On the gravitational redshift of cosmological objects

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Outline

- 1 Introduction – gravitational redshift
- 2 Theoretical formulation
- 3 Satellite galaxy （virialized in a halo）
- 4 Intracluster gas
- 5 galaxies of void
- 6 Summary and conclusions

1. Introduction– gravitational redshift

Relativistic effect in large scale structures

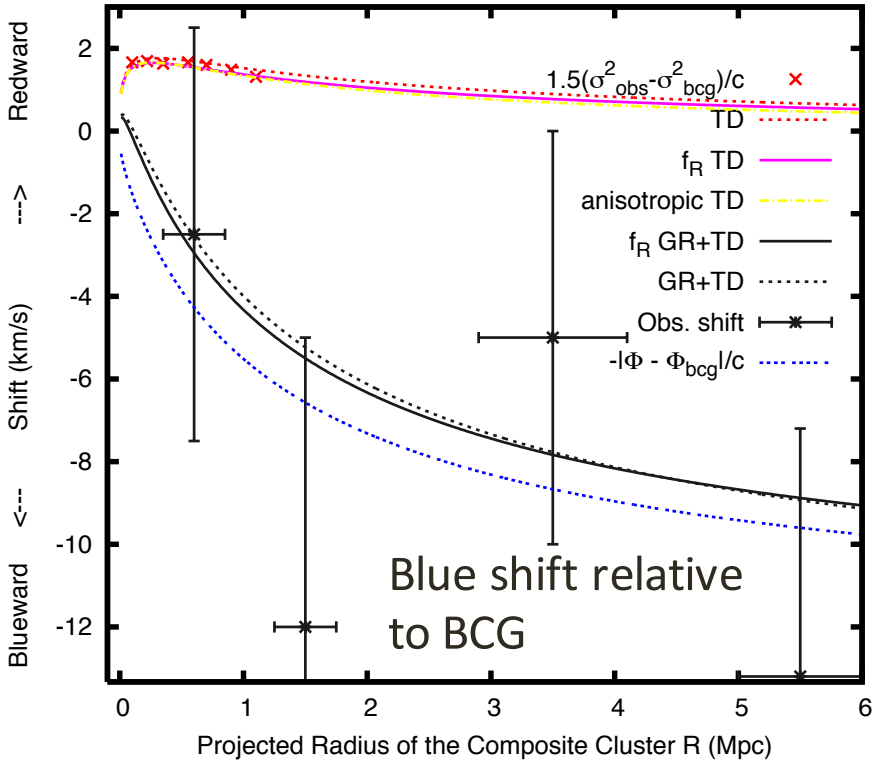
Taruya-san's talk

- Gravitational redshift in cluster of galaxies

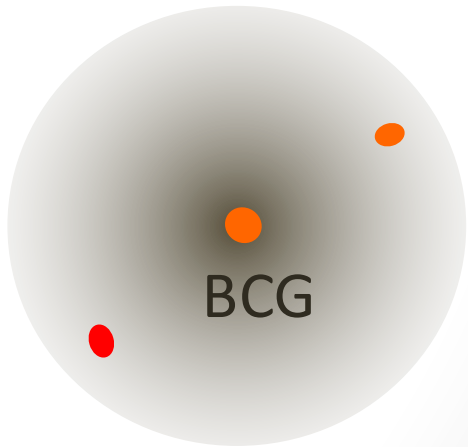
Zhao, Peacock, Li (2013)

Wojtak, et al, (2011)
Zhao, Peacock, Li (2013)
Kaiser, (2013)
Jimeno, et al. (2015)
Cai et al., (2016)

Negative value of δz



redshift relative to BCG



Signal has been detected.

Other possibility for detection?

Useful for a test of general relativity and gravity theories ?

2. Theoretical formulation

Newtonian gauge

$$ds^2 = a(\eta)^2[-(1+2\psi)d\eta^2 + (1+2\phi)d\vec{x}^2]$$

Geodesic equation: p is a physical energy for cosmological rest frame obs.

$$\frac{1}{p}\frac{dp}{d\eta} = -\mathcal{H} - \frac{\partial\phi}{\partial\eta} - \hat{p}^i\frac{\partial\psi}{\partial x^i}, \qquad \delta_{ij}\hat{p}^i\hat{p}^j = 1 \qquad |\psi|, |\phi| \ll 1$$

$$\frac{d\psi(\eta, x^i(\eta))}{d\eta} = \frac{\partial\psi}{\partial\eta} + \frac{dx^i}{d\eta}\frac{\partial\psi}{\partial x^i} \qquad \frac{dx^i}{d\eta} = \hat{p}^i(1+\psi-\phi)$$

Solution up to the first order of the metric perturbation

ISW term Potential term

$$\frac{1}{1+z_j} = \frac{p(\eta_0)}{p(\eta_j)} = \frac{a(\eta_j)}{a(\eta_0)}\exp\left\{-\int_{\eta_j}^{\eta_0}(\phi'(\eta)-\psi'(\eta))\,d\eta - \psi(\eta_0,\vec{x}(\eta_0)) + \psi(\eta_j,\vec{x}(\eta_j))\right\}$$

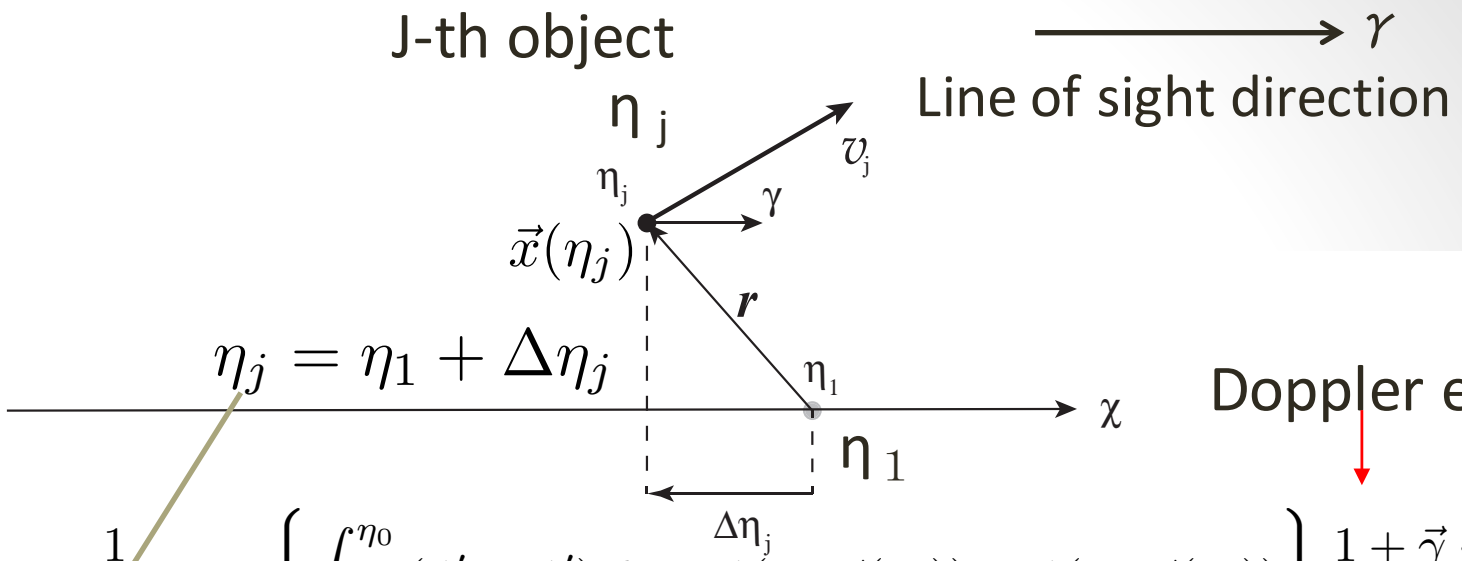
η_j Time at the emission of photon from the j-th object

η_0 Present epoch time $a(\eta_0) = 1$

$a(\eta_0) = 1$

η_0

Observer



$\eta_j = \eta_1 + \Delta\eta_j$

Doppler effect

$$1 + z_j = \frac{p(\eta_j)}{p(\eta_0)} = \frac{1}{a(\eta_j)} \exp\left\{ \int_{\eta_j}^{\eta_0} (\phi' - \psi')\, d\eta + \psi(\eta_0, \vec{x}(\eta_0)) - \psi(\eta_j, \vec{x}(\eta_j)) \right\} \frac{1 + \vec{\gamma}\cdot\vec{v}_j}{\sqrt{1 - \vec{v}_j^2}}.$$

$|\vec{v}| \ll 1$

$$\frac{1 + \vec{\gamma}\cdot\vec{v}_j}{\sqrt{1 - \vec{v}_j^2}} \simeq 1 + \vec{\gamma}\cdot\vec{v}_j + \frac{1}{2}\vec{v}_j^2$$ → Doppler term

$$\frac{1}{a(\eta_j)} \simeq \frac{1}{a(\eta_1)} \left\{ 1 - \mathcal{H}(\eta_1)\Delta\eta_j + \left(\mathcal{H}^2(\eta_1) - \frac{1}{2}\frac{a''(\eta_1)}{a(\eta_1)} \right) \Delta\eta_j^2 \right\}$$

$$1 + z_1 \equiv \frac{1}{a(\eta_1)}$$

Hubble terms

ISW term

$$\boxed{\begin{aligned} 1 + z_j \simeq (1 + z_1) \Big\{ & 1 - \mathcal{H}(\eta_1)\Delta\eta_j + \left(\mathcal{H}^2(\eta_1) - \frac{1}{2}\frac{a''(\eta_1)}{a(\eta_1)} \right) \Delta\eta_j^2 + \int_{\eta_j}^{\eta_0} (\phi' - \psi')\, d\eta \\ & + \psi(\eta_0, \vec{x}(\eta_0)) - \psi(\eta_j, \vec{x}(\eta_j)) + \vec{\gamma}\cdot\vec{v}_j + \frac{1}{2}|\vec{v}_j|^2 \Big\} \end{aligned}}$$

$|\psi|, |\phi| \ll 1$ Gravitational potential term

Doppler terms

We omit the ISW term because we consider objects of subhorizon scale and consider the relative gravitational redshift.

$$\int_{\eta_j}^{\eta_0} (\phi' - \psi')\, d\eta \simeq \int_{\eta_1}^{\eta_0} (\phi' - \psi')\, d\eta - (\phi'(\eta_1, \vec{x}(\eta_1)) - \psi'(\eta_1, \vec{x}(\eta_1)))\Delta\eta_j$$

$\psi(\eta_0, \vec{x}(\eta_0))$ Gravitational redshift of a observer does not contribute to the relative gravitational redshift.

