Limits on Statistical Anisotropy from Large-scale structure of the Universe

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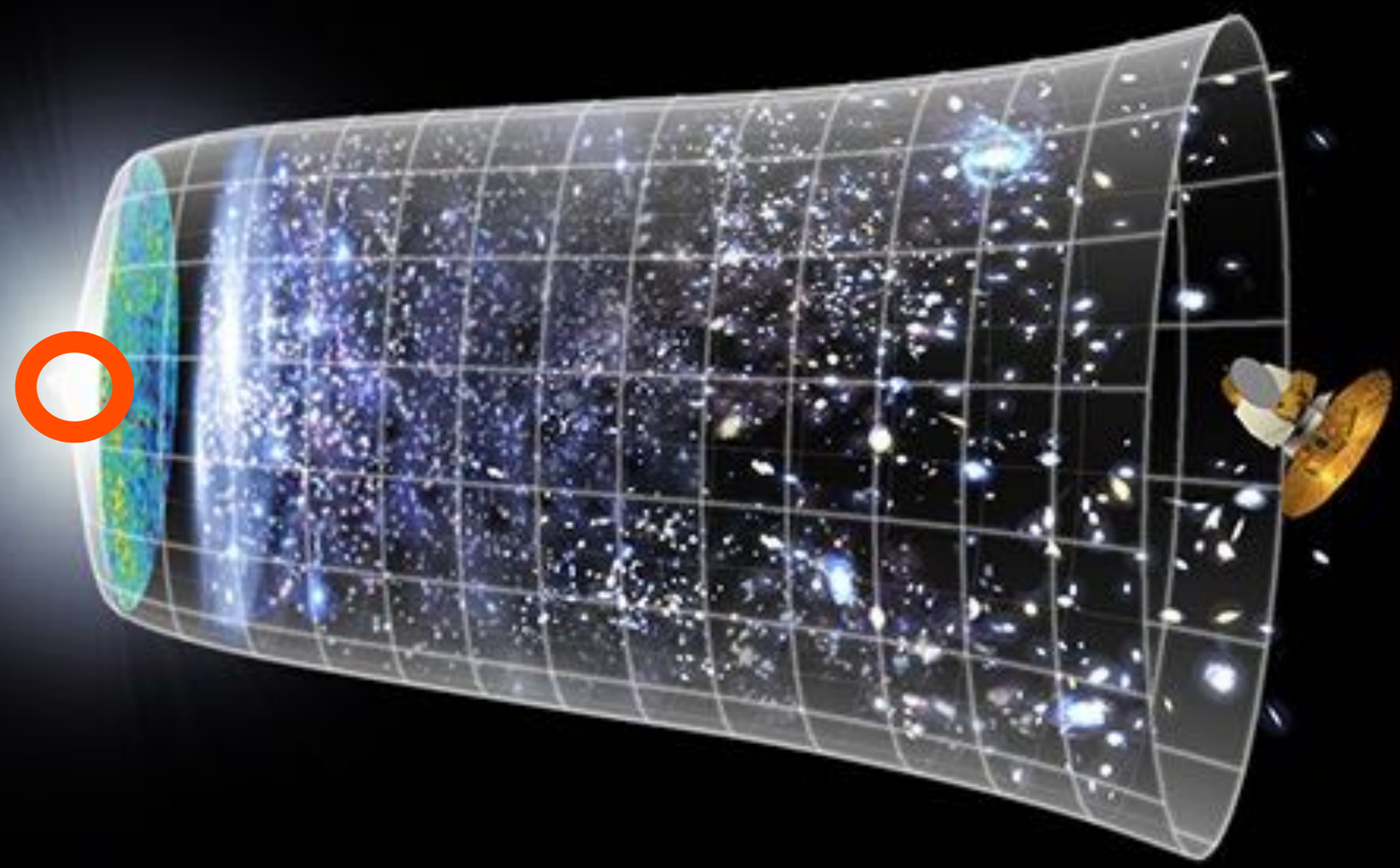
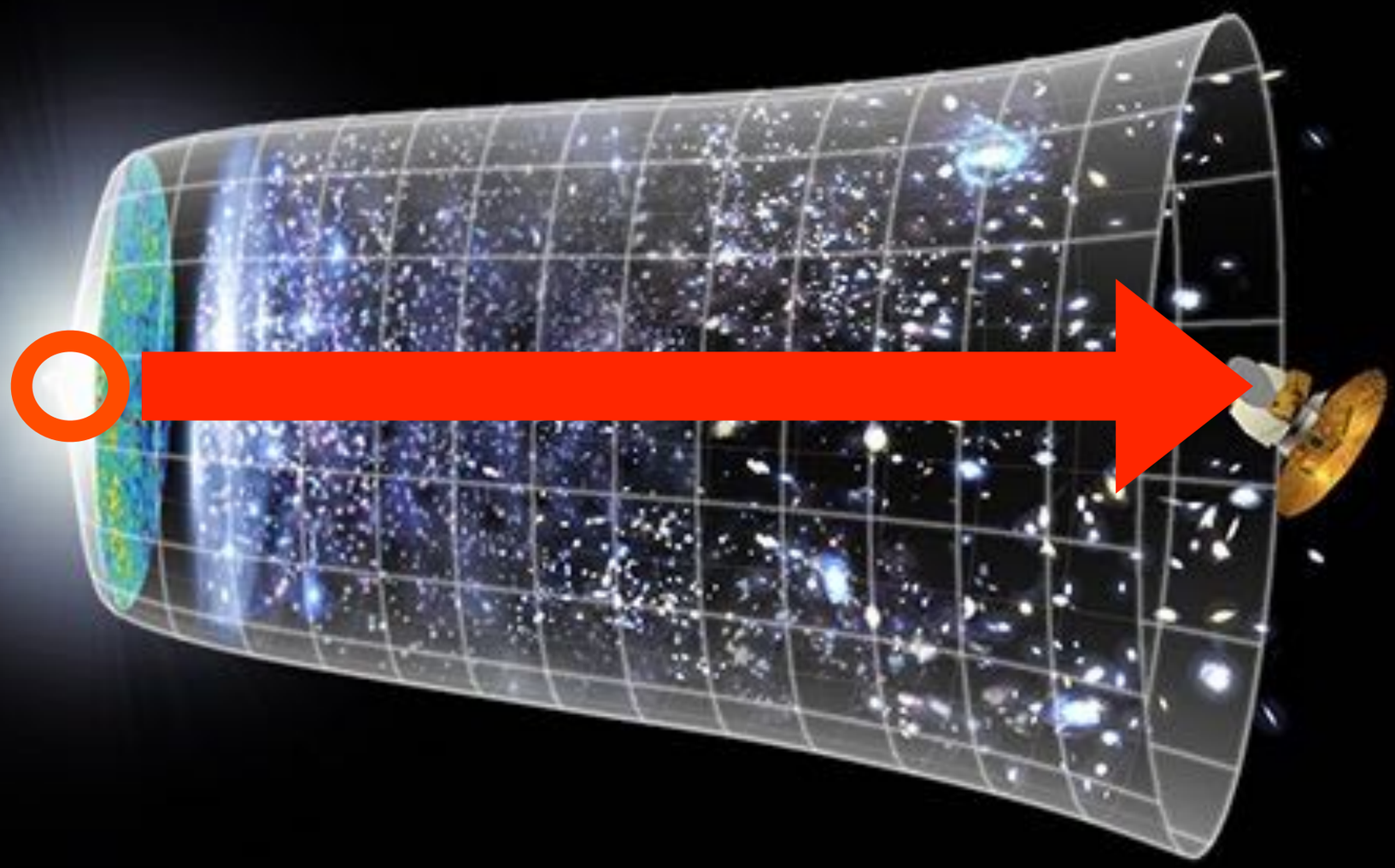
Next generation cosmology with large-scale structure:
CosKASI-ICG-NAOC-YITP joint workshop @ Kyoto Univ., Sep.7-8, 2017

The beginning of the Universe



The beginning of the Universe



The standard inflation scenario (Slow-roll single scalar field inflation)

Lagrangian

$$\mathcal{L} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi)$$

Energy and pressure

$$\begin{aligned} \bar{\rho} &= \frac{1}{2}\dot{\bar{\varphi}}^2 + V(\bar{\varphi}), \\ \bar{p} &= \frac{1}{2}\dot{\bar{\varphi}}^2 - V(\bar{\varphi}), \end{aligned}$$

The standard inflation scenario (Slow-roll single scalar field inflation)

If potential is flat (constant):

$$\bar{\rho} = \frac{1}{2}\dot{\bar{\varphi}}^2 + V(\bar{\varphi}),$$
$$\bar{p} = \frac{1}{2}\dot{\bar{\varphi}}^2 - V(\bar{\varphi}),$$

$$V(\varphi) = \text{const.}$$

The standard inflation scenario (Slow-roll single scalar field inflation)

If potential is flat (constant):

$$\bar{\rho} = \cancel{\frac{1}{2}\dot{\bar{\varphi}}^2} + V(\bar{\varphi}),$$

$$\bar{p} = \cancel{\frac{1}{2}\dot{\bar{\varphi}}^2} - V(\bar{\varphi}),$$

Cosmological constant.

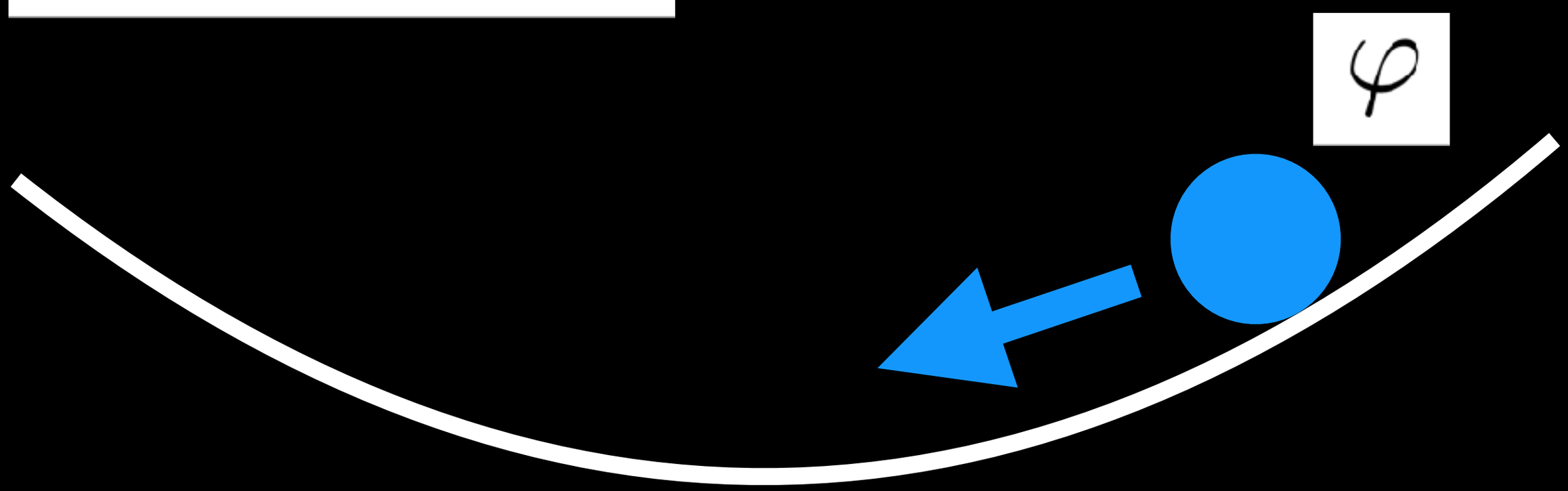
$$\rho = -p$$

$$V(\varphi) = \text{const.}$$

The standard inflation scenario (Slow-roll single scalar field inflation)

