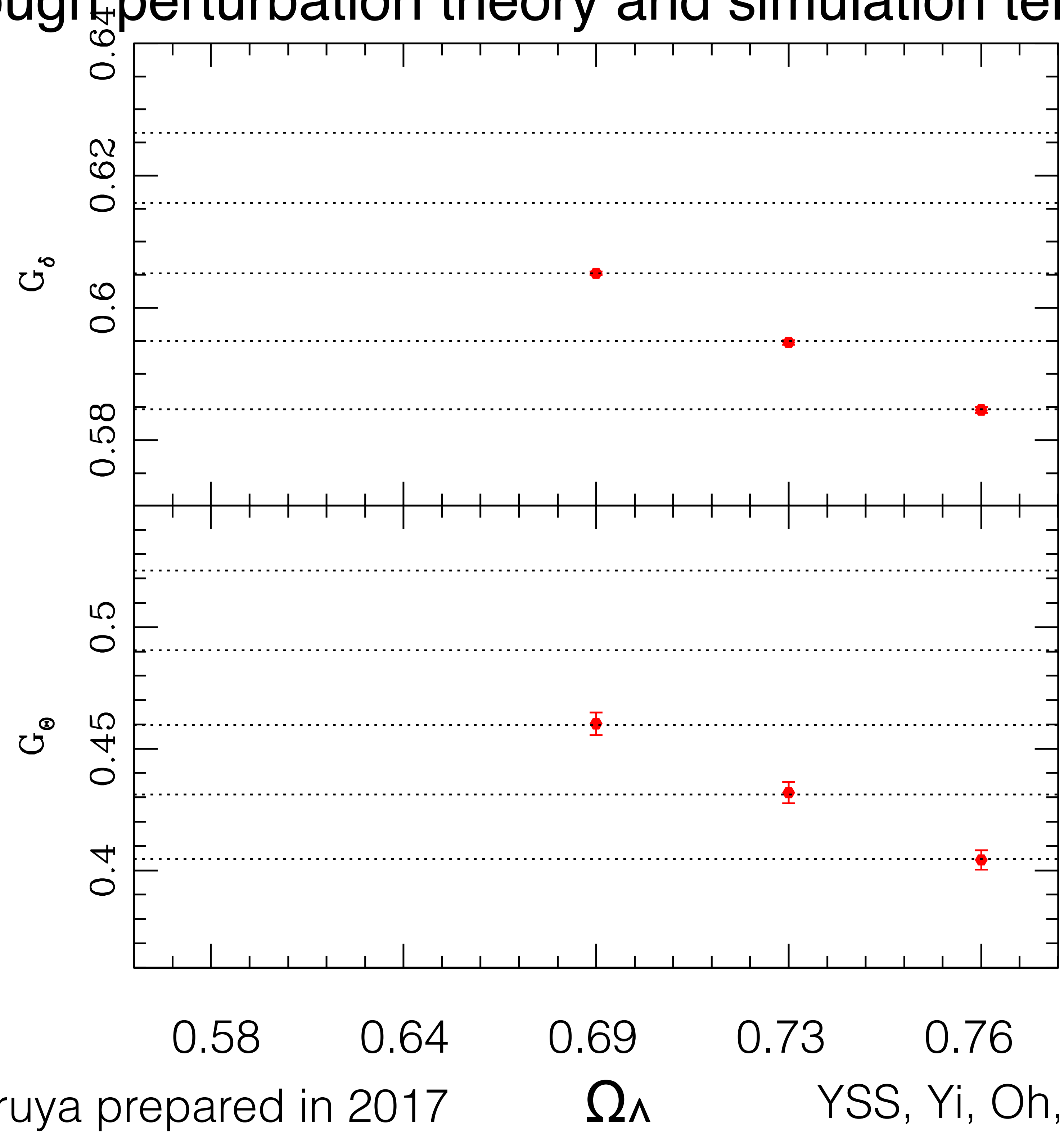

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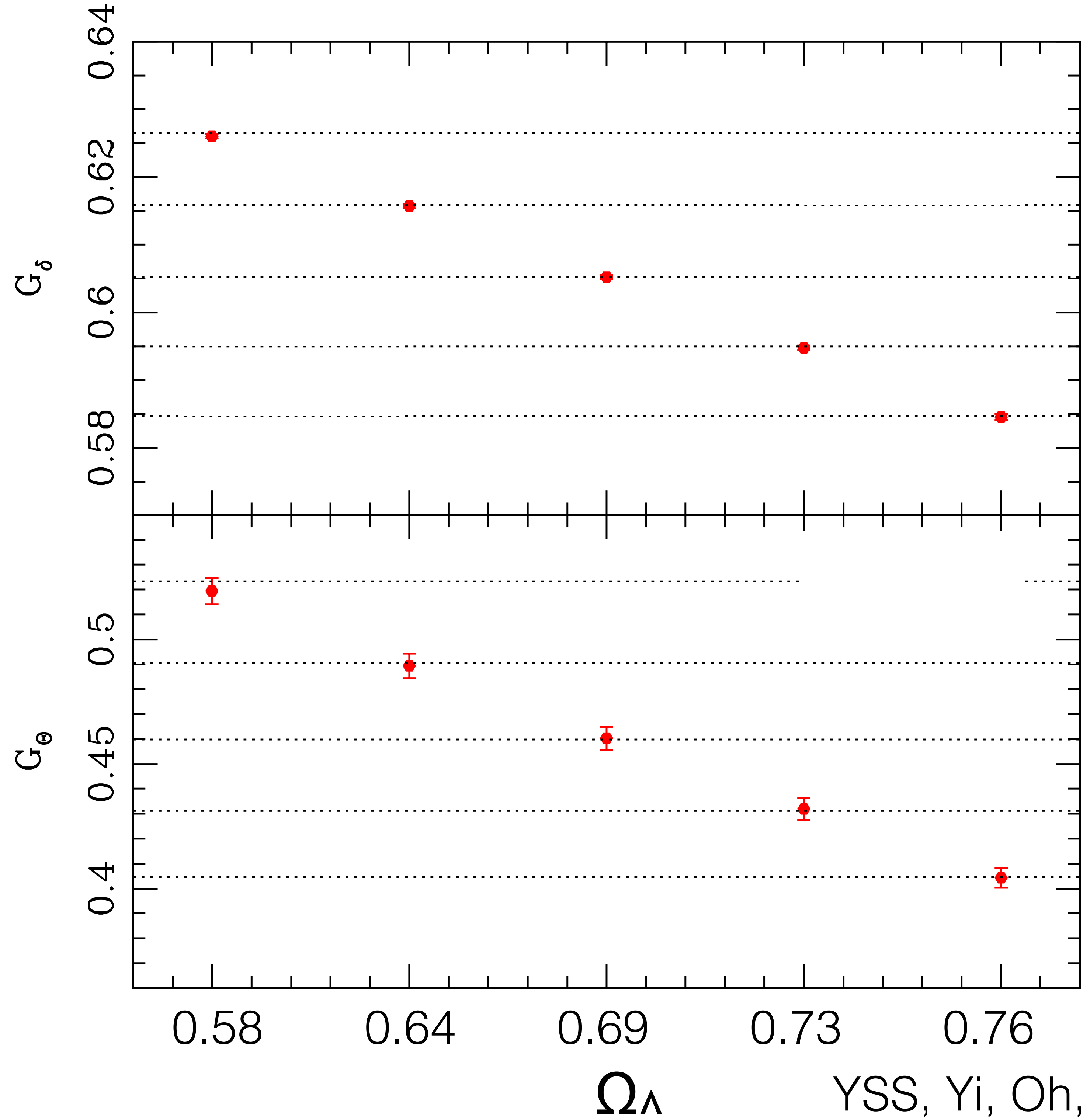

YSS, Yi, Oh, Taruya 2017 prepared

Precise determination on Ω_Λ

(D_A, H^{-1}, G_δ, G_Θ, FoG)

Standard model

Cold dark matter

Massless neutrino

Quintessence dark energy

Phantom dark energy

Λ

Plan for RSD emulator

- We provide RSD emulator based upon dark matter simulation measurements

$$\bar{P}_{XY}(k,z) = \bar{P}^{\rm th}_{XY}(k,z) + \bar{P}^{\rm res}_{XY}(k,z),$$

$$\bar{T}_1 = j_1^2 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle u_z(\boldsymbol{r}) u_z(\boldsymbol{r}) \delta(\boldsymbol{r}) \delta(\boldsymbol{r}')\rangle_c$$