Redshift of the j-th object

Hubble terms

$$1 + z_j \simeq (1 + z_1)\left\{1 - \mathcal{H}(\eta_1)\Delta\eta_j + \left(\mathcal{H}^2(\eta_1) - \frac{1}{2}\frac{a''(\eta_1)}{a(\eta_1)}\right)\Delta\eta_j^2 - \psi(\eta_j, \vec{x}(\eta_j)) + \vec{\gamma}\cdot\vec{v}_j + \frac{1}{2}|\vec{v}_j|^2\right\}$$

$$1 + z_1 \equiv \frac{1}{a(\eta_1)}$$

Gravitational potential term

Doppler terms

Cluster's gravitational redshift

Effect of the light-cone coordinate

Kaiser (2013)

Expectation value integrated over the velocity space

Distribution function on the light-cone coordinate

$$f(\vec{x},\vec{v}) = \underbrace{(1+(\vec{\gamma}\cdot\vec{v}))f_{\mathrm{RF}}(\vec{x},\vec{v})}$$

$$\delta z \supset (\vec{\gamma}\cdot\vec{v}) + \frac{1}{2}|\vec{v}|^2$$

$$\boxed{\langle\delta z\rangle = \frac{\int d^3x \int d^3v \delta z f(\vec{x},\vec{v})}{\int d^3x \int d^3v f(\vec{x},\vec{v})} \supset \langle(\vec{\gamma}\cdot\vec{v})^2\rangle + \frac{1}{2}\langle|\vec{v}|^2\rangle}$$

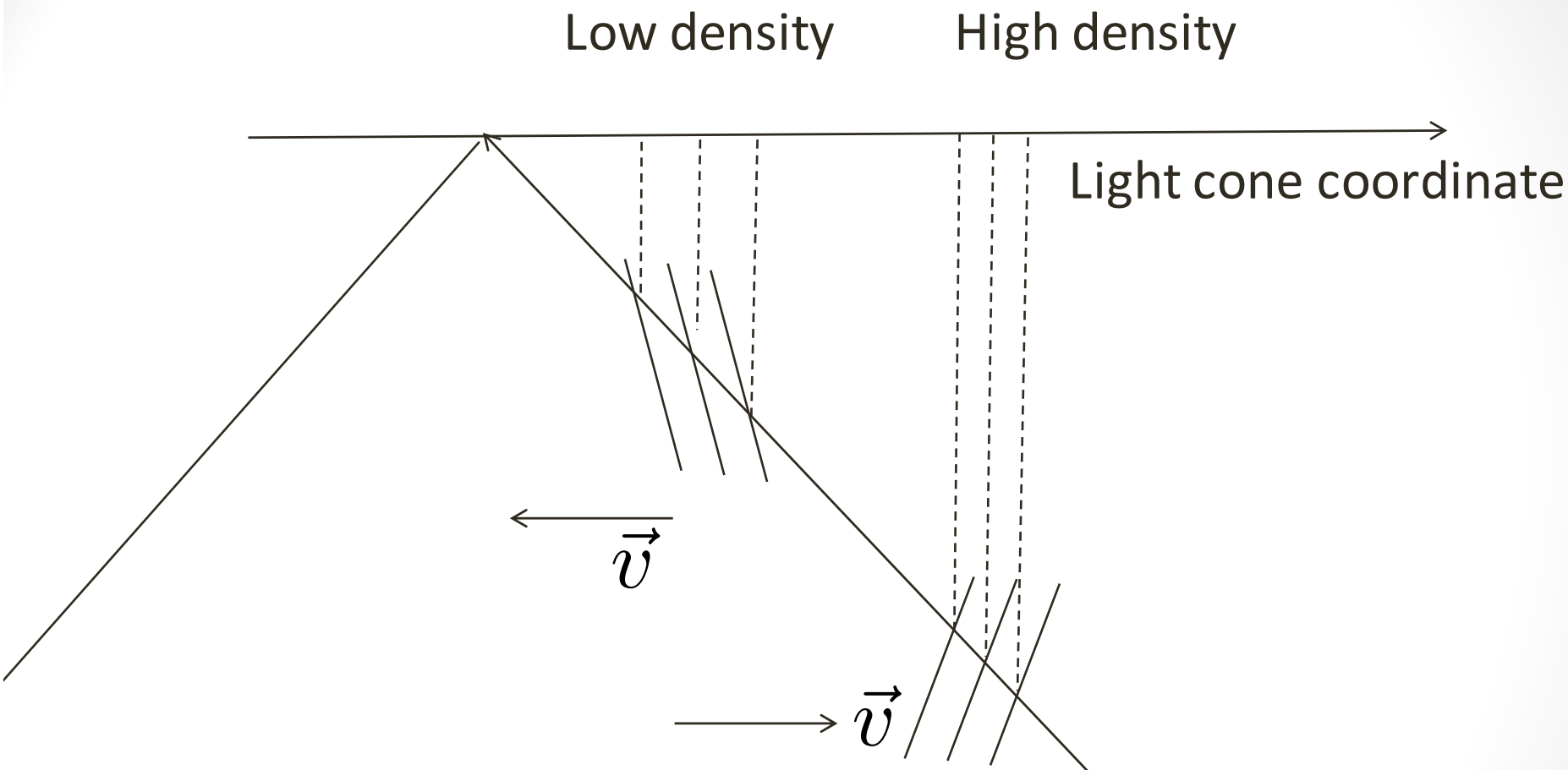
Additional second order Doppler term

Statistical isotropy in velocity space

$$\langle(\vec{\gamma}\cdot\vec{v})^2\rangle = \frac{1}{3}\langle|\vec{v}|^2\rangle \longrightarrow \langle(\vec{\gamma}\cdot\vec{v})^2\rangle + \frac{1}{2}\langle|\vec{v}|^2\rangle = \frac{5}{2}\langle(\vec{\gamma}\cdot\vec{v})^2\rangle$$

makes some contribution to the results

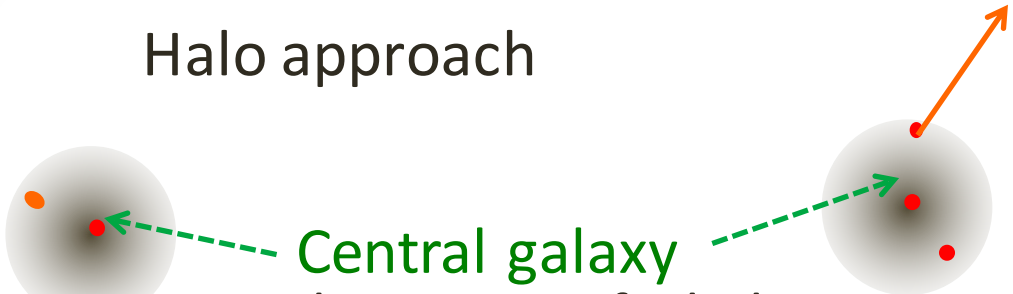
Phase space density on lightcone coordinate (Kaiser 2013)



$$f(\vec{x},\vec{v}) = (1 + (\vec{\gamma}\cdot\vec{v}))f_{\mathrm{RF}}(\vec{x},\vec{v})$$

3 Satellite galaxies virialized in halos

Halo approach



✓ Halo occupation distribution (HOD)