If potential is nearly flat (constant):

$$\bar{\rho} = \frac{1}{2}\dot{\bar{\varphi}}^2 + V(\bar{\varphi}),$$
$$\bar{p} = \frac{1}{2}\dot{\bar{\varphi}}^2 - V(\bar{\varphi}),$$



The standard inflation scenario (Slow-roll single scalar field inflation)

- Nearly scale-free power spectrum

$$\frac{k^3}{2\pi^2} P_{\delta\varphi}(k) \propto k^{n_\mathrm{s}-1}$$

- The spectral index is characterized by the derivative of potential:

$$n_\mathrm{s} - 1 = 2\eta - 6\varepsilon \qquad \eta = \frac{V''}{V} \qquad \varepsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2$$

- Departure from the scale-free:

Detection! $n_\mathrm{s} = 0.9652 \pm 0.0047$

Planck 2015

The standard inflation scenario (Slow-roll single scalar field inflation)

- **Nearly Gaussian:**

$$P(\delta\varphi) \sim e^{iS(\delta\varphi)}$$

$$S = S(\delta\varphi^2) + S(\delta\varphi^3) + \cdots$$

Linear **Non-Linear**

- **Departure from the Gaussian:**

No detection

$$f_{\rm NL}^{\rm local} = 0.8 \pm 5.0$$

The standard inflation scenario (Slow-roll single scalar field inflation)

- Adiabatic condition

$$S_{m\gamma} = \frac{1}{3}\delta_m - \frac{1}{4}\delta_\gamma = 0$$

- Multi-field inflation scenarios can break this

- Departure from adiabatic condition (Existence of isogurvature modes):

No detection

The standard inflation scenario (Slow-roll single scalar field inflation)

- **Parity-symmetry**

$$P_{\delta\varphi}(\vec{k}) = P_{\delta\varphi}(-\vec{k})$$

- **Departure from the parity-symmetry:**

No detection

The standard inflation scenario (Slow-roll single scalar field inflation)

- Translational invariance:

$$\langle \delta\varphi(\vec{k})\delta\varphi(\vec{k}')\rangle = (2\pi)^3 \delta_{\rm D}\left(\vec{k}+\vec{k}'\right) P_{\delta\varphi}(\vec{k})$$

- Departure from the translational invariance:

No detection

The standard inflation scenario (Slow-roll single scalar field inflation)

- **Rotational invariance:**

$$P_{\delta\varphi}(\vec{k}) = P_{\delta\varphi}(|\vec{k}|)$$

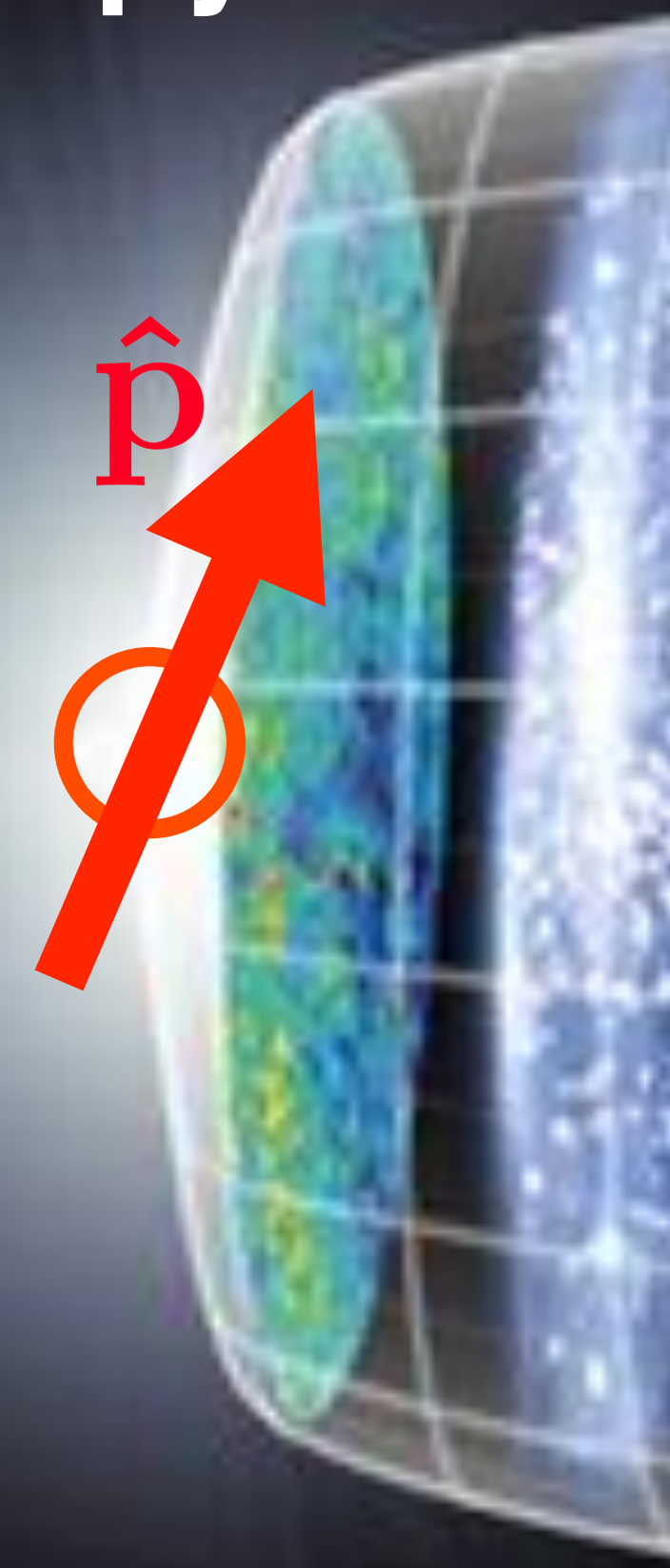
- **Vector field inflation theories can break this.**

- **Departure from the rotational invariance:**

No detection

Quadrupolar anisotropy

$$P_\zeta(\vec{k}) = P_{\rm iso}(k)\left[1 + g_*\left(\hat{k}\cdot\hat{p}\right)^2\right]$$



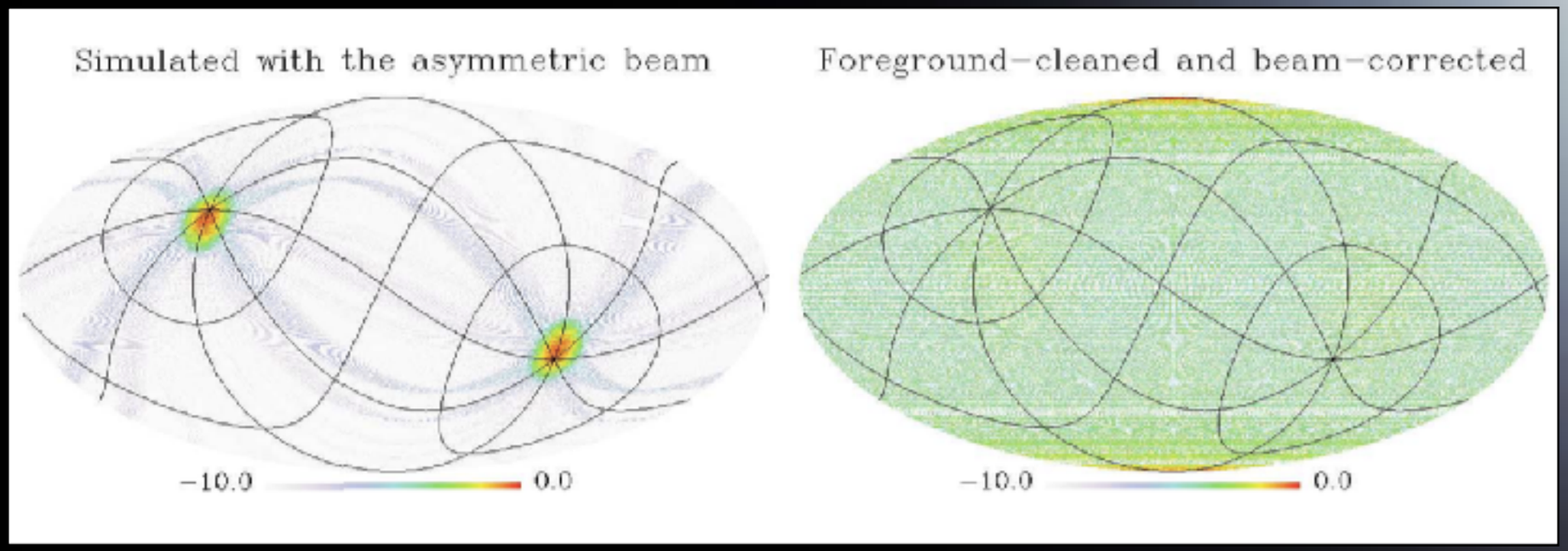
The simplest model
breaking statistical isotropy
with preserving
party-symmetry and
translational invariance.

