EXPLOSIVE NUCLEOSYNTHESIS IN GRB JETS ACCOMPANIED BY HYPERNOVAE
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ABSTRACT

Two-dimensional hydrodynamic simulations are performed to investigate explosive nucleosynthesis in a collapsar using the model of MacFadyen and Woosley. It is shown that $^{56}$Ni is not produced in the jet of the collapsar sufficiently to explain the observed amount in a hypernova when the duration of the explosion is $\sim 10$ s. Even though a considerable amount of $^{56}$Ni is synthesized if all the explosion energy is deposited initially, the opening angles of the jets become too wide to realize highly relativistic outflows. From these results, it is concluded that the origin of $^{56}$Ni in hypernovae associated with GRBs is not the explosive nucleosynthesis in the jet. We consider that the idea that the origin is the explosive nucleosynthesis in the accretion disk is more promising. We also show that the explosion becomes bipolar naturally because of the deformed progenitor. This fact suggests that the $^{56}$Ni is synthesized in the accretion disk and conveyed as outflows blown along the rotation axis, which will explain the line features of SN 1998bw and the double-peaked line features of SN 2003jd. Some fraction of the gamma-ray lines from $^{56}$Ni decay in the jet will appear without losing their energies as long as the jet is a relativistic flow, which may be observed as relativistically Lorentz-boosted line profiles in the future. We show that the abundance of nuclei whose mass number $\sim 40$ in the ejecta depends sensitively on the energy deposition rate. So it may be determined by observations of chemical composition in metal-poor stars which model is the proper one.

Subject headings: accretion, accretion disks — black hole physics — Galaxy: halo — gamma rays: bursts — nuclear reactions, nucleosynthesis, abundances — supernovae: general

1. INTRODUCTION

There has been growing evidence linking long gamma-ray bursts (GRBs; in this study, we consider only long GRBs, so we refer to long GRBs as GRBs hereafter for simplicity) to the death of massive stars. The host galaxies of GRBs are star-forming galaxies, and the positions of GRBs appear to trace the blue light of young stars (Vreeswijk et al. 2001; Bloom et al. 2002; Gorosabel et al. 2003). Also, “bumps” observed in some afterglows can be naturally explained as the contribution of bright supernovae (Bloom et al. 1999; Reichart 1999; Galama et al. 2000; Garnavich et al. 2003). Moreover, direct evidence of some GRBs accompanied by supernovae have been reported, such as the association of GRB 980425 with SN 1998bw (Galama et al. 1998; Iwamoto et al. 1998) and that of GRB 030329 with SN 2003jd (Hjorth et al. 2003; Price et al. 2003; Stanek et al. 2003).

It should be noted that these supernovae are categorized as a new type of supernova with large kinetic energy ($\sim 10^{52}$ ergs), nickel mass ($\sim 0.5 M_\odot$), and luminosity (Iwamoto et al. 1998; Woosley et al. 1999), so these supernovae are sometimes called hypernovae. Also, since GRBs are considered to be jetlike phenomena (Rhoads 1999; Stanek et al. 1999), it is natural to consider the accompanying supernova to be a jet-induced explosion (MacFadyen & Woosley 1999; Khokhlov et al. 1999). It is two radioactive nuclei, $^{56}$Ni and its daughter nucleus, $^{56}$Co, that brighten the supernova remnant and determine its bolometric luminosity. $^{56}$Ni is considered to be synthesized through explosive nucleosynthesis because its half-life is very short (5.9 days). Thus, it is natural to investigate explosive nucleosynthesis in jet-induced explosions to understand GRBs accompanied by hypernovae.

Nagataki et al. (1997) have done a numerical calculation of explosive nucleosynthesis taking account of effects of jet-induced explosion in the context of a normal core-collapse supernova explosion whose explosion energy is set to be $10^{51}$ ergs. They found that much $^{56}$Ni is produced in the jet region, which means that much explosive nucleosynthesis occurs around the jet region. Also, it was found that the velocity distribution of iron becomes double-peaked due to the aspherical explosion and explosive nucleosynthesis (Nagataki et al. 1998b; Nagataki 2000). They also showed that the explosion energy is set to be $10^{52}$ ergs. They also calculated profile, depends on the angle between our line of sight and the rotation axis (Nagataki 2000). Maeda et al. (2002) did a numerical calculation of explosive nucleosynthesis taking into account the effect of jet-induced explosion in the context of hypernovae whose explosion energy is set to be $10^{52}$ ergs. They showed that a mass of $^{56}$Ni sufficient to explain the observation of hypernova ($\sim 0.5 M_\odot$) can be synthesized around the jet region when the explosion energy is set to be $10^{52}$ ergs. They also calculated line profiles of [Fe ii] blend and of [O i] from one-dimensional non-LTE nebular code (Mazzali et al. 2001) and found that these line profiles depend on the angle between our line of sight and the direction of the jet. Recently, it was reported that an asymmetric hypernovae, SN 2003jd, revealed double-peaked profiles in the nebular lines of neutral oxygen and magnesium (Mazzali et al. 2005).