galaxy number in a halo with mass M

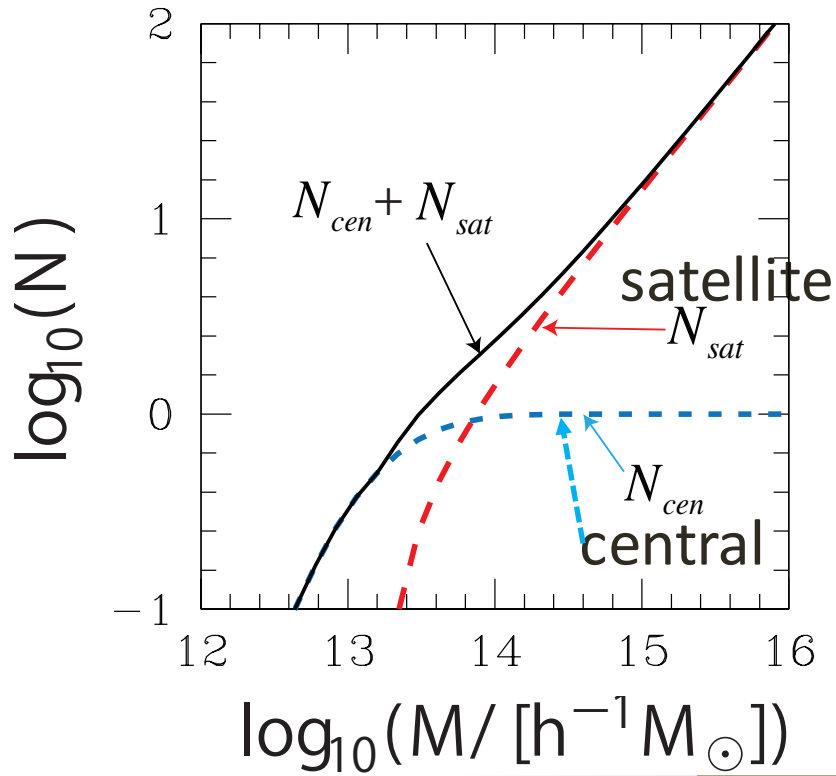
✓ Satellite galaxy random velocity

Virial random velocity variance

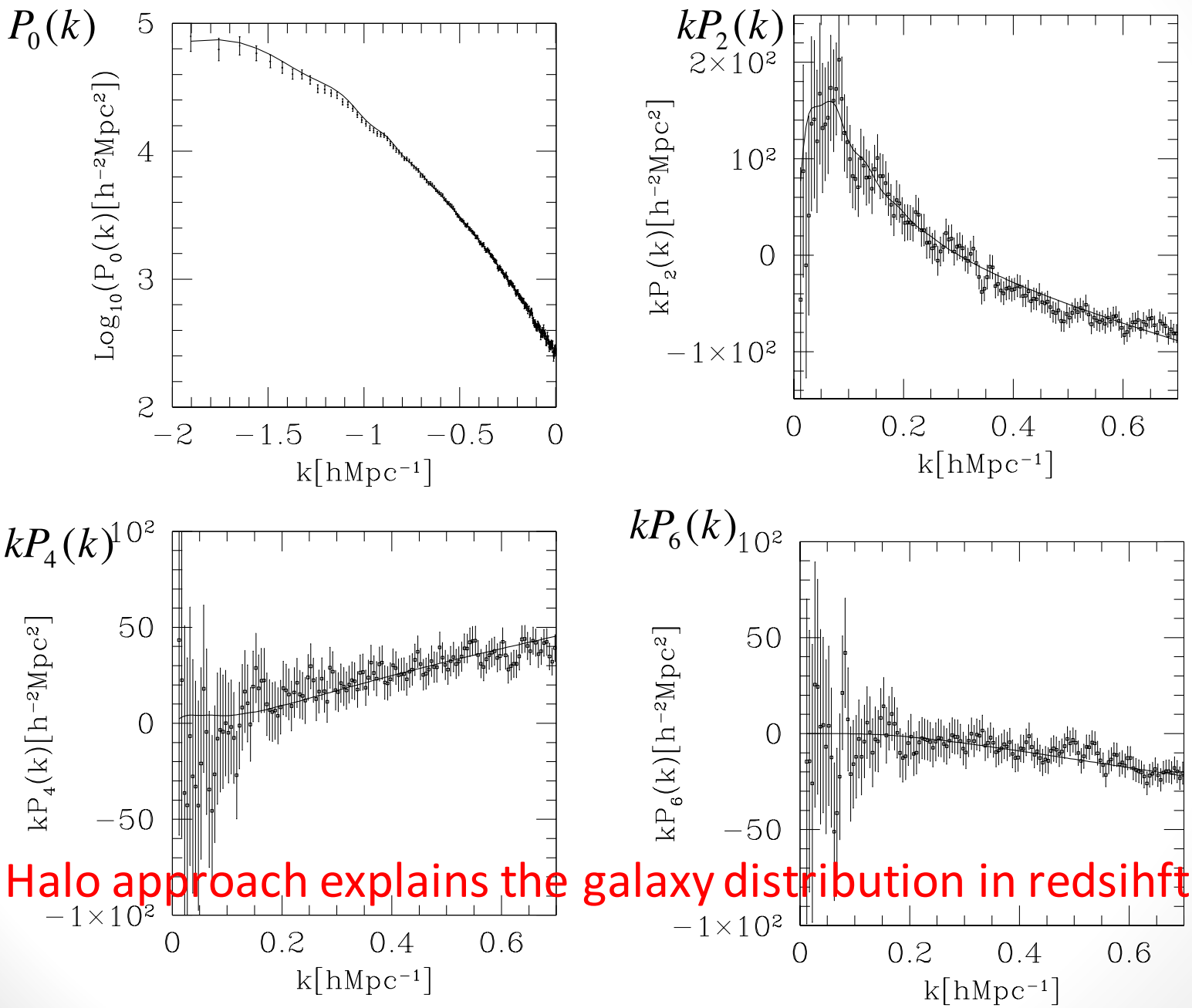
$$\sigma_{v,off}^2 = \frac{GM}{2r_{\rm vir}}$$

Finger of God effect

LOWZ sample (Parejko et al)



SDSS III LOWZ sample / multipole power spectrum



Halo approach explains the galaxy distribution in redsihft space

Redshift of satellites relative to central galaxy

$$\langle \delta z \rangle = \frac{\int d^3x \int d^3v_j \delta z_{jr} f(\vec{x}, \vec{v}_j)}{\int d^3x \int d^3v_j f(\vec{x}, \vec{v}_j)}$$

Assuming central galaxy's random velocity is zero

$$\langle (\vec{\gamma} \cdot \vec{v}_j)^2 \rangle = \frac{1}{3} \langle \vec{v}_j^2 \rangle$$

$$\boxed{\langle \delta z \rangle = \psi(0) - \langle \psi_s \rangle + \frac{5}{2} \langle (\vec{\gamma} \cdot \vec{v}_j)^2 \rangle = \langle \delta z(M) \rangle}$$

NFW density profile

$$\psi(0) = -4\pi G \rho_s r_s^2 = -\frac{GM_{\rm vir}}{r_{\rm vir}} \frac{c}{m(c)} \qquad m(c) = \ln(1+c) - c/(1+c)$$

$$\langle \psi_s \rangle = 4\pi \int_0^{r_{\rm vir}} dr r^2 \frac{\rho_{\rm NFW}(r)}{M_{\rm vir}} \psi(r) = -\frac{c}{m^2(c)} \frac{c - \log(1+c)}{1+c} \frac{GM_{\rm vir}}{r_{\rm vir}}$$

$$\langle (\vec{\gamma} \cdot \vec{v}_j)^2 \rangle = \sigma^2_{\rm v,off}(M_{\rm vir}) = \frac{GM_{\rm vir}}{2R_{\rm vir}}$$

$$\langle \delta z(M) \rangle = \frac{GM_{\rm vir}}{r_{\rm vir}} \left(-\frac{c}{m(c)} + \frac{c}{m^2(c)} \frac{(c - \log(1+c))}{1+c} + \frac{5}{4} \right)$$

Integration over halo mass distribution function multiplied by $\langle N_{sat} \rangle$

$$\langle \delta z \rangle = \frac{\int dM \frac{dn}{dM} \langle N_{\rm sat} \rangle \langle \delta z(M) \rangle}{\int dM \frac{dn}{dM} \langle N_{\rm sat} \rangle}$$

Result （general relativity）

	LRG (SDSS II)	LOWZ	CMASS
$\langle \delta z \rangle$	$-1.5 \times 10^{-5} (-4.6\mathrm{km/s})$	$-1.2 \times 10^{-5} (-3.5\mathrm{km/s})$	$-0.8 \times 10^{-5} (-2.5\mathrm{km/s})$
$\psi(0) - \langle \psi_s \rangle$	$-3.2 \times 10^{-5} (-9.7\mathrm{km/s})$	$-2.3 \times 10^{-5} (-7.0\mathrm{km/s})$	$-1.8 \times 10^{-5} (-5.4\mathrm{km/s})$
$\frac{5}{2}\langle (\vec{\gamma} \cdot \vec{v})^2 \rangle$	$+1.7 \times 10^{-5} (+5.1\mathrm{km/s})$	$+1.2 \times 10^{-5} (+3.5\mathrm{km/s})$	$+1.0 \times 10^{-5} (+3.0\mathrm{km/s})$
$\bar{r}_{\mathrm{vir}}$	$1.0\ h^{-1}\mathrm{Mpc}$	$0.85\ h^{-1}\mathrm{Mpc}$	$0.79\ h^{-1}\mathrm{Mpc}$

- ✓ Amplitude of signal is δz = 5～2 km/s, depending on the HOD
- ✓ Significant contribution from the 2nd order Doppler term

Consistent with the previous results Zhao et al (13) Jimeno et a l(15)