CMB experiments

Planck provides

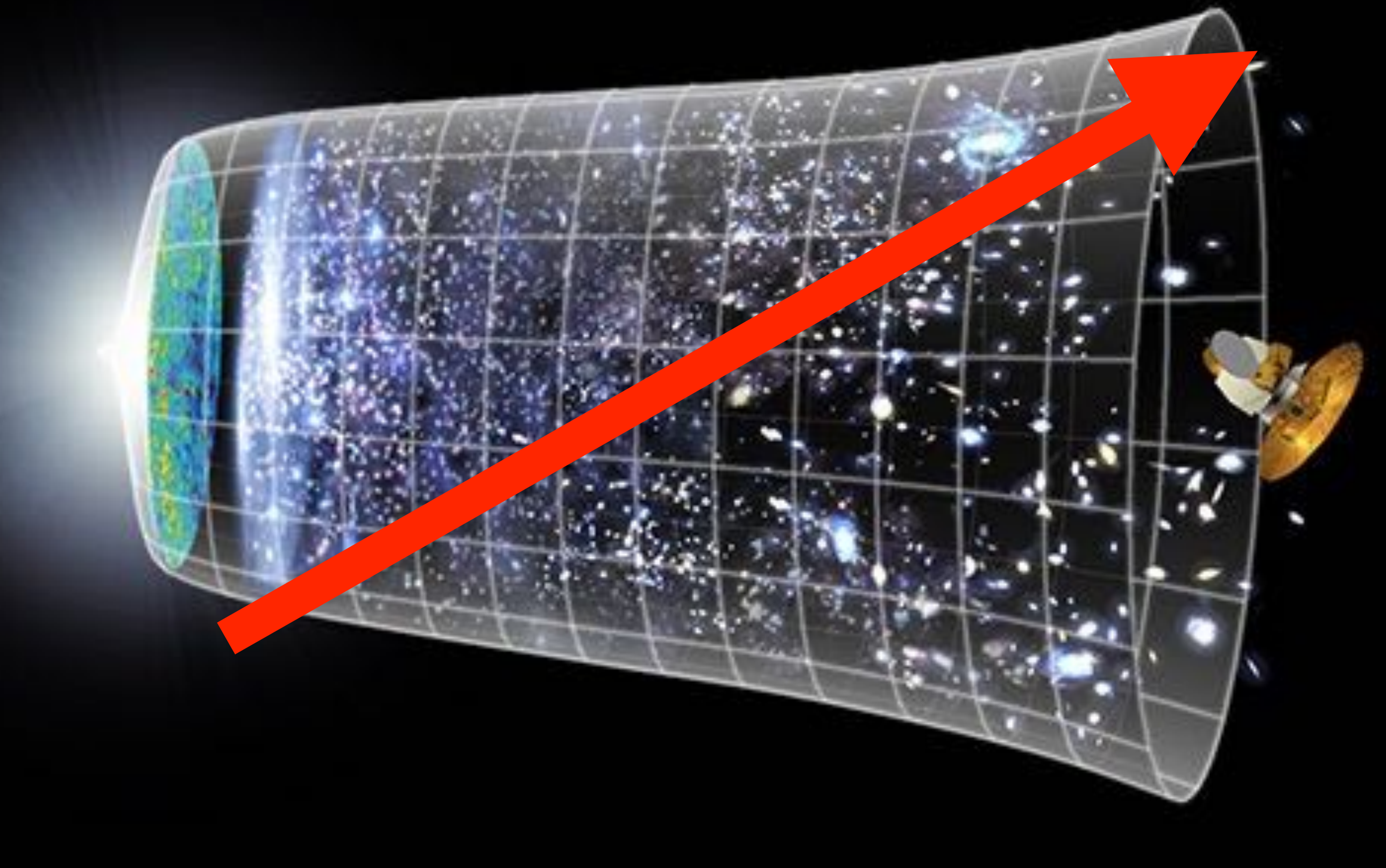
$$g_* = 0.002 \pm 0.016 \ (68\%\mathrm{CL})$$

(Kim and Komatsu 2014)

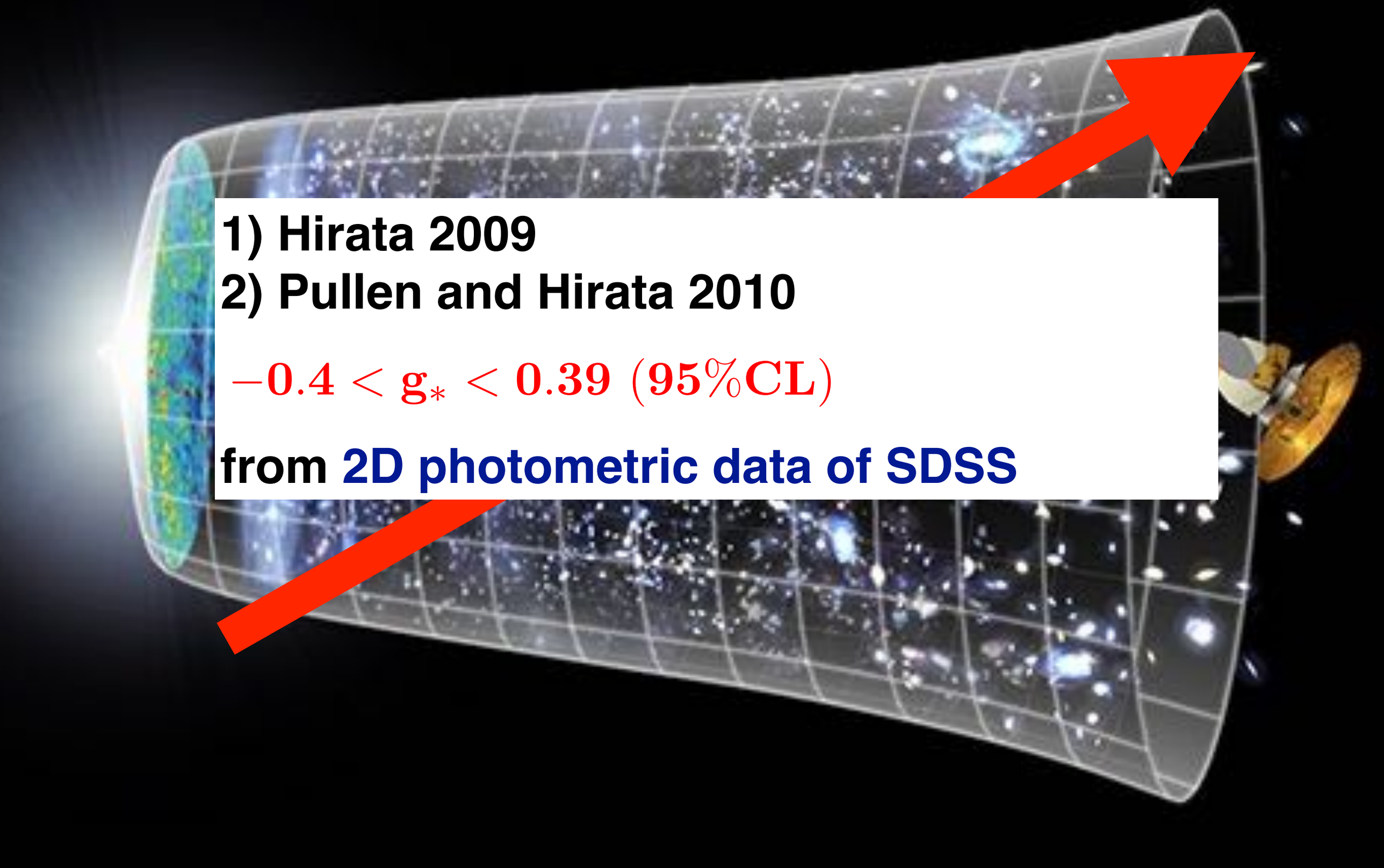


Before Beam Correction After Beam Correction

Large-scale structure



Large-scale structure

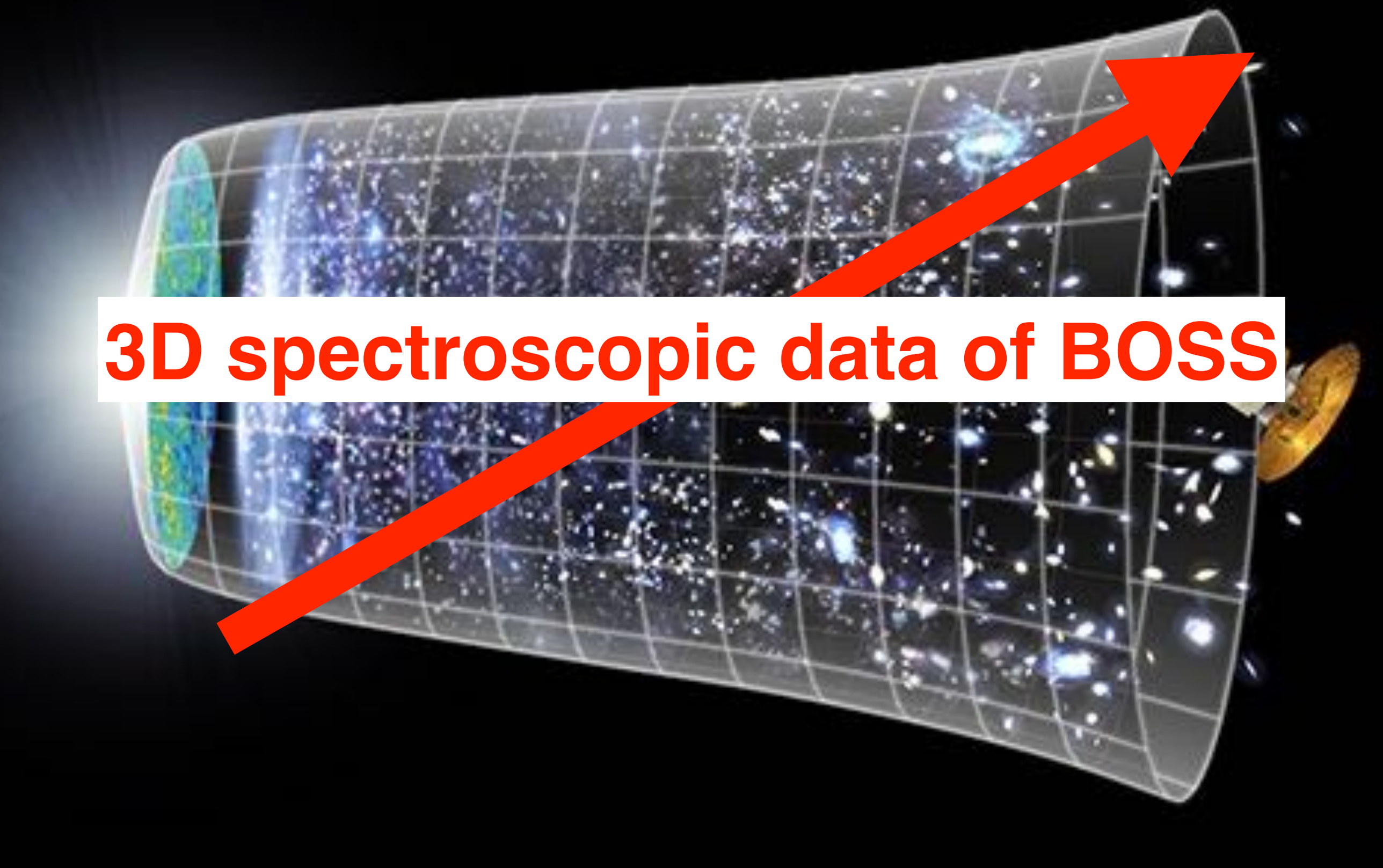


1) Hirata 2009
2) Pullen and Hirata 2010

$$-0.4 < g_* < 0.39 \ (95\%\mathrm{CL})$$

from 2D photometric data of SDSS

Large-scale structure



3D spectroscopic data of BOSS

Large-scale structure

1, Theory (M. Shiraishi, NS and T.Okumura, 2017)
— Bipolar spherical harmonics

2, Measurement (NS, M.Shiraishi and T.Okumura, 2017)
— BOSS
— Survey window corrections