However, there is one question: whether the difference between explosion energies of $10^{51}$ and $10^{52}$ ergs is important or not. This is, what does the different explosion energy mean? The answer is yes: it is impossible to overemphasize its importance because the explosion scenario has to be dramatically changed to explain the energetic explosion of $10^{52}$ ergs and to realize a GRB.
Nagataki et al. (1997) investigated explosive nucleosynthesis accounting for the effects of jet-induced explosion in the context of normal core-collapse supernova explosion, because there is a possibility that a normal core-collapse supernova becomes jet-like when rotation effects are taken into account (Yamada & Sato 1994; Shimizu et al. 1994; Kotake et al. 2003). In this scenario, the typical timescale of core-collapse supernovae is as short as ~500 ms (Wilson 1985), so the surrounding layers of the iron core do not collapse as much. As a result, the progenitor outside the iron core cannot be deformed due to the collapse. This fact supports the treatment of using a spherical progenitor when explosive nucleosynthesis is investigated. Note that the central iron core collapses to a neutron star, and it is enough to calculate explosive nucleosynthesis in a spherical outer layer such as Si-, O-, and He-rich layers in the case of normal core-collapse supernovae (Nagataki et al. 1997, 1998a, 1998b; Nagataki 2000). Also, to initiate the explosion, the explosion energy is deposited around the Si-rich layer as an initial condition in their works. This is justified because the timescale of explosion is very short.

On the other hand, the central engine of GRBs accompanied by hypernovae is not well known. But it is generally considered that normal core-collapse supernovae cannot cause an energetic explosion of the order of 10^{52} ergs. So another scenario has to be considered to explain the system of GRBs associated with hypernovae. One of the most promising is the collapsar scenario (Woosley 1993). In the collapsar scenario, a black hole is formed as a result of gravitational collapse. Also, rotation of the progenitor plays an essential role. Due to the rotation, an accretion disk is formed around the equatorial plane. On the other hand, the matter around the rotation axis falls into the black hole. It was pointed out that the jet-induced explosion along the rotation axis occurs due to the heating through neutrino-antineutrino pair annihilation emitted from the accretion disk. MacFadyen & Woosley (1999) demonstrated the numerical simulations of the collapsar, showing that the jet is launched ~7 s after the gravitational collapse and the duration of the jet is about 10 s, which is comparable to the typical observed duration of GRBs (Mazet et al. 1981; Kouveliotou et al. 1993; Lamb et al. 1993). This timescale is much longer than the typical timescale of normal core-collapse supernovae. As a result, the progenitor becomes deformed even at the Si-rich and O-rich layer in the collapsar model (MacFadyen & Woosley 1999). In particular, the density around the rotation axis becomes low because a considerable amount of the matter falls into the black hole, which is a good environment for producing a fireball (Woltjer 1966; Rees 1967).

Maeda et al. (2002) investigated explosive nucleosynthesis, taking account of effects of jet-induced explosion in the context of hypernovae whose energy is 10^{52} ergs using the spherical progenitor model and depositing explosion energy at the innermost region initially. However, this treatment seems to be incompatible with the collapsar scenario. The importance of the duration of explosion, ~10 s, is investigated in some papers (Nagataki et al. 2003; Maeda & Nomoto 2003), and it was concluded that the abundance of $^{56}$Ni synthesized during the explosion depends sensitively on the duration of the explosion (i.e., energy deposition rate) and $^{56}$Ni is not produced sufficiently to explain the observed amount, ~0.5 $M_\odot$ when the timescale of explosion becomes as long as 10 s. In fact, MacFadyen & Woosley (1999) discussed that not enough $^{56}$Ni is synthesized in the jet in the collapsar model. Rather, they pointed out the possibility that a substantial amount of $^{56}$Ni is produced in the accretion disk and a part of it is conveyed outward by the viscosity-driven wind (MacFadyen & Woosley 1999; Pruet et al. 2003). There is another question. Does all of the $^{56}$Ni produced in the jet of the collapsar model brighten the supernova remnant? If the jet becomes optically thin before $^{56}$Ni decays into $^{56}$Co and $^{56}$Co decays into $^{56}$Fe, these nuclei should result in emitting gamma rays rather than brightening the supernova remnant.

Let us summarize our motivation. We want to understand how collapsars produce a GRB jet and how collapsars eject enough $^{56}$Ni to explain the luminosity of hypernovae. We seek a self-consistent theory of the GRB-hypernova connection. As a first step, we consider in this work the consistency between the collapsar model of MacFadyen & Woosley (1999) and explosive nucleosynthesis in a hypernova jet. Can a hypernova jet cause a GRB jet and a sufficiently explosive nucleosynthesis to explain the luminosity of hypernova? Or should we instead consider the GRB jet to be different from the hypernova jet? Moreover, should we consider $^{56}$Ni to come from the explosive nucleosynthesis in the hypernova jet? Or should we instead consider $^{56}$Ni to come from a different site? Finding these answers is our motivation for this work.

Due to the motivation mentioned above, we investigate explosive nucleosynthesis in the context of the collapsar model. We use the collapsar model of MacFadyen & Woosley (1999), in which effects of rotation are included. As a result, the progenitor becomes deformed significantly, as mentioned above. We show that $^{56}$Ni is not produced sufficiently to explain the observed amount when the duration of the explosion is ~10 s, which is consistent with the previous works (Nagataki et al. 2003; Maeda & Nomoto 2003). A fine tuning is required to explain the amount of $^{56}$Ni by the explosive nucleosynthesis in the jet. This result brings us to the conclusion that the origin of $^{56}$Ni in hypernovae associated with GRBs is not the explosive nucleosynthesis in the jet but the one in the accretion disk. We also show that the explosion becomes bipolar naturally due to the effect of deformed progenitor. This fact suggests that the $^{56}$Ni synthesized in the accretion disk and conveyed as outflows could be blown along the rotation axis, which can explain the line features of SN 1998bw and double-peaked line features of SN 2003jd (Mazzali et al. 2005). Also we predict that some fraction of gamma-ray lines from $^{56}$Ni decays in the jet may show relativistically Lorentz-boosted line profiles, which might be observed in the future.