$$\bar{T}_2 = j_1^2 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle u_z(\boldsymbol{r}) u_z(\boldsymbol{r}) \delta(\boldsymbol{r})\, \nabla_z u_z(\boldsymbol{r}')\rangle_c$$

$$\bar{T}_3 = j_1^2 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle u_z(\boldsymbol{r}) u_z(\boldsymbol{r})\, \nabla_z u_z(\boldsymbol{r}) \delta(\boldsymbol{r}')\rangle_c$$

$$\bar{T}_4 = j_1^2 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle u_z(\boldsymbol{r}) u_z(\boldsymbol{r})\, \nabla_z u_z(\boldsymbol{r})\, \nabla_z u_z(\boldsymbol{r}')\rangle_c$$

$$\bar{T}_5 = \frac{1}{2} j_1^2 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle -2 u_z(\boldsymbol{r}') u_z(\boldsymbol{r}) \delta(\boldsymbol{r}) \delta(\boldsymbol{r}')\rangle_c$$

$$\bar{T}_6 = j_1^2 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle -2 u_z(\boldsymbol{r}') u_z(\boldsymbol{r}) \delta(\boldsymbol{r})\, \nabla_z u_z(\boldsymbol{r}')\rangle_c$$

$$\bar{T}_7 = \frac{1}{2} j_1^2 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle -2 u_z(\boldsymbol{r}') u_z(\boldsymbol{r})\, \nabla_z u_z(\boldsymbol{r})\, \nabla_z u_z(\boldsymbol{r}')\rangle_c$$

$$\bar{A}_1 = 2 j_1 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle \bar{u}_z(\boldsymbol{r}) \bar{\delta}(\boldsymbol{r}) \bar{\delta}(\boldsymbol{r}')\rangle_c$$

$$\bar{A}_2 = j_1 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle u_z(\boldsymbol{r}) \delta(\boldsymbol{r})\, \nabla_z u_z(\boldsymbol{r}')\rangle_c$$

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$$\bar{A}_4 = 2 j_1 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle u_z(\boldsymbol{r})\, \nabla_z u_z(\boldsymbol{r})\, \nabla_z u_z(\boldsymbol{r}')\rangle_c$$

$$\bar{A}_5 = j_1 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle -\delta(\boldsymbol{r}) u_z(\boldsymbol{r}')\, \nabla_z u_z(\boldsymbol{r}')\rangle_c$$

$$\bar{A}_6 = j_1 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle -\nabla_z u_z(\boldsymbol{r}) u_z(\boldsymbol{r}') \delta(\boldsymbol{r}')\rangle_c$$

$$\bar{B}_1 = j_1^2 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle -u_z(\boldsymbol{r}') \delta(\boldsymbol{r})\rangle_c \langle u_z(\boldsymbol{r}) \delta(\boldsymbol{r}')\rangle_c$$

$$\bar{B}_2 = j_1^2 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle -u_z(\boldsymbol{r}') \delta(\boldsymbol{r})\rangle_c \langle u_z(\boldsymbol{r})\, \nabla_z u_z(\boldsymbol{r}')\rangle_c$$

$$\bar{B}_3 = j_1^2 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle -u_z(\boldsymbol{r}')\, \nabla_z u_z(\boldsymbol{r})\rangle_c \langle u_z(\boldsymbol{r}) \delta(\boldsymbol{r}')\rangle_c$$

$$\bar{B}_4 = j_1^2 \int d^3\boldsymbol{x}\; e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\; \langle -u_z(\boldsymbol{r}')\, \nabla_z u_z(\boldsymbol{r})\rangle_c \langle u_z(\boldsymbol{r})\, \nabla_z u_z(\boldsymbol{r}')\rangle_c$$

Open new window to test cosmological models

(D_A, H^{-1}, G_δ, G_Θ, FoG, , New, …)

Standard model	New physics
Massive neutrino	Chameleon type gravity

?

Accurate measurement of growth function

- The template should be made independent of the types of biased tracers, and it is prepared using dark matter particle simulations

- The structure formation grows coherently from the last scattering surface to the present epoch in most dark energy models. We test whether we can exploit the fiducial template to generate different cosmological models which is different by growth functions

- Non-linear spectrum: we use the perturbative theory and the simulation measurement, in order to classify the different growth function dependences

- Higher order polynomials: we split different growth function dependent terms in pieces, and apply zeroth order growth function multiplication

- We keep the same Gaussian FoG with one single parameter σ_p