3, Future surveys

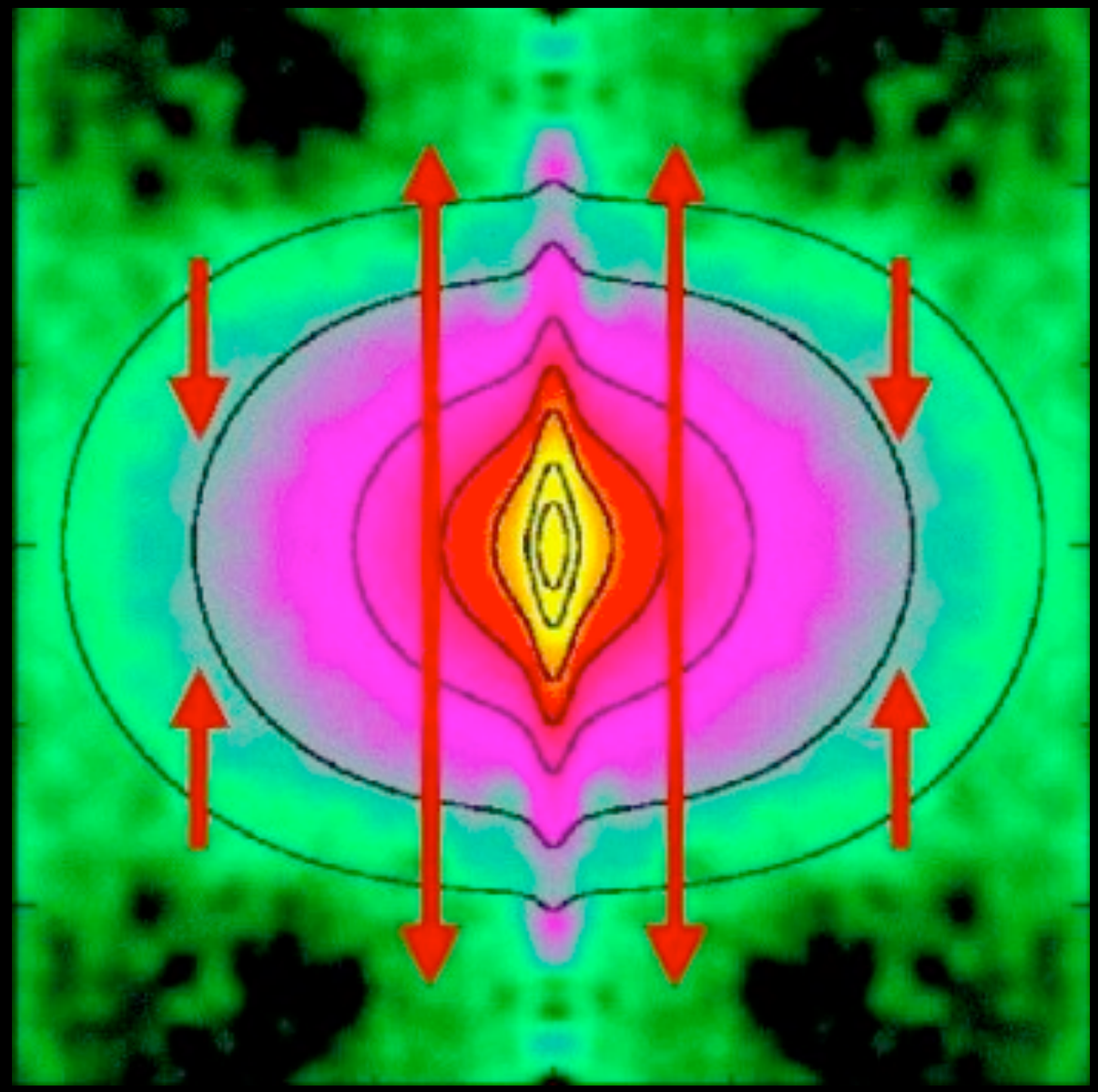
Spectroscopic galaxy surveys

We observe:

1) Right ascension
2) Declination
3) Redshift

Redshift space distortions:

$$1 + z = \frac{E_e}{E_o} \approx 1 + \bar{z} + \frac{\vec{v} \cdot \hat{n}}{c}$$



The observed radial distance of galaxies is distorted by peculiar velocities.

$$P_g(\vec{k}, \hat{n})$$

Under the assumption of statistical isotropy, Legendre polynomials decomposition:

$$P_{\mathrm{g}}(\vec{k}, \hat{n}) = \sum_{\ell} P_{\ell}(k)\, \mathcal{L}_{\ell}\left(\hat{k} \cdot \hat{n}\right)$$

Preferred direction

$$P_{\mathrm{g}}(\vec{k}, \hat{n}, \hat{p})$$

Quadrupolar anisotropy

Preferred direction

Linear theory:

$$P_{\rm g}(\vec{k},\hat{n},\hat{p}) = \left(b + f(\hat{k}\cdot\hat{n})^2\right)^2 \left\{1 + g_* \left[\left(\hat{k}\cdot\hat{p}\right)^2 - \frac{1}{3}\right]\right\} P_{\rm m}(k)$$

Kaiser factor

The Legendre decomposition is not sufficient any more.

Legendre decomposition

$$\int \frac{d^2\hat{n}}{4\pi} \int \frac{d^2\hat{k}}{4\pi} \underline{\mathcal{L}_\ell(\hat{k}\cdot\hat{n})} P_{\rm g}(\vec{k},\hat{n},\hat{p}) =$$

Linear theory:

$$P_0^s(k) = \left(b^2 + \frac{2}{3}bf + \frac{1}{5}f^2\right) P_{\delta\delta}^{\rm lin}(k),$$

$$P_2^s(k) = \left(\frac{4}{3}bf + \frac{4}{7}f^2\right) P_{\delta\delta}^{\rm lin}(k),$$

$$P_4^s(k) = \frac{8}{35}f^2 P_{\delta\delta}^{\rm lin}(k).$$

No primordial anisotropic contributions!!

Spherical harmonic decomposition

$$P_{\rm g}(\vec{k},\,\hat{n},\,\hat{p}) = \sum_{\ell m}\sum_{\ell' m'} P_{\ell\ell'}^{mm'}(k,\hat{p})\,Y_{\ell m}(\hat{k})\,Y_{\ell' m'}(\hat{n})$$

Bipolar Spherical harmonic decomposition

$$P_{\rm g}(\vec{k},\,\hat{n},\,\hat{p}) = \sum_{LM}\sum_{\ell\ell'} P_{\ell\ell'}^{LM}(k,\hat{p})\,\left(Y_\ell(\hat{k})\otimes Y_{\ell'}(\hat{n})\right)_{LM}$$

Angular momentum coupling

Bipolar Spherical harmonic decomposition

$$P_{\ell\ell'}^{LM}(k) = \sum_{mm'} \begin{pmatrix} \ell & \ell' & L \\ m & m' & M \end{pmatrix} P_{\ell\ell'}^{mm'}(k)$$

Wigner 3j symbol

Angular momentum coupling

Bipolar Spherical harmonic decomposition

$$P_{\ell\ell'}^{LM}(k) = \sum_{mm'} \begin{pmatrix} \ell & \ell' & L \\ m & m' & M \end{pmatrix} P_{\ell\ell'}^{mm'}(k)$$