2. METHOD OF CALCULATION

We present our method of calculation in this study. We account for some effects that had not been included in Nagataki et al. (2003). In this study, the effects of gravitation and rotation are included. We also adopt the realistic equation of state (EOS) of Blinnikov et al. (1996). Furthermore, we adopt an asymmetric progenitor model obtained by MacFadyen & Woosley (1999). Thus, we believe we have done more realistic calculation of explosive nucleosynthesis in this study compared with Nagataki et al. (2003).

We realize the jet-induced explosion by injecting thermal energy around the polar region in the same way as MacFadyen & Woosley (1999) and Aloy et al. (2000). After such a hydrodynamic calculation, we calculate the products of explosive nucleosynthesis as postprocessing. We explain our detailed method of calculation in the following subsections.

2.1. Hydrodynamics

2.1.1. The Scheme

We have done two-dimensional hydrodynamic simulations taking into account self-gravity and the gravitational potential of the central point mass. The calculated region corresponds to a quarter of the meridian plane under the assumption of
axisymmetry and equatorial symmetry. The spherical mesh with 250(r) \times 30(\theta) grid points is used for all the computations. The radial grid is nonuniform, extending from \(2.0 \times 10^7\) to \(3.0 \times 10^{11}\) cm with finer grids near the center, while the polar grid is uniform.

The basic equations in the following form are finite differenced in spherical coordinates:

\[
\frac{D\rho}{Dt} = -\rho \nabla \cdot \mathbf{v},
\]

\[
\frac{D\mathbf{v}}{Dt} = -\nabla p - \rho \nabla \Phi,
\]

\[
\frac{D}{Dt} \left( \frac{\epsilon}{\rho} \right) = -p \nabla \cdot \mathbf{v},
\]

where \(\rho\), \(\mathbf{v}\), \(p\), \(\Phi\), and \(\epsilon\) are density, velocity, gravitational potential, and internal energy density, respectively. The Lagrangian derivative is denoted as \(D/Dt\). The gravitational potential of the central point mass is modified to account for some of the effects of general relativity (Paczynski & Witt 1980) \(\phi = -GM/(r - r_s)\), where \(r_s = 2GM/c^2\) is the Schwarzschild radius. The ZEUS-2D code developed by Stone & Norman (1992) has been used with an EOS of an electron-positron gas, which is in thermal equilibrium with blackbody radiation and ideal gas of nuclei (Blinnikov et al. 1996). Since the ideal gas contribution of nuclei to the total pressure is negligible relative to those of electron-positron gas and thermal radiation, we fixed the mean atomic weight in the EOS to be 16 to calculate total pressure and temperature using this EOS.

2.1.2. Initial and Boundary Conditions

We adopt the 9.15 \(M_\odot\) collapsar model of MacFadyen & Woosley (1999). When the central black hole has acquired a mass of 3.762 \(M_\odot\), we map the model to our computational grid. The surface of the helium star is \(R_e = 2.98 \times 10^{10}\) cm. Electron fraction, \(Y_e\), is set to 0.5 throughout of this paper since neutrino process is not included.

To simulate the jet-induced explosion, we deposit only thermal energy at a rate \(\dot{E} = 10^{51}\) ergs s\(^{-1}\) homogeneously within a 30° cone around the rotation axis for 10 s. In the radial direction, the deposition region extends from the inner grid boundary located at 200 km to a radius of 600 km. This treatment is same as that of Aloy et al. (2000). We name this model E51. We consider this model to be the standard one. For comparison, we perform a calculation in which total explosion energy (10\(^{52}\) ergs) is put initially with the same deposition region as model E51. We also perform a calculation in which total explosion energy is initially deposited in a spherically symmetric way (from 200 to 600 km). We name these models E52 and E52S, respectively. We consider that these models represent extreme cases. The models in this study are summarized in Table 1.

As for the boundary condition in the radial direction, we adopt the inflow boundary condition for the inner boundary, while the outflow boundary condition is used for the outer boundary. That is, the flow toward the central black hole is prohibited at the inner boundary, and the inflow from the surface of the progenitor is prohibited at the outer boundary. This is because we consider the phenomenon of explosion, in which the free-fall timescale at the inner boundary will be longer than that of the explosion. Note also that we checked that results of explosive nucleosynthesis do not depend sensitively on the inner boundary condition by changing the inflow boundary condition to the outflow boundary condition for the inner boundary condition. As for the boundary condition in the zenith angle direction, the axis of symmetry boundary condition is adopted for the rotation axis, while the reflecting boundary condition is adopted for the equatorial plane.

2.2. Explosive Nucleosynthesis

2.2.1. Test Particle Method

Since the hydrodynamics code is Eulerian, we use the test particle method (Nagataki et al. 1997) in order to obtain the information on the time evolution of the physical quantities along the fluid motion, which is then used for the calculations of the explosive nucleosynthesis. Test particles are scattered in the progenitor and are set at rest initially. They move with the local fluid velocity at their own positions after the passage of the shock wave. The temperature and density that each test particle experiences at each time step are preserved.