- ✓ Modified gravity test ? $v^2 \propto G_{\mathrm{eff}}$

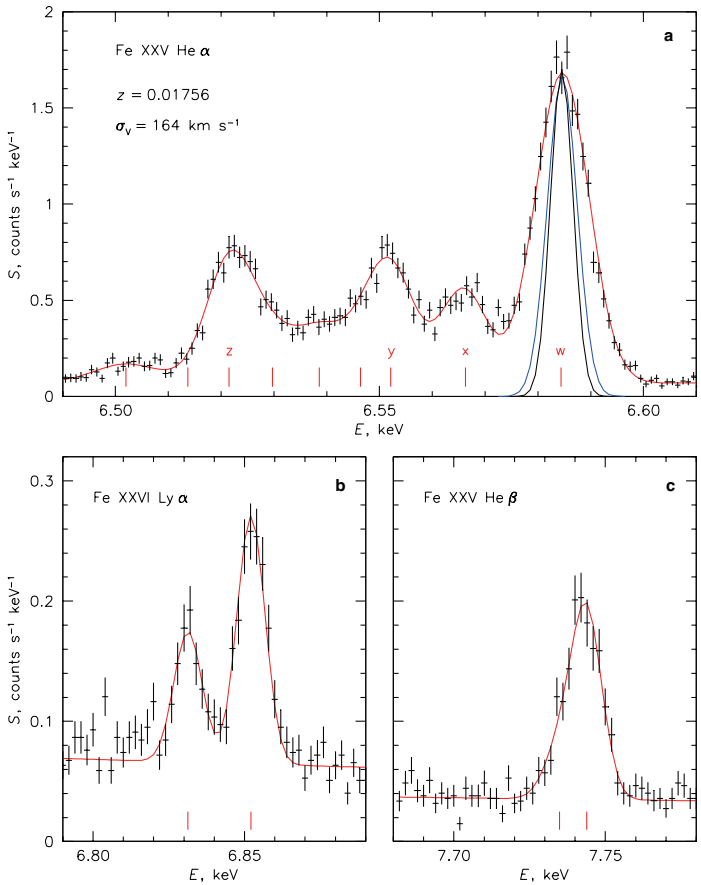
$$G_{\mathrm{eff}} = \frac{4}{3} G$$

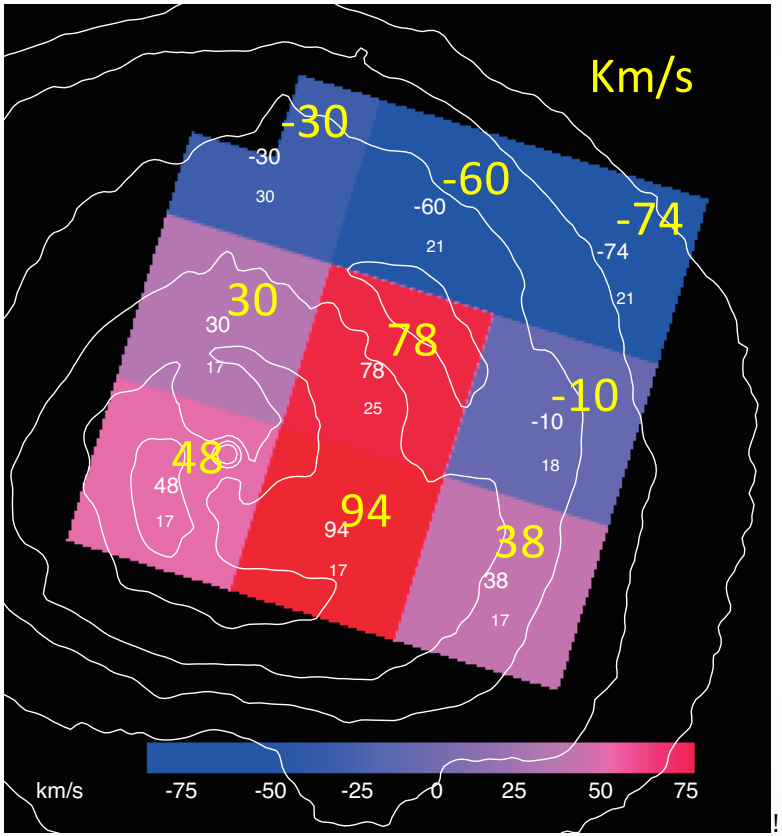
	LRG (SDSS II)	LOWZ	CMASS
$\langle \delta z \rangle$	$-0.9 \times 10^{-5} (-2.9\mathrm{km/s})$	$-0.7 \times 10^{-5} (-2.3\mathrm{km/s})$	$-0.5 \times 10^{-5} (-1.4\mathrm{km/s}$
$\psi(0) - \langle \psi_s \rangle$	$-3.2 \times 10^{-5} (-9.7\mathrm{km/s})$	$-2.3 \times 10^{-5} (-7.0\mathrm{km/s})$	$-1.8 \times 10^{-5} (-5.4\mathrm{km/s})$
$\frac{5}{2}\langle (\vec{\gamma} \cdot \vec{v})^2 \rangle$	$+2.3 \times 10^{-5} (+6.8\mathrm{km/s})$	$+1.6 \times 10^{-5} (+4.7\mathrm{km/s})$	$+1.3 \times 10^{-5} (+4.0\mathrm{km/s})$
$\bar{r}_{\mathrm{vir}}$	$1.0\ h^{-1}\mathrm{Mpc}$	$0.85\ h^{-1}\mathrm{Mpc}$	$0.79\ h^{-1}\mathrm{Mpc}$

potentially being an interesting test of modified gravity

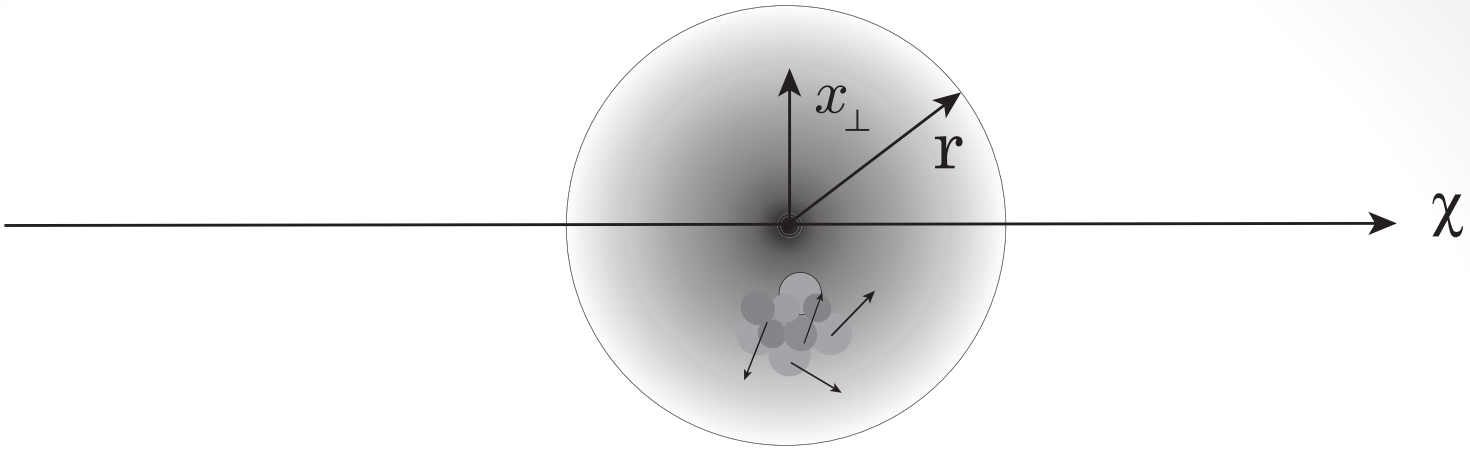
4 Intracluster gas

Perseus cluster gas motions observed by Hitomi X-ray satellite





The observation of the lines profile, the gas motions could be investigated with the accuracy of the order, 10～20 km/s.



Non-thermal pressure (unsolved problem in cluster physics)

Cosmological hydrodynamical numerical simulations

✓ Intracluster gas motions can be generated in the structure formation process, non-thermal random motions (bulk motion or turbulence) contributes to the non-thermal pressure. ～10 % ～ 30 % of the total pressure

This might cause discrepancy between

✓ Hydroequilibrium mass from X-ray, SZ effect v.s. Lensing mass

✓ Small scale random motions of gas → non-thermal pressure

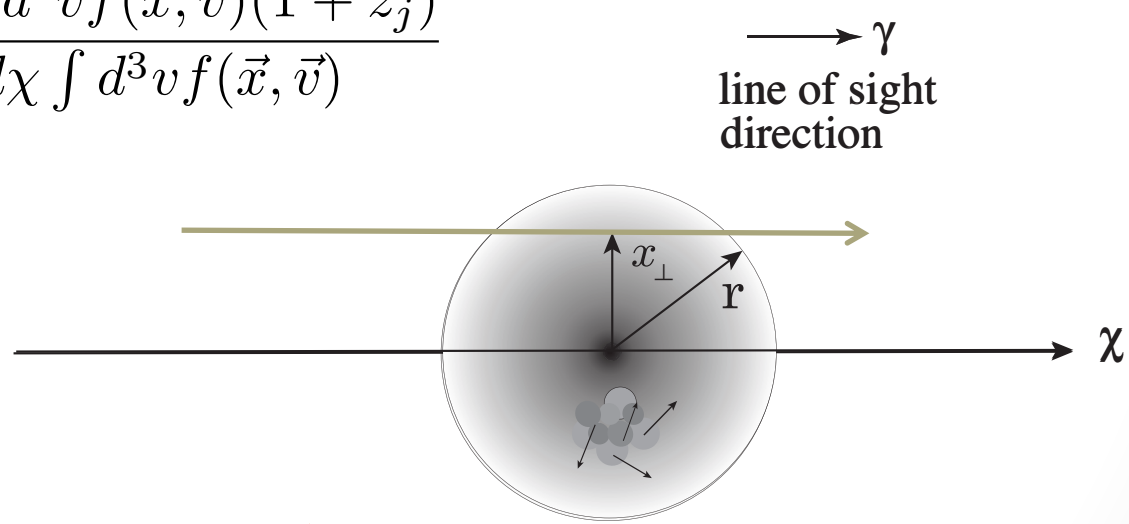
Formulation – reference frame distribution function of gas

$$f_{\rm RF}(\vec{x},\vec{v}) = n(\vec{x})\left(\frac{2\pi}{mT(\vec{x})}\right)^{3/2}\exp\left\{-m\frac{[\vec{v}-\vec{V}(\vec{x})]^2}{2T(\vec{x})}\right\}$$

Density Temperature Peculiar velocity (random)

$$1+z_j \simeq 1+z_1+(1+z_1)\left\{-\psi(\eta_j,\vec{x}(\eta_j))+\vec{\gamma}\cdot\vec{v}_j+\frac{1}{2}\vec{v}_j^2\right\}$$

$$1+\langle z(x_\perp)\rangle = \frac{\int d\chi \int d^3v f(\vec{x},\vec{v})(1+z_j)}{\int d\chi \int d^3v f(\vec{x},\vec{v})}$$



$$1+\langle z(x_\perp)\rangle$$

$$= 1+z_1+(1+z_1)\frac{\int d\chi\, n(\vec{x})\left\{\cancel{\vec{\gamma}\cdot\vec{V}(\vec{x})}+(\vec{\gamma}\cdot\vec{V}(\vec{x}))^2+\frac{1}{2}|\vec{V}(\vec{x})|^2+\frac{5}{2}\frac{T(\vec{x})}{m}-\psi(\vec{x})\right\}}{\int d\chi\, n(\vec{x})(1+\cancel{\vec{\gamma}\cdot\vec{V}})}$$

Statistically spherical symmetry $\int d\chi n(\vec{x})\gamma\cdot\vec{V}=0$

Isotropy of the random motion of gas statistically

$$3\langle(\vec{\gamma}\cdot\vec{V})^2\rangle = \langle|\vec{V}|^2\rangle = \vec{\sigma}^2_{\rm rnd}$$

Variance of random velocity of gas,

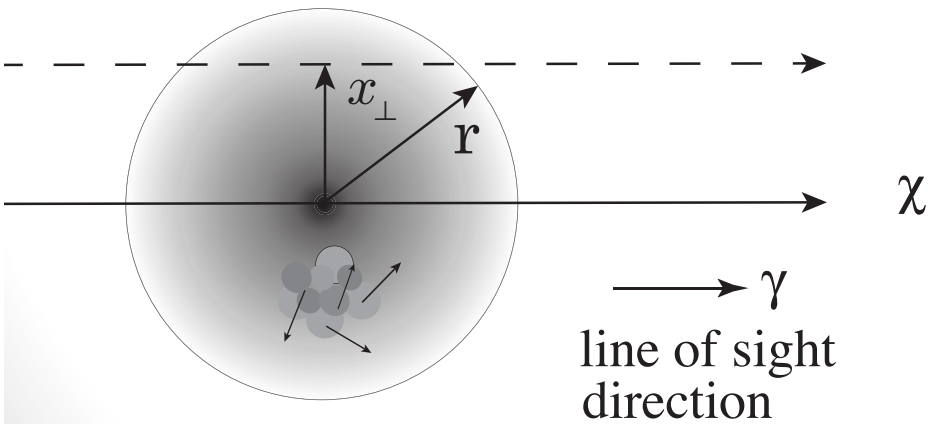
$$n \propto \rho_{\rm gas}$$

$$1 + \langle z(x_\perp)\rangle = 1 + z_1 + (1+z_1)\frac{\int d\chi \rho_{\rm gas}(\vec{x})\left(\frac{5}{6}\vec{\sigma}^2_{\rm rnd}(\vec{x}) + \frac{5}{2}\frac{T(\vec{x})}{m} - \psi(\vec{x})\right)}{\int d\chi \rho_{\rm gas}(\vec{x})}$$