Total angular momentum

L = 0 : Legendre polynomials

L > 0 : Statistical anisotropy

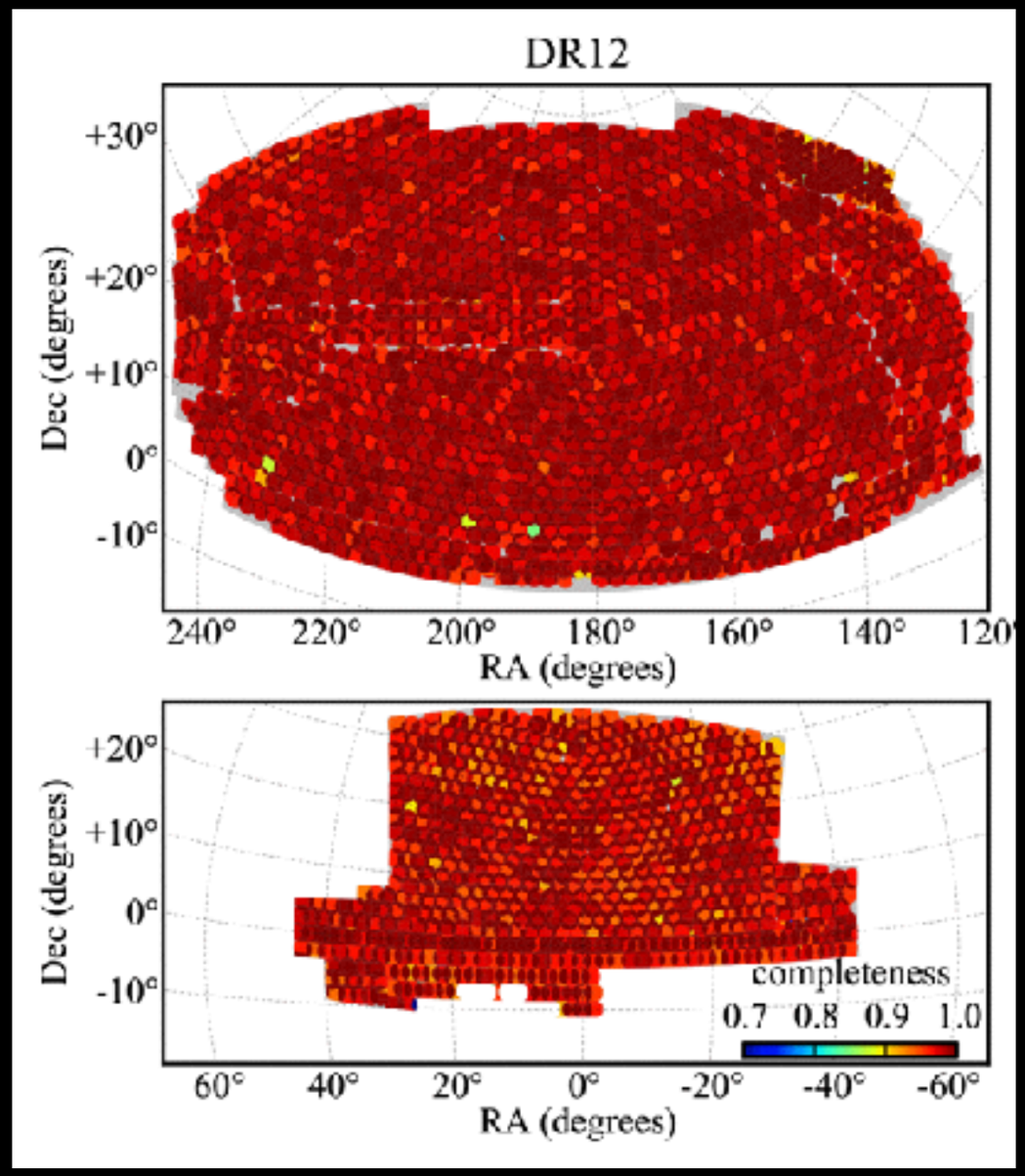
Quadrupolar anisotropy

No primordial anisotropic constributions

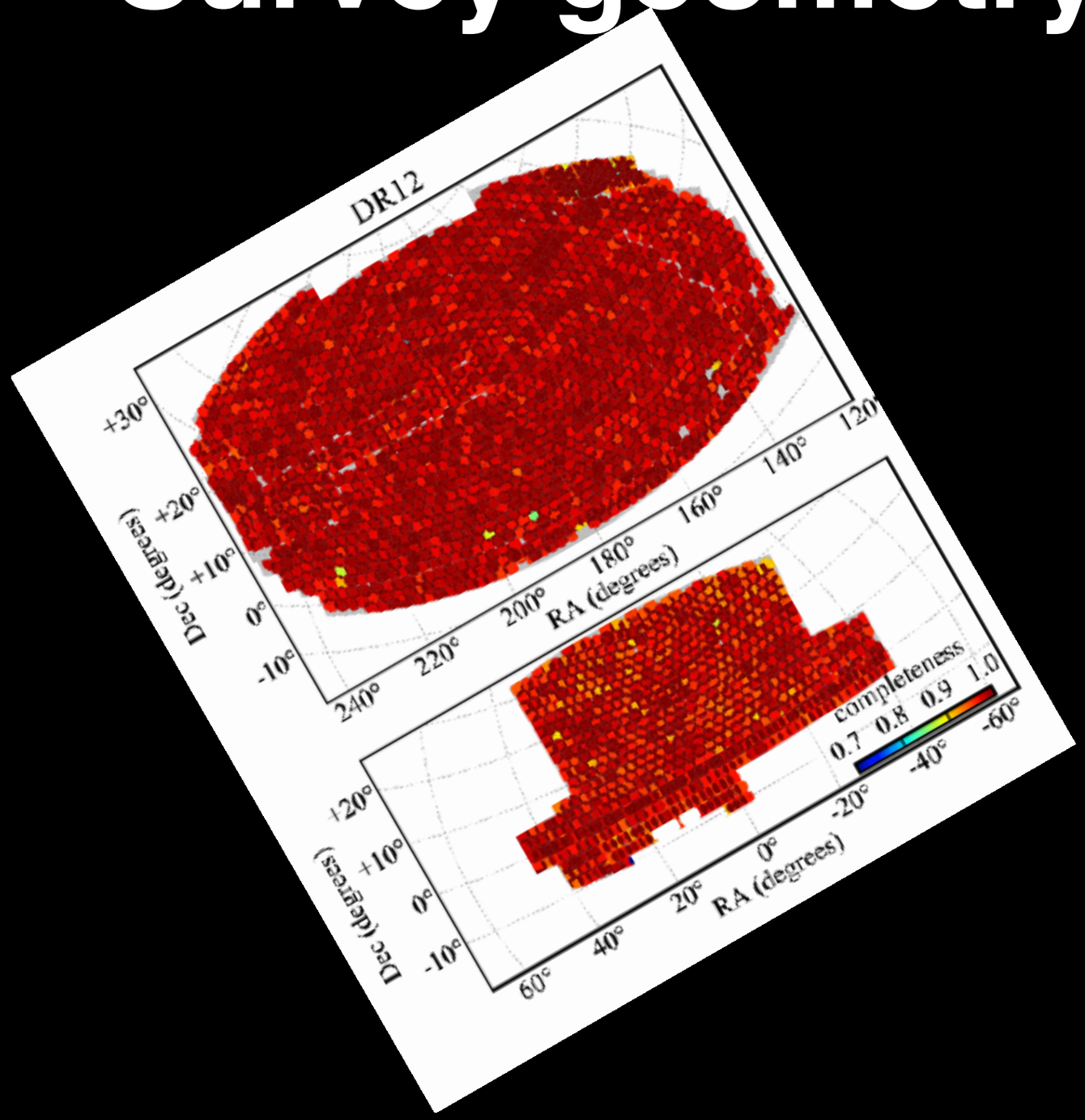
$$P_{\ell\ell'}^{L=0,M=0}(k) = \delta_{\ell\ell'} P_\ell(k)$$

$$P_{\ell\ell'}^{L=2,M}(k) \propto g_* Y_{2M}(\hat{p})$$

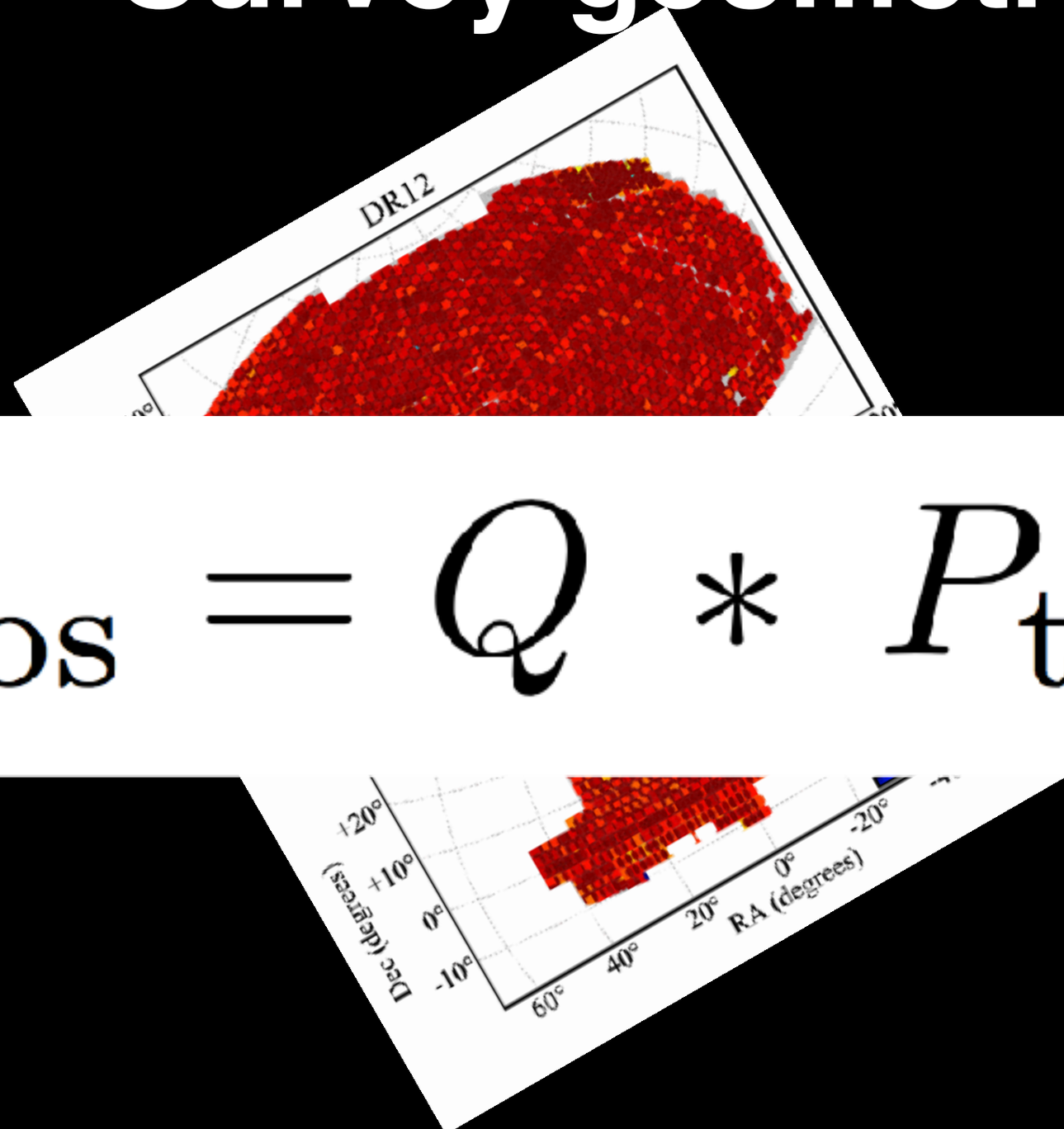
Survey geometry



Survey geometry



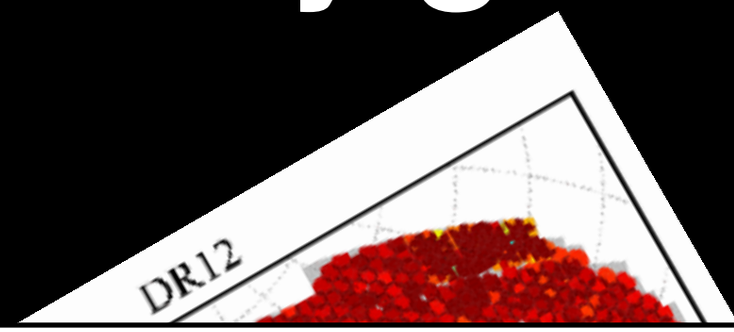
Survey geometry



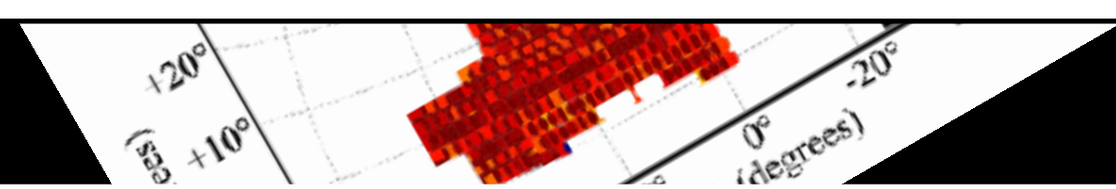
$$P_{\rm obs} = Q * P_{\rm theory}$$

Q: window function

Survey geometry



$$
\begin{aligned}
\left\langle \widehat{\xi}_{20}^{2M}(r)\big|_{\mathrm{A}} \right\rangle &= Q_0(r)\,\xi_{20}^{2M}(r) + Q_{20}^{2M}(r)\,\left[\xi_0(r) - \langle \bar{\delta}^2 \rangle\right] \\
&+ \frac{1}{5}\left[Q_{02}^{2M}(r) + Q_{22}^{2M}(r) + Q_{42}^{2M}(r)\right]\,\xi_2(r) \\
&+ \frac{1}{9}\left[Q_{24}^{2M}(r) + Q_{44}^{2M}(r)\right]\,\xi_4(r) \\
&+ \cdots,
\end{aligned}
$$