Calculations of hydrodynamics and explosive nucleosynthesis are performed separately, since the entropy produced during the explosive nucleosynthesis is much smaller (roughly a few percent) than that generated by the shock wave. In calculating the total yields of elements, we assume that each test particle has its own mass determined from their initial distribution, so that their sum becomes the mass of the layers where these are scattered. It is also assumed that the nucleosynthesis occurs uniformly in each mass element. These assumptions are justified, since the movement of the test particles is not chaotic (i.e., the distribution of test particles at the final time still reflects the given initial condition) and the intervals of test particles are sufficiently narrow to give a smooth distribution of the chemical composition in the ejecta. The number of the test particles is 1500. The test particles are put uniformly in the radial direction, extending from \(2.0 \times 10^7\) to \(3.0 \times 10^{10}\) cm with close separations near the center, while they are put uniformly in the polar direction.

2.2.2. Postprocessing

Since the chemical composition behind the shock wave is not in nuclear statistical equilibrium, the explosive nucleosynthesis has to be calculated using the time evolution of \((\rho, T)\) and a nuclear reaction network, which is called postprocessing. We use the data of \((\rho, T)\) coming with the matter obtained by the test particle method mentioned in § 2.2.1. The nuclear reaction network contains 250 species (see Table 2). We add some species around \(^{44}\)Ti to Hashimoto’s network, which contains 242 nuclei (Hashimoto et al. 1989), although it turned out that the result was not changed essentially by the addition.

3. RESULTS

First, initial density structure in our simulation is shown in Figure 1. This model is the 9.15 \(M_\odot\) collapsar model from MacFadyen & Woosley (1999). The mass of the central black hole is 3.762 \(M_\odot\) (Aloy et al. 2000). The surface of this helium star is \(R_e = 2.98 \times 10^{10}\) cm. The color represents the density (in g cm\(^{-3}\)) on a logarithmic scale. The polar axis represents the
rotation axis, while the horizontal axis represents the equatorial plane. The axis is written in units of cm. The arrows represent the velocity field in the \((r, \theta)\) plane. The region within \(10^{10}\) cm is shown in the left panel, while that within \(10^9\) cm is shown in the right panel. An accretion disk is clearly seen in the right panel. The typical specific angular momentum is \(\sim 10^{17}\) cm s\(^{-1}\) (MacFadyen & Woosley 1999), although this is not shown in Figure 1.

As explained in § 2.1.2, we deposit thermal energy to launch a jet from the central region of the collapsar. The density structure for models E51, E52, and E52S at \(t = 1.0\) s (left panel) and \(t = 1.5\) s (right panel) are shown in Figures 2, 3, and 4, respectively. It is clearly shown that a sharp, narrow jet propagates along the rotation axis in model E51, which is similar to Aloy et al. (2000). On the other hand, in the case of E52, a broad, deformed shock wave propagates in the progenitor. Also, in model E52S, the shock wave is deformed even though the thermal energy is deposited in a spherically symmetric way. This is due to the asymmetry of the density structure of the progenitor. That is, in the low-density region around the rotation axis, the injected thermal energy is mainly shared with electrons, positrons, and photons. On the other hand, at the high-density region around the equatorial plane, the injected energy is shared with radiations mentioned above and nuclei/nucleons. As a result, the pressure gradient at the energy-injected region becomes aspherical, which causes a bipolar flow along the rotation axis. It is also noted that the shock wave is more deformed in model E52S than model E52. This is because the energy density around the polar region is higher in model E52, making this region expand very strongly.

Forms of the mass cut (the boundary between ejecta and the matter that falls into the central black hole) are shown in Figure 5 for each model. Red particles represent the ones that can escape from the gravitational potential to infinity, while green particles represent the ones that are trapped in the gravitational potential. As a criterion for judging whether a test particle can escape or not, we calculated the total energy (summation of kinetic energy,

### Table 2

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Fig. 1.—Initial density structure in our simulation. This model is the 9.15 \(M_\odot\) collapsar model from MacFadyen & Woosley (1999). The mass of the central black hole is 3.762 \(M_\odot\). The surface of this helium star is \(R_c = 2.98 \times 10^{10}\) cm. The color represents the density (in g cm\(^{-3}\)) in logarithmic scale. The polar axis represents the rotation axis, while the horizontal axis represents the equatorial plane. The axis is written in units of cm. The arrow represents the velocity field in the \((r, \theta)\) plane. The region within \(10^{10}\) cm is shown in the left panel, while that within \(10^9\) cm is shown in the right panel.
thermal energy, and gravitational energy) of the test particles at the final stage of simulations ($t = 10 \text{ s}$). We judge that a test particle can escape if its total energy is positive, and vice versa.

Strictly speaking, we have to simulate for a much longer time to determine whether a test particle can really escape or not. In particular, there is a region around the equatorial plane ($r \geq 10^{10} \text{ cm}$, $\theta \geq 70^\circ$) where the shock wave does not reach even at the final stage of the simulations ($t = 10 \text{ s}$), because the propagation speed of the shock wave is slower around the equatorial plane compared with the polar region. In this study, we assumed that such a region where the shock wave does not reach even at the final stage can escape to infinity. This is because such a region is distant from the central black hole ($r \geq 10^{10} \text{ cm}$), so the gravitational potential is shallow. Moreover, as shown below, at such a distant region, explosive nucleosynthesis hardly occurs and the most important nucleus in this study, $^{56}\text{Ni}$, is not synthesized. So our results on the abundance of $^{56}\text{Ni}$ do not depend on this assumption.