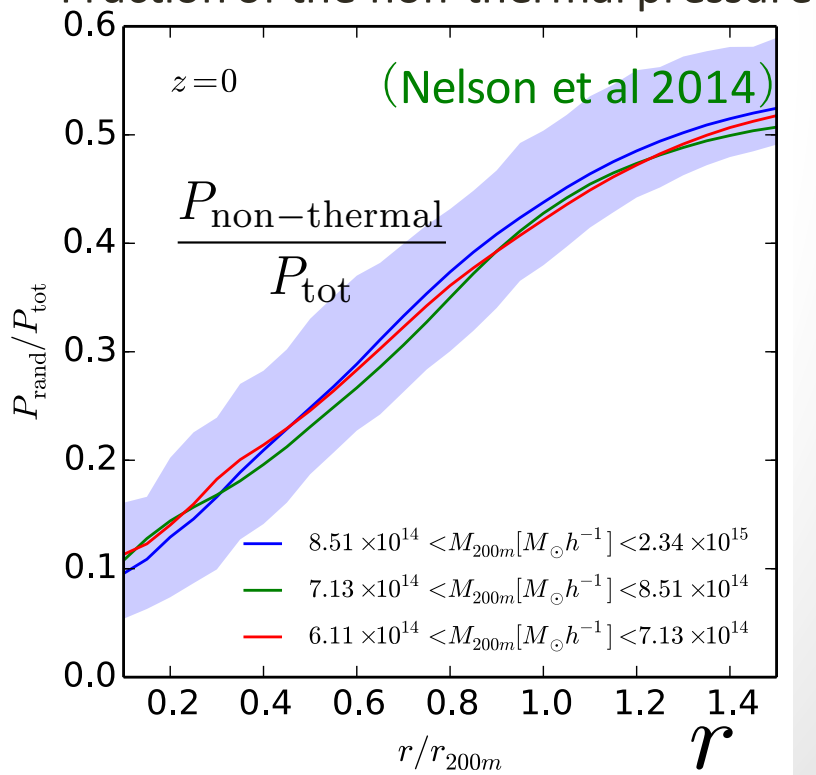
random motion of gas ⇄ non-thermal pressure

Shi et al (2014)