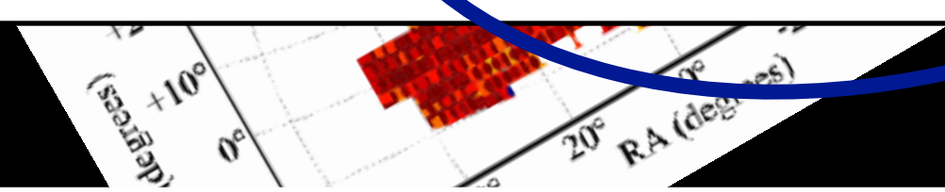
Bipolar Spherical harmonic
decomposition of the window function

Survey geometry

Window effects

Anisotropic signal

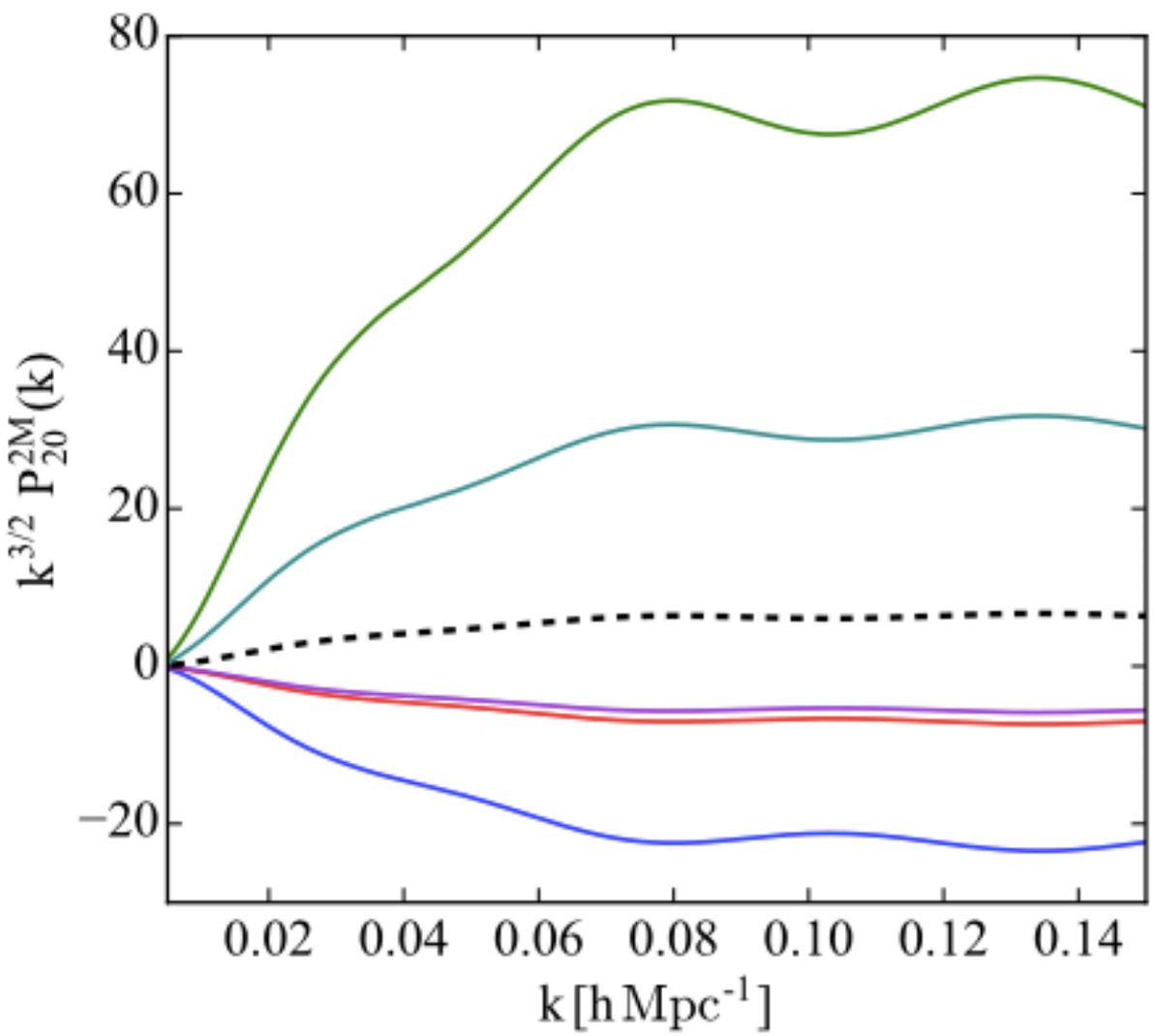
$$\left\langle \widehat{\xi}_{20}^{2M}(r)\big|_{\mathrm{A}} \right\rangle = Q_0(r)\,\xi_{20}^{2M}(r) + Q_{20}^{2M}(r)\,\left[\xi_0(r) - \langle \bar{\delta}^2 \rangle\right] + \frac{1}{5}\left[Q_{02}^{2M}(r) + Q_{22}^{2M}(r) + Q_{42}^{2M}(r)\right]\,\xi_2(r) + \frac{1}{9}\left[Q_{24}^{2M}(r) + Q_{44}^{2M}(r)\right]\,\xi_4(r) + \cdots,$$



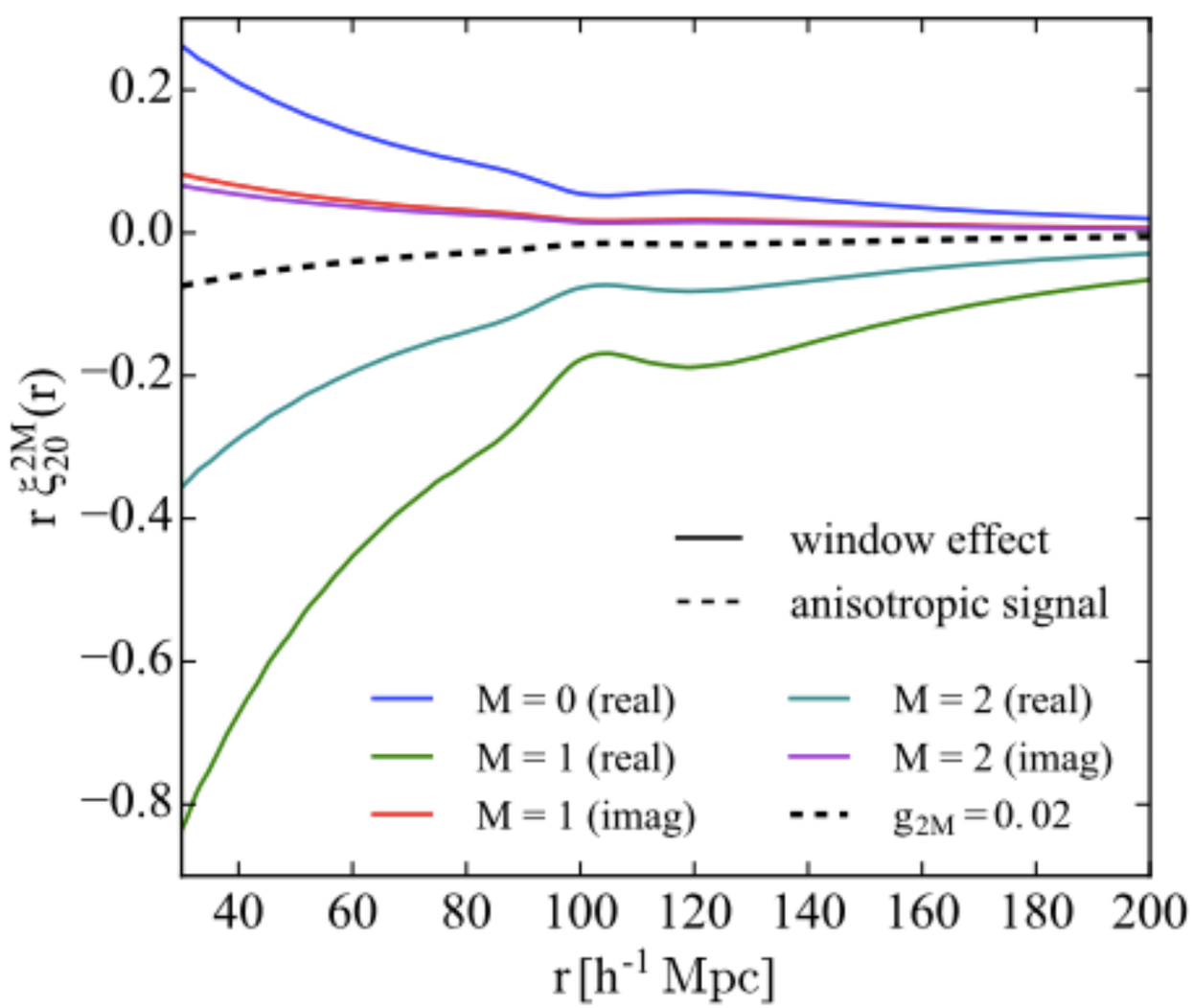
Bipolar Spherical harmonic
decomposition of the window function