To clearly show how test particles are ejected, we plot the positions of the test particles at $t = 0 \text{ s}$ (Fig. 6, top left panel), $3.11 \text{ s}$ (Fig. 6, top right panel), $3.69 \text{ s}$ (Fig. 6, bottom left panel), and $4.27 \text{ s}$ (Fig. 6, bottom right panel) for model E51. The particles colored green, red, and blue are the ones that are initially put around the rotation axis ($\theta \leq 30^\circ$), middle range ($30^\circ < \theta \leq 60^\circ$), and...
Fig. 4.—Same as Fig. 2, but for model E52S at $t = 1.0\, s$ (left panel) and $t = 1.5\, s$ (right panel). It is clearly shown that the shock wave is deformed even though the thermal energy is deposited in a spherically symmetric way as an initial condition. This is due to the asymmetry of the density structure of the progenitor.

Fig. 5.—Mass cut for models E51 (top panel), E52 (bottom left panel), and E52S (bottom right panel). Red particles represent the ones that can escape from the gravitational potential to infinity, while green particles represent the ones that are trapped in the gravitational potential.
and equatorial plane \((60^\circ \leq \theta \leq 90^\circ)\), respectively. The radius of the progenitor is \(2.98 \times 10^{10}\) cm. The (white) region where no test particle exists shows the shocked, low-density region. It is clearly shown that some fraction of the matter behind the shock wave composes the jet component around the rotation axis, while some fraction of it is pushed away toward the \(\theta\)-direction that composes supernova component.

We can find where and how much \(^{56}\text{Ni}\) is synthesized by doing postprocessing. Also, we can see how \(^{56}\text{Ni}\) is ejected, that is, how much \(^{56}\text{Ni}\) is ejected as the jet component or the supernova component. In Figure 8, positions of the ejected test particles for model E51 at \(t = 0\) s (left panel) and \(t = 4.27\) s (right panel) for model E51. The particles colored green, red, and blue are the ones that are put around the rotation axis initially \((\theta \leq 30^\circ)\), middle range \((30^\circ \leq \theta \leq 60^\circ)\), and equatorial plane \((60^\circ \leq \theta \leq 90^\circ)\), respectively, where \(\theta\) is the zenith angle. Radius of the progenitor is \(2.98 \times 10^{10}\) cm. The (white) region where no test particle exist shows the shocked, low-density region. It is clearly shown that some fraction of the matter behind the shock wave composes the jet component around the rotation axis, while some fraction of it is pushed away toward the \(\theta\)-direction that composes supernova component.

Before we show the results of explosive nucleosynthesis, we present a contour of entropy per baryon in units of Boltzmann constant \((k_B)\) for model E51 at \(t = 1.5\) s in the left panel of Figure 7. For comparison, positions of test particles at that time is shown in the right panel of Figure 7. As for the entropy per baryon, the range \(10^3 - 10^5\), which corresponds to the shocked region, is shown. It is clearly shown that test particles downstream of the shock wave are moving with the shock velocity, and no test particles are left inside the shocked region, where the entropy per baryon is quite high.
are shown that satisfy the condition that the mass fraction of $^{56}\text{Ni}$ becomes greater than 0.3 as a result of explosive nucleosynthesis. The total ejected mass of $^{56}\text{Ni}$ becomes $0.0439\, M_{\odot}$. In particular, the total mass of it in the supernova component is $0.0175\, M_{\odot}$, which is much smaller than the observed values of hypernovae. In Figures 9 and 10, the same values are shown as in Figure 8, but for models E52 and E52S. As mentioned above, the outflow becomes bipolar due to the asymmetry of the density structure. As a result, the jet component can be seen even in model E52S (Fig. 10). The total ejected mass of $^{56}\text{Ni}$ is $0.23\, M_{\odot}$ (model E52), which is comparable to the observed values of hypernovae. It is also noted that most of the synthesized $^{56}\text{Ni}$ is in the jet component (0.23 $M_\odot$), while a small amount of $^{56}\text{Ni}$ (0.00229 $M_\odot$) is in the supernova component. From this result, we can guess that some fraction of the gamma-ray lines from $^{56}\text{Ni}$ decays appear without losing their energies. This point is discussed in § 4 in detail. Of course, it is noted that this model will not explain the association of GRBs with hypernovae, since

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Fig. 7.—Left: Contour of entropy per baryon in units of $k_B$ for model E51 at $t = 1.5\, s$. Right: Positions of test particles for model E51 at $t = 1.5\, s$. For the entropy per baryon, the range shown is $10^0 - 10^5$, which corresponds to the shocked region. It is clearly shown that test particles in the downstream of the shock wave are moving with the shock velocity, and no test particles are left inside the shocked region where the entropy per baryon is quite high.