$$\rho_{\rm gas}\vec{\sigma}^2_{\rm rnd} = P_{\rm non-thermal}$$

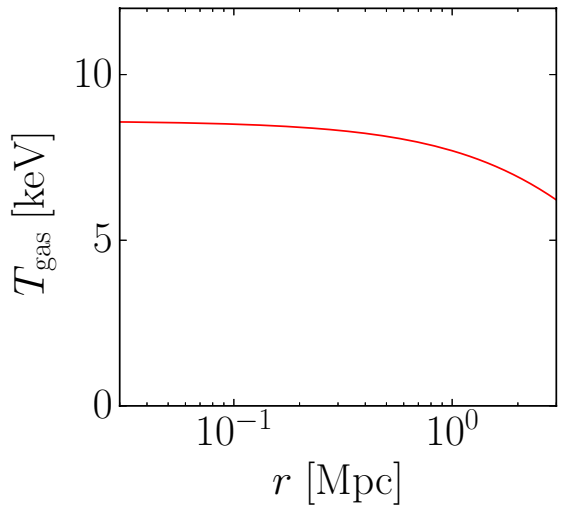


Fraction of the non-thermal pressure

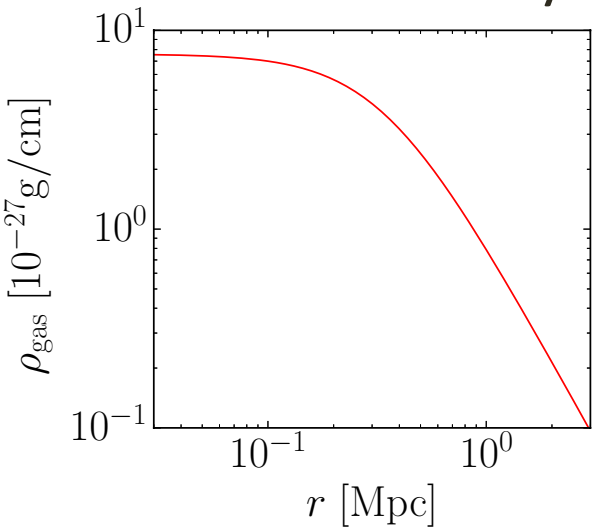


Coma cluster

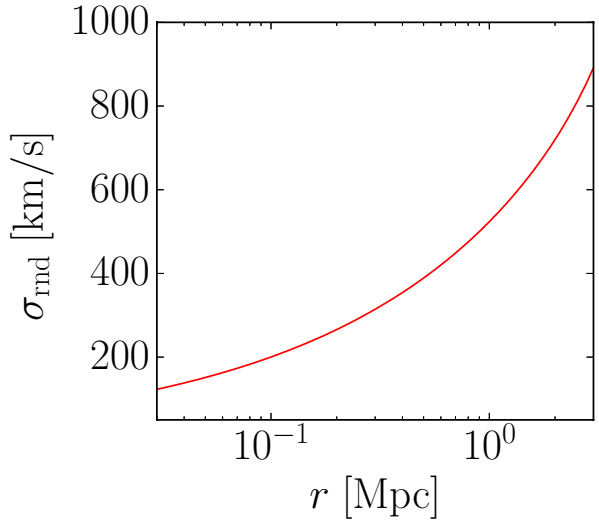
temperature

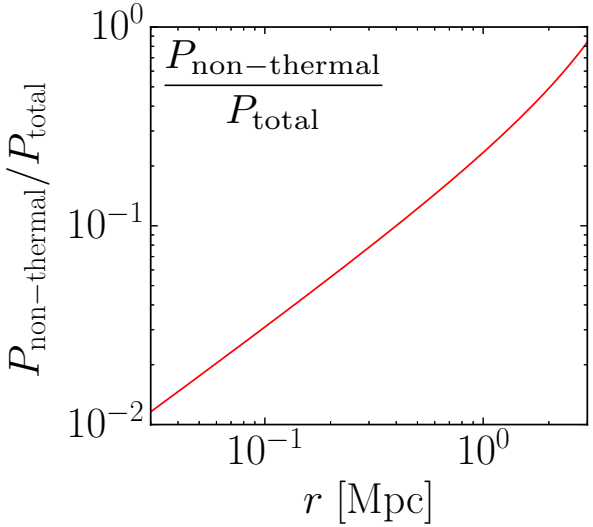


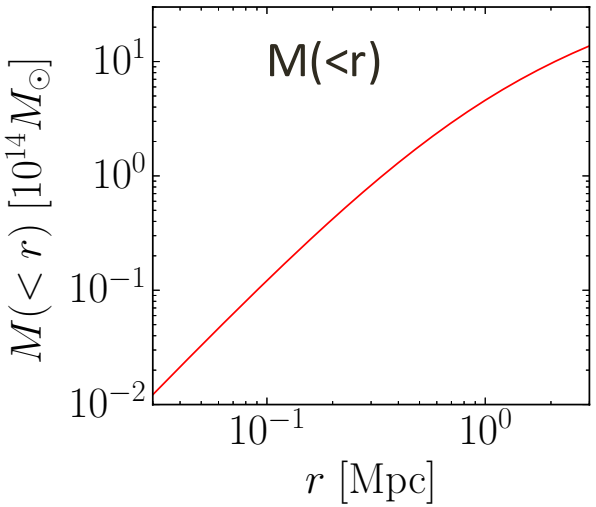
mass density

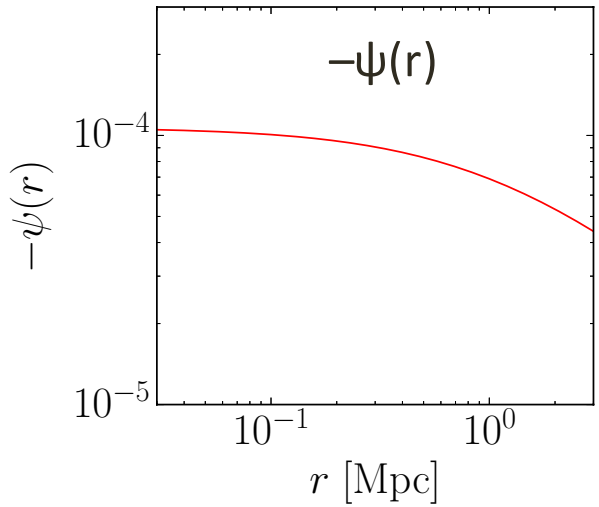


$\sigma^2_{\rm rnd}$

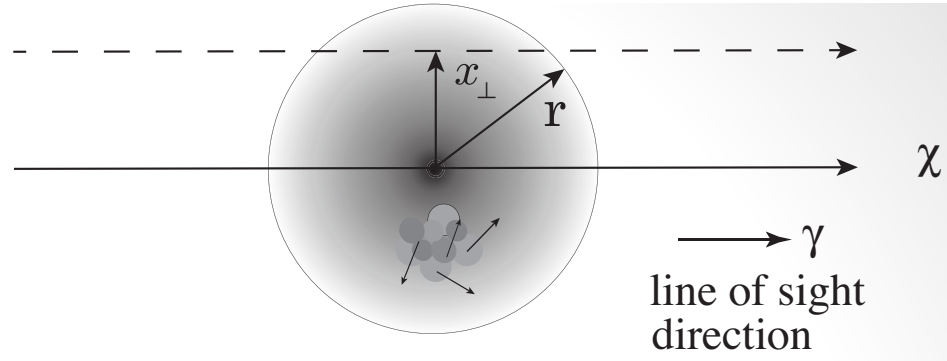




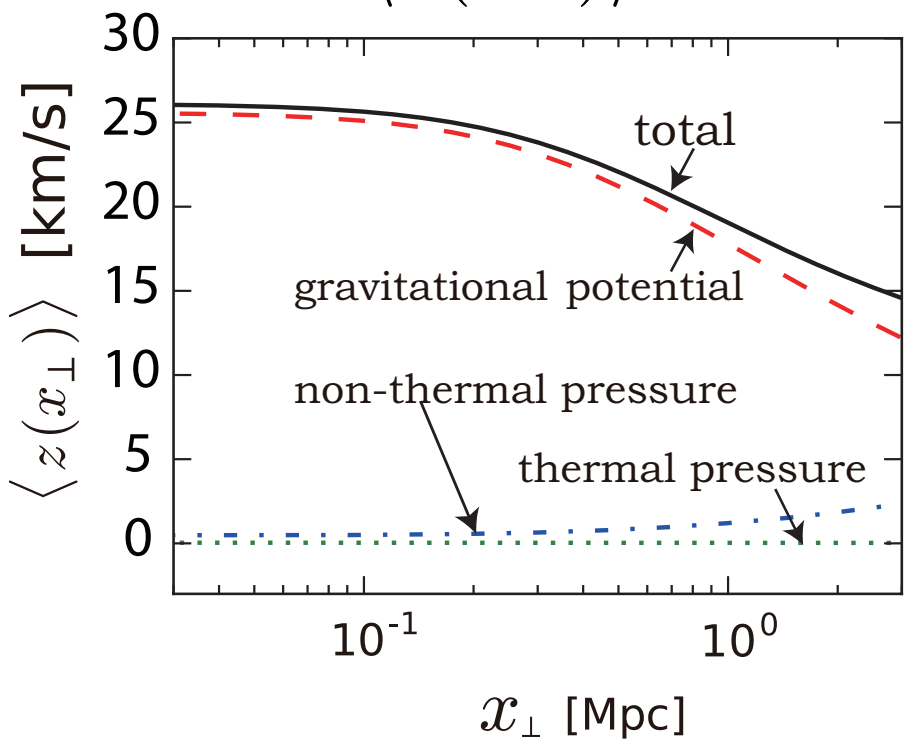




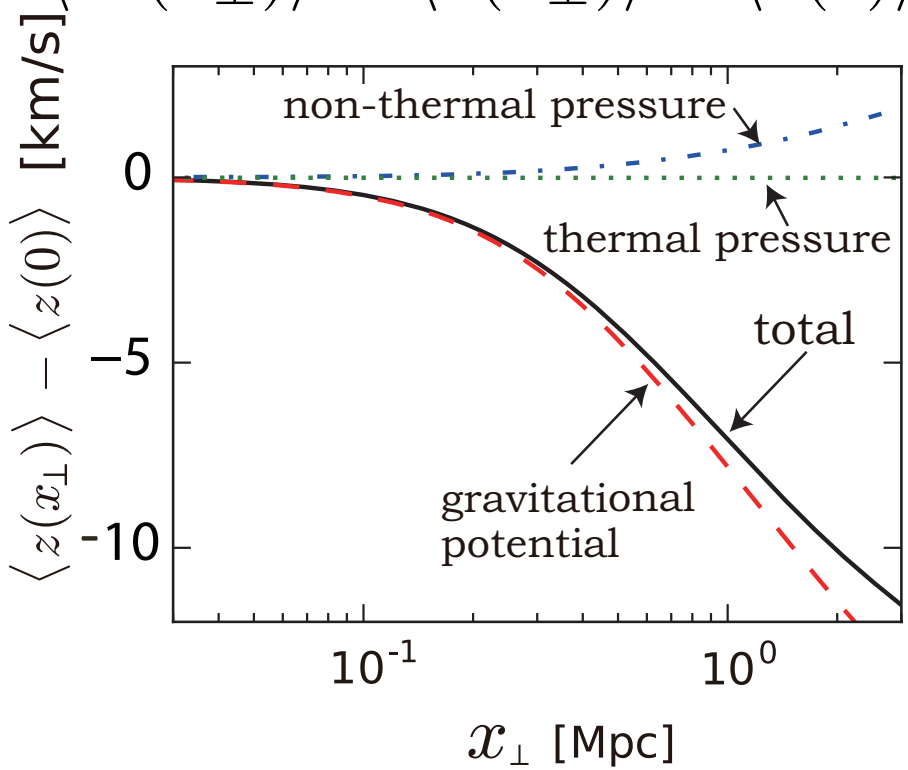
Coma cluster



$$\langle z(x_\perp)\rangle$$



$$\langle \delta z(x_\perp)\rangle = \langle z(x_\perp)\rangle - \langle z(0)\rangle$$



Amplitude of the signal is 5 ~ 10 km/s

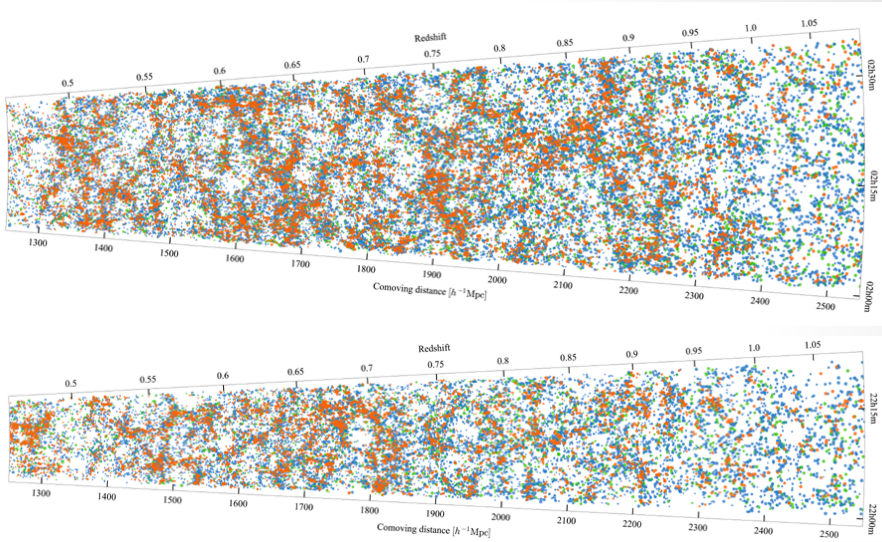
Gravitational redshift of intracluster gas

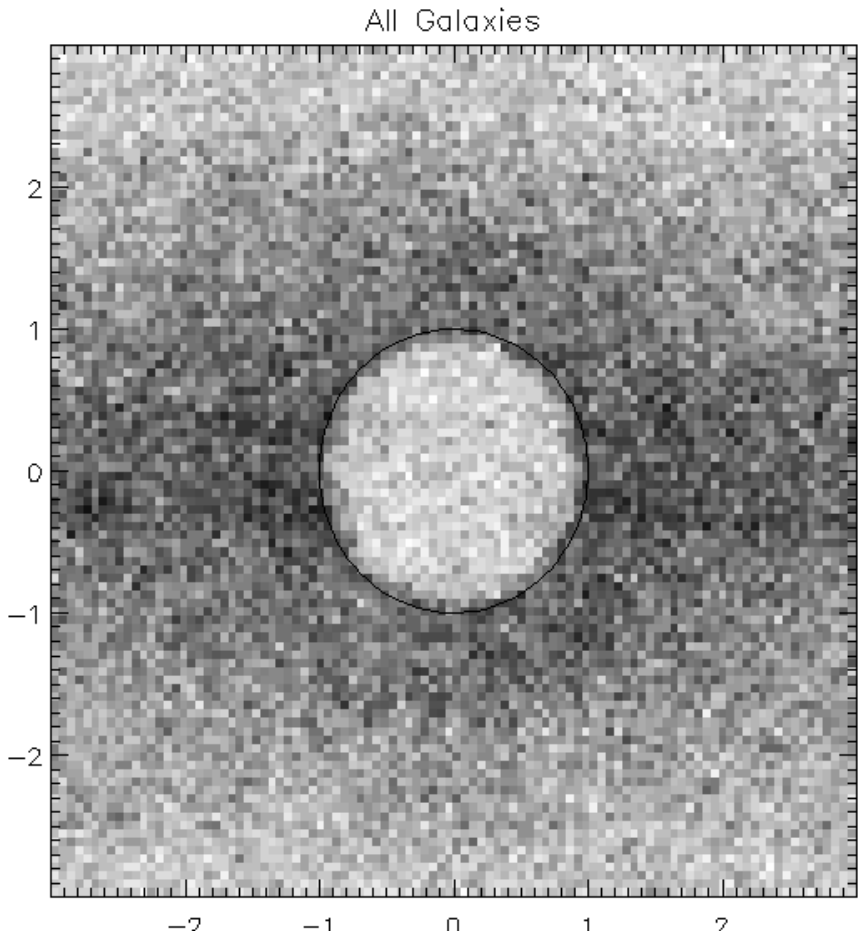
- ✓ The relative amplitude of signal is 5 ～ 10 km/s
- ✓ Gravitational potential tem makes a dominant contribution to the gravitational redshift.
- ✓ Random motion of gas of non thermal pressure makes a slight contribution to the gravitational redshift.
- ✓ Measurements of lines of out skirt region is necessary.

5 .Void

Hawken et al. (2016)

Stacked voids in VIPERS (VIMOS Public Extragalactic Redshift Survey)





34600 galaxies

0.55< z< 0.9

$1.6 \times 10^7 (h^{-1}\mathrm{Mpc})^3$

822 voids

galaxies inside void

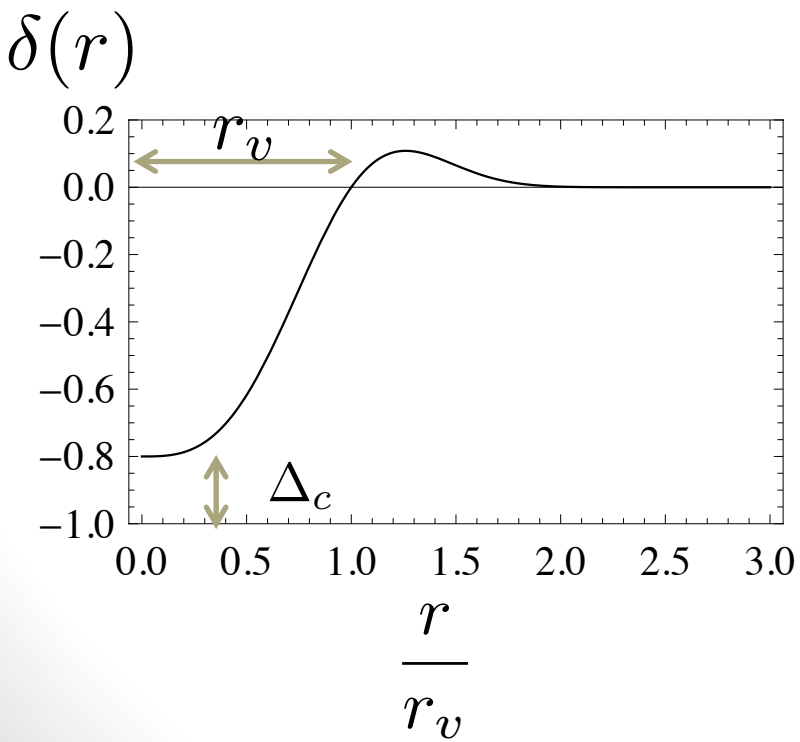
→ possible signal of gravitational redshift ?