Survey geometry

Power spectrum

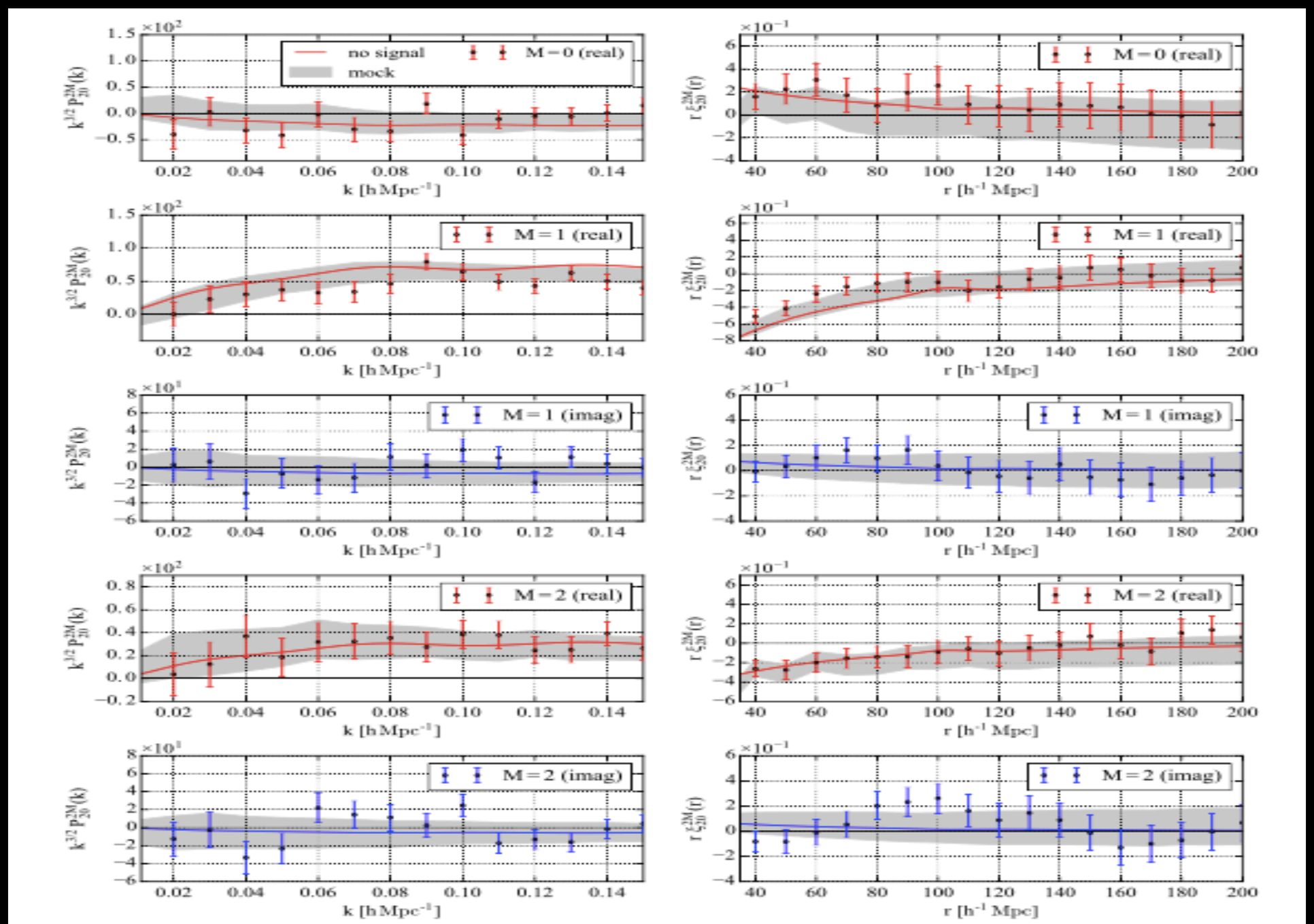


2pt correlation function



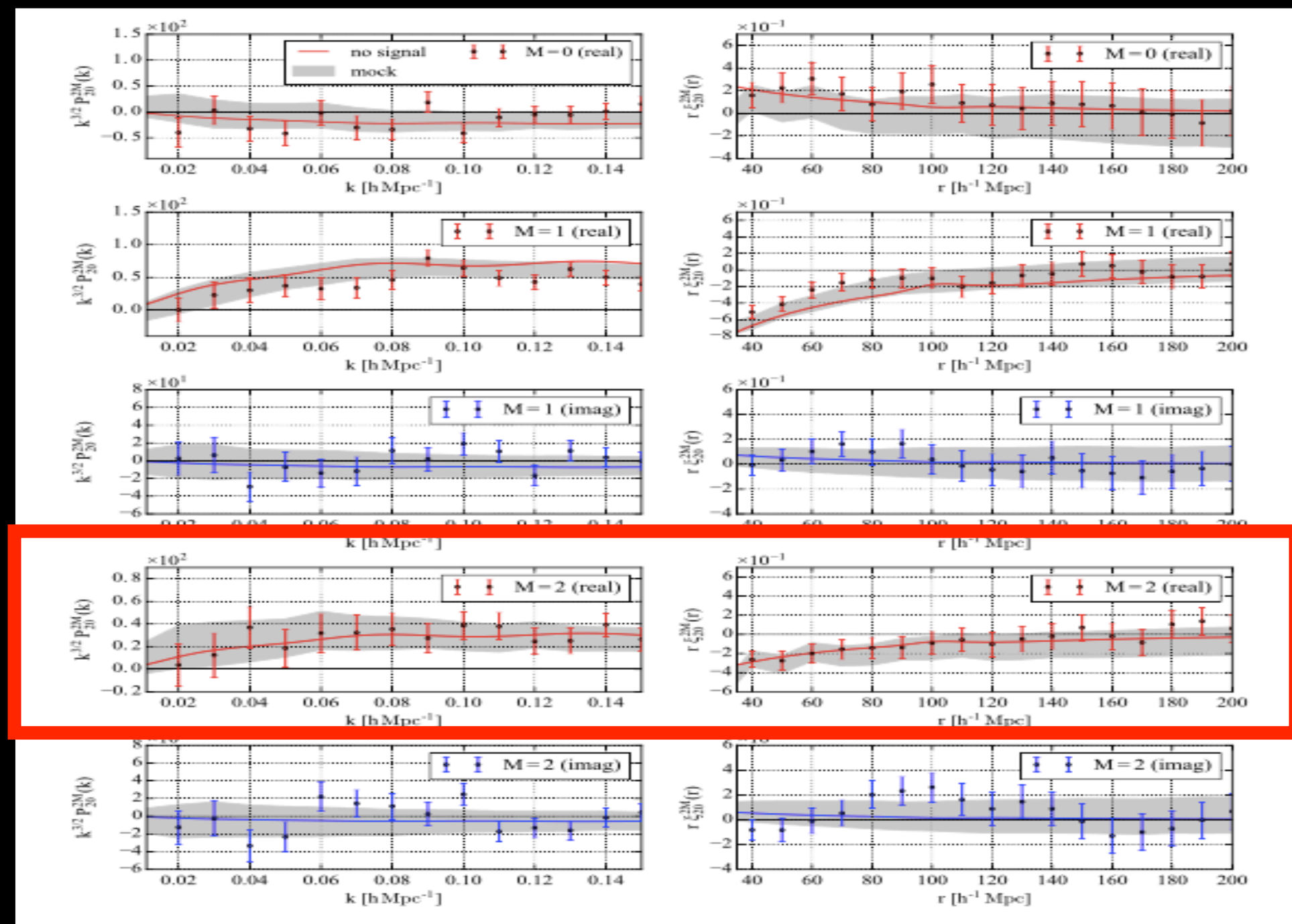
L=2
M=-2,1,0,1,2

Power spectrum **2pt function**



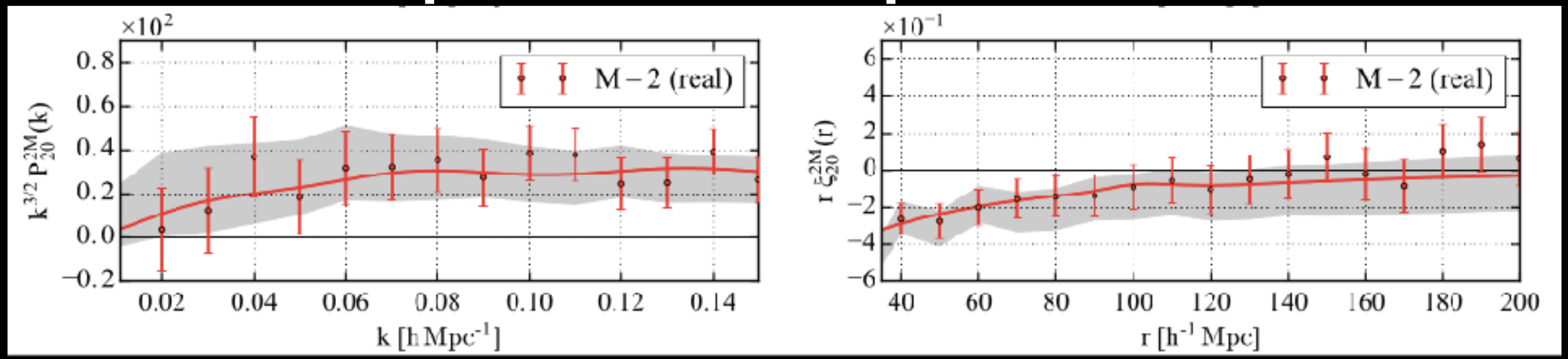
L=2
M=-2,1,0,1,2

Power spectrum **2pt function**



(L, M) = (2, 2)

Power spectrum **2pt correlation function**



Shaded regions:

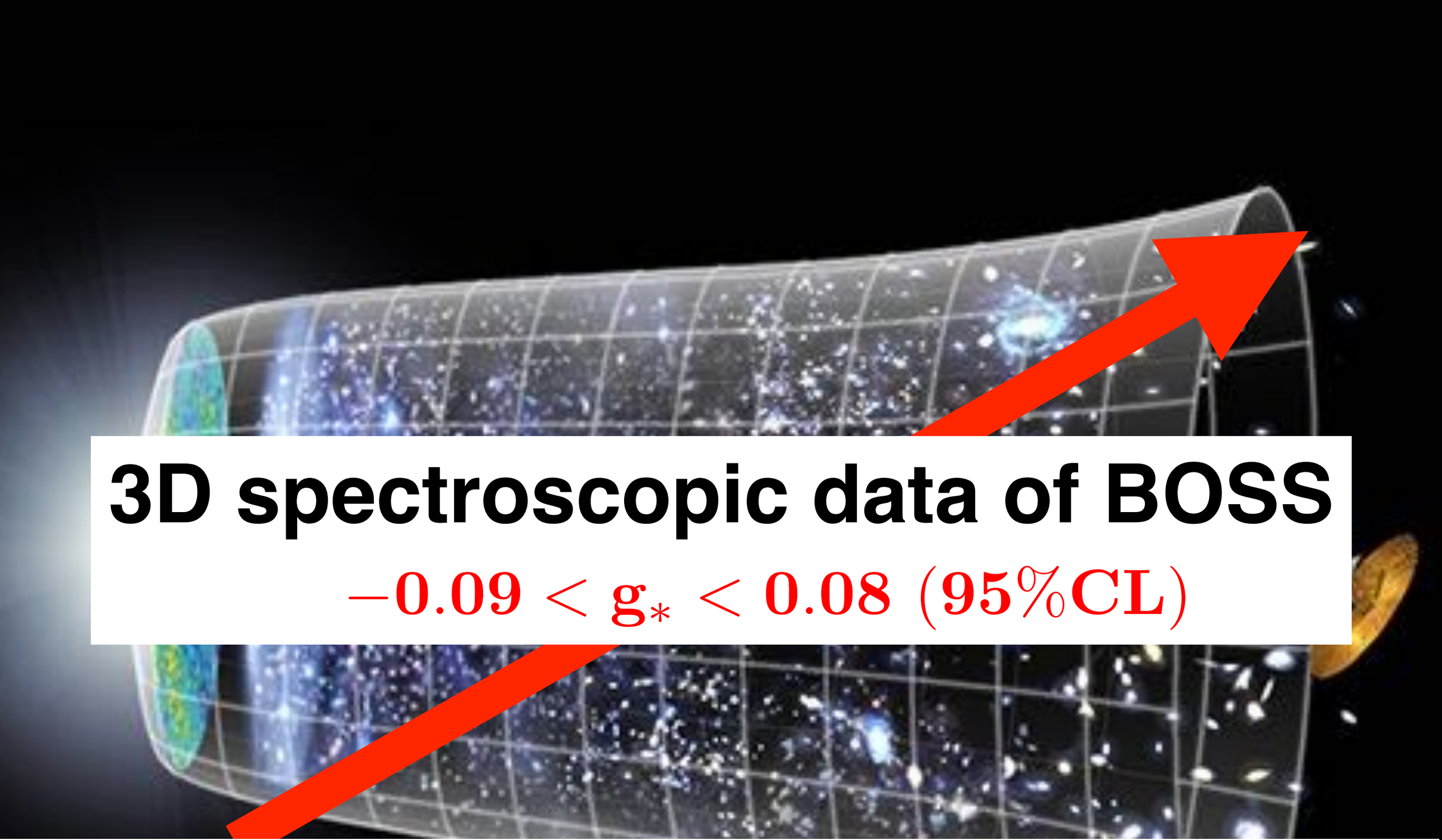
mocks without statistical anisotropic signal