Fig. 8.—Positions of the ejected test particles at $t = 0\, s$ (left) and $t = 4.27\, s$ (right) that meet the condition that the mass fraction of $^{56}\text{Ni}$ becomes greater than 0.3 as a result of explosive nucleosynthesis for model E51. The total ejected mass of $^{56}\text{Ni}$ becomes $0.0439\, M_{\odot}$. In particular, its total mass in the supernova component is $0.0175\, M_{\odot}$, which is much smaller than the observed value of hypernovae.
this model cannot cause highly relativistic jets. The features of model E52S are same as that of model E52. That is, the ejected mass of $^{56}$Ni in model E52S is $0.28 M_e$, which is comparable to that of model E52 and is much larger than that of model E51. From this result, it is shown that the total ejected mass of $^{56}$Ni depends sensitively on the energy deposition rate and not so sensitively on the mass of the heated region (energy-injection region), which is consistent with Nagataki et al. (2003). The reason is as follows: The criterion for the complete silicon burning is $T_{\text{max}} \geq 5 \times 10^9$ K (Thielemann et al. 1996). It is well known that the matter behind the shock wave is radiation-dominated, and $T_{\text{max}}$ can be estimated well by equating the supernova (hypernova) energy with the radiation energy inside the radius $r$ of the shock front:

$$E_{\text{HN}} = 10^{52} \left( \frac{E_{\text{HN}}}{10^{52} \text{ ergs}} \right) = \frac{11 \pi^3}{45} \frac{k^4}{h^2 c^3} r^3 T_{\text{max}}^4, \quad (4)$$

However, most of the synthesized $^{56}$Ni is in the jet component ($0.23 M_e$), while a small amount of $^{56}$Ni ($0.00229 M_e$) is in the supernova component.

Fig. 9.—Same as Fig. 8, but for model E52. The total ejected mass of $^{56}$Ni is $0.23 M_e$ (model E52), which is comparable to the observed value of hypernovae. However, most of the synthesized $^{56}$Ni is in the jet component ($0.23 M_e$), while a small amount of $^{56}$Ni ($0.00229 M_e$) is in the supernova component.

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Fig. 10.—Same as Fig. 8, but for model E52S (right panel). The total ejected mass of $^{56}$Ni is $0.28 M_e$ (model E52S), respectively, which is comparable to the observed value of hypernovae. However, as in model E52, most of the synthesized $^{56}$Ni is in the jet component ($0.195 M_e$), while a small amount of $^{56}$Ni ($0.0882 M_e$) is in the supernova component.
First, we discuss the formation of highly relativistic jets to realize GRBs. Of course, in the present study, we cannot investigate the acceleration of the jet to the relativistic regime since our numerical code is Newtonian. We have to investigate this topic using a relativistic hydrodynamic code. At present, we discuss this topic by introducing previous works that investigated GRB jets using relativistic hydrodynamic codes. Aloy et al. (2000) performed such a calculation using the collapsar model (MacFadyen & Woosley 1999). They showed that the jet is formed as a consequence of an assumed energy deposition rate in the range of $10^{50} - 10^{51}$ ergs s$^{-1}$ within a 30° cone around the rotation axis, which is a similar treatment to model E51 in this study. They reported that the maximum Lorentz factor of the jet is 44, which seems to be smaller than the required value to explain GRBs (~300; Piran 1999). Zhang et al. (2003) also calculated the propagation of the relativistic jet through the collapsar with constant energy deposition rate ($1 - 3 \times 10^{50}$ ergs s$^{-1}$). They set the location of the inner boundary to be $2 \times 10^8$ cm. They estimated that the terminal Lorentz factor, which is calculated by assuming that all internal energy is converted into kinetic energy, is $\sim 100$, although their calculated jet Lorentz factor is $\sim 50$ at most. Zhang & Woosley (2004) improved their code and demonstrated that the bulk Lorentz factor of the jet can reach $\sim 100$, although they

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E_{\text{IN}} = \frac{5 \times 10^9 \text{K}}{T_{\text{max}}} \left( \frac{E_{\text{IN}}}{10^{52} \text{ergs}} \right)^{1/3} \left( \frac{5.7 \times 10^8 \text{cm}}{r} \right)^{4/3}.
\]
set the inner boundary to be $10^{16}$ cm. From their work, we can understand that it is very difficult to realize a highly relativistic jet whose bulk Lorentz factor is larger than 100. They required a collimated, narrow jet by depositing explosion energy for $\sim 10$ s.

The importance of the collimation to realize a highly relativistic jet can also be understood by rough estimation. We can calculate the mass of the progenitor included within a cone around the rotation axis. The masses included within a cone with zenith angles $3^\circ$, $5^\circ$, $10^\circ$, and $15^\circ$ are $7.9 \times 10^{30}$, $3.1 \times 10^{31}$, $7.1 \times 10^{31}$, and $2.0 \times 10^{32}$ g, respectively. So if this matter is accelerated to the highly relativistic regime with bulk Lorentz factor $\Gamma$, the kinetic energies have to be $7.1 \times 10^{53}(\Gamma/100)$, $2.8 \times 10^{54}(\Gamma/100)$, $6.4 \times 10^{54}(\Gamma/100)$, and $1.8 \times 10^{55}(\Gamma/100)$ ergs, respectively, which shows the importance of collimation. The importance of collimation is also confirmed by our forthcoming paper (Mizuta et al. 2006). In models E52 and E52S, the opening angles of the jets are wider than that in model E51 (see Figs. 2, 3, 4), so we consider that highly relativistic jet will not be produced in models E52 and E52S even if relativistic hydrodynamic code is used. So we consider that models E52 and E52S cannot explain the phenomena of association of GRBs with hypernovae, even though much $^{56}$Ni is synthesized in these models.

