Scale free property of Wilson's numerical renormalization group

Kouichi Okunishi[1] and Tomotoshi Nishino[2]

[2] Department of Physics, Niigata University, Niigata 950-2181, JAPAN

[2] Department of Physics, Kobe University, Kobe 657-8501, JAPAN

It is well known that two numerical renormalization groups made great success in quantitative analysis of 1D quantum systems: Wilson's numerical renormalization group(NRG) for the Kondo impurity problem and density matrix renormalization group(DMRG). An interesting point is that these two methods have very contrasted features. DMRG is accurate for gapful 1D systems and its theoretical background is a variational optimization for the matrix product wavefunction, where maximization of the entanglement entropy plays an essential role. Thus, DMRG is different from the conventional renormalization group based on the coarse graining and the scale transformation of the lattice space. On the other hand, Wilson NRG is efficient for gapless excitation of an effective 1D system with the impurity sitting at the boundary. However, the reason for the success of Wilson NRG has not been discussed so much. For further developments in the numerical renormalization, it may be instructive to see how Wilson NRG can nicely control the energy scale of the gapless system.

In this talk, we would like to discuss the scale-free property of Wilson NRG.[1] The single-particle state of the effective Hamiltonian with a cutoff Λ is described by the wavepacket basis having the scale free property; The energy scale of the system can be controlled by the lattice translation of the wavepacket basis with no reference of rescaling of the lattice space. We also analyze the role of the Kondo interaction in the context of wavepacket basis and then discuss the scaling and renormalization of the Kondo coupling. In addition, we clarify the role of the edge state in the lowest energy scale of Wilson NRG. A generalization for one-dimensional quantum systems is also discussed.

References

[1] K. Okunishi and T. Nishino, arXiv:1001.2594; to appear in Phys. Rev. B.