Solid lines:

theory without statistical anisotropic signal

Points:

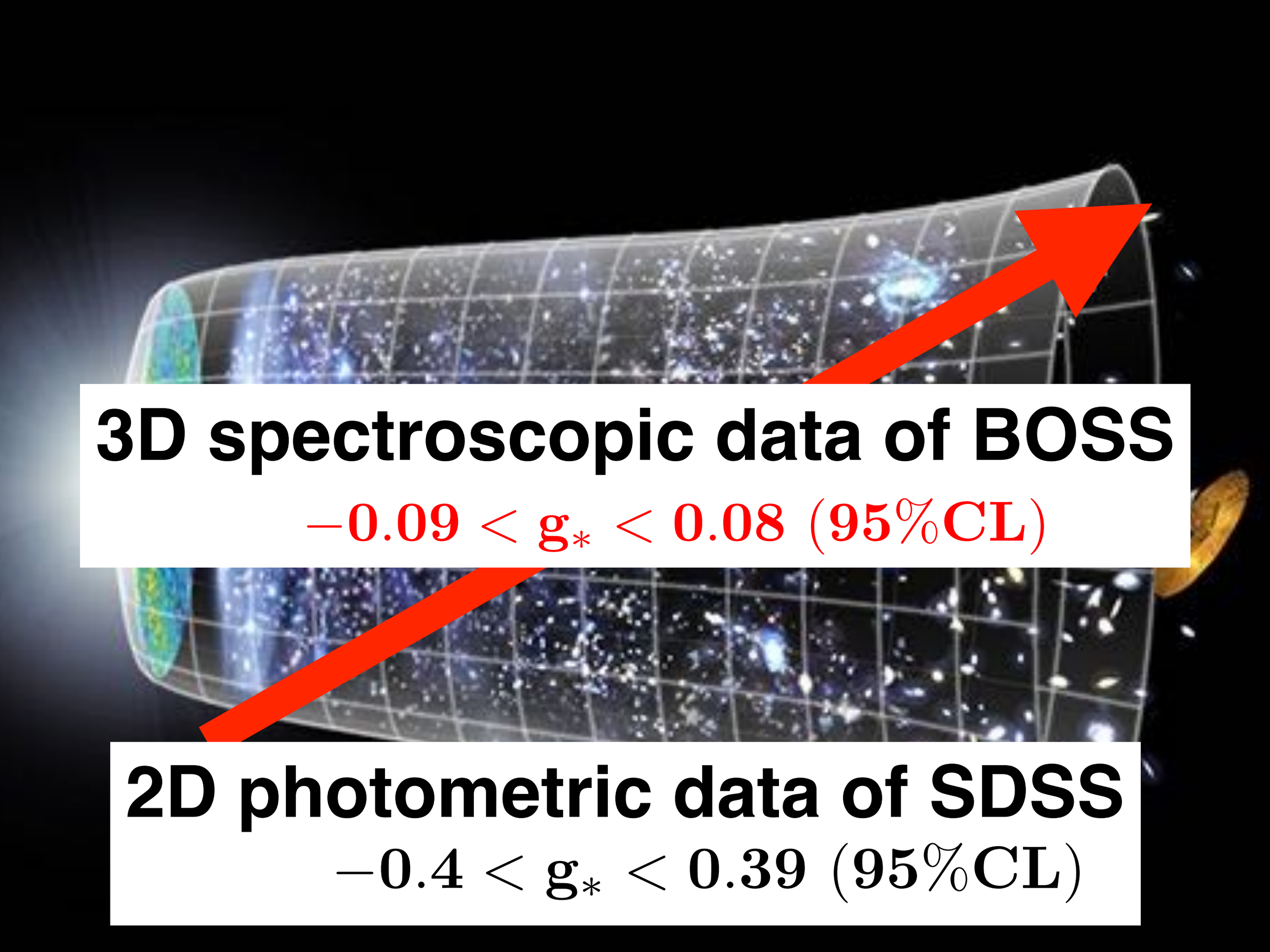
observed data



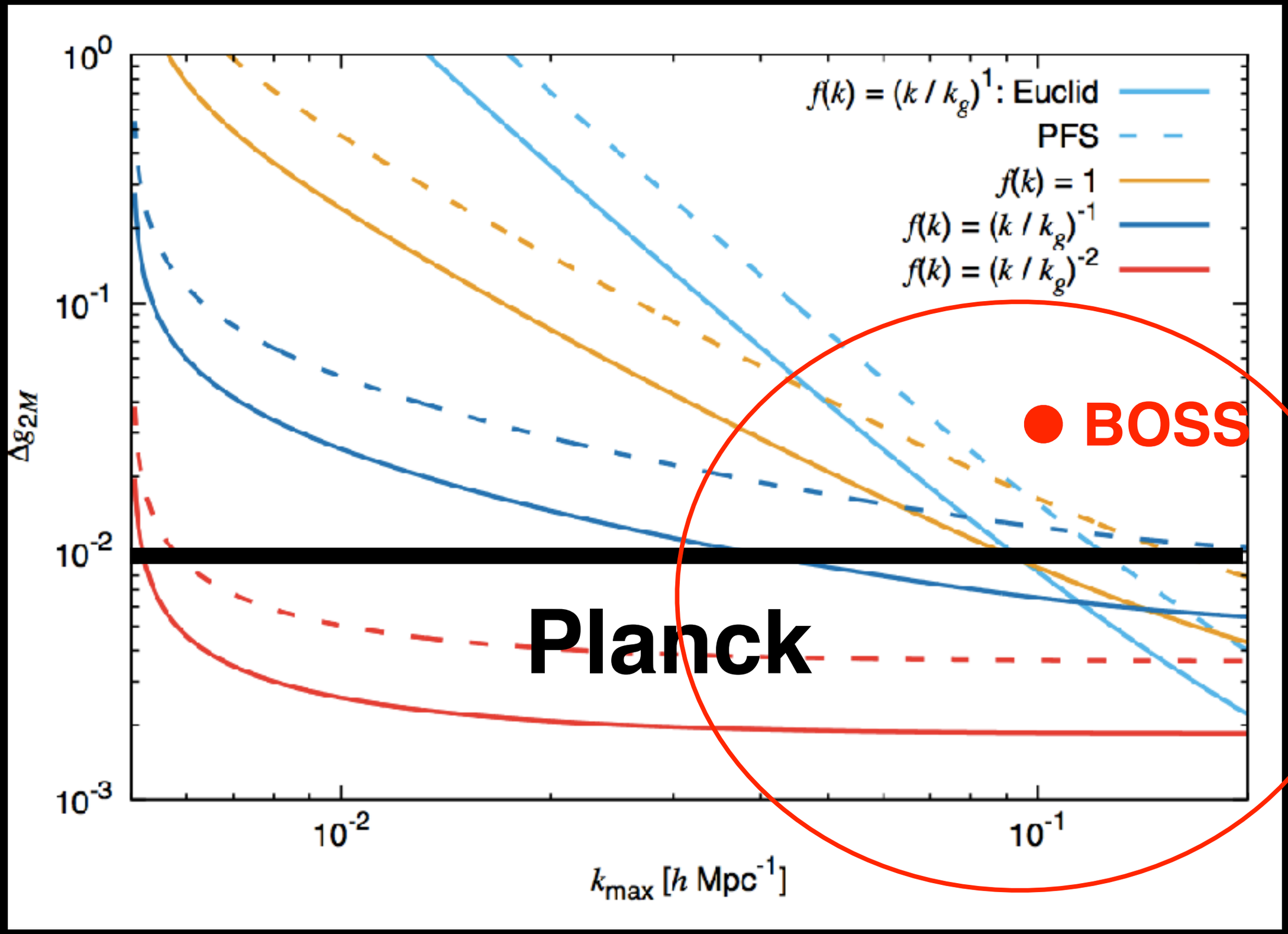
3D spectroscopic data of BOSS

$$-0.09 < g_* < 0.08 \ (95\%\text{CL})$$

	CMASS NGC	CMASS SGC	LOWZ NGC	LOWZ SGC	All
Power Spectrum	$-0.14 < g_* < 0.09$	$-0.14 < g_* < 0.18$	$-0.12 < g_* < 0.11$	$-0.18 < g_* < 0.19$	$-0.093 < g_* < 0.079$
Correlation function	$-0.14 < g_* < 0.10$	$-0.13 < g_* < 0.18$	$-0.16 < g_* < 0.13$	$-0.19 < g_* < 0.23$	$-0.088 < g_* < 0.085$


3D spectroscopic data of BOSS
$-0.09 < g_* < 0.08$ (95%CL)
2D photometric data of SDSS
$-0.4 < g_* < 0.39$ (95%CL)

Future surveys



Summary

- **First** limits on statistical anisotropy from BOSS
- Bipolar spherical harmonic decomposition
- Estimator of the bipolar coefficients
- Survey geometry

3D spectroscopic data of BOSS

$$-0.09 < g_* < 0.08 \ (95\%\mathrm{CL})$$

Extra slides

Non-linear information