There is another question. Does all of the $^{56}$Ni produced in the jet of the collapsar model brighten the supernova remnant? If the jet becomes optically thin before $^{56}$Ni decays into $^{56}$Co and $^{56}$Co decays into $^{56}$Fe, these nuclei should result in only emitting gamma rays and cannot brighten the supernova remnant. Colgate et al. (1980) consider the deposition of energy by gamma rays emanating from decay of $^{56}$Ni and $^{56}$Co. They found that the mass opacity of the gamma-ray absorption is about $0.029$ cm$^2$ g$^{-1}$, for either $^{56}$Ni or $^{56}$Co decay spectrum. The half-lives of $^{56}$Ni and $^{56}$Co are 5.9 and 77.1 days, respectively. Thus, we can roughly estimate the opacity for these gamma rays in the jet. We assume that the opening angle of the jet is $10^\circ$ and expansion velocity of the jet is the speed of light. Under these assumption, when substantial $^{56}$Ni decays into $^{56}$Co, the volume of the jet becomes $3.2 \times 10^{-3} R^2$ cm$^3$, where $R$ is the radius of the jet with $R = 1.5 \times 10^{16} \Gamma^3$ cm and $\Gamma$ is the bulk Lorentz factor of the jet. Since the mass included within a cone with zenith angle $10^\circ$ is $7.1 \times 10^{31}$ g, the density of the jet at that time becomes $6.6 \times 10^{-16}$ g cm$^{-3} \Gamma^{-3}$. So the mean free path of the gamma rays becomes $5.3 \times 10^{16} \Gamma^{-3}$ cm, which is comparable to or longer than the radius of the jet. Of course, the gamma rays can easily escape from the side of the cone. So we think most of the gamma rays produced in a jet will escape without depositing energy to the jet and supernova components, as long as the bulk speed of the jet is relativistic. A similar discussion can be adopted for the decays of $^{56}$Co into $^{56}$Fe. Thus, we can conclude that some fraction of gamma-ray lines from $^{56}$Ni decays in the jet may appear as relativistically Lorentz-boosted gamma-ray line profiles in future. To obtain a more firm conclusion, it will be necessary to perform multidimensional relativistic hydrodynamics with radiation transfer and calculate the light curve of hypernovae.

Here we have to comment on the resolution of numerical simulations in this study. It is shown in numerical simulations with high resolution (Zhang & Woosley 2004) that the jet propagates only mildly relativistically ($\sim c/2$) while in the star, and shocked gas can move laterally to form a cocoon, allowing the core of the jet to remain relativistic. Thus, the total mass accelerated relativistically ($\Gamma \sim 100$) by the jet is not the fraction of the star intercepted by the jet, although the importance of collimation can be understood by calculating mass within the cone of the jet as mentioned above. So, strictly speaking, when relativistic hydrodynamic simulations are performed, there are three components in a collapsar model, highly relativistic jet, cocoon, and supernova component. In this study, the highly relativistic jet and cocoon are called the “jet component.”

We concluded that the highly relativistic jet and mildly relativistic cocoon are optically thin against the gamma rays that come from decays of $^{56}$Ni and $^{56}$Co. As mentioned above, the mean free path of the gamma rays becomes $5.3 \times 10^{16} \Gamma^{-3}$ cm, which is comparable to or longer than the radius of the jet component, $1.5 \times 10^{16} \Gamma$ cm (note that these are comparable even if $\Gamma = 1$). Thus, the $^{56}$Ni in the highly relativistic jet and mildly relativistic cocoon should not contribute to the light curve of a supernova.
On the other hand, a subrelativistic cocoon will be optically thick against the gamma rays, so a subrelativistic cocoon may contribute to the optical light curve of hypernovae. In fact, it is clear that too much energy is required to accelerate all of $^{56}$Ni in the jet to relativistic speed. From the discussion of energetics mentioned above, the required energy (in ergs) becomes $9.0 \times 10^{53}(I/1)(M_{\text{Ni}}/0.5 \, M_\odot)$, where $M_{\text{Ni}}$ is the mass of $^{56}$Ni. Thus, we consider that there is a possibility that a part of the gamma-ray lines may appear without losing their energies. As mentioned above, numerical simulations of relativistic hydrodynamics with high resolution should be required to distinguish the highly relativistic jet from the cocoon, and to obtain precise distribution of burning products, which we are planning to simulate in the near future.

Let us emphasize our motivation in this work here. In this study, we want to consider the consistency between the collapsar model of MacFadyen & Woosley (1999) and explosive nucleosynthesis in a hypernova jet. The answer is as follows. From the discussions mentioned above, it seems difficult to explain the required amount of $^{56}$Ni ($\sim 0.5 \, M_\odot$) by the explosive nucleosynthesis in the jet. We think the other idea that the origin of $^{56}$Ni is the explosive nucleosynthesis in the accretion disk (MacFadyen & Woosley 1999; Pruet et al. 2003) is much simpler and adequate to explain the association of GRBs and hypernovae. In this scenario, it is not necessary for $^{56}$Ni to be synthesized in a short timescale, like with models E52 and E52S. As explained in § 1, in the collapsar scenario, the jet is launched $\sim 7$ s after the gravitational collapse and the duration of the jet is about 10 s, which is much longer than the typical timescale of normal core-collapse supernovae and comparable to the typical observed duration of GRBs. As a result of gravitational collapse in a long timescale, the density around the rotation axis becomes low, which is a good environment for producing a firewall. That is, a long timescale of the order of 10 s is essential for realizing a GB. As shown in this study, in such a case (model E51), not much $^{56}$Ni is synthesized in the jet. Rather, it will be natural to consider that the origin of $^{56}$Ni is the accretion disk around the black hole. In this scenario, $^{56}$Ni is also ejected on a long timescale of the order of 10 s. No requirement that $^{56}$Ni has to be produced in a short timescale exists in this scenario. Also, as shown in this study, the explosion becomes naturally bipolar in any case (even in model E52S) due to the aspherical density structure of the progenitor. So it is natural to consider that the $^{56}$Ni synthesized in the accretion disk and conveyed as outflows are blown along the rotation axis, which can explain the line features of SN 1998bw and double-peaked line features of SN 1998bw.