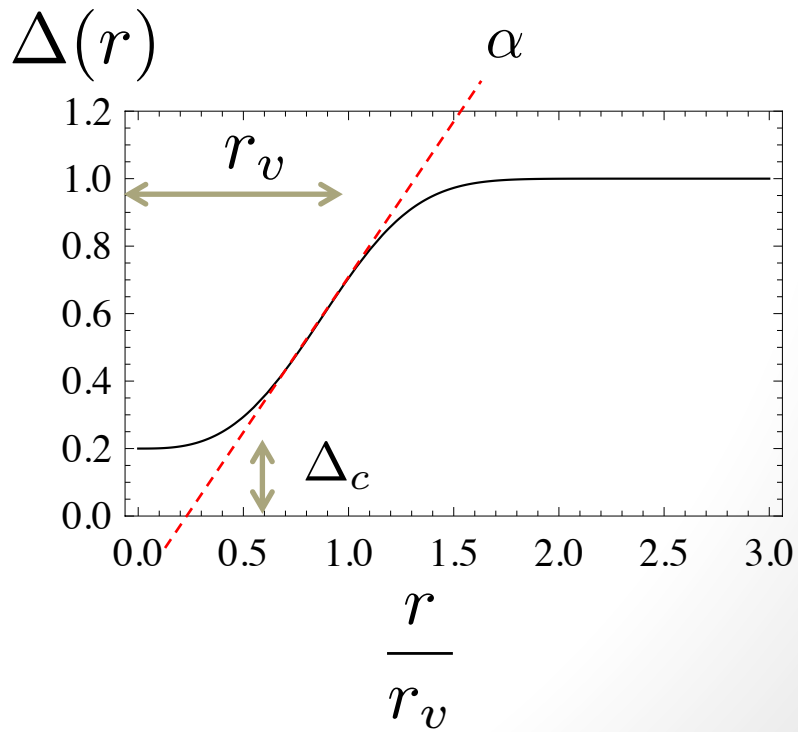
Simple void profile

Hawken et al 2016

$$\Delta(r) = \frac{M(<r)}{4\pi r^3/3} = \frac{3}{r^3}\int_0^r dr' r'^2 \delta(r') = \Delta_c e^{-(r/r_v)^{\alpha}}$$

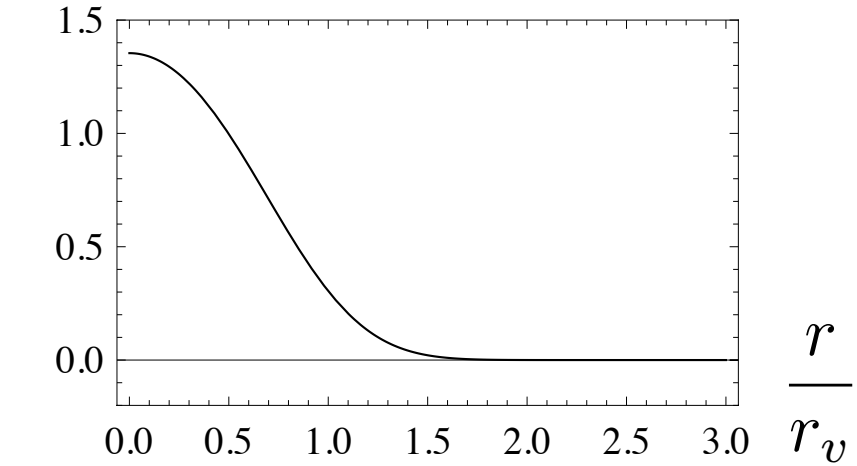
$$\delta(r) = \frac{1}{r^2}\frac{d}{dr}\left(\frac{r^3\Delta(r)}{3}\right) = \Delta_c\left(1 - \frac{\alpha}{3}\left(\frac{r}{r_v}\right)^{\alpha}\right)e^{-(r/r_v)^{\alpha}}$$





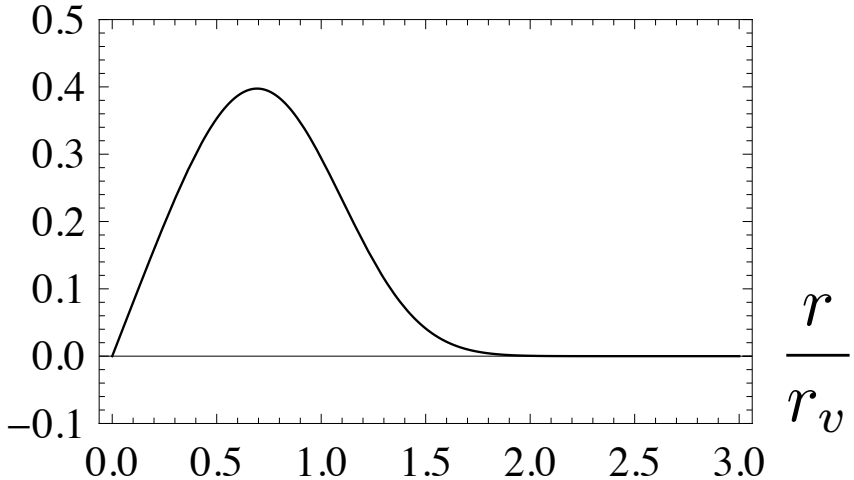
Gravitational potential

$$\psi(r) = -\frac{3\Omega_m}{2a} H_0^2 \int_r^\infty dr' r' \frac{\Delta(r')}{3} = -\frac{H_0^2 r_v^2}{2} \frac{\Omega_m \Delta_c}{\alpha a} \Gamma(2/\alpha, (r/r_v)^\alpha)$$



Radial velocity

$$V(r) = -\mathcal{H} r \Delta(r) \frac{f(a)}{3}$$



Gravitational redshift of galaxy of void

$$1+\langle z(x_\perp)\rangle = \frac{\int d\chi \int d^3v_j (1+z_j) f(\vec{x},\vec{v}_j)}{\int d\chi \int d^3v_j f(\vec{x},\vec{v}_j)}$$

$$= 1 + z_1 + (1+z_1)\frac{\int d\chi n_{\rm g}(\chi, x_\perp)\widetilde{\delta z}(\chi, x_\perp)}{\int d\chi n_{\rm g}(\chi, x_\perp)}$$

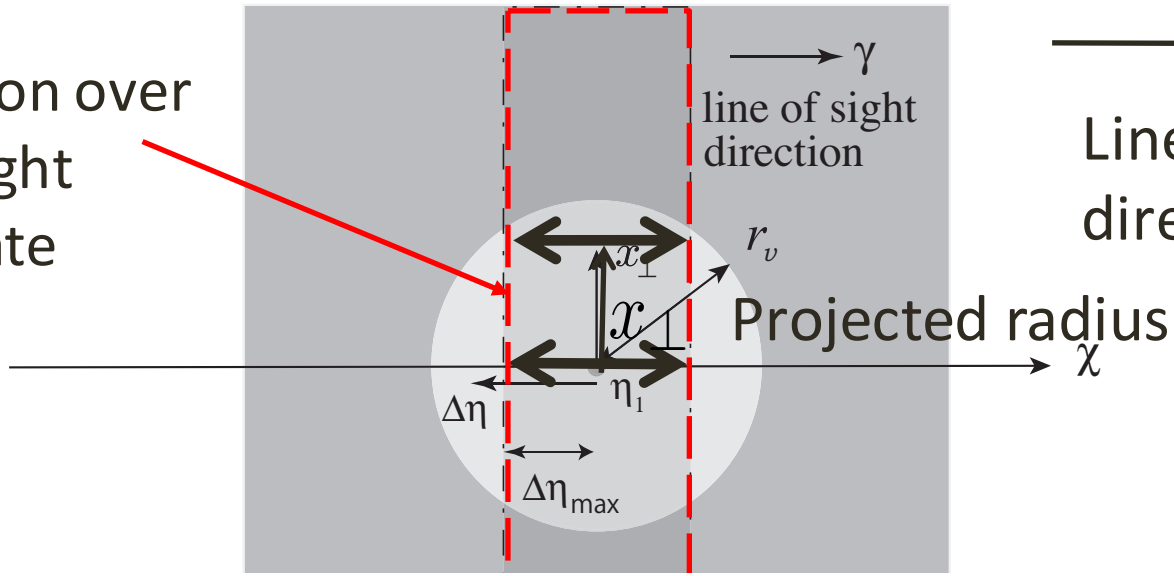
$$\widetilde{\delta z}(\chi, x_\perp) = \left\{ \left(\mathcal{H}^2(\eta_1) - \frac{1}{2}\frac{a''(\eta_1)}{a(\eta_1)} \right) \Delta\eta_j^2 - \psi(\eta_j, \vec{x}(\eta_j)) + (\vec{\gamma}\cdot\vec{V})^2 + \frac{1}{2}|\vec{V}|^2 \right\}$$

2nd order Hubble term

Gravitational potential

Doppler term

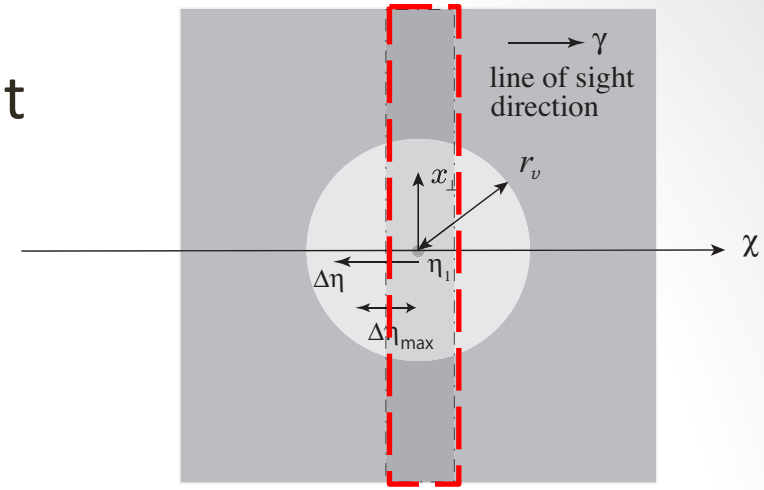
Integration over line of sight coordinate