$^{56}$Ni is a magnetar (Rees & Mészáros 2000; Takiwaki et al. 2004) or magnetized collapsar (Blandford & Znajek 1977; Blandford & Payne 1982; Proga et al. 2003; Koide 2003; Mizuno et al. 2004; Proga 2005; McKinney 2005a, 2005b), then the timescale of the explosion will be shorter than that of a collapsar, which will result in the different method of nucleosynthesis in this study. Further investigation is still required to understand the central engine of GRBs and origin of $^{56}$Ni in hypernovae.

In the early universe, where the metal content of gas is very low, the enrichment by a single supernova can dominate the preexisting metal contents (Audouze & Silk 1995). Since GRBs also occur in the early universe, there is a possibility that some fraction of metal-poor stars reflects the chemical abundance of GRBs accompanied by hypernovae (Maeda & Nomoto 2003). From Figure 11, we can see that there is an enhancement of nuclei whose mass number $\sim 40$ in models E52 and E52S compared with model E51. This is the result of incomplete silicon burning and alpha-rich freezeout (Nagataki et al. 1997, 1998b; Nagataki 2000; Maeda et al. 2002; Maeda & Nomoto 2003). In particular, in models E52 and E52S, $[\text{Ca}/\text{Si}] \equiv (X_{\text{Ca}}/X_{\text{Si}}) - (X_{\text{Ca}}/X_{\text{Si}})_0$ is larger than unity, where $X_i$ is the mass fraction of the $i$th element and $(X_{\text{Ca}}/X_{\text{Si}})_0$ is the solar value, which is in contrast with model E51 and previous works (Qian & Wasserburg 2002; Pruet et al. 2004). So it can be determined which model is the proper one as a model of hypernovae by observations of chemical composition in metal-poor stars (Ishimaru & Wanajo 1999; Umeda & Nomoto 2002, 2005; Tsujimoto 2004; Ishimaru et al. 2004).

It will be a good challenge to perform a calculation of the $r$-process and/or $p$-process nucleosynthesis in the GRB jet in this study, because this jet will also be able to satisfy a high-entropy condition enough to realize these processes (MacFadyen & Woosley 1999; Nagataki 2000, 2001; Nagataki & Kohri 2001; Wanajo et al. 2001, 2002; Suzuki & Nagataki 2005). As shown in Figure 7, there is really a region in the jet where high entropy per baryon is realized (it reaches $10^7$ at most). However, in this study, test particles in the downstream of the shock wave are moving with the shock velocity, and no test particles are left inside the shocked region where entropy per baryon is quite high. So another method is required to investigate the $r$-process and/or $p$-process nucleosynthesis in the jet. We are planning to perform such calculations. Results will be presented in the near future.

5. SUMMARY AND CONCLUSION

We have performed two-dimensional hydrodynamic simulations to investigate explosive nucleosynthesis in a collapsar using the model of MacFadyen & Woosley (1999). We have shown that $^{56}$Ni is not produced in the jet sufficiently to explain the observed amount of a hypernova such as SN 1998bw when the duration of the explosion is $\sim 10$ s (the standard model, E51). Even though a considerable amount of $^{56}$Ni is synthesized if all explosion energy is deposited initially (the extreme models, E52 and E52S), the opening angles of the jets become too wide to realize highly relativistic outflows and a GRB in such a case. From these results, we conclude that the origin of $^{56}$Ni in hypernovae associated with GRBs is not the explosive nucleosynthesis in the jet. We consider that the idea that the origin of $^{56}$Ni in hypernovae is the explosive nucleosynthesis in the accretion disk is more promising. We have also shown that the explosion becomes bipolar naturally due to the effect of the deformed progenitor. This fact suggests that the $^{56}$Ni synthesized in the accretion
investigation is still required to understand the central engine of a gamma-ray burst accompanied by a hypernova with rich freezeout. So it may be determined which is the proper model of a gamma-ray burst accompanied by a hypernova with observations of chemical composition in metal-poor stars.

Of course there is still uncertainty of observations and theories on the formation of GRBs accompanied by hypernovae. Further investigation is still required to understand the central engine of GRBs and origin of $^{56}$Ni in hypernovae.

Note added in manuscript.—After we had written this manuscript, we found recent papers by Maeda et al. (2006) and Maeda (2006) in which topics similar to that of this study are considered. Their conclusions are consistent with our prediction. That is, they calculated gamma-ray lines from $^{56}$Ni decays in the jet, although relativistically Lorentz-boosted line profiles are not calculated because they used a Newtonian code. Also, it is noted that they deposit all explosion energy as an initial condition, like models E52 and E52S in this study. That is why they can obtain enough $^{56}$Ni to explain the light curve of SN 1998bw, which is consistent with the results in this study. The reason why they obtained a little more $^{56}$Ni ($\sim0.4\,M_\odot$) may be the difference in the progenitor. They used a spherically symmetric progenitor, while we have used a collapsar model. As stated in this study, the density of the collapsar model around the polar region is lower than in the spherically symmetric progenitor. So the abundance of synthesized $^{56}$Ni may be smaller in this study.

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