Line of sight direction

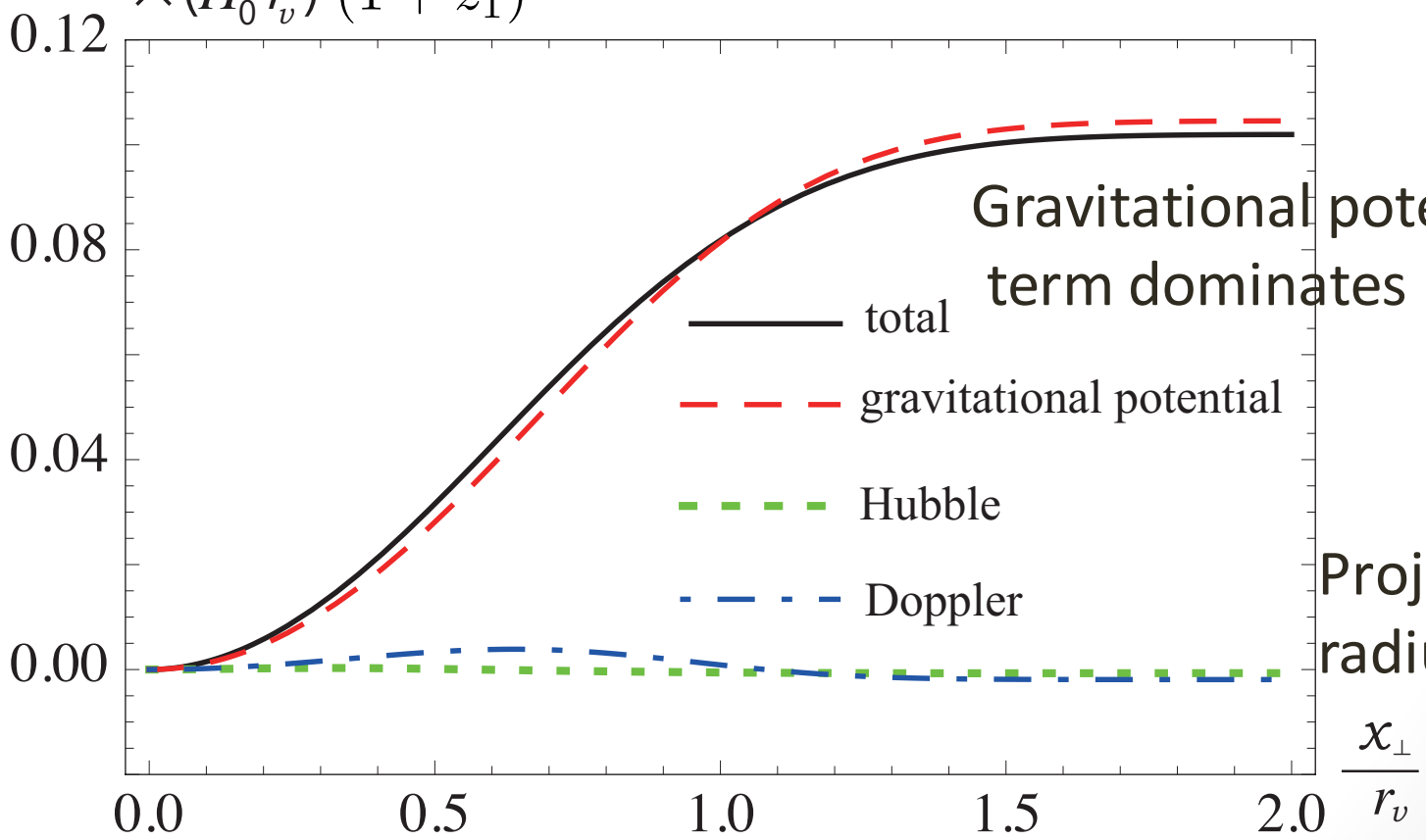
$\langle \delta z(x_\perp)\rangle = \langle z(x_\perp)\rangle - \langle z(0)\rangle$: Relative redshift

Line of sight direction



$$\langle \delta z(x_\perp) \rangle = \langle z(x_\perp) \rangle - \langle z(0) \rangle$$

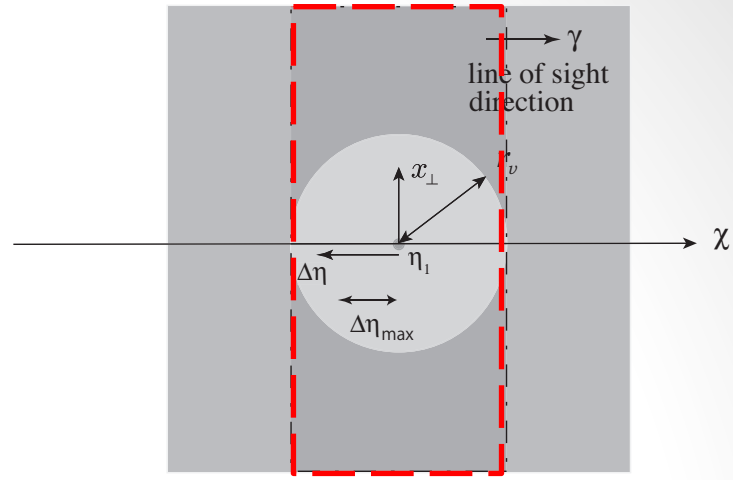
$$\times (H_0 r_v)^2 (1 + z_1)$$



Gravitational potential term dominates result

Projected radius

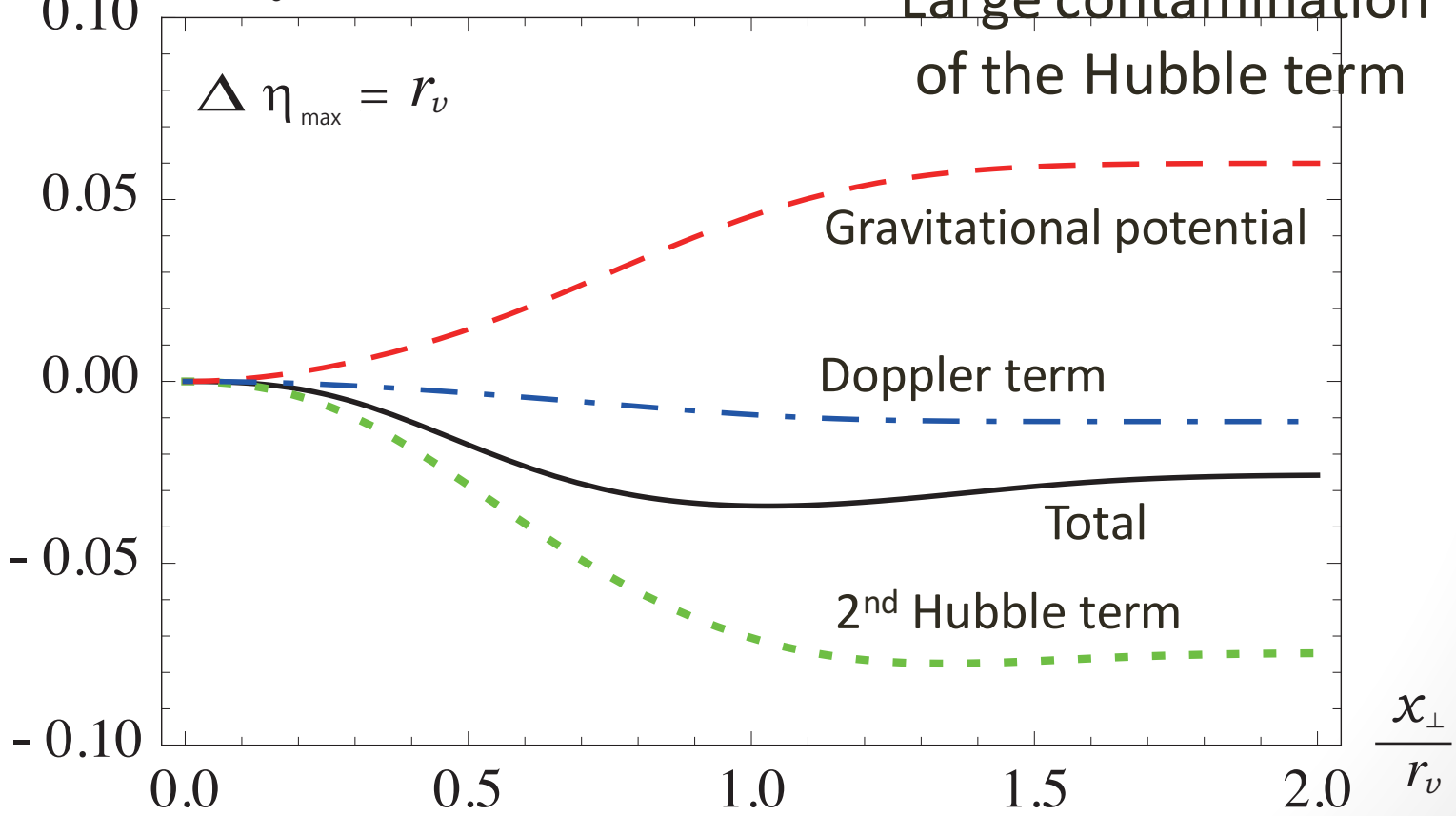
Line of sight direction



$$\langle \delta z(x_\perp) \rangle = \langle z(x_\perp) \rangle - \langle z(0) \rangle$$

$$\times (H_0 r_v)^2 (1 + z_1)$$

Large contamination of the Hubble term



Gravitational redshift of galaxies of voids

✓ The amplitude of signal is

$$\langle z(x_\perp)\rangle - \langle z(0)\rangle \sim -\mathcal{O}(10^{-6}) \times \left(\frac{r_v}{10h^{-1}\mathrm{Mpc}}\right)^2 (1+z_1)$$

✓ The 2nd order Hubble term makes a large contribution, when the range of projection over the line of sight coordinate is wide.

✓ Feasibility of the detection should be checked more carefully.

6. Conclusions

gravitational redshift, Simple theoretical model

✓ Satellite galaxy virialized in a halo with the HOD description

Amplitude of signal depends on the HOD,
Doppler term makes significant contribution, test of gravity

✓ Ingracluster gas

Gravitational potential tem makes a dominant contribution to the gravitational redshift, non-thermal pressure may make a contribution slightly.

✓ Void Possible signal of the gravitational redshift

$$\langle z(x_\perp)\rangle - \langle z(0)\rangle \sim -\mathcal{O}(10^{-6}) \times \left(\frac{r_v}{10h^{-1}\mathrm{Mpc}}\right)^2 (1+z_1)$$

The 2nd order Hubble term can make a large contribution,
when the range of projection over the line of sight direction is wide.

Thank you !